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Fair Gatekeeping in Digital Ecosystems

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Abstract. Do users receive their fair contribution to digital ecosystems? The frequent accusations of excessive platform fees and self-preferencing leveled at dominant gatekeepers raise the issue of the standard gatekeepers should be held to. The paper provides a framework to explain business strategies and assess regulatory proposals. It stresses the key role played by the zero lower bounds on core and app prices in the setting of privately and socially optimal platform fees. Finally, it derives a simple rule for regulating access conditions and analyses its implementation.

Keyworks. Platforms, ecosystems, fair access, price and non-price foreclosure, zero lower bounds.

JEL numbers. L12, L4.

1 Introduction

Platforms – the gatekeepers of the digital economy – control sellers', app developers' and advertisers' access to their consumers. They do so through consumers' use of their "core services", which, under EU law for instance, may signify a search engine, a marketplace, a social network, an app-store or a video-sharing platform. Recent regulatory guidelines, including the EU Digital Market Act (DMA) and the proposed American Innovation and Choice Act and Open App Markets Act, echo earlier public-utility regulations and the antitrust treatment of essential facilities; they require that business users be given equal access to the core services and receive a fair share of their contribution to the ecosystem. They reflect two key concerns.[1]

First, platforms operate markets, but also compete in them; there have been many allegations that platforms engage in self-preferencing. Relatedly, even pure platform players (like Airbnb or Booking, which operate markets, but do not compete in them) may enter "sweet deals" with

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[1]See Online Appendix A for a brief description of the antitrust cases related to access conditions and of the regulatory proposals.

specific business users. Guidelines therefore require platforms to offer a level playing field among all business users, internal or external. Second, the magnitude of the merchant fees has always raised eyebrows, from the 2 or 3% levied by numerous payment systems to the 30% demanded by Apple's app-store or Google Play. A number of ongoing antitrust cases (Epic Games v. Apple; Spotify v. Apple) concern 3$^{\text{rd}}$ party apps trying to circumvent app-store fees they deem unfairly high. The DMA's suggestion that access condition be fair, reasonable and non-discriminatory (FRAND) leaves open the question of what "fair and reasonable" conceptually means, even putting aside the measurement issue. Can we design good rules to constrain the fees charged to merchants and apps by digital platforms such as Apple, Amazon, Booking, Google or Facebook?

This paper provides a first answer to the question of how antitrust authorities can enable business users to get their "fair share" of their contribution to a digital ecosystem, where "fair" refers to 3$^{\text{rd}}$ party developers and merchants receiving their contribution to the ecosystem (we later discuss the robustness of our conclusions to alternative notions of fairness). While there is a rich and important literature on pure-player and hybrid platforms, which we will later review, this literature mostly ignores two key features of the digital economy: Both core services (search, marketplaces, etc.) and digital goods (let's call them apps) are often offered to consumers for free, implying that platforms and apps would like to pay for usage but are prevented from doing so by consumer arbitrage.[2]

As of July 2022, 97% and 94% of apps in Google Play and Apple's app-store were freely available.[3] These include some of the most common 3$^{\text{rd}}$ party apps (e.g., PayPal, Dropbox), as well as the competing in-house apps by Apple and Google (e.g., Apple Pay and Google Pay, iCloud and Google Drive, respectively). On the core side, most digital platforms, such as the major app-stores, e-commerce platforms, search engines and social networks, grant free access to consumers. As is well known, a zero price on the consumer side is not to be taken as granted, as it hinges on the platform's ability to monetize the consumer's data or the other sides of the market – namely, 3$^{\text{rd}}$ party sellers and advertisers.[4] One of our main contributions is precisely to show how a downward price constraint (which for convenience we label the "zero lower bound", ZLB) either on core services ("core ZLB") or on applications ("app ZLB") is essential to the understanding of platforms' business strategies and desirable policy interventions.

Our basic model considers a two-sided platform that connects sellers of digital goods with consumers patronizing the platform. The platform levies a per-unit access charge a on 3$^{\text{rd}}$ party app sales (our results are robust to ad-valorem fees), charges a non-negative access price to consumers, and operates as a hybrid marketplace: It sells its own in-house apps in competition with the 3$^{\text{rd}}$ party sellers' ones. A key feature is that apps' developers may receive ancillary benefits from attracting a consumer, such as advertising revenues (content providers), data

[2]Negative prices would attract fake consumers (say, bots) who would not buy on the platform, nor be receptive to advertising, and supply meaningless data.

[3]See, e.g., `https://www.statista.com/statistics/263797/number-of-applications-for-mobile-phones/`.

[4]For instance, after its ads-business has been severely hurt by Apple's ad-tracking changes on iOS, and following the recent broader pullback in digital ad-spending, Meta created a new division with the aim of building paid products across Facebook, Instagram and WhatsApp. See `https://www.theverge.com/2022/8/31/23331342/meta-plans-paid-features-facebook-instagram-whatsapp`.

collection (most apps), or ancillary and premium services (e.g., Dropbox, Spotify, Zoom); we capture these per-consumer benefits by a number $b > 0$. In the digital, low-or-zero-marginal-cost world, ancillary benefits imply that the app's opportunity cost of supplying a customer is negative, making competitive prices infeasible. The app ZLB then forces the developer of an inferior app to settle for a complimentary use of the app.

To understand why these ZLB-based departures from existing theory matter, it is useful to return to the old Chicago School critique of foreclosure theory, which can be stated for the platform context in the following way: "*Aside from efficiency motives, a platform (the monopoly segment) has no incentive to foreclose a 3rd party app (an independent player in the competitive market): A rich ecosystem benefits consumers in two ways, product variety and enhanced competition, and allows the platform to raise its consumer price to extract the associated increase in consumer surplus.*"

(a) Impact of the access charge.

To make our point in the starkest possible form, Section 2 develops the simplest possible model of complementary core and app segments, with zero marginal costs. We show that, if none of the two ZLBs is binding, the Chicago School's "rich ecosystem argument" prevails. Furthermore, the access charge levied by the platform on the 3$^{\text{rd}}$ party app sales is neutral: local changes in its level have no allocative or redistributive consequences. Novel insights emerge when either the app ZLB or the core ZLB bind:

App supranormal profit. For per-consumer access charges a smaller than the ancillary benefit ($a < b$), a superior 3$^{\text{rd}}$ party app developer obtains a supranormal profit, i.e. it receives more than the value it creates. The reason is that it does not feel the full competitive pressure from the in-house or other inferior apps that are constrained by the app ZLB. Because its wholesale activity is insufficiently profitable, the platform benefits from self-preferencing/foreclosing the 3$^{\text{rd}}$ party app.

Impossibility to cash in on a rich ecosystem. A binding ZLB on the core product price means that a zero price for consumer access to the platform is already too high; the platform would like to subsidize consumers' participation in the first place and therefore may not want to raise the core price above zero, even to reflect a richer ecosystem as the Chicago School argument would have it. We show that the core ZLB is binding when the access charge is high (but not so high that the 3$^{\text{rd}}$ party app exits the market); a high access charge implies high app prices, making it necessary for the platform to stop charging for the core product in order to maintain the consumers on the platform. The 3$^{\text{rd}}$ party app is then squeezed, in that it receives less than its contribution to the ecosystem.

The analysis also unveils a deep connection between the ZLBs, fairness, and Baumol and Willig's "Efficient Component Pricing Rule" (known through its acronym ECPR). The latter defines a cap for the access charge equal to the vertically integrated firm's equilibrium markup in the competitive segment. The region in which the platform wishes to foreclose the 3$^{\text{rd}}$ party app ($a < b$) coincides with the region in which the access fee lies below the ECPR level and the

3rd party app receives more than its contribution to the ecosystem. The competitive neutrality region exhibits an equality between the access charge and the ECPR level, and a fair reward for the 3rd party app. Finally, the squeeze region has an access charge above the ECPR level and an unfair compensation of the 3rd party app.

(b) Pigouvian regulation.

In the basic model, these two ZLBs do not play out in the same circumstances (as we will see, the two ZLBs bind simultaneously when a sufficiently elastic demand forces the monopoly platform to lower its core price, or under platform competition). The app ZLB arises for low access fees, when the platform has too little skin in the game when providing access and therefore wants to engage in self-preferencing. The core ZLB operates for high access fees, which entail a squeeze of the superior 3rd party app, which then receives only a fraction of its contribution to the ecosystem. Such a margin squeeze is profitable for the platform and emerges under laissez faire, i.e., when the platform chooses the access fee: The platform optimal access charge fully squeezes the 3rd party app. Capping the access charge to any level in the competitive neutrality region (i.e., the intermediate region where no ZLB binds) is thus needed to ensure that 3rd party developers precisely appropriate the social value of their innovations. Provided the 3rd party app already exists, however, all outcomes in which it is not foreclosed are ex-post equivalent from a social welfare standpoint, since the margin squeeze has purely redistributional effects. The unregulated outcome is ex-post socially efficient, so that one might argue from this limited analysis that there is little scope for regulation besides fairness concerns.

Section 3.1 introduces innovation/investment by the 3rd party app. We initially take an efficiency/welfare perspective: efficient innovation/entry requires that the 3rd party app developer develop the app if and only if the expected development cost is smaller than the contribution to the ecosystem. For that, the developer must receive its contribution to the ecosystem. A squeezed 3rd party seller has a suboptimal incentive to develop its app, while the supranormal profit made under a low access charge – assuming that foreclosure is monitored – encourages excessive entry. Overall $a = b$ (or for that matter any access fee in the neutrality region) provides proper investment incentives. We also explain why the message delivered by the efficiency perspective is reasonable even if one rather used a more consumer-oriented normative criterion.

Section 3.2 introduces heterogeneity in consumer preferences, either in the overall utility of apps or in the extra benefit brought about by the superior app relative to the in-house one. In both cases, the platform again benefits from foreclosure if and only if $a < b$. The robustness of this conclusion comes from the simple comparison between the platform's per-consumer app revenue when it sells the in-house app and the 3rd party one, respectively.

Section 3.2.1 assumes that consumers' demand for the platform's app or more generally for the platform's offering is elastic. The core ZLB may then bind regardless of the level of the access charge. In that case the 3rd party app is squeezed for any access charge $a > b$. Proper investment incentives require that $a = b$. When consumers prefer the 3rd party app but differ with respect to their preference for it (Section 3.2.2), the lower the price difference between the two apps, the more consumers will use the superior 3rd party app, and so the more efficient the allocation. Because cheap access ($a < b$) is not an option when non-price foreclosure is feasible, $a = b$ is again optimal.

Combining all the desiderata requires that, when foreclosure is not monitored, the access charge be set at the Pigouvian level: The regulated access fee should coincide with the ancillary benefit associated with app distribution: $\hat{a} = b$. The reason why this can be interpreted as a Pigouvian access charge is that the 3rd party app "steals" b from the in-house app when taking a consumer away from it, thereby setting $\hat{a} = b$ makes the independent app internalize this externality.

(c) Contested bottlenecks.

Section 4.1 considers platform competition under consumer single-homing. Fierce competition to become the bottleneck of access to consumers (a) implies that the core ZLB binds, and (b) incentivizes platforms to price in-house apps aggressively (i.e., below their opportunity cost). The latter feature prevents the superior 3rd party apps' price from increasing in response to higher access charges, whenever, individually, they cannot affect the allocation of single-homing consumers across platforms. As in the case of a monopoly platform, competing platforms fully squeeze the 3rd party apps. Furthermore, the core ZLB prevents the corresponding profits from being passed through to consumers. As a result, platform competition does not reduce the desirability of regulating/capping access charges.

Section 4.2 demonstrates that platform competition and app-store competition work very differently. While Section 4.1 showed that platform competition is too app unfriendly, Section 4.2 demonstrates that app-store competition is too app friendly. If, refining recent regulatory prescriptions, designated platforms must give access to external app-stores,[5] creating app-store rivalry that itself precludes any access charge paid by apps, there is no levy along the value chain on apps, which are then empowered. Superior apps make supranormal profits. Overall, we should not expect competition to remove inefficiencies in the digital world of ZLBs.

(d) Implementation.

Section 5 turns to the implementation of the Pigouvian rule, according to which the access charge should be equal to the ancillary benefit. Can this ancillary benefit be measured? In principle, merchant and advertising fees might be measurable, although the platform may try to prevent the regulator from observing b (subsidiaries abroad in charge of advertising, bundling with other services, etc.), which implies that some interventions may be needed on that front to secure measurability. It may be more difficult to value data and consumer lock-in benefits.

Section 5.1 shows that, under the absence of monitoring of foreclosure, the relevant information cannot be elicited from the platform, even if the regulator knows the distribution of ancillary benefits among the various app categories. The platform's incentive is to set high access charges in low ancillary-benefit markets, so as to profitably squeeze superior sellers in these markets, and

[5]It is unclear whether the DMA requests free access of app-stores to platforms or access at a FRAND price, whatever this means. Section 4.2 first looks at the case of free access, and then observes that FRAND access should be interpreted as the Pigouvian rule.

foreclose superior apps in markets where ancillary benefits are higher (where it is constrained to charge lower fees, for the distribution of access charges to mimic that of benefits). Section 5.2 in contrast shows that the information about the ancillary benefits can be obtained from app developers, provided the platform can deny them access (which prevents the app developers from being too greedy). Section 5.3 then discusses the robustness of this important, and perhaps counterintuitive, insight. It shows how an appeal procedure allows to implement the Pigouvian rule, even if the court's measurement of the ancillary benefit is imprecise.

Section 6 concludes. Omitted proofs and additional material can be found in the Online Appendix.

Relevant literature. There is a large literature on foreclosure practices and the essential facility doctrine (e.g., Hart and Tirole, 1990; Rey and Tirole, 2007), and on access pricing for one-sided markets (e.g., Laffont and Tirole, 1994) and for telecom and payment card markets (e.g., Armstrong, 1998; Laffont et al., 1998; Rochet and Tirole, 2002, 2011). The literature on access to regulated bottlenecks showed that the regulation of access is needed, as a vertically integrated incumbent has little incentive to provide access to competitors. A celebrated rule, the ECPR (or Baumol-Willig) rule states that the access charge should be no greater than the vertically integrated monopolist's lost margin in the competitive retail segment. Its properties are analysed in Laffont and Tirole (1994); obviously it just connects two prices and says little about their absolute level. Another classic implication of the theoretical analysis is that an access markup does not necessarily mean that competitors are disadvantaged, as the markup increases the opportunity cost of the vertically integrated firm and its rivals alike.[6]

Three papers study the regulation of platform fees when the consumer and the merchant can transact through multiple channels: the platform and another channel (direct purchases, other platforms, other payment methods in the case of a payment platform). Because the consumer chooses the channel, the welfare analysis is naturally grounded in the externalities associated with this choice.

Two papers suppose that the merchant offers the same price regardless of the channel (there is a most-favored-nation, MFN, clause); the merchant's revenue from a sale is then channel-independent, which does not mean that its markup is. The merchant may enjoy a convenience benefit from the platform channel, as in Rochet and Tirole (2011): A card payment may dominate cash and cheque in terms of expediency, fraud prevention, accounting, or absence of hold up. The socially optimal access charge corrects for externalities of consumer channel choice upon merchants, and the socially optimal access charge (which in payment networks is at least partially passed through by issuers to consumers) is equal to the merchant benefit from a card usage;[7] this internalization principle is the so-called *tourist test*. In Gomes and Mantovani (2021),

[6]This is important because marginal-cost pricing of access is not the right welfare benchmark. It jeopardizes the recovery of fixed costs for the essential infrastructure owner and it further incentivizes foreclosure ("self-preferencing" in modern parlance), requiring heavy investment in regulatory capacity: The vertically integrated firm cannot make money by selling access and therefore must make its money on the competitive segment.

[7]This socially optimal access charge is nonetheless smaller that the platform's preferred one.

the platform creates an informational and a convenience benefit for consumers; in particular, the platform offers products that they were unaware of. This improved-opportunities benefit of the platform is of course internalized by consumers. But, consumers' access to the platform being free, they do not directly reward the platform for it, which is a problem if the platform is created only if sufficiently profitable. The platform however can charge consumers indirectly through the competing merchants' access charge, then passed through to consumers. Gomes and Mantovani show that, provided the presence of the platform does not increase aggregate sales, the welfare-maximizing access fee equals the sum of these two benefits.[8]

Alternatively, there may be no MFN and so prices are lower on the platform, which displays tougher merchant competition than the direct sale channel. The consumers may then choose to transact through the platform not because they prefer this channel, but because the latter lowers merchants' markups, at least in part a redistributive effect (Wang and Wright, 2022). The privately optimal fee may now fall short of the socially efficient one, which equals the platform's marginal cost of implementing the transaction plus the amount by which the platform, by intensifying seller competition, decreases the merchants' margins. Again, the merchants' pass-through of the access charge is key to restoring proper consumer incentives.

In contrast with these three papers, which hinge on consumers' choice of channel to interact with merchants, we assume that consumers single-home, whether there is platform competition of not: the platform is a "gatekeeper". The set of potential externalities under consideration is then rather different: (a) a vertically integrated gatekeeping platform may use non-price instruments to prevent consumers from accessing the best product; (b) the platform may jeopardize the existence of superior 3^{rd} party apps by squeezing them through a high access charge; (c) the 3^{rd} party app enjoys supranormal profit when the app ZLB binds.[9] The welfare-maximizing access charge is then equal to the opportunity cost for the platform of letting 3^{rd} party sellers serve consumers, rather than to the benefits it brings to one or both sides of the market.

A number of recent papers examine platforms' incentive to vertically integrate, and the welfare effect of this vertical integration, in the presence of foreclosure and/or imitation concerns: see Anderson and Bedre-Defolie (2021), Etro (2021b, 2022a), Gutiérrez (2021), Hagiu et al. (2022) and Zennyo (2022). Yet, these works, as the ones on platform fees' regulation, assume non-negative opportunity costs (i.e., rule out an app ZLB) and do not consider access pricing on the consumer side.[10] To be certain, one may argue that the widespread assumption that platforms grant free access to consumers in these papers reflects a core ZLB.[11] However, they do not connect the validity of the underlying assumption with the level of seller access charges: They just assume that the access charge choice – whether by the platform or the regulator – is

[8] As the platform's profit-maximizing fee always exceeds this level, it is again optimal to cap platform fees.

[9] We also look at (d) externalities stemming from double marginalization.

[10] By considering access pricing both on consumer and seller side, our work relates to the literature on optimal pricing by two-sided platforms pioneered by Armstrong (2006), Caillaud and Jullien (2003) and Rochet and Tirole (2003, 2006), though we abstract from cross-group externalities. This literature however is not concerned with hybrid platforms and mostly ignores ZLB constraints.

[11] In other papers on hybrid platforms, including Etro (2022b) and Padilla et al. (2022), app-stores are bundled with physical devices, so that consumers are always charged a positive price.

unconstrained, which we show cannot be the case.

