Two-Sided Platforms and Biases in Technology Adoption*

Jay Pil Choi† Doh-Shin Jeon‡

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Abstract

We investigate the relationship between market structure and platforms’ incentives to adopt technological innovations in two-sided markets, where platforms may find it optimal to charge zero price on the consumer side and to extract surplus on the advertising side. We consider innovations that affect the two sides in an opposite way. We compare private incentives with social incentives and find that the bias in technology adoption depends crucially on whether the non-negative pricing constraint binds or not. Our results provide a rationale for a tougher competition policy to curb concentration if competition authorities put more weight on consumer surplus in welfare calculations.

JEL Codes: D4, L1, L5

Key Words: Technology Adoption, Two-Sided Platforms, Non-Negative Pricing Constraint, Pass-through

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†Department of Economics, Michigan State University, 220A Marshall-Adams Hall, East Lansing, MI 48824 -1038. E-mail: choijay@msu.edu.

‡Toulouse School of Economics, University of Toulouse Capitole. E-mail: dohshin.jeon@gmail.com.
1 Introduction

The public sentiment regarding digital platforms has changed recently since the revelation of the Cambridge Analytica scandal with people becoming more concerned about market concentration and big platforms’ market power (Kahn, 2016). There have been several initiatives proposing modification of the current framework of competition policy in order to promote competition and curb concentration of market power in the area of digital platforms; see for instance the ACCC report (2019), the CMA report (2020), the Furman report (2019), the Stigler report (Stigler Committee on Digital Platforms, 2019), the Vestager report (Crémer, de Montjoye and Schweitzer, 2019).

A challenge to competition policy in the area of digital platforms arises from the fact that many two-sided platforms provide free services to consumers and generate revenue by charging the other side such as advertisers or application developers (Rochet and Tirole, 2005; Amelio and Jullien, 2012; Choi and Jeon, forthcoming). When the service is free, consumer harm from the exercise of market power does not take the form of a higher price. Instead, consumer harm is likely to be manifested in terms of lower quality of service, more nuisance from advertisements or less privacy protection (Newman, 2015, 2016). Furthermore, the digital platform industry is a dynamic one in which innovations play a major role. Therefore, it is crucial to understand how market power shapes digital platforms’ incentives to innovate. In fact, all reports mentioned above commonly argue that a major harm from concentration of platform market power consists of distortions in innovation incentives. To quote the CMA report (2020, p.7),

"First, competition problems may inhibit innovation and the development of new, valuable services for consumers. ....This impact on innovation is likely to be the largest source of consumer harm."

However, to the best of our knowledge, there has been no formal investigation of the relationship between platform market power and innovation incentives. This paper attempts to fill this gap by addressing how market power affects platforms’ incentives to adopt technological innovations. More specifically, we consider two-sided platforms which may find it optimal to charge zero price on the consumer side and to extract surplus on the advertising side. Adoption of innovations can affect both consumer gross surplus and advertiser gross surplus. We characterize private incentives to trade off consumer surplus reduction (respectively, increase) with advertiser surplus increase (respectively,
reduction) and compare them to social incentives across different market structures: a monopoly platform and duopoly competitive bottleneck. In particular, we focus on which kind of biases in innovation adoption incentive arise due to platform market power. We endogenously derive conditions under which platforms provide free services and point out the importance of the non-negative price constraint (NPC) by showing that the bias in technology adoption depends crucially on whether the services are provided for free (i.e., the NPC is binding) or at a positive price (i.e., the NPC is not binding). This result suggests that the formulation of optimal antitrust policies towards the platform market can be substantially different for markets where services are provided for free.

Our analysis can also be reinterpreted as platforms’ incentives to adopt certain business policies that create trade-offs between the consumer and the advertiser side. First, a platform’s privacy policy can be interpreted as having a similar effect as a technology adoption in that collection of consumers’ sensitive information may impose privacy costs on the consumers, but may help increase advertising revenues. A second policy available to platforms concerns "ad load"; an increase in ad load would decrease consumer surplus but increases advertising revenues per consumer. "Search engines like Google can determine the overall limit on the number of ads that appear in search results and how these ads are presented alongside organic search results (the CMA Report, 2020, p. 229)." Similarly, "Facebook can directly set the ad load by determining the ad gap – the ratio of advertising to organic content users see when interacting with the platform (the CMA Report, 2020, p. 256)." Finally, big tech platforms can control the balance, in their ad auction mechanisms, between ad price and quality (i.e., relevance of ads shown to users) by choosing how much weight to place on quality metrics in determining the winning bid for ad slots; a lower weight on quality and relevance metrics induces higher bidding prices and generate more revenue, but reduces the quality of the platform services for users (the CMA Report, 2020, p. 230 and p. 256). The choices of platforms on these policy dimensions can also be analyzed in our theoretical framework.

In one-sided markets, one can consider either a quality-increasing innovation or a cost-reducing innovation and study private incentives to adopt it by incurring a fixed cost and compare it with a social planner’s incentives. What is interesting in a two-sided platform is that one can consider innovations that affect the two sides in an opposite way and study a platform’s incentive to trade-off the gain from one side against the loss from the other side without an explicit consideration of adoption costs. To illustrate this point, consider a potential technology without any fixed cost of adoption. If it affects both sides
in the same way, private incentives are aligned with social incentives: all platforms want to adopt innovations which increase surplus on both sides and no platforms wants to adopt innovations which reduce surplus on both sides. In contrast, for instance, when we consider an innovation which increases advertiser surplus but reduces consumer surplus, private incentives to adopt such innovation may not be aligned with social incentives. We characterize both the locus of technology adoption that yields the same profit to a platform and the one that yields the same welfare and compare the two. We say that a platform’s innovation incentive is biased toward a consumer-surplus increasing (hence, advertiser-surplus decreasing) technology when it adopts a consumer-surplus increasing technology which would not be adopted by a social planner. Symmetrically, we say that a platform’s innovation incentive is biased toward an advertiser-surplus increasing (hence, consumer-surplus decreasing) technology when it adopts an advertiser-surplus increasing technology which would not be adopted by a social planner.

To analyze potential market biases in technology adoption for platform markets, we develop a stylized model of two-sided markets with consumers on one side and advertisers on the other side. We consider a typical situation in which each consumer the platforms serve can generate additional surplus on the advertising side (represented by parameter $\beta$). We show that a zero price for consumers arises endogenously when $\beta$ is sufficiently large and show that the endogenously derived market price can be a critical determinant of the direction of biases in technology adoption.

We first consider a monopoly platform and analyze the benchmark case in which the platform is able to extract the whole advertising surplus. In such a case, we establish a key result of "pass-through rate equalization" when the NPC is not binding and the services are provided at a positive price. This result shows that consumer net benefit from an increase in the gross value of the services to consumers (and a corresponding price increase) is equal to the one from an increase in the gross surplus on the advertising side, which is passed over to consumers in terms of a reduced price. Even if the platform does not internalize these external effects on consumers, as the magnitudes of these two external effects are the same, there is no bias in technology adoption choice by the platform compared to the social optimum. This result immediately implies that if the platform cannot extract all the surplus from the advertising side, the platform would have biases toward consumer-surplus increasing technology.

In contrast, when the NPC is binding and the services are provided for free, the price does not respond any more to an increase in the value of services, which leads to
biases toward advertiser-surplus increasing technology. At first blush, this result may be considered obvious because there is no revenue from the consumer side with a zero price. However, recall that we derive zero price endogenously and this happens when the advertising revenue per consumer is sufficiently large. This means that when the NPC is binding, the platform has a higher incentive to attract additional consumers. As the price is already zero, the only way to attract more consumers is to increase the value of the services (represented by $u$) it provides. In fact, we show that the platform values an increase in $u$ relatively more than an increase in $\beta$ compared to the case where the services are provided at a positive price. Nonetheless, the platform’s technology adoption is biased toward $\beta$ because a social planner values an increase in $u$ even more than the platform does when the NPC binds.

We also extend our analysis to a duopoly model of competitive bottleneck (Armstrong, 2005, and Armstrong and Wright, 2007) to investigate implications of competition on technology adoption. When the NPC is not binding, we obtain a similar pass-through rate equalization result even for the duopoly case with strategic effects and show that technology adoption choice is biased toward the consumer-side as in the monopoly case. However, when the NPC is binding, additional strategic effects can overturn the result in the monopoly case. More specifically, we show that when competition is weak, we have a similar result to the monopoly case with technology adoption biased toward the advertiser side. In contrast, if competition is sufficiently strong, business stealing effects can lead to a bias toward the consumer side.

Our results allow us to make the following predictions regarding digital platforms which charge zero price to consumers as they monetize consumer attention. Initially when they are nascent and face fierce competition, they have strong incentives to innovate in order to increase the consumer side surplus. However, once the market tips to them or after their market power becomes entrenched, the same platforms, which were consumer advocates, have strong incentives to introduce innovations/policies that increase the advertiser side surplus to the detriment of consumer surplus. Therefore, our results provide a rationale for a tougher competition policy to curb concentration if competition authorities are more concerned with consumer surplus relative to the advertiser side surplus in welfare calculations.

Our paper is related to the literature on technology adoption. In contrast to our approach that investigates biases in technology adoption, the literature mostly focuses on dynamic diffusion process of new technologies. For instance, Fudenberg and Tirole (1985,
1987) focus on the strategic aspects of technology adoption and shows that the technology adoption pattern can be characterized by either preemptive adoption or delayed joint adoption depending on the profitability of the second-mover vs. the first mover and the speed of imitation. Gilbert and Newbery (1982) compare patenting incentives of an incumbent with those of a potential entrant and shows that the incumbent has more incentives to acquire the new substitute technology due to the efficiency effect when the new technology can be used as a vehicle for entry. Farrell and Saloner (1985) and Katz and Shapiro (1982) analyze incentives for technology adoption in the presence of network externalities. Their analyses, however, are in the framework of one-sided market and address a very different set of questions from ours. Elsewhere in Choi and Jeon (in progress), we analyze platforms’ incentives to improve the value of services for consumers and advertising technology in two-sided markets. In this analysis, we focus on whether the platforms have overinvestment or underinvestment in each side of the market compared to the socially optimal outcome rather than biases in the technology adoption decisions. Thus, these two papers investigate different aspects of technological competition and should be viewed complementary.

More recent work on the direction of innovation is also related to our paper in that the main focus is on the direction or research rather than the quantity of R&D. Bryan and Lemus (2017) show that "racing" and "underappropriation" distortions lead to inefficient allocation of resources even if the aggregate quantity of research is optimal. Choi and Gerlach (2014) analyze selection biases in the project choice of complementary technologies which allow innovating firms to hold up rivals who succeed in developing other system components with patents. They show that patents make innovation rewards independent of project difficulties and firms excessively cluster their R&D efforts on a relatively easier technology in order to preemptively claim stakes on component property rights. Finally, Chen, Pan, and Zhang (2018) investigates how patentability standards may affect the rate and direction of cumulative innovation in an industry where firms can conduct R&D in multiple directions. Our paper uncovers a different type of inefficiency that arises due to the two-sidedness of platform markets even in the absence of "racing" component which plays an important role in all three papers discussed here.

