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"Type-contingent Information Disclosure"

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Type-contingent Information Disclosure*

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Abstract

We study a mechanism design problem where the principal can also manipulate the agent's information about a payoff-relevant state. Jointly designing information and allocation rule is proved equivalent to certain multi-dimensional screening problem. Based on this equivalence, when the agent's types are positively-related, full disclosure is proved optimal under regularity conditions; while with negatively-related types, the optimal disclosure policy takes the form of a bad-state alert, which is in general a type-contingent disclosure policy. In a binary environment, we fully characterize the optimal mechanisms and discuss when type-contingent disclosure strictly benefits the principal and its welfare consequences.

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1 Introduction

Consider a retailer (a principal) of goods with various characteristics, and a potential buyer (an agent) whose willingness-to-pay is his private information. As in the standard mechanism design, the retailer desires to screen the agent's type and optimally allocate goods and charge payments, subject to the buyer's incentive compatibility constraint. However, because the goods' characteristics are (perhaps partially) unknown to the buyer, there is another dimension of the design variables for the retailer in addition to the prices, namely, the information about the goods' characteristics to the buyer.

The optimal provision of information has been an important topic in the field of industrial organization (in particular, in the informative advertisement literature), but the analyses are often under restrictive simplifying assumptions. For example, Lewis and Sappington (1991, 1994) and Johnson and Myatt (2006) consider particular forms of information structures,[1] and restrict attention to non-discriminatory disclosure policies; Anderson and Renault (2006) assume the agent has no private information, so that there is no room for discriminatory disclosure. Additionally, the associated pricing schemes considered in these papers are limited to common prices (i.e., no price discrimination) or non-signal-contingent prices (i.e., the price cannot depend on the realizations of the signal).

As another example, imagine an online platform (the principal) who matches sellers and buyers. Based on past transaction data, the platform has a good idea

[1]In Lewis and Sappington (1991, 1994)'s environment, the principal chooses the probability of sending an informative signal ("accuracy" of information structure); he sends an uninformative signal with the complementary probability. Thus, the feasible information structures can be totally ordered according to Blackwell informativeness. Johnson and Myatt (2006) cover this kind of information structure; moreover, they also study the case where the signal is normal-distributed conditional on each state.

about the connection between buyers' demographic data and their willingness-to-pay, which the seller (the agent) does not know. As in the previous example, the principal again can design the information of the agent, as well as the allocations and prices (e.g., registration fees, maximum allowed trade volume, etc.), through which the principal can extract the agent's private information (such as the opportunity cost of selling).

In this paper, we study the principal's problem of optimal design of information and allocation rules. Notice that each of these tools could be useful in extracting the agent's private information: for example, a mechanism may potentially propose different kinds of information to different types of the agent, in order for the agent to self-select the best kind of information depending on his type; and similarly regarding the allocation rules. An important general question is how those two tools are executed in the optimal mechanism. Another (related) question is efficiency: It is well-known that the incentive issue often makes the optimal allocation rule inefficient, due to some rent-efficiency tradeoff. Do we have to suffer from similar inefficiency in terms of the optimal information design? Or, are there some cases where the optimal mechanism is efficient in terms of the provided information or implemented allocation? How do the answers to these questions differ from the cases where only information or allocations are to be designed?

This paper contributes to a better understanding of the above set of important questions as follows. The first contribution is methodological: In Theorem 1, we show that the principal's problem of designing both information and allocation rule is equivalent to certain multi-dimensional screening problem (where only an allocation rule is designed). As explained more in detail in the corresponding part of the paper, this result (i) greatly simplifies the search of the optimal mechanism,

(ii) shows a robustness of our results to some alternative modeling (e.g., in terms of timing and information structures), and (iii) suggests that some known results in multi-dimensional screening, and more generally linear-programming tools, can be useful in identifying the optimal mechanism.

At the same time, as is well known, it is difficult in multi-dimensional screening to obtain general analytical / closed-form results. In this sense, our equivalence result may also be seen as (somewhat unfortunately) uncovering the limitation or difficulty in very general analyses of the environment without parametric restrictions. Therefore, the later sections consider some restricted environments, in order to illustrate some economic implications of the model.

Those substantive results in later sections (Theorems 3, 4) are our second contribution. First, consider the case with positively related types, that is, if a type of an agent has a higher valuation than another type in one state, then the same ranking holds in any other states. A possible interpretation may be that the type is a *vertical* characteristics of the agent. In this case, under certain regularity conditions, it is always optimal for the principal to fully disclose the state information to the agent. Therefore, there is no inefficiency in terms of information provision. Accordingly, the standard results regarding the optimal allocations hold "state by state".

Second, consider the case with negatively related types, that is, if a type of an agent has a higher valuation than another type in one state, the opposite ranking holds in the other state. A possible interpretation may be that the type is a *horizontal* characteristics of the agent. In this case, under certain conditions, the optimal mechanism assigns different information structures to different types. Roughly, those types who have low values in one state are assigned information structures that reveal that "bad" state (with the optimally chosen probability); and those types whose values do not vary much across states are assigned uninformative information structures. Inefficiency occurs for the former, and in particular, for some non-extreme types who are assigned *imprecise* bad-state-alerting information structures: With a positive probability, such a type does not receive a bad-state alert even if the state is indeed bad for him; and in that case, this type's ex post payoff can be negative. Conversely, sufficiently extreme types tend to receive fuller information, and the types whose values do not vary much with the state do not care about the information.

The above two cases allow for an arbitrary (finite) number of types but with parametric assumptions with which tractable analyses are possible. The other result (Theorem 2) provides a full characterization of the optimal mechanisms but in a binary environment. There, we show that the optimal mechanism involves either (i) full disclosure of the state information for both types, (ii) partial but non-type-contingent (i.e., "public") disclosure, or (iii) type-contingent disclosure. Though admittedly restrictive, the exhaustive characterization enables us to discuss a policy-relevant question as to whether / when type-contingent information disclosure is welfare-enhancing or not, and more importantly, the logic behind it. The advancement of the digital technology continuously reduces the cost of individualized / targeted advertisement and information disclosure, based on the data accumulated by large-scale platforms. As a consequence, the mode of advertisement has been experiencing some shift from the classical "public" advertisement to individualized / targeted advertisement. Those platforms often claim that such individualization / targeting technologies are welfare-enhancing, as each consumer can be assigned the most valuable information for him; on the other hand, this raises some general concerns, including the possibility that such individualization / targeting technologies may be new sources of consumers' rent extraction, possibly with inefficient information suppression. Our analysis shows that both of the arguments are relevant: Under some parameter values, the optimal disclosure policy is type-contingent, and it provides more precise information to the agent (e.g., the optimal non-type-contingent policy is partially informative (to both types), while the optimal type-contingent policy offers the same information structure to one of the types, and full information to the other type). Under alternative parameter values, the optimal disclosure policy is type-contingent, and it provides less precise information (e.g., the optimal non-type-contingent policy is fully informative (to both types), while the optimal type-contingent policy offers full information to only one of the types). Although which sets of parameter values are more relevant is an empirical question, our analysis sheds some light on this important policy discussion.

1.1 Related literature

Most papers in the mechanism design literature focus on the design of allocation rules but without the design of information; and most papers in the information design literature focus on the design of information but without the design of allocation rules. Our paper belongs to the small yet important intersection of those two strands of literature. Most of the papers in this intersection study the "sequential screening" problem, where the allocation (in particular the monetary transfer) cannot depend directly on the information disclosed by the principal to the agent.[2] See, for example, Bergemann and Pesendorfer (2007), Eső and Szentes (2007a), Krähmer and Strausz (2015), Li and Shi (2017), Bergemann, Bonatti, and Smolin (2018), Guo, Li, and Shi (2020), Wei and Green (2020), Zhu (2021). In our case, the allocation can be contingent on that disclosed information (in other words, that disclosed information is *contractible*). A possible interpretation is that the principal observes the same disclosed information as the agent, which is perhaps reasonable in certain applications such as informative advertisement and certification.[3] Note, however, that Proposition 1 in Eső and Szentes (2007b) is for the case where disclosed information is contractible. Another related paper is Yamashita (2018) who considers the same problem as ours but restricting attention to public / non-type-contingent disclosure policies. We show that some results in that paper carries over even if the principal can potentially offer private / type-contingent disclosure policies. Wei and Green (2020) show that their optimal mechanism continues to be optimal even when the disclosed information is contractible. That case (i.e., their original setting except with contractible disclosed information) is mathematically equivalent to ours with positively related information (Section 5.1), where full disclosure is shown to be optimal.[4]

Another potential application is a pricing problem of information ("data" or "advice"), as recently studied by Eső and Szentes (2007b), Babaioff, Kleinberg, and Paes Leme (2012), Bergemann and Bonatti (2015), Bergemann, Bonatti, and

[2]In other words, the information revealed to the agent cannot be observed by the principal, so that it becomes the agent's private information and can only be elicited through incentive compatible allocation rules.

[3]On the other hand, the sequential-screening modeling would be more appropriate in the context of experience goods, where the principal's information disclosure is through allowing the agent to "experience" the good before the final purchase decision.

[4]Thus, the mechanism we show optimal is different from what Wei and Green (2020) show optimal. This is not a contradiction, as those two mechanisms are both optimal in that case. Indeed, the realized allocations are equivalent in these two mechanisms.

Smolin (2018), and Yang (2020). These papers consider the situation where the principal is a revenue-maximizing seller of information and the agent is a potential user of that information (such as a retailer planning targeted advertisement based on consumer data). The difference from our environment is two-folds. First, they consider a rich set of actions that the agent takes after buying (or not buying) information. For example, Yang (2020) assumes that the agent is a seller who chooses a monopoly price given data about the demand function. Babaioff, Kleinberg, and Paes Leme (2012) and Bergemann, Bonatti, and Smolin (2018) consider general action spaces. Our model can be interpreted as a model with a binary action space (for example, our agent may be a retailer in a market with exogenously given price, who decides simply whether to serve the market or not, given data about the demand function). Second, in these papers, the agent's action is not contractible, that is, the price of information cannot depend on the agent's action. In our model, the agent's action is contractible: in the above interpretation that the agent is a retailer, the price of data may be paid if and only if he serves the market in the end (for example, as a result of ex post individual rationality or limited liability). In addition, these papers, except for Babaioff, Kleinberg, and Paes Leme (2012), assume that the price of information cannot vary with the content of information itself, and in this sense, those papers are more closely related to the sequential-screening literature (see above).[5] Bergemann, Bonatti, and

[5]The difference is crucial in the sense that the first-best outcome is possible in Bergemann, Bonatti, and Smolin (2018) if such contingent contracts are allowed in their setting. Yang (2020), on the other hand, shows that the optimal non-contingent mechanism in his setting remains to be optimal even among contingent ones. His setting with contingent mechanisms is essentially equivalent (modulo technical differences such as finite and continuous types) to ours with positively related information (Section 5.1), where full disclosure is shown to be optimal, a different mechanism from what Yang (2020) shows optimal. This is not a contradiction, as those two mechanisms are both optimal in that case.