Another closely related contribution to our paper is Choi and Jeon (2021). They show that tying may help a firm circumvent a non-negative price constraint in the tied (complementary) product market that prevents it from squeezing superior sellers in that market. Zero lower bounds do not usually emerge in standard models (e.g., Choi and Stefanadis, 2001, Carlton and Waldman, 2002), which assume that the tied market involves a positive marginal cost.[12] Unlike in this literature on tying, which does not consider access pricing, in our paper margin squeeze of superior 3rd party sellers by the platform does not necessarily occur via below-cost pricing in the tied (competitive) good market, but primarily via fees' extraction: In this case, it is the core ZLB, rather than the ZLB in the tied market (the app ZLB in our terminology), that binds.

In our model, below-cost pricing by a monopoly platform in the competitive segment occurs in equilibrium when allowing for heterogeneous consumers' value brought about by the 3rd party app (see Section 3.2.2). The mechanism is similar to the one at play in Chen and Rey (2012), who provide a rationale for loss leading by large retailers who face competition by more efficient, smaller, retailers, in a model where consumers have heterogeneous shopping costs.[13]

2 Impact of the access charge

2.1 Basic framework

Consider a two-sided platform (e.g., an app-store) that connects sellers of *digital goods* (hereafter, "apps") with consumers. Digital goods are assumed to entail negligible marginal costs of production and distribution. Rather, their usage by consumers brings several benefits for the app providers, such as advertising revenues, fees collected from merchants selling their products through the app, or consumers' data that can be monetized. Hence, their opportunity cost is negative. Sellers face a zero lower bound constraint because negative prices are subject to arbitrage: Bots and uninterested consumers may take advantage of the payment for usage, and yet bring no profit for merchants and advertisers and provide valueless data.

As described in Figure 1, the platform: (i) is a gatekeeping platform, and charges consumers a fixed access price; (ii) adopts an agency business model (app providers pay access fees for distributing their apps and set their prices); and (iii) operates a hybrid marketplace, i.e. also distributes its in-house app in the app-store.

Two apps, one in-house and another one supplied by a 3rd party, compete for the platform's

[12] For an earlier work on the effects of tying in two-sided markets where ZLB constraints may bind, see Amelio and Jullien (2012). They show that, in situations where a platform would like to set negative prices on one side of the market, tying serves as a mechanism to introduce implicit subsidies on that side. As a result, it can raise participation on both sides and benefit consumers.

[13] By pricing the competitive good below cost, and raising the price for the monopolized good (that is, consumers' access price) accordingly, the platform: (i) maintains the total price charged to consumers with low (extra-) willingness to pay for the 3rd party app (corresponding to one-stop shoppers in Chen-Rey), who buy the in-house app; (ii) increases the margin earned on those with higher willingness to pay, who buy the 3rd party app (Chen-Rey's multi-stop shoppers) in the monopolized segment; (iii) induces 3rd party sellers to reduce their prices (hence, squeezes their margin).

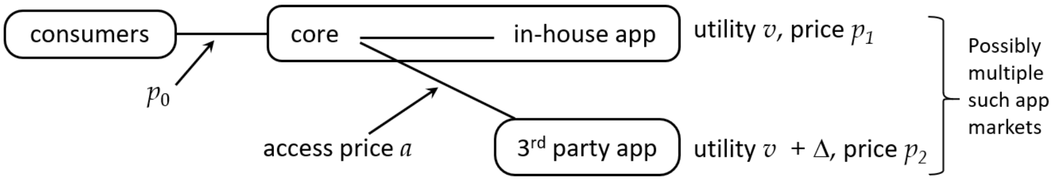

Figure 1: Two-sided market.

customers (there may more generally be multiple such app submarkets that depend on access to the platform). Consumers have unit demand, and benefit $v \geq 0$ (resp. $v + \Delta$, with $\Delta > 0$) when using the platform's in-house app (resp. the 3$^{\text{rd}}$ party app).[14] We assume that the platform brings no per-se value to consumers (independently of the consumption in the competitive segment). This assumption, which will be later relaxed, is a good approximation for app-stores (Apple's App Store and Google's Play Store), OTAs and other reservation systems (Booking, Airbnb and Uber), and e-commerce platforms (Amazon and eBay).

Let $x = 1$ if consumers buy the 3$^{\text{rd}}$ party app and $x = 0$ if they buy the in-house one. The platform levies a unit access charge $a \geq 0$ on apps distributed by the 3$^{\text{rd}}$ party provider (Online Appendix C.1 shows that our insights are unchanged considering instead ad-valorem access charges). In the following, $b \geq 0$ denotes the unit benefit accruing to the app provider (advertising, value of data, consumer lock-in), p_0 the consumers' access price to the app-store, and p_1 (resp. p_2) the price of the in-house app (resp. the 3$^{\text{rd}}$ party app). The platform's profit when the consumers patronize the platform can be written as the profit, $p_0 + a$, it would make as a pure platform, plus (if $x = 0$) the extra-profit, $p_1 + b - a$, it would obtain by capturing the app market as well:

$$\pi_1 \equiv p_0 + a + (1 - x)(p_1 + b - a),$$

while the 3$^{\text{rd}}$ party app developer's profit is:

$$\pi_2 \equiv x(p_2 + b - a).$$

We first assume that non-price foreclosure is impossible. We then allow the platform to make its competitor less attractive (i.e., to engage in self-preferencing), and to pick, at no cost, any 3$^{\text{rd}}$ party app's competitive advantage (or disadvantage if negative) $\delta \leq \Delta$. We will employ "foreclosure" and "self-preferencing" indifferently in our context. By contrast, $\delta = \Delta$ is the only option for the platform when a regulator can and does monitor non-price foreclosure. Throughout the analysis in this section, we take the access charge a as given (whether set by regulation or by the platform) and consider simultaneous pricing choices. The timing is given in Figure 2.

Before moving forward to the equilibrium analysis, we introduce some useful definitions:

Definition (competitive neutrality). The access charge a is competitively neutral in a range $[\underline{a}, \bar{a}]$

[14]The case of an inferior 3$^{\text{rd}}$ party app ($\Delta < 0$) is uninteresting as it would play no role in this framework.

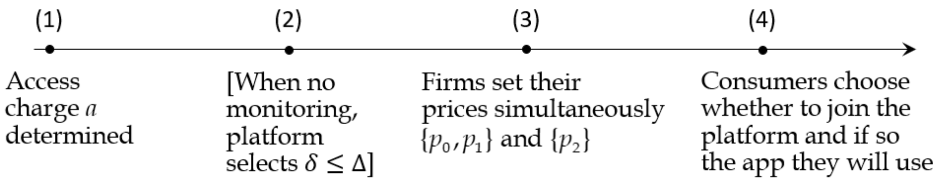

Figure 2: Timing.

if, in this range, (i) the platform has no incentive to use non-price instruments to foreclose the 3rd party app (even if it can), and (ii) the equilibrium profits, $\pi_1^*(a)$ and $\pi_2^*(a)$, and the allocation x are independent of a over the range.

Definition (fairness and squeeze). The 3rd party app developer receives its fair share of its contribution to the ecosystem if $\pi_2^*(a) = \Delta$. The 3rd party app developer is squeezed if (i) the platform does not foreclose it ($\delta = \Delta$), but (ii) $\pi_2^*(a) < \Delta$.

Definition (zero lower bounds). The app zero lower bound (ZLB) binds if $p_1^* = 0$. The core ZLB binds if $p_0^* = 0$.

Definition (ECPR level). The access charge is below (equal to, above) the Baumol-Willig efficient component pricing rule level if a is smaller than (equal to, higher than) the unit profit, $p_1 + b$, lost by the platform when the 3rd party app attracts a consumer.

Finally, the simultaneity of price choices gives rise to a multiplicity of equilibria when demand is inelastic, as is familiar from Nash demand games. We look for an equilibrium in which the platform is "pivotal" for consumers' participation:

Definition (platform pivotality). An equilibrium of the price subgame exhibits platform pivotality if the in-house and 3rd party apps play the Bertrand equilibrium $\{p_1^*, p_2^*\}$ in undominated strategies of the pure app-pricing game that would prevail were the consumers already present on the platform.

Intuitively, in a platform pivotality equilibrium, the 3rd party app does not feel responsible for attracting consumers to the platform.[15] To see what pivotality implies, suppose that the consumers' participation on the platform is a foregone conclusion. The app providers have the same opportunity cost $a - b$ and do not charge below this level in an equilibrium in undominated strategies.

- When this opportunity cost is negative ($a - b < 0$), the platform cannot charge an app price p_1 below 0 due to the app ZLB and therefore sets $p_1^* = 0$, while the 3rd party app is priced at $p_2^* = \Delta$. The superior 3rd party app does not feel the full competitive pressure

[15] This is always the case with a continuum of (independent) app markets (monopolistic competition, as in Etro, 2021a, 2022b, and Jeon and Rey, 2023), because the presence and price of an individual (superior) 3rd party app in one market has a negligible impact on consumers' overall utility from access to the platform. There may nonetheless be other equilibria in general, in which the platform engages in below-opportunity-cost pricing in all app markets: i.e., $p_1^* < a - b$ for all $a > b$. In these equilibria, a superior 3rd party app receives its fair share of its contribution to the ecosystem if and only if $a = b$.

from the in-house app, and so makes supranormal profit ($\pi_2^*(a) > \Delta$). The consumers obtain surplus $v > 0$ and so $p_0^* = v$.

- If the opportunity cost is non-negative ($a - b \geq 0$), the standard Bertrand equilibrium in undominated strategies has $p_1^* = a - b$ (the app ZLB does not bind) and $p_2^* = (a - b) + \Delta$, so that $\pi_2^* = \Delta$ (fair reward). A change in the access charge increases one-for-one the 3rd party app's marginal cost and raises, also one-for-one, the platform's opportunity cost of supplying the app in-house. Access charge pass-through makes charging the consumer or the 3rd party app for access perfect substitutes: Hence, the neutrality result.[16] This competitive neutrality region is the counterpart of the Chicago School theory that a dominant firm does not benefit from tying. It shows that the Chicago School conclusion requires an access price that exceeds the benefit of attracting consumers on the app. The neutrality region exhibits the familiar "seesaw property" of two-sided-market theory, in which an increase in the merchant fee translates (in our case one-for-one) into a decrease in the consumer fee.[17] However, such decrease is feasible in our model so long as the core ZLB does not bind; using the fact that consumers are in equilibrium indifferent between the in-house and the 3rd party app, $p_0^* = v - (a - b)$, and so it must be $a - b \leq v$.
- When $a > b + v$, the core ZLB binds ($p_0^* = 0$) and so the 3rd party app is constrained by consumers' willingness to pay for its app, which requires picking $p_2^* = v + \Delta < p_1^* + \Delta = (a - b) + \Delta$. App prices still form an undominated Nash equilibrium of the game in which consumers' presence on the platform is taken for granted (platform pivotality). But the access charge is no longer neutral, as the 3rd party app developer must absorb its increase to keep customers. Hence, the 3rd party seller's margin is squeezed ($\pi_2^*(a) < \Delta$), and the platform appropriates part of the 3rd party app's contribution to the ecosystem.

2.2 Absence of foreclosure

The analysis in Section 2.1 proves Proposition 1:

Proposition 1 (retail prices in the absence of foreclosure). *Suppose that the platform cannot use non-price strategies to foreclose the 3rd party app. Because consumers are homogeneous, their surplus is extracted ($p_0^* + p_2^* = v + \Delta$), and $\pi_1^*(a) + \pi_2^*(a) = b + v + \Delta$ for all $a \leq b + v + \Delta$.*

The app ZLB binds ($p_1^* = 0$) if and only if $a < b$. Furthermore,

- *when $a < b$:*

$$\begin{cases} p_1^* = 0 \quad \textit{and} \quad p_2^* = \Delta, \\ \textit{Supranormal app profit:} \quad \pi_2^*(a) = \Delta + (b - a) > \Delta. \end{cases}$$

- *when $b \leq a \leq b + v$:*

[16] For the relaxation of the one-for-one substitution assumption, see Gans and King (2005).

[17] Sullivan (2022) studies the impact of US city regulations that have capped the fee paid by restaurants to food delivery platforms to 15% (instead of the customary 30%). In the absence of MFN clauses, this cap resulted in a 9-22% increase in fees paid by consumers; while neutrality did not prevail for reasons studied by Sullivan, the pattern is a clear seesaw one.

$$\begin{cases} p_1^* = a - b \quad \textit{and} \quad p_2^* = p_1^* + \Delta, \\ \textit{Fair reward:} \quad \pi_2^*(a) = \Delta. \end{cases}$$

- *when* $b + v < a \leq b + v + \Delta$*:*

$$\begin{cases} p_1^* = a - b \quad \textit{and} \quad p_2^* = v + \Delta < p_1^* + \Delta, \\ \textit{Squeeze:} \quad \pi_2^*(a) = b + v + \Delta - a < \Delta. \end{cases}$$

- *when* $a > b + v + \Delta$*:*

$$\begin{cases} p_0^* + p_1^* = b + v, \\ \textit{Price foreclosure:} \quad \pi_2^*(a) = 0. \end{cases}$$

The link between ECPR and the ZLBs. The equilibrium characterization unveils a deep connection between the ZLBs and Baumol and Willig's ECPR rule:

Corollary 1 (ECPR)**.** *The access charge is*

- *below the ECPR level* ($a < p_1^* - (-b)$) *if and only if the app ZLB binds* ($a < b$)*;*
- *at the ECPR level* ($a = p_1^* - (-b)$) *if and only if no ZLB binds* ($b \leq a \leq b + v$)*;*
- *above the ECPR level* ($a > p_1^* - (-b)$) *if and only if the core ZLB binds* ($a > b + v$)*.*

Evidence on ZLBs. The basic model has the virtue of simplicity while capturing the main forces of the gatekeeping environment. Its simplicity may imply overly strong restrictions, though. Indeed, at first sight, the evidence on ZLBs seem to run counter two subordinate implications of the basic theory. First, the latter predicts that the core and app ZLBs cannot bind simultaneously, while the data shows that frequently $p_0 = p_1 = 0$. The latter coexistence of binding ZLBs can however be rationalized in a variety of ways that are consistent with the model. Indeed, Section 3.2.1 and 4 will show that it arises when the demand for the platform is downward-sloping, or when there are competing platforms, respectively.

The second subordinate implication of the basic theory is that superior apps should fetch a positive price ($p_2 > 0$); this however need not be the case when the demand for the 3rd party app is itself downward sloping (Section 3.2.2); in this case and even ignoring traditional explanations for low prices such as introductory pricing or installed base building in the presence of network externalities, the equilibrium price of superior apps can be zero.

Moreover, many apps adopt the so-called freemium model – i.e., a basic version (with limited functionalities) is made available for free, while consumers are charged a positive price for the premium version (which includes all functionalities). The freemium model obtains in our framework if (i) the basic versions of both in-house and 3rd party apps provide similar values to (heterogeneous) consumers; (ii) there is consumer lock-in and no commitment to premium versions' prices (the ancillary benefit from distributing the basic app then stems from the extraction of consumer value for the proprietary premium version); and (iii) the platform levies ad-valorem fees on 3rd party sales. The same conclusions, including the desirability of access charge regulation, apply to the freemium environment: see Online Appendix C.2.

2.3 Foreclosure

Let us augment the strategy space by letting the platform choose a non-price foreclosure strategy (the platform is left unmonitored). Without loss of generality, we can assume that $\delta = \Delta$ (no foreclosure) or $\delta = -v$ (full foreclosure). Intuitively, the platform's choice determines which among the in-house and 3$^{\text{rd}}$ party apps the consumers will select. In the former case, making the 3$^{\text{rd}}$ party app worthless involves no loss of generality. In the latter case, picking $\delta < \Delta$ creates social waste and reduces the platform's ability to monetize its ecosystem.

Proposition 2 (foreclosure)**.** *Left unmonitored, the platform engages in non-price foreclosure if and only if $a < b$.*

Proof of Proposition 2. When foreclosing, only the total price $p_0 + p_1$ matters, and so we can assume w.l.o.g. that $p_1 = 0$, and so $p_0 = v$. The platform can achieve profit $v + b$, i.e., the value it creates on a stand-alone basis. When not foreclosing, the platform makes profit $\pi_1^*(a) = v + a$ if there is no squeeze (and more when there is a squeeze, which is the case for $a > b + v$). So, the platform wants to foreclose if and only if $a < b$. ■

In a nutshell, for $a < b$ (i.e., when the app ZLB binds), the platform does not have enough skin in the game to want to give access to its rival. Figure 3 indicates the platform's and the 3$^{\text{rd}}$ party app developer's profit, with and without foreclosure.

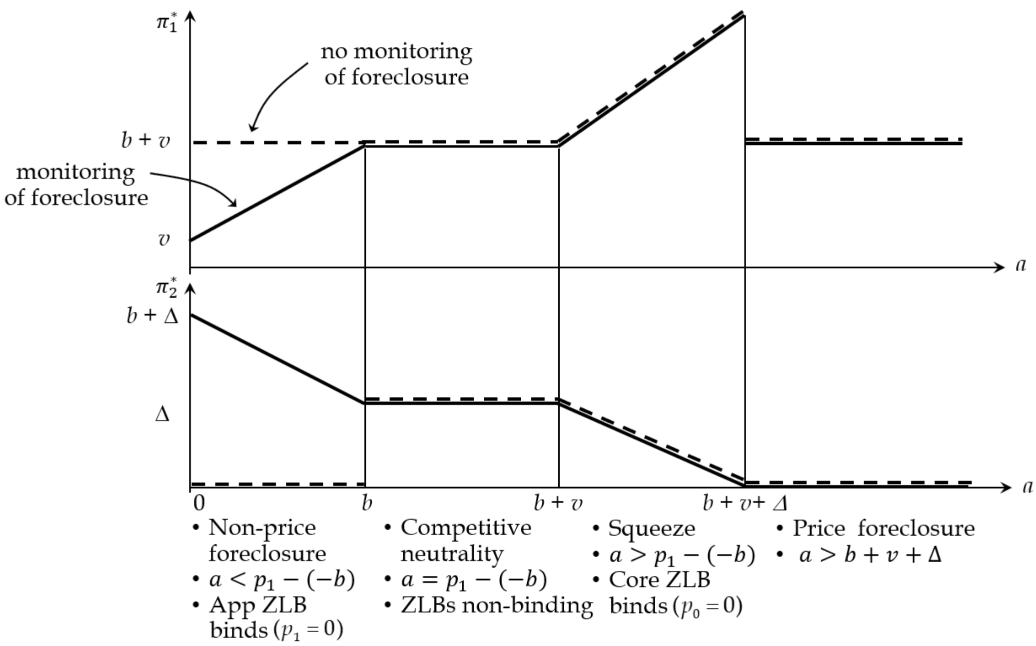

Figure 3: The dashed lines represent the profits when non-price foreclosure is feasible and the full lines the profits when it is not. They differ only when $a < b$.

2.4 Platform-optimal, welfare-optimal, and fair access pricing

We define social welfare W as the sum of consumer net surplus S and the firms' profit: $W = S + \sum_{i=1,2} \pi_i$.

