WORKING PAPERS

N° 1431

April 2023

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James K. Hammitt

Toulouse School of Economics

Consistent Valuation of a Reduction in Mortality Risk using Values per Life, Life Year, and Quality-Adjusted Life Year

James K Hammitt
Harvard University (Center for Risk Analysis & Center for Health Decision Science)
Toulouse School of Economics, University of Toulouse-Capitole
jkh@harvard.edu

April 2023

Abstract

The monetary value of a reduction in mortality risk can be accurately characterized using the alternative concepts of value per statistical life (VSL), value per statistical life year (VSLY), and value per quality-adjusted life year (VQALY). Typically, each of these values depends on the age and other characteristics of the affected individual; at most one of the values can be independent of age. The common practice of valuing a transient or persistent risk reduction using a constant VSL, VSLY, or VQALY yields systematic differences in the calculated monetary value that depend on the age at which the risk reduction begins, its duration, time path, and whether future lives, life years, or quality-adjusted life years are discounted. Mutually consistent, age-dependent VSL, VSLY, and VQALY are derived and the large differences in valuation of illustrative transient and persistent risk reductions that can result from assuming age-independent values of each of the three concepts are illustrated.

Keywords: Value per statistical life, value per statistical life year, quality-adjusted life year, mortality risk.

Acknowledgement: This work was supported by the U.S. National Science Foundation (award number 1824492) and the French National Research Agency (ANR) under the Investments for the Future program (Investissements d'Avenir, grant ANR-17-EURE-0010). I thank Mead Over, Jytte Nielsen, Sue Chilton, two anonymous referees, and the editor, John Wildman, for helpful comments.

1. Introduction

The monetary value of a reduction in mortality risk is often quantified using one of three alternative concepts: the "value per statistical life" (VSL), "value per statistical life year" (VSLY), and "value per quality-adjusted life year" (VQALY). The concept used tends to differ across application domains and there has been debate over which measure is most appropriate. Government guidance often recognizes all three concepts. For example, United States federal-government guidance for evaluating life-saving regulations encourages the use of both VSL and VSLY; it also supports the evaluation of regulations using cost-effectiveness analysis with reductions in health and mortality risks measured in quality-adjusted life years (QALYs) (U.S. OMB 2003). In the United Kingdom, treasury department guidance provides monetary values of VSL, VSLY,[1] and VQALY for use in evaluating government policies (H.M. Treasury 2020). In France, guidance from the office of the prime minister provides reference values for VSL and VSLY and encourages the calculation of cost-effectiveness ratios of euros spent per QALY gained (Commissariat général à la stratégie et la prospective 2013).

In typical applications, when calculating the monetary value of an intervention that reduces mortality risk the same unit value of either VSL, VSLY, or VQALY is used for all individuals whose risk is affected; the unit value does not depend on age, income, or other individual characteristics. As a result, both the calculated monetary value of a risk reduction and the relative monetary values of decreasing risk to different individuals can depend on which concept is used. For example, the calculated value of decreasing current mortality risk by a specified amount is the same for all individuals using a common VSL but is larger for younger than for older individuals using a common VSLY or VQALY (because remaining life expectancy typically decreases with age).

I show here that the monetary value of a reduction in mortality risk can be accurately described using any of the three alternative concepts. However, an individual's VSL, VSLY, and VQALY depend on her life expectancy and other characteristics. In general, each of the three values depends on the individual's age; at most one can be independent of age. Moreover, the appropriate values also depend on the time path of the risk reduction, e.g., whether it is short-term or continuing, whether it increases or decreases over time. Many policies that decrease mortality risk do so over an extended period, often the rest of an individual's lifetime. Yet most of the literature evaluates risk reductions over a relatively short period, such as a year.

[1] In the UK, these concepts are called the "value of a prevented fatality" (VPF) and "value of a life year" (VOLY), respectively.

The value of mortality-risk reduction for an individual is defined as her monetary value (compensating variation) for a specified decrease in her mortality risk. The value of a life-saving program to a population is simply the sum of the affected individuals' monetary values.

For any risk reduction, the appropriate individual VSL, VSLY, and VQALY are equal to the monetary value of the risk reduction divided by the corresponding mortality benefit. Specifically, the monetary value can be described as the product of VSL and "lives saved" (the decrease in the expected number of deaths in a specified period), the product of VSLY and "life years saved" (the increase in life expectancy), and the product of VQALY and "QALYs gained" (the increase in quality-adjusted life expectancy, QALE).[2] For a short-term (instantaneous) risk reduction, the mortality benefit (lives saved, life years saved, or QALYs gained) depends on the risk reduction and, for life years saved and QALYs gained, on life expectancy and QALE conditional on surviving the period, respectively. For a continuing risk reduction, the monetary value can be described using either of two approaches. First, it can be described as the product of an appropriate aggregate VSL, VSLY, or VQALY and the total mortality benefit (lives saved, life years saved, or QALYs gained). Alternatively, the continuing risk reduction can be represented as a series of short-term risk reductions and the monetary value as the expected sum of the discounted monetary values of the short-term risk reductions, where each is valued using age-specific unit values and mortality benefits.

Much of the empirical literature on the monetary value of reducing mortality risk estimates VSL (for recent meta-analyses, see Viscusi and Masterman 2017 and Masterman and Viscusi 2020). Comparatively few studies estimate VSLY or VQALY directly (exceptions include Johannesson and Johansson 1996, 1997, Johannesson et al. 1997, Morris and Hammitt 2001, Desaigues et al. 2011, Robinson et al. 2013, Pennington et al. 2015, Hammitt and Tunçel 2023). VSLY and VQALY are often estimated by dividing an estimate of average VSL by average life expectancy or quality-adjusted life expectancy in the study population (Hirth et al. 2000, Hubbell 2006, Minor et al. 2015, Watts et al. 2021, Robinson et al. 2022); VSLY is also estimated by dividing individual or age-dependent VSL by the corresponding life expectancy (Alberini et al. 2006, Aldy and Viscusi 2007, 2008, Viscusi and Hersch 2008). Mason et al. (2009) estimate VSLY by dividing age-dependent VSL by life expectancy and, alternatively, by interpreting the estimated change in VSL with age as a measure of the value of

[2] Valuing mortality risk using VQALY allows mortality and morbidity risks to be valued in an integrated framework. For this integration to be useful, the individual's preferences over health risks (holding wealth constant) must be consistent with maximizing expected QALYs, which implies her utility function for wealth, health, and longevity can be expressed as $a(w)\ Q + b(w)$ where Q is expected future QALYs and the terms $a(w) > 0$ and $b(w)$ are functions of wealth w (Hammitt 2013). Empirical studies suggest that marginal WTP is a decreasing function of the expected gain in QALYs and that it can depend on whether QALYs are increased through a change in severity or duration (Ryen and Svensson 2015, Hammitt 2017).

the associated decrease in remaining life expectancy (the second approach implies negative VSLY for ages at which estimated VSL increases with age). Calibrated simulation models have also been used to estimate age-dependent VSL, VSLY, and VQALY (e.g., Shepard and Zeckhauser 1984, Rosen 1988, Ng 1992, St-Amour 2022, Sweis 2022).

Estimates of VSL are usually obtained for short-term risk reductions. For example, hedonic-wage studies estimate the annual wage premium paid as compensation for annual occupational risk; many stated-preference studies elicit willingness to pay to reduce mortality risk for one or at most a few years (e.g., Alberini et al. 2004, Hammitt and Haninger 2010).

