Platform Design Biases in Ad-Funded Two-Sided Markets*

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Abstract

We investigate how platform market power affects platforms’ design choices in ad-funded two-sided markets, where platforms may find it optimal to charge zero price on the consumer side and to extract surplus on the advertising side. We consider design choices affecting both sides in opposite ways and compare private incentives with social incentives. Platforms’ design biases depend crucially on whether they can charge any price on the consumer side. We apply the framework to technology adoption, privacy, and ad load choices. Our results provide a rationale for a tougher competition policy to curb market power of ad-funded platforms with free services.

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1 Introduction

The public sentiment regarding digital platforms has changed recently since the revelation of the Cambridge Analytica scandal with people becoming more concerned about market concentration and big platforms’ market power (Kahn, 2016). There have been several initiatives proposing modification of the current framework of competition policy in order to promote competition and curb concentration of market power in the area of digital platforms; see for instance the ACCC report (2019), the CMA report (2020), the Furman report (2019), the Stigler report (Stigler Committee on Digital Platforms, 2019), the Vestager report (Crémer, de Montjoye and Schweitzer, 2019).

A challenge to competition policy in the area of digital platforms arises from the fact that many two-sided platforms use business models that provide free services to consumers and generate revenue by charging the other side such as advertisers or application developers (Rochet and Tirole, 2005; Amelio and Jullien, 2012; Choi and Jeon, 2021). When the service is free, consumer harm from the exercise of market power does not take the form of a higher price. Instead, consumer harm is likely to be manifested in terms of less innovation, lower quality of service, more nuisance from advertisements or less privacy protection (Newman, 2015, 2016). Therefore, it is crucial to understand how platform market power and business models shape platforms’ design choices along these dimensions.

As discussed below, there are recent papers that study various aspects of platform design issues. However, to the best of our knowledge, there has been no systematic formal investigation of the relationship between platform market power and platform design incentives. This article attempts to fill this gap by addressing how market power affects design choices of ad-funded platforms. More specifically, we consider two-sided platforms which may find it optimal to charge zero price on the consumer side and to extract surplus on the advertising side. To analyze a platform’s incentives to trade off consumer surplus reduction (respectively, increase) with advertiser surplus increase (respectively, reduction), we consider platform design choices that affect the two sides (the consumer and the advertiser sides) in opposite ways. To identify potential platform biases, we characterize both the locus of design choices that yields the same profit to a platform and the one that yields the same welfare for comparison. We say that a platform’s design incentive is CS-biased (i.e., biased toward the consumer side) when it adopts a consumer-surplus increasing (and hence advertiser-surplus decreasing) design policy which would not
be adopted by a social planner. Symmetrically, we say that a platform’s design incentive is AS-biased (i.e., biased toward the advertiser side) when it adopts an advertiser-surplus increasing (hence consumer-surplus decreasing) design policy which would not be adopted by a social planner.

To capture ad-funded business models with zero pricing, we develop a stylized model of two-sided markets with consumers on one side and advertisers on the other side. We consider a typical situation in which each consumer a platform attracts generates additional surplus on the advertising side. We further assume that the platform is in a competitive advertising market where it cannot adjust the price and captures a fixed proportion of the surplus on the advertising side.¹ We distinguish two cases: one in which the price on the consumer side is fixed at some level (for instance, at zero) and the other in which platforms can charge any price on the consumer side. The price constraint may be due to price regulations or platform business models. The constraint also can arise endogenously. The literature on two-sided markets shows that oftentimes below-cost pricing on one side naturally arises as an optimal pricing structure, because the loss from the below-cost pricing can be recouped on the other side of the market (see Armstrong (2006) and Rochet and Tirole (2006)). When the marginal cost is low as in digital markets, the optimal pricing strategy may entail negative prices. However, we can imagine situations in which negative prices may be impractical due to adverse selection and opportunistic behaviors by consumers (Farrell and Gallini (1988), Armstrong and Wright (2007), Amelio and Julien (2012) and Choi and Jeon (2021)). In this case, the price constraint takes the form of the non-negative price constraint (NPC), which is relevant to ad-funded business models.

We first study the baseline model of a monopoly platform and show that the platform’s design biases depend crucially on whether or not it can freely charge any price on the consumer side: without any price constraint, platform design is CS-biased whereas when the price is fixed at zero on the consumer side, platform design is AS-biased. After developing a general framework, we provide applications to platform choices regarding technology adoption, privacy, and ad load. We also extend our analysis to a duopoly model of competitive bottleneck (Armstrong, 2005, and Armstrong and Wright, 2007) to investigate implications of competition on platform design. In the absence of any price constraints on the consumer side, we find the same result as in the monopoly case: each

¹One important factor that affects the platform’s ability to appropriate the advertising side surplus is whether the platform has its own ad tech system or it has to rely on third-party ad intermediaries which charge "ad tech take" for their intermediation service (see Section 5 for more details).
platform’s design is CS-biased. However, in the presence of price constraints, the result in the monopoly case can be overturned: when competition is weak, platform design is AS-biased as in the monopoly case whereas if competition is sufficiently strong, business stealing effects lead to a bias toward the consumer side.

Our results that platform design biases crucially depend on the existence of price constraint imply that optimal regulatory or antitrust policies towards the platform market can be substantially different for markets where services are provided for free (i.e., the NPC is binding) from those for markets with a positive price (i.e., the NPC is not binding). In particular, our results allow us to make the following predictions regarding digital platforms which charge zero price to consumers as they monetize consumer attention. Initially when they are nascent and face fierce competition, they have strong incentives to increase consumer surplus in their platform design. However, once the market tips to them or after their market power becomes entrenched, the same platforms, which were consumer advocates, have strong incentives to introduce innovations/policies that increase the advertiser side surplus to the detriment of consumer surplus. This view resonates with the evolution of business strategies of Facebook. According to Srinivasan (2019), when Facebook entered the market against the then incumbent MySpace, it presented itself as a "privacy-centered alternative" with consumer privacy taken seriously. For instance, Facebook promised not to "use cookies to collect private information from any user (p. 49)" and provided users with "the ability to opt out of having their information being shared with third parties (p. 51)". However, once all meaningful competitors had exited the market and Facebook became a virtual monopoly by 2014, it faced no restraining forces of competition and started degrading privacy. Our results thus provide a rationale for a tougher competition policy to curb concentration in ad-funded platforms with free services if competition authorities are more concerned with consumer surplus relative to the advertiser side surplus in welfare calculations. For instance, competition authorities should factor in platforms’ design biases when they formulate a merger policy or a data sharing policy.

The main intuition for the design bias of a monopoly platform can be explained in the following way. Consider a simple case with inelastic demand for consumers. When there is no price constraint on the consumer side and the platform is free to charge any prices, the platform can fully capture any increase in consumer surplus by raising its price by the same amount. This implies that there is no bias in platform design if the platform is also able to extract the whole advertising surplus. However, if the platform cannot capture
all the advertising surplus, then it always prefers consumer biased design. It turns out this simple logic extends to elastic demand because the preferences of consumers and the platform are perfectly aligned. This is due to the "pass-through rate equalization" result, which states that consumers benefit equally from an increase in the consumer side surplus and the advertiser surplus. In contrast, when the consumer price constraint binds, the platform cannot freely adjust price to capture surplus which can result in the platform favoring advertising biased technology.²

In terms of our applications, one important and prototype example of platforms’ design choices is innovation adoption choices. In fact, all reports mentioned above commonly argue that a major harm from concentration of platform market power consists of distortions in innovation incentives. To quote the CMA report (2020, p.7),

"First, competition problems may inhibit innovation and the development of new, valuable services for consumers. ....This impact on innovation is likely to be the largest source of consumer harm."

In one-sided markets, one can consider either a quality-increasing innovation or a cost-reducing innovation and study private incentives to adopt it by incurring a fixed cost and compare it with a social planner’s incentives. What is interesting in a two-sided platform is that one can consider innovations that affect the two sides in an opposite way, study a platform’s incentive to trade-off the gain from one side against the loss from the other side and compare it with a social planner’s incentive. In this case, a bias in platform design can be interpreted as a bias in the direction of platform innovation. In particular, our result of AS-bias from the NPC implies that a purely ad-funded platform’s direction of innovation is biased toward the advertiser side. Hence, even if the platform spends a large amount of resources on innovations, there is a risk that its focus on advertiser-surplus increasing innovations may have negative consequences on consumer surplus.

Our analysis can also be applied to platforms’ other design policies that create trade-offs between the consumer and the advertiser side. A platform’s privacy policy, for instance, can be interpreted as having a similar effect as technology adoption in that collection of consumers’ sensitive information may impose privacy costs on the consumers, but may help increase advertising revenues. Our result of AS-bias from the NPC thus implies that a purely ad-funded platform collects too much data from consumers to increase advertiser surplus.

²As discussed in section 2, our model can also be applied to platforms with (direct) network effects such as social networking sites.
Another policy option available to platforms concerns "ad load"; an increase in ad load would decrease consumer surplus but increase advertising revenues per consumer. According to the CMA Report (2020), "[s]earch engines like Google can determine the overall limit on the number of ads that appear in search results and how these ads are presented alongside organic search results (p. 229)." Similarly, "Facebook can directly set the ad load by determining the ad gap – the ratio of advertising to organic content users see when interacting with the platform (p. 256)." Finally, big tech platforms can control the balance, in their ad auction mechanisms, between ad price and quality (i.e., relevance of ads shown to users) by choosing how much weight to place on quality metrics in determining the winning bid for ad slots; a lower weight on quality and relevance metrics induces higher bidding prices and generates more revenue, but reduces the quality of the platform services for users (p. 230 and p. 256). The choices of platforms on these policy dimensions can be analyzed with some modifications in our framework. More specifically, in contrast to the two previous applications where the platform is assumed to capture a fixed fraction of advertiser surplus, we put more structure on the advertiser side in the analysis of ad load. We consider a platform which has monopoly power on both sides such that the ad price is determined by the ad load it chooses. In this extension, we show that there is a force mitigating the AS-bias in the presence of the NPC. The AS-bias in the context of ad load is manifested in the form of too much ad as the platform does not fully internalize consumer nuisance costs from advertising. However, the exercise of the monopoly power in the advertising market induces the platform to reduce the ad load in order to raise the ad price. Therefore, depending on the elasticity of demand on each side, we can have too much or too little ad load when the NPC binds.

Our article contributes to the recently emerging literature on platform design that has studied the incentives of digital platforms on various key issues of governance as a gatekeeper. They include incentives to delist low-quality sellers (Casner, 2020) or IP-infringing sellers (Jeon, Lefouili and Madio, 2021), to curate apps (Etro, 2021), to introduce deceptive features (Johnen and Somogyi, 2021), to choose the intensity of seller competition (Johnson, et al. 2021, Teh, forthcoming), and to moderate content (Liu et al., 2021; Madio and Quinn, 2021) among others. Our article is most closely related to Teh (forthcoming) and Etro (2021). Teh (forthcoming) considers marketplace platforms that can play a regulatory role in running their platforms. He analyzes a platform’s non-price "governance designs" and shows that the platform’s governance designs can be distorted.

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3See also Chapter 6 of Belleflamme and Peitz (2021).
towards inducing insufficient or excessive seller competition, depending on the nature of
the fee instrument employed by the platform. His analysis thus is complementary to our
analysis, focusing on different aspects of platform design. Etro (2021) considers competi-
tion between a device-funded platform (Apple) and an ad-funded platform (Google) in a
two-sided market composed of a consumer side and an app side. The former is vertically
integrated into the device market (i.e., the consumer side) whereas the latter is not. He
finds that as the former can capture consumer surplus from app curation through its de-
vice price, it has more incentive (than the latter) to curate apps to raise consumer surplus
although this can reduce the revenue from the app side. We do not consider competition
between different business models. Instead, we isolate the effect of a price constraint in
the consumer side on the discrepancy between private and social incentives for trading-off
consumer surplus against advertiser surplus across market structures.

