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“Was Lucifer a Gambler? A Rational-Choice Hermeneutic of Peter Olivi’s Treatise on Demons”

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Abstract: This paper presents an interpretation of the main arguments used in Peter Olivi's *Treatise on Demons*, published *circa* 1295 in Narbonne, Languedoc, within a rational-choice framework. This book has been widely praised as a landmark in the philosophical literature on personhood and personal freedom, since it was (re)discovered about a century ago. In it, Olivi discusses most of the relevant classical and medieval literature on this topic before stating his own position. In the scholastic tradition, the book does not make for easy reading. Moreover, it is evidently a "work in progress", as pointed out by the translator. Many paragraphs end with "Ergo, etc.", suggesting that he planned to add something, but could not find time enough for that. He died in 1298, aged 50. This paper offers a simple game-theoretic model aimed at articulating Olivi's main arguments in a consistent rational choice framework, supported by many quotes translated from French into English by me. It suggests that the "fall of the devil" is used as a parable on human freedom and agency, given a set of incentives strategically chosen by "God" to minimize the number of "sinners", with some potential interference by "Lucifer".

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1. Introduction

A hermeneutic is a special type of exegesis using a distinctive methodology, rather than a thorough analysis mobilizing all available tools to extract the “truth” from a given text. The folk etymology derives hermeneutic from the ancient Greek god Hermes, the typical go-between establishing links between people, like traders and consumers, for example, across the *Agora*. The rational-choice hermeneutic deployed in the present paper aims at translating the main arguments used in Peter Olivi’s *Treatise on Demons*, published *circa* 1295 in Narbonne (Languedoc) into a language that will make sense to a reader equipped with a *modicum* of standard microeconomics and a pinch of game theory. Moreover, it sometimes requires the researcher to fill some gaps to clarify the consistency of some arguments where the author’s intuition is probably not precisely conveyed by his verbal expression.

A Short Biography of Peter Olivi¹

Peter Olivi was a Franciscan monk, born in Languedoc in 1248 in Sérignan, between Beziers and the Mediterranean Sea. He enrolled in the Franciscan order at the age of twelve. After studying in Paris in the late 1260s, he returned to the south after six or eight years, depending on the sources, to start a teaching career at Montpellier in 1272 or 1274, without having completed his bachelor’s degree. There, Kaye (2014) suggests that he very likely interacted on a regular basis with Arnau de Vilanova, a famous professor of medicine in the Galenic tradition. According to Kaye, Olivi became acquainted with the concept of balance, which is crucial in that tradition, during multiple discussions with Vilanova. This would provide a clue about the origin of Olivi’s bent towards scientific reasoning and the attention he pays to interactions between agents. Vilanova’s name would appear again at the turn of the century as an important member of the “Spirituals” movement, strongly rooted in Olivi’s writings (Burr, 2002). Leclair (2020) suggests that Olivi interacted also with the troubadour Peire Cardenal who was still alive at that time. The latter was probably the most politically acerbic troubadour of all, author of many *sirventès* poems denouncing the misbehavior of the clerics and the civilizational threat that the French invaders represented for the Languedocian society, in the wake of the Albigensian crusade. He also held *tensos* (political debates) on the same topics at various Languedocian friendly castles. After serving for decades at the Count’s

¹ Peter Olivi is his generally accepted name in the relevant literature. He is also often called Peter of John Olivi, because his Latin name, the only one we are sure of is *Petrus Johannis Olivi*. However, the family name Johannis was quite common in Languedoc at that time, meaning “son of John”, while Olivi means that he was part of the branch of the Johannis family from the village of Olieu, near Montpellier. Hence, a literal translation of his name into English would be Peter Johnson Olivian. Nobody would guess who this refers to.

Chancery in Toulouse, Cardenal sought refuge after 1249 in the King of Aragon's palace in Montpellier, which was an Aragonese enclave protected from the Inquisition at the time, under Jacme I the Conqueror. However, Cardenal was born in 1180, and thus was well above 92 years old at the time when Olivi might have met him in Montpellier. As such, this does not preclude any productive meeting between the two, but makes it unlikely, given the 62 years of age difference and Cardenal's defiance against clerics. Nevertheless, it is not unthinkable that Olivi was exposed to some of Cardenal's writings, may be by hearsay, which might have opened his mind to some of the societal threats of his time. Olivi's independence of mind triggered some hostility in the Franciscan hierarchy and beyond, and he was censored and his writings condemned to be burned a first time in 1283 for a short period, after a thorough examination of his work by a committee of Parisian Scholars. Once rehabilitated, he spent two years in Florence in 1287-89, before returning to Montpellier and then to the *studium* of Narbonne where he wrote the *Treatise on Demons*, the *Treatise on Contracts*, and other important books before his death in 1298, aged 50. Although Olivi spent a lot of his time in Florence at the Franciscan *studium* of the Church of *Santa Croce*, where Dante Alighieri is thought to have attended many classes (Lesnick, 1989), it is unlikely that the two met. Dante was then between 22 and 24 years old and was already deeply engaged in the troubled Florentine political environment. Nevertheless, many authors support the view that some of Olivi's books exerted a decisive influence on Dante's thinking. Forni (1999) presents a convincing discussion of this issue involving a rich review of this legacy of Olivi's, emphasizing his *Comments on the Apocalypse* written nearly a decade later. By contrast, Burr (2002) quotes from letters written by Ubertino da Casale (1259-1329), who was to play a key part in the Italian Spirituals movement, that Peter Olivi and him had important discussions in Florence, with a special focus on the Apocalypse and Joachim of Fiore's legacy. Burr (1997) and Kaye (2014) provide more detailed narratives of Olivi's life and peregrinations.

How to read the Treatise on Demons

Written in the medieval scholastic tradition, the book does not make for easy reading, the more so as it is evidently a "work in progress", as pointed out by the translator (Alain Boureau). Many paragraphs end with "Ergo, etc.", suggesting that he wanted to add something, but did not find time enough for that before his death three years later, or that he had lost his interest in it. The flow of the argument developed below tends to provide some support to the latter explanation. He used his final years to work among other things on some *Comments on the Apocalypse*, which also infuriated his hierarchy. Written in the Joachite tradition, after Joachim of Fiore (1135-1202), his comments announced the end of the Church

hierarchy that prevailed at the time. Burr (1993, 1997) provides a blow-by-blow account of this acrimonious debate. As a tool for ensuring consistency of a framework of analysis, the simple modelling exercise performed below may help such a trained reader (with a *modicum* of microeconomics and a pinch of game theory) to grasp the thrust of Olivi's argument in an articulate and consistent fashion, without searching the book for the bits and pieces scattered here and there in the different sections. I tried my best to quote the most relevant sentences that may justify the assumptions made in the model, showing that they are truthful to Olivi's writing.