There is limited evidence about how VSL, VSLY, and VQALY vary with age (Hammitt 2007, Krupnick 2007). Models and empirical studies often suggest that VSL increases then decreases with age, but the magnitudes of the increase and decrease and the age at which VSL peaks are uncertain (e.g., Shepard and Zeckhauser 1984, Ng 1992, Alberini et al. 2004, 2006, Aldy and Viscusi 2007, 2008, Cameron and DeShazo 2013, Ketcham et al. 2022). Because of this uncertainty, it is common practice to value a life-saving policy by assuming that one of these unit values is constant (independent of age and other individual attributes).

I examine the differences in the calculated values of illustrative short-term and continuing risk reductions when they are evaluated using a constant VSL, VSLY, or VQALY. These results assume that VSL at age 40 is well estimated and that VSLY and VQALY are derived to provide consistent valuation of a one-year reduction in mortality risk at age 40. Using realistic inputs, the calculated value of a short-term risk reduction at ages 0 and 80 years can differ from its value at age 40 by a factor as large as three; for a continuing risk reduction the difference can exceed a factor of ten.

The remainder of the paper is organized as follows. Section 2 presents a continuous-time model that describes the monetary value of any change in mortality risk to an individual. The risk change can be summarized as lives saved, life years saved, or QALYs gained and its value can be represented as the product of each of these terms with the corresponding VSL, VSLY, or VQALY. The model reveals how VSL, VSLY, and VQALY depend on the time paths of both the risk reduction and the individual's baseline mortality and health risks. Appendix A provides a complementary presentation of the main results using a simple two-period model. Section 3 presents examples that illustrate: (a) the effect of using a common VSL, VSLY, or VQALY to value either a transient or continuing risk reduction that affects an individual at different ages, and (b) the aggregate VSL, VSLY, and VQALY corresponding to alternative time paths of risk reduction. Conclusions are presented in Section 4. Appendix B provides an example showing that VSL can increase or decrease with life expectancy.

2. Consistent valuation using VSL, VSLY, and VQALY

Consider the valuation of a mortality-risk reduction to an individual. The value to a population is the sum of the affected individuals' values.

An individual's age-dependent mortality risk can be described using any of three functions: her hazard, survival, or probability distribution of age at death. Any two of these can be derived from the third. Let t denote the individual's age and time t denote the corresponding calendar time. The hazard function $h(t)$ is the probability of dying at age t conditional on having survived to that age, the survival function $s(t)$ is the probability of not having died before t, and the marginal probability of age at death $f(t)$ is the unconditional probability of dying at age t.[3] The three functions are illustrated in Figure 1. They are related as follows:

$$s(t) = exp\left[-\int_0^t h(\tau)d\tau\right] = 1 - \int_0^t f(\tau)d\tau \tag{2.1}$$

$$h(t) = \frac{f(t)}{s(t)} = -\frac{d}{dt}log[s(t)\,] \tag{2.2}$$

$$f(t) = h(t) \cdot s(t) = -\frac{d}{dt}s(t). \tag{2.3}$$

It is common in economic evaluation to discount future life years and QALYs to account for time preference. Let $\delta(t)$ be the discount factor for time t. With conventional exponential discounting, $\delta(t) = \exp[-\omega\,(t - x)]$, where ω is the discount rate and x is the time to which the value is discounted. The expected present value of future life years at age x is given by

$$LE(x) = \frac{1}{s(x)\delta(x)}\int_x^\infty s(t)\delta(t)dt. \tag{2.4}$$

Similarly, the expected present value of future QALYs is given by

$$QALE(x) = \frac{1}{s(x)\delta(x)}\int_x^\infty q(t)s(t)\delta(t)dt, \tag{2.5}$$

where $q(t)$ is the expected quality weight (health-related quality of life, HRQL) at age t.[4] Note that life expectancy and quality-adjusted life expectancy are special cases of equations (2.4) and (2.5) obtained with zero discounting, i.e., when $\delta(t) = 1$ for all t. In what follows, I refer to $LE(t)$ and

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[3] Note that the survival function $s(t)$ is one minus the cumulative distribution function for age at death, so $s(t)$ is one minus the cumulative probability of dying before age t as shown in the last expression of equation (2.1). Similarly, the probability density of age at death $f(t)$ is minus one times the derivative of $s(t)$ as shown in the last expression of equation (2.3).

[4] Throughout, I assume $q(t)$ is independent of any changes in mortality hazard.

$QALE(t)$ as life expectancy and quality-adjusted life expectancy with the understanding that any discounting is incorporated.[5]

Consider the monetary value of a "blip," a short-term decrease in mortality risk at age t (Johannesson et al. 1997). The blip decreases the individual's hazard by a small amount, $r(t)$, during the short interval from time t to $t + \mu$. The expected number of lives saved is the reduction in the expected number of deaths in the period,

$$\Delta L = r(t) \cdot \mu. \tag{2.6}$$

The increase in life expectancy or the expected number of life years saved equals the decrease in the expected number of deaths multiplied by life expectancy,

$$\Delta LE = \Delta L \cdot LE(t). \tag{2.7}$$

Similarly, the increase in quality-adjusted life expectancy or the expected number of QALYs gained equals the decrease in the expected number of deaths multiplied by quality-adjusted life expectancy,

$$\Delta QALE = \Delta L \cdot QALE(t). \tag{2.8}$$

Note that lives saved is a measure of gross risk reduction; it does not account for the induced increase in the probability of dying at an older age. Lives saved at age t is defined as the decrease in the expected number of deaths at age t due to a decrease in the hazard at t. In contrast, excess deaths at age t measures the net effect of a risk reduction that occurs at or before t. Excess deaths is defined as the difference between the expected number of deaths occurring at age t without the risk reduction and the expected number with the risk reduction, i.e., the change in $f(t)$ (equation (2.3)). For any specified period, cumulative lives saved is the gross decrease in the expected number of deaths in the period and the decrease in excess deaths is the net decrease. For a continuing risk reduction beginning at age x ($r(t) > 0$ for all $t \geq x$), cumulative lives saved after age x is positive and increases with age (although it cannot exceed one[6]). In contrast, the decrease in excess deaths at age $t \geq x$ is positive for ages close to x but negative for older ages; cumulative excess deaths from x through infinity equals zero. Moreover, the decrease in excess deaths at age $t \geq x$ is less than or

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[5] Jones-Lee at al. (2015) emphasize that the expected present value of longevity described in equation (2.4) is not equal to the present value of a constant stream of length $LE(t)$ (with zero discounting), though the latter expression is often used (incorrectly or as an approximation) in the literature.

[6] From equation (2.15) below, the expected number of lives saved over an individual's lifetime $E(\Delta L) = \int_0^\infty \Delta L(t) s(t) dt$. The risk reduction $\Delta L(t)$ cannot exceed the hazard $h(t)$. Because $\Delta L(t) \leq h(t)$, $E(\Delta L) \leq \int_0^\infty h(t) s(t) dt = \int_0^\infty f(t) dt = 1$, where $f(t)$ is the probability density of age at death (equation (2.3)).

equal to lives saved at t, because the risk reductions at times prior to t increase the probability of surviving to, and being at risk of dying at, age t. In contrast, life years saved and QALYs gained are both measures of the net effect of a risk reduction.

The monetary value v of the momentary risk reduction $r(t)$ can be expressed alternatively as the product of the expected number of lives saved, life years saved, or QALYs gained and the corresponding age-dependent VSL, VSLY, or VQALY, where

$$VSL(t) = \frac{v}{\Delta L}, \tag{2.9}$$

$$VSLY(t) = \frac{v}{\Delta LE} = \frac{VSL(t)\cdot\Delta L}{\Delta L\cdot LE(t)} = \frac{VSL(t)}{LE(t)}, \text{ and} \tag{2.10}$$

$$VQALY(t) = \frac{v}{\Delta QALE} = \frac{VSL(t)\cdot\Delta L}{\Delta L\cdot QALE(t)} = \frac{VSL(t)}{QALE(t)}. \tag{2.11}$$

The penultimate terms in equations (2.10) and (2.11) are obtained by substituting for v in the numerator (using equation (2.9)) and for ΔLE and $\Delta QALE$ in the denominator (using equations (2.7) and (2.8), respectively).