Weyl and Fabinger (2013) and Miklos-Thal and Shaffer (forthcoming) provide a general analysis of pass-through as an economic tool and provide applications to tax incidence, optimal procurement, third-degree price discrimination, etc. To our knowledge, the pass-through equalization result that we obtained both in the monopoly and the duopoly is
new as the existing literature does not compare the pass-through rate of the consumer side surplus to consumers with that of the advertiser side surplus to consumers. For instance, Weyl and Fabinger (2013) and Miklos-Thal and Shaffer (forthcoming) focus on the pass-through of the marginal cost to consumers, which is similar to the pass-through of advertiser surplus (with an opposite sign) in our framework as we set the marginal cost to zero. By contrast, they do not consider pass-through of consumer surplus. The same remarks apply to papers on two-sided markets that employ pass-through rate for a part of their analysis such as Bedre-Defolie and Calvano (2013) and Anderson and Peitz (2020).

The rest of the paper is organized in the following way. In section 2, we analyze a monopoly platform. We endogenously derive conditions under which services are provided for free and show that the bias in technology adoption runs in the opposite direction depending on whether the non-negative price constraint is binding or not. Section 3 considers a duopoly model of competitive bottleneck (Armstrong, 2005). We also illustrate the general results for the duopoly case by analyzing the Hotelling model and the logit model. Section 4 provides a summary with discussion. Section 5 provides concluding remarks.

2 Monopoly Platform

Consider a monopolistic platform in a two-sided market. Let $u$ and $p$ respectively denote the gross surplus per consumer and the price charged by the platform on the consumer side. The number of consumers on board depends on the net surplus $s$ provided by the platform, where $s = u - p$, and is represented by $D(s)$. We assume that $D(.)$ is strictly increasing and weakly concave.

When the platform attracts consumers, each additional consumer allows the platform to generate additional revenue from the other side. For instance, we can envision a situation in which the platform sells content to consumers and use the customer base to derive advertising revenues from advertisers who need access to consumers. Another source of revenue could be in-app purchases. For simplicity, we assume that the platform can generate a total surplus of $\beta$ per consumer on the advertiser side. We adopt a parsimonious reduced form modeling in that the platform can extract a $\tau$ proportion of the surplus, where $\tau \in (0, 1]$. In other words, on the advertising side, each consumer generates an ad revenue of $\tau \beta$ to the platform. We provide a microfoundation of this model in the Appendix.
A main reason for which we model the advertising market in a reduced form is that the boundary of the advertising market is much broader than that of a product market. Consider the case of the programmatic display advertising market, which sells display advertising inventories through real-time auctions. In this market, all kinds of publishers (including online newspapers) and content producers compete together with social media on the supply side. Most publishers and content producers rely on various advertising intermediaries to sell their advertising inventories to a large number of advertisers. Hence, even if a publisher is a monopolist in its product market, it has no or little market power on the advertising side of which the outcome is largely determined by the total supply and the total demand conditions. One important factor determining $\tau$ is what is called "ad tech tax", which represents the share taken by ad intermediaries from the advertising expenditure paid by advertisers. Small platforms such as online newspapers that rely on ad intermediaries have a smaller $\tau$ while big tech platforms that have built their own digital ad ecosystem and hence are not subject to ad tech tax would have a larger $\tau$ (See Section 4 for more details).

We assume that the marginal cost of serving a consumer is normalized to zero, without loss of generality. Hence, the platform’s profit is

$$\pi(p; u, \beta) = D(u - p)(p + \beta).$$

Maximizing it with respect to $p$ gives the following first order condition (F.O.C.):

$$\frac{\partial \pi(p; u, \beta)}{\partial p} = -D'(u - p)(p + \beta) + D(u - p) = 0. \quad (1)$$

Let the price that satisfies the above condition be denoted by $\tilde{p}$. As is typical in two-sided markets, because of the extra revenue that can be generated by the advertising side, the optimal price on the consumer side may entail below cost pricing (see Armstrong (2006) and Rochet and Tirole (2006)). When the marginal cost is low or even zero as in the digital markets, this implies that the optimal price can be negative. However, we impose the non-negative price constraint because negative prices can invite opportunistic behaviors by consumers due to various moral hazard and adverse selection reasons (Farrell and Gallini (1988), Armstrong and Wright (2007), Amelio and Jullien (2012) and Choi and Jeon (forthcoming)).\(^1\) Indeed, one of the main goals in this paper is to illustrate the

\(^1\)See Choi and Jeon (forthcoming) for more detailed discussion of the non-negative price constraint.
importance of the non-negative price constraint on the technology adoption choice.

With the non-negative price constraint, the optimal monopoly price is given by \( p^* = \max[\bar{p}, 0] \). The maximized profit, denoted by \( \pi^m(u, \beta) \), can be written as

\[
\pi^m(u, \beta) = \max_{p \geq 0} D(u - p)(p + \tau \beta) = D(u - p^*)(p^* + \tau \beta),
\]

where the superscript \( m \) represents monopoly. Let the aggregate consumer surplus (CS) be denoted by \( v(s) \), where \( v(.) \) satisfies the envelope condition \( v'(s) = D(s) \). We can also define the corresponding social welfare given \( (u, \beta) \).

\[
W^m(u, \beta) = \pi^m(u, \beta) + \underbrace{v(u - p^*)}_{\text{Consumer Surplus}} + \underbrace{D(u - p^*)(1 - \tau)\beta}_{\text{Advertiser Surplus}}.
\]

Consider now a technology adoption that changes \((u, \beta)\). To analyze this, consider a local locus of \((u', \beta')\) that would provide the same monopoly profit, where \( u' = u + du \) and \( \beta' = \beta + d\beta \). This locus can be derived by

\[
d\pi^m(u, \beta) = \underbrace{D'(u - p^*) (p^* + \tau \beta)}_{\text{PMB}_u} du + \underbrace{D(u - p^*) \tau d\beta}_{\text{PMB}_{\beta}} = 0,
\]

where \( \text{PMB}_u \) (\( \text{PMB}_{\beta} \)) refers to the private marginal benefit from an increase in \( u \) (in \( \beta \)). Note that we can ignore the indirect effects through \( p \) since the envelope theorem applies when \( p^* > 0 \) and there is no response in \( p \) when \( p^* = 0 \). The iso-profit curve for the monopolist, i.e., the locus of new technologies that would yield the same monopoly profit, is thus given by

\[
\frac{d\beta}{du} \bigg|_{\pi^m=0} = -\frac{\text{PMB}_u}{\text{PMB}_{\beta}} = -\frac{D'(u - p^*) (p^* + \tau \beta)}{D(u - p^*) \tau}
\]

Similarly, we can derive the technology adoption locus that yields the same social welfare, the iso-welfare curve, as follows:

\[
dW^m(u, \beta) = \frac{d\pi^m(u, \beta)}{du} \\
= \begin{cases} 
[v'(u - p^*) + D'(u - p^*)(1 - \tau)\beta] (du - dp) \\
+ D(u - p^*) (1 - \tau) d\beta
\end{cases}
\]

where \( \text{External Effects on CS} \) and \( \text{External Effects on AS} \).
where AS means advertiser surplus. The discrepancy between private and social incentives arises due to the external effects of the platform’s decision on consumers and advertisers.

The slope of the iso-welfare curve can be derived as

\[
\frac{d\beta}{du}|_{dW^m=0} = -\frac{SMB_u}{SMB_\beta},
\]

where SMB\(_u\) and SMB\(_\beta\) refer to the social marginal benefit from an increase in \(u\) and \(\beta\), respectively, and are given by

\[
SMB_u = D(u - p^*)(p^* + \tau \beta) + [D(u - p^*) + D'(u - p^*)(1 - \tau)\beta] \left(1 - \frac{dp}{du}\right)
\]

\[
SMB_\beta = D(u - p^*) - [D(u - p^*) + D'(u - p^*)(1 - \tau)\beta] \frac{dp}{d\beta}.
\]

To analyze private and social incentives for technology adoption and identify potential biases in the market outcome, we compare the slopes of the iso-profit and iso-welfare curves measured at the current level of \((u, \beta)\).

**Definition 1.** A platform’s technology adoption is CS-biased (respectively, AS-biased) if

\[
\left|\frac{d\beta}{du}|_{d\pi^m=0}\right| > \left|\frac{d\beta}{du}|_{dW^m=0}\right| \quad (respectively, \ if \ \left|\frac{d\beta}{du}|_{dW^m=0}\right| < \left|\frac{d\beta}{du}|_{d\pi^m=0}\right|) \quad (see \ Figures \ 1 \ and \ 2).
\]

As shown in Figure 1, when the slope of the iso-profit curve is steeper than that of the iso-welfare curve (that is, \(\left|\frac{d\beta}{du}|_{d\pi^m=0}\right| > \left|\frac{d\beta}{du}|_{dW^m=0}\right|\)), we can find two shaded areas in which private and social incentives conflict. The shaded area in the second quadrant (hurting consumers, but benefiting advertisers) represents technologies that would be socially beneficial, but would not be adopted by the monopolist. The shaded area in the fourth quadrant (benefiting consumers, but hurting advertisers), in contrast, represents technologies that would be welfare-reducing but would be adopted by the monopolist. In that sense, technology adoption incentives by the monopolist are biased towards the consumer side surplus. Similarly, when the slope of the iso-welfare curve is steeper than that of the iso-profit curve as in Figure 2, we can identify two areas that exhibit technology adoption incentives that are biased towards the advertiser side surplus.
When we compare the social marginal benefit of an increase in $u$ to the private one, we find

$$\text{SMB}_u - \text{PMB}_u = [D(u - p) + D'(u - p^*)(1 - \tau)\beta] \left(1 - \frac{dp}{du}\right) > 0$$

Note that $(\text{SMB}_u - \text{PMB}_u)$ contains two terms. First, an increase in $u$ directly increases both consumer surplus and advertiser surplus. Second, an increase in $u$ indirectly reduces, by increasing $p$, both consumer surplus and advertiser surplus. The net effect of the two is positive as long as the pass-through rate is smaller than one, i.e. $\frac{dp}{du} < 1$, which holds as we show below in Lemma 1.
Similarly, we find
\[ \text{SMB}_\beta - \text{PMB}_\beta = D(u - p)(1 - \tau) - [D(u - p) + D'(u - p^*)(1 - \tau)\beta] \frac{dp}{d\beta} > 0. \]

As in the case for a change in \( u \), (SMB\(_\beta\)−PMB\(_\beta\)) contains two terms. First, an increase in \( \beta \) directly increases advertiser surplus. Second, an increase in \( \beta \) indirectly increases, by reducing \( p \), both consumer surplus and advertiser surplus. Both effects are positive.\(^2\)

To analyze potential biases in technology adoption compared to the (second-best) social optimum where the price decision is left to the platform, we distinguish two cases depending on whether or not the non-negative price constraint (NPC) is binding. We show that the direction of the market biases crucially depend on whether the NPC is binding or not (i.e., whether the services to consumers are provided for free or not).

