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"Taxing Financial Transactions: A Mirrleesian Approach"

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Taxing Financial Transactions: A Mirrleesian Approach[1]

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Abstract:

Taxing financial transactions is often advocated for Pigouvian reasons, when financial speculation is supposed to generate inefficiencies. We adopt instead a Mirrleesian approach, and study the optimal taxation of financial transactions when financial markets are efficient, but the tax system is imperfect, due to asymmetric information. In our model, financial transactions are used by entrepreneurs to hedge shocks on their skills, in line with the New Dynamic Public Finance literature. Entrepreneurs privately observe their skills, but trades in financial markets are publicly observable. The optimal mechanism maximizes a convex combination of utilitarian welfare and Rawlsian criterion, subject to feasibility and incentive constraints. Entrepreneurial projects are subject to liquidity shocks, which can be smoothed by conducting financial transactions. Better skilled entrepreneurs' projects have larger expected profits, but also larger shocks. Trades therefore signal skills, implying it is optimal to tax financial transactions, in addition to capital income and wealth.

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1 Introduction

Governments facing large funding needs often consider taxing financial transactions. a well known historical example is when the British government instituted, in 1694, a stamp duty on share transfers to finance war against France. This tax is still in place today. More recently, after the Global Financial Crisis of 2007-09, G-20 members discussed introducing a financial transaction tax and some countries, including France and Italy, implemented it. But is it a good idea to tax financial transactions?

Financial transactions' taxation was originally suggested by Keynes (1936). In Keynes' proposal, as well as in more recent analyses (see Tobin 1978, Stiglitz 1989, Summers & Summers 1989, Dow & Rahi 2000, Davila 2021, Rüdiger & Sorensen 2021, and Dieler et al (2022)), financial transactions taxes are Pigovian. When financial markets are imperfect, some trades can actually impose negative externalities, which financial transactions taxation can curb.

Opponents of financial transaction taxes argue they impair, rather than improve, the functioning of markets. This point is made by Campbell & Froot (1994) and also Schwert & Seguin (1993) who write:

> "a tax on transactions would increase the cost of capital, reduce market liquidity and bring down security values... The economic and societal distortions resulting from taxation and avoidance would likely be large."

In fact, there is not much empirical support for the hypothesis that financial transaction taxation improves the workings of markets. On the contrary, studies such as Colliard Hoffman (2017) show that financial transaction taxes reduce liquidity.

We depart from this debate, by taking a Mirrleesian, rather than Pigovian approach. We recognize that taxes create distortions, but we argue these distortions should be put in balance with the social benefits of redistribution and public spendings. When information is symmetric, it is possible to raise personalized lump-sum taxes that have no distortionary effect. Under asymmetric information, in contrast, taxes create distortions, and optimal taxation seeks to mitigate these distortions. In line with the mechanism design approach of Mirleess (1971), we study optimal taxation under information asymmetry.

In our analysis, the key information asymmetry variable is the profits of private businesses. Private businesses are a large share of US top wealth. As explained by Smith, Yagan, Zidar, and Zwick (2021) these businesses are difficult to value and, since there are no observed transaction prices, valuation requires self-assessment and imprecise and arbitrary estimation methods. Moreover these firms can be very opaque. Smith, Yagan Zidar, and Zwick (2021) show that around one half of US busineses are organized as pass-throughs, via partnerships or S-corporations. This typically involves partnerships owned by partnership, owned by partnership, etc. Consequently, as noted by Smith, Yagan Zidar, and Zwick (2021), following money through partnerships proves challenging. This leaves ample room for information asymmetries between entrepreneurs and tax authorities. To model information asymmetry, we assume, in line with Bolton and Scharfstein (1990), DeMarzo and Fishman (2008), Rampini and Vishwanathan (2010), that entrepreneurs can unobservably divert and consume a fraction of the value created by their businesses.[2] As explained by Kopczuk and Zwick (2020, p 33)

> "the owner might choose to consume through the firm... fringe benefits ... might include meals, club memberships, travel expenses, technology, transportation, or even housing... The owner could also choose to give to charity through the firm - even a charity that the owner personally supervises... this form of income would not appear to have been paid to the owner because the firm would report these expenditures as business expenses or charitable contributions."

We analyse optimal taxation when agents choose between entrepreneurship and wage earning, and then can invest in primary financial markets and trade in secondary markets. We assume agents privately observe their skills at entrepreneurship. Better skilled agents can undertake entrepreneurial projects with larger profits. Our assumption that private businesses' profits reflect entrepreneurs' skills is in line with the empirical results of Smith et al (2019). In particular, Smith et al (2019) document that the profits of private "pass-through" businesses accrue to working-age owners in skill-intensive industries and fall dramatically when the owner retires or dies. We also assume that entrepreneurial profits can be hit by shocks, and our key assumption is that projects with larger expected cash flows are also subject to larger shocks. This is in line with the finding by Bach et al (2020) that idiosyncratic volatility of net wealth returns is maximal for the top percentiles of the wealth distribution.[3] For simplicity, we assume shocks have zero mean and are independent across agents, so that, in the aggregate they cancel out. We accordingly refer to them as liquidity shocks.

Our model features two important functions of financial markets. First, agents can save by initially investing in bonds, earning returns at the later period. Second, agents can trade these bonds in secondary markets at an interim date. By trading in secondary markets, agents attempt to smooth out the impact of liquidity shocks on their interim consumption. Since better skilled entrepreneurs run larger businesses, hit by larger shocks, they tend to conduct larger trades. Hence, larger secondary market trades tend to reflect larger skills. In contrast with private businesses, investment and trading in regulated financial markets as well as securities' income are assumed to be observed by the

[2] Landier and Plantin (2015) make a similar assumption in their analysis of redistributive taxation.

[3] Bach et al (2020) also find, as written on page 2707 that: "At the annual frequency the heterogeneity of returns is mostly driven by idiosyncratic risk and heterogeneous exposures to economy wide shocks."

government.[4] Since publicly observed trades convey a signal about privately observed skills, it is optimal to tax them.

Because they take advantage of the signal conveyed by trades, financial transaction taxes relax incentive compatibility constraints. The optimal mechanism, which involves financial transaction taxation, creates less distortions than a tax system constrained to rely on income and wealth tax only. Correspondingly, the optimal tax system, including financial transactions taxation, induces more entrepreneurship than the constrained tax system, excluding financial transactions taxation. While social welfare is improved by financial transactions taxation, the richest agents in the economy are better off if financial transactions taxation is precluded. By relaxing incentives constraints, financial transactions taxation reduces rents, which is costly for the agents with the highest skills and hence the largest rents.

Section 2 discusses the relation between our analysis and the literature. Section 3 presents the model. Section 4 analyzes the case in which entrepreneurial skills are perfectly observable (first best). Section 5 studies the case in which agents privately observe their entrepreneurial skills (second best). Section 6 studies the case in which skills is exponentially distributed. In that case the optimal mechanism can be implemented with affine taxes.

2 Literature

2.1 The Pigovian approach

Keynes (1936) introduced the idea that taxing financial transactions would curb excessive speculation. In line with Keynes (1936), Tobin (1978), Stiglitz (1989) and Summers and Summers (1989) called for financial transaction taxes, arguing they would reduce volatility. More recently, Subrahmaniam (1998), Dow and Rahi (2000), Davila (2017) and Sorensen (2019) emphasized the Pigovian role of financial transaction taxes in imperfect markets. In Dow and Rahi (2000) and Subrahmaniam (1998), informed agents scale back their trades in response to the tax. Dow and Rahi (2000) show that this can lead to larger (after tax) profits for informed traders than when there is no tax. Because in Dow and Rahi (2000) and in Sorensen (2019) all agents have well defined preferences and make optimal decisions, the impact of financial transaction taxes on welfare can be analysed. In this context, Dow and Rahi (2000) show that taxing financial transactions can increase welfare.[5]

[4]This contrasts with Golosov and Tsyinski (2010), who consider the case in which transactions are unobservable.

[5]While, as discussed above, financial transaction taxes can increase informed profits, their effect on hedgers depend on their consequences on hedging opportunities. By making prices less informative, financial transaction taxes can actually increase the expected utility of hedgers, because of the Hirshleifer (1971) effect. Thus, taxing financial transactions can lead to a Pareto improvement.

2.2 The Mirrleesian approach

A large fraction of literature on capital taxation followed the approach of Ramsey (1927) and focused on linear taxes.[6] Mirrlees (1971), however, pointed that, to optimally cope with information asymmetry, the social planner should consider general tax schedules, that can be nonlinear in the observables. As written by Golosov et al (2006):

> "Rather than starting with an exogenously restricted set of tax instruments, Mirrlees's (1971) starting point is an information friction that endogenizes the feasible set of tax instruments."

In line with Mirleess (1971), Diamond (1998), Diamond and Mirrlees (1978), and the "new dynamic public finance" literature (see, e.g., Golosov et al (2003), Farhi and Werning (2010), and Golosov et al (2006)), we adopt a mechanism design approach to the optimal taxation of financial markets. We characterize the optimal mechanism and then study the tax scheme implementing the optimal mechanism.

Optimal taxation models are often difficult to analyze. To gain tractability, we make the following two assumptions.

- First, in line with Piketty (1997), Diamond (1998), and Saez and Stancheva (2016), we consider quasi-linear preferences: while the utility of interim consumption is concave, which implies agents want to smooth out liquidity shocks, utility of final consumption is linear, which eliminates wealth effects.

