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"Decentralizing Cooperation through Upstream Bilateral Agreements"

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Decentralizing Cooperation through Upstream Bilateral Agreements*

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Abstract

We consider an industry with $n \geq 3$ firms owning upstream inputs and interacting noncooperatively in a downstream market. Under general conditions, upstream *bilateral* agreements giving firms access to one another's input lead to industry profit maximization. This decentralization result applies to various upstream agreements including cross-licensing agreements among patent-holding manufacturers, interconnection agreements among telecommunication companies, interbank payments for ATM networks, and data-sharing agreements among competitors or complementors.

Keywords: Bilateral oligopoly, upstream agreements, cooperation.

JEL Codes: L13, L41.

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1 Introduction

It is common for two firms that interact in product markets as competitors or complementors to sign an upstream bilateral agreement through which they provide some input to one another. The input can take the form of intellectual property rights, data, access to proprietary networks, etc. For instance, cross-licensing of patents is a widespread practice across many industries.[1] Similarly, bilateral agreements to provide mutual access to proprietary networks are signed among telecommunications operators (Armstrong, 1998; Laffont et al., 1998), among Internet backbone companies (Crémer et al., 2000) and among banks for the use of ATMs (Donze and Dubec, 2006). Additionally, as data are becoming a key input for marketing and innovation, it is expected that increasingly more firms will engage in data-sharing agreements.[2]

We consider a situation in which $n \geq 3$ firms engage in downstream interactions and study the outcome resulting from upstream bilateral agreements. Specifically, we consider a two-stage game in which every pair of firms simultaneously decides whether to sign an upstream bilateral agreement that provides each firm with access to the other firm's input before all n firms simultaneously choose, in a noncooperative way, their downstream actions. Our framework is general in that it allows for any number of firms and any form of asymmetry among them and that it covers both complementarity and substitutability in downstream interactions. We find that, under a very wide range of circumstances, equilibrium upstream bilateral agreements lead to industry profit maximization. In other words, upstream bilateral agreements allow the firms to implement the fully cooperative outcome in a decentralized way. We also consider two extensions. First, we show that equilibrium upstream bilateral agreements not only maximize the joint profit of all firms but also the profit of any subset of firms, which makes them robust to deviations by a coalition of any size. Second, we establish that our central result holds not only for private agreements (as in the baseline model) but also for public agreements.

We assume that two firms reaching an upstream bilateral agreement maximize their joint profit, which they can share through fixed payments. We consider a simple bilateral contract between firms i and j that specifies a fixed transfer and a pair of per-unit input prices $(r_{i\to j}, r_{j\to i})$ where $r_{i\to j}$ is paid by firm i to firm j per unit of the former's output. Joint profit maximization requires firms i and j to choose the pair $(r_{i\to j}, r_{j\to i})$ that induces each of them

[1] Taylor and Silberston (1973) report that cross-licensing accounts for a significant share of all licensing arrangements in many industries. In particular, cross-licensing in the semiconductor industry has received much attention in the literature (e.g., Grindley and Teece, 1997 and Hall and Ziedonis, 2001).

[2] See Arnaut et al. (2018) for a study of data sharing between companies in Europe. In their report for the European Commission, Crémer et al. (2019) emphasize the need to carry out economic research on data-sharing agreements.

to adopt the downstream actions x_i and x_j that maximize their joint profit. The key condition under which our main result holds is that input prices are *pairwise independent* instruments, in the sense that they allow firms i and j to achieve any local deviation in downstream actions through some local deviation in input prices.

The main intuition for our central result can be given as follows. Firm i maximizes its own profit when it chooses its downstream action. The upstream bilateral contract it signs with firm j essentially induces firm i to internalize the impact of its action on firm j's profit (and vice versa) through an adequate choice of the per-unit input prices $(r_{i\to j}, r_{j\to i})$. If firm i signs a bilateral agreement with all other firms, it ends up internalizing the impact of its action on all firms' profits. Therefore, a complete network of bilateral agreements leads to industry profit maximization. We find our central result surprising because it shows that simple bilateral contracts can enable firms to implement the fully cooperative outcome in a decentralized way.

Related literature. Our paper is related to the literature on bilateral monopoly/oligopoly pioneered by Crémer and Riordan (1987) and Horn and Wolinsky (1988), which recently expanded because many empirical papers embraced this framework.[3] Among the theoretical papers in that literature, the closest to ours are Nocke and Rey (2018), Collard-Wexler et al. (2019) and Rey and Vergé (2019), as they also consider settings with interlocking relationships. The main difference between these three papers and ours is that they mainly consider bilateral agreements between (pure) upstream firms and (pure) downstream firms, while we consider bilateral agreements among vertically integrated firms. Collard-Wexler et al. (2019) provide a noncooperative foundation for "Nash-in-Nash" bargaining. However, they restrict attention to lump-sum payments, thus focusing on the division of gains from trade, while we consider two-part tariffs. In that respect, our paper is closer to Nocke and Rey (2018) and Rey and Vergé (2019), as they both consider negotiations over (general) nonlinear tariffs. However, Nocke and Rey (2018) restrict attention to private upstream agreements and downstream Cournot competition, while we allow for both private and public agreements and consider a general model of downstream interactions. Moreover, their focus is on the effects of exclusive dealing and vertical integration, while ours is on whether upstream bilateral agreements among vertically integrated firms can lead to industry profit maximization.[4] Rey and Vergé (2019) also consider a specific form of downstream interaction, namely, competition in prices with differentiated products, and focus mainly on a setting with no vertically integrated

[3] See, e.g., Crawford and Yurukoglu (2012), Grennan (2013), Gowrisankaran et al. (2015), Ho and Lee (2017), and Crawford et al. (2018).

[4] Another difference between our paper and Nocke and Rey (2018) is that we do not use the same equilibrium concept.

firms.[5] Finally, our paper is related to de Fontenay and Gans (2014), who provide an analysis of a noncooperative pairwise bargaining game in a buyer-supplier network (as well as more general networks). However, they focus on contracts that are contingent on the set of realized agreements, while we assume that a contract depends only on the quantities produced by the firms signing it.

The idea that firms can implement the industry-profit-maximizing outcome through upstream bilateral agreements has been illustrated in a *duopoly* setting in the context of interconnection agreements between telecommunication companies and cross-licensing agreements between competitors. Armstrong (1998) and Laffont et al. (1998) show that termination fees can be used as a collusive device in a telecommunication industry with two competing networks, and Katz and Shapiro (1985) and Fershtman and Kamien (1992) obtain a similar result in a setting with two duopolists signing a cross-licensing agreement. In Jeon and Lefouili (2018), we consider a setting with *more than two* firms and show that a complete network of bilateral cross-licensing agreements among competing firms generates the industry-profit-maximizing outcome. The current paper substantially generalizes Jeon and Lefouili (2018) in several directions. While the latter focuses on cross-licensing of patents between symmetric Cournot oligopolists who sign private bilateral contracts, the current paper (i) considers a general class of two-stage games with any type of upstream input and any form of downstream interactions (including Bertrand and Cournot competition as well as strategic interactions between complementors), (ii) allows for asymmetric players, and (iii) considers both private and public contracts. These differences require us to use an approach fundamentally different from the one we used in Jeon and Lefouili (2018).

Finally, our paper is related to Watson (2018). Both papers show that private bilateral contracting among multiple players can lead to an outcome that maximizes (or almost maximizes) the joint payoff of all players. However, there are several key differences between the two papers that make them complementary. In particular, the type of networks, negotiations and contracts we consider differ from those in Watson (2018). Specifically, we consider a single round of simultaneous negotiations, allow each firm to sign a bilateral contract with any other firm (thus allowing for endogenously complete networks) and assume that payments in a bilateral contract between two firms i and j can be contingent only on the quantity of output produced by firm i and the output produced by firm j.[6] In contrast, Watson (2018) allows for

[5] They also consider an extension in which there is one vertically integrated firm interacting with independent suppliers and independent retailers.

[6] The output of firm i coincides with firm i's action only if the second-stage game is Cournot competition. Otherwise (for instance, in the case of price competition), the output of firm i is different from its action.

multiple rounds of (private) negotiations,[7] considers an exogenous network with potentially missing (direct) links and assumes that payments in a bilateral contract between two players can be contingent on actions taken by *other* players.

The remainder of the paper is organized as follows. Section 2 lays out the baseline model. Section 3 presents our main result. Section 4 extends our analysis in two directions. Section 5 concludes the paper.

2 Baseline Model

Setting and description of the game. Consider $n(\geq 3)$ firms, each owning an upstream input and interacting in a downstream market. The upstream input can be, for instance, a patent covering a product or a technology, a dataset, or a proprietary network, while the downstream interaction can be, for example, the production and marketing of substitutable or complementary goods. Each pair of firms decides whether to sign a bilateral contract that provides each firm with access to the other firm's upstream input. We assume that an upstream bilateral contract between firms i and j specifies a pair of *per-unit input prices* $(r_{i\to j}, r_{j\to i})$ as well as a (possibly negative) fixed transfer $f_{i\to j}$, where $r_{i\to j}$ refers to the input price per unit of firm i's output paid by firm i to firm j and $f_{i\to j}$ refers to the fixed fee paid by firm i to firm j.[8] In our baseline model, we assume that upstream agreements are private: whether an upstream contract is signed between two given firms and the terms of the contract if any are known only to these two firms.[9] For the sake of exposition, we use the term "upstream agreement" in a broader sense than the term "upstream contract": the former accounts for the possibility that a pair of firms choose the *null* upstream contract, i.e., not to sign any upstream contract. After firms decide whether they share access to their inputs, each firm chooses a downstream action noncooperatively. This action can be, for instance, a price or a quantity. Specifically, the timing of the game is as follows:

- Stage 1 (upstream bilateral agreements): Every pair of firms (i, j) with $1 \leq i, j \leq n$ simultaneously decides whether to sign a bilateral contract that provides each firm with access to the other firm's upstream input and the terms of the corresponding contract if they agree on one.

[7] Moreover, players are allowed to cancel the bilateral contracts agreed upon in previous rounds of negotiations.

[8] We assume that firm j charges firm i an input price per unit of firm i's output to fix ideas. However, our analysis would be qualitatively unchanged if we assumed, for instance, that firm j charges firm i an input price per unit of firm j's input.

[9] Section 4.2 provides an extension in which all upstream agreements are public and shows that our main finding holds in that alternative scenario.

- Stage 2 (downstream noncooperative actions): Each firm i chooses a (real-valued) downstream action x_i noncooperatively and simultaneously.

Notations and basic assumptions. Let $R \equiv (r_{i\to j})_{1\leq i,j\leq n}$ be the matrix of per-unit input prices, with the convention that $r_{i\to i} = 0$ for any $i \in \{1,...,n\}$ and $r_{i\to j} = 0$ for any pair of firms (i,j) that do not sign an upstream bilateral contract. Similarly, define $F \equiv (f_{i\to j})_{1\leq i,j\leq n}$ as the matrix of fixed fees paid by firms to each other, with the convention that $f_{i\to i} = 0$ for any $i \in \{1,...,n\}$ and $f_{i\to j} = 0$ for any pair of firms (i,j) that do not sign an upstream bilateral contract. Note that $f_{j\to i} = -f_{i\to j}$ for any pair of firms (i,j). Finally, let $\mathbf{x} = (x_1,...,x_n)$ denote the vector of firms' actions, and let $\mathbf{x}_{-i}$ (resp. $\mathbf{x}_{-ij}$) be the vector obtained from vector $\mathbf{x}$ by removing x_i (resp. by removing x_i and x_j).