Smolin (2018) explain that their non-contingent model would be more appropriate for some types of data selling practices such as *data appends*, while our contingent model would be more appropriate for other types of data selling practices such as *marketing lists*.[6]

2 The Model

We consider a single-good environment with one principal (seller) and one agent (buyer). The agent has a privately-known *type* $t \in T$, where $|T| < \infty$ and $F(t)$ denotes the probability of each type t. The information controlled by the principal, called the *state*, is denoted by $\theta \in \Theta$, where $|\Theta| < \infty$ and $F_0(\theta)$ denotes the probability of each state θ. We assume that t and θ are independently distributed.

The agent's valuation for the object is given by $v(\theta, t)$, while the principal's valuation is zero. Let $q \in [0, 1]$ be the probability of assigning the object to the agent, and $p \in \mathbb{R}$ be the transfer from the agent to the principal. Then, the principal's payoff is p, and the agent's payoff is $q \cdot v(\theta, t) - p$.

An information disclosure policy is defined as (M, G), where M collects all possible signals that the agent can receive, and $G(\theta)[m] : \Theta \to \Delta(M)$ is a measurable mapping which specifies the probability of sending signal m under state θ. Let $\Lambda \in \Delta(M)$ be the unconditional distribution of m induced by (M, G), where $\Lambda(m) = \sum_{\theta' \in \Theta} F_0(\theta')G(\theta')[m]$. Then, on observing any m (which occurs with strictly positive probability), one can form a posterior belief $\Psi_m(\theta) \in \Delta(\Theta)$

[6] Marketing lists is a type of data selling practices, which, for example, allows a retailer planning targeted advertisement to obtain a list of consumers who belong to a specific category.

about the state through Bayesian updating:

$$\Psi_m(\theta) = \frac{F_0(\theta)G(\theta)[m]}{\sum_{\theta' \in \Theta} F_0(\theta')G(\theta')[m]}, \quad \forall \theta \in \Theta.$$

The principal selects, and can commit to, different disclosure policies and allocation rules for different types of the agent, without knowing the state. The timing is as follows:

1. The principal commits to $\big((M_t, G_t), (q_t(m), p_t(m))_{m \in M_t}\big)_{t \in T}$.
2. The agent learns his own type t, and reports $\hat{t} \in T$.
3. The agent observes a signal $m \in M_{\hat{t}}$ with probability $\Lambda_{\hat{t}}(m)$, and is offered $(q_{\hat{t}}(m), p_{\hat{t}}(m))$.
4. The agent decides to accept or reject the offer.

Thus, the principal's problem, denoted by (P), is defined by:

$$(P) \qquad \max_{M,G,q,p} \sum_t \sum_\theta \left(\int_{m \in M_t} p_t(m) dG_t(\theta)[m] \right) F_0(\theta) F(t)$$

$$s.t. \quad \forall t, t' :$$

$$\int_{m \in M_t} \sum_\theta \big(q_t(m)v(\theta,t) - p_t(m)\big) \Psi_{m|t}(\theta) d\Lambda_t(m)$$

$$\geq \int_{m \in M_{t'}} \max \left\{ \sum_\theta \big(q_{t'}(m)v(\theta,t) - p_{t'}(m)\big) \Psi_{m|t'}(\theta), 0 \right\} d\Lambda_{t'}(m).$$

For $t \neq t'$, the constraint is called $BIC_{t \to t'}$. Note that the constraint with $t' = t$ corresponds to the (post-each-m) participation constraint. We denote the value of the problem by $V(P)$.

We underline that, in this problem, the agent's posterior belief about θ is conditional on both his report $\hat{t}$ and the realized signal $m \in M_{\hat{t}}$. On observing $m \in M_t \cap M_{t'}$, $\Psi_{m|t}$ could be different from $\Psi_{m|t'}$. This is because $\Psi_{m|\hat{t}}$ is jointly determined by F_0 and $G_{\hat{t}}$.

3 Equivalence to a Multi-Dimensional Screening Problem

One of our main theoretical findings is that the above problem is equivalent to a multi-dimensional screening problem (with $|T|$ types of the agent, where each type is a $|\Theta|$-dimensional vector).

Theorem 1. Let $V(P^+)$ denote the value of the following multi-dimensional screening problem:

$$\begin{aligned}
(P^+) \quad & \max_{x,\tau} && \sum_{t \in T} \tau(t) F(t) \\
& s.t. && \forall t,\ \forall t' \neq t: \\
& IC_{t \to t'} && \sum_{\theta} \big(x(\theta,t) w(\theta,t)\big) - \tau(t) \geq \sum_{\theta} \big(x(\theta,t') w(\theta,t)\big) - \tau(t'); \\
& IR_t && \sum_{\theta} \big(x(\theta,t) w(\theta,t)\big) - \tau(t) \geq 0; \\
& && 0 \leq x(\theta,t) \leq \mathbf{1}, \quad \tau(t) \in \mathbb{R},
\end{aligned}$$

where $w(\theta,t) = F_0(\theta) v(\theta,t)$. Then, we have $V(P^+) = V(P)$.

This result greatly simplifies problem (P). In the original problem, we are to decide (i) an information disclosure policy for each type, and (ii) an allocation for each type and each signal realization. Potentially, a signal space may be very complicated so that the choice of allocations in (ii) could be a daunting problem.

Our equivalence result shows that the dimension of the problem is much limited. The joint choice problem of (i) and (ii) reduces to a problem of assigning a single probability of buying for each type and each state, and the constraints are also standard-looking incentive compatibility, participation, and feasibility constraints. In particular, known useful techniques in the literature of (linear) multidimensional screening can be useful (Rochet, 1987; Vohra, 2011; Kos and Messner, 2013; Daskalakis, Deckelbaum, and Tzamos, 2013), and more generally, those in (finite-dimensional) linear programming.

Besides, as emphasized in the proof, Theorem 1 says that the principal can achieve the best possible payoff in terms of the agent's information structure with respect to θ. More specifically, the principal's expected payoff in the optimal mechanism coincides with the case where the allocation fully contingent on θ *without disclosing it to the agent*. This implies robustness of the optimal allocation with respect to some variations in timing/information structure, such as in the order/amount of communication before the buyer's purchase decision (e.g.: What if the principal can first disclose some public information and then the agent reports his type, which is followed by further type-contingent disclosure? Or more general dynamic communication schemes?), or in the timing of the buyer's purchase decision (e.g.: What if the principal can ask the agent to make partial commitment before disclosing information? What if the principal can ask some advance payment before signal realization?). The theorem states that they do not matter, as far as the principal's objective is concerned.[7]

[7] Of course, the practical implementation of the optimal mechanism may depend on those

On the other hand, this equivalence result shows a limit in the problem of jointly designing information disclosure and allocation rules: As is well-known for multi-dimensional screening, unless restrictive assumptions are made (such as small number of types (Armstrong and Rochet, 1999) or certain homogeneity structures (Armstrong, 1996)), obtaining closed-form solutions is in general prohibitive. In Section 4, we focus on a binary environment and fully characterize the optimal mechanism. Arguably it is a significant restriction, but nevertheless the results we obtain provide a rich set of economic insights in terms of optimal disclosure policies (in particular, whether/why it is optimal to disclose different information depending on reported types), optimal mechanisms (in particular, how it changes with the disclosed information), and economic welfare (in particular, under what conditions type-contingency is welfare-improving).

We finish this section by providing the proof of our equivalence result, followed by the remark explaining that Theorem 1 is not a mere consequence of some known "revelation principle" results. In particular, through an example we emphasize the role of independence between θ and t for Theorem 1.

Proof of Theorem 1. To show $V(P^+) \geq V(P)$, we consider a relaxed problem of (P) where the agent's participation constraints are at the interim stage (more precisely, at the stage where the agent knows t but the signal has not been realized yet). Then, type-t agent's expected payoff by pretending to be type t' can be

timing and information structures.

written as

$$\sum_\theta \Big(\int_{m\in M_{t'}} \big(q_{t'}(m)v(\theta,t) - p_{t'}(m)\big) dG_{t'}(\theta)[m] \Big) F_0(\theta)$$
$$= \sum_\theta \Big(v(\theta,t) \int_{m\in M_{t'}} q_{t'}(m) dG_{t'}(\theta)[m] - \int_{m\in M_{t'}} p_{t'}(m) dG_{t'}(\theta)[m] \Big) F_0(\theta).$$

We can see that, based on the agent's reported type t', the principal effectively chooses a probability of selling $x(\theta,t') := \int_{m\in M_{t'}} q_{t'}(m)dG_{t'}(\theta)[m] \in [0,1]$ and payment $\tau(\theta,t') := \int_{m\in M_{t'}} p_{t'}(m)dG_{t'}(\theta)[m]$ for each realization of θ. Thus, the relaxed problem is defined as follows:

$$\begin{aligned}
\max_{x,\tau} \quad & \sum_{\theta,t} \tau(\theta,t)F_0(\theta)F(t) \\
s.t. \quad & \forall t,\ \forall t' \neq t : \\
IC_{t\to t'} \quad & \sum_\theta \big(x(\theta,t)v(\theta,t) - \tau(\theta,t)\big)F_0(\theta) \geq \sum_\theta \big(x(\theta,t')v(\theta,t) - \tau(\theta,t')\big)F_0(\theta); \\
IR_t \quad & \sum_\theta \big(x(\theta,t)v(\theta,t) - \tau(\theta,t)\big)F_0(\theta) \geq 0; \\
& 0 \leq x(\theta,t) \leq \mathbf{1}, \quad \tau(\theta,t) \in \mathbb{R}.
\end{aligned}$$

Clearly, the value of this problem is an upper bound of the original problem. Moreover, this relaxed problem is equivalent to the problem (P^+) by simply setting $\tau(t) = \sum_\theta \tau(\theta,t)F_0(\theta)$: Because of quasi-linearity, it is enough to only consider the expected (in θ) payment.

To show $V(P^+) \leq V(P)$, we first examine Problem (P) more in detail, and obtain an equivalent problem (which we call Problem (P^*) below).

Suppose that in the solution to (P) we have $q_t(m) \in (0,1)$ for some $t \in T$ and some $m \in M_t$. We replace m by two signals m^0 and m^1 such that: for all $\theta \in \Theta$,

$$G_t(\theta)[m^0] = (1 - q_t(m))G_t(\theta)[m], \quad G_t(\theta)[m^1] = q_t(m)G_t(\theta)[m].$$

Then, we have (i) $\Psi_{m^0|t} = \Psi_{m^1|t} = \Psi_{m|t}$, and (ii) $\Lambda_t(m^0) = (1 - q_t(m))\Lambda_t(m)$, $\Lambda_t(m^1) = q_t(m)\Lambda_t(m)$. Let $q_t(m^0) = 0$, $p_t(m^0) = 0$, $q_t(m^1) = 1$, $p_t(m^1) = \frac{p_t(m)}{q_t(m)}$. One can easily check that this modification satisfies all constraints in (P), and achieves the same expected revenue. Thus, without loss of generality, we only need to consider the solution to (P) satisfying $q_t(m) \in \{0, 1\}$ for any t and any m. Let M_t^1 and M_t^0 collect all such m^1 and m^0, respectively.