- Second, instead of using welfare weights that depend on utility levels like most of the optimal tax literature, we use rank dependent welfare weights, in the spirit of Yaari (1997) or Simula and Trannoy (2022). This allows us to exploit the tractability from quasi-linear preferences and obtain transparent formulas for optimal tax rates. In line with Chichinilsky (1996), we use a particularly simple rank-dependent welfare function, equal to a convex combination of utilitarian welfare and a Rawlsian objective, equal to the minimum expected utility across all agents.

This modeling strategy allows us to obtain transparent formulas and clarify the trade off between rent extraction and efficiency arising in our model similarly to standard screening models (see for example Myerson 1981).

In Mirrlees (1971), Diamond and Mirrlees (1978), and the new public finance literature, there is asymmetric information about wage earners' skills. The optimal mechanism is therefore contingent on labour income, which is related to skills. In contrast, in our framework, information asymmetry bears on entrepreneurs' skills and the corresponding private business profits. Because

[6] See for example Chamley (1986) and Judd (1985). See also Straub and Werning (2020) who revisit the Chamley-Judd results and find a positive optimal capital income tax in the long term when the intertemporal elasticity of substitution is less than or equal to one.

we assume larger profits are hit by lager shocks, which agents can smooth out by trading in secondary financial markets, the implementation of the optimal mechanism involves financial transactions taxation.

In Golosov, Tsyvinski, and Werning (2007), privately observed wage earners' skills evolve stochastically. Agents use financial markets to hedge the corresponding risk. In that context it is optimal to tax capital income, since it is related to the variable of adverse selection. While the analysis of Golosov, Tsyvinski, and Werning (2007) rationalizes a capital income tax, it does not consider financial transactions taxation.

In a dynamic model of labour taxation, Laroque (2011) examines whether wealth taxation could complement income taxation. He finds that wealth taxation is optimal whenever the agents' permanent income is positively correlated with aggregate life time savings.

Atkinson and Stiglitz (1976) also study an optimal taxation problem in which agents privately observe their productivity. They show that, when an agent's utility is separable between consumption and labour, optimal taxes depend on labour income but not on consumption[7]. This result, in which one instrument (income tax) is enough to deal with one dimension of information asymmetry (productivity), differs from ours, in which several instruments (capital income tax and financial transaction tax) are necessary to optimally deal with one dimension of information asymmetry (wealth). This difference arises because, in Atkinson and Stiglitz (1976), for a given level of net income (reflecting productivity and income tax), agents' decisions (consumption) don't depend on the adverse selection variable (productivity), while in our analysis, for a given level of savings (and net capital income), agents decisions (financial transactions) still depend on the adverse selection variable (wealth), because financial transactions mitigate liquidity shocks which are increasing in wealth. In that respect, our analysis is in line with Saez (2002). In Saez (2002), commodity taxation improves welfare when high productivity agents have different preferences over commodities than low productivity agents. Thus, as in our analysis, while there is only one dimension of information asymmetry, it is optimal to use several instruments, contingent on different observable variables, because they all depend on the information asymmetry variable.

3 Model

Our model has three dates $t = 0, 1, 2$, features agents, firms and a government, and involves only one good, that can be consumed or invested. There is a mass one continuum of heterogeneous agents, with types y, distributed over $[y_{\min}, \infty)$ with c.d.f. F and density f. Firms have access to a long term technology producing, at time 2, $R > 1$ units of good per unit.They are competitive and identical. The government aims to design a tax system that allows to finance its expenditures in a way that maximizes a welfare function that incorporates redistributive objectives.

[7]This result is explained in a very clear way by Laroque (2005).

At time 0, the sequence of events is the following:

- After observing his type y, each agent makes an occupational choice, between wage-earning, yielding ω units of good, and entrepreneurship, yielding y units of good. While y varies across agents, reflecting different entrepreneurial skills, for simplicity ω is assumed constant across agents. The choice to become an entrepreneur is denoted by $\ell(y) = 1$, while the choice of wage-earning is denoted by $\ell(y) = 0$.
- Firms issue bonds for financing their long term investments.
- Finally, each agent decides how much of the good to store for consumption at time 1, and how much to invest in the bonds issued by the firms (primary market) allowing the agent to consume at time 2.

At time 1, entrepreneurial projects are hit by liquidity shocks, which can be positive with $\varepsilon = +1$ or negative with $\varepsilon = -1$. Liquidity shocks are i.i.d across agents. For each type of entrepreneurial project y, a fraction one half of the projects are hit by a positive shock, while the complementary fraction is hit by a negative shock. Thus, there is no aggregate risk. Entrepreneurs can trade bonds at time 1 (secondary market) to absorb part of their liquidity shocks. This framework enables us to capture in a very simple way the two basic functions of financial markets: channelling savings to productive investment opportunities, and enabling agents to smooth out liquidity shocks.

We assume the size of liquidity shocks increases with the type of entrepreneurial projects. More precisely, type-y projects are hit by shocks equal to $\varepsilon\sigma(y)$, where $\sigma(y)$ is assumed to be differentiable with $\sigma'(y) > 0$.

Consumption takes place at times 1 and 2, with time 1 consumption taking place after the realisation of the liquidity shocks, and time 2 consumption taking place after firms repay their bonds.

For simplicity, the discount factor is assumed to be one. The time 1 and time 2 consumptions of agent y hit by shock ε are denoted by $C_1^\varepsilon(y)$ and $C_2^\varepsilon(y)$, respectively.

In line with Piketty (1997), Diamond (1998), and Saez and Stancheva (2016), we assume quasi-linear preferences, which eliminates wealth effects and greatly enhances tractability. Thus, the expected utility at time 0 of an agent of type y is

$$U(y) \equiv E_\varepsilon \left[u(C_1^\varepsilon(y)) + C_2^\varepsilon(y) \right], \tag{1}$$

where the utility function u, the same for all agents, is such that $u' > 0$, $u'' < 0$ and $u''' > 0$.

In line with Chichinilsky (1996), we consider welfare function W defined as

$$W = (1 - \alpha) E_y[U(y)] + \alpha \min_y U(y) \tag{2}$$

where $0 < \alpha < 1$ and E_y denotes expectation with respect to the distribution F across types y. W is a convex combination of utilitarian welfare s and of a Rawlsian term, the minimum expected utility across all agents. The parameter

α captures the intensity of the redistributive objective of the government. In a sense, our welfare function captures a "minimal" version of redistributive objectives: the government only wants to protect the poorest agents but treats all other agents, including the richest, in the same way.

The resource constraint of the government is

$$E_{y,\varepsilon}\left[C_2^\varepsilon(y)\right] + G \leq RE_{y,\varepsilon}\left[(1-\ell(y))\omega + \ell(y)y - C_1^\varepsilon(y)\right], \tag{3}$$

where the left-hand side is the sum of aggregate time-2 consumption and public expenditures G, while the right-hand side is the aggregate output of the firms. For simplicity, we take G as exogenous, and do not explicit1ly model the social utility generated by these expenditures.

4 When entrepreneurial skills are publicly observable

Recall that the expected utility of agent y is given by (1) and that the objective of the government by (2). When the only constraint faced by the government is the resource constraint, the Rawlsian term implies that it is optimal to equalize $U(y)$ across all types y[8]. Thus $U(y) \equiv W, \forall y$, and (1) implies that

$$E_{\varepsilon,y}[C_2^\varepsilon(y)] = W - E_{\varepsilon,y}[u(C_1^\varepsilon(y))]. \tag{4}$$

Substituting (4) into (3), the resource constraint rewrites

$$W \leq E_{y,\varepsilon}\left[u(C_1^\varepsilon(y)) + R\left((1-\ell(y))\omega + \ell(y)y - C_1^\varepsilon(y)\right) - G\right]. \tag{5}$$

Maximizing W with respect to $C_1^\varepsilon(y)$, subject to (5), we obtain our first proposition:

Proposition 1 *When entrepreneurial skills are publicly observable, the optimal mechanism is such that types $y > \omega$ opt for entrepreneurship (and types $y \leq \omega$ opt for wage earning) while consumption is allocated such that*

$$\forall(y,\varepsilon), u'(C_1^\varepsilon(y)) \equiv R. \tag{6}$$

Proposition 1 states that, when types are publicly observable, occupational choices are efficient, utility is equalised across all agents (full redistribution), marginal rates of transformation are equal to the marginal rate of substitution between consumption at the two periods (investment efficiency), and consumption is independent of liquidity shocks ε (complete insurance). Proposition 1 also shows that when types are observable the optimal allocation does not depend on the weight α put on the Rawlsian term, as long as it is strictly positive.

[8] To see this, suppose by contradiction that one agent has lower utility than the average. Then W can be increased by redistributing towards this agent while keeping the average utility constant

As we will show below, this is no longer the case when entrepreneurial skills are privately observable.

As implied by the second welfare theorem, the optimal allocation can be implemented with complete financial markets and personalized lump sum taxes. All endowments (ω for wage earners, and y for entrepreneurs) are fully taxed, and all agents receive the same allocation, namely $C_1^* = u'^{-1}(R)$ at time 1 and $C_2^* = W - u(C_1^*$ at time 2. These taxes do not depend on the decisions of the agents, namely savings and financial transactions. Therefore they don't distort these decisions.

