WORKING PAPERS

N° 1359

September 2022

"Downside Risk Aversion vs Decreasing Absolute Risk Aversion: An Intuitive Exposition"

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Abstract

Downside risk aversion (downside RA) and decreasing absolute risk aversion (DARA) are different concepts that describe preferences for which the harm from bearing risk is lessened by an increase in wealth. This note presents some intuitive explanations of the difference between the two concepts using simple lotteries and graphical analysis. All risk-averse utility functions exhibit downside risk aversion, except those that exhibit sufficiently strong increasing absolute risk aversion (IARA). In a sense, downside RA is to be expected: adding downside risk to a baseline lottery is analogous to increasing risk while adding upside risk is analogous to decreasing risk. The difference between the two concepts can be attributed to the use of different measures of the harm from risk bearing: downside RA measures harm using the utility premium and DARA measures harm using the risk premium. The two premia can change at different rates and even in different directions as wealth increases.

Keywords: risk aversion, prudence, risk apportionment, utility premium

Acknowledgement: I acknowledge funding from the U.S. National Science Foundation (award number 1824492) and the French National Research Agency (ANR) under the Investments for the Future program (Investissements d'Avenir, grant ANR-17-EURE-0010). I thank Ellen Considine, a student who asked me to clarify the difference between downside RA and DARA, for stimulating this work. Loïc Berger, David Crainich, Louis Eeckhoudt, François Salanié, Nicolas Treich, the editor, Mohammad Abdellaoui, and an anonymous referee provided helpful suggestions, including the referee's recommendation to include the work of Jindapon et al. (2021).

1. Introduction

What is the relationship between downside risk aversion (downside RA) and decreasing absolute risk aversion (DARA)? Under expected utility, both properties imply the harm from risk bearing is smaller when the individual has greater wealth, but they are not equivalent. Downside RA (Menezes et al. 1980) is equivalent to prudence (Kimball 1990); it is characterized by a positive third derivative of the utility function.[1] DARA is characterized by the property that the risk premium for a lottery is a decreasing function of baseline wealth (Pratt 1964). DARA implies downside RA but the converse is not true. The difference between the concepts arises because they measure the harm from risk bearing differently. Downside RA measures the harm using the utility premium and DARA measures it using the risk premium.

To strengthen intuition about the two concepts, I compare downside RA and DARA from several perspectives. My analysis builds on the work of Eeckhoudt and Schlesinger (2006), who showed that downside RA can be described as a preference for adding risk to the better outcome rather than to the worse outcome of a binary, equal-probability lottery. The first perspective is a graphical derivation of the utility and risk premia, clarifying how the change in these premia with wealth depends on the rate of change of the curvature of the utility function, and hence on the third derivative of utility. The second perspective shows how adding risk to the better or worse branch of an equal-probability binary lottery is analogous to a mean-decreasing contraction or a mean-decreasing spread of the initial lottery, respectively. Since risk aversion is equivalent to a preference for mean-preserving contractions over spreads, adding risk to the upper branch will tend to be preferred.

The paper is organized as follows. Section 2 defines and graphically derives the utility premium and the risk premium. Section 3 illustrates how the utility and risk premia change with baseline wealth for some simple utility functions that differ in their third derivatives. Section 4 shows how adding risk to the better or worse outcomes of a binary lottery is analogous to contracting or spreading the outcomes of the lottery, and hence downside RA is related to an aversion to mean-preserving spreads. Section 5 applies the probability premium introduced by Jindapon et al. (2021) as a measure of the strength of downside RA. Conclusions are presented in Section 5.

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[1] More generally, downside RA is an aversion to increasing the concordance between two risks, one of which is first-order stochastically dominated (a shift of probability from better to worse outcomes) and the other is second-order stochastically dominated (a mean-preserving spread) (Gollier 2021).

2. Utility premium and risk premium

Let $u(x)$ denote an individual's utility of wealth x with $u' > 0$, $u'' < 0$, and u''' exists. He faces a baseline lottery P that yields wealth levels x_0 and x_1 ($x_0 < x_1$) with equal probability. He must add a mean-zero risk $\tilde{\varepsilon}$ to one of the two outcomes. Assume the risk $\tilde{\varepsilon}$ is small enough that changes in expected utility can be adequately approximated using second-order Taylor series.