However, I made one daring clarification by introducing in the model a concept that Olivi does not precisely use in this book, while he is the pathbreaker who introduced it in his other book written at about the same time, the *Treatise on Contracts*, namely the concept of probability. James Franklin (2015) supports the view that Olivi really was the first author to use this notion, which is often attributed to Girolamo Cardano in the sixteenth century. David (1962, 1998) discusses this ascription to Cardano with a rich historical background that sheds some exciting light on this intriguing character. In the *Contracts* book, Olivi endeavoured to defeat a lot of the arguments employed by some confessors to inflict fines, called "restitutions", on merchants and traders in the name of fighting against "usury". Confession had been made compulsory to all Christians, at least once a year, at the Lateran council in 1215 (Piron, 1997). Olivi tore to pieces the theory of usury that was widely used by the confessors by introducing two new concepts (i) "capital", defined as productive money lent for investment, as distinct from plain money just lent for consumption in the Aristotelian tradition, and (ii) the concept of "probability", to explain the price discount for risky assets. That way, he was able to explain why successful traders could sell a cargo of goods shipped across a rough sea at a higher price than the price paid for purchasing it, arguing rightly that in case of wreckage, with the complementary probability, they would have just lost their money. He argued that inflicting a "restitution" in case of success would provide the wrong incentive and damage the "common good" by discouraging much needed imports. In the *Demons* book, he uses once the concept of "probable reason" (p.187), directly borrowed from Roman Law, as well as the superlative "*probabiliter*", many times, as a way of expressing his relative appreciation of some arguments put forward by some authors that he quotes or discusses. He also uses this word (p.183) to describe a bandit's potential perception of the risk of getting hung. Similarly, Olivi also writes that the demons can learn by using "probable conjectures" (p.141). In the book, the reader can also find various expressions for describing the randomness of some events or outcomes by using words and expressions like "accidental",

“not indubitable belief” or “lack of certainty”, etc. Hence, the word probability would have found a friendly environment in this book, but it does not actually turn up in it. By contrast, when Olivi introduces the word “*probability*” in the *Contracts* book, he seems to be so pleased with this innovation, that he uses it several times per page in his whole discussion of capital asset pricing. This suggests, as seems likely from other details, that the *Contracts* book was written after the surviving copy of the *Demons* book that we have. I took the step of introducing it in my modelling exercise, as it seems very likely that Olivi would have introduced it, had he lived longer and recovered his interest in polishing his writing in the *Demons* book. This move clearly brings out this book’s true ambition: it really is a treatise on human freedom of choice within a “Divine Lottery” framework. The modelling performed below then highlights the richness of this philosophical contribution, as well as its potential shortcomings².

Contents of the Paper

The next section describes the basic decision to be made by the angels between playing safe and thus getting a modest reward, on the one hand, or taking a riskier path to get a bigger reward, at the risk of getting punished with some probability, on the other hand. The sin involved in the latter choice is linked in this literature with pride and selfishness, while the former choice involves a lower profile. Sinners are then randomly allocated as demons or tormented victims. The demons are thus playing a key part in providing the incentive to refrain from sinning. Section 3 assumes that “God”, i.e., the abstract pseudo-God modelled here (not the one you might believe in), chooses the number of demons for the sake of minimizing the number of sinners without directly coercing the angels. The “power to sin” (*potentia peccandi*) is thus kept as a basic component of their personal freedom. A diagrammatic analysis is derived from this basic mathematical framework to describe the optimal policy mix of demons and sinners and derive some comparative-static predictions. Section 4 marginally extends the model to try and accommodate a series of *obiter dicta* found in the *Demons* book, which give to “Lucifer” (the model’s one, not the one you might believe in) an effective operational role among the demons. This is first performed within a Kalai-Smorodinsky bargaining solution framework to show that there is a tension between the quite irenic tone of the simpler version of the model and the more chaotic one derived from the subtle extension giving to “Lucifer” some executive power. Neither one can accommodate consistently some of Olivi’s claims. A less glamorous approach is then offered that clears the

² Further justification of this hermeneutic strategy is provided in the appendix.

problem satisfactorily and dispels any suspicion of inconsistency. This rectification illustrates the power of the rational choice hermeneutic approach to reject misleading interpretations in a revealed preference framework. Section 5 concludes by suggesting that this model provides a parable on human freedom and agency that seems faithful to Peter Olivi's book, giving the whole exercise a broader reach than just a better understanding of a late 13th century book.

2. A Model of the Divine Lottery

As emphasized by Boureau (2011) and Bobillier (2020), most of the medieval authors discussing the fall of the devil do not regard the distinction between angels and humans as rigid and watertight. For example, the Franciscan monks believed that St Francis should be promoted to sit at the right of God, taking over the seat vacated by the fallen Lucifer. In the *Demons* book Olivi often uses directly some lessons drawn from his discussion of the angelic world to derive various positive or normative implications for us humans. In particular, he regards the *potentia peccandi* as a key component of both human and angelic freedom, against the tradition initiated by Anselm of Cantorbery (1033-1109). Therefore, it is not far-fetched to consider his discussion of the world of angels as a parable aimed at helping us to better understand human affairs, the more so because Olivi acknowledges that we necessarily analyse the angelic world using ideas and intuitions developed in contact with our own world: "all what we say about their glory or their fault, we say using familiar images borrowed from inferior realities" (p.139).

Micro-Foundations

The basic choice made by the angels in this model is between a proper god-loving life yielding a safe and modest welfare level b , the same for all, and a more risky one that offers a higher reward B that might be paid for by a punishment H , inflicted with probability π . This randomness of the punishment in case of sin plays a key part in Olivi's analysis of why the first angel³ took the risk to commit a sin. After discussing other information-related reasons, Olivi suggests that God is leaving some uncertainty about the actual implementation of the punishment. Talking about the "first angel", he writes: "he did not know nor believed without doubt that if he sinned, God would have no pity for him" (p.117). To reinforce his point, he quotes Anselm of Cantorbery⁴ who wrote: "Had he known it infallibly, then, he

³ This probably refers to Lucifer, but it is not certain from the context.

⁴ Anselm (1033-1109) was an Italian monk who became archbishop of Cantorbery, as it was spelt at the time and in all this literature.

would never have consented to a sin, as he would have refrained from any sin because of the plain fear of the punishment” (p.117). Olivi takes the trouble of spelling out that what he wrote for the first angel extends without doubt to the other angels: “as seen before, they did not know that God would sentence them irrevocably, if they sinned” (p.127). Ruling out preferential treatments or cognitive dissonance (Akerlof and Dickens, 1982) to remain faithful to the “rational choice” hermeneutic approach, implies that this punishment’s probability is the same for all: if the angels form rational expectations trusting the punishment to be allocated at random among the sinners, they thus all form the same probability π of being punished in that case. However, all angels are not the same, and their personal valuation B of the reward for choosing the risky path is assumed to be distributed across the angel population according to the following cumulative uniform probability distribution: $B \sim F(B) = B/B_H, B \in [0, B_H], B_H > b$, for the sake of simplicity.