Consider a graph (N,G) consisting of a set of nodes $N = \{1,...,n\}$ and an $n \times n$ matrix G with elements $g_{ij} \in \{0,1\}$, where $g_{ij} = 1$ (i.e., firms i and j are linked) if firm i and firm j sign an upstream bilateral contract and $g_{ij} = 0$ if they do not; we set $g_{ii} = 1$ without loss of generality. We will call (G,R,F) a network of upstream agreements.[10] Thus, an upstream agreement between firms i and j is represented by $(g_{ij}, r_{i\to j}, r_{j\to i}, f_{i\to j})$, where we do not write $f_{j\to i}$ as $f_{j\to i} = -f_{i\to j}$, and where $r_{i\to j} = r_{j\to i} = f_{i\to j} = 0$ whenever $g_{ij} = 0$.

Let $\Pi_i(G,R,F,\mathbf{x})$ represent player i's payoff function for a given network of upstream agreements (G,R,F) and given actions $\mathbf{x}$. We make the following assumptions regarding the effects of upstream agreements on payoffs:

A1 For any i with $1 \leq i \leq n$, there exists a function π_i differentiable with respect to the vector of actions $\mathbf{x}$ such that, for any $(G,R,F,\mathbf{x})$, $\Pi_i(G,R,F,\mathbf{x}) = \pi_i(G,R,\mathbf{x}) - \sum_{j\neq i} f_{i\to j}$.

A2 For any i,j such that $1 \leq i,j \leq n$ and $i \neq j$ and for any $(G,R,\mathbf{x})$, $\pi_i(G,R,\mathbf{x}) + \pi_j(G,R,\mathbf{x})$ does not depend on $r_{i\to j}$. Moreover, for any i,j,k such that $1 \leq i,j,k \leq n$, $i \neq j$ and $k \notin \{i,j\}$ and any $(G,R,\mathbf{x})$, $\pi_k(G,R,\mathbf{x})$ does not depend on $r_{i\to j}$.

Assumption **A1** states that $f_{i\to j}$ is a fixed fee for any i,j and provides a regularity condition with respect to the firms' downstream actions. Assumption **A2** means that $r_{i\to j}$ is a transfer within the coalition $\{i,j\}$, i.e., it affects neither the joint profit of firms i and j nor the profit of any third firm k for a *given* action profile. Note that we allow input prices $r_{i\to j}$ to take positive and negative values. This rules out noninterior equilibria, which simplifies our analysis by making it possible to rely on first-order conditions.

To define our equilibrium concept, we make the following assumption:

[10] Note that as agreements are private, node ij is not observable to any firm $k \neq i,j$. However, in equilibrium, any firm will correctly anticipate G.

A3 For any network of upstream agreements (G, R, F), any pair of firms (i, j), and any actions $\mathbf{x}_{-ij}$ of firms $k \neq i, j$, the two-player simultaneous game derived from the n-player downstream game by fixing the actions of firms $k \neq i, j$ to $\mathbf{x}_{-ij}$ has a unique Nash equilibrium, i.e., there is a unique pair of actions $(\tilde{x}_i (G, R, \mathbf{x}_{-ij}), \tilde{x}_j (G, R, \mathbf{x}_{-ij}))$ such that

$$\begin{cases} \tilde{x}_i (G, R, \mathbf{x}_{-ij}) \in \arg\max_{x_i} \pi_i (G, R, \mathbf{x}_{-ij}, \tilde{x}_j (G, R, \mathbf{x}_{-ij}), x_i) \\ \tilde{x}_j (G, R, \mathbf{x}_{-ij}) \in \arg\max_{x_j} \pi_j (G, R, \mathbf{x}_{-ij}, \tilde{x}_i (G, R, \mathbf{x}_{-ij}), x_j) \end{cases}.$$

Finally, to capture upstream deviations by a pair of firms, let $D_{ij} (G, R, F)$ denote the set of upstream network agreements that differ from a given network (G, R, F) only through the agreement between firms i and j, i.e., the set of networks $(G', R', F') \neq (G, R, F)$ such that $g'_{kl} = g_{kl}$, $r'_{k\to l} = r_{k\to l}$, and $f'_{k\to l} = f_{k\to l}$ for any $(k, l) \neq (i, j)$.

Equilibrium concept. We are now in a position to define our equilibrium concept. We adopt the contract equilibrium approach proposed by Crémer and Riordan (1987) and Horn and Wolinsky (1988) and used most recently by Collard-Wexler et al. (2019) and Rey and Vergé (2019). Following Rey and Vergé (2019), we allow for "balanced" bargaining and let $\alpha_{ij} \in [0, 1]$ denote the bargaining power of firm i in its bargaining with firm j. In equilibrium, each firm chooses the downstream action that maximizes its profit given the upstream agreements it is involved in and assuming that all other firms choose their equilibrium downstream actions. Moreover, in Stage 1, each pair of firms (i, j) chooses an upstream agreement that satisfies the following two conditions. First, it maximizes their joint profit given all other upstream agreements and the induced downstream actions of firms i and j, assuming that all other firms $k \neq i, j$ play their equilibrium downstream actions. Second, if firms i and j sign an upstream contract,[11] a share α_{ij} of the resulting gain from trade goes to firm i, while a share $\alpha_{ji} = 1 - \alpha_{ij}$ goes to firm j.

Definition 1 *(Bilaterally efficient equilibrium) A bilaterally efficient equilibrium* $(G^*, R^*, F^*, \mathbf{x}^*)$ *is an equilibrium network of upstream agreements* (G^*, R^*, F^*) *together with an equilibrium vector of actions* $\mathbf{x}^* = (x_i^*)_{1\leq i\leq n}$ *such that:*

- in Stage 2, the equilibrium action of each firm maximizes its profit given the actions of all other firms: $x_i^* \in \arg\max_{x_i} \pi_i (G^*, R^*, \mathbf{x}^*_{-i}, x_i)$.

- in Stage 1, the equilibrium upstream agreement between any pair of firms i and j maximizes their joint profit given the equilibrium actions of all other firms $k \neq i, j$, all other upstream agreements, and the induced downstream actions of firms i and j: for any $(i, j) \in$

[11] Recall that firms i and j may agree on *not* signing such a contract.

$\{1, ..., n\}^2$ *such that* $i \neq j$ *and any* $(G, R, F) \in D_{ij}(G^*, R^*, F^*)$,

$$(\pi_i + \pi_j)(G^*, R^*, \mathbf{x}^*) \geq (\pi_i + \pi_j)\left(G, R, \mathbf{x}^*_{-ij}, \tilde{x}_i(G, R, \mathbf{x}^*_{-ij}), \tilde{x}_j(G, R, \mathbf{x}^*_{-ij})\right).$$

Moreover, if firms i *and* j *agree on signing an upstream contract, then firm* i *(resp. firm* j*) obtains a share* α_{ij} *(resp.* $1 - \alpha_{ij}$*) of the additional profit generated by this contract.*

Rey and Vergé (2019) provide a noncooperative foundation for a similar equilibrium concept in a setting where (pure) upstream firms sell their inputs to (pure) downstream firms. Their microfoundation relies on a sequential noncooperative game of delegated negotiations, in which each firm sends different agents to negotiate with its potential partners, and for each negotiation, one of the two agents is randomly selected to make a take-it-or-leave-it offer. We expect that a noncooperative foundation based on delegated negotiations can also be provided for our setting, but establishing this formally is beyond the scope of this paper.

Illustrations. The general model above can serve to describe several settings in which firms can provide upstream inputs to each other before interacting in a downstream market. These include:

- Cross-licensing of patents: $r_{i \to j}$ is the per-unit royalty paid by patent-owning firm i to patent-owning firm j, and x_i is a price or a quantity chosen by i. Note that the model applies not only to the case in which cross-licensing firms produce substitutable goods but also to the case in which they produce complementary goods (see Jeon and Lefouili, 2018 for the special case of perfectly substitutable products sold by symmetric Cournot oligopolists and produced with constant marginal costs).

- Two-way access pricing in telecommunication networks: $r_{i \to j}$ is the per-unit access charge paid by network i to network j, and x_i is the linear retail price charged by network i to its customers (see Armstrong, 1998, and Laffont et al., 1998, for an analysis of this issue in a duopolistic setting).

- Interconnection among Internet backbone companies: $r_{i \to j}$ is the per-unit access charge paid by backbone company i to j in a transit agreement, and x_i is the capacity choice made by i (see Crémer et al., 2000).

- Interbank payments for the use of ATMs: $r_{i \to j}$ is the per-unit interchange fee paid by bank i to bank j, and x_i is the number of ATMs deployed by bank i (see Donze and Dubec, 2006, for a setting with a multilateral negotiation of the interchange fee).

- Data-sharing agreements: $r_{i\to j}$ is a per-unit fee paid by firm j to firm i, and x_i is the price or the quantity of the product sold by firm i.

3 Analysis and Main Result

In this section, we analyze our baseline model with private upstream agreements. We first provide two conditions under which an equilibrium network of upstream agreements is necessarily complete, i.e., all pairs of firms sign an upstream contract.[12]

C1 (Gain from trade) For any pair of firms $(i,j) \in \{1,..,n\}^2$ satisfying $i \neq j$, any network of upstream agreements (G,R,F) such that $g_{ij}=0$ and any vector of actions $\mathbf{x}$, the following holds for any upstream network $(G',R',F') \in D_{ij}(G,R,F)$ such that $g'_{ij}=1$:

$$\pi_i(G',R',\mathbf{x}) + \pi_j(G',R',\mathbf{x}) > \pi_i(G,R,\mathbf{x}) + \pi_j(G,R,\mathbf{x}).$$

This condition says that signing an upstream bilateral contract strictly increases the joint payoff of firms i and j for any *given* action profile. For instance, in the case of cross-licensing of patents, an upstream contract between two firms giving each firm access to the other firm's patented technology reduces production costs or increases product quality, which leads to higher profits (holding *fixed* all downstream actions). Similarly, data sharing can increase the quality of each firm's service and thus increase its profit.

C2 (Replicability) For any pair of firms $(i,j) \in \{1,..,n\}^2$ satisfying $i \neq j$, any network of upstream agreements (G,R,F) such that $g_{ij}=0$ and any vector $\mathbf{x}_{-ij}$ of actions of firms $k \neq i,j$, there exists a network of upstream agreements $(G',R',F') \in D_{ij}(G,R,F)$ with $g'_{ij}=1$ such that

$$\tilde{x}_i(G',R',\mathbf{x}_{-ij}) = \tilde{x}_i(G,R,\mathbf{x}_{-ij}) \quad \text{and} \quad \tilde{x}_j(G',R',\mathbf{x}_{-ij}) = \tilde{x}_j(G,R,\mathbf{x}_{-ij}).$$

This condition can be interpreted as follows. Consider a network of upstream agreements such that firms i and j do not sign an upstream contract. Condition **C2** means that there exists an upstream contract between these two firms that generates the same equilibrium actions of

[12] These assumptions are relevant only when bilateral agreements are voluntary, as in our game. There are alternative settings of interest in which upstream agreements are *mandated* by a regulator. This is for instance the case for interconnection agreements among mobile phone companies in several jurisdictions. If upstream agreements are mandated by a regulator (while the terms of the agreements are decided by the firms), then the network of bilateral agreements is necessarily complete, and therefore, Conditions **C1** and **C2** are not needed.

firms i and j in Stage 2 for fixed actions by firms $k \neq i, j$ and fixed upstream agreements by pairs of firms $(k, l) \neq (i, j)$.

