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“Buying and Selling Entrepreneurial Assets”

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Buying and Selling Entrepreneurial Assets*

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Abstract

How are the options to buy and sell a business relevant for entrepreneurs? Prospective entrepreneurs value the purchase of mature firms while incumbents want to recover both the tangible and intangible value of their businesses upon exit. We introduce a theory of entrepreneurial assets transfer consistent with empirical evidence and centered around a businesses for sale market that lets entrepreneurs trade the maturity components of their firms. We find that shutting that market down leads to a substantial drop in aggregate output and alters the pool of firms, incentives to enter and exit, and the wealth distribution.

Keywords: Entrepreneurship, Business transfers, Maturity value, Intangible assets.

JEL classification: E22, E23, D40, J24.

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1 Introduction

Prospective entrepreneurs can acquire a firm by either a new creation or the purchase of an existing business. However, empirical evidence shows a clear performance divide between recently created and purchased firms, giving this choice some significance. For instance, in the pool of entrepreneurs who recently acquired a business, those who purchased an existing business face half the failure rate of those who founded a new one.¹ Moreover, despite accounting for about 20% of the mass of recently acquired firms, purchased firms contribute to about 60% of the employment and the total sales. In fact, this disparity between founded and purchased firms extends over to the main components of heterogeneity the entrepreneurial literature generally considers, namely risk, financial conditions, and productivity. We relate this difference to the maturity of a firm: early-stage firms will face more stringent credit, productivity, and risk conditions as compared to mature firms.²

The importance of the maturity of a firm can be explained by the fact that entrepreneurial assets are not limited to tangible physical capital. According to [Bhandari and McGrattan \(2018\)](#), around 60% of business assets are in the form of intangible assets –customer base, client lists, brand value, organization, etc.– most of which, as opposed to tangible assets, cannot be bought directly and take time to accumulate. The option to purchase an existing business is a key factor in shortening that time and preserving the value of intangible assets in the economy. Given this, the question of transferring entrepreneurial assets appears particularly consequential. On the one hand, the exiting entrepreneur has to decide either to sell or liquidate her assets, conditioning whether the accumulated maturity of her firm will persist or not. On the other hand, the entering entrepreneur will find it desirable to purchase an existing mature business but will be subject to borrowing constraints. In this paper, we build a theory of entrepreneurial assets transfer consistent with empirical evidence and introduce a businesses for sale market that values the maturity of entrepreneurial firms.

The agenda of assessing and explaining the purchase and the sale of entrepreneurial assets presents a few challenges. Data on small and medium-sized enterprise (SME) transfers is scarce. Moreover, there is no theoretical framework in the literature to properly consider transfers in a standard entrepreneurial setting. Thus this paper makes two main contributions. First, we provide a theoretical framework with endogenous options to buy or found businesses on the entry side and sell and liquidate them on the exit side. Our model embeds a businesses for sale market allowing firm transfers and is designed to capture the frictions appearing on that market. We especially consider two distinct margins that interact with the decision to either purchase or found a business on the SME market: (i) the existence of sub-

¹Whenever possible, we define an entrepreneur as an individual actively managing a firm, deriving her primary source of income from it, and holding at least a part of the business assets. We, therefore, exclude passive business owners from the analysis.

²Indeed, those margins have been shown to be age-dependent. See, among others, [Dunne et al. \(1988\)](#), [Sakai et al. \(2010\)](#), or [Dyrda \(2015\)](#).

stantial differences between the foundation of a new business and a purchase that we capture with the concept of maturity, (ii) selling and purchasing frictions that limit business transfers in equilibrium. Second, we study the quantitative importance of those margins on aggregate and distributional outcomes.

Our baseline economy is a stylized life-cycle occupational choice model with heterogeneous agents. Individuals choose whether to work in a corporate sector or to be entrepreneurs. We introduce key endogenous choices: each period, an incumbent entrepreneur might need to sell her business and will face an equilibrium selling price as well as a probability to sell. Without an opportunity to sell, the incumbent will be forced to either continue her current activity or liquidate the business assets. Conversely, a prospective entrepreneur might enter the sector by endogenously choosing a firm size and either finding an existing business to buy or founding a new one, these decisions being subject to credit constraints and specific costs. A small and medium-sized enterprises for sale market (SMESM) aggregates selling and buying decisions. Its equilibrium price is designed as an abstract object to account for both the intertemporal and intangible value of a business. Outside this market, the value of intangible assets cannot be recovered. With the concept of maturity, we introduce a very parsimonious measure of intangible assets: for two firms with the same level of tangible assets, the difference in the sale value will reflect specific advantages provided by transferable intangible assets. As most intangibles cannot be directly bought and their accumulation is time-intensive, we argue that the maturity of a firm is a measurement of the intangibles it has built. We assume that all founded businesses are early-stage immature firms with low levels of intangibles whereas purchased ones are well-established and mature with high levels of intangibles. This assumption is supported by empirical evidence: controlling for characteristics of firms and owners, early-stage businesses are, on average, more likely to fail, make lower profits, are charged higher interest rates and have a tighter borrowing constraint.

We support our theoretical contribution with data from the Survey of Small Business Finances (SSBF), the Survey of Business Owners (SBO), and the Survey of Consumer Finances (SCF). With the above data, we first show that business buying and selling constitutes fundamental margins for entrepreneurs. Second, we document notable differences between business acquisition as a result of a purchase as opposed to a new creation and illustrate that, overall, the transfer of business assets over the SMESM results in more efficient acquisition patterns. We use key moments in the SCF, the SBO, and the SSBF data to discipline our model and show that our baseline setting provides a consistent aggregate and cross-sectional representation of the U.S. economy. We carefully validate the properties of our baseline, even outside of specific targets. For instance, the model plausibly accounts for entrepreneurial life-cycle patterns, the increasing survival rate relative to the preceding year as businesses age, and it furthermore generates a consistent concentration at the top of the wealth distribution.

Our results can be organized around four main points, all related to the significance of the SMESM and the maturity of firms. First, we demonstrate the aggregate importance of

the SMESM by shutting down that market: under our standard parameterization, the aggregate output drops by a substantial 10.5% with respect to our baseline. This drop is mostly due to an important decrease in the SME sector production. Aggregate savings also decline but at the general equilibrium, the interest rate increases and the wage rate falls, somewhat counteracting potential further output losses. At the same time, the fraction of entrepreneurs decreases despite being mitigated by higher incentives to enter entrepreneurship due to the combined effect of prices. However, the fraction of mature businesses clearly diminishes, changing the composition of the types of firms in the economy: trading on the SMESM generates larger businesses and the ability to transfer maturity preserves the higher survival rates, profitability, and better credit conditions of existing firms.

Second, we decompose the maturity of a firm into its components –namely failure rates, profit rates, and borrowing limit and interest rates– in order to understand the specific impact of each of them on aggregate outcomes. We find that the lower failure rate and higher profit rate of mature businesses are the most important elements embedded in the option value of purchasing a business relative to founding while the other components only have marginal effects. Without the contribution of the first two components, the fraction of business purchasers substantially reduces. Moreover, when the contribution of all components of maturity is removed, we show that there is nothing of value to transfer on the SMESM.

Third, we underline a completely new channel to match wealth concentration and inequality based on the heterogeneity of firms and which is furthermore consistent with empirical evidence. Our baseline model convincingly reproduces the U.S. wealth concentration but the novel aspect is due to the key role of the SMESM and maturity in producing that outcome. Indeed, mature firms accumulate higher returns and, because of lower failure rates, they do so over longer periods. In turn, the SMESM preserves the benefits of maturity between owners, concentrating more wealth into the hands of these individuals.

Finally, we find that matching frictions on the SMESM have a substantial impact on aggregate outcome and the wealth distribution. Increasing the probability to sell a business on the SMESM by one percentage point above our baseline increases the output in the entrepreneurial sector by 6.9% and the wealth Gini by 0.8%.

Related Literature This paper is related to the extensive literature on SMEs and entrepreneurship with a macroeconomic perspective. This literature generally depicts entrepreneurs as agents adjusting physical capital and hiring employees subject to idiosyncratic business shocks, entrepreneurial abilities, financial frictions, or unexpected capital destruction. Seminal papers in this literature are [Quadrini \(2000\)](#), [Cagetti and De Nardi \(2006\)](#), or [Buera and Shin \(2013\)](#): those especially focus on credit constraints and the role of entrepreneurship in shaping the wealth distribution. Along the lines of our paper, [Liang et al. \(2018\)](#) and [Engbom \(2019\)](#) also discuss the relation between age and the decision to enter entrepreneurship. Compared to the above literature, this paper introduces an empirically relevant theoretical framework that

accounts for the life-cycle properties of entrepreneurship and the underlying mechanisms of entry and exit while modeling explicitly the market frictions arising upon the transfer of business assets.

Many recent papers highlight the key role of the age of a firm. The argument follows [Jovanovic \(1982\)](#) and [Arkolakis et al. \(2018\)](#): firms acquire knowledge about their environment and learn about the demand addressed to them as they age, which is translated by a higher maturity and a larger stock of intangible assets. For example, among many other studies, [Dunne et al. \(1988\)](#) show that the exit hazard rate decreases with age. In [Clementi and Palazzo \(2016\)](#), this is the case because, on average, entrants are less productive than incumbents. Relatedly, [Warusawitharana \(2018\)](#) shows that profitability evolves with the age of a firm. Moreover, using panel data, [Sakai et al. \(2010\)](#) show that younger small businesses face higher borrowing costs since firms tend to accumulate *reputation* as they age. [Dyrda \(2015\)](#) and [Garcia-Macia \(2017\)](#) show that borrowing constraints faced by entrepreneurs are age-dependent and help to shape the heterogeneous business cycle responses of firms. The older the firm, the less stringent the constraint. The relation between the age of a firm and business performance is modeled, for instance, by [Garcia-Macia \(2017\)](#) and [Bhandari and McGrattan \(2018\)](#) through the accumulation of intangible assets. Compared to the above papers, we explicitly introduce and model the transfer of illiquid business assets. In our case, liquidating a firm lets entrepreneurs recover part of the tangible business assets while selling a (or part of a) business reproduces the transfer of both tangible and intangible assets. Finally, the literature has mainly focused on business transfers through inheritance or gifts, as in [Cagetti and De Nardi \(2009\)](#). This paper, however, shows that business transfers through a purchase are more common, accounting for over 70% of total business transfers.

The remaining of the paper is organized as follows. Section 2 documents empirical elements on business acquisition and transfers, the business for sale market, and the entrepreneurial life-cycle. In Section 3, we present our baseline model and Section 4 describes how we take the model to the data. We evaluate our model in Section 5 and in Section 6, we show the importance of the business for sale market. Section 7 concludes.

2 Business Transfers and the SME for Sale Market

This section details empirical evidence on business transfers and the relative performance of purchased versus founded businesses. We mainly gather disparate information from the 2007 Survey of Business Owners (SBO), the 2016 Annual Survey of Entrepreneurs (ASE), and the 2003 Survey of Small Business Finances (SSBF). We complement the above with evidence from the Survey of Consumer Finances (SCF), the National Longitudinal Survey of Youth 1979 (NLYS79) and the Panel Study of Income Dynamics (PSID). These datasets provide broad pictures of firm characteristics by acquisition type, and characteristics of purchasers with

respect to founders.³

2.1 Business Acquisition and Exit

The literature on entrepreneurship has long been interested in the behavior of incumbent entrepreneurs but has been somewhat silent on how businesses come to be in the first place.⁴ Throughout this paper, we argue that purchasing and selling a business are important components of entrepreneurship, as evidenced by the behavior of a non-negligible fraction of entrepreneurs in the data. Survey questions often define as *acquisition* the way the entrepreneur became the owner of the business: founding a new business or purchasing an existing one are two common alternative types of acquisition. Using the SBO (2007) survey data, [Table 1](#) provides estimates of the types of acquisition. One key information appears: about 20% of all entries into entrepreneurship are the result of the purchase of an existing business.⁵ We find very similar numbers using the SCF (2016), the ASE (2016), or the SSBF (2003). They account for about 70% of all business transfers, dwarfing gifts and inheritances (see [Appendix A.1](#) for details). Moreover, purchased firms account for a large fraction of the employment and total sales, especially in the pool of recently acquired firms.

Table 1. Business acquisition by type ^a

Group	Acquisition Type ^b	Metric		
		(%) Firms	(%) Employment	(%) Total sales
Of all firms	Purchased	19.6	39.6	42.7
	Founded	80.3	60.4	57.3
Of firms within 3 years of acquisition	Purchased	20.5	60.6	61.4
	Founded	79.5	39.4	38.6

^a Survey of Business Owners (2007). An entrepreneur is defined as an individual declaring that her business constitutes her primary source of income with an active management role, whenever possible.

^b These numbers exclude inheritance and gifts as they account for a minor fraction of reported transmissions. These results hold even when only the main/first owner is considered and for the pool of firms with employees.

Concerning the exit out of entrepreneurship, there is little detailed evidence in the literature despite an important body of papers focusing on this subject and its relation to life-cycle aspects. We find that a non-negligible fraction of active business owners sell their firms upon exit: 8% according to the SBO (2007) (varying from 7% for the main owner to 18% for the

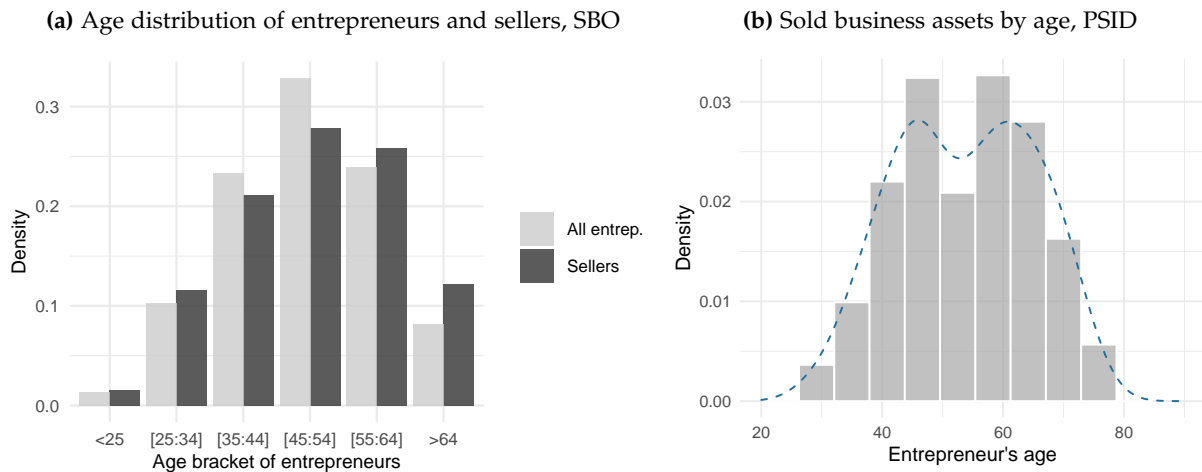
³For consistency reasons among the datasets and the model specification, we define an entrepreneur as an active self-employed business owner whenever possible and as a self-employed business owner otherwise. In the SBO, we additionally consider only individuals declaring that their businesses constitute their primary source of income. As ASE microdata are not publicly available, we report macro estimates for all business owners with at least one paid-employee.

⁴An adjacent literature has focused on firm dynamics (for example, [Clementi and Palazzo \(2016\)](#) for recent papers). That literature often abstracts from the firm's acquisition.