The next step is to show that, for each t, we only need one "buy" signal, denoted by m_t^1, to induce $q = 1$, and one "not buy" signal, denoted by m_t^0, to induce $q = 0$. Particularly, fixed arbitrary t, we define the following information disclosure policy: for all $\theta \in \Theta$,

$$G_t(\theta)[m_t^1] = \int_{m^1 \in M_t^1} dG_t(\theta)[m^1], \quad G_t(\theta)[m_t^0] = \int_{m^0 \in M_t^0} dG_t(\theta)[m^0].$$

Then, we have

$$\Lambda_t(m_t^1) = \int_{m^1 \in M_t^1} d\Lambda_t(m^1), \quad \Lambda_t(m_t^0) = \int_{m^0 \in M_t^0} d\Lambda_t(m^0);$$
$$\Psi_{m_t^1|t} = \frac{1}{\Lambda_t(m_t^1)} \int_{m^1 \in M_t^1} \Psi_{m^1|t} d\Lambda_t(m^1), \quad \Psi_{m_t^0|t} = \frac{1}{\Lambda_t(m_t^0)} \int_{m^0 \in M_t^0} \Psi_{m^0|t} d\Lambda_t(m^0).$$

Let $q_t(m_t^0) = 0$, $p_t(m_t^0) = 0$, $q_t(m_t^1) = 1$, $p_t(m_t^1) = \frac{1}{\Lambda_t(m_t^1)} \int_{m^1 \in M_t^1} p_t(m^1) d\Lambda_t(m^1)$.

Immediately, this modification doesn't change the total expected revenue. Because

$$
\begin{aligned}
&\int_{m\in M_t}\sum_\theta \big(q_t(m)v(\theta,t)-p_t(m)\big)\Psi_{m|t}(\theta)d\Lambda_t(m)\\
=&\int_{m^1\in M_t^1}\sum_\theta \big(1\cdot v(\theta,t)-p_t(m^1)\big)\Psi_{m^1|t}(\theta)d\Lambda_t(m^1)\\
=&\left(\sum_\theta 1\cdot v(\theta,t)\Psi_{m_t^1|t}(\theta)-p_t(m_t^1)\right)\Lambda_t(m_t^1),
\end{aligned}
$$

the modified solution satisfies all $BIC_{t\to t'}$ such that $t'\neq t$. The remaining is to show that agent with each $t'\neq t$ won't pretend to have type t in the modified solution. If $\sum_\theta 1\cdot v(\theta,t')\Psi_{m_t^1|t}(\theta)-p_t(m_t^1)<0$, then we have

$$
\max\left\{\sum_\theta 1\cdot v(\theta,t')\Psi_{m_t^1|t}(\theta)-p_t(m_t^1),0\right\}\Lambda_t(m_t^1)=0,
$$

which means $BIC_{t'\to t}$ is satisfied. If $\sum_\theta 1\cdot v(\theta,t')\Psi_{m_t^1|t}(\theta)-p_t(m_t^1)\geq 0$, because

$$
\begin{aligned}
&\int_{m\in M_{t'}}\sum_\theta \big(q_{t'}(m)v(\theta,t')-p_{t'}(m)\big)\Psi_{m|t'}(\theta)d\Lambda_{t'}(m)\\
\geq&\int_{m\in M_t}\max\left\{\sum_\theta \big(q_t(m)v(\theta,t')-p_t(m)\big)\Psi_{m|t}(\theta),0\right\}d\Lambda_t(m)\\
\geq&\int_{m\in M_t}\sum_\theta \big(q_t(m)v(\theta,t')-p_t(m)\big)\Psi_{m|t}(\theta)d\Lambda_t(m)\\
=&\left(\sum_\theta 1\cdot v(\theta,t')\Psi_{m_t^1|t}(\theta)-p_t(m_t^1)\right)\Lambda_t(m_t^1)\\
=&\max\left\{\sum_\theta 1\cdot v(\theta,t')\Psi_{m_t^1|t}(\theta)-p_t(m_t^1),0\right\}\Lambda_t(m_t^1),
\end{aligned}
$$

the modified solution also satisfies $BIC_{t'\to t}$. We conclude that it is without loss of generality to only consider a subset of candidate solutions where (i) each type t

is offered an information structure with two signals $\{m_t^1, m_t^0\}$, inducing posterior beliefs $\mu_t^1 := \Psi_{m_t^1|t}$ and $\mu_t^0 := \Psi_{m_t^0|t}$, respectively[8]; (ii) the allocation rule satisfies $q_t(m_t^1) = 1$ and $q_t(m_t^0) = 0$. Let $v(\mu, t) = \sum_\theta v(\theta, t)\mu(\theta)$ for any $\mu \in \Delta(\Theta)$. Then, problem (P) can be written as:

$$\begin{aligned}
\max_{p,\Lambda,(\mu_t^1,\mu_t^0)_{t\in T}} \quad & \sum_{t\in T} p_t(\mu_t^1)\Lambda_t(\mu_t^1)F(t) \\
s.t. \quad & \forall t, \forall t' \neq t: \\
& \big[v(\mu_t^1, t) - p_t(\mu_t^1)\big]\Lambda_t(\mu_t^1) \geq \max\big\{v(\mu_{t'}^1, t) - p_{t'}(\mu_{t'}^1), 0\big\}\Lambda_{t'}(\mu_{t'}^1); \\
& v(\mu_t^1, t) - p_t(\mu_t^1) \geq 0; \\
& \Lambda_t(\mu_t^1) + \Lambda_t(\mu_t^0) = 1, \quad \Lambda_t(\mu_t^1), \Lambda_t(\mu_t^0) \geq 0; \\
& \mu_t^1\Lambda_t(\mu_t^1) + \mu_t^0\Lambda_t(\mu_t^0) = F_0, \quad \mu_t^1, \mu_t^0 \in \Delta(\Theta).
\end{aligned}$$

Notice that the "max" function can be removed, and we get an equivalent problem:

$$\begin{aligned}
(P^*) \qquad \max_{\substack{p,\ \Lambda, \\ (\mu_t^1,\mu_t^0)_{t\in T}}} \quad & \sum_{t\in T} p_t(\mu_t^1)\Lambda_t(\mu_t^1)F(t) \\
s.t. \quad & \forall t,\ \forall t' \neq t: \\
BIC_{t\to t'} \quad & \big[v(\mu_t^1, t) - p_t(\mu_t^1)\big]\Lambda_t(\mu_t^1) \geq \big[v(\mu_{t'}^1, t) - p_{t'}(\mu_{t'}^1)\big]\Lambda_{t'}(\mu_{t'}^1); \\
EPIR_t \quad & v(\mu_t^1, t) - p_t(\mu_t^1) \geq 0; \\
& \Lambda_t(\mu_t^1) + \Lambda_t(\mu_t^0) = 1, \quad \Lambda_t(\mu_t^1), \Lambda_t(\mu_t^0) \geq 0; \\
& \mu_t^1\Lambda_t(\mu_t^1) + \mu_t^0\Lambda_t(\mu_t^0) = F_0, \quad \mu_t^1, \mu_t^0 \in \Delta(\Theta).
\end{aligned}$$

Now, we show that the value of this problem (P^*) is weakly higher than that of

[8]It is possible to have $\mu_t^1 = \mu_t^0$.

Problem (P^+). To see this, let $(\mathbf{x}, \tau)$ be a solution to (P^+). For any $t \in T$, we define $\Lambda_t(\mu_t^1) = \sum_{\theta \in \Theta} F_0(\theta) x(\theta, t)$, which belongs to $[0, 1]$. Let $\Lambda_t(\mu_t^0) = 1 - \Lambda_t(\mu_t^1)$. If $\Lambda_t(\mu_t^1) \in (0, 1)$, then for any $t \in T$ and $\theta \in \Theta$, we define $\mu_t^1(\theta) = \frac{F_0(\theta)x(\theta,t)}{\Lambda_t(\mu_t^1)}$, $\mu_t^0(\theta) = \frac{F_0(\theta)[1-x(\theta,t)]}{\Lambda_t(\mu_t^0)}$. It is easy to check that $\mu_t^1, \mu_t^0 \in \Delta(\Theta)$ and $\mu_t^1 \Lambda_t(\mu_t^1) + \mu_t^0 \Lambda_t(\mu_t^0) = F_0$. We also define $p_t(\mu_t^1) = \frac{\tau(t)}{\Lambda_t(\mu_t^1)}$. If $\Lambda_t(\mu_t^1) = 1$ (or 0), type t receives no additional information about θ, and gets the object with probability 1 (or 0) and pays the price $\tau(t)$ (or 0). Notice that for any t, $\hat{t}$ we have

$$\begin{aligned}
&\left[v(\mu_{\hat{t}}^1, t) - p_{\hat{t}}(\mu_{\hat{t}}^1)\right] \Lambda_{\hat{t}}(\mu_{\hat{t}}^1) \\
=&\Big[\sum_{\theta \in \Theta} \mu_{\hat{t}}^1(\theta) v(\theta, t) - p_{\hat{t}}(\mu_{\hat{t}}^1)\Big] \Lambda_{\hat{t}}(\mu_{\hat{t}}^1) \\
=&\Big[\sum_{\theta \in \Theta} \frac{F_0(\theta) x(\theta, \hat{t})}{\Lambda_{\hat{t}}(\mu_{\hat{t}}^1)} v(\theta, t) - \frac{\tau(\hat{t})}{\Lambda_{\hat{t}}(\mu_{\hat{t}}^1)}\Big] \Lambda_{\hat{t}}(\mu_{\hat{t}}^1) \\
=&\sum_{\theta \in \Theta} x(\theta, \hat{t}) w(\theta, t) - \tau(\hat{t}).
\end{aligned}$$

Then, due to $IC_{t \to t'}$ and IR_t in (P^+), the way we construct $p, \Lambda, (\mu_t^1, \mu_t^0)_{t \in T}$ satisfies all $BIC_{t \to t'}$ and $EPIR_t$ in (P^*). Thus, the value of (P^*) is weakly higher than the value of (P^+). □

Remark 1. From the proof of Theorem 1, we know that, under the independence assumption between θ and t, it is innocuous to focus on (1) restrictive disclosure policies with a single "buy" signal, and (2) interim participation constraints which are more tractable. It is worth noting that these "ideal" properties hinge on the independence assumption. Consider the following example:

Example 1. Assume $\theta, t \in \{1, 2\}$, and θ is almost (but not fully) perfectly correlated with t: $\Pr(\theta = k \mid t = k) = 1 - \varepsilon$ for $k = 1, 2$, for some positive ε. Assume

$v(\theta, t) = 1$ if $\theta = t = 1$; otherwise $v(\theta, t) = 2$.