It is straightforward that under Conditions **C1** and **C2**, any two distinct firms that have not signed an upstream contract can (strictly) increase their joint payoff by signing an agreement and, conversely, that an upstream bilateral deviation consisting of not signing a contract cannot be strictly (jointly) optimal for any deviating pair of firms. This generalizes the argument that Jeon and Lefouili (2018) use to show that in the case of cross-licensing and Cournot downstream competition, any pair of firms has an incentive to sign a cross-licensing agreement.[13] Therefore, denoting by G^c the complete network structure, i.e., the network structure such that all pairs of firms sign an upstream contract ($g_{ij} = 1$ for any i, j), we obtain the following result:

Lemma 1 *Under Assumptions **A1-A3** and Conditions **C1-C2**, an equilibrium network of upstream agreements (G^*, R^*, F^*) is necessarily such that $G^* = G^c$.*

In what follows, we focus on complete networks of upstream agreements (G^c, R, F). We first provide a necessary condition for a network of this type to be an equilibrium network. To this end, let us define for each $(i, j) \in \{1, ..., n\}^2$ such that $i \neq j$ the matrix

$$M_{ij}^{private}(R, \mathbf{x}_{-ij}) \equiv \begin{pmatrix} \frac{\partial \tilde{x}_i}{\partial r_{i \to j}} & \frac{\partial \tilde{x}_j}{\partial r_{i \to j}} \\ \frac{\partial \tilde{x}_i}{\partial r_{j \to i}} & \frac{\partial \tilde{x}_j}{\partial r_{j \to i}} \end{pmatrix},$$

where all partial derivatives are evaluated at $(G^c, R, \mathbf{x}_{-ij})$, and let us introduce the following condition:

C3 (Independence) $\det M_{ij}^{private}(R, \mathbf{x}_{-ij}) \neq 0$ for any (i, j) satisfying $i \neq j$ and any $(R, \mathbf{x}_{-ij})$.

This rank condition can be interpreted as follows. It means that the per-unit input prices paid by each firm to the other, $r_{i \to j}$ and $r_{j \to i}$, are pairwise independent instruments in the sense that any local downstream deviation can be obtained through a local upstream deviation. This ensures that the set of upstream instruments used by a given pair of firms is rich enough

[13] In Jeon and Lefouili (2018), licensing firm i's patents to firm j reduces the latter's marginal cost. Suppose that, initially, i does not license its patent to j. These two firms can (weakly) increase their joint profit if i licenses its patent to j by specifying the payment of a per-unit royalty $r_{j \to i}$ equal to the reduction $\Delta_{j \to i}$ in marginal cost generated by j's use of the technology covered by i's patent. Such a licensing agreement would not affect the firms' marginal costs of production but would allow them to save (jointly) $\Delta_{j \to i}$ per unit of output produced by j. It will therefore (weakly) increase their joint profit.

to implement any desired downstream actions of these two firms given the actions of all other firms (and given all other upstream agreements).

The following examples describe two environments in which Conditions **C1**, **C2**, and **C3** are satisfied. In Example 1, patent-holding firms producing substitutable or complementary goods need to decide whether they will sign bilateral cross-licensing agreements before interacting in the downstream market.[14] In Example 2, firms producing substitutable goods must decide whether they will sign data-sharing agreements before competing in the downstream market.

Example 1. Consider n firms $i = 1, ..., n$ producing goods that can be either (imperfectly) substitutable or complementary. Specifically, they face the following demand system:

$$q_i = \frac{1}{1-\gamma^2}\left[v\left(1-\gamma\right) - x_i + \gamma \sum_{j \neq i} x_j\right],$$

$i = 1, 2, ..., n$, where x_i is the price set by firm i, q_i is the demand addressed to firm i, and $\gamma \in (-1/(n-1), 1)$ measures the degree of substitution or complementarity between the products (products are substitutes if $\gamma > 0$ and complements if $\gamma < 0$).[15] Each firm owns one patent covering a cost-reducing technology and can obtain access to its rivals' technologies through cross-licensing agreements. For the sake of exposition, we assume that the patents are symmetric in the sense that the marginal cost of a firm depends only on the number of patents to which it has access. Let $c_i(l_i)$ be firm i's marginal cost when it has access to a number $l_i \in \{1, ..., n\}$ of patents with $c_i(n)(\equiv \underline{c}_i) < c_i(n-1) < ... < c_i(1)(\equiv \bar{c}_i)$. We show in the Appendix that Conditions **C1**, **C2** and **C3** are satisfied in this environment.

Example 2. Consider n firms producing differentiated goods at a constant marginal cost c_i and competing against each other in quantity. Each firm owns one dataset that can be used to improve the value of its product (and the other firms' products if they have access to it). Let $v_i(l)$ denote the value of the product produced by firm i when it has access to $l \in \{1, ..., n\}$ number of distinct datasets with $(\bar{v}_i \equiv) v_i(n) > v_i(n-1) > ... > v_i(1)(\equiv \underline{v}_i)$. Assume further that the demand system is linear. We show in the Appendix that Conditions **C1**, **C2** and **C3**

[14] This example is different from the special case of cross-licensing agreements among symmetric Cournot oligopolists studied in Jeon and Lefouili (2018) (Conditions **C1**, **C2**, and **C3** also hold in that scenario). Note that there is a major difference between the Bertrand game with cross-licensing and the Cournot game with cross-licensing. In the latter, a firm's best-response function only depends on the per-unit royalties that it pays (as its licensing revenues are fixed for given rivals' quantities), while in the former, a firm's best-response function also depends on the per-unit royalties it receives because a change in a rival's price affects its licensing revenues (through its impact on the demand addressed to all rival firms).

[15] We need to assume that $\gamma > -1/(n-1)$ for the problem to be well-behaved; see, e.g., Amir et al. (2017).

are satisfied in this environment.

We now provide a necessary condition for a complete network of upstream agreements to be an equilibrium network.

Lemma 2 *Suppose that Assumptions **A1-A3** and Condition **C3** hold. Then, a necessary condition for a complete network of upstream agreements* $(G^c, R, \mathbf{x})$ *to be an equilibrium network is that*

$$\frac{\partial \pi_i}{\partial x_j}(G^c, R, \mathbf{x}) = 0,$$

for any $(i,j) \in \{1, ..., n\}^2$ *such that* $i \neq j$.

Proof. See Appendix. ■

The intuition behind Lemma 2 is as follows. Under Condition **C3**, each pair of firms (i,j) is able to use upstream input prices to induce the downstream actions that maximize their joint profits (everything else being equal). Since firm j maximizes its profit with respect to its own downstream action, the impact of a marginal change in an input price on the profit of firm j through a change in firm j's downstream action is a *second-order* effect, while the impact on firm i's profit is a *first-order* effect. This implies that a bilaterally efficient agreement between firms i and j induces firm j to choose a downstream action that maximizes firm i's profit (for given actions of all firms $k \neq i, j$). In other words, it induces firm j to fully internalize the effect of its downstream action on firm i.

To state our main result, we first need to consider the benchmark situation in which all n firms maximize industry profits $\sum_{i=1}^{n} \Pi_i(G^c, R, F, \mathbf{x})$ under a complete network structure with respect to their downstream actions $\mathbf{x}$.[16] Note that by **A1-A2**, this joint payoff does not depend on R and F. We make the following assumption regarding the existence and uniqueness of the vector of actions that maximizes industry profits:

A4 There exists a unique vector $\mathbf{x}^c$ of downstream actions that maximizes industry profits $\sum_{i=1}^{n} \Pi_i(G^c, R, F, \mathbf{x})$ under a complete network of upstream agreements;[17] moreover, the industry profit function is differentiable at $\mathbf{x}^c$, and the latter is the unique solution to the corresponding system of first-order conditions.

We can now state our main result.

[16] Notice that assumption **C1** implies that a necessary condition for all firms to maximize their joint payoff is that all pairs of firms sign an upstream contract.

[17] Note that under Condition **C1**, industry profit maximization cannot be achieved unless all pairs of firms sign an upstream agreement, i.e., unless the network of upstream agreements is complete.

Proposition 1 *(i) Under Assumptions **A1-A4** and Conditions **C1-C3**, any bilaterally efficient equilibrium maximizes industry profits.*

*(ii) Moreover, under Assumptions **A1-A4** and Conditions **C1** and **C3**, any bilaterally efficient equilibrium involving a complete network of upstream agreements maximizes industry profits.*

Proof. See Appendix. ■

Providing sufficient conditions for the existence of an equilibrium in our general framework is a challenging task that we do not undertake in this paper.[18] However, Proposition 1 shows that whenever such an equilibrium exists, it maximizes industry profits. To understand why, recall that a bilaterally efficient network of upstream agreements induces each firm to internalize the impact of its downstream actions on the firms with which it has signed an upstream bilateral contract. If such a network is complete (as is the case under Conditions **C1** and **C2**), then each firm's downstream action maximizes the profits of *all other firms*. Since a firm's downstream action also maximizes its own profit, it follows that any bilaterally efficient equilibrium maximizes industry profits.

4 Extensions

4.1 Robustness to upstream deviations by coalitions

In this section, we show that under a wide range of circumstances, a bilaterally efficient upstream network not only maximizes industry profits but is robust to upstream deviations by coalitions of any size.[19] To formalize and show this, let us first define for any $s \in \{3, ..., n-1\}$ the set Γ_s of functions σ from $\{1, 2, ..., s\}$ to $\{1, 2, ..., n\}$ such that $\sigma(i) < \sigma(i')$ for any $i, i' \in \{1, 2, ..., s\}$ satisfying $i < i'$.[20] Then, the set $\{\{\sigma(1), \sigma(2), ..., \sigma(s)\}, \sigma \in \Gamma_s\}$ represents all the possible coalitions of size s. For a given coalition $\{\sigma(1), \sigma(2), ..., \sigma(s)\}$ and a given vector of actions $\mathbf{x}$, let $\mathbf{x}_{-\sigma}$ denote the vector of $n-s$ actions obtained from $\mathbf{x}$ by removing $x_{\sigma(1)}$,

[18] This amounts to providing sufficient conditions under which a given system of $n(n-1)$ potentially asymmetric and non-linear first-order equations with $n(n-1)$ unknown variables has at least one solution. Note, however, that Jeon and Lefouili (2018) demonstrate the existence of an equilibrium in the specific context of cross-licensing agreements among symmetric Cournot oligopolists.

[19] Our result that a bilaterally efficient network of upstream agreements maximizes the joint profit of all firms means that these agreements are robust to upstream deviations by the grand coalition. Moreover, by definition, a bilaterally efficient network of upstream agreements is robust to upstream deviations by pairs of firms.

[20] We assume in this section that $n \geq 4$. In the case of $n = 3$, Proposition 1 is sufficient to state that (complete) networks of bilaterally efficient agreements are robust to upstream deviations by coalitions of any size because the only coalition of size $n \neq 2$ in this case is the grand coalition.

