Market Information in Banking Supervision: The Role of Stress Test Design*

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Abstract

This paper studies the optimal degree of leniency in a bank stress test, when poorly capitalized banks engage in risk shifting and a banking supervisor can intervene to prevent it. The stress test directly provides the supervisor with noisy information about whether or not a bank is well capitalized. Furthermore, the stress test outcome affects a speculator's incentives to acquire costly information about the bank and trade in its shares. This in turn affects the amount of market information available to the supervisor when she takes her intervention decision. We show that a supervisor optimally distorts the stress test towards leniency for banks whose shares are relatively illiquid, and about whom the supervisor has little private information. When the supervisor has substantial private information about a bank whose shares are fairly liquid, it is optimal to apply a conservative stress test.

Keywords: Stress test, leniency, monitoring, feedback, risk shifting

JEL classification: G14, G28

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1 Introduction

There has been considerable interest in recent years in the question how information conveyed by prices in secondary financial markets feeds back into real decisions (see Bond, Edmans and Goldstein, 2012, for a survey). One application of that literature points to the importance of stock price information in guiding intervention decisions of regulators, for example, a supervisor who needs to decide whether to intervene in a troubled bank (Bond, Goldstein and Prescott, 2010, and Bond and Goldstein, 2015). As Flannery and Bliss (2019) argue: "We believe that market discipline can, potentially, complement and support official oversight of risky financial institutions, [...] by providing market signals that supervisors can use to motivate their own actions..." In parallel, a number of papers have investigated how supervisors themselves should produce and communicate information, for example via bank stress tests, in order to assess the need for intervention (e.g., Colliard, 2019, and Carletti, Dell'Ariccia and Marquez, 2020). It remains a largely open question how the two interact.

In this paper we ask how a bank supervisor should design her monitoring technology (the stress test) in light of the impact this will have on information reflected in financial markets. We view the monitoring technology as having two roles: firstly, it determines directly what the supervisor can learn, and secondly, it affects the incentives for a speculator to produce and trade on costly information. We have in mind a supervisor who can choose the "pass" hurdle of a stress test carried out on a bank, and then, based on the test result and any further information contained in the bank's share price, decide whether or not to intervene in the bank. The supervisor tries to learn about the value of the bank's assets, knowing that a low asset value would induce risk shifting by the bank. The supervisor can intervene and reduce the bank's risk exposure, for example by arranging its sale to a better capitalized bank that will not engage in risk shifting. Since a speculator can try to learn the value of the bank's assets, stock prices may contain additional information that is useful for the supervisor.

The specific question we ask is how lenient a supervisor's stress test should be? That is, unlike existing papers in the literature (for an overview, see Goldstein and Yang, 2017) we do not focus on information precision per se, but on how the supervisor should optimally trade off type I and type II errors. Under a lenient stress test, a bad bank is more likely to pass (type II error), while good banks are more likely to be subject to intervention under a conservative test (type I error). Similar to the literature on Bayesian persuasion (discussed in more detail below), we assume that the supervisor designs the monitoring technology (stress test) in a way that is publicly observable and then publishes the outcome of the test. Hence, there is no scope for *ex post* opportunism, for example in the form of the supervisor hiding or misreporting the test result.

Our main findings are the following. We show that a speculator's expected trading profits are higher for a bank that passes its stress test than for one that fails it. This is because a bank that fails the test is subject to intervention by the supervisor, which wipes out equity, leaving no profitable trading opportunity for the speculator. A supervisor therefore has an incentive to let more banks pass the test, as only for these banks market information will be produced. The optimal stress test design will therefore trade-off the increased information conveyed by stock prices under a more lenient test, against the cost of allowing more bad banks to continue without intervention.

We also consider the case where the supervisor may sometimes privately learn more from the stress test than the publicly observed pass/fail signal. Interestingly, the existence of such private information increases the speculator's trading profits following a failed test. This is because there is now a chance that the supervisor will have privately learned that a bank failed a test only by a small margin and therefore ignore the test result and allow the bank to continue anyway. Equity value will therefore not necessarily be wiped out following a failed test, which allows the speculator to trade profitably. Inducing information production following either test outcome requires distorting the stress test towards conservatism: The supervisor, being concerned with making trade profitable following a "fail" certification, may need to leave more trading profits to the speculator by making the "fail" outcome less informative. That can be achieved by generating erroneous "fail" certifications more frequently, i.e., by applying a more conservative test. This increases the informational advantage (and trading profits) of a speculator vis-a-vis the market maker who only observes the publicly announced test result.

Our model shows that lenient tests should be applied to banks for which it is particularly difficult to encourage the production of market information, i.e., banks whose shares are less liquid, or where information production is particularly costly for speculators. Moreover, when a supervisor can rely more heavily on additional information, beyond the publicly observable results of stress test, we would expect her to benefit from using a more conservative test design.

It would be an exaggeration to argue that supervisors in practice determine the leniency of their stress test design, solely on the basis of the trade-offs described in our model. Evidently, supervisors will also be guided by other concerns, and below we discuss some of the papers that have studied such concerns. Nevertheless, we believe it is important to be aware of the trade-offs we identify. One clear implication of our analysis is that a "one-size-fits-all" approach has costs, as for some banks it will likely result in a drop in the information quality on which intervention decisions are based. For example, a supervisor who adopts a lenient test design should be aware that this will have adverse consequences for the quality of stock price information for banks who failed the test. In a similar vein, an increase in conservatism of the test design, can actually *reduce* the unconditional probability of an intervention by the supervisor. This is, because conservatism may increase the amount of speculative information produced following a "fail" result, and this additional information can prevent unnecessary interventions in banks that are actually sound (but for which the test erroneously generated a "fail" verdict).

In addition we study the impact of the severity of the risk shifting problem on the usefulness of markets in producing information. As the risk shifting problem gets more severe, the stock market becomes less useful in providing information. This happens because the value of an equity claim, conditional on no intervention, becomes less sensitive to the underlying state of the world, undermining a speculator's incentives to produce information about it. On the one hand, in the low state of the world, the value of assets in place is low, reducing the value of equity. On the other hand, the bank engages in risk shifting in the low state of the world, and the accompanying expropriation of creditors increases equity value. We thus identify a new wedge between the private and social incentives to produce information. Private incentives are driven by the variability in value of the traded claim, which is equity. Social incentives, on the other hand, stem from the value that accrues to debt *and* equityholders together. A worsening risk shifting problem reduces private incentives of a share trader to produce information, but increases the social value of this information. This problem differs from the one identified in Bond, Goldstein and Prescott (2010) who show that the mapping from the states of the world to the price may not be invertible. The fact that the price of an equity claim sends a mixed message when risk shifting is a problem has been recognized intuitively by advocates of subordinated debt as generating market discipline (see Flannery and Bliss, 2019, for a review of the arguments). However, this point has not been taken up in the feedback literature, probably because most of the papers work in relatively abstract settings in which the link between regulatory intervention and the value of the traded claim is fixed by assumption.

We extend our analysis to allow for trade in debt claims and show that it may be the case that a speculator trades in shares when the stress test was passed, but trades in debt claims when a bank failed the test. Whether in practice the secondary market in debt claims provides an adequate venue for information aggregation is questionable. Debt markets are typically over the counter, making it harder for supervisors to learn from trades. Debt markets are also significantly less liquid than equity markets, and taking short positions in debt markets is more costly, reducing an informed trader's ability to profit. Finally, it is widely believed that in spite of recent regulatory changes, markets expect significant fractions of debt claims to be bailed out in case of a bank failure, limiting their exposure to the risk of failure (see Cutura, 2018, or Flannery and Bliss, 2019). The limited role that secondary debt markets are likely to play in practice justifies our paper's focus on stock markets.

There are a number of papers that have studied whether stress test results should be disclosed, e.g., Orlov, Zryumov and Skrzypacz, 2018, Bouvard, Chaigneau and de Motta, 2015, Goldstein and Leitner, 2015, Leitner and Williams, 2018, and Williams, 2017 (see also Goldstein and Sapra, 2014, for a review). Disclosure matters, as it may affect market discipline, the functioning of the interbank market, financial stability, bank lending behaviour and risk sharing. In this paper, we take for granted that stress test results are published, which corresponds to the practice that supervisors have converged to. The supervisor's choice of leniency, however, indirectly affects the quality of information that is publicly available, including the limiting case, where all banks always pass the test (or always fail it), which degenerates the stress test to have zero information. Given that our model allows for additional information to be available to the supervisor, our specification includes the case where a supervisor is privately informed and chooses not to provide information to the public by using a degenerate test. We show that in the limiting case, where the supervisor's private information is so good that she never learns anything from the stress test, it may indeed be optimal to design a degenerate test, and rely entirely on private information. We also study an extension where the supervisor learns from both the test and a noisy private signal, but can commit not to disclose the test result. We show that it can be optimal to commit to disclosure, particularly when banks are ex ante in relatively bad shape. In that case trading profits under a no-disclosure policy may be too low to warrant information production (since an intervention is ex ante quite likely). By committing to disclose, a supervisor can ensure that at least information production occurs for those banks that pass the test. The dependence of disclosure on the general "health" of the banking sector is reminiscent of Bouvard et al. (2015) and Wiliams (2017), although the underlying mechanisms are very different.

The papers closest to ours are Bond and Goldstein (2015) and Siemroth (2019) who study the interaction of a regulator's information (including a decision to disclose such information) with information revealed by share prices, when that information is in turn used by the regulator. They show that more public information may crowd out private information as it reduces the informational advantage of speculators. This effect is balanced by a crowding-in effect, as public information reduces the riskiness of speculators' trades, inducing them to take larger positions. Also related is Goldstein and Yang (2017) who study the interaction between public disclosure and market based information in a context where the decision maker learns from both, the public signal and market prices (unlike in Bond and Goldstein (2015) where the regulator has information regardless of whether or not it is made public). Goldstein and Yang (2017) focus on two dimensions of uncertainty and explore how disclosure affects the weight that traders put on one of the two private signals they possess. They show that when information is disclosed about the dimension of uncertainty that is relevant for the real decision, then this will reduce the weight that traders put on that dimension of their private signals. By crowding out information aggregation on the "useful" dimension, more public disclosure may reduce the overall amount of information relevant for the real decision.

Our focus is different in that we study the role of tilting the supervisor's information production technology towards either type I or II errors in a context where a speculator faces an information production cost. Unlike in Bond and Goldstein (2015) or Goldstein and Yang (2017), the information production decision depends sensitively on the *realization* of the public signal, with less information being produced following a negative public signal than following a positive one.

The effect that trading profits differ, depending on whether public information is positive or negative, is due to the fact that the supervisor's decision depends on public information in such a way as to reduce the residual uncertainty of the traded claim following negative public information. This is related to Dow, Goldstein and Guembel (2017) who show that speculators' information production may break down when firms' investment prospects are unfavorable, because such firms are unlikely to invest, undermining the incentive for speculators to produce information about those prospects.

When thinking about monitoring and reporting, we take an angle akin to that in the literature on Bayesian persuasion (see Kamenica and Gentzkow, 2011 and Szydlowski, 2020, among others). That is, we think of a supervisor as having to design an information technology up front, which determines how an underlying state is mapped into a publicly observable signal. In the Bayesian persuasion literature, a receiver acts once the signal is observed. Our paper differs in that the receiver (the speculator) has a role of producing additional information that is subsequently disseminated via a price mechanism. The supervisor thus chooses the information structure with a view to encouraging information production by the speculator, taking into account that the information structure affects both direct and indirect learning.

Our paper is also related to the literature on banking regulation which regards the bank's moral hazard problem as a central friction that regulation can address, for example, Bhattacharya (1982), Calzolari and Loranth (2011), Fecht, Inderst and Pfeil (2017), Gorton and Huang (2004), Hellmann, Murdock and Stiglitz (2000), Morrison and White (2005) or Rochet (1992). A number of papers have argued that in such a context a supervisor plays a role by intervening in bad banks at risk of failure by rendering their risky payoffs safe (see Calzolari, Colliard, and Loranth, 2018). In Carletti, Dell'Ariccia and Marquez (2020) banks take too much risk in a laissez-faire equilibrium and supervision is designed to reduce their risk exposure. The supervisor monitors and learns about the amount of a bank's capital (and its portfolio) and can then intervene so as to reduce risk exposure. When an intervention occurs, shareholders are expropriated. We share with that literature the focus on a bank's risk shifting incentives and the supervisor's role in curbing them. Our central point on the design of the supervisor's information and its interaction with market-based information is new to this literature.

Shapiro and Zeng (2018) look into the leniency of bank stress tests. As in our paper, stress tests serve to generate information for the supervisor. The leniency of the test is the supervisor's hidden choice and supervisors build a reputation over time. Leniency in their paper trades off information content (higher default probability) against a reduction in loans to the real sector. There may be multiple equilibria due to reputational concerns.

The remainder of the paper proceeds as follows. Section 2 explains the model set-up. In Section 3 we solve for the benchmark without a speculator. We then introduce a speculator and study a special parametric case in Section 4 while Section 5 solves for the general case. In Section 6 we study model extensions and Section 7 concludes. Most proofs are relegated to the Appendix.

2 The model

To guide the detailed model description, we begin with a brief overview. There are four dates $t = 0, ..., 3$. There is a bank, a supervisor, a speculator, a market maker and a liquidity trader. At date 0, the supervisor chooses a monitoring technology, i.e., a stress test that generates a public signal at date 1 about the value of the bank's assets. The speculator can observe the public signal and then decide whether to acquire costly (private) information and trade on it at date 2. The supervisor observes the bank's share price and possibly an additional signal before having to decide whether to intervene or allow the bank to continue. If the supervisor does not intervene, the bank can choose whether or not to engage in a risk shifting activity. At date 3, uncertainty is realized and all payoffs made.

There is a bank with assets in place whose value A_ω depends on an underlying state of the world $\omega \in \{l, h\}$. Each state is ex ante equally likely and chosen by nature at $t = 0$. For simplicity we assume $A_l = 0$ and $A_h > 0$. The bank also has on-going operations that generate risky cash flows in the future. Details on this follow below. The bank is financed with an amount of debt $D > A_h$, which we take as given, and is run by a manager whom we call the banker and who acts in the interest of the bank's shareholders. The banker is privately informed about ω, which is otherwise unobserved.[1]

[1]We exclude here the possibility of truthtelling contracts that might extract the banker's information. An older literature has considered how regulation, such as the pricing of deposit insurance, can induce truthtelling by the banker when the latter is also subject to an agency problem (e.g., Chan, Greenbaum and Thakor, 1992, Giammarino, Lewis and Sappington, 1993 or Freixas and Rochet, 1995). The subsequent literature's continued focus on the monitoring role of supervisors is arguably an implicit acknowldgement of the difficulty of implementing such schemes in practice. Moreover, having market information at her disposal, the supervisor could presumably reduce the informational rent left to the banker from a truthtelling mechanism. Hence, allowing for trutelling contracts does not necessarily

The supervisor has a monitoring technology which we call a stress test and that generates a noisy public signal $m \in \{fail, pass\}$ at date 1 about the state ω. At date 0 the supervisor can choose the degree of conservatism of the stress test, captured by the choice variable $s^* \in [0,1]$. This choice affects the conditional distribution of the public signal as follows,

$$\Pr(m = pass|\omega = h) = 1 - (s^*)^2, \tag{1}$$
$$\Pr(m = pass|\omega = l) = (1 - s^*)^2. \tag{2}$$

The specific assumptions embedded in (1) and (2), are convenient because they generate Bayesian updates that are linear in the choice variable s^*:

$$\Pr(\omega = h|m = pass) = \frac{1+s^*}{2}, \tag{3}$$
$$\Pr(\omega = l|m = fail) = 1 - \frac{s^*}{2}. \tag{4}$$

Intuitively, an increase in s^* makes the positive signal $m = pass$ more informative about the state $\omega = h$. At the same time, it reduces the information content of the negative signal $fail$. An increase in s^* can therefore be thought of as increasing conservatism: the negative signal $m = fail$ is generated more frequently, but is also less informative, while the positive signal $m = pass$ is awarded rarely, but when it is, it is highly informative. At the corner $s^* = 0$, observing a $pass$ signal is entirely uninformative about which is the true state, while a $fail$ signal perfectly reveals that the state of the world is $\omega = l$. At the opposite corner $s^* = 1$, the $pass$ signal perfectly reveals that the true state is $\omega = h$, while a $fail$ signal is now entirely uninformative. In the middle $\left(s^* = \frac{1}{2}\right)$, inferences are noisy and symmetric, i.e., $\Pr(\omega = h|m = pass) = \Pr(\omega = l|m = fail) = \frac{3}{4}$.

In addition to the public signal, we allow for the case where the supervisor may glean additional information during the monitoring process. We assume that such information is soft and cannot be revealed publicly, possibly because of its technical complexity. In order to model such additional information, suppose that the monitoring process generates an internal signal $s \in [0,1]$ drawn from the following distribution:

$$\begin{aligned} f(s|\omega = h) &= 2s, \\ f(s|\omega = l) &= 2(1-s). \end{aligned} \tag{5}$$

The internal signal could be thought of as information generated, for example, by applying a variety of scenarios or different valuation models to value a bank's assets. The internal signal s then generates the public signal m as follows

$$m(s) = \begin{cases} fail & \text{if} \quad s < s^* \\ pass & \text{if} \quad s \geq s^* \end{cases}. \tag{6}$$

Notice that (5) and (6) yield exactly the Bayesian updates given in (3) and (4). This captures the idea that by choosing s^*, the supervisor can apply tougher or softer scenarios (more or less generous valuation models) when monitoring the bank, and that this will

undermine the role of market information. However, taking these considerations fully on board would distract us from the paper's main message.

have an impact on the publicly observable assessment, m. In what follows we will refer to s^* interchangeably as the signal cut-off and the degree of leniency of the stress test.

Suppose that the supervisor directly observes the internal signal with probability $\theta \in [0,1]$ and with complementary probability does not observe it, i.e., she observes a signal

$$s_{\text{sup}} = \begin{cases} s, \text{ with probability } \theta \\ \varnothing, \text{ with probability } 1-\theta \end{cases} \tag{7}$$

θ captures the extent to which the supervisor does herself understand and include in her own decision making the information s. It has been argued that in a supervisory union, the local (national) supervisor has better information than the centralized (international) supervisor, and θ can be thought of as capturing such informational differences (e.g., Colliard (2019) or Carletti, Dell'Ariccia and Marquez (2020)). That is, it could be that a local supervisor observes s but the central supervisor only observes m. Alternatively, one could think of s as being complex, technical information, which the supervisor may be unable to use in its decision process, possibly because it is distributed across technicians (e.g., several derivatives pricing experts) whose information is not aggregated other than in the public signal m and who have no further influence on the supervisor's decision.

Once m is publicly communicated, the speculator can decide whether to acquire a private signal about ω.[2] Assume that he can neither directly observe s, nor whether the supervisor has observed s. If the speculator pays a cost c he receives a signal z, which fully reveals ω with a probability $\sigma \in (0,1]$ and is completely uninformative otherwise:

$$z = \begin{cases} \omega \text{ with probability } \sigma \\ \varnothing \text{ with probability } (1-\sigma) \end{cases}. \tag{8}$$

The speculator can then trade in the bank's shares at date 2. The market mechanism is based on Kyle (1985): the speculator can submit a market order to a risk neutral market maker. In addition to the speculator, there is a liquidity trader who buys or sells with equal probability a quantity n. The liquidity trader can be thought of as trading for non-information related reasons, such as taxes, consumption needs, insurance and hedging etc. The market maker observes each order separately but cannot tell which originates from the liquidity trader and which from the informed speculator. The market maker then sets a price P so as to break even in expectation.

Still at date 2, but after having observed m, s_{sup}, and P, the supervisor can choose whether or not to intervene in the bank. We denote the corresponding action by $a \in \{0,1\}$ where $a = 1$ means the supervisor allows the bank to continue without intervention. If the bank is allowed to continue, its operations generate cash flows at date 3, which can be high, R, medium, r, or low, 0 with $R > r > 0$. The banker can choose whether to act prudently or recklessly. If the banker is prudent, the probabilities of the respective outcomes are $\Pr(R) = \frac{p}{2}$, $\Pr(r) = 1-p$ and $\Pr(0) = \frac{p}{2}$. If the banker is reckless, the probabilities change to $\Pr(R) = \frac{p+\varepsilon}{2}$, $\Pr(r) = 1-(p+\varepsilon)$ and $\Pr(0) = \frac{p+\varepsilon}{2}$, where $\varepsilon \in (0, 1-p)$

[2]Our assumption that a speculator can learn something from the stress test results is in line with empirical evidence showing that stock prices react to stress test outcomes (Petrella and Resti, 2013, and Georgescu et al., 2017).

	Probability of outcomes	
Future cash flows	*Prudent*	*Reckless*
R	$\frac{p}{2}$	$\frac{p+\varepsilon}{2}$
r	$1-p$	$1-(p+\varepsilon)$
0	$\frac{p}{2}$	$\frac{p+\varepsilon}{2}$

We make several parametric assumptions, which are aimed at capturing the following: the banker who acts on behalf of shareholders behaves prudently as long as the bank's balance sheet is in good health (assets are worth A_h), but engages in risk shifting when its balance sheet deteriorates (assets are worth $A_l = 0$).