We can prove that with sufficiently small ε, the solution to (P) in this payoff environment extracts almost full surplus. Consider the following mechanism: if the agent reports $t = 1$, then for each realized θ, the agent observes a fully-revealing signal θ, and is offered the allocation $(q_1(\theta), p_1(\theta)) = (1, \theta)$ (i.e., the mechanism has *two* signal realizations at which the agent buys (at different prices)); if the agent reports $t = 2$, then he observes no signal and is assigned $(q_2, p_2) = (1, 2-\varepsilon)$. By reporting $t = 2$, type 1 will be offered a price $(2 - \varepsilon)$ which is higher than his expected valuation $(1 + \varepsilon)$, and thus will not buy and get zero payoff, which is the same as truth-telling. Type 2 gets ε by telling the truth, while gets $\varepsilon(2 - 1) + (1 - \varepsilon)(2 - 2) = \varepsilon$ by reporting $t = 1$. As a result, the mechanism is incentive compatible and achieves full-surplus extraction as ε vanishes.

We now see why we cannot restrict attention to mechanisms with a *single* "buy" signal in Example 1. In such a mechanism, in order to achieve the revenue not lower than the above, the agent who reports t must be recommended "buy" at price $\tau(t)$ with some large-enough probability. It follows that $\tau(1) = \tau(2) \leq 1 + \varepsilon$ in order to satisfy the incentive compatibility and post-each-signal participation constraints. As ε vanishes, the surplus left to type 2 converges to 1, making it impossible for the principal to achieve the same revenue as above.

To see why post-each-signal participation constraints cannot be moved to the interim stage when θ and t are correlated, let $\varepsilon = 0.1$ in Example 1. With interim participation constraints, the principal can extract full surplus by fully revealing the state and applying the Crémer-McLean mechanism. While with post-each-signal participation constraints, in any mechanism which extracts the surplus from type 1, there must be some signal $\tilde{m}$ which recommends "buy" at a price strictly less than 2. This means type 2 will earn some rent; otherwise pretending to be $t = 1$ and buying only at signal $\tilde{m}$ would be a profitable deviation.

To conclude, the coincidence of $V(P)$ and $V(P^+)$ holds under independence of θ and t, but not necessarily under their correlation.

4 Binary Case

In this section, we consider a binary environment, where $T = \Theta = \{0, 1\}$. To slightly simplify the notation, let $f = F(1)$ and $f_0 = F_0(1)$. Thanks to the simplicity brought by this assumption, we can fully characterize the optimal mechanism.[9] We also characterize the optimal mechanism with non-type-contingent (or "public") disclosure, and compare those mechanisms, especially in terms of the welfare.

Even in the binary case, the structure of the optimal mechanism is quite rich. In order to better understand it, we first consider two benchmark mechanisms. First, we characterize the optimal mechanism with full disclosure. With full disclosure, the problem essentially reduces to a mechanism design problem, separately for each realized θ. The optimal mechanism for each θ is given based on a standard argument. Next, we characterize the optimal mechanisms with non-type-contingent disclosure. The difference from the full-disclosure benchmark highlights the principal's motif of controlling information, in order to attain higher expected revenue. However, at least for certain parameters, we explain that the principal's power is

[9] In this section, we use the linear programming approach to characterize the optimal mechanism due to the equivalence result in Theorem 1. In the Online Appendix, we provide a *generalized concavification approach* to characterize the solution to (P), which is closely related to the graphical approach for solving the standard Bayesian persuasion model.

limited if he only consider non-type-contingent mechanisms: It may be better to disclose more information for some type, while less information for the other. Then, we would be ready to explain how the optimal mechanism with type-contingent disclosure can leverage the power of controlling individual information.

In what follows, we focus on the case where $v(\theta, 1) > v(\theta, 0)$ if $\theta = 1$ and $v(\theta, 1) < v(\theta, 0)$ if $\theta = 0$ (i.e., type t is the higher type in state $\theta = t$). That is, depending on θ, the order of types reverses. Without this order reversion, the optimal mechanism is always with full disclosure (Theorem 3 in Section 5.1).

4.1 Benchmark 1: Optimal mechanism with full disclosure

To explain the optimal mechanism we obtain later in Section 4.3, it is useful to first consider two benchmark cases. The first benchmark is the optimal mechanism with full disclosure, in the following sense. Imagine that the principal fully discloses θ, and then designs the optimal mechanism for each θ. This benchmark helps us understand some intuition in the optimal mechanism without the full-disclosure assumption.

The principal's optimal mechanism within this class is given as follows:[10]

$$\begin{aligned} \Pi^F \;=\; & \max_{(q_t(\theta), p_t(\theta))_{\theta,t}} && f_0[fp_1(1) + (1-f)p_0(1)] + (1-f_0)[fp_1(0) + (1-f)p_0(0)] \\ & \text{sub. to} && v(\theta,t)q_t(\theta) - p_t(\theta) \geq \max\{0, v(\theta,t)q_{t'}(\theta) - p_{t'}(\theta)\}, \;\; \forall \theta, t, t'. \end{aligned}$$

Clearly, the problem is fully separable with respect to θ. Thus, the solution is

[10]Note that, because of the restriction, the equivalence result in the previous section (Theorem 1) does not generally hold in this context. The same remark applies to the next benchmark case.

$(q_t^F(\theta), p_t^F(\theta))_{\theta,t}$, where for each θ, $(q_t^F(\theta), p_t^F(\theta))_t$ solves:

$$\begin{aligned}\Pi^F(\theta) &= \max_{(q_t,p_t)_t} \; fp_1 + (1-f)p_0 \\ &\quad \text{sub. to} \quad v(\theta,t)q_t - p_t \geq \max\{0, v(\theta,t)q_{t'} - p_{t'}\}, \; \forall t,t',\end{aligned}$$

and $\Pi^F = f_0\Pi^F(1) + (1-f_0)\Pi^F(1)$.

If $\theta = 1$, then $t = 1$ is the higher type, and thus, the optimal allocation is either (i) $(q_1, p_1) = (q_0, p_0) = (1, v(1, 0))$, (ii) $(q_1, p_1) = (1, v(1, 1))$ and $(q_0, p_0) = (0, 0)$, or (iii) $(q_1, p_1) = (q_0, p_0) = (0, 0)$.

If $\theta = 0$, then $t = 0$ is the higher type, and thus, the optimal allocation is either (i) $(q_1, p_1) = (q_0, p_0) = (1, v(0, 1))$, (ii) $(q_0, p_0) = (1, v(0, 0))$ and $(q_1, p_1) = (0, 0)$, or (iii) $(q_1, p_1) = (q_0, p_0) = (0, 0)$.

Thus, we obtain the following.

Proposition 1. The optimal mechanism with full disclosure attains:

$$\Pi^F = f_0 \max\{v(1,0), fv(1,1), 0\} + (1-f_0) \max\{v(0,1), (1-f)v(0,0), 0\}.$$

4.2 Benchmark 2: Optimal mechanism with non-type-contingent disclosure

Next, consider another benchmark case where the principal discloses some (not necessarily full) information about θ, regardless of the agent's type. Then, the principal assigns an allocation as a function of the agent's type report and the disclosed information. A full-disclosure policy is a special case. We provide an example in the appendix (Example 4) where the principal can save the agent's information rent by only partially disclosing information about θ.

The optimal mechanism with non-type-contingent disclosure is more complicated than the full-disclosure case. Still, it has the same kind of simplicity in the sense that the allocation part of the mechanism is fully separable with respect to the (non-type-contingent) information disclosed to the agent.

More specifically, fix an arbitrary non-type-contingent disclosure policy, denoted by $\Lambda \in \Delta([0,1])$. A signal realization $\mu \sim \Lambda$ is observed by the agent of any type. It is well-known that it is without loss to identify μ with the posterior belief for $\theta = 1$ given that signal μ. Moreover, with this identification, any feasible Λ can be represented as a distribution over $[0,1]$ such that $\int \mu d\Lambda(\mu) = f_0$.

For each μ, the optimal allocation given μ solves:

$$\begin{aligned} V^N(\mu) = \max_{(q_t(\mu),p_t(\mu))_t} \quad & f p_1(\mu) + (1-f)p_0(\mu) \\ \text{sub. to} \quad & v(\mu,t)q_t(\mu) - p_t(\mu) \geq \max\{0, v(\mu,t)q_{t'} - p_{t'}\}, \ \forall t, t', \end{aligned}$$

where $v(\mu,t) = \mu v(1,t) + (1-\mu)v(0,t)$, and the optimal Λ is given by:

$$\begin{aligned} V^N = \max_{\Lambda} \quad & \int V^N(\mu) d\Lambda(\mu), \\ \text{sub. to} \quad & \int \mu d\Lambda(\mu) = f_0. \end{aligned}$$

Let $\mu^* \in (0,1)$ be such that $v(\mu^*,1) = v(\mu^*,0) \equiv v^*$, that is, μ^* is the agent's belief with which both types have the same expected valuation. In what follows, we only consider the case with $\mu^* > f_0$, but a similar result holds with $\mu^* < f_0$. Let Λ^F denote the full-disclosure signal distribution (i.e., $\Lambda(1) = f_0$ and $\Lambda(0) = 1 - f_0$), and let Λ^* denote a binary-support distribution on $\{0, \mu^*\}$ with $\Lambda(\mu^*) = \frac{f_0}{\mu^*}$ and

$\Lambda(0) = 1 - \frac{f_0}{\mu^*}$.

Proposition 2. The optimal Λ is either Λ^F or Λ^*.

In case the optimal Λ is Λ^F, the optimal allocation coincides with the case under full disclosure, attaining Π^F.

In case the optimal Λ is Λ^*, the optimal allocation is as follows. (i) Given posterior $\mu = 0$, the optimal allocation coincides with the case under full disclosure with $\theta = 0$. (ii) Given posterior $\mu = \mu^*$, the optimal allocation is $(q_1, p_1) = (q_0, p_0) = (1, v^*)$.[11]

Proof. See Appendix A.2. □

4.3 Optimal mechanism with type-contingent disclosure

Now we are ready to explain the optimal mechanism with type-contingent disclosure. The following example shows that, in some cases, type-contingent disclosure can attain a strictly higher expected revenue than with full or non-type-contingent disclosure.

Example 2. Assume $f_0 = f = \frac{1}{2}$, and $v(\theta, t)$ satisfies:

$v(\theta, t)$	$\theta = 1$	$\theta = 0$
$t = 1$	3	-3
$t = 0$	2	2

This is the same setting as Example 4 in the appendix. Consider the following type-contingent disclosure policy:

- For $t = 0$: no disclosure, and $(q, p) = (1, 2)$.

[11] As is clear from the proof, the optimal Λ is not full disclosure only if $v^* \geq 0$.

- For $t = 1$: full disclosure, and $(q, p) = (1, 3)$ if $\theta = 1$ ($(0, 0)$ o.w.).

In this mechanism, it is clearly optimal for the agent with $t = 0$ to truthfully report his type and accepts the trade with price of 2. For $t = 1$, truth-telling implies expected payoff 0, while pretending to be $t = 0$ yields expected payoff:

$$\frac{1}{2}(3 - 2) + \frac{1}{2}(-3 - 2) = -2.$$

Therefore, the mechanism is also incentive compatible for $t = 1$.

