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# "Market Information in Banking Supervision: the Role of Stress Test Design"

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# Market Information in Banking Supervision: the Role of Stress Test Design*

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**Abstract**

This paper studies how a banking supervisor should design a bank stress test. The test directly provides public information about a bank's stress resilience, which is a noisy indicator of the bank's true capitalization level. Moreover, the test result affects incentives for information production in a financial market, and the resulting price signals the supervisor can use to determine whether to intervene in a bank. We show that for banks that fail the test, information production in the financial market collapses. This introduces a motive to make the test lenient, even though this sometimes implies that a poorly capitalized bank may escape intervention. We also show that the supervisor optimally makes the lowest *pass* grade coarse. Since a lenient test allows some sub-standard banks to receive a *pass* grade, the latter must also include some strictly above-standard banks to remain credible. In addition, the supervisor benefits from market information, precisely for banks that are "close calls." Making the test coarse around the intervention threshold optimally spurs information production. For high resilience levels, the value added from financial market information is limited and the supervisor therefore applies a more granular test.

*Keywords*: Feedback, risk shifting, information design
*JEL classification*: G14, G28

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# 1 Introduction

There has been considerable interest in recent years in the question how information conveyed by prices in secondary financial markets feeds back into real decisions (see Bond, Edmans and Goldstein (2012) and Goldstein (forthcoming) for surveys). One application of that literature points to the importance of stock price information in guiding intervention decisions of regulators, for example, a supervisor who needs to decide whether to intervene in a troubled bank (Bond, Goldstein and Prescott (2010), and Bond and Goldstein (2015)). As Flannery and Bliss (2019) argue: "[...] market discipline can, potentially, complement and support official oversight of risky financial institutions, [...] by providing market signals that supervisors can use to motivate their own actions..." In parallel, a number of papers have investigated how supervisors themselves should produce and communicate information, for example, via bank stress tests that allow them to assess the need for intervention (e.g., Colliard (2019), and Carletti, Dell'Ariccia and Marquez (2021)). When designing such tests in practice, supervisors need to decide how severe the adverse shock should be, which has led to some discussion whether stress tests were too lenient in some instances.[1] Other choice variables include whether to apply one or multiple adverse scenarios[2] and whether to apply different scenarios across institutions.[3] It remains a largely open question how the design of stress tests interacts with information produced by financial markets, a gap this paper aims to address.

In this paper we ask how a bank supervisor should design her monitoring technology (the stress test) in light of the impact this will have on information reflected in financial markets. We view the monitoring technology as having two roles: Firstly, it determines directly what the supervisor can learn. Secondly, it affects the incentives for a speculator to produce and trade on costly information. We have in mind a supervisor who can run an arbitrary number of stress scenarios and observes for each stress level, whether the bank can resist the stress. Based on the test result and any further information contained in the bank's share price, the supervisor decides whether to intervene in the bank. The supervisor tries to learn about the value of the bank's assets, knowing that a low asset value would induce risk shifting by the bank. The

[1] See, e.g., Raoul Ruparel, *European Bank Tests Not As Stressful As Hoped*, Forbes, October 27, 2014; and Francesco Guarascio, *EU's 2018 bank stress test too mild, spared weaker states: auditors*, Reuters, July 10, 2019; and Martha C. White, *Banks are acing their 'stress tests' but that's not necessarily a good thing*, BBC News, June 21, 2019.

[2] E.g., in the US banks are subject to two different stress tests, the Comprehensive Capital Analysis and Review (CCAR) and the Dodd-Franks Act Stress Test (DFAST). The two differ, among other things, in their assumptions about banks' capital distributions in the event of a shock. Roughly speaking, under DFAST capital distributions do not adjust to a shock, making the test harsher than under CCAR.

[3] The US Fed selectively applies, in addition to the adverse scenario applied to all tested banks, a global market shock and / or a counterparty default to banks with particularly large trading operations (see 2022 Stress Test Scenarios).

supervisor can intervene and reduce the bank's risk exposure, for example by arranging its sale to a better capitalized bank that will not engage in risk shifting. Since a speculator can try to learn the value of the bank's assets and trade on this information, stock prices may help guide the supervisor's intervention.

The specific question we study is how the supervisor should design the stress test, i.e., what levels of severity in terms of stress should the test apply, and how granular should the information be? In line with the literature on Bayesian persuasion, discussed in more detail below, we assume that the supervisor designs the monitoring technology (stress test) in a way that is publicly observable and then publishes the outcome of the test. Hence, there is no scope for *ex post* opportunism, for example in the form of the supervisor hiding or misreporting the test result.

Our main findings are the following. We show that a speculator's expected trading profits depend crucially on the intervention decision that the stress test induces. Test results that would induce an intervention in the absence of further information from the stock market, can be classified as *fail*. Conversely, those that induce no intervention can be labelled *pass*. In our model, an intervention wipes out the bank's equity. A trader can therefore not profit from acquiring information, if he anticipates that the bank will be subject to an intervention. That is, the trader produces no information for banks that fail the test. This provides a motive for the supervisor to make the test more lenient, i.e., award *pass* grades more generously. In addition, it forces the supervisor to make the most marginal *pass* grade a coarse category, i.e., even banks that comfortably pass the test will have to be lumped into the marginal grade. Coarseness is needed in order to render a lenient *pass* grade credible. For those banks that have a very high level of stress *resilience*, the optimal test will identify them as such and provide a very fine categorization. This is because the test is designed to maximize information production by speculators, precisely when market information is most useful to the supervisor. This in turn is the case for banks that are "close calls," that is banks that have resilience levels close to the intervention threshold and where the test itself is least conclusive.