### 2.1 NPC Not Binding (\( p^* > 0 \))

When the NPC is not binding (that is, \( p^* > 0 \)), the optimal price on the consumer side satisfies the F.O.C. (1). As a result, we have
\[ \frac{d\beta}{du}|_{\text{d}u^m=0} = -\frac{D'(u - p^*)(p^* + \tau \beta)}{D(u - p^*)\tau} = -\frac{1}{\tau}. \]

This states that in order to neutralize the profit increase from one extra unit of \( u \), the reduction in \( \beta \) should be \( 1/\tau \) unit, leading to the slope of the iso-profit curve equal to \(-1/\tau\).

In the non-binding NPC case, we have the following lemma which establishes some useful properties of the pass-through rates of \( u \) and \( \beta \).

**Lemma 2.** When \( p^* > 0 \), we have \( \frac{1}{2} < \frac{dp}{du} < 1 \), \( -\frac{1}{2} < \frac{1}{\tau} \frac{dp}{d\beta} < 0 \) and \( 1 - \frac{dp}{du} = -\frac{1}{\tau} \frac{dp}{d\beta} \).

**Proof.** By totally differentiating (1), we can derive,
\[ \frac{1}{2} < \frac{dp}{du} = \frac{D' - D''(p + \tau \beta)}{2D' - D''(p + \tau \beta)} < 1, \]
\[ -\frac{\tau}{2} < \frac{dp}{d\beta} = -\frac{D'\tau}{2D' - D''(p + \tau \beta)} < 0, \]

\(^2\)If we analyze R&D incentives to increase \((u, \beta)\), this analysis suggests that the monopolist has less incentives to do R&D compared to the social planner on both sides. See Choi and Jeon (in progress) for a related analysis on this.
because $D$ is assumed to be weakly concave and strictly increasing.

As a result, we have

$$1 - \frac{dp}{du} = \frac{D'}{D''(p + \tau \beta) - 2D'} = \frac{1}{\tau} \frac{dp}{d\beta}.$$ 

\hfill \Box

**Corollary 1.** When $\tau = 1$, we have the pass-through rate equalization in the following sense:

$$1 - \frac{dp}{du} = \frac{dp}{d\beta}$$

Pass-through rate to CS from an increase in $u$ Pass-through rate to CS from an increase in $\beta$

Consider first a benchmark in which the monopoly platform can fully extract advertiser surplus (i.e., $\tau = 1$). In this case, we show below that there is no bias in the platform’s technology adoption compared to the social optimum. This no bias result is a consequence of the pass-through equalization result in Corollary 1, which means that with full extraction of advertiser surplus by the platform, an increase in $u$ confers the same level of surplus to the consumer side as an increase in $\beta$ does.

With full extraction of surplus on the advertiser side, the discrepancy between the social incentive and the private one comes only from the effect on consumer surplus. As the magnitudes of external effects coming from an increase in $u$ and $\beta$ are exactly the same by the "pass-through rate equalization" result, there is no bias in private technology adoption decisions.

More precisely, when $\tau = 1$, we have:

$$\text{SMB}_u|_{\tau=1} - \text{PMB}_u|_{\tau=1} = [D(u - p)] \left(1 - \frac{dp}{du}\right);$$

$$\text{SMB}_\beta|_{\tau=1} - \text{PMB}_\beta|_{\tau=1} = D(u - p) \left(-\frac{dp}{d\beta}\right).$$

Hence, $1 - \frac{dp}{du} = -\frac{dp}{d\beta}$ implies that the gap between the social marginal benefit and the private one is equalized across the two sides. This together with $\text{PMB}_u|_{\tau=1} = \text{PMB}_\beta|_{\tau=1}$ from the first-order condition (1) implies

$$\frac{d\beta}{du}|_{\Delta w^m = 0} = \frac{d\beta}{du}|_{\Delta \pi^m = 0} = -1 \text{ when } \tau = 1.$$
Therefore, when the monopolist platform can fully extract advertiser surplus, it exhibits no bias in technology adoption.

Consider now the case of imperfect extraction of advertiser surplus, i.e., $\tau < 1$. Intuitively, in this case the private incentives would be CS-biased as the surplus given to the advertiser side with an increase in $\beta$ would not be internalized by the platform. Indeed, we have

**Lemma 3.** We have

$$\frac{d\beta}{du} \bigg|_{d\pi^m = 0} = -\frac{1}{\tau} + \frac{D(u - p)(\frac{1-\tau}{\tau})}{D(u - p) - D'(u - p)(p + \beta)\frac{dp}{d\beta}} \geq 0.$$ 

**Proof.** See the Appendix. \qed

**Corollary 2.** For $\tau \in (0,1)$, the platform’s technology adoption is CS-biased:

$$\frac{d\beta}{du} \bigg|_{d\pi^m = 0} > \frac{d\beta}{du} \bigg|_{d\pi^m = 0} = -\frac{1}{\tau} \text{ for } \tau \in (0,1).$$

Therefore, we find that imperfect extraction of advertiser surplus induces the platform’s technology adoption incentives biased against advertisers, which is very intuitive given that the platform does not internalize the surplus left to advertisers and that it exhibits no bias when it fully extracts advertiser surplus.

In summary, we have:

**Proposition 1.** Consider a monopolistic two-sided platform. If the non-negative pricing constraint does not bind on the consumer side, its technology adoption incentive is CS-biased unless it can fully extract advertiser surplus (in which case its incentive is unbiased).

2.2 NPC Binding ($p^* = 0$)

The non-negative price constraint is binding if the following condition holds:

$$\frac{\partial\pi(p; u, \beta)}{\partial p} \bigg|_{p=0} = -D'(u)\tau \beta + D(u) < 0.$$  \hspace{1cm} (5)

If the constraint is binding, the monopoly profit is given by
\[ \pi^m(u, \beta) = D(u)\tau\beta, \]

where \( \tau \in (0, 1) \). Therefore, the locus of technology adoptions that would provide the same monopoly profit is given by

\[ d\pi^m(u, \beta) = \frac{\tau D'(u)\beta du}{PMB_u} + \frac{\tau D(u)d\beta}{PMB_\beta} = 0. \]

Therefore, we have

\[ \frac{d\beta}{du}|_{d\pi^m=0} = \frac{-\frac{D'(u)\beta}{D(u)}}{\frac{PMB_u}{PMB_\beta}} < -\frac{1}{\tau}, \]

where the inequality is from (5). The iso-profit curve does not depend on \( \tau \) when the NPC is binding. Note that the R.H.S. of the inequality, \(-\frac{1}{\tau}\), is the slope of the iso-profit curve \( \frac{d\beta}{du}|_{d\pi^m=0} \) when the NPC does not bind (see (4)). Therefore, the absolute slope of the iso-profit curve becomes steeper, implying that the platform values an increase in \( u \) relatively more than an increase in \( \beta \) when the NPC is binding. This is because the NPC is binding when \( \beta \) is relatively large (see condition (5)), which induces the platform to value an increase in \( u \) relatively more to attract additional consumers.

Social welfare is given by

\[ W^m(u, \beta) = \pi^m(u, \beta) + v(u) + D(u)(1 - \tau)\beta. \]

Hence, the locus of technology adoptions that would provide the same welfare is given by

\[ dW^m(u, \beta) = [\underbrace{D(u) + D'(u)\beta}_{SMB_u}]du + \underbrace{D(u)d\beta}_{SMB_\beta} = 0. \]

Thus, we have

\[ \frac{d\beta}{du}|_{dW^m=0} = \frac{-\frac{D(u) + D'(u)\beta}{D(u)}}{\frac{SMB_u}{SMB_\beta}} < 0. \]

Note that the iso-welfare curve does not depend on \( \tau \) either.

When we compare the slopes of the indifference curves, we get

\[ \frac{d\beta}{du}|_{dW^m=0} = -1 + \frac{d\beta}{du}|_{d\pi^m=0}. \]

Since none of the two indifference curves depend on \( \tau \), in order to get intuition, we
can focus on $\tau = 1$. Then, we have

$$\text{SMB}_{u|\tau=1} - \text{PMB}_{u|\tau=1} = D(u);$$

$$\text{SMB}_{\beta|\tau=1} - \text{PMB}_{\beta|\tau=1} = 0.$$  

On the consumer side, the monopolist does not internalize the increase in consumer surplus and hence the private marginal benefit is smaller than the social one. In contrast, on the advertising side, the social marginal benefit is exactly equal to the private one. Therefore, when the non-negative price constraint is binding (i.e., $p^* = 0$), there is an unambiguous bias against the technology adoption that enhances consumer surplus.

Summarizing, we have:

**Proposition 2.** Consider a monopolistic two-sided platform. If the non-negative pricing constraint binds on the consumer side, its technology adoption incentive is always AS-biased (i.e. biased against a consumer-surplus increasing technology).

3 Competitive Bottleneck: Duopoly with Horizontal Differentiation

In this section, we analyze technology adoption incentives by competing platforms. Our model involves two (symmetric) horizontally differentiated platforms. In particular, we consider a competitive bottleneck situation in which the platforms compete to attract single-homing consumers and use the customer base to derive advertising revenues from advertisers who need access to consumers.

3.1 Duopoly Competition with Horizontal Differentiation

We consider two symmetric platforms 1 and 2. Let $s_i = u_i - p_i$ represent the net surplus a consumer obtains from platform $i = 1, 2$. The number of consumers for platform $i$ is given by $D^i(s_1, s_2)$. We consider a symmetric demand: $D^1(s, s') = D^2(s', s)$. Let subscripts denote partial derivatives such that $D^i_i = \frac{\partial D^i}{\partial s_i}$ and $D^i_j = \frac{\partial D^i}{\partial s_j}$, where $i, j = 1, 2$ and $i \neq j$.

The symmetric demand implies $D^1_1(s, s') = D^2_2(s', s)$ and $D^1_2(s, s') = D^2_1(s', s)$.

**Assumption 1.** (i) $D^i_i > 0$, $D^i_j < 0$, (ii) $D^i_i \geq |D^i_j|$, and (iii) $D^i_{ij} \leq 0$ for $i = 1, 2$ and $i \neq j$. 

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This is a standard assumption. A1(i) means that each platform’s demand increases in its surplus provided to consumers, while it decreases in the surplus provided to consumers by the rival platform. A1(ii) means that the own effect weakly dominates the cross effect. With the symmetry of demand, A1(ii) also captures the market expansion effect with $D^i_i + D^i_j \geq 0$ for $i = 1, 2$ and $i \neq j$; when $D^i_i + D^i_j = 0$, the overall market size is fixed and there is no market expansion. Finally, A1(iii) means that the demand is concave in its own surplus provided to consumers, which is a sufficient condition to satisfy the second order condition of the profit maximization. As long as the second order condition is satisfied, we can allow for $D^i_{ii} > 0$.