First, assume that the bank can repay its debt, if it generates cash flow R, regardless of the value of assets in place, i.e.,

$$R > D. \tag{9}$$

Second, assume that when cash flows are medium (r), the bank cannot repay its debt when assets have a low value $A_l = 0$, i.e.,

$$D > r. \tag{10}$$

The two inequalities (9) and (10) imply that a poorly capitalized bank (i.e., in state $\omega = l$) behaves recklessly. To see this, compare the expected equity value from being prudent $\frac{p}{2}(R - D)$ to that from being reckless $\frac{p+\varepsilon}{2}(R - D)$, the latter obviously being higher.

Moreover, we make the following parametric assumption, which ensures that a well capitalized bank chooses to behave prudently:

$$2r + A_h - R \geq D. \tag{11}$$

Note that (11) implies

$$\frac{p}{2}(R + A_h - D) + (1-p)(r + A_h - D) \geq \frac{p+\varepsilon}{2}(R + A_h - D) + (1-(p+\varepsilon))(r + A_h - D),$$

which means equity value is higher from being prudent (left-hand side of the inequality) than from being reckless. Inequality (11) also implies $r + A_h - D \geq R - r > 0$, so that a well capitalized bank can repay its debt when the medium cash flow r is realized.

Intervention ($a = 0$) is intended to limit the risk taking activities of the bank. Here, we assume that intervention takes the form of arranging an acquisition of the bank's ongoing activities by a better capitalized bank who will not engage in risk shifting. Alternatively, one could assume that the bank is liquidated following an intervention. We assume that the value of the bank's ongoing activities is reduced by the intervention so that the expected cash flow generated after an intervention is

$$\delta\left(\frac{p}{2}R + (1-p)\,r\right),$$

where $\delta < 1$. The loss in value can be thought of as stemming from the new owner's inferior ability to manage the failed bank's assets, or from a fire-sale discount when liquidating the bank's operations (or any other costs associated with intervention). We assume, without loss of generality, that the value of assets in place, A_ω is not affected by the intervention. Hence, the bank's total value following an intervention is $\delta\left(\frac{p}{2}R+(1-p)\,r\right)+A_\omega$.

Assume that assets in place are not enough to repay debt D in full when a high-value bank is liquidated:

$$D>\delta\left(\frac{p}{2}R+(1-p)\,r\right)+A_h. \tag{12}$$

This implies that an intervention by the supervisor fully wipes out the bank's shareholders.

It follows from (11) and (12), that

$$2r>R, \tag{13}$$

which implies that expected cash flows are higher when the bank is prudent. Hence, the model captures a situation, where a bank that experiences an adverse shock to its balance sheet ($A_l=0$) becomes undercapitalized (excessively levered), generating incentives to gamble. Since the supervisor's intervention can prevent excessive risk taking by the bank, it will be crucial for her to identify if the bank's assets have a high or low value.

Note that our model is rich enough to capture a forced recapitalization as an alternative way to think about a supervisor's intervention. A forced recapitalization would wipe out (or severly dilute) existing shareholders and require a potentially costly subsidy by the supervisor, providing her with an incentive to intervene only in state $\omega=l$. We can also include a cost of an ex-post bank failure (such as through negative spillovers on other banks) in the supervisor's objective function without this changing our results. In Appendix B, we show how this can be done.

We can define by V_ω^a the bank's overall expected value (debt plus equity), conditional on the state ω and the supervisor's action a. If the bank is allowed to continue, we know from before, that risk shifting will follow in the low state, but not the high state. We therefore have

$$V_h^1 = \frac{p}{2}R+(1-p)\,r+A_h, \tag{14}$$

$$V_l^1 = \frac{p+\varepsilon}{2}R+(1-p-\varepsilon)\,r \tag{15}$$

and

$$V_h^0 = \delta\left(\frac{p}{2}R+(1-p)\,r\right)+A_h, \tag{16}$$

$$V_l^0 = \delta\left(\frac{p}{2}R+(1-p)\,r\right). \tag{17}$$

Denote by $\Delta V_h\equiv V_h^1-V_h^0$, the gain from allowing a well capitalized bank to continue instead of intervening. By our assumption $\delta<1$ this is positive, i.e., $\Delta V_h>0$. Similarly, denote by $\Delta V_l\equiv V_l^0-V_l^1$ the gain from intervening in a poorly capitalized bank instead of allowing it to continue. Since we are interested in the case where intervention is valuable when the bank is poorly capitalized, we require $\Delta V_l>0$,which boils down to the parametric assumption

$$\delta>1-\frac{\frac{\varepsilon}{2}(2r-R)}{\frac{p}{2}R+(1-p)\,r}. \tag{18}$$

There is a lower bound on δ, because an intervention is costly. This cost needs to be balanced against the gain from curbing excessive risk taking by the bank, which is captured by $\frac{\varepsilon}{2}(2r-R)$. As ε increases, the risk shifting problem gets more severe, and hence the value from intervening increases, lowering the bound on δ that still leaves an intervention optimal.

3 Benchmark without a speculator

As a first step in the analysis it is useful to consider what the optimal degree of leniency s^* would be in the absence of a potentially informed speculator. This benchmark would apply to banks that are not listed and therefore the price as an information channel is absent. Suppose therefore that the supervisor chooses s^* so as to maximize the bank's *ex ante* expected value, knowing that the supervisor's information set will consist of m, and, potentially, s.

If the supervisor ends up observing s directly, the signal m contains no additional information. In this case, it is optimal for the supervisor to allow the bank to continue if and only if

$$\begin{aligned} & \Pr(\omega = h|s)\, V_h^1 + [1 - \Pr(\omega = h|s)]\, V_l^1 \\ \geq\; & \Pr(\omega = h|s)\, V_h^0 + [1 - \Pr(\omega = h|s)]\, V_l^0. \end{aligned}$$

Calculating $\Pr(\omega = h|s)$ by Bayesian updating, using (5) yields

$$\Pr(\omega = h|s) = s,$$

and therefore the supervisor's optimal policy is to allow the bank to continue if and only if

$$s \geq \widehat{s} \equiv \frac{\Delta V_l}{\Delta V_h + \Delta V_l}. \tag{19}$$

Using (14) to (17) the cut-off $\widehat{s}$ can be re-written as

$$\widehat{s} = 1 - (1-\delta)\, \frac{\frac{p}{2}R + (1-p)\, r}{\frac{\varepsilon}{2}(2r-R)}. \tag{20}$$

Using assumption (18), it follows that $\widehat{s} \in (0,1)$. We observe directly that $\hat{s}$ increases in both δ and ε. Intuitively, the supervisor is more prone to intervening when the bank's liquidation value is higher (δ is higher) and when the risk-shifting problem is more severe (a higher ε), implying that more value gets destroyed by allowing a bad bank to continue. This renders a more conservative intervention policy optimal, i.e., it "raises the bar" that will induce the supervisor to allow the bank to continue.

When the supervisor does not observe s, i.e., when $s_{\text{sup}} = \varnothing$, she needs to rely on m for her intervention decision. We first determine the optimal intervention decision, given the observed signal m, and then study the ex ante optimal choice of s^*. In principle, three intervention policies are possible. There are two uncontingent policies (always intervene or never intervene) and one contingent policy (intervene if and only if $m = fail$). If the degree of leniency is such that an unconditional policy would be *ex post* optimal, then the signal m would be altogether useless. It is therefore obvious that in the benchmark, an optimal cut-off will induce a conditional intervention policy.

It is useful to characterize the regions of the cut-off s^* such that the intervention policy of the supervisor will actually be contingent on the signal m. In order to highlight this region, we state the following straightforward result.[3]

Lemma 1 *In the absence of market information, the supervisor's intervention policy is contingent on m, if and only if*

$$2\widehat{s} - 1 \leq s^* \leq 2\widehat{s}. \tag{21}$$

For $s^* < 2\widehat{s} - 1$, the supervisor always intervenes and for $s^* > 2\widehat{s}$ the supervisor never intervenes, regardless of the realization of m.

Proof. Using the Bayesian updates calculated in (3) and (4) it is clear that intervention is optimal following a $fail$ signal if

$$\Pr(\omega = h | m = fail) = \frac{s^*}{2} \leq \widehat{s}.$$

On the other hand it is optimal not to intervene following a $pass$ signal if

$$\Pr(\omega = h | m = pass) = \frac{1 + s^*}{2} \geq \widehat{s}.$$

Rearranging the two inequalities yields (21). ■

In other words, the supervisor's intervention policy will be signal-contingent if the cut-off s^* satisfies (21). Note that for $\widehat{s} < \frac{1}{2}$ this effectively reduces to the interval $s^* \in [0, 2\widehat{s})$ and for $\widehat{s} \geq \frac{1}{2}$ to $s^* \in [2\widehat{s} - 1, 1]$. We can thus denote the lower and upper bounds on s^* such that intervention is contingent on the test result, by $\underline{s} \equiv \max\{0, 2\widehat{s} - 1\}$ and $\overline{s} \equiv \min\{2\widehat{s}, 1\}$, respectively. Intuitively, $\widehat{s} < \frac{1}{2}$ corresponds to the case where, in the absence of any signal m, it is optimal not to intervene. Hence, no intervention remains optimal after a *pass* signal, regardless of the choice of s^*, while a $fail$ signal will induce intervention as long as $s^* < 2\widehat{s}$. When $s^* \geq 2\widehat{s}$ the $fail$ signal is so uninformative that the supervisor will ignore it and allow the bank to continue. Conversely, when $\widehat{s} > \frac{1}{2}$, it is optimal to intervene when nothing further is known about the bank, so that intervention always remains optimal after a $fail$ signal. A *pass* signal prevents the supervisor from intervening when it is sufficiently informative ($s^* \geq 2\widehat{s} - 1$) but not otherwise.

The following Lemma characterizes the optimal cut-off s^*.

Lemma 2 *When there is no speculator, the optimal stress test features $s^* = \widehat{s}$.*

Proof. Assume there is no stock market. If the supervisor observes $s_{\text{sup}} = s$, the location of s^* is irrelevant. If the supervisor does not observe s, the public signal m is used to take a decision. Clearly an s^* that is optimal will induce a signal contingent intervention decision (otherwise m is completely useless) so that $a = 1$ following $m = pass$ and $a = 0$ otherwise.

[3] Suppose in case of indifference, the supervisor follows the public signal. This assumption is immaterial.

Using the Bayesian updates we can calculate expected bank value as a function of the cut-off s^* (the index $\varnothing$ indicates that the speculator does not produce any information)

$$\begin{aligned} V_{\varnothing}\left(s^* | s_{\text{sup}} = \varnothing\right) &= \frac{1}{2}\left[\left(1-(s^*)^2\right)V_h^1 + (s^*)^2 V_h^0\right] \\ &\quad + \frac{1}{2}\left[(1-s^*)^2 V_l^1 + s^*\left(2-s^*\right)V_l^0\right]. \end{aligned} \tag{22}$$

Since s^* is immaterial when $s_{\text{sup}} = s$, the supervisor chooses s^* to maximize (22). Taking the first-order condition and solving with respect to s^* yields $s^* = \widehat{s}$. ■

The result of Lemma 2 is very intuitive. Without an informed speculator, the only source of information for the supervisor is the signal generated by the monitoring technology. The optimal cut-off therefore maximizes the information content of that signal. The latter is most informative when knowing the s which generated the signal would lead to the same decision as observing the signal m only. This is the case when $s^* = \widehat{s}$.

Note that the optimal stress test generates different probabilities of type I and type II errors, corresponding to the economic cost of each of the errors. For example, when $\Delta V_h > \Delta V_l$, it is more costly to intervene in a high value bank (type I error), then to allow a low value bank to continue (type II error) and from (19), $\widehat{s} < \frac{1}{2}$. The probability of making a type I error is $\Pr\left(m = fail | \omega = h\right) = (s^*)^2$, while the probability of a type II error is $\Pr\left(m = pass | \omega = l\right) = (1-s^*)^2$. Clearly, if $s^* = \widehat{s} < \frac{1}{2}$, then, at the optimal policy, the probability of a type I error is lower than that of a type II error. The stress test therefore exhibits some bias towards leniency in that it generates more *pass* signals than *fail* ones.

We now want to focus on the case, where the presence of a stock market as a potential information channel may generate a bias in the supervisor's stress test design. We therefore define a stress test as being conservative (lenient), if it is more conservative (lenient) than the benchmark we just derived, i.e., conservatism corresponds to $s^* > \widehat{s}$ and leniency to $s^* < \widehat{s}$.

4 Supervisor has no private signal ($\theta = 0$)

We now analyze the case when there is a speculator. We start with the extreme case where the supervisor never has any private information. Doing so allows us to highlight the main economic mechanism that leads the supervisor to distort the optimal stress test towards leniency.[4] In Section 5 we study the general case $\theta \geq 0$.

In the first step we need to examine how the speculator's decision to acquire costly information depends on the supervisor's stress test design. This requires a characterization of the trading subgame and associated trading profits for the speculator. Since the speculator trades the bank's shares, we need to determine the bank's equity valuation, depending on the various combinations of the state ω and the supervisor's action a.

We denote by E_ω^a the bank's equity value that corresponds to the state of the world ω and the supervisor's decision a. From assumption (12) it is obvious that, if the supervisor

[4] The case analyzed here also corresponds to a situation in which the supervisor commits to following the stress test outcome, no matter what her private information. Such a commitment can be valuable if a supervisor would otherwise engage in forbearance.

intervenes,

$$E_h^0 = E_l^0 = 0.$$

This captures the idea that a regulatory intervention will generate a bail-in and drastically reduce the value of equity, in our case, all the way down to zero.[5] If the supervisor allows the bank to continue, equity value is given by

$$E_h^1 = \frac{p}{2}(R + A_h - D) + (1-p)(r + A_h - D), \tag{23}$$

$$E_l^1 = \frac{p+\varepsilon}{2}(R - D). \tag{24}$$

The value of equity under continuation depends on the state ω in two ways. First, the value of assets in place A_ω is higher in state $\omega = h$, lifting the value of equity. Second, the banker decides to be prudent in state $\omega = h$ but is reckless in state $\omega = l$. Gambling in the low state actually increases the value of equity as it expropriates creditors. We define $\Delta E^1 \equiv E_h^1 - E_l^1$ which can be re-written using (23) and (24)

$$\Delta E^1 = \left(1 - \frac{p}{2}\right) A_h - (1-p)(D - r) - \frac{\varepsilon}{2}(R - D).$$

It is easy to verify that our parametric assumptions (namely $1 - p - \varepsilon > 0$ and (11)) imply $\Delta E^1 > 0$. The following proposition presents the equilibrium and associated profits of the trading sub-game.

Proposition 1 *Suppose the speculator pays the information acquisition cost c. If he receives an uninformative signal $z = \varnothing$ he does not trade. He buys n units after observing $z = h$, and sells n units after observing $z = l$.*

Expected trading profits conditional on a positive public signal $m = pass$ are given by $n\pi_{pass}$, where

$$\pi_{pass} = \begin{cases} \frac{\sigma}{4}\Delta E^1 \left(1 - s^{*2}\right) & if \quad s^* \geq 2\widehat{s} - 1 \\ 0 & otherwise \end{cases}. \tag{25}$$

Expected trading profits conditional on a negative public signal $m = fail$ are given by $n\pi_{fail}$, where

$$\pi_{fail} = \begin{cases} \frac{\sigma}{4}\Delta E^1 s^* \left(2 - s^*\right) & if \quad s^* > 2\widehat{s} \\ 0 & otherwise \end{cases}. \tag{26}$$

Proof see Appendix.

The trading strategy is intuitive: the speculator buys on positive information, sells on negative information and does not trade when he is uninformed.

First, note that order flow is either fully revealing (when (n, n) or $(-n, -n)$) or entirely uninformative (when $(-n, n)$). When order flow is fully revealing, the supervisor ignores the public signal m and follows the information revealed through the trading process. Because order flow fully reveals ω the speculator cannot make a trading profit when order flow is either (n, n) or $(-n, -n)$. When order flow is uninformative, the supervisor

[5] Note that if one were to assume that the supervisor's intervention generates a liquidation value which is independent of ω, perhaps because the government nationalizes the bank at terms that are independent of ω, then we would directly get $E_h^0 = E_l^0 > 0$. This would give us exactly the same results, even though equity would only be partially wiped out following an intervention.

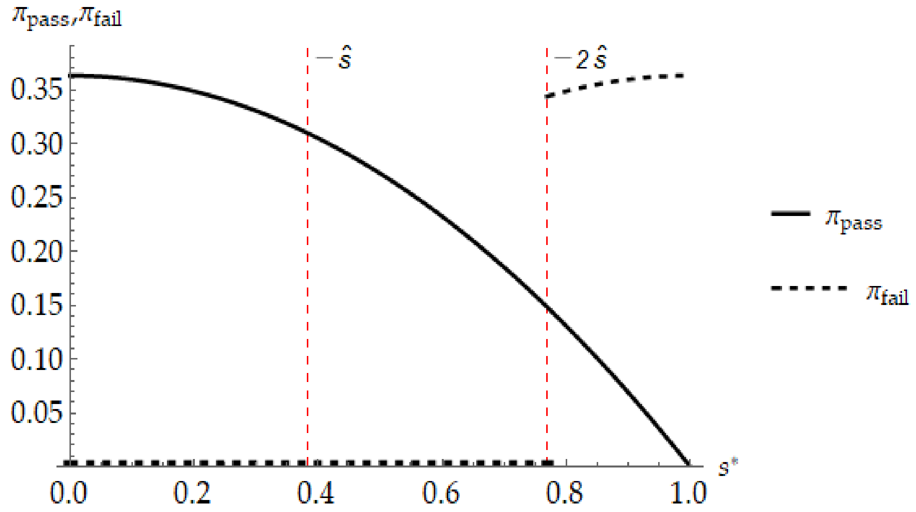

Figure 1: This Figure shows the speculator's expected trading profits, conditional on observing $m = pass$ (downward sloping line) or $m = fail$ (dashed line) as a function of the cutoff s^*. Parameter values are $A_h = 2.5$, $r = 7.5$, $R = 9$, $D = 8.15$, $p = 0.57$, $\epsilon = 0.13, \delta = 0.9585$.

makes her intervention decision contingent on m only. Expected trading profits therefore differ, depending on whether the public signal m conveys good or bad information, i.e., whether the bank's stress test generated a *pass* or a *fail* verdict.

Consider first trading profits following a *pass* signal. When $s^* \geq 2\widehat{s} - 1$ (and the supervisor learns nothing from the stock price), the supervisor will not intervene. The range of the bank's equity values is therefore ΔE^1 to which the speculator's profits are proportional.

When $s^* < 2\widehat{s} - 1$, the supervisor who learns $m = pass$ would ignore that information and intervene anyway. In that case the debtholders claim the liquidation value $\delta\left(\frac{p}{2}R + (1-p)\,r\right) + A_\omega$ and leave nothing to the equityholder. Speculators thus cannot make any trading profit. The supervisor's intervention eliminates all equity risk and thereby the value of the speculator's private information.

Note also that for s^* above $2\widehat{s} - 1$, expected trading profits are decreasing in s^*. This is because the information content of a *pass* signal is higher when s^* increases: the more conservative the stress test, the less likely is it that a *pass* signal will emerge. But if it does emerge, the signal $m = pass$ is a stronger indication that the true state is $\omega = h$. This, however, reduces the speculator's informational advantage vis-a-vis the market maker and therefore his expected trading profits.

Trading profits following the *fail* signal display some important differences. If the supervisor actually follows the *fail* signal and intervenes (whenever $s^* \leq 2\widehat{s}$), then expected trading profits are zero, i.e., $\pi_{fail}\left(s^* \leq 2\widehat{s}\right) = 0$. Trading profits can only be positive when the *fail* signal has become so uninformative that the supervisor chooses to ignore it and allow the bank to continue regardless (when $s^* > 2\widehat{s}$). In that region, trading profits are increasing in s^*, because the *fail* signal gets less and less informative when the stress test becomes increasingly conservative.

We can now determine the optimal stress test design. From the above, three broad options emerge. In the first two, the monitoring technology is chosen, such that it is uncontingent in the absence of information contained in the stock price (either always intervene, or never intervene). In the benchmark, such a policy was clearly inferior to a monitoring technology that induces a signal contingent intervention decision. This, however, cannot be taken for granted in the presence of a potentially informative stock price, since one needs to understand the effect of an uncontingent policy on information production by speculators. As the Proposition below shows, the direct positive effect of a contingent monitoring policy dominates, so that we can rule out uncontingent policies and focus on the interval $s^* \in [\underline{s}, \overline{s}]$.

