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Abstract

We study the design of pension benefits for male and female workers. Women live longer than men but have a lower wage. Individuals can be single or live in couples who pool their incomes. Social welfare is utilitarian but an increasing concave transformation of individuals' lifetime utilities introduces the concern for redistribution between individuals with different life-spans.

We derive the optimal direction of redistribution and show how it is affected by a gender neutrality rule. With singles only, a simple utilitarian solution implies redistribution from males to females. When the transformation is sufficiently concave redistribution may or may not be reversed. With couples only, the ranking of gender retirement ages is always reversed when the transformation is sufficiently concave.

Under gender neutrality pension schemes must be self-selecting. With singles only this implies distortions of retirement decision and restricts redistribution across genders. With couples, a first best that implies a lower retirement age for females can be implemented by a gender-neutral system. Otherwise, gender neutrality implies equal retirement ages and restricts the possibility to compensate the shorter-lived individuals. Calibrated simulations show that when singles and couples coexist, gender neutrality substantially limits redistribution in favor of single women and fully prevents redistribution in favor of male spouses.

Keywords: gender wage gap, gender gap in longevity, retirement systems.
JEL classification: H55, H31, H21.

1 Introduction

The longevity gap and the wage gap are two important factors in gender inequality, particularly when it comes to the retirement period. On average, women outlive men but, having earned less during their active life, they tend to have less savings when retiring. As a result, women are at greater risk of poverty in later life than men (Policy Department, European Parliament, 2019).

The longevity gap has been decreasing during the last decades, but it continues to be significant. Among OECD nations, the difference in life expectancy at birth is currently around four to six year (seven in Japan); see Goldin and Lleras-Muney (2019). Women have not only a longer life expectancy at birth, their mortality rates at every age are also lower. The explanation of these gender differences in mortality is subject to some debate, and there are several schools of thought, see Cullen et al. (2016) and references within. Theories range from those stipulating a selective female survival advantage on a "hard-wired" biologic basis to more sociological and behavioral based explanations.

Turning to the gender wage gap, the persisting and systematic gender differences in employment outcomes have been extensively studied; see Bertrand (2020) for a recent survey. The gap in earnings is synthesized by the gender wage gap: on average, women in the EU earn around 15 % less per hour than men (Eurostat 2020); see also Blau and Kahn (2018).

In the EU, pension systems manage to reduce these inequalities to some extent. Still, different earning histories and child care involvement continue to be reflected in a significant gender pension gap. In 2019, the average female pension income was 37% lower than that of men (European Parliament resolution of Jan.30, 2020). The redistributive elements of pensions and tax systems and the coverage of unemployment spells related to care activities mitigate the gender difference in labor market earnings. The importance of solidarity and redistribution has been recently confirmed by the Resolution of 14 June 2017 on the need for an EU strategy to end and prevent the gender pension gap. In addition, the European Parliament resolution of Jan.30, 2020 on the gender pay gap writes: "[...] in order to overcome pension gender inequalities and safeguard and increase pensions in general, *it is imperative that **social security systems** continue to exist within the public sphere and integrate the principles of solidarity and redistribution*". As a result, in many Member States, women are still granted pension rights for child care subject to certain conditions. The coverage or premium for child care, gender-specific retirement ages and work experience-based eligibility conditions continue to have an impact on the gender pension gap of some countries among both high and low pensions.

However, this differential treatment of women has been increasingly challenged by policymakers, particularly at the EU level. Directive 2006/54/EC (following Directive 96/97/EC), *promoting equal treatment in occupational social security schemes and prohibiting (gender) discrimination*, reduces the possibility to redistribute from men to women to compensate the time that women spent in child care activities during their working life. Specifically, the Directive's Chapter 2, article 9, states that "Provisions contrary to the principle of equal treatment shall include those based on sex, either directly or indirectly, [...] for fixing different retirement ages [...] and setting different conditions for the granting of benefits." Following the Directive, all Member States reduced the gender difference in pensionable ages and pension benefits. Some countries fully implemented gender equality of pensionable ages (Austria, Belgium, Denmark and Germany, among others); some other countries apply derogations in accordance with Article 141(4) of the Treaty and continue to compensate for women for the time they spent raising children (for example Bulgaria, France, Italy, Lithuania, Slovenia).[1]

Despite the relevance of this subject and the ongoing debates about pension reforms in many Member States, the underlying gender issues have received very little attention. Consequently some fundamental questions are currently not well understood. This is a serious omission because these problems are crucial in an aging society, where gender equality is becoming a key concern. Specifically, there is little guidance to what

[1] DIRECTIVE 2006/54/EC states that "[...] the principle of equal treatment does not prevent Member States from maintaining or adopting measures providing for specific advantages in order to make it easier for the under-represented sex to pursue a vocational activity or to prevent or compensate for disadvantages in professional careers. Given the current situation and bearing in mind Declaration No 28 to the Amsterdam Treaty, Member States should, in the first instance, aim at improving the situation of women in working life."

would be the appropriate "direction" and extent of redistribution between men and women in a society where women live longer but have lower labor incomes. Bommier et al. (2011) provide some partial insights by studying pension design when individuals differ in life-spans. However, they assume that all individuals have the same earning opportunities which means that their analysis cannot directly be applied to gender issues where the wage gap is significant. Furthermore, they do not account for the possibility that individuals may form couples and pool their resources.

Another open questions concerns the implication of "equal treatment" rules requiring gender neutrality of the pension scheme. Though appealing from a "horizontal equity" perspective, this is similar to "no tagging" conditions which have been studied in the optimal taxation literature; see for instance Cremer et. al (2012a). We know from this literature that imposing gender neutrality in a society where men and women differ in crucial characteristics like life expectancy and earning opportunities necessarily reduces overall welfare. But an interesting open question is how this requirement affects pension design, the induced allocation and particularly gender gaps in retirement ages and the extent of redistribution. Gender neutrality clearly comes at a cost and the relevant issue it then to know how much inefficiency a government must be ready to concede (at least in the short run) to promote the objective of gender parity in pensions.[2]

Our analysis aims at improving our understanding of these issues. We assume that men and women choose their retirement ages given the pension scheme. We determine the pension scheme that maximizes welfare accounting for individual's decisions. Women live longer than men but have lower earning opportunities. Gender and retirement ages are publicly observable, while individual consumption levels are not. This constraint is irrelevant when there are only singles, but it imposes a restriction with couples because

[2] Gender neutrality and other non-discrimination rules are common in insurance markets and in regulated industries. In private (imperfectly competitive) markets these rules are not necessarily inefficient. However, in the case of insurance policies, they amount to introducing adverse selection in markets which would otherwise yield actuarially fair contracts (and full insurance). Finkelstein et al. (2009) for instance, study the efficiency cost and the redistributive impact of banning gender specific annuity pricing in the UK. While their exercise bears some similarities with our paper it differs in several crucial aspects. In particular, they consider private markets where averse selection brings about Rothschild and Stiglitz or similar types of equilibria while we consider social insurance where the objective is redistribution. In addition, they do not consider couples.

the allocation of resources between spouses cannot be controlled. The solutions we refer to as first best (FB) is then effectively a constrained FB.

In the first part, we study the desirable direction of gender redistribution for singles and for couples. When individuals differ in their life-spans the definition of social welfare is both crucial and not trivial. A simple utilitarian welfare function fails to capture the possible concern for redistribution between long-lived and short-lived individuals. Indeed, it effectively puts a higher weight on the longer lived since their instantaneous utilities are added over more period. The specification of welfare we use is inspired by Bommier et al. (2011). Social welfare is utilitarian in that it is additive and puts the same weight on all individuals. However, the sum is taken over an increasing concave transformation of individuals' lifetime utilities. The concave transformation reflects society's aversion towards multiperiod inequality (Atkinson and Bourguignon, 1982, and Gottschalk and Spolaore, 2002) or risk aversion with respect to the life duration (Bommier, 2006).

With singles only, and absent of a concave transformation, redistribution from men to women is always optimal. In first best, men retire later and have a lower lifetime consumption. This is because per-period consumption is the same for all, while men have a shorter life-span. As a result, women receive a pension which exceeds their contributions while the opposite is true for men. When the transformation is sufficiently concave, redistribution *may* be reversed but the possibility that there continues to be redistribution from men to women cannot be ruled out even in the Rawlsian case when wages are sufficiently different.

The existence of couples does not affect the solution when no concave transformation is applied. However, it differs as soon as the transformation exhibits some concavity. Because of the consumption pooling, redistribution between gender can be achieved only via the retirement age. The concave transformation then has a "more drastic" impact on retirement ages than in the singles case. In particular when the transformation is sufficiently concave the ranking of the desired male and female retirement ages is *always* reversed, irrespective of the size of the wage gap.

In the second part of our analysis we introduce gender neutrality so that pension schemes cannot be explicitly conditioned (tagged) on gender. Formally this amounts to imposing a self-selection constraint: the government will offer a menu of incentive compatible pension schemes and men and women will choose the preferred one. The solution then depends on the pattern of binding incentive constraints.

With singles only, when the FB implies redistribution from men to women, gender neutrality entails distortion of the female retirement age. As in optimal tax models, the sign of the distortion depends on the comparison between the marginal rates of substitution of the mimicker and the mimicked. When wages are not too different female retirement age is reduced (compared to the FB). However, since indifference curves may not be single crossing, other patterns of distortions may arise.

When all individuals live in couples, gender neutrality may have an even more drastic impact. As long as the FB implies a lower retirement age for female partners, it can be implemented by a gender neutral pension system. However, a solution in which women retire later than men can no longer be implemented. Consequently when the transformation applied to utilities is sufficiently concave the gender neutral solution implies pooling and thus equal retirement ages. In that case gender neutrality restricts the possibility to compensate the shorter lived individuals.

Our theoretical analysis is completed by numerical simulation based on a calibrated model. The numerical results illustrate our analytical result and show which of the cases discussed are likely to arise with empirically relevant parameter values both for the FB and for the gender neutral second best. They also allow us to quantify the size of the overall welfare cost imposed to society by gender neutrality, as well as its impact on the different segments of the population: male and female singles and spouses. In addition, we also consider the more realistic case where singles and couples coexist. Since the policy can be conditioned on the marital status (there is tagging to this respect) this won't affect the qualitative results obtained within each group. However, there is now a global budget constraint allowing for cross subsidies among single individuals and married individuals. It is well known from the optimal tax literature that analytically these transfers cannot be signed except under very restrictive assumptions; see for instance Cremer et al. (2012b). However, the numerical solution of the calibrated model provides us with an empirically meaningful estimation of the directions of transfers. Last but not least, the numerical results provide a more explicit description of the pension schemes (both marginal and total).

2 The Model

Preferences over consumption c and labor ℓ, of an individual of age t can be expressed by an instantaneous utility function $V(t)$ assumed to be additively separable:

$$V(t) = u\left(c\left(t\right)\right) - r\left(t\right)\ell\left(t\right),$$

where $r\left(t\right)$ represents the instantaneous intensity of labor disutility. Utility of consumption $u(c)$ is strictly increasing and concave, while $r(t)$ is an increasing and convex function so that disutility of labor increases with age at an increasing rate, reflecting for instance a declining health status.

We concentrate on labor supply at the extensive margin using the following restriction: $\ell \in \{0, 1\}$. At any moment in time, individuals can either work a given number of hours, normalized to one, or not work at all, that is retire. Given that $r(t)$ is increasing, $V(t)$ can be rewritten as

$$\begin{aligned} V(t) &= u\left(c\left(t\right)\right) - r\left(t\right) \qquad &\text{if } t \leq \tau \\ &= u(c(t)) &\text{if } t > \tau, \end{aligned} \tag{1}$$

where τ denotes the retirement age, i.e. the length of working life.