The rest of the article is organized in the following way. In Section 2, we analyze
a monopoly platform. In Section 3, we apply the monopoly framework to technology
adoption, privacy, ad load policies. Section 4 considers a duopoly model of competi-
tive bottleneck and illustrates the general results for the duopoly case by analyzing the
Hotelling model and the logit model. Section 5 contains a summary of the main results
with discussions. Section 6 provides concluding remarks.

2 Monopoly Platform

In this section, we consider a monopolistic platform in a two-sided market and analyzes
its design incentives and compare them with those of a social planner.

2.1 The Baseline Model of Monopoly Platform

In the baseline model, we consider a monopolistic platform. Let \( u \) and \( p \) respectively
denote the gross surplus per consumer and the price charged by the platform on the
consumer side. The number of consumers on board depends on the net surplus \( s (= u - p) \)
provided by the platform and is represented by \( D(s) \). We assume that \( D(.) \) is strictly
increasing and weakly concave.

When the platform attracts consumers, each consumer allows the platform to generate
additional revenue from the other side. For instance, we can envision a situation in which
the platform sells content to consumers and use the customer base to derive advertising
revenues from advertisers who need access to consumers. Another source of revenue could
be in-app purchases. For simplicity, we assume that the platform can generate a total surplus of $\beta$ per consumer on the advertiser side. We adopt a parsimonious reduced form modeling in that the platform can extract a $\tau$ proportion of the surplus, where $\tau \in (0, 1]$. In other words, on the advertising side, each consumer generates an ad revenue of $\tau \beta$ to the platform. We provide a microfoundation of this model in the Appendix. In Section 3.3, we also introduce more structure on the advertiser side when we analyze the platform’s "ad load" policy as an application of this framework.

A main reason for why we model the advertising market in a reduced-form way is that the boundary of the advertising market is much broader than that of a product market. Consider the case of the programmatic display advertising market, which sells display advertising inventories through real-time auctions. In this market, all kinds of publishers (including online newspapers) and content producers compete together with social media on the supply side. Most publishers and content producers rely on various advertising intermediaries to sell their advertising inventories to a large number of advertisers. Hence, even if a publisher is a monopolist in its product market, it has no or little market power on the advertising side of which the outcome is largely determined by the total supply and the total demand conditions. One important factor determining $\tau$ is what is called "ad tech take", which represents the share taken by ad intermediaries from the advertising expenditure paid by advertisers. Small platforms such as online newspapers that rely on ad intermediaries have a smaller $\tau$ whereas big tech platforms that have built their own ad tech system and hence are not subject to ad tech take would have a larger $\tau$ (see Section 5 for more details).

We assume that the marginal cost of serving a consumer is normalized to zero, without loss of generality. Hence, the platform’s profit is

$$\pi(p; u, \beta) = D(u - p)(p + \tau \beta).$$

Maximizing it with respect to $p$ gives the following first order condition (F.O.C.):

$$\frac{\partial \pi(p; u, \beta)}{\partial p} = -D'(u - p)(p + \tau \beta) + D(u - p) = 0.$$

Let the price that satisfies the above condition be denoted by $\bar{p}$. As will be further explained below, we consider two scenarios depending on the existence of any price constraints. Let $p^*$ denote the platform’s (optimal) price. When there is no price constraint
and the platform can charge any prices, the monopolist would set the price of \( p^* = \hat{p} \) defined by (1). When the platform’s price is not flexible and constrained to be set at \( p \), we have \( p^* = \bar{p} \).

We can write the platform’s (maximized) profit as

\[
\pi^m(u, \beta) = D(u - p^*)(p^* + \tau \beta), \tag{2}
\]

where the superscript \( m \) represents monopoly. Let the aggregate consumer surplus (CS) be denoted by \( v(s) \), where \( v(.) \) satisfies the envelope condition \( v'(s) = D(s) \). We can also define the corresponding social welfare given \((u, \beta)\).

\[
W^m(u, \beta) = \pi^m(u, \beta) + v(u - p^*) + D(u - p^*)(1 - \tau) \beta. \tag{3}
\]

A comparison of (2) and (3) reveals that the platform’s and a social planner’s preferences over \((u, \beta)\), respectively represented by \( \pi^m(u, \beta) \) and \( W^m(u, \beta) \), are potentially misaligned because the platform does not take into account the effects of changes in \((u, \beta)\) on consumer and advertiser surpluses.

To analyze potential biases in platform’s design choice in comparison to the (second-best) social optimum where the price decision is left to the platform, we perform the following exercise. Consider any design changes from \((u, \beta)\) to \((u', \beta')\) that would provide the same monopoly profit, where \( u' = u + \Delta u \) and \( \beta' = \beta + \Delta \beta \). Suppose that a profit-neutral design choice entails a positive change in \( u \) (i.e., \( \Delta u > 0 \), which implies that \( \Delta \beta < 0 \) to keep the platform profit constant). If such a change leads to a decrease in social welfare, it implies that the platform’s private value of an increase in \( u \) is higher than social value in the sense that it is willing to sacrifice more aggregate advertiser surplus \( \beta \) for a unit increase of \( u \) than the social planner would. In this case, we say that the platform’s design choice is \textit{CS-biased} because it favors the consumer side. In contrast, if such a profit-neutral change leads to an increase in social welfare, we say that the platform’s design choice is \textit{AS-biased}. With differentiability, this idea can be formally stated as follows.

\textbf{Definition 1.} A platform’s design is \textit{CS-biased} (respectively, \textit{AS-biased}) if \( \frac{dW}{du} \bigg|_{\pi^m=0} < 0 \) (respectively, if \( \frac{dW}{du} \bigg|_{\pi^m=0} > 0 \)).

In our analysis, we distinguish two cases depending on the presence of (or lack of) any price constraint on the consumer side. There can be various reasons for the existence
of price constraints. For instance, the price constraint may arise due to government regulations that impose price ceilings. Alternatively, the monopolist may voluntarily have made a prior commitment to a certain price to relieve any concern for future opportunist behavior of the platform. For instance, Google has made a strategic decision to make its Android system available for “free” without any charges as an “open source” mobile operating system when it was first introduced in 2007.\footnote{The decision may have been necessary for market penetration and building an installed base of consumers to compete against alternatives such as Symbian and Windows Mobile.} Or the price constraint can result from platforms’ embracing ad-financed business models that provide services for free. Indeed, one important and very relevant case in platform markets is the "non-negative price constraint." As is typical in two-sided markets, because of the extra revenue that can be generated by the advertising side, the optimal price on the consumer side may entail below cost pricing (see Armstrong (2006) and Rochet and Tirole (2006)). When the marginal cost is low or even zero as in the digital markets, this implies that the optimal price can be negative. However, negative prices can invite opportunist behaviors by consumers due to various moral hazard and adverse selection reasons (Farrell and Gallini (1988), Armstrong and Wright (2007), Amelio and Jullien (2012), Choi and Jeon (2021) and Jeon, Menicucci and Nasr (2021)).\footnote{See Choi and Jeon (2021) for more detailed discussion of the non-negative price constraint.} In such a scenario, negative prices are impractical and the platform is constrained to set the price at zero. One of the main goals in this article is to identify the effect of the price constraint (in particular, the nonnegative price constraint) on biases in platform design.

2.2 No Price Constraint Case

In the absence of any price constraint (that is, the price can be any including a negative price), the optimal price on the consumer side satisfies the F.O.C. (1). For our analysis, it turns out that a change of variables with \( q = p + \tau \beta \) yields cleaner results, where \( q \) represents the total profit per-consumer (including advertising revenues) of the platform.\footnote{We thank an anonymous reviewer and Anton Sobolev for suggesting this alternative formulation of the problem.} With this change, we can rewrite the platform’s maximized profit and the corresponding social welfare, (2) and (3), as

\[
\pi^m(u, \beta) = Max_q D(u + \tau \beta - q)q = D(u + \tau \beta - q^*)q^*, \tag{4}
\]

\[
W^m(u, \beta) = \pi^m(u, \beta) + v(u + \tau \beta - q^*) + D(u + \tau \beta - q^*)(1 - \tau)\beta, \tag{5}
\]
where $q^* = p^* + \tau \beta$. Consider now a design choice that changes $(u, \beta)$. To analyze this, consider a local locus of $(u', \beta')$ that would provide the same monopoly profit, where $u' = u + du$ and $\beta' = \beta + d\beta$. From (4), it is clear that $u + \tau \beta$ should stay constant for the monopoly profit to be the same, which implies that $q^*$ remains the same.

As noted earlier, a comparison of (4) and (5) reveals that the platform’s and a social planner’s preferences over $(u, \beta)$, respectively represented by $\pi^m(u, \beta)$ and $W^m(u, \beta)$, are potentially misaligned because the platform does not take into account the effects of changes in $(u, \beta)$ on consumer and advertiser surpluses. However, it turns out that the preferences of consumers and the platform are perfectly aligned because consumer surplus stays the same along the locus of $(u_0, \beta_0)$ that would provide the same monopoly profit. This can be seen by the expression for consumer surplus $v(u + \tau \beta - q^*)$: the requirement that $u + \tau \beta$ be constant for the monopoly profit to be the same ensures that consumer surplus also stays constant.\(^7\) Thus, the only discrepancy is due to the effects on advertiser surplus. An inspection of (5) makes it clear that any changes in $(u, \beta)$ that provide the same monopoly profit (with $u + \tau \beta$ being constant) would lead to an increase in social welfare if the change entails a positive (respectively, negative) change in $\beta$ (respectively, $u$). This implies that the platform favors the consumer side in the sense that it is willing to sacrifice more aggregate advertiser surplus $\beta$ for a unit increase of $u$ than the social planner does unless it is able to appropriate the whole advertiser side surplus (i.e., $\tau = 1$, in which case the platform design choice is neutral and the platform incentives are aligned with the social planner’s).

In summary, we have:

**Proposition 1.** Consider a monopolistic two-sided platform. If there is no price constraint on the consumer side, its design choice incentive is CS-biased unless it can fully extract advertiser surplus (in which case its incentive is unbiased).

### 2.3 Price Constraint Case (with $p = \bar{p}$)

Consider now the case in which the platform’s price is fixed, for some reason, at a non-negative level (i.e., $p = \bar{p}(\geq 0)$). Although we consider a non-negative $\bar{p}$ for expositional simplicity, our analysis applies to a negative $\bar{p}$ as long as the per-consumer profit is positive (i.e., $\bar{p} + \tau \beta > 0$). Then, the monopoly profit is given by

\(^7\)The reason for this result is due to the "pass-through rate equalization," that is, the pass-through rates to consumer surplus from an increase in $u$ and $\tau \beta$ are the same. More precisely, as long as $u + \tau \beta$ is constant, the platform will choose the same $q (= p + \tau \beta)$. This implies, $1 - dp/du = |dp/d(\tau \beta)|$. 

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\[
\pi^m(u, \beta) = D(u - \bar{p})[\bar{p} + \tau \beta],
\]
where \(\tau \in (0, 1]\).

Social welfare is given by
\[
W^m(u, \beta) = \pi^m(u, \beta) + v(u - \bar{p}) + D(u - \bar{p})(1 - \tau)\beta
\]
\[
= v(u - \bar{p}) + D(u - \bar{p})[\bar{p} + \beta].
\]

Before a general analysis for any \(\bar{p}(\geq 0)\), we first consider one important class of a fixed price case in which the non-negative price constraint is binding with a zero price. This situation arises when the platform would like to charge a negative price (i.e., provide a subsidy to consumers) to attract consumers, but is unable to do so due to various adverse selection and/or moral hazard reasons. The non-negative price constraint is binding if the following condition holds:
\[
\frac{\partial \pi(p; u, \beta)}{\partial p} \bigg|_{p=0} = -D'(u)\tau \beta + D(u) < 0.
\]
(6)

In such a case, the platform is constrained to charge a zero price \((\bar{p} = 0)\). Then, it is immediate from (9) that the platform design choice is always AS-biased. To see this, note that with \(\bar{p} = 0\), the monopoly profit and social welfare are reduced to:
\[
\pi^m(u, \beta) = D(u)\tau \beta
\]
\[
W^m(u, \beta) = v(u) + D(u)\beta.
\]

It is clear that any changes in \((u, \beta)\) that keeps \(\pi^m(u, \beta) = D(u)\tau \beta\) unchanged also keep the second term in welfare, \(D(u)\beta\), unchanged because \(\tau\) is a constant parameter. This implies that any profit-neutral design choice that entails a positive change in \(u\) increases social welfare; the platform’s design choice is AS-biased. This result suggests that the formulation of optimal antitrust policies towards the platform market can be substantially different for markets where services are provided for free.