⁵Interestingly, purchasers are not more likely than founders to have a previous self-employment experience in the SBO (2007) and the SSBF (2003) (see [Appendix A.2](#) for details).

third and fourth owners and to 16.9% for entrepreneurs with paid employees) and 17% in the 2016 ASE (owners with paid employees). In the NLSY79 (2002-2016), pooling individuals with past ownership, 20% sold their businesses, 70% shut them down and the remaining were in an undefined alternative situation.⁶ This selling behavior is largely related to the entrepreneurial age profile and the decision to retire. Using SBO data, we first show in panel (a) of Figure 1 that the age distribution of sellers is further to the right with respect to the overall population of entrepreneurs: 38% of sellers are over 54 years old (in contrast, buyers are relatively young, with a mean age of 44 in the SCF (2007) and 44.5 in the SSBF (2003)). In panel (b) of Figure 1, we corroborate this evidence using PSID data. We find that the sale of business assets peaks at two age brackets: the 45-50 and the 60-65, close to the typical U.S. average retirement age. It is worth noting that this supports the fact that retirement is one of the main reasons to cease a business. In the ASE, 19% of businesses ceasing were explained by owners retiring.⁷

Figure 1. Entrepreneurial life cycle, acquisition and business selling



Source: SBO 2007 and PSID averaged over the waves from 1990 to 2015 (adjusted for inflation using the CPI index). The mean age of the distribution is 53.6 and the median is 54.

Finally, we find evidence suggesting substantial difficulties for transferring businesses on the small and medium-sized enterprises for sale market (SMESM). According to the 2016 ASE, among business owners with paid employees reporting how they planned to exit entrepreneurship, 50% were thinking of selling their businesses to a third-party and 10% to a

⁶Note that the NLSY79 included this question only after 2002. One explanation of the gap is that the SBO provides many different options to choose from for the main reason to cease. In contrast, the NLSY79 only offers three options: *selling the business*, *shutting it down* or *other*. For instance, it might be possible that retiring owners in the SBO reported *retirement* as the reason to cease even if the means of exit was selling of the business. Moreover, the SBO treats businesses and business owners differently. Therefore, it is possible that owners exit by selling their shares, while the associated businesses keep on operating. Finally, Appendix A.3 provides further evidence on the exit rate, especially by type of exit.

⁷We also find that around 20% of sellers declare selling for retirement purposes in detailed businesses transaction data. See Appendix A.4 for details.

family member. This is in stark contrast to the much lower number of businesses actually being sold that we report above. Moreover, among business owners with a firm of 16 years of age and more, i.e. businesses much less likely to close due to economic reasons, the main exit strategy is consistently the sale of the entire business (53%). However, even in this population, only 26% declared effectively selling their business *ex post* while 43% declared just retiring and 14% declared failing due to business conditions. Moreover, according to business transactions data from 2018 to 2019, selling a business is a long process: only 30% of businesses for sale are sold within a year and a non-negligible fraction remain unsold.⁸ This might reflect the importance of transaction delays (training, screening, etc.) and the existence of asymmetric information on business performance.

This paper provides two plausible explanations that could generate this *low* observed selling rate. First, selling a business requires the matching of the specific interests and skills of a potential buyer. We refer to this as *selling friction*. Second, purchasing a business requires a payment for its physical capital assets but also for the value of its intangible assets. Borrowing constraints could substantially limit the capacity of potential buyers to purchase existing businesses beyond their traditional effects described in the literature, for instance in [Quadrini \(1999, 2000\)](#) among others.

2.2 Sources of Heterogeneity and Maturity of a Business

We documented above that an important fraction of entrepreneurial firms were purchased by their current owner. There might be a number of reasons as to why a prospective entrepreneur will favor the purchase of a firm with respect to creating a new one. Nevertheless, the evidence we collected points to at least one significant reason explaining the purchase behavior: purchased and founded firms differ with respect to the main sources of heterogeneity that the entrepreneurial literature generally considers, namely, heterogeneity in risk, credit conditions, and profitability.

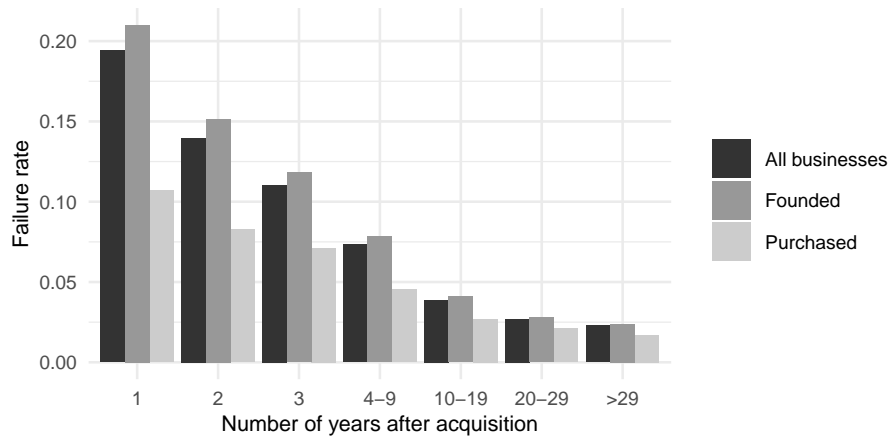
We start this discussion with [Figure 2](#) that displays the failure rate of firms in the 2007 SBO by the number of years after the acquisition. The failure rate can be seen as a measure of the risk the firm is facing.⁹ We observe a stark performance divide between founded and purchased firms benefiting the latter: for instance, one year after the acquisition, the failure rate of newly founded businesses is twice that of purchased ones. As the number of years after acquisition increases, the failure rate decreases and the difference between purchased and founded businesses dissipates. Interestingly, there is still a decreasing shape with respect to age for recently purchased firms, which might be linked to non-transferable intangible assets

⁸See [Appendix A.4](#) for details on this business transactions data.

⁹The literature sometimes uses the term *exit hazard rate* to cover most of what we call *failure rate*. As in [Dunne et al. \(1988\)](#), we define the failure rate as the ratio of firms exiting due to economic reasons between period t and $t + 1$ and the number of operating firms in time t . Results are robust to the use of all exiting reasons.

such as an entrepreneur’s talent and knowledge about its environment.¹⁰ In more general terms, without any controls, we find that purchased businesses systematically perform better with respect to all sources of firm heterogeneity that we consider.

Figure 2. Failure rate by acquisition type and for all businesses.



Source: author’s computation using the 2007 SBO. We compute the failure rates using ceasing option linked to either inadequate cash-flows or low sales and lack of business or personal loans/credit.

Using SSBF and SBO data and controlling for characteristics of firms and owners, we estimate the average difference distinguishing recently purchased and founded firms over five components: the failure rate, the credit line interest rate, the credit limit, the credit score, and the profit normalized by the average 2-digits sectoral profit.¹¹ We report the results in Table 2. The fourth and fifth columns, under the label Δ Conditional, display the conditional difference in the specific components. As a reference, the two first columns display the uncontrolled sample average, respectively for purchased and founded firms, while the third column is simply the difference between those two values. We compare our results between a pool of recent firms within 3 years of their acquisition and one of older firms over 15 years of theirs.

Starting with the failure rate, our estimates using the SBO show that purchased firms are a significant 6.3% less likely to fail. Importantly, contrarily to many surveys considering the acquisition and establishment date as equivalent, the SBO contains information about the true establishment year of a business.¹² Although imperfect, the establishment year provides us with a key element: it lets us control for the contribution of a firm established many years prior to its sale with respect to a recently established and sold firm. Interestingly, when controlling for the establishment age, the associated failure rate wedge falls to 2.4%

¹⁰Guiso et al. (ming) show that that type entrepreneurial knowledge is important for the decision to enter the sector.

¹¹Specifically, we use the following OLS regression: $C_i = \alpha + \beta D_i(\text{purchased}) + \gamma X_i + \epsilon_i$ with $D_i(\text{purchased})$ a dummy indicating whether the business was initially purchased, C_i the specific component and X the vector of controls. β that captures the difference associated with purchasing relative to founding a businesses.

¹²This information is available yearly between 2003 to 2007 and is bracketed prior to that as [2000:2002], [1990:1999], [1980:1989], and [before 1980].

(last column), implying that this age captures (part of) the difference between a founded and a purchased firm. A key consequence of this is that the true age of a firm, and thus how actually mature it is, appears to be a critical factor. This maturity and the associated advantages it provides might be what a prospective entrepreneur is seeking when purchasing an existing business. Quite contrastingly, in the pool of older firms, the wedge between purchased and founded businesses is virtually negligible, and the effect of the establishment date also disappears. Overall, our result points out that recently purchased firms are less likely to fail than recently founded firms. Consistently with [Dunne et al. \(1988\)](#), this difference is partly captured by the fact that purchased firms are in general older (conditional on size, and other characteristics). In a model of firm dynamics with an entry margin, older firms are those that survived and were selected over time. From the viewpoint of a new potential entrepreneur, this selection is not known *ex-ante*. Following [Jovanovic \(1982\)](#), we argue that an entrepreneur learns the characteristics and the potential of a new firm mostly *on the business* but directly observes the characteristics of older (and purchasable) firms.

Table 2. Key Heterogeneity Components by Acquisition Type

	Acquisition Type		Δ Unconditional	Δ Conditional	
	Purchased	Founded		Controls ^a	+Estab. date
Firms within 3 years of acquisition					
Risk: Failure rate	0.087	0.164	-0.077	-0.063*** (0.004)	-0.024*** (0.005)
Financial: Int. rate	10.89	12.12	-1.234	-1.98** (0.831)	
Financial: $\frac{\text{Granted loan}}{\text{Applied for}}$	0.994	0.950	0.043	0.061** (0.029)	
Financial: Credit score	3.316	2.964	0.352	0.526*** (0.173)	
Norm. Profitability	0.710	0.364	0.346	0.431* (0.232)	
Firms over 15 years after acquisition					
Risk: Failure rate	0.023	0.033	-0.010	-0.005*** (0.001)	-0.002*** (0.001)
Financial: Int. rate	12.29	12.38	-0.081	0.229 (0.548)	
Financial: $\frac{\text{Granted loan}}{\text{Applied for}}$	0.975	0.975	-0.002	0.006 (0.019)	
Financial: Credit score	4.024	3.913	0.111	0.155 (0.143)	
Norm. Profitability	1.616	1.176	0.420	0.030 (0.326)	

Note: * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$. Source: SBO, SSBF (2003). The sample comprises owners managing their firms and declaring they constitute their primary source of income. The first owner is used to select the firm acquisition year.

^a Firm and owner controls: depending on the survey, characteristics of owners include age, experience as entrepreneur, education, sex, home-based dummy, home equity and other net worth, and number of owners. Characteristics of firms: sector and FIPS dummies, employment, legal structure, equity, past bankruptcy indicator, urban dummy, payroll and franchise indicator.

Our findings for the failure rate extends to financial and productivity components. In the SSBF data, recently founded businesses tend to pay, on average, a higher interest rate on their credit line with respect to recently purchased businesses. The premium is non-negligible at around 2.0%. Concerning the credit limit, we find that recently purchased businesses obtain a higher fraction of the loan they applied for. This is also confirmed when using the credit score as a proxy for credit constraints: recently purchased businesses get a significantly higher score, of 0.5 on average, on a scale from 1 (lowest score) to 6 (highest score). These findings suggest that founded businesses face tighter financial constraints, consistently with [Sakai](#)

et al. (2010) and Dyrda (2015).¹³ Finally, regarding productivity, the ratio of profit relative to the average profit in the corresponding industry is higher by about 0.4 for purchased firms. This is consistent with a *mean-reverting productivity* argument also found in Clementi and Palazzo (2016) and Warusawitharana (2018), in which new firms enter with an, on average, lower productivity level and then converge to their long-run productivity level as they age.¹⁴ This remains consistent with the argument of the selection of the best firms through time: if purchased firms are in general older, they already passed this selection phase. Consistently, in the older pool of firms, the gap between purchased and founded firms either vanishes or is not significant for financial and productivity components.

In general, the importance of the maturity of a firm is difficult to distinguish from a best firms/entrepreneurs selection mechanism in cross-sectional data since the latter would also lead to, for instance, higher average productivity over time as under-performing firms are exiting. By distinguishing recently purchased from founded businesses, our results show that maturity components are crucial and allow potential purchasers to overcome the selection process. We argue that our findings concerning the performance divide between purchased and founded firms and the importance of the maturity of a business for a prospective entrepreneur is a key element to incorporate in a model examining business assets transfers. We relate these findings to a growing literature documenting the importance of intangible assets (customer bases, client lists, organizational methods, brand value, etc.) in explaining the market value of a firm. However, as evidenced by Bhandari and McGrattan (2018), the direct measurement of most intangible assets is a difficult task. But basically, as they cannot be directly bought and their accumulation is time-intensive, early-stage firms are immature with low levels of intangibles whereas well-established ones are mature with high levels of intangibles.¹⁵ We, here, adopt a parsimonious approach supported by our empirical evidence: an immature firm will face more stringent credit, productivity and risk conditions whereas a mature firm will have better perspectives on these components. From the point of view of a prospective entrepreneur, a business purchase will convey the specific value of maturity and the business for sale market will play a crucial role in transferring it between owners.¹⁶

¹³As additional evidence, in both the SCF and the SSBF, the main reason given to explain why early-stage businesses face difficulties in obtaining the credit they apply for is "*the firm was not in business long enough*" (see Appendix A.2 for details).

¹⁴Unfortunately, the SSBF data does not include the establishment date of firms, making it impossible to control by this element for financial and profit components.

¹⁵Bhandari and McGrattan (2018) also discuss the dynamics of intangible assets accumulation and show how it is related to the age of a firm. The relation with age is also consistent with the literature on firm dynamics (see Dunne et al. (1988)).

¹⁶This paper focuses solely on transferable assets, whether tangible or intangible. We abstract from non-transferable intangible asset, such as the managerial value of a specific retiring business owner.

3 Model

This section introduces a dynamic general equilibrium model with endogenous buying and selling decisions. The economy consists of a corporate sector and a unit measure of *ex post* heterogeneous agents. A fraction of the latter, called entrepreneurs, hold small and medium-sized businesses while the remaining, called workers, occupy wage-paying jobs in a corporate sector. Entry and exit into and out of the small business sector are subject to specific conditions. On the one hand, individuals entering the sector have to either found a new business or purchase an existing one. On the other hand, upon exit, entrepreneurs can either sell their business or liquidate the physical business assets. Purchasing and selling are subject to financial and selling frictions. Therefore, a *small and medium-sized enterprises for sale* market (SMESM) constitutes a pivotal piece of our model. Finally, a government levies a menu of taxes to cover for old-age pensions and other public expenditures.

3.1 Corporate Sector

The corporate sector output Y_t is produced by a single competitive representative firm using a Cobb-Douglas technology with capital share $\alpha \in (0, 1)$ and total factor productivity A , capital level $K_{c,t}$ and labor $L_{c,t}$, such that: $Y_t = F(K_{c,t}, L_{c,t}) = AK_{c,t}^\alpha L_{c,t}^{1-\alpha}$. Capital depreciates at rate δ in both the corporate and the SME sectors. The interest rate and the wage rate equalize their respective marginal products: $r_t = F'_{K_{c,t}}(K_{c,t}, L_{c,t})$ and $w_t = F'_{L_{c,t}}(K_{c,t}, L_{c,t})$.

3.2 Agents

We use a stylized life-cycle setup with aging and probabilistic death in the last age bracket. Households live through J stages of life and the total population, of unit mass, is divided among J generations indexed with $j \in [1; J]$. Groups 1 through $J - 1$ are called Juniors and have access to the labor market. The J^{th} group, called Seniors, is comprised of individuals beyond the retirement age. We assume that a fraction p_{die} of the Seniors pass away and exit the model.¹⁷ Over the life-cycle, households belong in an occupation $o \in \{o_e, o_w, o_r\}$. Junior households can be entrepreneurs (o_e) or occupied in the workforce (o_w) whereas Senior households are either retired (o_r) or are old age entrepreneurs.

Households have preference described by the life-time utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathcal{U}(c_t, j_t, o_t), \quad (1)$$

where we drop the dependence on time t in the following. The age argument in the utility function translates the fact that being active beyond the retirement age might generate disutility costs. The occupation argument relates to the assumption that entrepreneurship could generate non-pecuniary benefits, as argued by [Hurst and Pugsley \(2015\)](#).

¹⁷This assumption is widely used in the literature: see [Cagetti and De Nardi \(2006\)](#) in a related literature.