This heterogeneous valuation of the reward captures an essential part of Olivi’s concept of personhood based on the freedom of choice, which would be trivial if the angels were all the same with the same preferences. The angels are responsible for their preferences, not for producing them, but for activating them. They can strategically obey them or exert some restraint depending on their expected consequences. This distinction was overlooked by Anselm of Cantorbery, thus creating the *conundrum* that led him to exclude the power to sin from the freedom of choice (see Anselm, 2002). It can’t be disputed that Olivi’s framework clearly results in a consequentialist rational choice analysis much before its time. Olivi seems to follow the literature of the time by emphasizing pride and selfishness as the key components of B that are absent or negligible in b . However, the book also mentions other components that are described below. With the complementary probability $1 - \pi$, the sinners are assigned the task of tormenting the other sinners, i.e., they become the demons who do not torment one another, but will only be punished in eschatological times. Olivi claims that an endogenous cumulative process makes the latter more and more vicious as time passes: “although they knew that they could never recover the good entirely, they did not know necessarily that they would be so stripped of all sense of virtue and justice that they would never be able, even moderately, to repent for their evil, nor cooperate, even moderately, to their return” (p.127). This clearly is a lock-in mechanism that does not give a second chance to the demons. Olivi goes one step further in his analysis of the demons’ motivations in question 46 (p.281), in an intriguing sentence claiming that “the demons’ wills” ... “can freely and diversely give in to well received and desired temptations” ... “conceded by God to

their capacity, to some extent.” (p.177). Are these incentives aimed at boosting the demons’ effectiveness? Then, *ceteris paribus*, they are additional positive components of B , at least for some angels who value them.

The Angels’ Expected Utility Function and Decision

Let us assume that both H and π are functions of n_s and n_d , which denote respectively the numbers of sinners and demons, both treated as continuous variables. For the sake of simplicity, let us specify these functions as linear in the n_d/n_s ratio as $\pi = 1 - n_d/n_s, 0 \leq n_d \leq n_s$ and $H = \lambda n_d/n_s, \lambda > 0$. Hence, $\lambda > 0$ measures the demons “productivity” at inflicting torments on the other sinners, assumed constant across the demons’ population. This captures both the ideas (i) that the probability of a sinner being punished decreases, the larger the number of them that are exempted from punishment by being employed as demons, and (ii) that the pain inflicted increases with the number of demons available *per capita* to inflict the torments, each one of them having more time to devote to each tormented fallen angel. Hence, the risky choice to take the B path provides the angel with an expected utility:

$$U_s = B - \pi H = B - \lambda \left(1 - \left(n_d/n_s\right)\right) \left(n_d/n_s\right), \quad (1)$$

while exerting self-restraint and making the safe choice of b yields:

$$U_r = b. \quad (2)$$

It follows that the angel will choose the risky path if $U_s \geq U_r$, i.e., if:

$$B \geq b + \lambda \left(1 - \left(n_d/n_s\right)\right) \left(n_d/n_s\right), \quad (3)$$

and will refrain from it otherwise.

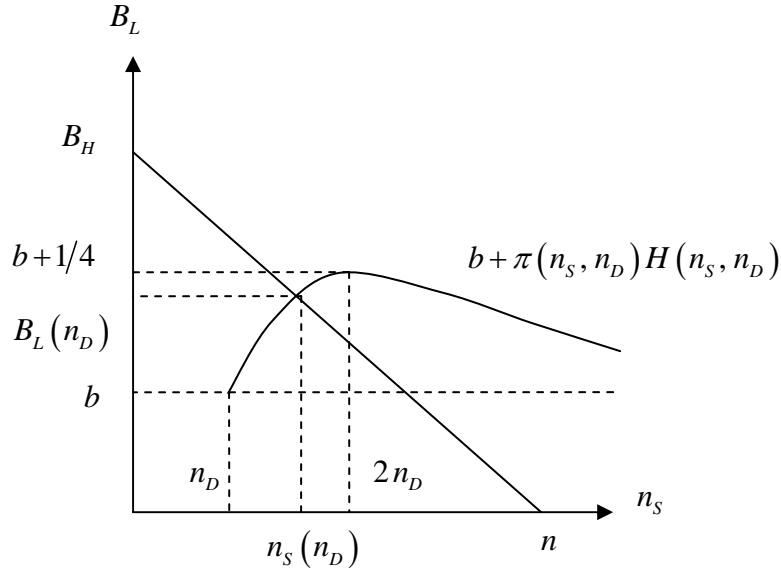


Figure 1: Partial Equilibrium Number of Sinners

Define $B_L(n_D)$ as the lowest value of B that is consistent with choosing the risky path, given n_D . Figure 1 depicts how this cutoff value $B_L(n_D)$ is determined in equilibrium for a given number of demons n_D . All the angels having a $B \geq B_L$ choose the risky path, while the others decide to play safe and choose b . Let n be the total number of angels facing the choice to make between the risky B path and the safe b one. Then, the uniform distribution assumed for B implies that the fraction n_s/n of sinners in the whole angel population is equal to $1 - B_L/B_H$. The downward-sloping straight line represents this relationship as $B_L = (1 - n_s/n)B_H$ over the relevant range. It intersects the x -axis where $n_s = n$ and the y -axis where $B_L = B_H$ and hence $n_s = 0$. The asymmetric hump-shaped curve depicts the total cost of making this choice, being the sum of the opportunity cost b of not playing safe and the expected pains endured in Hell $\pi H \geq 0$. The partial equilibrium $B_L(n_D)$ and $n_s(n_D)$ for a given number of demons n_D are found where the two curves intersect. Notice the kind of efficiency implied by this separating equilibrium, for a given n_D . The cutoff value $B_L(n_D)$ separates the cheapest ones to deter from sin, as measured in terms of demons, thanks to a low B , on the one hand, from the most expensive ones to deter that are left undeterred as sinners, because of a higher B , on the other hand. Lastly, because $\pi H \geq 0$,

figure 1 implies as well that $B_L(n_D) \geq b$, which in turn implies that $n_S < n$. This is because, using the fact that $B_L = (1 - n_S/n)B_H \geq b$, we can write $n_S \leq (1 - b/B_H)n < n$. Hence, in this model, it is not possible to induce all the angels to become sinners.