The simple analysis (with a change of variables) we have adopted cannot be applied for a more general price constraint case of \(\bar{p} > 0\). For a more general analysis, it turns out that a comparison of the slopes of the "iso-profit" and "iso-welfare" curves yields equivalent results and more convenient. More specifically, consider a local locus of \((u', \beta')\)
that would provide the same monopoly profit, where \( u' = u + du \) and \( \beta' = \beta + d\beta \). This iso-profit locus can be derived by

\[
d\pi^m(u, \beta) = \frac{\partial \pi^m}{\partial u} du + \frac{\partial \pi^m}{\partial \beta} d\beta = 0. \tag{7}
\]

The (absolute value of the) slope of the iso-profit curve is the platform’s willingness to trade-off \( \beta \) against \( u \), and hence represents the platform’s incentives for its design choice in \((u, \beta)\).

\[
\left| \frac{d\beta}{du} \right|_{d\pi^m=0} = \frac{\partial \pi^m}{\partial u} \frac{\partial \pi^m}{\partial \beta}
\]

In a similar manner, we can derive the design choice locus that yields the same social welfare, the iso-welfare curve, as follows:

\[
dW^m(u, \beta) = \frac{\partial W}{\partial u} du + \frac{\partial W}{\partial \beta} d\beta = 0.
\]

The slope of the iso-welfare represents the preference of a social planner taking consumer surplus and advertiser surplus into account.

\[
\left| \frac{d\beta}{du} \right|_{dW^m=0} = \frac{\partial W}{\partial u} \frac{\partial W}{\partial \beta}
\]

**Lemma 1.** A platform’s design is CS-biased (respectively, AS-biased) if and only if

\[
\left| \frac{\partial \beta}{\partial u} \right|_{d\pi^m=0} > \left| \frac{\partial \beta}{\partial u} \right|_{dW^m=0} \quad \text{(respectively, if and only if} \left| \frac{\partial \beta}{\partial u} \right|_{d\pi^m=0} < \left| \frac{\partial \beta}{\partial u} \right|_{dW^m=0} \text{)} (\text{see Figures 1 and 2}).
\]

**Proof.** See the Appendix □

In our definition of the platform’s biases in design choices, we have evaluated changes in social welfare along the iso-profit curve. The equivalence of this approach to the comparison of the slopes for the iso-profit and iso-welfare curves (established in Lemma 1) can be easily seen from Figures 1 and 2. As shown in Figure 1, when the slope of the iso-profit curve is steeper than that of the iso-welfare curve (that is, \( \left| \frac{\partial \beta}{\partial u} \right|_{d\pi^m=0} > \left| \frac{\partial \beta}{\partial u} \right|_{dW^m=0} \)), we can find two shaded areas in which private and social incentives conflict. The shaded area in the second quadrant (harming consumers, but benefiting advertisers) represents platform designs that would be socially beneficial, but would not be adopted by the monopolist. The shaded area in the fourth quadrant (benefiting consumers, but
harming advertisers), in contrast, represents platform designs that would be welfare-reducing but would be chosen by the monopolist. In that sense, design choice incentives by the monopolist are biased towards the consumer side surplus. Similarly, when the slope of the iso-welfare curve is steeper than that of the iso-profit curve as in Figure 2, we can identify two areas that exhibit design choice incentives that are biased towards the advertiser side surplus.

\textit{Figures 1 and 2 here}

We can easily verify that the slopes with a price constraint of \( p = \overline{p}(\geq 0) \) can be derived as follows.

\[
\left|\frac{d\beta}{du}\right|_{\delta \pi^m = 0} = \frac{D'(u - \overline{p})(\overline{p} + \tau \beta)}{\tau D(u - \overline{p})} = \frac{D'(u - \overline{p})}{D(u - \overline{p})} \left[\frac{\overline{p} + \tau \beta}{\overline{p}}\right],
\]

(8)

and

\[
\left|\frac{d\beta}{du}\right|_{\delta \pi^m = 0} = \frac{D(u - \overline{p}) + D'(u - \overline{p})[\overline{p} + \beta]}{D(u - \overline{p})} = 1 + \frac{D'(u - \overline{p})}{D(u - \overline{p})} [\overline{p} + \beta].
\]

Note that the iso-welfare curve does not depend on \( \tau \) because the welfare does not depend on \( \tau \).

When we compare the slopes of the indifference curves, we get

\[
\left|\frac{d\beta}{du}\right|_{\delta \pi^m = 0} - \left|\frac{d\beta}{du}\right|_{\delta \pi^m = 0} = 1 - \frac{D'(u - \overline{p})}{D(u - \overline{p})} \left[\frac{(1 - \tau)}{\tau} \overline{p}\right].
\]

(9)

Thus, we can confirm our earlier result that the platform’s design choice is AS-biased if \( \overline{p} = 0 \) because \( \left|\frac{d\beta}{du}\right|_{\delta \pi^m = 0} > \left|\frac{d\beta}{du}\right|_{\delta \pi^m = 0} \) always holds; otherwise (i.e., if \( \overline{p} > 0 \)), we have

\[
\left|\frac{d\beta}{du}\right|_{\delta \pi^m = 0} > \left|\frac{d\beta}{du}\right|_{\delta \pi^m = 0} \text{ if and only if } \tau > \frac{\eta}{1 + \eta},
\]

where \( \eta \) is the price elasticity of demand evaluated at the fixed price \( \overline{p} \).

Summarizing, we have:

\textbf{Proposition 2.} Consider a monopolistic two-sided platform. Suppose that the price on the consumer side is fixed at \( p = \overline{p}(\geq 0) \). Then, its design choice incentive is AS-biased if and only if \( \overline{p} = 0 \) holds or \( \overline{p} > 0 \) and \( \tau > \frac{\eta}{1 + \eta} \) hold.

Our model can also be applied to platforms with (direct) network effects such as social
networking sites. The standard way to incorporate direct network effects is to add network benefits to the stand-alone value in the consumer utility.\(^8\) Let us normalize that there is a mass 1 of potential consumers. With network effects, the value of joining the platform when a fraction \(n\) of consumers have joined is given by \(u + v(n) - p\), where \(u\) is the stand-alone value of the platform service and \(v(n)\) represents the network benefits with \(v(0) = 0\) and \(v'(.) > 0\). Then, the equilibrium number of consumers is implicitly defined by \(n^* = D(u + v(n^*) - p)\). If \(D'v' < 1\) (i.e., if network effects are not too strong), the number of consumers is uniquely determined. By totally differentiating this equilibrium condition, we can also easily verify that the number of consumers is increasing in \(u\) and decreasing in \(p\) as \(\frac{dn^*}{d(u-p)} = \frac{D'}{1-D'v'} > 0\). Our framework thus can accommodate direct network effects on the consumer side, which can be relevant to social media.

We have analyzed price constraints exogenously imposed. However, the non-negative price constraint may be imposed endogenously. In such a case, a change in platform design may induce a regime change from no price constraint to an endogenously derived zero price regime or vice versa. Then, we have to distinguish a local analysis from a global analysis. Our analysis is local in the endogenously derived price constraint case and implicitly assumes that the platform choice does not induce a regime change. More precisely, the non-negative price constraint binds when \(\frac{\partial n(p,u,\beta)}{\partial p}|_{p=0} = -D'(u)\tau\beta + D(u) < 0\), that is \(\beta\) is sufficiently large. We can define a (positively sloped) locus of \((u,\beta)\) where the constraint just binds: the locus represents combinations of \((u,\beta)\) where \(\frac{\partial n(p,u,\beta)}{\partial p}|_{p=0} = -D'(u)\tau\beta + D(u) = 0\). Below this locus the optimal price is positive whereas above the locus the non-negative price constraint is binding with the price of zero. If the platform’s initial point and the point after a design choice are on the same side of the locus, there is no regime change and our analysis applies. Even if we have a regime change, but the new \((u,\beta)\) stays near the constraint locus, our result about the bias is still valid as indifference curves for both private and social incentives are continuous (including the region around the locus). For instance, if the initial \((u,\beta)\) is below the locus and \(\tau < 1\), we have an area of \((u,\beta)\) representing the CS-bias between the initial point and the locus. By continuity, this area continues to exist on the other side of the locus as long as \((u,\beta)\) is close enough to the locus. However, if the design choice induces a regime change that is far away from the locus such that the two indifference curves that intersect at the initial point intersect again in the other side of the constraint locus, then we can have an area of \((u,\beta)\) representing the AS-bias starting from the second intersection point, which is consistent

\(^8\)See for instance Section II.A in Choi and Jeon (2021), which builds on Katz and Shapiro (1986).
with our local analysis.\textsuperscript{9}

3 Applications: Technology Adoption, Privacy and Ad Load Policies

In the previous section, we have proposed a basic framework to analyze the monopolistic platform’s design choices that may affect the two sides of the market in opposite ways and compare them against a social planner’s. In this section, we extend the analysis to show how the framework can be modified for applications to some of the design choices that have figured prominently in the recent policy discussions: technology adoption, privacy protection, and ad load. In the basic framework, we have analyzed the platform’s incentives towards any arbitrary combinations of design choices represented by \((u, \beta)\) without explicit cost considerations. Many design choices by the platform, however, change \((u, \beta)\) in a specific way with associated costs. We thus put more structure in the model that explicitly reflects this relationship and potential costs associated with such design choices. We present a detailed analysis for technology adoption to show how our basic framework can be adapted. For applications to privacy and ad load policies, we briefly describe the frameworks to analyze the relevant issues with a summary of the main results. The detailed derivations are in the Appendix.

3.1 Technology Adoption

The digital platform industry is a dynamic one in which innovations such as artificial intelligence (AI) play a major role. Therefore, it is crucial to understand how market power shapes digital platforms’ incentives to innovate. If we consider technology adoption choices that would affect the two sides in opposite ways but without additional costs, the analysis in the previous section immediately applies verbatim. We extend the previous framework by adding costs of technology adoption. More specifically, let \(C(u, \beta)\) be the cost of adopting technology that provides \((u, \beta)\) to the platform. We assume that \(C(u, \beta)\) is convex in \((u, \beta)\) with \(C_u > 0, C_{uu} > 0, C_\beta > 0, C_{\beta\beta} > 0\). This setup allows us to consider technology adoptions that entail both \(u\) and \(\beta\) in the same positive direction, but we show that the monopolistic platform exhibits the same type of biases in its technology adoption as in Section 2.