In our simple set-up with quasi-linear preferences and no aggregate risk, it is enough for the effective completeness of markets to have only one financial instrument, a bond, and two markets where this bond is traded against the physical good, at $t = 0$ and $t = 1$. Each bond delivers R units of consumption at date 2. We take the consumption good as the numéraire. Bonds are issued by competitive firms, investing the proceeds in the long term production technology. Competition between firms pins down the time-0 price of the bond to 1. Denote the time-1 bond price by B. At $t = 0$, agent y invests $S(y)$ in the primary bond market. At $t = 1$, after a liquidity shock ε, agents adjust their holdings by trading $\Delta^\varepsilon(y)$ bonds in the secondary market. Thus, time 1 consumption is

$$C_1^\varepsilon(y) = (1 - \ell(y))\omega + \ell(y)\,(y + \varepsilon\sigma(y)) - S(y) - B\Delta^\varepsilon(y),$$

and time 2 consumption is

$$C_2^\varepsilon(y) = R(S(y) + \Delta^\varepsilon(y)) - T(y).$$

At time 1, the type y agent in state ε chooses $\Delta^\varepsilon(y)$ such that

$$\Delta^\varepsilon(y) = \arg\max_\Delta u((1-\ell(y))\omega+\ell(y)\,(y + \varepsilon\sigma(y))-S(y)-\Delta B)+(S(y)+\Delta)R-T(y).$$

The first order condition is

$$Bu'(C_1^\varepsilon(y)) = R.$$

This implies that agents perfectly smooth out the impact of liquidity shocks, so that their time 1 consumption is the same after positive and negative shocks. Of course, in our simple setting, agents who opt for wage-earning are not subject to shocks, and therefore don't need to trade in the secondary market to smooth consumption. In contrast, agent y who went for the entrepreneurial project buys $\Delta(y) = \sigma(y)/B$ bonds after a positive shock and sells $\Delta(y)$ after a negative shock. Thus, the market clears since half the agents of each type are hit by a positive shock and buy, while the other half, hit by a negative shock, sell. Given that agents eliminate the impact of liquidity shocks, at time 0, they choose $C_1(y) = y - S(y)$ to maximize

$$u(C_1) + R(y - C_1) - T(y).$$

The first order condition with respect to $S(y)$ is

$$u'(C_1(y)) = R.$$

Comparing the two first order conditions shows that $B = 1$. Hence, the optimal choices of initial savings at time 0 and bond trades at time 1 imply that (6) holds. Thus, we obtain our next proposition:

Proposition 2 *The first best allocation can be achieved with a personalized lump sum tax and a competitive market for bonds. At time 0, agents with $y \geq \omega$ chose to undertake an entrepreneurial project while agents with $y < \omega$ choose wage earning. At time 1 the bond price is $B = 1$, and type $y \geq \omega$ buys (resp. sells) $\Delta(y) = \sigma(y)$ bonds after a positive (resp. negative) shock.*

The proposition is illustrated in Figure 1. The bond position of agent y net of financial transactions is $S(y) + \varepsilon\Delta(y)$ and can be interpreted as agent y's net savings at date 1. When $u'(\omega) < R$, which we will assume, wage earners also buy bonds at time 0 (for the amount $\omega - u'^{-1}(R)$ and hold them until time 2. By contrast,entrepreneurs (who save more than wage earners because they are richer) actively manage their bonds holdings, by rebalancing their portfolio at time 1, following their liquidity shocks.

5 When entrepreneurial skills are privately observable

We now turn to the case where entrepreneurial skills are privately observable[9]. In this case, allocations must satisfy incentive compatibility constraints. For simplicity we first assume private liquidity shocks ε are publicly observable. We then discuss how this assumption can be relaxed without altering our results. We also make the realistic assumption that occupational choices ($\ell(y)$) are publicly observable.

5.1 Incentive compatible mechanisms

The government does not observe y and elicits a report $\hat{y}$ from each agent y. The agent can decide to truthfully report y or to misreport it.[10] Wage earners have no scope for misreporting since ω is publicly observable, but entrepreneurs can potentially hide part of their wealth. If an entrepreneur misreports then he/she can secretely increase his/her consumption by $g(y - \hat{y})$, where g is a constant lower than or equal to one. The larger the parameter g, the more attractive it is to divert resources, the more severe the information asymmetry

[9] Since our focus is on the taxation of financial activities, we also assume that first period consumptions are not observable, otherwise a consumption tax would also be part of the optimal tax mix.

[10] This is in line with the cash diversion problem analyzed by Bolton and Scharfstein (1990), DeMarzo and Fishman (2008), Biais et al. (2007), and Landier and Plantin (2015).

problem. Resource diversion is inefficient iff $g < 1$. We assume that for all y, $g - \sigma'(y) > 0$, so that an entrepreneur's net wealth is increasing in his skills, even after diversion and a negative shock $\varepsilon = -1$.

By the revelation principle, we can restrict attention to direct mechanisms, mapping reported types $\hat{y}$ into occupational choices and consumptions. If agent y reports $\hat{y}$ his/her time 1 consumption is

$$C_1^\varepsilon(y, \hat{y})) = (1 - \ell(\hat{y}))\omega + \ell(\hat{y})\left(\hat{y} + g(y - \hat{y}) + \varepsilon\sigma(y)\right) - S^\varepsilon(\hat{y}), \tag{7}$$

where $g(y - \hat{y})$ is the additional consumption generated by diverting wealth $(y - \hat{y})$. Consumption at time 2 is entirely determined by observable returns from the productive technology and government transfers. Therefore, while it depends on $\hat{y}$, it does not depend on y. We denote it by $C_2^\varepsilon(\hat{y})$. The indirect utility function of agent y is still denoted by $U(y)$, but now we have

$$U(y) = \max_{\hat{y}} E_\varepsilon \left[u(C_1^\varepsilon(y, \hat{y}))) + C_2^\varepsilon(\hat{y})\right]. \tag{8}$$

The incentive compatibility condition is that truthful reporting is optimal

$$y \in \arg\max_{\hat{y}} E_\varepsilon[u(C_1^\varepsilon(y, \hat{y}))] + C_2^\varepsilon(\hat{y})]. \tag{9}$$

Using (7), the envelope theorem applied to the incentive compatibility condition implies that the derivative of the value function of the agent with respect to y is

$$U'(y) = E_\varepsilon\left[\ell(y)(g + \sigma'(y)\varepsilon)u'(C_1^\varepsilon(y))\right]. \tag{10}$$

Thus, for wage earners, with $\ell(y) = 0$, expected utility is constant while, for entrepreneurs, with $\ell(y) = 1$, expected utility is increasing in y, since $\sigma'(y) < g$. This differs from the first best, in which $U(y)$ was constant across all agents. The intuition is the following. Wage earners' income is observable, so there is no need to deviate from constant utility, since there is no incentive problem. Entrepreneurs, in contrast, privately observe their output y. To avoid underreporting of y, the social planner must give larger utility to agents who (truthfully) report larger y. This implies these agents earn rents, that are increasing in their entrepreneurial skills y. Condition (10) shows that, keeping everything else constant, the increase in utility needed to provide incentives for thruthful reporting is increasing in g. Larger diversion opportunies worsen asymmetric information problems and thus increase rents. Condition (10) also shows that for entrepreneurs the contribution of marginal utility to the expected rent is higher after a positive shock than after a negative shock. This will result in a higher distortion of consumption after a positive than a negative shock, hence incomplete insurance of liquidity shocks in the constrained optimal allocation.

5.2 Constrained optimal allocation

The planner still maximizes (2) subject to the resource constraint (3), but now, the incentive compatibility constraint must be satisfied. We first study the relaxed problem, in which the incentive compatibility constraint (9) is replaced by the weaker envelope condition (10). Then, we clarify under what conditions the solution of the relaxed problem is also the solution of the original problem.

Rent efficiency tradeoff: Truthful reporting and the definition of the indirect utility function of the agent, condition (8), imply that expected second period consumption is

$$E_{\varepsilon,y}\left[C_2^{\varepsilon}(y)\right] = E_{\varepsilon,y}\left[U(y) - u(C_1^{\varepsilon}(y))\right]. \tag{11}$$

Substituting (11) into the resource constraint (3), we have

$$E_{y,\varepsilon}\left[RC_1^{\varepsilon}(y) + U(y) - u(C_1^{\varepsilon}(y))\right] \leq RE_{y,\varepsilon}\left[(1-\ell(y))\omega + \ell(y)y\right] - G. \tag{12}$$

Thus, utilitarian welfare is such that

$$E_y\left[U(y)\right] \leq RE_{y,\varepsilon}\left[(1-\ell(y))\omega + \ell(y)y\right] - G + E_{y,\varepsilon}\left[u(C_1^{\varepsilon}(y)) - RC_1^{\varepsilon}(y)\right]. \tag{13}$$

This condition is similar to its first best counterpart (see (5)), but here the choice of the optimal consumption $C_1^{\varepsilon}(y)$ is subject to the incentive compatibility condition.[11] The incentive compatibility condition implies the envelope condition (10). Integrating by parts this condition yields

$$E_y\left[U(y)\right] = U(y_{\min}) + E_y\left[\ell(y)(g+\sigma'(y)\varepsilon)u'(C_1^{\varepsilon}(y))\frac{1-F(y)}{f(y)}\right]. \tag{14}$$

The second term on the right hand side of (14) is the aggregate rent. Substituting (14) into the resource constraint (13), and binding the latter, we have

$$U(y_{\min}) = E_{y,\varepsilon}\left[u(C_1^{\varepsilon}(y)) + R\left((1-\ell(y))\omega + \ell(y)y - C_1^{\varepsilon}(y)\right) - \ell(y)(g+\sigma'(y)\varepsilon)u'(C_1^{\varepsilon}(y))\frac{1-F(y)}{f(y)}\right] - G. \tag{15}$$

Equation (15) shows that the utility of the lowest type is equal to utilitarian welfare

$$E_{y,\varepsilon}\left[u(C_1^{\varepsilon}(y)) + C_2^{\varepsilon}(y)\right] = E_{y,\varepsilon}\left[u(C_1^{\varepsilon}(y)) + R\left((1-\ell(y))\omega + \ell(y)y - C_1^{\varepsilon}(y)\right)\right] - G,$$

minus aggregate rent. Since U is increasing, $\min_y U(y) = U(y_{\min}) = U(0)$. Hence, the objective of the social planner writes

$$W = (1-\alpha)E_y[U(y)] + \alpha U(0). \tag{16}$$

Binding the resource constraint (13) and substituting it, along with the envelope condition (15), into the program of the social planner (16), we obtain our next proposition:

[11] Also, in contrast with (5) which prevailed in the first best, informational rents depend on y, so we cannot have $U(y) \equiv W$.

