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Recycling under environmental, climate and resource constraints

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Abstract

We study the recycling opportunity of an industrial sector constrained by climate, resource and waste capacities. A final good is produced from virgin and recycled materials, and its consumption releases both waste and GHG emissions. We identify the optimal trajectories of resources use, mainly depending on the emission rates of each resource and on the relative scarcity of their stocks. Recycling is sometimes an opportunity to reduce the impact of consumption on primary resources and waste but can still affect the environment. We characterize the optimal recycling strategy and we show that, in some cases, the time pace of the recycling rate is inverted U-shaped. Last, we discuss the policy implications of our model by identifying and analyzing the set of optimal tax-subsidy schemes.

Keywords— Recycling; Resource extraction; Waste; GHG emissions.

JEL classifications— Q32, Q53, Q54.

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1. Introduction

1.1. Context and motivation

The principal motivation for recycling has been the saving of extracted resources for a long time, and has been an important focus of economists in early studies on secondary materials economics, up until the 21^{st} century and our current resources constraints [32]. To the opportunity of reducing resources constraints, we can add the mitigation of waste pollution and its increasing impacts on the environment. For these reasons, the concept of circular economy, for which recycling is one of the cornerstones, is a solution for a more sustainable economical model, as formalized by Braungart and McDonough in their book *Cradle to Cradle* [5]. This concept generated a significant amount of "grey literature", through many non-governmental organization like the *Ellen McArthur Foundation*. Besides, these grey literature and academic literature are especially concentrated in Europe and Asia, two areas with many implemented drivers in this field, both in academic domain and institutional initiatives [9].

The study of recycling can be extended when considering climate change as an additional externality. This extension of the problem leads to new arbitrations: in most cases, recycling is a way of reducing the use of resources with a high carbon footprint [13][32] but is still the source of greenhouse gas emissions thus having an impact on climate change. It has already been highlighted that circularity and environmental issues are connected in an industrial sector, with for instance used tires [23]. In France, studies of Federec [13] and ADEME [2] quantified different impacts of recycling in terms of GHG emissions, showing that industrial process are often highly carbon intensive compared to recycling industries (see Table 1). However, recycling does not appear to be the ideal clean substitute to regular production: it does produce emissions, there is a need in initial production from a regular source, recycling comes with a cost [31].

With this in mind, we want to examine three different balances in an industrial sector: a material balance in order to examine the saving of natural resources and the reduction of waste accumulation, a carbon balance for the topic of climate change and an economic balance for the evolution of consumption. We see in fact environmental objectives of recycling going

	Paper	PET Plastics	Aluminum	Glass[a]
Virgin material	297	3 270	9 827	923
Recycled material	317	202	513	409

Table 1: Examples of emissions rates in France (in kg CO_2e/t)
Note: Values obtained from Federec[13] and [a] from ADEME, "http://www.bilans-ges.ademe.fr"

in three different directions: the saving of natural resources while its shortage could lead to economic difficulties ; the reduction of waste accumulation that is costly to manage for public and private entities and poses a threat to the environment; the fight against climate change and especially the reduction of greenhouse gas emissions. The objective of this paper is to develop an approach embracing these issues simultaneously instead of detached problems.

1.2. Related literature

Early studies from the 70s-80s already tackled resource scarcity. For instance, Smith [29] puts forward social costs linked to waste accumulation and stocks diminution. He focuses on the dynamics of waste when recycling is under consideration, and he shows that there is a trade-off between private costs (labor, material) and social costs (waste accumulation, resource depletion). Dynamic models were able to draw the first economic guidelines motivating recycling and waste accumulation issues were soon added to the topic with various models intending to find the optimal level of pollution in an economy (see for instance Plourde [27], Forster [15], or Hoel [18]). Later, the work of Chakravorty *et al.* [7] focusing on the order of resources extraction gives many insights when resource depletion induces pollution, changing the extraction order, but does not include recycling in the model.

An important part of literature was later developed around the topic of green policies to promote recycling. Palmer *et al.* [25] use a static micro-economic model in order to analyze the effects of diverse economic incentives such as subsidies, waste tax and deposit-refunds. This approach gives many policy insights but only takes into account waste and recycling activities. They have been completed with environmental effects associated with recycling and resource extraction [16][24]. Going further in this type of analysis, Acuff and Kaffine [1] add carbon emissions to their model and show that the objective of reducing it is a strong incentive to increase recycling, and that green policies can be implemented with this goal. These articles add a significant contribution regarding public intervention linked to recycling activities. However this kind of static analysis omits the dynamic aspects of stocks mentioned above. A further analysis has to examine the arbitration between environmental externalities and resource depletion, for instance extending the Acuff and Kaffine model [1] (initially being an extension of the Palmer and Walls model [25]) to a dynamic system.

A rare example of this type of work is the article of Huhtala [21], one of the few to analyze the optimal use of an exhaustible resource considering at the same time issues of waste accumulation, resource depletion and pollutant emissions. She describes the best arbitration of labor between recycling and primary production, and designs a fitting tax-subsidy scheme to achieve it under a balanced budget. This work is completed by different recent studies on resources economics, with for instance Pittel, Amigues and Kuhn [26] who model a decentralized economy including a recycling activity and highlight the market failure resulting from the absence of a market for waste, despite their economical value. They provide an optimum by setting up a market for waste and subsidizing recycling activities. This dynamic approach is also the perspective of Di Vita [10][11] who assesses the possibility of an economic and welfare growth with a material constraint, thanks to investments in recycling. Sorensen[30] also considers a recycling technology in a Ramsey model that alleviates externalities due to resource extraction and consumption. These articles share the use of optimal control theory, but propose different models to represent a circular economy. Besides, Di Vita[10][11] proposes models that do not respect a material balance in the economy, in contrast to the physical reality of the use of secondary materials. With these propositions, he arrives to the result that recycling allows a stationary growth path.

An alternative modeling approach is given by Boyce [4] who chooses to specify a recycling stock separated from accumulated waste. He examines the dynamic of this stock when there is perfect substitution between virgin and recycled material. It enables him to describe economic trade-offs between the two material to manufacture a final good. With this Herfindhal analysis, he can depict economic paths and shows that recycling adds different sets of possible consumption paths, variations of the least-cost principle. In this paper, we follow this approach and extend the study by considering both recycling and climate change issues. We investigate different economic paths and the influence of environmental and climate constraints on the use of an exhaustible resource.

1.3. Sketch of the model

This paper presents a model taking into account exhaustible resource constraints, waste accumulation and GHG emissions in the context of an industrial sector with a recycling branch. We do not pretend to calibrate policies to implement but we try to highlight analytically the crossed effects and eventual synergies between the fight against climate change, preservation of resources and the limitation of waste disposal. We choose to build a continuous centralized model maximizing utility of consumption of a manufactured good, produced from virgin or recycled materials. This model displays two negative environmental externalities: waste accumulation that harms the economy through a specific damage function, and cumulative GHG emissions that are constrained by an exogenous budget. A third source of (positive) externality must also be considered as long as there is no waste market, as consumption of the final good provided a waste stream which can be reused thanks to an endogenous effort of recycling. We then characterize the main properties of the optimal trajectories of the model. In particular, we discuss the merit order in using each type of resource as well as their respective dynamics. We also analyze the optimal time pace of the recycling rate and show that, under some conditions, it can be inverted U-shaped. Last, we show that, in a market economy, this optimal outcome can be implemented by a set of tax-subsidy schemes and we discuss their policy implications depending on the identity of the tax payer or the subsidy beneficiary.

1.4. Outline

The remaining of this paper is organized as follows. Section 2 presents the model. Section 3 characterizes the optimal solution and describes the different possible scenarios of consumption and recycling. Section 4 studies the decentralized equilibrium outcome and provides some possible extensions of the model. Section 5 concludes.

2. Setup of the model

We model an industrial sector producing a quantity q of a final good from two different inputs: a virgin resource and a recycled one, of relative quantities v and r. We assume that there is perfect substitution between these two inputs, involving that $q(t) = v(t)+r(t)$ for any time t. Consuming q units of final good provides a gross surplus $u(q)$ to the final user, where function $u(.)$ follows the standard hypotheses: of class $\mathcal{C}^2$, increasing ($u' > 0$), concave ($u'' < 0$) and verifying the Inada conditions.[1] Here we do not differentiate utility drawn from the consumption of a good made out of virgin material and a good made out of recycled material, thus following the above hypotheses of perfect substitution in the production function.[2]

Input flows v and r come from two different primary sources, a virgin stock and a recycled stock. We denote respectively by c_v and c_r the marginal delivery costs of these two resources, which include the extraction, production and transportation costs, and which are assumed to be constant over time. We also state that producing input material from the recycled stock is more expensive than from the virgin stock: $c_r > c_v$. It follows the difference of maturity between these two materials and can be observed as the recycling branch is usually not favored when social costs are not internalized.[3]

The industry is initially endowed with a stock $V_0 > 0$ of virgin resource, but we suppose that recycling has never happened by the past, meaning that the initial stock of recycled material is zero: $R_0 = 0$. At any time t, the current levels of virgin and recycled stocks $V(t)$ and $R(t)$ are governed, respectively, by the following extraction processes:

$$\dot{V}(t) = -v(t) \quad , \quad V(0) = V_0 \tag{1}$$

$$\dot{R}(t) = -r(t) + \beta(t)q(t) \quad , \quad R(0) = 0\,, \tag{2}$$

where β, such that $0 \leq \beta \leq 1$, represents the endogenous recycling intensity, i.e. the share of production collected after use, sorted and incorporated into the recycled stock.[4] However,

[1] $\lim_{q\to 0^+} u'(q) = +\infty$ and $\lim_{q\to\infty} u'(q) = 0$.