This is the first-best mechanism for the principal, because the trade allocation is efficient and the entire surplus is extracted by the principal (i.e., the agent does not earn any information rent). As a comparison, in the optimal full-disclosure mechanism, the allocation is efficient but the agent earns positive information rent; in the optimal non-type-contingent mechanism, the agent earns zero information rent but the allocation is not efficient (due to non-full disclosure). □

Solving the optimal mechanism with type-contingent disclosure is fundamentally different from the two benchmark cases, because now the problem is non-separable. For instance, imagine that type t is assigned full disclosure. If t' is also assigned full disclosure, then we can separately solve the problem for each θ. However, if t' is assigned less disclosure (e.g., no disclosure), then his incentive compatibility is based on his average payoff in each state, which implies a restriction *jointly* on the two allocation rules (one for $\theta = 0$ and the other for $\theta = 1$). This means that we can no longer solve each problem separately.

However, thanks to the equivalence result (Theorem 1), we can characterize the optimal mechanism by solving a multi-dimensional screening problem.

Interestingly, the optimal mechanism with type-contingent disclosure has certain similar features to the one with non-type-contingent disclosure. In particular, the optimal disclosure policy Λ_t for each t assigns a positive probability only on two of $\mu \in \{0, \mu^*, 1\}$. Again, we focus on the case with $f_0 < \mu^*$, which implies that Λ_t must be supported on either $\{0, \mu^*\}$ or $\{0, 1\}$.

Theorem 2. Whether the optimal disclosure policy is type-contingent or not depends on the parameter.

In case the optimal mechanism is non-trivially type-contingent, it is $\Lambda_t = \Lambda^F$ for some t and $\Lambda_{t'} = \Lambda^*$ for $t' \neq t$. The allocation for type t is either $(q_t, p_t) = (1, v(\theta, t))$ or $(0, 0)$ when $\theta \in \{0, 1\}$ is disclosed. The allocation for type t' is (i) $(q_t, p_t) = (1, v(0, t))$ or $(0, 0)$ when $\mu = 0$ is disclosed, while (ii) $(q_t, p_t) = (1, v^*)$ when $\mu = \mu^*$ is disclosed.[12]

In case the optimal disclosure policy is de facto non-type-contingent, it coincides with what Proposition 2 describes, and so does the optimal allocation rule.

Proof. See Appendix A.3. □

4.4 Welfare consequence of type contingency

An interesting applied question is the potential welfare effect of type-contingent disclosure policies. Thanks to the advancement of the digital technology and the data accumulated by large-scale platforms, the cost of individualized / targeted advertisement and information disclosure is lowered. As a consequence, the mode of advertisement has been experiencing some shift from the classical "public" advertisement to individualized / targeted advertisement. It is an important policy-

[12] Recall that Λ^* has a binary support on $\{0, \mu^*\}$.

relevant question whether such individualization is welfare-enhancing. Theorem 2 suggests that it depends on the parameters, and provides a basic logic behind the welfare comparison.

To better understand the comparison, it is useful to observe that there are in general two kinds of inefficiency in the optimal mechanism (with or without type-contingency). The first kind of inefficiency happens when the buyer buys the good with a positive probability even though his ex post valuation is negative: we call it "*inefficient deal*". Such inefficiency can happen only if the buyer is assigned non-full disclosure, because otherwise the buyer would not buy the good as long as he is fully informed of the state where his valuation is negative. The second kind happens when the buyer does not buy the good even though his ex post valuation is positive (but is lower than the price), which is the standard "*inefficient exclusion*" in monopoly pricing.

The following example shows that type-contingency can be beneficial or adversarial to the total welfare, depending on the parameter values.

Example 3. Consider a variant of the previous example with $f = f_0 = \frac{1}{2}$ and:

$v(\theta, t)$	$\theta = 1$	$\theta = 0$
$t = 1$	3	-3
$t = 0$	2	x

where $x \in (-1, 1)$.

The optimal type-contingent mechanism assigns full disclosure to type 1, while partial disclosure to type 0. More precisely, this partial disclosure takes the form of a "0-alert", where the type-0 buyer's posterior is made either $\mu = \mu^*$ or $\mu = 0$. The buyer with type 1 buys the good if and only if $\theta = 1$. The buyer with type

0 buys the good given $\mu = \mu^*$, while he buys the good given $\mu = 0$ if and only if $v(0,0) = x \geq 0$.[13] Therefore, this mechanism achieves the first-best total welfare if $x \geq 0$, but induces an inefficient deal given $(\theta, t) = (0,0)$ if $x < 0$. More specifically, when $x < 0$, the buyer with $t = 0$ buys the good with a positive probability given $(\theta, t) = (0,0)$ even though $v(0,0) = x < 0$. On the other hand, exclusion inefficiency does not exist.

What about the optimal mechanism in the class of *non*-type-contingent mechanisms? If $x \geq 0$, then the optimal non-type-contingent mechanism assigns the same "0-alert" partial disclosure, but to both types. Therefore, an inefficient deal occurs for *type 1* here, while the first best efficiency is achieved under type-contingency.

If $x < 0$, then the optimal non-type-contingent mechanism assigns full disclosure to both types, and both types buy the good if and only if $\theta = 1$ (at price 2). Therefore, the mechanism achieves the first-best total welfare here, while an inefficient deal occurs under type-contingency. □

The parameters in Example 3 are carefully chosen so that the optimal mechanisms with or without type-contingency only exhibit "inefficient deals" but not "inefficient exclusion". More generally, with only the possibility of inefficient deals, the total welfare can simply be compared based on the information disclosed to the buyer: Under the parameters where the optimal non-type-contingent mechanism involves full disclosure for both types, type-contingency is (weakly) adversarial to the total welfare, because full disclosure happens only for one type there; while under the other parameters where the optimal non-type-contingent mechanism involves partial disclosure for both types, type-contingency is (weakly) welfare-enhancing (because one of the types is assigned full disclosure there).

[13] See Case 1.3 in the proof of Theorem 2.

With more general parametrization where the optimal mechanisms may exhibit *both* "inefficient deals" and "inefficient exclusion", the comparison is more subtle. Even if one mechanism provides more information than another, it is not clear if the total welfare is higher in the first mechanism. However, in the binary environment, this exclusion inefficiency basically makes type-contingency more likely welfare-enhancing. To explain this, consider the same parameter values as in Example 3 with $x < 0$, except that now type 1 is much more likely than type 0: $f > \frac{2}{3}$. It can be shown that the optimal type-contingent mechanism is the same as in Example 3 (with $f = \frac{1}{2}$). The optimal non-type-contingent mechanism still assigns full disclosure for both types, but now, it only trades with type 1 (with price 3) given $\theta = 1$, rather than trading with both types (with price 2). That is, the mechanism now induces *inefficient exclusion*. As a consequence, a trade happens if and only if $(\theta, t) = (1, 1)$. The optimal type-contingent mechanism achieves higher total welfare because $v^* > 0$[14] despite its inefficient deal issue.

The conclusion holds more generally in the binary environment. More precisely, if type-contingency (at least partially) addresses the inefficient exclusion issue in the optimal non-type-contingent mechanism, then it is welfare-enhancing even if this is achieved at the cost of causing the inefficient deal issue (as explained above). On the other hand, (at least partially) eliminating the inefficient deal in the optimal non-type-contingent mechanism always entitles type-contingency to improve the total welfare (as in Example 3). The only case for type-contingency to be welfare-reducing is when it purely causes the inefficient deal issue[15]

Proposition 3. Type-contingency can be adversarial to the total welfare, but

[14] Recall $v^* = \mu^* v(1, t) + (1 - \mu^*) v(0, t)$ for both $t = 0, 1$.

[15] It is worth noting that in the binary environment, type-contingency would not cause more severe inefficient exclusion issue than the optimal non-type-contingent mechanism.

only when (i) the optimal type-contingent mechanism exhibits inefficient deal, (ii) the optimal non-type-contingent mechanism eliminates it by fully-disclosing information (for both types), and (iii) the optimal non-type-contingent mechanism does not involve inefficient exclusion.[16]

The proof is by checking all the cases exhaustively, and is omitted.

5 More General Cases

As discussed before, because of the multi-dimensional screening nature, it is difficult to obtain closed-form solutions in more general environments. Some of the results in the binary case continue to hold, however, in non-binary cases. In this section, we summarize those results. Further investigation of general cases are important but beyond the scope of the paper.

5.1 Positively related cases

Here, we consider the case where T is totally ordered, and $t < t'$ implies $v(\theta, t) < v(\theta, t')$ for *all* θ. That is, there is no preference reversal: a higher type is always associated with the higher valuation for every θ. Although mathematically it is a restrictive assumption, many economic applications can be in this class. For example, imagine that the agent is an intermediary who buys a good from the principal (a seller) and sells it to end-consumers; θ represents the *vertical* quality of the good (whose information is controlled by the principal / seller); and t is a parameter representing noisy information about the end-consumers' demand.

[16]Note that, in this case, the optimal non-type-contingent mechanism achieves the first-best total welfare.

In this case, the optimal policy is full disclosure, under a standard regularity condition. As is clear in the proof, the argument is rather standard once we consider the problem (P^+). In this sense, the result illustrates the usefulness of Theorem 1, connecting the original problem (P) and the simplified problem (P^+).

Theorem 3. Assume that (i) T is totally ordered (denoted by $T = \{1, 2, \ldots, N\}$), (ii) $t < t'$ implies $v(\theta, t) < v(\theta, t')$ for all θ, and (iii) F admits monotone virtual values: for each θ, the agent's virtual valuation

$$\gamma(\theta, v) \equiv v(\theta, t) - (v(\theta, t+1) - v(\theta, t)) \frac{\sum_{\tau > t} F(\tau)}{F(t)}$$

is non-decreasing in t.

Then, full disclosure is an optimal disclosure policy. Furthermore, the optimal allocation given each θ sets $q(\theta, t) = 1$ if $\gamma(\theta, v) > 0$, while $q(\theta, t) = 0$ if $\gamma(\theta, v) < 0$.

Proof. See Appendix A.4. □

5.2 Negatively related cases

Here, we consider the case with binary $\Theta = \{0, 1\}$ (with $f_0 = F_0(1)$), and totally ordered T: $v(0, t)$ is strictly increasing in t, and $v(1, t)$ is strictly decreasing in t. Furthermore, we assume that they are "concavely" related: There is a concave function $f : \mathbb{R} \to \mathbb{R}$ such that $v(1, t) = f(v(0, t))$ for all t.

For example, imagine that a product seller (the agent) sells through a platform (the principal). The platform can provide some information about the consumers' *horizontal* characteristics (parameterized by $\theta \in \{0, 1\}$) to the seller. For simplicity, let R be the seller's (sufficiently large, exogenously fixed) revenue in case a trade happens. However, to make the trade happen, the seller must make some pre-trade investment in order to, for example, customize the product to meet the consumers' needs. Assume that such investment incurs a cost $c(|\theta - t|)$, where c is non-negative, increasing and convex, $t \in [0,1]$ denotes the seller's pre-investment position, and $|\theta - t|$ measures the amount of required adjustment. Then the net profit to the seller is $v(\theta, t) = R - c(|\theta - t|)$.