Moreover, expected bank value depends on the information contained in stock prices. Denote by $\mathcal{I} \in \{\varnothing, \{pass\}, \{fail\}, \{pass, fail\}\}$ the public signals following which the speculator acquires information. If the speculator produces and trades on information following the *pass* signal and $s^* \in [\underline{s}, \overline{s}]$, the expected value of the bank can be written as follows[6]:

$$\begin{aligned} V_{\{pass\}}\left(s^* < \widehat{s}\right) &= \frac{1}{2}\left\{\left(1-\left(s^*\right)^2\right)V_h^1 + \left(s^*\right)^2 V_h^0\right\} \\ &\quad + \frac{1}{2}\left\{s^*\left(2-s^*\right) + \frac{\sigma}{2}\left(1-s^*\right)^2\right\}V_l^0 \\ &\quad + \frac{1}{2}\left(1-s^*\right)^2\left(1-\frac{\sigma}{2}\right)V_l^1. \end{aligned} \tag{27}$$

The maximum of the above is reached at

$$s^{\max} = \frac{\left(1-\frac{\sigma}{2}\right)\widehat{s}}{1-\frac{\sigma}{2}\widehat{s}}. \tag{28}$$

It can easily be verified that $s^{\max} \in (2\widehat{s}-1, \widehat{s})$. Moreover, we define $s_{pass}^{\varnothing}$ as the smaller of the two solutions[7] to

$$V_{\{pass\}}\left(s^* = s_{pass}^{\varnothing}\right) = V_{\varnothing}\left(\widehat{s}\right),$$

where $V_{\varnothing}(\widehat{s})$ is given by (22)[8]. $s_{pass}^{\varnothing}$ is thus the value of s^* that makes the supervisor indifferent between having information provided by the stock market (after the *pass* signal), or using the benchmark monitoring technology without any information from the stock market. $s_{pass}^{\varnothing}$ can be interpreted as the maximum distortion the supervisor is willing to accept so as to induce speculator information production following the *pass* signal. We can calculate

$$s_{pass}^{\varnothing} = \widehat{s}\frac{\left[\left(1-\frac{\sigma}{2}\right)-\left(1-\widehat{s}\right)\left(\frac{\sigma}{2\widehat{s}}\right)^{\frac{1}{2}}\right]}{1-\frac{\sigma}{2}\widehat{s}}. \tag{29}$$

As σ approaches 1, $s_{pass}^{\varnothing}$ can be below $\underline{s}$ which corresponds to the case when $V_{\{pass\}}(\underline{s}) > V_{\varnothing}(\widehat{s})$, i.e., the supervisor prefers to distort s^* to the maximum (its lower bound $\underline{s}$) rather than forgo information production by the speculator.

[6] We show in the proof of Proposition 3 that $V_{\{pass\}}(s^* < \widehat{s}) > V_{\{pass\}}(s^* \geq \widehat{s})$.

[7] Clearly, $V_{pass}(\widehat{s}) > V_{\varnothing}(\widehat{s})$ and therefore there are two points s^* where $V_{pass}(s^*) = V_{\varnothing}(\widehat{s})$. Since the supervisor is only willing to distort the signal in order to induce information production by the speculator whose trading profits $\pi_{pass}(s^*)$ are decreasing in s^*, we are only interested in the smaller solution.

[8] When $\theta = 0$, the expected bank value under $\mathcal{I} = \varnothing$ is the same as $V_{\varnothing}(\widehat{s}|s_{\text{sup}} = \varnothing)$.

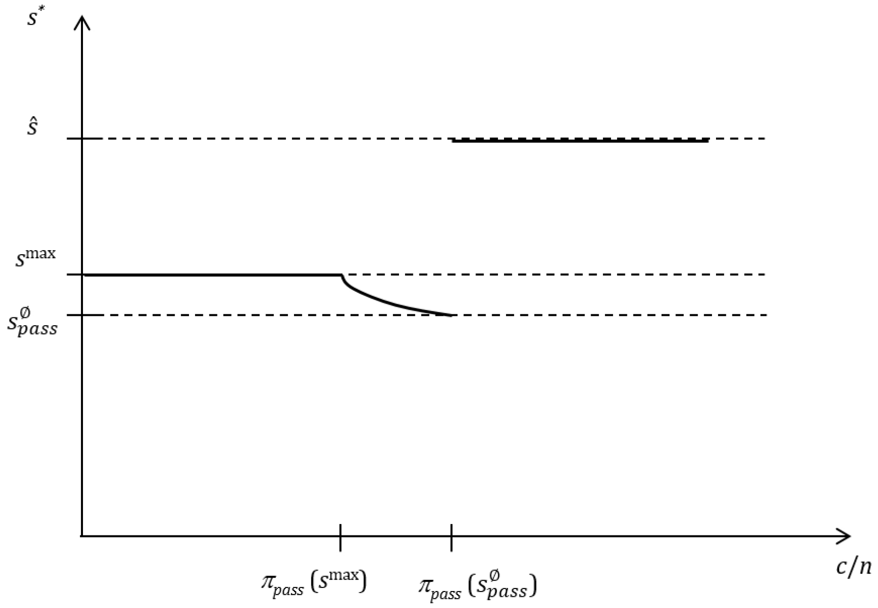

Figure 2: This Figure plots the optimal stress test s^* as a function of the ratio of information production cost over liquidity trader volume.

From the definitions of $s^{\max}$ and $s^{\varnothing}_{pass}$ it is clear that $s^{\varnothing}_{pass} < s^{\max}$. The following provides the supervisor's optimal choice of stress test.

Proposition 2 *(i) If* $\frac{c}{n} \leq \pi_{pass}\left(s^{\max}\right)$*, then* $s^* = s^{\max} < \widehat{s}$*.*

(ii) If $\pi_{pass}\left(s^{\max}\right) < \frac{c}{n} \leq \pi_{pass}\left(\max\left\{\underline{s}, s^{\varnothing}_{pass}\right\}\right)$ *, then* $\underline{s} \leq s^* < \widehat{s}$ *and* s^* *is given by the solution to* $\pi_{pass}\left(s^*\right) = \frac{c}{n}$

(iii) If $\pi_{pass}\left(\max\left\{\underline{s}, s^{\varnothing}_{pass}\right\}\right) < \frac{c}{n}$*, then* $s^* = \widehat{s}$*.*

Proof see Appendix.

First, note that the optimal stress test never induces information acquisition by the speculator following the $fail$ signal. Although the supervisor could, for some values of $\frac{c}{n}$, induce the production of speculative information by adopting a very conservative $(s^* > 2\widehat{s})$ or a very lenient $(s^* < 2\widehat{s} - 1)$ policy, this has a downside: it generates a direct loss of information from rendering the stress test very uninformative - as a matter of fact, so uninformative that the supervisor would ignore the stress test result entirely, unless the stock price reveals the bank's true asset value.

Second, the supervisor optimally distorts the monitoring policy towards leniency, i.e., $s^* < \widehat{s}$, unless information acquisition is too costly. The benefit of leniency can be understood from (27). A more lenient policy increases the *ex ante* likelihood that a *pass* signal will be generated. Since the speculator only produces information following the *pass* signal, a more lenient policy increases the likelihood that the speculator will produce information. Of course, this gain has to be traded off against the loss of direct information conveyed to the supervisor by the public signal. This loss manifests itself in too little intervention, namely when $\omega = l$ but $m = pass$, and order flow ends up being uninformative. The optimal cut-off is thus determined as an internal solution.

This effect drives leniency in region (i), i.e., when $\frac{c}{n} \leq \pi_{pass}(s^{\max})$. When $\frac{c}{n}$ increases above $\pi_{pass}(s^{\max})$ the supervisor may increase leniency even more, because it increases the trading profits the speculator can make. That is, a further distortion towards leniency occurs, because it allows the supervisor to keep the speculator in the market. However, at some point the distortion becomes so large that the supervisor prefers not to distort the stress test any further (when $s^* = s^{\varnothing}_{pass}$), or where any further distortion would render the *pass* signal useless ($s^* < \underline{s}$). At that point (region (iii)), there is no longer any reason to distort the stress test and the benchmark policy becomes optimal.

We thus expect leniency to be prevalent when n is large, i.e., when the bank's shares are fairly liquid, and when c is not too large. Moreover, if one measures the degree of leniency through $s^{\max}$, then leniency increases ($s^{\max}$ drops) when σ increases. That is, the more likely it is that the speculator's information production will generate useful information, the more it pays the supervisor to distort the stress test towards leniency.

5 The general case ($\theta \geq 0$)

We now analyze the general case where there is a speculator and the supervisor may have private information with a probability $\theta > 0$. We begin by characterizing the speculator's trading profits.

Lemma 3 *The speculator's trading strategy is described in Proposition 1. Expected trading profits are $n\pi_{pass}(s^*)$ where*

$$\pi_{pass}(s^*) = \begin{cases} \frac{\sigma}{4}\Delta E^1\left(1-s^{*2}\right) & \text{if } \widehat{s} \leq s^* \\ \frac{\sigma}{4}\left\{\Delta E^1\left[\left(1-s^{*2}\right)-\theta\left(\hat{s}^2-s^{*2}\right)\right]+2\theta\frac{(\hat{s}-s^*)(1-\hat{s})}{1-s^*}E^1_l\right\} & \text{if } 2\widehat{s}-1 \leq s^* \leq \widehat{s} \\ \frac{\sigma}{4}\theta\left[\Delta E^1\left(1-\widehat{s}^2\right)+2\frac{(\widehat{s}-s^*)(1-\widehat{s})}{1-s^*}E^1_l\right] & \text{if } s^* < 2\widehat{s}-1. \end{cases} \tag{30}$$

and

$$\pi_{fail}(s^*) = \begin{cases} \frac{\sigma}{4}\left(1-\frac{\hat{s}}{s^*}\right)\left\{\Delta E^1\left(2-s^*\right)\frac{s^{*2}-\theta\hat{s}^2}{(s^*-\hat{s})}+2\theta\hat{s}E^1_l\right\} & \text{if } 2\widehat{s} < s^* \\ \frac{\sigma}{4}\theta\left(1-\frac{\hat{s}}{s^*}\right)\left\{\Delta E^1\left(2-s^*\right)\left(s^*+\hat{s}\right)+2\hat{s}E^1_l\right\} & \text{if } \widehat{s} \leq s^* \leq 2\widehat{s} \\ 0 & \text{if } s^* < \widehat{s} \end{cases}. \tag{31}$$

Proof see Appendix.

Note that (per unit) trading profits π_{fail} are increasing in s^* and continuous, except in the point $s^* = 2\widehat{s}$ where π_{fail} jumps upwards, just like in the case previously analyzed. The speculator, however, can now make positive profits $\pi_{fail} > 0$ when $\widehat{s} \leq s^* \leq 2\widehat{s}$. This happens for the following reasons. When the stress test is conservative $(s^* > \widehat{s})$, a failed stress test is no longer a perfect predictor of a supervisory intervention: when the bank marginally fails a conservatively designed test and the supervisor knows that the failure was indeed only marginal, i.e., when $s_{\text{sup}} = s \in (\widehat{s}, s^*)$, then the supervisor optimally refrains from intervening. When that happens, the speculator can trade profitably. The higher the probability θ of such information, the higher the probability that the supervisor ignores a failed stress test, which increases the speculator's profit from trading in the bank's shares. As we will show below, this effect can induce the supervisor to adopt a

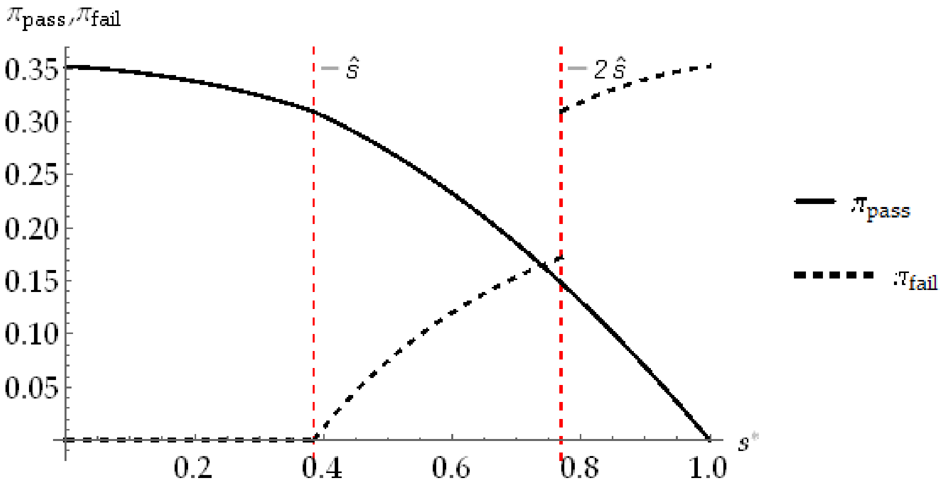

Figure 3: This Figure shows trading profits following a signal $m = pass$ (downward sloping line) and $m = fail$ (dashed line). Parameter values are the same as in Figure 1, except $\theta = 0.6$ instead of $\theta = 0$.

conservative stress test. By choosing a cut-off s^* above $\widehat{s}$ (and below $2\widehat{s}$) the speculator can now be induced to produce costly information following a $fail$ test, which was not the case when $\theta = 0$. It turns out that this will be important for the supervisor as it allows her, under some parameter values, to induce information production by the speculator following both, a passed or failed stress test.[9]

Trading profits π_{pass} following a positive signal $pass$ are decreasing in $s^* \in [\underline{s}, \overline{s}]$, which captures the, by now familiar, effect that a higher s^* reduces the speculator's informational advantage when the public has learnt $m = pass$. For s^* between $\underline{s}$ and $\widehat{s}$, an additional and opposing effect emerges. When $s_{\text{sup}} = s \in [s^*, \hat{s})$, the supervisor learns that a bank only passed the (lenient) stress test marginally and optimally intervenes - an action that reduces trading profits. An increase in θ increases the likelihood that the supervisor privately observes a counter-indication to the public signal $m = pass$ and π_{pass} is therefore decreasing in θ when $\underline{s} \leq s^* < \widehat{s}$. A jump occurs in π_{pass} in the point $s^* = 2\widehat{s} - 1$. For $s^* \geq 2\widehat{s} - 1$, the supervisor allows continuation after $m = pass$, unless $s_{\text{sup}} \in [s^*, \widehat{s})$. For $s^* < 2\widehat{s} - 1$ she always intervenes after $m = pass$ unless $s_{\text{sup}} \in [\widehat{s}, 1]$.

We can now state the main result on the optimal stress test design.

Proposition 3 *The supervisor's optimal stress test s^* is set as follows.*

(a) There is a threshold C_{con} such that for $\frac{c}{n} \in (0, C_{con}]$, the supervisor employs a conservative test design $s^* > \widehat{s}$ and the speculator produces information following both $pass$ and $fail$ tests. If $\theta > 0$ then $C_{con} > 0$ (and for $\theta = 0$, $C_{con} = 0$).

(b1) If $\frac{c}{n} \in (C_{con}, C_{\varnothing}]$, and $\widehat{s} < \frac{1}{2}$, then the optimal test design is lenient $s^* < \widehat{s}$, and the speculator only produces information following a pass test.

[9] When $s^* > 2\widehat{s}$, an increase in θ reduces trading profits following a $fail$ signal, because the supervisor may now deviate from the default of not intervening, to intervening when the private information is sufficiently negative. This effect, however, turns out not to be relevant, as such an extreme distortion is never optimal (see Proposition 3 below).

(b2) If $\frac{c}{n} \in (C_{con}, C_{\varnothing}]$, and $\widehat{s} > \frac{1}{2}$, the optimal test design may be conservative or lenient, depending on parameter values and the speculator produces information following a pass (fail) test when the policy is lenient (conservative).

(c) If $\frac{c}{n} > C_{\varnothing}$, then $s^* = \widehat{s}$.

Proof see Appendix.

As before, we can define by $V_{\{pass\}}(s^*)$ the expected bank value if the speculator produces information following $m = pass$ only and by $s^{\max}_{pass} < \widehat{s}$ the stress test design that maximizes expected bank value conditional on this behaviour. This turns out to be a lenient stress test, just like before. For low enough $\frac{c}{n}$ the supervisor can alternatively induce information production following either signal by distorting the stress test towards conservatism. Although the bank value $V_{\{fail,pass\}}(s^*)$ is maximized at $s^* = \widehat{s}$, the supervisor will have to set $s^* > \widehat{s}$ in order to induce information production following the $fail$ signal. As $\frac{c}{n}$ increases, a stronger distortion towards conservatism is required so as to leave more trading profits following $m = fail$.

Eventually, as the required disortion increases, several possibilities emerge. When θ is relatively low, the supervisor quickly reaches a point where a severe distortion would be necessary to induce information production following $m = fail$. The supervisor may then prefer to switch to a lenient policy $s^* = s^{\max}_{pass} < \widehat{s}$ and accept that information only be produced following $m = pass$. This happens at a threshold which we denote by $s^{pass}_{pass,fail}$. When θ is relatively large, $\pi_{fail}(s^*)$ increases more strongly in s^*. There can therefore be distortions $s^* \in (\widehat{s}, 2\widehat{s})$, that only induce information production following $m = fail$, but not following $m = pass$. Faced with the choice of being able to induce either $\mathcal{I} = \{fail\}$ or $\mathcal{I} = \{pass\}$ but not both, the supervisor would prefer $\mathcal{I} = \{pass\}$ if and only if $\widehat{s} < \frac{1}{2}$.

For $\widehat{s} < \frac{1}{2}$, the optimal policy then resembles the one described in Proposition 2: the supervisor switches to a lenient policy when $\frac{c}{n}$ exceeds a threshold C_{con}. This is feasible for any value of $\frac{c}{n}$ that would have allowed to induce $\mathcal{I} = \{fail\}$ because $\pi_{pass}(0) > \pi_{fail}(2\widehat{s})$.

For $\widehat{s} > \frac{1}{2}$ the preferred option is $\mathcal{I} = \{fail\}$ at a distortion $s^{\max}_{fail} > \widehat{s}$ towards conservatism.[10], as the expected bank value $V_{\{fail\}}(s^*)$ reaches the maximum at $s^* = s^{\max}_{fail}$ when information production occurs following $m = fail$ only. This option, however, is not necessarily attainable since $\pi_{pass}\left(s^{\max}_{pass}\right) > \pi_{fail}\left(s^{\max}_{fail}\right)$. For low enough θ, $V_{\{pass\}}\left(s^{\max}_{pass}\right) > V_{\{fail\}}(1)$, and $\pi_{pass}\left(s^{\max}_{pass}\right) > \pi_{fail}(1)$ so that for increasing $\frac{c}{n}$ the supervisor switches towards a lenient stress test $s^* \in \left[s^{\varnothing}_{pass}, s^{\max}_{pass}\right]$ and then remains there until $\frac{c}{n}$ has increased so much that $\mathcal{I} = \varnothing$ and $s^* = \widehat{s}$ becomes optimal. For θ in an intermediate range, it is possible that the supervisor may switch back and forth between leniency and conservatism more than once. This can happens when $\pi_{fail}(1) > \pi_{pass}(2\widehat{s} - 1)$ and $\pi_{pass}\left(s^{\max}_{pass}\right) > \pi_{fail}\left(s^{pass}_{fail}\right)$, where s^{pass}_{fail} is the maximum distortion towards conservatism that the supervisor prefers before implementing the optimal lenient distortion $s^{\max}_{pass}$. For θ high enough, $\pi_{pass}(2\widehat{s} - 1) < \pi_{fail}(1)$, and the supervisor optimally distorts towards conservatism $s^* \in \left[\widehat{s}, \min\left\{s^{\varnothing}_{fail}, 1\right\}\right]$ until $\frac{c}{n}$ is so high as to render the distortion too costly and $s^* = \widehat{s}$ becomes optimal, foregoing market information ($\mathcal{I} = \varnothing$).

We summarize the comparative statics in Corollary 1 which follows directly by taking the derivatives of the corresponding expressions given in the Appendix.

[10]The optimal distortion $s^{\max}_{fail}$ is larger than $\widehat{s}$ because the supervisor benefits from generating a $fail$ signal more often since market information is only produced for banks that fail the stress test.

Corollary 1 *As θ increases the optimal distortions become more extreme, i.e., conservative thresholds increase* $\frac{\partial s^{pass}_{fail,pass}}{\partial\theta} \geq 0$, $\frac{\partial s^{\max}_{fail}}{\partial\theta} \geq 0$ *and* $\frac{\partial s^{\varnothing}_{fail}}{\partial\theta} \geq 0$ *and lenient thresholds* $\frac{\partial s^{\max}_{pass}}{\partial\theta} \leq 0$ *and* $\frac{\partial s^{\varnothing}_{pass}}{\partial\theta} \leq 0$ *decrease.*

Given an increase in θ, the supervisor is more likely to ignore the public signal as a source of information since she can more often observe s directly. This has several implications. First, under a conservative stress test, the supervisor is more likely to let the bank continue despite a negative signal, when θ is higher. This increases the speculator's trading profits and therefore allows the supervisor to induce more easily information production following the $fail$ signal. Second, the direct cost of distorting the stress test is lower, since the supervisor needs to rely less on the information it generates. Hence, the negative impact of a further distortion gets weaker when θ increases. Both effects work in the same direction to increase the thresholds for conservative distortions and decrease those for lenient distortions.

In the extreme case where the supervisor is always privately informed ($\theta = 1$), we get $s^{\max}_{pass} = \underline{s}$ and $s^{\max}_{fail} = \overline{s}$. That is, the supervisor would be willing to distort the stress test to the point where it no longer contains any directly useful information. This limiting case captures a situation where the supervisor always has private information and making the publicly observable stress test uninformative, becomes (weakly) optimal as this maximizes the speculator's trading profits.

