WORKING PAPERS

N° 1227

June 2021

“Keeping the Agents in the Dark: Private Disclosures in Competing Mechanisms”

Andrea Attar, Eloisa Campioni, Thomas Mariotti and Alessandro Pavan

Toulouse School of Economics

Keeping the Agents in the Dark: Private Disclosures in Competing Mechanisms*

Andrea Attar[†] Eloisa Campioni[‡]
Thomas Mariotti[§] Alessandro Pavan[¶]

June 3, 2021

Abstract

We study games in which several principals contract with several privately-informed agents. We show that enabling the principals to engage in contractible private disclosures – by sending private signals to the agents about how the mechanisms will respond to the agents' messages – can significantly affect the predictions of such games. Our first result shows that private disclosures may generate equilibrium outcomes that cannot be supported in any game without private disclosures, no matter the richness of the message spaces and the availability of public randomizing devices. The result thus challenges the canonicity of the universal mechanisms of Epstein and Peters (1999). Our second result shows that equilibrium outcomes of games without private disclosures need not be sustainable when private disclosures are allowed. The result thus challenges the robustness of the "folk theorems" of Yamashita (2010) and Peters and Troncoso-Valverde (2013). These findings call for a novel approach to the analysis of competing-mechanism games.

Keywords: Incomplete Information, Competing Mechanisms, Private Disclosures, Signals, Universal Mechanisms, Folk Theorems.
JEL Classification: D82.

---

*We thank conference participants at the 2020 Econometric Society World Congress and the 2021 ASSA Virtual Annual Meeting for many useful discussions. This research has benefited from financial support of the ANR (Programme d'Investissement d'Avenir ANR-17-EURE-0010), the MIUR (PRIN 2015), and the research foundation TSE-Partnership.

[†]Toulouse School of Economics, CNRS, University of Toulouse Capitole, Toulouse, France, and Università degli Studi di Roma "Tor Vergata," Roma, Italy. Email: `andrea.attar@tse-fr.eu.`

[‡]Università degli Studi di Roma "Tor Vergata," Roma, Italy. Email: `eloisa.campioni@uniroma2.it.`

[§]Toulouse School of Economics, CNRS, University of Toulouse Capitole, Toulouse, France, CEPR, and CESifo. Email: `thomas.mariotti@tse-fr.eu.`

[¶]Northwestern University, Evanston, United States and CEPR. Email: `alepavan@northwestern.edu.`

1 Introduction

Classical mechanism design theory identifies the holding of private information by economic agents as a fundamental constraint on the allocations of resources (Hurwicz (1973)). How agents communicate their information in a mechanism then becomes crucial for the decisions that are implemented. In pure incomplete-information environments in which all payoff-relevant decisions are taken by the principal, private communication can be restricted to be one-directional, from the agents to the principal (Myerson (1979)). In that case, the principal may, without loss of generality, post a mechanism that specifies a (possibly random) decision for every profile of messages she may receive from the agents; hereafter, we refer to such communication protocols as *standard mechanisms.* Communication from the principal to the agents takes the form of the announcement of such public mechanism. Any private communication from the principal to the agents is redundant, in that it has no impact on the set of allocations that can be implemented.

In this paper, we show that these standard insights from classical mechanism design theory do not extend to settings with competing principals. Specifically, we show that, when several agents contract with several principals, private disclosures from the principals to the agents can significantly affect the set of equilibrium outcomes. We introduce such private disclosures by allowing the principals to send contractible private signals to the agents before receiving messages from them. We derive our results in pure incomplete-information environments in which only the principals take payoff-relevant decisions.

The two theoretical pillars of this literature can be described as follows. First, one can construct a space of universal mechanisms whereby every agent can communicate to every principal his endogenous market information – that is, the profile of mechanisms posted by the other principals – in addition to his exogenous type (Epstein and Peters (1999)). An analogue of the revelation principle then holds: any equilibrium outcome of any competing- mechanism game can be supported as an equilibrium outcome of the game in which the principals can only post universal mechanisms. Second, one can obtain an explicit characterization of the equilibrium outcomes and of the equilibrium payoffs for a large class of games (Yamashita (2010), Peters and Troncoso-Valverde (2013)). An analogue of the folk theorem then holds: any incentive-feasible allocation that yields every principal a payoff above a well-defined min-max-min bound can be supported in equilibrium.

From our perspective, the key point is that these two central theorems are derived in a framework in which only the agents can privately communicate with the principals, who post standard mechanisms. We challenge these findings by considering a richer class of communication protocols in which the principals are allowed to disclose some information privately to the agents. While doing so, we maintain two main informational assumptions of this literature. First, principals do not directly communicate among them. Second, principals' mechanisms cannot directly refer to each other, that is, a principal's mechanism cannot condition its allocations directly on other principals' mechanisms.

Our first result is that equilibrium outcomes of competing-mechanism games in which principals are allowed to post mechanisms with private disclosures – i.e. with signals sent by the principals to the agents before the agents send messages to the principals – need not be equilibrium outcomes in any game in which principals are restricted to standard mechanisms, no matter the richness of the message spaces. The reason is that private disclosures may help the principals correlate their decisions with the information privately held by the agents in a way that cannot be replicated by the principals responding to the agents' messages when the latter are based solely on the agents' common knowledge of the mechanisms and of the agents' exogenous private information. We establish the result by means of an example in which the equilibrium correlation between the principals' decisions and the agents' exogenous private information requires that (a) the agents receive information about a principal's decision and pass it on to another principal before the latter principal finalizes her own decision, and (b) such information not create common knowledge among the agents about the former principal's decision before they communicate with the latter principal. The example shows the necessity of both (a) and (b) when it comes to sustaining certain outcomes, the possibility to accomplish both (a) and (b) with private disclosures, and the impossibility to accomplish (a) and (b) with standard mechanisms, regardless of the richness of the message spaces and the availability of public randomizing devices. The result thus also implies that the universal mechanisms of Epstein and Peters (1999) are not canonical when principals can engage in private disclosures.

Our second result establishes the non-robustness of equilibria in standard mechanisms. We provide an example of an equilibrium outcome of a game in which principals compete in standard mechanisms (with rich message spaces) that cannot be sustained in a game where, in addition to the original mechanisms, the principals can also offer mechanisms with private disclosures. In the original game, standard mechanisms are sufficiently rich to include *recommendation* ones as in Yamashita (2010). In a recommendation mechanism, agents, in addition to reporting their exogenous payoff-relevant information, are asked to "vote" on the direct mechanism the principal should use. Any such game is known to be amenable to folk-theorem-type of results whereby any allocation that is incentive-compatible for the agents (in the usual sense), and yields each principal a payoff above an appropriately-defined min-max-min threshold can be sustained in equilibrium. We show that, by deviating to a mechanism that discloses information about her decisions privately and asymmetrically to the agents, a principal can ensure that the agents no longer have the incentives to carry out the punishments with the non-deviating principals necessary to make the deviation unprofitable. Furthermore, the equilibrium allocation that is not robust to the principals deviating to mechanisms with private disclosure is incentive-compatible and individually rational in the sense of Myerson (1979). The result thus implies that the equilibria characterized by Yamashita (2010) and by Peters and Troncoso- Valverde (2013) need not be robust to deviations to mechanisms with private disclosures.

Taken together, the above results challenge the existing modeling approach to competing-mechanism games and suggest that private disclosures from the principals to the agents should be central to the theory of competing mechanisms.

Related Literature.

This paper contributes to the theoretical foundations of competing-mechanism games, in which principals fully commit to mechanisms in the presence of privately-informed agents. McAfee (1993) is the first to pint out that equilibrium outcomes in such games may rely on agents reporting *all* their private information to the principals, i.e. their payoff-relevant exogenous types and the market information contained in the other principals' posted mechanisms. Epstein and Peters (1999) are the first to construct a space of universal mechanisms that permits one to establish the analog of the revelation principle for competing-mechanism games. Subsequent work by Yamashita (2010) and Peters and Troncoso-Valverde (2013) provides an explicit characterization of the equilibrium allocations and payoffs of such games and to show that the latter coincide with those that are incentive-compatible and individually rational in the sense of Myerson (1979). Our results indicate that such a characterization is sensitive to the assumption that principals are restricted to standard mechanisms and does not extend to settings where principals can engage in private but contractible disclosures.

As it is well known from classical mechanism design theory (Myerson (1982)), private communication from a single principal to the agents is key when agents take payoff-relevant actions. Such a communication, which takes the form of action recommendations, has been shown to serve as a correlating device between the players' decisions in several problems of economic interest, such as the partnership model of Rahman and Obara (2010). Perhaps surprisingly, however, private disclosures have been neglected in competing-mechanism settings *even when agents take actions*, as in the lobbying model of Prat and Rustichini (2003).

To the best of our knowledge, the only exception is the recent work of Attar, Campioni and Piaser (2019), which considers a complete-information game in which agents' actions are observable. They construct an example in which equilibrium allocations sustained by standard mechanisms fail to be robust against unilateral deviations to mechanisms with private recommendations. In equilibrium, a principal implements a correlation between her decisions and the agents' actions that cannot be sustained without sending private recommendations.

While the above insights can be rationalized in the light of the traditional approach to single-principal settings (Myerson (1982, 1986), Forges (1986)), this paper shows that principals' signals cannot be reduced to action recommendations to the agents. The new role we highlight, instead, suggests that signals help overcoming the lack of a direct communication channel among the principals. By disclosing information asymmetrically to the agents, a principal may be able to correlate her decisions with other principals' decisions and the agents' exogenous information in a way that is not replicable with standard mechanism without private disclosures.

In our setting, the correlation in the principals' decisions is generated by the agents' independent reports in the mechanisms of the principals, that *cannot* directly condition on the other principals' mechanisms. Instead, Kalai et al. (2010), Peters and Szentes (2012), Peters (2015), and Szentes (2015), consider settings in which players make commitments that are conditional on the commitments of others, which requires more demanding observability and verifiability assumptions.

The role of signals we document hinges on the presence of at least two agents. In the presence of a single agent, the menu theorems of Martimort and Stole (2002), Peters (2001) and Pavan and Calzolari (2009, 2010) guarantee that any equilibrium allocation of a game in which principals commit to arbitrary message-contingent decisions can be reproduced in the (canonical) game in which principals offer subset (menus) of their decisions to the agent and delegate to the latter the choice of the final allocations. In such settings, private disclosures play no role.