[2] Note that when focusing on the use of different resources, perfect substitution is a common assumption in the literature [3][4][6][7]. This particular assumption, as well as some other ones, will be discussed in the last part of the paper.

[3] In some model such as Hoel[18], the private cost of the virgin resource is even assumed to be zero. Besides, Di Vita[11] precises that an industry of secondary materials with the same properties as virgin inputs would bear very high cost, not sustainable with current market prices.

[4] While β does not exactly fit the common definition, we will also refer to it as "recycling rate" throughout the rest of the paper. Moreover, as we model recycled resource and waste as two different stocks, β must be a control variable in order to have an endogenous recycling. A similar approach is used by Boyce[4] as he uses a sorting cost and assigns to the social planner the choice of sorting waste or not (a binary decision). This choice of modeling two stocks allows to represent the material balance constraint in our economy, with convex costs forbidding total recycling.

note that recovering waste from the flow q and using it in production are two different actions that are only correlated by the cost arbitration. As a matter of fact, it can be optimal to use a recycled input in production while deciding not to redirect waste into the recycled stock ($\beta = 0$), as long as this stock is not empty.

In this model, recovering material from waste is costly and this cost is added to the other private cost relative to the recycled input. This cost depends on both the level of output that can be recycled, i.e. the flow of waste generated by consumption[5], and the rate of redirection from the waste flow: $F(q, \beta)$. To simplify, we assume that it is linear in q and convex in β to reflect constant returns in the volume of materials to be reprocessed, and decreasing returns in the recycling technology: $F(q, \beta) = qf(\beta)$, with $f(0) = 0$, $f'(\beta) > 0$ and $f''(\beta) > 0$.

The remaining share of final good which could not be collected and recycled yields a waste stream $(1-\beta)q$ that accumulates.[6] We denote by $W(t)$ the resulting stock of waste at time t, and by W_0 the initial stock. Moreover this waste stock is reduced by a bio-decomposition, or another natural resorption mechanism, at rate α, an exogenous and constant parameter such that $0 < \alpha < 1$. Then, we have:

$$\dot{W}(t) = [1 - \beta(t)]q(t) - \alpha W(t), \quad W(0) = W_0\,. \tag{3}$$

Waste accumulation generates local environmental degradation that harms the economy through a damage function $D(W)$, with $D'(W) > 0$ and $D''(W) \geq 0$. In order to simplify the analysis, we will consider a linear damage function, with a constant marginal damage denoted by c_W.

Another environmental externality comes from greenhouse gases (GHG) emissions due to both virgin and recycled resource uses. We assume that input flows v and r respectively contribute at rates δ_v and δ_r to these emissions.[7] Then, the cumulative GHG emissions $E(t)$

[5]We assume here that waste is a one-to-one co-product of the final consumption good. An alternative would be to consider that a constant share of the final good is destroyed by consumption, and then unrecoverable. In that case, consuming a quantity q of good would release a flow γq of waste, with $\gamma < 1$ being the waste content rate of the manufactured good. This would not change significantly the analysis but does not respect the physical law of material balance.

[6]Note that the share $(1-\beta)$ of the waste stream that is not redirected to the recycled stock at instant t cannot be recycled later in the program.

[7]We suppose for the moment that the contribution of the recycling activity represented by rate β to total emissions is included in rate δ_r given for the use of the recycled input. We will relax this assumption later.

at any time t evolves as:

$$\dot{E}(t) = \delta_v v(t) + \delta_r r(t), \quad E(0) = 0\,. \tag{4}$$

Contrary to waste accumulation which creates local damages, GHG emissions is a global externality which does not directly harm the industry. However, for this last externality to be binding in our model, we define a GHG stabilization cap beyond which damages are supposed to be too high to be supported on a global scale. Such cumulative emission targets are set by international environmental efforts in order to curb global warming: we suppose in our model that this global target has been divided between different countries, leading to national emissions objectives (or budgets) that can be dispatched between industries. We set this sectoral carbon budget to $\bar{E}$ so that, at any time t, the cumulative emissions cannot be superior:

$$E(t) \leq \bar{E}, \forall t\,. \tag{5}$$

Note that, as this model does not have any carbon-free backstop resource and the emissions stock cannot be reduced, we cannot consume any more inputs, virgin or recycled, once the cap is reached. With Inada conditions, it forces $q = 0$ as late as possible, simplifying hypothesis (5) to a final state condition. We set an exogenous time-limit T to the model, corresponding to long-term climate objectives and its associated emissions ceiling, involving the constraint:

$$E(T) \leq \bar{E}\,. \tag{6}$$

A general overview of the model is given by Figure 1.

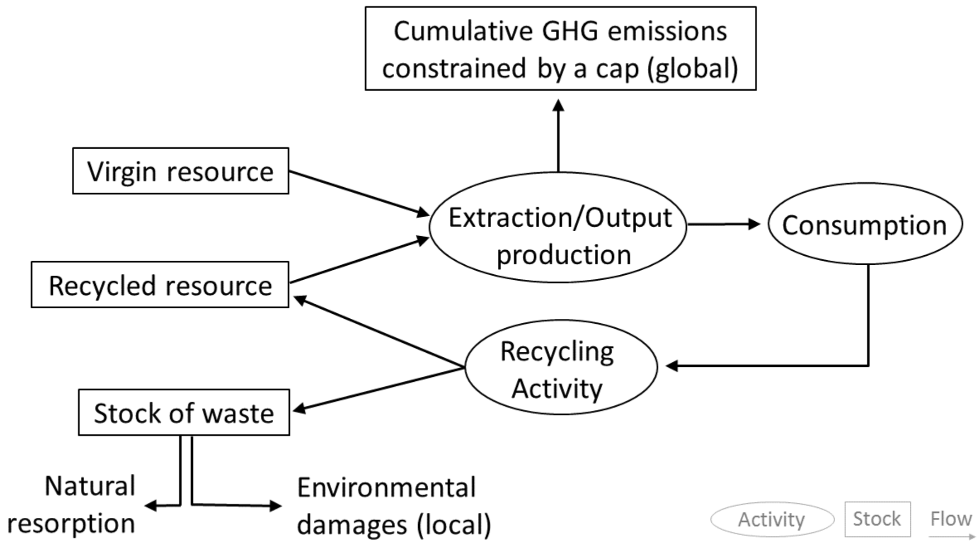

Figure 1: Global structure of the model, with stocks and flows

3. Social planner program

The objective of the central planner is to maximize the social welfare, evaluating trade-offs between consumption of goods, resources exhaustion and pollution accumulation. We consider a constant social discount rate ρ, $\rho > 0$. Formally, we want to solve the following problem:[8]

$$\max_{\{v,r,\beta\}} \int_0^T \left[u(v+r) - c_v v - c_r r - qf(\beta) - D(W)\right] \mathrm{e}^{-\rho t} dt \,, \tag{7}$$

subject to the dynamic constraints (1)-(4), to the carbon budget constraint (6), to the non-negativity constraints on v, r and to the inequality constraints $0 \leq \beta \leq 1$. This problem is solved by allocating the optimal amounts of inputs v and r and the optimal rate of waste recovery β at each instant. The Hamiltonian can be written as:

$$\begin{aligned}\mathcal{H} = u(v+r) - c_v v - c_r r - qf(\beta) - D(W) + \lambda_V(-v) + \lambda_R[\beta(v+r) - r] \\ - \lambda_W[(1-\beta)(v+r) - \alpha W] - \lambda_E(\delta_v v + \delta_r r) \,, \end{aligned} \tag{8}$$

[8]In order to simplify notation, we will hide the time subscript whenever convenient and it is clear from the context.

where λ_V, λ_R, $-\lambda_W$ and $-\lambda_E$ are the co-state variables attached to the virgin stock, the recycled stock, the waste stock and the emissions stock, respectively.[9]