\textsuperscript{9}More details that show this possibility with the Matlab code are available upon request.
3.1.1 No Price Constraint Case

Consider first the case in which the platform can charge any price \( p \) on the consumer side. Then, the platform’s profit taking the cost of adoption into account, \( \pi^m(u, \beta) \), is given by

\[
\pi^m(u, \beta) = \pi^m(u, \beta) - C(u, \beta) = D(u - p^*)(p^* + \tau \beta) - C(u, \beta)
\]

Any choice of \((u^*, \beta^*)\) that maximizes the platform’s profit should satisfy the following tangency condition

\[
\left. \left| \frac{d\beta}{du} \right|_{\pi^m=0} \right|_{(u, \beta)=(u^*, \beta^*)} = \frac{D'(u^* - \bar{p})(p^* + \tau \beta^*)}{D(u^* - \bar{p})\tau} = \frac{C_u(u^*, \beta^*)}{C_\beta(u^*, \beta^*)}
\]

The corresponding social welfare that accounts for the cost of adoption is given by

\[
\hat{W}^m(u, \beta) = W^m(u, \beta) - C(u, \beta) = \pi^m(u, \beta) + v(u - p^*) + D(u - p^*)(1 - \tau)\beta - C(u, \beta)
\]

Consider the locus of \((u, \beta)\) that would cost the same as \( C(u^*, \beta^*) = \overline{C} \). Note that our assumption of convex \( C(\cdot) \) implies that the iso-cost curve is concave and its absolute slope increases along the curve as we increase \( u \) (hence reduce \( \beta \)). Let \((u^o, \beta^o)\) represent the social planner’s choice of \((u, \beta)\) on this locus, which would satisfy

\[
\left. \left| \frac{d\beta}{du} \right|_{W^m=0} \right|_{(u, \beta)=(u^o, \beta^o)} = \frac{C_u(u^o, \beta^o)}{C_\beta(u^o, \beta^o)}.
\]

We know from Proposition 1 that \( \left. \left| \frac{d\beta}{du} \right|_{\pi^m=0} \right|_{(u, \beta)=(u^*, \beta^*)} \geq \left. \left| \frac{d\beta}{du} \right|_{W^m=0} \right|_{(u, \beta)=(u^*, \beta^*)} \), with the equality holding only when \( \tau = 1 \). The iso-cost curve is concave and the absolute value of its slope increases along the curve as we increase \( u \) (hence reduce \( \beta \)). This implies that for \( \tau \in (0, 1) \), \( u^* > u^o \) and \( \beta^* < \beta^o \), that is, the monopolist’s technology adoption choice is CS-biased when there is no restrictions on pricing whereas there is no bias (i.e., \( u^* = u^o \) and \( \beta^* = \beta^o \)) when \( \tau = 1 \).
3.1.2 Price Constraint Case (with $p = \bar{p}$)

Consider now the case in which the price on the consumer side is constrained to be $p (\geq 0)$. Then, the platform’s profit is given by

$$\tilde{\pi}^m(u, \beta) = \pi^m(u, \beta) - C(u, \beta)$$

$$= D(u - \bar{p})(\bar{p} + \tau \beta) - C(u, \beta)$$

Any choice of $(u^*, \beta^*)$ that maximizes the platform’s profit should satisfy the following condition

$$\left. \frac{d\beta}{du} \right|_{d\pi^m=0}^{(u, \beta)=(u^*, \beta^*)} = \frac{D'(u^* - \bar{p})}{D(u^* - \bar{p})} \left[ \bar{p} + \tau \beta^* \right] \frac{1}{\tau} = \frac{C_u(u^*, \beta^*)}{C_\beta(u^*, \beta^*)}$$

The corresponding social welfare that accounts for the cost of adoption is given by

$$\tilde{W}^m(u, \beta) = W^m(u, \beta) - C(u, \beta)$$

$$= \pi^m(u, \beta) + v(u - \bar{p}) + D(u - \bar{p})(1 - \tau)\beta - C(u, \beta)$$

Once again, consider the locus of $(u, \beta)$ that would cost the same as $C(u^*, \beta^*) = \bar{C}$. The social planner’s choice of $(u, \beta)$ on this locus would satisfy

$$\left. \frac{d\beta}{du} \right|_{d\tilde{W}^m=0}^{(u, \beta)=(u^*, \beta^*)} = \frac{C_u(u^*, \beta^0)}{C_\beta(u^*, \beta^0)}$$

We know from Proposition 2 that if $\bar{p} = 0$, $\left. \frac{d\beta}{du} \right|_{d\tilde{W}^m=0}^{(u, \beta)=(u^*, \beta^*)} > \left. \frac{d\beta}{du} \right|_{d\pi^m=0}^{(u, \beta)=(u^*, \beta^*)}$ holds; if $\bar{p} > 0$, we have

$$\left. \frac{d\beta}{du} \right|_{d\tilde{W}^m=0}^{(u, \beta)=(u^*, \beta^*)} > \left. \frac{d\beta}{du} \right|_{d\pi^m=0}^{(u, \beta)=(u^*, \beta^*)} \text{ if and only if } \tau > \frac{\eta}{1 + \eta},$$

where $\eta$ is the price elasticity of consumer demand. Thus, we can conclude that the platform’s technology adoption incentives are AS-biased with price constraints if and only if $\bar{p} = 0$ holds or $\bar{p} > 0$ and $\tau > \frac{\eta}{1 + \eta}$ hold.

The AS-bias that arises when the price is constrained to be zero implies that a purely ad-funded platform’s direction of innovation is biased toward the advertiser side. Hence, even if the platform spends a large amount of resources on innovations, there is a risk that its focus on advertiser-surplus increasing innovations may harm consumers.
3.2 Privacy Policy

We here consider an application to privacy policy. A platform’s privacy policy can be interpreted as a prime example of platform design that affects the two sides in opposite ways because collection of consumers’ sensitive personal information may impose privacy costs on the consumers, but may help increase advertising revenues through improved ad targeting. Let \( d \) represent the amount of personal data collected by the platform, \( c(d) \) the associated privacy cost to consumers, which is strictly increasing and convex, and \( \beta(d) \) the associated surplus on the advertising side, which is strictly increasing and concave because more data improves ad targeting.

With this formulation the platform’s profit and social welfare can be respectively written as follows.

\[
\pi(p, d) = D(u - c(d) - p)(p + \tau \beta(d))
\]

\[
W(p, d) = \pi(p, d) + v(u - c(d) - p) + D(u - c(d) - p)(1 - \tau)\beta(d),
\]

where the last term in social welfare captures the advertiser surplus. To investigate whether the platform’s data collection (\( d^* \)) is excessive (i.e., AS-biased) or insufficient (CS-biased) compared to the social optimum, we can inspect the sign of \( \frac{\partial W}{\partial d} \) evaluated at the privately optimal level of data collection \( \frac{\partial W}{\partial d} \bigg|_{d=d^*} \). In this framework, we can derive a result which parallels the one we obtained in the general analysis of platform design in Section 2 (see the Appendix for detailed analysis).

**Proposition 3.** Consider a monopoly platform which chooses the amount of personal data it collects for targeted advertising.

(i) In the absence of any pricing constraint on the consumer side, the platform collects too little data from social point of view for any \( \tau < 1 \) and collects the socially optimal amount for \( \tau = 1 \). In other words, the amount of collected data is CS-biased for \( \tau < 1 \) and unbiased for \( \tau = 1 \).

(ii) When the price on the consumer side is fixed at \( p(\geq 0) \), the platform collects too much data from social point of view if \( \bar{p} = 0 \) or if \( \bar{p} > 0 \) and \( \tau > \frac{\eta}{1+\eta} \). Hence, if the price constraint takes the form of the non-negative price constraint with \( \bar{p} = 0 \), the platform has an incentive to collect too much data for any \( \tau > 0 \).

Our result predicts that Apple would collect too little personal data whereas Google and Facebook collect too much data. Our result is also consistent with the Cambridge Analytica Scandal in which Cambridge Analytica took advantage of Facebook’s lax pri-
vacy policy, which enabled third-party developers to harvest not only data about their users but also data about their users’ friends.\(^\text{10}\)

3.3 Ad Load and Market Power on the Advertising Side

Dominant platforms such as Google and Facebook have market power not only in their consumer-facing products but also in the advertising market, and hence they can control the amount of ad shown and its price. We study the ad load choice by a platform with market power on both sides, which can be considered a platform design choice that affects both sides in opposite directions. In the case of search engine, for instance, a higher ad load by showing a greater proportion of ads relative to organic search results can increase the propensity of users to click on ads. However, the more ads are shown, the more likely it is that some ad content will be less relevant to the user search query, compromising the quality experienced by the user (the CMA Report, 2020, p. 223). In the case of display advertising, which is relevant to Facebook, a higher ad load can lead to a greater immediate financial reward, but inflicts more nuisance costs on consumers. To analyze this trade-off, we apply our framework to the platform’s ad load choice by endogenizing both the ad load and the ad price.

Let \(a\) denote the ad load per consumer chosen by the platform. Let \(c(a)\) denote the personal nuisance cost to consumers with \(c(0) = 0\), which is strictly increasing and convex. The ad price, \(r(a)\), that clears the ad market is strictly decreasing. The ad revenue per consumer thus is given by \(R(a) = ar(a)\). We assume that the marginal ad revenue \(R'(a)\) is strictly positive and strictly decreasing.

In this framework the platform’s profit and social welfare are given by

\[
\pi(p, a) = D(u - c(a) - p)(p + R(a))
\]

\[
W(p, a) = \pi(p, a) + v(u - c(a) - p) + D(u - c(a) - p) \int_0^a (r(x) - r(a))dx,
\]

where the last term in social welfare captures the advertiser surplus.

As in the privacy case, we ask whether the ad load chosen by the platform \((a^*)\) is

---

\(^{10}\) Cambridge Analytica created a personality test that would target American Facebook users. Two hundred seventy thousand people were paid one or two dollars each to take a test, which was designed to collect the personality traits of the test taker as well as data about friends and their Facebook activities. They had more than forty-nine million friends. See McNamee (2019).
socially excessive or not by investigating the derivative of the welfare at \( a = a^* \) (i.e., \( \frac{\partial W}{\partial a} \bigg|_{a=a^*} \)). We find that the ad load choice by the platform is CS-biased (i.e., too small from the welfare point of view) in the absence of any price constraints – once again, a result parallel to the one we obtained in the general analysis. This is because the platform does not internalize the surplus of advertisers. However, in the presence of price constraints, the result is more subtle because we assume market power on the advertising side in this application. In particular, the biases depend on the elasticity of demand on the advertising side.

When the elasticity of demand on the advertiser side is low, the platform’s exercise of its market power on the advertiser side leads to a high ad price with a low ad load. Hence, it is possible that the ad load is too small from the welfare perspective and the platform incentive is CS-biased. In contrast, a high elasticity of demand on the advertise side constrains the platform’s exercise of market power and thereby lowers the ad price with a large ad load, leading to an AS-bias. This logic can further be extended to platforms that are price takers with little market power on the advertising side. More precisely, consider a model in which \( R(a) = \tau \beta(a) \), where \( \beta(a) \) increases with \( a \) (as in the analysis of privacy policy in Section 3.2): the surplus on the advertiser side increases with ad load and the platform takes a constant fraction of this surplus, which is consistent with the platform being a price taker. Then, we obtain that the platform chooses an excessive ad load for any \( \tau \), leading to an AS-bias. Finally, even when the elasticity of demand on the advertising side is not high, the platform’s ad load can be excessive if the semi-elasticity of demand on the consumer side is sufficiently large (see the Appendix for more details).

Summarizing, we have:

**Proposition 4.** Consider a platform with monopoly power on both sides which chooses ad load per consumer.

(i) In the absence of any pricing constraint on the consumer side, the ad load chosen by the platform is too small from the social welfare point of view (i.e., the ad load is CS-biased).

(ii) In the presence of pricing constraint, the ad load chosen by the platform is socially excessive (i.e., the ad load is AS-biased) unless both the semi-elasticity of demand of consumers and the semi-elasticity of demand of advertisers are small enough.

(iii) In the presence of pricing constraint, the platform’s monopoly power on the advertiser side reduces the AS-bias.

The CMA (2020) finds that Google has been able to generate higher click-through rates
by increasing its ad load.\footnote{In 2016, Google removed right-hand side ads and increased from three to four the number of ads eligible to appear above the organic search results. Later in 2016, Google introduced ‘Expanded Text Ads’, which allows advertisers to enhance their ads with an optional third headline and a second description (the CMA report, 2020, p.233). In addition, several advertisers submitted to the CMA that recent changes to Google’s policies on ad load and the presentation of search advertising had the effect of increasing the propensity for users to click on ads rather than organic links (the CMA report, 2020, p.237).} The CMA (2020) also finds that the number of ad impressions served per hour on Facebook has increased from 40-50 in 2016 to 50-60 in 2019 and states that this increase in ad load partly explains why Facebook’s revenue per hour is greater than other platforms and has increased in the past four years (p. 259). These findings are consistent with our result: an ad-financed business model induces Google and Facebook to choose excessive ad load unless the elasticities of demand are small enough on both sides.