Among other comparative statics, we show that a more heavily indebted bank should be subject to a less lenient stress test. This is because information production in the financial market will be less effective for such a bank, reducing the benefit from distorting the stress test for the sake of generating market information. When the underlying risk shifting problem becomes more severe, a less lenient stress test is optimal. That is because with more severe risk shifting, it becomes more important for the supervisor to catch a poorly capitalized bank. Moreover, the value of the equity claim becomes less sensitive to the underlying state of the world, since risk shifting compensates for the low intrinsic value in the low state. The financial market therefore becomes a less effective producer of information, reducing the incentive to distort the stress test.

We study multiple extensions of the baseline model. First, we show that incentives to distort the stress test weaken, if the speculator can alternatively trade in either a debt or an equity claim. In this case, a supervisor may even wish to distort the test towards conservatism. Although this result is of theoretical interest, trading in debt instruments is arguably not a practical alternative due to the comparative lack of liquidity. Second, we allow for multiple sources of uncertainty in a setting where the supervisor can learn about a different piece of information than the financial market. We show that the central results of our paper hold, as long as the two sources of uncertainty are correlated.

It would be an exaggeration to argue that supervisors in practice determine the leniency of their stress test design, solely on the basis of the trade-offs described in our model. Evidently, supervisors will also be guided by other concerns, some of which have been highlighted in papers discussed below. Nevertheless, we believe it is important to be aware of the trade-offs we identify. One clear implication of our analysis is that a "one-size-fits-all" approach has costs, as for some banks it will likely result in a drop in the information quality on which intervention decisions are based. For example, a supervisor who adopts a more conservative test design should be aware that the implied increase in *fail* outcomes will reduce the quality of available market information.

The remainder of the paper proceeds as follows. Section 2 discusses the related literature. In Section 3 we describe the model, while the main results are presented in Section 4. Section 5 provides a microfoundation for bank value based on a classic risk-shifting problem and studies comparative statics. Section 6 presents the model extensions and Section 7 concludes. All the proofs are relegated to the Appendix.

## 2 Related Literature

There are a number of papers that have studied whether stress test results should be disclosed, e.g., Bouvard, Chaigneau and de Motta (2015), Williams (2015), Goldstein and Leitner (2018), Orlov, Zryumov and Skrzypacz (2021) and Leitner and Williams (forthcoming) (see also Goldstein and Sapra (2014) and Goldstein and Yang (2017) for a more general discussion and review). Disclosure matters, as it may affect market discipline, the functioning of the interbank market, financial stability, bank lending behaviour and risk sharing. In this paper, we take for granted that stress test results are published, which corresponds to the practice that supervisors have converged to.

The papers closest to ours are Bond and Goldstein (2015) and Siemroth (2019) who study the interaction of a regulator's information (including a decision to disclose such information) with information revealed by share prices, when that information is in turn used by the regulator. They show that more public information may crowd out private information as it reduces the informational advantage of speculators.[4] This effect is balanced by a crowding-in effect, as public information reduces the riskiness of speculators' trades, inducing them to take larger positions. Also related is Goldstein and Yang (2019) who study the interaction between public disclosure and market based information in a context where the decision maker learns from both, the public signal and market prices (unlike in Bond and Goldstein (2015) where the regulator has information regardless of whether or not it is made public). Goldstein and Yang (2019) focus on two dimensions of uncertainty and explore how disclosure affects the weight that traders put on one of the two private signals they possess. They show that when information is disclosed about the dimension of uncertainty that is relevant for the real decision, then this will reduce the weight that traders put on that dimension of their private signals. By crowding out information aggregation on the "useful" dimension, more public disclosure may reduce the overall amount of information relevant for the real decision.

Our focus is different in that we study the role of leniency and coarseness of the monitoring technology in a context where a speculator faces an information production cost. Unlike in Bond and Goldstein (2015) or Goldstein and Yang (2019), the information production decision depends sensitively on the *realization* of the public signal, with less information being produced following a negative public signal than following a positive one. From a methodological point of view, the papers by Bond and Goldstein (2015), Goldstein and Yang (2019) and Siemroth (2019), share their focus on the *intensity* with which speculators trade on their private information. The variants of the Grossman-Stiglitz framework they (and many others) use, assumes normal distributions and preserves quasi linearity of trades, which is a key property for tractability. However, that framework also has a very specific property, namely residual uncertainty from the speculator's perspective is a constant, independent of the signal realization. This property makes the framework arguably less well suited to studying trade in highly levered equity claims such as bank equity. Quite plausibly, residual uncertainty may be smaller for lower signal realizations (when the equity payoff is in the default region), thus inducing very different incentives to acquire information, depending which part of the payoff distribution a speculator expects to navigate.[5]

The effect that trading profits differ, depending on whether public information is positive or negative, is related to Dow, Goldstein and Guembel (2017) who show that speculators' information production may break down when firms' investment prospects are unfavorable. Such firms are unlikely to invest, and therefore undermine the incentive for speculators to produce

[4]Recent empirical evidence by Heitz and Wheeler (2022) supports the notion that information contained in stress tests does indeed crowd out information production by financial markets.