3. What Are Demons Good For?

Figure 1 has shown that there is an equilibrium relation $n_S(n_D)$ between the number of demons and the number of sinners. However, a glance at the right-hand side of (3) sends a warning that the slope of this relationship is not constant over the whole space as it can be written as a symmetric hump-shaped quadratic in (n_D/n_S) , which is zero for either $n_D = 0$ or $n_D = n_S$, and reaches a maximum for $n_D/n_S = 1/2$. This is translated into the asymmetric hump-shaped cost curve in figure 1 as a function of n_S . Therefore, it is not straightforward to guess confidently from the diagram any comparative static predictions about the impact of a change in the number of demons. The value of $B_L(n_D)$ could either shift upwards or downwards in response to a policy-change of n_D , depending on the initial conditions regarding b and n_D . Fortunately, some firmer findings can be obtained using a slightly more formal approach.

Equilibrium Sin-Deterrence Impact of Demons

This graphical insight can be taken on board more formally by writing the equilibrium condition as follows:

$$B_L = (1 - (n_S/n))B_H = b + \lambda(1 - (n_D/n_S))(n_D/n_S). \quad (4)$$

Rearranging the terms yields a very convenient equation for expressing this equilibrium condition, which determines the maximum possible value of n_S for any given n_D . It reads:

$$n_S^2((B_H - b) - n_S B_H/n) = \lambda(n_S - n_D)n_D. \quad (5)$$

Fortunately, there is no need to try and solve this equation as a simple diagram allows us to extract all the relevant qualitative information needed to characterize the equilibrium

relation between n_s and n_D and use it for successfully performing our analysis. From (5), one easily finds that the maximum number of sinners is reached when the following holds:

$$n_s = n(B_H - b)/B_H \text{ if either } n_D = 0 \text{ or } n_s = n_D, \quad (6)$$

This dual equation gives us the values of our two key variables at the two ends of the relevant range. Figure 2 only depicts the $n_s(n_D)$ curve within this range, namely the cone between the y-axis and the $n_s = n_D$ ray through the origin. No $\{n_s, n_D\}$ pair would make any sense outside this cone. It is a bit trickier to show that a third remarkable point of this equilibrium relation between n_s and n_D is found where $n_s = 2n_D$. We got a hint about it above when we found that the slope of the quadratic term in (3) was changing sign when this equality prevailed. For the more demanding reader, one can take the total differential of (5) in terms of dn_s and dn_D , and rearrange the terms to write:

$$\frac{dn_s}{dn_D} = \frac{\lambda(n_s - 2n_D)}{2n_s(B_H - b) - n_s^2 B_H/n - \lambda n_D} = 0 \text{ if } n_D = \frac{n_s}{2}. \quad (7)$$

This is the first-order condition of the following program:

$$\min_{n_D} n_s \text{ such that } n_s = n_s(n_D) \quad (8)$$

Substituting $n_s = 2n_D$ into (5) and rearranging the terms yields the minimum value of n_s as:

$$n_s = n(B_H - b - \lambda/4)/B_H \text{ if } n_s = 2n_D. \quad (9)$$

This value of n_s lies below the one given in (6), provided $\lambda > 0$, confirming that the $n_s(n_D)$ curve is U-shaped in the relevant cone as depicted in figure 2. This condition is easily accepted as $\lambda = 0$ would destroy the value of the model, making “God” impotent in his fight against sin, a statement that Olivi would strongly reject. Moreover, we need to assume that $\lambda \leq 4(B_H - b)/B_H$ to ensure that $n_s \geq 0$.

“God’s” Optimal Choice of the Demons’ Number

Figure 2 shows the optimal $\{n_s, n_D\}$ pair that minimizes the number of sinners at the intersection of the $n_s(n_D)$ curve and the $n_s = 2n_D$ ray through the origin. This means that in this model's specification, the total number of sinners should be split equally between the tormentors and their victims to provide the right incentives that minimize the number of sinners. This is obviously a model-dependent prediction, which can easily be modified as shown below. Before that, figure 2 may also be used to derive some comparative-static predictions regarding the impacts of the various parameters of the model on the optimal $\{n_s, n_D\}$ pair.

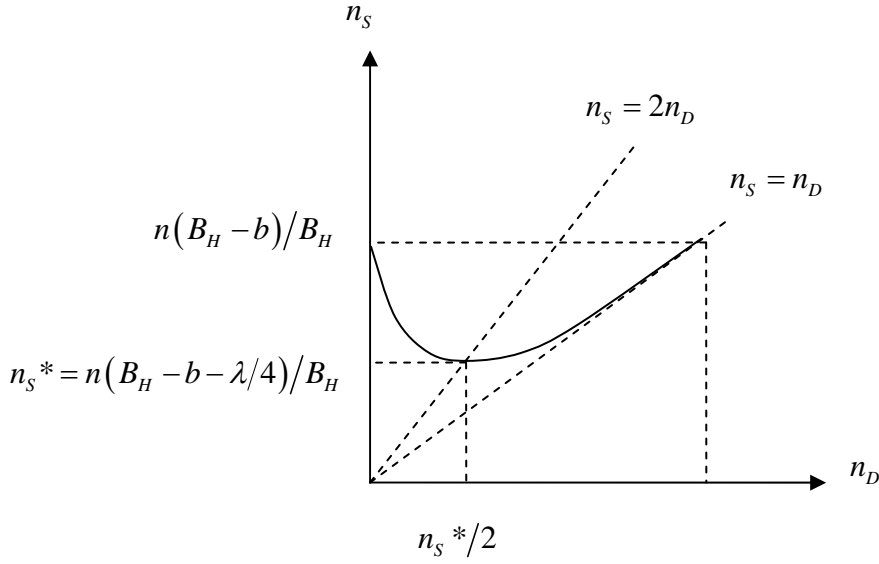


Figure 2: Number of Demons that Minimizes the Number of Sinners

Figure 2 is also allowing us to perform some comparative statics with respect to the four parameters $\{\lambda, B_H, b, n\}$. The first one in this list, λ , measures the harshness of the demons, i.e., their effectiveness at inflicting punishment and thus at deterring the sinners. An increase in λ stretches the $n_s(n_D)$ curve downwards, without changing its intersections with the two sides of the relevant cone. It follows that the optimal point slides along the $n_s = 2n_D$ ray so that both n_s and n_D fall, proving the effectiveness of “God’s” deterrence strategy. By contrast, an increase in B_H shifts the whole curve upwards, while the optimal point slides along the $n_s = 2n_D$ ray toward the north-east, showing that “God” would respond to an

increase in the angels' appetite for sin by increasing both n_s and n_D . Some improvement in the fate of the god-loving angels leading to a rise in b , making self-restraint more desirable, would instead shift the curve downwards, allowing "God" to cut both n_s and n_D . Lastly, an increase in the total number of angels n would shift the whole curve upwards, leading to increases in both n_s and n_D . This prediction makes pretty good sense but does not fit well with Olivi's discussion of the right number of angels to have in heaven. In several parts of the book, Olivi tackles this issue of the right number of angels. He has a very intricate discussion, which he uneasily concludes by advocating that their number should be kept constant by replacing the fallen ones by promoting some human beings to replace them. In particular, he stresses on several occasions that St Francis should be made an angel sitting to the right of God for replacing Lucifer. Figure 2 would point out instead that a fall in n would shift the curve downwards, leading to decreases in both n_s and n_D . This would be a good thing if reducing the incidence of sin among angels really was the ultimate goal to pursue. Hence, Olivi's conclusion might be a little tainted by some corporatist considerations in favor of the Franciscans, as our model's comparative statics would welcome the fall in the number of angels.