Proposition 3 (optimal access charges)**.**

(i) Welfare-optimal access charges. *Any access charge such that the 3rd party app is not foreclosed maximizes ex-post social welfare:* $a \in [b, b+v+\Delta]$ *if non-price foreclosure cannot be monitored,* $a \in [0, b+v+\Delta]$ *under monitoring of self-preferencing;*

(ii) Profit-maximizing access charge. *Platform's profit is maximized at the extractive access charge* $a^* = b+v+\Delta$*, implying* $\pi_2^*(a^*) = 0$*;*

(iii) Fair access charges. *The independent developer receives a fair reward for its contribution to the ecosystem if and only if* $a \in [b, b+v]$*.*

Proof of Proposition 3. In this basic model, consumer surplus is always extracted by the platform through the access price,[18] and $W^* = \pi_1^*(a) + \pi_2^*(a) = b+v+\Delta x$ is maximized whenever there is no price or non-price foreclosure, so that $x = 1$, from which (i) follows. For $a = a^*$: $\pi_1^*(a^*) = W^*$, which establishes (ii). Finally, the result in (iii) follows from the equilibrium profit $\pi_2^*(a)$ given in Proposition 1. ■

Because, for any a in the competitive neutrality or squeeze regions, the platform has no incentives to foreclose the 3rd party app, whether or not self-preferencing can be monitored does not affect welfare. However, while any access charge in the competitive neutrality region is such that the independent developer receives a fair reward for its contribution to the ecosystem, an unregulated platform would find it optimal to set a strictly higher access charge, such that the independent seller is fully squeezed.

2.5 Simple extensions

Physical goods. While our analysis mostly focuses on platforms' intermediating sales of digital goods (e.g., app-stores, search engines), it is also applicable to platforms hosting sellers of physical goods (e.g., e-commerce) or services (e.g., OTAs or ride-hailing platforms) that entail positive marginal costs.[19] Intuitively, the cost of physical goods makes the "app" ZLB less likely to bind and reduces concerns about non-price foreclosure. Let $\gamma_a \in (0, v)$ denote this unit cost. With ancillary benefit (repeat sales or consumer lock-in) $b > 0$, the net opportunity cost is then $\tilde{b} \equiv b - \gamma_a$. If this opportunity cost is non-negative, the non-price foreclosure region becomes $a \in [0, \tilde{b})$; otherwise it is empty.

Observation 1. *The previous analysis carries over to physical goods, whenever their net opportunity cost is non-negative* ($0 \leq \gamma_a < b$)*. Otherwise, there is no non-price foreclosure for any access charge.*

In both cases (supposing $\tilde{b} + v > 0$), the fair levels of the access charge $a \in [\max\{\tilde{b}, 0\}, \tilde{b} + v]$ still lie below the platform-optimal level $a^* = \tilde{b} + v + \Delta$.

[18] This will not be the case when the platform faces a downward-sloping demand: see Section 3.2.1.

[19] We here keep assuming that the platform is a digital good that provides no per se value. The case in which the platform is (bundled with) a physical device with positive stand-alone value and production cost is examined in Section 4.1.

Asymmetries in benefits from app distribution. Throughout the paper, we assume that platform and 3rd party seller(s) reap the same benefit b from app distribution. This need not be the case: First, the platform may obtain a share of benefits – e.g., from data[20] – when the 3rd party app is sold; second, the benefits from app distribution may depend on app-specific and/or provider-specific features.[21]

To accommodate both cases, suppose that platform and 3rd party seller obtain side benefits $b_1(x)$ and $b_2(x)$, respectively, depending on which app is purchased ($x \in \{0, 1\}$), with $b_1(0) > b_1(1) \geq 0$, and $b_2(1) > b_2(0) = 0$: Each firm obtains higher benefits when its app is distributed; and while the platform can get some benefits if consumers buy the 3rd party app, the converse looks implausible. Let

$$\tilde{b}_1 \equiv b_1(0) - b_1(1) > 0, \quad \text{and} \quad \tilde{b}_2 \equiv b_2(1).$$

In this model, the platform prefers selling its own app rather than distributing the competing one if and only if $p_1 + b_1(0) > a + b_1(1)$. Hence,

$$p_1^* = \max\{a - \tilde{b}_1, 0\} \quad \text{and} \quad p_2^* = \min\{p_1^* + \Delta, v + \Delta\}.$$

Suppose the 3rd party app brings a positive value, defined as the sum of the value Δ of its innovation and the (positive or negative) extra-benefits $(\tilde{b}_2 - \tilde{b}_1)$ its app distribution generates, compared with the in-house app: $\tilde{b}_1 - \tilde{b}_2 \leq \Delta$.[22] The foregoing analysis then goes through provided that b is replaced by $\tilde{b}_1$, which measures the extra-benefit for the platform deriving from its in-house app compared to 3rd party app distribution. That is, for all $a \in [\tilde{b}_1, \tilde{b}_1 + v]$: (i) no ZLB binds; (ii) the platform obtains the same profit as under foreclosure: $\pi_1^*(\tilde{b}_1) = v + b_1(0)$; and (iii) the 3rd party seller appropriates the social value generated by its app: $\pi_2^* = \Delta + (\tilde{b}_2 - \tilde{b}_1)$, which constitutes its contribution to the ecosystem, and can thus be regarded as its fair compensation.

Observation 2. *The previous analysis carries over for general ancillary benefits, provided that the 3rd party app creates net value ($\tilde{b}_1 - \tilde{b}_2 \leq \Delta$).*

Existing (GDPR, DMA) and forthcoming regulations aim at restricting the use of data, thereby reducing the ancillary benefits. How does it affect the platform's incentive to foreclose? A uniform decrease in b, say because data sets cannot be combined or resold, reduces the incentive for foreclosure, keeping the access charge constant; in contrast, the above analysis shows that the end of current arrangements in which the platform shares data with its apps would increase

[20] For instance, the platform may have unique or shared access to some data generated by consumers' usage of 3rd party apps on its app-store – either contractually or by processing in-app payments.

[21] On the one hand, b may positively depend on app quality – e.g., more user engagement, associated to higher-quality apps, generates more data and ad-revenues; on the other hand, compared with smaller independent developers, the platform may have more bargaining power vis-à-vis advertisers, or extract more value from user data from any app, as these can be combined with other data on the same consumers obtained from other services it offers.

[22] If, on the contrary, $\tilde{b}_1 - \tilde{b}_2 > \Delta$, then at any access charge for which the 3rd party app is viable, the platform would foreclose it. Intuitively, the platform must trade off two inefficiencies: the lower ability of the 3rd party app to generate ancillary benefits (no foreclosure) and the consumers' lower demand for the in-house app (foreclosure).

the incentive for foreclosure (by reducing $b_1(1)$ and so increasing $\tilde{b}_1$); put differently, such a move would have to be accompanied with augmented regulatory monitoring and/or reduced regulatory pressure on access charge setting.

Platform business model. We take as given throughout that the platform operates as a hybrid marketplace. Its business model can be easily endogenised in equilibrium, though. Suppose that in the status quo the platform is a pure player: Both apps in the marketplace are developed by 3$^{\text{rd}}$ party sellers. Then, asymmetric Bertrand competition still implies that the low-value app is priced at $\max\{a-b,0\}$. As app prices are as above, consumers' access price, and platform's and superior app's profits, are also unchanged, under the assumption of no-foreclosure.

From our analysis it then follows that, for all $a<b$, the platform has an incentive to vertically integrate by acquiring the low-value app at a negligible cost (given that the latter makes no profit in equilibrium), and practice self-preferencing to foreclose the superior app. For larger values of the access charge, the platform has no incentive to vertically integrate.[23]

Hence, absent access charge regulation, the platform could optimally operate as a pure marketplace and squeeze the superior seller's profit setting $a^* = b+v+\Delta$. Capping the access charge to any $a \in [b, b+v]$ prevents the platform from engaging in socially harmful vertical integration (combined with self-preferencing), and guarantees a fair reward to the innovative developer.[24]

Observation 3. *Propositions 1 and 3 carry over when the low-value app is owned by a 3rd party provider. The pure-player platform gains from becoming hybrid by purchasing the low-value app if and only if $a<b$ and foreclosure is not monitored.*

Location of value creation in ecosystem. We assumed that the ecosystem's value is created solely by its app(s), which is a reasonable approximation for app-stores and many other platforms; by contrast, consumers attach per-se value to Google's search engine or Facebook's social network, which are core products. More generally, let v_a and v_c denote the values of the (in-house) app and the core, and $v \equiv v_a + v_c$ denote the total value. The 3$^{\text{rd}}$ party app brings value $v_a+\Delta$ to the consumer.[25]

[23] Note that there is no scope for acquiring the high-value app given that the sum of platform's and high-value app provider's profit is always (excluding the uninteresting Pareto-dominated price-foreclosure region) $b+v+\Delta$, which coincides with the vertically integrated platform's profit.

[24] Hence, access charge regulation is likely to outperform outright prohibitions of vertical integration on the ground of fairness, given that: (i) if fees are left unregulated, the platform can optimally engage in margin squeeze of sellers, even if it is not vertically integrated; and (ii) vertical integration in some cases benefits consumers – e.g., through increased product variety and/or lower prices.

[25] Can platforms or apps escape a ZLB through enhanced quality? Consider for example a platform facing a ZLB and the case in which there is only an in-house app (the same reasoning holds with a superior 3$^{\text{rd}}$ party party app). Suppose that consumers' value for the platform service is distributed according to $F(v_c+v_a)$, with pdf $f(v_c+v_a)$; and that the quality of the core product can be increased at per consumer cost $c \geq 1$ (if $c<1$, the platform could create infinite value and monetize it); so starting at level v_c^0 the platform can deliver value v_c at per-consumer cost $c(v_c - v_c^0)$. The platform solves

$$\max_{p_0, v_c} [p_0 + b - c(v_c - v_c^0)][1 - F(p_0 - (v_c - v_c^0))].$$

Then, the core ZLB binds and yet no quality improvement is made if $b > \rho(0) \equiv [1-F(0)]/f(0)$. When, on the contrary, the improvement in value costs a fixed, increasing and convex $K(v_c - v_c^0)$, the condition becomes the

When the core brings value per se ($v_c > 0$), consumers can decide to patronize the ecosystem while dispensing with the app (*unbundling*). It is still the case that $p_1^* \leq p_1^* + \Delta$ and that a consumer in the ecosystem buys the (3rd party) app if and only if $p_2^* \leq v_a + \Delta$. To illustrate the difference it makes, consider the extreme case in which $v \equiv v_c$. For $a < b$, the platform still forecloses the 3rd party app. For $a > b$, the 3rd party app cannot appropriate its contribution Δ to the ecosystem without losing its customers: $\pi_2^*(a) = \Delta + b - a < \Delta$. More generally, the competitive neutrality region corresponds to $a \in [b, b + v_a]$. Moreover, the core ZLB never binds with homogeneous consumers as the platform always extracts the value v_c of the core: $p_0^* = v_c + v_a + \Delta - p_2^* \geq v_c$.[26]

Observation 4. *The lower the value v_a created by the in-house app, keeping v fixed, the smaller the competitive neutrality region:*

- *For $a - b \leq v_a$, app prices and profits are the same as in Proposition 1;*
- *The squeeze region corresponds to $a \in (v_a + b, a^*]$, where $a^* \equiv v_a + b + \Delta$ is the platform optimal fee;*
- *The platform charges consumers v_c on top of the core price characterized above: The possibility of consumer unbundling creates a v_c-lower bound constraint.*

When the in-house app is valueless ($v_a = 0$), the only access charge that (i) discourages foreclosure and (ii) provides a fair reward to the 3rd party app is $a = b$.[27]

2.6 Summing up

The results of the basic model, in the more general case where $v \equiv v_a + v_c$, are illustrated in Table 1.

3 Pigouvian regulation

In the simple model of Section 2, all outcomes in which there is no foreclosure are equivalent from a total welfare standpoint, since consumer surplus is entirely captured by the platform either directly via the consumer price p_0 or indirectly through the impact of the access charge a on the price of apps, and 3rd party seller's margin squeeze has only redistributional effects. Hence, even the fully extractive unregulated outcome ($a = a^*$) is socially efficient, so that one might argue that there is little scope for regulation.

In this section,[28] we first show that this conclusion is unwarranted if the introduction of a superior

conjunction of $b > \rho(0)$ and $K'(0) \geq 1 - F(0)$.

[26] The core ZLB may bind if consumers have heterogeneous willingness to pay for the core (see Online Appendix C.3). This is a fortiori true when, as in the case of search engines and social networks, the platform reaps ancillary benefits from consumers' participation even when they do not purchase apps.

[27] Even when there is no in-house app (hence, no foreclosure concerns), nor 3rd party competitor to the high-value app, $a = b$ is the unique level of the access charge ensuring that the superior 3rd party app developer receives a fair reward for its contribution to the ecosystem.

[28] We return to the case in which the value is created in the competitive segment ($v_a = v$). The analysis for the case in which consumers have heterogeneous valuations v_c for the core (and homogeneous valuations v_a and

0 — b — $b + v_a$ — $b + v_a + \Delta$ — access price a

Non-price foreclosure (access provider does not have sufficient skin in the game)	**Competitive neutrality** (Chicago School's "rich ecosystem argument" holds)	**Squeeze**	**Price foreclosure**
• a below ECPR level • Supranormal app profit (above Δ) if no foreclosure • App ZLB binds	• a at ECPR level • Fair reward (at Δ) • ZLBs do not bind	• a above ECPR level • Infranormal app profit (below Δ) • Core ZLB binds if $v_c = 0$	• a way above ECPR level
Self preferencing if no regulatory monitoring	Efficient ex-post allocation a neutral	a squeezes 3rd party app	Superior 3rd party app out of the market

Table 1: Access pricing's delicate balancing act.

3rd party app in the marketplace is endogenous, and depends on the independent developer's incentives to invest in product innovation (Section 3.1). We then extend the model allowing for heterogeneous consumers' valuations for the apps (Section 3.2), deriving a Pigouvian principle that underlies optimal access charge regulation in more general environments. Concretely, we show that the regulated access fee should coincide with the ancillary benefit associated with app distribution:

$$\hat{a} = b.$$

The reason why this can be interpreted as the Pigouvian level of the access charge is that the 3rd party app "steals" b from the in-house app when taking a consumer away from it, thereby setting $\hat{a} = b$ gives the independent seller incentives to internalize this externality.

The app ZLB prevents the platform from fully appropriating b under app competition for all $a < b$. For these values of the access charge, the only way for the platform to appropriate this benefit is to foreclose the 3rd party app. The Pigouvian level of the access charge makes the platform indifferent between in-house app and 3rd party app distribution to any consumer, and so also indifferent between engaging or not in self-preferencing. As we shall see, capping, by regulation, the access charge to that level allows the other ecosystem participants (i.e., independent developers and consumers) to fully reap the benefits from the presence of superior apps.

3.1 Endogenous innovation

Suppose that, absent innovation, both the platform's in-house app and the 3rd party app bring value v to consumers. In this case, the 3rd party seller makes zero profit, whereas the platform

Δ for the apps) is provided in Online Appendix C.3.

obtains $b+v$. Upon observing the access conditions, the independent developer decides whether to sink a cost $c > 0$ to introduce a superior version of the app, which brings an extra-value Δ to consumers. Suppose that the development cost c is distributed according to a smooth cdf $G(c)$, with density $g(c)$ and monotone hazard rate, and its realization is privately observed by the developer. Proposition 1 and 2 then imply:

Proposition 4 (fairness and innovation)**.** *With privately observed app developers' costs, welfare is maximized if and only if the access charge a lies in $[b, b+v]$, levels that are strictly lower than the platform's profit maximizing level a^*, given by*

$$a^* = b + v + \frac{G(v + \Delta + b - a^*)}{g(v + \Delta + b - a^*)}.$$

Squeezed 3[rd] party sellers have a suboptimal incentive to develop their apps. The impact of the access charge on the richness of the ecosystem is accounted for by the platform, but incompletely so. As a result, an inefficiently low amount of innovation takes place under laissez faire. Capping, by regulation, the access charge to any level in the competitive neutrality region (i.e., $a \in [b, b+v]$) is needed to maximize social welfare. Considering independent developers' innovation incentives unveils a natural link between fair access pricing and welfare maximization.

What is fair? Let us discuss the notion of "fairness". The call for reward Δ was made from the point of view of efficiency/welfare maximization. A consumer standard might seem to lead to a social demand for some "taxation" of innovation in the form of a squeeze on profits, provided that the increase in access charge is passed through to consumers via a reduction in the core price. But there is zero pass through in this model (the squeeze region coincides with the core ZLB one), and so the consumer standard and welfare maximization lead to the same conclusion on the innovation front. This conclusion is robust to platform or app-store competition (Section 4), because the core ZLB binds and so there is zero pass-through to consumers, and to consumer downward sloping demand for the platform (see Online Appendix C.3), because, whenever the core ZLB does not bind, there is limited pass-through.

The second point relates to the "excessive innovation" that arises when the access charge lies below the ancillary benefit ($a < b$) and foreclosure can be monitored. One may be suspicious of worries about excessive innovation. Yet, this possibility is natural in the digital economy. A me-too innovation on the app segment, bringing along a small improvement ϵ in app quality, allows the innovator to corner the app market, engendering profit $(b - a)$. Indeed it is easy to envision a free-entry-into-the-app-market equilibrium in which the supranormal profit is dissipated by competition for this rent.[29]

[29] Suppose that there are n innovators, that $a = 0$ (so that the app ZLB binds), and that any innovator can bring along an innovation of size $\Delta \in [0, b]$ (with $\Delta = 0$ for the in-house app), at increasing and convex cost $C(\Delta)$ (with $C(0) = 0$ and $C(b) > b$). Then, there exists a symmetric mixed-strategy equilibrium with no profit (all rents are dissipated into rent seeking). This mixed-strategy equilibrium is defined by a continuous distribution $Z(\Delta)$ with support $[0, \overline{\Delta}]$, where $\overline{\Delta}$ is given by $b + [\overline{\Delta} - \mathrm{E}_{n-1,Z}[\Delta_1]] = C(\overline{\Delta})$ and $\mathrm{E}_{n-1,Z}[\Delta_1]$ is the expectation of the first-order statistic when there are $(n-1)$ independent draws from the distribution Z.

3.2 Consumer heterogeneity

This section allows consumers to differ in the demand for apps (whether in-house or 3rd party)/ for the platform (Section 3.2.1) and in their demand for the superior app relative to the in-house one (Section 3.2.2). These extensions, as well the study of platform and app-store competition in Section 4, allow us to test the robustness of our insights; they also generate a positive consumer surplus and thereby introduce a meaningful distinction between the welfare standard and the consumer standard.