When $u_1 = u_2 = u$ and $\beta_1 = \beta_2 = \beta$, platform $i$’s profit is

$$\pi^i = (p_i + \tau \beta) D^i_i (u - p_i, u - p_j).$$

The F.O.C. is given by

$$\frac{\partial \pi^i}{\partial p_i} = -(p_i + \tau \beta) D^i_i + D^i_i = 0.$$

The second-order condition is satisfied under Assumption 1:

$$\frac{\partial^2 \pi^i}{\partial p_i^2} = (p_i + \tau \beta) D^i_{ii} - 2 D^i_i < 0$$

The non-negative price constraint (NPC) binds if

$$\frac{\partial \pi^i}{\partial p_i}\bigg|_{p_i = p_j = 0} = -D^i_i (u, u) \tau \beta + D^i_i (u, u) < 0.\quad (7)$$

In such a case, each platform charges zero price on the consumer side and each platform’s profit is $D^i_i (u, u) \tau \beta$ in a symmetric equilibrium.

### 3.2 Technology Adoption with the NPC Not Binding

As we consider symmetric equilibrium, let us consider platform 1 as the representative one. When the NPC is not binding and prices are positive, platform 1’s profit can be written as

$$\pi^1 (p; u, \beta) = (p_1 + \tau \beta_1) D^1_i (u_1 - p_1, u_2 - p_2),$$

where $p, u, \beta$ denote vectors of the associated variables.

Note that the first-order condition is given by
\[
\frac{\partial \pi^1}{\partial p_1} = 0 \implies D^1 = (p_1 + \tau \beta_1)D_1^1
\] (8)

We derive the locus of (unilateral) technology adoption choices which would yield the same profit for platform 1:

\[
d\pi^1(u, \beta) = D_1^1(p_1 + \tau \beta_1)du_1 + D^1 \tau d\beta_1
\]

\[
-(p_1 + \tau \beta_1)D_2^1 \left( \frac{dp_2}{du_1} + \frac{dp_2}{d\beta_1} \right).
\] (9)

where we suppress the arguments for demand functions. Then, we can derive the slope of the iso-profit curve for platform 1 as follows:

\[
\frac{d\beta_1}{du_1} \bigg|_{d\pi^1=0} = -\frac{(p_1 + \tau \beta_1) \left[ D_1^1 - D_2^1 \frac{dp_2}{du_1} \right]}{\tau D^1 - (p_1 + \tau \beta_1)D_2^1 \frac{dp_2}{d\beta_1}} \left( \equiv \frac{d\beta_1}{du_1} \bigg|_{d\pi^1=0} \right),
\]

where the superscript \(d\) represents duopoly. Recall that the locus associated with the monopoly is given by

\[
\frac{d\beta_1}{du} \bigg|_{d\pi^m=0} = -\frac{(p + \tau \beta)D'}{\tau D}
\]

If we compare the duopoly locus \(\left( \frac{d\beta_1}{du} \bigg|_{d\pi^d=0} \right)\) to the monopoly locus \(\left( \frac{d\beta_1}{du} \bigg|_{d\pi^m=0} \right)\), both the numerator and the denominator have one additional term which represents the strategic effect (i.e. \(- (p_1 + \tau \beta_1)D_2^1 \frac{dp_2}{du_1}\) in the numerator and \(- (p_1 + \tau \beta_1)D_2^1 \frac{dp_2}{d\beta_1}\) in the denominator).

By using the F.O.C. (8), Eq. (9) can be written as

\[
d\pi^1(u, \beta) = D^1 du_1 + D^1 \tau d\beta_1
\]

\[
-D^1 \frac{D_1^1}{D_2^1} \left[ \frac{dp_2}{du_1} + \frac{dp_2}{d\beta_1} \right] + D^1 \left[ 1 - \frac{D_1^1}{D_2^1} \frac{dp_2}{du_1} \right] du_1 + D^1 \left[ \tau - \frac{D_1^1}{D_2^1} \frac{dp_2}{d\beta_1} \right] d\beta_1,
\]

PMB_u

PMB_\beta

Thus, we have

\[
\frac{d\beta_1}{du_1} \bigg|_{d\pi^d=0} = -\frac{\text{PMB}_u}{\text{PMB}_\beta} = -\frac{1 - \frac{D_1^1}{D_2^1} \frac{dp_2}{du_1}}{\tau - \frac{D_1^1}{D_2^1} \frac{dp_2}{d\beta_1}}
\] (10)
Let social welfare be

\[
W(u, \beta) = v(u_1 - p_1, u_2 - p_2) + (p_1 + \tau \beta_1)D^1(u_1 - p_1, u_2 - p_2) + (1 - \tau)\beta_1 D^1(u_1 - p_1, u_2 - p_2) + (p_2 + \beta_2)D^2(u_1 - p_1, u_2 - p_2),
\]

where \(v(u_1 - p_1, u_2 - p_2)\) denotes consumer welfare function that represents the aggregate consumer surplus.

Let us assume that consumer welfare function can be written as \(v(s_1, s_2) = \bar{v}(g_1(s_1) + g_2(s_2))\), where \(\bar{v} > 0, g'_i > 0\). Then, we have \(D^1 = \bar{v}'(g_1(s_1) + g_2(s_2))g'_1(s_1)\) by the envelope theorem. The locus of technology adoption choices which would yield the same welfare can be derived as follows.

\[
dW(u, \beta) = \left[ (1 - \frac{dp_1}{du_1})du_1 - \frac{dp_1}{d\beta_1}d\beta_1 \right] [D^1 + (p_1 + \beta_1)D^1_1 + (p_2 + \beta_2)D^2_1] \\
- \left[ \frac{dp_2}{du_1}du_1 + \frac{dp_2}{d\beta_1}d\beta_1 \right] [(p_1 + \beta_1)D^1_2 + (p_2 + \beta_2)D^2_2] \\
+ (p_1 + \tau \beta_1)D^1_1 \left[ \frac{dp_1}{du_1}du_1 + \frac{dp_1}{d\beta_1}d\beta_1 \right] + D^1 d\beta_1 = 0 
\]

After collecting terms in (11) and using the F.O.C. (8), we can show that in a symmetric equilibrium (see the Appendix for the derivation)

\[
dW(u, \beta) = D^1 + \left[ 1 - \frac{dp_1}{du_1} - \frac{dp_2}{du_1} \right] [(p_1 + \beta_1)(D^1_1 + D^2_1)] du_1 \\
+ D^1 - \left( \frac{dp_1}{d\beta_1} + \frac{dp_2}{d\beta_1} \right) [(p_1 + \beta_1)(D^1_1 + D^2_1)] d\beta_1.
\]
The iso-welfare curve for platform 1’s technology adoption choices can be written as

\[
\frac{d\beta_1}{du_1}|_{dW^d=0} = -\frac{\text{SMB}_u}{\text{SMB}_\beta} = - \frac{D^1 + \left[1 - \frac{dp_1}{du_1} - \frac{dp_2}{du_1}\right]\left[(p_1 + \beta_1)(D^1_1 + D^2_1)\right]}{D^1 - \left(\frac{dp_1}{du_1} + \frac{dp_2}{du_1}\right)\left[(p_1 + \beta_1)(D^1_1 + D^2_1)\right]} \tag{12}
\]

The following two corollaries come immediately from (10) and (12).

**Corollary 3.** (convergence to the monopoly) Consider the limit case of no competition in the duopoly with no cross-firm demand and strategic effects; \(D_2^1 = D_1^2 = 0 = \frac{dp_2}{du_1} = \frac{dp_2}{du_1}\). Then,

\[
\frac{d\beta_1}{du_1}|_{dW^d=0} = \frac{d\beta}{du}|_{dW^m=0};
\]

\[
\frac{d\beta_1}{du_1}|_{dW^d=0} = \frac{d\beta}{du}|_{dW^m=0},
\]

where \(D^1\) and \(D^1\) can be considered as counterparts to \(D\) and \(D'\) in the monopoly, respectively.

**Corollary 4.** When there is no market expansion (i.e., \(D^1_i(s, s) + D^2_i(s, s) = 0\) and the NPC does not bind,

\[
\frac{d\beta_1}{du_1}|_{dW^d=0} = -1.
\]

As in the monopoly case, the following lemma on pass-through rates plays a key role in establishing the direction of technology adoption bias for the duopoly case.

**Lemma 4.** (Duopoly Pass-through Rates) In the case of duopoly, we have

(i)

\[
1 - \frac{dp_1}{du_1} = -\frac{1}{\tau} \frac{dp_1}{d\beta_1}; \quad \frac{dp_2}{du_1} = \frac{1}{\tau} \frac{dp_2}{d\beta_1}.
\]

(ii) 1 - \(\frac{dp_1}{du_1} - \frac{dp_2}{du_1}\) \(\geq 0\) if \(2D^1_1 + D^2_1 \geq \frac{D_1^1}{D_1^1}\) \([D^1_1(1 + D^2_1)\]

**Proof.** See the Appendix.

In Lemma 4(i), the first equation about the own effect is similar to what we have in the monopoly case in Lemma 2. The second is about the strategic effect. In Lemma 4(ii), \(2D^1_1 + D^2_1 \geq 0\) from A1. Hence, \(D^1_1 + D^1_2 \leq 0\) is a sufficient condition for \(1 - \frac{dp_1}{du_1} - \frac{dp_2}{du_1} \geq 0\), which we assume to hold.
Corollary 5. When $\tau = 1$, we have a double pass-through rate equalization:

$$
1 - \frac{d p_1}{d u_1} = \frac{d p_1}{d \beta_1}; \quad \frac{d p_2}{d u_1} = \frac{d p_2}{d \beta_1}.
$$

Pass-through rates via Own Effects \quad Pass-through rates via Strategic Effects

As in the monopoly case, with full extraction of advertiser surplus by the platforms, an increase in $u_i$ and an increase in $\beta_i$ confer the same level of surplus to the consumers of platform $i$. Furthermore, in the duopoly case, pass-through rates via strategic price effects are also equalized across the consumer and advertiser sides: an increase in $u_i$ and an increase in $\beta_i$ confer the same level of surplus to the consumers of platform $j$ through a change in $p_j$.

Using Lemma 4, it is straightforward to show that

$$
\left. \frac{d \beta_1}{d u_1} \right|_{d \pi^d = 0} = -\frac{1 - \frac{D_1^1 dp_2}{D_1^1 du_1}}{\tau - \frac{D_1^1 dp_2}{D_1^1 du_1}} = -\frac{1 - \frac{D_1^2 dp_2}{D_1^2 du_1}}{\tau - \frac{D_1^2 dp_2}{D_1^2 du_1}} = -\frac{1}{\tau},
$$

which replicates the result in the monopoly case. Furthermore, we have

$$
\left. \frac{d \beta_1}{d u_1} \right|_{d \pi^d = 0} - \left. \frac{d \beta_1}{d u_1} \right|_{d \pi^d = 0} = \frac{1 - \tau}{\tau} \frac{D_1^1}{D_1^1 + \tau \left[ 1 - \frac{d p_1}{d u_1} - \frac{d p_2}{d u_1} \right]} \left[ (D_1^1 + (1 - \tau) \beta)(D_1^1 + D_1^2) \right] \geq 0.
$$

Despite the presence of strategic price effects in the duopoly case, we can derive surprisingly parallel results to the monopoly case. In particular, we find that there is no bias in technology adoption when the platforms can extract full surplus from the advertiser side ($\tau = 1$). Once again, the duopoly version of the "pass-through rate equalization" result plays a key role for this outcome when $\tau = 1$. As in the monopoly case, the magnitudes of external effects coming from an increase in $u$ and $\beta$ on the consumer side are exactly the same, resulting in no bias in private technology adoption decisions even in the presence of the strategic effects with competition. Similarly, when $\tau < 1$, the platforms cannot extract the full surplus from the advertiser side, which leads to CS-biased technology adoption.