The optimal solution must satisfy the following first-order conditions:

$$u' \leq c_v + f(\beta) + \lambda_V - \beta\lambda_R + (1-\beta)\lambda_W + \delta_v\lambda_E, \quad (= \text{ if } v > 0) \tag{9}$$

$$u' \leq c_r + f(\beta) + (1-\beta)\lambda_R + (1-\beta)\lambda_W + \delta_r\lambda_E, \quad (= \text{ if } r > 0) \tag{10}$$

$$qf'(\beta) \gtreqless (\lambda_R + \lambda_W)q, \quad (= \text{ if } \beta \in]0,1[, \geq \text{ if } \beta = 0, \leq \text{ if } \beta = 1) \tag{11}$$

$$\dot{\lambda_V} = \rho\lambda_V \Leftrightarrow \lambda_V = \lambda_{V0}e^{\rho t} \tag{12}$$

$$\dot{\lambda_R} = \rho\lambda_R \Leftrightarrow \lambda_R = \lambda_{R0}e^{\rho t} \tag{13}$$

$$\dot{\lambda_E} = \rho\lambda_E \Leftrightarrow \lambda_E = \lambda_{E0}e^{\rho t} \tag{14}$$

$$\dot{\lambda_W} = (\rho + \alpha)\lambda_W - D'(W)\,, \tag{15}$$

completed by the transversality conditions:

$$\lambda_V(T)V(T)e^{-\rho T} = \lambda_R(T)R(T)e^{-\rho T} = \lambda_W(T)W(T)e^{-\rho T} = 0 \tag{16}$$

$$\lambda_E(T)[\bar{E} - E(T)]e^{-\rho T} = 0\,. \tag{17}$$

Conditions (9) and (10) compare the gross marginal surplus of using each type of resource with its full marginal cost and state that it is optimal to use the resource when both are equal. The full marginal cost (FMC) of each input v or r is composed of: the delivery cost (c_v or c_r) of the input; the unitary cost $f(\beta)$ of recovering material from the waste stream; the scarcity rent (λ_V or λ_R) of the resource stock; the social marginal cost $-\beta\lambda_R$ of replenishing the recycled stock (a negative cost, this stock being a good for society); the social marginal cost $(1-\beta)\lambda_W$ of waste accumulation; the social marginal cost λ_E of cumulative emissions weighted by the carbon intensity of each input.

We can already determine the motivation of recycling as implemented in this resource model: relaxing the resource constraint by replenishing the recycled stock, reducing waste accumulation and potentially reducing GHG emissions if the virgin input is more polluting than the recycled one, i.e. if $\delta_v > \delta_r$. However, the choice of using recycled inputs in

[9] As waste and GHG emissions are public bad, their shadow values are non-positive. For the purpose of simplifying the notations, we introduced positive shadow costs by considering formally $-\lambda_E$ and $-\lambda_W$ as co-state variables.

production here only relaxes pressure on virgin stock and sometimes GHG emissions. Damage due to waste accumulation is not impacted, as both inputs have the same impact on waste after consumption. This objective of reduction is devoted to the choice of the recycling rate β.

The optimal path for the recycling behavior is obtained from (11) by comparing the marginal cost $f'(\beta)$ per unit of final good with the marginal social gain $(\lambda_R + \lambda_W)$ of recycling. Indeed, for a given material stream q, increasing by $\Delta\beta$ the share of recycled material allows to increase by quantity $\Delta\beta q$ the stock of recycled resource, whose marginal shadow value is λ_R, and to reduce waste generation by this same unit, thus saving the marginal shadow cost λ_W of the waste stock.

Equations (12) to (15) rule the dynamics of the system. (12) and (13) illustrate in particular the Hotelling rule, the scarcity rents of the two resources growing at a rate equal to the social discount rate ρ. As shown in (14), the social cost of cumulative GHG emissions also follows the Hotelling rule, due to the depletion process of the remaining carbon budget $\bar{E} - E(t)$. We define as λ_{V0}, λ_{R0} and λ_{E0} the initial values of these social costs.

Equation (15), combined with the transversality condition (16), gives us the trajectory of the shadow cost of waste accumulation:[10]

$$\lambda_W(t) = \int_t^T D'(W) e^{-(\rho+\alpha)(s-t)} \mathrm{d}s \,. \tag{18}$$

We see here that the shadow cost of waste accumulation is equal to the intertemporal sum of the flows of marginal damages, discounted at rate $(\rho + \alpha)$ as waste is not only a flow but a stock that shrinks at rate α. With a constant marginal damage c_W, this expression can be simplified as follows:

$$\lambda_W(t) = \frac{c_W}{\rho + \alpha} \left[1 - \mathrm{e}^{-(\rho+\alpha)(T-t)}\right] . \tag{19}$$

[10] Solving the differential equations (3) and (15) yields respectively to: $W = e^{-\alpha t}[W_0 + \int_0^t (1-\beta) q e^{\alpha s} ds]$ and $\lambda_W = e^{(\rho+\alpha)t}[\lambda_{W0} - \int_0^t D'(W) e^{-(\rho+\alpha)s} ds]$. As $W_0 > 0$, the transversality condition (16) implies that $\lambda_{W0} = \int_0^T D'(W) e^{-(\rho+\alpha)t} dt$. The initial value of the waste social cost is independently given by conditions on waste, only taking into account the dynamic characteristics α of the stock and its cost $D(W)$ for society.

This shows that the cost is decreasing through time, as $\dot{\lambda_W}(t) = -c_W \mathrm{e}^{-(\rho+\alpha)(T-t)} < 0$. In fact, the cost λ_W accounts for the future damage due to waste, and the end of the program approaching, the constraint has less impact on the program.[11]

From first-order conditions (9) and (10), we can infer that the optimum is one or more consecutive phases of production from one input or the other, with in parallel a specific path for rate β ruled by equation (11). Perfect substitution and fixed private costs here do not allow simultaneous use of each input. Following the display of the model, we will be studying the switch from an input to the other, needed initial conditions and the optimal choice regarding the redirection of the waste flow to the recycled stock.

3.1. Relative scarcity of resource stocks and exhaustion of the carbon budget

From (1), (2) and (4), we obtain the expressions of the virgin, recycled and carbon stocks trajectories:

$$\begin{aligned} V(t) &= V_0 - V_c(t) && (20) \\ R(t) &= \int_0^t \beta(s) q(s) \mathrm{d}s - R_c(t) && (21) \\ E(t) &= \int_0^t [\delta_v v(s) + \delta_r r(s)] \mathrm{d}s = \delta_v V_c(t) + \delta_r R_c(t)\,, && (22) \end{aligned}$$

where $V_c(t) \equiv \int_0^t v(s)\mathrm{d}s$ and $R_c(t) \equiv \int_0^t r(s)\mathrm{d}s$ are the cumulative resource extraction at time t, from each resource stock. Besides, transversality conditions (16) and (17) give us a final state condition for the model. At final time T, stock V (resp. R and $\bar{E} - E$) is either a scarce resource, meaning that it is exhausted, $V(T) = 0$ (resp. $R(T) = 0$ and $E(T) = \bar{E}$), or it is an abundant resource and its shadow value is always nil, $\lambda_{V0} = 0$ (resp. $\lambda_{R0} = 0$ and $\lambda_{E0} = 0$). Combining these final conditions involves various scenarios. To narrow the study to the most plausible one, we make the following assumption:

Assumption 1. *The terminal states of the resource stocks are the following:*

- *The carbon budget is saturated $E(T) = \bar{E}$;*
- *The virgin resource is abundant $V(T) > 0$.*

[11] We discuss later in the paper the case where the damage caused by the left-over waste stock $W(T)$ still harms the economy after the terminal date T.

First, for the problem to be meaningful, we place our model in a situation where the carbon budget is set to answer to a pressing climate constraint, where there should be a significant change of behaviours in the economy. In other words, we assume that the carbon budget $\bar{E}$ is small enough, as compared with the initial endowment in carbon-emitting resources, so that it will be exhausted no later than time T. As there is no carbon-free option available in this economy, it is then optimal to postpone as much as possible the exhaustion of the budget, $E(T) = \bar{E}$, which implies a strictly positive shadow cost of emissions: $\forall t$, $\lambda_E(t) = \lambda_{E0}e^{\rho t} > 0$.

However exhausting the virgin resource would mean $V_c(T) = V_0$ hence $\delta_v V_0 \leq \bar{E}$ from equations (20) and (22). This situation is similar to what we would call a "business as usual" scenario, where imposing a carbon budget has no impact for resource exhaustion on a finite period. We will consider in the rest of the paper that the virgin resource is relatively abundant, meaning that $V(T) > 0$ and, from (16), that $\lambda_V(t) = 0$ $\forall t$.

