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Abstract

In order to encourage substitution of fossil fuels by cleaner renewables, regulatory agencies have generally chosen between two types of renewable energy standards. They have either mandated a minimum volume of renewable energy as in the case of ethanol in transport fuels, and for electricity in Texas and Iowa. Or they have specified a minimum blend (share) of renewables in the energy supply mix as in California, Michigan and many other states. This paper uses a simple model to compare the dynamic effects of these two policies. We show that a volume mandate leads to a lower energy price, induces a greater subsidy on clean energy and a smaller fossil fuel tax than the blend mandate. The volume mandate also leads to larger cumulative renewable energy use over the time horizon. We illustrate the model with plausible parameter values and show that the two energy mandates lead to large differences in fossil fuel taxes and clean energy subsidies.

Keywords – Renewable energy mandates; Fossil fuels; Energy transition; Subsidies; Carbon tax

JEL classifications – Q42, Q48, Q54

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1 Introduction

In order to promote clean energy and reduce fossil fuel use, many governments have implemented regulation that prescribes a minimal use of renewable energy in sectors such as transport and power generation. These Renewable Portfolio Standards (RPS) come in two forms – as volume or blend mandates. The volume mandate specifies a minimum *volume* of renewable energy that must be produced each period (*e.g.*, annually) and the blend mandate sets a minimum *share* of renewables in the total supply of energy that must be met each period. Figure 1 shows a map of the two types of standards in operation for power generation in various US states. Most states have implemented a blend mandate but Texas and Iowa have volume mandates. For biofuels, under the Energy Policy Act of 2005, the US EPA has a volume mandate – it requires that transport fuels must contain a minimum volume of renewable fuel such as ethanol (36 billion gallons by 2022).¹ On the other hand, the European Union has a blend mandate that prescribes a minimum 10 percent share of biofuels in transport.²

The objective of both types of mandates is to accelerate the transition from polluting fossil fuels such as oil and coal to cleaner renewable forms of energy. Apart from its contribution to climate policy (through lower carbon emissions per unit of energy), another stated motive for implementing these standards is to ensure the security of energy supply since the policy may encourage indigenous production of energy from land and other sources which substitute for fossil fuel imports (Brown and Huntington, 2010, Boeters and Koornneef, 2011).

These mandates have sharply different impacts on energy prices and the energy mix of dirty and clean fuels, as well as the time path of carbon taxes and subsidies. The goal of this paper is to compare these two mandates in a simple, dynamic framework.

We compare the two mandates in a setting where carbon emissions are produced by a single fossil fuel (*e.g.*, coal) and there is a perfectly substitutable clean resource (a backstop) such as solar energy or biofuels. We apply a carbon budget

¹See <http://www.epa.gov/otaq/fuels/renewablefuels/>

²See http://ec.europa.eu/energy/renewables/biofuels/doc/biofuels/com_2012_0595_en.pdf

which prescribes a cumulative volume of carbon that can be emitted to ensure that we do not cross an exogenously imposed atmospheric threshold such as a 2 degree Celsius rise in temperature (as in Fischer and Salant, 2017). In our framework this carbon budget can be achieved in three different ways: by (a) implementing a carbon tax without specifying any mandate on renewable energy³ (b) prescribing a minimum volume of renewable energy that must be used every time period (a volume mandate) or (c) specifying a minimum share of renewables in the energy mix (a blend mandate).

Our main results are as follows. The blend mandate leads to a higher energy price and lower energy consumption than a volume mandate, other things being equal. Both lead to the same carbon tax, but the blend mandate induces a lower subsidy on clean energy and a higher tax on the fossil fuel. The blend mandate also skews fossil fuel consumption towards the future, relative to the volume mandate and pushes solar use more towards the present. However cumulative use of solar energy over the entire planning horizon is always higher under the volume mandate.

How large might the differences in the effects of the two mandates be? We illustrate these results with a calibration exercise for the world electricity sector, using realistic parameter values, which suggests that the two mandates may have significantly different effects on energy consumption and welfare. The blend mandate leads to a 17% higher tax on the fossil fuel, coal. However the volume mandate leads to a 42% higher subsidy on the clean energy. Tax revenues are larger under the blend mandate by about 18% and subsidy payouts lower by 38%. That is, the fiscal burden of the volume mandate is greater for the government.

There is a sizable literature that examines the economic effects of renewable energy mandates. Most studies analyze these mandates in a static framework. The distributional impacts of renewable energy prices in terms of pass through to consumers has been studied by Borenstein and Davis (2016) and Reguant (2019). de Gorter and Just (2009) and Holland et al. (2009) discuss the environmental impacts of renewable fuel standards and find that these energy policies can often lead to an increase in carbon emissions. The general equilibrium effects of these

³We explore this case mainly to provide a benchmark that facilitates comparison with the two types of mandates.

clean energy standards through their interaction with the tax system has been examined by Goulder et al (2016). The static effects of renewable energy mandates on energy market prices and social welfare are addressed by Fisher (2010) and Lapan and Moschini (2011).⁴ Other studies use second-best models to investigate the economic rationale for using renewable energy subsidies or standards when the environmental externality is already internalized by a carbon tax (Eichner and Runkel, 2014, Fisher and Preonas, 2010, and Galinato and Yoder, 2011). The dynamics of energy policies have been studied by Greaker et al. (2014) who conclude that biofuel subsidies can speed up oil extraction and increase emissions. Using a calibrated model, Fischer and Newell (2008) compare the effectiveness of different policies for reducing carbon emissions such as emission quotas, fossil fuel taxes, mandates, and R&D subsidies. However, none of these studies focus on directly comparing the two types of mandates, especially their effect on energy prices, quantities and taxes, as well as welfare.

An interesting feature of the two types of mandates is that the volume mandate is an absolute mandate on the consumption of renewable energy, while the blend mandate is a share, hence links the use of the renewable to the use of the fossil fuel. This interdependency has efficiency implications for our analysis. Both mandates lead to the imposition of a carbon tax. However, the volume mandate only prescribes a subsidy on the clean fuel, while the blend mandate not only subsidizes the clean fuel but imposes an additional tax on the fossil fuel as well. Using an additional unit of the fossil fuel leads to increased use of the clean energy under the blend mandate, hence the fossil fuel must be taxed a second time. This leads to a higher total tax on the fossil fuel under the blend mandate.

From a policy point of view, these results suggest that the two types of mandates may lead to different second-best outcomes. The blend mandate increases energy prices but leads to a lower subsidy on clean fuels. The total tax on oil is higher as well. If the policy maker cares about learning by doing effects of renewable energy use, the volume mandate may be preferred because it leads to increased cumulative adoption of the renewable which could induce increased

⁴Specifically, Fisher shows that mandates can reduce energy prices depending upon the elasticity of energy supply from renewables relative to nonrenewables as well as the stringency of the mandate. Lapan and Moschini show that mandates dominate fuel subsidies from a welfare perspective.

learning and a faster transition to renewables. Government revenue is highest when there is no mandate because the government collects carbon taxes and does not provide any subsidy. Tax collections are lowest under a volume mandate, because larger volumes of the clean energy must be subsidized. The volume mandate, like the one on ethanol in the US, will generate a larger consumer surplus due to a low energy price and a higher subsidy. Taxes on the fossil fuel are lower so the fossil fuel industry benefits more under the volume mandate.

Section 2 presents the dynamic model. In section 3, we develop the main analytical results, first without any mandate which helps provide an useful benchmark, and then with the two types of mandates. In section 4, we compare the two mandates in the perfect competition case and, in section 5, we show how our results are affected when the fossil fuel is supplied by a regulated monopoly. The models are illustrated in Section 6. Section 7 concludes the paper.

2 The model

We consider a simple model of energy use with two perfectly substitutable sources of energy: a polluting fossil resource (say, coal) and a clean fuel available in abundant supply (assume that it is solar energy). Let the consumption of coal and solar energy at any time t be given by $x(t)$ and $y(t)$, respectively, and the total energy consumed be denoted by $q(t) = x(t) + y(t)$. The surplus $u(q)$ obtained from energy consumption is assumed to be increasing and concave.⁵ Let $p(q) = u'(q)$ be the price of energy. Its inverse is the usual downward-sloping demand function $q(p)$.

Coal is available in abundance but burning it leads to carbon emissions.⁶ The unit cost of coal is assumed to be a constant $c > 0$. Without loss of generality, we normalize emissions so that burning one unit of coal produces exactly one unit

⁵In order to simplify notation, we will hide the time subscript whenever convenient and it is clear from the context.

⁶Introducing scarce fossil fuel resources will not change our results, as long as the initial reserves are higher than the maximum cumulative extraction allowed. Most studies such as the successive IPCC Assessment Reports suggest that known reserves far exceed the quantity that can be used before catastrophic climate damages kick in, see Heede and Oreskes (2016).

of pollution. Let $Z(t)$ be the cumulative carbon emissions at time t . We set the initial stock of emissions to zero for simplicity. We then have

$$\dot{Z}(t) = x(t), \quad Z(0) = 0 \quad (1)$$

where $\dot{Z}(t)$ is the time derivative of $Z(t)$. We impose a carbon budget, *i.e.*, a limit on the aggregate stock of carbon emissions which can not be exceeded at any time. This carbon budget may be considered an upper bound on the stock of pollution beyond which damages are expected to be catastrophic. Let \bar{Z} denote this cap on the stock of carbon, giving us the following inequality constraint

$$\bar{Z} - Z(t) \geq 0. \quad (2)$$

If the carbon budget is completely exhausted at some time T , *i.e.*, $Z(T) = \bar{Z}$, then we can not burn any more coal beyond this time, so energy demand must be met by the clean alternative – which is solar energy with unit cost given by k , also assumed constant. The cost of solar is taken to be higher than that of coal, *i.e.* $k > c$.⁷ If only solar energy is used at any time, the price of energy must equal its unit cost, k . Let \hat{y} be the solar consumption at this price, *i.e.*, $u'(\hat{y}) = k$.