In general, the value v of the momentary risk reduction depends on the individual's lottery over future mortality, health, income, and other factors. For example, VSL at age t can depend on life expectancy conditional on surviving the current period; it can increase or decrease with life expectancy (see Appendix B). The value of a risk reduction at any time before t is simply the expected present value of its monetary value at t (i.e., the value at time $x < t$ equals $\frac{s(t)\delta(t)}{s(x)\delta(x)}$ times its value at time t).

Generalizing to persistent risk reductions, the value of any (small) risk reduction can be described as the expected present value of a stream of short-term risk reductions. Consider the value of a continuing risk reduction $r(t)$ beginning at age x or later (i.e., $r(t) = 0$ for $t < x$). The value of this risk reduction at age x, V, is given by

$$\begin{aligned} V &= \frac{1}{s(x)\delta(x)}\int_x^\infty VSL(t)r(t)s(t)\delta(t)dt \\ &= \frac{1}{s(x)\delta(x)}\int_x^\infty VSL(t)\Delta L(t)s(t)\delta(t)dt \end{aligned} \tag{2.12}$$

where $VSL(t)$ is the individual's age-dependent VSL. In the final expression, $\Delta L(t) = r(t)$ is the expected number of lives saved at age t conditional on survival to then. In words, the value of the risk reduction at time *t*, *VSL*(*t*) · *r*(*t*), is multiplied by the probability the individual is alive to benefit, *s*(*t*), and discounted by the factor $\delta(t)$.

Alternatively, *V* can be expressed as a function of age-dependent *VSLY*(*t*). Multiplying and dividing by *LE*(*t*) inside the first integral in equation (2.12) yields

$$V = \frac{1}{s(x)\delta(x)} \int_x^{\infty} \left[\frac{VSL(t)}{LE(t)} [r(t)LE(t)] \right] s(t)\delta(t)dt$$

$$= \frac{1}{s(x)\delta(x)} \int_x^{\infty} \left[VSLY(t)[\Delta LE(t)] \right] s(t)\delta(t)dt, \tag{2.13}$$

where *VSLY*(*t*) and Δ*LE*(*t*) are defined in equations (2.10) and (2.7), respectively.

Similarly, *V* can be expressed as a function of age-dependent *VQALY*(*t*). Multiplying and dividing by *QALE*(*t*) inside the first integral in equation (2.12) yields

$$V = \frac{1}{s(x)\delta(x)} \int_x^{\infty} \left[\frac{VSL(t)}{QALE(t)} [r(t)QALE(t)] \right] s(t)\delta(t)dt$$

$$= \frac{1}{s(x)\delta(x)} \int_x^{\infty} \left[VQALY(t)[\Delta QALE(t)] \right] s(t)\delta(t)dt, \tag{2.14}$$

where *VQALY*(*t*) and Δ*QALE*(*t*) are defined in equations (2.11) and (2.8), respectively.

Expressions (2.12), (2.13), and (2.14) are alternative methods of representing the value *V* of the continuing risk reduction *r*(*t*) using the concepts of VSL, VSLY, or VQALY. In each case, the value of VSL, VSLY, or VQALY may depend on age *t*. Although it is possible that one of the three values is independent of age, it is not possible for all three to be independent of age. Typically, the mortality hazard *h*(*t*) increases with age and health-related quality of life *q*(*t*) decreases with age, which implies that *LE*(*t*), *QALE*(*t*), and the ratio *QALE*(*t*)/*LE*(*t*) all decrease with age.

The value of a continuing risk reduction can alternatively be valued using an aggregate VSL, VSLY, or VQALY that applies to the total expected lives saved, life years saved, or QALYs gained from a risk reduction. The specific value depends on the time path of the risk reduction. From equation (2.12),

$$V = \frac{1}{s(x)\delta(x)} \int_x^{\infty} VSL(t)\Delta L(t)s(t)\delta(t)dt = \overline{VSL} \frac{1}{s(x)\delta(x)} \int_x^{\infty} \Delta L(t)s(t)\delta(t)dt$$

$$= \overline{VSL} \cdot EPV(\Delta L). \tag{2.15}$$

The total value *V* is the expected present value at time *x* of lives saved (*EPV*(Δ*L*)) multiplied by the corresponding aggregate VSL ($\overline{VSL}$). Similarly, from equation (2.13),

$$V = \frac{1}{s(x)\delta(x)} \int_x^\infty [VSLY(t)\Delta LE(t)]s(t)\delta(t)dt = \overline{VSLY} \frac{1}{s(x)\delta(x)} \int_x^\infty \Delta LE(t)s(t)\delta(t)dt$$

$$= \overline{VSLY} \cdot EPV(\Delta LE). \tag{2.16}$$

The total value is the expected present value at time *x* of life years saved (*EPV*(Δ*LE*)) multiplied by the corresponding aggregate VSLY ($\overline{VSLY}$).

Finally, from equation (2.14),

$$V = \frac{1}{s(x)\delta(x)} \int_x^\infty [VQALY(t)\Delta QALE(t)]s(t)\delta(t)dt = \overline{VQALY} \frac{1}{s(x)\delta(x)} \int_x^\infty \Delta QALE(t)s(t)\delta(t)dt$$

$$= \overline{VQALY} \cdot EPV(\Delta QALE). \tag{2.17}$$

The total value is the expected present value at time *x* of QALYs gained (*EPV*(*ΔQALE*)) multiplied by the corresponding aggregate VQALY ($\overline{VQALY}$).

3. Values of a risk reduction assuming a constant VSL, VSLY, or VQALY

The empirical literature provides many direct estimates of VSL and few direct estimates of VSLY or VQALY. Moreover, there is substantial uncertainty about how VSL, VSLY, and VQALY depend on age, life expectancy, future health, and other factors. As a result, it is common to derive age-independent estimates of VSLY and VQALY by dividing some age-independent estimate of VSL by population-average life expectancy and quality-adjusted life expectancy, respectively. Risk reductions are then valued by multiplying the expected number of lives saved, life years saved, or QALYs gained by a constant VSL, VSLY, or VQALY. The calculated values of a risk reduction so obtained differ, depending on the life expectancy and future health of the affected individuals. In the following subsections, I illustrate the systematic effects of valuing a risk reduction assuming that VSL, VSLY, or VQALY is independent of age, then derive aggregate unit values for illustrative time paths of risk reduction. Although the calculations are based on reasonable input values, my objective is to illustrate the direction and magnitude of the effects of assuming that each of the concepts is independent of age, not to recommend specific values for application.

3.1. Systematic differences in valuation

Applying a constant VSL, VSLY, or VQALY has systematic effects on the calculated values of a risk reduction for people of different ages and life expectancies. I illustrate these effects by calculating the value of alternative short-term and continuing risk reductions for an individual at age 0, 40, and

80 years. Throughout, the values of VSL, VSLY, and VQALY are chosen so that the calculated value of a one-year risk reduction at age 40 years is the same. This assumption is motivated by the fact that the average age of individuals in most studies that estimate VSL is around 40 years and so VSL is estimated more accurately at age 40 than at much younger and older ages. The calculations are from the perspective of an individual who is alive when the risk reduction begins.