Depending on its occupation, a household can possess liquid and/or illiquid assets. Liquid assets are akin to savings and are noted a . Illiquid (business) assets, noted k , are used to produce with the entrepreneurial technology. Both the corporate and the entrepreneurial sector produce a homogeneous consumption good. The liquid asset can be freely used to purchase it but not the illiquid asset. Our setup explicitly defines conditions to convert illiquid capital into liquid assets and conversely. To obtain liquid assets from illiquid assets, individuals have to either sell their firm contingent on finding a buyer or liquidate partially or totally subject to an adjustment cost. Conversely, acquiring illiquid capital using liquid capital is subject to an adjustment cost but can be also achieved by buying a firm with a specific illiquid capital amount. Investing or disinvesting in the illiquid capital are also subject to capital adjustment costs. These adjustment costs are generally noted $\mathcal{C}(k, k')$, with k' the next period capital.

The state-space for an entrepreneur are savings a , business capital k , and $\mathbf{x}_e = \{j, m\}$, where $m = \{0, 1\}$ indicates whether the business is mature. A newly founded firm is assumed to start immature ($m = 0$) and has a probability P_m to mature. Only mature firms can be sold on the SMESM. Thus, all purchased firms are preexisting mature businesses ($m = 1$).¹⁸ Maturity translates the accumulation of intangible assets and provides specific benefits. Entrepreneurs are precluded from possessing multiple firms. The state-space for a worker is a , and $\mathbf{x}_w = \{j, y, \iota\}$, with y her working productivity and ι her potential ability to manage a business. Both ι and y follow first-order Markov processes. We note $\mathcal{Y}(j, y)$ the income of a worker. The entrepreneurial income derives from entrepreneurial production using technology $f(k, m)$.¹⁹

For convenience, the full individual states vector is noted $\mathbf{X} := (a, k, j, m, \iota, y, o) \in \mathbb{X}$. The states of an entrepreneur are $\mathbf{X}^e := (a, k, j, m) \in \mathbb{X}^e$ and those of a worker are $\mathbf{X}^w := (a, y, j, \iota) \in \mathbb{X}^w$. Let $\{\Phi(\mathbf{X}), \Phi(\mathbf{X}^e), \Phi(\mathbf{X}^w)\}$ be measures over all agents and each occupation respectively.

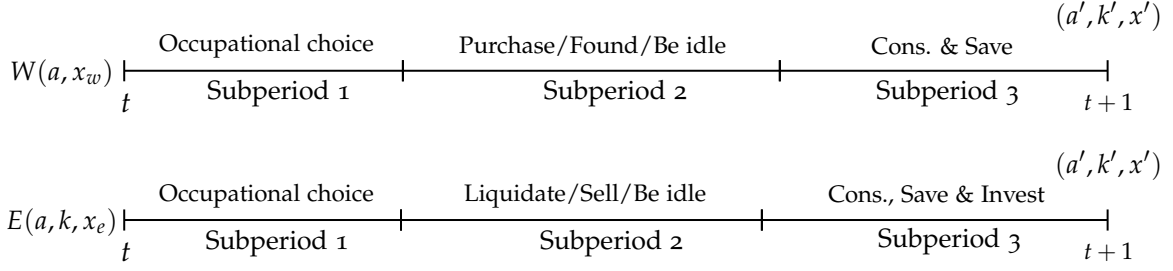
3.3 Dynamic Problem

We decompose an agent's intra-period decision process into a sequence of three subperiods. In the last subperiod, the consumption-saving and entrepreneurial investment problems are tackled. In the middle subperiod, the buying and selling problems are addressed contingent

¹⁸In detailed business transaction data, we find that only 15% of businesses for sale have been established in the 5 preceding years. In the SBO 2007, only about 23% of sellers declare selling a firm established in the 4 years preceding the sale. In practice, it would be straightforward to allow a fraction of firms to mature directly upon entry. We believe our results are robust to such an assumption.

¹⁹Note that for the sake of parsimony, we make a number of simplifying assumptions. First, we abstract from entrepreneurs hiring workers and leave it for a straightforward extension. Second, the fact that only mature firms can be sold on the SMESM reflects that in the data the average age of businesses for sale is much higher than the average of all firms. Using a dataset of business selling transactions detailed in Appendix A.4, we find that 85% of businesses for sale are older than 5 years. Moreover, as it takes time to create a valuable business, maturity captures the process of accumulating intangibles for early-stage firms.

Figure 3. Timing.



on occupational changes and the maturity of a business. Finally, in the first subperiod occupational choices are made. Given that $W(a, x_w)$ and $E(a, k, x_e)$ are respectively the value function of a worker and an entrepreneur, Figure 3 summarizes this decomposition. The intra-period problem is solved backwards and we detail below each subperiod problem.

3.3.1 The Last Subperiod: Consumption-Saving Problem

Depending on choices made in the previous subperiods, consumption and saving decisions in the last subperiod can be distinguished into those of workers either continuing or exiting their activity and those of entrepreneurs continuing or exiting theirs. For the sake of simplicity, continuing workers are subject to a no-borrowing constraint. Similarly to an incumbent entrepreneur, an exiting worker entering an entrepreneurial activity can borrow to invest in a level of business assets k , as long as a minimum amount $\theta(m)$ is pledged. Thus those individuals are subject to the following borrowing constraint:

$$a' \geq -\psi(k, m) \quad (2)$$

$$\psi(k, m) = (1 - \theta(m))[(1 - \delta)k - \mathcal{C}(k(1 - \delta), 0)] \quad (3)$$

where $\mathcal{C}(k(1 - \delta), 0)$ refers to the liquidation cost (i.e. the adjustment cost from reducing the capital level from $(1 - \delta)k$ to 0). We interpret $\theta(m)$ as a maturity-specific downpayment requirement translating the minimum fraction of business assets an entrepreneur has to provide in order to get a loan. This formulation implicitly assumes that, under a liquidation procedure, the creditor can only resell the business assets net of depreciation δ . An indebted entrepreneur faces an interest $r_b(m)$ that depends on her net worth, a , and the maturity of her business, m , such that $\tilde{r}(m) = \mathbb{1}_{a' \geq 0}r - \mathbb{1}_{a' < 0}r_b(m)$.

Continuing entrepreneurs An incumbent entrepreneur continuing her activity chooses next period's illiquid capital k' and saving a' given her current income $f(k, m)$. The consumption-saving problem of this entrepreneur is thus:

$$E^{cont}(a, k, \mathbf{x}_e) = \max_{c > 0, a' \geq -\psi(k'), k' \geq 0} \left\{ \mathcal{U}(c, j, o_e) + \beta \mathbb{E}_{j', m' | j, m} E(a', k', \mathbf{x}'_e) \right\} \quad (4)$$

$$s.t. \quad c + a' + k' = (1 + \tilde{r}(m))a + f(k, m) + k(1 - \delta) - \mathcal{C}(k(1 - \delta), k') \quad (5)$$

with E^{cont} the subperiod specific value function of this continuing entrepreneur and τ_w the tax rate on entrepreneurial income.

Exiting entrepreneurs When exiting, an entrepreneur has to choose savings a' subject to the no-borrowing constraint. The value function of an exiting entrepreneur depends on the exit option z : voluntarily or business failure liquidation ($z = 0$) or sale of the business ($z = 1$).

$$E_z^{exit}(a, k, \mathbf{x}_e) = \max_{c > 0, a' \geq 0} \left\{ \mathcal{U}(c, j, o_e) + \beta \mathbb{E}_{j', i' | j} \mathcal{W}(a', \tilde{\mathbf{x}}'_w) \right\} \quad (6)$$

$$\begin{aligned} \text{s.t. } c + a' &= (1 + \tilde{r}(m))a + f(k, m) \\ &+ (1 - z) \underbrace{\left[k(1 - \delta)(1 - \mathcal{C}(k(1 - \delta), 0)) \right]}_{\text{Business liquidation}} + z \underbrace{\left(\mathcal{P}(k(1 - \delta)) \right)}_{\text{Business sale}} \end{aligned} \quad (7)$$

with E^{exit} the subperiod specific value function of this entrepreneur and $\tilde{\mathbf{x}}'_w$ the specific exogenous worker state of an exiting entrepreneur.²⁰ Liquidating is identical to adjusting the business capital to zero by fully paying the corresponding adjustment cost $\mathcal{C}(k(1 - \delta), 0)$. Alternatively, by successfully selling the business the entrepreneur recovers the total amount $\mathcal{P}(k(1 - \delta))$.

Continuing workers Such a worker has to choose savings a' subject to the no-borrowing constraint and solves:

$$\mathcal{W}^{cont}(a, \mathbf{x}_w) = \max_{c > 0, a' \geq 0} \left\{ \mathcal{U}(c, j, o_w) + \beta \mathbb{E}_{j', y', i' | j, y, i} \mathcal{W}(a', \mathbf{x}'_w) \right\} \quad (8)$$

$$\text{s.t. } c + a' = (1 + r)a + \mathcal{Y}(j, y)(1 - \tau_w) \quad (9)$$

with \mathcal{W}^{cont} the subperiod specific value function of this worker and τ_w the tax on labor income.

Exiting workers An exiting worker enters entrepreneurship by either purchasing an existing mature business ($d = 1$ and $m' = 1$) and paying the total amount $\mathcal{P}(k')$ plus fixed cost F_b or by founding a new business ($d = 0$ and $m' = 0$) and paying the adjustment cost $\mathcal{C}(0, k')$.²¹ Depending on whether the agent is currently buying ($d = 1$) or founding ($d = 0$) a firm, her

²⁰The main specificity is the assumption that any new worker coming from the entrepreneurial sector starts with the lowest level of worker productivity. The argument is that the productivity state y is strongly related to the experience of a worker in a specific corporate job. This seniority on a job cannot be randomly obtained but has to be *earned*. Recall, however, that there is an age-component in the determination of the wage process. Finally, the entrepreneurial ability of a new worker is drawn from the invariant distribution of the associated process.

²¹We use the purchase specific fixed cost to bring the model closer to the data by capturing the fact that purchased firms are twice as large as founded ones in terms of start-up capital. A side effect is that buyers are prevented from purchasing very small businesses unless their credit constraint can afford this cost. The fixed cost could capture costs associated to brokerage, screening, negotiation or training.

problem is to choose the optimal next period capital size k' , savings a' and consumption c . Such a worker solves:

$$\mathcal{W}_d^{exit}(a, \mathbf{x}_w) = \max_{c>0, a' \geq -\psi(k'), k' \geq 0} \left\{ \mathcal{U}(c, j, o_w) + \beta \mathbb{E}_{j'|j} E(a', k', \mathbf{x}'_e) \right\} \quad (10)$$

$$s.t. \quad c + a' = (1+r)a + \mathcal{Y}(j, y)(1 - \tau_w) - \underbrace{d(\mathcal{P}(k') + F_b)}_{\text{Purchasing}} - (1-d) \underbrace{k'(1 + \mathcal{C}(0, k'))}_{\text{Founding}} \quad (11)$$

with \mathcal{W}^{exit} the subperiod specific value function.

3.3.2 The Middle Subperiod: Acquisition and Selling Problems

In the middle subperiod, the buying/founding and selling/liquidating problems are solved. When the sale of a business is unsuccessful, entrepreneurs can continue operating the business or may liquidate. Similarly, in the case of an unsuccessful business purchase, a buyer can remain a worker or found a new business.

The seller's problem An entrepreneur with a mature business ($m = 1$) can try to sell ($z = 1$) it on the *SMESM*. A buyer is found with probability h_s . Otherwise, the entrepreneur chooses whether to liquidate ($z = 0$) or to continue operating the business. Depending on whether the entrepreneur exits endogenously or is forced to exit, the following problem is solved:²²

$$\mathcal{S}^{ee}(a, k, \mathbf{x}_e) = \left(h_s E_{z=1}^{exit} + (1 - h_s) \max \{ E_{z=0}^{exit}, E^{cont} \} \right) (a, k, \mathbf{x}_e) \quad (\text{endogenous exit}) \quad (12)$$

$$\mathcal{S}^{fe}(a, k, \mathbf{x}_e) = \left(h_s E_{z=1}^{exit} + (1 - h_s) E_{z=0}^{exit} \right) (a, k, \mathbf{x}_e) \quad (\text{forced exit}) \quad (13)$$

where \mathcal{S}^{ee} and \mathcal{S}^{fe} are the subperiod specific value functions for the endogenous exit and the forced exit problems.

The buyer's problem A buyer has a probability h_b of finding a seller and purchasing a business ($d = 1$). Otherwise, she chooses whether to found a new business ($d = 0$) or to keep being a worker. Thus, the following problem is solved:

$$\mathcal{B}(a, \mathbf{x}_w) = \left(h_b \mathcal{W}_{d=1}^{exit} + (1 - h_b) \max \{ \mathcal{W}_{d=0}^{exit}, \mathcal{W}^{cont} \} \right) (a, \mathbf{x}_w) \quad (14)$$

with $\mathcal{B}(a, \mathbf{x}_w)$ the subperiod specific value function for this problem.

3.3.3 The First Subperiod: Occupational Choice and Exit Strategy

Worker A worker starts the period with states $\{a, \mathbf{x}_w\}$ and, provided she has an entrepreneurial ability (i.e. $\iota = 1$), chooses whether to try to purchase an existing business (with value

²²We introduce exogenous shocks to capture entrepreneurial exits unrelated to business failure: migration, death, divorce, etc.

$\mathcal{B}(a, \mathbf{x}_w)$), to found a new business ($d = 0$), or to remain a worker, such that:

$$\mathcal{W}(a, \mathbf{x}_w) = \left((1 - \iota) \mathcal{W}^{cont} + \iota \max \{ \mathcal{B}, \mathcal{W}_{d=0}^{exit}, \mathcal{W}^{cont} \} \right) (a, \mathbf{x}_w) \quad (15)$$

Entrepreneur An entrepreneur starts the period with states $\{a, k, \mathbf{x}_e\}$ and decides whether to sell, liquidate or continue her business endogenously unless she is forced to exit. $\chi(m)$ is the probability of entrepreneurial exit due to business failure, which is a function of the maturity of the business, and ζ is the exogenous exit probability, conditional on not failing. Only businesses that do not fail can be sold. In the end, the following problem is solved:

$$\begin{aligned} E(a, k, \mathbf{x}_e) = & \left(\underbrace{\chi(m) E_{z=0}^{exit}}_{\text{Failure}} + (1 - \chi(m)) \left[\underbrace{\zeta \left(m \max \{ \mathcal{S}^{fe}, E_{z=0}^{exit} \} + (1 - m) E_{z=0}^{exit} \right)}_{\text{Exogenous exit}} \right. \right. \\ & \left. \left. + (1 - \zeta) \left(m \max \{ \mathcal{S}^{ee}, E_{z=0}^{exit}, E^{cont} \} + (1 - m) \max \{ E_{z=0}^{exit}, E^{cont} \} \right) \right] \right) (a, k, \mathbf{x}_e) \\ & \underbrace{\hspace{15em}}_{\text{Endogenous exit/continue decision}} \end{aligned} \quad (16)$$

Contingent on the entrepreneur not failing (with probability $(1 - \chi(m))$), she has a probability ζ to be forced to exit, and a probability $(1 - \zeta)$ to choose whether to stay entrepreneur, liquidate the business or sell the business if the business is mature ($m = 1$).