Defining the two types of energy mandate

We define a volume mandate set by the regulator as a minimum volume of solar energy $\underline{y} > 0$ that must be consumed at each point in time, *i.e.*,

$$y(t) - \underline{y} \geq 0. \quad (3)$$

When the carbon budget is completely exhausted in our model, coal can not be used any more, so all energy must be supplied by solar, given by \hat{y} . In order for the mandate to bind before coal is completely exhausted, we assume that the volume mandate prescribes a lower level of solar energy consumption than \hat{y} , *i.e.* $\underline{y} < \hat{y}$.

⁷Even though solar energy costs for electricity are declining rapidly in sectors such as transportation, solar may actually be more expensive, for example, given the relatively high cost of electric vehicles or the cost of reliable back-up generation or storage when the sun does not shine. Of course, with existing subsidies, solar costs often compare favorably with fossil alternatives. But studies generally suggest that solar energy in the form of photovoltaic cells may be significantly more expensive than fossil fuels even when externality costs are accounted for (see *e.g.*, Borenstein 2012).

On the other hand, the blend mandate specifies that the share of clean energy in the energy mix must have a lower bound, defined as σ , with $0 < \sigma < 1$. This generates the inequality $y \geq \sigma(x + y)$ that must bind each time period. We can simplify this expression by defining $\theta = \sigma/(1 - \sigma)$, the ratio of energy generation from renewables to nonrenewables, so that the blend mandate can be re-written as:

$$y(t) - \theta x(t) \geq 0. \quad (4)$$

Let $r > 0$ be the discount rate. The social planner chooses consumption of the fossil fuel and clean energy to maximize the sum of the discounted net surplus under either the volume mandate or the blend mandate. That is, the planner solves:

$$\max_{\{x,y\}} \int_0^\infty [u(x + y) - cx - ky] e^{-rt} dt, \quad (5)$$

subject to the carbon ceiling constraints (1) and (2), and either (3) or (4), depending on which mandate is being considered.⁸ Let λ be the shadow cost of the stock of pollution attached to condition (1), η the multiplier on the carbon budget given by (2) and μ the multiplier for the volume mandate (3) or the blend mandate (4), which will be clear from the context. Since coal is cheaper than solar, the former must be used until the carbon budget is exhausted. Thus there is a time T when the stock of pollution reaches the atmospheric limit \bar{Z} , beyond which no coal is used, and solar supplies all of the energy consumed.

3 Energy prices and quantities under the two mandates

The model with no mandate

Before investigating the model with the volume and blend mandates, it is useful to make note of the solution of the benchmark case when there is only a cap on cumulative carbon emissions but no mandate. That is, we solve problem (5) subject to constraints (1) and (2). The Hamiltonian can be written as

$$\mathcal{H} = u(x + y) - cx - ky - \lambda x + \eta(\bar{Z} - Z)$$

⁸Note that the two mandate constraints and the assumption on marginal utility together imply that there must be some solar use at all times, $y(t) > 0$.

giving us the following necessary conditions:

$$p \leq c + \lambda \quad (= \text{ if } x > 0) \quad (6)$$

$$p \leq k \quad (= \text{ if } y > 0) \quad (7)$$

$$\dot{\lambda} = r\lambda - \eta \quad (\eta = 0 \text{ if } \bar{Z} - Z > 0), \quad (8)$$

together with the usual transversality condition $\lim_{t \rightarrow \infty} \lambda(t)e^{-rt}Z(t) = 0$. Note that $\lambda(t) = \lambda_0 e^{rt}$ for $t < T$, where $\lambda_0 \equiv \lambda(0)$ and $\lambda(t)$ may be discontinuous at the instant when the carbon budget is reached at time T .

It is easy to interpret the above conditions. Equation (6) suggests that when coal is used, its price must equal its unit cost plus a shadow marginal cost of the externality given by λ . This externality cost is induced by the fact that burning one unit of coal emits one unit of pollution, thus reducing the available carbon budget by an equal amount. This cost rises exponentially at the rate of discount r , as seen from (8).⁹ It can be interpreted as the carbon tax required to implement the optimal solution in a market economy. Condition (7) suggests that the price of the renewable must equal its unit cost. In the absence of any minimal requirement of solar energy, the energy demand is initially only supplied by coal and, starting from a level lower than the unit cost of solar k , the energy price increases over time as shown in Figure 2 (top panel). When it reaches the trigger price k at transition time T , solar energy becomes competitive and it entirely substitutes for coal (bottom panel).¹⁰

Energy use and prices under the volume mandate

Under the volume mandate, we solve problem (5) with the constraints (1), (2) and (3). We can write the Hamiltonian as:

$$\mathcal{H} = u(x + y) - cx - ky - \lambda x + \eta(\bar{Z} - Z) + \mu(y - \underline{y}),$$

⁹Note that $\eta = 0$ except when the constraint (2) binds.

¹⁰Two conditions must be satisfied at time T : the energy price must equal the unit cost of solar and the carbon budget must be exhausted. These can be written as:

$$c + \lambda_0 e^{rT} = k, \quad (9)$$

$$\int_0^T x(t)dt = \int_0^T q(c + \lambda_0 e^{rt})dt = \bar{Z}. \quad (10)$$

The variables λ_0 and T solve this system of two equations.

and, for $x > 0$, the necessary conditions as:

$$p = c + \lambda \quad (11)$$

$$p = k - \mu \quad (12)$$

$$\dot{\lambda} = r\lambda - \eta. \quad (13)$$

Compared to the no-mandate case, only (7) changes to (12). Now, the price of the renewable must equal its unit cost net of the shadow marginal cost of the mandate. In a perfectly competitive economy, the optimal path can be implemented by using two instruments: a carbon tax of λ per unit of coal that ensures that the carbon budget is respected and a subsidy μ to solar energy induced by the volume mandate, which pays for the gap between the energy price and the solar cost k until solar becomes competitive, *i.e.* $\mu = k - (c + \lambda)$.¹¹

Figure 3 shows prices (top panel) and quantities (bottom panel) under the volume mandate. Energy prices rise over time, because of the shadow cost of carbon emissions, which increases over time. As previously, the price of coal is given by the sum of the marginal cost of coal plus the externality cost. However, since the mandate forces some use of solar \underline{y} at each time, and the price of coal and solar must be equal since they are perfect substitutes, there is a subsidy to solar energy μ as shown in the top panel. The subsidy equals the gap between the unit cost of solar and the social marginal cost of coal. It shrinks over time as the stock of carbon approaches the maximum level allowed. At time T , the carbon budget is exhausted and beyond this time, no more coal is used. The carbon tax jumps down to zero and solar energy becomes competitive without the subsidy. Until time T , solar use is constant and equal to the mandated level \underline{y} while coal consumption declines over time, as shown in the bottom panel. At time T , the use of solar jumps up to \hat{y} to satisfy demand at the price k and coal consumption jumps down to zero.¹²

¹¹A technical appendix available with the authors provides underlying details for this model.

¹²As before, λ_0 and T must satisfy the two conditions: the two prices must be equal at time T and the carbon budget must be exhausted:

$$c + \lambda_0 e^{rT} = k, \quad (14)$$

$$\int_0^T x(t)dt = \int_0^T q(c + \lambda_0 e^{rt})dt - \underline{y}T = \bar{Z}. \quad (15)$$

Compared to the case with only a carbon budget and no mandate, the time at which the

Energy use and prices under the blend mandate

With the blend mandate given by (4), the Hamiltonian is written as:

$$\mathcal{H} = u(x + y) - cx - ky - \lambda x + \eta(\bar{Z} - Z) + \mu(y - \theta x),$$

which yields the necessary conditions:

$$p = c + \lambda + \theta\mu \quad (16)$$

$$p = k - \mu \quad (17)$$

$$\dot{\lambda} = r\lambda - \eta. \quad (18)$$

Compared to the volume mandate, the only difference here is condition (16). Coal consumption in this case is explicitly tied to the production of solar energy because the mandate prescribes a minimum share of the renewable. The social marginal cost of coal now includes an additional term, $\theta\mu$ which depends on the shadow cost of the mandate, given by μ . For every additional unit of coal used, the mandate requires the use of an extra θ units of the renewable, which is subsidized at the rate μ so the additional cost is $\theta\mu$.

Figure 4 shows energy use and prices under the blend mandate. In the bottom panel, note that because coal use must decline as the cost of burning coal increases over time due to a tighter carbon budget, solar use must also decrease in tandem. The shape of the price path is similar to the previous case with a volume mandate. However, since the price of the two resources must be equal, as both are being used concurrently, we can substitute for μ from (16) and (17), which yields:

$$p = \sigma k + (1 - \sigma)(c + \lambda). \quad (19)$$

That is, the price of energy can be written as the weighted sum of the social marginal cost of the two resources, where the weights are the shares of the resources in the energy mix, given respectively by $x = (1 - \sigma)q$ and $y = \sigma q$.

We still need to tax coal at the rate λ , as in the volume mandate. However, there is an additional surcharge on coal, given by $\theta\mu$. Unlike in the volume mandate where the quantity of solar energy is fixed, here the use of solar energy

complete energy transition occurs is delayed and the carbon tax is lower. This can be checked by comparing (9)-(10) with (14)-(15).

is explicitly tied to how much coal we use, hence there is an additional tax on coal. The positive term $\theta\mu$ can thus be interpreted as the penalty for burning an additional unit of coal. As in the volume mandate case, the pollution tax increases over time, but this latter penalty decreases over time because the subsidy to solar decreases. Hence the total tax on coal given by τ is equal to $\lambda + \theta\mu$. Applying (16) and (19) we obtain:

$$\tau = (1 - \sigma)\lambda + \sigma(k - c). \quad (20)$$

The total tax on coal is the weighted sum of the additional unit cost ($k - c$) of using solar energy at the mandated level and the carbon tax, the weights being the share of the two sources in each unit of energy. As shown in the top panel of Figure 4, this tax τ increases over time (since it equals $p - c$) because the rising carbon tax more than compensates for the decline in the cost of the mandate.