Therefore, the foregoing mathematical modelling exercise suggests a neat interpretation of this medieval literature on the fall of the angels suggesting that "God" chooses the number of demons to provide the right incentives for minimizing the total number of sinners without restricting the angels' freedom of choice. This means that the rules of this game ensure equality of opportunities for all the angels, even if differentiated outcomes are resulting from the working of the divine lottery so devised, whose parameters are the same for all. Kaye (2014) emphasizes how such *aequalitas ad iustitia* became the dominant concept of equality in the 13th century, making the latter more flexible than in the Aristotelian tradition. Pasnau (1999) provides further discussion of Olivi's views on human freedom.

Antipathy for the Devil

Other assumptions about "God's" motivations could easily be analysed within this framework. For example, "He" could be assumed to minimize a loss function that involves an additional subjective cost of the demons' activity on top of their initial subjective cost to him as sinners, may be because of some compassion for the harmless sinners that they torment on duty. This could be modelled as follows:

$$\min_{n_D} n_s + \gamma(n_D), \gamma'(n_D) > 0, \text{ such that } n_s = n_s(n_D) \quad (10)$$

Such preferences could be represented by an indifference map with slope:

$$\frac{d n_s}{d n_D} = -\gamma'(n_D) < 0. \quad (11)$$

In this case, depending on the weight assigned to n_D , “God’s” chosen point could be located somewhere along the downward-sloping part of the $n_s(n_D)$ curve on the left, or even at the corner point where $n_D = 0$ and $n_s = n(B_H - b)/B_H$. However, as Olivi puts it: “God accomplishes all his works with the utmost consistency” (p.197), while this prediction of $n_D = 0$ clashes obviously with the actual number of demons that Olivi believed to be large. This suggests that the present model should not put too much emphasis on $\gamma'(n_D)$, keeping it low enough so that n_D would be far enough from 0. Moreover, this “antipathy for the devil” could arguably be viewed as a distortion against the equality of opportunities for all. Therefore, this mild extension could be forgotten without much loss, and consequently, “ $\min_{n_D} n_s$ such that $n_s = n_s(n_D)$ ” seems a pretty good specification of “God’s” objective function in terms of Olivian predictions.

4. Was Lucifer the Demons’ Collective-Action Leader?

A recurrent point in Olivi’s *Demons* book, as well as elsewhere in the medieval literature on the fall of the Devil, is that Lucifer was in fact the charismatic leader of the demons, showing them the way to the path of sin. This point is clearly made in the following quote, about why many angels followed him: “Each one was incomparably feeling grander with him than without him, and mainly because splitting from him, as he was the leader of all, amounted to splitting with all his subjects, like also splitting from the King’s grace amounts to splitting with the kingdom” (p.107). Notice that the positive externality felt by some the angels when they cooperate with “Lucifer”, as described in this quote, must be counted as an additional component of their value index B . The previous section modelled instead the fall of a large number of angels as a spontaneous response to the incentives given by “God”, while the latter was the undisputed master of the game, choosing the number of demons among the sinners with a view to minimize the latter’s number. In that framework, “Lucifer’s” grandeur could simply be captured by assuming that he had the largest B of all, making him a kind of

role model, true to his name meaning “light maker”. However, the image of the King used by Olivi in the above quote may hint that there was something more, in the minds of these authors, i.e., that Lucifer really exerted some executive leadership over the demons, namely in this framework, that “he” was able to somehow determine their number. This assumption is clearly a recurrent one in the long tradition of the myth of the fall of Lucifer, which Marx (2000) traces back to Sumerian times, in a very rich review of the “prehistory of a myth”. A simple extension of the model enables us to take such a mobilizing role for “Lucifer” into account, using elementary bargaining theory, to bring out first some key predictions that can be tested against Olivi’s claims made in the *Demons* book. It reveals a potential tension between some of them. Then, a subtle revision of the specification brings out the key assumption required to dispel that tension and establish full consistency.

A Clash of Policy Objectives

Assume now that “Lucifer” wanted to maximize his ego rent, for the sake of his pride and selfishness that are widely pointed out in many parts of the book and elsewhere in this medieval literature, by maximizing the number of demons n_D that would join him under the constraints that $n_D \leq n_S$ and $n_S = n_S(n_D)$. However, this number is controlled by “God” in the model’s specification analyzed above, and “Lucifer” must be able to tamper with the incentive structure to influence the number of sinners and thus, indirectly, the number of demons chosen by “God”. This section investigates “Lucifer’s” potential role as a collective-action leader managing to mobilize the demons to reduce their effectiveness at tormenting sinners, as captured by a fall in λ . The range of potential values of λ that Lucifer and his followers can implement by this mechanism is restricted as follows: $\lambda \in [\lambda_L, \lambda_H]$ such that $0 < \lambda_L \leq \lambda \leq \lambda_H$. The comparative-static analysis performed above has shown that such a cut would shift the $n_S(n_D)$ curve upwards, except at the two end points where it hits the borders of the relevant cone. “God’s” policy response is given by the $n_S = 2n_D$ ray through the origin, along which “God” chooses n_D that minimizes n_S , his objective function in this game, along the $n_S(n_D)$ curve. It follows that the best outcome that “Lucifer” could reach by minimizing λ , measured in terms of number of demons created, is located at the intersection of $n_S = 2n_D$ and $n_S = n(B_H - b - \lambda_L/4)/B_H$ in figure 3. Assume also that λ was initially at its maximum value, as seems fitting given “God’s” objective function, so that “Lucifer” can only reduce it by mobilizing the demons for reducing their harshness.