Summarizing, we have:

Proposition 3. Consider a competitive bottleneck model of duopoly platforms with hor-
izontal differentiation. If the NPC does not bind on the consumer side, a platform’s unilateral technology adoption incentive is CS-biased if it cannot fully extract advertiser surplus; its incentive is unbiased if it fully extracts advertiser surplus.

3.3 Technology Adoption with the NPC Binding

Consider the case where the non-negative price constraint is binding. Recall that the NPC is binding when (7) holds. In this case,

\[ \pi^1(p = 0; u, \beta) = \tau \beta_1 D^1(u_1, u_2). \]

The locus of technology adoptions that would provide the same profit for platform 1 is given by

\[ d\pi^1(u, \beta) = \tau \beta_1 D^1(u_1, u_2) du_1 + D^1(u_1, u_2) \tau d\beta_1 = 0 \]

Therefore, in a symmetric equilibrium we have

\[ \frac{d\beta_1}{du_1}|_{d\pi^1=0} = -\frac{D^1(u, u)}{D^1(u, u)} \beta < -\frac{1}{\tau}, \tag{13} \]

where the inequality is from (7). Note first that (13) is exactly the same as the corresponding formula in the monopoly case (6). In particular, the iso-profit curve does not depend on \( \tau \). In addition, the R.H.S. of the inequality \((-\tau)^{-1}\) is \( \frac{d\beta_1}{du_1}|_{d\pi^1=0} \) when the NPC does not bind. Thus, as in the monopoly case, the absolute slope of the iso-profit curve becomes steeper when the NPC becomes binding, implying that the platform values an increase in \( u \) relatively more than an increase in \( \beta \) when the NPC is binding.

Social welfare with the NPC binding is given by

\[ W(u, \beta) = v(u_1, u_2) + \tau \beta_1 D^1(u_1, u_2) + (1 - \tau)\beta_1 D^1(u_1, u_2) + \beta_2 D^2(u_1, u_2). \]

Hence, the locus of technology adoptions by firm 1 that would provide the same welfare is given by

\[ dW(u, \beta) = [D^1 + \beta_1 D^1 + \beta_2 D^2] du_1 + D^1 d\beta_1. \]

In a symmetric equilibrium, \( D^1 = D^2, D^1_1 = D^2_1, D^1_2 = D^2_1, \) \( p_1 = p_2 = 0, \beta_1 = \beta_2 = \beta, \) we...
have

\[ \frac{d\beta_1}{du_1}|_{dW^d=0} = - \frac{D^1(u, u) + (D^1_1 + D^2_1) \beta}{D^1(u, u)}. \] (14)

As in the monopoly case, the iso-welfare curve does not depend on \( \tau \).

**Corollary 6.** When there is no market expansion (i.e., \( D_i(s, s) + D_i^1(s, s) = 0 \)) and the NPC binds,

\[ \frac{d\beta_1}{du_1}|_{dW^d=0} = -1. \]

Corollary 4 and 6 together imply that when there is no market expansion, \( \frac{d\beta_1}{du_1}|_{dW^d=0} = -1 \) regardless of whether the NPC binds or not.

To analyze potential biases in technology adoption, we rewrite (14) as

\[ \frac{d\beta_1}{du_1}|_{dW^d=0} = \frac{d\beta_1}{du_1}|_{dx^d=0} - \left( 1 + \frac{D^1_2 \beta}{D^1(u, u)} \right) \]

Business Stealing Effect

(15)

Thus, the comparison of the private incentive with the social one depends on the relative magnitude of \( \left| \frac{d\beta_1^2}{D^1(u, u)} \right| \) vs. 1. If competition is weak and the demand is more or less independent, that is, \( D^2_1 \approx 0 \), then \( \frac{d\beta_1}{du_1}|_{dx^d=0} > \frac{d\beta_1}{du_1}|_{dW^d=0} \), as in the case of the monopoly platform. However, if competition is intense (\( D^2_1 \) is a large negative number) and \( \beta \) is sufficiently large, we could have \( \frac{d\beta_1}{du_1}|_{dx^d=0} < \frac{d\beta_1}{du_1}|_{dW^d=0} \).

Summarizing, we have:

**Proposition 4.** Consider a competitive bottleneck model of duopoly platforms with horizontal differentiation. If the NPC binds on the consumer side, its technology adoption incentive is AS-biased as in the monopoly case if competition is weak whereas its incentive is CS-biased if competition is intense and the business stealing effect is strong.

In the following two subsections, we illustrate our results on the duopoly case for the Hotelling and logit models.

### 3.4 Hotelling Model

In the Hotelling model, the optimal prices given symmetric \((u, \beta)\) can be derived in the following way (assuming that \( u \) is sufficiently large and the consumer side market is

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covered). Given \((u, p)\), firm \(i\)'s demand can be written as
\[
D_i(u, p) = \frac{1}{2} + \frac{(u - p_j - (u - p_j)}{2t} = \frac{1}{2} + \frac{s_i - s_j}{2t}.
\]

Hence, the demand function \(D_i(s_i, s_j)\) satisfies A1: in particular, we have \(D_i = |D_j|\).

Firm \(i\) solves the following problem.
\[
\max_{p_i} \left( p_i + \tau \beta_i \right) \left[ \frac{1}{2} + \frac{(u_i - p_i) - (u_j - p_j)}{2t} \right].
\]
The first order conditions for each firm yield the following reaction functions.
\[
p_i = R_i(p_j; u, \beta_i) = t + \frac{(u_i - u_j) - \tau \beta_i + p_j}{2}.
\]
Note that the profit is strictly concave in \(p_i\) and hence the second-order condition is satisfied. By solving the two reaction functions simultaneously, we can derive the Nash equilibrium prices as
\[
p_i = t + \frac{(u_i - u_j) - (2\tau \beta_i + \tau \beta_j)}{3}
\]
In what follows, we consider a symmetric equilibrium.

Consider first the case where \(p_1^* = p_2^* = p^* > 0\), which occurs iff \(t > \tau \beta\). Then, we can easily confirm our earlier general result that
\[
\frac{d\beta_1}{du_1}\big|_{du_4=0} = -\left[1 - \frac{1}{3}\right] = -\frac{1}{\tau}
\]
We also know from the previous general analysis that when the NPC is binding (that is, when \(t < \tau \beta\)), we have:
\[
\frac{d\beta_1}{du_1}\big|_{du_4=0} = -\frac{\beta}{t} < -\frac{1}{\tau}
\]
If \(D_1 + D_2\) is constant like in the Hotelling model with full market coverage, we have \(D_1 + D_2 = 0\). Hence, from Corollary 4 and 6, we find
\[
\frac{d\beta_1}{du_1}\big|_{dW_4=0} = -1,
\]
regardless of whether the NPC binds or not.
Therefore, for $\tau < 1$, we have

$$\frac{d\beta_1}{du_1}|_{dW=0} > \frac{d\beta_1}{d\pi^1}|_{d\pi^1=0}$$

regardless of whether the NPC binds or not. But when $\tau = 1$,

$$\frac{d\beta_1}{du_1}|_{dW=0} = -1 = \frac{d\beta_1}{d\pi^1}|_{d\pi^1=0}$$

if the NPC is not binding; otherwise, we have $\frac{d\beta_1}{du_1}|_{dW=0} > \frac{d\beta_1}{d\pi^1}|_{d\pi^1=0}$.

**Proposition 5.** Consider the Hotelling model. For $\tau \in (0, 1)$, each platform’s technology adoption incentive is CS-biased regardless of whether or not the NPC binds; if $\tau = 1$, each platform’s technology adoption incentive is unbiased if the NPC does not bind and is CS-biased if the NPC binds.

The intuition for the result can be given as follows. When the NPC does not bind, the technology adoption is CS-biased regardless of the market structure. When the NPC binds, we have $t < \tau \beta$ and hence the platforms compete aggressively to attract consumers in order to generate advertising revenues, which generates a bias toward the consumer side.

### 3.5 Logit Demand Model

Consider a discrete choice model of price competition with logit demand

$$D_i(u_i - p_i, u_j - p_j) = \frac{\exp[(u_i - p_i)/t]}{\sum_{k=0}^{2} \exp[(u_k - p_k)/t]} = \alpha_i,$$

where the outside good, good 0, has a utility of $u_0$ with price zero (i.e. $p_0 = 0$) and $t(> 0)$ represents the degree of product differentiation. The total number of consumers is normalized to 1 and $\alpha_i$ is the proportion of consumers who use platform $i$.

It can be shown that the F.O.C. for profit maximization (8) with logit demand can be written as
\[
(p_1 + \beta \tau) = \frac{t}{\exp[(u_1 - p_1)/t]} \frac{1}{\sum_{k=0}^{2} \frac{\exp[(u_k - p_k)/t]}{\exp[(u_0 - p_0)/t] + \exp[(u_2 - p_2)/t]}} = t \frac{1}{1 - \alpha_1}.
\]

3.5.1 NPC Not Binding

In a symmetric equilibrium in which \(\alpha_1 = \alpha_2 = \alpha \in (0, 1/2)\) and \(\alpha_0 = 1 - 2\alpha > 0\), we verify in the Appendix that the general results on pass-through rates in Lemma 4(i) are valid in the logit model. In addition, we show the following result (see the Appendix for details).

\[
\left. \frac{d\beta_1}{du_1} \right|_{dW^d=0} - \left. \frac{d\beta_1}{du_1} \right|_{d\tau^d=0} = \frac{1 - \tau}{\tau} \frac{t}{(1-\alpha)} + \frac{\tau}{(1-\alpha)^2} \frac{1}{\tau} \frac{t}{(1-\alpha)} \frac{\beta_0}{1-\alpha} > 0
\]

Hence, the technology adoption incentive is always CS-biased for all \(\tau \in (0, 1)\). When \(\tau = 1\), the private incentive coincides with the social one.

3.5.2 NPC Binding

In the logit demand case, the equilibrium price would be zero (i.e., the NPC binding) if \(\beta \tau \geq \frac{t}{1-\alpha_i}\) for \(i = 1, 2\). When the NPC binds in a symmetric equilibrium, we have

\[
\left. \frac{d\beta_1}{du_1} \right|_{dW^d=0} - \left. \frac{d\beta_1}{du_1} \right|_{d\tau^d=0} = -\left(1 + \frac{D^2_1 \beta}{D^1} \right) = -\left(1 - \frac{\alpha}{t} \beta \right),
\]

where \(\alpha = \frac{\exp \frac{u_2}{t} + 2 \exp \frac{u_1}{t}}{\exp \frac{u_2}{t} + 2 \exp \frac{u_1}{t}} < 1/2\).