[5]The cost of moving away from from the normal, quasi-linear framework, is that we cannot study the speculator's trading intensity.

information about those prospects. In this paper we focus on the *ex ante* information design problem when information production in financial markets depends on the trader's belief about fundamentals. Moreover, in our setting there is a risk shifting problem which drives a wedge between the objective of the policy maker (to maximize bank value) and the traded claim (levered equity). As the risk shifting problem gets more severe, the stock market becomes less useful in providing information. This happens because the value of an equity claim, conditional on no intervention, becomes less sensitive to the underlying state of the world, undermining a speculator's incentives to produce information about it. On the one hand, in the low state of the world, the value of assets in place is low, reducing the value of equity. On the other hand, the bank engages in risk shifting in the low state of the world, and the accompanying expropriation of creditors increases equity value. While we are by no means the first to point out that private incentives for information production differ from social value (see Paul (1992)), we identify a new wedge between the two. Private incentives are driven by the variability in value of the traded claim, which is equity. Social incentives stem nevertheless from the value that accrues to debt *and* equity holders together. A worsening risk shifting problem reduces private incentives of a share trader to produce information, but increases the social value of this information.[6] This problem differs from the one identified in Bond, Goldstein and Prescott (2010) who show that the mapping from the states of the world to the price may not be invertible. The fact that the price of an equity claim sends a mixed message when risk shifting is a problem has been recognized intuitively by advocates of subordinated debt as generating market discipline (see Flannery and Bliss (2019), for a review of the arguments). However, this point has not been taken up in the feedback literature, probably because most of the papers work in relatively abstract settings in which the link between regulatory intervention and the value of the traded claim is fixed by assumption.

Davis and Gondhi's (forthcoming) work analyses risk shifting with informational feedback from the stock market. Their focus is on how an agency conflict between debt and equity holders interacts with the endogenous information available in the stock market. They show that the relationship depends crucially on whether investment distortions are of a risk shifting or a debt-overhang type. As risk shifting in their model increases speculators' incentives to produce information, the feedback mechanism mitigates the inefficiency from the agency problem. One key difference between their set-up and ours is that in their model, uncertainty and information is about the project payoff which implies that risk shifting increases the information sensitivity of equity. In our model uncertainty is about assets in place. Negative news about assets in place

[6] Lenkey and Song (2017) also analyze in a feedback model the social value of private information, where a breakdown of bilateral trades due to information asymmetry may itself transfer socially valuable signals to a third party and hence induce value-enhancing actions.

therefore induces risk shifting, which reduces the information sensitivity of the equity claim.

When thinking about monitoring and reporting, we take an angle akin to that in the literature on Bayesian persuasion and information design (see, e.g., Kamenica and Gentzkow (2011), Bergemann and Morris (2019) for the technical underpinnings, and Szydlowski (2020) for a recent finance application). That is, we think of a supervisor as having to design an information technology up front, which determines how an underlying state is mapped into a publicly observable signal. In the Bayesian persuasion literature, a receiver acts once the signal is observed. Our paper differs in that the receiver (the speculator) has a role of producing additional information that is subsequently disseminated via a price mechanism. The supervisor thus chooses the information structure with a view to encouraging information production by the speculator, taking into account that the information structure affects both direct and indirect learning.

Other recent papers have modeled stress test design as a Bayesian persuasion problem, hence analyzing the optimal disclosure policy when this is chosen before the realization of the relevant information. Inostroza and Pavan (2021) look into optimal disclosure in a global games framework in which the policy is designed to minimize the probability of regime change. The optimal policy coordinates all market participants on the same course of action, but without fully revealing the state. Under some conditions the optimal policy is a "pass/fail" stress test. Orlov, Zryumov and Skrzypacz (2021) show the optimality of "pass/fail" tests failing all weak and some strong banks in order to limit the stigma of failure. Inostroza (2019) studies stress test under the scrutiny of multiple audiences finding that the optimal policy is opaque when the bank has high-quality assets, and transparent when the bank has poor-quality assets. A common finding among these papers is that stress tests should involve a degradation of public information. We add to this literature by identifying a new rationale for the coarsening of information, namely the encouragement of private information production by financial markets.

Shapiro and Zeng (2020) look into the leniency of bank stress tests. As in our paper, stress tests serve to generate information for the supervisor. The leniency of the test is the supervisor's hidden choice and supervisors build a reputation over time. Leniency in their paper trades off information content (higher default probability) against a reduction in loans to the real sector. There may be multiple equilibria due to reputational concerns.