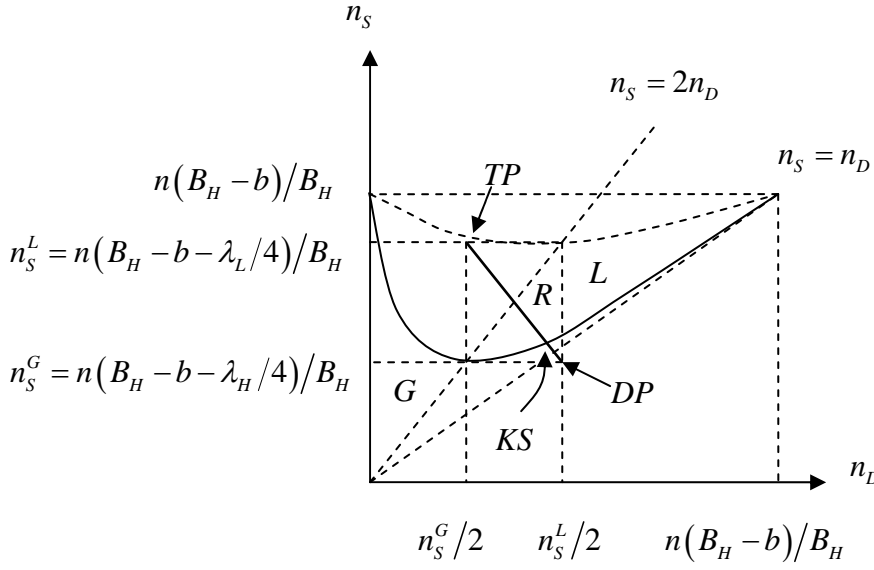


Figure 3: Kalai-Smorodinsky Bargaining Solution and Nash Equilibrium

Let us now enrich figure 2 to produce figure 3 by first labelling the two points preferred by the two players. Quite naturally, G represents the optimum point for “God” analysed in the previous section. As assumed above, “Lucifer” is seeking to maximize the number of demons, i.e., to reach the highest possible point on “God’s” $n_S = 2n_D$ policy-response function. This Northeastern-most possible point on the diagram is labelled L as it is “Lucifer’s” bliss point. However, they can’t both get their preferred outcomes simultaneously, as they involve incompatible numbers. Hence, the two players must find a way to cut the Gordian knot. Suppose they cannot just fight it violently between them, because such a behavior might seriously damage their self-image and destroy their charisma. Between them, assume first that there are only two dignified solutions (i) they can first toss a fair coin, or play with another fair lottery, and the winner simply chooses his preferred outcome, the loser getting then what’s left, i.e., his worst one, or (ii) they bargain to find a solution⁵. Ceccarelli (1999) shows how Olivi made a major contribution in considering a game of chance like the tossing of a fair coin as a contract, thus recognizing convincingly its undisputable lawfulness. This entails that he would not have raised any objection against this coin-tossing solution, were the two players credible enough to fulfil their contractual obligations, while the “sin” would be to breach this contract.. Let us first analyse why none of these dignified solutions is

⁵ See Mas Colell et al. (1995) (p.844) for this interpretation.

up to reconciling Olivi's observations about the rivalry between "God" and "Lucifer", before introducing below a less glamorous one that fits the alleged facts better.

"Tossing a coin" might be understood as staging a decisive battle in Heaven between the demons and the God-loving angels despite its very uncertain outcome. In the coin-tossing case, there are two outcomes with different winner and loser. If "God" wins, he will get $n(B_H - b - \lambda_H/4)/B_H$ sinners, the smallest number he could get as seen above, given the initial λ , while "Lucifer" will only get $n(B_H - b - \lambda_H/4)/2B_H$, the smallest number of demons that he could get. If "Lucifer" wins, he will get $n(B_H - b - \lambda_L/4)/2B_H$ demons, the largest possible number of demons along "God's" policy-response function $n_s = 2n_d$, and "God" will get $n(B_H - b - \lambda_L/4)/B_H$ the largest possible number of sinners, i.e., a terrible loss for him. Hence the differences between the two players' best rewards and worst outcomes can be measured by the sides of the rectangle drawn in figure 3 between G and L . The horizontal sides measure the loss affecting "Lucifer" if he loses instead of winning the toss. The vertical ones measure the difference between "God's" best reward, at G if he wins, and his worst outcome, on the horizontal line through L , otherwise. In terms of figure 3, the pair of *ex ante* expected payoffs from such a fair toss would thus be found exactly half way between the two bliss points G and L , at the point labelled R inside the rectangle along its diagonal.

Can "God" and "Lucifer" Bargain Successfully?

The second dignified solution announced above is (ii) to bargain a way of sharing the gains, without wasting any possible improvement for at least one of the players. The two worst outcomes for both are found together at the northwestern corner of the rectangle. This point is labelled "Threat point" (TP for short) because it represents the pair of worst outcomes that one or the other will get with probability $1/2$ if they fail to strike a bargain and thus resort to tossing a coin instead as described above. Let's define DP as the impossible "Dream Point" where each player gets his most-preferred outcome. Then, the segment joining TP to DP is the downward-sloping diagonal such that $n_s = n_s^L + n_s^G - 2n_d$, $n_d \in [n_s^G/2, n_s^L/2]$, that intersects the upward-sloping diagonal $n_s = 2n_d$ at the point R defined above, which represents the pair of expected payoffs from the tossing of the fair coin.

A glance at the diagram shows that the R allocation is Pareto-dominated because of the convex curvature of the $n_s(n_d)$ curve. There remain "100 \$ bills on the pavement" to be picked up that could be pocketed by rational players. By slightly cutting the number of sinners

and increasing slightly the number of demons relative to the values expected at R , i.e., by bartering a little concession on one's objective against a little concession on the other one's objective, both players could improve their payoffs, and go on doing so up to the point where they hit the U-shaped $n_s(n_d)$ curve, which delimits the outer possibility frontier for the $\{n_s, n_d\}$ pair. When they do, it is obvious that there are an infinity of points located in the southeastern rectangular cone from R such that all the points of the $n_s(n_d)$ curve that are inside this cone are Pareto optimal, meaning that you can't improve the payoff of one of the players without reducing that of the other along that line, and *vice versa*. The challenge is to find an agreeable method to devise a satisfactory sharing rule to allocate the benefit of the move between the two players, i.e., to choose a point on that Pareto optimal line.

Kalai and Smorodinsky (1975) have devised a simple and intuitively appealing solution to the problem of picking up a satisfactory sharing rule for both in this type of problem. The solution is based on the idea of proportion, which has been emphasized by Kaye (2014) as rising to a high ideological status during the 12th and 13th centuries. Moving from the threat point to R by the kind of barter described above takes place along a diagonal that reflects each player's contribution to their joint betterment. Kalai and Smorodinsky (1975) argue that the same proportion should govern the sharing of the additional gains obtained by moving from R to the U-shaped $n_s(n_d)$ curve⁶. Doing this, i.e., moving along the same diagonal, yields in figure 3 the *KS* point, which Pareto dominates R and exhausts the possible improvements for both, as the U-shaped $n_s(n_d)$ curve may be regarded as the outer possibility frontier. Such a fixed proportion associated with proportional increases in the "goods" that each one values seems a convincing way to settle a bargain and avoid gambling on a nearly all-or-nothing allocation by tossing a coin that is fairly congenial to the spirit of Olivi's time⁷. This pragmatically bargained allocation between "God" and "Lucifer" presents the additional advantage of providing some insurance to both, as a kind of risk-pooling agreement where both players cannot be surprised by an unfavorable toss. Moreover, this rule does not infringe on the personal freedom of the individual angels, it simply changes the proportion between the demons and the sinners relative to the cases where either "God" or

⁶ For a more elaborate axiomatic discussion of this assumption see Holler (1986) and Kalai (1977).