Thus, we can conclude that each platform’s incentive to adopt technology is AS-biased (CS-biased) if \(t > \alpha \beta\) (\(t < \alpha \beta\)).

3.5.3 Summary for Logit Demand

Note that the NPC binds if and only if \(t < (1-\alpha)\beta \tau\). Taken together, we can have two cases depending on the relative magnitudes of \((1-\alpha)\tau\) and \(\alpha\). Because \(\alpha < 1/2\),
we have a $\tau^* \in (0,1)$ such that $(1 - \alpha)\tau \geq \alpha$ if and only if $\tau \geq \tau^*$. For $\tau < \tau^*$, the NPC binding implies $t < \alpha \beta$: the NPC binding implies that the adoption incentive is CS-biased. For $\tau > \tau^*$, when the NPC binds, we can have both CS-bias and AS-bias depending on the sign of $t - \alpha \beta$. The logit demand model with market expansion thus reveals further insight on the importance of the share of the advertising surplus that each platform captures (represented by $\tau$).

In summary, we have

**Proposition 6.** In the logit model, there exists a $\tau^* \in (0,1)$ such that

(i) When $\tau < \tau^*$ (that is, $(1 - \alpha)\beta \tau < \alpha \beta$), a duopolistic platform’s technology adoption incentives are always CS-biased regardless of whether the NPC is binding or not.

(ii) When $\tau \geq \tau^*$ (that is, $(1 - \alpha)\beta \tau > \alpha \beta$), a duopolistic platform’s technology adoption incentives are CS-biased if $t > (1 - \alpha)\beta \tau$ (with the NPC not binding) or $t < \alpha \beta$ (with the NPC binding). However, if $t$ is in the intermediate range (i.e., $\alpha \beta < t < (1 - \alpha)\beta \tau$), then the NPC is binding and incentives are AS-biased. When $\tau = 1$, there is no bias if the NPC does not bind.

Therefore, when $\tau \geq \tau^*$, we may have non-monotonicity in technology adoption incentives. When $t < \alpha \beta$, the NPC is binding and competition to attract consumers is intense, which leads to CS-biased technology adoption. As $t$ becomes larger than $\alpha \beta$, the NPC is still binding, but competition is relaxed. As a result, technology adoption patterns exhibit AS-bias. Once $t$ becomes very large and exceeds $(1 - \alpha)\beta \tau$, the NPC does not bind any more and platform incentives revert back to CS-bias.

### 4 Summary and Discussions

We have found that the direction of biases in innovation adoption crucially depends on whether the non-negative price constraint is binding or not. More specifically, when the NPC is not binding and the equilibrium prices fully respond to changes in $(u, \beta)$, the market equilibrium in technology adoption is CS-biased both in monopoly and duopoly. If the NPC is binding and the services are provided for free, the market equilibrium is AS-biased if the market structure is monopolistic or competition is weak in duopoly. However, if competition is intense and the business stealing effect is strong, the bias can be reversed and becomes CS-biased in contrast to the monopoly case. The following table summarizes and compares technology adoption incentives for the monopoly and the duopoly platform cases.
Biases in Technology Adoption

The duopoly analysis of the logit model and the Hotelling model confirms the general finding. Furthermore, it generates additional useful insights. First, the analysis of the Hotelling model shows that in a mature market with little market expansion, the technology adoption incentive is always CS-biased. More importantly, the analysis of the logit model, which applies to a market with market expansion possibility, identifies the key role played by \( \tau \), the share of the advertising surplus that each platform captures. If \( \tau \) is small, then the platforms’ incentives are always CS-biased. In other words, a necessary condition for technology adoption to be biased against consumers is that \( \tau \) is not too small.

A wide range of publishers (including online newspapers) sell their advertising inventory to a wide range of advertisers through a complex chain of intermediaries that run real-time auctions on behalf of the publishers and advertisers. The intermediation ecosystem has evolved into a complex vertical chain of specialized providers such as publisher ad servers, SSPs (supply side platforms) including ad exchanges, DSPs (demand side platforms), advertiser ad servers. Google is dominant at each layer of intermediary. Various studies estimated what is called "ad tech tax",\(^3\) the share taken by ad intermediaries from the advertising expenditure paid by advertisers. For instance, according to the CMA report (2020), a lower bound of the ad tech tax is 35 percent, meaning that on average publishers receive at best 65% of the initial advertising revenue paid by advertisers. By contrast, large platforms such as Google and Facebook have built their own advertising ecosystem and hence are not subject to ad tech tax when they sell their advertising inventories. Therefore, in the context of our model, \( \tau \) of large platforms is much larger than \( \tau \) of publishers who rely on ad intermediaries, implying that the former’s technology adoption incentive is much more likely to be AS-biased than the latter's incentive.

Our analysis can also be interpreted as platforms’ incentives to adopt policies that create trade-offs between the consumer side and the advertiser side: privacy policy, ad load

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\(^3\)See for instance, ANA (2017), Plum (2019) and the CMA report (2020).
policy, the weight to place on quality (i.e. relevance of ads to consumers) over prices in determining the winning bid in auction mechanisms, etc. For instance, a platform’s privacy policy can be interpreted as having a similar effect as a technology adoption in that collection of consumers’ sensitive information may impose privacy costs on the consumers, but may help increase advertising revenues. A monopolistic platform (or a duopolistic platform facing weak competition) providing free services would have an excessive incentive to adopt a privacy policy that harms consumers in favor of advertising revenues. This is consistent with the Cambridge Analytica Scandal in which Cambridge Analytica took advantage of Facebook’s lax privacy policy, which enabled third-party developers to harvest not only data about their users but also data about their users’ friends. Our results suggest that the lax privacy policy can be a consequence of the exercise of Facebook’s market power.

The choice of ad load or the relative weight placed on quality metrics over bid prices in determining the winning bid can be analyzed in a similar way. More specifically, in the case of a search engine, a higher ad load by showing a greater proportion of ads relative to organic search results can increase the propensity of users to click on ads. However, the more ads are shown, the more likely it is that some ad content will be less relevant to the user search query, compromising the quality experienced by the user (the CMA Report, 2020, p. 223). The CMA (2020) finds that Google has been able to generate higher click-through rates by increasing its ad load. In the case of display advertising, a higher ad load can lead to a greater immediate financial reward, but inflicts more nuisance costs on consumers. The CMA (2020) also finds that the number of impressions served per hour on Facebook has increased from 40-50 in 2016 to 50-60 in 2019 and states that this increase in ad load partly explains why Facebook’s revenue per hour is greater than other platforms and has increased in the past four years (p. 259).

4 Cambridge Analytica created a personality test that would target American Facebook users. Two hundred seventy thousand people were paid one or two dollars each to take a test, which was designed to collect the personality traits of the test taker as well as data about friends and their Facebook activities. They had more than forty-nine million friends. See McNamee (2019).

5 In 2016, Google removed right-hand side ads and increased from three to four the number of ads eligible to appear above the organic search results. Later in 2016, Google introduced ‘Expanded Text Ads’, which allows advertisers to enhance their ads with an optional third headline and a second description (the CMA report, 2020, p.233). In addition, several advertisers submitted to the CMA that recent changes to Google’s policies on ad load and the presentation of search advertising had the effect of increasing the propensity for users to click on ads rather than organic links (the CMA report, 2020, p.237).
5 Concluding Remarks

In this paper, we have analyzed how market power affects platforms’ incentives to adopt technological innovations. As many two-sided platforms provide free services to consumers and generate revenues by charging the other side, our analysis in particular focuses on two-sided platforms which may find it optimal to charge zero price on the consumer side and to extract surplus on the advertising side. In such a framework, we consider innovations that affect both sides in an opposite way and study a platform’s incentives to trade-off the gain from one side with the loss from the other side. We compare private incentives with social incentives across different market structures (monopoly platform and duopoly competitive bottleneck) in order to identify biases in innovation generated by platform market power.

We have found that the direction of biases in technology adoption crucially depends on whether the non-negative price constraint is binding or not. When it is not binding, adoption incentive is biased toward consumer-surplus-increasing technology both in monopoly and duopoly. If the NPC is binding, adoption incentive is biased toward advertiser-surplus-increasing technology if the market structure is monopolistic or competition is weak in duopoly but biased toward consumer-surplus-increasing technology if competition is intense. Our results thus provide a rationale for a tougher competition policy to curb concentration if competition authorities put more weight on consumer surplus in welfare calculations.

Our analysis has relied on a differential technique and focused on technology adoptions that are local in nature. However, the driving force in our analysis would apply to technology adoptions that are discrete as long as the sign of the derivatives in our analysis remains the same along the path.
References


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Appendix

A Microfoundation of the Advertising Side Market

We provide a microfoundation of the advertising side that would yield the model assumed in the main text. Let us assume that there are two categories of products. Each consumer demands products from only one category. A priori, each category of products is equally likely to be demanded by each consumer. In each category, there is a measure of varieties, each of which is produced by monopolistic producers. To sell the product, each firm needs to advertise to inform consumers of the existence and price of the good as in Anderson and Coate (2005) and Choi (2006). Platforms provide such a channel and allow them to be matched with consumers. Let us assume that only a mass of monopoly producers of new goods can be matched with a consumer. This may be due to the advertising space limitation or consumer’s limited attention. New goods are produced with a constant marginal cost of zero without any loss of generality.

We consider a two-tier matching process between a consumer and advertisers. First, the platform transmits to the advertisers the data about the consumer’s profile and its prediction about the category the consumer is interested in. In addition, the platform announces the number of advertising slots. Second, based on the profile and the predicted category, advertisers estimate their willingness to pay for a slot. The slots are allocated according to the second-price auction: the winning bidders pay the highest losing bid.

Within a category, each new product is characterized by a parameter \( \alpha \in [0, 1] \), which represents the probability that the product will appeal to the consumer. If a product appeals to the consumer, the consumer is willing to pay \( \varpi \). We assume that \( \alpha \) is distributed according to \( F(\cdot) \). We assume that \( F \) is increasing and continuously differentiable.

When a consumer is matched with a product in the wrong category, the consumer has no demand for it. Since a consumer will pay \( \varpi \) or zero, each new producer’s optimal price is \( \varpi \). The platform attempts to match a consumer with the right category products, but the match is not perfect. The platform’s ability to match a consumer with the right product is represented by a probability of match \( \varphi(>1/2) \). A producer belonging to the category predicted by the platform has a willingness to pay to be advertised via the platform given by \( \varphi \alpha \varpi \). Let us define \( \alpha^* \) by

\[
z = 1 - F(\alpha^*)
\]

We assume that \( \alpha^* > \frac{1-\varphi}{\varphi} \). This condition implies that it is optimal for the platform to fill all advertising slots for a consumer with products from the category that is more likely to
suit the consumer. We also assume that the advertising slot is limited and it is optimal to fill all slots. This condition is given by \( \alpha^* > \alpha_m \), where \( \alpha_m = \arg \max a^F(1 - F(\alpha)) \).