If the individual is downside RA, he will prefer to add the risk to the better outcome x_1 rather than to x_0 (Eeckhoudt and Schlesinger 2006). At baseline, the individual's expected utility is $\mathrm{E}u(P) = 1/2\ u(x_0) + 1/2\ u(x_1)$. If he adds the risk $\tilde{\varepsilon}$ to the outcome x, his expected utility decreases by $1/2\ \theta(x)$, where $\theta(x) = u(x) - \mathrm{E}u(x + \tilde{\varepsilon}) > 0$ is the utility premium for the risk $\tilde{\varepsilon}$ when added to x (Friedman and Savage 1948, Hanson and Menezes 1971). Using a second-order Taylor-series approximation, $\theta(x) \approx -(\sigma^2/2)\ u''(x)$, where σ^2 is the variance of $\tilde{\varepsilon}$. If $u''(x)$ is constant (utility is quadratic), the utility premium is independent of the branch of the baseline lottery to which $\tilde{\varepsilon}$ is added; if $u'''(x) > 0$ (downside RA), the utility premium is smaller if $\tilde{\varepsilon}$ is added to the branch yielding the larger wealth x_1.

The risk premium for $\tilde{\varepsilon}$ is a monetary measure of the harm from bearing risk (it is the equivalent surplus of $\tilde{\varepsilon}$ when added to x). It is defined as the difference between the expected value and the certainty equivalent. When $\tilde{\varepsilon}$ is added to x, the risk premium $\pi(x) = x - \hat{x} > 0$, where the certainty equivalent $\hat{x}$ is defined by $u(\hat{x}) = \mathrm{E}u(x + \tilde{\varepsilon})$.

Using a second-order Taylor-series approximation, the risk premium is $\pi(x) \approx (\sigma^2/2)\ r(x)$ where $r(x) = -u''(x)/u'(x)$ is the Arrow-Pratt measure of risk aversion at x (Pratt 1964). It decreases with wealth if and only if u is DARA, i.e., $r'(x) < 0$. Noting that $r'(x) = r(x)\ [r(x) - \rho(x)]$, DARA is equivalent to $\rho(x) > r(x)$, where $\rho(x) = -u'''(x)/u''(x)$ is the measure of absolute prudence (Kimball 1990). Hence DARA implies downside RA, but the converse is not true; a utility function can be downside RA without being DARA.[2] For example, for exponential (constant absolute risk aversion, CARA) utility, $u''' > 0$ but $\rho(x) = r(x)$.

Because the risk premium is measured in units of wealth, it is the same for all strategically equivalent utility functions; i.e., for all utility functions representing the same preference ordering over lotteries.[3] In contrast, the utility premium is measured in utility units and so its value can differ

[2] Note also that downside RA is equivalent to $u'(x)$ is convex and DARA is equivalent to $\log[u'(x)]$ is convex. Because $\log(\cdot)$ is concave, DARA is a sufficient but not a necessary condition for downside RA.
[3] Strategically equivalent utility functions are related as positive affine transformations, i.e., u and v are strategically equivalent if and only if $v(x) = h\ u(x) + j$ where $h > 0$ and j are constants.

among strategically equivalent utility functions (although the sign of its change as x increases cannot).

The relationship between downside RA and DARA can be illustrated using Figure 1. Let the risk $\tilde{\varepsilon}$ be a binary lottery with payoffs ε and $-\varepsilon$ with equal probability ($\varepsilon > 0$). The expected utility of $\tilde{\varepsilon}$ added to x equals 1/2 $u(x - \varepsilon)$ + 1/2 $u(x + \varepsilon)$. Graphically, it is the utility at the midpoint of the arc between the points $(x - \varepsilon, u(x - \varepsilon))$ and $(x + \varepsilon, u(x + \varepsilon))$, i.e., the utility where the arc intersects a vertical line at x, illustrated by the dotted lines (for x_0 and x_1).

The risk premium is the change in wealth that has the same effect on utility as the utility premium. Hence the two premia are related by the marginal utility of wealth u'; specifically, $\pi(x) \approx \theta(x)/u'(x)$. Equivalently, the difference in utility associated with a small difference in wealth equals the wealth difference multiplied by the marginal utility of wealth ($\theta(x) \approx \pi(x) \cdot u'(x)$). Although the utility premium depends only on $u''(x)$, the risk premium depends on both $u''(x)$ and $u'(x)$. The risk premia of the risk $\tilde{\varepsilon}$ added alternatively to x_0 and to x_1 are illustrated by the dashed lines in Figure 1.