3.2.1 Elastic platform demand

Assume that consumers' willingness to pay for the in-house app, $v \geq 0$, has wide support, and is distributed according to a smooth cdf $F(v)$ with density $f(v)$ on $\mathbb{R}^+$, and monotone (inverse) hazard rate $\rho(v) \equiv [1 - F(v)]/f(v)$. If there is no 3rd party app (or if the 3rd party app is foreclosed), both the core and app ZLBs bind whenever[30]

$$\arg\max_{p_0+p_1} (p_0 + p_1 + b)[1 - F(p_0 + p_1)] \leq 0 \iff b \geq \rho(0).$$

As this is a novel feature of this model compared with the foregoing analysis (where at most one ZLB binds),[31] in the remainder of this section we restrict attention to this region of parameters. To ensure that the platform does not foreclose the 3rd party app also for values of the access charge exceeding the ancillary benefit, we also assume that the rival app's (type-independent) quality advantage satisfies $\Delta \geq b$.[32]

Proposition 5 (double marginalization)**.** *Let consumers have heterogeneous valuations for the apps, distributed for the in-house app according to $F(v)$ with support $\mathbb{R}^+$ (with a quality advantage Δ for the 3rd party app). Suppose that foreclosure is not monitored, that the core ZLB binds under foreclosure ($b \geq \rho(0)$), and that $\Delta \geq b$ (to avoid systematic foreclosure). Then,*

- *The platform forecloses the 3rd party app if and only if $a < b$.*
- *Any access charge $a \in [b, b - \rho(0) + \Delta]$ maximizes consumer surplus and social welfare.*[33]
- *These access charges lie weakly below the platform's profit maximizing level: $a^* \geq b -$*

[30] As shown in Gans (2022), a firm may set a zero price even when negative prices are feasible, provided there is a positive mass of consumers who derive no value from the good (i.e., $v = 0$; negative valuations are ruled out based on a free-disposal assumption).

[31] For $b < \rho(0)$, the results are as in the base model: Competitive neutrality arises for any $a \geq b$ not too large, such that the core ZLB does not bind and the platform offsets the increase in price by the 3rd party seller, as a grows larger, through a reduction in the core price.

[32] If Δ were lower, there could be an equilibrium where $p_2^* = a - b + \Delta$ for $a > b$. As the 3rd party app appropriates the whole extra-value its app brings to any consumer, any $a > b$ would inefficiently reduce demand, and so the platform would be better off under foreclosure. In these cases, $\hat{a} = b$, at which the platform has no strict incentive to foreclose the 3rd party app, would straightforwardly be welfare-optimal.

[33] The same holds when foreclosure is monitored. For $a \in [\rho(0), b)$, as the 3rd party app extracts the extra value it brings to consumers ($p_2^* = \Delta$), a consumer surplus standard would view foreclosure with indifference. That is, even if foreclosure cannot be monitored, the set of consumer surplus maximizing access charges is $a \in [\rho(0), b - \rho(0) + \Delta]$. This coincides with the set of welfare optimal access charges if foreclosure can be monitored: lower access charges imply $p_0^* > 0$ and $p_0^* + p_2^* > \Delta$, and so inefficiently reduce demand.

$\rho(0) + \Delta$ *(strictly so if and only if* $b + \Delta < \rho(0)[3 + \rho(0)^2 f'(0)]$*).*

- *The 3rd party app developer receives a fair reward for its contribution to the ecosystem if and only if* $\hat{a} = b$*: Any higher access charge leads to a squeeze, any lower charge to a supranormal profit or foreclosure.*

The last bullet point in the Proposition implies that capping the access charge at the Pigouvian level is needed to maximize welfare when considering endogenous innovation. When the core ZLB binds and $a \in [\rho(0), b - \rho(0) + \Delta]$, the 3rd party app does not pass through increases in fees to consumers so as to avoid reducing demand.[34] If the access charge exceeds the ancillary benefit, the 3rd party app's markup is lower than its contribution to the ecosystem: $p_2^* - (a - b) < \Delta$. So the 3rd party app is always squeezed if the Pigouvian rule does not prevail.

3.2.2 Heterogeneous quality valuations

This section introduces heterogeneity with respect to the perceived extra quality of the 3rd party app.[35]

Proposition 6 (efficient choice of app)**.** *Suppose that consumers have the same* v*, but have heterogeneous valuations* Δ *for the 3rd party app (distributed according to a smooth cdf with support* $\mathbb{R}^+$ *and a monotone hazard rate), and that non-price foreclosure cannot be monitored. The platform forecloses the 3rd party app if and only if* $a < b$*. Consumer surplus and social welfare are maximized at* $\hat{a} = b < a^*$*.*

By encouraging an excessive consumption of the in-house app, platform's below-opportunity-cost pricing in the competitive segment, which emerges in equilibrium for all $a > b$, harms both consumers and the 3rd party seller. Thus, under no monitoring of non-price foreclosure, even neglecting fairness considerations, optimal access charge regulation must follow a Pigouvian principle.[36]

4 Contested bottlenecks

4.1 Platform competition

Does platform competition improve the lot of consumers? That of developers? Does competition eliminate the scope for access charge regulation? To address these questions, this section returns to the framework of Section 2 and considers multiple competing platforms, single-homing consumers and multi-homing 3rd party app-developers. This is consistent with evidence that consumers commonly use one device, where they access only one app-store (Apple Store on

[34] Incomplete pass-through occurs for $a > b - \rho(0) + \Delta$. In these circumstances, $p_0^* + p_2^* > \Delta$, and fewer consumers patronize the platform (double marginalization).

[35] One might think of the 3rd party app as adding a functionality relative to the in-house one. Consumers value this extra functionality diversely.

[36] The above results imply that, if non-price foreclosure could be monitored, then consumer surplus and social welfare would be maximized by granting free access to the marketplace (i.e., for $a = 0$), though this would raise fairness concerns on the platform side, and excessive innovation on the app side.

iPhone, Google's Play Store on Android-powered devices, though recent regulations aim to induce app-store competition: see Section 4.2),[37] whereas most common apps are available on both Apple's and Google's stores (Bresnahan et al., 2015). We maintain the platform pivotality assumption: the 3[rd] party app takes as given consumers' demand on all the platforms when setting its prices.

Consider $N \geq 2$ (symmetric) competing platforms, indexed by i, and let $U^i \equiv u^i - p_0^i$ denote consumers' net value from access to platform i's ecosystem, where $u^i \equiv \max\{v - p_1^i,\ v + \Delta - p_2^i,\ 0\}$, and (p_0^i, p_1^i, p_2^i) are consumers' access price, in-house and 3[rd] party app prices on platform i, respectively. To analyse platform competition in the starkest way, we consider perfect competition. That is, we suppose that all consumers patronize the platform offering the highest net value U^i. As a tie breaking condition, we assume that platforms offering the same utility split equally the demand, though this does not affect our results. The timing is the same as with a single platform: (i) The platforms select their access conditions (a^i, δ^i);[38] (ii) The platforms and the apps select their prices $\{p_0^i, p_1^i\}$ and p_2^i; (iii) Consumers choose their platform, and their app on that platform.

Perfect competition implies that in equilibrium all platforms offer the same net utility $U^* = v$ to consumers, and that the core ZLB binds. Whenever the access charge on platform i is such that $a^i \leq b$, since a platform optimally prices its app at zero under platform monopoly, a fortiori it must do so under competition. However, the presence of platform competition acts as a commitment device for platforms to price their in-house app at zero even if $a^i > b$. Indeed, because in equilibrium consumers are indifferent between the in-house and the 3[rd] party app, any $p_1^i > 0$ would give room for platform i to undercut its rivals. By the same reasoning, the core price p_0^i must be equal to 0. The analysis is similar to that with a monopoly platform in which the core ZLB binds; indeed, it is optimal for each platform to fully squeeze the 3[rd] party app: $a^* = b + \Delta$. The only difference with the monopoly platform case is a transfer of value v from the platform to the consumers.

Proposition 7 (platform competition). *Consider $N \geq 2$ identical competing platforms, indexed by i.*

(i) Laissez-faire. *In the laissez-faire equilibrium, both ZLBs are binding ($p_0^i = p_1^i = 0$) and $p_2^i = \Delta$. All platforms select access charge $a^* = b + \Delta$ and make profit $(b + \Delta)/N$ each. The core ZLB prevents platforms' total profit $b + \Delta$ from being competed away. Consumers receive net surplus v each, and 3[rd] party apps are fully squeezed.*

(ii) Access charge regulation. *A regulator concerned with fairness optimally sets $\hat{a} = b < a^*$, yielding per-platform profit b/N and 3[rd] party app profit Δ. Consumers still receive net surplus v each.*

The laissez-faire result, illustrated in Figure 4, aligns with the conventional wisdom in platform

[37] Similarly, most consumers primarily use a single search engine (e.g., Evans, 2008).

[38] It is straightforward to see that, in this simple model, whether these first-stage decisions are observed by rival platforms is immaterial to the results.

economics[39] that the multi-homing side does not benefit from platform competition, while the single-homing one (the competitive bottleneck) does, because the platform is the gatekeeper for users on the single-homing side: Platform competition allows consumers to get positive net surplus v. The novel feature of our framework is that perfectly competing platforms earn high profits $(b + \Delta)$ under laissez faire. The first component of this unit profit is the benefit from app distribution, which can be appropriated also through foreclosure; the second component is the value brought about by superior sellers, which is extracted through the access charge. Both revenues are not competed away by price competition because of the core ZLB.

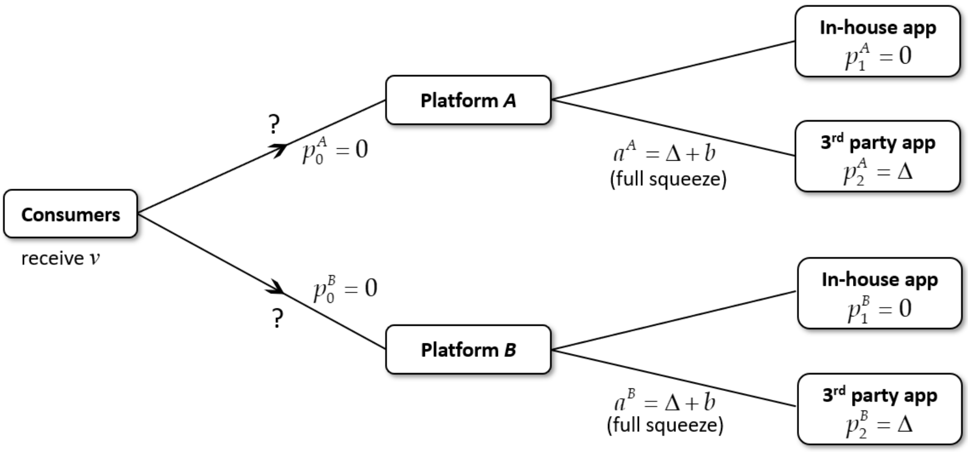

Figure 4: Laissez faire equilibrium with competing platforms under consumer single-homing.

Thus, enforcing the Pigouvian rule through a cap on access charges is needed to guarantee proper incentives to invest by independent app developers, even in the presence of fierce platform competition.[40]

Platform viability and entry. Assume that the social welfare function is $U + \omega\Pi$, where Π is total profit (platforms and apps) and $\omega \in (0, 1)$ is the weight on industry profits relative to consumer surplus.[41] Suppose that there is (sequential) free entry into the platform segment, with entry cost J. Suppose further that non-price foreclosure cannot be monitored, and so the access charge must be no lower than b. We then have:[42]

Observation 5. *Because the core ZLB prevents platform profits from being competed away,*

[39] See Caillaud and Jullien (2003), Armstrong (2006), Armstrong and Wright (2007) and, more recently, Teh et al. (2023). Only Armstrong and Wright (2007) explore the implications of a ZLB constraint on the access price charged to the single-homing side, which competing platforms would like to subsidize.

[40] This result hinges on the assumption that platforms are vertically integrated in the app segment: If also the low-value apps are offered by 3rd party providers, then $a^* = b$ would prevail in the laissez-faire equilibrium, which would eliminate the scope for regulation. The reason is that (as $p_1^i = \max\{a^i - b, 0\}$ and $p_2^i = \min\{p_1^i + \Delta, v + \Delta\}$) the superior app is priced at $p_2^i = \Delta$ for all $a^i \leq b$, whilst any larger access charge implies $p_2^i > \Delta$ and so $U^i < v$ and no customer for platform i.

[41] Under a social welfare standard ($\omega = 1$), platform competition just entails socially wasteful duplicative entry costs: welfare maximization dictates $N = 1$. On the contrary, under a consumer surplus standard ($\omega = 0$), as a monopolist brings zero net value to consumers, entry by any number $N \geq 2$ of platforms would be optimal (i.e., there is never excessive entry in equilibrium from consumers' standpoint).

[42] See Online Appendix C.4 for the complete analysis.

socially excessive entry prevails when the entry cost is low and foreclosure cannot be monitored. By contrast, for high entry costs, setting access charges above the Pigouvian level is desirable to spur platform entry, if no other instrument is available (as we saw, $a > b$ introduces distortions).

These results suggest that, while access charge regulation is an effective instrument to achieve fairness, thereby promoting efficient entry and investment decisions in the app segment, it may not be a jack of all trades, able to take on extra tasks such as ensuring contestability of the core segment.

Smartphones. As mentioned above, Apple's and Google's app-stores are accessed by consumers only upon purchase of a physical device (smartphone). Consider a device with stand-alone value v_c, produced at cost $\gamma_c > 0$. In the case of a *monopoly* platform that is also (vertically integrated with) the device manufacturer, by interpreting the core as the bundle of device and app-store, the results in Observation 4 apply, which calls for implementing the Pigouvian rule.[43]

However, with *fierce inter-platform competition*, increases in fees above the Pigouvian level could be passed through to consumers via a lower price for the devices, which would be sold below cost.[44] That is, as each app-store earns a per consumer (provided the 3rd party app is viable: $a \leq b + \Delta$), devices including access to the app-stores would be sold at $\gamma_c - a$ in equilibrium, as long as the core ZLB does not bind (or, more generally, the device price is not so low as to attract users that are not interested in the apps). In these cases, consumers would reap the benefits from the margin squeeze of the 3rd party seller. Of course, the regulator should always take into account the dynamic inefficiencies of allowing margin squeezes (namely, reduced entry/investments in the app segment).[45]

Observation 6. *When platforms are built around costly devices, competing platforms can pass through to consumers, via a below-cost price of their devices, the profits earned by squeezing 3rd party apps through an access charge $a^* = b + \Delta$: As long as the core ZLB does not bind, access charges above the Pigouvian level in this case benefit consumers. However, under the welfare oriented criterion with endogenous app entry, the optimal access charge is $\hat{a} = b$.*

[43]These conclusions hold so long as $\gamma_c < v_a + v_c + b$, so that the foreclosure profit is positive (otherwise, and provided $\gamma_c < v_a + v_c + b + \Delta$, at least some squeeze of the superior 3rd party apps is needed for the platform to be viable). If, instead, the device is produced by independent, competing, manufacturers, then it is priced at cost, and the platform charges p_0^* characterized in Section 2 (with v_a in place of v) for access to the app-store – equivalently, a device including access to the app-store is sold at $\gamma_c + p_0^*$, provided $\gamma_c + p_0^* + p_2^* \leq v_c + v_a + \Delta$ (which always holds if $\gamma_c \leq v_c$). As also app prices are unchanged, the Pigouvian principle still applies.

[44]This is certainly true for vertically integrated platforms. Vertically disintegrated platforms may similarly circumvent the core ZLB constraint on the app-store by subsidizing device manufacturers under exclusive dealing.

[45]Note that we have assumed that the 3rd party app already exists. If not, the prospect of being fully squeezed will discourage it from entering, even for a small entry cost. To remedy this, platforms may voluntarily cap their business users' access charges. Assuming that such a commitment is feasible, it still would not bring about a fair access charge. As long as app developers have negligible multi-homing costs, 3rd party app entry is a public good from the point of view of platforms, and free riding would be expected (this would necessarily be the case if Δ were random): see Jeon and Rey (2023).

4.2 App-store competition on a platform

The DMA and the proposed Open App Markets Act require Apple and Google to guarantee 3rd party app-stores access to their respective devices. As the regulatory texts are silent as to the access conditions (see Online Appendix A), we look at a benchmark in which 3rd party app-stores must be given free access to the platform (FRAND access is discussed at the end of this section). Does the availability of competing app-stores on a single device eliminate the scope for access charge regulation?

We now have a sequence of "platforms", so we must be clear on terminology. In the following, "platform" will keep designating the gatekeeper to the consumer, "app-stores" will be the entities interacting with business users: see Figure 5. Consider a (vertically integrated) monopoly device manufacturer, hereafter denoted by M. As above, its device brings value v_c to consumers and is produced at marginal cost $\gamma_c > 0$. Let p_0 denote its price. On its app-store, whose access is priced at p_0^M,[46] consumers can find its in-house app valued v_a and a superior 3rd party app valued $v_a + \Delta$, at prices p_1^M and p_2^M respectively. M's in-house app-store faces competition by 3rd party app-stores, indexed by j and priced at p_0^j, where consumers can find the respective in-house apps, bringing value v_a, at prices p_1^j, and the same, multi-homing 3rd party app available on M's store at prices p_2^j.

Suppose consumers multi-home across app-stores that can be accessed for free (which is always the case in equilibrium).[47] Then the superior 3rd party app would optimally serve all consumers on the least expensive platform: App-stores de facto engage in Bertrand competition for the 3rd party app, which dissipates their profits – i.e., $a^* = 0$ in equilibrium (Figure 5).[48]

Proposition 8 (app store competition)**.** *Suppose that the regulator mandates app-store competition on each device, with app-stores enjoying free access to the device. Suppose further that consumers multi-home on free app-stores on their device. Bertrand competition among app-stores induces them to charge nothing for consumer access to the app-store and to levy no app-store access charge on the 3rd party app, as the latter steers the consumer to the lowest access charge app-store, that is the most profitable for the superior app. The 3rd party app then makes supranormal profit $\Delta + b$. The Pigouvian access charge ($\hat{a} = b$, where now b is a floor rather than a cap) is needed to ensure fairness and avoid over-entry in the app market.*[49]

In this simple model, the fair outcome can be alternatively achieved by allowing the platform

[46] When multiple app-store compete for consumers on the same device, its vertically integrated manufacturer is forced to unbundle its two core products (the device and the app-store), charging two different prices. In what follows, we refer to the app-stores as the core products.

[47] If instead consumers always single-home (because of, e.g., memory constraints, each downloads at most one app-store), then, as all app-stores are equally constrained by the core ZLBs, $p_0^M \geq 0$ and $p_0^j \geq 0$, the analysis is as in Section 4.1 (with the only difference that the monopoly manufacturer appropriates consumer surplus charging $p_0 = v_a + v_c$ for the device). Pigouvian regulation is thus still needed to fairly reward the superior 3rd party app provider.

[48] While here the monopoly device manufacturer extracts consumer surplus through the device price $p_0 = v_a + v_c$, this value would be appropriated by consumers in the presence of platform competition.

[49] This conclusion would be supported also by a consumer surplus standard in a model where consumers have heterogeneous valuations v_c for the device, because M could react to the reduced profitability of the app-store (due to competition) by increasing the device price p_0.