The rest of the paper is organized as follows. Section 2 introduces a general model of competing mechanisms under incomplete information. Sections 3 and 4 present the results. Section 5 discusses the different roles of private disclosures. Section 6 concludes.

2 The Model

We consider a pure incomplete-information setting in which several principals, indexed by $j = 1, \ldots, J$, contract with several agents, indexed by $i = 1, \ldots, I$.

Information Every agent i (he) possesses some exogenous private information summarized by his type ω^i, which belongs to some finite set Ω^i. Thus the set of exogenous states of the world $\omega \equiv (\omega^1, ..., \omega^I)$ is $\Omega \equiv \Omega^1 \times \ldots \times \Omega^I$. Principals and agents commonly believe that the state ω is drawn from Ω according to the distribution $\mathbf{P}$.

Decisions and Payoffs Every principal j (she) takes a decision x_j in some finite set X_j. We let $v_j : X \times \Omega \to \mathbb{R}$ and $u^i : X \times \Omega \to \mathbb{R}$ be the payoff functions of principal j and of agent i, respectively, where $X \equiv X_1 \times \ldots \times X_J$ is the set of possible profiles of decisions for the principals. Agents take no payoff-relevant decisions. An *allocation* is a function $z : \Omega \to \Delta(X)$ assigning a lottery over the set X to every state of the world. The *outcome* induced by an allocation z is the restriction of z to the set of states occurring with positive probability under $\mathbf{P}$.[1]

Mechanisms with Signals A *mechanism with signals* for a principal consists, first, of a probability distribution over the signals that the principal privately sends to the agents, and, second, of a decision rule that assigns a lottery over the principal's decisions to every profile of signals sent to the agents and every profile of messages received from them. Formally, a mechanism with signals for principal j is a pair $\gamma_j \equiv (\sigma_j, \phi_j)$ such that

1. $\sigma_j \in \Delta(S_j)$ is a Borel probability measure over the profiles of signals $s_j \equiv (s_j^1, \ldots, s_j^I)$ principal j sends to the agents, where $S_j \equiv S_j^1 \times \ldots \times S_j^I$ for some collection of Polish spaces S_j^i of signals from principal j to every agent i.

2. $\phi_j : S_j \times M_j \to \Delta(X_j)$ is a Borel-measurable function assigning a lottery over principal j's decisions to every profile of signals $s_j \in S_j$ sent to the agents and every profile of messages $m_j \equiv (m_j^1, \ldots, m_j^I) \in M_j$ received from them, where $M_j \equiv M_j^1 \times \ldots \times M_j^I$ for some collection of Polish spaces M_j^i of messages from every agent i to principal j.

We assume that $\Omega^i \subset M_j^i$ for all i and j, so that the language through which agent i communicates with principal j is rich enough for him to report his type to her. A (potentially indirect) *standard mechanism* for principal j is a special case of a mechanism with signals in which S_j^i is a singleton for all i; hereafter, we will often simplify the notation by

[1] The distinction between an allocation and an outcome is relevant when the agents' types are correlated.

omitting σ_j and representing a standard mechanism solely by a Borel-measurable function $\phi_j : M_j \to \Delta(X_j)$ describing the principal's response to the messages she receives from the agents. The requirement that signal and message spaces be Polish entails no loss of generality; in particular, the universal standard mechanisms of Epstein and Peters (1999) involve uncountable Polish message spaces.

Admissibility A general requirement for defining expected payoffs in the game to be described below is that, for each j, the evaluation mapping $(\phi_j, s_j, m_j) \mapsto \phi_j(s_j, m_j)$ be measurable. To do so, we must define a measurable structure on the space of admissible functions ϕ_j. If S_j and M_j are countable, we can take this space to be $\Delta(X_j)^{S_j \times M_j}$, endowed with the product Borel σ-field. If S_j or M_j are uncountable, however, there is no measurable structure over the space of all Borel-measurable functions $\phi_j : S_j \times M_j \to \Delta(X_j)$ such that the evaluation mapping for principal j is measurable (Aumann (1961)); in that case, there is no choice but to restrict the space of admissible functions ϕ_j. Admissibility can be shown to coincide with the requirement that this space be of bounded Borel class (Aumann (1961), Rao (1971)), which still allows for a rich class of mechanisms for our analysis. We hereafter fix an admissible space Φ_j, endowed with a σ-field $\mathcal{F}_j$, so that $\Gamma_j \equiv \Delta(S_j) \times \Phi_j$ is the space of admissible mechanisms for principal j, endowed with the product σ-field $\mathcal{G}_j$ generated by the Borel subsets of $\Delta(S_j)$ and the elements of $\mathcal{F}_j$. When attention is restricted to standard mechanisms, the set of admissible mechanisms is simply denoted by Φ_j, with the understanding that signal spaces are singletons.

Timing and Strategies The competing-mechanism game G^{SM} unfolds in four stages:

1. the principals simultaneously post mechanisms, observed by all agents;
2. the principals' mechanisms simultaneously and privately send signals to the agents;
3. the agents simultaneously send messages to the principals;
4. the principals' decisions are implemented and all payoffs accrue.

A mixed strategy for principal j is a probability measure $\mu_j \in \Delta(\Gamma_j)$ over $\mathcal{G}_j$. A strategy for agent i is a measurable function $\lambda^i : \Gamma \times S^i \times \Omega^i \to \Delta(M^i)$ that assigns to every profile of mechanisms $\gamma \equiv (\gamma_1, ..., \gamma_J) \in \Gamma \equiv \Gamma_1 \times \ldots \times \Gamma_J$ the principals may post, to every profile of signals $s^i \equiv (s^i_1, ..., s^i_J) \in S^i \equiv S^i_1 \times \ldots \times S^i_J$ agent i may receive, and to every type $\omega^i \in \Omega^i$ of agent i a Borel probability measure over the profiles of messages $m^i \equiv (m^i_1, ..., m^i_J) \in M^i \equiv M^i_1 \times \ldots \times M^i_J$ sent by agent i, where $\Gamma \times S^i \times M^i$ is endowed with the appropriate product σ-field. The allocation $z_{\mu,\lambda} : \Omega \to \Delta(X)$ induced by the strategies $(\mu, \lambda) \equiv (\mu_1, \ldots, \mu_J, \lambda^1, \ldots, \lambda^I)$ is then defined by

$$z_{\mu,\lambda}(x \,|\, \omega) \equiv \int_\Gamma \int_S \int_M \prod_{j=1}^{J} \phi_j(s_j, m_j)(x_j) \bigotimes_{i=1}^{I} \lambda^i(\mathrm{d}m^i \,|\, \gamma, s^i, \omega^i) \bigotimes_{j=1}^{J} \sigma_j(\mathrm{d}s_j) \bigotimes_{j=1}^{J} \mu_j(\mathrm{d}\gamma_j)$$

for all $(\omega, x) \in \Omega \times X$, where $S \equiv S_1 \times \ldots \times S_J$ and $M \equiv M_1 \times \ldots \times M_J$. For every profile of mechanisms γ, a behavioral strategy for agent i in the subgame γ played by the agents is a Borel-measurable function $\beta^i : S^i \times \Omega^i \to \Delta(M^i)$ assigning a Borel probability measure over the profile of messages $m^i \in M^i$ she sends to the principals to every profile of signals $s^i \in S^i$ she may receive and to every realization $\omega^i \in \Omega^i$ of her type. We let $z_{\gamma,\beta}$ be the allocation induced by the profile of behavior strategies $\beta \equiv (\beta^1, \ldots, \beta^I)$ in the subgame γ; the latter is defined in the same way as $z_{\mu,\lambda}$, except that γ is fixed and $\lambda^i(\cdot \,|\, \gamma, s^i, \omega^i)$ is replaced by $\beta^i(\cdot \,|\, s^i, \omega^i)$ for all i.

A special case of the game G^{SM} arises when S^i_j is a singleton for all i and j, so that the principals can only post standard mechanisms. To distinguish this situation, we denote by G^M the corresponding competing-mechanism game *without signals*; the games studied by Epstein and Peters (1999) and Yamashita (2010) are prominent examples.

Equilibrium The strategy profile (μ, λ) is a perfect Bayesian equilibrium (PBE) of G^{SM} whenever

1. for each $\gamma \in \Gamma$, $(\lambda^1(\gamma), \ldots, \lambda^I(\gamma))$ is a Bayes–Nash equilibrium (BNE) of the subgame γ played by the agents;
2. given the continuation equilibrium strategies λ, μ is a Nash equilibrium of the game played by the principals.

Notice that, in any subgame γ, the beliefs of the agents are pinned down by the prior $\mathbf{P}$ and the signal distributions $(\sigma_1, \ldots, \sigma_J)$ to which the principals are committed through the mechanisms γ. An allocation z is *incentive-compatible* if, for all i and $\omega^i \in \Omega^i$,

$$\omega^i \in \arg\max_{\hat{\omega}^i \in \Omega^i} \sum_{x \in X} \sum_{\omega^{-i} \in \Omega^{-i}} \mathbf{P}[\omega^{-i} \,|\, \omega^i] z(x \,|\, \hat{\omega}^i, \omega^{-i}) u^i(x, \omega^i, \omega^{-i}).$$

It follows from the definition of a BNE in any subgame played by the agents that any allocation $z_{\mu,\lambda}$ supported by a PBE (μ, λ) of G^{SM} is incentive-compatible; otherwise, some type ω^i of some agent i would be strictly better off mimicking the strategy $\lambda^i(\cdot \mid \cdot, \cdot, \hat{\omega}^i)$ of some other type $\hat{\omega}^i$—this is an instance of the revelation principle (Myerson, 1982).

This observation implies that, when there is a single principal, any allocation that can be implemented by a mechanism with signals can also be implemented via a direct revelation mechanism; as agents take no payoff-relevant actions, such direct revelation mechanisms involve no private disclosures from the principal to the agents. As we show below, the situation is markedly different when several principals contract with several agents.

3 Non-universality of Standard Mechanisms

In this section, we establish our first result. We provide an example of an equilibrium outcome of a game in which principals post mechanisms with signals that cannot be sustained as an equilibrium outcome of *any* game with standard mechanisms, irrespective of the richness of the message spaces. The example shows that, with private signals, a principal can make the agents' messages to other principals depend on information that correlates with her own decision. In turn, this allows the principals to correlate their decisions with the agents' exogenous private information in a way that cannot be sustained by the principals randomizing over their mechanisms and/or by the agents randomizing over the messages they send to the principals.