$x_{\sigma(2)}, ..., x_{\sigma(s)}$. Finally, for a given network of upstream agreements (G, R, F) and a given $\sigma \in \Gamma_s$, let $D_\sigma(G, R, F)$ denote the set of networks of upstream agreements $(G', R', F') \neq (G, R, F)$ such that $g'_{kl} = g_{kl}$, $r'_{k\to l} = r_{k\to l}$ and $f'_{k\to l} = f_{k\to l}$ for any $(k, l) \notin \{\sigma(1), \sigma(2), ..., \sigma(s)\}^2$.

The following assumption extends Assumption **A3** to coalitions of any size $s \in \{3, ..., n-1\}$.

A5 For any network of upstream agreements (G, R, F), any $s \in \{3, ..., n-1\}$, any $\sigma \in \Gamma_s$, and any actions $\mathbf{x}_{-\sigma}$ of firms $k \notin \{\sigma(1), \sigma(2), ..., \sigma(s)\}$, the s-player simultaneous game derived from the n-player downstream game, by fixing the actions of firms $k \notin \{\sigma(1), \sigma(2), ..., \sigma(s)\}$ to $\mathbf{x}_{-\sigma}$, has a unique Nash equilibrium, i.e., there is a unique vector of actions $\left(\breve{x}_{\sigma(i)}(G, R, \mathbf{x}_{-\sigma})\right)_{1\leq i\leq s}$ such that

$$\breve{x}_{\sigma(i)}(G, R, \mathbf{x}_{-\sigma}) \in \arg\max_{x_{\sigma(i)}} \pi_i\left(G, R, \mathbf{x}_{-\sigma}, \left(\breve{x}_{\sigma(j)}(G, R, \mathbf{x}_{-\sigma})\right)_{1\leq j\leq s,\ j\neq i}, x_{\sigma(i)}\right).$$

We also make the following assumption that extends Assumption **A4** to coalitions of any size $s \in \{3, ..., n-1\}$.

A6 For any complete network (G^c, R, F), any $s \in \{3, ..., n-1\}$, any $\sigma \in \Gamma_s$, and any actions $\mathbf{x}_{-\sigma}$ of firms $k \notin \{\sigma(1), \sigma(2), ..., \sigma(s)\}$, there exists a unique vector of actions $\left(x^c_{\sigma(i)}(\mathbf{x}_{-\sigma})\right)_{1\leq i\leq s}$ that maximizes the joint payoff of the firms in the coalition $\{\sigma(1), \sigma(2), ..., \sigma(s)\}$; moreover, the joint payoff of these firms is differentiable at $\left(x^c_{\sigma(i)}(\mathbf{x}_{-\sigma})\right)_{1\leq i\leq s}$, and the latter is the unique solution to the corresponding system of first-order conditions.

We can now state the main result of this section.

Proposition 2 *Consider $s \in \{3, ..., n-1\}$.*

*(i) Under Assumptions **A1-A6** and Conditions **C1-C3**, in any bilaterally efficient equilibrium $(G^c, R^*, F^*, \mathbf{x}^*)$, the equilibrium upstream agreements between all pairs of firms within a coalition of size s maximizes the joint profit of the coalition given all the equilibrium upstream agreements involving at least one firm outside the coalition, the equilibrium actions of all firms outside the coalition, and the induced downstream actions of the firms within the coalition: for any $\sigma \in \Gamma_s$ and any $(G, R, F) \in D_\sigma(G^*, R^*, F^*)$,*

$$\sum_{1\leq i\leq s} \pi_{\sigma(i)}(G^*, R^*, \mathbf{x}^*) \geq \sum_{1\leq i\leq s} \pi_{\sigma(i)}\left(G, R, \mathbf{x}^*_{-\sigma}, \left(\breve{x}_{\sigma(j)}\left(G, R, \mathbf{x}^*_{-\sigma}\right)\right)_{1\leq j\leq s}\right).$$

*(ii) Under Assumptions **A1-A6** and Conditions **C1** and **C3**, the same holds for any bilaterally efficient equilibrium involving a complete network of upstream agreements.*

Proof. See Appendix. ■

This proposition shows that, in any complete network of bilaterally efficient upstream agreements, an upstream deviation by a coalition of firms is unprofitable for any coalition of size $s \in \{3, ..., n-1\}$. This, combined with the definition of bilateral efficiency and Proposition 1 implies that a bilaterally efficient complete network is robust to upstream deviations by any coalition of firms. The intuition behind this result is that in a bilaterally efficient equilibrium involving a complete network, a firm's downstream action maximizes *each* other firm's profit (see Lemma 2). This implies that a firm's downstream action maximizes the joint profit of *any* coalition of firms. Therefore, a bilaterally efficient equilibrium involving a complete network is robust to upstream deviations by any coalition, as the only purpose of such deviations is to affect downstream actions.

4.2 Public agreements

In this section, we suppose that (first-stage) upstream agreements become *public* after they are made, i.e., they are observed by all firms before they choose their (second-stage) downstream actions.

We maintain Assumptions **A1-A2** on the payoff functions but replace assumption **A3** with the following assumption that takes account of the public nature of upstream agreements:

A3' For any network of upstream agreements (G, R, F), the n-player downstream game has a unique Nash equilibrium, i.e., there exists a unique vector of actions $\hat{\mathbf{x}}(G, R)$ such that

$$\hat{x}_i(G, R) \in \arg\max_{x_i} \pi_i(G, R, \hat{\mathbf{x}}_{-i}(G, R), x_i)$$

for all $i \in \{1, ..., n\}$.

We also need to adapt our equilibrium concept as follows.

Definition 2 *(Bilaterally efficient equilibrium: public contracting) A bilaterally efficient equilibrium in the case of public upstream contracting is an equilibrium network of public upstream agreements* (G^*, R^*, F^*) *together with an equilibrium vector of actions* $\mathbf{x}^* = (x_i^*)_{1 \leq i \leq n}$ *, such that:*

- in Stage 2, the equilibrium action of each firm maximizes its profit given the actions of all other firms: $x_i^* \in \arg\max_{x_i} \pi_i\left(G^*, R^*, \mathbf{x}_{-i}^*\right)$*;*

- in Stage 1, the equilibrium upstream agreement between any pair of firms i *and* j *maximizes their joint profit given all other upstream agreements and the induced downstream actions*

of all firms: for any $(i,j) \in \{1,...,n\}^2$ *such that* $i \neq j$ *and any* $(G,R,F) \in D_{ij}(G^*,R^*,F^*)$,

$$(\pi_i + \pi_j)(G^*,R^*,\mathbf{x}^*) \geq (\pi_i + \pi_j)(G,R,\hat{\boldsymbol{x}}(G,R)).$$

Finally, while we maintain Condition **C1** in its current form, we need to modify Condition **C2** to account for the fact that an upstream bilateral deviation now leads to a (potential) change in the actions of *all* firms and not only the deviating pair of firms. Specifically, we replace it with the following condition:

C2' For any pair of firms $(i,j) \in \{1,..,N\}$ satisfying $i \neq j$ and for any network of upstream agreements (G,R,F) such that $g_{ij} = 0$, there exists a network of upstream agreements $(G',R',F') \in D_{ij}(G,R,F)$ with $g'_{ij} = 1$ such that $\hat{x}_k(G',R') = \hat{x}_k(G,R)$ for any $k \in \{1,...,n\}$.

Condition **C2'** seems *a priori* more demanding than Condition **C2** because it requires that the downstream equilibrium actions of *all* firms be preserved. However, this is not necessarily the case. To illustrate that, let us consider the special, but relevant, scenario in which for any $(G,R,\mathbf{x})$ and any i,j,k such that $i \neq j$ and $k \notin \{i,j\}$, $\pi_k(G,R,\mathbf{x})$ does not depend on g_{ij}. In other words, for *given* downstream actions, a firm's payoff does not depend on whether two other firms have signed an upstream bilateral contract. This is, for instance, the case if access to an upstream input affects only a firm's cost function (as is the case for a cost-reducing patented innovation). In this scenario, if moving from network (G,R,F) to a network $(G',R',F') \in D_{ij}(G,R,F)$ does not affect the equilibrium downstream actions of firms i and j, then it does not affect the downstream equilibrium actions of *any* firm.[21] This implies that **C2'** is not more demanding than **C2** in the sense that it is sufficient that a network $(G',R',F') \in D_{ij}(G,R,F)$ preserves the equilibrium downstream actions of firms i and j for **C2'** to be satisfied.

It is straightforward to derive the following lemma, which is the counterpart of Lemma 1 for the case of public upstream agreements.

Lemma 3 *Under Assumptions **A1, A2** and **A3'** and Conditions **C1** and **C2'**, an equilibrium network of public upstream agreements* (G^*,R^*,F^*) *is necessarily such that* $G^* = G^c$.

[21] To see why, note first that Assumptions **A1-A2** combined with the assumption that for *given* downstream actions, a firm's payoff does not depend on whether two other firms have signed a bilateral upstream contract, imply that the best-response function of a firm $l \notin \{i,j\}$ does not depend on the agreement between firms i and j, i.e., it does not depend on $(g_{ij}, r_{i\to j}, r_{j\to i})$. This implies that moving from network (G,R,F) to a network $(G',R',F') \in D_{ij}(G,R,F)$ affects the equilibrium downstream action of a firm $l \notin \{i,j\}$ only through the effect on the equilibrium actions of firms i and j. Thus, if the latter are preserved, then the equilibrium actions of all firms remain unchanged.

As in the case of private agreements, we focus on complete networks, and we provide a necessary condition for a network of this type to be an equilibrium network. To this end, we need to adapt the rank condition **C3**, meaning that per-unit input prices are pairwise independent from the public nature of upstream agreements. For this purpose, let us define $M^{public}(R)$ as the $n^2 \times n^2$ matrix whose elements are given by

$$M^{public}_{n(i-1)+j,n(l-1)+k}(R) = \begin{cases} \frac{\partial \hat{x}_k}{\partial r_{i \to j}} & \text{if} \quad l = i \text{ and } i \neq j \\ \frac{\partial \hat{x}_k}{\partial r_{i \to j}} & \text{if} \quad l = j \text{ and } i \neq j \\ 1 & \text{if} \quad l = k = i = j \\ 0 & \text{otherwise} \end{cases}$$

for any $i, j, l, k \in \{1, ..., n\}$ where the elements of the matrix are all evaluated at (G^c, R). The counterpart of Condition **C3** in the context of public agreements is the following:

C3' $\det M^{public}(R) \neq 0$ for any R.

Similar to the case of private upstream agreements, this condition ensures that any local downstream deviation by a pair of firms can be obtained through a local upstream deviation in the per-unit input prices they pay each other. The reason that Condition **C3'** is less simple than Condition **C3** is that a two-firm coalition contemplating a deviation now has to take into account the responses of the firms *outside* the coalition in the second stage of the game. Example 3 in the Appendix shows that Conditions **C1**, **C2'** and **C3'** are satisfied in the cross-licensing setting considered in Example 1 for all of the values of the substitutability parameter but one when upstream agreements are public and the number of firms is $n = 3$.

The following lemma is the counterpart of Lemma 2 when upstream agreements are public.

Lemma 4 *Suppose that Assumptions **A1**, **A2** and **A3'** and Condition **C3'** hold. Then, a necessary condition for a complete network of upstream agreements $(G^c, R, \mathbf{x})$ to be an equilibrium network is that*

$$\frac{\partial \pi_i}{\partial x_j}(G^c, R, \mathbf{x}) = 0,$$

for any $(i, j) \in \{1, ..., n\}^2$ such that $i \neq j$.