Our paper is also related to the literature on banking regulation which regards the bank's moral hazard problem as a central friction that regulation can address, for example, Bhattacharya (1982), Rochet (1992), Hellmann, Murdock and Stiglitz (2000), Gorton and Huang (2004), Morrison and White (2005), Calzolari and Lóránth (2011), or Fecht, Inderst and Pfeil (2022).[7] A number of papers have argued that in such a context a supervisor plays a role by

[7]High leverage associated with a poorly capitalized bank could also lead to debt overhang. In this context Philippon and Schnabl (2013) analyze the efficient design of a recapitalization when the regulator does not know

intervening in bad banks at risk of failure by rendering their risky payoffs safe (see Calzolari, Colliard, and Lóránth (2019)). In Carletti, Dell'Ariccia and Marquez (2021) banks take too much risk in a laissez-faire equilibrium and supervision is designed to reduce their risk exposure. The supervisor monitors and learns about the amount of a bank's capital (and its portfolio) and can then intervene so as to reduce risk exposure. When an intervention occurs, shareholders are expropriated. We share with that literature the focus on a bank's risk shifting incentives and the supervisor's role in curbing them. Our central point on the design of the supervisor's information and its interaction with market-based information is new to this literature.

Also somewhat related are models on the design of credit ratings agencies' evaluation scheme. Goldstein and Huang (2020) predict that CRAs inflate ratings in a model where creditors' heterogenous beliefs affect credit market conditions, which in turn generates a feedback loop from the CRA to the firm's actual investment decisions.[8] Apart from the difference in focus, Goldstein and Huang (2020) have in mind a CRA without commitment power over its ratings announcements, so ratings are subject to ex post opportunism by the CRA. Morevoer, in their paper the issue of information production by speculators or other market participants does not arise, as creditors have an exogenous information endowment. Piccolo and Shapiro (2022) look at a CRA who is subject to a moral hazard problem in information production. Informative stock prices serve to mitigate the agency problem. Higher ratings precision reduces information production.[9]

# 3 Model Set-up

To guide the detailed model description, we begin with a brief overview. There are five dates $t = 0, ..., 4$. There is a bank, a supervisor, a speculator, a market maker and a liquidity trader. We model the stress test design problem akin to a Bayesian persuasion game (see, e.g., Kamenica and Gentzkov (2011) and Bergemann and Morris (2019)) between a benevolent supervisor (sender) and a financial market (receiver) composed of the speculator, market maker, and the liquidity trader. At date 0, the supervisor chooses a monitoring technology, i.e., a stress test that generates a public signal about the value of the bank's assets. At date 1, the speculator can observe the public signal and decide whether to acquire costly (private) information and trade on it at date 2. The supervisor observes the bank's share price in addition to the date 1 public signal and then decides, at date 3, whether to intervene or allow the bank to continue without intervention.

the bank's type.

[8] Terovitis (2020) models a similar feedback loop from credit rating to project financing where managers have private information about the project quality.

[9] Note that private information in the loan market may also be transmitted through interest rates, which act to coordinate banks' actions in supplying credit to the real economy (see Shen (2021)).

At date 4, uncertainty is realized and all payoffs made.

There is an unobservable state of the world $\omega \in \{l, h\}$, with $\Pr(\omega = h) = 1/2$, that determines the value of the bank's assets in place.[10] In state $\omega = l$ the bank's assets have a low value, i.e., the bank is undercapitalized, while they have a high value in state $\omega = h$. The supervisor aims at maximizing the expected date 4 value of the bank and has at her disposal two distinct policy tools: an intervention decision at date 3 and a stress test at date 0. The stress test affects the information available to the supervisor and market participants, and will be described in more detail below. We denote the intervention decision by $a \in \{0, 1\}$, where $a = 0$ means that the supervisor intervenes and $a = 1$ means that she allows the bank to continue without intervention. We define $V_\omega^a$ as the final value of the bank (equity plus debt) in state $\omega$ subsequent to the intervention decision $a$, and we assume that the supervisor wishes to intervene ($a = 0$) when the bank is under-capitalized ($\omega = l$): $V_l^0 > V_l^1$. The supervisor prefers to let the bank continue without intervention in state $\omega = h$, i.e., $V_h^1 > V_h^0$. We define

$$\begin{aligned} \Delta V_h &\triangleq V_h^1 - V_h^0 > 0, \\ \Delta V_l &\triangleq V_l^0 - V_l^1 > 0. \end{aligned} \tag{1}$$

In Section 5 we provide a microfoundation for bank values $V_\omega^a$ based on the following mechanism. The bank is subject to a moral hazard problem in its choice of risk exposure. When it is undercapitalized ($\omega = l$) it chooses to take excessive risk. However, the supervisor can prevent such risk shifting by liquidating the bank or arranging the sale of its assets ($a = 0$) to a better capitalized bank who will not engage in risk shifting.[11] Liquidations incur a deadweight loss and are therefore only optimal in the low state, i.e., when the bank would otherwise engage in risk shifting, but not in the high state.

To inform her intervention decision, the supervisor designs and commits to a stress test at date 0. Suppose that the supervisor cannot directly observe the value of the bank's assets. This could be because the bank has complex derivative exposures, the true value of which the supervisor cannot directly assess, or because banks manage to obfuscate the true value of their assets. However, the supervisor can learn something about the value of assets by running experiments whereby she proposes a scenario of "stress", and learns whether the bank can "withstand" the stress, as defined, for example, by a maximum loss. Let the possible scenarios be normalized on an interval $\mathcal{S} = [0, 1]$ and assume that a bank's ability to withstand stress is itself random and correlated with the underlying value of the assets. Denote the stress level that a bank can withstand by $s \in \mathcal{S}$, called the banks' *resilience*, and assume its type dependent

[10] Our results hold for a general prior, see Appendix B. Assuming an even prior significantly simplifies computations.