3. Special cases: quadratic, cubic, and exponential utility

Consider some special cases. First, let $u(x)$ be quadratic.[4] Then $u''(x)$ and the utility premium $\theta(x)$ are independent of x and $u'''(x) = 0$. Quadratic utility is downside risk neutral; the utility premia are equal ($\theta(x_0) = \theta(x_1)$) and the individual is indifferent between adding $\tilde{\varepsilon}$ to x_0 or to x_1. Because $u'(x)$ decreases with x, the risk premium is increasing ($\pi(x_0) < \pi(x_1)$). Quadratic utility is increasing absolute risk averse (IARA), not DARA.

Second, let $u(x)$ be exponential with constant absolute risk aversion (CARA); specifically, $u(x) = -e^{-kx}$ with $k > 0$. Then $u''(x) = -k^2 e^{-kx} = k^2 u(x)$ is decreasing in absolute value as x increases, so the utility premium $\theta(x)$ decreases with x. The marginal utility $u'(x) = k e^{-kx} = -k\, u(x)$ is proportional to $u''(x)$, and hence both the measure of risk aversion $r(x)$ and the risk premium $\pi(x)$ are independent of x. CARA utility is downside RA but not DARA.

For CARA utility, absolute prudence equals absolute risk aversion, $\rho(x) = r(x)$. Hence if a CARA utility function is modified by slightly increasing $u'''(x)$ for all values of x, prudence will exceed risk aversion and the modified utility function will be DARA.

To strengthen intuition, consider a risk-averse utility function $u(x)$ (with $u' > 0$ and $u'' < 0$). Without changing the function at x_0, increase $u'''(x)$ for all values of $x > x_0$ by a small amount. This has the

[4] Quadratic utility is the upward sloping half of a concave (downward-opening) parabola. The criterion $u'(x) > 0$ implies that x must be restricted to values less than some threshold.

effect (for all $x > x_0$) of increasing $u''(x)$ (making it smaller in absolute value and making $u(x)$ straighter) and increasing $u'(x)$ (making $u(x)$ steeper and larger). At x_1, these changes decrease the utility premium $\theta(x_1)$ (because $u''(x_1)$ is smaller in absolute value). Moreover, they decrease the risk premium $\pi(x_1)$ because the utility premium $\theta(x_1)$ is smaller and is divided by a larger marginal utility $u'(x_1)$. Hence increasing $u'''(x)$ increases both the degree of downside RA (measured by $\theta(x_0) - \theta(x_1)$) and the degree of DARA (measured by $\pi(x_0) - \pi(x_1)$).

To illustrate, consider $x < 0$ and let $u(x) = - b\, x^2 + c\, x + (b + c)$ with $b > 0$ and $c > 0$. Marginal utility $u'(x) = - 2\, b\, x + c$ (> 0), $u''(x) = - 2\, b$ (< 0), and $u''' = 0$. Let $x_0 = - 1$. Construct a new utility function $v(x)$ such that $v(x_0) = u(x_0)$, $v'(x_0) = u'(x_0)$, and $v''(x_0) = u''(x_0)$, but $v'''(x) = a > 0$ for all x. The (cubic) utility function v is tangent and osculatory to u at x_0, but its third derivative is larger than $u'''(x)$ for all x. Solve sequentially for $v''(x)$, $v'(x)$, and $v(x)$ using the formula $f(x) = f(x_0) + \int_{x_0}^{x} f'(t)dt$ (by setting $f(x) = v''(x)$, $v'(x)$, and $v(x)$, respectively). This yields $v''(x) = a\, x + (a - 2\, b)$, $v'(x) = a/2\, x^2 + (a - 2\, b)\, x + (a/2 + c)$, and $v(x) = a/6\, x^3 + (a/2 - b)\, x^2 + (a/2 + c)\, x + (a/6 + b + c)$.