Our results imply that the bank supervisor should be more conservative when a bank's shares are fairly liquid ($\frac{c}{n}$ small). If however bank shares are illiquid, a lenient policy helps to encourage information production in the financial market. Moreover, an increase in θ renders conservatism optimal for a larger parameter set. This is because for low values of $\frac{c}{n}$ it is always optimal to use a conservative stress test and as θ increases the first switching point where conservatism ceases to be optimal is reached at higher values of $\frac{c}{n}$. Some researchers have argued that national supervisors are better informed about their local banks than a supra-national supervisor. To the extent that this is the case, one should expect national supervisors to be more conservative than a supra national supervisor.[11] This provides a counterveiling incentive for national supervisors than that highlighted by Carletti et al. (2020) who argue that national supervisors are more inclined to engage in forbearance than a supra-national supervisor and therefore would prefer to hide bad news about local banks.

Although our analysis is normative in spirit, there are some empirical predictions we wish to highlight. First, we would expect the information content of bank's share prices to differ depending on whether a bank passed or failed a stress test. Typically, share prices will be more informative for banks that pass the test compared to those who fail (assuming that the degree of leniency is uniform across banks). The opposite may be true only when banks are *ex ante* likely to be subject to intervention ($\widehat{s} > \frac{1}{2}$) and the supervisor applies a strongly conservative test. The more conservative the test, the smaller we would expect the discrepancy in the information content of stock prices between banks that passed and banks that failed the test to be.

Moreover, we showed that whenever $\frac{c}{n} > 0$, an undistorted stress test ($s^* = \widehat{s}$) would result in no information production for banks that failed the test. Suppose $\widehat{s} > \frac{1}{2}$, so that

[11] However, supra national supervisors tend to deal with large banks, hence they will tend to have more liquid shares, so the sample is not quite comparable.

the supervisor would intervene in the bank in the absence of any further information. This implies that a bank that failed the test would be shut-down with certainty. Suppose instead that the supervisor applies a conservative stress test ($s^* > \widehat{s}$) so as to induce the production of market information for banks that failed the test. This implies that there is a chance for well capitalized banks that failed the test to have this information reflected in their stock price and thereby prevent intervention. A more conservative stress test can therefore, somewhat paradoxically, reduce the probability that a bank will be subject to intervention.[12]

Let us now consider the effect that an increase in ε, i.e., a worsening risk shifting problem, has. As explained in Section 3, an increase in ε makes the benchmark stress test more conservative, because allowing bad banks to continue would destroy more value. Note that the impact of ε on the various threshold values of s^* are fully captured via a change in $\widehat{s}$. In this sense, the risk shifting problem does not in itself affect the distortions away from the benchmark. However, ε affects expected trading profits:

Lemma 4 *Expected trading profits $\pi_{pass}(s^*)$ and $\pi_{fail}(s^*)$ decrease in ε when the stress test is conservative ($s^* > \widehat{s}$).*

When ε increases, risk shifting generates more value for equityholders. An increase in ε therefore reduces the sensitivity of equity value with respect to the underlying state of the world ($\frac{\partial \Delta E^1}{\partial \varepsilon} < 0$). This reduces trading profits and the speculator's incentives to acquire private information. When the stress test is lenient the previous argument is no longer sufficient, because $\pi_{pass}(s^*)$ not only depends on ΔE^1 but also, positively, on E_l^1. This is because the expected value of equity, conditional on $m = pass$, is now also affected by the supervisor's private information, which will sometimes lead her to intervene in spite of a *pass* test (when $s \in [s^*, \widehat{s}]$). Since the signal is informative, the supervisor is more likely to intervene when the bank is poorly capitalized ($\omega = l$) pushing expected equity value conditional on $\omega = l$ more strongly below E_l^1 than expected equity value conditional on $\omega = h$ is pushed below E_h^1. This effect ends up generating a counterveiling effect of ε on trading profits.[13]

On the other hand, the benefit ΔV_l of identifying a poorly capitalized bank and intervening in it, increases in ε.[14] The private value of information (to the speculator and equity holders) can therefore drop below the social value of information. It is well known that, in general, the private value of information can be larger or smaller than the social value of information.[15] Although our finding is in the same spirit, we identify a novel reason for the discrepancy: The claims that are traded (equity) do not reflect the social value of the information, which is measured by the impact on total bank value, i.e.,

[12] There is of course a counterveiling effect, namely that a more conservative test generates more fail results and the corresponding banks are subject to intervention when market prices reflect no information. However, for values of s^* close enough to $\widehat{s}$ this effect is dominated.

[13] In numerical simulations we found that $\pi_{pass}(s^* < \widehat{s})$ is decreasing in ε for most, but not all, parameter constellations.

[14] This is immediate from (15) and (17). Note that ΔV_h is independent of ε, since a well capitalized bank never engages in risk shifting.

[15] For example, information may have a speculative value because it allows trading profits, but the information may not help allocate resources more efficiently, in which case the private value is above the social value. However, the opposite may also happen, when speculators cannot make money (e.g., because prices reveal their information), although their information would help improve resource allocation.

debt plus equity. In the context of a risk shifting problem, the impact of information on equity value may be small, although its effect on total "firm" value may be large. This, of course, raises the question whether it would be better to derive market information by having another claim than equity be traded. We address this question in Section 6.2.

6 Extensions

6.1 Disclosure policy

We assume in our main analysis that the supervisor is committed to disclosing the stress test results. We now relax this assumption and explore the possible outcomes when the supervisor can choose whether to disclose stress test results. We assume that m is hard information, i.e., the supervisor can choose whether or not to disclose the realization of m, but she cannot pretend that a realization $m = fail$ was a *pass*, and vice versa. First, let us consider the case where the supervisor can choose *ex post* whether to disclose the signal m. Given that it is in the supervisor's interest to encourage information production in the stock market, she would naturally want to disclose a *pass* signal and (weakly) prefer not to disclose a $fail$ signal. Other players can then perfectly infer a $fail$ signal from the supervisor's action not to disclose. Consequently, the choice of *ex post* disclosure corresponds to always disclosing. The case analysed in the paper can therefore also be thought of as arising from an inability by the supervisor to commit not to disclose her information *ex post*.

Next, we consider what happens if the supervisor commits *ex ante* to either always or to never disclosing. We analyzed in Section 5 the general case in which the supervisor always discloses the test result, so we directly turn to the case of no disclosure. If the supervisor never discloses the signal m, the threshold s^* only affects direct learning by the supervisor, and she optimally sets $s^* = \hat{s}$ (see Lemma 2). Having no access to the test result (m), the speculator's strategy is no longer contingent on signal m. We denote his expected trading profit in this case by $\pi_{ND}(\hat{s})$ and provide its characterization.

Lemma 5 *If the supervisor never discloses the stress test result, and the speculator acquires and trades on information, his expected trading profits are*

$$\pi_{ND}(\hat{s}) = \frac{1}{4}\sigma\left[\left(1-\hat{s}^2\right)\Delta E^1 + 2\hat{s}\left(1-\hat{s}\right)E_l^1\right]. \tag{32}$$

If the cost of information production is low enough ($\pi_{ND}(\hat{s}) \geq \frac{c}{n}$), the speculator will produce and trade on information when the stress test result is not disclosed. In that case the supervisor gains nothing from disclosing the stress test result. Note that $\pi_{ND}(\hat{s}) > \pi_{pass}(\widehat{s})$, because no disclosure increases the informational advantage of the speculator vis-a-vis the market maker. The supervisor may therefore be worse off from disclosing the stress test result - at least for an undistorted stress test.

On the other hand, we know that $\pi_{pass}(s^*)$ can be increased by distorting the test towards leniency and this can generate $\pi_{pass}(s^*) > \pi_{ND}(\hat{s})$.[16] There may therefore be values of $\frac{c}{n} > \pi_{ND}(\hat{s})$, such that no market information would be available under a no

[16] There is, however, no value $s^* \in [\underline{s}, \overline{s}]$ such that $\pi_{fail}(s^*) > \pi_{ND}(\hat{s})$.

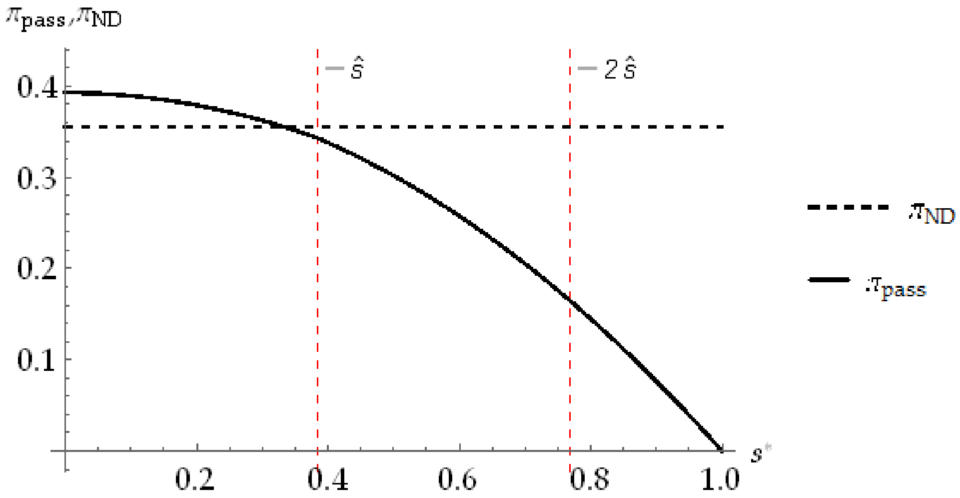

Figure 4: This Figure shows trading profits as functions of s^*. The solid line presents the trading profit conditional on a *pass* message while the dashed line shows the unconditional profit under the no-disclosure policy. We keep the same parameter values from the example used in Figure 2 except for A_h, D and θ, which are set at $A_h = 3$, $D = 8.7$ and $\theta = 0.2$. Note that the value of $\hat{s}$ remains unchanged.

disclosure policy, but it would be following a sufficiently lenient *pass* test. Figure 4 illustrates that distorting the stress test towards leniency and disclosing its result may generate enough trading profits so as to yield more market information.

Whether it is worth distorting (and disclosing) the stress test, or not distorting it and foregoing market information corresponds to one of the cases analysed in Proposition 3 (notably the threshold $s^{\varnothing}_{pass}$). This allows in principle a characterization of when committing not to disclose might or might not be optimal. From (32) we can see that $\pi_{ND}(\widehat{s})$ is a decreasing function of $\widehat{s}$ when $\widehat{s}$ is above $\frac{E_l^1}{E_h^1+E_l^1}(< \frac{1}{2})$. Thus, for banks that are more likely to require an intervention, it becomes harder to induce the production of market information without the disclosure of further information, making it more worthwhile to commit to disclosing. This finding is akin to Bouvard et al. (2015) or Williams (2017), although the underlying reasons are entirely different.

6.2 Debt trading

As shown above, the equity claim may become relatively insensitive to private information when the risk shifting problem is severe. In the bad state of nature, the lower asset value is somewhat compensated for from the perspective of equity holders, because the bank expropriates creditors by taking more risk. In this extension we allow the speculator to decide, after observing the stress test result m, to trade either a risky debt claim or an equity claim. To keep things simple, we assume that the speculator can only trade in one claim and thus has to decide whether to trade in debt or equity, based on the expected trading profit conditional on signal m. This assumption can be justified on the basis of position limits that might be imposed on the speculator, for example, because of financial constraints. We assume that a liquidity trader is present in the debt market and buys or sells quantities n_D with equal probability. The liquidity trader's demand is independent across the two markets.

We first determine the bank's debt valuation, depending on the state of the world ω as well as the supervisor's action. Following the notations previously used for equity value, we denote by D_ω^a the bank's debt value that corresponds to ω and the supervisor's decision a. If the supervisor intervenes, the debtholder collects a value $A_\omega + \delta \frac{p}{2} R + (1-p)\, r$.

$$\begin{aligned} D_h^0 &= \delta \left(\frac{p}{2} R + (1-p)\, r \right) + A_h \\ D_l^0 &= \delta \left(\frac{p}{2} R + (1-p)\, r \right) \end{aligned}$$

and thus

$$\Delta D^0 = D_h^0 - D_l^0 = A_h$$

If the supervisor allows the bank to continue, debt value is given by

$$D_h^1 = \frac{p}{2} D + (1-p)\, D + \frac{p}{2} A_h. \tag{33}$$

$$D_l^1 = \frac{p+\varepsilon}{2} D + (1-p-\varepsilon)\, r. \tag{34}$$

We denote the difference between D_h^1 and D_l^1 by ΔD^1 and write

$$\begin{aligned} \Delta D^1 &= D_h^1 - D_l^1 \\ &= (1-p)\,(D-r) + \frac{p}{2} A_h + \frac{\varepsilon}{2}\,(2r - D)\,. \end{aligned}$$

We observe immediately that debt value is always higher in the high state of the world, whether or not the intervention takes place, $\Delta D^0 > 0$ and $\Delta D^1 > 0$.[17] The debtholder collects a higher value A_h from the asset in place if the supervisor intervenes in the high state of the world. The speculator can thus profit from trading on his private information about the state ω even if the bank supervisor intervenes. If the supervisor allows the bank to continue, the debtholder benefits in the high state not only from a higher asset value but also from the banker's prudent behavior (no risk-shifting). We show the speculator's expected profit from trading in debt in the general case ($\theta > 0$):

Lemma 6 *The speculator's trading strategy is described in Proposition 1. Expected trading profits are* $n_D \pi_{D,pass}(s^*)$ *where*

$$\pi_{D,pass}(s^*) = \begin{cases} \frac{\sigma}{4} \Delta D^1 \left(1 - s^{*2}\right) & \textit{if} \quad \widehat{s} < s^* \leq 1 \\ \frac{\sigma}{4} \left\{ \begin{array}{c} \Delta D^1 \left[\left(1 - s^{*2}\right) - \theta \left(\hat{s}^2 - s^{*2}\right) \right] + \left(\hat{s}^2 - s^{*2}\right) \theta \Delta D^0 \\ -2\theta \frac{(\hat{s} - s^*)(1-\hat{s})}{1-s^*} \left(D_l^0 - D_l^1\right) \end{array} \right\} & \textit{if} \quad 2\widehat{s} - 1 \leq s^* \leq \widehat{s} \\ \frac{\sigma}{4} \left\{ \begin{array}{c} \theta \Delta D^1 \left(1 - \hat{s}^2\right) - 2\theta \frac{(\hat{s} - s^*)(1-\hat{s})}{1-s^*} \left(D_l^0 - D_l^1\right) \\ + \left[1 - s^{*2} - \theta \left(1 - \hat{s}^2\right) \right] \Delta D^0 \end{array} \right\} & \textit{if} \quad s^* < 2\widehat{s} - 1 \end{cases} \tag{35}$$

[17] Note that $D_l^0 > D_l^1$, i.e., creditors prefer the bank to be liquidated in the bad state, rather than be continued.

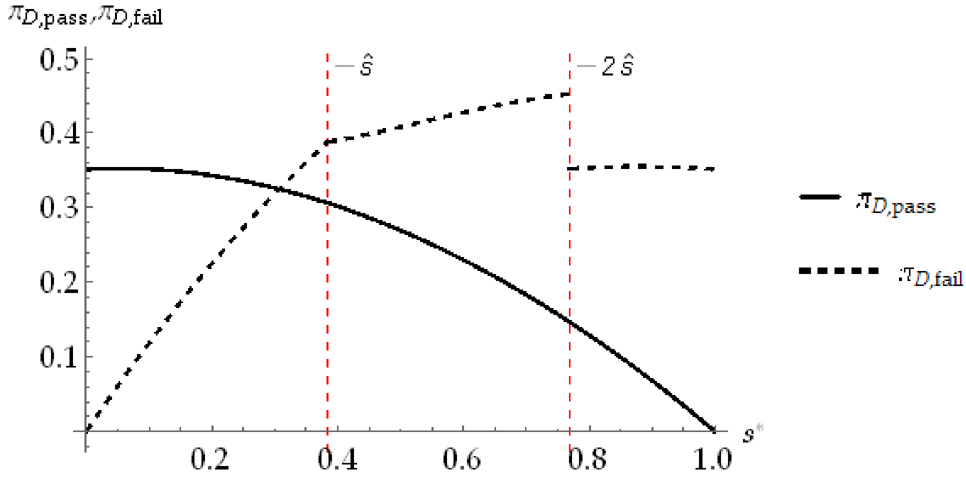

Figure 5: This Figure shows the speculator's profits from trading the debt claim following a message $m = pass$ (downward sloped line) and a message $m = fail$ (non-continuous dashed line). Parameter values are the same as in Figure 3.

and

$$\pi_{D,fail}(s^*) = \begin{cases} \frac{\sigma}{4s^*}\begin{bmatrix} \Delta D^1(2-s^*)(s^{*2}-\theta\hat{s}^2)+(2-s^*)\theta\hat{s}^2\Delta D^0 \\ -2\theta\hat{s}(s^*-\hat{s})(D_l^0-D_l^1) \end{bmatrix} & if \quad 2\widehat{s} < s^* \leq 1 \\ \frac{\sigma}{4}\left\{ \begin{array}{c} \theta\left(1-\frac{\hat{s}}{s^*}\right)\begin{bmatrix} (\Delta D^1-\Delta D^0)(2-s^*)(s^*+\hat{s}) \\ -2\hat{s}(D_l^0-D_l^1) \end{bmatrix} \\ +s^*(2-s^*)\Delta D^0 \end{array} \right\} & if \quad \widehat{s} \leq s^* \leq 2\widehat{s} \\ \frac{\sigma}{4}s^*(2-s^*)\Delta D^0 & if \quad s^* < \widehat{s} \end{cases} . \tag{36}$$

Note that, similarly to equity trading in Section 5, $\pi_{D,fail}$ is continuous in s^*, except at the point $s^* = 2\widehat{s}$ where the supervisor changes her intervention policy to always allowing the bank to continue following a $fail$ signal except when she privately observes $s \in [0, \widehat{s}]$. When ΔD^1 is higher than ΔD^0, implying that trade in debt is more profitable if the bank is allowed to continue, $\pi_{D,fail}$ jumps upwards in the point $s^* = 2\widehat{s}$. Otherwise, as shown in Figure 5, $\pi_{D,fail}$ jumps downwards.

Under the assumption $n_D = n$,[18] a direct comparison of Lemma 6 to the speculator's profit from trading shares presented in Lemma 3, allows us to determine which security the speculator prefers to trade in.

When the stress test is lenient ($s^* < \widehat{s}$), the speculator would always prefer to trade debt following a failed test. This is because a $fail$ signal under a lenient stress test conveys such negative information that the bank's equity is wiped out with certainty and it is therefore impossible to make a profit from trading in equity. Following a $pass$ signal, the speculator may prefer to trade a debt or an equity claim, depending on parameter values. When the risk shifting problem becomes more severe (ε increases), the value of the

[18] In practice, debt markets tend to be significantly less liquid than stock markets. The assumption $n_D = n$ is therefore not made for realism, but rather to provide analytical focus on the factors affecting the choice of the traded security. Obviously, liquidity is itself one such factor.

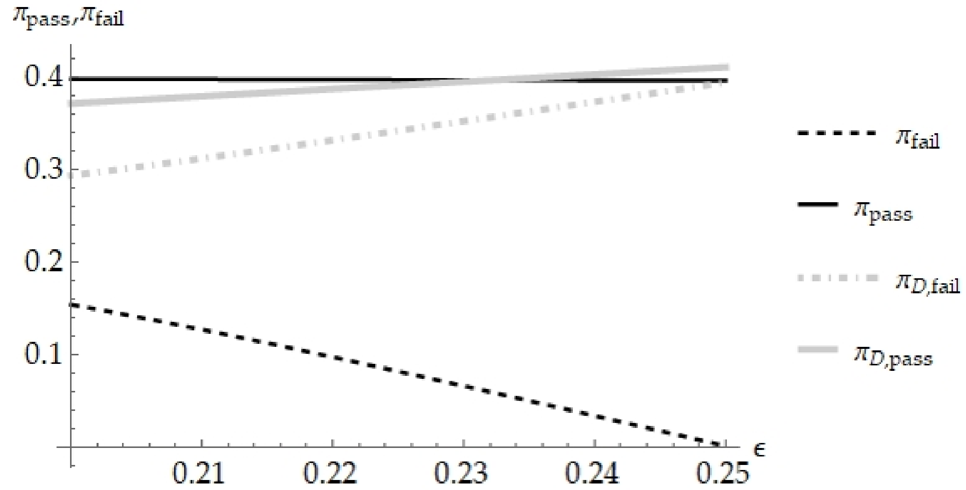

Figure 6: This Figure shows the speculator's profits from trading either the debt claim following a *pass* signal (grey solid line) and a *fail* signal (grey dot-dashed line), or the equity following *pass* (black solid line) and *fail* (black dashed line). For a better presentation, we have modified parameter values to : $\delta = 0.9$, $r = 7.5$, $R = 8.2$, $D = 7.9$, $A_h = 2.8$, $p = 0.57$, $s^* = 0.34$. The value of $\hat{s}$ varies from 0.174 to 0.338 for $\varepsilon \in [0.20, 0.25]$ so that s^* is always between $\hat{s}$ and $2\hat{s}$.

equity claim becomes less sensitive to the underlying state of the world. At relatively low levels of ε the speculator may therefore prefer to trade in equity, following a *pass* signal, but switch to trading the debt claim when ε increases. That is, we have $\Delta E^1 > \Delta D^1$ only if ε is sufficiently small, namely for

$$\varepsilon < \frac{2(1-p)\left(r - D + \frac{1}{2}A_h\right)}{r - D + \frac{R}{2}}. \tag{37}$$

We illustrate this effect by providing a numerical example in Figure 6. We discuss the details in the proof of Corollary 2. More generally, an increase in ε tends to make trading in a debt claim relatively more profitable, regardless of whether such a trade follows a *pass* or *fail* signal and regardless of whether the stress test is lenient or conservative. This is formally stated in the following Corollary:

Corollary 2 *The difference in trading profits from trading a debt instead of an equity claim is (weakly) increasing in ε, both following a pass and following a fail signal.*

Relating to Proposition 3, we see that allowing trade in a risky debt claim can encourage more information production by the speculator, particularly when the risk-shifting problem is relatively severe (high ε). As implied by Lemma 4, a high ε dampens the speculator's incentive to trade shares (for $\widehat{s} \le s^* \le 2\widehat{s}$), and consequently the supervisor is less likely to choose a conservative policy. The possibility of trading debt helps to relax the speculator's participation constraint and reduces the extent of policy distortion. In other words, the policy s^* can be set closer to the benchmark $\widehat{s}$ and hence the quality of the stress test is improved. This ultimately improves the supervisor's intervention decision.