Example 1 Let $I = J \equiv 2$. We denote the principals by P1 and P2, and the agents by A1 and A2. The decision sets are $X_1 \equiv \{x_{11}, x_{12}, x_{13}, x_{14}\}$ for P1 and $X_2 \equiv \{x_{21}, x_{22}\}$ for P2. A2 has two types, with $\Omega^2 \equiv \{\omega_L, \omega_H\}$, while A1's type space is a singleton and hence omitted to ease notation. We assume that the states ω_L and ω_H occur with probabilities $\mathbf{P}[\omega_L] = 1/4$ and $\mathbf{P}[\omega_H] = 3/4$, respectively.

The players' payoffs are represented in Tables 1 and 2 below, in which the first payoff is that of P2 and the last two payoffs are those of A1 and A2, respectively. We let $\zeta < 0$ be an arbitrary loss for P2. P1's payoff is constant over X and hence omitted.

	x_{21}	x_{22}
x_{11}	$\zeta, 4, 1$	$\zeta, 8, 3.5$
x_{12}	$\zeta, 2, 5$	$\zeta, 9, 8$
x_{13}	$10, 3, 3$	$\zeta, 5.5, 3.5$
x_{14}	$\zeta, 1, 3.5$	$10, 7.5, 7.5$

Table 1: Players' payoffs when A2 is of type ω_L.

	x_{21}	x_{22}
x_{11}	$\zeta, 1, 6$	$10, 7.5, 5$
x_{12}	$10, 3, 9$	$\zeta, 5.5, 6$
x_{13}	$\zeta, 8, 7$	$\zeta, 4.5, 7$
x_{14}	$\zeta, 9, 6$	$\zeta, 3, 9$

Table 2: Players' payoffs when A2 is of type ω_H.

3.1 An Equilibrium with Private Signals

To illustrate the key ideas in the simplest possible manner, we consider a specific game with signals in which only P2 can send signals to the agents, and these signals are binary; that is, we let $S_1^1 = S_1^2 \equiv \{\emptyset\}$ and $S_2^1 = S_2^2 \equiv \{1, 2\}$. To allow the agents to report their information to the principals, we let $M_1^i \equiv \Omega^i \times S_2^i$ and $M_2^i \equiv \Omega^i$.[2] We refer to this game as G_1^{SM}. The following result then holds.[3]

Claim 1 *For $\alpha = \frac{2}{3}$, the following is a PBE outcome of G_1^{SM}:*

$$z(\omega_L) \equiv \alpha\delta_{(x_{13},x_{21})} + (1-\alpha)\delta_{(x_{14},x_{22})}, \tag{1}$$

$$z(\omega_H) \equiv \alpha\delta_{(x_{12},x_{21})} + (1-\alpha)\delta_{(x_{11},x_{22})}, \tag{2}$$

that yields to P2 an equilibrium payoff equal to 10.

Proof. Let P2 post the mechanism $\gamma_2^* \equiv (\sigma_2^*, \phi_2^*)$ such that,

$$\sigma_2^*(s_2) \equiv \begin{cases} \frac{\alpha}{2} & \text{if } s_2 = (1,1), \\ \frac{\alpha}{2} & \text{if } s_2 = (2,2), \\ \frac{1-\alpha}{2} & \text{if } s_2 = (1,2), \\ \frac{1-\alpha}{2} & \text{if } s_2 = (2,1), \end{cases} \tag{3}$$

and, for each $(s_2, m_2) \in S_2 \times M_2$,

$$\phi_2^*(s_2, m_2) \equiv \begin{cases} \delta_{x_{21}} & \text{if } s_2 \in \{(1,1),(2,2)\}, \\ \delta_{x_{22}} & \text{if } s_2 \in \{(1,2),(2,1)\}, \end{cases} \tag{4}$$

irrespective of the messages $m_2 \in M_2$ she receives from the agents. A key feature of this mechanism is that, regardless of the signal he receives from P2, every agent's posterior distribution about P2's decision coincides with his prior distribution. That is, each agent believes that P2 takes decision x_{21} with probability α and decision x_{22} with probability $1-\alpha$.

[2] The result in Claim 1 does not depend on the specific assumptions on the agents' message sets and extends to $M_1^i \supset \Omega^i \times S_2^i$ and $M_2^i \supset \Omega^i$.

[3] For any finite set A and each $a \in A$, δ_a is the Dirac probability measure over A that assigns probability 1 to a.

For the same reason, every agent believes that the other agent received the same signal as his with probability α and a different signal with probability $1-\alpha$. Thus γ_2^* completely leaves both agents in the dark.

As for P1, let her post the deterministic mechanism $\gamma_1^* \equiv (\delta_\emptyset, \phi_1^*)$ such that, for each $(m_1, m_2) \in M_1 \times M_2$,

$$\phi_1^*(\emptyset, m_1) \equiv \begin{cases} \delta_{x_{13}} & \text{if } m_1 \in \{(1,\omega_L,1),(2,\omega_L,2)\}, \\ \delta_{x_{14}} & \text{if } m_1 \in \{(1,\omega_L,2),(2,\omega_L,1)\}, \\ \delta_{x_{12}} & \text{if } m_1 \in \{(1,\omega_H,1),(2,\omega_H,2)\}, \\ \delta_{x_{11}} & \text{if } m_1 \in \{(1,\omega_H,2),(2,\omega_H,1)\}, \end{cases} \tag{5}$$

in which, for instance, $(1, \omega_L, 1)$ stands for $m_1^1 = (1)$ and $m_1^2 = (\omega_L, 1)$, that is, A1 reporting signal 1 to P1, and A2 reporting type ω_L and signal 1 to P1. Observe from (4)–(5) that the outcome (1)–(2) is implemented in the subgame (γ_1^*, γ_2^*) if every agent truthfully reports to P1 his type and the signal he receives from P2. We now show that these behaviors constitute a BNE in the subgame (γ_1^*, γ_2^*). The proof consists of two steps.

Step 1 Consider first A1's incentives, under the belief that A2 is truthful to P1. Because A1 has only one type, the only incentives we have to take into account concern his report to P1 of the signal he receives from P2.

If A1 truthfully reports his signal to P1, then, regardless of the signal he receives from P2, his expected payoff is

$$\frac{1}{4}\left[\alpha u^1(x_{13}, x_{21}, \omega_L) + (1-\alpha)u^1(x_{14}, x_{22}, \omega_L)\right] \\ + \frac{3}{4}\left[\alpha u^1(x_{12}, x_{21}, \omega_H) + (1-\alpha)u^1(x_{11}, x_{22}, \omega_H)\right] = 3\alpha + 7.5(1-\alpha).$$

If A1 misreports his signal to P1, then, regardless of the signal he receives from P2, his expected payoff is

$$\frac{1}{4}\left[\alpha u^1(x_{14}, x_{21}, \omega_L) + (1-\alpha)u^1(x_{13}, x_{22}, \omega_L)\right] \\ + \frac{3}{4}\left[\alpha u^1(x_{11}, x_{21}, \omega_H) + (1-\alpha)u^1(x_{12}, x_{22}, \omega_H)\right] = \alpha + 5.5(1-\alpha),$$

which is strictly less than his payoff from reporting the received signal truthfully to P1 for all $\alpha \in [0, 1]$.

Step 2 Consider next A2's incentives, under the belief that A1 is truthful to P1.
We first consider the behavior of A2 when he is of type ω_L. If he truthfully reports both his type and his signal to P1, then, regardless of the signal he receives from P2, his expected payoff is equal to

$$\alpha u^2(x_{13}, x_{21}, \omega_L) + (1-\alpha)u^2(x_{14}, x_{22}, \omega_L) = 3\alpha + 7.5(1-\alpha). \tag{6}$$

If, instead, A2 truthfully reports his type but misreports his signal to P1, then, regardless of the signal he receives from P2, his expected payoff is equal to

$$\alpha u^2(x_{14}, x_{21}, \omega_L) + (1-\alpha)u^2(x_{13}, x_{22}, \omega_L) = 3.5,$$

which is smaller or equal to his payoff from reporting both ω^2 and s_2^2 truthfully to P1 if $\alpha \leq \frac{8}{9}$.

If A2 misreports his type to P1 but reports the signal received from P2 truthfully, his expected payoff is equal to

$$\alpha u^2(x_{12}, x_{21}, \omega_L) + (1-\alpha)u^2(x_{11}, x_{22}, \omega_L) = 5\alpha + 3.5(1-\alpha), \tag{7}$$

which is no greater than his payoff from reporting both ω^2 and s_2^2 truthfully if $\alpha \leq \frac{2}{3}$.

Finally, if A2 misreports both his type and the signal received from P2, his expected payoff is equal to

$$\alpha u^2(x_{11}, x_{21}, \omega_L) + (1-\alpha)u^2(x_{12}, x_{22}, \omega_L) = \alpha + 8(1-\alpha),$$

which is smaller or equal to his payoff from reporting both ω^2 and s_2^2 truthfully if $\alpha \geq \frac{1}{5}$.

We next consider the behavior of A2 when he is of type ω_H. If he truthfully reports both his type and his signal to P1, then, regardless of the signal he receives from P2, his expected payoff is equal to

$$\alpha u^2(x_{12}, x_{21}, \omega_H) + (1-\alpha)u^2(x_{11}, x_{22}, \omega_H) = 9\alpha + 5(1-\alpha). \tag{8}$$

If, instead, A2 truthfully reports his type but misreports his signal to P1, then, regardless of the signal he receives from P2, his expected payoff is equal to

$$\alpha u^2(x_{11}, x_{21}, \omega_H) + (1-\alpha)u^2(x_{12}, x_{22}, \omega_H) = 6,$$

which is smaller or equal to his payoff from reporting both ω^2 and s_2^2 truthfully if $\alpha \geq \frac{1}{4}$.

If A2 misreports his type to P1 but reports the signal received from P2 truthfully, his expected payoff is equal to

$$\alpha u^2(x_{13}, x_{21}, \omega_H) + (1-\alpha)u^2(x_{14}, x_{22}, \omega_H) = 7\alpha + 9(1-\alpha), \tag{9}$$

which is smaller or equal to his payoff from reporting both ω^2 and s_2^2 truthfully if $\alpha \geq \frac{2}{3}$.

Finally, if A2 misreports both his type and the signal received from P2, his expected payoff is equal to

$$\alpha u^2(x_{14}, x_{21}, \omega_H) + (1-\alpha)u^2(x_{13}, x_{22}, \omega_H) = 6\alpha + 7(1-\alpha), \tag{10}$$

which is smaller or equal to his payoff from reporting both ω^2 and s_2^2 truthfully if $\alpha \geq \frac{2}{5}$.