Let date 0 denotes entrance to the labor force and T the maximum life-span. The interest rate and the discount factor are constant, equal and normalized to 0. Lifetime utility is therefore given by

$$U = \int_0^T V(t)dt = \int_0^T u(c(t))dt - \int_0^\tau r(t)dt. \tag{2}$$

Separability between utility from consumption and disutility from labor, concavity of the instantaneous utility function, perfect capital markets and certain lifetimes all together imply that individuals will set their level of consumption equal in all periods.

Consequently, lifetime utility can be rewritten as

$$U = Tu(c) - R(\tau), \tag{3}$$

where $R(\tau) = \int_0^\tau r(t)dt$ is the lifetime disutility from labor.

We consider a population with men and women born in equal proportions. Individuals may remain single or form couples. In the analytical part, we concentrate on the cases where all individuals are singles or where they all live in couples. The results would not change if both types of living arrangements were to coexist, as long as the policy can be tagged on the marital status. We illustrate this case through a numerical example in Section 6.

Throughout the paper we concentrate on a single generation in the steady state with a stationary population. Since the population growth rate and the interest rate are equal, in the model funded and pay as you go pension systems are equivalent.

Singles. Utility of a single individual of gender $j = f, m$, female or male, is given by

$$U_j^s (c_j, \tau_j; T_j) = T_j u(c_j) - R(\tau_j). \tag{4}$$

Given retirement age τ_j, an individual's lifetime labor income is $w_j \tau_j$, where w_j reflects labor market conditions. Recall that women currently earn on average 15% less than man and live on average 4 to 6 years longer.[3] Consequently, we assume that

Assumption 1: $w_f \leq w_m$ and $T_f \geq T_m$.

We refer to Δw and ΔT as the degrees of gender wage and life-span heterogeneity in the society. To concentrate on redistribution across genders we assume that each group is homogenous: all men are characterized by the same wage and life-span and the same for women.[4]

[3] Longevity and income are positively correlated and the gender gap in longevity decreases with education and other socioeconomic characteristics (Bohàcek et al, 2020). Sheshinski and Caliendo (2021) study the design of a progressive pension system when individuals with larger income also live longer. They disregard gender gaps and the direction of desirable redistribution is obvious in their case: from high-income/long-lived individuals to low-income/short-lived ones.

[4] Both women and men in the EU can expect to live in good health until the age of 64 (European Institute for Gender Equality, 2019). Given that no specific gender gaps in health are observed for individuals in working age, we assume that the function $R(\cdot)$ is the same for both genders; see also Britton and French (2020).

Following Becker et al. (2005) we assume that utility when dead is normalized to zero and that for a given level of lifetime resources, individuals are always better off when living longer for any given retirement age $\bar{\tau} < T$, which requires :

$$\textbf{Assumption 2}: u(c) - R(\tau) > 0 \text{ for any } c \text{ and } \tau \text{ and } \frac{cu'(c)}{u(c)} < 1 \ \forall c.$$

Couples. We assume that couples are unitary and maximize the sum of spouses' utilities, so that they pool their resources. The utility of a couple is thus given by

$$U^c(c_f, c_m, \tau_f, \tau_m; T_f, T_m) = T_f u(c_f) + T_m u(c_m) - R(\tau_m) - R(\tau_f). \tag{5}$$

We assume throughout the paper that a couples' allocation of consumption across spouses is not publicly observable. Consequently, couples will always allocate their resources I^c so as to maximize $T_f u(c_f) + T_m u(c_m)$ subject to $T_f c_f + T_m c_m = I^c$, which implies $c_f = c_m = c$. In other words, spouses' instantaneous consumption levels are always equalized. This assumption also applies to the allocation referred to as FB and which is thus, effectively, a constrained FB. This allocation is the relevant benchmark to infer the direction of redistribution and to assess the second-best allocation achieved under gender neutrality.

3 The *laissez-faire*

3.1 Retirement decision of singles

Singles choose their lifetime consumption and retirement age maximizing (4) under the budget constraint

$$T_j c_j^s = w_j \tau_j^s, \tag{6}$$

where the superscript s refers to "single". Substituting $c_j^s = w_j \tau_j^s / T_j$ into (4), the objective function can be rewritten as:

$$T_j u(\tau_j^s w_j / T_j) - R(\tau_j^s).$$

The FOC with respect to τ_j^s is:

$$w_j u'(c_j^s) - R'\left(\tau_j^s\right) = 0. \tag{7}$$

Men and women differ in wages and in their life-span. To compare their retirement ages we have to study the impact of these two variables. From (7):

$$\frac{d\tau_j^s}{dT_j} = \frac{w_j^2}{T_j^2}\frac{u''(c_j^s)}{\text{SOC}} > 0, \tag{8}$$

where

$$\text{SOC} = \left(\frac{w_j^2}{T_j}u''(c_j^s) - R''(\tau_j^s)\right) < 0$$

is the second-order condition. In words, when two single individuals have the same wage, the one with a longer life-span will retire later.

While the effect of T is simple and unambiguous, the wages have a more complex impact on retirement. Differentiating (7) yields

$$\frac{d\tau_j^s}{dw_j} = -\frac{u'(c_j^s) + c_j^s u''(c_j^s)}{\text{SOC}} = \frac{c_j^s u''(c_j^s)}{\text{SOC}}\left(\varepsilon\left(c_j^s\right) - 1\right), \tag{9}$$

where $\varepsilon(c_j^s)$ is the intertemporal elasticity of substitution of individual j. Recall that, from Assumption 1, $T_f \geq T_m$ and $w_f \leq w_m$. In addition, from (8), $d\tau_j/dT_j > 0$. Two cases are then possible:

$$\begin{array}{lllll} \varepsilon(c_j^s) > 1 & \Rightarrow & \frac{d\tau_j^s}{dw_j} > 0 & \text{and} & \tau_f^{s*} \lesseqgtr \tau_m^{s*}, \\ \varepsilon(c_j^s) \leq 1 & \Rightarrow & \frac{d\tau_j^s}{dw_j} \leq 0 & \text{and} & \tau_f^{s*} > \tau_m^{s*}. \end{array}$$

In words, when the intertemporal elasticity of substitution is smaller than 1, women always retire later than men in the *laissez-faire*. However, when the intertemporal elasticity of substitution is larger than 1 then (8) and (9) are of opposite sign and the comparison between retirement ages is ambiguous and depends on which effect prevails. This in turn depends on the relative magnitudes of ΔT and Δw. The case where women retire later is more likely to occur when $\Delta T/\Delta w$ is relatively high, whereas men are more likely to retire later when when $\Delta T/\Delta w$ is low.

Male and female consumption levels are most easily compared when $\tau_f^{s*} < \tau_m^{s*}$. In that case, it follows directly from the budget constraint (6) together with Assumption 1 that $c_m^{s*} > c_f^{s*}$. The female single has a lower lifetime income and lives longer so that her consumption level must be lower. Interestingly we also obtain a lower female consumption even when $\tau_f^{s*} > \tau_m^{s*}$. From (7) we then have

$$\frac{R'(\tau_f^s)}{w_f} = u'(c_f^s) > u'(c_m^s) = \frac{R'(\tau_m^s)}{w_m},$$

implying again $c_m^{s*} > c_f^{s*}$.

3.2 Retirement decision of couples

Couples maximize (5) subject to

$$T_f c_f^c + T_m c_m^c = w_f \tau_f^c + w_m \tau_m^c.$$

where a superscript c refers to "couple". The solution implies $c_f^{*c} = c_m^{c*} = c^{c*}$ and

$$w_m u'(c^{c*}) = R'(\tau_m),$$
$$w_f u'(c^{c*}) = R'(\tau_f),$$

so that $\tau_m^{c*} > \tau_f^{c*}$. As anticipate after expression (5), the couple's disposable income is equally shared between spouses, but men retire later than their partners.[5] We observe cross-subsidies (or redistribution) from men to women.[6]

The results obtained so-far are summarized in the following proposition.

Proposition 1 (*Laissez-faire*) *1) Single women always consume less than single men ($c_f^{s*} < c_m^{s*}$).*

2) When $\varepsilon(c_j^s) \leq 1$, single women retire later ($\tau_f^{s*} > \tau_m^{s*}$) than single men. When $\varepsilon(c_j^s) > 1$, single women may retire later or earlier than single men.

3) A couple's disposable income is equally shared between partners, $c_f^{c*} = c_m^{c*} = c^{c*}$, but men retire later than their spouses $\tau_m^{c*} > \tau_f^{c*}$.

Note that the resource pooling along with the absence of uncertainty implies that women's inheritance of pension rights is perfect and there are no "poor widows": married

[5] All women are employed in our model. Housewives could be incorporated by letting w_f tend to zero, which would imply a corner solution for the retirement age of female spouses.

[6] The assumption that spouses enter the labor market at the same age implies that they *have* the same age. Differences in spouses' age can be incorporated in the model by changing the longevity gap ΔT. Specifically, an increase in the longevity gap would capture the situation in which female spouses are younger than their partners.

women have a constant consumption flow over time, extending into widowhood. In our setting "poor single women" are more relevant and are likely to represent the target of redistribution.

4 First best

The government maximizes the following social welfare function:

$$SW = \varphi\left(U_f\right) + \varphi(U_m),$$

where φ is an increasing and concave function. For example, social welfare can be specified as

$$SW = \frac{1}{1-\nu}\sum_j (U_j)^{1-\nu};\ j = f, m. \tag{10}$$

In this specification ν measures the degree of concavity of φ. A larger level of ν implies a larger degree of inequality aversion. Special cases include the traditional utilitarian solution (linear φ) for $\nu = 0$, and a Rawlsian welfare function for $\nu \to \infty$. When φ is linear we return to an utilitarian welfare and there is no concern for redistributing *between individuals of different life-spans.* The only redistributive concern is the one across income levels which is brought about by the concavity of U. A strictly concave φ introduces a countervailing effect tending to compensate men for living a shorter life and, when in couples, also for retiring later.

The resource constraint is imposed for the considered generation. Since the economy is stationary and the population growth and the interest rate are both zero, this is equivalent to imposing a "per period" budget constraint in an overlapping generations model.

We first consider the optimal allocation for singles and then for couples.

4.1 Singles

4.1.1 Allocation

The Lagrangian expression associated with the maximization of social welfare is given by

$$\begin{aligned}\mathcal{L}=&\varphi[T_f u(c_f^s) - R(\tau_f^s)] + \varphi[T_m u(c_m^s) - R(\tau_m^s)]\\&+ \mu[w_f \tau_f^s + w_m \tau_m^s - T_f c_f^s - T_m c_m^s]. \end{aligned} \tag{11}$$

Rearranging the FOCs, presented in Appendix B, yields

$$\frac{R'(\tau_j^{sFB})}{u'(c_j^{sFB})} = w_j \text{ for } j = f, m, \tag{12}$$

$$u'(c_f^{sFB})\varphi'(U_f^{sFB}) = u'\left(c_m^{sFB}\right)\varphi'(U_m^{sFB}). \tag{13}$$

We prove the following Proposition in Appendix C.

Proposition 2 (FB allocation with singles) *The first best allocation is described by (12) and (13) and always implies* $U_f^{sFB} \geq U_m^{sFB}$ *and* $c_m^{sFB} \geq c_f^{sFB}$.

(i) When φ *is linear, we have* $c_f^{sFB} = c_m^{sFB}$ *and* $\tau_f^{sFB} < \tau_m^{sFB}$.

(ii) When φ *is Rawlsian, we have* $\tau_f^{sFB} > \tau_m^{sFB}$ *if* $w_m = w_f$ *and* $T_f > T_m$.