3.4 The Small and Medium Sized Enterprises for Sale Market (SMESM)

On the *SMESM*, businesses sellers and buyers meet in a frictional decentralized market where transaction failures may result in business liquidation on the side of sellers and, on the other side, may compel prospective entrepreneurs into founding their businesses. For tractability, we make a number of assumptions:

Assumption 1. *The market is intermediated by passive brokers on both the selling and the buying side.*

Assumption 2. *Firms are valued and exchanged in this market using the business pricing function $\mathcal{P}(k)$. When selling a firm, an entrepreneur sells tangible illiquid capital assets at their liquidation value $k - \mathcal{C}(k, 0)$, and sells the intangible value of the business as relative expected cash flow units, such that:*

$$\mathcal{P}(k) = \underbrace{p [\mathcal{V}(k, 1) - \mathcal{V}(k, 0)]}_{\text{Maturity value}} (1 - \tau_s \mathbb{1}_{sell}) + \underbrace{k - \mathcal{C}(k, 0)}_{\text{Tangible value}} \quad (17)$$

$$\mathcal{V}(k, 1) = \sum_{t=0}^{\infty} \left(\frac{1 - \chi(1)}{1 + r} \right)^t \pi(k, 1) = \frac{1 + r}{r + \chi(1)} \pi(k, 1) \quad (18)$$

$$\mathcal{V}(k, 0) = \frac{1 + r}{r + \chi(0)(1 - P_m) + P_m} \left(\pi(k, 0) + P_m \frac{1 - \chi(0)}{r + \chi(1)} \pi(k, 1) \right) \quad (19)$$

with τ_s the capital gains tax, $\pi(k, m) = f(k, m) - \delta k$ a profit unit and p the equilibrium price of a relative expected cash flow unit.²³

Assumption 3. Sellers, buyers, and brokers are price takers. Upon meeting a broker, a seller agrees on an after-tax selling price $\mathcal{P}(k)$. Symmetrically, upon meeting a broker, a buyer agrees on a buying price $\mathcal{P}(k)$. Consequently, brokers are making no profit.

With these assumption, we avoid the challenging multidimensional dynamic sorting problem of the direct matching between heterogeneous buyers and sellers, which may require each individual to forecast the dynamics of the entire distribution of sellers and purchasers. Instead, with the above assumptions, the equilibrium condition requires the price p to clear the exchange of maturity value on the SMESM:

$$\begin{aligned} \int_{\mathbf{X}^e} z(\mathbf{X}^e) h_s [\mathcal{V}(k(\mathbf{X}^e), 1) - \mathcal{V}(k(\mathbf{X}^e), 0)] d\Phi(\mathbf{X}^e) \\ = \int_{\mathbf{X}^w} d(\mathbf{X}^w) h_b [\mathcal{V}(k'(\mathbf{X}^w), 1) - \mathcal{V}(k'(\mathbf{X}^w), 0)] d\Phi(\mathbf{X}^w) \end{aligned} \quad (20)$$

with h_s and h_b the respective frictions (or mismatch probability) on the sellers' and buyers' side of the market.

In this specification, cash flow units are indistinguishable. Therefore, selling a firm is here consistent with providing to the market all cash flow units and tangible business assets owned by the entrepreneur at the same time. Conversely, buying a firm is equivalent to collecting available cash flow units until the endogenously decided capital size k is attained and then paying the total price $\mathcal{P}(k)$. We argue that a number of elements support this specification. First, it lets us recover in a stylized manner that entire businesses are exchanged without changing global value. Second, it stresses that businesses can be bought not only by a single individual but by several individuals associated together.

These assumptions let us capture the fact that selling a business cannot be reduced to selling only its tangible assets. Instead, the value recovered after a transaction should cover the discounted value of future profits.²⁴ We convey this idea here through the fact that the price p is an abstract object. It is determined at the global equilibrium between cash flow units sold and bought translating at the same time the intertemporal (since holding businesses provide an expected stream of future profits) and intangible (with maturity affecting the relative value of buying versus founding) values of a business.

Finally, this pricing specification ensures that the value associated with selling a business is always higher than the liquidation value. A price $p = 0$ would mean that businesses are

²³Our price specification closely resemble the pricing found on business selling marketplaces where the selling price is computed based on total cash flow added to the asset value. Further details on business selling transaction data are provided in Appendix A.4.

²⁴Using the ValuSource business for sale transaction data, we estimate a ratio of intangible assets over the business price of about 38% for the median and 54% for the mean. Moreover, [Bhandari and McGrattan \(2018\)](#) find that there is little cross-sectional dispersion in intangible assets valuation, supporting our choice of a single price p for all cash flow units.

sold at their liquidation value and that the market does not price maturity. Consequently, in equilibrium, it should be that $p > 0$ whenever the mass of sellers and buyers is positive.

3.5 Demography and Bequest

The model features multiple generations of individuals. An individual in the last age bracket has a probability p_{die} to die. In such a case, the individual is assumed to be reborn as a worker with age $j = 0$, with the ownership of the net of estate taxation bequest. Estate taxation is defined by the tax rate τ_a on every unit of a bequest left to the descendant. When an entrepreneur dies, we assume that the business is liquidated and that the debt is reimbursed. The remaining becomes initial wealth for the newly born worker.²⁵

3.6 Government

The government collects revenues from labor income taxes and pensions (defined as the amount $\mathcal{Y}(J, y)$), as well as from estate taxation and taxes on the sale of businesses. Government expenditures comprise an exogenous government spending proportional to aggregate output, $G = \bar{G}Y$ and pensions. The government budget constraint is:

$$\begin{aligned} & \int_{\mathbf{X}^w} \left(\mathcal{Y}(y, j) \tau_w + \int_{\mathbf{X}^e} \left(z(\mathbf{X}^e) p [\mathcal{V}((1 - \delta)k(\mathbf{X}^e)) - \mathcal{V}((1 - \delta)k(\mathbf{X}^e), 0)] \tau_s \right) d\Phi(\mathbf{X}^e) \right) \\ & + \int_{\mathbf{X}} \mathbb{1}_{j=J} p_{die} \tau_a a(\mathbf{X}) d\Phi(\mathbf{X}) = \bar{G}Y + \int_{\mathbf{X}^w} \mathcal{Y}(J, y) d\Phi(\mathbf{X}^w) \end{aligned} \quad (21)$$

3.7 Equilibrium

A recursive competitive equilibrium consists of value functions for entrepreneurs and workers $\{E^{cont}(\mathbf{X}^e), E_z^{exit}(\mathbf{X}^e), S^{ee}(\mathbf{X}^e), S^{fe}(\mathbf{X}^e), E(\mathbf{X}^e)\}$, $\{\mathcal{W}^{cont}(\mathbf{X}^w), \mathcal{W}_d^{exit}(\mathbf{X}^w), \mathcal{B}(\mathbf{X}^w), \mathcal{W}(\mathbf{X}^w)\}$, decision rules $\{a'(\mathbf{X}), k'(\mathbf{X}), d(\mathbf{X}^w), z(\mathbf{X}^e), c(\mathbf{X}^e)\}$ and occupational choices, factor prices $\{w, r\}$, a price p for a unit of business profit, and government spending \bar{G} such that:

1. Household optimize value functions and decision rules by solving problems (4)-(16).
2. The labor and capital markets clear. Total labor demand by the corporate sector equals household labor supply. The wage is determined by the marginal productivity of labor in the corporate sector, such that $L_c = \int_{\mathbf{X}^w} h(j) y d\Phi(\mathbf{X}^w)$. Corporate capital and the total entrepreneurial capital equate total agent's net worth in the economy: $K_c + \int_{\mathbf{X}^e} k(\mathbf{X}^e) d\Phi(\mathbf{X}^e) = \int_{\mathbf{X}} a(\mathbf{X}) d\Phi(\mathbf{X})$. The interest rate is determined by the marginal productivity of capital in the corporate sector.²⁶

²⁵We also studied a version with a voluntary bequest motive in which older individuals value the utility of their descendants with a *warm-glow* utility function of the form $\mathcal{V}(a)$. The results are qualitatively similar.

²⁶By a no arbitrage condition factor prices are identical in the entrepreneurial and the corporate sectors.

3. The government budget constraint in (21) is balanced with \bar{G} .²⁷
4. The *SMESM* clears such that the price p in the pricing function (17) equates the value of relative expected cash flow units sold to those bought in equilibrium.
5. The distribution of agents $\Phi(\mathbf{x})$ is induced by decision rules and exogenous shocks and is summarized by the transition matrix of the system $M(\mathbf{X}', \Phi' | \mathbf{X}, \Phi)$. A steady state implies a stationary measure $\Phi(\mathbf{X})$.

This problem has no analytical solution and has to be solved numerically. Two main computational challenges arise. First, the dimensionality of the problem with two assets is large and fast optimization methods are required. Second, due to the presence of both discrete (occupational choices) and continuous choices, first-order conditions are no longer sufficient while still necessary. Our computation strategy follows a version of the Discrete Continuous Endogenous Grid Method (DC-EGM) developed in [Iskhakov et al. \(2017\)](#) with taste shocks to smooth kinks. Our specific algorithm is discussed in [Appendix B.1](#).

4 Parameterization

We parameterize the model to match microdata on the purchasing and selling margins, occupational choices, and life-cycle patterns. We compute the moments using the Current Population Survey (CPS) averaged from 2000 to 2008, the SCF averaged over the 2001, 2004 and 2007 waves, and finally the 2007 SBO. We pin down a number of parameters by normalizing them or by relying on values widely used in the literature. We then jointly set the rest of the parameters to match key moments in the data with their model counterpart.

4.1 Fixed Parameters

Demography and preferences We set $J = 9$, with 8 stages to represent adult working life, of 5 years each, and a last bracket to capture all ages beyond the retirement threshold. We use the following utility function:

$$\mathcal{U}(c, j, o) = \frac{(c^{1-\sigma} - 1)}{1 - \sigma} - \mathbb{1}_{j=J} u_R + \mathbb{1}_{o=o_e} u_E \quad (22)$$

with relative risk aversion $\sigma = 1.5$ and u_R and u_E are jointly endogenously calibrated.²⁸ Senior households face an additional utility cost u_R when operating a business, in order to translate the difficulty of still being active in old age.

²⁷In the benchmark economy, we set τ_w and let \bar{G} adjust. In counterfactual experiments, we keep \bar{G} to its benchmark value and adjust the tax rate τ_w .

²⁸Since a complete characterization of preference heterogeneity is outside the scope of this paper, we assume a unique IES-risk aversion parameter σ . However, risk aversion has been shown to have a key role on entrepreneurial decisions (see for instance [Herranz et al. \(2015\)](#)). In our setup, due to maturity-specific risk, high risk aversion individuals would rather purchase than found. We leave this very relevant issue for future research.

Earnings and retirement The labor income process is particularly important for the decision to become an entrepreneur as it lets workers accumulate sufficient wealth to run valuable businesses when they are endowed with the entrepreneurial ability.²⁹

We define labor earnings as a function of the wage level w , an age-dependent component $h(j)$ and a persistent stochastic process for labor productivity y such that:

$$\log(\mathcal{Y}_{i,t}(j, y)) = \log(w_t) + \log(y_{i,t}) + \log(h_{i,t}(j)) \quad \forall j \in \{0, \dots, J\} \quad (23)$$

$$\log(y_{i,t}) = \rho_y \log(y_{i,t-1}) + \epsilon_{i,t}^y; \quad \epsilon_{i,t}^y \sim \mathcal{N}(0, \sigma_y) \quad (24)$$

We discretize the process for y by setting $\rho_y = 0.96$ and adjusting the variance to $\sigma_y = 0.2$ to generate an earnings Gini of 0.36. When $j = J$, $h(j)$ defines the retirement pension that we set to 40% of the average income. Once retired, an individual keeps the same component y forever and her offsprings' productivity is drawn from the invariant distribution.

Otherwise, the components $h(j)$ for $j \in \{1, \dots, J - 1\}$ are chosen in order to replicate the average lifetime earning profile within each earning percentile as in [Güvenen et al. \(2015\)](#).³⁰ Additionally, the probability of dying, p_{die} , is set to 0.091 (corresponding to an expected retirement period of 11 years). The benchmark labor tax rate τ_w is set to 0.15.

Adjustment costs and liquidation value Incumbent and entering business owners pay a cost $\mathcal{C}(k, k')$ to adjust entrepreneurial capital from k to k' . For tractability, we assume those adjustment costs are linear, with ϕ_u the per-unit cost.³¹ At the other end, there are transaction costs when selling business assets. In particular, we assume that for each unit of business capital liquidated outside of the SMESM, entrepreneurs recover only a fraction $(1 - \phi_d)$. To summarize, we have:

$$\mathcal{C}(k, k') = \begin{cases} \phi_u(k' - k) & k' > k \\ \phi_d(k - k') & k < k' \end{cases} \quad (25)$$

We set ϕ_d to 30%, corresponding to a business capital recovery rate of the of 70%, which is in the range of the average liquidation costs reported in [Alderson and Betker \(1995\)](#). For the sake of parsimony, we normalize ϕ_u to 0.³²

Business maturity and intangible value This paper quantifies the importance of the SMESM when owners can sell both the physical capital assets and the intangible value of their firm. To quantify the importance of the SMESM, we must consistently match that intangible value,

²⁹Three saving motives arise in the model. A precautionary one due to the inherent productivity risk, a life-cycle one, and an entrepreneurial motive in order to acquire and run a larger, more profitable firm.

³⁰We provide the values in Appendix B.2.

³¹Assuming linear cost greatly simplify the complexity of the problem by avoiding to keep track of the past capital level when investing or disinvesting.

³²We perform sensitivity analysis on the effect of this parameter in Appendix C.

translated in the model by maturity components. An immature business switches from early-stage to mature with a yearly probability of 20% (about 5 years in operation). Maturity allows businesses to be sold and implies some additional benefits: (i) a lower interest rate charged on the debt, (ii) a higher borrowing limit, (iii) higher profitability, and (iv) a lower probability to fail. All these elements have been highlighted in Section 2 and we discuss in Section 6.2 the importance of each component for the intangible value of a business.

A mature business pays a lower interest rate on its financing, translating the higher amount of information that a creditor has access to (i.e. history of past transactions, client lists, etc.), which is intrinsically part of the intangible business value. We therefore define the debtor interest rate as $r_b(m) = r + v_s + v_m \mathbb{1}_{m=0}$, where v_s is a wedge common to all businesses while v_m is the additional interest rate premium charged on early-stage businesses. We set $v_s = 2\%$, the usual value used in the literature. The wedge charged on immature firms is set to 1.5% in line with our estimates.

The borrowing limit tightness $\theta(m = 1)$ is set to 0.3: entrepreneurs have to provide a down payment of 30%. In total, entrepreneurs can borrow up to $(1 - \theta(0))(1 - \phi_d) = 49\%$ of the business assets k , and therefore provide the remaining 51%, which is close to the 50% assumed in [Herranz et al. \(2015\)](#).³³ We estimate that for firms within 4 years of their purchase or foundation, the former ones are granted about 3-4 percentage points higher loans than the latter. Accordingly, we choose $\theta(m = 0) = 0.35$, corresponding to a borrowing limit of 0.455%.

We adopt a conservative 10% profit rate wedge which is lower than our estimate from the data but is closer to the wedge in [Clementi and Palazzo \(2016\)](#) for new entrants relative to old incumbents.³⁴ We therefore normalize $f(k, 0) = \gamma(0)k^\nu$ and $f(k, 1) = \gamma(1)k^\nu$, with $\nu < 1$ and $\gamma(1) = 1.1\gamma(0)$. Parameters ν and $\gamma(0)$ are part of the joint calibration.

Finally, we discussed in Section 2 that a key advantage of purchasing an existing business is a substantial reduction in the probability of failure in the first years after the acquisition. We convey this idea in the model by pinning down $\chi(m)$, the probability of failure. In the 2016 ASE, the fraction of early-stage business owners (within 5 years of acquisition) exiting for reasons related only to business conditions account for 50% of total exits.³⁵ We, therefore, set $\chi(0)$ to 50% of the average exit rate of newly created businesses, the latter being around 20 to 25%. Consequently, $\chi(0) = 0.12$. Then, using the 2007 SBO, we estimate a difference of 7 percentage points in the likelihood to fail of mature firms with respect to early-stage ones for recently acquired businesses. We thus set $\chi(1) = 0.05$. We then endogenously adjust ζ , the

³³In Appendix C we provide sensitivity analyses on this parameter and show that the model properties are broadly unaffected by reasonable changes to this parameter.

³⁴In Appendix C, we double the wedge associated to this component as a robustness exercise. All the quantitative results are magnified following the larger gap in maturity value between early-stage and mature firms.

³⁵The ASE survey question regarding the reasons to cease has multiple choices. This number is the total of *business failure* responses over the total number of responses excluding *other reasons*.

probability of exogenous entrepreneurial exit (independently of business maturity), to match a realized entrepreneurial exit rate of 15%.