The subsidy on solar can be seen from (17) as $k - p$ which upon substitution from (19) yields $\mu = (1 - \sigma)(k - c - \lambda)$. It is the extra cost of using one unit of solar energy instead of one unit of coal (net of the coal penalty) times the share of coal in the energy mix. Note that, as $\theta = \sigma/(1 - \sigma)$, the total solar subsidy payment at any time, *i.e.* the unit subsidy μ times the solar quantity σq is exactly compensated by the total coal surcharge, *i.e.* the unit additional tax on coal $\theta\mu$ times the coal quantity $(1 - \sigma)q$. That is, this tax and subsidy mechanism is revenue-neutral for the policy-maker.

Finally, after time T , as under a volume mandate, the price equals k and energy is exclusively supplied by solar energy.¹³

Effect of a smaller carbon budget and a larger solar energy mandate

Whatever the mandate, an exogenous decrease in the carbon budget \bar{Z} will lead to a higher tax on coal, which increases its price and decreases use in every period.

¹³As before, λ_0 and T must satisfy the following two conditions:

$$\sigma k + (1 - \sigma)(c + \lambda_0 e^{rT}) = k \Leftrightarrow c + \lambda_0 e^{rT} = k, \quad (21)$$

$$\int_0^T x(t)dt = (1 - \sigma) \int_0^T q[\sigma k + (1 - \sigma)(c + \lambda_0 e^{rt})]dt = \bar{Z}. \quad (22)$$

The price path starts higher and reaches the price of the renewable earlier in time, leading to a quicker transition time T . The subsidy to solar also declines. Aggregate welfare also declines because of the lower volume of carbon allowed for use.¹⁴

Figure 5 illustrates the effect of a smaller carbon budget for a volume mandate. In the top panel, the price of coal rises, and thus leads to a shorter transition time T . Since the volume mandate is unchanged, solar energy use does not change, but coal use declines and less coal is used in the aggregate. Total energy use declines as well.

Next, it is easy to see that a tightening of either of the mandates (a larger mandated volume of the renewable or its share for the blend mandate) will lead to coal being used over a longer period of time and a delayed transition time T . The carbon budget also lasts for a longer time period, and so its shadow price λ must always be lower, *i.e.*, the carbon tax will be lower. It will take longer for the price of coal to reach the backstop price k .

For a larger volume mandate, as the same cumulative quantity of coal must be used up at the end (due to the carbon budget) and as the quantity of solar energy is higher at any given time, this implies that the aggregate consumption of energy is higher and the energy price is lower. Since the marginal cost k of solar energy – the price at which solar energy becomes competitive – does not depend on the mandate, the subsidy provided to the solar industry will be higher. Coal use must decline since it is now spread over a longer time horizon. These results are illustrated in Figure 6.

The effect of a larger blend mandate is more nuanced, because of the proportional relationship between the two energy sources. The effect on energy quantity, price, total tax on coal and subsidy to solar energy are indeterminate. We will discuss these issues for some plausible parameter values later in the calibration section.

¹⁴A proof is provided in the Technical Appendix.

4 Comparing the two mandates

Defining a benchmark for comparison

Since the parameters that specify the volume and blend mandates are independent of each other, we need a yardstick for comparing the two. For this paper, we choose the two mandates such that under both, the carbon budget \bar{Z} is exhausted at the same time T . Figure 7 illustrates this rule for a particular transition time T^* . Each of the two curves shows the relationship between the level of a mandate and the transition time: tightening the mandate by increasing the volume or blend share of the renewable (traveling away from the origin on either side) postpones the transition to clean energy. For simplicity, the figure depicts this relationship as linear but it can be represented by any continuous increasing function. Given a carbon budget \bar{Z} and a transition time T , there is a unique value of the volume mandate \underline{y} and the blend mandate σ .¹⁵

We use the superscripts v and b to denote the values of the variables for the volume and blend mandates, respectively. Since the total time taken to exhaust the carbon budget is assumed to be the same across the two mandates, the shadow prices in the two models must be equal since both must rise exponentially and equal the price of the backstop solar at the common time T . Hence $\lambda_0^v = \lambda_0^b = \lambda_0$. That is, the carbon tax on coal is the same for both mandates. After the transition time T , only solar supplies energy and satisfies demand at its marginal cost. We therefore restrict the comparison to the period before time T .

Taxes and Subsidies

A comparison of the taxes and subsidies under the two mandates is best done graphically, as shown in Figure 8. Recall that the energy price to the consumer under the volume mandate is $p^v = c + \lambda$, and for the blend mandate, it is $p^b = c + \lambda + \theta\mu$. This yields $p^b(t) - p^v(t) = \theta\mu > 0$. As shown in the figure, the blend energy price is higher than the price under the volume mandate by the coal surcharge, $\theta\mu$. Both subsidies equal the respective difference between the unit cost

¹⁵There may be alternative criteria for comparison of the two mandates, such as targeting a specific value of a welfare measure (aggregate surplus) that must be attained at transition, or maintaining the same government revenue as in Durrmeyer and Samano (2018).

of solar and the energy price. Thus, the subsidy to solar is lower under the blend mandate than for the volume mandate, as shown. $k = p^v + s^v = p^b + s^b$ which implies that $s^v - s^b = \theta\mu > 0$.

The same carbon tax is applied under both mandates. However, there is an additional surcharge to coal under the blend mandate which leads to a higher tax on coal for the blend mandate. This surcharge also drives a wedge between the solar subsidies under the two mandates, as seen in the figure. The higher fossil fuel tax under the blend mandate is exactly compensated by the higher subsidy to the renewable under the volume mandate.

Energy use

A higher energy price under the blend mandate implies that aggregate energy use is lower at each instant, and cumulative energy use is lower as well, since transition time is the same. That is, $\int_0^T q^b dt < \int_0^T q^v dt$. But the total amount of coal burnt must be equal during the interval $[0, T]$ since we exhaust the carbon budget in both cases. Therefore, cumulative solar energy use must be higher under the volume mandate, $\int_0^T y^v dt > \int_0^T y^b dt$.

Note that $x^v = q^v - \underline{y}$ and $x^b = (1 - \sigma)q^b$, we can take time derivatives to get $|\dot{x}^b(t)| = (1 - \sigma)|\dot{q}^b| < |\dot{q}^b| < |\dot{q}^v| = |\dot{x}^v|$. That is, coal is extracted at a faster rate under the volume mandate. From Figure 8, it is easy to observe that $\dot{p}^v > \dot{p}^b$. Consequently, $|\dot{q}^v| > |\dot{q}^b|$. As equal volumes of carbon must be emitted over time, the extraction paths must cross, as shown in Figure 9. Because the blend mandate uses lower quantities of clean energy as coal use declines over time, more coal is extracted under this mandate later in time.

We know that under the volume mandate, solar energy use is fixed at a given level, $y^v = \underline{y}$. Under the blend mandate, it declines over time, $y^b = \theta x^b = \sigma q^b$. But as we have shown earlier, cumulative solar energy consumption is larger under the volume mandate. Then, two things can happen. Either solar energy use is higher at the beginning for the blend mandate but declines, such that aggregate solar use is lower. Or solar use is lower throughout, relative to the volume mandate. *Ex ante*, it is difficult to say which case is more likely, without making specific functional assumptions.

Welfare analysis

Under perfect competition, the solar and coal producers get zero profit because of constant returns to scale. This is true for both mandates. The consumer surplus is given by $CS \equiv u(q) - qp = u(q) - qu'(q)$. Since at any time, consumption under a volume mandate is higher, consumers enjoy a larger surplus with a volume mandate than with a blend mandate.

The government revenue at any instant GR is the total tax income net of the subsidy payout: $GR \equiv \tau x - sy$. Comparing these revenues under each mandate leads to ambiguous results. Since the total tax under the blend mandate is higher, $\tau^b > \tau^v$ and the subsidy to solar is lower, $s^b < s^v$, the government revenue per unit of delivered energy is always higher with a blend mandate. However, as the level of coal consumption under the blend mandate is smaller at the beginning of the planning horizon, and higher at the end, it is not possible to say which policy yields a larger aggregate government revenue over the planning horizon. Finally, we also have ambiguity in the comparison of total current surplus $W = u(q) - cx - ky$. The calibrated example developed in the next section will address this issue.

We now summarize the main insights from the comparison of the two mandates:

Proposition: For mandates with the same carbon budget \bar{Z} exhausted at the same time T , the blend mandate induces: i) a higher energy price and lower aggregate energy consumption; ii) the same carbon tax but a lower subsidy to solar and a higher total tax on coal; iii) the same cumulative coal use but less at the beginning and more during later periods; and iv) a lower cumulative solar energy use.

5 Coal is supplied by a regulated monopoly

One may ask whether our results will be radically different if coal was supplied by a monopoly which is regulated to produce at the optimal level. This regulated monopoly would choose the quantity of coal x to maximize $(u'(x + y) - c - \tau)x$ where τ is the tax imposed by the planner. The model is the same as before except that the supplier is a monopoly. This yields the necessary condition

$$p = u'(q) = c + \tau - xu''(q) \quad (23)$$

which suggests that the monopoly receives a subsidy equal to $-xu''(q) > 0$ per unit of coal to produce at the optimal level. The monopoly price is the sum of the marginal extraction cost and the total tax on coal (including the carbon tax and the coal surcharge in case of a blend mandate), net of this subsidy. At equilibrium, this price must be equal to the marginal surplus of the consumer. The price of energy is same as before, only the transfer to the monopoly changes. The producer surplus $PS = -x^2u''(q)$ is now strictly positive.

Comparing (23) with (11) for the volume mandate, and (16) for the blend mandate (while taking into account that $\mu = (1 - \sigma)(k - c - \lambda)$), we obtain the unit transfers to the coal producer under the two mandates:

$$\tau^v = \lambda + x^v u''(q^v) \quad \text{with } x^v = q^v - \underline{y}, \quad (24)$$

$$\tau^b = \sigma(k - c) + (1 - \sigma)\lambda + x^b u''(q^b) \quad \text{with } x^b = (1 - \sigma)q^b. \quad (25)$$

This net transfer incorporates both the distortion from market power and the environmental externality. If the former is larger, the the monopoly receives a positive net transfer (*i.e.*, $\tau < 0$), otherwise the monopolist pays a positive net tax (*i.e.* $\tau > 0$). The sign of τ is indeterminate. However, compared to a competitive coal producer, τ is smaller for the monopolist, because the unrestricted monopoly always produces a quantity less than the competitive market.