The analysis above implies that it is a BNE for A1 and A2 to report truthfully to P1 in the subgame (γ_1^*, γ_2^*) if and only if $\alpha = \frac{2}{3}$. In this continuation equilibrium, P2 obtains her maximal payoff of 10. Because P1's payoff is constant, there exists a PBE of G_1^{SM} in which P1 and P2 post the mechanisms γ_1^* and γ_2^* on the equilibrium path, and A1 and A2 play any BNE in any subgame following a deviation by P1 or P2—the existence of such an equilibrium being guaranteed by the fact that all the subgames are finite. Hence the result. ■

Observe for future reference that, in equilibrium, the expected payoff to A1 is 4.5, while A2 obtains 4.5 if he is of type ω_L and $\frac{23}{3}$ if he is of type ω_H.

The above construction relies on the fact that, although the mechanism with signals γ_2^* is publicly disclosed to both agents, A1 and A2 receive different signals from P2 (private disclosures). In the example, such signals are uninformative of P2's decision. If P2 were to inform the agents of her decision, then after learning that P2 takes decision x_{21}, A2 of type ω_L would not be willing to induce the decision x_{13} with P1 .

The construction also reveals that, for P2 to obtain her maximal payoff of 10 while respecting the agents' incentives, it is essential that both principals randomize over their decisions but do so in a perfectly correlated manner. Whereas it is technically feasible to implement the equilibrium correlation between P1 and P2's decision by letting the agents randomize over their messages to the principals and committing to respond deterministically to the received messages, such a delegation is not incentive compatible. It is thus essential that the randomization be carried out by the principals themselves. The correlation in the principals' decisions then requires that some information be passed on from one principal to the other, which is possible only with private disclosures. The analysis in the next subsection (as well as the discussion in Section 5) confirms the above intuition by establishing the indispensability of the signals, no matter the richness of the message spaces.

3.2 The Indispensability of Private Signals

We now show that the outcome (1)–(2) for $\alpha = \frac{2}{3}$ cannot be supported in any PBE of any game without signals in which principals are constrained to post standard mechanisms, irrespective of the richness of message spaces. That is, private signals are indispensable to support the outcome in (1)–(2). To this end, we consider a general competing-mechanism game without signals in which every principal j can only post a standard mechanism ϕ_j :

$M_j \to \Delta(X_j)$. This general formulation notably allows us to capture the case where every principal j's message spaces M_j^1 and M_j^2 are large enough—namely, uncountable Polish spaces—to encode the agents' information about the mechanism posted by her opponent, as in Epstein and Peters (1999). We generically refer to such a game as G_1^M.

Claim 2 *There exists no PBE of G_1^M that supports the outcome (1)–(2) for $\alpha = \frac{2}{3}$.*

Proof. The proof more generally shows that there is no joint probability measure $\mu \in \Delta(\Phi_1 \times \Phi_2)$ over $\mathcal{F}_1 \otimes \mathcal{F}_2$ and no continuation equilibrium $\lambda = (\lambda^1, \lambda^2)$ that support the outcome (1)–(2) for any value of $\alpha \in [0, 1]$. The proof is by contradiction, and consists of four steps.

Step 1 Observe first that, with probability 1, μ must select a pair of mechanisms $\phi \equiv (\phi_1, \phi_2)$ such that, in the subgame ϕ, the equilibrium behavior strategies $(\lambda^1(\phi), \lambda^2(\phi))$ support an outcome of the form

$$z^\phi(\omega_L) \equiv \alpha_L^\phi \delta_{(x_{13}, x_{21})} + (1 - \alpha_L^\phi)\delta_{(x_{14}, x_{22})},$$
$$z^\phi(\omega_H) \equiv \alpha_H^\phi \delta_{(x_{12}, x_{21})} + (1 - \alpha_H^\phi)\delta_{(x_{11}, x_{22})},$$

for some $(\alpha_L^\phi, \alpha_H^\phi) \in [0, 1] \times [0, 1]$. Otherwise, with positive probability, P2 would incur a loss ζ, and his overall expected payoff would be strictly less than 10, a contradiction. Thus, for μ-almost every ϕ and for $(\lambda^1(\phi), \lambda^2(\phi))$-almost every message profile (m^1, m^2) sent by the agents under the equilibrium behavior strategies $(\lambda^1(\phi), \lambda^2(\phi))$, the lotteries $(\phi_1(m_1), \phi_2(m_2))$ over the principals' decisions must be degenerate.

Step 2 Consider then a subgame ϕ as in Step 1. We first claim that $\alpha_L^\phi \leq \alpha_H^\phi$. To see this, notice that, as A1 does not know which state prevails, the type-dependent behavior strategies $\lambda^2(\phi)(\cdot\,|\,\omega_L)$ and $\lambda^2(\phi)(\cdot\,|\,\omega_H)$ of A2 induce $z^\phi(\omega_L)$ and $z^\phi(\omega_H)$, respectively, given the equilibrium behavior of A1. Then, for type ω_L of A2 to induce $z^\phi(\omega_L)$ instead of $z^\phi(\omega_H)$, it must be that

$$3\alpha_L^\phi + 7.5(1 - \alpha_L^\phi) \geq 5\alpha_H^\phi + 3.5(1 - \alpha_H^\phi). \tag{11}$$

Similarly, for type ω_H of A2 to induce $z^\phi(\omega_H)$ instead of $z^\phi(\omega_L)$, it must be that

$$9\alpha_H^\phi + 5(1 - \alpha_H^\phi) \geq 7\alpha_L^\phi + 9(1 - \alpha_L^\phi). \tag{12}$$

The two inequalities are satisfied only if $\alpha_L^\phi \leq \alpha_H^\phi$. The claim follows. Because

$$\int \alpha_L^\phi \, \mu(\mathrm{d}\phi) = \alpha = \int \alpha_H^\phi \, \mu(\mathrm{d}\phi), \tag{13}$$

we obtain that, with μ-probability 1, $\alpha_L^\phi = \alpha_H^\phi$. Substituting $\alpha_L^\phi = \alpha_H^\phi$ into (11)–(12), it follows that, with μ-probability 1, $\alpha_L^\phi = \alpha_H^\phi = \frac{2}{3}$ and, hence, by (13), that $\alpha = \frac{2}{3}$. It follows that, with probability 1, μ must select a pair of mechanisms ϕ such that the agents' equilibrium behavior strategies $(\lambda^1(\phi), \lambda^2(\phi))$ implement the outcome (1)–(2) for $\alpha = \frac{2}{3}$, yielding types ω_L and ω_H of A2 expected payoffs of 4.5 and $\frac{23}{3}$, respectively.

Step 3 Now, observe that for $\lambda^2(\phi\,|\,\omega_H) \otimes \lambda^2(\phi\,|\,\omega_H)$-almost every pair of message profiles $(m^2, \hat{m}^2)$ sent by type ω_H of A2, if he sends the message profile $(m_1^2, \hat{m}_2^2)$, then, given that A1 randomizes according to $\lambda^1(\phi)$, the mechanisms (ϕ_1, ϕ_2) induce the following marginal distributions over $(x_{11}, x_{12}, x_{21}, x_{22})$:

$$\Pr(x_{11}, x_{21}) + \Pr(x_{11}, x_{22}) = \frac{1}{3}, \tag{14}$$

$$\Pr(x_{12}, x_{21}) + \Pr(x_{12}, x_{22}) = \frac{2}{3}, \tag{15}$$

$$\Pr(x_{11}, x_{21}) + \Pr(x_{12}, x_{21}) = \frac{2}{3}, \tag{16}$$

$$\Pr(x_{11}, x_{22}) + \Pr(x_{12}, x_{22}) = \frac{1}{3}. \tag{17}$$

It is easy to check that this system has not full rank, and admits a continuum of solutions indexed by $p \equiv \Pr(x_{11}, x_{21}) = \Pr(x_{12}, x_{22})$, which allows us to write $\Pr(x_{11}, x_{22}) = \frac{1}{3} - p$ and $\Pr(x_{12}, x_{21}) = \frac{2}{3} - p$. Now, if type ω_L of A2 were to send the messages $(m_1^2, \hat{m}_2^2)$, he would obtain an expected payoff of

$$p + 8p + 5\left(\frac{2}{3} - p\right) + 3.5\left(\frac{1}{3} - p\right) = 4.5 + 0.5p.$$

Because this must at most be his equilibrium payoff of 4.5, it follows that $p = 0$. This implies that, for $\lambda^1(\phi)$-almost every m_1, we have

$$(\phi_1(m_1^1, m_1^2), \phi_2(m_2^1, \hat{m}_2^2)) \in \{(x_{11}, x_{22}), (x_{12}, x_{21})\}.$$

But, for $\lambda^1(\phi)$-almost every m_1, we have

$$(\phi_1(m_1^1, m_1^2), \phi_2(m_2^1, m_2^2)) \in \{(x_{11}, x_{22}), (x_{12}, x_{21})\},$$

$$(\phi_1(m_1^1, \hat{m}_1^2), \phi_2(m_2^1, \hat{m}_2^2)) \in \{(x_{11}, x_{22}), (x_{12}, x_{21})\},$$

and thus, as decisions are perfectly correlated,

$$(\phi_1(m_1^1, m_1^2), \phi_2(m_2^1, m_2^2)) = (\phi_1(m_1^1, \hat{m}_1^2), \phi_2(m_2^1, \hat{m}_2^2)). \tag{18}$$

Because this property is satisfied for $\lambda^2(\phi\,|\,\omega_H) \otimes \lambda^2(\phi\,|\,\omega_H)$-almost every $(m^2, \hat{m}^2)$, we can conclude from Fubini's theorem that (18) holds for $\lambda^1(\phi) \otimes \lambda^2(\phi\,|\,\omega_H) \otimes \lambda^2(\phi\,|\,\omega_H)$-almost every $(m^1, m^2, \hat{m}^2)$. From Fubini's theorem again, it follows that for $\lambda^1(\phi)$-almost every m_1, (18) holds for $\lambda^2(\phi \mid \omega_H) \otimes \lambda^2(\phi \mid \omega_H)$-almost every $(m_2, \hat{m}_2)$, so that the mapping $(m_1^2, m_2^2) \mapsto (\phi_1(m_1^1, m_1^2), \phi_2(m_2^1, m_2^2))$ is constant over a set of $\lambda^2(\phi)$-measure 1.