For specificity, let $b = 1$, $c = 2$, and consider alternative values $a = 1/2$ and $a = 1$. The functions and their Arrow-Pratt measures of risk aversion $r(x)$ are plotted in Figure 2. For all $x > x_0 = - 1$, the utility $v(x)$ is straighter, steeper, and larger than the utility $u(x)$. For this example, u is IARA ($r(x)$ increases with x). With $a = 1/2$, v is IARA but with $a = 1$, v is DARA ($r(x)$ decreases with x).[5]

To evaluate the utility and risk premia, consider a small risk $\tilde{\varepsilon}$ with variance $\sigma^2 = 1/10{,}000$.[6] If $\tilde{\varepsilon}$ is added to x_0, the utility premium $\theta(x_0)$ and the risk premium $\pi(x_0)$ are approximately the same for u and for v (because the functions have the same slope and curvature at x_0). As shown in Table 1, for both u and for v, $\theta(x_0) = 10 \cdot 10^{-5}$ and $\pi(x_0) = 2.5 \cdot 10^{-5}$. In contrast, if $\tilde{\varepsilon}$ is added to a larger value of x, e.g., $x_1 = - 0.1$, then the utility and risk premia differ between the utility functions. For u, the utility premium is the same as at x_0, because u is quadratic and downside risk neutral. For v, the utility premium at x_1 is smaller than at x_0 and decreases as $v''' = a$ increases; it equals $7.8 \cdot 10^{-5}$ for $a = 1/2$ and $5.5 \cdot 10^{-5}$ for $a = 1$. In contrast, the risk premia change when moving from x_0 to x_1, but not necessarily in the same direction. For u, $\pi(x_1) = 4.5 \cdot 10^{-5}$, larger than $\pi(x_0)$ because u is IARA. With $a = 1/2$, v is IARA (but less so than u) and $\pi(x_1) = 3.2 \cdot 10^{-5}$, larger than $\pi(x_0)$. With $a = 1$, v is DARA and $\pi(x_1) = 2.1 \cdot 10^{-5}$, smaller than $\pi(x_0)$.

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[5] A cubic utility function can be DARA over some values of x and IARA over other values. The statements about v are valid for the values of x considered here, between -1 and 0.

[6] For example, if $\tilde{\varepsilon}$ is a binary lottery with equal chances of gaining or losing 1/100, its variance is 1/10,000.

4. Equivalent lotteries between certainty equivalents

For another perspective on how downside RA implies a preference for adding a risk to the better rather than the worse outcome, consider the (equal-probability) lotteries illustrated in Figure 3. Let P_0 denote the lottery formed by adding $\tilde{\varepsilon}$ to the lower payoff x_0 of the baseline lottery P and P_1 denote the lottery formed by adding $\tilde{\varepsilon}$ to the higher payoff x_1. The expected utility of the lower branch of P_0, $Eu(x_0 + \tilde{\varepsilon})$, equals the utility of its certainty equivalent, $u(x_0 - \pi(x_0))$. Hence the individual is indifferent between P_0 and Q_0, a binary, equal-probability lottery between $x_0 - \pi(x_0)$ and x_1. Similarly, the expected utility of the upper branch of P_1, $Eu(x_1 + \tilde{\varepsilon})$, equals the utility of its certainty equivalent, $u(x_1 - \pi(x_1))$. Hence the individual is indifferent between P_1 and Q_1, a binary, equal-probability lottery between x_0 and $x_1 - \pi(x_1)$. The lotteries Q_0 and Q_1 are equal-probability lotteries between the certainty equivalents of the upper and lower branches of P_0 and P_1, respectively.

Note that Q_0 can be obtained from P by combining a mean-preserving spread (shifting the two endpoints away from each other) with a decrease in mean. In contrast, Q_1 can be obtained from P by combining a mean-preserving contraction (shifting the two endpoints toward each other) with a decrease in the mean.[7]

Substituting Q_0 for P harms the individual in two ways: it decreases the mean payoff and increases risk. In contrast, substituting Q_1 for P decreases the mean payoff but decreases risk. Because risk aversion is equivalent to a preference for mean-preserving contractions (Rothschild and Stiglitz 1970), this suggests that Q_1 is preferred to Q_0 unless the decrease in mean payoff with Q_1 is enough larger than the decrease in mean payoff with Q_0 to offset the difference in risk. The decrease in mean payoff (from P to Q_i) equals one-half the risk premium from adding $\tilde{\varepsilon}$ to x_i, i.e., $E(P) - E(Q_i) = 1/2\ \pi(x_i)$ for $i = 1, 2$. With CARA utility, $\pi(x_0) = \pi(x_1)$, the decreases in mean payoff are equal, Q_0 is a mean-preserving spread of Q_1, and Q_1 is preferred to Q_0. With DARA utility, $\pi(x_0) > \pi(x_1)$, the decrease in mean payoff is larger for Q_0 than for Q_1, and Q_1 is preferred to Q_0 because it is both less risky and has a higher mean payoff.