Proof. See Appendix. ■

Maintaining Assumption **A4** as it is, we can now extend our main result to the case of public upstream agreements.

Proposition 3 *(i) Under Assumptions **A1**, **A2**, **A3'**, and **A4** and Conditions **C1**, **C2'**, and **C3'**, any bilaterally efficient equilibrium maximizes industry profits.*

*(ii) Moreover, under Assumptions **A1**, **A2**, **A3'**, and **A4** and Conditions **C1** and **C3'**, any bilaterally efficient equilibrium involving a complete network of upstream agreements maximizes industry profits.*

Proof. Similar to the proof of Proposition 1. ■

Thus, our main result does not depend on the private nature of upstream bilateral agreements. Public bilateral agreements can also allow them to achieve the fully cooperative outcome if the input prices are independent instruments. It can also be readily shown that this result extends to an environment in which some of the agreements are private while others are public, as well as a setting in which each agreement becomes public with a certain probability (because of a leakage, for instance).[22]

5 Conclusion

This paper shows that under a wide range of circumstances, upstream bilateral agreements among firms interacting in a downstream market can allow them to achieve the industry-profit-maximizing outcome. This result has been shown to hold under relatively mild conditions, independent of the nature of downstream interactions, regardless of whether the agreements are public or private and regardless of whether the firms are symmetric.

We have focused in our model on the scenario wherein all firms own an upstream input. However, our analysis can be extended in a straightforward way to a situation in which only a subset of firms own upstream inputs and can engage in (private) upstream bilateral agreements. In that case, it can be shown that under conditions of the same nature as those used in our analysis, upstream bilateral agreements between these (vertically integrated) firms allow them to achieve an outcome that maximizes the joint profits of all of them, given the downstream actions of all firms that do not own an upstream input.

Another (perhaps simpler) way for firms to achieve the outcome that maximizes industry profits is to sign a *multilateral* upstream agreement involving all of them. However, such agreements are sometimes regarded with suspicion by regulators. For instance, a cross-licensing agreement involving *more than two* firms in a given industry cannot benefit from the safe harbor provided by the European Commission's Technology Transfer Block Exemption Regulation. Our main result suggests that policymakers should not regard a complete network of

[22] The matrix $M^{public}(R)$ needs, however, to be adapted to these alternative settings.

upstream bilateral agreements between firms interacting in *both* upstream and downstream markets to be fundamentally different from that of a multilateral upstream agreement involving all of them. In particular, this implies that if such a multilateral agreement is deemed undesirable by policymakers, e.g., because the fully cooperative outcome reduces social welfare, then it should also be the case for a complete network of upstream bilateral agreements.

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Appendix

Example 1. Access by any firm to the technology of another firm leads to a strict decrease in its marginal cost. This implies that Condition **C1** is satisfied.

Let us now show that Condition **C2** holds. Let (G, R, F) be a network of upstream bilateral agreements such that $g_{ij} = 0$. Let $S_i(G)$ denote the set of firms that signed a cross-licensing contract with firm i, i.e., firms $k \neq i$ such that $g_{ik} = 1$, and let $m_i(G)$ denote the number of firms in $S_i(G)$. Similarly, let $S_j(G)$ denote the set of firms that signed a cross-licensing contract with firm j, and $m_j(G)$ denote the number of firms in $S_j(G)$. Moreover, let $\Delta_i(G) \equiv c_i(m_i(G)) - c_i(m_i(G)+1)$ and $\Delta_j(G) \equiv c_j(m_j(G)) - c_j(m_j(G)+1)$.

Under (G, R, F) and for a given vector $\mathbf{x}_{-ij}$ of actions of firms $k \neq i, j$, the actions of firms i and j satisfy the following equalities:

$$x_i = \frac{1}{2}\left[v(1-\gamma) + \gamma \sum_{k \neq i} x_k + c_i(m_i(G)) + \sum_{k \in S_i(G)} r_{i \to k} + \gamma \sum_{k \in S_i(G)} r_{k \to i}\right] \tag{1}$$

$$x_j = \frac{1}{2}\left[v(1-\gamma) + \gamma \sum_{k \neq j} x_k + c_j(m_j(G)) + \sum_{k \in S_j(G)} r_{j \to k} + \gamma \sum_{k \in S_j(G)} r_{k \to j}\right] \tag{2}$$

For any network $(G', R', F') \in D_{ij}(G, R, F)$ with $g'_{ij} = 1$ and for a given vector $\mathbf{x}_{-ij}$ of actions of firms $k \neq i, j$, the actions of firms i and j satisfy the following equalities:

$$x_i = \frac{1}{2}\left[v(1-\gamma) + \gamma\sum_{k\neq i} x_k + c_i(m_i(G)+1) + \sum_{k\in S_i(G)} r_{i\to k} + r'_{i\to j} + \gamma\sum_{k\in S_i(G)} r_{k\to i} + \gamma r'_{j\to i}\right] \quad (3)$$

$$x_j = \frac{1}{2}\left[v(1-\gamma) + \gamma\sum_{k\neq j} x_k + c_j(m_j(G)+1) + \sum_{k\in S_j(G)} r_{j\to k} + r'_{j\to i} + \gamma\sum_{k\in S_j(G)} r_{k\to j} + \gamma r'_{i\to j}\right] \quad (4)$$

To show that **C2** holds, it is sufficient to show that there exist $(r'_{i\to j}, r'_{j\to i})$ such that equations (1) and (3) are identical and equations (2) and (4) are identical. This is the case if and only if

$$c_i(m_i(G)) = c_i(m_i(G)+1) + r'_{i\to j} + \gamma r'_{j\to i}$$

and

$$c_j(m_j(G)) = c_j(m_j(G)+1) + r'_{j\to i} + \gamma r'_{i\to j},$$

or equivalently,

$$(r'_{i\to j}, r'_{j\to i}) = \left(\frac{\Delta_i - \gamma\Delta_j}{1-\gamma^2}, \frac{\Delta_j - \gamma\Delta_i}{1-\gamma^2}\right).$$

Hence, Condition **C2** holds.

Finally, let us show that Condition **C3** holds. Under a complete network of upstream agreements, firm i's profit (gross of fixed payments) is given by

$$\begin{aligned}
\pi_i(G^c, R, \mathbf{x}) &= \left(x_i - \underline{c}_i - \sum_{j\neq i} r_{i\to j}\right) q_i + \sum_{j\neq i} r_{j\to i} q_j \\
&= \frac{1}{1-\gamma^2}\left(x_i - \underline{c}_i - \sum_{j\neq i} r_{i\to j}\right)\left[v(1-\gamma) - x_i + \gamma\sum_{j\neq i} x_j\right] \\
&\quad + \frac{1}{1-\gamma^2}\sum_{j\neq i} r_{j\to i}[v(1-\gamma) - x_j + \gamma\sum_{k\neq j} x_k].
\end{aligned}$$

Differentiating firm i's profit with respect to x_i yields

$$x_i = \frac{1}{2}\left[v(1-\gamma) + \gamma\sum_{j\neq i} x_j + \underline{c}_i + \sum_{j\neq i} r_{i\to j} + \gamma\sum_{j\neq i} r_{j\to i}\right].$$

The pair $(\tilde{x}_i(G^c, R, \mathbf{x}_{-ij}), \tilde{x}_j(G^c, R, \mathbf{x}_{-ij}))$ is the solution of the following two-equation system:

$$\begin{cases} x_i = \frac{1}{2}\left[v(1-\gamma) + \gamma x_j + \gamma \sum_{k\neq i,j} x_k + \underline{c}_i + r_{i\to j} + \sum_{k\neq i,j} r_{i\to k} + \gamma r_{j\to i} + \sum_{k\neq i,j} r_{k\to i}\right] \\ x_j = \frac{1}{2}\left[v(1-\gamma) + \gamma x_i + \gamma \sum_{k\neq i,j} x_k + \underline{c}_j + r_{j\to i} + \sum_{k\neq i,j} r_{j\to k} + \gamma r_{i\to j} + \sum_{k\neq i,j} r_{k\to j}\right] \end{cases}$$

and therefore,

$$\begin{cases} \frac{\partial \tilde{x}_i}{\partial r_{i\to j}} = \frac{1}{2}\left[\gamma \frac{\partial \tilde{x}_j}{\partial r_{i\to j}} + 1\right] \\ \frac{\partial \tilde{x}_j}{\partial r_{i\to j}} = \frac{1}{2}\left[\gamma \frac{\partial \tilde{x}_i}{\partial r_{i\to j}} + \gamma\right] \end{cases}$$

which leads to

$$\frac{\partial \tilde{x}_i}{\partial r_{i\to j}} = \frac{2+\gamma^2}{4-\gamma^2} \text{ and } \frac{\partial \tilde{x}_j}{\partial r_{i\to j}} = \frac{3\gamma}{4-\gamma^2},$$

hence, the following expression for $M_{i,j}^{private}$:

$$M_{ij}^{private} = \begin{pmatrix} \frac{2+\gamma^2}{4-\gamma^2} & \frac{3\gamma}{4-\gamma^2} \\ \frac{3\gamma}{4-\gamma^2} & \frac{2+\gamma^2}{4-\gamma^2} \end{pmatrix}.$$

This implies that

$$\det M_{ij}^{private} = \frac{\gamma^4 - 5\gamma^2 + 4}{(4-\gamma^2)^2} = \frac{1-\gamma^2}{4-\gamma^2} \neq 0,$$

i.e., Condition **C3** holds.

Example 2. Let $\mathbf{v} \equiv (v_1, ..., v_N)$ be the vector representing the value of each firm's product after Stage 1 (for a given network of upstream agreements). Let $\underline{v} \equiv \min_{1\leq i \leq \leq n} \underline{v}_i$.

We define Cournot competition for a given $\mathbf{v} \equiv (v_1, ..., v_N)$ as follows. Each firm i simultaneously chooses its quantity x_i. Given $\mathbf{v} \equiv (v_1, ..., v_N)$, $\mathbf{x} \equiv (x_1, ..., x_N)$ and $X = x_1 + ... + x_n$, the quality-adjusted equilibrium prices are determined by the following two conditions: (i) the indifference condition:

$$v_i - p_i = v_j - p_j \qquad \text{for all } (i,j) \in \{1, ..., N\}^2;$$

and (ii) the market clearing condition:

$$X = D(p) \text{ where } p_i = p + v_i - \underline{v}.$$

The market clearing condition means that this price is adjusted to make the total supply equal to demand. The indifference condition implies that the price each firm charges is adjusted such that all consumers who buy any product are indifferent among all products. A microfoundation of this setup can be provided as follows. There is a mass one of consumers. Each consumer has unit demand and hence buys at most one unit among all products. A consumer's gross utility from having a unit of the product of firm i is given by $u + v_i$, where u is specific to the consumer while v_i is common to all consumers. Let $F(u)$ represent the cumulative distribution function of u. Then, by the construction of quality-adjusted prices, any consumer is indifferent among all products, and the marginal consumer indifferent between buying any product and not buying is characterized by $u + \underline{v} - p = 0$, implying

$$D(p) = 1 - F(p - \underline{v}).$$

In equilibrium, p is adjusted such that $1 - F(p - \underline{v}) = X$. Let $P(X)$ be the inverse demand function, and let us assume that it is linear, i.e., $P(X) = a - X$. Hence, denoting by $S_i(G)$ the set of firms that signed an upstream contract with firm i, i.e., firms $k \neq i$ such that $g_{ik} = 1$, and $m_i(G)$ the number of firms in $S_i(G)$, firm i's profit (gross of fixed payments) is given by

$$\pi_i\left(G, R, \mathbf{x}\right) = \left(a - X + v_i\left(G\right) - \underline{v} - c_i - \sum_{k \in S_i(G)} r_{i \to k}\right) x_i + \sum_{k \in S_i(G)} r_{k \to i} x_k.$$

where we now make the dependence of v_i upon G explicit. Since access to an additional dataset strictly increases the value of its product, it is straightforward that Condition **C1** is satisfied.