[11] An alternative mechanism would be a forced recapitalization, which we develop in Appendix B.

distribution is as follows:

$$\begin{aligned} f(s|\omega = h) &= 2s, \\ f(s|\omega = l) &= 2(1-s), \end{aligned} \tag{2}$$

with corresponding cumulative distributions $F(s|\omega) \triangleq F_\omega(s)$. A stress test then consists of a sequence of stress scenarios that the supervisor can apply. Assume that the supervisor can costlessly run as many scenarios as she desires. Suppose, for example, that the supervisor chooses a stress test that consists of two scenarios, say $s_1$ and $s_2$ where $0 < s_1 < s_2 < 1$. The supervisor can then first apply the stress scenario $s_1$ which will result either in a pass or a fail.[12] If the bank failed the stress scenario $s_1$ the supervisor knows that the bank's underlying resilience is $s < s_1$ in which case the second scenario becomes superfluous. If the bank passed the stress scenario $s_1$ the supervisor can apply the more adverse scenario $s_2$. If the bank passed scenario $s_1$ but fails scenario $s_2$ the supervisor knows that the bank's resilience $s$ is in the interval $[s_1, s_2)$. If, however, the bank passes both scenarios, the supervisor knows that resilience is $s \geq s_2$. Note that from (2) it follows that all banks, even $\omega = l$ types, pass the most lenient stress scenario 0 and all banks, including the $\omega = h$ types, fail the most stringent scenario, given by 1. Formally, the stress test, and its outcome, are defined as follows.

**Definition 1** (Stress Test and Outcome)**.** *A stress test is a partition $\mathcal{P} = \{s_0, s_1, \ldots, s_n\}$ of the $\mathcal{S}$ space with $0 = s_0 < s_1 < \cdots < s_n = 1$. An outcome of the stress test $\mathcal{P}$ is a public signal $m_i \triangleq s \in [s_{i-1}, s_i)$ for $i \in \{1, \ldots, n\}$.*

Using (2) it is easy to show that a stress test $\mathcal{P}$ induces a distribution of posterior beliefs

$$\begin{aligned} \Pr(m_i) &= s_i - s_{i-1} \triangleq \nu_i, \\ \Pr(\omega = h|m_i) &= \frac{s_{i-1} + s_i}{2} \triangleq \mu_i. \end{aligned} \tag{3}$$

satisfying Bayes-plausibility, i.e. $\sum_{i=1}^n \nu_i \mu_i = \Pr(\omega = h) = 1/2$, with $\sum_{i=1}^n \nu_i = 1$.

After the outcome of the stress test $m_i$ is publicly observed, at date 1 the speculator can acquire private information about the underlying state and trade in the bank's shares at date 2. Information acquisition generates a signal $z \in \{l, h, \varnothing\}$. The signal is fully informative ($z = \omega$) with probability $\sigma$ and uninformative ($z = \varnothing$) otherwise. The speculator can choose $\sigma$, i.e., how much information to acquire subject to the cost $\frac{1}{2}\tau\sigma^2$ (with $\tau > 0$) this incurs.

[12] In practice applying the stress scenario may convey more information than just pass / fail, as the supervisor may also observe the amount of any shortfall. We ignore this potential complication for two reasons. Firstly, since payoffs to derivatives are highly non-linear, the amount of a short fall may itself not be very informative about the magnitude of the stress that can be withstood at a given level of acceptable loss. Second, since the supervisor can costlessly run as many scenarios as she wishes, she does not need the quantity of the short fall to pin down the bank's actual resilience level $s$ if she wishes to find it out.

We denote with $E^a_\omega$ the bank's equity value in state $\omega$ after the intervention decision $a$. Moreover, we assume that the supervisor's regulatory intervention wipes out equityholders who see the value of their claims reduced to zero, $E^0_l = E^0_h = 0$. On the other hand, if the supervisor allows the bank to continue, equity value $E^1_\omega$ depends on the state $\omega$ :[13]

$$\Delta E \triangleq E^1_h - E^1_l > 0. \tag{4}$$

The market mechanism is based on Kyle (1985). The speculator can submit a market order $x_s$ to a risk neutral market maker. In addition to the speculator, there is a noise trader who buys or sells, $x_n \in \{1, -1\}$, with equal probability, $\Pr(x_n = 1) = 1/2$. The noise trader can be thought of as trading for non-information-related reasons, such as taxes, consumption needs, insurance, hedging, etc. The market maker observes both orders $X = (x_n, x_s)$ but cannot tell which order originates from whom. He then sets a price $p$, publicly observed, to break even in expectation.

Finally, at date 3, the supervisor takes an intervention decision given all the available public information, i.e., the outcome of the stress test $m_i$ and the share price $p$. Payoffs realize at date 4.

The timing of the game is reported in Figure 1.