Matching probabilities Buyers and sellers are subject to selling frictions captured by the respective probabilities of finding a seller (h_b) and a buyer (h_s). Given the scarcity of business transactions data, measuring those probabilities pose a challenge, in particular for small and middle-sized businesses. On the seller side, we circumvent this issue by relying on a new dataset of business selling transactions from a leading U.S. online marketplace. This dataset includes more than 90,000 observations and provides various business-specific characteristics such as age, size, cash-flow, EBITDA, the fraction of fixed assets, the number of employees, etc. In contrast to other business transactions data, we continuously observe businesses for sale and closed transactions over time, allowing us to construct a panel dataset of businesses for sale. We provide a detailed overview of this dataset in Appendix A.4.

We use the above data to infer the probability that a business is sold within a year by constructing a daily panel of businesses for sale between 2018 and 2019. We then construct cohorts of those businesses and compute the total number of sold businesses over time. We exclude from the cohorts all the businesses that were removed from the listings without resulting in a sale.³⁶ Then, a year after the first listing dates, we compute the fraction of sold businesses relative to the total initial number of businesses for sale within the cohort. The resulting indicator provides the fraction of businesses for sale that is actually sold after a year. Using this indicator, we find that the average probability of selling a business is about 30% after a year. This probability displays a slight size-dependence. Taking the price as a proxy for size, we find that firms with a listing price below 500K dollars (resp. above 1000K dollars) have a probability of being sold of 35% (resp. 27%) after a year. Therefore, we pick a conservative estimate for the probability of finding a buyer with $h_s = 0.3$.

Other parameters The corporate sector features a constant returns to scale Cobb-Douglas production function with capital share $\alpha = 0.34$. Total factor productivity is normalized to $A = 1$ and capital depreciates at rate $\delta = 0.07$ in both sectors. The estate taxation is set to 30%, consistently with the statutory tax rate in the U.S. and the value used in Cagetti and De Nardi (2009).

In the U.S., capital gains following the sale of a business are taxed at a statutory tax rate between 0% and 20%. We choose a benchmark tax of 10%. Finally, we calibrate the transition probability of the entrepreneurial ability process ι : we endogenously determine $p_\iota = P(\iota' = 1 | \iota = 0)$ and we restrict $P(\iota' = 0 | \iota = 1) = \chi(0)$.

³⁶Results are qualitatively similar if we include those businesses, while the magnitude of the probabilities is lowered by around 15-20%. We posit that excluding vanishing businesses from the stock of businesses for sale lets us exclude businesses that are not performing well from the sample.

Table 3. Fixed parameters

Parameter	Value	Description
σ	1.5	Risk-aversion coefficient
$\{\delta, \alpha\}$	$\{0.07, 0.34\}$	Depreciation rate, Corporate returns to scale
$\{\rho_y, \sigma_y\}$	$\{0.96, 0.2\}$	Earnings process
$h(j)$	See Appendix B.2	Life-cycle earnings
p_{die}	0.091	Probability of dying during retirement
h_s	0.3	Probability of selling the business within a year
ϕ_d	30%	Liquidation recovery rate
$\{\tau_s, \tau_a\}$	$\{10\%, 30\%\}$	Selling and estate tax rates
P_m	20%	Probability of maturing
$\{v_s, v_m\}$	$\{2\%, 1.5\%\}$	Interest rate wedge for immature/mature businesses
$\chi(m)$	$\{12\%, 5\%\}$	Exogenous probability to fail
$\gamma(m=1)$	$1.1\gamma(m=0)$	Profitability wedge (10%)

4.2 Joint Parameterization

The remaining nine parameters are chosen jointly so that the model matches nine moments of the U.S. economy related to the small business market, entrepreneurship, and the wealth distribution. The discount factor β helps to match a capital-output ratio of 3.2, computed using the Penn World Table 9.1. The probability of being endowed with an entrepreneurial ability p_i captures the fraction of entrepreneurs in the working-age population, which ranges between 7% to 12% in the data, depending on the survey, the period considered and the definition. We choose a target of 11%. The probability to fail for exogenous reasons ζ helps to match the exit rate of entrepreneurs, which is equal to 15% in the PSID, according to [Mankart and Rodano \(2015\)](#). Entrepreneurial ability scale $\gamma(0)$ helps to match a share of small business GDP of 46%, as reported in [Kobe \(2012\)](#) for 2008, while the return to scale ν helps to match the wealth Gini coefficient of 0.81. The purchasing fixed cost captures the ratio of the mean capital of purchased business relative to founded ones which is 2.2 in the SSBF, and h_b helps to recover a fraction of purchased businesses of 22% (SCF) upon entry.³⁷ Finally, preference parameters u_E and u_R help to capture the ratio of the median net worth between entrepreneurs and workers of 7.0 which is closely what is observed in the SCF, and the about 5% of entrepreneurs in the last age bracket.

Our model is exactly identified, with nine parameters used to pin down nine moments. The resulting parameter values are reported in [Table 4](#).

³⁷The mean capital of purchased business relative to founded ones is computed by comparing firms within 5 years of acquisition.

Table 4. Model parameters calibrated within the model ^a

Description	Symbol	Value	Data	Model	Source/Main moment ^c
Discount factor	β	0.910	3.20	3.24	Capital-output ratio (Penn World Table 9.1)
Returns to scale priv. bus.	ν	0.845	0.81	0.81	Wealth Gini coefficient
Buyer's matching friction	h_b	0.331	22.0	24.0	% purchasing bus. (SCF)
Prob. to fail for exo reasons	ζ	0.081	15.0	14.9	% exiting self-employed (PSID)
Disutility of working (retired)	u_R	1.601	4.87	4.84	% retired entrepreneurs (SCF)
Non-pecuniary benefits	u_E	1.490	7.00	6.80	Ratio median net worth E/W
Buying fixed cost	F_b	1.271	2.20	2.22	Ratio mean K buying/founding (SSBF)
Probability entrep. ability ^b	p_l	0.020	11.0	11.3	% share of entrepreneurs to workers (SCF)
Entrepreneurial ability scale	$\gamma(0)$	0.570	46.0	45.1	% share of small business GDP, SBA

^a The main moments are indicative. Changing one endogenous parameter affects the whole equilibrium. All targets are matched within an interval lower than 10%.

^b Computations using the CPS are averaged from 2001 to 2008.

^c The share of GDP attributable to small businesses (less than 250 employees) in the U.S. is taken from the OECD estimates.

5 Properties of the Baseline Model

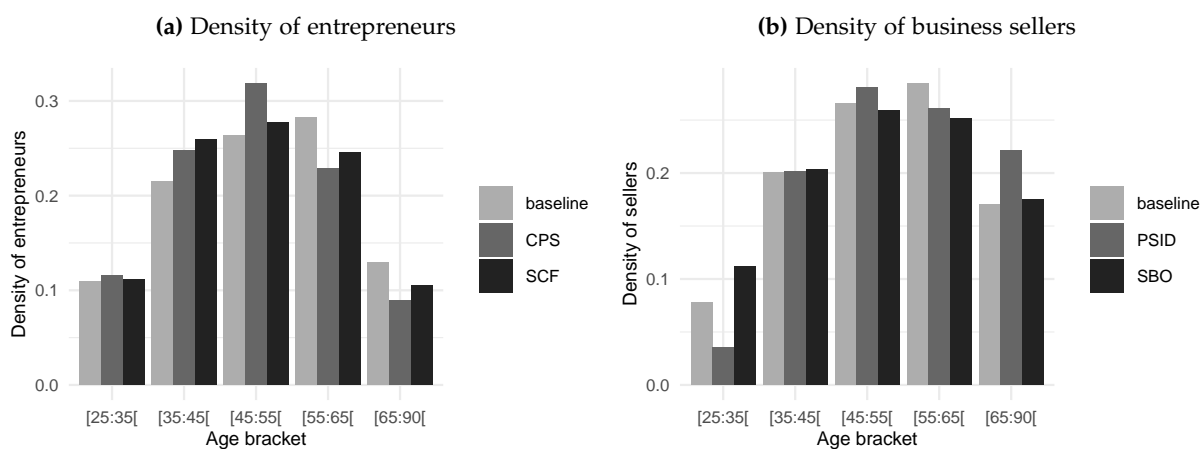
5.1 Model Validation

In this section, we validate our framework by reporting key model generated statistics that were not targeted in the joint parameterization. In our baseline model, the entrepreneurial sector holds around 47% of total capital which is slightly higher than the 40% reported in [Quadrini \(2000\)](#). We find a fraction of mature businesses of 66%. This is comparable to the statistics reported by the Bureau of Labor Statistics: 64% of businesses were 5 years or more in 2003 and the corresponding number is 62% in 2010. This fraction is generated by the probability to mature (P_m) and the fraction of mature businesses that are transferred between individuals through the SMESM. The fraction of agents with zero net worth in the population is 14%, against 12% in [Quadrini \(2000\)](#).

While we pin down the probability of selling a business within a year, the baseline generates a ratio of sellers to exiting entrepreneurs of about 9.9%, against 7% to 20% in the 2007 SBO, the 2014-2016 ASE, and the NLSY79. Regarding life-cycle characteristics, [Figure 4a](#) displays the baseline density of entrepreneurs by age bracket compared to the distribution in the 2007 SCF. Similarly, [Figure 4b](#) compares the baseline density by age bracket of entrepreneurs exiting by selling their business to both the 2007 SBO and to business assets in the PSID averaged over the 1989-2015 waves. The model replicates reasonably well the life-cycle patterns of an average entrepreneur in the economy. It is especially relevant here as we are interested in characterizing who sells and buys businesses. We also find that the ratio of business assets sold in the last age bracket (65 and over) relative to the total business assets being sold is about 44% in the PSID averaged over the 1989-2016 waves. The corresponding number in the baseline is 33%. However, note that in the PSID, we can not distinguish between business assets sold as part of the sale of entire businesses and liquidations of fractions of business

assets. Our baseline number is only about entire business sales. Regarding the age of entry into entrepreneurship, we find a mean age of 44 for both founders and purchasers in the 2007 SCF while it is respectively 45 and 46.8 in the model.³⁸ On the exit side, the mean age of business asset sellers is 53.7 in the PSID, while it is 52.4 in the model. Thus, consistently with the data, the entrepreneurial life-cycle appears to be a key component of both the model and the SMESM. In an alternative economy where we double the disutility cost u_R associated with working while in retirement, we observe a decrease in the fraction of individuals in the last age bracket from 4.8% in the baseline to 1.4%. At the same time, the fraction of business sellers substantially increases to reach 12.3% and the business price p reduces from 0.17 to 0.16 (see Appendix C for details).

Figure 4. Life-cycle pattern of entrepreneurship



Note: we report the survey weighted density for the 2007 Survey of Consumer Finance (SCF) and the 2007 Survey of Business Owners (SBO) and the PSID (averaged from 1989 to 2015, deflated). Baseline reports the exact same densities in the model.

Finally, the baseline model is also able to closely reproduce wealth concentration and inequality statistics. We delay this discussion to Section 6.4.

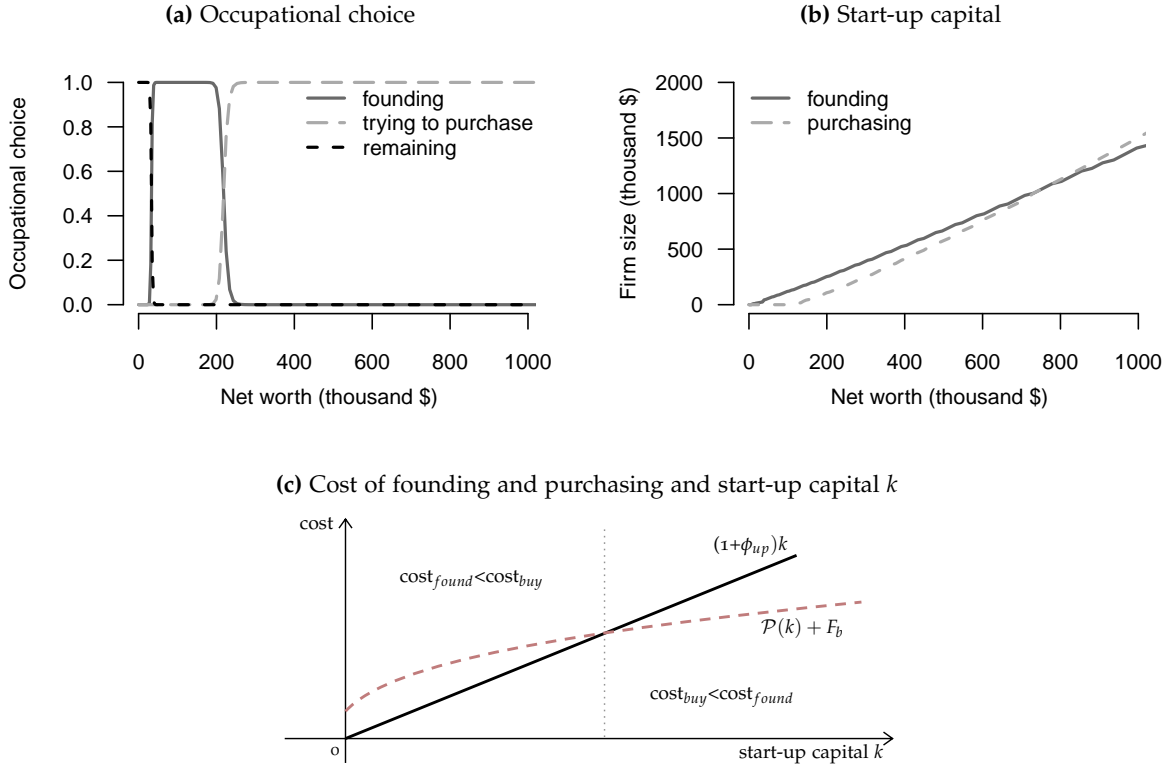
5.2 The Decision to Enter Entrepreneurship

The selection in and out of entrepreneurship is a key element of our model. The main drivers leading individuals to select into entrepreneurship is wealth and entrepreneurial ability. Following the literature, non-pecuniary benefits appear as an additional driver. Concerning the type of acquisition, fixed costs F_b lead wealth-poor individuals to enter entrepreneurship by founding instead of purchasing. Figure 5a displays the model-based decision to enter entrepreneurship and the type of acquisition as a function of wealth for an entrant with av-

³⁸Our comparison point is new entrepreneurs within 4 years of firm ownership. Notice that in the CPS, the average entry age into entrepreneurship is 43 in 1996 and 48 in 2016. Using the SSBF, we find no clear difference between the age of buyers and founders, as shown in table 11 of Appendix A.2. In the model, we use the midpoint within an age bracket to compute the mean age.

erage working productivity. Conditional on always finding an existing business, wealthy individuals would rather purchase an existing mature business instead of founding a new one. However, the existence of matching frictions ($h_b > 0$) suggests that only 33% of potential purchasers will match a seller. Consistently with the fact that purchasers are in general wealthier, the ratio of the mean net worth between recent purchasers and founders is about 2.8 in the SCF (2007). In the model, this ratio is about 2.1.

Figure 5. Occupational choice and start-up capital as a function of wealth for new entrants.



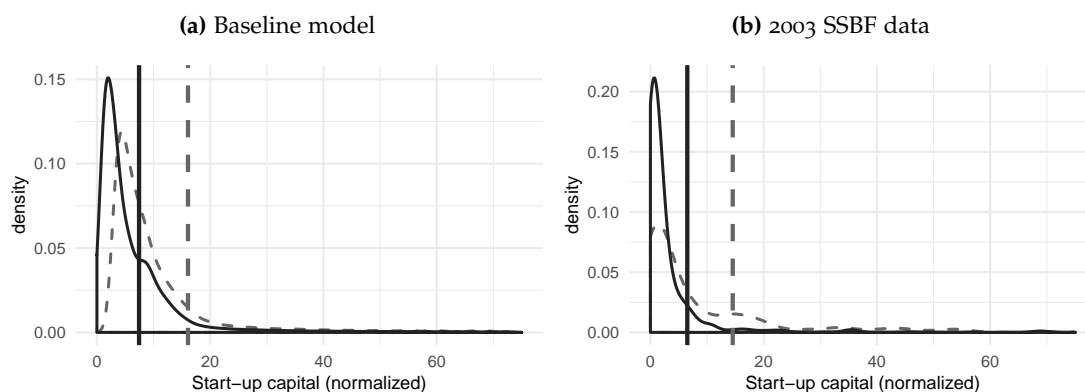
Note: panel (a) displays the behavior of an hypothetical worker ($j = 4$, $y = 2$ and $\iota = 1$) when faced with the alternatives of entering entrepreneurship either by founding or purchasing and remaining a worker. Panel (b) displays the behavior of an hypothetical entrant with $j = 4$. Notice that the probability to switch occupations is not binary due to the perturbation method used to smooth the kinks generated by the occupational choices.