(iii) When φ *is Rawlsian we have* $\tau_f^{sFB} < \tau_m^{sFB}$ *if* $w_m > w_f$ *and* $T_m = T_f$.

Two general properties emerge from the first-best allocation. First, women always benefit from a higher life cycle utility than men (except of course in the case where φ is Rawlsian in which case $U_f^{sFB} = U_m^{sFB}$). Second, men always benefit from a per period consumption that is at least as large as that of women. The implications of these results for the redistribution across gender will become clear in the next subsection where we study the implementation via a pension system.

4.1.2 Implementation

Omitting superscript s to alleviate notation, the pensions system applied to singles is represented by the *net* benefit functions $P_j(\tau_j)$. It indicates the pension received minus the contributions and is conditioned on the retirement age. Given this benefit rule, singles maximize

$$T_j u(c_j) - R(\tau_j);$$

subject to

$$w_j \tau_j + P_j(\tau_j) - T_j c_j = 0.$$

The FOC yields:

$$\frac{R'(\tau_j)}{u'(c_j)} = w_j + P_j'(\tau_j). \tag{14}$$

Implementing the FB thus requires $P'(\tau_j) = 0$; the pension scheme is "flat" in the sense that there is no *marginal* distortion of the individual pension decision. The level of P_j^{FB} then follows from the individual budget constraint:

$$P_f^{FB} = T_f c_f^{FB} - w_f \tau_f^{FB},$$
$$P_m^{FB} = T_m c_m^{FB} - w_m \tau_m^{FB}.$$

Note that $P_f^{FB} + P_m^{FB} = 0$ so that redistribution from men to women entails $P_f^{FB} > 0 > P_m^{FB}$. In words, women receive pension benefits exceeding their overall contributions while the opposite occurs for men. We are now in a position to discuss the redistributive implications of Proposition 2.

When φ is linear, the government is only concerned about redistribution between agents with different yearly labor income. As a result, both men and women receive the same consumption level and women retire earlier. Since women live longer, this can only be achieved by redistribution from men to women so that $P_f^{sFB} > 0 > P_m^{sFB}$ (point (i) of Proposition 2).

Consider now the case where φ is strictly concave so that the government is also concerned about redistribution from short to long-lived agents. When wages are equal ($w_f = w_m$) we return to the setting considered by Bommier et al. (2011) and we know from (13) that $c_f^{sFB} < c_m^{sFB}$ as soon as φ is strictly concave, so that it certainly holds in the Rawlsian case. From equation (12) this implies $\tau_f^{sFB} > \tau_m^{sFB}$. Consequently to achieve $U_f^{FB} = U_m^{FB}$, as implied by the Rawlsian solution we must have $P_f^{sFB} < 0 < P_m^{sFB}$. This shows that when wages are equal the direction of redistribution is effectively reversed at the Rawlsian solution. By continuity this property continues to hold when φ is sufficiently concave. Similarly it continues to hold when yearly incomes are not too different. But when w_f is much smaller than w_m, we can no longer conclude. In that case we cannot rule out the possibility that even in the Rawlsian case we have $P_f^{sFB} > 0 > P_m^{sFB}$. Note that this is necessarily true when w_f is close to zero while w_m is sufficiently large. These results are summarized in the following proposition.

Proposition 3 (FB pensions with singles) *With a utilitarian SW function redistribution from men to women is always optimal. When the transformation of the SW function is sufficiently concave, redistribution may be reversed. However, the possibility that redistribution from men to women remains optimal cannot be ruled out, even in the Rawlsian case, when the wage gap is sufficiently high.*

4.2 Couples

We now turn to the case where all individuals live in couples. As mentioned above we characterize a constrained FB in which couples pool their consumption so that $c_f^c = c_m^c = c^c$.[7] The social planner maximizes

$$SW = \varphi\left(T_f u(c^c) - R(\tau_f)\right) + \varphi(T_m u(c^c) - R(\tau_m)) \tag{15}$$

subject to

$$w_m \tau_m + w_f \tau_f - (T_f + T_m)c^c = 0. \tag{16}$$

Denoting the multiplier associated with the resource constraint by μ we have the following FOCs

$$\frac{\partial \mathcal{L}}{\partial c^c} = \varphi'(U_f)T_f u'(c^c) + \varphi'(U_m)T_m u'(c^c) - \mu(T_f + T_m) = 0, \tag{17}$$

$$\frac{\partial \mathcal{L}}{\partial \tau_f} = -\varphi'(U_f)R'(\tau_f) + \mu w_f = 0, \tag{18}$$

$$\frac{\partial \mathcal{L}}{\partial \tau_m} = -\varphi'(U_m)R'(\tau_m) + \mu w_m = 0. \tag{19}$$

[7] This assumption is made to concentrate on pension design. The unrestricted FB would in general require different consumption levels. However since couples pool their ressources, this solution can only be implemented when benefits are conditioned not just on retirement ages but also on spouses' consumption levels. In other words, the pension system would have to be associated with an implicit or explicit tax on spouses' consumption levels. But this is ruled out by our assumption that the allocation of disposable income within a couple is not publicly obsevable.

When φ is linear we return to the solution with singles and

$$c_f^{cFB} = c_m^{cFB} = c^{cFB};$$
$$\tau_f^{cFB} < \tau_m^{cFB}.$$

Thus we have redistribution from men to women and the female spouse is better off. Note that unlike in the singles case the FB coincides with the *laissez-faire*. The equalization of consumption levels, which in the singles case required transfers, is automatically achieved with couples because they pool their resources.[8]

When $\varphi'' < 0$ the solutions with singles and couples differ. This follows because couples pool their incomes so that $c_f^c = c_m^c = c^c$ applies by definition, while consumption levels will in general differ for singles. Formally this is as if we impose an extra constraint so that social welfare with couples will be lower. It also means that the results of Bommier et al. (2011) no longer apply even when $w_f = w_m$. Combining (17) and (18) yields

$$\frac{R'(\tau_f^{cSB})}{u'(c^{cSB})} = \frac{w_f}{\varphi'(U_f^{cSB})} \left[\frac{T_f \varphi'(U_f^{cSB}) + T_m \varphi'(U_m^{cSB})}{T_f + T_m} \right], \tag{20}$$

while (17) and (19) imply

$$\frac{R'(\tau_m^{cSB})}{u'(c^{cSB})} = \frac{w_m}{\varphi'(U_m^{cSB})} \left[\frac{T_f \varphi'(U_f^{cSB}) + T_m \varphi'(U_m^{cSB})}{T_f + T_m} \right], \tag{21}$$

so that

$$\frac{R'(\tau_f^{cSB})\varphi'(U_f^{cSB})}{w_f} = \frac{R'(\tau_m^{cSB})\varphi'(U_m^{cSB})}{w_m}. \tag{22}$$

From this condition we obtain the following Proposition (see the proof in Appendix D).

Proposition 4 (FB allocation with couples) *A first-best allocation is described by (20) and (21).*

(i) It always implies $U_f^{cFB} \geq U_m^{cFB}$ irrespective of the degree of concavity of φ.

(ii) When φ is linear we have $\tau_f^{cFB} < \tau_m^{cFB}$ but $\tau_f^{cFB} > \tau_m^{cFB}$ always obtains when φ is sufficiently concave.

[8] And in this case our constrained FB is effectively the same as the unrestricted FB, given that the latter requires equal consumption levels anyway.

In words, in the FB, the female spouse is always better off than the male irrespective of the concavity of the transformation φ. But this does not tell us anything about the spouses' retirement ages. Recall that in the *laissez-faire* and in the FB with linear φ, women always retire earlier than man. We now examine if we can have $\tau_f > \tau_m$ when φ is sufficiently concave. With the Rawlsian welfare function $SW = \min[U_f, U_m]$, the solution implies $U_f^{cFB} = U_m^{cFB}$. The interesting result is that, with couples, $U_f^{cFB} = U_m^{cFB}$ implies immediately $\tau_m^{cFB} < \tau_f^{cFB}$. This was not necessarily true with singles only but since spouses pool their incomes and women live longer, utilities can only be equalized when men retire earlier. By continuity $\tau_m^{cFB} < \tau_f^{cFB}$ also obtains when φ is sufficiently concave. To sum up, in the case with couples only, we can say for sure that the ranking of gender retirement ages is reversed when φ is sufficiently concave.

4.2.1 Implementation

In the case of singles we have concentrated on the sign of the net pensions $P_f^s(\tau_f^s)$ and $P_m^s(\tau_m^s)$ because this showed the direction in which the system redistributes. With couples only, the levels of the pensions are no longer relevant for the redistribution between spouses. Recall that couples pool their resources and their total *net* pension is by definition always equal to zero (see below). Hence, with couples only, redistribution can only take place via the retirement ages. These in turn depend on the derivative of the benefit function $P_j^c(\tau_j^c)$, $j = f, m$. A couple maximizes

$$(T_f + T_m)u(c^c) - R(\tau_f^c) - R(\tau_m^c);$$

subject to

$$w_f\tau_f^c + w_m\tau_m^c + P_f^c(\tau_f^c) + P_m^c(\tau_m^c) - (T_f + T_m)c^c = 0;$$

where $P_f^c(\tau_f) + P_m^c(\tau_m) = 0$ because we have identical couples. Omitting superscript c to alleviate notation, the FOCs are given by

$$\frac{R'(\tau_f)}{u'(c)} = w_f + \frac{\partial P_f^{cFB}(\tau_f)}{\partial \tau_f}, \tag{23}$$

$$\frac{R'(\tau_m)}{u'(c)} = w_m + \frac{\partial P_m^{cFB}(\tau_m)}{\partial \tau_m}, \tag{24}$$

which are the counterparts to expression (14) in the case of singles. Using the FOCs (20) and (21) and after some rearrangement, we obtain that the implementing benefit function must satisfy

$$\frac{\partial P_f^{cFB}(\tau_f)}{\partial \tau_f} = w_f \left[\frac{T_m(\varphi'(U_m^{cFB}) - \varphi'(U_f^{cFB}))}{\varphi'(U_f^{cFB})(T_f + T_m)} \right] \tag{25}$$

for τ_f^{cFB}. Proceeding in the same way, we obtain

$$\frac{\partial P_m^{cFB}(\tau_m)}{\partial \tau_m} = w_m \left[\frac{T_f(\varphi'(U_f^{cFB}) - \varphi'(U_m^{cFB}))}{\varphi'(U_m^{cFB})(T_f + T_m)} \right] \tag{26}$$

for τ_m^{cFB}.

When φ is linear, from (25) and (26) we have

$$\frac{\partial P_f^{cFB}}{\partial \tau_f} = \frac{\partial P_m^{cFB}}{\partial \tau_m} = 0.$$

As mentioned above the couples' income pooling imposes no extra constraint here, quite the opposite, it ensures that the *laissez-faire* corresponds to the FB. In the general case, when $\varphi'' < 0$ together with Lemma 4, equations (25)–(26) imply:

$$\frac{\partial P_f^{cFB}}{\partial \tau_f} > 0 > \frac{\partial P_m^{cFB}}{\partial \tau_m}.$$

In words, when φ is strictly concave, the pension scheme will induce the couple to increase the retirement age of the female spouse and decrease that of the male spouse. This is in line with the results presented above and particularly the property that when φ is sufficiently concave we will have $\tau_f^{cFB} > \tau_m^{cFB}$. Intuitively the concave φ calls for redistribution towards the shorter-lived male. Since consumption levels are equal, pension *levels* are ineffective for this purpose and the only way to mitigate the longevity effect is to increase female retirement age and decrease the male one. To sum up we have:

Proposition 5 (FB pensions with couples) *Because of the couples' consumption pooling, redistribution can be achieved only by distorting retirement ages. With a utilitarian SW function, redistribution from men to women is not only optimal but also* *achieved in a decentralized way in the* laissez-faire *allocation. When the transformation is not too concave, redistribution from men to women remains optimal. When the transformation is sufficiently concave, redistribution from women to men becomes optimal irrespective of the size of the wage gap. It requires the ranking of gender retirement ages to be reversed.*

5 Gender neutrality

As mentioned in the Introduction, according to Directive 2006/54/EC, social security schemes should treat men and women equally, in particular with regard to their pension benefits. So far we have assumed that the benefit scheme can be conditioned (tagged) on the gender. The optimal tax literature has shown that tagging, that is conditioning the transfer function on an exogenous and observable variable, is in general welfare improving; see Cremer et al. (2012a) and the references provided there. However, it may violate the principle of horizontal equity and thus be considered as unacceptable as it may imply more or less arbitrary discrimination.