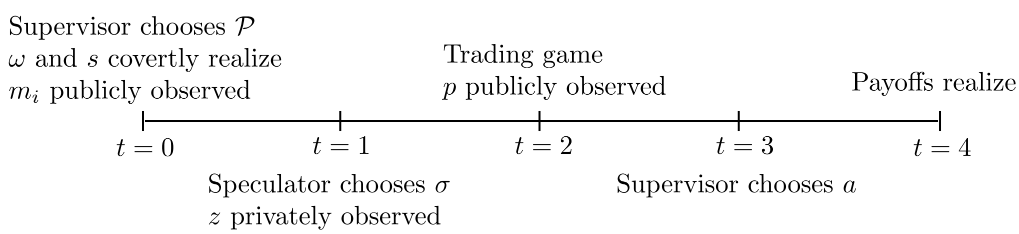


Figure 1: Timing

Before proceeding with the analysis a few comments about our assumptions are in order. Note that our definition of stress test $\mathcal{P}$ is quite flexible. In particular, we allow for complete learning of $s$, i.e. fully granular grades ($n \to \infty$), no information about $s$ ($n = 1$), or learning $s$ noisily ($n$ finite).[14] Nevertheless, we impose two notable restrictions. First, we require the stress test to be monotone, i.e. two disjoint intervals cannot produce the same test outcome.

[13] See Section 5 for a microfoundation of equity values. By way of generality, note that our assumptions $V^1_h > V^0_h$ and $V^0_l > V^1_l$ imply $V^1_h - V^1_l > V^0_h - V^0_l$. That is, the range of values the bank can take is larger if it is continued than when it is subject to an intervention. It also implies that $E^1_h - E^1_l > E^0_h - E^0_l$, which is the key property we need for our main results. For additional tractability we make the stronger assumption $E^0_h - E^0_l = 0$.

[14] Strictly speaking, with $n$ being an integer, the stress test cannot fully reveal $s$ which is a real number. However, since the limiting case is not materially affected by this distinction, we prefer to avoid complicating the notation in a way that would be required to formally take on board this subtlety.

This assumption is motivated by the fact that the stress test consists of a sequential application of stress scenarios. This way of learning is plausible and rules out that an observer may believe that resilience can be high or low, but not in the middle. Second, contrary to the Bayesian persuasion literature, the supervisor cannot condition the public signal $m_i$ on the state of the world $\omega$.[15] This assumption captures in a simple way the fact that the supervisor faces an upper bound on the informativeness of the stress test. In particular, the most informative stress test ($n \to \infty$) perfectly reveals any realization $s$ to the supervisor, but still leaves uncertainty about the underlying state $\omega$.[16]

## 4 Optimal Stress Test

**Supervisor's Intervention Decision** We solve the game by backward induction. At date 3, the information set of the supervisor includes the outcome $m_i$ of the stress test and the market price $p$ of the bank's shares. Denote the supervisor's information set by $\mathcal{I}$ which is described in more detail below once we discuss the trading game.

We can now pin down the posterior belief $\Pr(\omega = h|\mathcal{I})$ for which it is optimal for the supervisor to allow the bank to continue:

$$\Pr(\omega = h|\mathcal{I})V_h^1 + \big(1 - \Pr(\omega = h|\mathcal{I})\big)V_l^1 \geq \Pr(\omega = h|\mathcal{I})V_h^0 + \big(1 - \Pr(\omega = h|\mathcal{I})\big)V_l^0.$$

Equivalently,

$$\Pr(\omega = h|\mathcal{I}) \geq \frac{\Delta V_l}{\Delta V_l + \Delta V_h} \triangleq \widehat{s}.$$

Thus, the optimal intervention decision of the supervisor is

$$a(\mathcal{I}) = \begin{cases} 1 & \text{if } \Pr(\omega = h|\mathcal{I}) \geq \widehat{s} \\ 0 & \text{if } \Pr(\omega = h|\mathcal{I}) < \widehat{s}. \end{cases} \tag{5}$$

If the supervisor has sufficiently positive evidence of the bank being in good financial health, she allows the bank to continue, otherwise, she intervenes.

**Trading Game** At date 2, the speculator knows the outcome of the stress test $m_i$ and, if he acquired information, he also knows his private signal $z \in \{h, \varnothing, l\}$. The following lemma

[15]Note that, if the supervisor was able to condition the public signal directly on the state, she would choose a fully revealing stress test and the analysis would be trivial.

[16]In practice, supervisor's need to rely on risk models put forward by the banks themselves. As Leitner and Yilmaz (2019) argue, more intense monitoring by the supervisor may lead to a reduction in the informativeness of the bank's internal model. This puts a limit on how much a supervisor can learn, even if the supervisor's monitoring technology could be arbitrarily precise.

characterizes the trading strategy of the speculator and the associated trading profits for a given level $\sigma$ of information production.