Finally, as from (9)-(11), v, r and β are function of $(\lambda_{R0}, \lambda_{E0})$, these two variables must be endogenously determined from the two following terminal conditions:

$$\lambda_R(t) = 0 \text{ or } R(T) = 0 \quad \Leftrightarrow \quad \int_0^T [r - \beta(v + r)]dt = 0 \tag{23}$$

$$E(T) = \bar{E} \quad \Leftrightarrow \quad \int_0^T (\delta_v v + \delta_r r)dt = \bar{E}\,. \tag{24}$$

3.2. Arbitration on resources use

In order to get the optimal consumption path, we have to compare the full marginal costs of using each specific input (FMC_i, with $i \in \{v; r\}$), given by the right-hand side of (9) and (10). We also define ΔFMC as the difference between both FMCs:

$$\begin{aligned}
FMC_v &= c_v - \beta\lambda_R + \delta_v\lambda_E + (1-\beta)\lambda_W + f(\beta) && (25)\\
FMC_r &= c_r + (1-\beta)\lambda_R + \delta_r\lambda_E + (1-\beta)\lambda_W + f(\beta) && (26)\\
\Delta FMC &\equiv FMC_v - FMC_r = -(c_r - c_v) + [(\delta_v - \delta_r)\lambda_{E0} - \lambda_{R0}]\mathrm{e}^{\rho t}\,. && (27)
\end{aligned}$$

Examining the sign of this last expression gives us arbitration at stakes when applying Herfindhal least-cost principle analysis [17]. First we note that neither recycling nor the waste stock come into play. In fact, products made out of recycled or virgin materials have the same impact on waste accumulation and the activity of waste recovery does not depend on the origin of the input flow. Equation (27) shows two different effects when comparing costs.

The static term $-(c_r - c_v)$ is the impact of the private marginal cost of producing with a specific input, with the hypothesis $c_r > c_v$. Depending on endogenous initial values of shadow costs, this effect can be compensated by the dynamic term, evolving exponentially at rate ρ. It is a combined impact of externalities linked to the depletion of stock R (remind that stock V is assumed to be abundant), and the emissions accumulation E. The arbitration between those externalities and private costs can lead to a switch between inputs, mathematically materialized by the change in sign of ΔFMC before the end of the program. Formally, if $\Delta FMC < 0$ then we produce from the virgin input, if $\Delta FMC > 0$ then we produce from the recycled input.

From this analysis, Proposition 1 below describes the main characteristics of the optimal regime of input use.

Proposition 1. *Regardless of the recycling rate $\beta(t)$, the optimal path is such that:*

1. *It must start with a phase of production from extraction of virgin resource in all scenarios.*
2. *There can be at most one switch of input during the program. Let us call this time of switch $\tilde{T}$, whose existence within $[0; T]$ is not guaranteed:*

$$\exists! \tilde{T} > 0 \text{ such that } FMC(t) < 0 \text{ for any } t < \tilde{T} \text{ and } FMC(\tilde{T}) = 0.$$

Proof. (1.) For any $t \in [0, \tilde{T}[$, if $v(t) = 0$ then, from (21), $R(\tilde{T}) = -\int_0^{\tilde{T}} (1-\beta) r ds \leq 0$ which is not possible. (2.) Differentiating (27) with respect to time yields $\Delta \dot{FMC} = \rho[(\delta_v - \delta_r)\lambda_{E0} - \lambda_{R0}]e^{\rho t}$. As this expression is constant in sign, ΔFMC is monotonic and there can be at most one switch of inputs during the optimal program of the model. □

The first result highlights the need for initial conventional production: recycling can only be achieved when there has been enough extracted input consumed and then collected from the waste stream. As, by assumption, the recycled stock is empty at the beginning, there must always be a first phase of virgin production to replenish the recycled stock before being able to use it.

The second result implies that, if it exists, the time of the switch $\tilde{T}$ is the solution of the following equation $(\delta_v - \delta_r)\lambda_E(\tilde{T}) = \lambda_R(\tilde{T}) + (c_r - c_v)$. A single instant solution to $\Delta FMC = 0$ also means that a simultaneous use of virgin and recycled resource cannot occur, due to perfect substitution and constant marginal costs. Moreover, as trajectories with initial phase of recycled production are not possible and should be ruled out, λ_{R0} and λ_{E0} must satisfy the following condition:

$$(\delta_v - \delta_r)\lambda_{E0} - \lambda_{R0} < c_r - c_v \, . \tag{28}$$

Proposition 2. *Regardless of the recycling rate path $\beta(t)$, we can summarize the different optimal scenarios to the following ones:*

1. *When recycling is more emitting than extracting ($\delta_v \leq \delta_r$), we only use the virgin resource and FMCs diverge;*
2. *When extracting is more emitting than recycling ($\delta_r < \delta_v$), FMCs converge at first and:*
   (a) *We only use the virgin resource if the carbon ceiling is not constraining enough or the private costs difference is too high, meaning that $(\delta_v - \delta_r)\lambda_{E0}\mathrm{e}^{\rho T} \leq c_r - c_v$;*
   (b) *We use the virgin resource and then the recycled one, meaning that $c_r - c_v < [(\delta_v - \delta_r)\lambda_{E0} - \lambda_{R0}]\mathrm{e}^{\rho T}$. In this case the instant of switch $\tilde{T}$ is defined by:*

$$\mathrm{e}^{\rho\tilde{T}} = \frac{c_r - c_v}{(\delta_v - \delta_r)\lambda_{E0} - \lambda_{R0}} \, . \tag{29}$$

Proof. The different optimal paths can be drawn from examining the positivity conditions for ΔFMC. (1.) Given that $\delta_v \leq \delta_r$, the difference between the marginal costs expressed in (27) is always negative and decreasing (meaning that FMCs diverge): the recycled resource can never be used. Consequently $\lambda_R = 0$ during the program. Conversely, diverging FMCs and initial condition $\Delta FMC(0) < 0$ means that the recycled resource can never be used. It leads to $(\delta_v - \delta_r)\lambda_{E0} < 0$: the virgin resource is less polluting than the recycled one.

(2.) When the ΔFMC increases (meaning that FMCs converge at first, as $\Delta FMC(0) < 0$), different paths can occur depending on the speed of convergence. In the first case, the speed of convergence is not high enough and ΔFMC remains under zero during the whole program: it is never optimal to use the recycled resource, then $\lambda_R = 0$, and we know that $\Delta FMC(T) < 0$, thus proving (2.a). Finally, there can be the case where constraint on emissions is strong enough and leads to a switch toward the recycled input. In this case, a first phase with $\Delta FMC(t) < 0$ consists of the extraction of the resource, until an instant $\tilde{T}$ such that $\Delta FMC(\tilde{T}) = 0$, thus proving equation (29). This moment happens before the end of the program: $\mathrm{e}^{\rho\tilde{T}} < \mathrm{e}^{\rho T}$, thus proving (2.b). $\square$

In scenario 1, using the recycled resource does not show any interest as it is the worst choice both in terms of carbon emissions and of private costs. Scenario 2.a represents the case where, while using the virgin resource is more emitting, the constraint induced by the carbon budget is not strong enough to induce a switch in production towards the recycled resource. Last, scenario 2.b is the most intuitive when investigating the optimal use of resources, as the environmental effect is causing the switch, while having a binding constraint on the stock of recycled resource. It is especially the case for highly emitting industries despite high private costs for recycling, such as metals and plastics. As private costs spur extraction, a low-enough carbon budget can be useful to force the optimal path toward recycled inputs.

3.3. Recovering waste

The optimal path of the recycling rate β is determined from equation (11). In order to simplify the analysis, we define the function $\Phi_\beta \equiv \lambda_R + \lambda_W$ as the marginal social gain of the recycling effort. Recycling intensity results from the arbitration between its marginal cost and this marginal gain (both in avoiding accumulation of waste and replenishing the stock of recycled resource). This involves that there can be recycling $\beta > 0$ while the virgin resource is not used: recycling only alleviates the pressure on waste accumulation in this case.

Formally, the recycling path can be composed of three types phases depending on the value of Φ_β: a no-recycling phase ($\beta = 0$) when $\Phi_\beta \leq f'(0)$; a full-recycling phase ($\beta = 1$) when $\Phi_\beta \geq f'(1)$; and a phase of partial recycling $\beta \in]0; 1[$ when $\Phi_\beta = f'(\beta)$.

Studying the optimal behavior regarding redirecting the waste flow to the recycled stock means evaluating function $\Phi_\beta(t)$ inside the rectangle of exogenous boundaries $[0; T] \times [f'(0); f'(1)]$. Values $f'(0)$ and $f'(1)$ appear to be saturation bounds, beyond which recycling is not profitable ($\Phi_\beta < f'(0)$) or recycling cannot be improved ($\Phi_\beta > f'(1)$).

Assumption 2. *Full recycling is not attainable:* $\beta \in [0; 1)$.

In our model, we assume that full recycling is not possible, as there should always be a part of the waste that cannot be recovered, at any cost. It can be due to manufacturing process where the original resource is fatally transformed, or very dispersive use of the resource when it is technologically impossible to retrieve all of it after consumption. This assumption can be easily obtained with a divergent cost function f such as $\lim_{\beta \to 1} f'(\beta) = +\infty$.[12]

The optimal recycling program can be described by three instants with characteristics detailed in the following propositions. The recycling rate $\beta(t)$ reaches a maximum at time $\hat{T}_\beta$.

[12] This hypothesis reflects the dispersive use of many materials as well as complex applications at mass production scales. In fact, thermodynamic limitations to recycling can involve important economic costs, or even impracticability.[31]

This might be completed by an initial period without recycling until $\underline{T}_\beta$ and a final period without recycling starting at $\overline{T}_\beta$.