Q_0 can be preferred to Q_1 only if the decrease in mean payoff for Q_1 is enough larger than the decrease in mean payoff for Q_0 to offset the difference in risk, which requires that utility is sufficiently IARA. Even for the degree of increasing absolute risk aversion associated with quadratic utility, the individual is indifferent between Q_1 and Q_0. For larger degrees of IARA (for which $u''' < 0$)

[7] To obtain Q_0 from P, add $\pi(x_0)/2$ to x_1 and subtract it from x_0, then subtract this amount from both endpoints. To obtain Q_1 from P, subtract $\pi(x_1)/2$ from x_1 and add it to x_0, then subtract this amount from both endpoints.

the harm from bearing a risk at the endpoint x_1 measured by the utility premium $\theta(x_1)$ will exceed the harm from bearing that risk at x_0, $\theta(x_0)$, so Q_0 will be preferred to Q_1 and the individual will be upside RA (downside risk seeking). Although a utility function can be upside RA for some values of *x*, upside RA cannot hold globally. Menegatti (2001) shows that if $u' > 0$, $u'' < 0$, and u''' does not change sign, then $u''' > 0$. Hence if $u''' < 0$ for some values of *x*, it must be greater than zero for other values.

5. Probability premium

The previous sections describe the conditions under which a utility function exhibits downside RA or DARA but not the strength of these properties. Jindapon et al. (2021) propose a probability premium as a measure of the strength of preference for risk apportionment of order n. In the context of downside RA (i.e., risk apportionment of order 3), a necessary condition for this premium to be larger for one utility function than another is for the first utility function to be locally more prudent than the other; a sufficient condition is for the first utility function to be more prudent than the other as characterized by a generalization of greater Ross risk aversion.[8]

For measuring downside RA, the Jindapon et al. probability premium can be defined by modifying the lotteries P_0 and P_1 described in the previous section. Define $P_0'(s)$ and $P_1'(s)$ as lotteries obtained by shifting an amount of probability *s* from the lower to the higher outcomes of P_0 and P_1, respectively. The probability premium *p* is the value of *s* such that the individual is indifferent between $P_0'(s)$ and $P_1'(s)$. Clearly, *p* is between 0 and 1/2: if $s = 0$, $P_1'(0) = P_1$ is preferred to $P_0'(0) = P_0$, and if $s = 1/2$, $P_0'(1/2) = x_1$ is preferred to $P_1'(1/2) = x_1 + \tilde{\varepsilon}$. Setting $Eu[P_0'(p)] = Eu[P_1'(p)]$ implies

$$\left(\tfrac{1}{2}+p\right)u(x_1)+\left(\tfrac{1}{2}-p\right)Eu(x_0+\tilde{\varepsilon})=\left(\tfrac{1}{2}+p\right)Eu(x_1+\tilde{\varepsilon})+\left(\tfrac{1}{2}-p\right)u(x_0).$$

Solving for *p* yields

$$p=\frac{1}{2}\frac{\theta(x_0)-\theta(x_1)}{\theta(x_0)+\theta(x_1)}$$

where $\theta(x) = u(x) - Eu(x + \tilde{\varepsilon})$ is the utility premium defined in Section 2.

[8] Let $u(x)$ and $v(x)$ be utility functions with strictly positive first and third derivatives and strictly negative second derivatives for all $x \in [a, b]$. Let p^u and p^v be the probability premiums for *u* and *v*, respectively (which depend on baseline wealth, sure losses, and mean-zero risks). Then, for all $x, y \in [a, b]$, baseline wealth, sure losses, and mean-zero risks, $-u'''(x)/u''(y) \geq -v'''(x)/v''(y)$ implies $p^u \geq p^v$, which implies $-u'''(x)/u''(x) \geq -v'''(x)/v''(x)$. See Jindapon et al. (2001), Theorem 2.