Consumption of coal and solar are still the same as before. The solar suppliers still earn zero profit. The subsidy on solar remains unchanged. However the net tax on coal changes. From (24) and (25), the difference in total coal taxes for the volume and blend mandates is $\tau^v - \tau^b = -\sigma(k - c - \lambda) + x^v u''(q^v) - x^b u''(q^b)$, whose sign cannot be determined analytically. The difference in the surplus accruing to the coal producer $PS = -x^2u''(q)$ and in government revenues $GR = \tau x - sy$ are indeterminate as well.

6 Numerical illustration

In this section, we use a simple calibration model to illustrate the effect of the two mandates on energy prices and consumption. We compare them under the

condition that a common carbon budget must be exhausted at the same time. Energy quantities are measured in terawatt-hour (TWh), CO₂ emissions in tons (tCO₂), prices in dollars per megawatt (\$/MWh), and carbon taxes in dollars per ton of CO₂ (\$/tCO₂). The base year is 2016 and the model is run until the year 2065.

We only model the world electricity sector and make the simple assumption that electricity is supplied by coal and solar energy. The part of demand supplied by hydro and other sources is taken out of the model. According to the International Energy Agency, electricity generation is responsible for 42% of global CO₂ emissions.¹⁶ Of this, 73% can be attributed to coal-fired power plants. Thus, coal contributes to global emissions by about 30%.

The unit delivery cost of each energy source is given by the levelized cost of electricity (LCOE) which is the average total cost per unit of generation. They include capital and investment costs, fuel costs, and other fixed and variable operations and maintenance (O&M) costs. Using data from the U.S. Energy Information Administration, mean LCOE for conventional coal and solar PV are set at $c = \$95/\text{MWh}$ and $k = \$130/\text{MWh}$, respectively.¹⁷

We assume a quadratic utility function of the form $u(q) = \alpha q - (\beta/2)q^2$, which yields a linear demand function $q(p) = (\alpha - p)/\beta$, with an upper bound on energy consumption, $q < \alpha/\beta$. The corresponding price elasticity is given by $\epsilon \equiv |pq'(p)/q(p)| = p/(\alpha - p)$. The demand function is then calibrated as follows. First, absent any climate and energy policy, electricity is generated only by coal and its price equals $p = c = \$95$. As in the DICE model (Nordhaus, 2015), we assume that this initial state is characterized by a price elasticity of demand equal to $\epsilon = 0.65$. This yields $\alpha = p(1 + \epsilon)/\epsilon = 241$. Parameter β is calibrated from the initial world electricity production from coal in 2016, which according to the International Energy Agency, equals 9,594 TWh so that $\beta = (\alpha - p_0)/q_0 = (241 - 95)/(9594 \times 10^6) = 1.52 \times 10^{-8}$.¹⁸

For convenience, the initial level of cumulative pollution is normalized to zero. We use a carbon dioxide emissions factor for conventional coal of 0.32

¹⁶See <https://www.inec.org/statistics/co2emissions/>

¹⁷Annual Energy Outlook (2019), world markets data: <http://www.eia.gov/outlooks/aeo/>

¹⁸<https://www.iea.org/statistics/>

tCO₂/MWh.¹⁹ In the analytical model earlier, this parameter was taken as one. The Intergovernmental Panel on Climate Change estimates a remaining global carbon budget of 570 GtCO₂ for a 66% probability of limiting global warming to 1.5°C above the pre-industrial level, and of 1,320 GtCO₂ for a 2°C rise with the same probability.²⁰ Here we focus on the 2°C scenario. As coal-fired electricity generation accounts for around 30% of global emissions, we consider a sector-specific carbon budget of 396 GtCO₂.²¹ The social discount rate is set at 3%, which is in the standard range 0 – 5% generally used in climate economic models.

We consider the following three cases: no mandate, a "low" mandate (either a 1,250 TWh volume mandate or a 15% blend mandate) and a "high" mandate (either a 2,700 TWh volume mandate or a 30% blend mandate). The no mandate case assumes the model has a carbon budget but does not specify a solar mandate. Each of the low and high mandates (either volume or blend) are designed to satisfy the comparison rule illustrated by Figure 7.²²

Calibration results are summarized in Table 1. The results specific to the monopoly coal sector are given in Table 2. We only discuss the high mandate case, although the solution for both high and low mandates is shown in Table 1.

1. Note that the time to energy transition T is the same for either type of mandate. However a larger mandate extends the use of the fossil fuel from 2043 to 2049. Without a mandate, the carbon budget (396 GtCO₂) is exhausted by 2039.

¹⁹See MIT Units and Conversions Fact Sheet: http://web.mit.edu/mit_energy

²⁰IPCC Special Report on Global Warming of 1.5°C: <https://www.ipcc.ch/sr15/>

²¹<https://www.iea.org/statistics/co2emissions/>

²²We identify the set of mandates $\{\underline{y}, \sigma\}$ such that a common carbon budget \bar{Z} is exhausted at the same time T . Introducing a carbon emission factor ζ for coal and replacing $q(p)$ by $(\alpha - p)/\beta$ into (15) (respectively, (22)) while using (14) and ((21)), we get: $(\alpha - c - \beta\underline{y})T = \frac{(k - c - \zeta\lambda_0)}{r} + \frac{\beta\bar{Z}}{\zeta}$ and $[\alpha - \sigma k - (1 - \sigma)c]T = \frac{(1 - \sigma)(k - c - \zeta\lambda_0)}{r} + \frac{\beta\bar{Z}}{(1 - \sigma)\zeta}$. As the initial shadow cost of carbon emissions λ_0 (which is equal to $(k - c)e^{-rT}$ in both cases) must be the same under each mandate, the two last equations imply, after simplification:

$$\underline{y} = \left(\frac{\sigma}{1 - \sigma} \right) \left[\frac{(2 - \sigma)\bar{Z}}{(1 - \sigma)\zeta T} - \frac{(\alpha - k)}{\beta} \right],$$

which defines the set of all values of $\{\underline{y}, \sigma\}$ that meet our comparison criteria.

2. Without a mandate, the initial carbon tax is $\$57/\text{tCO}_2$, growing at 3% per year. The carbon tax is lower by about 25% with a high mandate ($\$43/\text{tCO}_2$).

3. Compared to a volume mandate, the blend mandate leads to a higher tax on coal, by about 17% from $\$22$ to $\$26/\text{MWh}$. Solar subsidies are higher for the volume mandate by about 42% from $\$8.9$ to $\$12.7/\text{MWh}$.

4. Tax revenues from the mandate policy are higher under the blend mandate by about 18%. Subsidy payouts are lower as well by about 38%. So the government surplus for the blend mandate is higher, by about 52%. This is because the subsidy to the renewable is lower and the tax on the fossil fuel is higher for the blend mandate.

5. The initial price of energy is about 6% higher under the blend mandate, relative to the volume mandate. Note that the terminal prices must be equal since they all equal the solar price, albeit at different times. This higher price under the blend mandate leads to a lower consumer surplus as well.

6. Cumulative solar use is higher by about 12% under the volume mandate, since under the blend mandate, as the price of energy goes up, the mandate declines.

The dynamic behavior of prices is shown in Figure 10. Energy prices are lower under the two volume mandates relative to the case with no mandates (panel (a)). The fall in price is larger when the mandate is larger.²³ However the blend mandates skew the price distribution – larger mandates raise prices today but lower them in the future (panel (b)). A larger blend mandate increases the surcharge on coal (panel (c)). Finally a higher blend mandate raises the initial solar subsidy while lowering it in the future (panel (d)).

In Figure 11 we show the dynamic effects from the volume and blend mandates for the high case. Note how the volume mandate energy price starts lower and approaches the blend price at the time of energy transition (panel (a)). However, since aggregate coal use must be constant, the coal use under the volume mandate is higher at the beginning and lower at the end (panel (b)). Solar energy use is higher under the volume mandate throughout (panel (c)).

²³The role of subsidies in lowering energy prices has been noted empirically by Liski and Vehvilainen (2016) for the Nordic electricity market.

Finally we show results when the producer of coal is a monopoly (see Table 2). Most results go through with the introduction of market power in the coal market. Producer surplus increases under the volume mandate because the tax on coal is lower. The mean tax on coal net of the subsidy is about 26% higher, and tax revenue accruing to the government is 28% higher under the blend mandate.

7 Concluding remarks

Two types of renewable energy mandates are commonly observed in practice – a volume mandate that prescribes a certain volume of renewable energy use and a blend mandate, which is a share of the total energy mix that must be sourced from renewable energy. In this paper we compare the dynamic effects of these two mandates, both in a simple analytical framework and through a calibration with realistic data.

Using coal and solar energy as the dirty and clean sources of energy, we show that the blend mandate leads to a higher price of energy, a lower subsidy to solar and a higher tax on coal, relative to a volume mandate. The blend mandate also leads to lower coal consumption at the beginning but higher in later time periods, than a volume mandate. Because the blend instrument ties solar to coal use, it also leads to a lower cumulative solar consumption than the volume mandate.

When the two types of mandates have the same goals in terms of the carbon budget to be exhausted and the time of complete transition to renewable supply, the blend mandate imposes a higher tax on coal and a lower subsidy to the renewable. It also skews coal use by pushing more of it to the future, and leads to lower cumulative use of the renewable.

A simple calibration with realistic data shows that the differences in the two mandates may be significant. The blend mandate leads to a 17% higher tax on coal. On the other hand, the volume mandate induces a 42% higher subsidy on solar. Tax revenues for the government are greater under the blend mandate by about 18% and subsidy payouts are lower by about 38%. The blend mandate taxes the fossil fuel more and subsidizes the renewable less, ensuring a lower fiscal burden for the government. The price of energy is slightly higher for the blend

mandate as well. Cumulative solar use is higher under the volume mandate since the blend mandate is diluted over time as the energy mix uses less of the fossil fuel due to its rising price.