Proposition 3 *When entrepreneurial skills y are privately observable, the objective of the social planner becomes*

$$W = E_{y,\varepsilon}\left[u(C_1^\varepsilon(y)) + R\left((1-\ell(y))\omega + \ell(y)y - C_1^\varepsilon(y)\right)\right] - \alpha E_{y,\varepsilon}\left[\ell(y)(g+\sigma'(y)\varepsilon)\frac{1-F(y)}{f(y)}u'(C_1^\varepsilon(y))\right]. \tag{17}$$

The proposition highlights the rent-efficiency tradeoff between social surplus, first term in brackets in (17), and agency rent, second term in (17), the difference between the two being the "virtual surplus" (see Myerson, 1981). Rents are costly for the social planner because, as mentioned above in the discussion of (15), they reduce the utility of the lowest type. This cost increases with the weight placed on the Rawlsian criterion (α).

Optimal mechanism: An advantage of the formulation of the program given in Proposition 3 is that it enables to find the optimum by pointwise maximisation of the planner's objective. For each (ε, y) the planner solves

$$\max_{C_1} u(C_1) + R\left((1-\ell(y))\omega + \ell(y)y - C_1\right) - \alpha\ell(y)(g+\sigma'(y)\varepsilon)\frac{1-F(y)}{f(y)}u'(C_1). \tag{18}$$

The first optimality condition with respect to consumption is that the consumption of type y hit by shock ε must be such that

$$u'(C_1^\varepsilon(y)) = R + \alpha\ell(y)(g+\sigma'(y)\varepsilon)\frac{1-F(y)}{f(y)}u''(C_1^\varepsilon(y)). \tag{19}$$

Comparing (19) to its first best counterpart (6), we see that there is an additional term on the right hand side

$$\alpha\ell(y)(g+\sigma'(y)\varepsilon)\frac{1-F(y)}{f(y)}u''(C_1^\varepsilon(y)), \tag{20}$$

reflecting the distortion, or wedge, induced by information asymmetry.[12] This wedge is equal to the weight placed by the planner on the Rawlsian criterion (α) multiplied by the rent of type y. The wedge in (20) implies that savings are lower than in the first best.

Moreover, for each y the planner solves

$$\max_{\ell} E_\varepsilon\left[u(C_1^\varepsilon(y)) + R\left((1-\ell)\omega + \ell y - C_1^\varepsilon(y)\right) - \alpha\ell(g+\sigma'(y)\varepsilon)\frac{1-F(y)}{f(y)}u'(C_1^\varepsilon(y))\right]. \tag{21}$$

The optimality condition with respect to occupational choice is

$$\ell(y) = 1 \iff y \geq y_{\min} \equiv \omega + \frac{\alpha}{R}E_\varepsilon\left[(g+\sigma'(y)\varepsilon)\frac{1-F(y)}{f(y)}u'(C_1^\varepsilon(y))\right]. \tag{22}$$

[12] Ths is in line with the new dynamic public finance literature shows that under asymmetric information there typically is a wedge between the marginal rate of substitution and the marginal rate of transformation (R in our notations), see, e.g., Proposition 3 in Golosov, Tsyinski and Werning (2006).

Again, there is a distortion: Because of information asymmetry the threshold value of y above which agents undertake the entrepreneurial project is higher than in the first best. That is, there is less entrepreneuship in the information constrained case than in the first best.

Define the auxiliary function $C^*(d)$ as

$$C^*(d) \equiv \arg\max_C \left[u(C) - RC - du'(C)\right]. \tag{23}$$

Relying on this notation, we can state our next proposition (whose proof is in the appendix):

Proposition 4 *We assume that* $\sigma'' \leq 0$ *and*

$$[\frac{1-F(y)}{f(y)}]' \leq \frac{1}{\alpha}. \tag{24}$$

then the solution of the relaxed problem is also the solution of the full problem, and is characterized by

$$\ell(y) = 1\left(y \geq \omega + \frac{\alpha}{R}(g + \sigma'(y)\varepsilon)\frac{1-F(y)}{f(y)}u^{''}(C_1^\varepsilon(y))\right), \tag{25}$$

where $1(.)$ *is the indicator function, and*

$$C_1^\varepsilon(y) = C^*\left(\alpha\ell(y)(g + \sigma'(y)\varepsilon)\frac{1-F(y)}{f(y)}\right). \tag{26}$$

Condition (26) implies that for wage earners, consumption does not vary with y, and is equal to its first best level. For entrepreneurs, since C^* is increasing and $g + \sigma'(y)\varepsilon \geq 0$, a sufficient condition for (24) is that the inverse hazard rate be non increasing and $\sigma(y)$ affine in y. (26) implies that entreperneurs' consumption is larger after a positive shock than after a negative shock:

$$C_1^+(y) > C_1^-(y), \forall y.$$

So, entrepreneurs are exposed to liquidity shocks, in contrast with the first best, in which they are fully insured against liquidity shocks. While exposing entreperneurs to liquidity shocks reduces utilitarian welfare, it relaxes the incentive compatibility condition, because shocks are increasing in wealth.

Comparative statics: Since $C^*(.)$ is increasing, we can use Proposition 4 to obtain comparative statics results. First, as α increases, date 1 entrepreneurs' consumption increases and thus savings decrease further below the first best level.[13] That is, the larger the weight placed by the planner on the Rawlsian criterion (α), the larger the distortion in the allocation of consumption through time. Since the utility that can be given to the lowest type decreases with

[13]In Figure 2, an increase in α implies downward shifts in the dotted lines.

the informational rents (as made clear by (15)), an increase in α induces the planner, when faced with the rent efficiency tradeoff highlighted in Proposition 3, to reduce efficiency in order to lower rents, to raise $U(y_{\min})$. When $\alpha = 0$, there is no distortion: first period consumption is the same in the first best and the second best.

A second comparative statics result is that entrepreneurs' date 1 consumption increases when g increases.[14] Indeed when g increases, the ability of agents to divert resources increases. This worsens the asymmetric information problem, which, other things equal, increases rents. To curb rents (in order to maintain $U(y_{\min})$ high enough) the principal increases distortions.

5.3 When liquidity shocks are not observable

So far, we have assumed that ε was publicly observable. We now exhibit a natural condition under which our results are unchanged if ε is privately observed by the agent. Equation (7) shows that agent y's consumption when he/she reports the truth $\hat{y} = y$ is the difference between his/her endowment net of the liquidity shock, $y + \sigma(y)\varepsilon$, and net savings, $S^{\varepsilon}(y)$:

$$C_1^{\varepsilon}(y, y) = y + \varepsilon\sigma(y) - S^{\varepsilon}(y). \tag{27}$$

Building on this notation, we obtain our next proposition (whose proof is in the appendix):

Proposition 5 *If the mechanism that is optimal when ε is publicly observable is such that*

$$S^{+}(y) \geq S^{-}(y), \tag{28}$$

then it delivers the same consumption profile as the mechanism that is optimal when ε is privately observed by agents.

Condition (28) is quite natural: It means that, in the optimal mechanism, agents with positive liquidity shocks, giving them additional resources at time 1, save more than agents with negative shocks, whose time 1 resources are lower. Since agents hit by a positive (resp. negative) shock are happy to save more (resp. less), when the optimal mechanism obtained when ε is observable is such that (28) holds, the allocation corresponding to that mechanism remains incentive compatible (and therefore constrained optimal) when ε is privately observed.

5.4 Can the second best be implemented without a financial transactions tax?

So far, this section has characterized the optimal mechanism in terms of consumption profiles. In the remainder of the paper, we study how this optimal mechanism can be implemented by a tax system. First, we investigate whether

[14]Similarly, in Figure 2, an increase in g implies downward shifts in the dotted lines.

the optimal mechanism can be implemented without taxing financial transactions. We obtain the following proposition, whose proof is in the appendix.

Proposition 6 *Under (24) the only case in which the optimal mechanism can be implemented without taxing financial transactions is when the inverse hazard rate* $\frac{1-F(y)}{f(y)}$ *is exactly proportional to the size of the liquidity shocks* $\sigma(y)$*. Apart from that non generic case, implementing the optimal mechanism implies taxing financial transactions.*

The condition that the hazard rate $\frac{1-F(y)}{f(y)}$ be proportional to the size of the interim shock $\sigma(y)$ is not natural because it imposes a special relation between two unrelated quantities, the size of liquidity shocks $(\sigma(y))$ and the distribution of wealth $(F(y))$.

The intuition behind the proposition is the following. Agents have decreasing marginal utility of consumption at time 1. Hence, when they are exposed to liquidity shocks, they seek to smooth out the impact of these shocks on their consumption. In the first best they do so by selling bonds after negative shocks, and buying bonds after positive shocks (see Proposition 2). Now, the larger their initial wealth y, the larger their shock $\sigma(y)$ or $-\sigma(y)$, the more bonds they want to buy or sell after being hit by a liquidity shock. Hence, trades in financial markets convey information about agent's initial wealth. Except when the distribution of initial wealth is related to liquidity shocks in a very special way, the government finds it optimal to use both signals, the one conveyed by savings and the one conveyed by trades, and thus to tax financial transactions as well as savings.