For the calculations, the hazard function (illustrated in Figure 1) is for American females[7] and the age-dependent expected health-related quality of life is for American females using the SF-6D (Hanmer et al. 2016).[8] Life expectancy and quality-adjusted life expectancy are reported in Table 1 for ages 0, 40, and 80 years, with and without discounting. The discount rate is 3 percent per year.[9] Table 2 reports unit values of VSL, VSLY, and VQALY. VSL is set equal to $10 million, roughly consistent with current U.S. values (Masterman and Viscusi 2020). Unit values of VSLY and VQALY are obtained by dividing VSL by life expectancy or quality-adjusted life expectancy, using a discount rate of either zero or 3 percent per year, to produce the same value of a one-year risk reduction at age 40 using each concept. For example, VSLY is calculated as VSL ($10 million) divided by life expectancy at age 40 (42.52 years without discounting, 23.09 with discounting), which yields $235,000 (without discounting) and $433,000 (with discounting).

Table 3 reports the value of a one-year risk reduction of 5/100,000. Using a VSL of $10 million, the value of the risk reduction is $500, independent of age and discounting. Using VSLY, the value of a one-year risk reduction at age 40 is the same as the value calculated using VSL, by construction. In contrast, the value of a one-year risk reduction is larger at younger ages and smaller at older ages. Compared with the value at age 40, the value of a one-year risk reduction at age 0 is 1.91 times larger and the value at age 80 is 0.23 times as large. When life years are discounted, the effect of age on value is diminished; the values of one-year risk reductions at ages 0 and 80 are 1.29 and 0.34 times the value of the risk reduction at age 40, respectively. Using VQALY (with or without discounting), results are similar to using VSLY except the value of a risk reduction is larger at age 0,

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[7] The hazard is for the U.S. Social Security area population as used in the 2022 Trustees Report; it reports annual hazards from birth through age 120 (the horizon used for all calculations in this paper). Available at https://www.ssa.gov/oact/STATS/table4c6.html.

[8] Hanmer et al. (2016) report average HRQL by 10 year age group between ages 20 and 89. I use their values for all ages within an age group and assume health-related quality of life for ages 0-19 equals its value for ages 20-29 and for ages 90 and older equals its value for ages 80-89.

[9] An annual discount rate of 3 percent is recommended by widely cited guidance for cost-effectiveness analysis (Gold et al. 1996, Neumann et al. 2016) and (together with a rate of 7 percent) is specified in U.S. guidance for benefit-cost analysis of federal regulations (U.S. OMB 2003). Using a different (positive) rate would not qualitatively affect the results but using a larger or smaller discount rate would respectively increase or decrease the difference between the discounted and undiscounted results.

and smaller at age 80, because expected health-related quality of life decreases with age. As with VSLY, discounting moderates the effect of age on the value of a one-year risk reduction.

The value of a continuing risk reduction depends on how it changes with age. To illustrate, consider two examples: an additive risk reduction, in which the annual hazard is decreased by subtracting a constant in each year, and a proportional risk reduction, in which the annual hazard is decreased by subtracting a constant fraction of the baseline hazard in each year. (The constants are chosen so the value of each continuing risk reduction beginning at age 40, without discounting, is the same as the value of a one-year risk reduction at age 40, in each case calculated using VSL.[10]) The proportional risk reduction might be produced by a decrease in a hazard that increases with age such as some types of cardiovascular disease and cancer. The additive risk reduction might be produced by a decrease in a hazard that is approximately independent of age, perhaps transportation accidents or domestic fires.

Because the annual hazard increases sharply with age, these continuing risk reductions produce different time paths of risk reduction. Figure 2 illustrates the risk reductions as a function of age and Figure 3 illustrates the expected number of lives saved by a continuing risk reduction beginning at age 0 as a function of time.[11] For the additive risk reduction, lives saved grows nearly linearly with age until about age 80, after which it grows at a rapidly decreasing rate because the probability of surviving beyond about age 80 decreases sharply. In contrast, for the proportional risk reduction lives saved remain small until about age 50, because the baseline hazard, and hence the risk reduction, are small at younger ages. The number of lives saved increases at an increasing rate from about age 50 to age 90, after which the rate of increase declines and lives saved plateaus near age 100 because the probability of surviving to experience these risk reductions is small.

Table 4 reports the value of the continuing additive risk reduction depending on whether it begins at age 0, 40, or 80. Using VSL, the values beginning at ages 0 and 80 are 1.91 and 0.23 times as large as the value if it begins at age 40. These ratios are the same as for the value of a one-year risk reduction evaluated using a constant VSLY (Table 3) because the expected number of lives saved with the continuing additive risk reduction is proportional to life expectancy (the expected number of years over which the individual survives to experience the risk reduction). When lives saved are discounted, the values are smaller (because fewer discounted lives are saved) but the effect of the

[10] For the additive risk reduction, the annual hazard is decreased by subtracting 1.175989e-6. For the proportional risk reduction, the annual hazard is multiplied by 1 – 4.761802e-5.

[11] Cumulative lives saved by an intervention beginning at age $x > 0$ can be calculated from Figure 3. Subtracting cumulative lives saved at x yields cumulative lives saved from the perspective of a newborn. Dividing the result by $s(x)$ yields cumulative lives saved from the perspective of an individual at age x.

starting age is parallel to the effect of age for the one-year risk reduction using VSLY for discounted life years, for the same reason.

Using VSLY, the value of a continuing additive risk reduction is much larger if it begins at younger than at older ages. The ratios of the values of the continuing additive risk reduction beginning at age 0 and at age 80 are 3.50 and 0.062 times as large, respectively, as the value if it begins at age 40. This follows because the life years gained by decreasing risk are much greater at younger than at older ages. Discounting life years again moderates the effect of beginning the risk reduction at younger or older ages. Using VQALY, results are similar to using VSLY except the effects of the age at which the risk reduction begins are somewhat larger, again because years lived at younger ages produce more QALYS than those lived at older ages.

For the continuing proportional risk reduction, the results differ substantially. As shown in Table 5, using VSL the value of a continuing risk reduction is almost unaffected by the age at which it begins. The expected number of lives saved is only slightly decreased by postponing the start of the risk reduction from age 40 to 80 and it is increased by postponing the start of the risk reduction from age 0 to 40.[12] If lives saved are discounted, however, the value of the continuing proportional risk reduction is much larger if it begins at older rather than younger ages; most of the lives saved occur at older ages and their value is diminished by discounting from the age at which the risk reduction begins. With discounting, the ratios of the value of the continuing proportional risk reduction beginning at ages 0 and 80 are 0.33 and 2.44 times as large as the value if it begins at age 40.

Using VSLY, the value of the continuing proportional risk reduction is larger if it begins at younger rather than older ages. The effect of beginning the risk reduction at age 0 rather than 40 is modest (a 9 percent increase) but the effect of beginning at age 80 rather than 40 is proportionally larger (it decreases the value by almost half). This difference reflects the larger proportional decrease in life expectancy with age at older ages. Discounting life years reverses this pattern: the value of a continuing proportional risk reduction is larger if it begins at older ages than younger; compared with beginning at age 40, the value of the risk reduction is 0.38 and 1.24 times as large if it begins at ages 0 and 80, respectively. Because most of the life years saved are saved at older ages, discounting from a younger starting age has a large effect on the total value. Using VQALY, results are similar to using VSLY. Because expected health-related quality of life decreases with age, the value of beginning the continuing proportional risk reduction is slightly increased if it begins at younger ages

[12] Recall that the calculations are from the perspective of an individual alive when the risk reduction begins. For an individual at any age, postponing the start of a continuing risk reduction cannot increase the expected number of lives saved. (See footnote 11.)

and slightly decreased if it begins at older ages (relative to beginning at age 40), compared with the values calculated using a constant VSLY.

3.2. Aggregate unit values for alternative risk reductions

As described in Section 2, the value of any specified risk reduction can be accurately characterized using an aggregate VSL, VSLY, or VQALY. These aggregate values depend on how the risk reduction is distributed over time. In this subsection I present aggregate values for the one-year, additive, and proportional risk reductions beginning at ages 0, 40, and 80 years of age.