Step 4 We are now ready to complete the proof. The upshot from Step 3 is that A1 can force the decision when the state is ω_H. This implies that M^1 should include a message profile allowing A1 to implement (x_{11}, x_{22}) regardless of the message sent in equilibrium by A2. By sending this message, A1 can achieve a payoff of 7.5 when the state is ω_H. Thus, he can guarantee himself an expected payoff of at least $\frac{3}{4} \times 7.5$, which is higher than his equilibrium payoff of 4.5, a contradiction. Hence the result. ■

The reason why the outcome (1)–(2) cannot be supported with standard mechanisms is the following. First, because the principals' decisions are perfectly correlated in both states, the mechanisms offered by the principals must respond deterministically to the messages sent by the agents (Step 1 in the proof of Claim 2). Second, because only A2 observes the state, and because the distribution over the principals' decisions varies with the state, A2 must weakly prefer the distribution over the messages he is supposed to send in each state to the one he is supposed to carry out in the other state. This constraints the joint distributions that can be sustained in the two states (Step 2 in the proof of Claim 2). Third, for A2 to prefer the distribution over the principals' decisions he is supposed to induce in state ω_L to the one that he can induce by "mis-matching" the principals' decisions while preserving the marginal distributions he induces in state ω_H, it must be that, given the mechanisms offered in equilibrium, the messages he sends in state ω_H are not influential when combined with those sent with positive probability by A1 (Step 3 in the proof of Claim 2). But then A1 has a profitable deviation (Step 4 in the proof of Claim 2).

It should be noted that the result in Claim 2 holds no matter the richness of the message spaces. Hence, it also applies to the Epstein and Peters (1999) class of universal mechanisms. Specifically, because these mechanisms are standard ones, the two claims above jointly imply the following result:

Proposition 1 *There exist allocations that can be supported in a PBE of a competing-mechanism game in which principals offer mechanisms with signals but that cannot be supported in any PBE of any competing-mechanism game in which principals are restricted to offering standard mechanisms (including universal mechanisms)—and this, even if the principals or the agents mix in equilibrium.*[4]

[4]Epstein and Peters (1999, Theorem 4.1) restrict attention to PBEs in which principals play pure strate-

The result suggests that the universal mechanisms of Epstein and Peters (1999) fail to be canonical when principals can engage in private disclosures, that is, when they can send private signals to the agents about their decisions as a way of correlating their decisions with those of other principals and with the agents' exogenous private information.

Remark The proof of Claim 2 does not make use of the property that the principals' randomizations are independent. It also allows for the possibility that the agents observe the realization of a common randomization device that they use to correlate their messages. In other words, the result in Claim 2 carries over to the case where G_1^M is augmented by arbitrarily rich public randomizing devices.

4 Non-robustness of Equilibria in Standard Mechanisms

Example 1 shows that competing-mechanism games in which principals can engage in private disclosures before the agents send their messages admit equilibria whose outcomes may not be sustainable in games in which principals are restricted to offered standard mechanisms, no matter the richness of the message spaces.

In this section, we address the dual question of whether equilibria in games in which principals are restricted to standard mechanisms (with a rich message space) are robust to the possibility that principals deviate and offer mechanisms with signals. This issue is especially relevant in light of the fact that, as shown by Yamashita (2010) and Peters and Troncoso-Valverde (2013), competing-mechanism games without private disclosures typically lend themselves to folk-theorem-type of results.

The construction in Yamashita (2010) exploits the idea that a principal's equilibrium mechanism can be made sufficiently flexible to punish possible deviations by other principals by allowing the agents to recommend to her which direct mechanism to use. Formally, a (deterministic) *direct mechanism* for principal j is a mapping $d_j : \Omega \to X_j$ associating a decision to every profile of reported types she may receive from the agents; we denote by D_j the finite set of all such mechanisms. Then consider a competing-mechanism game without signals in which every message set M_j^i is sufficiently rich to allow agent i to recommend a mechanism in D_j to principal j and to make a report about his type, that is, $D_j \times \Omega^i \subset M_j^i$ for all i and j. Formally, a *recommendation mechanism* ϕ_j^r for principal j stipulates that, if

gies.

every agent i sends a message $m_j^i \equiv (d_j^i, \omega^i) \in D_j \times \Omega^i$, then

$$\phi_j^r(m_j^1, \ldots, m_j^I) \equiv \begin{cases} d_j(\omega^1, \ldots, \omega^I) & \text{if card}\{i : d_j^i = d_j\} \geq I-1, \\ \overline{x}_j & \text{otherwise,} \end{cases} \tag{19}$$

where $\overline{x}_j$ is some fixed decision in X_j; if, instead, some agent i sends a message $m_j^i \notin D_j \times \Omega^i$, then ϕ_j^r treats agent i's message as if it coincided with some fixed element $(\overline{d}_j, \overline{\omega}^i)$ of $D_j \times \Omega^i$. Recommendation mechanisms provide a flexible system of punishments against unilateral deviations by other principals. Specifically, Theorem 1 in Yamashita (2010) states that, if there are at least three agents, every deterministic incentive-compatible allocation that yields every principal a payoff above a well-defined min-max-min bound can be supported in equilibrium.[5]

Below we show that the introduction of private signals challenges this characterization result. We exhibit an example of an equilibrium outcome of a competing-mechanism game à la Yamashita (2010) that does not survive when principals can deviate to mechanisms with private disclosures. Equilibrium outcomes sustained by standard mechanisms may thus not be robust.

Example 2 Let $J = 2$ and $I = 3$. We denote the principals by P1 and P2, and the agents by A1, A2, and A3. The decision sets are $X_1 \equiv \{x_{11}, x_{12}\}$ for P1 and $X_2 \equiv \{x_{21}, x_{22}\}$ for P2. A1 and A2 each have two types, with $\Omega^1 = \Omega^2 \equiv \{\omega_L, \omega_H\}$; A3 has only one type, which we drop from all the notation to ease the exposition. A1's and A2's types are perfectly correlated: only the states (ω_L, ω_L) and (ω_H, ω_H) can occur with positive probability under $\mathbf{P}$.

The players' payoffs are represented in Tables 7 and 8 below, in which the first payoff is that of P2 and the last two payoffs are those of A1 and A2, respectively. P1's and A3's payoffs are constant over X and hence omitted.

	x_{21}	x_{22}
x_{11}	$5, 8, 8$	$5, 1, 1$
x_{12}	$6, 4.5, 4.5$	$6, 4.5, 4.5$

Table 3: Players' payoffs in state (ω_L, ω_L).

[5] As pointed out by Peters (2014), these bounds, however, typically depend on the message spaces M_j^i. Attar, Campioni, Mariotti, and Piaser (2021) also show that this and related folk theorems crucially require that every agent participates and communicates with every principal regardless of the profile of posted mechanisms.

	x_{21}	x_{22}
x_{11}	$6, 4.5, 4.5$	$6, 4.5, 4.5$
x_{12}	$5, 1, 1$	$5, 8, 8$

Table 4: Players' payoffs in state (ω_H, ω_H)

4.1 An Equilibrium in Recommendation Mechanisms

We first characterize an equilibrium outcome supported by recommendation mechanisms, as in Yamashita (2010). Thus consider a general competing-mechanism game without signals but with rich message spaces. In particular, assume that $D_j \times \Omega^i \subset M_j^i$ for all i and j, so that recommendation mechanisms are feasible. To guarantee the existence of a BNE in every subgame $\phi \equiv (\phi_1, \phi_2)$, assume that all the message spaces M_j^i are finite. We generically refer to such a game as G_2^M. The following result then holds.

Claim 3 *The outcome*

$$z(\omega_L, \omega_L) \equiv \delta_{(x_{11}, x_{21})}, \quad z(\omega_H, \omega_H) \equiv \delta_{(x_{12}, x_{22})} \tag{20}$$

is a PBE outcome of G_2^M. In any equilibrium supporting such an outcome, P2 obtains a payoff of 5.

Proof. First we show that, if both principals post recommendation mechanisms, then there is a continuation equilibrium implementing the outcome (20). Next we show that one can construct a complete strategy profile $\lambda = (\lambda_1, \lambda_2, \lambda_3)$ for the agents which is a BNE in each subgame $\phi = (\phi_1, \phi_2)$ and such that, when P1 offers the equilibrium recommendation mechanism, P2 does not have profitable deviations. The result then follows from the above properties along with the fact that P1's payoff is constant over X and hence does not have profitable deviations.[6]

On Path. Suppose that both P1 and P2 post recommendation mechanisms. We claim that, in the corresponding subgame (ϕ_1^r, ϕ_2^r), it is a BNE for the three agents to recommend the direct mechanisms (d_1^*, d_2^*) defined by

$$d_1^*(\omega) \equiv \begin{cases} x_{11} & \text{if } \omega = (\omega_L, \omega_L), \\ x_{12} & \text{otherwise,} \end{cases} \qquad d_2^*(\omega) \equiv \begin{cases} x_{21} & \text{if } \omega = (\omega_L, \omega_L), \\ x_{22} & \text{otherwise,} \end{cases}$$

for all $\omega \equiv (\omega^1, \omega^2) \in \Omega^1 \times \Omega^2$, and for A1 and A2 to report their types truthfully to both principals. To see this, simply observe that these strategies implement the outcome (20),

[6] Consistently with what mentioned above, for simplicity, we denote a generic profile of standard mechanisms by ϕ instead of γ.

which yields A1 and A2 their maximum payoff of 8 in every state; because A3's payoff is constant, these strategies thus form a BNE of the subgame (ϕ_1^r, ϕ_2^r).

Off Path. Because P1's payoff is constant, she has no profitable deviations. Suppose then that P2 deviates to some arbitrary standard mechanism $\phi_2 : M_2 \to \Delta(X_2)$, and let $p(m_2)$ be the probability that the lottery $\phi_2(m_2)$ assigns to decision x_{21} when the agents send the messages $m_2 \equiv (m_2^1, m_2^2, m_2^3) \in M_2$ to P2. Now, let

$$\overline{p} \equiv \max_{m_2 \in M_2} p(m_2) \tag{21}$$

and then let $\overline{m}_2 \equiv (\overline{m}_2^1, \overline{m}_2^2, \overline{m}_2^3)$ be a message profile that achieves the maximum in (21); similarly, let

$$\underline{p} \equiv \min_{(m_2^1, m_2^2) \in M_2^1 \times M_2^2} p(m_2^1, m_2^2, \overline{m}_2^3) \tag{22}$$

and denote by $(\underline{m}_2^1, \underline{m}_2^2) \in M_2^1 \times M_2^2$ a profile of messages for A1 and A2 that, given $\overline{m}_2^3$, achieves the minimum in (22).[7] That $\overline{p}$, $\overline{m}_2$, $\underline{p}$, and $(\underline{m}_2^1, \underline{m}_2^2)$ are well-defined follows from the fact that M_2 is finite. We now prove that there exist BNE strategies for the agents in the subgame (ϕ_1^r, ϕ_2) such that P2 obtains a payoff of 5, so that the deviation is not profitable. We consider two cases in turn.