This negatively-related and concave structure of the payoff environment guarantees that, the solution to problem (P^+) possesses what we called a "rotating" property. Based on this observation, again thanks to Theorem 1, we can show that the resulting optimal disclosure policy exhibits some intuitive property.

Let Λ be a binary distribution over $[0,1]$ such that $\sum_\mu \mu\Lambda(\mu) = f_0$. As before, we say that Λ is a θ-*alert* information structure if $\theta \in \text{supp}\Lambda$. Note that any two θ-alert information structures can be ordered with respect to its Blackwell informativeness: the one with a higher probability of signal θ is more informative (and also, obviously, more informative than the fully uninformative one, identified by Λ with $\Lambda(f_0) = 1$).[17]

Theorem 4. Assume that $\Theta = \{0,1\}$, $T = \{1, \ldots, N\}$, $v(0,t)$ is strictly increasing in t, $v(1,t)$ is strictly decreasing in t, and that there is a concave function f such that $v(1,t) = f(v(0,t))$ for all t.

In the optimal mechanism, (i) every t is assigned either a 0-alert, 1-alert, or fully-uninformative information structure, and there exists t^* such that (ii) for $t < t' \leq t^*$, t is assigned more informative information structure than t' is; and (iii) for $t > t' \geq t^*$, t is assigned more informative information structure than t' is.

[17] An information structure is more *(Blackwell-)informative* than another information structure if the latter is obtained by adding some noise ("garbling") to the former.

In the associated allocation, if type t is assigned a fully-uninformative information structure, then he always buys; while if assigned a 0-alert (1-alert) information structure, then he buys if and only if the signal based on Λ_t is not $\mu = 0$ ($\mu = 1$).

Proof. See Appendix A.5. □

The key first step of the proof examines the simplified problem (P^+), and shows that (i) only local incentive compatibility constraints are relevant (i.e, the non-local incentive constraints can be ignored without loss); and (ii) in the solution, either $x(0,t)$ or $x(1,t)$ (or both) is 1 (Lemma 1). Once this step is established, the rest is to verify that the information disclosure policies and allocations in Theorem 4 can be constructed from the solution to (P^+). For example, if $x(0,t) = 1$ and $x(1,t) < 1$ for some t, then that type is assigned a 1-alert; while $x(0,t) = x(1,t) = 1$ corresponds to a fully-uninformative information structure.

6 Conclusion

We study a mechanism design problem where the principal can also control the agent's knowledge about a payoff relevant state. We analyze how the revenue-maximizing principal can properly manipulate the information disclosure policy and the allocation rule so as to screen the agent's type and extract the surplus as best possible. We show that the principal's problem is equivalent to a multi-dimensional screening problem (where only an allocation rule is designed), which contributes methodologically to the literature by greatly simplifying the search for optimal mechanisms, and, at the same time, sheds light on the essence of joint design of information and allocation.

We also study the features of optimal disclosure policies in more restrictive but economically interesting payoff environments. With positively-related types (such as vertical characteristics of the agent), full disclosure is optimal under regularity conditions; while with negatively-related types (such as horizontal characteristics of the agent), the optimal disclosure policies take the form of bad-state alerts which are type-contingent in general. Furthermore, we provides a full characterization of the optimal mechanisms in a binary environment, which facilitates the analyses as to when type-contingent disclosure strictly benefits the principal, and the welfare consequences of jointly using information structure and allocation rule as screening tools.

A Omitted materials

A.1 Example in Section 4.2

Example 4. Assume $f_0 = f = \frac{1}{2}$, and $v(\theta, t)$ satisfies:

$v(\theta,t)$	$\theta = 1$	$\theta = 0$
$t = 1$	3	-3
$t = 0$	2	2

Assuming full disclosure, the optimal allocation for $\theta = 0$ is $(q_0, p_0) = (1, 2)$ and $(q_1, p_1) = (0, 0)$, yielding $\Pi^F(0) = 1$; and the optimal allocation for $\theta = 1$ is $(q_t, p_t) = (1, 2)$ for both t, yielding $\Pi^F(1) = 2$. Thus, the principal's expected revenue is $\frac{3}{2}$. Note that the allocation is efficient (i.e., maximizing the total surplus), and the agent earns some information rent (when $(\theta, t) = (1, 1)$).

Now consider the following "partial" non-type-contingent disclosure. If $\theta = 1$, then the principal discloses $\mu = \frac{5}{6}$ to the agent, regardless of the agent's type t; if $\theta = 0$, then the principal discloses either $\mu = 0$ (with probability $\frac{4}{5}$) or $\mu = \frac{5}{6}$ (with probability $\frac{1}{5}$), regardless of the agent's type t. Note that μ can interpreted as the buyer's posterior for $\theta = 1$ given the principal's message μ: If $\mu = 0$ is sent, then it must be that $\theta = 0$; while if $\mu = \frac{5}{6}$ is sent, then the posterior for $\theta = 1$ is $\frac{\frac{1}{2} \cdot 1}{\frac{1}{2} \cdot 1 + \frac{1}{2} \cdot \frac{1}{5}} = \frac{5}{6}$.

Given $\mu = 0$, the principal offers $(q_0, p_0) = (1, 2)$ and $(q_1, p_1) = (0, 0)$, yielding $\Pi^F(0) = 1$. Given $\mu = \frac{5}{6}$, the principal offers $(q_t, p_t) = (1, 2)$ for both t, yielding revenue 2. Notice that, with $\mu = \frac{5}{6}$, the agent's expected valuation for the good is 2 *regardless of his type* t. Non-full disclosure is useful for the principal to extract the agent's rent. Indeed, the agent loses all the information rent in this mechanism. The ex ante expected revenue is:

$$\frac{1}{2}\frac{4}{5} \cdot 1 + (1 - \frac{1}{2}\frac{4}{5}) \cdot 2 = \frac{8}{5} (> \frac{3}{2}).$$

In return to this higher revenue, the allocation becomes inefficient, as the agent buys the good with a strictly positive probability given $(\theta, t) = (0, 1)$ even though $v(0, 1) < 0$. $\square$

A.2 Proof of Proposition 2

Proof. For $\mu < \mu^*$, we have $v(\mu, 0) > v(\mu, 1)$, and in this sense, $t = 0$ is the higher type. Thus, the optimal allocation given μ is either (i) $(q_0, p_0) = (q_1, p_1) = (1, v(\mu, 1))$, (ii) $(q_0, p_0) = (1, v(\mu, 0))$ and $(q_1, p_1) = (0, 0)$, or (iii) $(q_0, p_0) =$

$(q_1, p_1) = (0, 0)$. The expected revenue given μ is thus:

$$V^N(\mu) = \max\{v(\mu, 1), (1 - f)v(\mu, 0), 0\}.$$

For $\mu > \mu^*$, $t = 1$ is the higher type, and the optimal allocation given μ is either (i) $(q_0, p_0) = (q_1, p_1) = (1, v(\mu, 0))$, (ii) $(q_1, p_1) = (1, v(\mu, 1))$ and $(q_0, p_0) = (0, 0)$, or (iii) $(q_0, p_0) = (q_1, p_1) = (0, 0)$. The expected revenue given μ is thus:

$$V^N(\mu) = \max\{v(\mu, 0), fv(\mu, 1), 0\}.$$

What is the optimal Λ? As this is essentially a Bayesian persuasion problem, the solution is given by the concavification of $V^N(\mu)$. However, notice that $V^N(\mu)$ is convex on $\mu \in (0, \mu^*)$, and then on $\mu \in (\mu^*, 1)$, possibly kinked at $\mu = \mu^*$. Therefore, it is enough to consider Λ which has a binary support on $\{0, \mu^*, 1\}$. Given the condition that $\mu^* > f_0$, the support of Λ must be $\{0, \mu^*\}$ or $\{0, 1\}$. □

A.3 Proof of Theorem 2

Proof. Recall that the principal's expected payoff in the optimal disclosure policy is given by:

$$\begin{aligned}
\max_{x,\tau} \quad & f\tau_1 + (1 - f)\tau_0 \\
\text{sub. to} \quad & x_t(1)w(1, t) + x_t(0)w(0, t) - \tau_t \\
& \geq \max\{0, x_{t'}(1)w(1, t) + x_{t'}(0)w(0, t) - \tau_{t'}\}, \ \forall t, t' \\
& x_t(\theta) \in [0, 1], \ \forall t, \theta,
\end{aligned}$$

where $w(\theta, t) = F_0(\theta)v(\theta, t)$.

We only consider the case with $\mu^* > f_0$, or equivalently, $w(1,1) - w(1,0) < w(0,0) - w(0,1)$. The other case is similar.

Let $\lambda_{t,t'}(\geq 0)$ be the multiplier for the incentive constraint for type t (not to pretend to be type t'), $\lambda_t(\geq 0)$ be the multiplier for the participation constraint for type t, and $\phi_t(\theta)(\geq 0)$ be the multiplier for $x_t(\theta) \leq 1$.

Then, the dual problem is given by:

$$\begin{aligned}
\min_{\lambda,\phi\geq 0} \quad & \sum_{t,\theta} \phi_t(\theta) \\
\text{sub. to} \quad & \lambda_1 = f - \lambda_{10} + \lambda_{01} \\
& \lambda_0 = 1 - \lambda_1 \\
& \phi_1(1) \geq (\lambda_{10} + \lambda_1)w(1,1) - \lambda_{01}w(1,0) \\
& \phi_1(0) \geq (\lambda_{10} + \lambda_1)w(0,1) - \lambda_{01}w(0,0) \\
& \phi_0(1) \geq (\lambda_{01} + \lambda_0)w(1,0) - \lambda_{10}w(1,1) \\
& \phi_0(0) \geq (\lambda_{01} + \lambda_0)w(0,0) - \lambda_{10}w(0,1).
\end{aligned}$$

In the solution, we have

$$\begin{aligned}
\phi_1(1) &= \max\{0, fw(1,1) + \lambda_{01}(w(1,1) - w(1,0))\} \\
\phi_1(0) &= \max\{0, fw(0,1) - \lambda_{01}(w(0,0) - w(0,1))\} \\
\phi_0(1) &= \max\{0, (1-f)w(1,0) - \lambda_{10}(w(1,1) - w(1,0))\} \\
\phi_0(0) &= \max\{0, (1-f)w(0,0) + \lambda_{10}(w(0,0) - w(0,1))\},
\end{aligned}$$

and therefore, the problem reduces to:

$$\begin{aligned}\min_{\lambda_{01},\lambda_{10}} \quad & \max\{0, fw(1,1)+\lambda_{01}(w(1,1)-w(1,0))\} \\ & +\max\{0, fw(0,1)-\lambda_{01}(w(0,0)-w(0,1))\} \\ & +\max\{0,(1-f)w(1,0)-\lambda_{10}(w(1,1)-w(1,0))\} \\ & +\max\{0,(1-f)w(0,0)+\lambda_{10}(w(0,0)-w(0,1))\} \\ \text{sub. to} \quad & \lambda_{10},\lambda_{01}\geq 0,\ \lambda_{01}-\lambda_{10}\in[-f,1-f].\end{aligned}$$

In what follows, we characterize the solution to all the parameters under three assumptions. First, $w(1,1) > w(1,0)$ and $w(0,0) > w(0,1)$, that is, type $t = \theta$ is the high type given state θ. Second, for each t, there is some θ such that $w(\theta, t) > 0$. Otherwise, the problem becomes trivial. Finally, $\mu^* > f_0$ ($\Leftrightarrow w(1,1) - w(1,0) < w(0,0) - w(0,1)$). The analyses of the other cases are similar.