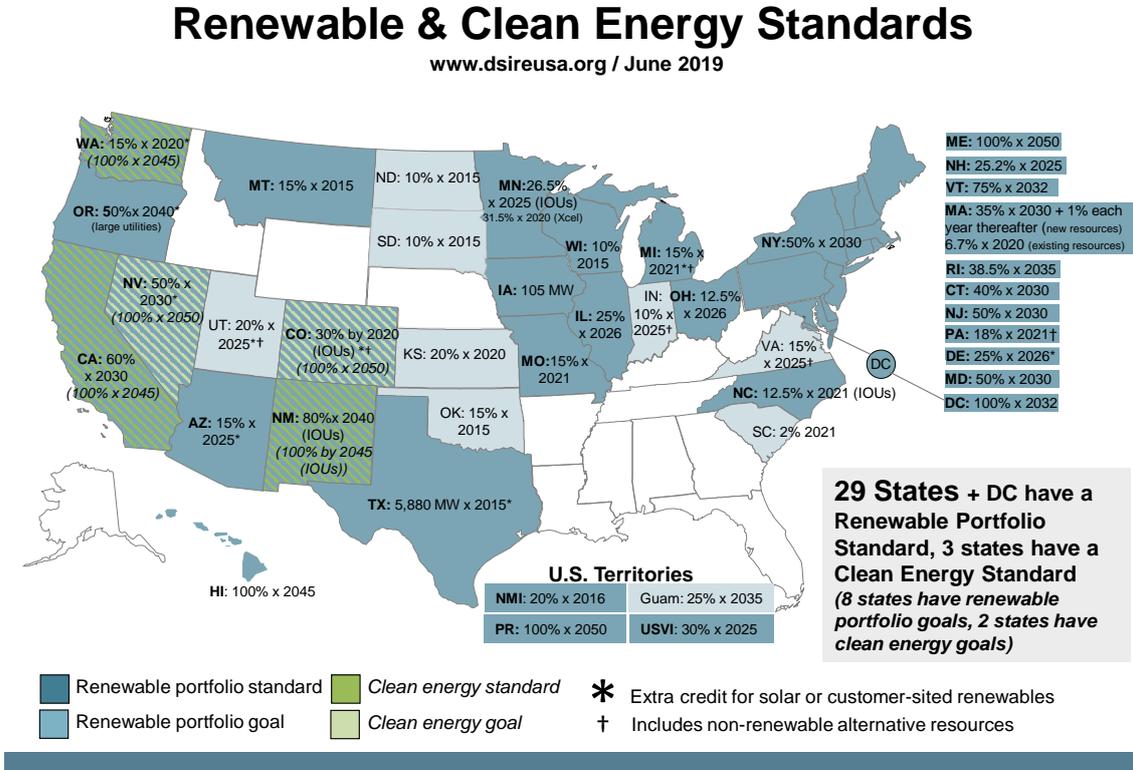
In general, mandates subsidize renewable energy and reduce carbon taxes and the price of energy. The larger the mandate the greater is the subsidy. However, the volume mandate leads to a larger subsidy for the renewable and a lower tax on the fossil fuel. These results have political economy implications that should be considered in future work. For example, do jurisdictions which have stronger fossil fuel lobbies prefer volume mandates? An important issue we do not explicitly model is learning-by-doing, especially in newer technologies such as solar energy. However, if the degree of learning is proportional to the size of the renewables market, then the volume mandate may lead to larger learning effects, since unlike the blend mandate, solar consumption does not decline over time. The mandates differ in the distribution of the energy mix over time, with the blend mandate pushing fossil fuel use to future time periods, and the volume mandate using more of the polluting fuel in early periods. To the extent that the timing of carbon emissions matters for the environment and not just the aggregate pollution, the blend mandate may produce less damage because it postpones carbon emissions to future periods. More work needs to be done to understand these differential effects.

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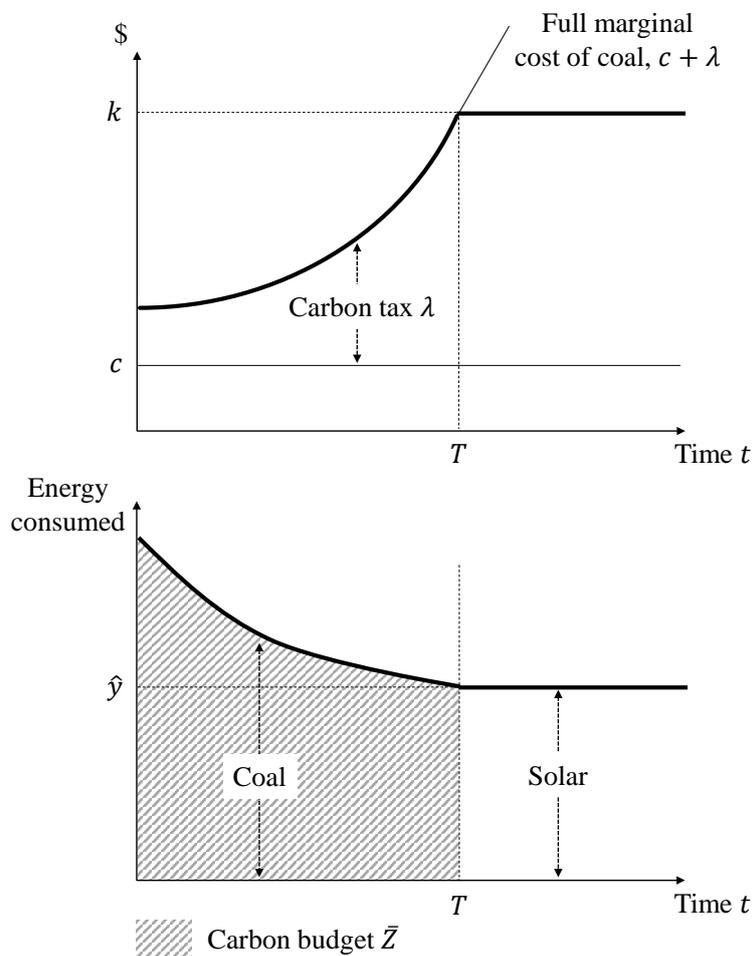
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Figure 1: Both volume and blend mandates are in effect in the US



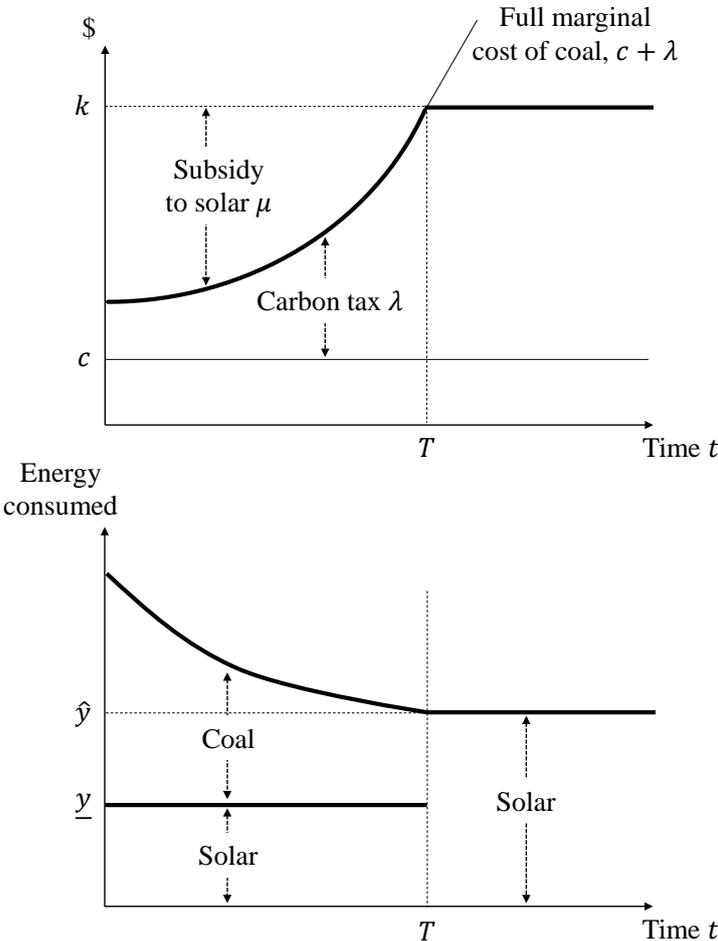
Note: Texas and Iowa have volume mandates while many other states have blend mandates.

Figure 2: Energy use and prices without any mandate



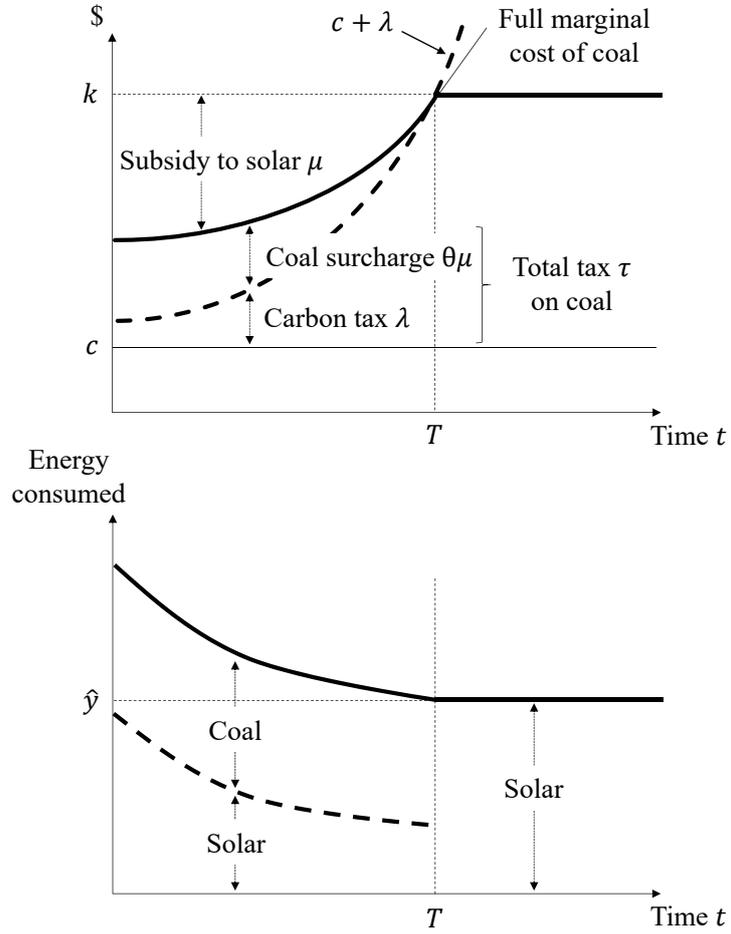
Note: The top panel shows that energy prices increase over time until they reach the solar price. In the bottom panel, only coal is used until exhaustion at T , followed by solar energy. The shaded area represents the total carbon budget that must be exhausted.