Our findings are thus broadly in line with existing proposals to introduce sub-ordinated debt in a bank's capital structure as a means to make market information about risk taking available to bank supervisors (see, for example, Flannery and Bliss, 2019). A full analysis of this question would, however, require a richer set-up. In this simple extension, allowing risky debt to be traded is like a free option, that is, risky debt would at worst be useless. As such it is not surprising that it may sometimes help. More interesting is the observation that it will help for the banks that failed a stress test, but it may not help for those banks that passed it. Of course, this ignores many relevant complications: We assumed that liquidity of debt is just the same as of equity and the introduction of one security does not affect the liquidity of the other. Both of these assumptions are questionable. Relatedly, if a trader can acquire information and then decide in which instruments to trade (possibly in both), this will obviously affect information revelation. Fully taking these issues on board is beyond the scope of this paper.

7 Conclusion

This paper develops a theory of how a bank supervisor's degree of leniency in designing stress tests for banks affects information produced via financial markets. We show that the supervisor may optimally lower the hurdle for banks to pass the test in order to encourage information production in the stock market. Our model shows that such lenient tests should be applied to banks whose shares are less liquid, or where information production is particularly costly for speculators. Moreover, a supervisor may find it optimal to use a conservative test design when she is more likely to be privately informed. We also point out that the stock market may become a less suitable conduit for information when the risk-shifting problem worsens.

While our model is framed in a setting where bank supervisors learn about the bank types from prices and decide whether to intervene, we believe many of the paper's insights apply more broadly. For example, a credit rating agency can be more or less lenient in its ratings, which, together with information learned from stock prices, will affect the availability and quality of credit. Again, we would expect an interaction between the credit rating agency's degree of leniency and the usefulness of stock markets in producing additional information about a firm's prospects.

8 Appendix A

Proof of Proposition 1 : We first show that it is optimal for the speculators to submit an order with a fixed size n. The market maker observes anonymous orders and we denote the actual order as a pair,

$$X = (x_n, x_s)$$

in which x_n is the size of the order submitted by the liquidity trader and x_s the size of the order by the speculator. The order observed by the market maker X' can be thought of as a random reshuffling so that $X' = (x_n, x_s)$ or $X' = (x_s, x_n)$ with equal probability. Let the market maker's out of equilibrium belief be that any buy (sell) order that is not of size n $(-n)$ is due to a positively (negatively) informed speculator. If the speculator were to submit an order of any size other than n, he would immediately reveal his identity and thus his private information to the market trader. Therefore, the speculator cannot make any trading profit unless he trades in the same size as the liquidity trader, in which case the speculator may hide his private information if $X = (-n, +n)$ or $X = (+n, -n)$. When $X = (-n, -n)$, the market knows that speculator submitted a sell order and sets a corresponding low price, reflecting $\omega = l$. Similarly, the market maker will set a high price upon observing $X = (+n, +n)$. When the speculator has no information he would lose on average if he trades based on an uninformative signal.

We next prove the speculator's trading direction and profit conditional on the signal $m = pass$. First, when s^* is chosen at any level above $2\widehat{s} - 1$, the supervisor will let the bank continue following good news ($m = pass$) unless the share price reveals $\omega = l$. The bank's equity value is thus E_h^1 if $\omega = h$ and E_l^1 otherwise. Anticipating that, the market maker sets the following price $P_{pass}^{-n,n}$ upon receiving an uninformative order flow, as the weighted average of E_h^1 and E_l^1:

$$\begin{aligned} P_{pass}^{-n,n} &= \Pr\left(\omega = h | s > s^*\right) E_h^1 + \Pr\left(\omega = l | s > s^*\right) E_l^1 \\ &= \frac{1+s^*}{2} E_h^1 + \frac{1-s^*}{2} E_l^1. \end{aligned} \tag{38}$$

We can then compute, for the case with $s^* \geq 2\widehat{s} - 1$, the speculator's per unit trading profit conditional on $m = pass$, after observing the signal $z \in \{l, h\}$ and trading in direction $q_\omega \in \{-1, 0, 1\}$:

$$\begin{aligned} \pi_{pass}\left(s^* \geq 2\widehat{s} - 1\right) &= \frac{\sigma}{2} \Pr\left(\omega = h | m = pass\right) \left(E_h^1 - P_{pass}^{-n,n}\right) q_h \\ &\quad + \frac{\sigma}{2} \Pr\left(\omega = l | m = pass\right) \left(E_l^1 - P_{pass}^{-n,n}\right) q_l. \end{aligned} \tag{39}$$

Since $E_h^1 > P_{pass}^{-n,n} > E_l^1$ it is optimal to buy after $\omega = h$ $(q_h = 1)$ and sell after $\omega = l$ $(q_l = -1)$, yielding total trading profits

$$n\pi_{pass} = n\frac{\sigma}{4}\left(E_h^1 - E_l^1\right)\left(1 - s^{*2}\right).$$

If $s^* < 2\widehat{s} - 1$, the supervisor intervenes following both $m = pass$ and $m = fail$ if the share price reveals no additional information, in which case the equity value of the bank is zero. The speculator would thus have no incentive to acquire information and trade, i.e., $\pi_{pass}\left(s^* < 2\widehat{s} - 1\right) = 0$.

Consider now the speculator's trading profit conditional on signal $m = fail$. When s^* is chosen at any level above $2\widehat{s}$, the supervisor would allow the bank to continue following $m = fail$ unless the share price reveals $\omega = l$. The market maker thus sets the following price $P_{fail}^{-n,n}$ after receiving an uninformative order flow,

$$\begin{aligned} P_{fail}^{-n,n} &= \Pr\left(\omega = h | s < s^*\right) E_h^1 + \Pr\left(\omega = l | s < s^*\right) E_l^1 \\ &= \frac{s^*}{2} E_h^1 + \left(1 - \frac{s^*}{2}\right) E_l^1 \end{aligned} \tag{40}$$

The speculator's per unit trading profit conditional on $m = fail$ for the case when $s^* > 2\widehat{s}$ is

$$\pi_{fail}\left(s^* > 2\widehat{s}\right) = \frac{\sigma}{2} q_h \Pr\left(\omega = h | m = fail\right)\left(E_h^1 - P_{fail}^{-n,n}\right) \tag{41}$$
$$+ q_l \Pr\left(\omega = l | m = fail\right)\left(E_l^1 - P_{fail}^{-n,n}\right). \tag{42}$$

As before, it is optimal to set $q_h = n$ and $q_l = -n$ since $E_h^1 > P_{fail}^{-n,n} > E_l^1$. Hence,

$$\pi_{fail}\left(s^* > 2\widehat{s}\right) = \frac{\sigma}{4}\Delta E^1 s^* \left(2 - s^*\right).$$

Similarly, for the case with $s^* \leq 2\widehat{s}$, the supervisor always intervenes following $m = fail$ if the share price does not reveal other information, in which case $\pi_{fail}\left(s^* \leq 2\widehat{s}\right) = 0$. QED.

Proof of Proposition 2 : See proof of Proposition 3.

Proof of Lemma 3 : We first check the speculator's trading profits conditional on $m = pass$. Note that he makes profits only when his private information is hidden in the order flow received by the market maker. If s^* is chosen above $\hat{s}$, the supervisor allows the bank to continue when learning $m = pass$ regardless of whether she is privately informed (any private information in this range would only confirm the optimality of continuation). The speculator's trading profit is then $\frac{\sigma}{4}\Delta E^1\left(1 - \left(s^*\right)^2\right)$, as in the corresponding case of Proposition 1. If s^* is chosen between $2\widehat{s} - 1$ and $\widehat{s}$, the supervisor would also continue following $m = pass$, except when she learns privately $s \in \left(s^*, \hat{s}\right)$. Anticipating the supervisor's strategy, the market maker sets the trading price as:

$$\begin{aligned}
P_{pass}^{-n,n} = & \Pr\left(\hat{s} \leq s < 1 \cap \omega = h | s \geq s^*\right) E_h^1 + \Pr\left(\hat{s} \leq s < 1 \cap \omega = l | s \geq s^*\right) E_l^1 \\
& + \Pr\left(s^* \leq s < \hat{s} \cap \omega = h | s \geq s^*\right)\left[\theta E_h^0 + \left(1 - \theta\right) E_h^1\right] \\
& + \Pr\left(s^* \leq s < \hat{s} \cap \omega = l | s \geq s^*\right)\left[\theta E_l^0 + \left(1 - \theta\right) E_l^1\right]
\end{aligned}$$

and thus

$$\begin{aligned}
P_{pass}^{-n,n} = & \frac{1 - \hat{s}^2}{2\left(1 - s^*\right)} E_h^1 + \frac{\left(1 - \hat{s}\right)^2}{2\left(1 - s^*\right)} E_l^1 + \left(1 - \theta\right)\frac{\hat{s}^2 - s^{*2}}{2\left(1 - s^*\right)} E_h^1 \\
& + \left(1 - \theta\right)\frac{2\left(\hat{s} - s^*\right) - \left(\hat{s}^2 - s^{*2}\right)}{2\left(1 - s^*\right)} E_l^1.
\end{aligned} \tag{43}$$

We then compute the speculator's expectation of the bank's equity value given its private signal z, $z = \omega$, and uninformative order flows

$$\begin{aligned}
E[E_\omega^a | pass, \omega] = & \Pr\left(\hat{s} \leq s < 1 | s \geq s^* \cap \omega\right) E_\omega^1 \\
& + \Pr\left(s^* \leq s < \hat{s} | s \geq s^* \cap \omega\right)\left[\theta E_\omega^0 + \left(1 - \theta\right) E_\omega^1\right]
\end{aligned}$$

We have therefore

$$E[E_\omega^a | pass, h] = \frac{1 - \hat{s}^2}{1 - s^{*2}} E_h^1 + \left(1 - \theta\right)\frac{\hat{s}^2 - s^{*2}}{1 - s^{*2}} E_h^1 \tag{44}$$

$$E[E_\omega^a | pass, l] = \frac{\left(1 - \hat{s}\right)^2}{\left(1 - s^*\right)^2} E_l^1 + \left(1 - \theta\right)\frac{2\left(\hat{s} - s^*\right) - \left(\hat{s}^2 - s^{*2}\right)}{\left(1 - s^*\right)^2} E_l^1 \tag{45}$$

The speculator buys when receiving $z = h$ and sells when $z = l$, by comparing the market maker's price in (43) to the speculator's conditional expectation of the bank's equity value as in, respectively (44) and (45). This then allows us to calculate the per unit trading profit following $m = pass$.

$$\begin{aligned}\pi_{pass}\left(s^* \in [\underline{s}, \overline{s}]\right) &= \frac{\sigma}{2}\Pr\left(\omega = h | m = pass\right)\left(E[E_\omega^a | pass, h] - P_{pass}^{-n,n}\right) \\ &\quad + \frac{\sigma}{2}\Pr\left(\omega = l | m = pass\right)\left(P_{pass}^{-n,n} - E[E_\omega^a | pass, l]\right)\end{aligned} \tag{46}$$

$$= \frac{\sigma}{4}\left(1 - s^{*2}\right)\left(E[E_\omega^a | pass, h] - E[E_\omega^a | pass, l]\right)$$

$$= \frac{\sigma}{4}\left\{\Delta E^1\left[\left(1 - s^{*2}\right) - \theta\left(\hat{s}^2 - s^{*2}\right)\right] + 2\theta\frac{\left(\hat{s} - s^*\right)\left(1 - \hat{s}\right)}{1 - s^*}E_l^1\right\}$$

$$= \frac{\sigma}{4}\left(1 - s^{*2}\right)\left\{\left(1 - \theta\right)\Delta E^1 + \theta\left[\frac{1 - \widehat{s}^2}{1 - s^{*2}}E_h - \frac{\left(1 - \widehat{s}\right)^2}{\left(1 - s^*\right)^2}E_l\right]\right\}.$$

π_{pass} decreases in $s^* > 0$, since

$$\frac{\partial \pi_{pass}}{\partial s^*} = -\frac{\sigma}{2}\left[s^*\left(1 - \theta\right)\Delta E^1 + \frac{\left(1 - \widehat{s}\right)^2}{\left(1 - s^*\right)^2}\right] < 0.$$

Finally if s^* is chosen below $2\widehat{s} - 1$, the supervisor will always intervene following a *pass* message unless she observes privately $s > \widehat{s}$. We derive the market maker's price when learning $m = pass$ as well as the speculator's expectation of the bank's equity value based on both m and z.

$$\begin{aligned}P_{pass}^{-n,n} &= \Pr\left(\hat{s} \le s < 1 \cap \omega = h | s \ge s^*\right)\left[\theta E_h^1 + \left(1 - \theta\right)E_h^0\right] \\ &\quad + \Pr\left(\hat{s} \le s < 1 \cap \omega = l | s \ge s^*\right)\left[\theta E_l^1 + \left(1 - \theta\right)E_l^0\right] \\ &\quad + \Pr\left(s^* \le s < \hat{s} \cap \omega = h | s \ge s^*\right)E_h^0 \\ &\quad + \Pr\left(s^* \le s < \hat{s} \cap \omega = l | s \ge s^*\right)E_l^0,\end{aligned}$$

which can be simplified to

$$P_{pass}^{-n,n} = \frac{1 - \hat{s}^2}{2\left(1 - s^*\right)}\theta E_h^1 + \frac{\left(1 - \hat{s}\right)^2}{2\left(1 - s^*\right)}\theta E_l^1 \tag{47}$$

$$E[E_\omega^a | pass, h] = \frac{1 - \hat{s}^2}{1 - s^{*2}}\theta E_h^1 \tag{48}$$

$$E[E_\omega^a | pass, l] = \frac{\left(1 - \hat{s}\right)^2}{\left(1 - s^*\right)^2}\theta E_l^1 \tag{49}$$

We can then compute the trading profit following a *pass* message for the case of choosing $s^* < 2\widehat{s} - 1$,

$$\pi_{pass}\left(s^* < 2\widehat{s} - 1\right) = \frac{\sigma}{4}\left(1 - s^{*2}\right)\left(\frac{1 - \hat{s}^2}{1 - s^{*2}}\theta E_h^1 - \frac{\left(1 - \hat{s}\right)^2}{\left(1 - s^*\right)^2}\theta E_l^1\right)$$

which can be simplified to

$$\pi_{pass}\left(s^* < 2\widehat{s} - 1\right) = \frac{\sigma}{4}\theta\left[\left(1 - \widehat{s}^2\right)\Delta E^1 + 2\left(\widehat{s} - s^*\right)\frac{1-\widehat{s}}{1-s^*}E_l^1\right]. \tag{50}$$

We now check the speculator's trading profits conditional on $m = fail$. If $s^* < \hat{s}$, the supervisor will intervene upon its own information. Trading thus generates the same profit π_{fail} as in (26) (for $s^* < 2\widehat{s}$). If $\widehat{s} \leq s^* \leq 2\widehat{s}$, the supervisor always intervene except when she is informed of the value of s and $s \in [\hat{s}, s^*]$. As in the previous case, we compute the market maker's price,

$$\begin{aligned} P_{fail}^{-n,n} &= \Pr\left(s < \hat{s} \cap \omega = l | s < s^*\right) E_h^0 + \Pr\left(s < \hat{s} \cap \omega = h | s < s^*\right) E_l^0 \\ &\quad + \Pr\left(\widehat{s} \leq s < s^* \cap \omega = h | s < s^*\right)\left[\theta E_h^1 + (1-\theta) E_h^0\right] \\ &\quad + \Pr\left(\hat{s} \leq s < s^* \cap \omega = l | s < s^*\right)\left[\theta E_l^1 + (1-\theta) E_l^0\right] \end{aligned}$$

which can be simplified to

$$P_{fail}^{-n,n} = \theta\left[\frac{s^{*2} - \hat{s}^2}{2s^*}E_h^1 + \frac{2(s^* - \hat{s}) - (s^{*2} - \hat{s}^2)}{2s^*}E_l^1\right], \tag{51}$$

We then compute the speculator's conditional expectation of the bank's equity value,

$$E[E_\omega^a | fail, h] = \frac{s^{*2} - \hat{s}^2}{s^{*2}}\theta E_h^1 \tag{52}$$

$$E[E_\omega^a | fail, l] = \frac{2(s^* - \hat{s}) - (s^{*2} - \hat{s}^2)}{2s^* - s^{*2}}\theta E_l^1. \tag{53}$$

By comparing the market maker's price in (51) to the speculator's conditional expectation of the bank's equity value as in (52) and (53) respectively, we know that the speculator buys if his private signal $z = h$ and sells otherwise. We then compute the speculator's expected per unit trading profit following $m = fail$ for $\widehat{s} \leq s^* \leq 2\widehat{s}$,

$$\begin{aligned} \pi_{fail} &= \frac{\sigma}{4}s^*\left(2 - s^*\right)\left(E[E_\omega^a | fail, h] - E[E_\omega^a | fail, l]\right) \\ &= \frac{\sigma}{4}\theta\left(1 - \frac{\hat{s}}{s^*}\right)\left[\Delta E^1\left(2 - s^*\right)\left(s^* + \hat{s}\right) + 2\hat{s}E_l^1\right] \end{aligned}$$

which increases in $s^* \in [\widehat{s}, 2\widehat{s}]$.