Case 1: $w(0,0) < 0$ **(hence** $w(0,1) < 0$**):** Note that our parameter assumption above implies $w(1,1) > w(1,0) > 0$. Thus, $fw(1,1) + \lambda_{01}(w(1,1) - w(1,0)) > 0$ and $fw(0,1) - \lambda_{01}(w(0,0) - w(0,1)) < 0$, implying $x_1(1) = 1$ and $x_1(0) = 0$. The problem becomes:

$$\begin{aligned}\min_{\lambda_{01},\lambda_{10}} \quad & fw(1,1)+\lambda_{01}(w(1,1)-w(1,0)) \\ & +\max\{0,(1-f)w(1,0)-\lambda_{10}(w(1,1)-w(1,0))\} \\ & +\max\{0,(1-f)w(0,0)+\lambda_{10}(w(0,0)-w(0,1))\} \\ \text{sub. to} \quad & \lambda_{10},\lambda_{01}\geq 0,\ \lambda_{01}-\lambda_{10}\in[-f,1-f].\end{aligned}$$

It is optimal to set $\lambda_{01} = \max\{0, \lambda_{10} - f\}$, implying:

$$\begin{aligned}
\min_{\lambda_{01}} \quad & fw(1,1) + \max\{0, (\lambda_{10} - f)(w(1,1) - w(1,0)))\} \\
& + \max\{0, (1-f)w(1,0) - \lambda_{10}(w(1,1) - w(1,0))\} \\
& + \max\{0, (1-f)w(0,0) + \lambda_{10}(w(0,0) - w(0,1))\} \\
\text{sub. to} \quad & \lambda_{10} \geq 0.
\end{aligned}$$

Case 1.1: $v^* < 0$ and $w(1,0) < fw(1,1)$ Note that this case is equivalent to:

$$\frac{(1-f)w(1,0)}{w(1,1) - w(1,0)} < \min\{\frac{(1-f)(-w(0,0))}{w(0,0) - w(0,1)}, f\}.$$

Then, any $\lambda_{10} \in (\frac{(1-f)w(1,0)}{w(1,1)-w(1,0)}, \min\{\frac{(1-f)(-w(0,0))}{w(0,0)-w(0,1)}, f\})$ makes all the max terms zero, which is obviously a solution. The objective is $fw(1,1)$. It is easy to derive $x_0(1) = x_0(0) = 0$ and $\tau_1 = w(1,1)$ and $\tau_0 = 0$. This allocation is clearly achieved with full disclosure.

Case 1.2: $w(1,0) > fw(1,1)$ and $-w(0,0) > f(-w(0,1))$ Note that this case is equivalent to:

$$f < \min\{\frac{(1-f)(-w(0,0))}{w(0,0) - w(0,1)}, \frac{(1-f)w(1,0)}{w(1,1) - w(1,0)}\}.$$

Then, any $\lambda_{10} \in (f, \min\{\frac{(1-f)(-w(0,0))}{w(0,0)-w(0,1)}, \frac{(1-f)w(1,0)}{w(1,1)-w(1,0)}\})$ is a solution, making the objective $w(1,0)$. It is easy to derive $x_0(1) = 1$ and $x_0(0) = 0$ and $\tau_1 = \tau_0 = w(1,0)$. This allocation is clearly achieved with full disclosure.

Case 1.3: $v^* > 0$ and $-w(0,0) < f(-w(0,1))$ Note that this case is equivalent to:

$$\frac{(1-f)(-w(0,0))}{w(0,0)-w(0,1)} < \min\{f, \frac{(1-f)w(1,0)}{w(1,1)-w(1,0)}\}.$$

Then, it is optimal to set $\lambda_{10} = \frac{(1-f)(-w(0,0))}{w(0,0)-w(0,1)} > 0$ (hence, the incentive constraint for type $t = 1$ holds with equality). As $\lambda_{01} = 0$, we have $\lambda_0, \lambda_1 > 0$ (hence, the participation constraints for both types hold with equality). Finally, as $(1-f)w(1,0) - \lambda_{10}(w(1,1) - w(1,0)) > 0$, we have $x_0(1) = 1$. These binding constraints imply $x_0(0) = \frac{w(1,1)-w(1,0)}{w(0,0)-w(0,1)}$, and the objective attained is $fw(1,1) + (1-f)\frac{w(1,1)w(0,0)-w(1,0)w(0,1)}{w(0,0)-w(0,1)}$. This allocation is achieved by the following type-contingent disclosure:

- For $t = 1$: The principal fully discloses θ, and allocates $(q,p) = (1, v(1,1))$ if $\theta = 1$ and $(0,0)$ if $\theta = 0$;

- For $t = 0$: The principal sends $\mu = \mu^*$ with probability 1 given $\theta = 1$ and probability $x_0(0) = \frac{w(1,1)-w(1,0)}{w(0,0)-w(0,1)}$ given $\theta = 0$, and sends $\mu = 0$ with probability $1 - x_0(0)$ given $\theta = 0$; The allocation is $(q,p) = (1, v^*)$ if $\mu = \mu^*$ is sent, and is $(0,0)$ if $\mu = 0$.

Case 1': $w(1,1) < 0$ (hence $w(1,0) < 0$): Similar to Case 1, and hence is omitted.

Case 2: $w(0,0) > 0 > w(0,1)$ and $w(1,1) > 0 > w(1,0)$ Because $fw(1,1) + \lambda_{01}(w(1,1) - w(1,0)) > 0$ and $(1-f)w(0,0) + \lambda_{10}(w(0,0) - w(0,1)) > 0$, we obtain $x_1(1) = x_0(0) = 1$. Because $fw(0,1) - \lambda_{01}(w(0,0) - w(0,1)) < 0$ and

$(1-f)w(1,0) - \lambda_{10}(w(1,1) - w(1,0)) < 0$, we obtain $x_1(0) = x_0(1) = 0$. The objective is $fw(1,1) + (1-f)w(0,0)$, and it is easy to derive $\tau_1 = w(1,1)$ and $\tau_0 = w(0,0)$ (where both types' participation constraints are satisfied).

This allocation is clearly achieved with full disclosure.

Case 3: $w(0,0) > 0 > w(0,1)$ **and** $w(1,1) > w(1,0) > 0$ Because $fw(1,1) + \lambda_{01}(w(1,1) - w(1,0)) > 0$ and $(1-f)w(0,0) + \lambda_{10}(w(0,0) - w(0,1)) > 0$, we obtain $x_1(1) = x_0(0) = 1$. Because $fw(0,1) - \lambda_{01}(w(0,0) - w(0,1)) < 0$, we obtain $x_1(0) = 0$.

Thus, the problem becomes:

$$\begin{aligned}
\min_{\lambda_{01},\lambda_{10}} \quad & fw(1,1) + \lambda_{01}(w(1,1) - w(1,0)) \\
& + \max\{0, (1-f)w(1,0) - \lambda_{10}(w(1,1) - w(1,0))\} \\
& +(1-f)w(0,0) + \lambda_{10}(w(0,0) - w(0,1)) \\
\text{sub. to} \quad & \lambda_{10}, \lambda_{01} \geq 0,\ \lambda_{01} - \lambda_{10} \in [-f, 1-f].
\end{aligned}$$

It is clearly optimal to set $\lambda_{01} = \max\{0, \lambda_{10} - f\}$, and thus:

$$\begin{aligned}
\min_{\lambda_{10}} \quad & fw(1,1) + \max\{0, (\lambda_{10} - f)(w(1,1) - w(1,0))\} \\
& + \max\{0, (1-f)w(1,0) - \lambda_{10}(w(1,1) - w(1,0))\} \\
& +(1-f)w(0,0) + \lambda_{10}(w(0,0) - w(0,1)) \\
\text{sub. to} \quad & \lambda_{10} \geq 0.
\end{aligned}$$

Because the objective is strictly increasing in λ_{10}, it is optimal to set $\lambda_{10} = 0$, and thus $x_0(1) = 0$ and $\lambda_{01} = 0$ (hence, both types' participation constraints hold with equality, and no incentive constraint is binding). This allocation is achieved by the following type-contingent disclosure:

- For $t = 1$: The principal fully discloses θ, and allocates $(q, p) = (1, v(1, 1))$ if $\theta = 1$ and $(0, 0)$ if $\theta = 0$;

- For $t = 0$: The principal sends $\mu = \mu^*$ with probability 1 given $\theta = 1$ and probability $x_0(0) = \frac{w(1,1)-w(1,0)}{w(0,0)-w(0,1)}$ given $\theta = 0$, and sends $\mu = 0$ with probability $1 - x_0(0)$ given $\theta = 0$; The allocation is $(q, p) = (1, v^*)$ if $\mu = \mu^*$ is sent, and is $(1, v(0, 0))$ if $\mu = 0$.[18]

Case 3': $w(0, 0) > w(0, 1) > 0$ and $w(1, 1) > 0 > w(1, 0)$ Similar to Case 3, and hence is omitted.

Case 4: $w(0, 0) > w(0, 1) > 0$ and $w(1, 1) > w(1, 0) > 0$ Because $fw(1, 1) + \lambda_{01}(w(1, 1) - w(1, 0)) > 0$ and $(1 - f)w(0, 0) + \lambda_{10}(w(0, 0) - w(0, 1)) > 0$, we obtain $x_1(1) = x_0(0) = 1$. Given this, the objective is strictly increasing in λ_{10}, and thus, it is optimal to set $\lambda_{10} = \max\{0, \lambda_{01} + f - 1\}$.