The previous sections have shown that, when this is possible, differential treatment of genders may indeed by optimal. In the singles case the implementing pension schemes differed across genders and with couples, spouses retirement choices affect pension in a gender-specific way. We now examine how the solution would be affected if gender neutrality is imposed in the sense that tagging is no longer possible. This does not mean that men and women retire at the same age nor that they must obtain the same net benefits. It does mean, however, that they must be offered the same options. Consequently we do not rule out differentiation across genders but the allocation must be incentive compatible: the same menu of contracts must be offered to men and women who then self-select. In other words, gender neutrality effectively means that the policy has to be designed as if gender were not observable.

We look at contracts in the (τ, P) space, where P is the pension *net of contributions.* We assume that these are the observable variables. Note that we assume that per-period consumption c's is not observable or rather cannot be specified in the contract because this would violate gender neutrality.[9] Define

$$U_j^s = T_j u\left(\frac{w_j\tau_j^s + P_j^s}{T_j}\right) - R(\tau_j^s), \text{ with } j = m, f, \tag{27}$$

for singles and

$$U^c = \sum_{j=m,f}\left[T_j u\left(\sum_{j=m,f}\left(w_j\tau_j^c + P_j^c\right) / \sum_{j=m,f} T_j\right) - R(\tau_j^c)\right] \tag{28}$$

for couples. We assume that pensions, notwithstanding gender neutrality, can be conditioned on marital status.

5.1 Singles only

Indifference curves in the (τ, P) space may not be monotonic and it is not possible to establish a single-crossing property, except in the special case where $w_f = w_m$. To see that, consider Figure 1 that illustrates the profile of women and men's indifference curves in this case. The slope of an indifference curve in the (τ, P) space is the MRS obtained by differentiation of (27):

$$MRS = \left.\frac{dP_j}{d\tau_j}\right|_{U^s} = \frac{R'(\tau_j)}{u'\left(\frac{w_j\tau_j + P_j}{T_j}\right)} - w_j, \text{ for } j = f, m. \tag{29}$$

Equation (A.5) in Appendix E implies that when wages are equal we have $MRS^f < MRS^m$ at any given point. Hence, single crossing holds in this special case. However, expressions (A.4)–(A.5) show that this inequality *may* be reversed when wages differ. Furthermore no *general* single-crossing property can be established.

[9] Directly controlling individual consumption levels would bring us back to gender tagging as c reveals T and thus gender.

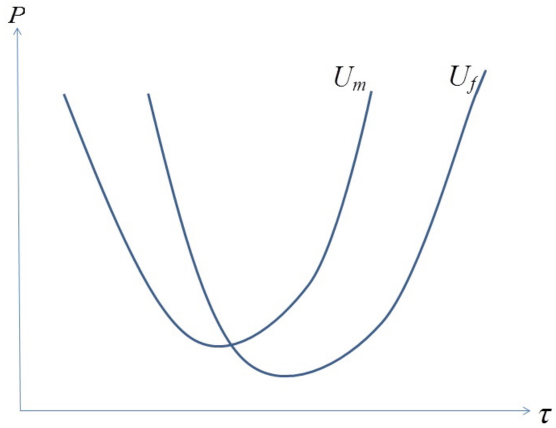

Figure 1: Men and women's indifference curves in the (τ, P) space for $T_f > T_m$ and $w_f = w_m$.

5.1.1 Implementing gender neutrality for singles

The optimal allocation is obtained by solving

$$\max_{P_f^s, P_m^s, \tau_f^s, \tau_m^s} \quad \varphi\left[T_f u\left(\frac{w_f\tau_f^s + P_f^s}{T_f}\right) - R(\tau_f^s)\right] + \varphi\left[T_m u\left(\frac{w_m\tau_m^s + P_m^s}{T_m}\right) - R(\tau_m^s)\right] \tag{30}$$

$$\text{s.t.} \quad P_m^s + P_f^s = 0, \tag{31}$$

$$T_f u\left(\frac{w_f\tau_f^s + P_f^s}{T_f}\right) - R(\tau_f^s) \geq T_f u\left(\frac{w_f\tau_m^s + P_m^s}{T_f}\right) - R(\tau_m^s), \qquad (\lambda_f^s) \tag{32}$$

$$T_m u\left(\frac{w_m\tau_m^s + P_m^s}{T_m}\right) - R(\tau_m^s) \geq T_m u\left(\frac{w_m\tau_f^s + P_f^s}{T_m}\right) - R(\tau_f^s), \qquad (\lambda_m^s), \tag{33}$$

where λ_j^s, $j = f, m$, denotes the Lagrangian multiplier associated with the incentive compatibility constraint of type j.

From Proposition 3 we know that the FB allocation implies redistribution from male to female singles when the social welfare function is utilitarian. We can thus conjecture that, when φ is close to linear, (33) is binding, so that we have $\lambda_m^s > 0$

and $\lambda_f^s = 0$. To see this let us denote by subscripts mf and fm mimicking individuals' consumption bundle; for instance $c_{mf}^s = (w_m \tau_f^s + P_f^s)/T_m$. When φ is linear, the FB allocation entails $c_f^{sFB} = c_m^{sFB}$ and $\tau_f^{sFB} < \tau_m^{sFB}$, which implies $c_{mf}^s \geq c_m^{sFB}$, so that the incentive constraint (33) is violated. By continuity this will remain true when φ is not too concave. On the other hand, when φ is sufficiently concave redistribution absent of gender neutrality *may* be reversed so that we would have $\lambda_m^s = 0$ and $\lambda_f^s > 0$ because the FB allocation now violates the female incentive constraint.

In addition, we know from equation (14) that the implementing pension rule must satisfy

$$P_j'(\tau_j^s) = \frac{R'(\tau_j^s)}{u'\left(\frac{w_j \tau_j^s + P_j^s}{T_j}\right)} - w_j = MRS_j^s. \tag{34}$$

5.1.2 Properties of the solution

We concentrate on the case where a single incentive constraint binds. We prove the following Proposition in Appendix F.

Proposition 6 (Gender neutrality with singles) *(i) If* $\lambda_m^s > 0$ *and* $\lambda_f^s = 0$ *then* $MRS_m^{sSB} = 0$ *so that* $P'(\tau_m^{sSB}) = 0$ *and either (ia) or (ib) realizes:*

(ia) $0 > MRS_f^{sSB} > MRS_{mf}^{sSB}$

$\Leftrightarrow P'(\tau_f^{sSB}) < 0,\ c_f^{sSB} < c_{mf}^s < c_m^{sSB},\ \tau_f^{sSB} < \tau_m^{sSB}$ *and* $U_m^{sSB} > U_f^{sSB}$;

(ib) $MRS_{mf}^{sSB} > MRS_f^{sSB} > 0 \Leftrightarrow P'(\tau_f^{sSB}) > 0,\ c_{mf}^s > c_m^{sSB}$ *and* $\tau_f^{sSB} > \tau_m^{sSB}$.

(ii) If $\lambda_f^s > 0$ *and* $\lambda_m^s = 0$, *then* $MRS_f^{sSB} = 0$ *so that* $P'(\tau_f^{sSB}) = 0$ *and either (iia) or (iib) realizes:*

(iia) $0 > MRS_m^{sSB} > MRS_{fm}^{sSB} \Leftrightarrow P'(\tau_m^{sSB}) < 0,\ c_f^{sSB} > c_{fm}^{sSB}$ *and* $\tau_f^{sSB} > \tau_m^{sSB}$;

(iib) $MRS_{fm}^{sSB} > MRS_m^{sSB} > 0$

$\Leftrightarrow P'(\tau_m^{sSB}) > 0,\ c_f^{sSB} < c_{fm}^s < c_m^{sSB} \Leftrightarrow \tau_f^{sSB} < \tau_m^{sSB} \Leftrightarrow U_f^{sSB} > U_m^{sSB}$.

where $MRS_{mf}^s = R'(\tau_f^s)/u'(c_{mf}^s) - w_m$ *and* $MRS_{fm}^s = R'(\tau_m^s)/u'(c_{fm}^s) - w_f$ *with* $c_{fm}^s = \left(w_f \tau_m^s - P_f^s\right)/T_f$ *and* $c_{mf}^s = \left(w_m \tau_f^s + P_f^s\right)/T_m$.

The proposition shows that gender neutrality implies a distortion of the retirement decisions. We have two possible regimes *(i)* and *(ii)* depending on the binding incentive constraint. The regime determines whose choices will be distorted; for the other gender we have the traditional no distortion at the top property. As usual in tax theory the sign of the distortion hinges on the comparison of the *MRS* between mimicker and mimicked individuals. However, its determination is more complex than in the standard model. Since the indifference curves are not monotonic they may intersect the increasing or the decreasing part and this determines which subcase of the regime applies, *(a)* or *(b)*.

As mentioned above regime *(i)* occurs when φ is linear or not too concave. When φ is sufficiently concave we *may* have regime *(ii)*. As stated in the proposition, in regime *(i)* the no distortion at the top property applies for the male. The sign of the distortion for the female is ambiguous. However, from Figure 1 and expression (A.5) we know that when $w_f = w_m$ and $\lambda_m^s > 0$ case *(ib)* obtains. In words gender neutrality here leads to a retirement age for female workers that is marginally larger than otherwise optimal. In Appendix F, we also show that women retire later than men in this case. This remains true by continuity when wages differ slightly but for a significant wage difference the sign of the distortion does not appear to be unambiguous. Then case *(ia)* may be relevant and the marginal distortion of female retirement age is negative. In addition we show that women then actually retire earlier then men and that men have a higher lifetime utility.

To sum up in both cases gender neutrality limits the possibilities of redistributing from male to female singles. In case *(ib)* this leads to a larger retirement age for females but the comparison between genders' lifetime utilities is not clear. In case *(ia)*, on the other hand gender neutrality has an even more drastic effect. While women continue to retire earlier, they end up with a lower lifetime utility than men.

Let us now turn to regime *(ii)* which may be relevant when φ is sufficiently concave. We know from the results presented in Subsection 4.1 that, when wages are equal, it will occur for sure in the Rawlsian case (or when φ is sufficiently concave). Figure 1, or more formally equation (A.5), then imply that subcase *(iia)* applies. The proposition here shows that the direction of the marginal distortion (which applies now to the males) is reversed compared to case *(ia)* (which instead applies when wage are equal and social welfare is linear) and women continue to retire later than men. In case *(iib)* when indifference curves intersect in the different direction, all inequalities are again reversed.