First, consider the case where the value of a risk reduction is consistent with an age-independent VSL of $10 million. Table 6 reports the corresponding aggregate VSL, VSLY, and VQALY for the one-year, additive, and proportional risk reductions beginning at ages 0, 40, and 80 years of age. For each of the risk reductions included in the table, the corresponding aggregate VSL is exactly the VSL used to value that reduction, $10 million.

For the one-year risk reductions beginning at ages 0, 40, and 80 years, the corresponding aggregate VSLY equals $123,000, $235,000, and $1,036,000, respectively. These values are equal to the VSL ($10 million) divided by life expectancy at ages 0, 40, and 80 years. Similarly, the aggregate VQALY for the one-year risk reduction beginning at ages 0, 40, and 80 equals $160,000, $315,000, and $1,485,000, equal to VSL divided by QALE at each age. For the additive continuing risk reduction, the aggregate VSLY equals $241,000, $442,00, and $1,611,000 for risk reductions beginning at ages 0, 40, and 80 years, respectively. The aggregate VQALY for these risk reductions are larger than the aggregate VSLY, with the proportionate difference increasing with age as the expected health-related quality of life is less than one and decreases with age. For the proportional continuing risk reduction, the aggregate VSLY and VQALY are larger than for the additive risk reduction because the proportional risk reduction saves fewer life years and QALYs than does the additive risk reduction.

Table 7 presents analogous results for the case where each risk reduction is correctly valued using a constant VSLY of $235,000. Accordingly, the corresponding aggregate VSLY for all of the risk reductions equals $235,000. For a one-year risk reduction, the corresponding aggregate VSL is equal to the VSLY multiplied by the expected number of life years gained by the one-year risk reduction at each age. The aggregate VSL for the continuing risk reductions are smaller than for the one-year risk reductions because the continuing risk reductions save some lives at older ages, with smaller life expectancies, and so the number of life years gained per life saved is smaller than for the one-year risk reductions. Similarly, the aggregate VSL for the proportional risk reduction is smaller than for the additive risk reduction because the proportional risk reduction disproportionately saves lives at older ages, with smaller life expectancies. Finally, the aggregate VQALY is modestly larger than the VSLY for all the risk reductions, because the expected number of QALYs gained is modestly smaller than the number of life years saved in each case.

4. Conclusion

The value of a reduction in mortality risk, whether transient or persistent, can be accurately expressed using the concepts of VSL, VSLY, or VQALY. However, the appropriate unit value for each concept depends on the individual's age, the details of the risk reduction (including its duration and time path), and on the individual's future lottery on mortality, health, income, and other factors. Because life expectancy and health-related quality of life tend to decrease with age, at most one of the three unit values can be independent of age; it is likely that none of the three are independent of age for any individual.

Given that there are few direct estimates of VSLY or VQALY and substantial uncertainty about how VSL, VSLY, and VQALY depend on age, it is common practice to evaluate life-saving programs by assuming that one of these values is independent of age and to derive unit values of VSLY and VQALY by dividing an estimate of VSL by average life expectancy or quality-adjusted life expectancy in the population. This practice leads to systematic and sometimes large differences in the calculated value of a risk reduction depending on its duration, time path, and the individual's age at which it begins. The illustrative calculations in Section 3 suggest that the relative value of a short-term risk reduction at very young or old ages (0 and 80 years) can differ from its value at middle age (40 years) by a factor as large as three (Table 3); for a continuing additive risk reduction the difference can exceed a factor of 10 (Table 4). The proportional differences are generally larger when applying an age-40 value to older than to younger individuals. Values calculated using an age-independent VSLY or VQALY are typically larger for risk reductions that begin at younger ages, although if the risk reduction increases with baseline risk its value can be larger for risks that begin at older ages, especially if future effects are discounted.

Regardless of whether VSL, VSLY, or VQALY is chosen, accurate valuation of risk reduction is likely to require using age-dependent values. Using a value that is independent of age is simpler, easier to communicate, and likely to be more readily accepted by decision makers and the public. If an age-independent value is to be applied, it would be useful to compare the induced errors and biases in valuation, which depend on the ages of the affected individuals and the time paths of continuing risk reductions. If VSL first rises then falls over the lifecycle (consistent with much of the literature), an age-independent VSL will overestimate the value of risk reductions at young and old ages and underestimate the value at middle ages. In contrast, an age-independent VSLY or VQALY may more-accurately represent the value of risk reduction at older ages but will increase the overestimation at younger ages. If age-dependent values are applied, the calculation using VSL appears more straightforward but the identical result can be obtained using age-dependent values of VSLY and VQALY.

Appendix A. Consistent valuation illustrated with a two-period model

This appendix illustrates the valuation of short-term and continuing risk reductions in a two-period model. The results parallel those of the continuous-time model presented in Section 2; this complementary perspective may provide clearer understanding.

A.1. Two-period model

Assume an individual can live for at most two periods, each having duration 1. If the individual is alive at the beginning of period t, her probability of surviving to the end of the period is $p_t = 1 - h_t$, where h_t is her mortality hazard in period t. The individual experiences the utility of living in period t, $u(c_t)$, if she survives the period and zero otherwise (assume death can occur only at the beginning of a period). Utility of consumption $u(c_t)$ is strictly increasing and concave, i.e., $u' > 0$ and $u'' < 0$ (where single and double primes denote first and second derivatives, respectively). Consumption in period t equals c_t. Assume the individual optimizes consumption subject to her budget constraint or, alternatively, that consumption in each period is exogenous (i.e., she cannot borrow or save to shift consumption between periods). To simplify notation, let $u_t = u(c_t)$. The individual discounts future utility by the discount factor $\delta > 0$.

Expected discounted utility at the beginning of period 1 ("time 0") equals

$$U = p_1 u_1 + \delta p_1 p_2 u_2. \tag{A.1}$$

At the beginning of period t, the expected discounted value of future longevity ("discounted life expectancy") is

$$LE_1 = p_1 (1 + \delta p_2) \tag{A.2}$$

and

$$LE_2 = p_2. \tag{A.3}$$

Let $r_t > 0$ denote a small decrease in period t hazard and ΔL_{st} denote the expected decrease in the discounted number of deaths ("lives saved") in period t as viewed from the beginning of period s. Then

$$\Delta L_{11} = r_1, \tag{A.4}$$

$$\Delta L_{22} = r_2, \tag{A.5}$$

and

$$\Delta L_{12} = r_2 \delta p_1. \tag{A.6}$$

Note that delaying the start of a continuing risk reduction cannot increase the expected number of lives saved from the perspective of time 0 but can increase lives saved from the perspective of the time the risk reduction begins (as illustrated in Table 5 by the increase in value using a constant VSL from postponing the start of the proportional risk reduction from age 0 to 40). Cumulative lives saved viewed from time 0 equals $r_1 + \delta p_1 r_2$. Viewed from the start of period 2, cumulative lives saved equals r_2, which can be larger than lives saved viewed from time 0 if $r_1 < r_2 (1 - \delta p_1)$, i.e., if r_2 is sufficiently large and r_1, δ, and p_1 are sufficiently small.

Let ΔLE_{st} denote the increase in discounted life expectancy at the beginning of period *s* due to the decrease in period *t* risk, r_t. Then[13]

$$\Delta LE_{11} = r_1 \frac{\partial LE_1}{\partial p_1} = r_1(1 + \delta p_2), \tag{A.7}$$

$$\Delta LE_{22} = r_2 \frac{\partial LE_2}{\partial p_2} = r_2, \tag{A.8}$$

and

$$\Delta LE_{12} = r_2 \frac{\partial LE_1}{\partial p_2} = r_2 \delta p_1. \tag{A.9}$$

Finally, let q_t be health-related quality of life in period *t* and assume $0 \le q_t \le 1$.[14] At the beginning of period *t*, expected future discounted QALYs (discounted quality-adjusted life expectancy QALE) is given by

$$QALE_1 = p_1 (q_1 + \delta p_2 q_2) \tag{A.10}$$

and

[13] ΔLE_{11} can be expressed as $r_1 + r_1 \delta p_2$. Nielsen et al. (2010) describe the first term as a safety effect (that increases the probability of surviving the current period) and the second term as a survival effect (that increases the probability of surviving later periods). In the continuous-time model the safety effect is not explicitly represented in equation (2.7) because the duration of the period is zero.