The values of the probability premium for the example introduced at the end of Section 3 are reported in Table 1. Because the utility function *u* is downside risk neutral, the probability premium is 0. For *v*, the probability premium is positive and increases with the degree of downside risk aversion, which increases with the value of the parameter *a*. For *a* = 1/2, the probability premium is 0.063, which is smaller than the probability premium with *a* = 1, which is 0.145.

6. Conclusion

Though downside RA and DARA both describe conditions under which the harm from risk bearing is lessened by an increase in wealth, the concepts differ. The difference can be understood as arising from the use of alternative measures of harm. Downside RA measures harm by the utility premium and DARA measures harm by the risk premium. The Jindapon et al. (2021) probability premium measures the strength of downside RA as a normalized difference between the utility premia associated with adding a risk to different wealth levels. The utility premium depends on the curvature of *u* (measured by its second derivative *u*″) while the risk premium depends on the ratio of the curvature to the slope (the marginal utility *u*′). If *u*‴ > 0, then *u*″ increases (becomes smaller in absolute value) as *x* increases, hence the utility premium decreases with wealth and the utility function is downside RA. The risk premium may decrease, remain constant, or even increase with wealth depending on how quickly the marginal utility decreases with wealth. If *u*‴ is very small, the marginal utility decreases rapidly with wealth, the risk premium increases, and *u* is IARA. If *u*‴ is large, the marginal utility decreases slowly with wealth, the risk premium decreases, and *u* is DARA.[9] For the intermediate case where *u*″ is proportional to *u*′, the marginal utility decreases at the same rate as *u*″ so the risk premium is independent of wealth and *u* is CARA. Hence downside RA is a necessary but not sufficient condition for DARA.

Downside RA can be understood as closely related to risk aversion, characterized as aversion to mean-preserving spreads. Adding risk to the worse outcome of a binary lottery is equivalent to decreasing the certainty equivalent of the worse outcome, which spreads the certainty equivalents of the two outcomes apart and increases risk. In contrast, adding risk to the better outcome contracts the distance between the certainty equivalents of the two outcomes and decreases risk. The less risky lottery between certainty equivalents that results from adding the risk to the better outcome is preferred, except when the decrease in expected value is large enough to offset the

[9] The effect of an increase in *u*‴ on the rate at which the risk premium $\pi(x)$ declines is reinforced by the fact that the utility premium $\theta(x)$ decreases more rapidly with *x* when *u*‴ is larger.

greater risk that results from adding the risk to the worse outcome; this requires that utility is upside RA ($u''' < 0$), which is a sufficient (but not necessary) condition for the utility function to be IARA.

Funding and competing interests

The author acknowledges funding from the U.S. National Science Foundation (award number 1824492) and the French National Research Agency (ANR) under the Investments for the Future program (Investissements d'Avenir, grant ANR-17-EURE-0010). He has no relevant financial or non-financial interests to disclose.