⁷ Other bargaining solutions would do as well, but the key point for the present discussion is that they all require a credible commitment capacity.

“Lucifer” rules alone as the law maker, by settling for the middle way. It does not infringe either on the angels’ equality of opportunity found in the previous case, with one exception, i.e., the promotion of “Lucifer” as a collective decision maker dealing indirectly with “God”.

However, it seems that Olivi did not consider the option of a bargain between God and Lucifer. He claims that the demons “when they are fought in a virile and triumphant way by the saints or diversely outwitted by God, they are saddened and irritated more. But when they triumph over us as they please, they enjoy themselves that much” (p.179). On the face of it, it seems that Olivi understands the battle between God and Lucifer as an ongoing one, rather than a decisive coin-tossing contest as described above. This suggests that neither point R , in expected value terms, nor the deterministic Pareto Optimal point KS would satisfactorily describe the outcome. This active fighting might result from “Lucifer’s” incapacity to tie his own hands to commit credibly to the deal. Blattman (2020) discusses very clearly the literature that ascribes violent fighting to such a commitment failure, starting with Azam (1995) and Fearon (1995). “Lucifer” would then seize the first opportunity to break the deal, by increasing the number of demons beyond the KS point *via* a cut in λ . Knowing this, “God” would then refrain from striking such a deal with “Lucifer”. This is a neat application of Olivi’s understanding of games of chance as contracts, so that the sin is not to gamble but to breach a contract, as discussed by Ceccarelli (1999). Who would believe “Lucifer’s” word anyway? Not even demons would, probably. Notice that this commitment failure also kills the coin-tossing approach, as “God” would not trust “Lucifer” to accept the potentially adverse outcome drawn using any decisive fair-lottery, and “He” would guess that “he” would challenge it by all means. This kills the hope of explaining Olivi’s observations within the dignified Kalai-Smorodinsky framework, and probably also any other bargaining framework. However, a third and less dignified solution seems more promising, involving neither coin-tossing nor bargaining.

Guerrilla Warfare vs. Kalai-Smorodinsky

Hence, the subtle change of assumption that gives “Lucifer” some leverage to determine the number of demons by tampering with the incentive structure, tainted by his inability to commit credibly, seems to change the model’s predictions from irenic to chaotic. Two assumptions must be changed to bypass this dead end. First, by contrast with the coin-tossing solution discussed above, “Lucifer” does not really want to win a war against “God” by staging a decisive battle. Instead, “his” objective is to have as many demons admiring him as possible, and for this “he” needs to sabotage “God’s” sin-deterrence apparatus. This is the key insight that ensures Olivi’s theory’s consistency announced in the introduction. A guerilla

warfare's "hit-and-run" strategy, where no violent encounter is ever decisive, turns out to be more appropriate for "Lucifer's" purpose, and to better fit Olivi's observation that the war against "Lucifer" is an ongoing one quoted above. It is then rational to think that waging such a war in Heaven is one of "Lucifer's" handles to divert the demons' time and energy from tormenting the sinners, i.e., to keep λ low and thus n_s and n_d high. Hence, "Lucifer" is not an inveterate gambler after all as he chooses undecisive guerilla warfare rather than "tossing a coin". Second, keeping the demons busy this way does not require "Lucifer" to make any promise that would not be believed anyway, what killed the bargaining solution examined above. His charisma mentioned above would be enough to enroll a large enough number of demons. Therefore, point L is the true (Nash) equilibrium point of this ongoing-guerilla game, with probability 1. It is reached when "Lucifer" and his demons have reduced λ to such an extent that the U-shaped $n_s(n_d)$ curve is shifted to the dashed position, its minimum point being now at L . Notice that "Lucifer's" inability to commit credibly is rewarded by a small increase in n_d relative to the KS point while "God" pays a large price for that. This Nash equilibrium is thus clearly inefficient, like so many of them are. As the demons' collective-action leader, "Lucifer" is thus able to corner "God" in a defensive position that just prevents the number of demons from getting out of control. In game-theoretic terms, "Lucifer" is in fact the "Stackelberg Leader" in the game⁸. This conclusion, which is not derived so clearly by Olivi, but probably perceived by him in some intuitive fashion, might explain why he never managed to complete the *Demons* book (as far as we know), and turned instead his attention to *Usus Pauper*, on the one hand, and the *Apocalypse*, on the other hand. It is only in the *Apocalypse* Ap 12 that the end of this ongoing war in Heaven is announced as the visionary sees the "grand dragon, the ancient snake called Devil or Satan", ... "thrown down to Earth, his angels ... with him", at "the close of a fight that opposed him to Michael and his angels" (cited in Marx, 2000, p.171, my translation). Then, Lucifer is irreversibly defeated, in eschatological times. May be, that's why Olivi focused on the *Apocalypse* in the last few years of his life. In the meantime, the subtext of the *Usus Pauper* literature, on the grassroot Franciscan side, might just be something like: 'OK, if God and Lucifer cannot settle their accounts in heaven, let us take the issue in our own hands and solve it by self-discipline',

⁸ A Muslim friend of mine, seeing this implication, asked me why an all powerful god would then create such a wicked and charismatic angel. I answered that this was in fact a commitment device to make freedom of choice credible for angels and humans. But why should there be any freedom of choice? Without it, people would be reduced to the role of "toys", and it would make no sense to be god. This is the message delivered by Olivi's story.

i.e., making b as large as possible by enforcing a stricter and more contemplative rule. Burr (2002) suggests that even such an agenda was neither easy nor consensual to implement.