6 The affine exponential case

In general, the tax schedule implementing the optimal allocation cannot be characterized explicitly. However, this section shows that, when the distribution of y is exponential, with constant hazard rate $\frac{1}{\mathcal{A}}$, i.e.,

$$f(y) = \frac{1}{\mathcal{A}} \exp^{-\frac{y}{\mathcal{A}}},$$

and $\sigma(y)$ is affine, i.e., there are two positive constants σ_0 and σ_1 such that

$$\sigma(y) = \sigma_0 + \sigma y,$$

then a simple implementation of the optimal allocation can be readily characterized.

6.1 Characterization of the optimal allocation

Replacing the inverse hazard rate by $\mathcal{A}$ and the derivative of shock size by σ in Proposition 4, we obtain a simple characterization of the second best allocation in the exponential case:

Proposition 7 *When y is exponentially distributed and $\sigma(y)$ affine, the threshold value of y above which agents opt for entrepreneurship is*

$$y_{\min}(\sigma) \equiv \omega + \frac{1}{R} E_{\varepsilon} \left[\alpha \mathcal{A}(g + \sigma\varepsilon) u^{'}(C^{*}(\alpha \mathcal{A}(g + \sigma\varepsilon))) \right], \qquad (29)$$

and the consumption arising in the second best allocation is $C_1^{\varepsilon}(y) = u'^{-1}(R)$ for $y < y_{\min}$, while for $y \geq y_{\min}$ it is

$$C_1^{\varepsilon}(y) = C^{*}(\alpha(g + \sigma\varepsilon)\mathcal{A}). \qquad (30)$$

As can be seen in equation (18), for a given realization of the liquidity shock, the rent efficiency tradeoff faced by the social planner depends on the agent's type (y) only via the inverse hazard rate and the occupational choice ($\ell(y)$). When the hazard rate is constant, for a given occupational choice the rent efficiency tradeoff, and correspondingly the optimal consumption, are the same for all agents hit by the same liquidity shock. In this context, for $y < y_{\min}$, consumption at date 1 is constant, as in the first best. For $y > y_{\min}$, consumption is constant across types, but different after negative and positive shocks, in contrast with the first best. Also in contrast with the first best, there is a jump in consumption at $y_{\min}$, with

$$u'^{-1}(R) < C_1^{-}(y) < C_1^{+}(y), \forall y \geq y_{\min},$$

where the left hand side ($u'^{-1}(R)$) is the time 1 consumption of types $y < y_{\min}$. Finally note that, when y is exponentially distributed and $\sigma(y)$ affine, (24) holds, i.e., the solution of the relaxed problem is also the solution of the original problem.

6.2 Implementation

We now prove that, in this simple case, the optimal mechanism can be implemented with a bond market, lump sum transfers z_0 for wage earners and z_1 for entrepreneurs,[15] and constant marginal tax rates on wealth (T), capital income (t) and financial transactions (τ) for entrepreneurs.[16] In other words, the government can set the lump sum transfers z_0 and z_1, and the linear tax rates T, t and τ, such that agents' optimal decisions and equilibrium prices in the bond market give rise to the same allocation as in the optimal mechanism.

To characterize the implementation of the optimal mechanism, we first write down: i) the objective of agents, ii) their first order conditions, iii) the bond market equilibrium conditions, and iv) the condition under which agents sell bonds after negative shocks and purchase bonds after positive shocks. We then use these conditions to solve for the optimal tax rates and explain how the lump sum taxes can be used to implement optimal occupational choices.

[15] These lump sum transfers can be positive or negative depending in particular on the size of public expenditures G.

[16] Note that optimality does not require to tax transactions in the primary bond market.

Agents' objective: Consider the optimal actions of an agent who chose to become an entrepreneur. This agent must choose how much initial wealth to report to the tax authority ($\hat{y}$), how much to invest at time 0 in the primary bond market ($S(y)$), and how much to trade at time 1 in the secondary bond market ($\Delta^{\varepsilon}(y)$), to maximise

$$\begin{aligned}&E_{\varepsilon}[u(\hat{y}+g(y-\hat{y})+\varepsilon\sigma(y)-S(y)-B\Delta^{\varepsilon}(y))\\&+R(S+\Delta^{\varepsilon}(y))-tR(S+\Delta^{\varepsilon}(y))-\tau|\Delta^{\varepsilon}(y)|-T\hat{y}-z_1,\end{aligned}$$

where B is the price of the bond at date 1.

First order conditions of the agent's problem: Still focus on agents who opted for entrepreneurship. The first order condition with respect to $S(y)$ is

$$E_{\varepsilon}[u'(C_1^{\varepsilon})]=R(1-t),$$

which pins down the income tax rate, as a function of C_1^{ε}, which is constant across y for agents who opted for entrepreneurship.

The first order condition with respect to $\Delta^{\varepsilon}(y)$ is

$$Bu'(C_1^{\varepsilon})=R(1-t)-\varepsilon\tau, \tag{31}$$

where the term ε multiplying τ comes from the fact that the FTT is proportional to the absolute value of the trade. This term captures the fact that the FTT creates a wedge between consumption after a positive shock and after a negative shock.

The first order condition with respect to $\hat{y}$ is

$$(1-g)E_{\varepsilon}[u'(C_1^{\varepsilon})]=T.$$

Substituting the first order condition with respect to $S(y)$ into the first order condition with respect to $\hat{y}$ yields

$$(1-g)R(1-t)=T, \tag{32}$$

which pins down the wealth tax rate relative to the income tax rate.

Equilibrium in the bond market: By symmetry, $\Delta^{\varepsilon}(y)=\varepsilon\Delta(y)$. Half of type y agents are hit by a positive shock and buy $\Delta(y)$ bonds in the secondary market, while the other half are hit by a negative shock and sell $\Delta(y)$ bonds. Thus the market clears. Taking the expectation with respect to ε in the first order condition with respect to $\Delta^{\varepsilon}(y)$ and comparing it with the first order condition with respect to $S(y)$ we have $B=1$.[17]

[17] This comes from the fact that the FTT is the same for buyers and sellers. If only the buyers pay it, the price goes down at date 1, but the allocation is the same.

Condition for sales after negative shocks and purchases after positive shocks: For agents who opted for entrepreneurship, consumption at time 1 writes as

$$C_1^{\varepsilon} = y + \varepsilon\sigma(y) - S(y) - \varepsilon\Delta(y). \tag{33}$$

Multiplying both sides of (33) by ε, bearing in mind that $\varepsilon^2 = 1$, and taking expectations gives

$$\Delta(y) = \sigma(y) - E_{\varepsilon}[\varepsilon C_1^{\varepsilon}] = \sigma_0 + \sigma_1 y - \frac{C_1^{+} - C_1^{-}}{2}. \tag{34}$$

We focus on the case in which parameters are such that $\Delta(y) > 0$ for all y. Since the right hand side of (34) is increasing in y, $\Delta(y) > 0$ iff $\Delta(y_{\min}) \geq 0$. A sufficient condition is

$$\sigma_0 > \frac{C^*(\alpha(g+\sigma)\mathcal{A}) - C^*(\alpha(g-\sigma))\mathcal{A})}{2}. \tag{35}$$

By Proposition 6, under condition (35) the optimal mechanism is implementable even if liquidity shocks are privately observed.

Solving for optimal taxes: For the optimal mechanism to be implemented, it must be that the first order conditions with respect to $\Delta(y)$ and $S(y)$ are satisfied for the constrained optimal consumptions: $C^*(\alpha(g+\sigma)\mathcal{A})$ and $C^*(\alpha(g-\sigma))\mathcal{A})$. Substituting the optimal value of time-1 consumption and $B = 1$ into the first order condition with respect to $\Delta(y)$ (31) and evaluating the condition at $\varepsilon = 1$ and $\varepsilon = -1$, we obtain a system of two linear equations in two unknowns, t and τ,

$$u'(C^*(\alpha(g+\sigma)\mathcal{A})) = R(1-t) - \tau, \tag{36}$$

$$u'(C^*(\alpha(g-\sigma\varepsilon)\mathcal{A})) = R(1-t) + \tau. \tag{37}$$

Condition (36) states that, after a positive shock, the marginal opportunity cost of purchasing bonds at time 1 ($u'(C^*(\alpha(g+\sigma)\mathcal{A}))$) is equal to the time 2 net marginal benefit. Similarly, Condition (37) states that the marginal utility of bond sales at time 1 is equal to their marginal cost at time 2. Solving (36) and (37) we obtain the optimal marginal tax rates on capital income

$$t = 1 - \frac{1}{R} E_{\varepsilon}\left[u'(C^*(\alpha(g+\sigma\varepsilon)\mathcal{A}))\right], \tag{38}$$

and financial transactions

$$\tau = -E_{\varepsilon}\left[\varepsilon u'(C^*(\alpha(g+\sigma\varepsilon)\mathcal{A}))\right]. \tag{39}$$

(39) implies

$$\tau = \frac{u'\left(C^*(\alpha(g-\sigma)\mathcal{A})\right) - u'\left(C^*(\alpha(g+\sigma)\mathcal{A})\right)}{2}. \tag{40}$$

Since u is concave, $C^*(\alpha(g-\sigma)\mathcal{A}) < C^*(\alpha(g+\sigma)\mathcal{A})$ implies that $\tau > 0$, i.e., the optimal financial transaction tax is strictly positive. Note that, in our affine-exponential specification, optimal tax rates are constant.

Implementing second best occupational choices: So far, we have not used the lump sum taxes on wage earners (z_0) and entrepreneurs (z_1) for the implementation, but we have not yet tackled the implementation of the second best occupational choices. It is easy to see that the threshold y_{min} can be implemented by an appropriate choice of the difference $z_1 - z_0$. Finally, the budget constraint of the government gives another condition which completely determines z_1 and z_0.