Figure 3: Energy use and prices under the volume mandate



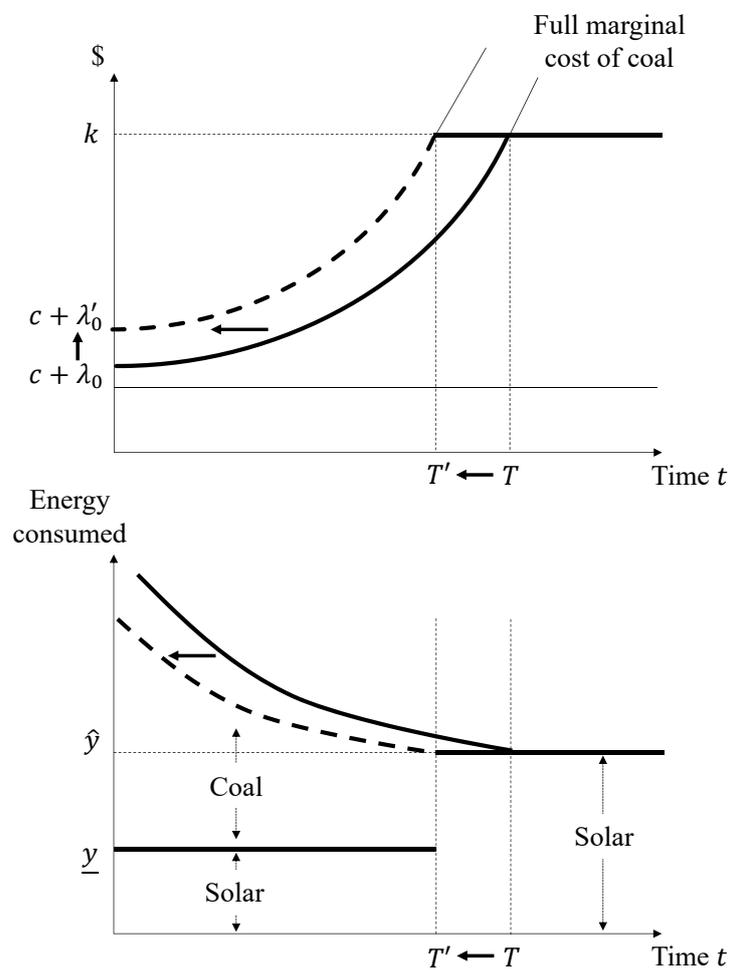
Note: The carbon tax increases over time and the subsidy to solar decreases. Coal use declines while solar use is fixed by the mandate.

Figure 4: Energy use and price under the blend mandate



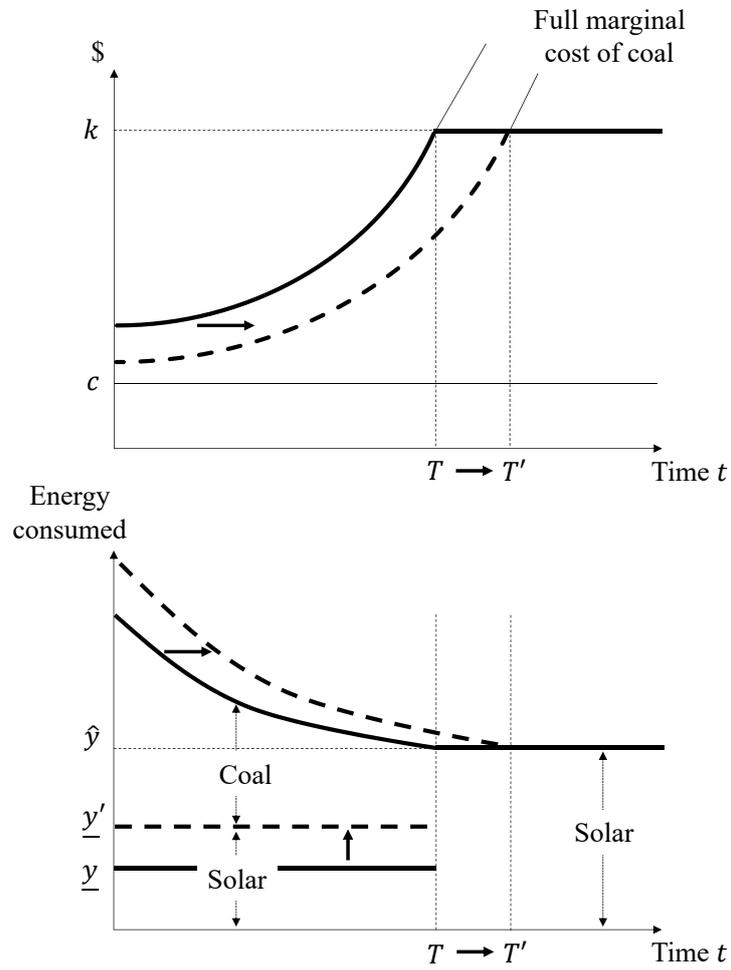
Note: There is a carbon tax on coal plus a surcharge because the blend mandate induces additional solar deployment for each unit of coal used.

Figure 5: Effect of a smaller carbon budget with a volume mandate



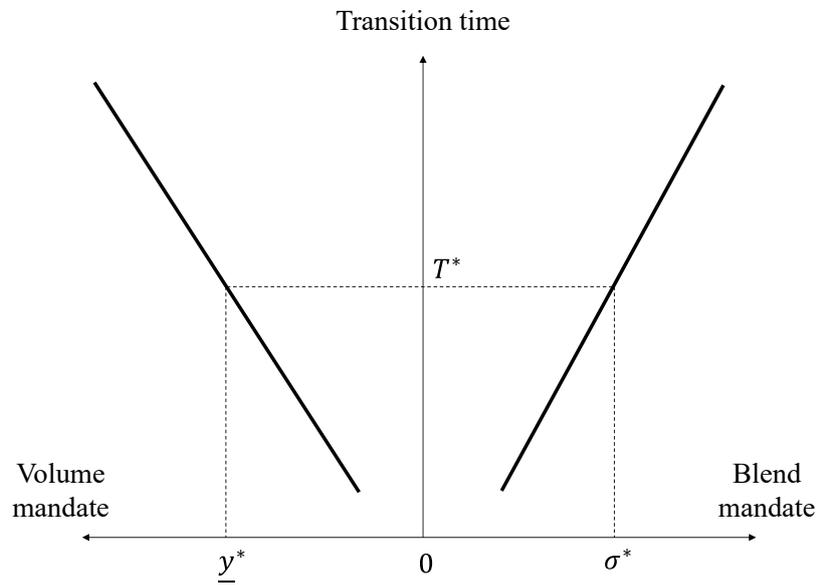
Note: A smaller carbon budget leads to an increase in the energy price. Total coal use declines and there is a quicker transition to clean energy.

Figure 6: Effect of a larger volume mandate



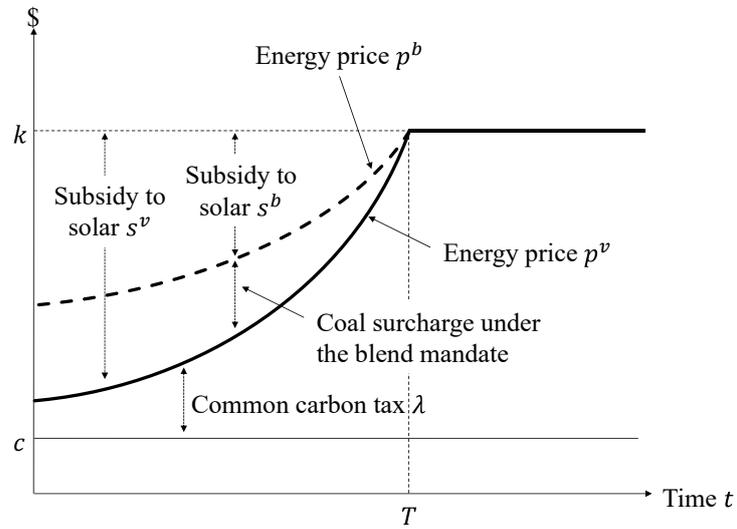
Note: Mandate \underline{y} shifts up to \underline{y}' . The price of energy decreases, the tax on coal decreases and aggregate energy consumption increases. Solar use increases and coal use must decline because it is spread over a higher time period.

Figure 7: Criteria for comparing volume and blend mandates



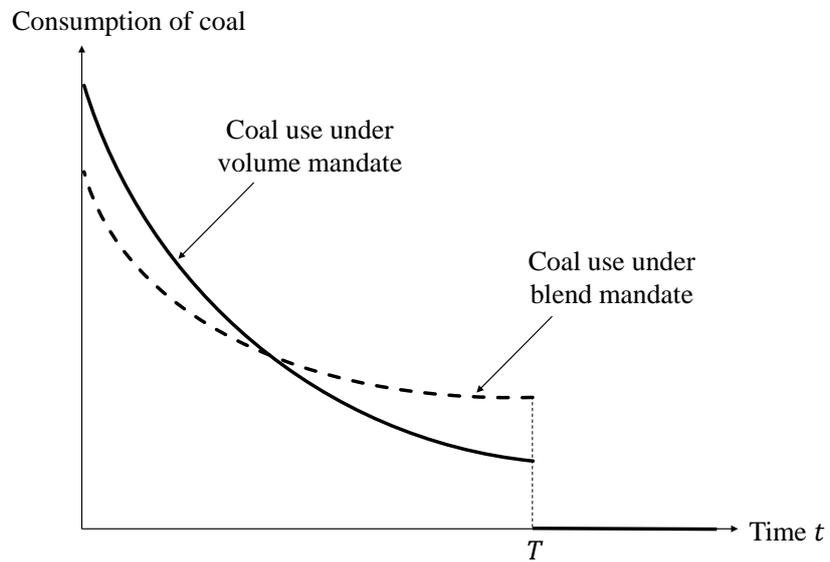
Note: We compare volume and blend mandates that use the same carbon budget over the same time period.