5. Conclusion: A Parable on Human Freedom and Agency

In the Rational-Choice hermeneutic of Olivi's *Treatise on Demons* presented in this paper, "God" chooses the number of demons and of sinners, using the former as a tool to minimize the latter, may be under stress on "Lucifer's" part leading to a more complex outcome, closely fitting Olivi's description. A simple extension of the model has thus shown how "Lucifer" can tamper with the incentive structure to increase the number of demons that would join him, thus enhancing his ego rent, by reducing their sin-deterrence efficiency to induce "God" to increase both the numbers of sinners and demons. This can be achieved by diverting some of the demons' tormenting capacity into a hit-and-run guerilla strategy against "God" where no encounter is ever decisive before doomsday. However, neither "God" nor "Lucifer" determines who will become what, and they only determine, alone (section 3) or jointly (section 4) by interacting strategically, the right incentives or "rules of the game" to simultaneously (i) minimize the number of sinners or at least keep it under control, while (ii) also maximizing the number of demons, in a mutually compatible way. Kaye (2014) emphasizes that "God is the Law" in the dominant ideology emerging in the 13th century. Similarly, in this model, the "God-and-Lucifer" pair may jointly choose the number of demons needed to achieve that objective in the Nash equilibrium, but neither one makes any "*ad hominem*" decisions, letting the random draws determine the individual outcomes. Faced with the appropriate incentives, each person is free to choose what position to take in the "Divine Lottery" as a function of her idiosyncratic preferences and rational understanding of the incentive structure. In this model, the latter is made fuzzier as "Lucifer", whose ability to commit credibly is doubtful, is having some policy handle for influencing the number of demons chosen by "God". Then a more chaotic regime arises, disturbed by an ongoing undecisive hit-and-run guerilla warfare in Heaven between "God" and "Lucifer" which disturbs the rules of the game by reducing the demons' sin-deterrence efficiency. This step in the analysis plays the key role in accommodating Olivi's claims consistently within the rational-choice hermeneutic approach. As mentioned in the introduction, in the medieval literature so analysed, from Anselm of Cantorbery to Peter Olivi, the lessons derived from the study of the world of angels and demons are meant to be transposed to human beings and their society. Angels and demons are thus used as a kind of simulation model for understanding

human social interactions, cutting some of the human world's complexity away by wisely using Ockham's razor. Olivi and the Franciscan "Spirituals" that followed in his wake had chosen "*usus pauper*" (poverty and low profile) as a disciplining device to play safe in that lottery, i.e., to ensure that $B < b$ for all of them. Olivi goes as far as stating that this is the only path to being a true Christian.

In his work on the Apocalypse that shortly preceded his death, thoroughly analysed in context by Burr (1993), Olivi coined the expression "Carnal Church" to refer to those confessors and inquisitors who were extracting very large fines and "restitutions" from merchants and traders or other productive agents like farmers and tenants, the revenues of which were accruing to the Bishoprics and some monasteries. In a Joachite three-slot time-line perspective, he predicted that they would disappear in the "Age of the Spirit", the final age that he predicted would follow the current "Age of the Son", whose beginning of the end he thought he was witnessing. The latter would occur in a near future that would lead eventually in due course to a festive Apocalypse in the year 2000 or about. Some authors even announce that a large banquet would take place in Jerusalem where the Christians and the Jews would fraternize irreversibly. According to Forni (1999), it is *via* this piece of work that Olivi exerted a strong influence on Dante. By exposing the confessors and inquisitors to such a harsh criticism as he does, one wonders whether Olivi was not influenced to some extent by his own analysis of the fallen angels and the demons. About twenty years after his death, he was censored a second time and his grave desecrated in 1318 because it attracted by word-of-mouth many grassroots pilgrims coming from many different and even distant places who claimed that he was a non-canonized saint, as proved by the miracles that they claimed were occurring around it. Many of them, including some monks and clerics, were burned at the stake after Olivi's excommunication. Biget (1999), Burnham (1999) and Burr (1997, 2002) describe in detail the Olivian cult that developed in Languedoc at the turn of the 14th century and its repression. Burnham (1999) and Burr (1997, 2002) analyse in depth the very touching case of Na Prous Boneta, probably burned at the stake for being a virgin and wanting to remain so. Nevertheless, she had a very deep and articulate understanding of Olivi's theses, as reflected in the records of the Inquisition (see Appendix in Burnham, 2000, and Burr, 2002), thus fatally violating an important stereotype against lay women and their cultural standards prevailing at the time. Burr (2002) mentions that some of Olivi's books were translated from Latin into the Languedocian vernacular shortly after his death, thus enhancing his impact on ordinary people.

Fortunately, from our point of view, instead of being burned as they were supposed to be since 1326 in the wake of Olivi's excommunication, some copies of some of his books were hidden by fearless people, and new copies made. Most of these copies were anonymized and used under cover in the following centuries, in particular by the street preacher San Bernardino da Siena (1380-1444), among others. The latter marked his surviving own copy of the *Contracts* book, found in his personal archives in Siena, with the initials *P. I.*, for *Petrus Johannis*. For the survival of brilliant ideas, plagiarism is obviously better than neglect. It is quite ironic that San Bernardino da Siena was canonized for the preaching and writing that he produced under the influence of an author that he very well knew was excommunicated and his books sentenced to be burnt. This is a nice "nose-thumbing" by delegation at the religious authorities that excommunicated Olivi. Some new copies of some of the latter's books were made when a window of opportunity opened during the 1471-1484 period, when Pope Sixte IV lifted the ban on Olivi (Burr, 1997). These books were (re)discovered and properly ascribed only less than a century ago and then made openly available to a broader readership. Some people would probably interpret as an eschatological sign the fact that his name mostly disappeared underground in secrecy for more than six centuries and then reappeared in the 1930s, triggering a slowly growing yet very active and well-deserved research activity about his work.

5. Appendix: Legitimacy of Modelling a Medieval Book's Argument

This way of translating a book into a mathematical framework seems shocking to many readers, especially those trained in the pure medieval history tradition who would call it "anachronic". However, it is a standard practice in economics where, for instance John Hicks (1936) translated Keynes's *General Theory* into the highly successful *IS-LM* model, which was the main tool for teaching macroeconomic theory for decades and is still taught in good universities at the introductory level. Luigi Pasinetti (1960) did the same for David Ricardo's work, with a significant following in the 1960s and 1970s. Similarly, Paul Samuelson (1971) devised a model of Karl Marx's labor theory of value and exploitation to bring out its dead ends. Duncan Foley (1984) presents a rich review of the many attempts made at modelling various aspects of Marx's theories, confirming that you can't have both a pure labor theory of value and a theory of exploitation based on class struggle at the society's level. This would entail a different rate of profit in different sectors, what even Marx did not accept as a realistic prediction in *Capital's* Vol. 3. In a similar spirit, the simple modelling performed above brings out a tension in Olivi's thought, as reflected plausibly in his failure to complete the

Demons book. However, that tension is relieved above by introducing a subtle assumption that brings out the key insight required to secure Olivi's theory's consistency. Closer to the topic of this paper, Laurence Iannaccone (1995) made a breakthrough in the analysis of religion by giving a rational choice, mathematically articulated account of Blaise Pascal's wager, among other contributions. His whole paper emphasizes the close links between religion and risk. This was a winning move, as many authors engaged in the research avenue so opened. The present paper shares some similarities with that line of research, but its ambition is restricted to a hermeneutic presentation of one book written in Languedoc in the 13th century. Nevertheless, it also focuses on the role of some forms of gambling in religious matters, like Pascal's wager.

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