A platform’s advertising revenue per consumer is given by \( \varphi \alpha^* \varpi = \varphi F^{-1}(1 - z) \varpi \) and the advertisers’ net surplus is given by \( \varphi \varpi \int_{\alpha^*}^{1}(\alpha - \alpha^*) dF(\alpha) \). Then, we can set

\[
\beta = \varphi F^{-1}(1 - z) \varpi + \varphi \varpi \int_{\alpha^*}^{1}(\alpha - \alpha^*) dF(\alpha) = \varphi \varpi \left[ zF^{-1}(1 - z) + \int_{\alpha^*}^{1}(\alpha - \alpha^*) dF(\alpha) \right]
\]

\[
\tau = \frac{\int_{\alpha^*}^{1}(\alpha - \alpha^*) dF(\alpha)}{zF^{-1}(1 - z) + \int_{\alpha^*}^{1}(\alpha - \alpha^*) dF(\alpha)}
\]

We can interpret an increase in \( \beta \) comes from a platform’s better targeting technology in matching a consumer with the right product category, that is, an increase in \( \varphi \). Notice that \( \tau \) is independent of \( \varphi \) (and \( \beta \)).

**Proof of Lemma 3**

From the F.O.C. (1), we have

\[
D'(u - p)(p + \beta) = D(u - p) + D'(u - p)(1 - \tau) \beta.
\]

We have

\[
\frac{d\beta}{du}_{\varpi=0} = -\left\{ D'(u - p)(p + \beta) + D(u - p) - [D(u - p) + D'(u - p)(1 - \tau) \beta \frac{dp}{du}] \right\}
\]

\[
\left\{ D(u - p)(1 \frac{dp}{du}) - D'(u - p)(1 - \tau) \beta \frac{dp}{du} \right\}
\]

\[
= -\left\{ D(u - p) - D'(u - p)(p + \beta)(1 - \frac{dp}{du}) \right\}
\]

\[
\left\{ D(u - p) - D'(u - p)(p + \beta) \frac{dp}{du} \right\}
\]

\[
= -\left\{ D(u - p)(\frac{1}{\tau} + \frac{1}{\tau}) - D'(u - p)(p + \beta) \frac{dp}{du} \right\}
\]

\[
\left\{ D(u - p)(\frac{1}{\tau}) - D'(u - p)(p + \beta) \frac{dp}{du} \right\}
\]

\[
= -\frac{1}{\tau} + \frac{D(u - p)(1 - \frac{dp}{du})}{\tau} \geq 0
\]
where for the second equality, we use (16) and for the third equality, we use $1 - \frac{dp}{du} = \frac{1}{\tau} \left| \frac{dp}{d\beta} \right|$ from Lemma 2.

**Derivation of the Iso-Welfare Curve in the Duopoly Case**

We have

$$dW(u, \beta) = \left[ (1 - \frac{dp_1}{du_1})du_1 - \frac{dp_1}{d\beta_1}d\beta_1 \right] \left[ D^1 + (p_1 + \beta_1)D^1_1 + (p_2 + \beta_2)D^2_1 \right]$$

$$- \left[ \frac{dp_2}{du_1}du_1 + \frac{dp_2}{d\beta_1}d\beta_1 \right] [(p_1 + \beta_1)D^1_2 + (p_2 + \beta_2)D^2_2]$$

$$+ (p_1 + \tau \beta_1)D^1_1 \left[ \frac{dp_1}{du_1}du_1 + \frac{dp_1}{d\beta_1}d\beta_1 \right] + D^1 d\beta_1$$

In a symmetric equilibrium, $D^1 = D^2, D^1_1 = D^2_1, D^1_2 = D^2_2, p_1 = p_2 = p, \beta_1 = \beta_2 = \beta$

$$dW(u, \beta) = \left[ (1 - \frac{dp_1}{du_1})du_1 - \frac{dp_1}{d\beta_1}d\beta_1 \right] \left[ D^1 + (p_1 + \beta_1)(D^1_1 + D^2_1) \right]$$

$$- \left[ \frac{dp_2}{du_1}du_1 + \frac{dp_2}{d\beta_1}d\beta_1 \right] [(p_1 + \beta_1)(D^1_1 + D^2_1)]$$

$$+ (p_1 + \tau \beta_1)D^1_1 \left[ \frac{dp_1}{du_1}du_1 + \frac{dp_1}{d\beta_1}d\beta_1 \right] + D^1 d\beta_1$$

By using the F.O.C. (8), we have

$$dW(u, \beta) = \left[ (1 - \frac{dp_1}{du_1})du_1 - \frac{dp_1}{d\beta_1}d\beta_1 - \left[ \frac{dp_2}{du_1}du_1 + \frac{dp_2}{d\beta_1}d\beta_1 \right] \right] [(p_1 + \beta_1)(D^1_1 + D^2_1)]$$

$$+ D^1 \left[ (1 - \frac{dp_1}{du_1})du_1 - \frac{dp_1}{d\beta_1}d\beta_1 + \left[ \frac{dp_1}{du_1}du_1 + \frac{dp_1}{d\beta_1}d\beta_1 \right] \right] d\beta_1$$

$$= \left[ (1 - \frac{dp_1}{du_1})du_1 - \frac{dp_1}{d\beta_1}d\beta_1 - \left[ \frac{dp_2}{du_1}du_1 + \frac{dp_2}{d\beta_1}d\beta_1 \right] \right] [(p_1 + \beta_1)(D^1_1 + D^2_1)]$$

$$+ D^1 [du_1 + d\beta_1]$$

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Hence,

\[
dW(u, \beta) = D_1 + \left[1 - \frac{dp_1}{du_1} - \frac{dp_2}{du_1}\right] \left[(p_1 + \beta_1)(D_1^1 + D_2^3)\right] du_1 + D_1^1 - \left(\frac{dp_1}{d\beta_1} + \frac{dp_2}{d\beta_1}\right) \left[(p_1 + \beta_1)(D_1^1 + D_2^2)\right] d\beta_1
\]

\[\text{SMB}_a\]

\[\text{SMB}_b\]

**Proof of Lemma 4**

Fully differentiating the F.O.C. (8) with respect to \(u_1\) yields

\[
D_1^1(1 - \frac{dp_1}{du_1}) - D_2^2 \frac{dp_2}{du_1} = D_1^1 \frac{dp_1}{du_1} + (p_1 + \tau \beta_1) \left[D_{11}^1(1 - \frac{dp_1}{du_1}) - D_{12}^1 \frac{dp_2}{du_1}\right]
\]

\[
= D_1^1 \frac{dp_1}{du_1} + \frac{D_1^1}{D_1^1} \left[D_{11}^1(1 - \frac{dp_1}{du_1}) - D_{12}^1 \frac{dp_2}{du_1}\right],
\]

where the second equality comes from the F.O.C. Similarly, fully differentiating the F.O.C. (8) with respect to \(u_2\) gives us

\[
-D_1^1 \frac{dp_1}{du_2} + D_2^1(1 - \frac{dp_2}{du_2}) = D_1^1 \frac{dp_1}{du_2} + \frac{D_1^1}{D_1^1} \left[-D_{11}^1 \frac{dp_1}{du_2} + D_{12}^1(1 - \frac{dp_2}{du_2})\right].
\]

At the symmetric equilibrium, we have \(\frac{dp_1}{du_1} = \frac{dp_2}{du_2} = x\) and \(\frac{dp_2}{du_2} = \frac{dp_1}{du_1} = y\). By summing up the two conditions above, we have

\[
D_1^1(1 - 2x - 2y) + D_2^1(1 - x - y) = \frac{D_1^1}{D_1^1} \left[D_{11}^1 + D_{12}^1\right](1 - x - y)
\]

Hence,

\[
(x + y) = \frac{D_1^1 + D_2^1 - \frac{D_1^1}{D_1^1} [D_{11}^1 + D_{12}^1]}{2D_1^1 + D_2^1 - \frac{D_1^1}{D_1^1} [D_{11}^1 + D_{12}^1]}
\]

By proceeding in a similar manner for \(\beta_1\) and \(\beta_2\), we derive two corresponding condi-
tions to the above.

\[-D_1^{\frac{d p_1}{d \beta_1}} - D_2^{\frac{d p_2}{d \beta_1}} = D_1^{\frac{d p_1}{d \beta_1} + \tau} + D_1^{\frac{d p_1}{d \beta_1} - \frac{D_1^{d p_1}}{D_1^{d \beta_1}} - D_1^{d p_2} D_1^{d \beta_1}}\]

\[-D_1^{\frac{d p_1}{d \beta_2}} - D_2^{\frac{d p_2}{d \beta_2}} = D_1^{\frac{d p_1}{d \beta_2} + \frac{D_1^{d p_1}}{D_1^{d \beta_2}} - D_2^{d p_2} D_2^{d \beta_2}}\]

At the symmetric equilibrium, we have \(\frac{d p_1}{d \beta_1} = \frac{d p_2}{d \beta_2} = w\) and \(\frac{d p_1}{d \beta_1} = \frac{d p_2}{d \beta_2} = z\). Summing up the two conditions above gives us

\[-(2w + 2z + \tau)D_1^1 - (w + z)D_2^1 = -(w + z)\left(D_1^{d p_1} D_1^{d \beta_1} + D_1^{d p_2} D_1^{d \beta_1}\right),\]

Hence,

\[(w + z) = \frac{-\tau D_1^1}{2D_1^1 + D_2^1 - \frac{D_1^{d p_1}}{D_1^{d \beta_1}} [D_1^{d p_1} + D_1^{d \beta_1}]}\]

We also have

\[1 - (x + y) = \frac{2D_1^1 + D_2^1 - \frac{D_1^{d p_1}}{D_1^{d \beta_1}} [D_1^{d p_1} + D_1^{d \beta_1}] - \left(D_1^1 + D_2^1 - \frac{D_1^{d p_1}}{D_1^{d \beta_1}} [D_1^{d p_1} + D_1^{d \beta_1}]\right)}{\left[2D_1^1 + D_2^1 - \frac{D_1^{d p_1}}{D_1^{d \beta_1}} [D_1^{d p_1} + D_1^{d \beta_1}]\right]}\]

As a result, we have

\[\tau (1 - x - y) = -(w + z)\]

\[1 - x - y \geq 0\] if \(2D_1^1 + D_2^1 \geq \frac{D_1^{d p_1}}{D_1^{d \beta_1}} [D_1^{d p_1} + D_1^{d \beta_1}].\]

Let us consider a candidate solution, which is \(z = \tau y, w = -\tau (1 - x)\). We can easily verify that this candidate solution satisfies all equations above simultaneously. This proves that the pass-through rates in the duopoly model satisfies the following relationships.

\[\tau (1 - \frac{d p_1}{d u_1}) = -\frac{d p_1}{d \beta_1}; \tau \frac{d p_2}{d u_1} = \frac{d p_2}{d \beta_1}.\]

By solving equations above simultaneously, we can also derive that\(^6\)