Case 1. Suppose first that ϕ_2 is such that $\overline{p} \geq \frac{1}{2}$. We claim that there exists a BNE of the subgame (ϕ_1^r, ϕ_2) with the following properties: (i) all agents recommend d_1^* to P1, as if P2 did not deviate; (ii) A1 and A2 report truthfully their types to P1; (iii) A3 sends the message $\overline{m}_2^3$ to P2; (iv) P2 obtains a payoff of 5. As for (i), the argument is that unilaterally sending a different recommendation to P1 is of no avail as no agent is pivotal. Consider now (ii)–(iii). Clearly, for A3 sending the message $\overline{m}_2^3$ to P2 is optimal given that his payoff is constant over X. Thus consider A1 and A2 . Suppose first that the state is (ω_L, ω_L). Because $\overline{p} \geq \frac{1}{2}$, $8\overline{p} + (1 - \overline{p}) \geq 4.5$. From Table 3, and by definition of d_1^* and $\overline{m}_2$, it is then clear that, if A2 reports ω_L to P1 and sends $\overline{m}_2^2$ to P2, and if A3 sends $\overline{m}_2^3$ to P2, then A1 best responds by reporting ω_L to P1 and sending $\overline{m}_2^1$ to P2; the argument for A2 is identical. Suppose next that the state is (ω_H, ω_H). If either A1 or A2 truthfully reports his type to P1, then, by definition of d_1^*, the other informed agent A2 or A1 cannot induce P1 to take a decision other than x_{12}. The above properties, along with the finiteness of M_2, then imply existence of a BNE for the subgame (ϕ_1^r, ϕ_2) satisfying properties (i)-(iii) above. Under such a BNE, P1 takes the decision x_{11} in state (ω_L, ω_L) and the decision x_{12} in state (ω_H, ω_H), yielding a payoff of 5 to P2, as claimed in (iv).

[7] Clearly, $\overline{p}$, as well as $\overline{m}_2$ and $(\underline{m}_2^1, \underline{m}_2^2)$ depend on the mechanism ϕ_2. We drop the dependence to ease the notation.

Case 2. Suppose next that ϕ_2 is such that $\overline{p} < \frac{1}{2}$. We claim that there exists a BNE of the subgame (ϕ_1^r, ϕ_2) satisfying the following properties: (i) all agents recommend the direct mechanism

$$d_1(\omega) \equiv \begin{cases} x_{12} & \text{if } \omega = (\omega_H, \omega_H), \\ x_{11} & \text{otherwise} \end{cases}$$

to P1; (ii) A1 and A2 report their types truthfully to P1; (iii) A3 sends the message $\overline{m}_2^3$ to P2; (iv) P2 obtains a payoff of 5. The arguments for (i), (iii) and (iv) are the same as in Case 1. Thus consider (ii). Suppose first that the state is (ω_L, ω_L). If either A1 or A2 truthfully reports his type to P1, then, by definition of d_1, the other informed agent A2 or A1 cannot induce P1 to take a decision other than x_{11}. Suppose next that the state is (ω_H, ω_H). Because $\underline{p} \leq \overline{p} < \frac{1}{2}$, $\underline{p} + 8(1 - \underline{p}) \geq 4.5$. From Table 4, and by definition of d_1 and $(\underline{m}_2^1, \underline{m}_2^2)$, it is then clear that, if A2 reports ω_H to P1 and sends the message $\underline{m}_2^2$ to P2, and if A3 sends $\overline{m}_2^3$ to P2, then A1 best responds by reporting ω_H to P1 and $\underline{m}_2^1$ to P2; the argument for A2 is identical. The above properties, together with the finiteness of M_2 then imply existence of a BNE satisfying properties (i)-(iv) above, as claimed. ■

The proof of Claim 3 relies on the same intuition as in Yamashita (2010, Theorem 1). The possibility for the agents to recommend a different direct mechanism to P1 for every mechanism posted by P2 allows them to implement punishments contingent on P2's deviations. In particular, the argument in Case 2 shows that any deviation by P2 to a mechanism that implements x_{21} with a probability strictly less than $\frac{1}{2}$ is blocked by recommending to P1 to use the direct mechanism d_1 which is different from the equilibrium mechanism d_1^*. Observe that, although principals may post stochastic mechanisms, the threat of agents choosing a deterministic direct mechanism is sufficient to yield P2 her min-max-min payoff of 5 in equilibrium, as in Yamashita (2010).

Remark In related work, Peters and Troncoso-Valverde (2013) establish a folk theorem in a generalized version of Yamashita (2010). In the game they consider, any outcome corresponding to an allocation that is incentive-compatible and individually rational in the sense of Myerson (1979) can be supported in equilibrium provided there are at least seven players. It is straightforward to check that the outcome (20) satisfies these conditions, which guarantees that it can also be supported in equilibrium in their framework.[8]

[8] The requirement on the number of players can always be met in our example by adding additional agents identical to A3.

4.2 Non-robustness to Private Signals

We now show that the outcome in (20) cannot be supported in any equilibrium of an enlarged game in which the principals can send private signals to the agents. To this end, we consider a general competing-mechanism game with signals, in which $M_j^i \supset D_j \times \Omega^i$. To show that the result is not driven by possible non-existence of equilibria, we restrict the message spaces M_j^i and the signal spaces S_j^i to be finite, for all j and i.[9] We generically refer to such a game as G_2^{SM}. The following result then holds.

Claim 4 *G_2^{SM} admits a PBE. Moreover, if* $\operatorname{card} S_2^1 \geq 2$, *then P2's payoff in any PBE of G_2^{SM} is strictly higher than 5.*

Proof. We first establish the second part of the result, assuming that the set of PBE of G_2^{SM} is non-empty. We then show that G_2^{SM} admits at least one PBE.

To prove the second part of the result, we exhibit a mechanism with signals that guarantees P2 an expected payoff strictly above 5, regardless of the mechanism posted by P1 and of the agents' continuation equilibrium. That is, we show that there exists a mechanism $\gamma_2 \in \Gamma_2$ such that

$$\inf_{\gamma_1 \in \Gamma_1} \inf_{\beta \in B^*(\gamma_1, \gamma_2)} \sum_{\omega \in \Omega} \mathbf{P}[\omega] \sum_{x \in X} z_{\gamma_1, \gamma_2, \beta}(x \mid \omega) v_2(x, \omega) > 5, \tag{23}$$

where $B^*(\gamma_1, \gamma_2)$ is the set of BNE of the subgame (γ_1, γ_2), and where $z_{\gamma_1, \gamma_2, \beta}(x \mid \omega)$ is the probability that the decision profile x is selected when the agents' private information is ω, the principals' mechanisms are (γ_1, γ_2) and the agents play according to β. The proof consists of two steps.

Step 1 We first establish that, for each $\gamma_2 \in \Gamma_2$, there exists a pair (γ_1, β) that achieves the two infima in (23). Because the signal and message spaces are finite, $\Gamma_1 \equiv \Delta(S_1) \times \Delta(X_1)^{S_1 \times M_1}$ is compact and every subgame (γ_1, γ_2) is finite; moreover, the agents' information structures and payoffs in this subgame are continuous functions of γ_1. Hence the correspondence $B^*(\cdot, \gamma_2) : \Gamma_1 \twoheadrightarrow \prod_{i=1}^{3} \Delta(M^i)^{S^i \times \Omega^i}$ is upper hemicontinuous with nonempty compact values (Milgrom and Weber (1985, Theorem 2)). Because $z(\gamma, \beta)$ is multilinear in β, this implies that the infimum with respect to $\beta \in B^*(\gamma_1, \gamma_2)$ in (23) is attained and that the value of this infimum is a lower semicontinuous function of γ_1 (Aliprantis and Border (2006, Lemma 17.30)). Because Γ_1 is compact, this, in turn, implies that the infimum with respect to $\gamma_1 \in \Gamma_1$ in (23) is attained.

[9] It should be clear from the arguments below that the result in Claim 4 extends to any infinite game G_2^{SM} for which the set of PBE is not empty.

Step 2 Now, suppose that card $S_2^1 \geq 2$, and, without loss of generality, let $\{1,2\} \subset S_2^1$.[10] Fix some $\alpha \in (\frac{1}{2}, 1)$ and consider the following mechanism with signals for P2, $\overline{\gamma}_2$, in which:

- with probability α, P2 sends signal $s_2^1 = 1$ to A1 and signal $s_2^2 = s_2^3 = \emptyset$ to A2 and A3 and commits to take decision x_{21} regardless of the messages she receives from the agents;
- with probability $1 - \alpha$, P2 sends signal $s_2^1 = 2$ to A1 and signal $s_2^2 = s_2^3 = \emptyset$ to A2 and A3 and commits to take decision x_{22} regardless of the messages she receives from the agents.

Given the private signals received from P2, A1 knows exactly P2's decision, whereas A2 and A3 remain uninformed, that is, believe that P2 takes decision x_{21} with probability α and decision x_{22} with probability $1 - \alpha$, but know that A1 knows P2's decision. We establish that $\overline{\gamma}_2$ satisfies (23).

Indeed, suppose that $\overline{\gamma}_2$ does not satisfy (23). Then, because 5 is the lowest payoff that P2 can obtain in the game, it follows from Step 1 that there exists $(\gamma_1, \beta) \in \Gamma_1 \times B^*(\gamma_1, \overline{\gamma}_2)$ such that posting $\overline{\gamma}_2$ yields P2 a payoff exactly equal to 5. Observe that the mechanism $\overline{\gamma}_2$ implements decisions in X_2 that are independent of any messages P2 may receive from the agents and, hence, of any signals sent by γ_1. Thus the only role that signals in γ_1 could play, given $\overline{\gamma}_2$, would be to affect the distribution over P1's decisions induced by the agents; but it follows from standard arguments (Myerson (1982)) that messages are enough to this end, and thus that signals are redundant. We can thus assume that γ_1 is equivalent to a standard mechanism ϕ_1, involving no signals.