Proposition 3. *When the recycling rate $\beta(t)$ is not always zero, it reaches a maximum at time $\hat{T}_\beta$. This instant can take the following values:*

$$\hat{T}_\beta = \begin{cases} 0 & , \textit{if } \mathrm{e}^{(\rho+\alpha)T} \leq \frac{c_W}{\rho\lambda_{R0}} \textit{ or } \lambda_{R0} = 0 \\ T & , \textit{if } \frac{c_W}{\rho\lambda_{R0}} \leq \mathrm{e}^{\rho T} \\ \frac{1}{\alpha}\left[\ln\left(\frac{\rho\lambda_{R0}}{c_W}\right) + (\rho+\alpha)T\right] \in (0,T) & , \textit{if } \mathrm{e}^{\rho T} < \frac{c_W}{\rho\lambda_{R0}} < \mathrm{e}^{(\rho+\alpha)T}. \end{cases} \tag{30}$$

Proof. The cost function associated to the recycling rate $f(\beta)$ is increasing and convex. From (11), we can then study the dynamics of β by studying the evolution of Φ_β:[13]

$$\Phi_\beta(t) = \lambda_{R0}\mathrm{e}^{\rho t} + \frac{c_W}{\rho+\alpha}\left(1 - \mathrm{e}^{-(\rho+\alpha)(T-t)}\right).$$

In the case of an abundant recycled resource ($\lambda_{R0} = 0$), $\Phi'_\beta(t) = \dot{\lambda}_W = -c_W\mathrm{e}^{-(\rho+\alpha)(T-t)} \leq 0$ and then, β is decreasing and reaches its maximum, if not always zero, at the beginning of the program. In the more general case, the function $\Phi_\beta(t)$ defined on $\mathbb{R}^+$ is proved to be increasing from $\Phi_\beta(0) = \lambda_{R0} + \frac{c_W}{(\rho+\alpha)}[1-e^{-(\rho+\alpha)T}]$ to its maximum $\Phi_\beta(\hat{T}_\beta)$, and then declining, with $\lim_{t\to\infty}\Phi_\beta(t) = -\infty$. The existence of $\hat{T}_\beta$ in the boundaries of the program $[0;T]$, as illustrated by Figure 2, directly follows the study of the equation $\Phi'_\beta(t) = 0$ and gives the results of Proposition 3. □

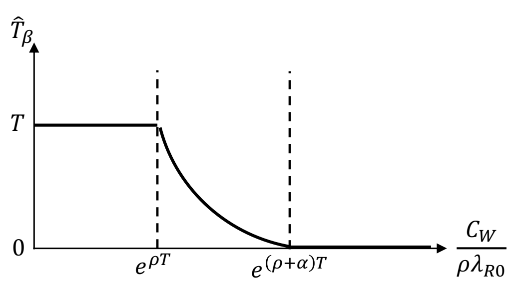

Figure 2: Maximum time $\hat{T}_\beta$

Proposition 4. *There can be zero, one or two phases where the recycling rate β saturates and is zero. These phases can either be:*

1. *No saturation phase (we always recycle);*

[13] For the purpose of this analysis, we extend the definition of $\Phi_\beta(t)$ to $\mathbb{R}^+$, and then discuss the existence of solutions in $[0,T]$.

2. One phase that is the full program $[0;T]$ (we never recycle);
3. One phase $[0;\underline{T}_\beta]$ or $[\overline{T}_\beta;T]$ where the values of times $\underline{T}_\beta$ or $\overline{T}_\beta$ are the solution in $[0;T]$ of equation $\Phi_\beta(t) = f'(0)$;
4. Two phases $[0;\underline{T}_\beta]$ and $[\overline{T}_\beta;T]$ where the values of times $\underline{T}_\beta$ and $\overline{T}_\beta$ are respectively the lower solution and the higher or only solution in $[0;T]$ of equation $\Phi_\beta(t) = f'(0)$.

Proof. Following the study of function Φ_β illustrated by Figure 3, we see that the equation $\Phi_\beta(t) = f'(0)$ can have zero, one or two solutions in $\mathbb{R}^+$: zero solution when $\forall t \in \mathbb{R}^+, \Phi_\beta(t) < f'(0)$, one solution when $f'(0) \leq \frac{c_W}{\rho+\alpha}$ and two solutions otherwise.[14] We can determine $\underline{T}_\beta$ and $\overline{T}_\beta$ as the results of this equation, when they exist in $[0;T]$, giving the result of Proposition 4. $\square$

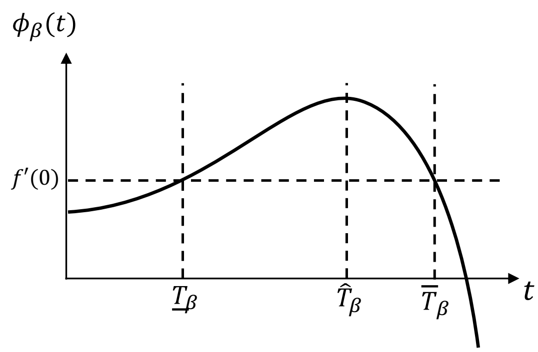

Figure 3: Saturation of β

The previous study of the dynamics of recovering waste shows that the permanent arbitration with the marginal social benefit of recycling can induce phases with $\beta = 0$. When this phase comes first, it means that the resource constraint on stock R is not high enough to start recycling. The social planner only initiates it after $\underline{T}_\beta$. On the other hand, there can also be a final phase without recycling as the shrinking social cost of waste accumulation makes it no more optimal. This is visible from the evolution of maximum time exposed in Figure (30). We see that instant $\tilde{T}_\beta$ is given by the comparison between the sum of discounted marginal waste damage $\frac{c_W}{\rho} = \int_0^\infty c_W e^{-\rho t} dt$, and the initial scarcity of the recycled resource λ_{R0}. For high damage (or a low constraint on the recycled resource), the maximum of recycling occurs at the beginning ($\tilde{T}_\beta \to 0$).

[14] We intentionally omit the specific case when there is a tangential solution for the equation, such as $\max_{\mathbb{R}^+} \Phi_\beta = f'(0)$.

3.4. Dynamics of resource flows

While no closed-form expression for the consumption trajectories of resources can be determined analytically in this program, first-order conditions can give us some insights on the different phases of production, as seen previously, and the fluctuations of the material flows. For that we consider a phase during which $v > 0$ or $r > 0$. We look at the dynamics of $v(t)$ and $r(t)$ by examining the time derivatives of conditions (9) and (10). Using (11), we obtain for any value of $\beta(t)$:

$$\begin{aligned} F\dot{M}C_v &= \left[\rho\delta_v\lambda_{E0} - \rho\beta\lambda_{R0} - (1-\beta)c_W \mathrm{e}^{-(\rho+\alpha)T}\mathrm{e}^{\alpha t}\right]\mathrm{e}^{\rho t}\,, & (31)\\ F\dot{M}C_r &= \left[\rho\delta_r\lambda_{E0} + \rho(1-\beta)\lambda_{R0} - (1-\beta)c_W \mathrm{e}^{-(\rho+\alpha)T}\mathrm{e}^{\alpha t}\right]\mathrm{e}^{\rho t}\,. & (32)\end{aligned}$$

For this we define functions Φ_v and Φ_r as follow:

$$\begin{aligned} \Phi_v(t) &\equiv [1-\beta(t)]\mathrm{e}^{\alpha t} + \beta(t)\mathrm{e}^{\alpha \hat{T}_\beta}\,, & (33)\\ \Phi_r(t) &\equiv [1-\beta(t)]\mathrm{e}^{\alpha t} - [1-\beta(t)]\mathrm{e}^{\alpha \hat{T}_\beta}\,, & (34)\end{aligned}$$

where, from (30), $\hat{T}_\beta$ is such that $e^{\alpha\hat{T}_\beta} = \frac{\rho\lambda_{R0}e^{(\rho+\alpha)T}}{c_W}$.