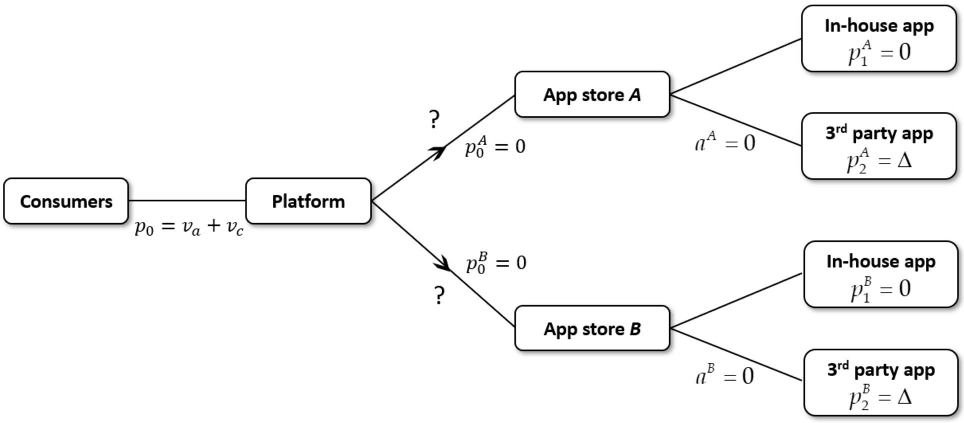

Figure 5: Laissez faire equilibrium with competing app-stores.

to levy on 3rd party app-stores a unit access charge α for each app sold through their stores. As Bertrand competition among app-stores with opportunity cost α implies that they will in turn charge $a^* = \alpha$ to the 3rd party app, setting by regulation $\hat{\alpha} = b$ ensures fairness. Thus, whether for apps or for app-stores, FRAND access pricing boils down to the Pigouvian principle.

5 Implementation

The paper's primary objective has been to derive principles to guide the regulation of access. In this respect, it showed that the optimal access charge is equal to the ancillary benefit obtained by the apps when acquiring a customer. Like for optimal taxes in public finance, the theoretical benchmark must be completed with an empirical methodology to measure the relevant data, here the ancillary benefit. A full treatment of this second step lies outside the scope of the paper, but we nonetheless shed some light on the relevant considerations.

A first measurement approach consists in the regulator's estimating the ancillary benefit. In practice, app categories differ substantially in terms of the benefit their distribution generates. For instance, there are data-poor and data-rich markets – e.g., social media and food delivery apps sell much more personal data to 3rd party advertisers than videoconferencing apps.[50] The industry has private information about these values that is hardly available to the regulator. Alternatively, the regulator can try to elicit the value of the benefit b from the industry. Sections 5.1 and 5.2 consider schemes that aim at obtaining the information from the platform and from the 3rd party developer, respectively.

5.1 Eliciting the information from the platform

We here consider an elicitation of ancillary benefits from the platform. We already know that, with one app category, asking the platform about the value of the ancillary benefit would be to no avail: the access charge recommended by the platform would fully squeeze the 3rd party app.

[50] See `https://www.pcloud.com/it/invasive-apps`.

We now show that one cannot have a much better luck with a large number of app categories for which one would know the distribution of ancillary benefits, but not the individual realization for each category. Suppose that the platform gives access to a continuum of mass 1 of (independent and heterogeneous) app markets, indexed by $k \in [0,1]$. In each market, there is a in-house platform app, valued $v^k \geq 0$ by all consumers, and a superior 3rd party app, valued $v^k + \Delta^k$, with $\Delta^k \geq 0$. We assume that each 3rd party developer sells in only one market. The ancillary benefit is denoted b^k.[51]

To examine how the regulator's limited knowledge of market-specific ancillary benefits affects access charge regulation in the simplest possible model, we consider the best-case scenario in which the regulator knows their cumulative distribution $K(b)$ in the population of apps.[52] Of course, in order for the Pigouvian access charge to be implemented in all markets, the distribution of (observed) access charges must equal the distribution of benefits.

Proposition 9 (impossibility of elicitation from the platform)**.** *Suppose the regulator knows the distribution $K(b)$ of ancillary benefits and lets the platform choose $(a^k)_{k\in[0,1]}$ subject to the constraint that the distribution of access charges mimics that of benefits (i.e., follows $K(a)$). Then, if non-price foreclosure cannot be monitored, setting $a^k = b^k$ for all $k \in [0,1]$ is not incentive-compatible for the platform.*

If non-price foreclosure cannot be monitored, the Pigouvian rule is not implementable in all markets even if the regulator knows the distribution of b, and so can require that a and b have the same distribution $K(\cdot)$. The reason is that, rather than charging $a^k = b^k$ in all markets, the platform can profitably charge higher fees in markets where b is lower, so as to squeeze 3rd party developers' margins in these markets, and foreclose developers in markets where b is higher, where it is constrained to set lower fees, which allows it not to lose profits in these markets. Thus, under no monitoring of non-price foreclosure, market-specific fees cannot be enforced under asymmetric information, and the regulator faces a trade-off between preventing foreclosure of developers in high-b markets and allowing margin squeeze (hence, dampening innovation incentives) of sellers in low-b ones.

5.2 Eliciting the information from app developers

The previous impossibility result hinged on the assumption that fee setting is delegated to the platform. The next proposition reverses the roles in access fee setting:

Proposition 10 (information light regulation)**.** *Regardless of the regulator's information, the Pigouvian rule can be implemented in all markets by letting 3rd party app developers pick their access charge subject to the threat of foreclosure.*

[51] The results of the baseline model easily extend to each app market. In particular, any $a^k \in [b^k, b^k + v^k]$ is a fair access charge, whilst the platform's profit is maximized at the extractive access charge $a^{k*} = b^k + v^k + \Delta^k$. Hence, if b^k were observed by the regulator, whilst v^k were not, and were distributed with a cdf $L(v^k)$ such that $L(v^k) > 0$ for all $v^k > 0$, then the regulator could guarantee fair access pricing in all app markets by imposing $\hat{a}^k = b^k$.

[52] We can assume that the regulator does not observe (v^k, Δ^k) either. Whether or not it knows their distribution is immaterial to the results.

Proof of Proposition 10. If foreclosure is not monitored, then choosing $a^k \in [b^k, b^k + v^k]$ is optimal for 3rd party sellers, as they are foreclosed for $a^k < b^k$, and squeezed for $a^k > b^k + v^k$. ■

If, on the contrary, the platform were not able to foreclose the 3rd party app (i.e., under monitoring of self-preferencing), then independent sellers would choose $a^k = 0$: This would over-incentivize 3rd party apps to enter and would further expropriate the platform's investments, thereby dampening its innovation incentives.[53]

Competing 3rd party apps. Let us assume that the platform has no in-house app and that there are, say, two 3rd party apps, app 1 with value v and app 2 with value $v + \Delta$. Can the Pigouvian outcome still be implemented by letting each app provider pick its own access charge? Suppose that the timing goes as follows: (i) the inferior and superior 3rd party apps propose access charges a_1 and a_2; (ii) the platform gives access to no, one or the two app providers; (iii) the platform and the (non foreclosed) app providers set their prices $\{p_0, p_1, p_2\}$; (iv) consumers take their consumption choice.

The inferior app knows that it can win consumers if and only if it is the more rewarding app from the platform's standpoint: In this case, the platform would foreclose the superior app, and the inferior app would set $p_1 = v$ and make profit $\pi_1 = v + b - a_1$. Hence, asymmetric Bertrand competition in access charges implies $a_1^* = b + v$ in equilibrium. Then, the high-value app optimally proposes the same fee ($a_2^* = b + v$), and sets $p_2^* = p_1^* + \Delta = v + \Delta$, so that the platform optimally let it serve consumers:[54]

Observation 7. *The Bertrand equilibrium has 3rd party app providers selecting access charges $a_1^* = a_2^* = b + v$. The platform lets both apps operate, and app prices are $p_1^* = v$ and $p_2^* = v + \Delta$. Hence, consumers patronize solely app 2, the platform sets $p_0^* = 0$ and receives $b + v$, and the superior 3rd party app obtains its fair contribution to the ecosystem ($\pi_2^* = p_2^* + b - a_2^* = \Delta$).*

5.3 Adding appeals

While Proposition 10 and Observation 7 are encouraging results, they rely on the platform maximizing its profit in each app submarket. However, if there are multiple app submarkets, the platform might engage in three forms of predation, that is sacrifice profit for later recoupment based on a reputation for aggressive behaviour: (a) use non-price foreclosure against the 3rd party app; (b) price below opportunity cost in the app market; (c) use either of these against a 3rd party app developer who would refer to the regulator for an arbitrage (see below). To illustrate form (a) of predation, contemplate sequential entry of superior apps in distinct but identical app submarkets; suppose that the discount factor β exceeds 1/2 and consider the mechanism described in Proposition 10, in which the 3rd party app sets the access charge. The platform can fully squeeze the 3rd party apps by foreclosing a 3rd party app whenever the latter sets an

[53] Moreover, the ability to foreclose ensures that the platform keeps controlling access to consumers, which protects them from fraudulent apps. Under monitoring of foreclosure, and provided the regulator knows the distribution $K(b)$ of benefits, the Pigouvian rule can also be implemented by delegating fee setting to the platform under the constraint that the distributions of a and b be the same.

[54] See Online Appendix C.5 for the complete analysis.

access charge below $b + v + \Delta$.

In the following we make the most pessimistic assumption on the monitoring of foreclosure: The regulator has no information, even a noisy one, on whether the platform engages in foreclosure. We will however allow the regulator to provide a noisy measure of the ancillary benefit if called upon by a party as part as the designed regulatory process. More precisely, suppose that the *platform* chooses the access conditions (a, δ) (of which the regulator observes only a), then prices are set, and finally the 3$^{\text{rd}}$ party app chooses whether to appeal against a high access charge. In this appeal procedure, the authority observes a noisy, but unbiased, version $\tilde{b}$ of the ancillary benefit, with cdf $R(\tilde{b})$ on $\mathbb{R}$ such that $\int_{\mathbb{R}} \tilde{b} \mathrm{d}R(\tilde{b}) = b$. If $a > \tilde{b}$, then the access charge is assessed to be unfair, and the defendant (the platform) must pay a fine $t(a - \tilde{b})$ (with $t > 0$) to the plaintiff (the 3$^{\text{rd}}$ party app); and vice versa if $a \leq \tilde{b}$. The outcome of the appeal procedure interferes neither with the platform's choice of whether to foreclose the 3$^{\text{rd}}$ party app, nor with access and app prices chosen by the firms before the appeal. This implies that the 3$^{\text{rd}}$ party app will appeal whenever $\int_{\mathbb{R}} t(a - \tilde{b})\mathrm{d}R(\tilde{b}) > 0 \Leftrightarrow a > b$.

Moving backwards to the pricing stage, $p_1 = 0$ and $p_2 = \Delta$ is the worst-case scenario for the 3$^{\text{rd}}$ party app. Then, for all $a \leq b + \Delta$ (no access-price foreclosure), the 3$^{\text{rd}}$ party app's profit, absent non-price foreclosure, is at least $\pi_2 = \Delta + b - a$. Because total profit is at most $v + b + \Delta$, the platform's maximal expected profit from setting any $a > b$ and therefore being challenged is

$$v + a - \int_{\mathbb{R}} t(a - \tilde{b})\mathrm{d}R(\tilde{b}),$$

which is decreasing in a provided $t > 1$. In contrast, the platform makes profit $\pi_1 = v + b$ either by setting $a < b$ and foreclosing the superior app, or by choosing $a = b$, while any other (a, δ)-choice yields strictly lower profit. Therefore, the Pigouvian principle can always be implemented by giving the platform a tiny advantage in the appeal procedure – e.g., the appeal benefits the 3$^{\text{rd}}$ party app if and only if $a > \tilde{b} + \epsilon$ for a small positive ϵ, so that a small squeeze is tolerated and the platform strictly prefers not to foreclose.

The observation that the 3$^{\text{rd}}$ party app appeals for any $a > b(+\epsilon)$ crucially hinges on the fact that appealing has no impact on its market profit. This would not be the case if the platform had the possibility (and the incentive) to foreclose it *post* appeal. When such a post-appeal foreclosure threat is credible, the 3$^{\text{rd}}$ party app does not appeal whenever $\Delta + b - a \geq \int_{\mathbb{R}} t(a - \tilde{b})\mathrm{d}R(\tilde{b})$, or equivalently $a \leq a^{\dagger} \equiv b + \frac{\Delta}{t+1}$. If t is large enough relative to the platform's discount factor β, however, such reputation building strategy can be prevented. To see this, suppose for simplicity that, by foreclosing after $a = a^{\dagger}$ is appealed in the first market, the platform is able to secure profit $v + a^{\dagger}$ forever after, which implies a discounted extra profit $\frac{\beta\Delta}{(1-\beta)(1+t)}$ from future markets relative to the profit $v + b$ it obtains by proposing $a = b$,[55] at an expected loss $\int_{\mathbb{R}} t(a^{\dagger} - \tilde{b})\mathrm{d}R(\tilde{b}) = t\frac{\Delta}{1+t}$ from the appeal. Therefore, setting $t \geq \frac{\beta}{1-\beta}$ prevents such reputation building strategy.

[55]For $t > 1$, this profit in turn exceeds the profit from proposing $a^{\dagger}$ in the first market and not foreclosing after being challenged, thereby failing to build a reputation for foreclosing future apps.

To sum up, we have:

Observation 8. *Let the 3^{rd} party app have the right to appeal against a high access charge chosen by the platform. If the regulator can produce a noisy measure $\tilde{b}$ of the ancillary benefit, and impose sufficiently large fines to the platform if it loses the appeal (namely, $t(a-\tilde{b})$ if $a > \tilde{b}$, with $t \geq \frac{\beta}{1-\beta}$), then the Pigouvian rule can be implemented even though the platform could build a reputation for engaging in below-opportunity-cost pricing or foreclosing after being challenged.*

6 Conclusion

Gatekeeping platforms control businesses' access to us. Novel to the literature, two zero lower bounds, on the pricing of apps and of core products, were shown to play a key role regarding business users' access to platforms. These non-negative-price constraints generate undesirable outcomes, respectively undue business-user profits and the concomitant incentive for non-price foreclosure for low access charges, and the squeezing of business users for high ones. The paper then stressed that laissez-faire – in the sense of a lack of interference with the platforms' preferred access policies – breeds unfair access conditions for these business users. Furthermore, we should not expect competition to solve the gatekeeping problem in the digital world of ZLBs. Indeed, the core price constraint prevents platform competition from disciplining access policies. We also showed that platform competition and app-store competition work very differently. While platform competition is too business-user unfriendly, app-store competition is too business-user friendly.

The overall picture is therefore a need for overseeing the terms and conditions offered by platforms to business users. In this we concur with recent regulatory developments. The latter however remain nebulous when it comes to specific recommendations, and the occasional invocation of the need for "fair, reasonable and non-discriminatory" terms is not helpful. The paper's second main insight relates to the social benefits of setting the access charge at the ancillary benefit associated with acquiring a customer. This level discourages non-price foreclosure and thereby spares intrusive assessment of whether access conditions are actually fair; it also provides app developers with a fair return and therefore a proper incentive to innovate; finally, it minimizes double marginalization conditional on intrusive regulation being infeasible or too costly.

Despite these clear theoretical messages, meeting the empirical challenge of regulating platforms' access policies remains as difficult as it is essential. The task of answering whether a 20% or 30% merchant fee is appropriate is marred with asymmetric information. We made real progress on the question of how to implement the theoretical benchmark; but we feel that more work is necessary to properly tame the gatekeeping platforms while not preventing them from offering innovative services to consumers and businesses alike. This question should remain on our priorities.

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Online Appendix

A Antitrust cases and regulation

A.1 Antitrust cases

Self-preferencing. In 2017 the European Commission fined Google for giving an unfair advantage to its own shopping comparison service. The Commission argued that Google abused its dominance in the search engine market by systematically giving prominent placement to its own comparison shopping service, and demoting rival comparison shopping services in its search results. See `https://ec.europa.eu/commission/presscorner/detail/en/IP_17_1784` for the details.

In 2020, the same Commission launched an investigation into Amazon's Buy-Box, concerning the possible preferential treatment of Amazon's own retail offers and those of marketplace sellers that use Amazon's logistics and delivery services. See `https://ec.europa.eu/commission/presscorner/detail/en/ip_20_2077`.

App-store commissions. In August 2021, Apple removed Epic's Fortnite game from its app-store because it circumvented Apple's 30% fee by offering an external payment option. A federal judge in California in September 2021 ruled that Apple must allow developers to route customers to 3rd party payment options and not force them to pay the app-store's fees for in-app purchases. See, e.g., `https://www.forbes.com`.

In the same year, in the EU, following-up on a complaint by Spotify, the Commission informed Apple of its preliminary view that it abused its dominant position for the distribution of music streaming apps through its app-store. The Commission is concerned by the mandatory use of Apple's own in-app purchase mechanism imposed on music streaming app developers to distribute their apps via Apple's App Store. Executive Vice-President Margrethe Vestager said: "Our preliminary finding is that Apple is a gatekeeper to users of iPhones and iPads via the App Store. With Apple Music, Apple also competes with music streaming providers. By setting strict rules on the App store that disadvantage competing music streaming services, Apple deprives users of cheaper music streaming choices and distorts competition. This is done by charging high commission fees on each transaction in the App store for rivals and by forbidding them from informing their customers of alternative subscription options." See `https://ec.europa.eu/commission/presscorner/detail/en/ip_21_2061`.

In January 2022, the Competition Commission of India stated that the 30% commission Apple charges developers unfairly pushes up costs for both app makers and consumers, and also acts as a barrier to entry for new developers. See, e.g., `https://nypost.com/2022/01/03/apple-hit-with-antitrust-probe-in-india-over-app-store-fees/`.

A.2 Regulatory proposals

EU Digital Markets Act. The DMA, which entered into force on November 1, 2022, has the purpose "to contribute to the proper functioning of the internal market by laying down harmonised rules ensuring for all businesses, contestable and fair markets in the digital sector across the Union where gatekeepers are present, to the benefit of business users and end users" (Article 1.1).

Inter alia, it is concerned with excessive access charges that gatekeeping platforms impose on retailers: In paragraph 62 (page L 265/16), it is stated: "Pricing or other general access conditions should be considered unfair if they lead to an imbalance of rights and obligations imposed on business users or confer an advantage on the gatekeeper which is disproportionate to the service provided by the gatekeeper to business users or lead to a disadvantage for business users in providing the same or similar services as the gatekeeper."

Vertically-integrated gatekeeping platforms also bring up concerns for non-price foreclosure practices: "Gatekeepers are often vertically integrated and offer certain products or services to end users through their own core platform services, or through a business user over which they exercise control which frequently leads to conflicts of interest. [...] When offering those products or services on the core platform service, gatekeepers can reserve a better position, in terms of ranking, and related indexing and crawling, for their own offering than that of the products or services of third parties also operating on that core platform service. [...] instances are those of software applications which are distributed through software application stores" (paragraph 51, L 265/13); "In such situations, the gatekeeper should not engage in any form of differentiated or preferential treatment in ranking on the core platform service, and related indexing and crawling, whether through legal, commercial or technical means, in favour of products or services it offers itself or through a business user which it controls" (paragraph 52, L 265/14).

Finally, the DMA promotes app-store competition: "To ensure contestability, the gatekeeper should furthermore allow the third-party [...] software application stores to prompt the end user to decide whether that service should become the default and enable that change to be carried out easily" (paragraph 50, L 265/13).