Figure 8: Higher price of energy under the blend mandate



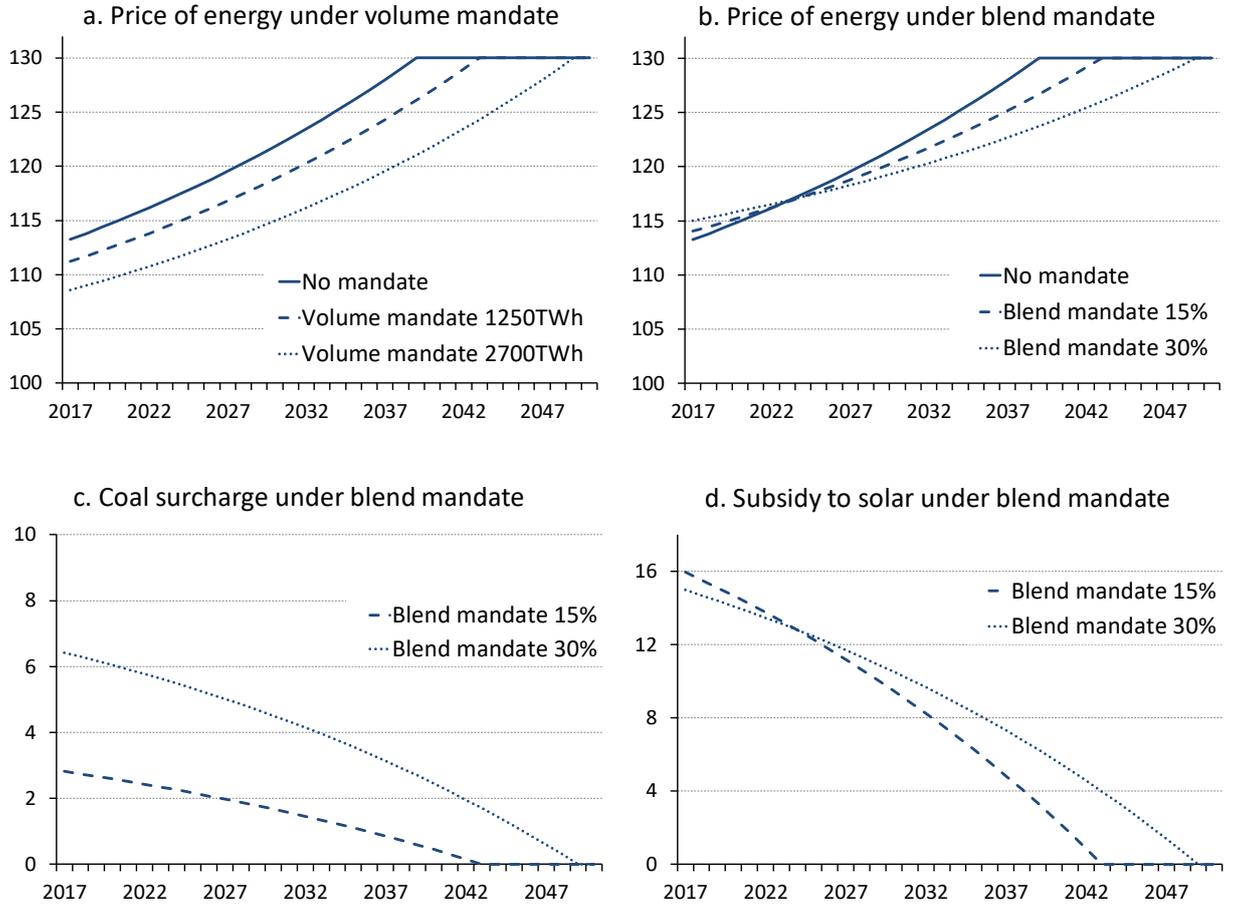
Note: The price of energy is higher under a blend mandate because of the additional surcharge on coal. The total tax on coal is also higher, and the subsidy on solar energy is lower. The surcharge on coal for the blend mandate exactly compensates for the additional subsidy to solar under the volume mandate.

Figure 9: Initial coal use is higher under the volume mandate, and aggregate energy use is lower.



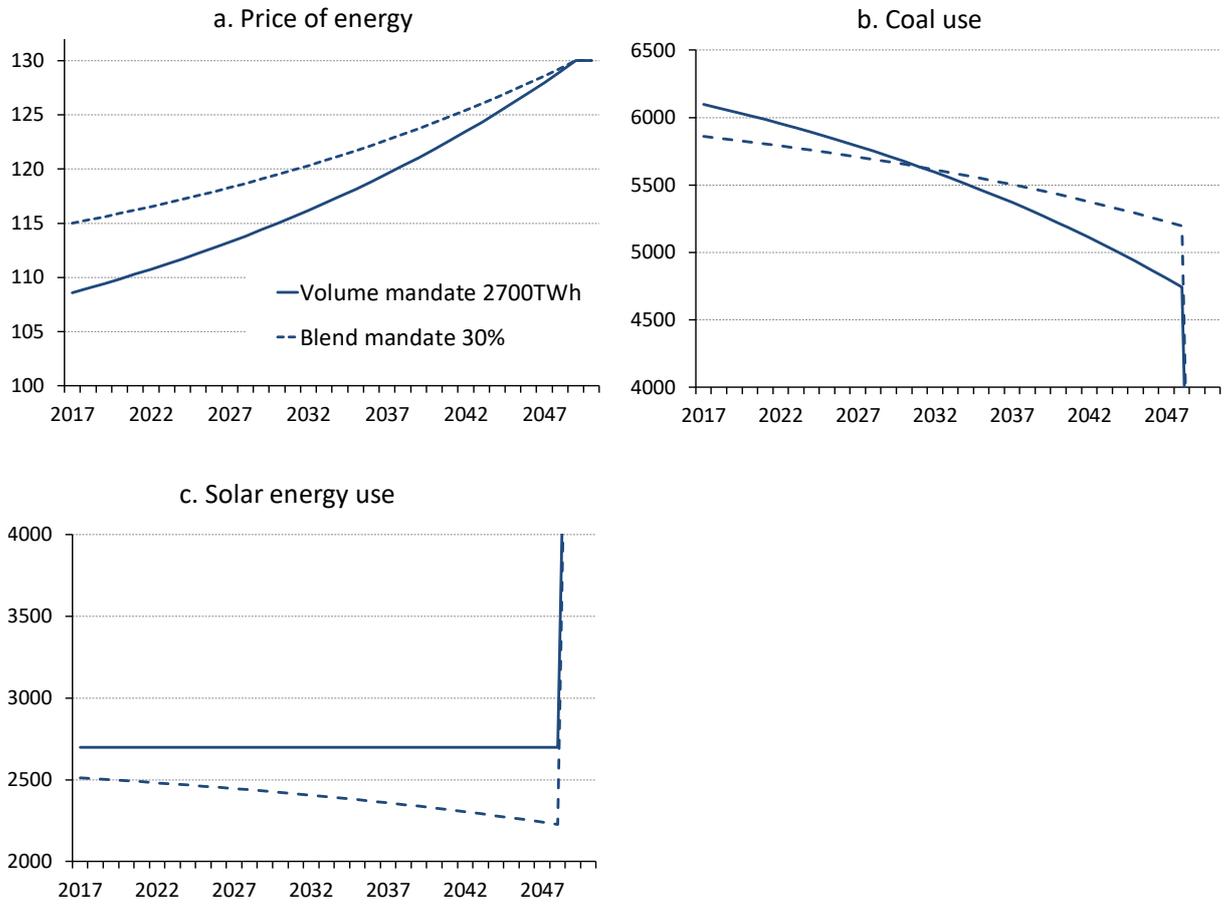
Note: The blend mandate pushes coal use to future periods.

Figure 10: Effect of a larger mandate



Note: All prices are in \$/MWh. Raising the volume mandate reduces the price of energy in all periods, but raising the blend mandate increases current prices and decreases future prices. Raising the blend mandate increases the coal surcharge but shifts the subsidy on solar to future periods.

Figure 11: Comparison of volume and blend mandates



Note: The blend mandate raises energy prices, postpones coal use to the future and uses less solar energy.

Table 1: Model results with competitive coal supply

<i>Model outcome variables</i>	No mandate	Low mandate		High mandate	
		Volume 1250TWh	Blend 15%	Volume 2700TWh	Blend 30%
Time to energy transition T	2039	2043	2043	2049	2049
Initial carbon tax (\$/tCO ₂)	57.46	51.05	51.05	42.76	42.76
Initial price of energy (\$/MWh)	113.27	111.23	114.04	108.59	115.01
<i>Mean values</i>					
<i>over the planning horizon</i>					
Initial coal use (TWh)	8288	7156	7013	5863	5745
Final coal use (TWh)	7746	6550	6576	5176	5408
Solar energy use (TWh)	0	1250	1199	2700	2390
Cumulative solar use (TWh)	0	33750	32632	89100	79394
Subsidy to solar (\$/MWh)	0	10.94	9.29	12.7	8.89
Surcharge on coal (\$/MWh)	0	0	1.64	0	3.81
Total tax on coal (\$/MWh)	25.35	24.06	25.71	22.30	26.11
Tax revenue (billion)	203.27	164.91	174.67	123.1	145.61
Total subsidy payment (billion)	0	13.67	11.14	34.29	21.25
<i>Discounted aggregate surplus</i>					
Consumer surplus (billion)	12471	12860	12593	13474	12748
Government revenue (billion)	3174	2507	2769	1481	2260
Total (billion)	15645	15367	15362	14955	15008

Note: Initial coal use is the mean annual use for the first half of the planning horizon, and final coal use is the mean for the second half. Other values are averaged over the whole planning horizon. Aggregate surplus is discounted at 3%.