Proposition 5. *The full marginal cost FMC_v (resp. FMC_r) of producing with the virgin resource (resp. recycled) can reach an extremum at time $\hat{T}_v$ (resp. $\hat{T}_r$). Moreover, in the specific case of a non-monotonous recycling rate, $0 < \hat{T}_\beta < T$:*

1. *An extremum exists if $\Phi_v(0) < \frac{\rho\delta_v\lambda_{E0}}{c_W}\mathrm{e}^{(\rho+\alpha)T} < \Phi_v(T)$ (resp. $\Phi_r(0) < \frac{\rho\delta_r\lambda_{E0}}{c_W}\mathrm{e}^{(\rho+\alpha)T} < \Phi_r(T)$). These instants correspond to maxima of the FMCs and are determined by:*

$$\Phi_v(\hat{T}_v) = \frac{\rho\delta_v\lambda_{E0}}{c_W}\mathrm{e}^{(\rho+\alpha)T} \text{ and } \Phi_r(\hat{T}_r) = \frac{\rho\delta_r\lambda_{E0}}{c_W}\mathrm{e}^{(\rho+\alpha)T}.$$

2. *If true and if the $\hat{T}$'s exist, we have the following inequalities:*

$$\begin{gathered}(\delta_v - \delta_r)\lambda_{E0} > \lambda_{R0}\,,\\ \hat{T}_\beta < \hat{T}_r < \hat{T}_v\,.\end{gathered}$$

Proof. (1.) The full marginal cost FMC_v will reach an extremum when $F\dot{M}C_v = 0$. Using (31) and (33), this condition becomes: $F\dot{M}C_v = \left[\frac{\rho\delta_v\lambda_{E0}}{c_W}\mathrm{e}^{(\rho+\alpha)T} - \Phi_v\right]c_W\mathrm{e}^{\rho t}\mathrm{e}^{-(\rho+\alpha)T} = 0$. In addition, Φ_v is strictly increasing: $\Phi_v'(t) = \left(e^{\alpha\hat{T}_\beta} - e^{\alpha t}\right)\dot{\beta} + (1-\beta)e^{\alpha t} > 0$ for any t, as $\hat{T}_\beta$ is the maximum instant of β. Then, if $F\dot{M}C_v = 0$, the extremum of FMC_v is a maximum. The same reasoning is applied to FMC_r, with $F\dot{M}C_r = \left[\frac{\rho\delta_r\lambda_{E0}}{c_W}\mathrm{e}^{(\rho+\alpha)T} - \Phi_r\right]c_W\mathrm{e}^{\rho t}\mathrm{e}^{-(\rho+\alpha)T}$.

(2.) First, from the expression of Φ_r, condition $\Phi_r(\hat{T}_r) = \frac{\rho\delta_r\lambda_{E0}}{c_W}\mathrm{e}^{(\rho+\alpha)T}$ implies that $\hat{T}_\beta < \hat{T}_r$. Next, from (27), we can write $\Delta \dot{FMC} = \rho[\Delta FMC + (c_r - c_v)]$. As $\tilde{T}$ exists, we know that there is a second phase during which the recycled input is used, meaning $\Delta FMC > 0$ for $t > \tilde{T}$, which involves $\Delta \dot{FMC} > 0$. This result implies $[(\delta_v - \delta_r)\lambda_{E0} - \lambda_{R0}] > 0$. As it is independent of time, we can generalize with $\Delta \dot{FMC} > 0$, $\forall t$. Hence $\dot{FMC}_v > \dot{FMC}_r$. By rewriting expressions of $\dot{FMC}_i$ with Φ_i $(i \in \{v; r\})$, which are proved to be increasing functions of time, we get that $FMC_i > 0$ before $\hat{T}_i$ and $FMC_i < 0$ after. This gives us that $\hat{T}_r < \hat{T}_v$. □

Listing all the possible combinations of FMCs time paces, given their characteristics described in Propositions 1, 2 and 5, allows us to determine the following typical trajectories of the gross marginal surplus $u'(q)$:

- strictly increasing if, $\forall \hat{T}_v$, $T \leq \hat{T}_r$;
- increasing and then decreasing, with a maximum attained after the time $\tilde{T}$ of the switch, if, $\forall \hat{T}_v$, $\tilde{T} < \hat{T}_r < T$;
- increasing and then decreasing, with a maximum attained at the time $\tilde{T}$ of the switch, if, $\forall \{\hat{T}_v, \hat{T}_r\}$, $\hat{T}_r \leq \tilde{T} \leq \hat{T}_v$;
- increasing and then decreasing, with a maximum attained before the time $\tilde{T}$ of the switch, if, $\forall \hat{T}_r$, $0 < \hat{T}_v < \tilde{T}$;
- strictly decreasing if, $\forall \hat{T}_r$, $\hat{T}_v \leq 0$.

By concavity of u, resource extraction, either from the virgin stock or from the recycled stock, decreases (resp. increases) over time when this marginal surplus increases (resp. decreases). As usual, when the Hotelling effect prevails, the extraction path must be declining. But as previously explained, this scarcity effect (on the resource stocks or on the carbon budget) can be counterbalanced by a recycling effect aiming at increasing the resource use in order to provide raw materials to be recycled.

This analysis tells us that the total flow of material can increase at the end of the program (after time $\hat{T}_i$) in some cases. This constitutes a catch-up phase: the lower pressure on waste (as the end of the program approaches) and a higher appreciation of the replenishment of the recycled stock counterbalance GHG externalities. The constraint $\hat{T}_\beta < \hat{T}_r < \hat{T}_v$ follows when $\hat{T}_\beta$ comes before the end of the program. A catch-up phase with an increasing production comes after a phase when the interest in recovering waste is lowered $\dot{\beta} < 0$, meaning that the social cost of waste accumulation declines more than the social cost of the resource.

4. Discussion and extensions

4.1. Market economy and policy implications

In order to analyze environmental policy tools to promote recycling and the reduction of carbon emissions, there is a need to discuss the decentralized version of our model. Transfer functions for the different actors in the economy are introduced to correct for the externalities previously developed in this paper.

We assume that the economy is composed of four agents: the final consumer, the producer of the manufactured good, the virgin resource sector and the recycled resource sector, which manages both the recycling activity and the exploitation of the recycled stock.[15] Property rights of each resource stocks are correctly defined so that each extracting sector is owner of its reserves. These agents can take actions on four markets, assumed to be perfectly competitive: the virgin resource market (price p_v), the recycled resource (price p_r), the manufactured good (price p_q) and the waste market, provided that such a market exists (price p_w). Initially, we omit the waste market and we introduce it in a second time. [16]

The policy-maker can influence private decisions on each market thanks to a set of monetary transfer functions, denoted by $T_C(\cdot)$, $T_P(\cdot)$, $T_V(\cdot)$ and $T_R(\cdot)$, to the consumer, the producer, the virgin resource sector and the recycling sector, respectively.

4.1.1. Equilibrium in the absence of a waste market

The consumer determines the quantity q she will consume in order to maximize her instantaneous surplus function $S_C \equiv [u(q) - p_q q + T_C(q)]$.[17] The producer of the manufactured good chooses the quantity of inputs v and r in order to maximize instantaneous profits $S_P \equiv [p_q q - p_v v - p_r r + T_P(v, r)]$ subject to the technological constraint $q = v + r$. The virgin

[15]We can obtain similar results with a less developed model, where the production sector is omitted. In this case, the final consumer directly consumes the two types of resources, which are perfect substitutes for her.

[16]Note that by considering this market economy, we only apply a partial decentralization process, in the sense that we do not develop explicitly the financial market. We assume that private agents discount their monetary flows at the rate ρ.

[17]There is no dynamic budget constraint, as we do not explicit the financial market.

resource owner chooses the extraction rate v maximizing profits during an exogenous finite time T, expressed by $S_V \equiv \int_0^T [p_v v - c_v v + T_V(v)]e^{-\rho t}dt$ subject to (1). Last, the problem of the recycling sector is to determine both the share β of final consumption good to be recycled, and the recycled resource extraction r that maximize $S_R \equiv \int_0^T [p_r r - c_r r - F(q,\beta) + T_R(r,\beta)]e^{-\rho t}dt$ subject to (2). Note that we directly focus here on interior values of β (the conditions for corner solution has been discussed previously in the centralized economy).

Proposition 6. *Given λ_i^*, $i \in \{E; W\}$ the shadow costs for an optimal trajectory, the set of policy instruments that restores optimality in the absence of a market for waste must satisfy the following conditions:* [18]

$$\begin{aligned} \frac{1}{q}\frac{\partial T_R(r,\beta)}{\partial \beta} &= \lambda_W^* && (35)\\ T_V'(v) + \frac{\partial T_P(v,r)}{\partial v} + T_C'(q) &= \pi(\beta) - \delta_v \lambda_E^* - \lambda_W^* && (36)\\ \frac{\partial T_R(r,\beta)}{\partial r} + \frac{\partial T_P(v,r)}{\partial r} + T_C'(q) &= \pi(\beta) - \delta_r \lambda_E^* - \lambda_W^*\,, && (37) \end{aligned}$$

with $\pi(\beta) \equiv \beta f'(\beta) - f(\beta) \geq 0$ for any β.