Finally, we compute the trading profit following $m = fail$ when s^* is chosen above $2\widehat{s}$. In this case, the supervisor would never intervene upon learning $m = fail$ except when it observes $s < \widehat{s}$. Using the same algorithm, we compute below the market maker's price and then compare it to the speculator's expectation of the bank's equity value conditioning on his own signal z,

$$\begin{aligned} P_{fail}^{-n,n} &= \left[\Pr\left(s < \hat{s} \cap \omega = l | s < s^*\right) E_l^1 + \Pr\left(s < \hat{s} \cap \omega = h | s < s^*\right) E_h^1\right](1-\theta) \qquad (54) \\ &\quad + \Pr\left(\widehat{s} \leq s < s^* \cap \omega = h | s < s^*\right) E_h^1 + \Pr\left(\hat{s} \leq s < s^* \cap \omega = l | s < s^*\right) E_l^1 \\ &= \left[\frac{\hat{s}^2}{2s^*}E_h^1 + \frac{2\hat{s} - \hat{s}^2}{2s^*}E_l^1\right](1-\theta) + \frac{s^{*2} - \hat{s}^2}{2s^*}E_h^1 + \frac{2(s^* - \hat{s}) - (s^{*2} - \hat{s}^2)}{2s^*}E_l^1 \end{aligned}$$

$$E[E_\omega^a|fail,h]=\left[\frac{\hat{s}^2}{s^{*2}}(1-\theta)+\frac{s^{*2}-\hat{s}^2}{s^{*2}}\right]E_h^1 \tag{55}$$

$$E[E_\omega^a|fail,l]=\left[\frac{2\hat{s}-\hat{s}^2}{2s^*-s^{*2}}(1-\theta)+\frac{2(s^*-\hat{s})-(s^{*2}-\hat{s}^2)}{2s^*-s^{*2}}\right]E_l^1. \tag{56}$$

We compute the per unit trading profit as

$$\begin{aligned}\pi_{fail} &= \frac{\sigma}{4}s^*(2-s^*)\left(E[E_\omega^a|fail,h]-E[E_\omega^a|fail,l]\right)\\ &= \frac{\sigma}{4}\left(1-\frac{\hat{s}}{s^*}\right)\left[\Delta E^1(2-s^*)\frac{s^{*2}-\theta\hat{s}^2}{(s^*-\hat{s})}+2\theta\hat{s}E_l^1\right]\end{aligned}$$

which increases in s^* and peaks when $s^*=1$. QED

Proof of Proposition 3: Preliminaries: We state the value functions $V_{\mathcal{I}}(s^*)$ valid on the interval $s^*\in[\underline{s},\overline{s}]$, i.e., when s^* is sufficiently informative for the supervisor to follow the test result in the absence of any further information.

$$\begin{aligned}V_{\{fail,pass\}}(s^*) &= \frac{1}{2}\left(V_h^1+V_l^0\right)\\ &+\frac{1}{2}\left(1-\frac{\sigma}{2}\right)\left\{-(s^*)^2\Delta V_h-(1-s^*)^2\Delta V_l+\theta\left(\widehat{s}-s^*\right)^2(\Delta V_h+\Delta V_l)\right\},\end{aligned} \tag{57}$$

which reaches its maximum at $s^*=\widehat{s}$. Concerning $V_{pass}(s^*)$ we need to distinguish the case $s^*\geq\widehat{s}$ from $s^*<\widehat{s}$.

$$\begin{aligned}V_{\{pass\}}(s^*\geq\widehat{s}) &= \frac{1}{2}\left[\widehat{s}^2V_h^0+\left(2\widehat{s}-\widehat{s}^2\right)V_l^0\right]\\ &+\frac{1}{2}\left[(s^*)^2-\widehat{s}^2\right]\left[\theta V_h^1+(1-\theta)V_h^0\right]\\ &+\frac{1}{2}\left\{\left(2s^*-2\widehat{s}-(s^*)^2+\widehat{s}^2\right)\left[\theta V_l^1+(1-\theta)V_l^0\right]\right\}\\ &+\frac{1}{2}\left[1-(s^*)^2\right]V_h^1+\frac{1}{2}(1-s^*)^2\left[\frac{\sigma}{2}V_l^0+\left(1-\frac{\sigma}{2}\right)V_l^1\right]\end{aligned}$$

which is maximized at $s^*=\widehat{s}$. Moreover,

$$\begin{aligned}V_{\{pass\}}(s^*<\widehat{s}) &= \frac{1}{2}\left\{\left(1-(s^*)^2\right)V_h^1+(s^*)^2V_h^0\right\}\\ &+\frac{1}{2}\left\{s^*(2-s^*)+\frac{\sigma}{2}(1-s^*)^2\right\}V_l^0\\ &+\frac{1}{2}(1-s^*)^2\left(1-\frac{\sigma}{2}\right)V_l^1\\ &+\frac{1}{2}\theta\left(1-\frac{\sigma}{2}\right)\left\{\left[(1-s^*)^2-(1-\widehat{s})^2\right]\Delta V_l-\left[\widehat{s}^2-(s^*)^2\right]\Delta V_h\right\}.\end{aligned} \tag{58}$$

Using $\widehat{s}=\frac{\Delta V_l}{\Delta V_h+\Delta V_l}$ we can find the value of s^* that maximizes $V_{\{pass\}}(s^*)$. Defining $s_{pass}^{\max}\equiv\arg\max V_{\{pass\}}(s^*)$, we get

$$s_{pass}^{\max}=\max\left\{\frac{(1-\theta)\left(1-\frac{\sigma}{2}\right)\widehat{s}}{(1-\theta)\left(1-\frac{\sigma}{2}\right)+\frac{\sigma}{2}(1-\widehat{s})},2\widehat{s}-1\right\}<\widehat{s}. \tag{59}$$

Since $V_{\{pass\}}(s^*)$ is continuous in $s^* = \widehat{s}$, $V_{\{pass\}}(s^*)$ reaches its maximum on the interval $[\underline{s}, \overline{s}]$ at $s_{pass}^{\max}$. Note that (27) and (28) are special cases of (58) and (59), respectively.

Expected bank value when the speculator only acquires information following a *fail* test is given by

$$\begin{aligned} V_{\{fail\}}(s^* \geq \widehat{s}) &= \frac{1}{2}\left\{\left[1-(s^*)^2\right]V_h^1 + \widehat{s}^2\left[\frac{\sigma}{2}V_h^1 + \left(1-\frac{\sigma}{2}\right)V_h^0\right]\right\} \\ &+\frac{1}{2}\left\{\left[(s^*)^2 - \widehat{s}^2\right]\left[\left(\frac{\sigma}{2} + \left(1-\frac{\sigma}{2}\right)\theta\right)V_h^1 + (1-\theta)\left(1-\frac{\sigma}{2}\right)V_h^0\right]\right\} \\ &+\frac{1}{2}\left\{(1-s^*)^2 V_l^1 + \left[1-(1-\widehat{s})^2\right]V_l^0\right\} \\ &+\frac{1}{2}\left\{\left[(1-\widehat{s})^2 - (1-s^*)^2\right]\left[\left(\frac{\sigma}{2} + \left(1-\frac{\sigma}{2}\right)(1-\theta)\right)V_l^0 + \left(1-\frac{\sigma}{2}\right)\theta V_l^1\right]\right\} \end{aligned}$$

which is maximized at $s_{fail}^{\max} \equiv \arg\max V_{\{fail\}}(s^*)$ given by

$$s_{fail}^{\max} = \min\left\{\frac{\widehat{s}\left[\frac{\sigma}{2} + (1-\theta)\left(1-\frac{\sigma}{2}\right)\right]}{\widehat{s}\frac{\sigma}{2} + (1-\theta)\left(1-\frac{\sigma}{2}\right)}, 2\widehat{s}\right\} > \widehat{s}. \tag{60}$$

Note that $V_{\{fail\}}(s^* < \widehat{s})$ is irrelevant as $\mathcal{I} = \{fail\}$ is never induced for any $s^* < \widehat{s}$ when $\frac{c}{n} > 0$.

Finally,

$$\begin{aligned} V_{\varnothing}(s^* \geq \widehat{s}) &= \frac{1}{2}\left\{\left(1-(s^*)^2\right)V_h^1 + \left((s^*)^2 - \widehat{s}^2\right)\left[\theta V_h^1 + (1-\theta)V_h^0\right] + \widehat{s}^2 V_h^0\right\} \quad (61) \\ &+\frac{1}{2}\left\{(1-s^*)^2 V_l^1 + \left((1-\widehat{s})^2 - (1-s^*)^2\right)\left[\theta V_l^1 + (1-\theta)V_l^0\right]\right\} \\ &+\frac{1}{2}\left(2\widehat{s} - \widehat{s}^2\right)V_l^0 \quad (62) \end{aligned}$$

reaches its maximum at $s^* = \widehat{s}$. An analogous expression obtains for $V_{\varnothing}(s^* \leq \widehat{s})$ which also reaches its maximum at $s^* = \widehat{s}$. Hence, the optimal policy is $s^* = \widehat{s}$, and the value $V_{\varnothing}(s^* = \widehat{s})$ is independent of θ and given by

$$V_{\varnothing}(\widehat{s}) = \frac{1}{2}\left\{\left(1-\widehat{s}^2\right)V_h^1 + \widehat{s}^2 V_h^0 + (1-\widehat{s})^2 V_l^1 + \left(2\widehat{s} - \widehat{s}^2\right)V_l^0\right\}.$$

Denote by $V_{pass}^{\max} \equiv V_{\{pass\}}\left(s_{pass}^{\max}\right)$ and $V_{fail}^{\max} \equiv V_{\{fail\}}\left(s_{fail}^{\max}\right)$. From these we note the following properties. If $\widehat{s} < \frac{1}{2}$ then $V_{\{pass\}}(s^* = 0) > V_{\{fail\}}(s^* = 2\widehat{s})$ and $V_{pass}^{\max} > V_{fail}^{\max}$. If $\widehat{s} > \frac{1}{2}$ then $V_{\{pass\}}(s^* = 2\widehat{s} - 1) < V_{\{fail\}}(s^* = 1)$ and $V_{pass}^{\max} < V_{fail}^{\max}$. Obviously, we also have $V_{\{pass,fail\}}(\widehat{s}) \geq V_{pass}^{\max} \geq V_{\varnothing}(\widehat{s})$ and $V_{\{pass,fail\}}(\widehat{s}) \geq V_{fail}^{\max} \geq V_{\varnothing}(\widehat{s})$ with strict inequalities when $\sigma > 0$.

Moreover, denote by $s_{\mathcal{I}}^{\mathcal{I}'}$ the maximum distortion that the supervisor is willing to incur so as to ensure that information is produced following messages $m \in \mathcal{I}$ instead of a different set $\mathcal{I}'$. Hence, the corresponding indifference points can be calculated as follows:[19]

$$
\begin{aligned}
V_{\{pass,fail\}}\left(s^{pass}_{pass,fail}\right) &= V^{\max}_{pass}, \\
s^{pass}_{pass,fail} &= \hat{s}\left[1+\left(\frac{(1-\hat{s})\,\sigma}{2-\theta\,(2-\sigma)-\sigma\hat{s}}\right)^{\frac{1}{2}}\right] > \widehat{s} \qquad (63)\\
V_{\{pass\}}\left(s^{\varnothing}_{pass}\right) &= V_{\varnothing}\left(\widehat{s}\right), \\
s^{\varnothing}_{pass} &= \widehat{s}\frac{(1-\theta)\left(1-\frac{\sigma}{2}\right)-(1-\widehat{s})\left(\frac{\sigma\left[1-\theta\left(1-\frac{\sigma}{2}\right)\right]}{2\widehat{s}}\right)^{\frac{1}{2}}}{(1-\theta)\left(1-\frac{\sigma}{2}\right)+\frac{\sigma}{2}\left(1-\widehat{s}\right)} < \widehat{s} \qquad (64)\\
V_{\{fail\}}\left(s^{\varnothing}_{fail}\right) &= V_{\varnothing}\left(\widehat{s}\right), \\
s^{\varnothing}_{fail} &= \widehat{s}\frac{1-\theta\left(1-\frac{\sigma}{2}\right)+\left\{\frac{\sigma}{2}\left(1-\widehat{s}\right)\left[1-\theta\left(1-\frac{\sigma}{2}\right)\right]\right\}^{\frac{1}{2}}}{(1-\theta)\left(1-\frac{\sigma}{2}\right)+\frac{\sigma}{2}\widehat{s}} > \widehat{s} \qquad (65)\\
V_{\{fail\}}\left(s^{pass}_{fail}\right) &= V^{\max}_{pass}. \\
s^{pass}_{fail} &= \widehat{s}\frac{1-\theta\left(1-\frac{\sigma}{2}\right)+\left[\frac{(1-\widehat{s})(2\widehat{s}-1)(1-\theta)\left(1-\frac{\sigma}{2}\right)\sigma\left[1-\theta\left(1-\frac{\sigma}{2}\right)\right]}{2\widehat{s}\left[1-\theta\left(1-\frac{\sigma}{2}\right)-\widehat{s}\frac{\sigma}{2}\right]}\right]^{\frac{1}{2}}}{1-\theta\left(1-\frac{\sigma}{2}\right)-\left(1-\widehat{s}\right)\frac{\sigma}{2}} > \widehat{s} \qquad (66)
\end{aligned}
$$

Note that s^{pass}_{fail} only exists for $\widehat{s} \geq \frac{1}{2}$ (otherwise $V^{\max}_{pass} > V^{\max}_{fail}$). It can be shown that $\pi_{pass}\left(s^{\max}_{pass}\right) > \pi_{fail}\left(s^{\max}_{fail}\right)$ which implies that the point s^{fail}_{pass} defined by the solution to $V_{\{pass\}}\left(s^{fail}_{pass}\right) = V^{\max}_{fail}$ is irrelevant (the supervisor would not choose $s^* < s^{\max}_{pass}$ unless $\frac{c}{n} > \pi_{pass}\left(s^{\max}_{pass}\right)$ and for such values of $\frac{c}{n}$, it is never possible to induce $I = \{fail\}$ at $s^{\max}_{fail}$). Note also that the point $s^{fail}_{pass,fail}$ defined by $V_{\{pass,fail\}}\left(s^{fail}_{pass,fail}\right) = V^{\max}_{fail}$ is also irrelevant since the required distortion to induce either $\mathcal{I} = \{pass, fail\}$ or $\mathcal{I} = \{fail\}$ go in the same direction (conservatism) and $\mathcal{I} = \{pass, fail\}$ obviously dominates $\mathcal{I} = \{fail\}$ (hence any switch from $\mathcal{I} = \{pass, fail\}$ to $\mathcal{I} = \{fail\}$ would occur because $\mathcal{I} = \{pass, fail\}$ has become infeasible, but not because it is dominated by $\mathcal{I} = \{fail\}$).

Note that we have $s^{\varnothing}_{pass} \leq s^{\max}_{pass} < \widehat{s} < s^{\max}_{fail} < s^{pass}_{fail} \leq s^{\varnothing}_{fail}$. Moreover, (29) is a special case of (64) for $\theta = 0$.

We start by showing that it is never optimal to set $s^* > 2\widehat{s}$ (relevant when $\widehat{s} < \frac{1}{2}$) or $s^* < 2\widehat{s} - 1$ (relevant when $\widehat{s} > \frac{1}{2}$). Consider first $\widehat{s} < \frac{1}{2}$ and $s^* > 2\widehat{s}$. Since a $fail$ test is *per se* no longer sufficiently informative to induce intervention, expected bank value for a given set $\mathcal{I}$ drops in s^* around $s^* = 2\widehat{s}$. The only potential advantage of setting $s^* > 2\widehat{s}$ would be to improve the information production by the speculator, i.e., change the set $\mathcal{I}$. $\pi_{pass}\left(s^*\right)$ is decreasing in s^* so an increase of s^* will only tighten the constraint on information production following $m = pass$. $\pi_{fail}\left(s^*\right)$ is increasing in s^*, so the only advantage of setting $s^* > 2\widehat{s}$ would be to induce information production following $m = fail$. Comparing the set $\mathcal{I}$ induced when $s^* \in [\underline{s}, \overline{s}]$ and the set $\mathcal{I}'$ induced by setting $s^* > 2\widehat{s}$, several configurations are possible. (a) $\mathcal{I} = \{pass\}, \mathcal{I}' = \{pass, fail\}$, (b) $\mathcal{I} = \{pass\}, \mathcal{I}' = \{fail\}$, (c) $\mathcal{I} = \varnothing, \mathcal{I}' = \{fail\}$. In case (a) the bank's expected

[19] The solutions (63) - (66) can generate numbers outside the admissible interval for s^*, in which case it is understood that the supervisor is never indifferent between the corresponding two options.

value at $\mathcal{I}' = \{pass, fail\}$ is:

$$\begin{aligned} V_{\{fail,pass\}}(s^* > 2\widehat{s}) &= \frac{1}{2}\left\{\widehat{s}^2\left[\frac{\sigma}{2}V_h^1 + \left(1-\frac{\sigma}{2}\right)\left(\theta V_h^0 + (1-\theta)V_h^1\right)\right] + \left[1-\widehat{s}^2\right]V_h^1\right\} \\ &+\frac{1}{2}\left(2\widehat{s}-\widehat{s}^2\right)\left[\frac{\sigma}{2}V_l^0 + \left(1-\frac{\sigma}{2}\right)\left(\theta V_l^0 + (1-\theta)V_l^1\right)\right] \\ &+\frac{1}{2}\left(1-\widehat{s}\right)^2\left[\frac{\sigma}{2}V_l^0 + \left(1-\frac{\sigma}{2}\right)V_l^1\right] \end{aligned} \quad (67)$$

which is independent of s^*. It can shown that $V_{pass}^{\max} > V_{\{fail,pass\}}(s^* > 2\widehat{s})$ and hence it is better to set $s^* = s_{pass}^{\max}$. Since $\pi_{pass}(s^*)$ is a decreasing function, it follows that under configuration (a) when $\mathcal{I}' = \{pass, fail\}$ is feasible for $s^* > 2\widehat{s}$, $\mathcal{I} = \{pass\}$ is feasible for any $s^* < 2\widehat{s}$, so that $V_{pass}^{\max}$ can actually be attained. For cases (b) and (c), note that $\pi_{pass}(s^* = 0) = \pi_{fail}(s^* = 1)$ and therefore whenever $\mathcal{I}' = \{fail\}$ is feasible by setting $s^* > 2\widehat{s}$ it is also feasible to induce $\mathcal{I} = \{pass\}$ by setting s^* low enough and inducing the value $V_{\{pass\}}(s^*)$. Moreover, we can calculate expected bank value when $\mathcal{I}' = \{fail\}$ and $s^* > 2\widehat{s}$ (and hence the $fail$ test is ignored by the supervisor who allows the bank to continue in the absence of further information):

$$\begin{aligned} V_{\{fail\}}(s^* > 2\widehat{s}) &= \frac{1}{2}\left\{\left(1-\widehat{s}^2\right)V_h^1 + \widehat{s}^2\left[\left(\frac{\sigma}{2} + \left(1-\frac{\sigma}{2}\right)(1-\theta)\right)V_h^1 + \left(1-\frac{\sigma}{2}\right)\theta V_h^0\right]\right\} \\ &-\frac{1}{2}\left(1-s^*\right)^2\frac{\sigma}{2}\Delta V_l + \frac{1}{2}\left(1-\widehat{s}\right)^2\left[\frac{\sigma}{2}V_l^0 + \left(1-\frac{\sigma}{2}\right)V_l^1\right] \\ &+\frac{1}{2}\left[1-\left(1-\widehat{s}\right)^2\right]\left[\left(\frac{\sigma}{2} + \left(1-\frac{\sigma}{2}\right)\theta\right)V_l^0 + \left(1-\frac{\sigma}{2}\right)(1-\theta)V_l^1\right], \end{aligned}$$

which is maximized at $s^* = 1$. Note that $V_{\{fail\}}(s^* = 1) = V_{\{pass\}}(s^* = 0)$ and since $V_{pass}^{\max} > V_{\{pass\}}(s^* = 0)$ and $V_{\{pass\}}(s^* = 0)$ is increasing in the point $s^* = 0$ there are points $0 < s^* < \widehat{s}$ that are strictly preferred to $s^* = 1$ and that are also feasible. It follows that cases (b) and (c) are strictly dominated, except in the limit case $\theta = 1$, when the supervisor is indifferent between setting $s^* = 0$ or $s^* = 1$.

Consider next $\widehat{s} > \frac{1}{2}$ and $s^* < 2\widehat{s} - 1$. Similar to the previous case, the only potential benefit of choosing $s^* < 2\widehat{s} - 1$ is to induce $\mathcal{I}' = \{pass\}$ (It is impossible to induce $\mathcal{I}' = \{pass, fail\}$ at $s^* < 2\widehat{s} - 1$ since $\pi_{fail}(s^* < \widehat{s}) = 0$). Using the expected trading profits given in (30), we can show that, for

$$\theta > \theta_\pi \equiv \frac{4\widehat{s}\Delta E^1}{4\widehat{s}\Delta E^1 + E_l^1\left(2\widehat{s}-1\right)}, \quad (68)$$

$\pi_{pass}(s^*)$ is maximized at $s^* = 0$, and for $\theta \leq \theta_\pi$, $\pi_{pass}(s^*)$ is maximized at $s^* = 2\widehat{s} - 1$. If $\theta \leq \theta_\pi$, then setting $s^* < 2\widehat{s} - 1$ does not help in inducing information production, so there is clearly no benefit in doing so. If $\theta > \theta_\pi$, then setting $s^* < 2\widehat{s} - 1$ can potentially help inducing information production following the $pass$ signal. Like above, setting $s^* = 0$ is optimal on the interval $[0, 2\widehat{s} - 1)$. Since we have again $\pi_{pass}(s^* = 0) = \pi_{fail}(s^* = 1)$ and at $\widehat{s} > \frac{1}{2}$ we have $V_{fail}^{\max} > V_{\{fail\}}(s^* = 1) = V_{\{pass\}}(s^* = 0)$ it follows that inducing information production is weakly more valuable by distorting s^* towards 1, as $m = fail$ is still informative in this case, rather than setting $s^* = 0$.

Proof of part (a): Since $\pi_{fail}(\widehat{s}) = 0$, $\mathcal{I} = \{pass, fail\}$ is not feasible at $s^* = \widehat{s}$ for any $\frac{c}{n} > 0$. For $\theta > 0$, we have $\pi_{fail}(\widehat{s} > s^*) > 0$ and increasing in s^*. For small enough values of $\frac{c}{n}$ it is therefore possible to induce $\mathcal{I} = \{pass, fail\}$ by setting $s^* > \widehat{s}$ where s^* given by $\pi_{fail}(s^*) = \frac{c}{n}$. By continuity of the value functions, there exist small enough values of $\frac{c}{n}$ (and hence s^* close enough to $\widehat{s}$), such that $V_{\{pass,fail\}}(s^*) > V_{\{pass\}}\left(s^{\max}_{pass}\right)$. As $\frac{c}{n}$ increases, the s^* that solves $\pi_{fail}(s^*) = \frac{c}{n}$ also increases. Eventually, $\frac{c}{n}$ reaches a point that we denote by C_{con}, where it becomes either (i) impossible to induce $\mathcal{I} = \{pass, fail\}$ or (ii) undesirable. (i) happens when s^* is so high such that information production cannot be induced following either test result. Since π_{pass} is increasing and π_{fail} decreasing in s^*, denote by $s^{=}$ the highest value of s^* such that $\pi_{fail}(s^*) \leq \pi_{pass}(s^*)$ (since π_{fail} is discontinuous in $2\widehat{s}$ the two functions may not intersect). C_{con} is then given by $C_{con} = \pi_{fail}(s^{=})$. (ii) happens when the required distortion s^*given by the solution to $\pi_{fail}(s^*) = \frac{c}{n}$ is so strong as to imply that $V_{\{pass,fail\}}(s^*) < V^{\max}_{pass}$. (Note that at any s^* that is set so as to induce information production following $fail$, we never have $V_{\{pass,fail\}}(s^*) < V_{\{fail\}}(s^*)$.) In this case $C_{con} = \pi_{fail}\left(s^{pass}_{pass,fail}\right)$.