### 4 Competitive Bottleneck: Duopoly with Horizontal Differentiation

In this section, we analyze design incentives of competing platforms. Our model involves two (symmetric) horizontally differentiated platforms. In particular, we consider a competitive bottleneck situation in which the platforms compete to attract single-homing consumers and use the customer base to derive advertising revenues from advertisers who need access to consumers.

#### 4.1 Duopoly Competition with Horizontal Differentiation

We consider two symmetric platforms 1 and 2. Let $s_i = u_i - p_i$ represent the net surplus a consumer obtains from platform $i = 1, 2$. The number of consumers for platform $i$ is given by $D_i(s_1, s_2)$. We consider a symmetric demand: $D_1(s, s') = D_2(s', s)$. Let subscripts denote partial derivatives such that $D_i^i = \frac{\partial D_i}{\partial s_i}$ and $D_i^j = \frac{\partial D_i}{\partial s_j}$, where $i, j = 1, 2$ and $i \neq j$.

The symmetric demand implies $D_1^1(s, s') = D_2^2(s', s)$ and $D_1^2(s, s') = D_2^1(s', s)$.

**Assumption 1.** (i) $D_i^i > 0$, $D_j^i < 0$, (ii) $D_i^i \geq |D_j^i|$, and (iii) $D_{ii}^i \leq 0$ for $i = 1, 2$ and $i \neq j$.

This is a standard assumption. A1(i) means that each platform’s demand increases in its surplus provided to consumers, whereas it decreases in the surplus provided to
consumers by the rival platform. A1(ii) means that the own effect weakly dominates the cross effect. With the symmetry of demand, A1(ii) also captures the market expansion effect with $D_i^i + D_i^j \geq 0$ for $i = 1, 2$ and $i \neq j$; when $D_i^i + D_i^j = 0$, the overall market size is fixed and there is no market expansion. Finally, A1(iii) means that the demand is concave in its own surplus provided to consumers, which is a sufficient condition to satisfy the second order condition of the profit maximization. As long as the second order condition is satisfied, we can allow for $D_i^i > 0$.

When we derive the iso-profit curve of platform $i$, we consider its unilateral design choice of $(u_i, \beta_i)$ given $(u_j, \beta_j)$, which is followed by a pricing game only if there is no price constraint. When there is no price constraint, platform $j$ observes $(u_i, \beta_i)$ chosen by platform $i$ and both platforms simultaneously choose their prices. The pricing game does not apply when both platforms’ prices are fixed.

4.2 No Price Constraint Case

As we consider symmetric equilibrium, let us consider platform 1 as the representative one. In the absence of any price constraints, platform 1’s profit can be written as

$$\pi^1(p; u, \beta) = (p_1 + \tau \beta_1)D^1(u_1 - p_1, u_2 - p_2),$$

where $p, u, \beta$ denote vectors of the associated variables.

Let social welfare be

$$W(u, \beta) = v(u_1 - p_1, u_2 - p_2) + (p_1 + \tau \beta_1)D^1(u_1 - p_1, u_2 - p_2) + (1 - \tau)\beta_1D^1(u_1 - p_1, u_2 - p_2) + (p_2 + \beta_2)D^2(u_1 - p_1, u_2 - p_2),$$

where $v(u_1 - p_1, u_2 - p_2)$ denotes consumer welfare function that represents the aggregate consumer surplus.

As in the monopoly case, the analysis is simplified with changes of variables:

$$q_i = p_i + \tau \beta_i.$$

With changes of variables, we have

$$\hat{\pi}^1(q; u, \beta) = q_1D^1(u_1 + \tau \beta_1 - q_1, u_2 + \tau \beta_2 - q_2) \quad (10)$$
and

\[
W(u, \beta) = v(u_1 + \tau \beta_1 - q_1, u_2 + \tau \beta_2 - q_2) + q_1 D^1(u_1 + \tau \beta_1 - q_1, u_2 + \tau \beta_2 - q_2) + (1 - \tau) \beta_1 D^1(u_1 + \tau \beta_1 - q_1, u_2 + \tau \beta_2 - q_2) + q_2 D^2(u_1 + \tau \beta_1 - q_1, u_2 + \tau \beta_2 - q_2)
\]  

(11)

By following the same logic in the monopoly case, we can easily verify that

\[
\left| \frac{d\beta_1}{du_1} \right|_{d\tau^d=0} = \frac{1}{\tau},
\]

where the superscript \( d \) means duopoly. (12) replicates the result in the monopoly case.

In addition, when \( u_1 + \tau \beta_1 \) stays the same to make platform 1’s profit constant, consumer surplus and the combined profits of platform 2 and its advertisers stay constant.\(^{12}\) Thus, the only discrepancy that induces platform 1’s design bias is due to the effects on the surplus of platform 1’s advertisers. As a result, we can derive surprisingly parallel results to the monopoly case despite the presence of potential strategic price effects in the duopoly case, as is clear from a comparison of (10) and (11).

In particular, we find that there is no bias in platform design choices when the platforms can extract full surplus from the advertiser side (\( \tau = 1 \)). Similarly, when \( \tau < 1 \), the platforms cannot extract the full surplus from the advertiser side, which leads to CS-biased design choices.

Summarizing, we have:

**Proposition 5.** Consider a competitive bottleneck model of duopoly platforms with horizontal differentiation. In the absence of any price constraints on the consumer side, a platform’s design choice incentive is CS-biased if it cannot fully extract advertiser surplus; its incentive is unbiased if it fully extracts advertiser surplus.

### 4.3 Price Constraint Case (with \( p = \bar{p} \))

Consider the case where the platforms’ price is fixed at \( p = \bar{p}(\geq 0) \). In this case,

\[
\pi^1(p = \bar{p}; u, \beta) = D^1(u_1 - \bar{p}, u_2 - \bar{p}) \left[ \bar{p} + \tau \beta_1 \right].
\]

\(^{12}\)As in the monopoly case, the reason for this result is due to the "pass-through rate equalization" which also holds in the duopoly case. In fact, we have here a double pass-through equalization: (i) the pass-through rates from an increase in \( u_i \) and \( \tau \beta_i \) to surplus of consumers using platform \( i \) are the same as in the monopoly case and (ii) the pass-through rates from an increase in \( u_j \) and \( \tau \beta_j \) to surplus of consumers using platform \( i \) are the same (i.e., \( dp_i/du_j = dp_i/d(\tau \beta_j) \)).
The locus of design choices that would provide the same profit for platform 1 is given by
\[
d\pi^1(u, \beta) = D_1^1(u_1 - \bar{p}, u_2 - \bar{p}) [\bar{p} + \tau \beta_1] du_1 + D_1^1(u_1 - \bar{p}, u_2 - \bar{p}) \tau d\beta_1 = 0
\]

Therefore, in a symmetric equilibrium we have
\[
\left| \frac{d\beta_1}{du_1} \right|_{d\pi^d=0} = \frac{D_1^1(u - \bar{p}, u - \bar{p}) \left[ \bar{p} + \tau \beta \right]}{\tau D_1^1(u - \bar{p}, u - \bar{p})} = \frac{D_1^1(u - \bar{p}, u - \bar{p}) \left[ \bar{p} + \tau \beta \right]}{D_1^1(u - \bar{p}, u - \bar{p})} \frac{1}{\tau} \quad (13)
\]

Note first that (13) is exactly the same as the corresponding formula in the monopoly case (8).

Social welfare with the price constraint is given by
\[
W(u, \beta) = v(u_1 - \bar{p}, u_2 - \bar{p}) + D_1^1(u_1 - \bar{p}, u_2 - \bar{p}) [\bar{p} + \tau \beta_1] + (1 - \tau) \beta_1 D_1^1(u_1 - \bar{p}, u_2 - \bar{p}) + D_2^1(u_1 - \bar{p}, u_2 - \bar{p}) [\bar{p} + \beta_2].
\]

Hence, the locus of design choices by platform 1 that would provide the same welfare is given by
\[
dW(u, \beta) = \left[ D^1 + \beta_1 D_1^1 + \beta_2 D_2^1 + \bar{p}(D_1^1 + D_2^1) \right] du_1 + D^1 d\beta_1 = 0
\]

In a symmetric equilibrium, $D_1 = D_2$, $D_1^1 = D_2^1$, $D_1^2 = D_2^2$, $p_1 = p_2 = \bar{p}$, $\beta_1 = \beta_2 = \beta$, we have
\[
\left| \frac{d\beta_1}{du_1} \right|_{dW^d=0} = \frac{D_1^1(u - \bar{p}, u - \bar{p}) + (D_1^1 + D_2^1) (\bar{p} + \beta)}{D_1^1(u - \bar{p}, u - \bar{p})} = 1 + \frac{(D_1^1 + D_2^1)}{D_1^1(u - \bar{p}, u - \bar{p})} (\bar{p} + \beta). \quad (14)
\]

As in the monopoly case, the iso-welfare curve does not depend on $\tau$.

To analyze potential biases in platform design, we rewrite (14) as
\[
\left| \frac{d\beta_1}{du_1} \right|_{dW^d=0} - \left| \frac{d\beta_1}{du_1} \right|_{d\pi^d=0} = 1 + \frac{(D_1^1 + D_2^1)}{D_1^1(u - \bar{p}, u - \bar{p})} (\bar{p} + \beta) - \frac{D_1^1(u - \bar{p}, u - \bar{p})}{D_1^1(u - \bar{p}, u - \bar{p})} \frac{\bar{p} + \tau \beta}{\tau} \frac{1}{\tau}
\]
\[
= \left( 1 - \frac{D_1^1(u - \bar{p}, u - \bar{p})}{D_1^1(u - \bar{p}, u - \bar{p})} \left[ \frac{(1 - \tau)}{\tau} \right] + \frac{D_2^1(\bar{p} + \beta)}{D_1^1} \right). \quad (15)
\]
Therefore, we have

\[
\left| \frac{d\beta}{du_1}\Big|_{dW^d=0} \right| - \left| \frac{d\beta}{du_1}\Big|_{dn^d=0} \right| = \begin{cases} 
1 - \frac{(1-\tau)\eta_1}{\tau} + \frac{D_1^2(\bar{p} + \beta)}{D_1}, & \text{if } \bar{p} > 0, \\
1 - \frac{D_1^2}{D_1} \beta, & \text{if } \bar{p} = 0, 
\end{cases}
\]

where \( \eta_1 \) is the price elasticity of platform 1’s consumer demand. Compared to the monopoly condition (9), condition (16) has one additional term, which can be considered the business stealing effects and favors the consumer side. In particular, consider the case where the price constraint takes the form of the non-negative price constraint with \( \bar{p} = 0 \). In such a case, the platform’s design choice is always AS-biased in the monopoly case (Corollary 1). However, the bias can be reversed with competition. When \( \bar{p} = 0 \), the comparison of the private incentive with the social one depends on the relative magnitude of \( \frac{D_1^2}{D_1} \beta \) vs. 1. If competition is weak and the demand is more or less independent, that is, \( D_1^2 \approx 0 \), then \( \left| \frac{d\beta}{du_1}\Big|_{dW^d=0} \right| > \left| \frac{d\beta}{du_1}\Big|_{dn^d=0} \right| \), as in the case of the monopoly platform. However, if competition is intense \( (D_1^2 \) is a large negative number) and \( \beta \) is sufficiently large, we could have \( \left| \frac{d\beta}{du_1}\Big|_{dW^d=0} \right| < \left| \frac{d\beta}{du_1}\Big|_{dn^d=0} \right| \).

Summarizing, we have:

**Proposition 6.** Consider a competitive bottleneck model of duopoly platforms with horizontal differentiation. Suppose that the price on the consumer side is fixed at \( p = \bar{p}(\geq 0) \). Then, a platform’s design choice incentive is AS-biased as in the monopoly case if competition is weak whereas its incentive is CS-biased if competition is intense and the business stealing effect is strong.

In the next section, we illustrate our results on the duopoly case for the Hotelling model when the price constraint takes the form of the non-negative price constraint. The Appendix provides another illustration of our results with a discrete choice model with logit demand.