The full text is available at: `https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32022R1925&from=EN`.

US Innovation and Choice Act and Open App Markets Act. The Innovation and Choice Act, passed in the Judiciary Committee on January 20, 2022, aims to prevent dominant hybrid platforms from self-preferencing their own products. Sec. 2(a) states: "It shall be unlawful for a [...] platform [...] to engage in any conduct [...] that advantages the platform operator's own products, services, or lines of business over those of another business user". The full text is available at: `https://www.congress.gov/bill/117th-congress/house-bill/3816/text`.

On February 3, 2022, the Senate Judiciary Committee advanced the legislation of the Open App Markets Act, which would prevent app-stores from requiring developers to directly use their in-app payment systems and mandate reduced fees for developers. Sec 3(a) states that a covered company (platform) shall not: "(1) require developers to use or enable an in-app payment system owned or controlled by the covered company or any of its business partners as a condition of the distribution of an app on an app store or accessible on an operating system; [...] take punitive action or otherwise impose less favorable terms and conditions against a developer for using or offering different pricing terms or conditions of sale through another in-app payment system or on another app store". Finally, it promotes app-store competition: "A covered company that controls the operating system or operating system configuration on which its app store operates shall allow and provide readily accessible means for users of that operating system (1) choose third-party apps or app stores as defaults for categories appropriate to the app or app store; (2) install third-party apps or app stores through means other than its app store; and (3) hide or delete apps or app stores provided or preinstalled by the app store owner or any of its business partners" (Sec. 3(d)). The full text is available at: `https://www.congress.gov/bill/117th-congress/senate-bill/2710/text`.

Caps on access charges. Many local governments in the US introduced caps on the commissions that food delivery platforms charge to restaurants after the beginning of the COVID-19 pandemic. Several jurisdictions then made their commission caps permanent. The major platforms (Uber Eats, Grubhub and DoorDash) typically charge a 30% fee, and most governments capped these commissions to 15%. See Sullivan (2022) for the details.

B Omitted proofs

B.1 Proof of Proposition 4

For $a < b$, either the anticipation of being foreclosed gives the 3rd party developer no incentive to innovate, or the supranormal profit obtained if foreclosure is monitored implies that some socially inefficient innovations – namely, those with development cost $c \in (\Delta, \Delta + b - a]$ – are undertaken. For $a \in [b, b + v]$, the innovation takes place if and only if $c \leq \Delta$, i.e. whenever it is socially optimal, and, regardless of whether or not the superior app is developed, the platform earns profit $\pi_1 = v + b$. While this outcome is socially optimal, the platform has incentives to charge a higher access fee, which inefficiently hampers innovation. Indeed, for any $a \geq b + v$, the 3rd party developer optimally innovates if and only if $c \leq v + \Delta + b - a$, so that the platform's expected profit is

$$\pi_1^*(a) = v + b + [a - (b + v)]G(v + \Delta + b - a).$$

Maximizing it with respect to a then yields

$$a^* = b + v + \frac{G(v + \Delta + b - a^*)}{g(v + \Delta + b - a^*)},$$

with $a^* > b + v$. This implies that, under laissez faire, some socially optimal innovations – namely, those with development cost $c \in (v + \Delta + b - a^*, \Delta]$ – will not be undertaken. ■

B.2 Proof of Proposition 5

Suppose that the app market is served by the 3rd party app (there is no foreclosure). In the platform pivotality equilibrium, $p_1^* = \max\{a - b, 0\}$, and

$$\pi_1 = (p_0 + a)[1 - F(p_0 + p_2 - \Delta)],$$

whereby

$$\frac{\partial \pi_1}{\partial p_0} \propto \rho(p_0 + p_2 - \Delta) - (p_0 + a).$$

Let us first look at equilibria where the core ZLB binds. We have

$$p_0^* = 0 \iff a \geq \rho(p_2 - \Delta).$$

Since $p_2^* \geq \Delta$ whenever $p_0 = 0$ (because all consumers have a non-negative valuation – i.e., $F(0) = 0$ – there is no point charging $p_2 < \Delta$), the core ZLB binds for all $a \geq \rho(0)$.

For any such value of the access charge, the 3rd party seller solves

$$\begin{cases} \max\limits_{p_2 \geq 0} [p_2 - (a - b)][1 - F(p_2 - \Delta)] \\ \text{s.t.} \quad p_2 \leq \max\{a - b, 0\} + \Delta \end{cases}$$

Ignoring the constraint, the first-order condition is

$$\frac{\partial \pi_2}{\partial p_2} \propto \rho(p_2 - \Delta) - [p_2 - (a - b)].$$

Can one have a corner solution at Δ? The answer is yes, provided that the access charge not be too much above the ancillary benefit:

$$p_2^* = \Delta \iff \Delta - (a - b) \geq \rho(0).$$

A consequence of this characterization is that, for all $a \in [\rho(0), b + [\Delta - \rho(0)]]$, the platform's profit is $[1 - F(0)]a$, and $[1 - F(0)]b$ when it forecloses the 3rd party app. So foreclosure is again optimal for the platform if and only if $a < b$, given that foreclosure is a fortiori optimal for $a < \rho(0)$, where $p_0^* > 0$ and $p_0^* + p_2^* > \Delta$.[56]

For $a > b + \Delta - \rho(0)$, the 3rd party seller's problem admits an interior solution $p_2^*(a) \in (\Delta, a -$

[56] For $a < \rho(0)$, as $p_0^* > 0$, we have either $p_2^* = \Delta$, and so $p_0^* + p_2^* > \Delta$, or $p_2^* \in (0, \Delta)$, with (p_0^*, p_2^*) being the solution of the system of FOCs $\frac{\partial \pi_1}{\partial p_0} = 0$ and $\frac{\partial \pi_2}{\partial p_2} = 0$, which can be rewritten as

$$\begin{cases} p_0 + a = p_2 - a + b \\ p_0 + a = \rho(2(p_0 + a) - b - \Delta) \end{cases}$$

Also in this equilibrium $p_0^* + p_2^* > \Delta$. To see this, recall that it must be $p_0^* + p_2^* \geq \Delta$, and suppose by contradiction that $p_2^* = \Delta - p_0^*$. Then, the first equation of the system would yield $p_0 = \frac{1}{2}(\Delta + b) - a$ which, substituted into the second equation, would give $\frac{1}{2}(\Delta + b) = \rho(0)$, which is not possible given $b \geq \Delta > \rho(0)$. We prove below that in any equilibrium where the core ZLB does not bind the platform is better off foreclosing the 3rd party app.

$b+\Delta$),[57] and the monotone hazard rate assumption implies

$$0<\frac{\partial p_2^*}{\partial a}<1.$$

Thus, increases in the access charge generate double marginalization, thereby inefficiently reducing demand. The platform optimal access charge solves $\max_a a[1-F(p_2^*(a)-\Delta)]$, and so

$$a^*\geq b+\Delta-\rho(0).$$

At $a\to[b+\Delta-\rho(0)]^+$, since $p_2^*\to\Delta$:

$$\left.\frac{\partial\pi_1^*}{\partial a}\right|_{a\to[b+\Delta-\rho(0)]^+}>0\iff(b+\Delta-\rho(0))\left.\frac{\partial p_2^*}{\partial a}\right|_{a\to[b+\Delta-\rho(0)]^+}<\rho(0)\iff b+\Delta<3\rho(0)+\rho(0)^2\frac{f'(0)}{f(0)},$$

which, as $F(0)=0$, rewrites as $b+\Delta<\rho(0)[3+\rho(0)^2f'(0)]$. Under this condition, $a^*>b+\Delta-\rho(0)$.

Can there be equilibria where $p_0>0$ and the superior app is not foreclosed? In any such equilibrium, it must be

$$p_0+a=\rho(p_0+p_2-\Delta).$$

Next, note that $p_0+p_2\geq\Delta$ (since θ is distributed on $\mathbb{R}^+$), and so $\rho(p_0+p_2-\Delta)\leq\rho(0)$. Hence,

$$\pi_1=(p_0+a)[1-F(p_0+p_2-\Delta)]=\rho(p_0+p_2-\Delta)[1-F(p_0+p_2-\Delta)]\leq\rho(0)[1-F(0)]\leq b[1-F(0)],$$

and so the platform is better off foreclosing the 3rd party app.[58]

Absent foreclosure, social welfare is given by

$$W^*=\int_{v\geq p_0^*+p_2^*-\Delta}(b+\Delta+v)dF(v).$$

Thus, it is decreasing in consumers' total price $p_0^*+p_2^*$, and the access charge affects social welfare only through its impact on equilibrium prices.

If the 3rd party app is foreclosed, $p_0^F=0$ implies that

$$W^F=b+\mathbb{E}[v],$$

[57] An equilibrium where $p_2^*=a-b+\Delta$ would exist if and only if

$$\frac{\partial\pi_2}{\partial p_2}|_{p_2=a-b+\Delta}\geq 0\iff\Delta\leq\rho(a-b),$$

which is not compatible with the parametric restriction $\Delta\geq b\geq\rho(0)$.

[58] Even if foreclosure is monitored, equilibria with $p_0>0$ cannot exist for $a>b$. This is because

$$p_0+a>p_0+b>b\geq\rho(0)\geq\rho(p_0+p_2-\Delta),$$

which contradicts the above first-order condition for $p_0>0$.

which may be higher than W^* if demand is inefficiently low in the no-foreclosure equilibrium.

For all $a \in [b, b+\Delta-\rho(0)]$, as the core ZLB binds and $p_2^* = \Delta$, consumers' demand is the same with or without foreclosure. As foreclosure reduces social surplus by Δ, any such fee is preferable to $a < b$ under no-monitoring of foreclosure. If non-price foreclosure can be monitored, then all fees $a \in [\rho(0), b)$ are optimal as well, given that for any such a, in equilibrium $p_0^* + p_2^* = \Delta$ (by contrast, for lower fees $a < \rho(0)$, $p_0^* + p_2^* > \Delta$, and so social welfare is strictly lower). For $a > b+\Delta-\rho(0)$, the core ZLB keeps binding but $p_2^* > \Delta$ increases in a, which inefficiently reduces demand. Yet, from the platform's viewpoint, whenever $b+\Delta < \rho(0)[3+\rho(0)^2 f'(0)]$, this reduction in demand is more than compensated by the increased revenue from sellers' margin squeeze for all $a < a^*$. Hence, in these circumstances social welfare maximization requires imposing a cap on access charges.

Turning to consumer surplus, with foreclosure consumers get net value v, and so $S^F = \mathbb{E}[v]$. As, absent foreclosure, $p_0^* + p_2^* \geq \Delta$ for all $a < b$, $S^* \leq S^F$, with equality if and only if $a \in [\rho(0), b)$. The same surplus $S^F = \mathbb{E}[v]$ is obtained for all $a \in [b, b+\Delta-\rho(0)]$, as the core ZLB binds and they purchase the high-value app at $p_2^* = \Delta$. For larger values of a, $p_2^* > \Delta$ and so $S^* < S^F$.

While consumer surplus and social welfare are maximized for a compact set of access charges, $\pi_2^*(a) = \Delta$ if and only if $\hat{a} = b$. Indeed, for $a < b$, either the platform forecloses it, or the 3rd party seller makes a supranormal profit $\pi_2^*(a) > \Delta$;[59] while any $a > b$ entails some margin squeeze, given that $p_2^* < a - b + \Delta$, and so $\pi_2^*(a) < \Delta$. ■

B.3 Proof of Proposition 6

For $p_0 + p_1 \leq v$, all consumers buy one app. Since a consumer with type Δ prefers the 3rd party app if and only if $\Delta \geq p_2 - p_1$, letting $H(\cdot)$ denote the cdf of consumers' type Δ, firms' profits are

$$\pi_1^* = p_0 + a + H(p_2 - p_1)(p_1 + b - a),$$

and

$$\pi_2^* = [1 - H(p_2 - p_1)](p_2 + b - a).$$

Since π_1^* is increasing in p_0, the platform optimally sets $p_0 = v - p_1 \in [0, v]$, so that all consumers buy one app, and those buying the in-house app are left with no surplus. By doing so, it achieves a higher profit compared with the one attainable setting any larger prices so that $p_0 + p_1 > v$.[60] For any given a, the equilibrium app prices and firms' profits are as follows:

[59] We have $\pi_2^* = \Delta + b - a > \Delta$ for $a \in [\rho(0), b]$, and a weakly higher profit for lower access charges (as the the 3rd party app equilibrium profit is non-decreasing in a).

[60] For $p_0 + p_1 > v$, only the 3rd party app is bought in equilibrium, and firms' profits are

$$\pi_1 = [1 - H(p_0 + p_2 - v)](p_0 + a),$$

and

$$\pi_2 = [1 - H(p_0 + p_2 - v)](p_2 + b - a).$$

For $p_0 = v - p_1$: $p_0 + p_2 - v = p_2 - p_1$, and so

$$\pi_1^* = [1 - H(p_0 + p_2 - v)](p_0 + a) + H(p_0 + p_2 - v)(b + v) > \pi_1.$$

Lemma B.3.1. *Non-price foreclosure is optimal for the platform if and only if $a < b$. There are two thresholds $(\underline{a}, \overline{a})$, with $b < \underline{a} < b + v < \overline{a}$, such that:*

- *For $a \leq \underline{a}$, the app ZLB binds: $p_1^* = 0 \leq p_2^*$, with p_2^* being strictly positive and increasing in a for all $a \in [b, \underline{a}]$, and $p_0^* = v$.*
- *For $a \in (\underline{a}, \overline{a})$: $0 < p_1^* < a - b < p_2^*$, and $p_0^* = v - p_1^* > 0$, with $(p_2^* - p_1^*)$ and firms' profits being constant when a varies.*
- *For $a \geq \overline{a}$, the core ZLB binds ($p_0^* = 0$) and $p_1^* = v < p_2^*$, with p_2^* being strictly increasing in a. The platform's profit is maximized at $a^* > \overline{a}$.*

Proof of Lemma B.3.1. Denoting by $h(\cdot)$ the pdf of consumers' type and by $\rho_\Delta(\tilde{\Delta}) \equiv [1 - H(\tilde{\Delta})]/h(\tilde{\Delta})$ the inverse hazard rate, which we assumed is decreasing, we have

$$\frac{\partial \pi_1^*}{\partial p_1} = -h(p_2 - p_1)(p_1 + b - a) - 1 + H(p_2 - p_1) = 0 \iff a - b - p_1 = \rho_\Delta(p_2 - p_1), \quad \text{(B.1)}$$

and

$$\frac{\partial \pi_2^*}{\partial p_2} = -h(p_2 - p_1)(p_2 + b - a) + 1 - H(p_2 - p_1) = 0 \iff p_2 - (a - b) = \rho_\Delta(p_2 - p_1). \quad \text{(B.2)}$$

As Δ is distributed on $\mathbb{R}^+$, $p_2^* \geq p_1^*$ in any equilibrium, with strict inequality whenever the app ZLB does not bind. First, consider an equilibrium where $p_1^* = 0 \leq p_2^*$. By (B.1), this is the case if and only if

$$\left.\frac{\partial \pi_1^*}{\partial p_1}\right|_{p_1=0} \leq 0 \iff a - b \leq \rho_\Delta(p_2) \leq p_2 - (a - b) \iff p_2 \geq 2(a - b), \quad \text{(B.3)}$$

where the second inequality uses (B.2), which holds with equality as long as $p_2 > 0$. Hence, in equilibrium $p_1^*(a) = p_2^*(a) = 0$ if $\frac{\partial \pi_1^*}{\partial p_1}|_{p_1=p_2=0} \leq 0$ and $\frac{\partial \pi_2^*}{\partial p_2}|_{p_1=p_2=0} \leq 0$, which gives $a < b - \rho_\Delta(0)$. In turn, from (B.2),

$$p_2 \geq 2(a - b) \iff a - b \leq \rho_\Delta(2(a - b)), \quad \text{(B.4)}$$

which, as the LHS (resp. RHS) is increasing (resp. decreasing) in a, is satisfied if and only if $a \leq \underline{a}$, with $\underline{a} > b$. The platform's profit is

$$\pi_1^*(a) = v + a + H(p_2^*)(b - a).$$

For $a \in [0, b - \rho_\Delta(0)]$, as $p_2^* = p_2^* = 0$ and $H(0) = 1$ (i.e., all consumers buy the 3rd party app), $\pi_1^*(a) = v + a < v + b \equiv \pi^F$, with π^F denoting the platform's profit under foreclosure. For $a \in (b - \rho_\Delta(0), \underline{a}]$, $p_2^* > 0$, and, by the implicit function theorem,

$$\frac{\partial \pi_1^*}{\partial a} = h(p_2^*)\frac{\partial p_2^*}{\partial a}(b - a) - H(p_2^*) + 1 > 0 \iff \frac{\partial p_2^*}{\partial a}(a - b) < \rho_\Delta(p_2^*) = p_2^* - (a - b),$$

Therefore, $p_0^* = v - p_1$ is set so that all consumers access the platform in equilibrium, whereby the platform pivotality condition is satisfied.

which is satisfied for all $a < b$, as $\frac{\partial p_2^*}{\partial a} > 0$ (since $p_2^* > 0 > a - b$), and for $a \in [b, \underline{a}]$ as well, by (B.3), as $\frac{\partial p_2^*}{\partial a} < 1$ (the monotone hazard rate assumption implies $\frac{\partial p_2^*}{\partial a} \in (0,1)$). Therefore, we can conclude that $\frac{\partial \pi_1^*}{\partial a} > 0$ for all $a \in [0, \underline{a}]$. Given that $\pi_1^*(b) = \pi^F$, it then follows that non-price foreclosure is optimal for the platform if and only if $a < b$.

Next, consider an equilibrium where $p_2^* > p_1^* \in (0, v)$. In this equilibrium, (B.1)-(B.2) imply

$$a - b - p_1 = \rho_\Delta(p_2 - p_1) = p_2 - (a - b) \iff p_1 + p_2 = 2(a - b). \tag{B.5}$$

As $p_2^* > p_1^*$, it must be $p_1^* < a - b < p_2^*$. Using (B.5), (B.1) rewrites as

$$a - p_1 - b = \rho_\Delta(2(a - p_1 - b)). \tag{B.6}$$

As the LHS (resp. RHS) is decreasing (resp. increasing) in p_1, this equilibrium exists if and only if

$$p_1^* > 0 \iff a - b > \rho_\Delta(2(a - b)) \iff a > \underline{a},$$

and, using (B.2),

$$p_1^* < v \iff a - b - v < \rho_\Delta(2(a - b - v)) \iff a < \overline{a},$$

where, comparing the two above inequalities, it follows that $\overline{a} > \underline{a}$. From (B.6) it follows that $p_1^* - a$ is constant varying a. Since $p_1 + p_2 = 2(a - b)$ is equivalent to $p_1 - a = a - p_2 - 2b$, this implies that $a - p_2^*$ is constant in a as well, and so also $p_2^* - p_1^*$ does not vary with a. This shows a neutrality result: $\pi_1^*(a) = [1 - H(p_2^* - p_1^*)](v - p_1^* - a) + H(p_2^* - p_1^*)(b + v)$ is independent of a in this range. However, $\pi_1^*(a) > \pi^F$ since $p_1^* < a - b$.