For $\theta = 0$, $\pi_{fail}(s^* \in [\underline{s}, \overline{s}]) = 0$ and hence $\mathcal{I} = \{pass, fail\}$ is never feasible, nor is $\mathcal{I} = \{fail\}$. We therefore get $C_{con} = 0$ and are in region (b) for any $\frac{c}{n} > 0$.

Proof of part (b1): We know that for $\widehat{s} < \frac{1}{2}$, $V_{\{pass\}}(s^* = 0) > V_{\{fail\}}(s^* = 2\widehat{s})$ and $V^{\max}_{pass} > V^{\max}_{fail}$. Using (60) it can be shown that for θ below a cut-off, $s^{\max}_{fail}$ lies at the interior of its admissible interval, i.e., $s^{\max}_{fail} \in (\widehat{s}, 2\widehat{s})$. For $\widehat{s} < \frac{1}{2}$ we always have $s^{\max}_{pass} \in [0, \widehat{s})$. Using (30) and (31) it can be shown that $\pi_{pass}\left(s^{\max}_{pass}\right) > \pi_{fail}\left(s^{\max}_{fail}\right)$ when both $s^{\max}_{pass}$ and $s^{\max}_{fail}$ are interior solutions. Hence, if it is feasible to induce $\mathcal{I} = \{fail\}$ at $s^{\max}_{fail} \in (\widehat{s}, 2\widehat{s})$ it is also feasible to induce $\mathcal{I} = \{pass\}$ at $s^{\max}_{pass}$, which the supervisor prefers. For θ above the cut-off, we get $s^{\max}_{fail} = 2\widehat{s}$. It can be shown that for $\widehat{s} < \frac{1}{2}$, $\pi_{pass}\left(s^{\max}_{pass}\right) > \pi_{fail}(2\widehat{s})$ and since $V_{\{pass\}}(s^* = 0) > V_{\{fail\}}(s^* = 2\widehat{s})$, where $V_{\{pass\}}(s^* = 0)$ is the lower bound of what can be implemented if the supervisor switches to $\mathcal{I} = \{pass\}$, the supervisor prefers $\mathcal{I} = \{pass\}$ for any necessary distortion $s^* < s^{\max}_{pass}$.

As $\frac{c}{n}$ increases, eventually, $\mathcal{I} = \{pass\}$ becomes either infeasible (when $\frac{c}{n} > \pi_{pass}(0)$) or undesirable (when the required distortion s^* is so strong as to imply $V_{\varnothing}(\widehat{s}) > V_{\{pass\}}(s^*)$). Whichever point is reached first pins down the cut-off $C_{\varnothing}$.

Proof of part (b2): We know that for $\widehat{s} > \frac{1}{2}$, $V_{\{pass\}}(s^* = 2\widehat{s} - 1) < V_{\{fail\}}(s^* = 1)$ and $V^{\max}_{pass} < V^{\max}_{fail}$. Regarding which $\mathcal{I}$ is feasible, it is useful to distinguish the following two cases. As $\pi_{pass}(0) = \pi_{fail}(1)$ the threshold θ_π defined in (68) is also the threshold such that (i) for $\theta \leq \theta_\pi$, , $\pi_{pass}(2\widehat{s} - 1) \geq \pi_{fail}(1)$ and (ii) for $\theta > \theta_\pi$, $\pi_{pass}(2\widehat{s} - 1) < \pi_{fail}(1)$. In case (i) we can further distinguish between the case (ia) where $\pi_{pass}\left(s^{\max}_{pass}\right) \geq \pi_{fail}(1)$ and (ib) where $\pi_{pass}\left(s^{\max}_{pass}\right) < \pi_{fail}(1)$ (there is again a cut-off in θ such that for sufficiently small values of θ case (ia) occurs. For brevity we do not provide the exact value of this threshold here).

Moreover, there are thresholds

$$
\begin{aligned}
\theta_1 &= \max\left\{0, \frac{1 - \widehat{s} - \widehat{s}\frac{\sigma}{2}}{(1 - \widehat{s})\left(1 - \frac{\sigma}{2}\right)}\right\}, \\
\theta_2 &= \frac{1 - \sigma\widehat{s}}{1 - \frac{\sigma}{2}},
\end{aligned}
$$

where $\theta_2 > \theta_1$ such that: for $\theta < \theta_1$, $s^{\max}_{pass} > 2\widehat{s} - 1$ and $s^{pass}_{fail} < 1$, for $\theta \in [\theta_1, \theta_2]$, $s^{\max}_{pass} > 2\widehat{s} - 1$ and $s^{pass}_{fail} = 1$ and finally for $\theta > \theta_2$, $s^{\max}_{pass} = 2\widehat{s} - 1$ and $s^{pass}_{fail} = 1$. Note that, depending on the values of σ and $\widehat{s}$, θ_1 can take any value between zero and one and θ_1 can be smaller or larger than θ_π.

When $\theta < \theta_1$, then $s_{pass}^{\max}$ is an interior solution and $\pi_{pass}\left(s_{pass}^{\max}\right) > \pi_{fail}\left(s_{fail}^{\max}\right)$. Thus there are values $\frac{c}{n} \in \left(\pi_{fail}\left(s_{fail}^{\max}\right), \pi_{pass}\left(s_{pass}^{\max}\right)\right)$ where it is infeasible to induce $\mathcal{I} = \{fail\}$ even though it is the preferred option. When $\theta = 0$, $\pi_{fail}\left(s^*\right) = 0$ and the supervisor sets $s^* \leq \widehat{s}$ for any $\frac{c}{n} > 0$. In this case, the supervisor optimally induces $\mathcal{I} = \{fail\}$ by setting s^* to the level that solves $\frac{c}{n} = \pi_{fail}\left(s^*\right)$ until $\frac{c}{n}$ reaches $\pi_{fail}\left(s_{fail}^{pass}\right)$. At that point the supervisor switches to $s^* = s_{pass}^{\max}$. When $\frac{c}{n} > \pi_{pass}\left(s_{pass}^{\max}\right)$ there are several possibilities. If θ is low enough so that $\pi_{pass}\left(s_{pass}^{\max}\right) \geq \pi_{fail}\left(1\right)$ (case (ia)) then feasibility requires that s^* is set to solve $\pi_{pass}\left(s^*\right) = \frac{c}{n}$ until $\frac{c}{n} = \pi_{pass}\left(s_{pass}^{\varnothing}\right)$ beyond which the supervisor sets $s^* = \widehat{s}$ and accepts $\mathcal{I} = \varnothing$. If θ is high enough so that $\pi_{pass}\left(s_{pass}^{\max}\right) < \pi_{fail}\left(1\right)$ (case (ib)), the supervisor may optimally switch back to conservatism, i.e., to an $s^* > \widehat{s}$ given by $\pi_{fail}\left(s^*\right) = \frac{c}{n}$ and $\mathcal{I} = \{fail\}$ is induced and then stay at the s^* that solves $\frac{c}{n} = \pi_{fail}\left(s^*\right)$ until $\frac{c}{n}$ hits $\min\left\{\pi_{fail}\left(s_{fail}^{\varnothing}\right), \pi_{fail}\left(1\right)\right\}$.

When $\theta \geq \theta_1$ the supervior prefers to stick to $\mathcal{I} = \{fail\}$ for all necessary distortions up to $s^* = 1$ rather than to induce $\mathcal{I} = \{pass\}$ by setting $s^* = s_{pass}^{\max}$. In case (ii) this is feasible and so s^* is determined by $\frac{c}{n} = \pi_{fail}\left(s^*\right)$ until $C_\varnothing = \pi_{fail}\left(1\right)$. In case (i) the supervisor also distorts to the point $s^* = 1$, but for $\frac{c}{n} > \pi_{fail}\left(1\right)$ may switch to $s^* = s_{pass}^{\max}$ until the required distortion is $s^* = s_{pass}^{\varnothing}$, which pins down $C_\varnothing$. QED

Proof of Lemma 4 : We use $\pi_{pass}\left(s^*\right)$ defined in (30) to compute the partial derivative of $\pi_{pass}\left(s^*\right)$ with respect to ε for the case in which the policy is conservative ($s^* \in (\hat{s}, \overline{s})$).

$$\frac{\partial \pi_{pass}\left(s^*\right)}{\partial \varepsilon} = \frac{\sigma}{4}\frac{\partial \Delta E^1}{\partial \varepsilon}\left(1 - s^{*2}\right).$$

Since $\frac{\partial \Delta E^1}{\partial \epsilon} = -\frac{R-D}{2} < 0$, $\pi_{pass}\left(s^*\right)$ decreases in ε.

Similarly, using the trading profit $\pi_{fail}\left(s^*\right)$ defined in Lemma 3, we compute the partial derivative for $s^* \in (\hat{s}, \overline{s})$:

$$\begin{aligned}
\frac{\partial \pi_{fail}\left(s^*\right)}{\partial \varepsilon} &= \frac{\sigma}{4}\theta\left\{\left(1 - \frac{\hat{s}}{s^*}\right)\left[\Delta E^1\left(2 - s^*\right)\left(s^* + \hat{s}\right) + 2\hat{s}E_l^1\right]\right\}' \\
&= \frac{\sigma}{4}\theta\left(1 - \frac{\hat{s}}{s^*}\right)\left[\frac{\partial \Delta E^1}{\partial \varepsilon}\left(2 - s^*\right)\left(s^* + \hat{s}\right) + 2\hat{s}\frac{\partial E_l^1}{\partial \varepsilon}\right] + \frac{\partial \pi_{fail}\left(s^*\right)}{\partial \hat{s}}\frac{\partial \hat{s}}{\partial \varepsilon} \\
&= \frac{\sigma}{4}\theta\left(s^* - \hat{s}\right)\frac{R-D}{2}\left(-2 + s^* + \hat{s}\right) + \frac{\partial \pi_{fail}\left(s^*\right)}{\partial \hat{s}}\frac{\partial \hat{s}}{\partial \varepsilon}.
\end{aligned}$$

We have $\left(-2 + s^* + \hat{s}\right) < 0$, $\frac{\partial \hat{s}}{\partial \varepsilon} > 0$ and

$$\frac{\partial \pi_{fail}\left(s^*\right)}{\partial \hat{s}} = \frac{\theta\sigma}{2s^*}\left[\Delta E^1\hat{s}\left(-2 + s^*\right) - E_l^1\left(2\hat{s} - s^*\right)\right].$$

$\frac{\partial \pi_{fail}(s^*)}{\partial \hat{s}}$ is negative for $s^* < 2\hat{s}$. We thus have $\frac{\partial \pi_{fail}(s^*)}{\partial \varepsilon} < 0$. QED

Proof of Lemma 5 Knowing that the test result (m) is now private, the speculator's strategy is no longer contingent on m. Given $s^* = \hat{s}$, the expected equity value conditional on ω is,

$$E[E_\omega^a|\omega] = \Pr\left(\hat{s} \leq s \leq 1|\omega\right)E_\omega^1 + \Pr\left(0 \leq s < \hat{s}|\omega\right)E_\omega^0$$

and the market maker's price is

$$\begin{aligned} P^{-n,n} &= \Pr(\hat{s} \leq s < 1 \cap \omega = h) E_h^1 + \Pr(\hat{s} \leq s < 1 \cap \omega = l) E_l^1 \\ &\quad + \Pr(s < \hat{s} \cap \omega = h) E_h^0 + \Pr(s < \hat{s} \cap \omega = l) E_l^0. \end{aligned}$$

which can be simplified to

$$P^{-n,n} = \Pr(\omega = h) E[E_\omega^a | \omega = h] + \Pr(\omega = l) E[E_\omega^a | \omega = l]$$

The expected trading profit $\pi_{ND}(s^*)$ for $s^* = \hat{s}$ is thus

$$\begin{aligned} \pi_{ND}(\hat{s}) &= \frac{1}{4}\sigma (E[E_\omega^a|\omega = h] - E[E_\omega^a|\omega = l]) \\ &= \frac{1}{4}\sigma \left[\left(1 - \hat{s}^2\right) E_h^1 - (1 - \hat{s})^2 E_l^1 \right] \\ &= \frac{1}{4}\sigma \left[\left(1 - \hat{s}^2\right) \Delta E^1 + 2\hat{s}(1 - \hat{s}) E_l^1 \right] \end{aligned}$$

QED

Proof of Lemma 6 : We first calculate the speculator's trading profits conditional on $m = pass$. If s^* is set above $\hat{s}$, the supervisor chooses to allow the bank to continue when learning $m = pass$ regardless of whether they are privately informed. The bank's debt value is thus D_h^1 if $\omega = h$ and D_l^1 otherwise. Anticipating that, the market maker sets the following price $P_{pass}^{-n,n}$ upon receiving an uninformative order flow:

$$\begin{aligned} P_{pass}^{-n,n} &= \Pr(\omega = h|s \geq s^*) D_h^1 + \Pr(\omega = l|s \geq s^*) D_l^1 \qquad (69) \\ &= \frac{1 + s^*}{2} D_h^1 + \frac{1 - s^*}{2} D_l^1. \end{aligned}$$

We can then compute the speculator's per unit trading profit conditional on $m = pass$ after observing the signal $z \in \{l, h\}$:

$$\begin{aligned} \pi_{D,pass} &= \frac{\sigma}{2} \Pr(\omega = h|m = pass) \left(D_h^1 - P_{pass}^{-n,n}\right) \\ &\quad - \frac{\sigma}{2} \Pr(\omega = l|m = pass) \left(D_l^1 - P_{pass}^{-n,n}\right) \qquad (70) \\ &= \frac{\sigma}{4} \left(D_h^1 - D_l^1\right) \left(1 - s^{*2}\right) \qquad (71) \end{aligned}$$

If however s^* is chosen between $2\widehat{s} - 1$ and $\widehat{s}$, the supervisor would not intervene except when it learns privately $s \in [s^*, \hat{s})$. Anticipating the supervisor's strategy, the market maker sets the trading price as below,

$$\begin{aligned} P_{pass}^{-n,n} &= \Pr(\hat{s} \leq s \leq 1 \cap \omega = h|s \geq s^*) D_h^1 + \Pr(\hat{s} \leq s \leq 1 \cap \omega = l|s \geq s^*) D_l^1 \qquad (72) \\ &\quad + \Pr(s^* \leq s \leq \hat{s} \cap \omega = h|s \geq s^*) \left[\theta D_h^0 + (1 - \theta) D_h^1\right] \\ &\quad + \Pr(s^* \leq s \leq \hat{s} \cap \omega = l|s \geq s^*) \left[\theta D_l^0 + (1 - \theta) D_l^1\right] \end{aligned}$$

We then compute the speculator's expectation of the bank's debt value given its private signal z, $z = \omega$, and uninformative order flows

$$\begin{aligned} E[D_\omega^a|pass, \omega] &= \Pr(\hat{s} \leq s \leq 1|s \geq s^* \cap \omega) D_\omega^1 \\ &\quad + \Pr(s^* \leq s < \hat{s}|s \geq s^* \cap \omega) \left[\theta D_\omega^0 + (1 - \theta) D_\omega^1\right] \end{aligned}$$

We have therefore

$$E[D_\omega^a|pass,h] = \frac{1-\hat{s}^2}{1-s^{*2}}D_h^1 + \frac{\hat{s}^2 - s^{*2}}{1-s^{*2}}\left[\theta D_h^0 + (1-\theta)\,D_h^1\right] \tag{73}$$

$$E[D_\omega^a|pass,l] = \frac{(1-\hat{s})^2}{(1-s^*)^2}D_l^1 + \frac{2(\hat{s}-s^*) - (\hat{s}^2 - s^{*2})}{(1-s^*)^2}\left[\theta D_l^0 + (1-\theta)\,D_l^1\right] \tag{74}$$

Comparing the market maker's price given by (72) to the speculator's conditional expectation of the bank's debt value as in respectively (73) and (74), we infer that buying (selling) when $\omega = h$ $(\omega = l)$ is optimal. This then allows us to calculate the per unit trading profit following $m = pass$.

$$\begin{aligned}
\pi_{D,pass} &= \frac{\sigma}{2}\Pr(\omega = h|m = pass)\left(E[D_\omega^a|pass,h] - P_{pass}^{-n,n}\right) \\
&\quad + \frac{\sigma}{2}\Pr(\omega = l|m = pass)\left(P_{pass}^{-n,n} - E[D_\omega^a|pass,l]\right) \\
&= \frac{\sigma}{4}\left(1 - s^{*2}\right)\left(E[D_\omega^a|pass,h] - E[D_\omega^a|pass,l]\right)
\end{aligned}$$

$$\begin{aligned}
= \; & \frac{\sigma}{4}\left\{\Delta D^1\left[\left(1-s^{*2}\right) - \theta\left(\hat{s}^2 - s^{*2}\right)\right] - 2\theta\frac{(\hat{s}-s^*)(1-\hat{s})}{1-s^*}\left(D_l^0 - D_l^1\right)\right\} \\
& + \frac{\sigma}{4}\left(\hat{s}^2 - s^{*2}\right)\theta\Delta D^0
\end{aligned}$$

which decreases in s^*.

Finally if however s^* is chosen below $2\widehat{s}-1$, the supervisor would always intervene except when she learns privately $s \in [\hat{s}, 1]$. Anticipating the supervisor's strategy, the market maker sets the trading price as below,

$$\begin{aligned}
P_{pass}^{-n,n} &= \Pr(\hat{s} \le s \le 1 \cap \omega = h|s \ge s^*)\left[\theta D_h^1 + (1-\theta)\,D_h^0\right] \\
&\quad + \Pr(\hat{s} \le s \le 1 \cap \omega = l|s \ge s^*)\left[\theta D_l^1 + (1-\theta)\,D_l^0\right] \\
&\quad + \Pr(s^* \le s < \hat{s} \cap \omega = h|s \ge s^*)\,D_h^0 + \Pr(s^* \le s < \hat{s} \cap \omega = l|s \ge s^*)\,D_l^0.
\end{aligned} \tag{75}$$

The speculator's expectation of the bank's debt value is, given his private signal z, $z = \omega$, and uninformative order flow

$$\begin{aligned}
E[D_\omega^a|pass,\omega] &= \Pr(\hat{s} \le s \le 1|s \ge s^* \cap \omega)\left[\theta D_\omega^1 + (1-\theta)\,D_\omega^0\right] \\
&\quad + \Pr(s^* \le s < \hat{s}|s \ge s^* \cap \omega)\,D_\omega^0.
\end{aligned}$$

Therefore,

$$E[D_\omega^a|pass,h] = \frac{1-\hat{s}^2}{1-s^{*2}}\left[\theta D_h^1 + (1-\theta)\,D_h^0\right] + \frac{\hat{s}^2 - s^{*2}}{1-s^{*2}}D_h^0 \tag{76}$$

$$E[D_\omega^a|pass,l] = \frac{(1-\hat{s})^2}{(1-s^*)^2}\left[\theta D_l^1 + (1-\theta)\,D_l^0\right] + \frac{2(\hat{s}-s^*) - (\hat{s}^2 - s^{*2})}{(1-s^*)^2}D_l^0 \tag{77}$$

We then compute the per unit trading profit following $m = pass$ for $s^* < 2\widehat{s}-1$,

$$\pi_{D,pass} = \frac{\sigma}{4}\left(1 - s^{*2}\right)\left(E[D_\omega^a|pass,h] - E[D_\omega^a|pass,l]\right)$$

$$
\begin{aligned}
&= \frac{\sigma}{4}\left\{\theta \Delta D^1\left(1-\hat{s}^2\right)-2\theta\frac{\left(\hat{s}-s^*\right)\left(1-\hat{s}\right)}{1-s^*}\left(D_l^0-D_l^1\right)\right\} \\
&\quad +\frac{\sigma}{4}\left[1-s^{*2}-\theta\left(1-\hat{s}^2\right)\right]\Delta D^0
\end{aligned}
$$

In the second part of this proof, we compute the speculator's profits in trading debt following a signal $m = fail$. If $s^* < \hat{s}$, the supervisor will always intervene, regardless whether they are privately informed. The bank's debt value is thus D_h^0 if $\omega = h$ and D_l^0 otherwise. Anticipating that, the market maker sets the following price $P_{fail}^{-n,n}$ upon receiving an uninformative order flow:

$$
\begin{aligned}
P_{fail}^{-n,n} &= \Pr\left(\omega = h | s < s^*\right) D_h^0 + \Pr\left(\omega = l | s < s^*\right) D_l^0 \\
&= \frac{s^*}{2} D_h^0 + \frac{2-s^*}{2} D_l^0.
\end{aligned}
\tag{78}
$$

We can then compute the speculator's per unit trading profit conditional on $m = fail$ after observing the signal $z \in \{l, h\}$:

$$
\begin{aligned}
\pi_{D,fail} &= \frac{\sigma}{2}\Pr\left(\omega = h | m = fail\right)\left(D_h^0 - P_{pass}^{-n,n}\right) \\
&\quad -\frac{\sigma}{2}\Pr\left(\omega = l | m = fail\right)\left(D_l^0 - P_{pass}^{-n,n}\right) \\
&= \frac{\sigma}{4}s^*\left(2-s^*\right)\left(D_h^0 - D_l^0\right)
\end{aligned}
$$