### 4.4 Hotelling Model

In the Hotelling model, the optimal prices given symmetric \((u, \beta)\) can be derived in the following way (assuming that \( u \) is sufficiently large and the consumer side market is
covered). Given \((u, p)\), platform \(i\)'s demand can be written as

\[
D^i(u, p) = \frac{1}{2} + \frac{(u_i - p_i) - (u_j - p_j)}{2t} = \frac{1}{2} + \frac{s_i - s_j}{2t}.
\]

Hence, the demand function \(D^i(s_i, s_j)\) satisfies Assumption 1: in particular, we have \(D^i = |D^j|\).

Platform \(i\) solves the following problem.

\[
\max_{p_i} (p_i + \tau \beta_i) \left[ \frac{1}{2} + \frac{(u_i - p_i) - (u_j - p_j)}{2t} \right].
\]

The first order conditions for each platform yield the following reaction functions.

\[
p_i = R_i(p_j; u, \beta_i) = \frac{t + (u_i - u_j) - \tau \beta_i + p_j}{2}.
\]

Note that the profit is strictly concave in \(p_i\) and hence the second-order condition is satisfied. By solving the two reaction functions simultaneously, we can derive the Nash equilibrium prices as

\[
p^*_i = t + \frac{(u_i - u_j) - (2\tau \beta_i + \tau \beta_j)}{3}.
\]

In what follows, we consider a symmetric equilibrium. Note that \(p^*_1 = p^*_2 = p^* > 0\) iff \(t > \tau \beta\).

When there is no price constraint on the consumer side, the general analysis in Section 4.2 applies to the Hotelling model as the Hotelling demand satisfies Assumption 1. Therefore, we focus on the case in which the price is fixed at zero (i.e., \(p_1 = p_2 = 0\)) and hence assume \(t < \tau \beta\).\(^{13}\) Then, from (13) in the previous general analysis, we have:

\[
\left| \frac{d\beta_1}{du_1} \right|_{d\pi^i=0} = \frac{\beta}{t},
\]

which is strictly larger than \(1/\tau\) from \(t < \tau \beta\).

If \(D^1 + D^2\) is constant like in the Hotelling model with full market coverage, we have

\(^{13}\)Assuming that the prices are fixed simplifies our analysis. If we allow each firm to choose its price subject to the NPC, there can exist a boundary case. For instance, when \(t\) is sufficiently close to \(\tau \beta\), depending on \((u_1, \beta_1)\), the best response price of firm 1 to \(p^*_2 = 0\) may be strictly positive or zero. This boundary case is not our main interest.
\[ D_1^1 + D_1^2 = 0. \] Hence, from (14), we find
\[
\left| \frac{d \beta_1}{d u_1} \right|_{dV^d = 0} = 1.
\]

Therefore, for any \( \tau \in (0, 1] \), we have
\[
\left| \frac{d \beta_1}{d u_1} \right|_{d \pi^d = 0} > \frac{1}{\tau} \geq \left| \frac{d \beta_1}{d u_1} \right|_{dV^d = 0},
\]
which means that the platform design incentive is CS-biased. Basically, \( t < \tau \beta \) implies that the platforms compete aggressively to attract consumers in order to obtain advertising revenues, which generates a bias toward the consumer side.

Summarizing, we have:

**Proposition 7.** Consider the Hotelling model.

(i) When there is no price constraint on the consumer side, each platform’s design choice incentive is CS-biased for any \( \tau \in (0, 1) \) and is unbiased for \( \tau = 1 \).

(ii) When each platform’s price on the consumer side is fixed at zero and \( t < \tau \beta \) holds, each platform’s design choice incentive is CS-biased for any \( \tau \in (0, 1] \).

## 5 Summary and Discussions

We have found that the direction of biases in platform design crucially depends on whether or not there is a price constraint on the consumer side. More specifically, when there is no price constraint and the equilibrium prices fully respond to changes in \((u, \beta)\), the market equilibrium in platform design is CS-biased both in monopoly and duopoly. When there is a price constraint on the consumer side such that the price is fixed, the market equilibrium in platform design is AS-biased if the market structure is monopolistic or competition is weak in duopoly. However, if competition is intense and the business stealing effect is strong, the bias can be reversed and becomes CS-biased in contrast to the monopoly case. The following table summarizes and compares platform design incentives for the monopoly and the duopoly platform cases.
Biases In Platform Design

The duopoly analysis of the Hotelling model and the logit model confirms the general finding. Furthermore, it generates additional insights regarding the case in which the consumer-side price is fixed at zero. First, the market expansion effect in general raises the marginal social benefit from increasing $u$ and thus can create an AS-bias if a platform does not fully internalize it. However, in the Hotelling model, there is no market expansion effect and thus we find that the platform design incentive is CS-biased even when the consumer-side price is fixed at zero. More importantly, the analysis of the logit model, which applies to a market with market expansion possibility, identifies the key role played by $\tau$, the share of the advertising surplus that each platform captures. If $\tau$ is small, then the platforms’ design incentives are always CS-biased. In other words, a necessary condition for platform design to be AS-biased is that $\tau$ is not too small.

A large number of publishers (including online newspapers) sell their display advertising inventory to a wide range of advertisers through a chain of intermediaries that run real-time auctions on behalf of the publishers and advertisers. The intermediation ecosystem has evolved into a complex vertical chain of specialized providers such as publisher ad servers, SSPs (supply side platforms) including ad exchanges, DSPs (demand side platforms), advertiser ad servers.\textsuperscript{14} Google is dominant at each layer of intermediation. Various studies estimated what is called "ad tech take",\textsuperscript{15} the share taken by ad intermediaries from the advertising expenditure paid by advertisers. For instance, according to the CMA report (2020), a lower bound of the ad tech take is 35 percent, meaning that on average publishers receive at best 65% of advertisers’ expenditure. By contrast, large platforms such as Google and Facebook have built their own ad tech system and hence are not subject to ad tech take when they sell their own advertising inventories. Therefore, in the context of our model, $\tau$ of large platforms is much larger than that of

\begin{table}[h]
\centering
\begin{tabular}{|l|c|c|}
\hline
         & No Price Constraint & Price Constraint \\
\hline
Monopoly & CS-Biased           & AS-Biased            \\
\hline
Duopoly  & CS-Biased           & AS-Biased If Competition Is Weak \\
         &                     & CS-Biased If Competition Is Strong \\
\hline
\end{tabular}
\end{table}

\textsuperscript{14}See the CMA (2020) report, Jeon (2021), Scott Morton and Dinielli (2020) and Srinivasan (2020) for more details about the ad intermediation market.

\textsuperscript{15}See for instance, ANA (2017), Plum (2019) and the CMA report (2020).
publishers who rely on third-party ad intermediaries, implying that the former’s platform design incentive is much more likely to be AS-biased than the latter’s incentive.

Even though it is beyond the scope of our article to provide a model of the vertical chain of ad intermediation and to determine $\tau$ in an endogenous way, we think that Google and Facebook have an incentive to abuse their market power in ad intermediation and in ad inventory in order to increase their $\tau$,\footnote{Jeon (2021), Scott Morton and Dinielli (2020) and Srinivasan (2020) describe how Google abuses its market power in ad intermediation and in ad inventory.} which in turn induces them to increase their ad load. This can explain why Google and Facebook increased the ad load, as described in Section 3.3. We should be also cognizant that potentially harmful effects of the abuse of market power in the advertising market can spill over to the consumer side. An increased price of advertising through their market power can drive up the costs of merchants and ultimately consumer prices of goods. Our model does not capture this indirect channel of spillover, but this possibility should be recognized by policymakers.

Policies that reduce market power and increase transparency in ad intermediation will reduce ad tech take rates (see the CMA report (2020) for such policy proposal)\footnote{The Digital Service Act and the Digital Market Act proposed by the European Commission also intend to increase transparency in the digital advertising market.}. This will lower $\tau$ of vertically integrated platforms which sell their ad inventory through their own ad tech systems (like Google and Facebook). According to the results from the logit model, such policies can generate a change from an AS-bias to a CS-bias in design choices of those platforms. By contrast, the same policies will increase $\tau$ for platforms (like newspapers) who rely on third-party ad intermediation. Even if this may generate a change from a CS-bias to an AS-bias, it seems to be less likely as they typically face strong competition on the consumer side.

We have shown in Section 3 that our analysis can be applied to platforms’ incentives to design policies that create trade-offs between the consumer side and the advertiser side: technology adoption policy, privacy policy, ad load policy. It can be also applied to the design choice concerning the relative weight to place on quality (i.e. relevance of ads to consumers) over prices in determining the winning bid in ad auction mechanisms.

### 6 Concluding Remarks

In this article, we have analyzed how platform market power and business models shape ad-funded two-sided platforms’ design choices. We consider design choices that affect
both sides in opposite ways and study a platform’s incentives to trade-off the gain from one side with the loss from the other side. We compare private incentives with social incentives across different market structures (monopoly platform and duopoly competitive bottleneck) in order to identify biases in platform design generated by market power. We find that biases in platforms’ design choices depend crucially on whether or not they can freely charge any price on the consumer side. Without any price constraints, the platform design incentive is CS-biased. In contrast, with a price constraint, it is AS-biased unless there is strong competition.

Many two-sided platforms provide free services to consumers and generate revenues by charging the advertising side. Our analysis of platform design with price constraints is particularly relevant to such ad-funded two-sided platforms which may find it optimal to charge zero price on the consumer side due to the non-negative price constraint. According to our analysis, ad-funded platforms with market power exhibit an AS-bias in their design choices. This AS-bias can be manifested in terms of a bias in the direction of innovation (i.e., platforms’ innovation incentives are biased towards increasing the advertiser side surplus to the detriment of consumer surplus), excessive collection of personal data or excessive ad load and nuisance to consumers.

Our results allow us to make the following predictions regarding the evolution of ad-funded platforms. Initially when they are nascent and face fierce competition, they have strong incentives to increase consumer surplus in their platform design. However, once the market tips to them or after their market power becomes entrenched, the same platforms, which were consumer advocates, have strong incentives to introduce innovations/policies that increase the advertiser side surplus to the detriment of consumer surplus. This is exactly what happened with the evolution of business strategies of Facebook, according to Srinivasan (2019). Our analysis thus provides a rationale for a tougher competition policy to curb market power of ad-funded platforms with free services if competition authorities put more weight on consumer surplus in welfare calculations.
References


Appendix

A Microfoundation of the Advertising Side Market

We provide a microfoundation of the advertising side that would yield the model assumed in the main text. Let us assume that there are two categories of products. Each consumer demands products from only one category. A priori, each category of products is equally likely to be demanded by each consumer. In each category, there is a measure 1 of varieties, each of which is produced by monopolistic producers. To sell the product, each firm needs to advertise to inform consumers of the existence and price of the good as in Anderson and Coate (2005) and Choi (2006). Platforms provide such a channel and allow them to be matched with consumers. Let us assume that only a mass \( z \) of monopoly producers of new goods can be matched with a consumer. This may be due to the advertising space limitation or consumer’s limited attention. New goods are produced with a constant marginal cost of zero without any loss of generality.

We consider a two tier matching process between a consumer and advertisers. First, the platform transmits to the advertisers the data about the consumer’s profile and its prediction about the category the consumer is interested in. In addition, the platform announces the number of advertising slots. Second, based on the profile and the predicted category, advertisers estimate their willingness to pay for a slot. The slots are allocated according to the second-price auction: the winning bidders pay the highest losing bid.