**Lemma 1** (Trading Profits)**.** *The speculator's optimal trading strategy is*

$$x_s(z) = \begin{cases} 1 & \text{if } z = h \\ 0 & \text{if } z = \varnothing \\ -1 & \text{if } z = l. \end{cases}$$

*The associated expected trading profits, excluding information acquisition costs, are*

$$\pi(\mu_i) = \begin{cases} \sigma \Delta E \mu_i (1-\mu_i) & \text{if } \mu_i \geq \widehat{s} \\ 0 & \text{if } \mu_i < \widehat{s}. \end{cases} \tag{6}$$

*Proof.* See the Appendix. □

The trading strategy is intuitive: the speculator buys on positive information, sells on negative information and does not trade when he is uninformed. First, note that the order flow either fully reveals the speculator's private information (when $x_s = x_n = 1$ or $x_s = x_n = -1$) or is entirely uninformative (when $x_s = 0$ or $x_s = -x_n$). When the order flow is fully revealing, the speculator cannot make a trading profit as the market maker possesses the same information. Moreover, when the order flow reveals $\omega$, the supervisor ignores $m_i$, the outcome of the stress test, and follows the information revealed through the trading process ($\mathcal{I} = \omega$). On the other hand, when the order flow is uninformative, the supervisor makes her intervention decision contingent on the outcome of the stress test only ($\mathcal{I} = m_i$) and expected trading profits therefore differ, depending on whether the outcome conveys good or bad information. If the outcome induces pessimistic beliefs $\mu_i < \widehat{s}$ such that the supervisor intervenes, the bank's equity is wiped out ($E_h^0 = E_l^0 = 0$), and the speculator's expected trading profits are zero. If instead, the outcome induces an optimistic belief $\mu_i \geq \widehat{s}$ that leads the supervisor to condition her action on share prices, the range of the bank's equity values is $\Delta E$, to which the speculator's profits are proportional.

Finally, notice that conditional on positive news, the speculator's trading profits are increasing in the residual uncertainty, $\xi(\mu_i) = \mu_i(1-\mu_i)$, and are maximal at $\mu_i = 1/2$. This happens because the greater is the uncertainty, the higher is the information advantage of the speculator vis-à-vis the market maker and, consequently, his trading profits.

**Speculator's Information Acquisition** At date 1, the speculator knows the realization of the public signal $m_i$ and can choose the precision $\sigma$ of his private signal $z$. The speculator's optimal information acquisition can be found simply by considering the first-order condition with respect to $\sigma$ of the net trading profits $\pi(\mu_i) - \frac{1}{2}\tau\sigma^2$. This yields

$$\sigma(\mu_i) = \begin{cases} \frac{1}{\tau}\Delta E \mu_i(1-\mu_i) & \text{if } \mu_i \geq \widehat{s} \\ 0 & \text{if } \mu_i < \widehat{s}. \end{cases} \tag{7}$$

Similar to trading profits, information acquisition increases with the residual uncertainty: the greater the uncertainty, the higher the trading profits, the larger the benefits from acquiring information.

In what follows, we assume

$$0 < \frac{1}{\tau}\Delta E < 4, \tag{8}$$

which ensures that information acquisition in (7) is a non-degenerate probability.

**Supervisor's Stress Test Design Problem** We turn next to the supervisor's stress test design problem at date 0. For each belief induced by the result of a stress test, we know the amount of information produced by the speculator from (7) and, using (5), the probability that the supervisor intervenes. For a given stress test $\mathcal{P}$, we can thus compute the expected value of the bank following any sub-game starting with the realization of the public signal $m_i$. Taking expectations over $m_i$ then yields the supervisor's expected payoff associated with the stress test $\mathcal{P}$. For ease of exposition we do not present here the supervisor's general optimization problem. Instead, we provide an equivalent, but simpler representation in the next lemma, and relegate the exposition of the full problem to the proof.

**Lemma 2** (Stress Test Design Problem)**.** *The supervisor solves the following stress test design problem:*

$$\begin{aligned} \max_{\mathcal{P}} \; & V(\mathcal{P}) = \chi(s_1) + \Sigma(\mathcal{P}) \\ \text{s.t. } & \mu_1 < \widehat{s} \\ & \mu_2 \geq \widehat{s}, \end{aligned} \tag{9}$$

*where*

$$\begin{aligned} \chi(s_1) &= \frac{1}{2}\Big(F_h(s_1)V_h^0 + \big(1 - F_h(s_1)\big)V_h^1 + F_l(s_1)V_l^0 + \big(1 - F_l(s_1)\big)V_l^1\Big), \\ \Sigma(\mathcal{P}) &= \frac{1}{2\tau}\Delta E \Delta V_l \sum_{i=2}^{n} \nu_i \mu_i (1-\mu_i)^2. \end{aligned} \tag{10}$$

*Proof.* See the Appendix. □

The objective function involves two terms. The first term, $\chi(s_1)$, is the direct value of information embedded in the stress test, i.e., ignoring the effect of the stress test on market information. Accordingly, the test provides statistical evidence $m_i$ about the state $\omega$ and allows the supervisor to tailor her intervention decision. Since this information is useful to the extent that it guides the binary decision of the supervisor, all the realizations of $s$ inducing the same intervention decision can be pooled in a unique message. It follows that $\chi(s_1)$ depends only on one threshold $s_1$ such that the supervisor, if she were to observe only the outcome of the stress test, would choose to intervene if $s < s_1$ but would not intervene when $s \geq s_1$. In other words, if one labels a stress test outcome as a *fail* whenever it induces supervisory intervention when no further (market) information becomes available, then it is sufficient to restrict attention to stress tests that have only a single fail message. It is useless to know just by how much the bank failed a test. At this stage we view this mainly as a technical result allowing us to simplify the supervisor's optimization problem.[17]