[14] Values of health-related quality of life less than zero are excluded for simplicity.

$$QALE_2 = p_2\, q_2. \tag{A.11}$$

Let $\Delta QALE_{st}$ denote the increase in QALE at the beginning of period *s* due to the risk reduction in period *t*, r_t. Then

$$\Delta QALE_{11} = r_1 \frac{\partial QALE_1}{\partial p_1} = r_1(q_1 + \delta p_2 q_2), \tag{A.12}$$

$$\Delta QALE_{22} = r_2 \frac{\partial QALE_2}{\partial p_2} = r_2 q_2, \tag{A.13}$$

and

$$\Delta QALE_{12} = r_2 \frac{\partial QALE_1}{\partial p_2} = r_2 \delta p_1 q_2. \tag{A.14}$$

A.2. Consistent valuation using VSL, VSLY, or VQALY

The monetary value in period t of a specified risk reduction is the individual's compensating variation, i.e., the decrease in c_t that compensates for the decrease in risk, holding expected lifetime utility constant. For small risk reductions, the value is approximately equal to the product of the risk reduction and the individual's marginal rate of substitution of c_t for the risk reduction. This value can be represented alternatively using VSL, VSLY, or VQALY as shown in the following subsections.

A.2.1. VSL

Define VSL_{st} as the marginal rate of substitution of period *s* consumption for a reduction in period *t* hazard:

$$VSL_{11} = -\frac{dc_1}{dp_1} = \frac{\frac{\partial U}{\partial p_1}}{\frac{\partial U}{\partial c_1}} = \frac{u_1 + \delta p_2 u_2}{p_1 u_1{}'}, \tag{A.15}$$

$$VSL_{22} = -\frac{dc_2}{dp_2} = \frac{\frac{\partial U}{\partial p_2}}{\frac{\partial U}{\partial c_2}} = \frac{u_2}{p_2 u_2{}'}, \tag{A.16}$$

and

$$VSL_{12} = -\frac{dc_1}{dp_2} = \frac{\frac{\partial U}{\partial p_2}}{\frac{\partial U}{\partial c_1}} = \frac{\delta u_2}{u_1{}'}. \tag{A.17}$$

At time 0, the value V_{1t} of a marginal decrease in period *t* hazard r_t is given by

$$V_{11} = \Delta L_{11} \cdot VSL_{11} \tag{A.18}$$

and

$V_{12} = \Delta L_{12} \cdot VSL_{12}$. (A.19)

Hence the value at time 0 of a continuing risk reduction (r_1, r_2) is

$$V_{11} + V_{12} = r_1 \frac{u_1 + \delta p_2 u_2}{p_1 u_1'} + r_2 \frac{\delta u_2}{u_1'} = \frac{r_1 u_1 + \delta(r_1 p_2 + r_2 p_1) u_2}{p_1 u_1'}. \tag{A.20}$$

This value can be described using an aggregate VSL, obtained by dividing the value of the risk reduction (V_{11} + V_{22}) by the expected discounted number of lives saved in periods 1 and 2 (ΔL_{11} + ΔL_{12}), i.e.,

$$\overline{VSL} = \frac{V_{11} + V_{12}}{\Delta L_{11} + \Delta L_{12}} = \frac{r_1 u_1 + \delta(r_1 p_2 + r_2 p_1) u_2}{p_1 u_1' [r_1 + \delta p_1 r_2]}. \tag{A.21}$$

Clearly $\overline{VSL}$ depends on the relative magnitudes of the risk reductions r_1 and r_2 as well as the baseline survival probabilities p_1 and p_2.

A.2.2. VSLY

Define $VSLY_{st}$ as the marginal rate of substitution of period *s* consumption for an increase in discounted life expectancy at the beginning of period *s*, where the increase in life expectancy is due to a decrease in period *t* hazard:

$$VSLY_{st} = -\frac{dc_s}{dLE_t} = \frac{-\frac{dc_s}{dp_t}}{\frac{\partial LE_s}{\partial p_t}} = \frac{VSL_{st}}{\frac{\partial LE_s}{\partial p_t}}. \tag{A.22}$$

Hence

$$VSLY_{11} = \frac{VSL_{11}}{\frac{\partial LE_1}{\partial p_1}} = \frac{u_1 + \delta p_2 u_2}{p_1 u_1' (1 + \delta p_2)}, \tag{A.23}$$

$$VSLY_{22} = \frac{VSL_{22}}{\frac{\partial LE_2}{\partial p_2}} = \frac{u_2}{p_2 u_2'}, \tag{A.24}$$

and

$$VSLY_{12} = \frac{VSL_{12}}{\frac{\partial LE_1}{\partial p_2}} = \frac{u_2}{p_1 u_1'}. \tag{A.25}$$

Note that $VSLY_{st}$ equals VSL_{st} divided by life expectancy at the beginning of period *s* conditional on surviving period *s*.[15]

[15] Recall that death can occur only at the beginning of a period, so decreasing mortality risk in period *t* increases the chance of surviving period *t* and all later periods.

At time 0, the value Y_{1t} of a marginal increase in life expectancy ΔLE_{1t} due to a decrease in period t hazard is the product of the increase in life expectancy and the corresponding VSLY,

$$Y_{11} = \Delta LE_{11} \cdot VSLY_{11} = r_1(1+\delta p_2)\frac{u_1+\delta p_2 u_2}{p_1 u_1'(1+\delta p_2)} = r_1 \frac{u_1+\delta p_2 u_2}{p_1 u_1'} \tag{A.26}$$

$$Y_{12} = \Delta LE_{12} \cdot VSLY_{12} = r_2 \delta p_1 \frac{u_2}{p_1 u_1'} = r_2 \frac{\delta u_2}{u_1'}\,. \tag{A.27}$$

Note that these values are identical to the values calculated using VSL, i.e., $Y_{11} = V_{11}$ and $Y_{12} = V_{12}$. From this, it is clear that the value of a continuing risk reduction calculated using VSLY, $Y_{11} + Y_{12}$, equals the value calculated using VSL, $V_{11} + V_{12}$.

Aggregate VSLY for the continuing risk reduction can be defined by dividing the value of the risk reduction ($Y_{11} + Y_{22}$) by the expected increase in discounted life expectancy ($\Delta LE_{11} + \Delta LE_{12}$), i.e.,

$$\overline{VSLY} = \frac{Y_{11}+Y_{12}}{\Delta LE_{11}+\Delta LE_{12}} = \frac{r_1 u_1 + \delta(r_1 p_2 + r_2 p_1)u_2}{p_1 u_1'[r_1 + \delta(r_1 p_2 + r_2 p_1)]}. \tag{A.28}$$

A.2.3. VQALY

Define $VQALY_{st}$ as the marginal rate of substitution of period s consumption for an increase in discounted quality-adjusted life expectancy at the beginning of period s due to a decrease in period t hazard:

$$VQALY_{st} = -\frac{dc_s}{dQALE_t} = \frac{-\frac{dc_s}{dp_t}}{\frac{\partial QALE_s}{dp_t}} = \frac{VSL_{st}}{\frac{\partial QALE_s}{dp_t}}. \tag{A.29}$$

Then

$$VQALY_{11} = \frac{VSL_{11}}{\frac{\partial QALE_1}{dp_1}} = \frac{u_1+\delta p_2 u_2}{p_1 u_1'(q_1+\delta p_2 q_2)}, \tag{A.30}$$

$$VQALY_{22} = \frac{VSL_{22}}{\frac{\partial QALE_2}{dp_2}} = \frac{u_2}{p_2 q_2 u_2'}, \tag{A.31}$$

and

$$VQALY_{12} = \frac{VSL_{12}}{\frac{\partial QALE_1}{dp_2}} = \frac{u_2}{p_1 q_2 u_1'}. \tag{A.32}$$

Parallel to VSLY, $VQALY_{st}$ equals VSL_{st} divided by expected future QALYs conditional on surviving period s.