Alongside the decision to buy or found, a prospective entrepreneur also chooses the amount of start-up capital. In Figure 5b, we display a typical model-based start-up capital policy function for a new entrepreneur deciding either to buy an existing business or found a new one. There is a threshold below which agents found and above which they purchase. The exact position of the threshold is state-dependent. We illustrate the case of an individual in a middle-age bracket for whom the threshold is at a net worth around 750K dollars. Purchasing a size k business is more expensive than founding one of the same size. This is due to the intangible value embedded in a purchased firm which incentivizes entrepreneurs to buy an existing mature business even when the initial size is smaller. Finally, the slope difference between the purchasing and the founding curves is generated by two components. First, founders face a tighter borrowing constraint because $\theta(0) > \theta(1)$. Second, the nature of

the business price leads to decreasing returns to scale when buying larger businesses. This is due to the concavity of the production function reflected in the maturity value that enters the pricing formula $\mathcal{P}(k)$ together with the fact that the liquidation value is $(1 - \phi_d) < (1 + \phi_u)$. As a consequence, as shown in Figure 5c, the average price $\mathcal{P}(k)/k$ declines as the purchased business capital increases, leading to higher start-up capital when entrepreneurs are able to purchase larger businesses.

We finally show in Figures 6a and 6b the distribution of start-up capital as a function of business acquisition type: foundation or purchase. The model generates a consistent right-skewed distribution of start-up capital. Moreover, the distribution of purchased businesses is shifted to the right relative to that of founded ones.³⁹ To further relate the above points to empirical observations, in the SSBF, among firms within 3 years of their acquisition, the ratio of total firms assets between purchased and founded firms is about 3.7. In terms of the number of employees, profit and total sales, these ratios are respectively 2.1, 2.1 and 3.8 (see Appendix A.2 for details). We observe similar evidence using the median ratio. From this, we infer that purchased businesses are indeed substantially larger upon acquisition, a feature that the model captures well.

Figure 6. Density of start-up capital by type of acquisition.



Note: we normalize start-up capital by the median net worth in the baseline model and the data. The straight line corresponds to founded businesses while the dashed line to purchased businesses. The corresponding vertical lines indicate the mean normalized start-up capital for each density.

5.3 The Decision to Exit Entrepreneurship

Our baseline model is consistent with several features of entrepreneurial exit. Most notably, it is able to capture the exit behavior of firms with respect to their maturity. In Table 5, we compare the survival rates of firms in our model to both the Bureau of Labor Statistics and

³⁹Notice however that our resulting distribution of start-up capital for purchased businesses is less dispersed relative to the data. An extension with an heterogeneous fixed cost F_b in the range of Clementi and Palazzo (2016) would generate such a distribution. For instance, increasing F_b to 2.0 raises the average firm size of purchased businesses from 20.2 in the baseline to 22.4.

Knaup and Piazza (2007) data. We start with the survival rate relative to the establishment year: our baseline replicates it well in the first years but underestimates it after 6 years. This comes from the fact that for mature businesses, the failure rate, given by $\zeta + \chi(1)$ is somewhat constant.⁴⁰ However, a model without maturity and constant exit rate, such as the one in Cagetti and De Nardi (2006), would underestimate the survival rate even more: after 5 years, such a model has a survival rate lower by 5 percentage points relative to our baseline. Moreover, the model captures quite well the increasing survival rate as firms mature.⁴¹ In Section 6, we discuss the central role of maturity in our model, relating it to the intangible value of a firm embedded in the purchasing option. Interestingly, in the data, the survival rate relative to the preceding year substantially increases between 4 and 5 years, which may indicate that firms start to be well-established after that. This supports our calibration of the probability to switch from immature to mature with an average of 5 years in the early-stage.

Table 5. Survival rate: model versus data

	Number of years						
	1y	2y	3y	4y	5y	6y	10y
Survival rate relative to first year							
U.S. data (BLS)	80.1	68.7	60.2	52.6	46.8	43.2	33.8
U.S. data (Knaup and Piazza (2007))	81.2	65.8	54.3	44.4	38.3	34.4	–
Baseline model	80.8	66.4	55.3	46.5	39.5	33.7	19.6
Zero probability to mature ^a	80.8	65.4	52.8	42.7	34.5	27.9	11.9
Survival rate relative to the preceding year							
U.S. data (BLS)	80.1	85.8	87.6	87.4	89.0	92.3	94.1
U.S. data (Knaup and Piazza (2007))	81.2	81.0	82.6	81.7	86.3	89.9	–
Baseline model	80.8	82.1	83.2	84.1	84.9	85.4	86.7
Zero probability to mature ^a	80.8	80.8	80.8	80.8	80.8	80.8	80.8

^a This is the survival rate when entrepreneurs are not allowed to mature using the same panel of entrepreneurs.

Overall, the baseline model’s ability to appropriately replicate a number of key features of the data that were not targeted during the joint parameterization seems sound to us.

⁴⁰In a recent paper, Fairlie et al. (2018) document survival rate differences between start-ups with and without employees, as well as the dependence on the legal form. Our estimates are in the range of theirs.

⁴¹The fact that the model survival rate is lower as compared to the BLS data can be explained by their specific definition: their survival rate is constructed using establishment openings (new businesses consisting of both establishments that are created and establishments that are reopening, including establishments that open on a seasonal basis). Moreover, the difference with the data is quite large after 10 years. Nevertheless, the model does match the difference in survival rates one year after acquisition between founded businesses and purchased ones, which is more likely to be relevant for agents deciding between those two options. Moreover, the data reports the survival rate of firms with at least one employee. In practice, the model also accounts for self-employed businesses that may have lower survival rates. See Knaup and Piazza (2007) for further details.

6 Businesses for Sale Market: Quantitative Analysis

This model is the first to feature a businesses for sale market (SMESM) letting entrepreneurs transfer their business assets to a different owner. Thus, this section presents our quantitative assessment of the importance of that market and the associated maturity components. We show that the ability to build a firm’s maturity is a critical component of that market and its outcomes. We emphasize two dimensions: the aggregate outcomes and the cross-sectional implications on occupational choices, the distribution of firms, and wealth inequality.

6.1 Assessing the Importance of Business Transfers

We first investigate the counterfactual in which the SMESM is missing. In this alternative economy, all new entrants must found a new immature business and all exiting entrepreneurs must liquidate their assets. We consider two situations: a general equilibrium (GE) case where prices and labor tax τ_w adjust and a partial equilibrium (PE) case where prices and taxes are kept at the baseline level. [Table 6](#) reports the results.

Without the SMESM and, thus, in the absence of any business transfers, the steady-state decline in aggregate output is substantial at 10.5%. This drop is mostly due to the 18.7% decrease in the SME sector production, accounting only for 41% of total production against 45% in the baseline. As a side effect, aggregate savings also decline by almost 15%, which leads to a corporate sector production drop of 3.8%. Overall, 80% of the loss is attributable to the decrease in the SME sector production, while 20% is coming from a lower corporate output.

Interestingly, the decline in aggregate savings has important effects on prices: the interest rate is higher and the wage rate is lower in equilibrium.⁴² These GE effects somewhat counteract the potential corporate output loss: in the PE case, aggregate savings losses are much larger. In the absence of price adjustments, the steady-state aggregate output would decrease by 13% with a 21% drop of aggregate savings.

Table 6. Aggregate outcomes with and without the SMESM

	Y	$\frac{\Delta Y}{Y}$	Y_c	Y_{SME}	K	$\frac{Y_{SME}}{Y}$	$\frac{K}{Y}$	r_s	w	τ_w
U.S. data ^a	–	–	–	–		46.0	3.20	–	–	–
Baseline with SMESM	2.38	–	1.31	1.07	7.70	45.1	3.24	4.9 %	1.18	15.0 %
No SMESM (GE)	2.13	-11%	1.26	0.87	6.56	40.8	3.07	5.6 %	1.15	16.0 %
No SMESM (PE)	2.07	-13%	1.21	0.86	6.07	41.4	2.94	4.9 %	1.18	15.0 %

^a The U.S. share of output produced in the SME sector is taken from [Kobe \(2012\)](#). Y_c and Y_{SME} refer respectively to the corporate and the SME sector output.

In [Table 7](#), we compare our baseline to the alternatives described above but along the

⁴²Moreover, notice that the labor income tax increases since a higher fraction of older individuals are not working and there are no government revenues from business transfers.

lines of occupational decisions and firm size distribution. Without the SME for sale market, the fraction of entrepreneurs diminishes by about 0.3 percentage points. A larger drop is partly offset by higher incentives to enter entrepreneurship coming from lower wages and a higher interest rate, the latter letting workers accumulate more capital. Surprisingly, even without price changes, the entry rate increases. This is directly linked to the absence of a SMESM. Indeed, we find that entrepreneurial entry decisions are not comparable with and without the SMESM. One key reason is the natural incentive to wait for a future opportunity after an unsuccessful attempt to buy an existing business. Such a mechanism is absent in current entrepreneurial models in the literature. As we illustrate in [Figure 7](#), this mechanism conditions the nature of entry in the sector: the dashed (blue) line reports the probability to enter the sector by founding a business in a setting without the SMESM whereas the dotted (green) line reports the probability to enter by founding conditional on not finding a business to purchase in a setting with a SMESM. With respect to net worth, there is a significant gap between these two lines relating to the incentive to wait. Upon not finding a business to purchase, for a large range of net worths, prospective entrepreneurs will choose to rather wait for a future purchase opportunity. For the same range of net worth, without a SMESM, prospective entrepreneurs can only found and enter resulting in an appreciable difference in the type of entrepreneurial firms that can be generated by the two settings. In this range of net worth, the setting with a SMESM would produce mature firms entry corresponding to business transfers instead of new firms of similar sizes resulting in a diverging pool of firms in the economy. As a comparison point, the solid (red) line indicates the baseline probability to purchase instead of founding. For similar reasons, [Table 7](#) shows that the exit rate increases: in the baseline sellers who are unable to find a buyer might postpone their exit. In the alternative, exiting entrepreneurs can only liquidate their assets and all exits are immediate. This feature also leads old owners (above 65 years of age) to exit entrepreneurship earlier. The fraction of entrepreneurs in the last bracket falls from 4.83% in the baseline to 4.58% without the SMESM.

Table 7. Occupational choice and distribution of firms with and without a SMESM

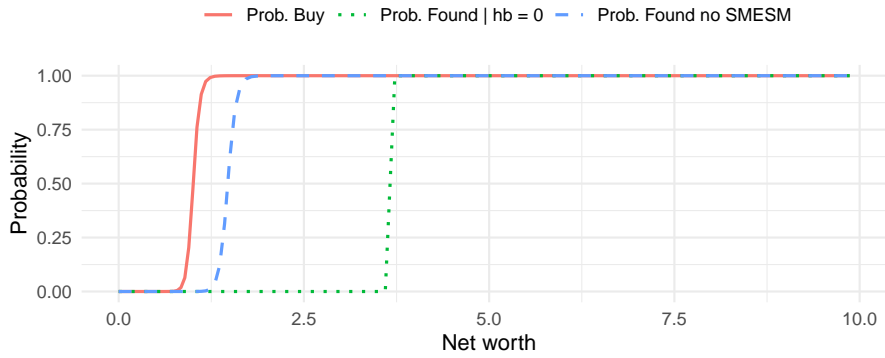
	$\% \frac{E}{W+E}$	Mature ^a %	Entry ^b %	Exit ^b %	Average size		
					<i>k</i>	<i>k</i> entry	<i>k</i> exit
Baseline with SMESM	11.27	66.0	1.797	14.94	45.0	11.8	41.3
No SMESM (GE)	10.99	53.2	1.838	15.74	36.1	9.1	32.0
No SMESM (PE)	10.90	53.2	1.824	15.74	35.9	8.5	31.7

^a This column measures the share of mature ($m = 1$) businesses in the economy.

^b The entry rate is computed as the ratio of workers entering entrepreneurship relative to the total population of workers. The exit rate is computed as the ratio of entrepreneurs exiting entrepreneurship relative to the total population of entrepreneurs.

Without the SMESM, valuable existing businesses are liquidated instead of being transferred, resulting in two especially remarkable findings concerning the distribution of firms. First, the share of mature businesses falls by a considerable 13 percentage points, implying

Figure 7. Entrepreneurial entry decision



that the SMESM alone is a key market in transferring overall business maturity. As a result, the economy is faced with a substantial loss in intangible assets due to the impossibility to transfer them. Second, there is a significant decrease in the average firm size due to both the overall increase in the failure rate and the lower average start-up capital upon entry. Indeed, without the SMESM, immature businesses are founded resulting in a higher failure rate as compared to mature ones. Consequently, a smaller number of businesses are accumulating productive capital over time, reducing aggregate output. As smaller firms enter, the overall distribution of firm sizes is shifted to the left and the average firm size upon exit is also significantly lower (by about 22%). In other words, the possibility to transfer business assets through the SMESM creates an environment where firms expand without losing intangible value.

6.2 Decomposing Maturity Effects on the Businesses for Sale Market

In this section, we further detail the importance of maturity on aggregate outcomes and entry decisions. To that end, based on our finding documented in Section 2, we decompose the wedges generating components of maturity: (i) the profit rate, (ii) the interest rate charged on the debt, (iii) the failure rate and (iv) the borrowing constraint tightness. Our approach is to set each maturity component to its average value in the benchmark economy.⁴³ To help our decomposition, we distinguish the following alternative economies with respect to our baseline case: case (1) removes the SMESM entirely, case (2) removes all maturity effects making the SMESM inoperative and cases (3) through (6) remove each of the specific maturity components one at a time. Table 8 displays the results of this decomposition.

We find the failure rate and the profit rate to be the most significant components of maturity as emphasized by case (6) and then (5). When removing those two components, the proportion of business buyers in the population of new entrepreneurs is substantially re-

⁴³For instance, concerning the failure rate, as 53% of businesses are mature and 47% are immature without differential failure rate, we set $\chi(1) = \chi(0) = 0.915$ such that we broadly recover the same failure rate as in the benchmark economy.

duced. While the profit rate component substantially increases the returns associated with running a business, the lower failure rate of a mature business largely decreases the risks associated with entrepreneurial activities. The latter generates *higher returns* while the former increase the *persistence* of entrepreneurial returns. Consistently, our results confirm that the option to buy an existing business offers an appealing and empirically relevant mechanism through which prospective entrepreneurs can reduce the risks associated with early-stage entrepreneurship. As a consequence, the fraction of mature businesses in the economy falls, which, in turn, lowers aggregate output. In contrast, the interest rate and borrowing limit components have marginal aggregate effects despite slightly reducing the fraction of buyers and the average business size. In fact, the lower average firm size coming from more stringent borrowing constraints and financial conditions does not significantly impact aggregate outcomes. Consistently, in case (2), where we remove all four of the components above, we find substantial deviations from the baseline. This supports the fact that not taking into account the importance of firm maturity and the accumulation of intangibles would significantly lower aggregate outcomes as well as the fraction of entrepreneurs in the economy.⁴⁴

Overall, maturity components and their potential transfer on a market is a fundamental interaction supporting the underlying mechanisms of entry and exit in the entrepreneurial sector, the firm distribution and aggregate outcomes.