Summarizing the above discussion, we obtain our next proposition:

Proposition 8 *When the distribution of y is exponential, $\sigma(y)$ is affine and (35) holds, the optimal mechanism can be implemented with lump sum transfers for wage earners (z_0) and for entrepreneurs (z_1), combined with the constant marginal tax rates on wealth (T), capital income (t) and financial transactions ($\tau > 0$) given in (32), (38) and (39) respectively.*

As stated in Proposition 4, in the optimal mechanism consumption is larger after a positive shock than after a negative shock. Proposition 9 shows the key role of the financial transaction tax in implementing this outcome. By increasing the cost of trades, the financial transaction tax prevents agents from fully smoothing out the impact of liquidity shock, which results in larger consumption after positive shocks than after negative ones.

6.3 Comparative statics

Relying on the above analysis, one can perform a comparative statics analysis to shed light on the determinants of the optimal taxes. The derivative of the right-hand side of (38) with respect to α is

$$-\frac{\mathcal{A}}{R}E_\varepsilon\left[(g+\sigma\varepsilon)C^{*\prime}(\alpha(g+\sigma\varepsilon)\mathcal{A})u^{''}(C^*(\alpha(g+\sigma\varepsilon)\mathcal{A}))\right].$$

Since $g > \sigma$, $C^{*'} > 0$ and $u^{''} < 0$, this derivative is positive. Similarly, the derivative of the right-hand side of (38) with respect to g is also positive. So we can state our next proposition:

Proposition 9 *When the distribution of y is exponential and $\sigma(y)$ affine, the optimal capital income tax rate is increasing with the efficiency of diversion g and the weight of the Rawlsian criterion α.*

Other things equal, the larger the weight of the Rawlsian criterion the more the government wants to tax to redistribute towards the poorest agents, the larger the capital income taxes. On the other hand, the easier it is to divert wealth and thus the less it is possible to rely on wealth taxes, the larger the capital income tax rate.

From (40), the derivative of τ with respect to the size of the liquidity shock σ is

$$\frac{d\tau}{d\sigma} = -\alpha\mathcal{A}\frac{u^{''}(C^*(\alpha(g-\sigma)\mathcal{A})) + u^{''}(C^*(\alpha(g+\sigma)\mathcal{A}))}{2}, \tag{41}$$

which is positive when $u^{'''} > 0$. Similarly, $\frac{d\tau}{dg} > 0$ when $u^{'''} > 0$. So we can state our next proposition:

Proposition 10 *When the distribution of y is exponential, $\sigma(y)$ is affine, and $u^{'''} > 0$ the optimal rate of the financial transaction tax increases with the size of liquidity shocks and the easiness of wealth diversion.*

When wealth diversion is easy, the government has limited ability to tax wealth. To make up for the foregone tax revenue, the government has to rely more heavily on the taxation of financial transactions. Moreover, when liquidity shocks are large, agents are eager to trade in financial markets to smooth out the impact of shocks on their consumption. The optimal tax mechanism takes advantage of that eagerness by raising the financial transactions' tax.

Finally, the envelope theorem applied to W in (17) implies that, in the linear exponential case, the derivative of W with respect to σ is

$$\frac{dW}{d\sigma} = -\alpha \mathcal{A} E_\varepsilon \left[\varepsilon u'(C^*(\alpha(g + \varepsilon\sigma)\mathcal{A})) \right]. \tag{42}$$

Since $C^*(\alpha(g + \sigma)\mathcal{A}) > C^*(\alpha(g - \varepsilon\sigma)\mathcal{A})$ and u is concave, (42) implies that W is increasing in σ. So we can state our next proposition.

Proposition 11 *In the second best allocation, welfare W increases with σ.*

This result might seem surprising, but one must bear in mind that there is no aggregate risk in our model. Thus, σ does not quantify the size of aggregate risk but the size of idiosyncratic risks. Since agents are risk averse, they seek insurance against these idiosyncratic risks. The principal can use this to extract rents from agents. Agents underreporting their type get less insurance than when they truthfully reveal their type. This makes underreporting unattractive and thus relaxes the incentive constraint.

6.4 Impact of the financial transaction tax

When there is no financial transaction tax: To better understand the role of financial transaction taxes, consider the "third best" case in which the financial transaction tax is ruled out, so that $\tau = 0$.

When there is no financial transaction tax, agent $y \geq y_{\min}$ chooses how much to trade at time 1, $\Delta^\varepsilon(y)$, to maximise

$$u(y + \varepsilon\sigma(y) - S(y) - \Delta^\varepsilon(y)) + R(1 - t)(S + \Delta^\varepsilon(y)) - Ty - z_1,$$

The first order condition with respect to $\Delta^\varepsilon(y)$ is

$$u'(C_1^\varepsilon(y)) = R(1 - t),$$

which implies $C_1^\varepsilon(y)$ does not depend on ε. Thus, agents set $\Delta^\varepsilon(y) = \varepsilon\sigma(y)$ to perfectly smooth out liquidity shocks. Consequently, everything is as if there was no shock ($\sigma = 0$). Combining this observation with Proposition 11, stating that welfare increases with σ, we have that welfare is larger with a financial transaction tax than without, in line with Proposition 6. We now establish two surprising results, namely that if government expenditures G are held constant, ruling out the financial transaction tax actually **discourages** entrepreneurship and **reduces** overall investment.

An optimal FTT encourages entrepreneurship: To further evaluate the impact of financial transaction taxation, we now compare occupational choices when $\sigma > 0$ and when $\sigma = 0$. In the linear exponential case, the threshold value $y_{\min}(\sigma)$ is given (29). To compare $y_{\min}(\sigma > 0)$ to $y_{\min}(\sigma = 0)$, we will rely on the following lemma, proved in the appendix:

Lemma 2: *If u is CRRA then*

$$E_\varepsilon \left[\alpha\mathcal{A}(g + \sigma\varepsilon) u^{'} (C^*(\alpha\mathcal{A}(g + \sigma\varepsilon))) \right]$$

decreases with σ.

Lemma 2 implies that $y_{\min}(\sigma > 0) < y_{\min}(\sigma = 0)$, that is the threshold above which agents opt for entrepreneurship is lower with financial transactions taxation than without. Thus we can state our next proposition:

Proposition 12 *When the distribution of y is exponential, $\sigma(y)$ is affine, and u is CRRA, there is more entrepreneurship with financial transactions taxation than without.*

Of course, if we compare with the first best allocation, there is less entrepreneurship in the second best allocation. However, for a given level of government expenditures G, taxing both capital income and financial transactions is more "balanced" than taxing only capital income, and thus introduces less distortions on occupational choices. We now show that this is also true for overall investment.

An optimal financial transaction tax stimulates investment This surprising result is a consequence of the following lemma, also proved in the appendix:

Lemma 3: *If u is CRRA, the function C^* is concave.*

An immediate consequence of Lemma 3 is that, when the FTT is banned, the average consumption of entrepreneurs $C^*(\alpha\mathcal{A}g)$ is higher than when the FTT is allowed, namely $E_\varepsilon [C^*(\alpha\mathcal{A}(g + \sigma\varepsilon))]$. Combined with the observation that entrepreneurship is stimulated by the FTT, we obtain:

Proposition 13 *When the distribution of y is exponential, $\sigma(y)$ is affine, and u is CRRA, there is more investment with an optimal financial transaction tax than without.*

The richest agents are better-off without a financial transaction tax: We have established that, for a given level of government expenditures G, social welfare is increased by taxing financial transactions, which allows to reducing capital income taxes). However this is not a Pareto improvement: we show now that the richest agents are always worse-off when the optimal transaction tax is introduced. To see this, it is useful to consider a geometric representation of agents' expected utilities. The expected utility of type $y \geq y_{\min}$ is:

$$U(y) = U(y_{\min}) + \int_{y_{\min}}^{y} U'(z)dz.$$

By the envelope condition

$$U'(y) = E_\varepsilon \left[\ell(y)(g + \sigma\epsilon)u'(C^*(\alpha(g + \sigma\varepsilon)\mathcal{A}))\right].$$

When the distribution of y is exponential, for agents with $y \geq y_{\min}$, since $C^*(\alpha(g + \sigma\varepsilon)\mathcal{A})$ does not depend on y, $U'(y)$ is constant. Thus, as depicted in Figure 3, indirect utility is affine in y

$$U(y) = U(y_{\min}) + (y - y_{\min})^+ s(\sigma), \forall y, \tag{43}$$

with slope

$$s(\sigma) = E_\varepsilon \left[(g + \sigma\varepsilon)u'(C^*(\alpha\mathcal{A}(g + \sigma\varepsilon)))\right]. \tag{44}$$

Lemma 2 directly implies that when utility is CRRA the slope $s(\sigma)$ is decreasing in σ. As can be seen in Figure 3, the properties that $s(\sigma)$ is decreasing and $y_{\min}(\sigma)$ decreasing in σ yield the next proposition.

Proposition 14 *When the distribution of y is exponential, $\sigma(y)$ is affine, and u is CRRA, there is a threshold of wealth y^* such that agents with $y > y^*$ are better off without financial transaction tax.*

Although the optimal transaction tax encourages entrepreneurship and increases social welfare,, the richest agents are better off without such a tax. This is because the tax enables to better extract rents, which are particularly high for the richest agents, in order to redistribute more and encourage entry into entrepreneurship, two things which don't benefit the richest agents.

6.5 Tax avoidance

The financial transaction tax is based on financial markets transactions. To avoid that tax, agents could try to smooth out consumption without actually conducting financial market transactions. To achieve this, banks could "internalize" transactions, offering agents contracts replicating the trades they would have conducted in secondary markets in the absence of financial transactions taxation. In this context, internalizing banks would offset the flows towards agents hit by negative shocks with flows from agents hit by positive shocks.