We now come to the bulk of the argument. Notice that the only way P2's payoff can be equal to 5 is for P1 to take decision x_{11} in state (ω_L, ω_L) and x_{12} in state (ω_H, ω_H) with probability 1. From Table 3, in state (ω_L, ω_L), upon receiving $s_2^1 = 2$ from P2, A1 wants to minimize the probability that P1 takes decision x_{11}. Because this probability must be equal to 1, it must be that $\phi_1(m_1^1, m_1^2, m_1^3) = \delta_{x_{11}}$ for any message $m_1^1 \in M_1^1$ that A1 can send to P1 and any profile of messages $(m_1^2, m_1^3) \in M_1^2 \times M_1^3$ that A2 and A3 send with positive probability to P1 in state (ω_L, ω_L). Similarly, from Table 4, in state (ω_H, ω_H), upon receiving $s_2^1 = 1$ from P2, A1 wants to maximize the probability that P1 takes decision x_{11}. Because this probability must be equal to 0, it must be that $\phi_1(m_1^1, m_1^2, m_1^3) = \delta_{x_{12}}$ for any message $m_1^1 \in M_1^1$ that A1 can send to P1 and any profile of messages $(m_1^2, m_1^3) \in M_1^2 \times M_1^3$ that A2 and A3 send with positive probability to P1 in state (ω_H, ω_H).

[10] Because the labels of the signals play no role, if card $S_2^1 \geq 2$, it is without loss of generality to assume that two signals in S_2^1 are labelled 1 and 2.

The upshot of this discussion is that, given the other agents' strategies, A1's message to P1 must have no influence on P1's decision. Moreover, because A3 does not observe the state, and because P2 obtains a payoff of 5 only when P1's decision varies with the state, the message that A3 sends to P1 under the putative BNE must provide A2 with full control over P1's decision. That is, there must be sets of messages $M_1^2(x_{11}) \subsetneq M_1^2$ and $M_1^2(x_{12}) \subsetneq M_1^2$ such that, in the subgame $(\phi_1, \overline{\gamma}_2)$, when A3 sends any message $m_1^3 \in supp[\beta^3]$, by sending any message $m_1^2 \in M_1^2(x_{11})$ (alternatively, any message $m_1^2 \in M_1^2(x_{12})$), A2 induces x_{11} (alternatively, x_{12}) with certainty, for any message $m_1^1 \in M_1^1$ that A1 can possibly send to P1. Furthermore, β^2 must be such that, for any $s^2 \in S^2$,

$$\beta^2(M_1^2(x_{11}) \times M_2^2 | s^2, \omega_L) = 1 = \beta^2(M_1^2(x_{12}) \times M_2^2 | s^2, \omega_H).$$

That is, the messages that A2 sends to P1 in state ω_L (alternatively, in state ω_H) must be such that, when paired with the equilibrium messages sent by A3 to P1 , they induce P1 to select x_{11} (alternatively, x_{12}) with certainty, for any possible message sent by A1 to P1.

However, because P2 takes decision x_{21} with probability $\alpha > \frac{1}{2}$, it is clear from Table 4 that A2 has a profitable deviation: when observing ω_H she is better off inducing P1 to take decision x_{11} instead of x_{12}, thus contradicting the assumption that $\beta \in B^*(\gamma_1, \overline{\gamma}_2)$. Hence, $\overline{\gamma}_2$ satisfies (23). The second part of the claim then follows.

To complete the proof, there remains to show that G_2^{SM} admits a PBE. Because P1's payoff is constant, we can assume that, for any profile of mechanisms (γ_1, γ_2) posted by the principals, the agents play an equilibrium $\beta^*(\gamma_1, \gamma_2)$ in the subgame (γ_1, γ_2) that maximizes P2's expected payoff, that is,

$$\beta^*(\gamma_1, \gamma_2) \in \underset{\beta \in B^*(\gamma_1, \gamma_2)}{\arg\max} \sum_{\omega \in \Omega} \mathbf{P}[\omega] \sum_{x \in X} z_{(\gamma_1, \gamma_2, \beta)}(x \mid \omega) v_2(x, \omega). \tag{24}$$

By the measurable maximum theorem (Aliprantis and Border (2006, Theorem 18.19)), $\beta^*(\gamma_1, \gamma_2)$ can be selected so as to be measurable in (γ_1, γ_2); for each i, the corresponding strategy for agent i in G_2^{SM} is then defined by $\lambda^{i*}(m^i \,|\, \gamma_1, \gamma_2, s^i, \omega^i) \equiv \beta^{i*}(\gamma_1, \gamma_2)(m^i \,|\, s^i, \omega^i)$. It follows along the same lines as in Step 1 that the value of the maximum in (24) is upper semicontinuous in (γ_1, γ_2), and hence attains itself a maximum at some $\gamma^* \equiv (\gamma_1^*, \gamma_2^*)$ in the compact set $\Gamma_1 \times \Gamma_2$. Because P1's payoff is constant, the strategies (γ^*, λ^*) form a PBE of G_2^{SM}. Hence the result. ■

The proof of Claim 4 crucially exploits the fact that, by posting a mechanism with signals, P2 can make the agents asymmetrically informed about her decision. Specifically, the mechanism $\overline{\gamma}_2$ we construct is such that, when communicating with P1, A1 is perfectly informed of P2's decision, while A2 and A3 are kept in the dark. Such an asymmetry in the received information among the agents, which is possible only with private disclosures, is what guarantees that, no matter the mechanism offered by P1 and the selection of the equilibrium in the subgame among the agents, P2 obtains a payoff strictly above her min-max-min value of 5. To see this, note that the only way to keep P2's expected payoff down to 5 is for P1 to take decision x_{11} in state (ω_L, ω_L) and decision x_{12} in state (ω_H, ω_H). By informing A1 of her decision, P2 can exploit the fact that, given x_{22}, A1's preferences over X_1 are perfectly aligned with P2's (in each state) and guarantee that, if A1 can influence P1's decision in state (ω_L, ω_L), she would induce x_{12} with positive probability, bringing P2's payoff strictly above 5. Hence, given the other agents' messages, A1 must not be able to influence P1's decision in state (ω_L, ω_L). A similar argument implies that, given the messages sent by the other agents in state (ω_H, ω_H), A1 must also not be able to influence P1's decision in that state as well.[11] Because A3 does not observe the state, his message to P1 must be the same in each state. But this implies that, de facto, A2 has full control over P1's decision. However, when P2 is expected to select x_{21} with probability $\alpha > \frac{1}{2}$, A2, without receiving further information from P2, strictly prefers to induce x_{11} in both state. Hence, if she has the possibility to do so, which we just argued is the case, she does not induce the distribution over X_1 that delivers the max-min-max payoff of 5 to P2.

Because the agents' preferences are perfectly aligned, the reader may wonder why P2 informs the agents asymmetrically. The reason is that, when they have the same information, they can discipline each other, thus implementing incentive-compatible punishments for P2, as in Yamashita (2010)'s construction. For example, when they are perfectly informed of x_2, there exists a mechanism for P1 along with an equilibrium in the subgame among the agents that implements the distribution over X_1 that delivers 5 to P2. The possibility to inform the agents asymmetrically of her decision is precisely what permits P2 to avoid that the agents select the direct mechanism that would punish her deviation.

Jointly, Claims 3 and 4 imply the following result:

Proposition 2 *PBEs of competing-mechanism games in which principals are restricted to offering standard mechanisms need not be robust to the possibility that principals deviate to mechanisms with signals. Furthermore, equilibrium payoffs vectors that can be sustained in competing-mechanism games with rich message spaces—that is, such that* $M^i_j \supset D_j \times \Omega^i$ *for*

[11] Otherwise, in state (ω_H, ω_H), upon learning that $x_2 = x_{21}$, A1 would induce P1 to select x_{11} with positive probability, bringing P2 's payoff again strictly above 5.

all i and j—need not be sustainable in games in which principals can offer mechanisms with signals.

Remark Examples 1–2 illustrate how private disclosures (that is, signals sent by the principals to the agents before the latter send their messages to the principals) can affect the set of equilibrium outcomes in competing-mechanism games. For this to be the case, it is crucial that such signals be received by the agents before they send messages to the principals. For instance, in Example 2, if P1 could guarantee that agents send their messages to her after seeing P2's mechanism but before receiving private signals from P2, then the outcome (20) could be supported in equilibrium, yielding P2 a payoff of 5. Intuitively, this could be done by P1 posting a recommendation mechanism, as in the proof of Claim 3. However, in practice, it seems unlikely that principals have such a perfect control over the timing of communication with the agents necessary to neutralize the effects of private disclosures.

5 Discussion

In this section, we discuss the different roles played by private disclosures in our two examples. As shown in Example 1, private signals, while necessary to sustain certain outcomes, need not modify the agents' beliefs. In fact, in that example, signals work as pure encryption keys that, in isolation, are completely uninformative about a principal's decision, but that, when combined with the keys given to other agents, reveal a principal's decision. In contrast, the profitable deviations identified in Example 2 that undermine the robustness of equilibria in standard mechanisms hinge on the possibility for the principals to change the agents' beliefs off-path, before the agents communicate with other principals.

As discussed above, the impossibility to sustain the outcome in (1)–(2) in Example 1 extends to settings in which the principals and the agents have access to rich public randomization devices (aka "sunspots"). In contrast, the signals could be dispensed with if the principals had access to correlation devices that are either realized after the agents send their messages or that are observed privately by the principals but not by the agents. What the example shows is that, even in the absence of such devices, the principals can correlate their decisions by randomizing over the signals they send to the agents and have the agents pass the information on to other principals. In this respect, it is worth pointing out that the role that the signals play in Example 1 are different from the role that they play in single-principal settings. In the latter, signals are used to correlate the agents' behavior, when the agents have primitive decisions that cannot be taken by the principals directly. In Example 1, instead, the role of the signals is to pass information on from one principal to another so as to facilitate the correlation between the principals' decisions and the agents' exogenous private information, while respecting incentive compatibility.

Because signals play a broader role than correlation devices and/or recommendations for the actions to take in other mechanisms, an open question for this literature is what structure for the signal and message space is fully canonical, meaning that it (a) permits one to sustain all equilibrium allocations of rich competing-mechanism games with private disclosures, and (b) guarantees that the equilibrium outcomes of the corresponding competing-principal games are robust to the possibility for the principals to deviate to mechanisms with richer message and signal spaces.