Proof. Maximizing surplus functions of the consumer, the producer, the resource owner and the recycling sector, we get the following first-order conditions:

$$\begin{aligned} u'(q) &= p_q - T_C'(q) && (38)\\ p_q &= p_v - \frac{\partial T_P}{\partial v} = p_r - \frac{\partial T_P}{\partial r} && (39)\\ p_v &= c_v + \tilde{\lambda}_V - T_V'(v), \quad \text{with } \tilde{\lambda}_V = \tilde{\lambda}_{V0}e^{\rho t} && (40)\\ p_r &= c_r + \tilde{\lambda}_R - \frac{\partial T_R}{\partial r}, \quad \text{with } \tilde{\lambda}_R = \tilde{\lambda}_{R0}e^{\rho t} && (41)\\ f'(\beta) &= \tilde{\lambda}_R + \frac{1}{q}\frac{\partial T_R}{\partial \beta}\,. && (42) \end{aligned}$$

Given Eq. (38)-(41), the clearing-market conditions that characterize an equilibrium are:

$$\begin{aligned} u'(q) &= c_v + \tilde{\lambda}_V - T_V'(v) - \frac{\partial T_P}{\partial v} - T_C'(q) && (43)\\ u'(q) &= c_r + \tilde{\lambda}_R - \frac{\partial T_R}{\partial r} - \frac{\partial T_P}{\partial r} - T_C'(q), && (44) \end{aligned}$$

combined with Condition (42). Remind that (9)-(11) give the first-order conditions for optimal interior solutions in the central planner model. Comparing these conditions with (42)-(44), we get equations (35) to (37). □

[18] Here, as we follow a partial equilibrium approach, we do not consider the budget balance equation of the policy-maker. In particular, there is no reason at all to suppose that the net sum of all these monetary transfers must be equal to zero.

Simple economic policies can be illustrated when considering linear and additive separable transfer functions, i.e. tax-subsidy schemes: $T_C(q) = T_C \times q$, $T_P(v, r) = T_{Pv} \times v + T_{Pr} \times r$, $T_V(v) = T_V \times v$ and $T_R(r, \beta) = T_{Rr} \times r + T_{R\beta} \times \beta q$. In that case, first-best implementation requires four policy instruments. The first one is a unit subsidy $\pi(\beta)$ (always non-negative given the properties of function $f(.)$) on the flow of produced/consumed goods. For the case of an interior solution ($\beta > 0$), from (11), this subsidy can be rewritten as follows: $\pi(\beta) = [\beta q \Phi_\beta - F(q, \beta)]/q$. This expression reads as the average net social gain of recycling the share β of the flow q of final good (remind that Φ_β is the marginal social benefit of the recycling effort and $F(q, \beta) = qf(\beta)$ is the recycling cost function). The policy maker must implement such a subsidy in order to correct for the positive externality generated by the waste production for free in the absence of waste market. A second unit subsidy λ_W^* on the flow of recycled good βq is required due to the avoided waste accumulation and it is always attributed to the recycling industry.

Finally, the policy maker enforces two sets of taxes corresponding to the two environmental negative externalities, GHG emissions and waste accumulation. From (36) and (37), we can write: $T_{Rr} + T_{Pr} - T_V - T_{Pv} = (\delta_v - \delta_r)\lambda_E^*$. Then, a uniform carbon tax λ_E^*, weighted by the carbon content rates of each primary material, must be imposed either to the producer of the final good or to the extraction sectors. The second unit tax λ_W^* focuses on the potential waste generation from production/consumption of the final good. Last, we can notice that the two monetary transfers associated with waste accumulation, the tax and the subsidy presented earlier, represent together at the aggregated level a single tax on the effective waste accumulation $-(1 - \beta)\lambda_W^*$.

Table (2) illustrates various possibilities depending on who pays the taxes λ_E^* and λ_W^*, and who receives the subsidy π.[19] Note that all these options are revenue-equivalent for the policy-maker.

These examples show how the carbon tax should be distributed between the producer (T_P) and resource managers (T_V and T_R). The implementation of this tax is the subject of a wide range of literature, and is not the subject of this paper. While existing policies

[19] As usual, the transfer of the tax burden may depend on the price elasticity of demand and supply functions. It is not the purpose here to develop this aspect.

Transfers to:	**Example 1**	**Example 2**	**Example 3**
Final consumer (T_C)	$\pi(\beta)q$	$(\pi(\beta) - \lambda_W^*)q$	$-\lambda_W^* q$
Producer (T_P)	$-\lambda_W^*(v + r)$	$-\lambda_E^*(\delta_v v + \delta_r r)$	$-\lambda_E^*(\delta_v v + \delta_r r)$
Virgin resource (T_V)	$-\lambda_E^* \delta_v v$	0	$\pi(\beta)v$
Recycling sector (T_R)	$\lambda_W^* \beta q - \lambda_E^* \delta_r r$	$\lambda_W^* \beta q$	$\pi(\beta)r + \lambda_W^* \beta q$

Table 2: Examples of policy-mix

are often developed to favor the recycling branch, our model advocates for a carbon tax on this industry, at the level of the GHG content of the sector. Help for this industry comes separately through specific tax-subsidy schemes. It is interesting to note that whatever the transfer structure, there is a subsidy on the recovered flow of material for the recycling sector. The combination of this subsidy with a taxation on products is often promoted in economic literature, as a deposit-refund scheme.[16][8] Besides, this implementation depends on the specific characteristics of the industrial sector like emissions rates, private and social costs of waste management.[1] This specific system is still hardly implemented, however there are already several subsidy schemes for recycling industries as well as taxation on waste. In our examples, taxation relies on the global flow of material, subject to a potential waste damage. It is the case of Extended Producer Responsibility (EPR) systems, where producers have to assume financial responsibility for the potential environmental damage of their products during their whole life-cycle (through an eco-contribution). In example 2 and 3, this cost is assumed by the final consumer. However, we can note that a tax on the producer will eventually be reflected on the price of the product and be assumed by the consumer as well. EPR instruments are implemented by industrial sector, thus with a wide range of amounts. In France in 2020, Citeo (agency organizing REP for packaging industries) fixes a baseline level for eco-contributions of 16,53 ct€/kg for paper and cardboard, and 28,88 ct€/kg for light PET (colourless).[20] Note that almost 80% of these amounts are redistributed for the recycling sector (collecting, sorting and resource production), thus covering the subsidy part

[20]This amount is completed by a unit based amount and a bonus-penalty system depending on the eco-design of the product.

of our monetary transfers (Citeo, 2019).

In the case of example 3, note that subsidy $\pi(\beta)$ is applied to resources producers instead of the final consumer. This reflects the prospective activity of recycling, as consumed resources are potential recycled resources as well as potential waste.

4.1.2. Existence of a waste market

Let us now assume that the final consumer can sell her flow of waste q at price p_w on a specific market. Her surplus must now includes the revenue from the waste sale $p_w q$: $S_C = [u(q) + (p_w - p_q)q + T_C(q)]$. The new expression of the intertemporal surplus of the recycling sector is $S_R = \int_0^T [p_r r - c_r r - F(q,\beta) - p_w q + T_R(r,\beta,q)]e^{-\rho t} dt$, which must be maximized subject to (2). The private marginal cost of recycling now also includes waste purchase. Behaviors of the other agents are unchanged.

Proposition 7. *Given λ_i^*, $i \in \{E; W\}$ the shadow costs for an optimal trajectory, the set of policy instruments that restores optimality with a market for waste must satisfy the following conditions:*

$$\frac{1}{q}\frac{\partial T_R}{\partial \beta} = \lambda_W^* \tag{45}$$

$$T_V'(v) + \frac{\partial T_R}{\partial q} + \frac{\partial T_P}{\partial v} + T_C'(q) = -\delta_v \lambda_E^* - (1-\beta)\lambda_W^* \tag{46}$$

$$\frac{\partial T_R}{\partial r} + \frac{\partial T_R}{\partial q} + \frac{\partial T_P}{\partial r} + T_C'(q) = -\delta_r \lambda_E^* - (1-\beta)\lambda_W^* \,. \tag{47}$$

Proof. Programs of the final good producer and of the virgin resource extracting sector are not affected by the existence of a waste market. Then, Eq. (39) and (40) still hold. First-order conditions (41) and (42) are unchanged for the recycling sector. However another condition appears for this sector as well as a changed condition for the consumer:

$$u'(q) + p_w = p_q - T_C'(q). \tag{48}$$

$$f(\beta) + p_w = \beta\tilde{\lambda}_R + \frac{\partial T_R}{\partial q}\,, \tag{49}$$

From these equations, compared with first-order conditions for optimal interior solutions in the central planner model given by (9)-(11), we get equations (45) to (47). □

As expected, the waste market allows to fully internalize the positive externality associated with the waste generation by the final consumer.