In our optimal mechanism approach, such internalization strategies would alter outcomes only if they were unobservable. If, in contrast, banks must disclose the payments received by their customers, then the contracts they offer to replicate secondary market trading are just as observable, and hence taxable, as financial markets transactions.[18] With disclosure, if banks offer contracts to replicate financial markets transactions, then the government can tax these contracts, thus replicating financial transactions taxation. One way to implement this would be to put in place appropriate payment taxes based on the payments induced by the internalization contracts between banks and investors.

7 Conclusion

Tax authorities face a tradeoff between the social benefits of public spendings and redistribution and the distortions induced by taxes. As pointed out by Mirrlees (1971), tax induced distortions arise because of information asymmetry between the tax authority and private agents.[19] In line with Mirrlees (1971) and the new public finance literature (see Golosov, Tsyvinski and Werning, 2006), we assume private agents' skills are unobservable. In contrast with previous literature, we assume privately observed skills affect entrepreneurial profits, which are themselves privately observed. In addition, we assume entrepreneurial profits are subject to liquidity shocks, and that entrepreneurs with better skills undertake projects with larger expected profits, but also larger shocks. Entrepreneurs trade in financial markets to smooth out the impact of shocks on their consumption. Better skilled entrepreneurs, subject to larger shocks, conduct larger trades in financial markets. Hence, these trades provide a signal about privately observed entrepreneurs' skills. This is why the optimal tax system involves a financial transaction tax, complementing capital income tax and wealth tax.

Our analysis partially agrees with opponents of financial transactions taxation, who argue that it reduces savings and liquidity. We just point that other taxes, such as income tax or wealth tax also induce distortions. Therefore, the decision whether to include financial transactions taxes or not reflects the comparison between the marginal distortion induced by those taxes and the marginal distortion induced by other taxes.

In our framework, taxing financial transactions relaxes incentive compatibility constraints and thus reduces distortions. Correspondingly, the optimal tax

[18] Observability of transfers to or from financial institutions is enhanced by the Foreign Account Tax Compliance Act (FATCA), passed by the US Congress in 2010, and later adopted by most OECD countries. This act requires that foreign financial institutions report on the assets held by their account holders. Our emphasis on observability of transfers to or from financial institutions is in line with the point made by Saez and Zucman (2019, page 480): "The key to successful modern income taxation is information reporting by third parties such as employers and financial institutions." While their point applies to income taxation, our analysis shows it is also relevant for financial transactions' taxation.

[19] Otherwise tax revenues can be raised without distortions by personalized lump sum taxes independent of agents' actions.

system, involving financial transactions taxation, induces more entrepreneurship and more investment than a tax system constrained to exclude financial transactions taxation. Yet, while social welfare is improved by financial transactions taxation, the richest agents in the economy are better off if financial transactions taxation is precluded. Because it relaxes incentives constraints, financial transactions taxation reduces rents. This is costly for the richest agents, whose rents are the largest. This may explain the political opposition to financial transaction taxes, which is often very strong in countries like the USA or Switzerland.

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Appendix

Lemma 1: $0 < C^{*\prime} < 1$. **Proof of Lemma 1:** The single crossing property, i.e., the fact that the cross derivative of the right hand side of (23) with respect to d and C is positive implies that $C^{*\prime} > 0$. By differentiating with respect to d the first order condition associated to (23) we obtain

$$u'' C^{*\prime} = u'' + du'''.$$

Since $u''' > 0$, $d > 0$ and $u'' < 0$, this implies that $C^{*\prime} < 1$ and the proof of the lemma is complete.

Proof of Proposition 4: Let

$$U(y, \hat{y})$$

be the expected utility of an entrepreneur of type y who reports $\hat{y}$. We have

$$U(y, \hat{y}) = E_{\varepsilon}\left[u(g(y - \hat{y}) + (\sigma(y) - \sigma(\hat{y}))\varepsilon + C_1^{\varepsilon}(\hat{y})) + C_2^{\varepsilon}(\hat{y})\right].$$

Its derivative with respect to y is

$$U_1(y, \hat{y}) = E_{\varepsilon}\left[(g + \sigma'(y)\varepsilon) u'(g(y - \hat{y}) + (\sigma(y) - \sigma(\hat{y}))\varepsilon + C_1^{\varepsilon}(\hat{y}))\right]$$

and the cross derivative is

$$U_{12}(y, \hat{y}) = E_{\varepsilon}\left[(g + \sigma'(y)\varepsilon)(C_1^{\varepsilon\prime}(\hat{y}) - (g + \sigma'(\hat{y})\varepsilon))u''\right],$$

where we have omitted the argument of u'' for the sake of readibility. Now we compute the derivative of $C_1^{\varepsilon}(\hat{y})$:

$$C_1^{\varepsilon\prime}(\hat{y}) = \alpha C^{*\prime}[(g + \sigma'(y)\varepsilon)[\frac{1 - F(\hat{y})}{f(\hat{y})}]' + \varepsilon\sigma''(\hat{y})\frac{1 - F(\hat{y})}{f(\hat{y})}]$$

Since $0 < C^{*\prime} < 1$ and $\sigma'' \leq 0$, condition (24), namely $[\frac{1-F(\hat{y})}{f(\hat{y})}]' < \frac{1}{\alpha}$, implies that

$$C_1^{\varepsilon\prime}(\hat{y}) \leq \alpha(g + \sigma'(\hat{y})\varepsilon)\left(\frac{1 - F(\hat{y})}{f(\hat{y})}\right)' C^{*\prime} < g + \sigma'(\hat{y})\varepsilon.$$

This implies that $U_{12}(y, \hat{y}) > 0$ for all $y, \hat{y}$.

Now define the function

$$\delta(y, \hat{y}) = U(y, y) - U(y, \hat{y}),$$

for arbitrary $y, \hat{y}$. We can write it as

$$\delta(y, \hat{y}) = \int_{\hat{y}}^{y} \left[U_1(s, y) - U_1(s, \hat{y})\right] ds,$$

and

$$\delta(y,\hat{y}) = \int_{\hat{y}}^{y}\int_{\hat{y}}^{s} U_{12}(s,t)dsdt.$$

Since $U_{12}(y,\hat{y}) > 0$, this double integral is always positive. Thus, for all $y, \hat{y}$, $U(y,y) \geq U(y,\hat{y})$, which establishes Proposition 4.

Proof of Proposition 5: Without loss of generality, and by analogy with the implementation of the first best, we can define two functions

$$S(y) = \frac{S^{+}(y) + S^{-}(y)}{2}.$$

and

$$\Delta(y) = \frac{S^{+}(y) - S^{-}(y)}{2}.$$

so that $S^{\varepsilon}(y)$ rewrites as

$$S^{\varepsilon}(y) = S(y) + \varepsilon\Delta(y). \tag{45}$$

$\varepsilon\Delta(y)$ can be interpreted as the adjustment in savings at time 1, after the realization of the shock ε.

Equations (27) and (45) imply that time 1 consumptions after shocks ε and $-\varepsilon$ are

$$C_1^{\varepsilon}(y) = y + \varepsilon\sigma(y) - S(y) - \varepsilon\Delta(y),$$

and

$$C_1^{-\varepsilon}(y) = y - \varepsilon\sigma(y) - S(y) + \varepsilon\Delta(y), \tag{46}$$

respectively.

The incentive compatibility condition with respect to ε is

$$U^{\varepsilon}(y) = u(y+\sigma(y)\varepsilon - S(y) - \varepsilon\Delta(y)) + C_2^{\varepsilon}(y) \geq u(y+\sigma(y)\varepsilon - S(y) + \varepsilon\Delta(y)) + C_2^{-\varepsilon}(y).$$

This is equivalent to

$$U^{\varepsilon}(y) \geq u(2\sigma(y)\varepsilon + C_1^{-\varepsilon}(y)) + C_2^{-\varepsilon}(y).$$

Thus

$$U^{+}(y) = u(C_1^{+}(y)) + C_2^{+}(y) \geq u(2\sigma(y) + C_1^{-}(y)) + C_2^{-}(y),$$

and

$$U^{-}(y) = u(C_1^{-}(y)) + C_2^{-}(y) \geq u(-2\sigma(y) + C_1^{+}(y)) + C_2^{+}(y).$$

By adding the two conditions and symplifying we obtain

$$u(C_1^{+}(y)) + u(C_1^{-}(y)) \geq u(2\sigma(y) + C_1^{-}(y)) + u(-2\sigma(y) + C_1^{+}(y)).$$

That is

$$u(C_1^{+}(y)) - u(C_1^{+}(y) - 2\sigma(y)) \geq u(C_1^{-}(y) + 2\sigma(y)) - u(C_1^{-}(y)). \tag{47}$$

Since the only other constraint on U^ε is $E_\varepsilon[U^\varepsilon] = U$, as soon as (47) holds it is possible to find U^ε such that the incentive constraints relative to ε are satisfied.

In the last step of the proof, we show that (47) holds iff $\Delta(y) \geq 0$. Define the function $h(x) = u(x + 2\sigma) - u(x)$. We have $h'(x) = u'(x + 2\sigma) - u'(x)$. This is negative because u is concave and $\sigma > 0$. Using the function $h(\cdot)$, (47) can be written

$$h(C_1^+(y) - 2\sigma(y)) \geq h(C_1^-(y)).$$

Since $h(\cdot)$ is decreasing, (47) holds if

$$C_1^+(y) - 2\sigma(y) \leq C_1^-(y).$$

Substituting (46) this is equivalent to

$$y + \sigma(y) - S(y) - \Delta(y) - 2\sigma(y) \leq y - \sigma(y) - S(y) + \Delta(y).$$

That is

$$\Delta(y) \geq 0,$$

which concludes the proof.