Denoting $a_i(G) = a + v_i(G) - \underline{v} - c_i$, firm i's profit (gross of fixed payments) can then be written as

$$\pi_i\left(G, R, \mathbf{x}\right) = \left(a_i(G) - \sum_{k \in S_i(G)} r_{i \to k} - X\right) x_i + \sum_{k \in S_i(G)} r_{k \to i} x_k.$$

Assume now that $g_{ij} = 0$ for some $(i, j) \in \{1, ..., n\}^2$. The FOCs associated with the maximization of $\pi_i\left(G, R, \mathbf{x}\right)$ with respect to x_i and the maximization of $\pi_j\left(G, R, \mathbf{x}\right)$ with respect to x_j are respectively given by

$$x_i = \frac{1}{2}\left(a_i(G) - \sum_{k \in S_i(G)} r_{i \to k} - x_j - \sum_{k \neq i,j} x_k\right) \tag{5}$$

and

$$x_j = \frac{1}{2}\left(a_j(G) - \sum_{k\in S_j(G)} r_{j\to k} - x_i - \sum_{k\neq i,j} x_k\right) \tag{6}$$

Similarly, for any network $(G', R', F') \in D_{ij}(G, R, F)$ with $g'_{ij} = 1$, the FOCs associated with the maximization of $\pi_i(G', R', \mathbf{x})$ with respect to x_i and the maximization of $\pi_j(G', R', \mathbf{x})$ with respect to x_j are respectively given by

$$x_i = \frac{1}{2}\left(a_i(G') - r'_{i\to j} - \sum_{k\in S_i(G)} r_{i\to k} - x_j - \sum_{k\neq i,j} x_k\right) \tag{7}$$

and

$$x_j = \frac{1}{2}\left(a_j(G') - r'_{j\to i} - \sum_{k\in Sj(G)} r_{j\to k} - x_i - \sum_{k\neq i,j} x_k\right). \tag{8}$$

This shows that a necessary and sufficient condition for equations (5) and (7) to be identical and equations (6) and (8) to be identical is that

$$r'_{i\to j} = a_i(G') - a_i(G) \ \text{ and } r'_{j\to i} = a_j(G') - a_j(G),$$

which proves that Condition **C2** holds.

Let us now show that Condition **C3** also holds. Considering a complete network of upstream agreements (G^c, R, F), we have

$$\pi_i(G^c, R, \mathbf{x}) = \left(a_i(G^c) - \sum_{k\neq i} r_{i\to k} - X\right) x_i + \sum_{k\neq i} r_{k\to i} x_k.$$

The pair $(\tilde{x}_i(G^c, R, \mathbf{x}_{-ij}), \tilde{x}_j(G^c, R, \mathbf{x}_{-ij}))$ is the solution of the following two-equation system:

$$\begin{cases} x_i = \frac{1}{2}\left(a_i(G^c) - \sum_{k\neq i,j} x_k - \sum_{k\neq i} r_{i\to k} - x_j\right) \\ x_j = \frac{1}{2}\left(a_j(G^c) - \sum_{k\neq i,j} x_k - \sum_{k\neq j} r_{j\to k} - x_i\right) \end{cases}$$

Therefore,

$$\begin{cases} \frac{\partial \tilde{x}_i}{\partial r_{i\to j}} = -\frac{1}{2}\left(\frac{\partial \tilde{x}_j}{\partial r_{i\to j}} + 1\right) \\ \frac{\partial \tilde{x}_j}{\partial r_{i\to j}} = -\frac{1}{2}\frac{\partial \tilde{x}_i}{\partial r_{i\to j}} \end{cases}$$

which yields

$$\frac{\partial \tilde{x}_i}{\partial r_{i\to j}} = -\frac{2}{3} \text{ and } \frac{\partial \tilde{x}_j}{\partial r_{i\to j}} = \frac{1}{3}.$$

Hence,

$$M_{ij}^{private} = \begin{pmatrix} -\frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} \end{pmatrix}.$$

This implies that

$$\det M_{ij}^{private} = \frac{1}{3} \neq 0,$$

i.e., Condition **C3** holds.

Proof of Lemma 2. Suppose that Assumptions **A1-A3** and Condition **C3** hold, and assume that the complete network of upstream agreements $(G^c, R^*, \mathbf{x}^*)$ is an equilibrium network. Then, for any given $(i,j) \in \{1,...,n\}^2$ such that $i \neq j$, it must hold that

$$(\pi_i + \pi_j)(G^c, R^*, \mathbf{x}^*) \geq (\pi_i + \pi_j)\left(G^c, R, \tilde{x}_i(G^c, R, \mathbf{x}^*_{-ij}), \tilde{x}_j(G^c, R, \mathbf{x}^*_{-ij}), \mathbf{x}^*_{-ij}\right).$$

for any $(G^c, R, F) \in D_{ij}(G^c, R^*, F^*)$.

From **A3** and our assumption that $(G^c, R^*, \mathbf{x}^*)$ is an equilibrium network, it follows that $\tilde{x}_i(G^c, R^*, \mathbf{x}^*_{-ij}) = x_i^*$ and $\tilde{x}_j(G^c, R^*, \mathbf{x}^*_{-ij}) = x_j^*$, which implies that

$$(\pi_i + \pi_j)\left(G^c, R^*, \tilde{x}_i(G^c, R^*, \mathbf{x}^*_{-ij}), \tilde{x}_j(G^c, R^*, \mathbf{x}^*_{-ij}), \mathbf{x}^*_{-ij}\right) = (\pi_i + \pi_j)(G^c, R^*, \mathbf{x}^*).$$

Hence,

$$\begin{aligned}&(\pi_i + \pi_j)\left(G^c, R^*, \tilde{x}_i(G^c, R^*, \mathbf{x}^*_{-ij}), \tilde{x}_j(G^c, R^*, \mathbf{x}^*_{-ij}), \mathbf{x}^*_{-ij}\right)\\ &\geq (\pi_i + \pi_j)\left(G^c, R, \tilde{x}_i(G^c, R, \mathbf{x}^*_{-ij}), \tilde{x}_j(G^c, R, \mathbf{x}^*_{-ij}), \mathbf{x}^*_{-ij}\right)\end{aligned}$$

for any $(G^c, R, F) \in D_{ij}(G^*, R^*, F^*)$. Considering the special case of matrices R that are different from R^* only through $r_{i\to j}$, the latter inequality implies that $(\pi_i + \pi_j)\left(G^c, R, \tilde{x}_i(G^c, R, \mathbf{x}^*_{-ij}), \tilde{x}_j(G^c, R, \mathbf{x}^*_{-ij}), \mathbf{x}^*_{-ij}\right)$ is maximized at $r_{i\to j} = r^*_{i\to j}$, which requires that[23]

$$\frac{d(\pi_i + \pi_j)}{dr_{i\to j}}\left(G^c, R, \tilde{x}_i(G^c, R, \mathbf{x}^*_{-ij}), \tilde{x}_j(G^c, R, \mathbf{x}^*_{-ij}), \mathbf{x}^*_{-ij}\right) = 0.$$

Since $r_{i\to j}$ affects $(\pi_i + \pi_j)\left(G^c, R, \tilde{x}_i(G^c, R, \mathbf{x}^*_{-ij}), \tilde{x}_j(G^c, R, \mathbf{x}^*_{-ij}), \mathbf{x}^*_{-ij}\right)$ only through its effect

[23] We use the notation $\frac{d(\pi_i+\pi_j)}{dr_{i\to j}}$ instead of $\frac{\partial(\pi_i+\pi_j)}{\partial r_{i\to j}}$ to emphasize that we consider the total derivative of $\pi_i + \pi_j$ with respect to $r_{i\to j}$.

on $\tilde{x}_i(G^c, R, \mathbf{x}^*_{-ij})$ and $\tilde{x}_j(G^c, R, \mathbf{x}^*_{-ij})$ (this follows from **A2**), the latter can be rewritten as

$$\frac{\partial \pi_i}{\partial x_i}\frac{\partial \tilde{x}_i}{\partial r_{i\to j}} + \frac{\partial \pi_j}{\partial x_i}\frac{\partial \tilde{x}_i}{\partial r_{i\to j}} + \frac{\partial \pi_i}{\partial x_j}\frac{\partial \tilde{x}_j}{\partial r_{i\to j}} + \frac{\partial \pi_j}{\partial x_j}\frac{\partial \tilde{x}_j}{\partial r_{i\to j}} = 0, \tag{9}$$

where $\frac{\partial \tilde{x}_i}{\partial r_{i\to j}}$ and $\frac{\partial \tilde{x}_j}{\partial r_{i\to j}}$ are evaluated at $\left(G^c, R^*, \mathbf{x}^*_{-ij}\right)$, and $\frac{\partial \pi_i}{\partial x_i}$, $\frac{\partial \pi_i}{\partial x_j}$, $\frac{\partial \pi_j}{\partial x_i}$, $\frac{\partial \pi_j}{\partial x_j}$ are evaluated at $\left(G^c, R^*, \tilde{x}_i(G^c, R^*, \mathbf{x}^*_{-ij}), \tilde{x}_j(G^c, R^*, \mathbf{x}^*_{-ij}), \mathbf{x}^*_{-ij}\right) = (G^c, R^*, \mathbf{x}^*)$.

Moreover, since $(G^c, R^*, \mathbf{x}^*)$ is an equilibrium network, it holds that

$$\frac{\partial \pi_i}{\partial x_i}(G^c, R^*, \mathbf{x}^*) = \frac{\partial \pi_j}{\partial x_j}(G^c, R^*, \mathbf{x}^*) = 0. \tag{10}$$

Combining (9) and (10) leads to

$$\frac{\partial \tilde{x}_j}{\partial r_{i\to j}}\frac{\partial \pi_i}{\partial x_j} + \frac{\partial \tilde{x}_i}{\partial r_{i\to j}}\frac{\partial \pi_j}{\partial x_i} = 0, \tag{11}$$

where $\frac{\partial \tilde{x}_j}{\partial r_{i\to j}}$ and $\frac{\partial \tilde{x}_i}{\partial r_{i\to j}}$ are evaluated at $\left(G^c, R^*, \mathbf{x}^*_{-ij}\right)$ and $\frac{\partial \pi_i}{\partial x_j}$ and $\frac{\partial \pi_j}{\partial x_i}$ are evaluated at $(G^c, R^*, \mathbf{x}^*)$.