At time 0, the value Z_{1t} of a marginal increase in QALE due to a decrease in period t hazard is the product of the increase in life expectancy and the corresponding VQALY,

$$Z_{11} = \Delta QALE_{11} \cdot VQALY_{11} = r_1(q_1 + \delta p_2 q_2)\frac{u_1+\delta p_2 u_2}{p_1 u_1{}'(q_1+\delta p_2 q_2)} = r_1 \frac{u_1+\delta p_2 u_2}{p_1 u_1{}'} \tag{A.33}$$

$$Z_{12} = \Delta QALE_{12} \cdot VQALY_{12} = r_2 \delta p_1 q_2 \frac{u_2}{p_1 q_2 u_1{}'} = r_2 \frac{\delta u_2}{u_1{}'}. \tag{A.34}$$

These values are identical to the values calculated using either VSL or VSLY, i.e., $Z_{11} = Y_{11} = V_{11}$ and $Z_{12} = Y_{12} = V_{12}$.

The aggregate VQALY for the continuing risk reduction is the ratio of the value of the risk reduction ($Z_{11} + Z_{12}$) to the total increase in discounted QALE ($\Delta QALE_{11} + \Delta QALE_{12}$), i.e.,

$$\overline{VQALY} = \frac{Z_{11}+Z_{12}}{\Delta QALE_{11}+\Delta QALE_{12}} = \frac{r_1 u_1+\delta(r_1 p_2+r_2 p_1)u_2}{p_1 u_1{}'[r_1 q_1+\delta(r_1 p_2+r_2 p_1)q_2]}. \tag{A.35}$$

As with the aggregate VSL and VSLY, $\overline{VQALY}$ depends on the magnitudes of the risk reductions and of the baseline survival probabilities in the two periods.

Appendix B. VSL as a function of life expectancy

VSL at any age depends on the individual's lottery over future longevity, income, health, and other factors. In general, the relationship between VSL and life expectancy is ambiguous.

Consider a simple two-period model. The individual seeks to maximize expected utility

$$U = p_1\, u_1(c_1) + p_1\, p_2\, u_2(c_2) \tag{B.1}$$

where p_t is the probability of surviving period t conditional on being alive at the start of the period. The individual gains utility from consumption $u_t(c_t)$ if and only if she survives period t. The period utility functions are strictly increasing and concave, i.e., $u' > 0$ and $u'' < 0$ (where single and double primes denote first and second derivatives, respectively).

If the duration of each period is 1, life expectancy at the start of the first period is $p_1\,(1 + p_2)$ and, conditional on surviving the first period, life expectancy at the start of the second period is p_2. Consumption is subject to the budget constraint

$$c_1 + \frac{p_2}{1+r} c_2 = y \tag{B.2}$$

where y is wealth (the expected present value of lifetime income) and r is the interest rate at which she can borrow or save. The budget constraint assumes that fair annuities are available so it is more stringent when p_2 is larger.

Optimal consumption requires that the marginal utility of first-period consumption equals the expected present value of the marginal utility of second-period consumption,

$$u'_1(c_1^*) = (1+r)u'_2(c_2^*), \tag{B.3}$$

where c_t^* is optimal consumption in period t.

VSL at the beginning of the first period is given by

$$VSL = -\frac{dy}{dp_1} = \frac{\frac{\partial U}{\partial p_1}}{\frac{\partial U}{\partial y}} = \frac{u_1(c_1^*)+p_2u_2(c_2^*)}{p_1\left[u'_1(c_1^*)\frac{dc_1^*}{dy}+p_2u'_2(c_2^*)\frac{dc_2^*}{dy}\right]}. \tag{B.4}$$

Life expectancy at the beginning of the first period is increasing in both p_1 and p_2. Yet an increase in p_1 decreases VSL because it does not affect the numerator of equation (B.4) but increases the denominator (the "dead-anyway effect," Pratt and Zeckhauser 1996). In contrast, an increase in p_2, which equals life expectancy conditional on surviving the first period, can increase or decrease VSL at the start of the first period. An increase in p_2 increases both the numerator and the denominator of equation (B.4) and the effect on the ratio is ambiguous.

As an example, let the interest rate r = 0 and $u_t(c_t) = \sqrt{c_t} + z_t$, where z_t can be interpreted as the component of the utility of living in period t that is independent of consumption (assume $z_t \geq 0$ for t = 1, 2). Consider the limiting cases p_2 = 0 and p_2 = 1. With p_2 = 0, life expectancy equals p_1, c_1^* = y and VSL(LE=p_1) = $\frac{2y}{p_1} + \frac{2\sqrt{y}}{p_1}z_1$. With p_2 = 1, life expectancy equals 2p_1, c_1^* = c_2^* = y/2 and VSL(LE=2p_1) = $\frac{2y}{p_1} + \frac{2\sqrt{y/2}}{p_1}(z_1 + z_2)$. The relative magnitude of VSL(LE=p_1) and VSL(LE=2p_1) depends on z_1 and z_2. Specifically, VSL(LE=2p_1) – VSL(LE=p_1) = $\frac{2\sqrt{y/2}}{p_1}\left[\left(1-\sqrt{2}\right)z_1 + z_2\right]$. If z_1 = 0 and z_2 > 0, VSL is larger when life expectancy equals 2p_1 than when it equals p_1. Conversely, if z_1 > 0 and z_2 = 0, VSL is larger when life expectancy equals p_1 than when it equals 2p_1. More generally, the difference in VSL is a decreasing function of z_1 and an increasing function of z_2. The increase as z_2 increases is obvious because the individual cannot experience z_2 if p_2 = 0 (and life expectancy equals p_1). The decrease as z_1 increases follows because the opportunity cost of spending (to increase the chance of surviving the first period to experience z_1) is larger when p_2 > 0 (and life expectancy exceeds p_1).

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Table 1. Life expectancy and quality-adjusted life expectancy by age (years)

	Discounted	Age 0	Age 40	Age 80
LE	No	81.03	42.52	9.65
LE	Yes	29.80	23.09	7.88
QALE	No	62.66	31.71	6.74
QALE	Yes	23.63	17.41	5.50

Note: Discount rate = 3 percent per year.

Table 2. Unit values of VSL, VSLY, and VQALY ($)

Measure	Future effects: Not discounted	Future effects: Discounted
VSL	10,000,000	10,000,000
VSLY	235,000	433,000
VQALY	315,000	574,000

Note: Unit values are normalized to yield the same value of a one-year risk reduction at age 40.

Table 3. Value of one-year risk reduction by age ($)

Measure	Discounted	Age 0	Age 40	Age 80
VSL	No	500	500	500
		(1)	(1)	(1)
VSL	Yes	500	500	500
		(1)	(1)	(1)
VSLY	No	953	500	113
		(1.91)	(1)	(0.23)
VSLY	Yes	645	500	171
		(1.29)	(1)	(0.34)
VQALY	No	988	500	106
		(1.98)	(1)	(0.21)
VQALY	Yes	679	500	158
		(1.36)	(1)	(0.32)

Note: Values in parentheses are ratios of value at specified age to value at age 40 in the same row. Risk reduction = 5e-5.