Next, consider Example 2. In that example, the possibility for P2 to guarantee herself a payoff strictly above her min-max-min payoff hinges on informing the agents asymmetrically about her decisions. Indeed, the main thrust of private signals in Example 2 is the destabilizing role they play out of equilibrium.

We already argued in Section 4 that, if P2 were to perfectly inform all agents of her decisions, or, more generally, of the allocations selected in response to the agents' messages, then it would be possible for P1 to offer a mechanism that brings P2's payoff back to the min-max-min value. Below we show that the same is true under any signal structures that leaves the agents' in the dark. Formally, we show that the analogue of Claim 4 would not hold if P2 were restricted to mechanisms in which private signals are uninformative encryption keys, as in the proof of Claim 1. To see this, consider the game G_2^{SM} studied in Claim 4; moreover, as in Claim 3, assume that $D_j \times \Omega^i \subset M_j^i$ for all i and j, so that recommendation mechanisms are feasible, and that all the message spaces M_j^i are finite. We say that a mechanism $\gamma_2 \equiv (\sigma_2, \phi_2)$ of P2 has *uninformative signals* if (a) σ_2 is a product measure over S_2, and (b) for all i, $s_2^i \in S_2^i$, $m_2 \in M_2$, and $x_2 \in X_2$,

$$\sum_{s_2^{-i} \in S_2^{-i}} \sigma_2^{-i}(s_2^{-i})\phi_2(x_2 \,|\, s_2^i, s_2^{-i}, m_2) = \sum_{s_2 \in S_2} \sigma_2(s_2)\phi_2(x_2 \,|\, s_2, m_2), \tag{25}$$

where σ_2^{-i} is the marginal of σ_2 over S_2^{-i}.[12]

The first condition states that the private signal P2 sends to any agent is uninformative about the private signals she sends to the other agents. The second condition states that, given any profile of messages the agents may send to P2, the private signal P2 sends to any

[12]That S_2 is a finite set plays no role in the following analysis. In particular, Claim 5 below remains valid if S_2 is an arbitrary Polish space.

agent is uninformative about her decision. The following result then holds.

Claim 5 *In Example 2, if P1 posts a recommendation mechanism ϕ_1^r, then, for every mechanism γ_2 of P2 that has uninformative signals, there exists a BNE of the subgame (ϕ_1^r, γ_2) that yields P2 a payoff of 5.*

Proof of Claim 5. Because A3's payoff is constant and A1 and A2's payoff functions are identical, we can focus on A1's incentives. Suppose that, in the subgame (ϕ_1^r, γ_2), A2 and A3 play behavior strategies β^2 and β^3 that prescribe the same play for any signals s_2^2 and s_2^3 they may receive from P2, respectively; that is, for each $\omega^2 \in \Omega^2$, $\beta^2(\cdot \mid s_2^2, \omega^2)$ is independent of s_2^2, and $\beta^3(\cdot \,|\, s_2^3)$ is independent of s_2^3. Then, because every signal A1 receives from P2 is uninformative, A1 may as well best respond by playing a behavior strategy β^1 that prescribes the same play for any signal s_2^1 he may receive from P2; that is, for each $\omega^1 \in \Omega^1$, $\beta^1(\cdot \,|\, s_2^1, \omega^1)$ is independent of s_2^1. Because all the message spaces M_j^i are finite, this implies that there exists a BNE of the subgame (ϕ_1^r, γ_2) in which all agents play behavior strategies that prescribe the same play for any signals they may receive from P2. According to (25), any such BNE of the subgame (ϕ_1^r, γ_2) can be straightforwardly turned into a BNE of the subgame $(\phi_1^r, \hat{\phi}_2)$ in which P1 posts the recommendation mechanism ϕ_1^r and P2 posts the standard mechanism $\hat{\phi}_2$ such that, for all $m_2 \in M_2$ and $x_2 \in X_2$,

$$\hat{\phi}_2(x_2 \,|\, m_2) \equiv \sum_{s_2 \in S_2} \sigma_2(s_2)\phi_2(x_2 \,|\, s_2, m_2).$$

Notice that, by construction, the same outcome is implemented in each case. Conversely, any BNE of the subgame $(\phi_1^r, \hat{\phi}_2)$ can be straightforwardly turned into a BNE of the subgame (ϕ_1^r, γ_2) in which all agents play behavior strategies that prescribe the same play for any signals they may receive from P2, and which implements the same outcome. To conclude, observe that, as $\hat{\phi}_2$ is a standard mechanism, we know from Claim 3 that there exists a BNE of the subgame $(\phi_1^r, \hat{\phi}_2)$ that yields P2 a payoff of 5. Hence the result. ■

This result reflects that, if P1 posts a recommendation mechanism ϕ_1^r and P2 posts a mechanism γ_2 with uninformative signals, there exists a one-to-one correspondence between the babbling BNEs of the subgame (ϕ_1^r, γ_2) and the BNEs of the subgame $(\phi_1^r, \hat{\phi}_2)$ in which P2 posts the standard mechanism $\hat{\phi}_2$ obtained by averaging γ_2 over the profiles of signals s_2. Because P2's payoff can be kept down to 5 in the latter case, this must also be the true in the former case. Notice that there is no tension between this result and Claim 1, which showed the power of mechanisms with uninformative signals in the context of Example 1;

indeed, the key step in the proof of Claim 1 was precisely to construct a non-babbling BNE of the agents' subgame in which they truthfully report to P1 the uninformative signals they receive from P2. Yet Claim 5 suggests that, from P2's perspective, a potential drawback of mechanisms with uninformative signals is that they accommodate for babbling equilibria that may keep her payoff down to 5. This contrasts with the mechanism for P2 constructed in Claim 4, in which disclosing her decision to the agents in an asymmetric way allows P2 to guarantee herself a payoff strictly above her min-max-min bound of 5, regardless of the mechanism posted by P1 and of the agents' continuation equilibrium.

6 Conclusions

The results above show that private disclosures, which have been ignored in previous work, have profound effects on the equilibrium allocations of competing-mechanism games. They question the canonicity of the universal mechanisms discussed in the literature and the validity of the "folk theorems" established for such games.

The examples also suggest that signals in competing-mechanism games play a richer role than in standard single-principal games (where they can be reduced to recommendations for non-contractible actions). Identifying a canonical extensive form and a universal class of mechanisms for competing-mechanism games remains an open question for future research.

References

[1] Aliprantis, C.D., and K.C. Border (2006): *Infinite Dimensional Analysis*: *A Hitchhiker's Guide*, Berlin, Heidelberg, New York: Springer.

[2] Attar, A., E. Campioni, T. Mariotti, and G. Piaser (2021): "Competing Mechanisms and Folk Theorems: Two Examples," *Games and Economic Behavior*, 125, 79–93.

[3] Attar, A., E. Campioni, and G. Piaser (2019): "Private Communication in Competing Mechanism Games," *Journal of Economic Theory*, 183, 258–283.

[4] Aumann, R.J. (1961): "Borel Structures for Function Spaces," *Illinois Journal of Mathematics*, 5(4), 614–630.

[5] Epstein, L.G., and M. Peters (1999): "A Revelation Principle for Competing Mechanisms," *Journal of Economic Theory*, 88(1), 119–160.

[6] Forges, F. (1986): "An Approach to Communication Equilibria," *Econometrica*, 54(6), 1375–1385.

[7] Hurwicz, L. (1973): "The Design of Mechanisms for Resource Allocation," *American Economic Review*, 63(2), 1–30.

[8] Kalai, A., Kalai, T., Lehrer, E., and D. Samet (2010): "A commitment folk theorem," *Games and Economic Behavior.*, 69(1), 127–137.

[9] Martimort, D. and L. Stole (2002): "The Revelation and Delegation Principles in Common Agency Games," *Econometrica*, 70(4), 1659-1673.

[10] McAfee, R. (1993): "Mechanism Design by Competing Sellers," *Econometrica*, 61(6), 1281–1312.

[11] Milgrom, P.R., and R.J. Weber (1985): "Distributional Strategies for Games with Incomplete Information," *Mathematics of Operations Research*, 10(4), 619–631.

[12] Myerson, R.B. (1979): "Incentive Compatibility and the Bargaining Problem," *Econometrica*, 47(1), 61–73.

[13] Myerson, R.B. (1982): "Optimal Coordination Mechanisms in Generalized Principal-Agent Problems," *Journal of Mathematical Economics*, 10(1), 67–81.

[14] Myerson, R.B. (1986): "Multistage Games with Communication," *Econometrica*, 54(2), 323–358.

[15] Peck, J. (1997): "A Note on Competing Mechanisms and the Revelation Principle," Mimeo, Ohio State University.

[16] Pavan, A. and G. Calzolari (2009): "Sequential Contracting with Multiple Principals," *Journal of Economic Theory*, 144(2), 503–531.

[17] Pavan, A. and G. Calzolari (2010): "Truthful Revelation Mechanisms for Simultaneous Common Agency Games," *American Economic Journal: Microeconomics*, 2(2), 132–190.

[18] Peters, M. (2001): "Common Agency and the Revelation Principle," *Econometrica*, 69(5), 1349–1372.

[19] Peters, M. (2014): "Competing Mechanisms," *Canadian Journal of Economics*, 47(2), 373–397.

[20] Peters, M. (2015): "Reciprocal Contracting" *Journal of Economic Theory*, 158(PA), 102–126.

[21] Peters, M., and B. Szentes (2012): "Definable and Contractible Contracts," *Econometrica*, 80(1), 363–411.

[22] Peters, M., and C. Troncoso-Valverde (2013): "A Folk Theorem for Competing Mechanisms," *Journal of Economic Theory*, 148(3), 953–973.

[23] Prat, A., and A. Rustichini (2003): "Games Played Through Agents," *Econometrica*, 71(4), 989-1026.

[24] Rahman, D., and I. Obara (2010): "Mediated Partnerships," *Econometrica*, 78(1), 285–308.

[25] Rao, B.V. (1971): "Borel Structures for Function Spaces," *Colloquium Mathematicum*, 23(1), 33–38.

[26] Szentes, B., (2015): "Contractible Contracts in Common Agency Problems," *The Review of Economic Studies*, 82(1), 391–422.

[27] Yamashita, T. (2010): "Mechanism Games With Multiple Principals and Three or More Agents," *Econometrica*, 78(2), 791–801.