Table 8. SME for sale market, maturity effects and intangible value decomposition

	$\frac{E}{W+E}$ %	Buy %	Sell %	Mature %	Avg. size	Υ	$\frac{Y_{SME}}{Y}$	$\frac{K}{Y}$	r_s
Baseline with SMESM	11.3%	24.0	9.96	66	45	2.4	0.45	3.24	4.9%
(1) No SMESM	11.0%	–	–	53	36	2.1	0.41	3.07	5.6%
(2) No maturity components	11.1%	–	–	49	27	1.9	0.35	2.93	6.0%
(3) No int. rate component r_b	11.3%	23.0	9.89	66	43	2.3	0.44	3.21	5.0%
(4) No borr. cst component θ	11.3%	22.6	9.74	65	43	2.3	0.44	3.21	5.0%
(5) No profit rate component γ	11.4%	19.6	9.59	64	38	2.2	0.42	3.15	5.4%
(6) No failure rate component χ	11.0%	10.7	7.6	55	31	2.0	0.38	3.02	5.6%

6.3 Businesses for Sale Market and Matching Efficiency

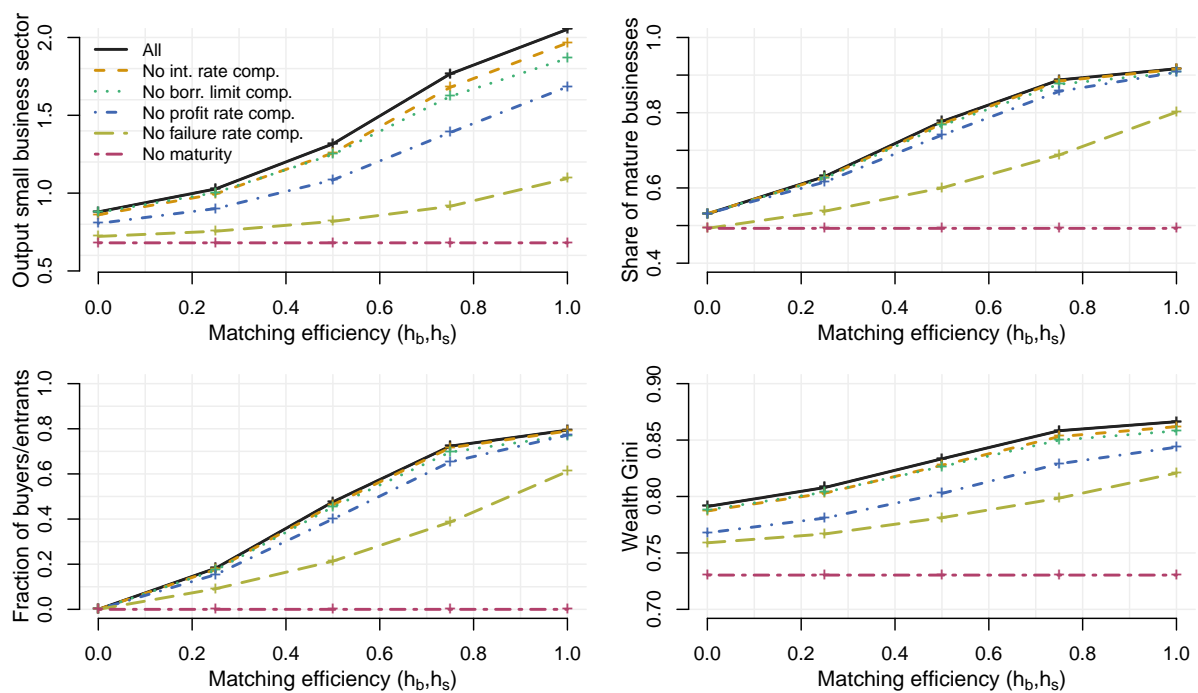
In our parameterization section, we provided evidence of selling frictions on the SMESM. In the model, both the actions of buying and selling a business are dependent on the occurrence of a match on the SMESM. Thus, we investigate the role of the matching efficiency on the propensity to transfer businesses between owners. [Figure 8](#) displays the equilibrium outcomes implied by the counterfactual experiment of increasing the matching probabilities (h_b, h_s) simultaneously and in the same proportion. Furthermore, as we stressed above that

⁴⁴This finding is related to [Bhandari and McGrattan \(2018\)](#) and a number of other papers showing the importance of intangibles when measuring contributions to aggregate output. Moreover, we solely focus on transferable intangible assets. In practice, non-transferable intangible assets might have large additional effects on output.

the effects of maturity were significant, we provide a decomposition by either removing all maturity components or each one, one by one.

We find that lowering the frictions on the SMESM by increasing the matching efficiency yields large aggregate outcomes. In our baseline with all maturity effects, increasing the probability to sell businesses by one percentage point on the SMESM increases output in the entrepreneurial sector by 6.9% and the wealth Gini by 0.8%. This is due to the large increase in the number of mature businesses in the economy. Moreover, consistently with our previous findings, most of the effect on the output of a higher matching efficiency comes from lower failure rates and higher profit rate.⁴⁵ Notice that even in the absence of mismatches between buyers and sellers (i.e. $h_b = h_s = 1$) the fraction of buyers to entrants is capped at 80%. This emphasizes the importance of both borrowing constraints and the fixed cost F_b in reducing the ability to purchase businesses. Interestingly, the last panel of Figure 8 shows that wealth concentration is sensitive to the component of the maturity and matching efficiency. We discuss the role of maturity in shaping wealth inequality in the next section.

Figure 8. The importance of the matching efficiency on the SME for sale market



6.4 Wealth Concentration and Inequality

In this section, following the important literature relating entrepreneurship and wealth concentration, we investigate the impact of maturity and the SMESM on wealth inequality. In most survey data but also in general business valuation approaches, the entrepreneur is re-

⁴⁵In the details, there are important general equilibrium effects. We underline them by comparing with a partial equilibrium model in a Supplementary Appendix available upon request.

quired to value her business assets based on their market value.⁴⁶ In the model, depending on the maturity m of a firm, we value business assets using the following approach:

$$\begin{aligned} \text{Business assets}(k, m) &= \text{Tangible assets}(k) + \text{Transferable intangible value}(k, m) \\ &= (1 - m)[k - C(k, 0)] + m\mathcal{P}(k) \end{aligned} \quad (26)$$

where only mature businesses possess a transferable intangible value.

Given the above, we find that the SMESM and maturity effects have remarkable consequences on wealth concentration and inequality. First, in [Table 9](#) we compare wealth (defined as net worth) distribution statistics in the model and in the SCF data. The baseline model with entrepreneurs matches the U.S. wealth concentration extremely well, while a comparable model without entrepreneurs is unable to do so. Previous entrepreneurial models, for instance [Cagetti and De Nardi \(2006\)](#), were also able to match the wealth distribution. However, the novel aspect here is that the SMESM and the maturity value of businesses enhance wealth concentration in our model. We illustrate this by comparing our baseline to an alternative without the SMESM in case (1). Two results emerge. First, wealth concentration is more pronounced in the baseline case because business owners transfer the value of maturity on the SMESM. As such, the average firm size is higher and entrepreneurs are richer. This also helps in better reproducing the wealth Gini with respect to the data. Second, the absence of a SMESM substantially impacts the ratio of median wealth between workers and entrepreneurs as then businesses are valued only based on their tangible assets.⁴⁷

Table 9. Wealth concentration and inequality

	Ratio E/W medians	Gini	Wealth share of top				
			1%	5%	10%	20%	40%
U.S. data ^a	7.0	0.810	33.6	60.3	71.4	83.4	94.6
Baseline with SMESM	6.8	0.814	35.2	62.8	73.8	83.7	93.9
Baseline without entrepreneurs	-	0.522	5.1	18.2	32.1	53.2	80.1
(1) No SMESM	4.1	0.782	32.4	58.5	69.1	80.2	92.0
(2) No maturity	3.6	0.730	24.8	49.9	61.9	75.2	90.0
(3) No interest rate component	6.7	0.808	34.3	62.0	72.9	83.3	93.7
(4) No borr. cst component	6.5	0.809	34.3	62.1	73.0	83.3	93.7
(5) No profit rate component	6.0	0.785	31.1	58.3	69.5	80.4	92.4
(6) No failure rate component	5.1	0.770	30.0	55.8	67.4	79.0	91.8

^a We report values for the U.S. wealth shares from [Benhabib et al. \(2019\)](#).

We further explore the maturity components, by applying the decompositions above to wealth concentration and inequality. As illustrated by case (2), in the absence of any maturity

⁴⁶For instance, the SCF evaluates business assets based on their market value using the following question: "What could you sell it for?".

⁴⁷Notice that inequality would still diminish if we were to assume that businesses keep the same valuation of maturity even in the absence of a SMESM.

effects (and thus of an operative SMESM), the ratio of median wealth between entrepreneurs and workers decreases appreciably, translating that the share of wealth held by entrepreneurs is significantly reduced. Wealth concentration and inequality drop by a sizable margin as evidenced by both the share of wealth held by the top percentiles and the wealth Gini. For instance, the wealth held by the top 1% falls from 35.2% in the baseline case to 24.8%. Comparing case (1) and (2), we note that removing maturity elements are enough to generate most of the effects on wealth concentration. Interestingly, the literature has provided a number of mechanisms to match wealth concentration, from the introduction of entrepreneurs into a worker-based economy ([Quadrini \(2000\)](#)), to heterogeneous patience between individuals ([Krusell and Smith \(1998\)](#)) or the existence of voluntary bequest motives ([Cagetti and De Nardi \(2006\)](#)).⁴⁸ We underline a completely new channel based on the heterogeneity of firms absent in the literature and that is furthermore consistent with the behavior of individuals in the economy and empirical evidence: maturity effects can help shape a significant portion of wealth concentration beside their importance for the SMESM. Maturity generates two key features: (i) more dispersed returns and income inequality between entrepreneurs with early-stage small businesses and those running large mature ones, (ii) more income persistence for entrepreneurs with mature businesses as the exit rate (the failure rate) is reduced. Those elements, in turn, translate into more wealth inequality and wealth concentration in the hands of very few individuals. Consequently, the effects of removing the SMESM (and hence the possibility to transfer maturity) on inequality is larger when the accumulation of intangible assets over time translates into more maturity, as shown by comparing our baseline and case (1).

To close our analysis of wealth concentration and inequality, we decompose the effects of maturity along the components mentioned above in case (3) through (6). Again the components on profits and failure rates reported in cases (5) and (6) are the most striking, with a significant reduction in the wealth Gini and wealth concentration at the top percentiles. This result mirrors our previous one on aggregate outcomes: most of the maturity effects come from those two margins.

6.5 Sensitivity Analyses

To conclude our quantitative analysis of the businesses for sale market, we perform a number of sensitivity and robustness exercises regarding model parameters. For reasonable parameter value changes, we find that the properties of the model remain relatively stable and that the main results of the paper are valid. The details of these exercises are discussed in [Appendix C](#).

⁴⁸For a more general discussion on relevant margins to generate wealth concentration consistent with social mobility in the U.S., we refer to [Benhabib et al. \(2019\)](#).

7 Conclusion

In this paper, we build a life-cycle heterogeneous agents model with occupational choices and introduce two key margins: prospective entrepreneurs must either buy or found their businesses upon entering the sector while incumbents must either sell or liquidate theirs upon exit. At the equilibrium, an endogenous business price clears a small and medium-sized enterprises for sale market (SMESM). The option to purchase lets entrepreneurs acquire well-established mature businesses, with a lower probability to fail, higher profits, and better financial conditions. We argue that maturity relates to the intangible value of a firm and show why it is a key component of entrepreneurship and entrepreneurial entry and exit.

Our baseline model provides a consistent cross-sectional and aggregate representation of the U.S. economy. We first demonstrate that the SMESM has important implications for aggregate outcomes. By allowing the transfer of the value of the maturity of a firm between owners, overall survival rates, firm sizes, and aggregate production are increased. Without this market, aggregate production drops by about 10% and the consequences on aggregate savings and prices are also severe. Second, we find that the decreasing failure rate over time is the most important component embedded in the option value of a purchase relative to that of a new firm creation. Third, we show that entrepreneurial life-cycle and matching frictions are important determinants of entry and exit. While the literature has evaluated the former, we establish that the latter can significantly shape aggregate outcomes and the composition of the entrepreneurial pool. Finally, we uncover a novel channel to match wealth concentration and inequality that is consistent with individual behavior and empirical evidence and is furthermore directly linked to the SMESM and business maturity.

Our contributions might be particularly relevant for future research on the aging of entrepreneurs and the decline of the start-up rate where we expect business transfers to play a predominant role.

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Appendix

A Empirical appendix

In this section of the Appendix, we provide additional empirical evidence supporting the fact that recently purchased businesses are substantially different from newly founded ones.

A.1 Acquisition type

Table 10 provides estimates of the proportion of firms by acquisition type for various U.S. survey data. Broadly speaking, one out of five entrepreneurs enter the sector through the purchase of an existing business. This number remains consistent across survey data and time.

Table 10. Business acquisition by type in U.S. surveys

Survey ^a	Year	Sample selection ^b	Acquisition (%)		Transmission (%)	
			Founded	Purchased	Inherited ^c	Other/Gift ^c
SCF	2016	All entrepreneurs	74.4	18.2	3.5	3.9
ASE	2016	Only employers	68.1	20.8	4.0	7.1
SSBF	2003	All entrepreneurs	79.8	16.7		– 3.5 –
SSBF	2003	Entrepreneurs (< 5y)	77.4	20.8		– 1.8 –
SBO	2007	All entrepreneurs	74.6	18.2	2.3	4.9
SBO	2007	Entrepreneurs (≤ 3y)	74.4	19.1	1.2	5.3

^a An entrepreneur is defined as an individual declaring that her business constitutes her primary source of income (with an active management role, whenever possible). The ASE reports macro data for all business owners with at least one paid employee.

^b The estimates are based on self-employed entrepreneurs defining themselves as business owners. Early-stage entrepreneurs are those who acquired their businesses within the last 5 years.

^c When possible, we distinguish the acquisition type between gift/other and inheritance.

A.2 Business performances and owner characteristics

Table 11 shows characteristics of firms within 3 years of their acquisition and the characteristics of their owners. Purchased firms systematically perform better than their founded counterparts: the former display 4 times the average total assets and total sales with respect to the latter and twice the average number of employees and profit. Concerning the owners of recently purchased or founded firms, they appear very similar in terms of age, years of experience as entrepreneurs and education. To complement, in the SBO 2007, entrepreneurs were asked *Whether the owner previously owned a business or had been self-employed*. 60% of founders reported having no prior experience compared to 63% for the purchasers.

The above statistics slightly differ from those in the 2007 SCF. In that survey, for a sample of entrepreneurs within 4 years of the acquisition of their firms, we find a mean age of about

44 for both founders and purchasers. The fraction of purchasers (resp. founders) with a degree above high school is 84% (resp. 66%). Finally, the ratio of the mean net worth held by purchasers relative to founders is about 2.8.

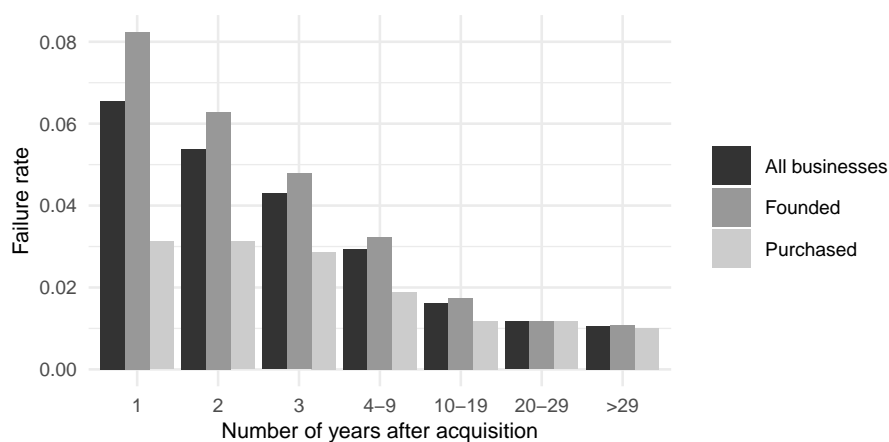
Table. 11. Characteristics of firms within 3 years of acquisition by type

	Mean			Median		
	Purchased	Founded	Ratio	Purchased	Founded	Ratio
Firms						
Total assets (USD)	766K	191K	4.0	74K	26K	2.8
Avg. number employees	9.2	4.1	2.2	5	2	2.5
Profit (USD)	118K	52K	2.3	20K	2K	10
Total sales (USD)	1093K	264K	4.1	300K	61K	4.9
Owners						
Age	44.4	44.9	1.0	44	45	1.0
Years experience	10.30	10.13	1.0	6	6	1.0
≥High school deg. (%)	64	63	1.0			

Source: Survey of Small Business Finances (2003)

Figure 9 complements Figure 2 in Section 2 on the failure rate of purchased versus founded firms. The evidence reported here only concern firms with paid employees. Broadly speaking, our results appear consistent when focusing on this group.