Table 2: Coal supplied by regulated monopoly

<i>Model outcome variables</i>	No mandate	Low mandate		High mandate	
		Volume 1250TWh	Blend 15%	Volume 2700TWh	Blend 30%
<i>Mean values:</i>					
Total tax on coal (\$/MWh)	25.35	24.06	25.71	22.30	26.11
Subsidy to coal (\$/MWh)	12.08	10.33	10.24	8.32	8.40
Net tax on coal (\$/MWh)	13.28	13.74	15.47	13.98	17.71
Tax revenue (billion)	106.43	94.15	105.05	77.19	98.75
<i>Discounted aggregate surplus:</i>					
Coal producer (billion)	1715	1405	1372	1054	1044
Government revenue (billion)	1459	1102	1398	427	1216

Technical Appendix provided only for manuscript review

Volume mandate

Under a volume mandate, λ_0 and T must solve the following system of equations:

$$c + \lambda_0 e^{rT} = k \quad (26)$$

$$\int_0^T q(t) dt - \underline{y}T = \bar{Z}, \quad (27)$$

where q solves $u'(q) = p = c + \lambda_0 e^{rt}$, which implies that $dq = \frac{dp}{u''(q)} = \frac{e^{rt}}{u''(q)} d\lambda_0$. Taking c , k and r as given, totally differentiating (26)-(27) and expressing in matrix form, we obtain:

$$\begin{bmatrix} r\lambda_0 & 1 \\ (\hat{y} - \underline{y}) & \int_0^T \frac{e^{rt}}{u''(q)} dt \end{bmatrix} \begin{bmatrix} dT \\ d\lambda_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} d\bar{Z} + \begin{bmatrix} 0 \\ T \end{bmatrix} d\underline{y}. \quad (28)$$

Let Δ be the determinant of the 2×2 matrix in the left-hand-side of (28), we can write:

$$\Delta = r\lambda_0 \int_0^T \frac{e^{rt}}{u''(q)} dt - (\hat{y} - \underline{y}),$$

which is negative as $u'' < 0$ and $\hat{y} > \underline{y}$ by assumption.

We first check the sensitivity to the carbon budget level. From (28), we get:

$$\frac{dT}{d\bar{Z}} = \frac{1}{\Delta} \begin{vmatrix} 0 & 1 \\ 1 & \int_0^T \frac{e^{rt}}{u''(q)} dt \end{vmatrix} = -\frac{1}{\Delta} > 0 \quad (29)$$

$$\frac{d\lambda_0}{d\bar{Z}} = \frac{1}{\Delta} \begin{vmatrix} r\lambda_0 & 0 \\ (\hat{y} - \underline{y}) & 1 \end{vmatrix} = \frac{r\lambda_0}{\Delta} < 0. \quad (30)$$

From (30), we directly obtain $dp/d\bar{Z} < 0$ and $dq/d\bar{Z} > 0$. Next, given that $x = q - \underline{y}$ and $\mu = k - p$, we conclude that $dx/d\bar{Z} = dq/d\bar{Z} > 0$ and $d\mu/d\bar{Z} = -dp/d\bar{Z} > 0$. Denoting by V the optimal value function of the program and applying standard dynamic programming methods (under exponential discounting), the Bellman equation writes: $rV(Z) = \max \{u(x + y) - cx - ky + xV'(Z)\} \forall t$, with $V'(Z) = -\lambda$. A simple expression of V is thus given by the Hamiltonian of the program evaluated at time $t = 0$:

$$rV = u(q_0) - (c + \lambda_0)x_0 - k\underline{y}. \quad (31)$$

Differentiating (31) and noting that $u'(q_0) = c + \lambda_0$, we get:

$$r \frac{dV}{d\bar{Z}} = u'(q_0) \frac{dq|_{t=0}}{d\bar{Z}} - (c + \lambda_0) \frac{dx|_{t=0}}{d\bar{Z}} - x_0 \frac{d\lambda_0}{d\bar{Z}} = -x_0 \frac{d\lambda_0}{d\bar{Z}} > 0.$$

For the effect of a larger volume mandate, (28) implies:

$$\frac{dT}{dy} = \frac{1}{\Delta} \begin{vmatrix} 0 & 1 \\ T & \int_0^T \frac{e^{rt}}{u''(q)} dt \end{vmatrix} = -\frac{T}{\Delta} > 0 \quad (32)$$

$$\frac{d\lambda_0}{dy} = \frac{1}{\Delta} \begin{vmatrix} r\lambda_0 & 0 \\ (\hat{y} - y) & T \end{vmatrix} = \frac{r\lambda_0 T}{\Delta} < 0, \quad (33)$$

which suggests that $dp/d\underline{y} < 0$, $dq/d\underline{y} > 0$ and $d\mu/d\underline{y} > 0$. The effects of a larger \underline{y} on coal consumption and on social welfare are ambiguous and may depend in particular on the characteristics of the demand function:

$$\begin{aligned} \frac{dx}{dy} &= \frac{dq}{dy} - 1 = \underbrace{\frac{e^{rt}}{u''(q)} \frac{d\lambda_0}{dy}}_{>0} - 1 \begin{matrix} \leq \\ > \end{matrix} 0 \\ r \frac{dV}{dy} &= -\underbrace{\left[x_0 \frac{d\lambda_0}{dy} \right]}_{<0} + \underbrace{[k - c - \lambda_0]}_{>0} \begin{matrix} \leq \\ > \end{matrix} 0. \end{aligned}$$

Blend mandate

Under a blend mandate, $\{\lambda_0, T\}$ is solution to

$$c + \lambda_0 e^{rT} = k \quad (34)$$

$$(1 - \sigma) \int_0^T q(t) dt = \bar{Z}, \quad (35)$$

where q satisfies $u'(q) = p = (1 - \sigma)(c + \lambda_0 e^{rt}) + \sigma k$, which implies that

$$dq = \frac{e^{rt}}{u''(q)} (1 - \sigma) d\lambda_0 + \left[\frac{k - c - \lambda_0 e^{rt}}{u''(q)} \right] d\sigma. \quad (36)$$

As before, we totally differentiate the system (34)-(35) as before, use (36) and rearrange to get

$$\begin{bmatrix} r\lambda_0 & 1 \\ \hat{y} & \int_0^T \frac{(1-\sigma)e^{rt}}{u''(q)} dt \end{bmatrix} \begin{bmatrix} dT \\ d\lambda_0 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{(1-\sigma)} \end{bmatrix} d\bar{Z} + \begin{bmatrix} 0 \\ \Lambda \end{bmatrix} d\sigma, \quad (37)$$

where $\Lambda \equiv \frac{\bar{Z}}{(1-\sigma)^2} - \int_0^T \frac{(k-c-\lambda_0 e^{rt})}{u''(q)} dt$ is positive as $u'' < 0$ and $k > c + \lambda_0 e^{rt}$ for $t \in [0, T)$. The determinant of the 2×2 matrix on the left-hand-side of (37) is given by:

$$\Delta = r\lambda_0 \int_0^T \frac{(1-\sigma)e^{rt}}{u''(q)} dt - \hat{y} < 0.$$

From (37), differentiating with respect to \bar{Z} yields:

$$\frac{dT}{d\bar{Z}} = \frac{1}{\Delta} \begin{vmatrix} 0 & 1 \\ \frac{1}{(1-\sigma)} & \int_0^T \frac{(1-\sigma)e^{rt}}{u''(q)} dt \end{vmatrix} = -\frac{1}{(1-\sigma)\Delta} > 0 \quad (38)$$

$$\frac{d\lambda_0}{d\bar{Z}} = \frac{1}{\Delta} \begin{vmatrix} r\lambda_0 & 0 \\ \hat{y} & \frac{1}{(1-\sigma)} \end{vmatrix} = \frac{r\lambda_0}{(1-\sigma)\Delta} < 0. \quad (39)$$

Equation (39) directly implies $dp/d\bar{Z} < 0$ and $dq/d\bar{Z} > 0$. As $x = (1-\sigma)q$ and $y = \sigma q$, we also have $dx/d\bar{Z} > 0$ and $dy/d\bar{Z} > 0$. The subsidy to solar energy is $\mu = k - p = (1-\sigma)(k - c - \lambda)$ so that $d\mu/d\bar{Z} = -(1-\sigma)d\lambda/d\bar{Z} > 0$. The total tax on coal $\tau = \lambda + \theta\mu = (1-\sigma)\lambda + \sigma(k - c)$ and $d\tau/d\bar{Z} = (1-\sigma)d\lambda/d\bar{Z} < 0$. That is, if the carbon budget increases, the consequent decrease in the carbon tax is greater than the increase in the tax on coal from the blend mandate so that the total tax on coal is lower. Lastly, the value function V is given by

$$rV = u(q_0) - [(1-\sigma)(c + \lambda_0) + \sigma k]q_0, \quad (40)$$

which yields $rdV/d\bar{Z} = -(1-\sigma)q_0d\lambda_0/d\bar{Z} > 0$.

Finally, (37) implies

$$\frac{dT}{d\sigma} = \frac{1}{\Delta} \begin{vmatrix} 0 & 1 \\ \Lambda & \int_0^T \frac{(1-\sigma)e^{rt}}{u''(q)} dt \end{vmatrix} = -\frac{\Lambda}{\Delta} > 0 \quad (41)$$

$$\frac{d\lambda_0}{d\sigma} = \frac{1}{\Delta} \begin{vmatrix} r\lambda_0 & 0 \\ \hat{y} & \Lambda \end{vmatrix} = \frac{r\lambda_0\Lambda}{\Delta} < 0. \quad (42)$$

That is, the effect of a larger blend mandate σ on energy prices and quantities, and taxes cannot be determined without specifying functional forms.