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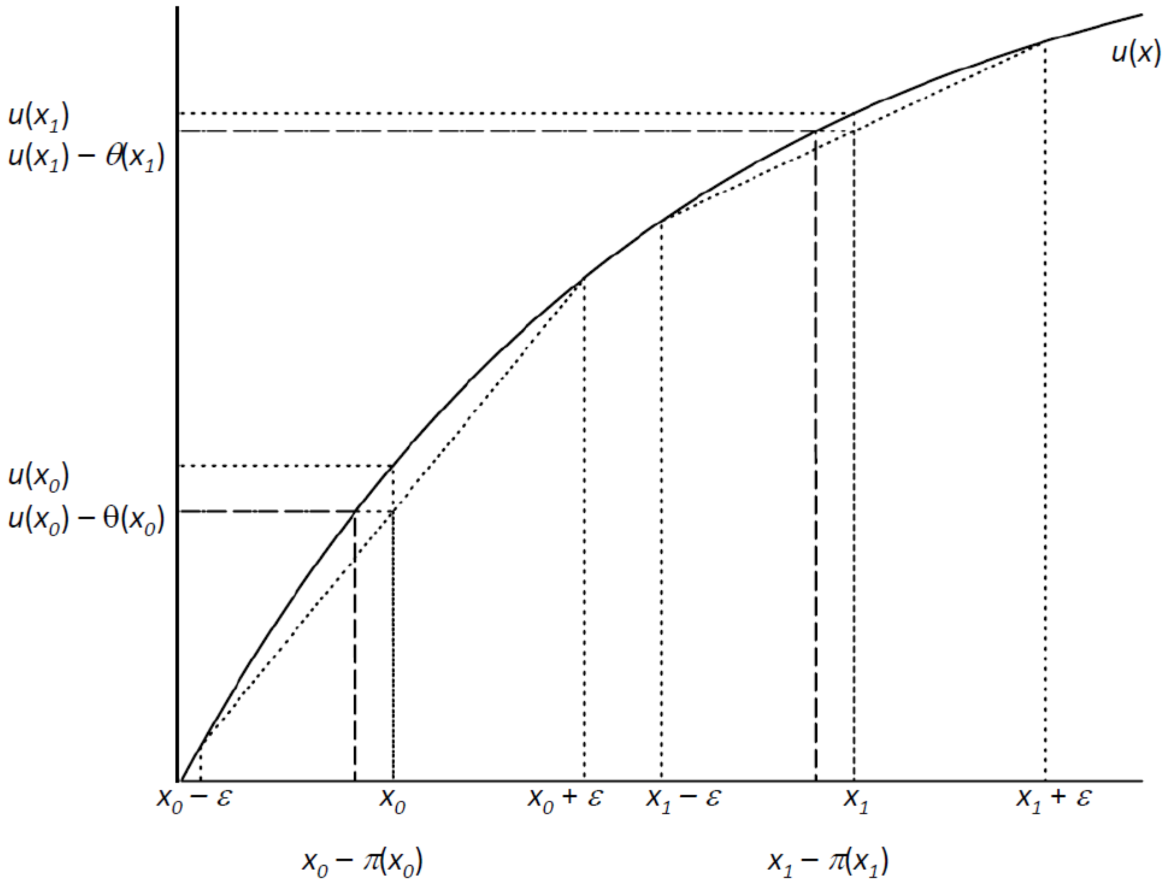


Figure 1. Utility premia (vertical axis) and risk premia (horizontal axis) for adding risk to x_0 and x_1.