4.2. Scrap value for waste

Here, we assume that waste accumulation is no longer damaging from time T onward: $D(W(t)) = 0, \forall t \geq T$. From (3), given that extraction/consumption flows are nil, the remaining stock of wastes evolves as: $W(t) = W(T)e^{-\alpha(t-T)}$, for $t \geq T$. A more realistic approach consists in attaching a scrap value Ω to $W(T)$ in case where the damage caused by the left-over waste stock remains after the end of the exploitation period of time:

$$\Omega(W(T)) = \int_T^\infty D\left(W(T)e^{-\alpha(t-T)}\right) e^{-\rho t} dt. \tag{50}$$

This scrap value must be added to the value function (7) of the social planner program, which slightly modifies the tranversality condition relative to stock W:

$$\left[\lambda_W(T) - \Omega'(W(T))\right] W(T)\mathrm{e}^{-\rho T} = 0\,. \tag{51}$$

Using (15), (50) and (51), the shadow cost of the waste stock at any point in time can be expressed as:

$$\begin{aligned}
\lambda_W(t) &= e^{(\rho+\alpha)t}\left[\Omega'(W(T))e^{-(\rho+\alpha)T} + \int_t^T D'(W)e^{-(\rho+\alpha)s}ds\right] \\
&= \int_t^T D'(W)e^{-(\rho+\alpha)(s-t)}ds + e^{-\rho T}\int_T^\infty D'(W)e^{-(\rho+\alpha)(s-t)}ds\,.
\end{aligned} \tag{52}$$

At any time t, the shadow marginal cost of waste contains two components: the sum from t to T of the marginal damage, discounted at rate $(\rho + \alpha)$ to reflect the marginal absorption process of the stock of waste by the environment, and the value at time T of the sum from T onward of the discounted marginal damage, i.e. the scrap value term. With constant marginal damage, this shadow cost becomes:

$$\lambda_W(t) = \frac{\theta}{(\rho+\alpha)}\left(1 - (1 - e^{-\rho T})e^{-(\rho+\alpha)(T-t)}\right)\,, \tag{53}$$

and we still have $\dot{\lambda}_W < 0$. The scrap value intervenes by adding an exogenous factor $1-\mathrm{e}^{-\rho T} \in (0, 1)$ to the decreasing term of the shadow cost. The remaining shadow cost for waste at the end of the program is the marginal scrap value $\frac{c_W}{\rho+\alpha}\mathrm{e}^{-\rho T} = \Omega'(W(T))$, instead of zero for the model previously discussed.

This additional assumption does not change the change the arbitration in the model, as the first-order conditions (9)-(15) remain the same. However, the optimal recycling rate follows a different time path, according to the following proposition.

Proposition 8. *When the recycling rate $\beta(t)$ is not always zero and considering a scrap value for the waste stock damage, it reaches a maximum at time $\hat{T}_\beta$. This instant can take the following values:*

$$\hat{T}_\beta = \begin{cases} 0 & , \textit{if } \frac{\mathrm{e}^{(\rho+\alpha)T}}{(1-\mathrm{e}^{-\rho T})} \leq \frac{c_W}{\rho\lambda_{R0}} \textit{ or } \lambda_{R0} = 0 \\ T & , \textit{if } \frac{c_W}{\rho\lambda_{R0}} \leq \frac{\mathrm{e}^{\rho T}}{(1-\mathrm{e}^{-\rho T})} \\ \frac{1}{\alpha}\left[\ln\left(\frac{\rho\lambda_{R0}}{c_W(1-\mathrm{e}^{-\rho T})}\right) + (\rho+\alpha)T\right] \in]0, T[& , \textit{if } \frac{\mathrm{e}^{\rho T}}{(1-\mathrm{e}^{-\rho T})} < \frac{c_W}{\rho\lambda_{R0}} < \frac{\mathrm{e}^{(\rho+\alpha)T}}{(1-\mathrm{e}^{-\rho T})} . \end{cases} \tag{54}$$

Proof. Now the marginal benefit of recycling is:

$$\Phi_\beta(t) = \lambda_{R0}\mathrm{e}^{\rho t} + \frac{c_W}{\rho+\alpha}\left(1 - \left(1 - \mathrm{e}^{-\rho T}\right)\mathrm{e}^{-(\rho+\alpha)(T-t)}\right)$$

Following the same analysis as the general case, we get Proposition (8). □

4.3. Decoupling emissions of the recycling branch

Another possible extension of the model is to consider a decoupling of the emissions related to the recycling process. In that case, parameter δ_r only concerns the transformation and use of the recycled input, while we add a constant rate δ_β for the activity of collecting and sorting from the waste flow: $\delta_\beta\beta(t)q(t)$. This leads to the following new dynamic equation for emissions accumulation:

$$\dot{E}(t) = [\delta_v + \beta(t)\delta_\beta]v(t) + [\delta_r + \beta(t)\delta_\beta]r(t),\ E(0) = 0\,. \tag{55}$$

Practically, this modification in the model does not change the arbitration between the two resources, as for any input, this activity happens: the term relative to this type of emissions is disappears with the Full Marginal Costs difference, the same way it does for terms relative to waste accumulation, recycled stock replenishment and the cost of recycling. However, there is an impact on the first-order condition relative to recycling, with a new function Φ_β:

$$\Phi_\beta = \lambda_R + \lambda_W - \delta_\beta\lambda_E \tag{56}$$

This more complex expression of the marginal social gain of recycling can be easily reinterpreted: we now recycle to replenish stock R and alleviate the cost of stock W but it costs emissions to the industrial sector, at rate δ_β. Developing this expression, we obtain: $\Phi_\beta = (\lambda_{R0} - \delta_\beta\lambda_{E0})\mathrm{e}^{\rho t} + \frac{c_W}{\rho+\alpha}\left[1 - \mathrm{e}^{-(\rho+\alpha)(T-t)}\right]$. The same analysis as before can be done for the evolution of the recycling rate. However we can also add the following Proposition:

Proposition 9. *It is never optimal to start recycling under the two following conditions:*

- *There is a higher initial social cost for emissions than the depletion of the recycled stock, i.e.* $\lambda_{R0} \leq \delta_\beta \lambda_{E0}$*;*
- *Damage of waste accumulation is such as* $\frac{c_W}{\rho+\alpha} \leq f'(0)$*.*

In this case, the recycled resource is never used, $\lambda_{R0} = 0$*.*

Proof. Given the new expression of Φ_β, we see that it is decreasing when $\lambda_{R0} \leq \delta_\beta \lambda_{E0}$. As $\lim_{-\infty} \Phi_\beta = \frac{c_W}{\rho+\alpha}$, equation $\Phi_\beta = f'(\beta)$ does not have a solution when $\frac{c_W}{\rho+\alpha} \leq f'(0)$, proving the necessary condition of the Proposition. As there is no recycled stock initially, we never use the recycled resource. Note that a more constraining condition for the absence of recycling is $\Phi_\beta(0) \leq f'(0)$. □

By adding an emission term for recycling activities, we can highlight a simple condition on the parameters of the model for which recycling is never optimal. The social cost of emitting with the recycling activity is too high, while the avoided cost of waste accumulation is too low. Finally, remark that we also observe the rebound effect for the use of inputs with this extension of the model.

4.4. Perfect substitution

Perfect substitution is a strong hypothesis here used in several models tackling recycling [4][19][20]. However it is known that for some raw materials, recycling lowers its quality, sometimes because different materials are mixed in the collecting process, producing a hybrid material with lower properties. Loss of quality is in fact an important topic for the academic research on circular economy, especially when the efficiency of a process is studied [14]. It is for instance the case in the paper and cardboard industry, when each recycling loop lowers the quality of the pulp and only allows a limited amount of cycles depending on the needed quality (usually 7 and 8 cycles are technically possible, but fiber is rather used 3.5 times on average in Europe [12]). There is also a common mistrust towards material coming from a waste flow, even for the same level of quality. To a certain extent, it explains the price structure of recycled material for which there is a systematic discount compared to the price of the virgin product, regardless of the cost structure. However, the manufacturer can also have a preference for recycled materials, for marketing reasons or a general pro-environment trend in the industry, leading to specific choices in favor of recycled material [28]. These two opposite motivations for the producer justify the choice of perfect substitutes. Moreover, technological progress in recycling tends to reach the same quality for virgin and recycled inputs [22].

5. Conclusion

We provided in this paper a discussion on optimal use of resources when climate change and waste are both additional constraints for the social planner. Depending on initial characteristics of the industrial sector, the optimal path can be divided into phases of virgin and recycled production, with a potential switch of inputs if recycling becomes socially more profitable. This change in the sector intervenes when the difference between emissions rates is high enough, especially compared to the fixed difference between private costs of inputs. In parallel, the social planner will elaborate a recycling (or waste recovering) strategy, such as it alleviates the cost of waste accumulation and allows a production from a recycled input. This shows the duality of the activity of sorting waste: a practical goal being the reduction of costs and a speculative goal as the use of recycled input can occur in the future. This observation can lead to behaviors where it is optimal increase recycling, then decrease it. Moreover, when the marginal cost of recycling is too high, it leads to phases where the recycling rate is zero (while it is possible to produce from the recycled input at the same time). In parallel, under certain conditions, production shows catch-up phases at the end of the program, as full marginal costs start decreasing (involving an increase in production). Our model gives a better understanding of the complementary between climate change objectives and resource issues: recycled inputs, while often a cleaner option for production, still come with GHG emissions.

While the central planner program gives many insights regarding optimal recycling activities, it does not give empirical results for policy making. The decentralized model of the sector allows to specify the need of public intervention. This new model highlights a possible tax/subsidy scheme in order to implement a first-best solution. A taxation based on GHG emissions must be introduced in both sectors, weighed by the carbon intensity of the branch (higher for virgin extraction in most cases). This comes in addition to a tax-subsidy system based on waste and stock replenishment, reflected in Extended Produced Responsibility programs currently implemented for specific products.

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