The bottom line is summarized by the following statement:

Proposition 7 (Redistribution accross singles under gender neutrality) *With singles only, gender neutrality hurts the gender towards whom redistribution is targeted at the first-best allocation.*

5.2 Couples only

5.2.1 Implementing gender neutrality for couples

We now return to the case where all individuals live in couples. The optimal gender neutral allocation is obtained by solving.

$$\max_{P_f^c, P_m^c, \tau_f^c, \tau_m^c} \quad \varphi \left[T_f u \left(\frac{1}{(T_m + T_f)} \left(w_m \tau_m^c + w_f \tau_f^c + P_m^c + P_f^c \right) \right) - R(\tau_f^c) \right]$$

$$+ \varphi \left[T_m u \left(\frac{1}{(T_m + T_f)} \left(w_m \tau_m^c + w_f \tau_f^c + P_m^c + P_f^c \right) \right) - R(\tau_m^c) \right], \qquad (35)$$

$$\text{s.t.} \quad P_m^c + P_f^c = 0, \qquad (36)$$

$$(T_f + T_m) u \left(\frac{1}{(T_m + T_f)} \left(w_m \tau_m^c + w_f \tau_f^c + P_m^c + P_f^c \right) \right) - R(\tau_f^c) - R(\tau_m^c) \geq$$

$$(T_f + T_m) u \left(\frac{1}{(T_m + T_f)} (w_m + w_f) \tau_m^c + 2P_m^c \right) - 2R(\tau_m^c), \qquad (\lambda_f^c) \quad (37)$$

$$(T_f + T_m) u \left(\frac{1}{(T_m + T_f)} \left(w_m \tau_m^c + w_f \tau_f^c + P_m^c + P_f^c \right) \right) - R(\tau_f^c) - R(\tau_m^c) \geq$$

$$(T_f + T_m) u \left(\frac{1}{(T_m + T_f)} (w_m + w_f) \tau_f^c + 2P_f^c \right) - 2R(\tau_f^c), \qquad (\lambda_m^c) \quad (38)$$

The objective function and the incentive constraints all account for the fact that couples pool their incomes. The problem differs from its first-best counterpart in that we have added two incentive constraints which ensure gender neutrality. The first constraint (37), with multiplier λ_f^c, is that of the female spouse while (38), with multiplier λ_m^c, applies to the male spouse.

5.2.2 Properties of the solution

The following proposition is established in Appendix G.

Proposition 8 (Gender neutrality with couples) *(i) The first-best allocation described in Proposition 4 is incentive compatible iff* $\tau_f^{cFB} \leq \tau_m^{cFB}$.

(ii) If $\tau_f^{cFB} > \tau_m^{cFB}$*, the second-best allocation entails the two incentive compatibility constraints to be binding with* $\tau_f^{cSB} = \tau_m^{cSB}$ *and* $P_f^{cSB} = P_m^{cSB} = 0$.

Before turning to the interpretation recall that, since the couple pools its resources, redistribution across genders is only possible via the retirement ages. In the *laissez-faire,* men retire later than women. This remains true at the FB when φ is not too concave but the retirement pattern will be reversed when φ is sufficiently concave.

Note that, as part of the proof, we show in Appendix G that there are two possible patterns of binding incentive constraints. We have $\lambda_f^c = \lambda_m^c = 0$ so that none of the constraints binds and this corresponds to part *(i)* of the proposition. Alternatively, we can have $\lambda_f^c > 0$ and $\lambda_m^c > 0$ in which case both incentive constraints bind and we have "pooling". This corresponds to point *(ii)* of the proposition.

Intuitively these results are easily understood. When the FB implies $\tau_f^{cFB} \leq \tau_m^{cFB}$, the couples' allocation differs from the *laissez-faire* solution (as long a $\varphi'' < 0$), but the couple prefers this allocation to one where the female spouse would have to retire later at τ_m^c or the male spouse would have to retire earlier at τ_f^c, that is the one achieved by "switching" retirement ages. Not surprisingly the proof shows that this would only bring the couple further away from its preferred (*laissez-faire*) retirement ages.

On the other hand, when the FB implies $\tau_f^{cFB} > \tau_m^{cFB}$, which reverses the *laissez-faire* ranking, it cannot be implemented. Given the budget constraint, we have one degree of freedom, namely $P_f^{cSB} = -P_m^{cSB}$ but this is not in general sufficient to make sure that the two incentive constraints are satisfied as equality; roughly we have two equations with one unknown. The only feasible solution then implies $\tau_f^{cSB} = \tau_m^{cSB}$ in which case both incentives constraints are satisfied in a trivial way.

The main conclusion is that, with couples' consumption pooling, gender neutrality limits the possibilities to redistribute towards the shorter-lived male partner which here can only take place via retirement ages. The order of gender retirement ages cannot be reversed under gender neutrality even when this would be otherwise desirable to redistribute towards the shorter-lived individuals. When the FB calls for such a reversal, the gender neutral solution implies pooling and thus equal retirement ages. As a result:

Proposition 9 (Redistribution under gender neutrality with couples) *As long as optimal redistribution in favor of the (shorter-lived) men is small so that the FB entails a lower retirement age for female spouses, it can be implemented by a gender neutral pension system. However, when optimal redistribution in favor of men is so large that the FB entails a larger retirement age for women, gender neutrality restricts the possibility to redistribute in favor of men.*

5.2.3 Implementation

A direct implication of Proposition 8 is that in case *(i)* the results obtained for the FB also apply here. Consequently we have

$$\frac{\partial P_f^{cSB}}{\partial \tau_f^c} = \frac{\partial P_f^{cFB}}{\partial \tau_f^c} > 0 > \frac{\partial P_m^{cSB}}{\partial \tau_m^c} = \frac{\partial P_m^{cFB}}{\partial \tau_m^c} \tag{39}$$

as long as $\varphi'' < 0$. Recall that when φ is linear the first best is implementable with $\partial P_f^c / \partial \tau_f = \partial P_m^c / \partial \tau_m = 0$, and this result also applies here. Turning to case *(ii)*, it also follows directly from equations (25) and (26) that inequality (39) continues to apply. To sum up, as long as φ is strictly concave there is always an upward distortion on the retirement age of women and a downward distortion on that of men.

6 Numerical results

6.1 Calibration of the model

The following simulations illustrate our analytical results, provide a more precise description of the pension system and show which of the cases discussed are likely to arise with empirically relevant parameter values both for the FB and for the gender-neutral second best. We also quantify the size of transfers across groups and the impact of gender neutrality on the different segments of the population: male and female singles and spouses. In addition, we now also study the case where singles and couples coexist. The SW function is then given by the sum of (30) and (35) weighted by the proportions of singles and couples in society. For the gender neutral solution we impose all the incentive constraints considered before: (32)-(33) and (37)-(38). Since the policy can be conditioned on individuals' marital status (there is tagging) this won't affect the qualitative results obtained within each group. However, the budget constraint is now "global", so that the solution may imply a transfer between singles and couples.

To calibrate the model, we proceed as follows. We assume that individuals start their career at age 25. We set $T_f = 60$, $T_m = 55$, $w_m = 45000$. Consequently women and men live respectively until ages 85 and 80, with a longevity gap of five years.

To calibrate the share of individuals living in couples, we use estimates and projections from the UN Population Division. Worldwide in 2020, 64% of women of reproductive age (15 to 49 years) were either married or in a cohabiting union; see Ortiz-Ospina and Roser (2020). The percentage is lower for OECD countries. With life-spans starting at the age of 25 the proportion of couples reported by the UN is likely to be underestimated. Consequently, we approximate the share of couples and singles, by 70% and 30% respectively. Hence, the budget constraint writes

$$0.3\left(P_m^s\left(\tau_m^s\right) + P_f^s(\tau_f^s)\right) + 0.7\left(P_m^c\left(\tau_m^c\right) + P_f^c(\tau_f^c)\right) = 0.$$

The utility is specified as follows:

$$U = Tu\left(c\right) - bR\left(\tau\right)$$

where

$$u\left(c\right) = \alpha + \left(1/\left(1 - \frac{1}{\varepsilon}\right)\right) c^{1-\frac{1}{\varepsilon}}, \tag{40}$$

$$R\left(\tau\right) = \left(1/\left(1 + \frac{1}{\eta}\right)\right) \tau^{\left(1+\frac{1}{\eta}\right)} \tag{41}$$

so that ε is the constant intertemporal elasticity of consumption, while η is the constant Frish elasticity of labor supply. We set $\varepsilon = 1.2$ and $\eta = 1$ following respectively Murphy and Topel (2006) and Blundell et al. (2016).

This leaves us with b and α to calibrate. We proceed in two steps. We first calibrate b so that a single man is indifferent between retiring at $\tau_m^s = 40$ and $\tau_m^s = 41$, which requires that b solves the following equation

$$T_m u\left(\frac{40w_m}{T_m}\right) - bR(40) = T_m u\left(\frac{41w_m}{T_m}\right) - bR(41),$$

in which α cancels out. Solving yields $b = 0.19$.

Second, in order to calibrate α, we first calculate the optimal retirement age for a single man with a life-span of T_m and a lifetime labor income of $w_m\tau_m - e$, where e denotes the willingness to pay for an additional year of life at age 25; see below. For such a worker, optimal retirement τ_m^* is given by

$$\tau_m^*\left(T_m, e\right) = \arg\max_{\tau_m} T_m u\left(\frac{w_m\tau_m - e}{T_m}\right) - bR(\tau_m),$$

which yields the following indirect utility function

$$V\left(T_m, e\right) = T_m u\left(\frac{w_m\tau^*\left(T_m, e\right) - e}{T_m}\right) - bR(\tau^*\left(T_m, e\right)).$$

Now, e represents the willingness to pay for an additional year of life at age 25 if it solves:

$$V\left(T_m, e\right) = V\left(T_m + 1, 0\right). \qquad (42)$$

We calibrate e to be 200000; see Murphy and Topel (2006). From (40) we obtain

$$\alpha = u\left(\frac{w_m\tau^*\left(T_m, e\right) - e}{T_m}\right) - u\left(\frac{w_m\tau^*\left(T_m + 1, 0\right)}{T_m + 1}\right) \\ - bR(\tau^*\left(T_m, e\right)) + bR(\tau^*\left(T_m + 1, 0\right)).$$

Substituting from (40) and (41) and then solving yields $\alpha = -14.13$.

We consider two scenarios concerning the degree of concavity of the SW function specified by (10). In Scenario 1 we set $v = 0.5$ which is a relatively low value. It implies that the concern for redistributing in favor of women, who are characterized by a lower yearly income, is relatively stronger. In Scenario 2 we have $v = 2$; social welfare is more concave and implies that the concern for redistributing in favor of the short-lived men becomes relatively stronger.

As mentioned in the Introduction, the gender wage gap amounts to 15% in Europe (Eurostat 2020). Given that we set $w_m = 45000$, this translates into a yearly female income of $w_f = 38000$. The 15% gap is expressed on a hourly basis and is thus likely to underestimate the yearly gender gap in income, because women have a lower employment rate than men, work part-time more often and, have career interruptions due to childbearing and child care. In other words, we set the wage gap in the low range of the possible estimates. The results bear out that even with this low value the wage gap turns out to be the dominant source of heterogeneity. In particular even with the more concave SW function we continue to have redistribution from single males to single women. This explains that an alternative setting with a wage gap of 20% yields similar results.[10]

6.2 Scenario 1: $\nu = 0.5$

The results for the cases of singles and couples only are reported in Table 1. We omit the levels of men's pension benefits, P_m, which by the budget constraint, are simply equal to $-P_f$.