Finally, we consider an equilibrium where $p_1^* = v < p_2^*$ (and so $p_0^* = 0$). By (B.1) and (B.2), this is the case if and only if

$$\left.\frac{\partial \pi_1}{\partial p_1}\right|_{p_1 = v} \geq 0 \iff a - b - v \geq \rho_\Delta(p_2 - v) = p_2 - (a - b) \iff a - b - v \geq \rho_\Delta(2(a - b - v)),$$

which holds if and only if $a \geq \overline{a}$, with $\overline{a} > b + v$ implying $p_1^* = v < a - b$. The platform's profit is

$$\pi_1^*(a) = H(p_2^* - v)(b + v - a) + a.$$

We then have:

$$\frac{\partial \pi_1^*}{\partial a} = h(p_2^* - v)\frac{\partial p_2^*}{\partial a}(b + v - a) - H(p_2^* - v) + 1,$$

where, by the monotone hazard rate assumption, $\frac{\partial p_2^*}{\partial a} \in (0,1)$ is characterized using the implicit function theorem. We then obtain:

$$\frac{\partial \pi_1^*}{\partial a} > 0 \iff a - b - v < 2(p_2^* + b - a) + \frac{h'(p_2^* - v)}{h(p_2^* - v)}(p_2^* + b - a)^2. \tag{B.7}$$

At $a = \overline{a}$, $p_2^* = 2(a-b) - v$. Substituting into (B.7) and simplifying gives

$$\frac{h'(2(a-b-v))}{h(2(a-b-v))}(a-b-v) = \frac{h'(2(a-b-v))}{h(2(a-b-v))}\rho_\Delta(2(a-b-v)) > -1,$$

where the equality follows from the definition of $\overline{a}$. This inequality is always satisfied as it is equivalent to the assumption of decreasing inverse hazard rate. Therefore, the platform's equilibrium profit is still increasing at $a = \overline{a}$, and so $a^* > \overline{a}$. ■

As $p_0^* + p_1^* = v$, consumers purchasing the platform's in-house app have zero surplus, and consumer surplus writes as

$$S^* = \int_{\Delta \geq p_2^* - p_1^*} [\Delta - (p_2^* - p_1^*)] dH(\Delta) > 0,$$

if there is no foreclosure, and $S^F = 0$ with foreclosure. Social welfare is given by

$$W^* = b + v + \int_{\Delta \geq p_2^* - p_1^*} \Delta dH(\Delta),$$

if there is no foreclosure, and $W^F = b + v < W^*$ with foreclosure.

Hence, both consumer surplus and social welfare are lower under foreclosure: If non-price foreclosure cannot be monitored, it must be that $a \geq b$. Moreover, both welfare objectives are decreasing in the relative price $p_2^* - p_1^*$. The access charge thus affects S^* and W^* only through its impact on the equilibrium prices. Then:

- for $a \in [0, \underline{a}]$, as p_2^* is increasing in a (strictly so for $a > b - \rho_\Delta(0)$) and $p_1^* = 0$, S^* and W^* are decreasing in a (strictly so for $a > b - \rho_\Delta(0)$);
- for $a \in (\underline{a}, \overline{a})$, $p_2^* - p_1^*$, and hence S^* and W^*, are constant when a varies;
- for $a \geq \overline{a}$, as p_2^* is strictly increasing in a and $p_1^* = v$, S^* and W^* are strictly decreasing in a.

Hence, if monitoring non-price foreclosure is not feasible, the optimal access charge, both from a consumer-surplus and a total-welfare standpoint, is $\hat{a} = b$. If, on the contrary, non-price foreclosure could be monitored, then any $a \in [0, b - \rho_\Delta(0)]$ maximizes both welfare objectives. ■

B.4 Proof of Proposition 7

As $u^i \equiv \max\{v - p_1^i,\ v + \Delta - p_2^i,\ 0\}$, and 3rd party sellers always charge $p_2^{i*} \geq \Delta$, we have $u^i \in [0, v]$ for all i. As $p_0 \geq 0$ and consumers' outside option is zero, also $U^i \in [0, v]$. Next, take two platforms i' and i'' and suppose they offer different utility levels to consumers $v \geq U^{i'} > U^{i''} \geq 0$, with $U^{i'} = \max_j\{U^j\}$ so that platform i' has strictly positive market share. Then, platform i'' would face no demand and make zero profits. By foreclosing the 3rd party app and setting prices $p_0^i + p_1^i \leq v - U^{i'}$, it would offer utility $U^{i''} \geq U^{i'}$ and make a positive profit.

As a result, all platforms must offer the same utility U^* in equilibrium. Hence, their profit is $\pi_1^i = \frac{1}{N}(p_0^i + p_1^i + b)$ with foreclosure, with $p_0^i + p_1^i = v - U^*$ and $\pi_1^i = \frac{1}{N}(p_0^i + a^i)$ without foreclosure, with $p_0^i = v + \Delta - p_2^* - U^*$. If $p_0^i > 0$ for some i, then, no matter whether it forecloses or not the 3rd party app, platform i would find it optimally to deviate, charging a slightly lower access price to consumers to serve all demand. Therefore, $p_0^{1*} = \ldots = p_0^{N*} = 0$ in equilibrium.

Whenever its rivals are expected to provide $U^* = v$ in equilibrium, any platform i has no profitable deviation to $U^i \neq U^*$: offering $U^i < v$ drives its profit to zero, and, as shown above, it is never possible to provide $U^i > v$. As $U^* = v$ can always be provided by foreclosing the 3rd party app and setting $p_0^i = p_1^i = 0$, it follows that an equilibrium where $U^* = v$ always exists. We next characterize the corresponding subgame perfect equilibrium prices for any given $(a^i, \delta^i = \Delta)_{i=1,\ldots,N}$ (no foreclosure). Suppose that in equilibrium $p_1^i > 0$. Then, as $p_2^{i*} = \min\{p_1^i + \Delta, v + \Delta\}$, $u^i = \max\{v - p_1^i, 0\} < v$. Given that rival platforms offer higher value $U^* = v$, the considered platform makes no profit. It has therefore a strictly profitable deviation: It can set $p_1^i = 0$ and thus, by selling its in-house app, offer value $U^i = v$ to consumers, so as to attract some of them and make positive profits (given that the marginal cost is negative). Hence, the app ZLB binds: $p_1^{i*} = 0$ for all i and $(a^i, \delta^i = \Delta)_{i=1,\ldots,N}$. Anticipating this, the (non-foreclosed) 3rd party seller must set $p_2^{i*} = \Delta$ to sell its app. It optimally does so whenever selling its app yields positive profit (i.e., as long as $\Delta + b - a^i \geq 0$).

As each platform's profit $\pi_1^i = \frac{a^i}{N}$ is increasing in the access charge, unregulated platforms set the highest a subject to the 3rd party app's participation constraint: $a^* = b + \Delta$. Hence, $\pi_1^* = \frac{b+\Delta}{N}$ exceeds the foreclosure profit $\frac{b}{N}$ (given that a platform foreclosing the superior app can provide utility $U^* = v$ to consumers and serve a share $1/N$ of them only by setting $p_0 = p_1 = 0$). By contrast, as it makes $\Delta + b - a^i$ per-consumer on any platform i, for the 3rd party app to receive its fair reward, the access charge must be capped by regulation at the Pigouvian level ($\hat{a} = b$), at which $\pi_1 = b/N$ coincides with the foreclosure payoff.[61] ■

B.5 Proof of Proposition 8

As in Proposition 7, app-store competition implies $p_0^{j*} = p_1^{j*} = 0$, for all a^j and app-stores j, including the in-house app-store. As all app-stores can be accessed for free, consumers multi-home. An equilibrium where the 3rd party app is foreclosed by all app-stores cannot exist: Given that consumers would prefer to buy the superior app at any price $p_2^j \leq \Delta$, any app-store would deviate by granting access to the superior app at the extractive access charge $a = b + \Delta$.

Therefore, the 3rd party provider can sell its app to all consumers on any app-store j at any price $p_2^j \leq \Delta$. Therefore, it will optimally sell at a price (slightly below) Δ on the app-store charging the lowest access fee. Whenever $a^j \geq a^{j'} > 0$ and app-store j' attracts some 3rd party app sales, app-store j can profitably undercut its rival (i.e., set $a^j = a^{j'} - \epsilon$) so as to induce the

[61] Under laissez faire, the game may also admit other equilibria with $U^* \in [0, v)$. Yet, for $\hat{a} = b$ it must be that $p_1^* = 0$ (below-opportunity-cost pricing is infeasible because of the app ZLB), and so $p_2^* = \Delta$ and $U^* = u^* = v$. Thus, when access charges are set by regulation at the Pigouvian level, the game admits a unique equilibrium outcome, which features fairness.

3^{rd} party provider to rather serve all consumers through its app-store. It then follows that in equilibrium $a^* = 0$ for all app-stores, and so $\pi_2^* = \Delta + b$, while app-stores make zero profits.

As the equilibrium apps' and app-stores' prices are the same for all values of the access charge (provided the 3^{rd} party app is viable), $\pi_2^* = \Delta$ if and only if $a = b$ (so that app-stores collectively make profit b).

Finally, both under laissez faire and with regulated access charges, consumers get net value v_a from the app-stores and v_c from the core, and so the monopoly platform optimally sets $p_0 = v_a + v_c$. ■

B.6 Proof of Proposition 9

With multiple heterogeneous app markets, consumers' utility from accessing the app-store is

$$U \equiv \int_{k\in[0,1]} u^k \; dk - p_0,$$

with p_0 again denoting consumers' access price, and u^k being the utility obtained from app market $k \in [0,1]$:

$$u^k \equiv \max\{v^k - p_1^k,\; v^k + \Delta^k - p_2^k,\; 0\},$$

where p_1^k and p_2^k denote the prices for in-house app and 3^{rd} party app, respectively, in the considered market k.

We first show that the results of Section 2 extend to this setting with multiple heterogeneous app markets. In the platform pivotality equilibrium,

$$p_1^{k*} = \max\{a^k - b^k, 0\}, \quad p_2^{k*} = \min\{p_1^{k*} + \Delta^k, v^k + \Delta^k\},$$

whenever $a^k \leq b^k + v^k + \Delta^k$ (any larger access charge implies access-price foreclosure of the 3^{rd} party app). Then, p_0 is set so as to satisfy consumers' participation constraint with equality ($U = 0$):

$$p_0^* = \int_{k\in[0,1]} u^{k*} \; dk.$$

Hence, denoting $x^k = 0$ (resp. $x^k = 1$) if the in-house app (resp. 3^{rd} party app) is sold in market k, platform's profit writes

$$\pi_1^* = p_0^* + \int_{\{k:\, x^k=0\}} (p_1^{k*} + b^k) \; dk + \int_{\{k:\, x^k=1\}} a^k \; dk = \int_{\{k:\, x^k=0\}} \pi_1^k(x^k = 0) \; dk + \int_{\{k:\, x^k=1\}} \pi_1^k(x^k = 1) \; dk,$$

where

$$\pi_1^k(x^k = 0) \equiv v^k + b^k, \qquad \pi_1^k(x^k = 1) \equiv v^k + \Delta^k - p_2^{k*} + a^k,$$

are the per-market profits with and without foreclosure, respectively (inclusive of the revenues from optimally setting consumers' access price).

If not foreclosed, the 3rd party seller in market k makes

$$\pi_2^{k*} = p_2^{k*} + b^k - a^k.$$

In any market k where $a^k < b^k$, absent foreclosure, consumers purchase the 3rd party app at $p_2^{k*} = \Delta^k$ and obtain utility $u^{k*} = v^k$. As this is the same utility that they would obtain under foreclosure and $p_1^{k*} = 0$, it follows that by foreclosing 3rd party rivals in any such market the platform can charge the same access price p_0^* to consumers, but obtains higher unit revenues $b^k > a^k$. In any market k where $a^k \in [b^k, b^k + v^k]$, absent foreclosure, consumers purchase the 3rd party app at $p_2^{k*} = a^k - b^k + \Delta^k$ and obtain utility $u^{k*} = v^k - (a^k - b^k) > 0$. From any such market, the platform obtains profit $\pi_1^k(x^k = 1) = v^k + b^k = \pi_1^k(x^k = 0)$, and so is indifferent between foreclosing or not. The 3rd party seller gains $\pi_2^{k*} = \Delta^k$. Finally, in any market k where $a^k \in (b^k + v^k, b^k + v^k + \Delta^k]$, absent foreclosure, consumers purchase the 3rd party app at $p_2^{k*} = v^k + \Delta^k < a^k - b^k + \Delta^k$ and obtain utility $u^{k*} = 0$. From any such market, the platform obtains profit $\pi_1^k(x^k = 1) = a^k > v^k + b^k = \pi_1^k(x^k = 0)$, and so is strictly better off than under foreclosure. The 3rd party app is squeezed: $\pi_2^{k*} = v^k + \Delta^k + b^k - a^k < \Delta^k$. Clearly, the profit maximizing fee in market k is $a^* = v^k + \Delta^k + b^k$.

We can now turn to establish Proposition 9. Suppose the regulator knows the distribution $K(b)$ of ancillary benefits and lets the platform choose $(a^k)_{k\in[0,1]}$ subject to the constraint that the distribution of access charges mimics that of benefits (i.e., follows $K(a)$). To show that setting $a^k = b^k$ is not incentive-compatible for the platform, take two markets k' and k'' such that $v^{k'} \leq b^{k''} - b^{k'} \leq v^{k'} + \Delta^{k'}$. By the above analysis, if the platform sets $a^k = b^k$ for $k \in \{k', k''\}$, it obtains profit $\pi_1^k = v^k + b^k$ from each of these markets. By setting instead $a^{k'} = b^{k''}$ and $a^{k''} = b^{k'}$, and foreclosing the 3rd party app in market k'', it still obtains profit $\pi^{k''} = v^{k''} + b^{k''}$ in the higher-b market, but now makes a larger profit $\pi^{k'} = a^{k'} = b^{k''} > b^{k'} + v^{k'}$ from the lower-b market.[62] ∎

C Further material

C.1 Ad-valorem access charges

Throughout the paper we considered for simplicity linear (per-unit) access charges. Here we show that our results are robust when considering instead ad-valorem fees (which are more often employed in reality): For each app sold by the 3rd party seller at price p_2, the platform gets tp_2 and the seller $(1-t)p_2$, with $t \in [0,1]$. In what follows, to make things interesting we posit $b < \Delta$ (as otherwise the platform would optimally foreclose the 3rd party app for all $t \in [0,1]$).

[62] Note that this deviation is not profitable if the platform cannot foreclose the 3rd party app in the high-b market k''. Indeed, the deviation profit equals now $\pi^{k''} = v^{k''} + b^{k'}$ in market k'', and so $\pi^{k'} + \pi^{k''} = b^{k'} + b^{k''} + v^{k''}$ is lower than the equilibrium profit $(b^{k'} + b^{k''} + v^{k'} + v^{k''})$. This suffices to prove that, if foreclosure can be monitored, then, under the constraint that the distribution of a mimic that of b, setting $a^k = b^k$ for all $k \in [0,1]$ is incentive-compatible for the platform.

Lemma C.1.1. *For any ad-valorem access charge $t \in [0,1]$, the equilibrium has the following features:*

- *For $t \in [0, \frac{b}{\Delta})$: $p_1^* = 0$, $p_2^* = \Delta$, and $p_0^* = v$; hence, $\pi_1^*(t) = v + t\Delta < v + b \equiv \pi^F$: the platform is better off foreclosing the 3rd party app; if foreclosure can be monitored, $\pi_2^*(t) = (1-t)\Delta + b > \Delta$.*
- *For $t \in \left[\frac{b}{\Delta}, \frac{b+v}{v+\Delta}\right]$, $p_1^* = \frac{t\Delta - b}{1-t}$, $p_2^* = \frac{\Delta - b}{1-t}$, and $p_0^* = v - \frac{t\Delta-b}{1-t}$; for any such t, $\pi_1^*(t) = \pi^F$ and $\pi_2^*(t) = \Delta$ (neutrality).*
- *For $t \in (\frac{b+v}{v+\Delta}, 1]$: $p_1^* = t(v+\Delta) - b$, $p_2^* = v + \Delta$, and $p_0^* = 0$; hence, $\pi_1^*(t) = t(v+\Delta) > \pi^F$ and $\pi_2^*(t) = (1-t)(v+\Delta) + b < \Delta$ (squeeze).*

Proof of Lemma C.1.1. In this setting, the platform prefers selling its own app as long as $tp_2 < b + p_1$. Hence, given the ZLB constraints, in the platform pivotality equilibrium, app prices are

$$p_1^* = \max\{0, tp_2^* - b\} \quad \text{and} \quad p_2^* = \min\{p_1^* + \Delta, v + \Delta\}.$$

As long as app providers are unconstrained by consumers' willingness to pay, the equilibrium app prices are

$$\begin{cases} p_1^* = \frac{t\Delta - b}{1-t}, p_2 = \frac{\Delta - b}{1-t} & \text{if } t \geq \frac{b}{\Delta} \\ p_1^* = 0, p_2 = \Delta & \text{if } t < \frac{b}{\Delta} \end{cases}$$

Hence, for $t < \frac{b}{\Delta} \in (0,1)$, $p_0^* = v$, and foreclosure is optimal:[63] $\pi_1^*(t) = v + t\Delta < v + b \iff t < \frac{b}{\Delta}$. For $t \geq \frac{b}{\Delta}$, $p_0^* = v + \Delta - p_2^* = v + \frac{b - t\Delta}{1-t}$, and so $\pi_1^*(t) = v + b = \pi^F$ and $\pi_2^*(t) = v$. This is the equilibrium outcome as long as $p_2^* < v + \Delta$, which requires $t < \frac{b+v}{v+\Delta}$. For $t \geq \frac{b+v}{v+\Delta}$, $p_2^* = v + \Delta$, and so $p_1^* = t(v+\Delta) - b \in (v, p_2^* - \Delta)$ and $p_0^* = 0$. In this case, $\pi_1^*(t) = t(v+\Delta) > \pi^F$ and $\pi_2^*(t) = (1-t)(v+\Delta) + b < \Delta$. ■

The equilibrium characterization thus mirrors the one under unit fees: For low (resp. high) values of the access charge, the app (resp. core) ZLB binds, and the platform is strictly better off foreclosing (resp. not foreclosing) the 3rd party app. For intermediate values of t, no ZLB binds, and the neutrality result holds. Accordingly, it is easy to derive the following results:

Proposition C.1.1 (optimal access charges)**.**

(i) Welfare-optimal access charges. *Any access charge such that the 3rd party app is not foreclosed maximizes ex-post social welfare: $t \in [\frac{b}{\Delta}, 1]$ if non-price foreclosure cannot be monitored, $t \in [0,1]$ under monitoring of self-preferencing;*

(ii) Profit-maximizing access charge. *Platform's profit is maximized at $t^* = 1$;*

(iii) Fair access charges. *The independent developer receives a fair reward for its contribution to the ecosystem if and only if $t \in \left[\frac{b}{\Delta}, \frac{b+v}{v+\Delta}\right]$.*

Proof of Proposition C.1.1. As consumer surplus is always extracted by the platform through the access price, social welfare is simply $W^* = \pi_1^*(t) + \pi_2^*(t) = b + v + \Delta x$, and so is maximized

[63] This result would hold for all $t \in [0,1]$ when $b > \Delta$.

whenever there is no price or non-price foreclosure, so that $x = 1$, from which (i) follows. Platform's profit is continuous, non-decreasing in t for $t \leq \frac{b+v}{v+\Delta}$, and strictly increasing for larger values of t, hence it is maximized at $t^* = 1$, which establishes (ii). Finally, the result in (iii) follows from the equilibrium profit $\pi_2^*(t)$ given in Lemma C.1.1. ■

Note that the lowest assess charge such that the platform has no incentives to practice self-preferencing, $\hat{t} = \frac{b}{\Delta}$, is such that $p_2^*(\hat{t}) = \Delta$, so that the platform obtains $\hat{t}p_2^*(\hat{t}) = b$ from distributing the 3rd party app. Hence, optimal access charge regulation still follows a Pigouvian principle: The superior seller must internalize that, for each app it sells, it "steals" b from the platform.