Within a category, each new product is characterized by a parameter \( \alpha \in [0, 1] \), which represents the probability that the product will appeal to the consumer. If a product appeals to the consumer, the consumer is willing to pay \( \varpi \). We assume that \( \alpha \) is distributed according to \( F(.) \). We assume that \( F \) is increasing and continuously differentiable. When a consumer is matched with a product in the wrong category, the consumer has no demand for it. Because a consumer will pay \( \varpi \) or zero, each new producer’s optimal price is \( \varpi \). The platform attempts to match a consumer with the right category products, but the match is not perfect. The platform’s ability to match a consumer with the right product is represented by a probability of match \( \varphi(>1/2) \). A producer belonging to the category predicted by the platform has a willingness to pay to be advertised via the platform given by \( \varphi \alpha \varpi \). Let us define \( \alpha^* \) by

\[
z = 1 - F(\alpha^*)
\]
We assume that $\alpha^* > \frac{1-\varphi}{\varphi}$. This condition implies that it is optimal for the platform to fill all advertising slots for a consumer with products from the category that is more likely to suit the consumer. We also assume that the advertising slot is limited and it is optimal to fill all slots. This condition is given by $\alpha^* > \alpha^m$, where $\alpha^m = \arg\max(1 - F(\alpha))\alpha$.

A platform’s advertising revenue per consumer is given by $z'F(1 - z)\varphi$ and the advertisers’ net surplus is given by $\varphi\int_{\alpha^*}^{1}(\alpha - \alpha^*)dF(\alpha)$. Then, we can set

$$
\beta = z\varphi F^{-1}(1 - z)\varphi + \varphi\int_{\alpha^*}^{1}(\alpha - \alpha^*)dF(\alpha) = \varphi\int_{\alpha^*}^{1}(\alpha - \alpha^*)dF(\alpha)
$$

$$
\tau = \frac{\int_{\alpha^*}^{1}(\alpha - \alpha^*)dF(\alpha)}{zF^{-1}(1 - z) + \int_{\alpha^*}^{1}(\alpha - \alpha^*)dF(\alpha)}
$$

We can interpret an increase in $\beta$ as a consequence from a platform’s better targeting technology in matching a consumer with the right product category, that is, an increase in $\varphi$. Notice that $\tau$ is independent of $\varphi$.

**Proof of Lemma 1**

For the definition of the platform’s design choice bias, we analyze changes in social welfare arising from the platform’s choice of $(u, \beta)$ that would keep its profits unchanged. With differentiability, this means we investigate how social welfare changes along the locus of the iso-profit curve.

$$
dW|_{d\pi^m=0} = \left[\frac{\partial W}{\partial u} du + \frac{\partial W}{\partial \beta} d\beta\right]|_{d\pi^m=0}
$$

$$
= \frac{\partial W}{\partial u} du + \frac{\partial W}{\partial \beta} \left(-\frac{\partial \pi^m}{\partial u}\frac{\partial \pi^m}{\partial \beta}\right) du = \left[\frac{\partial W}{\partial u} du - \frac{\partial W}{\partial \beta} \left(\frac{\partial \pi^m}{\partial u}\frac{\partial \pi^m}{\partial \beta}\right)\right] du
$$

Thus, we have

$$
\frac{\partial W}{\partial u}|_{d\pi^m=0} = \frac{\partial W}{\partial u} - \frac{\partial W}{\partial \beta} \left(\frac{\partial \pi^m}{\partial u}\frac{\partial \pi^m}{\partial \beta}\right).
$$

It is immediate that $\frac{\partial W}{\partial u}|_{d\pi^m=0} < 0$ and the platform’s design choice is CS-biased if and only if $\left|\frac{d\beta}{du}|_{d\pi^m=0}\right| > \left|\frac{d\beta}{du}|_{dW=0}\right|$. Similarly, the platform’s design choice is AS-biased if and only if $\left|\frac{d\beta}{du}|_{d\pi^m=0}\right| < \left|\frac{d\beta}{du}|_{dW=0}\right|$. 

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Analysis of Privacy Policy

Consider first the case in which the platform can charge any price $p$ on the consumer side. The platform’s profit is given by

$$\pi(p, d) = D(u - c(d) - p)(p + \tau \beta(d)).$$

After using a change of variable $q = p + \tau \beta(d)$, where $q$ represents the total revenue per consumer, we obtain the following expression of the profit:

$$\hat{\pi}(q, d) = D(u + \tau \beta(d) - c(d) - q)q.$$

From the first-order conditions, the profit-maximizing $q^*$ and data collected $d^*$ are respectively given by:

$$q^* = \frac{D}{D'}$$

and

$$\tau \beta'(d^*) = c'(d^*)$$

The first condition above is the well-known Lerner formula with $MC = 0$ (i.e., $1 = 1/\eta$) where $\eta$ is the elasticity of demand on the consumer side. The second one states that the marginal ad revenue per consumer is equal to the marginal privacy cost per consumer. In particular, note that $d^*$ is independent of $q$. In other words, for any given total revenue per consumer $q$, the profit-maximizing amount of data collection is given by $d^*$. To provide an intuition, consider $d < d^*$. Then it is optimal for the platform to slightly increase $d$ while reducing $p$ to keep $q$ unchanged as this increases $\tau \beta(d) - c(d)$ and thereby increases $D$. The reverse holds for $d > d^*$.

Consider now welfare, which is given as follows:

$$W(p, d) = \pi(p, d) + v(u - c(d) - p) + D(u - c(d) - p)(1 - \tau)\beta(d).$$

After the change of variable $q = p + \tau \beta(d)$, the welfare is given by:

$$\hat{W}(q, d) = \hat{\pi}(q, d) + v(u + \tau \beta(d) - c(d) - q)$$

$$+ D(u + \tau \beta(d) - c(d) - q)(1 - \tau)\beta(d).$$
Given that the platform chooses the price according to (17), we ask whether $d^*$ chosen by the platform is socially excessive or not. Given $q = q^*$, the derivative of the welfare at $d = d^*$ is given by

$$
\frac{\partial W}{\partial d}_{q=q^*,d=d^*} = D(u + \tau \beta(d^*) - c(d^*) - q^*)(1 - \tau)\beta'(d^*) \geq 0.
$$

Therefore, the platform collects too little data from welfare point of view as long as $\tau < 1$ whereas collecting the socially optimal amount for $\tau = 1$. This result is consistent with the result previously obtained when we studied platform’s design and accordingly we can say that the data collection choice is CS-biased for any $\tau < 1$ and unbiased for $\tau = 1$. The bias arises because the platform does not internalize the surplus of advertisers.

Consider now the case in which the price on the consumer side is fixed at $p(\geq 0)$. Then, the platform’s profit is given by

$$
\pi^m(p, d) = D(u - p - c(d))[p + \tau \beta(d)].
$$

The profit-maximizing amount of data $d^*$ is implicitly defined by

$$
D'c'[p + \tau \beta(d^*)] = D\tau \beta'(d^*). \quad (19)
$$

As the price is fixed, the privacy cost plays the role of a price on the consumer side such that the L.H.S. captures the reduction in revenue due to the decrease in the number of consumers after a marginal increase in privacy cost. The R.H.S. captures the increase in the marginal ad revenue times the number of consumers.

Social welfare is given by

$$
W(p, d) = \pi^m(p, d) + v(u - p - c(d)) + D(u - p - c(d))(1 - \tau)\beta(d).
$$

The derivative of the welfare with respect to $d$ evaluated at $d = d^*$ is simply given by

$$
\frac{\partial W(p, d)}{\partial d}_{d=d^*} = -c'(d^*) + D'c'(d^*) (1 - \tau)\beta(d) + D(1 - \tau)\beta'(d^*)
$$

$$
= -Dc'(d^*) + (1 - \tau) [D\beta'(d^*) - D'c'(d^*)\beta(d^*)]
$$

$$
= -Dc'(d^*) + \frac{(1 - \tau)}{\tau}D'c'(d^*)p,
$$

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where the second equality comes from the envelope condition $v' = D$ and the third equality is due to the first order condition on $d^*$ (19). We can rewrite the above equation as

$$\frac{\partial W(\bar{p}, d)}{\partial d} \bigg|_{d=d^*} = -Dc'(d^*) \left[ 1 - \frac{(1 - \tau)D'}{D'} \right]$$

Therefore, the platform collects too much data from welfare point of view if $\bar{p} = 0$ or if $\bar{p} > 0$ and $\tau > \frac{\eta}{1+\eta}$ where $\eta$ is the price elasticity of consumer demand. This result is the same as the one we obtained when we studied platform’s design in general. In particular, if we have the non-negative price constraint with $\bar{p} = 0$, we always have $\frac{\partial W(\bar{p}, d)}{\partial d} \bigg|_{d=d^*} = -Dc'(d^*) < 0$, implying too much data collection for all $\tau > 0$. In other words, the data collection choice is AS-biased for any $\tau > 0$.

**Analysis of Ad-Load Policy**

Consider first the case in which the platform can charge any price $p$ on the consumer side. Then, the platform’s profit is given by

$$\pi(p, a) = D(u - c(a) - p)(p + R(a))$$

After using a change of variable $q = p + R(a)$, where $q$ represents the total revenue per consumer, we obtain the following expression of the profit:

$$\hat{\pi}(q, a) = D(u + R(a) - c(a) - q)q.$$  

From the first-order conditions, the profit-maximizing $q^*$ and ad load $a^*$ are given by

$$q^* = \frac{D}{D'},$$

which is the same as (17) in the application on privacy policy, and

$$c'(a^*) = \frac{R'(a^*)}{\text{Marginal Nuisance Cost}} \quad \text{Marginal Revenue}$$  

where the marginal ad nuisance per consumer is equal to the marginal ad revenue per consumer. For any given total revenue $q$, the profit-maximizing ad load is $a^*$, which is independent of $q$.  

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Social welfare can be written as follows:

\[ W(p,a) = \pi(p,a) + v(u - c(a) - p) + D(u - c(a) - p) \int_0^a (r(x) - r(a))dx, \]

where the last term captures the advertiser surplus. After changing variables from \( p \) to \( q = p + R(a) \), the welfare is given by:

\[ \hat{W}(q,a) = \hat{\pi}(q,a) + v(u + R(a) - c(a) - q) \]
\[ + D(u + R(a) - c(a) - q) \int_0^a (r(x) - r(a))dx. \]

Given that the platform chooses \( q \) according to (20), we ask whether \( a^* \) chosen by the platform is socially excessive or not. Given \( q = q^* \), the derivative of the welfare at \( a = a^* \) is given by

\[ \left. \frac{\partial \hat{W}}{\partial a} \right|_{q=q^*,a=a^*} = -D(u + R(a^*) - c(a^*) - q^*)a^*r'(a^*) > 0. \]

Therefore, the ad load chosen by the platform is too small from the welfare point of view; in other words, the ad load choice is CS-biased in the absence of any price constraints on the consumer side.

Consider now the case in which the price on the consumer side is fixed at \( \bar{p}(\geq 0) \). Then, the platform’s profit is given by

\[ \pi^m(\bar{p},a) = D(u - \bar{p} - c(a))[\bar{p} + R(a)]. \]

The profit-maximizing ad load \( a^* \) is implicitly defined by

\[ D'c'[\bar{p} + R(a^*)] = D(u - \bar{p} - c(a))R'(a^*). \tag{22} \]

As the price is fixed at \( \bar{p} \), there is no price adjustment by the platform to counter changes in ad nuisance costs. The L.H.S. of (22) captures the marginal reduction in ad revenue due to the decrease in the number of consumers after a marginal increase in ad load and its associated nuisance cost. The R.H.S. captures the marginal increase in ad revenue.
Social welfare is given by

\[ W(p, a) = \pi^m(p, a) + v(u - \bar{p} - c(a)) + D(u - \bar{p} - c(a)) \int_0^a (r(x) - r(a)) dx. \]

The derivative of the welfare with respect to \( a \) evaluated at \( a = a^* \) is given by

\[
\frac{\partial W(p, a)}{\partial a} \bigg|_{a=a^*} = -v'c' - D'c' \int_0^{a^*} (r(x) - r(a)) dx - Da^* r'
\]

where the first two terms are negative whereas the last term is positive. The increase in nuisance reduces the consumer surplus and it also reduces the advertiser surplus by reducing the number of consumers who are exposed to the advertising. But given the number of consumers, the increase in ad load increases the advertiser surplus by lowering the ad price.