Table 4. Value of continuing additive risk reduction by age at start ($)

		Age at start		
Measure	Discounted	0	40	80
VSL	No	953	500	113
		(1.91)	(1)	(0.23)
VSL	Yes	350	272	93
		(1.29)	(1)	(0.34)
VSLY	No	931	266	17
		(3.50)	(1)	(0.062)
VSLY	Yes	382	199	22
		(1.92)	(1)	(0.11)
VQALY	No	940	261	16
		(3.60)	(1)	(0.059)
VQALY	Yes	392	195	20
		(2.01)	(1)	(0.10)

Note: Values in parentheses are ratios of value at specified age to value at age 40 in the same row. Risk reduction = 1.175989e-6.

Table 5. Value of continuing proportional risk reduction by age at start ($)

		Age at start		
Measure	Discounted	0	40	80
VSL	No	498	500	491
		(0.995)	(1)	(0.98)
VSL	Yes	50	151	368
		(0.33)	(1)	(2.44)
VSLY	No	120	110	59
		(1.09)	(1)	(0.53)
VSLY	Yes	23	60	74
		(0.38)	(1)	(1.24)
VQALY	No	116	106	55
		(1.10)	(1)	(0.52)
VQALY	Yes	22	58	69
		(0.39)	(1)	(1.20)

Note: Values in parentheses are ratios of value at specified age to value at age 40 in the same row. Risk reduction = hazard * 4.761802e-05.

Table 6. Aggregate unit values for constant VSL ($1,000)

		Age		
Measure	Risk reduction	0	40	80
VSL	One-year	10,000	10,000	10,000
VSL	Additive	10,000	10,000	10,000
VSL	Proportional	10,000	10,000	10,000
VSLY	One-year	123	235	1,036
VSLY	Additive	241	442	1,611
VSLY	Proportional	973	1,065	1,958
VQALY	One-year	160	315	1,485
VQALY	Additive	320	604	2,307
VQALY	Proportional	1,356	1,492	2,806

Note: Table shows aggregate unit value that yields same total value as constant VSL = $10,000,000.

Table 7. Aggregate unit values for constant VSLY ($1,000)

		Age		
Measure	Risk reduction	0	40	80
VSL	One-year	19,060	10,000	2,270
VSL	Additive	9,768	5,317	1,460
VSL	Proportional	2,416	2,209	1,201
VSLY	One-year	235	235	235
VSLY	Additive	235	235	235
VSLY	Proportional	235	235	235
VQALY	One-year	304	315	337
VQALY	Additive	312	321	337
VQALY	Proportional	328	330	337

Note: Table shows aggregate unit value that yields same total value as constant VSLY = $235,000. The exact values calculated using VQALY for risk reductions beginning at age 80 are not identical.

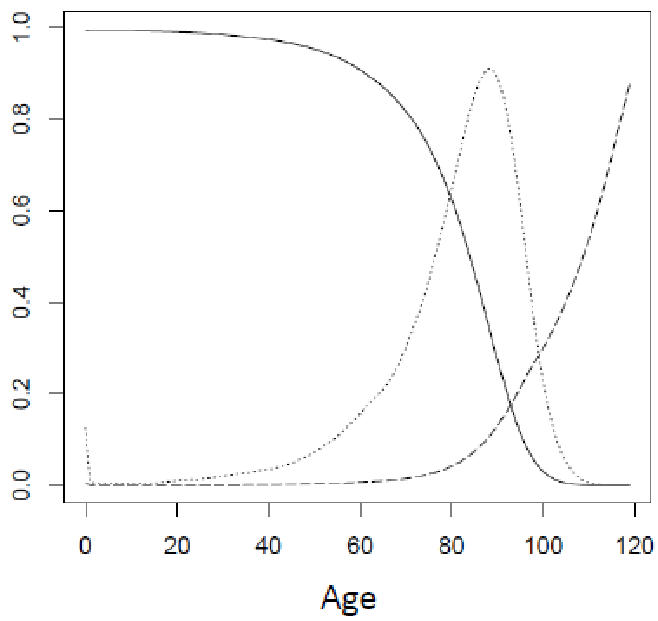

Figure 1. Survival, hazard, and probability density of age at death. Solid line = survival, dashed line = hazard, dotted line = probability density (rescaled: multiplied by 25).

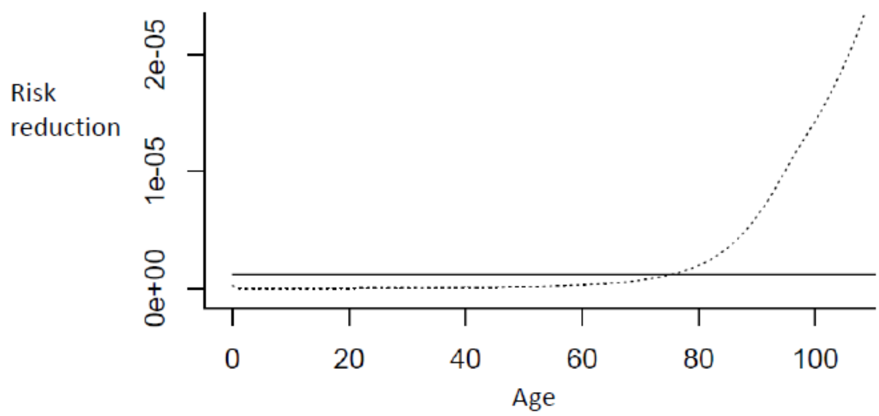

Figure 2. Risk reduction by age. Solid line = additive risk reduction, dotted line = proportional risk reduction.

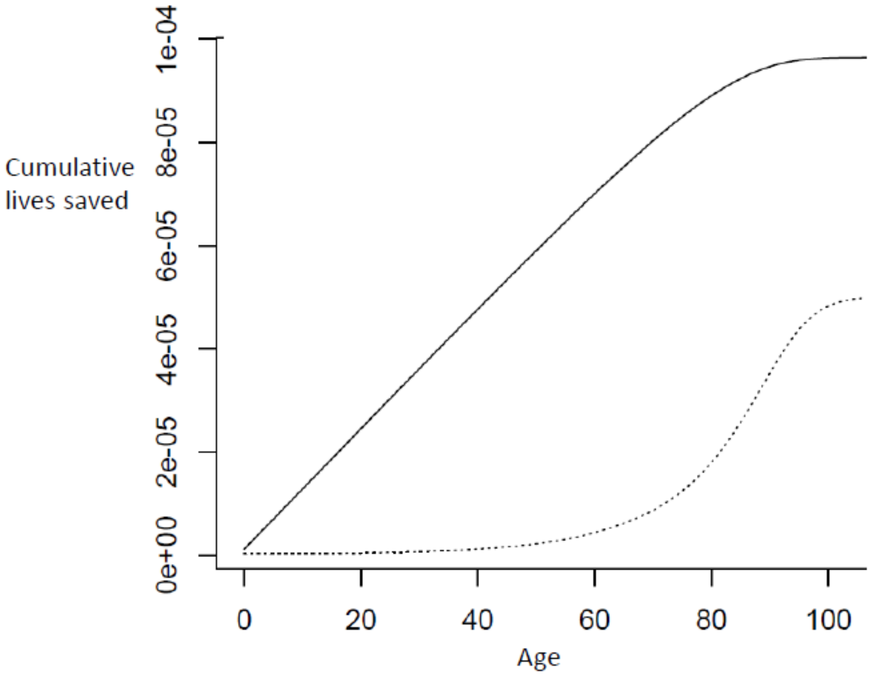

Figure 3. Lives saved by age for continuing risk reduction beginning at age 0. Solid line = additive risk reduction, dotted line = proportional risk reduction.