C.2 Freemium apps

Many apps adopt a *freemium* model: A basic version (with limited functionalities) is made available for free, while consumers are charged a positive price for the premium version (which includes all functionalities). This outcome can be easily obtained by augmenting our model.

Suppose that both firms sell a basic version and a premium version of their apps. The apps deliver the same value v for the basic service; prices for the basic service are p_1 for the in-house app and p_2 for the 3rd party app. The premium service will add extra value, V for the in-house app and $V + \Delta$ for the 3rd party app. Only those who have selected the basic service of a provider can enjoy its premium service (the two are complements).

There is no commitment as to the price of the premium service. The idea is that, while app providers can set a price for the current version of the premium service, they may start with no premium service or else will introduce new functionalities or new characters. And so the app providers charge the captive consumers' willingness to pay for the premium service. The consumers, anticipating that they will not receive surplus from the premium version, just compare the basic versions when selecting their app provider.

Suppose that $v \geq 0$ follows a distribution $F(v)$ with support $\mathbb{R}^+$ and monotone (inverse) hazard rate $\rho(v)$, and that (absent foreclosure) the 3rd party app always corners consumers whenever it sets $p_2 = 0$ (it cannot be undercut, but more on this below). The platform levies an ad-valorem tax $t \in [0, 1]$ on sales of the 3rd party app. Finally, to make our point in the simplest setting, we rule out exogenous ancillary benefits ($b = 0$).

Proposition C.2.1. *Suppose $V + \Delta > \rho(0)$. Then, both basic apps are priced at zero ("freemium apps") in equilibrium for all $t \in [0, 1]$, and the platform is strictly better off not foreclosing the 3rd party app for all $t \in (\frac{V}{V+\Delta}, 1]$.*

Proof of Proposition C.2.1. As its premium app is priced at $V + \Delta$ and bought by all consumers who first buy its basic version, the 3rd party app solves

$$\max_{p_2 \in [0, p_1]} [1 - F(p_0 + p_2)](1 - t)(p_2 + V + \Delta),$$

and so, under the assumption that (absent foreclosure) the 3rd party app always corners consumers by setting $p_2 = 0$, for all $t \in [0,1]$:

$$p_2^* = 0 \iff V + \Delta > \rho(p_0).$$

To demonstrate the possibility of the freemium (free basic apps) without-foreclosure outcome in equilibrium, we need to show that, for some $t \in [0,1]$, the platform is strictly better off not foreclosing. In this case, the in-house app does not play any role, so that we can posit w.l.o.g. $p_1 = 0$.

So long as $p_2^* = 0$, no matter whether the 3rd party app is foreclosed or not, the platform solves

$$\max_{p_0 \geq 0} [1 - F(p_0)](b_1 + p_0),$$

with $b_1 = V$ under foreclosure, and $b_1 = t(V + \Delta)$ without foreclosure. Hence, foreclosure is not optimal whenever

$$t(V + \Delta) \geq V \iff t \geq \frac{V}{V + \Delta} \in (0,1),$$

which concludes the proof. ■

Given that the same ad-valorem fee is levied on both basic and premium app sales, the 3rd party app is willing to price its app at zero for all t whenever the benefit that it obtains from consumer lock-in under no (or, more generally, limited) commitment is large enough. The platform is then better off not foreclosing whenever the unit tax revenue $t(V + \Delta)$ from the superior app sales exceeds the value V from selling to consumers the premium version of its in-house app.

Remark. How strong is the assumption that the 3rd party app corners the market when $p_1 = p_2 = 0$? After all, the consumers are indifferent and there would be no way for the 3rd party app to give them a bit more if the allocation of consumers to apps were different. But if the consumers' preferences for the premium version were heterogeneous, then the expected consumer net surplus from the upgrade (the relevant consideration if the consumers have no information about their preference for the premium version before trying the basic version) is (a) strictly positive, (b) under a reasonable assumption larger for the 3rd party app, implying that the only equilibrium allocation has all consumers go for the basic version of the 3rd party app (this extra surplus plays the role of Δ in our model). More formally, suppose that the distribution of V is $F(V)$ for the in-house app and $F(V - \xi)$ for the 3rd party app, where $\xi > 0$. Then the in-house price of the upgrade, P_1, solves $\max_{P \geq 0}[1 - F(P)]$ while that for the 3rd party app, P_2, is obtained from $\max_{P \geq 0}[1 - F(P - \xi)]$. Under the monotone hazard rate condition for $F(V)$, we have $P_2 > P_1 > P_2 - \xi$ (the "net price" is smaller for the superior app) and so the expected net surplus is higher for the 3rd party app.

The following proposition provides the regulatory implications of our analysis:

Proposition C.2.2. *Suppose that $V + \Delta > \rho(0)$, and that non-price foreclosure cannot be* *monitored. Then, any access charge* $t \geq \max\{\frac{\rho(0)}{V+\Delta}, \frac{V}{V+\Delta}\} \in (0,1)$*, such that the core ZLB binds and there is no foreclosure, maximizes consumer surplus and social welfare. The 3rd party app receives a fair compensation for its contribution* Δ *to the ecosystem if and only if* $V \leq \rho(0)$ *and* $\hat{t} = \frac{V}{V+\Delta}$.

Proof of Proposition C.2.2. As in Proposition 5, consumer surplus and social welfare are decreasing in the total price $p_0^* + p_2^*$. Provided the freemium equilibrium emerges, the core ZLB binds if and only if

$$\arg\max_{p_0 \geq 0}[1 - F(p_0)](t(V + \Delta) + p_0) \leq 0 \iff t \geq \frac{\rho(0)}{V + \Delta}.$$

Any such access charge is such that $p_0^* = p_2^* = 0$, and hence maximizes both welfare objectives, provided that either foreclosure can be monitored, or the platform has no incentive to foreclose, i.e. $V \leq \rho(0)$. For $V > \rho(0)$, preventing foreclosure (which would strictly decrease social welfare) requires raising the access charge to $t \geq \frac{V}{V+\Delta} > \frac{\rho(0)}{V+\Delta}$.

Fairness requires

$$\pi_2^* = [1 - F(p_0)](1 - t)(V + \Delta) = \Delta \iff t = 1 - \frac{\Delta}{[1 - F(p_0)](V + \Delta)}.$$

Provided that the core ZLB binds (i.e., $1 - F(p_0^*) = 1 - F(0) = 1$), this yields $t = \frac{V}{V+\Delta}$, at which the platform has no strict incentives to foreclose. If, however, $V > \rho(0)$, then fair compensation would require $t < \frac{V}{V+\Delta}$, which cannot be implemented without monitoring of foreclosure. ■

Fair compensation requires capping the access charge at the value such that the platform is indifferent between foreclosing or not the 3rd party app, i.e. obtains a unit revenue from 3rd party app distribution which exactly equals its opportunity cost from letting it serve consumers: $\hat{t}(V + \Delta) = V$. The Pigouvian principle is thus robust also when considering freemium apps.

C.3 Heterogeneous valuations for the core

In the models of Section 3.2.1 and 3.2.2, we have assumed that the platform is per-se valueless. This section introduces consumer heterogeneity with respect to the valuations $v_c \geq 0$ of the core (assuming that instead consumers have homogeneous valuations v_a and Δ for the apps).

Denoting by $F(v)$ the cdf of consumers' willingness to pay for the overall platform's services (core plus in-house app), and by $\rho(v)$ the inverse hazard rate (assumed to be decreasing), we have:

Lemma C.3.1. *App prices are given by* $p_1^* = \max\{a - b, 0\}$ *and* $p_2^* = \min\{p_1^* + \Delta, v_a + \Delta\}$. *Moreover:*

- *for* $a < b$ *(i.e., when the app ZLB binds),* p_0^* *is decreasing in* a*, and the platform is better off foreclosing the 3rd party app (if feasible);*
- *for* $a \in [b, b + v_a]$*, there is competitive neutrality:* $p_0^* + a$ *and* $p_2^* - a$ *are constant when* a

varies, and the platform is indifferent between practising foreclosure or not;

- *for $a \in (b+v_a, b+v_a+\Delta]$, the platform gains strictly more by squeezing the 3rd party app provider than by foreclosing it, and*
  - *for $a \in (b+v_a, \rho(0))$, if this region of parameters is non-empty, $p_0^* > 0$ is decreasing in a,*
  - *for $a \in [\max\{b+v_a, \rho(0)\}, b+v_a+\Delta]$, if this region of parameters is non-empty, the core ZLB binds.*

Proof of Lemma C.3.1. Under foreclosure, heterogeneity in v_c yields a demand $1-F(p_0+p_1-v_a)$. The platform can set $p_1 = 0$ and gets profit

$$\pi^F \equiv \max_{p_0 \geq 0} \, (p_0 + b)[1 - F(p_0 - v_a)].$$

Taking the first-order condition of the platform's problem gives

$$p_0^F + b = \rho(p_0^F - v_a),$$

and so the core ZLB is never binding under foreclosure ($p_0^F > 0$), given that $v_c + v_a \geq 0$ for all consumers.

Without foreclosure, as consumers' valuations in the app segment are homogeneous, under platform pivotality app prices (p_1^*, p_2^*) are as in the basic model. The demand is then $1 - F(p_0 + p_2^* - v_a - \Delta)$, and we have:

- For $a < b$, the platform's profit,

  $$\max_{p_0 \geq 0} \, (p_0 + a)[1 - F(p_0 - v_a)] < \pi^F,$$

  is maximized at

  $$p_0^* + a = \rho(p_0^* - v_a),$$

  from which, by the monotone hazard rate assumption, it follows that $\frac{\partial p_0^*}{\partial a} \in (-1, 0)$.

- For $a \in [b, b+v_a]$,

  $$\max_{p_0 \geq 0} \, (p_0 + a)[1 - F(p_0 + a - b - v_a)] = \max_{\tilde{p}_0 > 0} \, (\tilde{p}_0 + b)[1 - F(\tilde{p}_0 - v_a)] = \pi^F,$$

  where the equality follows because the core ZLB is not binding under foreclosure. In this case, there is competitive neutrality: $p_0^* + a$, as well as $p_2^* - a$, are constant when a varies.

- for $a \in (b+v_a, b+v_a+\Delta]$,

  $$\max_{p_0 \geq 0} \, (p_0 + a)[1 - F(p_0)] = \max_{\tilde{p}_0 > 0} \, (\tilde{p}_0 + b)[1 - F(\tilde{p}_0 - (a - b))] > \pi^F,$$

  given that $a - b > v_a$. The first-order condition for an interior optimum of the platform's problem gives

$$p_0^* + a = \rho(0),$$

from which it follows that, in an interior optimum, $\frac{\partial p_0^*}{\partial a} \in (-1, 0)$, and also that, in this region of parameters, the core ZLB binds if and only if $a \geq \rho(0)$. ■

Consider $b < \rho(0) - v_a$. Then, for $a \in (b + v_a, \rho(0))$ as in the basic model the 3rd party seller is constrained by consumers' willingness to pay for its app, and so cannot pass through increased fees to consumers, but here the core ZLB is not yet binding. Therefore, as the access charge grows larger, the platform gains higher unit profits from the app-store (by squeezing the 3rd party seller), which gives it an incentive to increase consumers' participation by reducing the access price ($\frac{\partial p_0^*}{\partial a} < 0$ so long as the core ZLB does not bind). As a result, consumer total price $p_0^* + v_a + \Delta$ is decreasing in a in this range: Higher access charges in this case reduce, rather than exacerbate, double marginalization, and so, provided the superior 3rd party already exists, the lowest among welfare-optimal access charges is $a = \rho(0) > b$. Otherwise, the Pigouvian access charge is still welfare-optimal.[64] In sum:

Proposition C.3.1. *Regardless of whether foreclosure is monitored or not:*

- *If $\rho(0) \leq b + v_a$, then any $a \geq b$ maximizes consumer surplus and social welfare;*
- *If instead $\rho(0) > b + v_a$, then consumer surplus and social welfare are maximized for any $a \geq \rho(0)$.*

Proof of Proposition C.3.1. Both consumer surplus,

$$S = \int_{p_0 + p_2 - (v_a + \Delta)}^{\infty} (v_c + v_a + \Delta - (p_0 + p_2)) dF(v_c),$$

and social welfare,

$$W = \int_{p_0 + p_2 - (v_a + \Delta)}^{\infty} (v_c + v_a + \Delta + b) dF(v_c),$$

are decreasing in the total price $p_0 + p_2$. From Lemma C.3.1, we have:

- for $a < b$, $p_0^* + p_2^* = p_0^* + \Delta$ is decreasing in a;
- for $a \in [b, b + v_a]$, $p_0^* + p_2^*$ is constant when a varies;
- for $a \in (b + v_a, b + v_a + \Delta]$, $p_0^* + p_2^* = p_0^* + v_a + \Delta$ is decreasing in a as long as the core ZLB does not bind, and constant in a otherwise.

Hence, the welfare objectives are always maximized when the core ZLB binds, i.e. for $a \geq \rho(0)$. However, if $\rho(0) \leq b + v_a$, then the core ZLB always binds in the squeeze region, and so any $a \geq b$ is optimal. ■

[64] If we allow some consumers to have a negative value from joining the platform ($v_c < 0$, because of, e.g., privacy costs, or hassle of installing and learning how to use the platform), then the core ZLB may bind under foreclosure: precisely, it does so whenever $b \geq \rho(-v_a)$. In these cases, as seen above for the cases $b \geq \rho(0) - v_a$, the Pigouvian access charge is welfare-optimal.

C.4 Platform viability and entry

This section details the analysis on platform entry, whose results are summarized in Observation 5.

The socially optimal number of platforms is at most two, because extra platforms beyond $N = 2$ do not alter the consumer surplus and variable profit, but add entry costs and so are necessarily suboptimal if $N > 2$. Given that, with $N = 3$, each entrant makes $\frac{b}{3}$ under the Pigouvian rule, and higher profits under laissez-faire, for all $J \le \frac{b}{3}$ there is always too much entry into the platform segment, which, without monitoring of non-price foreclosure (or a ban of the hybrid platform model), cannot be prevented by access charge regulation.

Under a monopoly platform, the consumers obtain no surplus ($U = 0$). A second entrant increases consumer surplus by v, at the expense of platform total profit, but also entails a socially wasteful entry cost J. Formally, $U + \omega\Pi = v + \omega(b + \Delta - 2J)$ under duopoly and $U + \omega\Pi = \omega(b + v + \Delta - J)$ under monopoly. Hence, a duopoly is preferred to monopoly if and only if $(1-\omega)v \ge \omega J$, or $J \le \frac{1-\omega}{\omega}v$.

Thus, assuming that the access charge is set, by regulation, at the Pigouvian level, for $J \in (\max\{\frac{1-\omega}{\omega}v, \frac{b}{3}\}, \frac{b}{2}]$ there is again too much entry, as two platforms enter but it would be optimal to have one. The region of parameters where excessive entry prevails of course expands when platforms are free to set access charges, as the absence of regulation increases their profits. If, on the contrary, $J \in (\frac{b}{2}, \min\{\frac{1-\omega}{\omega}v, b + v\}]$, then spurring the welfare-maximizing second entry requires setting the access charge above the Pigouvian level. Similarly, if $J \in (b + v, b + v + \Delta]$, there is a potential trade-off between the first platform's viability, which requires a squeeze in the app's profit, and app viability, which calls for staying away from the squeeze region to obtain the proper level of innovation.

C.5 Implementation with competing 3rd party apps

This section details the analysis on implementation of the Pigouvian rule with competing 3rd party apps, whose results are summarized in Observation 7.

Suppose that the platform gives access to a single app. It will do so only with an app that can make money and thus serve the market. Thus, we can without loss of generality focus on proposed access charges $a_1 \in [0, b + v]$ and $a_2 \in [0, b + v + \Delta]$. In the platform pivotality equilibrium, app 1, if selected alone, chooses $p_1 = v$ and makes profit $v + b - a_1$. App 2, if selected alone, chooses $p_2 = v + \Delta$ and makes profit $v + b + \Delta - a_2$. In either case, the consumer surplus from the app is extracted and so $p_0 = 0$. Thus, when foreclosing, the platform is better off foreclosing the lower access charge app.

Next, let us look at what happens when there is no foreclosure. Consider $a_2 \le a_1 + \Delta$, so that, if none of the apps is foreclosed, the superior app takes the app market, which must be the case in equilibrium.[65] The app prices are $p_1 = \max\{a_1 - b, 0\}$ and $p_2 = p_1 + \Delta$. Because the price

[65] An equilibrium where $a_2 > a_1 + \Delta$, and so the inferior app drives the superior one out of the market, cannot

p_2 would be even higher in the absence of app 1, there is no point foreclosing app 1. Does the platform want to foreclose the superior app? A necessary and sufficient condition for this is

$$p_0 + a_2 < a_1 \Longleftrightarrow a_1 - a_2 > v - \max\{a_1 - b, 0\}.$$

The Bertrand equilibrium in access prices has consumers patronizing solely app 2 and the app providers selecting access charges:[66]

$$a_1^* = a_2^* = b + v.$$

Note that, for these specific access charges, the platform is indifferent between selecting an app provider and letting both operate; in all cases the platform selects $p_0 = 0$ and receives $b + v$. If app 2 raises a_2 above $b + v$, it lowers its profit; if it decreases a_2 below $b + v$, it is foreclosed by the platform as $a_1 > a_2$. As for app 1, an increase in a_1 makes it non viable, while a reduction in a_1 keeps its profit at 0.

exist because the superior app would always profitably deviate and match the offer of the inferior one. By proposing $a_2 = a_1$ (say, plus ϵ), the platform has no incentives to foreclose it: The condition for no foreclosure, $a_1 - a_2 \leq v - \max\{a_1 - b, 0\}$ (see below), yields $0 \leq v - \max\{a_1 - b, 0\}$, which holds for all $a_1 \in [0, b + v]$. The superior app then wins consumers and obtains strictly positive profits.

[66] The considered equilibrium is unique when restricting attention to equilibria in undominated strategies.