By symmetry, we also have

$$\frac{\partial \tilde{x}_i}{\partial r_{j\to i}}\frac{\partial \pi_j}{\partial x_i} + \frac{\partial \tilde{x}_j}{\partial r_{j\to i}}\frac{\partial \pi_i}{\partial x_j} = 0, \tag{12}$$

where $\frac{\partial \tilde{x}_i}{\partial r_{j\to i}}$ and $\frac{\partial \tilde{x}_j}{\partial r_{j\to i}}$ are evaluated at $\left(G^c, R^*, \mathbf{x}^*_{-ij}\right)$ and $\frac{\partial \pi_i}{\partial x_j}$ and $\frac{\partial \pi_j}{\partial x_i}$ are evaluated at $(G^c, R^*, \mathbf{x}^*)$.

Denoting $y_{ij} = \frac{\partial \pi_i}{\partial x_j}(G^c, R^*, \mathbf{x}^*)$ and $y_{ji} = \frac{\partial \pi_j}{\partial x_i}(G^c, R^*, \mathbf{x}^*)$, (11) and (12) can be rewritten as a linear system of two equations in y_{ji} and y_{ij} :

$$\begin{cases} \frac{\partial \tilde{x}_i}{\partial r_{i\to j}} y_{ji} + \frac{\partial \tilde{x}_j}{\partial r_{i\to j}} y_{ij} = 0 \\ \frac{\partial \tilde{x}_i}{\partial r_{j\to i}} y_{ji} + \frac{\partial \tilde{x}_j}{\partial r_{j\to i}} y_{ij} = 0 \end{cases}$$

Condition **C3** says that the determinant of this system is different from zero, which implies that the unique solution is $y_{ji} = y_{ij} = 0$.

Therefore, we can conclude that

$$\frac{\partial \pi_i}{\partial x_j}(G^c, R^*, \mathbf{x}^*) = 0,$$

for any $(i, j) \in \{1, ..., n\}^2$such that $i \neq j$.

Proof of Proposition 1. Suppose that **A1-A4**, **C1** and **C3** hold and assume that $(G^c, R^*, F^*, \mathbf{x}^*)$ is a bilaterally efficient equilibrium (recall that under additional assumption **C2**, any equilibrium network of upstream agreements is complete). From Lemma 2, it follows that

$$\sum_{i=1, i\neq j}^{n} \frac{\partial \pi_i}{\partial x_j}(G^c, R^*, \mathbf{x}^*) = 0.$$

This, combined with $\frac{\partial \pi_j}{\partial x_j}(G^c, R^*, \mathbf{x}^*) = 0$ (which results from $\mathbf{x}^*$ being an equilibrium vector of actions), yields

$$\sum_{i=1}^{n} \frac{\partial \pi_i}{\partial x_j}(G^c, R^*, \mathbf{x}^*) = 0.$$

Since $\mathbf{x}^c$ is the unique maximizer of industry profits and is characterized by the corresponding first-order conditions (by **A4**), we can conclude that $\mathbf{x}^* = \mathbf{x}^c$, which means that the considered equilibrium maximizes industry profits.

Proof of Proposition 2. Suppose that **A1-A6**, **C1** and **C3** hold, and assume that $(G^c, R^*, \mathbf{x}^*)$ is an equilibrium network of upstream agreements (recall again that under additional assumption **C2**, any equilibrium network of upstream agreements is complete). Consider $s \in \{3, ..., n-1\}$ and $\sigma \in \Gamma_s$. From Lemma 2, we know that

$$\frac{\partial \pi_i}{\partial x_j}(G^c, R^*, \mathbf{x}^*) = 0$$

for any $(i, j) \in \{1, ..., n\}^2$ with $i \neq j$. This, combined with $\frac{\partial \pi_i}{\partial x_i}(G^c, R^*, \mathbf{x}^*) = 0$ (which results from $\mathbf{x}^*$ being an equilibrium vector of actions), implies that for any $s \in \{3, ..., n-1\}$, any $\sigma \in \Gamma_s$ and any $l \in \{1, ..., s\}$,

$$\sum_{k=1}^{s} \frac{\partial \pi_{\sigma(k)}}{\partial x_{\sigma(l)}}(G^c, R^*, \mathbf{x}^*) = 0,$$

or equivalently,

$$\sum_{k=1}^{s} \frac{\partial \pi_{\sigma(k)}}{\partial x_{\sigma(l)}}\left(G^c, R^*, (x^*_{\sigma(i)})_{1\leq i\leq s}, \mathbf{x}^*_{-\sigma}\right) = 0.$$

Then, from Assumption **A6**, it follows that $(x^*_{\sigma(i)})_{1\leq i\leq s}$ maximizes the joint profit of the firms in the coalition $\{\sigma(1), ..., \sigma(s)\}$ given that the actions of the firms outside the coalition are given by $\mathbf{x}^*_{-\sigma}$. In other words,

$$\sum_{1\leq k\leq s} \pi_{\sigma(k)} \left(G^c, R^*, \mathbf{x}^*\right) \geq \sum_{1\leq k\leq s} \pi_{\sigma(k)} \left(G^c, R^*, (x'_{\sigma(i)})_{1\leq i\leq s}, \mathbf{x}^*_{-\sigma}\right)$$

for any $(x'_{\sigma(i)})_{1\leq i\leq s}$. In particular,

$$\sum_{1\leq k\leq s} \pi_{\sigma(k)} \left(G^c, R^*, \mathbf{x}^*\right) \geq \sum_{1\leq k\leq s} \pi_{\sigma(k)} \left(G^c, R^*, \left(\breve{x}_{\sigma(i)} \left(G, R, \mathbf{x}^*_{-\sigma}\right)\right)_{1\leq i\leq s}, \mathbf{x}^*_{-\sigma}\right)$$

for any $(G, R, F) \in D_\sigma \left(G^c, R^*, F^*\right)$. Moreover, for any $(G, R, F) \in D_\sigma \left(G^c, R^*, F^*\right)$, we have

$$\begin{aligned} \sum_{1\leq k\leq s} \pi_{\sigma(k)} \left(G^c, R^*, \left(\breve{x}_{\sigma(j)} \left(G, R, \mathbf{x}^*_{-\sigma}\right)\right)_{1\leq j\leq s}, \mathbf{x}^*_{-\sigma}\right) &= \sum_{1\leq k\leq s} \pi_{\sigma(k)} \left(G^c, R, \left(\breve{x}_{\sigma(i)} \left(G, R, \mathbf{x}^*_{-\sigma}\right)\right)_{1\leq i\leq s}, \mathbf{x}^*_{-\sigma}\right) \\ &\geq \sum_{1\leq k\leq s} \pi_{\sigma(k)} \left(G, R, \left(\breve{x}_{\sigma(i)} \left(G, R, \mathbf{x}^*_{-\sigma}\right)\right)_{1\leq i\leq s}, \mathbf{x}^*_{-\sigma}\right) \end{aligned}$$

where the equality follows from **A2** and the inequality from **C1**. Therefore, for any $(G, R, F) \in D_\sigma \left(G^c, R^*, F^*\right)$,

$$\sum_{1\leq k\leq s} \pi_{\sigma(k)} \left(G^c, R^*, \mathbf{x}^*\right) \geq \sum_{1\leq k\leq s} \pi_{\sigma(k)} \left(G, R, \left(\breve{x}_{\sigma(i)} \left(G, R, \mathbf{x}^*_{-\sigma}\right)\right)_{1\leq i\leq s}, \mathbf{x}^*_{-\sigma}\right).$$

Example 3. Consider the cross-licensing setting presented in Example 1 with $n = 3$ firms, and assume that upstream agreements are public.

We have already shown that Condition **C1** is satisfied in that setting. Let us now show that Condition **C2'** holds. To this end, consider an upstream network of agreements (G, R, F) such that $g_{12} = 0$. The equilibrium downstream $\hat{\mathbf{x}}(G, R)$ solves for the following system of equations:

$$\begin{cases} x_1 = \frac{1}{2}\left[v(1-\gamma) + \gamma\left(x_2 + x_3\right) + c_1(1 + \mathbf{1}_{g_{13}=1}) + (r_{1\to 3} + \gamma r_{3\to 1})\mathbf{1}_{g_{13}=1}\right] \\ x_2 = \frac{1}{2}\left[v(1-\gamma) + \gamma\left(x_1 + x_3\right) + c_2(1 + \mathbf{1}_{g_{23}=1}) + (r_{2\to 3} + \gamma r_{3\to 2})\mathbf{1}_{g_{23}=1}\right] \\ \quad x_3 = \frac{1}{2}\left[v(1-\gamma) + \gamma\left(x_1 + x_2\right) + c_3(1 + \mathbf{1}_{g_{31}=1} + \mathbf{1}_{g_{32}=1}) \right. \\ \quad\quad \left. +(r_{3\to 1} + \gamma r_{1\to 3})\mathbf{1}_{g_{31}=1} + (r_{3\to 2} + \gamma r_{2\to 3})\mathbf{1}_{g_{32}=1}\right] \end{cases}$$

Consider now a network of upstream agreements $(G', R', F') \in D_{12}(G, R, F)$ with $g'_{12} = 1$. It is straightforward that the corresponding equilibrium downstream actions $\hat{\mathbf{x}}(G', R')$ solves the set of equations above if and only if

$$\begin{cases} c_1(2+\mathbf{1}_{g_{13}=1}) + r'_{1\to 2} + \gamma r'_{2\to 1} = c_1(1+\mathbf{1}_{g_{13}=1}) \\ c_2(2+\mathbf{1}_{g_{23}=1}) + r'_{2\to 1} + \gamma r'_{1\to 2} = c_2(1+\mathbf{1}_{g_{23}=1}) \end{cases}$$

which holds if

$$\begin{cases} r'_{1\to 2} = \frac{c_1(1+\mathbf{1}_{g_{13}=1}) - c_1(2+\mathbf{1}_{g_{13}=1}) - \gamma\left[c_2(1+\mathbf{1}_{g_{23}=1}) - c_2(2+\mathbf{1}_{g_{23}=1})\right]}{1-\gamma^2} \\ r'_{2\to 1} = \frac{c_2(1+\mathbf{1}_{g_{23}=1}) - c_2(2+\mathbf{1}_{g_{23}=1}) - \gamma\left[c_1(1+\mathbf{1}_{g_{13}=1}) - c_1(2+\mathbf{1}_{g_{13}=1})\right]}{1-\gamma^2} \end{cases}.$$

This shows that Condition **C2'** is satisfied.