Then, the problem becomes:

$$\begin{aligned}
\min_{\lambda_{01}} \quad & fw(1,1) + \lambda_{01}(w(1,1) - w(1,0)) \\
& + \max\{0, fw(0,1) - \lambda_{01}(w(0,0) - w(0,1))\} \\
& + (1-f)w(0,0) + \max\{0, (\lambda_{01} + f - 1)(w(0,0) - w(0,1))\} \\
& + \max\{0, (1-f)w(1,0) - \max\{0, (\lambda_{01} + f - 1)(w(1,1) - w(1,0))\}\} \\
\text{sub. to} \quad & \lambda_{01} \geq 0,
\end{aligned}$$

[18]The same objective is achieved if there is no disclosure for $t = 0$. In fact, there exist multiple optimal disclosure policies. However, in any case, information disclosure must be type-contingent.

where

$$\max\{0,(1-f)w(1,0)-\max\{0,(\lambda_{01}+f-1)(w(1,1)-w(1,0))\}\}$$
$$= \begin{cases} (1-f)w(1,0) & \text{if} \quad \lambda_{01} < 1-f \\ (1-f)w(1,0)-\lambda_{01}(w(1,1)-w(1,0)) & \text{if} \quad 1-f < \lambda_{01} < \frac{(1-f)w(1,0)}{w(1,1)-w(1,0)} \\ 0 & \text{if} \quad \lambda_{01} > \frac{(1-f)w(1,0)}{w(1,1)-w(1,0)}. \end{cases}$$

Case 4.1: $w(0,1) > (1-f)w(0,0)$ Because we have $1-f < \frac{fw(0,1)}{w(0,0)-w(0,1)}$ in this case, the objective is strictly decreasing in λ_{01} if $\lambda_{01} < 1-f$ and otherwise strictly increasing. Therefore, $\lambda_{01} = 1-f$ is optimal. This implies $\lambda_{10} = 0$ (hence the incentive constraint for $t = 0$ and the participation constraint for $t = 0$ hold with equality), and $x_1(0) = x_0(1) = 1$. We have $\tau_1 = \tau_0 = w(1,1) + w(0,1)$, and the objective becomes $w(1,1) + w(0,1)$.

This allocation can be achieved by the following non-type-contingent disclosure policy:

- For both t, the principal discloses $\mu = \mu^*$ with probability 1 given $\theta = 1$ and probability $\frac{w(1,1)-w(1,0)}{w(0,0)-w(0,1)}$ given $\theta = 0$; and he discloses $\mu = 0$ otherwise. Given $\mu = \mu^*$ disclosed, the allocation is $(q_1, p_1) = (q_0, p_0) = (1, v^*)$; while given $\mu = 0$ disclosed, it is $(q_1, p_1) = (q_0, p_0) = (1, v(0,1))$.

Case 4.2: $w(0,1) < (1-f)w(0,0)$ In this case, the objective is strictly decreasing in λ_{01} if $\lambda_{01} < \frac{fw(0,1)}{w(0,0)-w(0,1)}$ and strictly increasing otherwise. Thus, it is optimal to set $\lambda_{01} = \frac{fw(0,1)}{w(0,0)-w(0,1)}$. Then we have $\lambda_{10} = 0$ (hence the incentive constraint for $t = 0$ and both types' participation constraints hold with equality), and $x_0(1) = 1$. It is then implied that $x_1(0) = \frac{w(1,1)-w(1,0)}{w(0,0)-w(0,1)}$, $\tau_1 = w(1,1) + w(0,1)x_1(0)$

and $\tau_0 = w(0,0) + w(1,0)$. The objective becomes $f(w(1,1) + w(0,1)x_1(0)) + (1 - f)(w(0,0) + w(1,0))$.

This allocation can be achieved by the following non-type-contingent disclosure policy:

- For both t, the principal discloses $\mu = \mu^*$ with probability 1 given $\theta = 1$ and probability $\frac{w(1,1)-w(1,0)}{w(0,0)-w(0,1)}$ given $\theta = 0$; and he discloses $\mu = 0$ otherwise. Given $\mu = \mu^*$ disclosed, the allocation is $(q_1, p_1) = (q_0, p_0) = (1, v^*)$ (for both types); while given $\mu = 0$ disclosed, it is $(q_0, p_0) = (1, v(0,0))$ and $(q_1, p_1) = (0, 0)$.

Summary In sum, among Cases 1,2,3 and 4, the optimal disclosure policy is type-contingent in Case 1.3 and 3; otherwise it is non-type-contingent. □

A.4 Proof of Theorem 3

Proof. For the problem P^+, consider its relaxed version where only $IC_{t\to t-1}$ for $t > 1$ (i.e., t's local and downward incentive compatibility) and IR_1 (i.e., the lowest type's participation constraint) are considered. After the standard calculation, this relaxed problem becomes a virtual value maximization:

$$\begin{aligned} \max_x \quad & \sum_t \sum_\theta \gamma(\theta,t)x(\theta,t)F_0(\theta)F(t) \\ \text{s.t.} \quad & \sum_\theta \big(x(\theta,t) - x(\theta,t')\big)\big(v(\theta,t) - v(\theta,t')\big)F_0(\theta) \geq 0, \quad \forall t, t', \\ & 0 \leq x(\theta,t) \leq \mathbf{1}. \end{aligned}$$

Ignoring the monotonicity condition, the pointwise maximization implies $x(\theta,t) =$

1 if $\gamma(\theta, v) > 0$, while $x(\theta, t) = 0$ if $\gamma(\theta, v) < 0$. Under the monotone virtual value assumption, we have $\big(x(\theta,t) - x(\theta,t')\big)\big(v(\theta,t) - v(\theta,t')\big) \geq 0$ for each θ and $t \neq t'$, and thus the ignored constraints are automatically satisfied. Furthermore, this optimal value of the objective can be achieved under full disclosure because $x(\theta, t) \in \{0, 1\}$ for all θ and t. □

A.5 Proof of Theorem 4

Proof. The key step is to establish the following lemma:

Lemma 1. Assume that $\Theta = \{0, 1\}$, $T = \{1, \ldots, N\}$, $v(0, t)$ is strictly increasing in t, $v(1, t)$ is strictly decreasing in t, and that there is a concave function f such that $v(1, t) = f(v(0, t))$ for all t.

Then, the solution to (P^+) satisfies: (i) only local incentive compatibility constraints are relevant; (ii) as t changes from 1 to N, $\mathbf{x}(t)$ clockwise rotates along the upper and right boundaries, i.e., $\big([0,1] \times \{1\}\big) \cup \big(\{1\} \times [0,1]\big)$.

Proof of the lemma. Let $\lambda_{t,t'}$ be the multiplier for $IC_{t \to t'}$, λ_t be the multiplier for IR_t, $\phi_t(\theta)$ be the multiplier for $x(\theta, t) \leq 1$, and $\eta_t(\theta)$ be the multiplier for $x(\theta, t) \geq 0$. Let $\boldsymbol{\phi}_t = \big(\phi_t(\theta)\big)_{\theta \in \Theta} \in \mathbb{R}^{|\Theta|}$, and $\boldsymbol{\eta}_t = \big(\eta_t(\theta)\big)_{\theta \in \Theta} \in \mathbb{R}^{|\Theta|}$. Then, the dual problem of (P^+) is given by

$$
\begin{aligned}
&\min_{\lambda, \boldsymbol{\phi}, \boldsymbol{\eta} \geq 0} \sum_{t \in T} \boldsymbol{\phi}_t \cdot \mathbf{1} \\
&s.t. \quad F(t)\mathbf{w}(t) + \sum_{t' \neq t} \lambda_{t',t}\big[\mathbf{w}(t) - \mathbf{w}(t')\big] = \boldsymbol{\phi}_t - \boldsymbol{\eta}_t, \quad \forall t \\
&\qquad\quad F(t) - \sum_{t' \neq t} \lambda_{t,t'} + \sum_{t' \neq t} \lambda_{t',t} \geq 0, \quad \forall t.
\end{aligned}
$$

For all $t = 1, \ldots, N-2$, we have

$$\frac{w(1,t) - w(1,t+1)}{w(0,t) - w(0,t+1)} = \frac{F_0(1)}{F_0(0)} \cdot \frac{v(1,t) - v(1,t+1)}{v(0,t) - v(0,t+1)}$$
$$> \frac{F_0(1)}{F_0(0)} \cdot \frac{v(1,t+1) - v(1,t+2)}{v(0,t+1) - v(0,t+2)} = \frac{w(1,t+1) - w(1,t+2)}{w(0,t+1) - w(0,t+2)},$$

which means the agent's types are located on a decreasing concave curve if the horizontal (or vertical) axis denotes $w(0,t)$ (or $w(1,t)$).

Thus, given each t, for any $\big(\lambda_{t',t}\big)_{t' \neq t} \geq \mathbf{0}$, the vector $\sum_{t' \neq t} \lambda_{t',t}\big[\mathbf{w}(t) - \mathbf{w}(t')\big]$ will never be in the third quadrant. It follows that we always have either $\phi_t(0) > 0$ or $\phi_t(1) > 0$ or both. By the complementary slackness condition, we have either $x(0,t) = 1$ or $x(1,t) = 1$ or both.

Pick any pair of local incentive compatible constraints, $IC_{t \to t+1}$ and $IC_{t+1 \to t}$. We have $\big(\mathbf{x}(t) - \mathbf{x}(t+1)\big) \cdot \big(\mathbf{w}(t) - \mathbf{w}(t+1)\big) \geq 0$. Then, the solution to (P^+) must be one of the three cases:

(1) $x(0,t) \leq x(0,t+1)$, $x(1,t) = x(1,t+1) = 1$;

(2) $x(0,t) = x(0,t+1) = 1$, $x(1,t) \geq x(1,t+1)$;

(3) $x(0,t) < 1$, $x(1,t) = 1$, $x(0,t+1) = 1$, $x(1,t+1) < 1$.

Thus, $\mathbf{x}(t)$ clockwise rotates along the upper and right boundaries.

Using the above result, we can show that only local incentive compatibility constraints are relevant. This is because for arbitrary $t < t' < t''$, we have

$$\begin{aligned}
&\mathbf{x}(t) \cdot \mathbf{w}(t) - \tau(t) \geq \mathbf{x}(t') \cdot \mathbf{w}(t) - \tau(t') \\
&= \mathbf{x}(t') \cdot \mathbf{w}(t') - \tau(t') + \mathbf{x}(t') \cdot \big[\mathbf{w}(t) - \mathbf{w}(t')\big] \\
&\geq \mathbf{x}(t'') \cdot \mathbf{w}(t') - \tau(t'') + \mathbf{x}(t') \cdot \big[\mathbf{w}(t) - \mathbf{w}(t')\big] \\
&= \mathbf{x}(t'') \cdot \mathbf{w}(t) - \tau(t'') + \big[\mathbf{x}(t') - \mathbf{x}(t'')\big] \cdot \big[\mathbf{w}(t) - \mathbf{w}(t')\big] \\
&\geq \mathbf{x}(t'') \cdot \mathbf{w}(t) - \tau(t''),
\end{aligned}$$

where the first (or second) inequality is due to $IC_{t \to t'}$ (or $IC_{t' \to t''}$), and the third inequality is because the vector $\big(\mathbf{x}(t') - \mathbf{x}(t'')\big)$ is always in the second quadrant (or on its boundary). □

From Lemma 1, we know there exists some t^* such that $\mathbf{x}(t^*) = (1, 1)$. For $t < t' < t^*$, we have $x(0, t) \leq x(0, t')$ and $x(1, t) = x(1, t') = 1$. For $t > t' > t^*$, we have $x(1, t) \leq x(1, t')$ and $x(0, t) = x(0, t') = 1$. Then, it is easy to verify that the combination of the information disclosure policies and allocations in the statement achieves the resulted solution in the first step. We omit this part. □

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