\(^6\)Details available upon request.
Technology Adoption Bias with Logit Demand when the NPC is Not Binding

The F.O.C. can be written as

\[
x = \frac{dp_1}{du_1} = \frac{[2D_1^1 - D_{11}^1 \frac{d^1}{d^1 u_1}] [D_1^1 - \frac{d^1}{d^1 u_1} D_{11}^1] - [D_2^1 - \frac{d^1}{d^1 u_1} D_{12}^1]^2}{[2D_1^1 - \frac{d^1}{d^1 u_1} D_{11}^1] - [D_2^1 - \frac{d^1}{d^1 u_1} D_{12}^1]^2}
\]

\[
y = \frac{dp_2}{du_1} = \frac{D_1^1 [D_1^1 - \frac{d^1}{d^1 u_1} D_{12}^1]}{[2D_1^1 - \frac{d^1}{d^1 u_1} D_{11}^1] - [D_2^1 - \frac{d^1}{d^1 u_1} D_{12}^1]^2}
\]

\[
w = \frac{dp_1}{d\beta_1} = -\tau \frac{D_1^1 [2D_1^1 - D_{11}^1 \frac{d^1}{d^1 u_1}]}{[2D_1^1 - \frac{d^1}{d^1 u_1} D_{11}^1] - [D_2^1 - \frac{d^1}{d^1 u_1} D_{12}^1]^2}
\]

\[
z = \frac{dp_2}{d\beta_1} = \tau \frac{D_1^1 [D_2^1 - \frac{d^1}{d^1 u_1} D_{12}^1]}{[2D_1^1 - \frac{d^1}{d^1 u_1} D_{11}^1] - [D_2^1 - \frac{d^1}{d^1 u_1} D_{12}^1]^2}
\]

The F.O.C. can be written as

\[
(p_1 + \beta_1 \tau) \exp[(u_0 - p_0)/t] + \exp[(u_2 - p_2)/t] = t \sum_{k=0}^2 \exp[(u_k - p_k)/t]
\] (17)

Fully differentiating (17) with respect to \(u_1\) gives

\[
\frac{dp_1}{du_1} \exp[(u_0 - p_0)/t] + \exp[(u_2 - p_2)/t]
\]

\[-(p_1 + \beta_1 \tau) \frac{dp_2}{du_1} \frac{1}{t} \exp[(u_2 - p_2)/t]
\]

\[= (1 - \frac{dp_1}{du_1}) \exp[(u_1 - p_1)/t] - \frac{dp_2}{du_1} \exp[(u_2 - p_2)/t],
\]

which is equivalent to, at symmetric equilibrium,

\[
\frac{dp_1}{du_1} (1 - \alpha) - \frac{\alpha}{1 - \alpha} \frac{dp_2}{du_1} = \alpha \left[ 1 - \frac{dp_1}{du_1} - \frac{dp_2}{du_1} \right].
\]

Hence,

\[
\frac{dp_1}{du_1} = \alpha + \frac{dp_2}{du_1} \frac{\alpha^2}{1 - \alpha}.
\]
Fully differentiating (17) with respect to $u_2$ gives
\[
\frac{dp_1}{du_2} \left[ \exp[(u_0 - p_0)/t] + \exp[(u_2 - p_2)/t] \right] + (p_1 + \beta \tau)(1 - \frac{dp_2}{du_2}) \frac{1}{t} \exp[(u_2 - p_2)/t]
\]
\[
= -\frac{dp_1}{du_2} \exp[(u_1 - p_1)/t] + (1 - \frac{dp_2}{du_2}) \exp[(u_2 - p_2)/t],
\]
which is equivalent to, at symmetric equilibrium,
\[
\frac{dp_1}{du_2} (1 - \alpha) + \frac{\alpha}{1 - \alpha} (1 - \frac{dp_2}{du_2}) = \alpha \left[ 1 - \frac{dp_1}{du_2} - \frac{dp_2}{du_2} \right].
\]
Hence,
\[
\frac{dp_1}{du_2} = -\frac{\alpha^2}{1 - \alpha} (1 - \frac{dp_2}{du_2}).
\]
Since, because of the symmetry, we have $\frac{dp_1}{du_1} = \frac{dp_2}{du_2}$ and $\frac{dp_2}{du_1} = \frac{dp_1}{du_2}$, we find
\[
\frac{dp_1}{du_1} = \frac{\alpha(1 - \alpha)^2 - \alpha^4}{(1 - \alpha)^2 - \alpha^4} > 0, \quad \frac{dp_1}{du_2} = -\frac{\alpha^2(1 - \alpha)^2}{(1 - \alpha)^2 - \alpha^4} < 0.
\]
Fully differentiating (17) with respect to $\beta_1$ gives
\[
\left( \frac{dp_1}{d\beta_1} + \tau \right) \left[ \exp[(u_0 - p_0)/t] + \exp[(u_2 - p_2)/t] \right] - (p_1 + \beta \tau) \frac{dp_2}{d\beta_1} \frac{1}{t} \exp[(u_2 - p_2)/t]
\]
\[
= -\frac{dp_1}{d\beta_1} \exp[(u_1 - p_1)/t] - \frac{dp_2}{d\beta_1} \exp[(u_2 - p_2)/t],
\]
which is equivalent to, at symmetric equilibrium,
\[
\left( \frac{dp_1}{d\beta_1} + \tau \right)(1 - \alpha) - \frac{\alpha}{1 - \alpha} \frac{dp_2}{d\beta_1} = -\alpha \left( \frac{dp_1}{d\beta_1} + \frac{dp_2}{d\beta_1} \right).
\]
Hence,
\[
\frac{dp_1}{d\beta_1} = -\tau (1 - \alpha) + \frac{\alpha^2}{d\beta_1 1 - \alpha}.
\]
Fully differentiating (17) with respect to $\beta_2$ gives
\[
\frac{dp_1}{d\beta_2} \left[ \exp[(u_0 - p_0)/t] + \exp[(u_2 - p_2)/t] \right] - (p_1 + \beta \tau) \frac{dp_2}{d\beta_2} \frac{1}{t} \exp[(u_2 - p_2)/t]
\]
\[
= -\frac{dp_1}{d\beta_2} \exp[(u_1 - p_1)/t] - \frac{dp_2}{d\beta_2} \exp[(u_2 - p_2)/t],
\]
which is equivalent to, at symmetric equilibrium,
\[
\frac{dp_1}{d\beta_2} (1 - \alpha) - \frac{\alpha}{1 - \alpha} \frac{dp_2}{d\beta_2} = -\alpha \left( \frac{dp_1}{d\beta_2} + \frac{dp_2}{d\beta_2} \right).
\]
Hence,
\[
\frac{dp_1}{d\beta_2} = \frac{dp_2}{d\beta_2} \frac{\alpha^2}{1 - \alpha}.
\]
Since, because of the symmetry, we have \( \frac{dp_1}{d\beta_1} = \frac{dp_2}{d\beta_2} \) and \( \frac{dp_2}{d\beta_1} = \frac{dp_1}{d\beta_2} \), we find
\[
\frac{dp_1}{d\beta_1} = -\frac{\tau (1 - \alpha)^3}{(1 - \alpha)^2 - \alpha^4} < 0, \quad \frac{dp_1}{d\beta_2} = -\frac{\tau \alpha^2 (1 - \alpha)^2}{(1 - \alpha)^2 - \alpha^4} < 0
\]
We thus can confirm Lemma 4(i)
\[
\tau (1 - \frac{dp_1}{du_1}) = -\frac{dp_1}{d\beta_1}; \quad \tau \frac{dp_1}{du_2} = \frac{dp_1}{d\beta_2}
\]
In addition, we have
\[
\frac{D_2}{D_1} = \frac{\alpha}{1 - \alpha}.
\]
Therefore,
\[
\frac{d\beta_1}{du_1} \bigg|_{d\pi^*=0} = -\left[ 1 - \frac{\alpha^2 (1 - \alpha)^2}{(1 - \alpha)^2 - \alpha^4} \right] \frac{D_2/D_1}{du_1/d\beta_1} \tau - \frac{\alpha^2 (1 - \alpha)^2}{(1 - \alpha)^2 - \alpha^4}
\]
\[
= -\left[ 1 - \frac{\alpha^2 (1 - \alpha)^2}{(1 - \alpha)^2 - \alpha^4} \right] \frac{\alpha^3 (1 - \alpha)}{(1 - \alpha)^2 - \alpha^4}
\]
\[
= -\left[ 1 - \frac{\alpha^3 (1 - \alpha)}{(1 - \alpha)^2 - \alpha^4} \right] \tau - \frac{\alpha^3 (1 - \alpha)}{(1 - \alpha)^2 - \alpha^4}
\]
\[
= -\frac{1}{\tau}
\]
\[ \frac{d\beta_1}{du_1} \bigg|_{dW^4=0} = - \frac{\text{SMB}_u}{\text{SMB}_\beta} \]

\[ = \frac{D_1 + \left[ 1 - \frac{dp_1}{du_1} - \frac{dp_2}{du_1} \right] [(p_1 + \beta_1)(D_1^1 + D_1^2)]}{D_1 - \left( \frac{dp_1}{du_1} + \frac{dp_2}{du_1} \right) [(p_1 + \beta_1)(D_1^1 + D_1^2)]} \]

\[ = \frac{(p_1 + \tau \beta_1)D_1^1 + \left[ 1 - \frac{dp_1}{du_1} - \frac{dp_2}{du_1} \right] [(p_1 + \beta_1)(D_1^1 + D_1^2)]}{(p_1 + \tau \beta_1)D_1^1 - \left( \frac{dp_1}{du_1} + \frac{dp_2}{du_1} \right) [(p_1 + \beta_1)(D_1^1 + D_1^2)]} \]

\[ = \frac{(p_1 + \tau \beta_1) + \left[ 1 - \frac{dp_1}{du_1} - \frac{dp_2}{du_1} \right] [(p_1 + \beta_1 + (1 - \tau)\beta_1)(1 + \frac{D_1^2}{D_1^1})]}{(p_1 + \tau \beta_1) - \left( \frac{dp_1}{du_1} + \frac{dp_2}{du_1} \right) [(p_1 + \beta_1 + (1 - \tau)\beta_1)(1 + \frac{D_1^2}{D_1^1})]} \]

\[ = - \frac{\frac{t}{1-\alpha} + \frac{(1-\alpha+\alpha^2)(1-\alpha)^2}{(1-\alpha)^2 - \alpha^4} \left[ \left( 1 - \tau \right) \beta_1 \right]^- 1-2\alpha \right]}{\frac{t}{1-\alpha} + \frac{\alpha \left( 1-\alpha+\alpha^2 \right)(1-\alpha)^2}{(1-\alpha)^2 - \alpha^4} \left[ \left( 1 - \tau \right) \beta_1 \right]^- 1-2\alpha \right]} \]

Hence, if \( \tau = 1 \),

\[ \frac{d\beta_1}{du_1} \bigg|_{dW^4=0} = -1 = \frac{d\beta_1}{du_1} \bigg|_{dx^4=0}. \]