Let us start from the *laissez-faire* for singles. Women retire later than men but consume less. The first best implies redistribution from men to women. In the first best women receive an implicit transfer and $P_f^{sFB} = -P_m^{sFB} > 0$. Moreover, single women retire earlier and single men retire later than in the *laissez-faire.* When gender neutrality is imposed, one can check that, as expected, IC_{mf} binds for singles meaning that single men are the mimickers. In the second-best allocation women receive a lower implicit transfer than otherwise optimal. Retirement age is not distorted for single men but it is upward distorted for women so that we obtain $\tau_f^{s1SB} > \tau_m^{s1SB}$, where the superscript $s1SB$ indicates the second-best result of scenario 1 for singles only. This implies that case (*ib*) of Proposition 6 applies. We conclude that, in this scenario, gender neutrality dramatically impairs single women by decreasing the amount of feasible redistribution. Indeed, the second best does not significantly improve welfare

[10] We do not report them to avoid tedious repetitions but they can be obtained from the authors upon request.

	LF_s	FB_s	SB_s	LF_c	FB_c	SB_c
c_f	26277	28654	26353	29722	29624	29624
P_f	0	252383	613	0	0	0
P'_f	0	0	187	0	1276	1276
c_m	33135	30712	33129	29722	29624	29624
P'_m	0	0	0	0	-1540	-1540
τ_f	41.49	38.60	41.59	37.44	38.80	38.80
τ_m	40.49	43.14	40.50	44.33	42.93	42.93
U_f	951	1001	951	1022	1011	1011
U_m	936	892	936	872	882	882
SWF	122.89	123.03	122.89	123	123	123

Table 1: Scenario 1 (low concavity of the SWF): singles only and couples only.

compared to the *laissez-faire*, because the benefit single women obtain from the slightly larger per-period consumption is almost fully offset by the disutility from labor supply generated by the increase in their retirement age. To sum up, *in Scenario 1, with singles only, gender neutrality is highly detrimental to women.*

Let us now move to couples. As expected, in the *laissez-faire* spouses have the same per-period consumption but women retire earlier than their partners. First best requires redistribution from women to men that can only be achieved trough an adjustment of retirement ages: female retirement age increases while male retirement age decreases compared to the *laissez-faire.* However, in the first best we continue to have $\tau_f^{c1FB} < \tau_m^{c1FB}$ which implies that the first best is incentive compatible (Part *(i)* of Proposition 8 applies). In other words, the first-best allocation can be implemented by a gender neutral pension scheme. As a result, *in Scenario 1 for couples only, gender neutrality does not limit the extent of redistribution in favor of male spouses that the government wants to achieve.* In second best, female spouses are worse off while male spouses are better off with respect to the *laissez-faire.* However this is not due to gender neutrality but to the concavity of social welfare which calls for redistribution from female to male spouses.

Let us move to an economy where singles and couples coexist so that both transfers between men and women and transfers between singles and couples are possible. Within this context, both pension benefits for single men and single women are relevant because $P_f^{sFB} \neq -P_m^{sFB}$. In addition, for couples the total pension benefit $P^c (= P_f^c + P_m^c)$ matters here. Recall that before we had $P^c = 0$, but when couples and singles coexist the total pension benefit for couples can be positive or negative: $P^c \lesseqgtr 0$. The *laissez-faire* remains of course the same as in the economy with only singles and only couples; see Table 1.

The first best implies that implicit transfers are paid by single men mainly in favor of single women and to a lesser extent in favor of couples: $P_f^{s1FB*} = 251479$ and $P^{c1FB*} = 803$ are both positive while $P_m^{s1FB*} = -253307$ is negative. Here the star in the superscript (i.e. $s1FB*$) indicates that singles and couples coexist in scenario 1. However, in second best, couples receive a negative net pension benefit and are the ones who subsidize pensions of both single women and single men; we have indeed $P^{c1SB*} = -14255$, $P_f^{s1SB*} = 17206$ and $P_m^{s1SB*} = 15265$. This occurs because, under gender neutrality, IC_{mf} binds for single men and this prevents the desirable redistribution from single men to single women and to couples. In addition, no incentive constraint binds in couples; thus the optimal redistribution in the couple can take place. Hence, female spouses retire later while male spouses retire earlier than in the *laissez-faire*. On the contrary, the retirement decisions of singles in the second best are similar to the ones in *laissez-faire*: the retirement age of single women is slightly distorted upwards, while the retirement age of single men is not distorted but slightly decreases with respect to the laissez-faire because of the implicit transfer single men receive in the second best. To conclude, *in Scenario 1, in a mixed economy gender neutrality impairs both single women and, to a lower extent, male spouses: it fully prevents optimal redistribution from single men to couples and limits redistribution in favor of single women substantially*.

6.3 Scenario 2: $\nu = 2$

We now consider a more concave social welfare function which reflects a larger concern for redistribution in favor of short-lived men. Obviously, this does not affect the *laissez-faire* which remains the same as in Scenario 1.

	FB	SB
c_f^s	28645	26502
τ_f^s	38.61	41.39
$P_f'^s$	0	181
P_f^s	251479	17206
τ_m^s	43.15	40.33
c_m^s	30703	33295
$P_m^{s\prime}$	0	0
P_m	-253353	16055
τ_f^c	38.80	38.88
$P_f^{c\prime}$	1275	1277
τ_m^c	42.93	43.01
$P_m^{c'}$	-1539	-1542
c	29628	29557
P_c^s	803	-14255
U_f^s	1001	954
U_m^s	892	939
U_f^c	1011	1009
U_m^c	882	881
SWF	123.028	122.98

Table 2: Scenario 1 (low concavity of the SWF): singles and couples together.

	LF_s	FB_s	SB_s	LF_c	FBc	SB_c
c_f	26277	27406	26344	29722	29542	29536
P_f	0	122075	598	0	0	0
P'_f	0	0	164	0	3793	3317
c_m	33135	31943	33129	29722	29542	29536
P'_m	0	0	0	0	-4054	-3682
τ_f	41.49	40.06	41.58	37.44	41.38	40.92
τ_m	40.49	41.75	40.50	44.33	40.54	40.92
U_f	951	975	951	1022	990	994
U_m	936	915	936	872	900	898
SWF	-0.00211892	-0.00211682	-0.0021189	-0.00212477	-0.00211941	-0.00211948

Table 3: Scenario 2 (high concavity of the SWF): either singles only or couples only.

Let us start with singles. Given that the concern for heterogeneity in life-span is stronger, in the first-best allocation single men are better off while single women are worse off with respect to Scenario 1. Specifically, single women should receive a lower net pension benefit ($P_f^{s2FB} = 122075 (= -P_m^{s2FB}) < P_f^{s1FB} = 252383$), enjoy a lower per-period consumption and retire later than in Scenario 1. The opposite holds for single men. Nevertheless, redistribution from men to women continues to be desirable and this explains why, adding gender neutrality, single men are the mimicker and IC_{mf} still binds. As a result, *in Scenario 2 for singles only, gender neutrality limits the extend of feasible redistribution across singles individuals substantially and, in second best, women receive an extremely low and suboptimal net pension benefit.*

Moving to couples only, as expected, we observe that the desired level of redistribution from women to their spouses is larger than in Scenario 1. Now, the optimal increase in female retirement age and the decrease in male retirement age are much more pronounced than before and $\tau_f^{c2FB} > \tau_m^{c2FB}$ holds. Part (ii) of Proposition 8 applies because both incentive constraints bind for couples here. Hence, gender neutrality requires that both spouses retire at the same age ($\tau_f^{c2SB} = \tau_m^{c2SB}$) and thus makes female spouses better off with respect to the FB. To conclude, *in Scenario 2 for couples only, gender neutrality limits the extent of redistribution in favor of male spouses that can be achieved.*

The FB with singles and couples is similar to its counterpart in Scenario 1, except that the desired redistribution in favor of single women is lower, while the desired redistribution in favor of male spouses is higher than in Scenario 1 because here the concern for the short-lived man is stronger. As a result, in second best, male spouses are better off in the second scenario than in the first one while the opposite holds for single women. In the second-best setting, not much changes for singles with respect to the first scenario: single women continue to retire later than men, and the difference between retirement ages is the same in the two scenarios. However, in the second scenario, both single men and women receive a lower transfer from couples than in the first scenario. The incentive constraint of single men continues to be binding.

In the first best, couples's net pension benefit is positive and amounts to $P_f^{c2FB*} = 2586$ but, in the second best, it decreases dramatically and becomes negative, $P_f^{c2SB*} = -4608$. Like in the first scenario couples should be net recipients in the first best but they are net contributors in the second best. The only significant difference between the two scenarios is that the desired retirement age for female spouses in now larger than that for male spouses in first best ($\tau_f^{c2FB*} > \tau_m^{2cFB*}$). Consequently, we obtain the pooling regime described in Proposition 8 *(ii)* with $\tau_f^{c2SB*} = \tau_m^{2cSB*}$.

The qualitative conclusions of the first scenario continue to apply, but effects are here mitigated by the more concave SW function. *In Scenario 2, in the mixed economy gender neutrality impairs single women and male spouses. It benefits single men. It partially prevents redistribution in favor of single women and reverses the direction of the transfer between couples and singles resulting in negative net benefits for couples.*

7 Concluding comments

This paper has studied the design of pension schemes for male and female workers. Pension benefits net of contributions depend on the retirement age, which individuals choose given the benefit rule. Women live longer than men but have a lower wage. Individuals can be single or live in couples. Couples pool their incomes and equalize spouses' per-period consumption. Social welfare is utilitarian but the sum is taken

	FB	SB
c_f^s	27379	26392
τ_f^s	40.09	41.51
$P_f^{\prime s}$	0	161
P_f^s	119195	5962
τ_m^s	41.78	40.45
c_m^s	31913	33183
P_m^s	-125230	4791
$P_m^{s\prime}$	0	0
τ_f^c	41.37	40.94
$P_f^{c\prime}$	3793	3317
τ_m^c	40.53	40.94
$P_m^{c'}$	-4054	-3682
c	29555	29514
P_c	2586	-4608
U_f^s	975	952.
U_m^s	915	937
U_f^c	990	993
U_m^c	901	897
SWF	-0.002118	-0.00219

Table 4: Scenario 2 (high concavity of the SWF): singles and couples together.

over an increasing concave transformation of individuals' lifetime utilities in order to introduce the concern for redistribution between individuals with different life-spans.

The following main lessons have emerged. First, the social planner's concern for redistribution between individuals with different life-spans plays a crucial role in determining the desired direction of redistribution across genders and, thus, male and female optimal retirement ages. Second, resource pooling by couples limits the possibilities of gender redistribution via the level of net pension benefits. Instead, retirement ages, as determined by the specific design of the benefit rules now play an even more crucial role. Third, gender neutrality, though appealing on grounds of horizontal equity comes at a welfare cost. It limits the possibilities to redistribute across genders and may imply distortions of retirement ages. Depending on the specific case they may be lower or higher than otherwise optimal.

Elaborating on the third point, we have shown that because gender neutrality limits redistribution it negatively affects the group towards which redistribution is targeted. In turn, the gender and the marital status of the beneficiaries of redistribution depend on the concavity of the social welfare function. When the theoretical result is ambiguous, we used our calibrated numerical examples to obtain empirically relevant predictions. The impact of gender neutrality is most notable when there are singles only. In this case singles women are the target of redistribution and we conclude that gender neutrality limits such redistribution substantially.

Conversely, with a strictly concave social welfare function and a society only populated by couples, male spouses are always the target of redistribution and may therefore be adversely affected by gender neutrality. When the first-best level of redistribution towards male spouses is small, it can also be implemented under gender neutrality via an appropriate adjustment of retirement ages. Here gender neutrality has no cost. When instead the desired level of redistribution is so large that female workers would have to retire later than their spouses, it cannot be implemented under gender neutrality. The constrained pension scheme then involves pooling of retirement ages and redistribution is limited by gender neutrality.