Proof of Proposition 6: Consider the case in which there are no liquidity shocks, i.e., $\sigma = 0$, and net savings are increasing in wealth. Consumption in the optimal mechanism is given by (26), which simplifies to

$$C_1(y) = C^* \left(\alpha g \frac{1 - F(y)}{f(y)} \right).$$

Define

$$b(y) \equiv \alpha g \frac{1 - F(y)}{f(y)}.$$

Consumption in the optimal mechanism is such that

$$u^{'}(C_1(y)) = R + b(y) u^{''}(C_1(y)),$$

The tax $t(y)$ paid by type y to the government at time 2 is equal to the difference between the agent's capital income ($RS(y)$) and consumption ($C_2(y)$). Since $C_2(y) = U(y) - u(C_1(y))$, we have:

$$t(y) = RS(y) + u(C_1(y)) - U(y). \tag{48}$$

Since $S(\cdot)$ is increasing, there is a function $T(\cdot)$ of capital income $RS(y)$ that coincides with $t(y)$:[20]

$$T(RS(y)) = t(y), \forall y. \tag{49}$$

Thus, the optimal mechanism can be implemented with just a capital income tax, defined by (49), and without taxing financial transactions.

[20]Moreover T is unique on the relevant range.

Now turn to the case in which agents are exposed to liquidity shocks, but still focus on tax schedules that only depend on capital income, $T(RS^{\varepsilon})$. Can such schedules implement the optimal mechanism? The optimization program of type y agent, in state ε, confronted with $T(RS^{\varepsilon})$ is:

$$\max_{S^{\varepsilon}}[u(gy+\varepsilon\sigma(y)-S^{\varepsilon})+RS^{\varepsilon}-T(RS^{\varepsilon})].$$

The first order condition is:

$$u'(gy+\varepsilon\sigma(y)-S^{\varepsilon})=R[1-T'(RS^{\varepsilon})]. \tag{50}$$

Consider now two types who have the same net savings, s, but face opposite shocks. Type $y^{+}(s)$ is hit by a positive shock and chooses net savings $S^{+}(y^{+}(s))$. Type $y^{-}(s)$ is hit by a negative shock and chooses net savings $S^{-}(y^{-}(s))$. $y^{+}(s)$ and $y^{-}(s)$ are such that

$$S^{+}(y^{+}(s))=S^{-}(y^{-}(s))=s. \tag{51}$$

Since both types face the same marginal tax rate $T'(Rs)$, the first order condition above implies that their consumptions at date 1 must be the same, i.e., $C_1^{+}(y^{+})=C_1^{-}(y^{-})$. By adding consumption and savings, we conclude that their net wealth must also be equal:

$$gy^{+}(s)+\sigma_0+\sigma y^{+}(s)=gy^{-}(s)-\sigma_0-\sigma y^{-}(s). \tag{52}$$

This implies an affine relationship between $y^{+}(s)$ and $y^{-}(s)$:

$$y^{+}(s)\equiv\frac{(g-\sigma)y^{-}(s)-2\sigma_0}{g+\sigma}. \tag{53}$$

If we are to implement the second best allocation it must be that

$$u'(C_1^{\varepsilon}(y))=R+(g+\varepsilon\sigma)\,\alpha\frac{1-F(y)}{f(y)}u''(C_1^{\varepsilon}(y)). \tag{54}$$

Since $C_1^{+}(y^{+})=C_1^{-}(y^{-})$, this implies $\frac{1-F(y^{+})}{f(y^{+})}(1+\sigma)=\frac{1-F(y^{-})}{f(y^{-})}(1-\sigma)$.

Another implication of $C_1^{+}(y^{+})=C_1^{-}(y^{-})$ is

$$(g+\sigma)b(y^{+}(s))=(g-\sigma)b(y^{-}(s)). \tag{55}$$

Finally, since net savings and consumptions are the same for the two agents, their net endowments are also equal, i.e.,

$$(g+\sigma)y^{+}+\sigma_0=(g-\sigma)y^{-}-\sigma_0.$$

That is

$$y^{+}=\frac{(g-\sigma)y^{-}-2\sigma_0}{g+\sigma}\equiv\psi(y^{-}),\forall y^{-}. \tag{56}$$

Substituting (56) into (55),

$$b(\psi(y)) = \frac{g-\sigma}{g+\sigma} b(y), \forall y. \tag{57}$$

Differentiating both sides and using $\psi'(y) = \frac{g-\sigma}{g+\sigma}$,

$$b'(\psi(y)) = b'(y), \forall y.$$

This implies

$$b'(\psi^n(y)) = b'(y), \forall (y, n).$$

Since $\psi(\cdot)$ is a contraction, $\psi^n(y)$ converges to its fixed point $-\frac{\sigma_0}{\sigma}$ as n goes to ∞. By continuity of b' we have

$$\forall y, b'(y) = b'(-\frac{\sigma_0}{\sigma}),$$

which implies that $b(\cdot)$ is affine. Define b_1 and b_0 as follows

$$b_1 = b'(-\frac{\sigma_0}{\sigma}), b_0 = \frac{\sigma_0}{\sigma} b_1.$$

By (57), we have

$$b(y) = b_0 + b_1 y = \frac{\sigma_0}{\sigma} b_1 + b_1 y = \frac{b_1}{\sigma}(\sigma_0 + \sigma y) = \frac{b_1}{\sigma}\sigma(y).$$

That is, $b(y)$ is proportional to the interim shock $\sigma(y)$, which establishes Proposition 7.

Proof of Lemmas 2 and 3: We want to show that $C^*(\cdot)$ is concave and that

$$\psi(\sigma) \equiv E_\varepsilon \left[\alpha \mathcal{A}(g+\sigma\varepsilon) u^{'}(C^*(\alpha \mathcal{A}(g+\sigma\varepsilon)))\right]$$

decreases with σ. Define

$$A^\varepsilon \equiv (g+\sigma\varepsilon)\mathcal{A}.$$

We have

$$\psi(\sigma) = E_\varepsilon \left[\alpha A^\varepsilon u'(C^*(\alpha A^\varepsilon))\right].$$

Define

$$\phi(A) \equiv \alpha A u'(C^*(\alpha A)).$$

We have

$$\psi(\sigma) = E_\varepsilon \left[\phi(A^\varepsilon)\right].$$

Taking the derivative with respect to σ,

$$\psi'(\sigma) = E_\varepsilon \left[\alpha \varepsilon \mathcal{A} \phi'(A^\varepsilon)\right] = \alpha \mathcal{A} \frac{\phi'(A^+) - \phi'(A^-)}{2},$$

which is negative if ϕ is concave.

To conclude the proof, we now show that ϕ is concave. To do so, we take

$$u(C) = \frac{C^{1-\gamma}}{1-\gamma},$$

where $\gamma > 0$. By the definition of $C^*(.)$ in equation (23)

$$C^*(A) = \arg\max_C \left[\frac{C^{1-\gamma}}{1-\gamma} - RC - \mathcal{A}C^{-\gamma}\right].$$

The first order condition is

$$C^{-\gamma} - R + \gamma \mathcal{A} C^{-\gamma-1} = 0.$$

Denoting by $A^*(C)$ the inverse of $C^*(A)$, we have

$$A^*(C) = \frac{1}{\gamma}\left[RC^{1+\gamma} - C\right] > 0.$$

Thus A^* is convex and increasing, which implies that its inverse C^* is concave, and hence establishes Lemma 3.

We now rely on this expression for to prove that ϕ' is decreasing (so that ϕ is concave). By the definition of C^* and of ϕ,

$$\frac{\phi'(A)}{\alpha} = u'(C^*(A)) + Au''(C^*(A))\frac{dC^*}{dA}.$$

Using the function A^*, which is the inverse of $C^*(A)$, we can write the terms on both sides of the equality as functions of C

$$\begin{aligned}
\frac{\phi'(A^*(C))}{\alpha} &= C^{-\gamma} - \gamma \frac{A^*(C)C^{-\gamma-1}}{\frac{dA}{dC^*}} \\
&= C^{-\gamma} - \gamma \frac{\left[RC^{1+\gamma} - C\right]C^{-\gamma-1}}{(1+\gamma)RC^{\gamma} - 1} \\
&= \frac{(1+\gamma)R - C^{-\gamma} - \gamma R + \gamma C^{-\gamma}}{(1+\gamma)RC^{\gamma} - 1} \\
&= \frac{R - (1-\gamma)C^{-\gamma}}{(1+\gamma)RC^{\gamma} - 1}.
\end{aligned}$$

So

$$\frac{\phi'(A^*(C))}{\alpha}(1+\gamma) = \frac{1}{C^{\gamma}}\frac{(1+\gamma)RC^{\gamma} - (1-\gamma^2)}{(1+\gamma)RC^{\gamma} - 1}.$$

That is

$$\begin{aligned}
\frac{\phi'(A^*(C))}{\alpha}(1+\gamma) &= \frac{1}{C^{\gamma}}\left[\frac{(1+\gamma)RC^{\gamma} - 1}{(1+\gamma)RC^{\gamma} - 1} + \frac{\gamma^2}{(1+\gamma)RC^{\gamma} - 1}\right] \\
&= C^{-\gamma} + \frac{\gamma^2 C^{-\gamma}}{(1+\gamma)RC^{\gamma} - 1},
\end{aligned}$$

which is decreasing in C. Since $A^*(C)$ is increasing in C, $\phi'(A)$ is decreasing.

QED