Figure 9. Failure rate by acquisition type and for all businesses, employer group



Source: author's computation using the 2007 SBO. We compute the failure rates using ceasing option linked to either inadequate cash-flows or low sales and lack of business or personal loans/credit.

Finally, we show in Table 12 and Table 13 additional evidence on the financial constraints faced by the group of recent purchasers and founders (within 3 years of acquisition). First, founded firms are more likely to be denied a loan they required. Second, the main reason why recent founded firms are denied access to credit is related their status as *not in business long enough*, implying that, for creditors, the sales history is an important piece of information influencing credit conditions.

Table 12. Loan acceptance rate for firms within 4 years of acquisition by type (%)

Acquisition type	Access to loan is		
	Always accepted	Always denied	Sometimes accepted or denied
Purchased	85.7	12.1	2.2
Founded	71.5	21.4	7.1

Source: Survey of Small Business Finances (2003). We use 4 years in order to increase the number of observations.

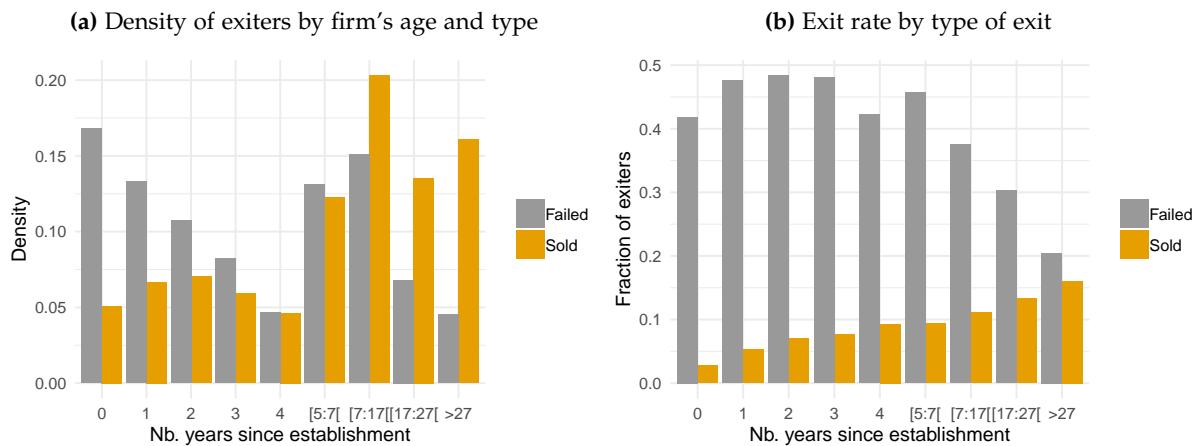
Table 13. Main reasons why credit was denied (%)

Acquisition type	Main reason for denial was			
	Not in business long enough	Credit history	Insufficient collateral	Other
Purchased	0.0	26.2	24.2	49.6
Founded	34.2	19.6	7.7	38.5

Source: Survey of Small Business Finances (2003). We take 4 years in order to increase the number of observations.

A.3 Exit rate and type of exit

We display in this subsection the exit rate by exit option: business failure versus successful sale. First, we note in [Figure 10](#) panel (a) that sold businesses are generally older. This point is also confirmed in [Appendix A.4](#) using business for sale transaction data. Second, in [Figure 10](#) panel (b), as opposed to early-stage business owners, old business owners are substantially more likely to exit after a successful sale of their firm rather than business failure. This additional evidence suggest that sold businesses are generally older well-established ones.

Figure 10. Type of exit and establishment data

A.4 Small and medium-sized enterprises for sale market

In order to characterize the selling frictions, we collected transactions data from the online platform *Bizbuysell.com* (hereafter BBS), one of the oldest and largest online marketplace dedicated to business selling transactions in the U.S.. The available data correspond to over 90,000 observations of businesses for sale or sold. In the details, we have two sets of data: (i) a panel of businesses for sale from 2018 to 2020 including any changes in business information; (ii)

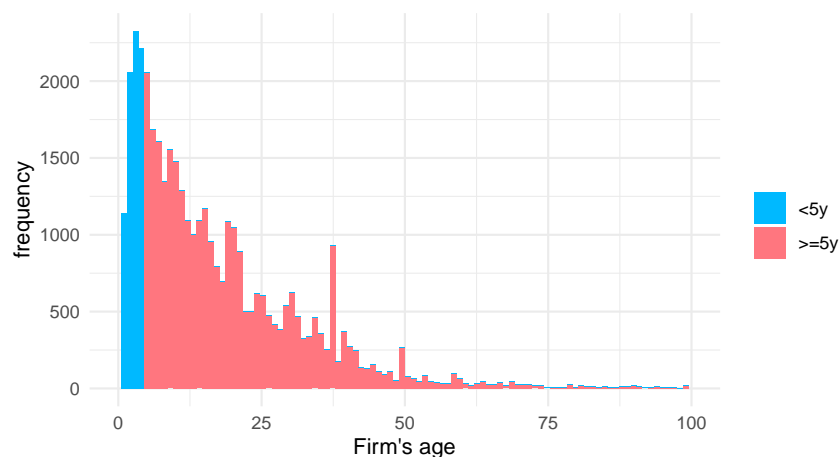
a collection of closed transactions from 2010 to 2020. Table 14 provide summary statistics regarding this BBS database.

Table. 14. Descriptive statistics: BBS data on businesses for sale (2018-2020)

Statistic	N	Min	Pctl(25)	Median	Pctl(75)	Max	Mean	St. Dev.
Listing price	93,270	1,100	120,000	250,000	595,000	620,000,000	713,556	4,124,977
Cash flow	57,925	0	70,000	120,812	221,000	50,000,000	200,146	405,272
Gross revenue	69,267	0	260,000	520,500	1,066,730	9,969,000	910,338	1,161,825
EBITDA	9,842	12	62,549	120,000	265,878	435,000,000	314,871	4,423,594
Nb. employees	51,855	0	2	4	9	913	8	15
Inventory	35,065	4	5,000	15,000	54,728	35,000,000	88,503	486,542
Ceasing for retirement	24,643	-	-	-	-	-	0.20	-

Figure 11 shows that only around 15% of businesses for sale are younger than 5 years. This confirms that many businesses for sale are actually well-established mature businesses.

Figure 11. Age profile of businesses for sale

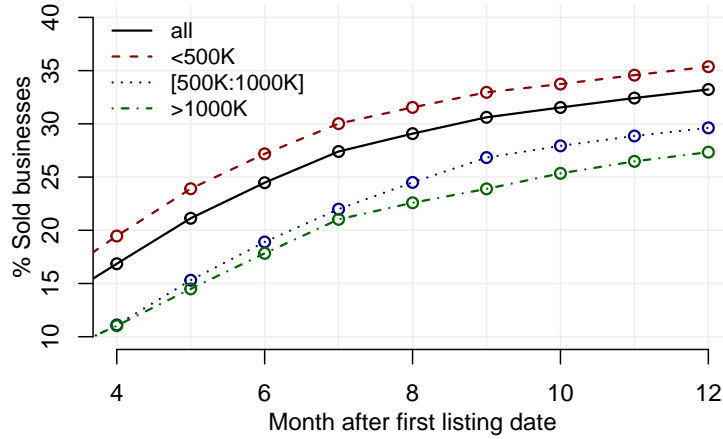


Source: Authors' own computation using BBS data.

Given the remarkably low number of entrepreneurs reporting having sold their business, especially with respect to initial intentions, important failures when trying to sell are potentially occurring. The inability of SME owners to reach any business selling deal results in liquidations: the sale at very low prices and usually restricted to tangible assets. Thus, to investigate whether there are mismatches on the SMESM, we use the BBS data to infer the probability of selling a business as well as the time needed to sell.⁴⁹

⁴⁹Concerning the comparability of the BBS dataset with respect to existing surveys, we note that 23% to 25% of sold businesses were sold because the owner(s) retired, against 19% in the ASE (2016). About the distribution of listing prices, the mean price is 495K USD and the median is 165K USD in BBS against respectively 682K USD and 95K USD in the PSID; meaning that the BBS price distribution is comparatively shifted to the right. However, this comparison is indicative: there are only 357 observations concerning sold businesses in the PSID (1990 to 2015) against 80,000 in BBS (and 93,000 businesses for sale). Moreover, many BBS listings are broker mediated and announcers have to pay a monthly premium membership to list their entry. This might be constraining enough for very small businesses. Overall, we believe that the BBS dataset provides a reasonable representation of U.S.

Figure 12. Probability to sell with listing price as a proxy for size.



To be read as follows: *after a year, 32% of all businesses have been sold.* The price corresponds to that at the initial listing date. *Source:* Authors' own computation using collected BBS data.

Using the same strategy as in the core of the paper, [Figure 12](#) displays the probability to sell a business after a number of months from the listing date and by listing price brackets. After a full year, only around 25%-35% of businesses for sale are actually sold. While this number is fairly similar for any business size (as proxied by the listing price), it seems to be slightly easier to sell a smaller business.

B Model: further details

B.1 Numerical solution method: discrete-continuous model with the endogenous grid method (DC-EGM) under taste shocks

To tackle the issue of the high dimensionality of our problem, we adapt the recently developed DC-EGM solution method introduced in [Iskhakov et al. \(2017\)](#): it solves the occupational choice problem while still accommodating the fast endogenous grid method developed in [Carroll \(2006\)](#). On top of a substantially increased computation speed, this method is also well-adapted to the context of occupational choices. As shown in [Hurst and Pugsley \(2011\)](#), the decision to enter entrepreneurship is also driven by non-pecuniary benefits that are, to some extent, not observable by the econometrician. In the model, we, therefore, introduce taste shocks to both smooth the value functions when applying the DC-EGM algorithm, as well as to get closed-form expressions for the probability to switch from one occupation to another through the available options of continuing, selling, purchasing, founding or liquidating a business.

Our algorithm has been implemented in C/C++. The details of the computation are

business transactions.

provided in the Online Appendix.

B.2 Calibration details

Earning process We take the life-cycle average earning profile in [Guvenen et al. \(2015\)](#). We fit a life-cycle earning profile with a third order polynomial. [Table 15](#) provides the corresponding values.

Table. 15. Life cycle earning profile

[25 : 30[[30 : 35[[35 : 40[[40 : 45[[45 : 50[[50 : 55[[55 : 60[[60 : 65[65 and over
$h(1)$	$h(2)$	$h(3)$	$h(4)$	$h(5)$	$h(6)$	$h(7)$	$h(8)$	$h(9)$
0.58	0.72	0.84	0.93	0.98	0.97	0.90	0.75	0.40

C Sensitivity Analyses

The baseline model embeds a number of features that let us match empirical elements. In this section, we investigate the importance of key features interacting with the option values of selling and purchasing a business that might have an impact on aggregate outcomes. In the following, prices clear the markets and taxes are adjusted to balance the government budget constraint. Results are displayed in [Table 16](#).

Table. 16. Sensitivity analyses on the SME market and aggregate outcomes.

	$\frac{E}{\bar{W}}$	Buy %	Sell %	Mature %	Avg. size	Y	$\frac{Y_{SME}}{Y}$	$\frac{K}{Y}$	p	r_s	τ_w
U.S. data ^a	11.0%	21.0	7-14	67	–	–	0.40	3.2	–	4-6%	–
Baseline model	11.3%	24.0	10.0	66	45	2.4	0.45	3.2	0.17	4.9%	15.0%
Adj. cost, $\phi_{up} = 0.1\Delta k$	11.0%	18.9	9.8	63	22	1.8	0.33	2.8	0.41	6.4%	16.9%
Doubling u_R cost	10.0%	26.3	12.3	65	38	2.1	0.39	3.2	0.16	5.1%	16.3%
No fixed cost $F_b = 0$	11.7%	51.0	13.1	79	48	2.5	0.47	3.3	0.27	4.8%	14.7%
Doubling profit rate wedge	11.0%	27.0	10.4	68	43	2.3	0.43	3.2	0.18	5.1%	15.2%
Borr. limit, $\begin{bmatrix} \theta(0) \\ \theta(1) \end{bmatrix} = \begin{bmatrix} 0.50 \\ 0.45 \end{bmatrix}$	11.1%	22.0	9.8	65	32	2.1	0.39	3.1	0.20	5.4%	15.9%
No sales tax $\tau_s = 0$	11.2%	24.1	10.0	66	45	2.4	0.45	3.2	0.17	4.9%	15.2%

^a We report value for the U.S. economy using Penn World Table 9.1, treasury bond interest rate, Current Population Survey (1998:2008), Survey of Consumer Finance (2001:2007) and the 2007 Survey of Business Owners.

We first investigate the role of adjustment costs. In the baseline model, we normalized the upsizing cost ϕ_{up} to zero. Thus the main source of capital illiquidity came from the downsizing cost of liquidating business assets. We test the sensitivity of increasing ϕ_{up} to 10%. That is, for each unit of capital k bought, the cost is $(1 + 0.1)k$. The resulting equilibrium implies a lower average business size. With the increase in the relative cost of founding a new business relative to purchasing an existing one, business price level increases and the fraction of buyers able to purchase business drops. Due to fewer business transfers and a lower level

of accumulated business capital, aggregate output drops. In the end, it comes as no surprise to us that adjustment costs have important equilibrium effects, but the main mechanisms of the model remain similar.

As we argued in Section 2, the entrepreneurial life-cycle matters since an important fraction of entrepreneurs sell their business assets upon retirement. In the model, we capture this behavior by having a disutility cost u_R of working in the retirement age bracket. Doubling this utility cost reduces the fraction of entrepreneurs in the last age bracket from 4.8% to 1.4%, and the fraction of entrepreneurs from 11.5% to 10.0%. The fraction of sellers substantially increases, since many old entrepreneurs are now trying to sell, lowering the business price and therefore increasing the fraction of buyers in the economy. Because older entrepreneurs want to exit earlier, they accumulate fewer capital assets, and the average firm size and production fall. We argue that the aging of entrepreneurs, accelerating since the 2010s, might be a first-order concern on the SMESM.

Buying a business incurs the payment of a fixed cost F_b . Our third sensitivity test sets F_b to zero. We find that the share of entrepreneurs increases by 0.3 percentage points and that the share of buyers rises from 21% to 51%. As purchasing a business is now less costly with respect to founding, the share of mature businesses increases significantly. Business price level increases since now a larger fraction of entrepreneurs are able to buy an existing business. Overall, this leads to a substantial increase in aggregate output.

Next, we benchmark the effect of the profit margin by doubling the profit component between immature and mature firms (we let $\gamma(0) = 0.9\gamma^{\text{benchmark}}(0)$ and we keep $\gamma(1) = 1.1\gamma^{\text{benchmark}}(0)$ such that the difference in terms of profit rate is about 20%). This leads to an increase in the propensity of new entrant to buy an existing business. As mature firms are now more valuable, more of them are bought and their proportion in the economy increases, leading to an increase in business price. However, due to the lower profit rate of early-stage businesses, aggregate output decreases..

Finally, additional sensitive tests include lowering the tightness of the borrowing constraint and setting the sales tax τ_s to zero. Those two margins are shown to have only marginal effects.