Finally, let us establish that Condition **C3'** also holds. The matrix M^{public} is equal to

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\partial \hat{x}_1}{\partial r_{1\to 2}} & \frac{\partial \hat{x}_2}{\partial r_{1\to 2}} & \frac{\partial \hat{x}_3}{\partial r_{1\to 2}} & \frac{\partial \hat{x}_1}{\partial r_{1\to 2}} & \frac{\partial \hat{x}_2}{\partial r_{1\to 2}} & \frac{\partial \hat{x}_3}{\partial r_{1\to 2}} & 0 & 0 & 0 \\ \frac{\partial \hat{x}_1}{\partial r_{1\to 3}} & \frac{\partial \hat{x}_2}{\partial r_{1\to 3}} & \frac{\partial \hat{x}_3}{\partial r_{1\to 3}} & 0 & 0 & 0 & \frac{\partial \hat{x}_1}{\partial r_{1\to 3}} & \frac{\partial \hat{x}_2}{\partial r_{1\to 3}} & \frac{\partial \hat{x}_3}{\partial r_{1\to 3}} \\ \frac{\partial \hat{x}_1}{\partial r_{2\to 1}} & \frac{\partial \hat{x}_2}{\partial r_{2\to 1}} & \frac{\partial \hat{x}_3}{\partial r_{2\to 1}} & \frac{\partial \hat{x}_1}{\partial r_{2\to 1}} & \frac{\partial \hat{x}_2}{\partial r_{2\to 1}} & \frac{\partial \hat{x}_3}{\partial r_{2\to 1}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial \hat{x}_1}{\partial r_{2\to 3}} & \frac{\partial \hat{x}_2}{\partial r_{2\to 3}} & \frac{\partial \hat{x}_3}{\partial r_{2\to 3}} & \frac{\partial \hat{x}_1}{\partial r_{2\to 3}} & \frac{\partial \hat{x}_2}{\partial r_{2\to 3}} & \frac{\partial \hat{x}_3}{\partial r_{2\to 3}} \\ \frac{\partial \hat{x}_1}{\partial r_{3\to 1}} & \frac{\partial \hat{x}_2}{\partial r_{3\to 1}} & \frac{\partial \hat{x}_3}{\partial r_{3\to 1}} & 0 & 0 & 0 & \frac{\partial \hat{x}_1}{\partial r_{3\to 1}} & \frac{\partial \hat{x}_2}{\partial r_{3\to 1}} & \frac{\partial \hat{x}_3}{\partial r_{3\to 1}} \\ 0 & 0 & 0 & \frac{\partial \hat{x}_1}{\partial r_{3\to 2}} & \frac{\partial \hat{x}_2}{\partial r_{3\to 2}} & \frac{\partial \hat{x}_3}{\partial r_{3\to 2}} & \frac{\partial \hat{x}_1}{\partial r_{3\to 2}} & \frac{\partial \hat{x}_2}{\partial r_{3\to 2}} & \frac{\partial \hat{x}_3}{\partial r_{3\to 2}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

The equilibrium downstream actions $\hat{\mathbf{x}}(G^c, R)$ for a given R solve the following system of equations:

$$\begin{cases} x_1 = \frac{1}{2}\left[v(1-\gamma) + \gamma(x_2+x_3) + \underline{c}_1 + r_{1\to 2} + r_{1\to 3} + \gamma(r_{2\to 1} + r_{3\to 1})\right] \\ x_2 = \frac{1}{2}\left[v(1-\gamma) + \gamma(x_1+x_3) + \underline{c}_2 + r_{2\to 1} + r_{2\to 3} + \gamma(r_{1\to 2} + r_{3\to 2})\right] \\ x_3 = \frac{1}{2}\left[v(1-\gamma) + \gamma(x_1+x_2) + \underline{c}_3 + r_{3\to 1} + r_{3\to 2} + \gamma(r_{1\to 3} + r_{2\to 3})\right] \end{cases}$$

Summing the three equations and denoting $X = x_1 + x_2 + x_3$, we obtain

$$X = \frac{3}{2}v(1-\gamma) + \gamma X + \frac{1}{2}(\underline{c}_1 + \underline{c}_2 + \underline{c}_3) + \frac{1}{2}(1+\gamma)(r_{1\to 2} + r_{1\to 3} + r_{2\to 1} + r_{3\to 1} + r_{2\to 3} + r_{3\to 2})$$

and therefore,

$$X = \frac{1}{2(1-\gamma)}\left[3v(1-\gamma) + (\underline{c}_1 + \underline{c}_2 + \underline{c}_3) + (1+\gamma)(r_{1\to 2} + r_{1\to 3} + r_{2\to 1} + r_{3\to 1} + r_{2\to 3} + r_{3\to 2})\right] \tag{13}$$

Moreover, multiplying the equations of the system above by $\left(1+\frac{\gamma}{2}\right)$ and rearranging terms leads to

$$\begin{cases} x_1 = \frac{1}{2+\gamma}\left[v(1-\gamma)+\gamma X+\underline{c}_1+r_{1\to 2}+r_{1\to 3}+\gamma\left(r_{2\to 1}+r_{3\to 1}\right)\right] \\ x_2 = \frac{1}{2+\gamma}\left[v(1-\gamma)+\gamma X+\underline{c}_2+r_{2\to 1}+r_{2\to 3}+\gamma\left(r_{1\to 2}+r_{3\to 2}\right)\right] \\ x_3 = \frac{1}{2+\gamma}\left[v(1-\gamma)+\gamma X+\underline{c}_3+r_{3\to 1}+r_{3\to 2}+\gamma\left(r_{1\to 3}+r_{2\to 3}\right)\right] \end{cases} \tag{14}$$

Differentiating (13) and (14) with respect to $r_{1\to 2}$, $r_{1\to 3}$, $r_{2\to 1}$, $r_{3\to 1}$, $r_{2\to 3}$, $r_{3\to 2}$ yields

$$\frac{\partial \hat{x}_1}{\partial r_{1\to 2}} = \frac{\partial \hat{x}_1}{\partial r_{1\to 3}} = \frac{\partial \hat{x}_2}{\partial r_{2\to 1}} = \frac{\partial \hat{x}_2}{\partial r_{2\to 3}} = \frac{\partial \hat{x}_3}{\partial r_{3\to 1}} = \frac{\partial \hat{x}_3}{\partial r_{3\to 2}} = \frac{2-\gamma+\gamma^2}{2(1-\gamma)(2+\gamma)},$$

$$\frac{\partial \hat{x}_1}{\partial r_{2\to 1}} = \frac{\partial \hat{x}_1}{\partial r_{3\to 1}} = \frac{\partial \hat{x}_2}{\partial r_{1\to 2}} = \frac{\partial \hat{x}_2}{\partial r_{3\to 2}} = \frac{\partial \hat{x}_3}{\partial r_{1\to 3}} = \frac{\partial \hat{x}_3}{\partial r_{2\to 3}} = \frac{3\gamma-\gamma^2}{2(1-\gamma)(2+\gamma)},$$

$$\frac{\partial \hat{x}_1}{\partial r_{2\to 3}} = \frac{\partial \hat{x}_1}{\partial r_{3\to 2}} = \frac{\partial \hat{x}_2}{\partial r_{1\to 3}} = \frac{\partial \hat{x}_2}{\partial r_{3\to 1}} = \frac{\partial \hat{x}_3}{\partial r_{1\to 2}} = \frac{\partial \hat{x}_3}{\partial r_{2\to 1}} = \frac{\gamma+\gamma^2}{2(1-\gamma)(2+\gamma)},$$

which determines all the coefficients of M^{public}. Computing the determinant of this matrix, we find that

$$\det M^{public} = \frac{16\left(1-\gamma\right)^2\left(2\gamma-1\right)^3\left(1+2\gamma\right)\left(2+3\gamma\right)^2}{\left[2(1-\gamma)(2+\gamma)\right]^9},$$

which is different from 0, for all values $\gamma \in \left(-\frac{1}{2},1\right)\setminus\{\frac{1}{2}\}$.[24]

Proof of Lemma 4. Suppose that Assumptions **A1**, **A2** and **A3'** and Condition **C3'** hold, and assume that the complete network of public upstream agreements $(G^c, R^*, \mathbf{x}^*)$ is an equilibrium network. It must then hold that

$$\left(\pi_i+\pi_j\right)\left(G^c, R^*, \hat{\mathbf{x}}\left(G^c, R^*\right)\right) = \left(\pi_i+\pi_j\right)\left(G^c, R^*, \mathbf{x}^*\right) \geq \left(\pi_i+\pi_j\right)\left(G^c, R, \hat{\mathbf{x}}\left(G^c, R\right)\right)$$

for any $(G^c, R, F) \in D_{ij}\left(G^c, R^*, F^*\right)$.

Considering the special case of matrices R that are different from R^* only through $r_{i\to j}$, the latter inequality implies that $\left(\pi_i+\pi_j\right)\left(G^c, R, \hat{\mathbf{x}}\left(G^c, R\right)\right)$ is maximized at $r_{i\to j} = r^*_{i\to j}$, which requires that[25]

$$\frac{d\left(\pi_i+\pi_j\right)}{dr_{i\to j}}\left(G^c, R^*, \hat{\mathbf{x}}\left(G^c, R^*\right)\right) = 0.$$

Since $r_{i\to j}$ affects $\left(\pi_i+\pi_j\right)\left(G^c, R^*, \hat{\mathbf{x}}\left(G^c, R^*\right)\right)$ only through its effect on $\hat{\mathbf{x}}\left(G^c, R^*\right)$ (by **A2**),

[24] Recall that we need to restrict γ to be above $-1/(n-1) = -1/2$ when $n = 3$.

[25] We again use the notation $\frac{d(\pi_i+\pi_j)}{dr_{i\to j}}$ instead of $\frac{\partial(\pi_i+\pi_j)}{\partial r_{i\to j}}$ to emphasize that we consider the total derivative of $\pi_i+\pi_j$ with respect to $r_{i\to j}$.

the latter can be rewritten as

$$\sum_{k=1}^{n} \frac{\partial \hat{x}_k}{\partial r_{i \to j}} (G^c, R^*) \frac{\partial \pi_i}{\partial x_k} (G^c, R^*, \hat{\mathbf{x}} (G^c, R^*)) + \sum_{k=1}^{n} \frac{\partial \hat{x}_k}{\partial r_{i \to j}} (G^c, R^*) \frac{\partial \pi_j}{\partial x_k} (G^c, R^*, \hat{\mathbf{x}} (G^c, R^*)) = 0,$$

or equivalently,

$$\sum_{k=1}^{n} \frac{\partial \hat{x}_k}{\partial r_{i \to j}} (G^c, R^*) \frac{\partial \pi_i}{\partial x_k} (G^c, R^*, \mathbf{x}^*) + \sum_{k=1}^{n} \frac{\partial \hat{x}_k}{\partial r_{i \to j}} (G^c, R^*) \frac{\partial \pi_j}{\partial x_k} (G^c, R^*, \mathbf{x}^*) = 0.$$

By symmetry, the first-order condition with respect to $r_{j \to i}$ can be written as

$$\sum_{k=1}^{n} \frac{\partial \hat{x}_k}{\partial r_{j \to i}} (G^c, R^*) \frac{\partial \pi_i}{\partial x_k} (G^c, R^*, \mathbf{x}^*) + \sum_{k=1}^{n} \frac{\partial \hat{x}_k}{\partial r_{j \to i}} (G^c, R^*) \frac{\partial \pi_j}{\partial x_k} (G^c, R^*, \mathbf{x}^*) = 0.$$

Hence, the first-order conditions associated with the suboptimality of upstream deviations (with respect to both $r_{i \to j}$ and $r_{j \to i}$) give rise to $n(n-1)$ conditions. Adding these to the n first-order conditions $\frac{\partial \pi_i}{\partial x_i} (G^c, R^*, \mathbf{x}^*) = 0$ associated with the downstream Nash equilibrium, we end up with a system of n^2 equations. The latter can be represented as a *linear* system $M^{public} Y = 0$, where Y is a $n^2 \times 1$ matrix whose elements (which are the "unknown variables") are defined as follows:

$$Y_{n(l-1)+k} = \frac{\partial \pi_l}{\partial x_k} (G^c, R^*, \mathbf{x}^*).$$

Since $\det M^{public} \neq 0$, this linear system has a unique solution given by

$$\frac{\partial \pi_l}{\partial x_k} (G^c, R^*, \mathbf{x}^*) = 0,$$

for any $k, l \in \{1, ..., n\}$, which completes the proof.