If $\widehat{s} \leq s^* \leq 2\widehat{s}$, the supervisor always intervenes except when she is informed of the value of s and $s \in [\hat{s}, s^*]$. As in the previous case, we compute the price and the speculator's conditional expectation of the bank's debt value,

$$
\begin{aligned}
P_{fail}^{-n,n} &= \Pr\left(s < \hat{s} \cap \omega = l | s < s^*\right) D_h^0 + \Pr\left(s < \hat{s} \cap \omega = h | s < s^*\right) D_l^0 \\
&\quad + \Pr\left(\widehat{s} \leq s < s^* \cap \omega = h | s < s^*\right)\left[\theta D_h^1 + (1-\theta) D_h^0\right] \\
&\quad + \Pr\left(\hat{s} \leq s < s^* \cap \omega = l | s < s^*\right)\left[\theta D_l^1 + (1-\theta) D_l^0\right]
\end{aligned}
$$

and

$$
E[D_\omega^a | fail, h] = \frac{\hat{s}^2}{s^{*2}} D_h^0 + \frac{s^{*2}-\hat{s}^2}{s^{*2}}\left[\theta D_h^1 + (1-\theta) D_h^0\right] \tag{79}
$$

$$
E[D_\omega^a | fail, l] = \frac{2\hat{s}-\hat{s}^2}{2s^*-s^{*2}} D_l^0 + \frac{2(s^*-\hat{s})-(s^{*2}-\hat{s}^2)}{2s^*-s^{*2}}\left[\theta D_l^1 + (1-\theta) D_l^0\right] \tag{80}
$$

We then compute the speculator's expected per unit profit from trading debt when $m = fail$ for the case of $\widehat{s} \leq s^* \leq 2\widehat{s}$,

$$
\begin{aligned}
\pi_{D,fail} &= \frac{\sigma}{4}s^*\left(2-s^*\right)\left(E[D_\omega^a | fail, h] - E[D_\omega^a | fail, l]\right) \\
&= \frac{\sigma}{4}\left\{\theta\left(1-\frac{\hat{s}}{s^*}\right)\left[\left(\Delta D^1 - \Delta D^0\right)\left(2-s^*\right)\left(s^*+\hat{s}\right) + 2\hat{s}\left(D_l^1 - D_l^0\right)\right]\right\} \\
&\quad +\frac{\sigma}{4}s^*\left(2-s^*\right)\Delta D^0.
\end{aligned}
\tag{81}
$$

Finally we compute the trading profit following $m = fail$ when $s^* > 2\widehat{s}$. In this case, the supervisor would never intervene upon learning $m = fail$ except when it observes

$s < \widehat{s}$. This yields

$$\begin{aligned} P_{fail}^{-n,n} &= \Pr\left(s < \hat{s} \cap \omega = l | s < s^*\right)\left[\theta D_l^0 + (1-\theta) D_l^1\right] \\ &\quad + \Pr\left(s < \hat{s} \cap \omega = h | s < s^*\right)\left[\theta D_h^0 + (1-\theta) D_h^1\right] \\ &\quad + \Pr\left(\widehat{s} \leq s < s^* \cap \omega = h | s < s^*\right) D_h^1 + \Pr\left(\hat{s} \leq s < s^* \cap \omega = l | s < s^*\right) D_l^1, \end{aligned}$$

and

$$E[D_\omega^a | fail, h] = \frac{s^{*2} - \hat{s}^2}{s^{*2}} D_h^1 + \frac{\hat{s}^2}{s^{*2}}\left[\theta D_h^0 + (1-\theta) D_h^1\right], \tag{82}$$

$$E[D_\omega^a | fail, l] = \frac{2(s^* - \hat{s}) - (s^{*2} - \hat{s}^2)}{2s^* - s^{*2}} D_l^1 + \frac{2\hat{s} - \hat{s}^2}{2s^* - s^{*2}}\left[\theta D_l^0 + (1-\theta) D_l^1\right]. \tag{83}$$

Per unit trading profits are thus

$$\begin{aligned} \pi_{D,fail} &= \frac{\sigma}{4} s^* (2 - s^*) \left(E[D_\omega^a | fail, h] - E[D_\omega^a | fail, l]\right) \\ &= \frac{\sigma}{4s^*} \Delta D^1 (2 - s^*) \left(s^{*2} - \theta \hat{s}^2\right) \\ &\quad - \frac{\sigma}{4s^*}\left[2\theta \hat{s}(s^* - \hat{s})\left(D_l^0 - D_l^1\right) - (2 - s^*)\theta \hat{s}^2 \Delta D^0\right]. \end{aligned}$$

Hence,

$$\pi_{D,fail}\left(s^* > 2\widehat{s}\right) = \frac{\sigma}{4}\left[\theta\left[(1-\widehat{s})\hat{s}\left(\Delta D^0 - \Delta D^1\right) + \hat{s}\left(D_l^1 - D_l^0\right)\right] + 4\widehat{s}(1-\widehat{s})\Delta D^1\right] \tag{84}$$

Comparing (81) to (84), we obtain the magnitude of the jump at the point $s^* = 2\hat{s}$ shown in Figure 5, which is

$$-\sigma \hat{s}(1-\hat{s})(1-\theta)\left(\Delta D^1 - \Delta D^0\right).$$

Hence, $\pi_{D,fail}$ jumps upwards at $s^* = 2\hat{s}$ iff $\Delta D^1 > \Delta D^0$. QED

Proof of Corollary 2 : In this proof we skip the case of having $s^* < 2\widehat{s} - 1$ in which the monitoring policy can never be optimal. When s^* is chosen below $\widehat{s}$, we know already that trading in equity following a $fail$ signal cannot be profitable, in which case the speculator will always choose to trade debt claims (whenever feasible) and his profit is independent of ε based on Lemma 6. In the situation where s^* is set between $\widehat{s}$ and $2\widehat{s}$ and the bank passes the stress test, we know from Lemma 3 and Lemma 6 that the magnitude of $\pi_{D,pass}(s^*) - \pi_{pass}(s^*)$ depends completely on $\Delta D^1 - \Delta E^1$, which increases in ε. The intuition is discussed in the main text.

We next check two other cases. The first one is when the bank fails the stress test given that s^* is set between $\widehat{s}$ and $2\widehat{s}$, and the second is when the bank passes the test for s^* chosen below $\widehat{s}$. The other possibilities are discussed in the main text. For the first case, we need to compute the difference between $\pi_{D,fail}(s^*)$ in Lemma 6 and $\pi_{fail}(s^*)$ in Lemma 3. The difference is

$$\begin{aligned} &\pi_{D,fail}\left(s^*\right) - \pi_{fail}\left(s^*\right) \qquad (85) \\ = &\frac{\sigma(2-s^*)}{4s^*}\theta\left[\left(\Delta D^1 - \Delta E^1\right)(s^{*2} - \hat{s}^2) + \Delta D^0 \hat{s}^2 + \frac{2\left(D_l^1 - E_l^1 - D_l^0\right)\hat{s}(s^* - \hat{s})}{2 - s^*}\right] \\ &+ \frac{\sigma(2-s^*)}{4s^*}\Delta D^0 (1-\theta) s^{*2} \end{aligned}$$

The first-order derivative of (85) with respect to ε is,

$$\begin{aligned}
&\frac{\sigma}{4}\theta\frac{2r+R-2D}{2}\left(s^{*}-\hat{s}\right)\left(2-s^{*}-\hat{s}\right) \\
&+\frac{\sigma}{4}\theta\frac{\partial\hat{s}}{\partial\varepsilon}\left[\left(D_{h}^{1}-E_{h}^{1}-D_{h}^{0}\right)\frac{2\left(s^{*}-2\hat{s}\right)}{s^{*}}-\left(\Delta D^{0}-\Delta D^{1}-\Delta E^{1}\right)2\left(1-\hat{s}\right)\right]
\end{aligned} \tag{86}$$

where the first term is obviously positive as $\frac{2r+R-2D}{2}>0$ and $s^{*}>\hat{s}$. We can show that

$$\Delta D^{0}-\Delta D^{1}-\Delta E^{1}=\frac{\varepsilon}{2}\left(R-2r\right)$$

and

$$D_{h}^{1}-E_{h}^{1}-D_{h}^{0}=\left(2-p\right)\left(D-A_{h}\right)-\left(1+\delta\right)\left[\frac{p}{2}R+\left(1-p\right)r\right]$$

Using the property of δ given in (18) and $\frac{\partial\hat{s}}{\partial\varepsilon}=\frac{1-\hat{s}}{\varepsilon}$, we can rewrite (86) and show it is higher than

$$\begin{aligned}
&\frac{\sigma}{4}\theta\left(R-D\right)\left(s^{*}-\hat{s}\right)\left(2-s^{*}-\hat{s}\right)+ \\
&\left[\left(2-p\right)\left(D-A_{h}-r\right)+p\left(r-R\right)\right]\frac{2\left(s^{*}-2\hat{s}\right)}{s^{*}}+\varepsilon\left(2r-R\right)\left(2\frac{s^{*}-\hat{s}}{s^{*}}-\hat{s}\right)
\end{aligned}$$

which is positive. Hence, the speculator may find it more profitable to trade debt when the risk-shifting problem is sufficiently severe (high ε).

Next, we check the case in which s^{*} is chosen below $\widehat{s}$. Upon a signal *pass*, the speculator compares $\pi_{D,pass}$ if he trades debt with a profit π_{pass} if he trades equity. Using the results from Lemma 3 and 6, we can compute the difference in trading profits,

$$\begin{aligned}
&\pi_{D,pass}-\pi_{pass} \\
=\;&\frac{\sigma}{4}\left(\Delta D^{1}-\Delta E^{1}\right)\left(1-s^{*2}\right) \\
&+\frac{\sigma}{4}\theta\left(\hat{s}-s^{*}\right)\left(s^{*}+\hat{s}\right)\left(D_{h}^{0}-D_{l}^{1}+E_{h}^{1}\right) \\
&-\frac{\sigma}{4}\theta\left(-\hat{s}^{2}+\left(s^{*}\right)^{2}-2\frac{\left(\hat{s}-s^{*}\right)\left(1-s^{*}\right)}{1-s^{*}}\right)\left(-D_{l}^{1}+E_{l}^{1}+D_{l}^{0}\right)
\end{aligned} \tag{87}$$

As in the previous case, the first term $\left(\Delta D^{1}-\Delta E^{1}\right)\left(1-s^{*2}\right)$ increases in ε and becomes positive when ε is above the threshold given in (37). In the second term, we have

$$\begin{aligned}
&D_{h}^{0}-D_{l}^{1}+E_{h}^{1} \\
=\;&\delta\left(\frac{p}{2}R+\left(1-p\right)r\right)+A_{h}+\left(1-\frac{p}{2}\right)\left(A_{h}-D\right)+\frac{p}{2}\left(R-D\right)+\frac{1}{2}\left(2r-D\right)\varepsilon
\end{aligned}$$

which also increases in ε as we know $2r-D>0$ from assumption (9) and (13). Finally, $\left(-D_{l}^{1}+E_{l}^{1}+D_{l}^{0}\right)$ in the third term increases in ε, and $-\left(-\hat{s}^{2}+\left(s^{*}\right)^{2}-2\frac{\left(\hat{s}-s^{*}\right)\left(1-s^{*}\right)}{1-s^{*}}\right)'=\frac{2\left(1+s^{*}\right)\left(1-\hat{s}\right)}{1-s^{*}}$ which is positive. Hence, the speculator should prefer to trade in debt if ε is sufficiently high. QED

9 APPENDIX B

In this Appendix we show how the supervisor's intervention can be interpreted as a bail-out, whereby a capital shortfall is made up by an equity issue plus government money. Let us also suppose that the government finds it costly to have the bank default on its creditors *ex post*, maybe because such a bank failure exerts negative externalities on the rest of the banking system. This will provide an additional motive for the supervisor to inject government money into a poorly capitalized bank.

The bank's incentive to behave prudently when its balance sheet is impaired can be restored by injecting fresh equity capital A_h. The value of the bank's equity, after a bank has raised capital A_h in state $\omega = l$ is

$$\frac{p}{2}\left(R + A_h - D\right) + (1-p)\left(r + A_h - D\right).$$

Given our prior parametric assumptions, even if the new equity holders are given 100% of the ownership of the bank, the debt overhang is so severe that they cannot break even, that is

$$\frac{p}{2}\left(R + A_h - D\right) + (1-p)\left(r + A_h - D\right) < A_h.$$

Recapitalizing the bank therefore requires (i) wiping out the old equity holders entirely, and (ii) an injection of government funds of magnitude

$$\begin{aligned} b &\equiv A_h - \frac{p}{2}\left(R + A_h - D\right) + (1-p)\left(r + A_h - D\right) \\ &= \frac{p}{2}\left(2r + A_h - R - D\right) + D - r > 0. \end{aligned}$$

Note that in this specification, there is a cost to the supervisor of bailing out the bank. One could imagine that some of the cost is recovered because the supervisor holds claims against the bank's future cash flows. Suppose for simplicity, that the bail-out money is a pure subsidy.

Suppose that a bank default exerts an externality of magnitude k. The supervisor maximizes the expected value of the bank minus the expected cost of a bank failure and minus the subsidy. Note that the new money raised by fresh equity washes out of the planner's objective function since this money was in the economy regardless. As before, a well capitalized bank will not engage in risk shifting and will only default if its cash flow is 0.[20] Hence, the supervisor's payoff from allowing a bank to continue without intervention in the high state $\omega = h$ is:

$$V_h^1 = \frac{p}{2}R + (1-p)r + A_h - \frac{p}{2}k,$$

while her payoff from re-capitalizing a bank in the good state is

$$V_h^0 = \frac{p}{2}R + (1-p)r + A_h - \frac{p}{2}k - b,$$

i.e., the supervisor loses from providing a subsidy to a well capitalized bank and $V_h^1 > V_h^0$.

[20] Assume that $2A_h < D$ so that even a good bank that was recapitalized erroneously will default in this case.

The supervisor's payoff when failing to recapitalize a poorly capitalized bank is

$$V_l^1 = \frac{p+\varepsilon}{2}R + (1-p-\varepsilon)r - \left(1-p-\varepsilon+\frac{p+\varepsilon}{2}\right)k,$$

while the payoff from recapitalizing is

$$V_l^0 = \frac{p}{2}R + (1-p)r - \frac{p}{2}k - b.$$

Hence, it is beneficial to recapitalize in the low state if $V_l^0 > V_l^1$, i.e., when

$$\frac{p}{2}R + (1-p)r > b - \left(1-p-\varepsilon+\frac{\varepsilon}{2}\right)k + \frac{p+\varepsilon}{2}R + (1-p-\varepsilon)r.$$

In other words, the benefit from a recapitalization is now two-fold. First, it increases bank value because a recapitalized bank refrains from risk shifting, and second, it reduces the failure externality. The cost of recapitalizing comes in the form of the social cost of the subsidy to the bank. One can now redo the previous analysis with the new expressions for V_ω^a.

References

[1] Bhattacharya, S., 1982, "Aspects of Monetary and Banking Theory and Moral Hazard," *Journal of Finance*, 37(2), 371 - 384.

[2] Bond, P., A. Edmans and I. Goldstein, 2012, "The Real Effects of Financial Markets," *Annual Reviews of Financial Economics*, 4, 339-360.

[3] Bond, P., and I. Goldstein, 2015, "Government Intervention and Information Aggregation by Prices," *Journal of Finance*, 70, 2777-2811.

[4] Bond, P., I. Goldstein and E.S. Prescott, 2010, "Market Based Corrective Actions," *Review of Financial Studies*, 23, 781 - 820.

[5] Bouvard, M., P. Chaigneau and A. de Motta, 2015, "Transparency in the Financial System: Rollover Risk and Crises," *Journal of Finance*, 70(4), 1805-1837.

[6] Calzolari, G., J.-E. Colliard, and G. Loranth, 2019, "Multinational Banks and Supranational Supervision," *Review of Financial Studies*, 32, 2997-3035.

[7] Carletti, E., G. Dell'Ariccia and R. Marquez, 2020, "Supervisory Incentives in a Banking Union," forthcoming, *Management Science*.

[8] Chan, Y., S. Greenbaum and A. Thakor, 1992, "Is Fairly Priced Deposit Insurance Possible?" *Journal of Finance*, 47, 227-245.

[9] Colliard, J.-E., 2019, "Strategic Selection of Risk Models and Bank Capital Regulation," *Management Science*, 65, 2591-2606.

[10] Curry, T.J., G.S. Fissel, and G.A. Hanweck, 2008, "Equity Market Information, Bank Holding Company Risk, and Market Discipline," *Journal of Banking and Finance*, 32, 807-819.

[11] Cutura, A., 2018, "Debt Holder Monitoring and Implicit Guarantees: Did the BRRD Improve Market Discipline?" unpublished manuscript, Goethe University Frankfurt.

[12] Dewatripont, M. and J. Tirole, 1994, "A Theory of Debt and Equity: Diversity of Securities and Manager-Shareholder Congruence," *Quarterly Journal of Economics*, 109, 1027-1054.

[13] Dow, J., I. Goldstein and A. Guembel, 2017, "Incentives for Information Production in Markets where Prices Affect Real Investments," *Journal of the European Economic Association*, 15(4), 877-909.

[14] Fecht, F. R. Inderst and S. Pfeil, 2017, "A Theory of the Boundaries of Banks with Implications for Financial Integration and Regulation," Unpublished manuscript.

[15] Flannery, M.J and R.R. Bliss, 2019, "Market Discipline in Regulation: Pre- and Post Crisis", in *Oxford Handbook of Banking.*

[16] Freixas, X. and J.-C. Rochet, 1995, "Fair Pricing of Deposit Insurance. Is it Possible? Yes. Is it Desirable? No." Finance and Banking discussion paper #4, Universitat Pompeu Fabra.

[17] Freixas, X., G. Loranth, and A.D. Morrison, 2007, Regulating Financial Conglomerates," *Journal of Financial Intermediation*, 16, 479 - 514.

[18] Georgescu, O.-M., M. Gross, D. Kapp and C. Kok, 2017, "Do Stress Tests Matter? Evidence from the 2014 and 2016 Stress Tests," European Central Bank working paper #2054.

[19] Giammarino, R.M., T.R. Lewis and D. Sappington, 1993, "An Incentive Approach to Banking Regulation," *Journal of Finance*, 48, 1523-1542.

[20] Goldstein, I. and Y. Leitner, 2018, "Stress Tests and Information Disclosure," *Journal of Economic Theory*, 177, 34-69.

[21] Goldstein, I. and L. Yang, 2017, "Information Disclosure in Financial Markets," *Annual Review of Financial Economics*, 9, 101-125.

[22] Goldstein, I. and L. Yang, 2017, "Good Disclosure, Bad Disclosure," forthcoming, *Journal of Financial Economics.*

[23] Goldstein, I. and H. Sapra, 2014, "Should Banks' Stress Test Results be Disclosed? An Analysis of the Costs and Benefits," Foundations and Trends in Finance, 8, 1-54.

[24] Gorton, G. and L. Huang, 2004, "Liquidity, Efficiency and Bank Bailouts," *American Economic Review*, 94(3), 455-483.

[25] Hellmann, T.F., K.C. Murdock, and J.E. Stiglitz, 2000, "Liberalization, Moral Hazard in Banking, and Prudential Regulation: Are Capital Requirements Enough?" *American Economic Review*, 90(1), 147 - 165.

[26] Holmström, B., & J. Tirole, 1993, "Market liquidity and performance monitoring", *Journal of Political Economy*, 101(4), 678-709.

[27] Kamenica, E. and M. Gentzkow, 2011, "Bayesian Persuasion," *American Economic Review*, 101(6), 2590-2615.

[28] Kyle, A., 1985, "Continuous Auctions and Insider Trading," *Econometrica*, 53, 1315 - 1335.

[29] Leitner, Y and B. Williams, 2018, "Model Secrecy and Stress Tests," unpublished manuscript.

[30] Monnet, C. and E. Quintin, 2017, "Rational Opacity," *Review of Financial Studies*, 30, 4317–4348.

[31] Morrison, A. and L. White, 2005, "Crises and Capital Requirements in Banking," *American Economic Review*, 95(5), 1548 - 1572.

[32] Orlov, D., P. Zryumov and A. Skrzypacz. 2018, "Design of Macro-Prudential Stress Tests," Stanford University Graduate School of Business Research Paper #17-41.

[33] Petrella, G. and A. Resti, 2013, "Supervisors as Information Producers: Do Stress Tests Reduce Bank Opaqueness?" *Journal of Banking and Finance*, 37(12), 5406-5420.

[34] Rochet, J.-C., 1992, "Capital Requirements and the Behaviour of Commercial Banks," *European Economic Review*, 36, 1137 - 1170.

[35] Shapiro, J. and J. Zeng, 2018, "Stress Testing and Bank Lending," unpublished manuscript.

[36] Siemroth, C., 2019, "The Informational Content of Prices When Policy Makers React to Financial Markets," *Journal of Economic Theory*, 179, 240-274.

[37] Szydlowski, M., 2020, "Optimal Financing and Disclosure," forthcoming *Management Science.*

[38] Williams, B., 2017, "Stress Tests and Bank Portfolio Choice," unpublished manuscript, New York University.