Let \( \sigma_a = -[ar'(a)]^{-1} \) and \( \sigma_c = -[\partial D/\partial p]/D \) denote the semi-elasticities of demand on the advertising side and the consumer side, respectively. By using \( v' = D \), we can rewrite (23) as

\[
\frac{\partial W^m(p, a)}{\partial a} \bigg|_{a=a^*} = -D \left\{ c'(a^*) - \frac{1}{\sigma_a} \right\} + c'(a^*) \sigma_c \int_0^{a^*} (r(x) - r(a)) dx \right\}.
\]

The platform’s choice of ad load is socially excessive (i.e., AS-biased) if the expression in the curly bracket of Eq. (24) is positive. As the last term in the curly bracket is positive, a sufficient condition to have a socially excessive ad load is

\[ c'(a^*) \geq \frac{1}{\sigma_a}, \]

which is satisfied if \( \sigma_a \) is sufficiently large.

Even when condition (25) fails, the platform’s ad load can be excessive if the semi-
elasticity of demand on the consumer side \( \sigma_c \) is sufficiently large, that is, if

\[
\sigma_c \geq \frac{1 - \sigma_a c'(a^*)}{\sigma_a c'(a^*) \int_0^a (r(x) - r(a)) \, dx}.
\]

**Logit Demand Model**

Consider a discrete choice model of price competition with logit demand

\[
D_i(u_i - p_i, u_j - p_j) = \frac{\exp[(u_i - p_i)/t]}{\sum_{k=0}^2 \exp[(u_k - p_k)/t]} = \alpha_i,
\]

where the outside good, good 0, has a utility of \( u_0 \) with price zero (i.e. \( p_0 = 0 \)) and \( t(> 0) \) represents the degree of product differentiation. The total number of consumers is normalized to 1 and \( \alpha_i \) is the proportion of consumers who use platform \( i \).

Consider first the case without any price constraint on the consumer side. The F.O.C. for profit maximization of platform 1 can be written as

\[
(p_1 + \beta_1 \tau) = t \frac{\exp[(u_1 - p_1)/t]}{\exp[(u_1 - p_1)/t] \sum_{k=0}^2 \exp[(u_k - p_k)/t] \exp[(u_0 - p_0)/t] + \exp[(u_2 - p_2)/t]} \tag{26}
\]

which can be rewritten as

\[
(p_1 + \beta_1 \tau) \left[ \exp[(u_0 - p_0)/t] + \exp[(u_2 - p_2)/t] \right] = t \left[ \sum_{k=0}^2 \exp[(u_k - p_k)/t] \right] \tag{27}
\]

Fully differentiating (27) with respect to \( u_1 \) gives

\[
\frac{dp_1}{du_1} [\exp((u_0 - p_0)/t) + \exp((u_2 - p_2)/t)] - (p_1 + \beta \tau) \frac{dp_2}{du_1} \frac{1}{t} \exp((u_2 - p_2)/t)
\]

\[
= (1 - \frac{dp_1}{du_1}) \exp((u_1 - p_1)/t) - \frac{dp_2}{du_1} \exp((u_2 - p_2)/t)
\]

In a symmetric equilibrium in which \( \alpha_1 = \alpha_2 = \alpha \in (0, 1/2) \) and \( \alpha_0 = 1 - 2\alpha > 0 \), we have

\[
\frac{dp_1}{du_1} (1 - \alpha) - \frac{\alpha}{1 - \alpha} \frac{dp_2}{du_1} = \alpha \left[ 1 - \frac{dp_1}{du_1} - \frac{dp_2}{du_1} \right].
\]
Hence,
\[
\frac{dp_1}{du_1} = \alpha + \frac{dp_2}{du_1} \frac{\alpha^2}{1 - \alpha}.
\]

In a similar way, fully differentiating (27) with respect to \(u_2\) yields the following condition at symmetric equilibrium:
\[
\frac{dp_1}{du_2} = -\frac{\alpha^2}{1 - \alpha}(1 - \frac{dp_2}{du_2}).
\]

As we have \(\frac{dp_1}{du_1} = \frac{dp_2}{du_2}\) and \(\frac{dp_2}{du_1} = \frac{dp_1}{du_2}\) due to the symmetry, we find
\[
\frac{dp_1}{du_1} = \frac{\alpha(1 - \alpha)^2 - \alpha^4}{(1 - \alpha)^2 - \alpha^4} > 0, \quad \frac{dp_1}{du_2} = -\frac{\alpha^2(1 - \alpha)^2}{(1 - \alpha)^2 - \alpha^4} < 0
\]

Fully differentiating (27) with respect to \(\beta_1\) and \(\beta_2\), respectively, gives
\[
\frac{dp_1}{d\beta_1} = -\tau(1 - \alpha) + \frac{dp_2}{d\beta_1} \frac{\alpha^2}{1 - \alpha}.
\]

\[
\frac{dp_1}{d\beta_2} = \frac{dp_2}{d\beta_2} \frac{\alpha^2}{1 - \alpha}.
\]

As we have \(\frac{dp_1}{d\beta_1} = \frac{dp_2}{d\beta_2}\) and \(\frac{dp_2}{d\beta_1} = \frac{dp_1}{d\beta_2}\) due to the symmetry, we find
\[
\frac{dp_1}{d\beta_1} = -\frac{\tau(1 - \alpha)^3}{(1 - \alpha)^2 - \alpha^4} < 0, \quad \frac{dp_1}{d\beta_2} = -\frac{\tau\alpha^2(1 - \alpha)^2}{(1 - \alpha)^2 - \alpha^4} < 0
\]

We thus have the following result:
\[
\tau(1 - \frac{dp_1}{du_1}) = -\frac{dp_1}{d\beta_1}; \quad \frac{dp_1}{du_2} = \frac{dp_1}{d\beta_2}.
\]

The result implies pass-through equalization when \(\tau = 1\). A marginal increase in \(u_1\) (respectively, \(u_2\)) has the same impact on the surplus of platform 1 consumers as a marginal increase in \(\beta_1\) (respectively, \(\beta_2\)).

In addition, we have
\[
\frac{D_1}{D_1} = -\frac{\alpha}{1 - \alpha}.
\]
Therefore,

\[
\frac{d\beta_1}{du_1}\bigg|_{du^d=0} = -\left[1 - \frac{D_t^1 \, dp_2}{D_t^1 \, du_1}\right] \left[\tau - \frac{D_t^1 \, dp_2}{D_t^1 \, du_1}\right] = \left[1 - \frac{\alpha}{1-\alpha} \frac{1}{(1-\alpha)^2-\alpha^2}\right] = \frac{-1}{\tau}
\]

\[
\frac{d\beta_1}{du_1}\bigg|_{du^d=0} = -D_t^1 \left[1 - \frac{dp_1}{du_1} - \frac{dp_2}{du_1}\right] \left[p_1 + \beta_1\right] \left[D_t^1 + D_t^2\right] = -\frac{p_1 + \tau \beta_1}{D_t^1} \left[1 - \frac{dp_1}{du_1} - \frac{dp_2}{du_1}\right] \left[p_1 + \beta_1\right] \left[D_t^1 + D_t^2\right]
\]

\[
= -\frac{p_1 + \tau \beta_1}{D_t^1} \left[p_1 + \beta_1\right] \left[1 - \frac{dp_1}{du_1} - \frac{dp_2}{du_1}\right] \left[p_1 + \beta_1\right] \left[D_t^1 + D_t^2\right]
\]

\[
= -\frac{p_1 + \tau \beta_1}{D_t^1} \left[p_1 + \beta_1\right] \left[1 - \frac{dp_1}{du_1} - \frac{dp_2}{du_1}\right] \left[p_1 + \beta_1\right] \left[D_t^1 + D_t^2\right]
\]

\[
= -\frac{t}{1-\alpha} + \frac{(1-\alpha+\alpha^2)(1-\alpha)^2}{(1-\alpha)^2-\alpha^2} \left[\left(\frac{t}{1-\alpha} + (1-\tau)\beta_1\right)^{\frac{1-2\alpha}{1-\alpha}}\right] > 0
\]

In a symmetric equilibrium in which \(\alpha_1 = \alpha_2 = \alpha \in (0, 1/2)\) and \(\alpha_0 = 1 - 2\alpha > 0\), we thus have

\[
\frac{d\beta_1}{du_1}\bigg|_{du^d=0} - \frac{d\beta_1}{du_1}\bigg|_{du^d=0} = \frac{1 - \tau}{\tau} \left[\frac{t}{1-\alpha} + \frac{(1-\alpha+\alpha^2)(1-\alpha)^2}{(1-\alpha)^2-\alpha^2} \left[\left(\frac{t}{1-\alpha} + (1-\tau)\beta_1\right)^{\frac{1-2\alpha}{1-\alpha}}\right]\right]
\]

Hence, each platform’s design incentive is CS-biased for all \(\tau \in (0, 1)\). When \(\tau = 1\), the private incentive coincides with the social one because

\[
\frac{d\beta_1}{du_1}\bigg|_{du^d=0} = -1 = \frac{d\beta_1}{du_1}\bigg|_{du^d=0}.
\]

This confirms the finding in Proposition 5.

Now consider the case in which each platform’s price on the consumer side is fixed at zero. From (26), a symmetric equilibrium price would be zero if \(t < (1 - \alpha)\beta\). Hence,
we assume $t < (1 - \alpha)\beta \tau$ in this case. We have

$$\frac{d\beta_1}{du_1}|_{dW^d=0} - \frac{d\beta_1}{du_1}|_{dW^d=0} = -\left(1 + \frac{D^2_1}{D^1}\right) = -\left(1 - \frac{\alpha}{t}\right),$$

where $\alpha = \frac{\exp \frac{u}{t}}{\exp \frac{u_0}{t} + 2 \exp \frac{u}{t}} < 1/2.$

Thus, each platform’s design incentive is AS-biased (CS-biased) if $t > \alpha \beta$ ($t < \alpha \beta$).

Therefore, when $t < (1 - \alpha)\beta \tau$ holds, we can have two cases depending on the relative magnitudes of $(1 - \alpha)\tau$ and $\alpha$. Because $\alpha < 1/2$, there exists a $\tau^* \in (0, 1)$ such that $(1 - \alpha)\tau \geq \alpha$ if and only if $\tau \geq \tau^*$. For $\tau < \tau^*$, $t < (1 - \alpha)\beta \tau$ implies $t < \alpha \beta$; then if each platform’s consumer price is fixed at zero, the platform design is always CS-biased. For $\tau > \tau^*$, if each platform’s consumer price is fixed at zero, we can have both CS-bias and AS-bias depending on the sign of $t - \alpha \beta$. Therefore the logit demand model with market expansion reveals further insight on the importance of the share of the advertising surplus that each platform captures (represented by $\tau$).

In summary, we have

Proposition 8. In the logit model,

(i) When there is no price constraint on the consumer side, each platform’s design choice incentive is CS-biased for any $\tau \in (0, 1)$ and is unbiased for $\tau = 1$.

(ii) When each platform’s price on the consumer side is fixed at zero and $t < (1 - \alpha)\beta \tau$ holds, there exists a $\tau^* \in (0, 1)$ such that

(a) for $\tau < \tau^*$ (that is, $(1 - \alpha)\beta \tau < \alpha \beta$), each platform’s design choice incentive is always CS-biased;

(b) for $\tau \geq \tau^*$ (that is, $(1 - \alpha)\beta \tau \geq \alpha \beta$), each platform’s design choice incentive is CS-biased (respectively, AS-biased) if $t < \alpha \beta$ (respectively, $t \geq \alpha \beta$).

Therefore, when $\tau \geq \tau^*$, each platform’s design incentive is non-monotonic in $t$ in the following sense. When $t < \alpha \beta$ and the NPC is binding, competition to attract consumers is intense, which creates a CS-bias. As $t$ becomes larger than $\alpha \beta$ but the NPC is still binding, competition is relaxed, generating a AS-bias. Once $t$ becomes very large and exceeds $(1 - \alpha)\beta \tau$ such that the NPC does not bind any more, each platform’s incentive reverts back to a CS-bias.
Figure 1: CS-Biased Platform Design Choice

Figure 2: AS-Biased Platform Design Choice