Finally, we have shown though our simulations that, in a setting with singles and couples, gender neutrality impairs both single women and male spouses: it dramatically limits redistribution in favor of single women and reverses the direction of the transfer between couples and singles resulting in negative net benefits for couples. Overall female spouses are not much affected by gender neutrality because it has two opposite effects. On the one hand, it limits redistribution from women to men inside the couple, thus making female spouses better off. On the other hand, it prevents redistribution in favor of couples thus making female spouses worse off. To conclude, gender neutrality is greatly advantageous to single men who should be the "net contributors" in this setting with singles and couples, but end up being "net beneficiaries".

Gender neutrality adds an extra constraint for the design of pension systems. In a purely normative model such a constraint can only reduce overall welfare. However, in reality "horizontal equity" requirements are often imposed as a safeguard against arbitrary discrimination, particularly when policy decisions are determined by some political process. From that perspective the advocates of gender neutrality may well be inspired by the motivation to prevent arbitrary gender discrimination. Quite ironically, though, our analysis has shown that gender neutrality is often detrimental to those it allegedly is supposed to protect, and particularly to single women.

Redistribution across genders is motivated by both the longevity gap and the wage gap. The two gaps may be decreasing but, as long as they continue to exist, some redistribution across genders is welfare improving and gender neutrality brings about the unintended consequences that our results have highlighted. Hence, until some redistribution across genders is desirable, the call for gender neutrality appears to be premature. As we mentioned in the Introduction some derogations to gender neutrality have been allowed to compensate for women's disadvantages in their professional life and for the time they devoted to childcare. Such derogations mitigate the problem but they may not be sufficient and represent just a patch for some of the issues. Anyway, they remain mostly hypothetical as in reality only few EU Countries appear to have adopted them.

This paper presents just a "first pass" at studying the overall issue. It needs to be completed in different directions. In particular, we have neglected complementarities in leisure between spouses which in reality are likely to affect couples' retirement decisions. Furthermore, we have assumed that all persons of a given gender have the same wage. In reality, however, pension schemes often redistribute also across income groups. When wage heterogeneity is introduced gender and income redistribution become intertwined problems and gender neutrality can be expected to have an even more drastic impact. All these issue are on our research agenda.

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A Appendix

B First-best allocation, singles only: first-order conditions

$$\frac{\partial \mathcal{L}}{\partial c_f} = \varphi'(U_f^s)T_f u'(c_f^s) - \mu T_f = 0,$$
$$\frac{\partial \mathcal{L}}{\partial c_m} = \varphi'(U_m^s)T_m u'(c_m^s) - \mu T_m = 0,$$
$$\frac{\partial \mathcal{L}}{\partial \tau_f} = -\varphi'(U_f^s)R'(\tau_f^s) + \mu w_f = 0,$$
$$\frac{\partial \mathcal{L}}{\partial \tau_m} = -\varphi'(U_m^s)R'(\tau_m^s) + \mu w_m = 0.$$

C Proof of Proposition 2

The proof is by contradiction. Assume $U_f^{sFB} < U_m^{sFB}$ so that $\varphi'(U_f^{sFB}) > \varphi'(U_m^{sFB})$ which, using (13), implies $c_f^{sFB} > c_m^{sFB}$. Using (12), together with $w_f \leq w_m$, this implies that $\tau_f^{sFB} < \tau_m^{sFB}$. But $c_f^{sFB} > c_m^{sFB}$ and $\tau_f^{sFB} < \tau_m^{sFB}$ implies $U_f^{sFB} > U_m^{sFB}$. A contradiction. The first best solution thus involves $U_f^{sFB} \geq U_m^{sFB}$ which by (13) implies $c_m^{sFB} \geq c_f^{sFB}$.

Point (i). When φ is linear, (13) implies $c_f^{sFB} = c_m^{sFB}$, so that by (12) it yields $\tau_f^{sFB} < \tau_m^{sFB}$.

Now when φ is Rawlsian, individual utilities are equalized i.e.:

$$U_f^{sFB} = T_f u\left(c_f^{sFB}\right) - R\left(\tau_f^{sFB}\right) = U_m^{sFB} = T_m u\left(c_m^{sFB}\right) - R\left(\tau_m^{sFB}\right). \tag{A.1}$$

Point (ii). Assume that $\tau_f^{sFB} \leq \tau_m^{sFB}$ and φ is Rawlsian, then if $w_f = w_m$, (12) yields $c_m^{sFB} \leq c_f^{sFB}$ and thus $U_f^{sFB} > U_m^{sFB}$. This contradicts $(A.1)$.

Point (iii). Assume $\tau_m^{sFB} \leq \tau_f^{FB}$ and φ is Rawlsian, then if $T_f = T_m = T$, (12) yields $c_m^{sFB} > c_f^{sFB}$. Since $(A.1)$ implies $R\left(\tau_f^{sFB}\right) - R\left(\tau_m^{sFB}\right) = T\left[u\left(c_f^{sFB}\right) - u\left(c_m^{sFB}\right)\right] \geq 0$ i.e. $c_m^{sFB} \leq c_f^{sFB}$ so that we have a contradiction.

D Proof of Proposition 4

The proof is by contradiction. Assume

$$U_f < U_m \Longleftrightarrow \varphi'(U_f) > \varphi'(U_m). \tag{A.2}$$

Since $T_f \geq T_m$ and $c_m = c_f = c$, $U_f < U_m$ implies

$$\tau_f \geq \tau_m. \tag{A.3}$$

But with $w_f \leq w_m$, $\varphi'(U_f) > \varphi'(U_m)$ implies

$$\frac{w_f}{\varphi'(U_f)} < \frac{w_m}{\varphi'(U_m)}.$$

From (22), the previous inequality implies $R'(\tau_f) < R'(\tau_m)$ and thus $\tau_f < \tau_m$, which contradicts $(A.3)$.

E Indifference curves of singles

Differentiating (29) we observe that indifference curves have the following properties

$$\frac{\partial MRS}{\partial w} = -1 - \frac{\tau}{T}\frac{R'(\tau)u''(c)}{(u'(c))^2} \gtrless 0, \tag{A.4}$$

$$\frac{\partial MRS}{\partial T} = \frac{c}{T}\frac{R'(\tau)u''(c)}{(u'(c))^2} < 0, \tag{A.5}$$

$$\frac{\partial MRS}{\partial \tau} = \frac{R''(\tau)u'(c) - wR'(\tau)u''(c)/T}{(u''(c))^2} > 0. \tag{A.6}$$

Because of (A.6), the indifference curves are U-Shaped in the (τ, P) space. Since $\partial MRS/\partial T < 0$, when $w_f = w_m$ the two curves cross only once at a point where $MRS_f < MRS_m$. Moreover, the point at which $MRS_f = MRS_m = 0$ lies south east for female relative to male.

F Proof of Proposition 6

Using the resource constraint (31), one has $P_m^s = -P_f^s$ so that the problem of the government can be rewritten as:

$$\max_{P_f^s,\tau_f^s,\tau_m^s} \quad \varphi\left[T_f u\left(\frac{w_f\tau_f^s + P_f^s}{T_f}\right) - R(\tau_f^s)\right] + \varphi\left[T_m u\left(\frac{w_m\tau_m^s - P_f^s}{T_m}\right) - R(\tau_m^s)\right], \quad \text{(A.7)}$$

$$T_f u\left(\frac{w_f\tau_f^s + P_f^s}{T_f}\right) - R(\tau_f^s) \geq T_f u\left(\frac{w_f\tau_m^s - P_f^s}{T_f}\right) - R(\tau_m^s), \quad \text{(A.8)}$$

$$T_m u\left(\frac{w_m\tau_m^s - P_f^s}{T_m}\right) - R(\tau_m^s) \geq T_m u\left(\frac{w_m\tau_f^s + P_f^s}{T_m}\right) - R(\tau_f^s). \quad \text{(A.9)}$$

Denoting λ_f^s and λ_m^s the Lagrange multipliers respectively associated with A.8 and A.9, the FOCs with respect to P_f^s, τ_f^s and τ_m^s write

$$\begin{aligned} & u'\left(c_f^s\right)\varphi'\left[U_f^s\right] - u'\left(c_m^s\right)\varphi'\left[U_m^s\right] \\ & +\lambda_f^s\left[u'\left(c_f^s\right) + u'\left(c_{fm}^s\right)\right] - \lambda_m^s\left[u'\left(c_m^s\right) + u'\left(c_{mf}^s\right)\right] = 0; \quad \text{(A.10)} \\ & \left[w_f u'\left(c_f^s\right) - R'\left(\tau_f^s\right)\right]\varphi'\left[U_f^s\right] \\ & +\lambda_f^s\left[w_f u'\left(c_f^s\right) - R'\left(\tau_f^s\right)\right] - \lambda_m^s\left[w_m u'\left(c_{mf}^s\right) - R'\left(\tau_f^s\right)\right] = 0; \quad \text{(A.11)} \\ & \left[w_m u'\left(c_m^s\right) - R'\left(\tau_m^s\right)\right]\varphi'\left[U_m^s\right] \\ & -\lambda_f^s\left[w_f u'\left(c_{fm}^s\right) - R'\left(\tau_m^s\right)\right] + \lambda_m^s\left[w_m u'\left(c_m^s\right) - R'\left(\tau_m^s\right)\right] = 0, \quad \text{(A.12)} \end{aligned}$$

where $c_{fm}^s = \left(w_f\tau_m^s - P_f^s\right)/T_f$ and $c_{mf}^s = \left(w_m\tau_f^s + P_f^s\right)/T_m$. Using the definition (29), the three FOCs can be respectively rewritten as

$$\begin{aligned} & u'\left(c_f^s\right)\left(\varphi'\left[U_f^s\right] + \lambda_f^s\right) - u'\left(c_m^s\right)\left(\varphi'\left[U_m^s\right] + \lambda_m^s\right) \\ & +\lambda_f^s u'\left(c_{fm}^s\right) - \lambda_m^s u'\left(c_{mf}^s\right) = 0, \quad \text{(A.13)} \\ & u'\left(c_f^s\right) MRS_f\left(\varphi'\left[U_f^s\right] + \lambda_f^s\right) \\ & -\lambda_m^s u'\left(c_{mf}^s\right) MRS_{mf} = 0, \quad \text{(A.14)} \\ & u'\left(c_m^s\right) MRS_m\left(\varphi'\left[U_m^s\right] + \lambda_m^s\right) \\ & -\lambda_f^s u'\left(c_{fm}^s\right) MRS_{fm} = 0, \quad \text{(A.15)} \end{aligned}$$

where $MRS_{mf}^s = R'(\tau_f^s)/u'(c_{mf}^s) - w_m$ and $MRS_{fm}^s = R'\left(\tau_m^s\right)/u'(c_{fm}^s) - w_f$.