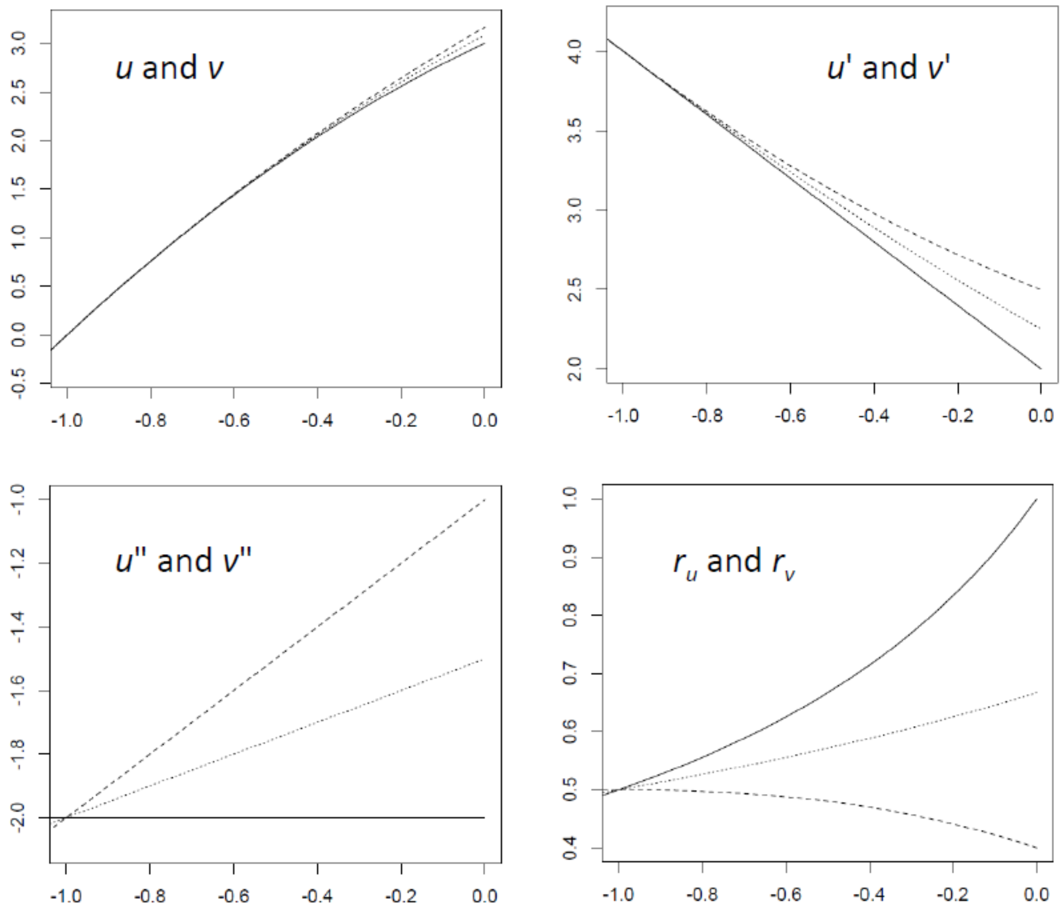
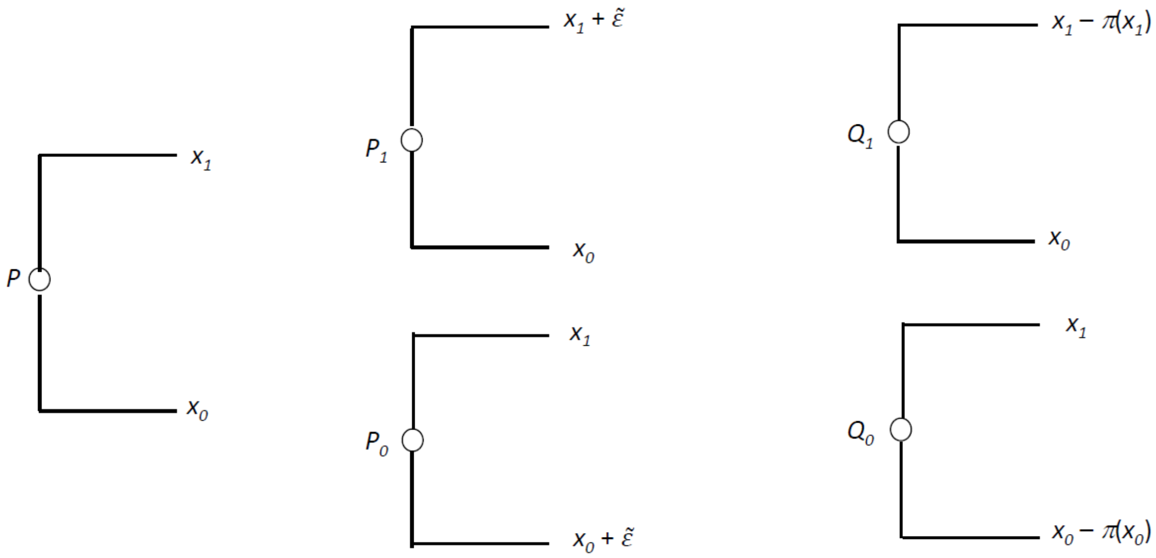


Figure 2. Utility *u*(*x*) (top left), marginal utility *u*'(*x*) (top right), curvature *u*"(*x*) (bottom left), and absolute risk aversion *r*(*x*) = – *u*"(*x*)/*u*'(*x*) (bottom right) for utility function *u* (solid lines), *v* (with *a* = 1/2, dotted lines), and *v* (with *a* = 1, dashed lines).



Note: In all lotteries the probability of each branch is 1/2.

Figure 3. Baseline and modified lotteries.

Table 1. Utility, risk, and probability premia for alternative utility functions

	u	v (a = 1/2)	v (a = 1)
Utility premium			
$\theta(x_0)$	$10 \cdot 10^{-5}$	$10 \cdot 10^{-5}$	$10 \cdot 10^{-5}$
$\theta(x_1)$	$10 \cdot 10^{-5}$	$7.8 \cdot 10^{-5}$	$5.5 \cdot 10^{-5}$
Risk premium			
$\pi(x_0)$	$2.5 \cdot 10^{-5}$	$2.5 \cdot 10^{-5}$	$2.5 \cdot 10^{-5}$
$\pi(x_1)$	$4.5 \cdot 10^{-5}$	$3.2 \cdot 10^{-5}$	$2.1 \cdot 10^{-5}$
Probability premium	0	0.063	0.145