F.1 Proof of point (i)

Suppose that $\lambda_m^s > 0$ and $\lambda_f^s = 0$. From $(A.15)$, $MRS_m^s = 0$. Moreover, combining $(A.13)$ with $(A.14)$ yields:

$$MRS_f^s = \left(1 - \frac{u'(c_m^s)(\varphi'[U_m^s] + \lambda_m^s)}{u'(c_f^s)\varphi'\left[U_f^s\right]}\right) MRS_{mf}^s,$$

where $\left(1 - u'(c_m^s)(\varphi'[U_m^s] + \lambda_m^s)/u'(c_f^s)\varphi'\left[U_f^s\right]\right) \in]0,1[$ by $(A.13)$ so that

$$\frac{MRS_f^s}{MRS_{mf}^s} < 1.$$

Recall that $MRS_{mf}^s = R'(\tau_f^s)/u'(c_{mf}^s) - w_m$ which using $MRS_m^s = 0$ yields

$$MRS_{mf}^s = \frac{R'(\tau_f^s)}{u'(c_{mf}^s)} - \frac{R'(\tau_m^s)}{u'(c_m^s)}. \tag{A.16}$$

Moreover, the binding self selection constraint $(A.9)$ implies

$$R(\tau_f^s) - R(\tau_m^s) = T_m\left[u(c_{mf}^s) - u(c_m^s)\right], \tag{A.17}$$

so that two cases are possible:

(ia) either $c_{mf}^s < c_m^s$ so that by $(A.17)$, $\tau_f^s < \tau_m^s$ which using $(A.16)$ yields $MRS_{mf}^s < 0$ (and thus $MRS_f^s < 0$).

(ib) or $c_{mf}^s > c_m^s$ so that by $(A.17)$, $\tau_f^s > \tau_m^s$ which using $(A.16)$ yields $MRS_{mf}^s > 0$ (and thus $MRS_f^s > 0$).

Note that in case (ia), using $(A.20)$, one has $c_m^s > c_{mf}^s > c_f^s$. Moreover, by $(A.13)$, one has $u'(c_f^s)\varphi'\left[U_f^s\right] > u'(c_m^s)\varphi'[U_m^s]$ so that $U_m^s > U_f^s$.

F.2 Proof of point (ii)

Suppose that $\lambda_f^s > 0$ and $\lambda_m^s = 0$. From $(A.14)$, $MRS_f^s = 0$. Moreover, combining $(A.13)$ with $(A.15)$ yields:

$$MRS_m^s = MRS_{fm}^s\left[1 - \frac{u'(c_f^s)\left(\varphi'\left[U_f^s\right] + \lambda_f^s\right)}{u'(c_m^s)\varphi'[U_m^s]}\right],$$

where $1-u'(c_f^s)\left(\varphi'\left[U_f^s\right]+\lambda_f^s\right)/u'\left(c_m^s\right)\varphi'\left[U_m^s\right]\in\left]0,1\right[$ by $(A.13)$ so that

$$\frac{MRS_m^s}{MRS_{fm}^s}<1.$$

Remember that $MRS_{fm}^s=R'\left(\tau_m^s\right)/u'(c_{fm}^s)-w_f$ which using $MRS_f^s=0$ yields

$$MRS_{fm}^s=\frac{R'\left(\tau_m^s\right)}{u'(c_{fm}^s)}-\frac{R'(\tau_f^s)}{u'(c_f^s)}. \tag{A.18}$$

Moreover, the binding self selection constraint $(A.8)$ implies

$$R(\tau_f^s)-R(\tau_m^s)=T_f\left[u(c_f^s)-u(c_{fm}^s)\right], \tag{A.19}$$

and by definition:

$$c_{mf}^s=\frac{w_m\tau_f^s+P_f^s}{T_m}>\frac{w_f\tau_f^s+P_f^s}{T_f}=c_f^s, \tag{A.20}$$

$$c_m^s=\frac{w_m\tau_m^s-P_f^s}{T_m}>\frac{w_m\tau_m^s-P_f^s}{T_m}=c_{fm}^s, \tag{A.21}$$

so that 2 cases are possible:

(iia) Either $c_f^s>c_{fm}^s$, so that by $(A.19)$, $\tau_f^s>\tau_m^s$ which using $(A.18)$ yields $MRS_{fm}^s<0$ (and thus $MRS_m^s<0$).

(iib) or $c_f^s<c_{fm}^s$, so that by $(A.19)$, $\tau_f^s<\tau_m^s$ which using $(A.18)$ yields $MRS_{fm}^s>0$ (and thus $MRS_m^s>0$). Note that in case (iib), using $(A.21)$, one has $c_m^s>c_{fm}^s>c_f^s$. Moreover, by $(A.13)$, one has $u'(c_f^s)\varphi'\left[U_f^s\right]<u'\left(c_m^s\right)\varphi'\left[U_m^s\right]$ so that $U_f^s>U_m^s$.

G Proof of Proposition 8

We prove the proposition by a succession of lemmas. In Lemma 1, we show that any first-best allocation can be implemented as long as $\tau_f^{cFB}\leq\tau_m^{cFB}$ with $P_m^{cFB}=P_f^{cFB}=0$. We then show in Lemma 2 that a constrained solution involves both incentive constraints to be binding. Then Lemma 3 proves that the constrained allocation cannot be implemented with $\tau_f>\tau_m$ so that a constrained solution implies $\tau_f^{cSB}=\tau_m^{cSB}$ and $P_f^{cSB}=0$.

Lemma 1 *Any first-best allocation described in Proposition 4 is implementable if* $\tau_f^{cFB}\leq\tau_m^{cFB}$ *and* $P_f^{cFB}=P_m^{cFB}=0$.

Proof. Recall that

$$U = Tu\left(\frac{w_m\tau_m + w_f\tau_f}{T}\right) - R(\tau_m) - R(\tau_f),$$

where $T = T_m + T_f$. Consider a variation $d\tau_f = -d\tau_m$, so that the female spouse chooses a higher retirement age and men spouse a lower one. Differentiation yields

$$\frac{dU}{d\tau_f} = w_f u'(c) - R'(\tau_f) - w_m u'(c) + R'(\tau_m). \tag{A.22}$$

Note that this expression is always negative when $\tau_f > \tau_m$. In other words, at any allocation with $\tau_f > \tau_m$, couples would prefer a lower retirement age for women and a higher one for men. More specifically, rewriting $(A.22)$ as follow:

$$\frac{dU}{d\tau_f} = u'(c)\left[w_f - w_m + \frac{R'(\tau_m)}{u'(c)} - \frac{R'(\tau_f)}{u'(c)}\right],$$

and substituting $R'(\tau_m)/u'(c)$ and $R'(\tau_f)/u'(c)$ by their first-best counterparts given by (20) and (21), one has

$$\frac{dU^{cFB}}{d\tau_f} = u'\left(c^{cFB}\right)\left[w_f - w_m + w_m\frac{T_f\varphi'(U_f^{cFB}) + T_m\varphi'(U_m^{cFB})}{\varphi'(U_m^{cFB})\,(T_f + T_m)} - w_f\frac{T_f\varphi'(U_f^{cFB}) + T_m\varphi'(U_m^{cFB})}{\varphi'(U_f^{cFB})\,(T_f + T_m)}\right],$$

which after some rearrangements yields

$$\frac{dU^{cFB}}{d\tau_f} = u'\left(c^{cFB}\right)\left[w_f\left(\frac{T_m}{T}\frac{\varphi'(U_f^{cFB}) - \varphi'(U_m^{cFB})}{\varphi'(U_f^{cFB})}\right) + w_m\left(\frac{T_f}{T}\frac{\varphi'(U_f^{cFB}) - \varphi'(U_m^{cFB})}{\varphi'(U_m^{cFB})}\right)\right].$$

Thus, by Proposition 4, $dU^{cFB}/d\tau_f < 0$ at any first best allocation. Again, couples would prefer a lower retirement age for women and a higher one for men. ■

Lemma 2 *There are only two possible regimes. Regime 1 in which $\lambda_f^c = \lambda_m^c = 0$ and Regime 2 in which $\lambda_f^c > 0$ and $\lambda_m^c > 0$.*

Proof. The problem can be simplified in a drastic way by substituting for the budget constraint into the problem, which amounts for instance to replacing P_m by $-P_f$. Omitting the superscripts to simplify notation, the problem can then be rewritten as follows

$$\max_{P_f,\tau_f,\tau_m} \quad \varphi\left[T_f u\left(\frac{1}{T_m+T_f}\left(w_m\tau_m+w_f\tau_f\right)\right)-R(\tau_f)\right]$$
$$+\varphi\left[T_m u\left(\frac{1}{T_m+T_f}\left(w_m\tau_m+w_f\tau_f\right)\right)-R(\tau_m)\right],$$
$$\text{s.t.} \quad (T_f+T_m)u\left(\frac{1}{T_m+T_f}\left(w_m\tau_m+w_f\tau_f\right)\right)-R(\tau_f)-R(\tau_m)\geq$$
$$(T_f+T_m)u\left(\frac{1}{T_m+T_f}(w_m+w_f)\tau_m-2P_f\right)-2R(\tau_m); \tag{A.23}$$
$$(T_f+T_m)u\left(\frac{1}{T_m+T_f}w_m\tau_m+w_f\tau_f\right)-R(\tau_f)-R(\tau_m)\geq$$
$$(T_f+T_m)u\left(\frac{1}{T_m+T_f}(w_m+w_f)\tau_f+2P_f\right)-2R(\tau_f). \tag{A.24}$$

Denoting λ_f^c and λ_m^c the Lagrange multipliers respectively associated with $(A.23)$ and (**??**), the FOC with respect to P_f then reduces to

$$\lambda_f^c 2u'(c_{fm}^c)-\lambda_m^c 2u'(c_{mf}^c)=0,$$

where

$$c_{fm}=(w_m+w_f)\tau_m/(T_m+T_f)-2P_f, \tag{A.25}$$
$$c_{mf}=(w_m+w_f)\tau_f/(T_m+T_f)+2P_f. \tag{A.26}$$

Consequently, we cannot have a solution where only one of the multipliers is strictly positive. The two possible regimes are then Regime 1: $\lambda_f^c=\lambda_m^c=0$ which is the first best outcome. The regime 2 involves $\lambda_f^c>0$ and $\lambda_m^c>0$. ■

Lemma 3 *In Regime 2, one necessarily has* $\tau_f^{cSB}\leq\tau_m^{cSB}$.

Proof. Assume by contradiction that $\tau_f^{cSB}>\tau_m^{cSB}$. Since the two incentive compatibility constraints $(A.23)$ and $(A.24)$ are binding, omitting the superscripts to simplify notation one has:

$$R(\tau_f)-R(\tau_m)=(T_f+T_m)\left[u\left(c\right)-u\left(c_{mf}\right)\right];$$
$$R(\tau_f^c)-R(\tau_m^c)=(T_f+T_m)\left[u\left(c_{fm}\right)-u\left(c\right)\right];$$

so that

$$c_{mf} < c < c_{fm} \tag{A.27}$$

and

$$u(c) = \frac{u(c_{fm}) + u(c_{mf})}{2}. \tag{A.28}$$

Using $(A.25)$ and $(A.26)$ and rearranging yields:

$$u(c) = \frac{u(c + x - 2P_f) + u(c - y + 2P_f)}{2},$$

where

$$x = w_f(\tau_f - \tau_m)/(T_m + T_f) \leq y = w_m(\tau_f - \tau_m)/(T_m + T_f). \tag{A.29}$$

By concavity of $u(.)$ and inequality $(A.27)$, equation $(A.28)$ is satisfied if and only if $c_{fm} - c > c - c_{mf}$ which implies

$$x > y.$$

The previous inequality contradicts $(A.29)$. ■

To sum up, we have shown that when $\tau_f^{cFB} < \tau_m^{cFB}$, the first best can be decentralized with $P_m^{cSB} = P_f^{cSB} = 0$. Whereas, when the two incentive compatibility constraints bind, $\tau_m^{cSB} = \tau_f^{cSB}$ holds, which necessarily implies $c_{mf} = c_{fm}$ in order to satisfy $(A.28)$ so that $P_f^{cSB} = 0$. This implies that when $\tau_f^{cFB} > \tau_m^{cFB}$, the two incentive compatibility constraints bind and $\tau_m^{cSB} = \tau_f^{cSB}$ with $P_f^{cSB} = 0$. This completes the proof of Proposition 8.