The second term, $\Sigma(\mathcal{P})$, is the additional expected benefit from information acquisition by the speculator: $\mathbb{E}_{m_i}\left(\frac{1}{2}\sigma(\mu_i)\Delta V_l|\omega = l\right)$. The benefit accrues to the supervisor by providing market information about poorly capitalized banks that erroneously passed the test. With probability $\frac{1}{2}\sigma(\mu_i)$ the speculator's private signal publicly reveals the state, allowing the supervisor to revise her intervention decision and gain additional value $\Delta V_l$. As a consequence, $\Sigma(\mathcal{P})$ is proportional to the gain from revising the supervisor's decision $\Delta V_l$ and to the profitability of trading $\frac{1}{\tau}\Delta E$, which affects the speculator's choice of information production. Note that the financial market does not help in preventing interventions in sound banks who were unlucky to display poor resilience to the test. This is because the speculator cannot benefit from acquiring information about banks with stress test results that induce sufficiently pessimistic beliefs ($\mu_i < \widehat{s}$) so as to trigger an intervention. Hence, the supervisor cannot learn from the market about the true health of such banks.

**Optimal Stress Test** As a first step in the analysis, it is useful to consider what the optimal stress test would be, in the absence of a potentially informed speculator. This benchmark would apply, for example, to banks that are not listed.

**Lemma 3** (Optimal Stress Test without Financial Markets)**.** *Without information acquisition by financial markets ($\frac{1}{\tau}\Delta E \to 0$) the (weakly) optimal stress test is $\mathcal{P} = \{0, \widehat{s}, 1\}$, i.e., a pass-fail experiment, that partitions $\mathcal{S}$ into two regions (buckets) such that the bank receives*

- *a fail grade $m_1$ if $s \in [0, \widehat{s})$,*

[17] This result is not without loss of generality as we will show in Section 6.1. When market information may be induced following a "fail" outcome, then multiple fail outcomes can add value, as will be the case if debt claims can be traded.

- *a pass grade* $m_2$ *if* $s \in [\widehat{s}, 1]$.

*Proof.* See the Appendix. □

The result is very intuitive. Without an informed speculator, the supervisor only cares to maximize the direct value of information $\chi(s_1)$. Since the supervisor's action is binary, it is sufficient to know whether $s$ is above or below the intervention threshold $\widehat{s}$. This stress test is not uniquely optimal since running more stress scenarios (i.e., generating a finer partition) has neither cost (by assumption) nor any benefit. Note, however, that introducing even a small cost of running a stress test would render the simple *pass/fail* partition strictly optimal.

We now analyze the optimal stress test when there is a speculator.

**Proposition 1** (Optimal Stress Test with Financial Markets)**.** *With information acquisition by financial markets ($\frac{1}{\tau}\Delta E > 0$) the optimal stress test $\mathcal{P}$ contains two buckets, followed by fully granular grades for resilience levels above the buckets. That is, it features:*

- *a fail grade* $m_1$ *if* $s \in [0, s_1)$ *where* $s_1 < \widehat{s}$,
- *a pass grade* $m_2$ *if* $s \in [s_1, s_2)$ *where* $s_2 \in [2\widehat{s} - s_1, 1)$,
- *fully granular grades* $s$ *for* $s \in [s_2, 1]$.

*Proof.* See the Appendix. □

When market information matters, the stress test is optimally distorted toward leniency ($s_1 < \widehat{s}$). That is, it awards *pass* grades to some banks whose resilience level is so low that, absent any further information, they ought to be subject to an intervention. This is the case for banks with resilience levels $s \in [s_1, \widehat{s})$.

Why does the supervisor wish to apply a more lenient *pass* threshold? After all, leniency has a cost, because it increases the likelihood of allowing undercapitalized banks to continue without intervention. The answer is that by virtue of being lenient, the stress test generates more *pass* grades. Since a *pass* grade is a precondition for the speculator to acquire information, leniency encourages the production of market information. This information will subsequently help the supervisor identify undercapitalized banks. The trade-off between the direct information value of the stress test, and its role in encouraging information production by the speculator implies that the optimal cut-off is determined as an internal solution, $s_1 \in (0, \widehat{s})$.

Since a *pass* grade is intended to convey sufficiently positive information to incentivize information production, the lowest *pass* grade must be somewhat coarse and include levels of resilience strictly above the benchmark failure threshold $\widehat{s}$. That is, $s_2$ must be sufficiently above $\widehat{s}$ so that observing $m_2$ actually induces information production by the speculator. This constraint can be viewed as a requirement for the *pass* grade to be credible. If $s_2$ were too close to $\widehat{s}$ the speculator would realize that the belief induced by the *pass* grade is too negative to warrant continuation without intervention. That is, it would not really be a *pass* grade. This insight can also be interpreted in light of the *ex post* versus *ex ante* optimal intervention policy. In order to stimulate information production by the speculator, the supervisor should intervene less often than she finds *ex post* optimal. Blurring the information she has at her disposal *ex post* allows the supervisor to implement an *ex ante* optimal, lenient intervention policy.[18]

Why does the supervisor need more than two grades, given that the induced action is binary? That is, why does the supervisor not simply set $s_2 = 1$? This may be surprising as one might expect a coarser test to leave more residual uncertainty and hence induce more information production. To illustrate, consider the following numerical example. Suppose there is a single (coarse) grade $m_2$ for resilience levels $s \in [\frac{1}{2}, 1]$ inducing a belief $\mu_2 = \frac{3}{4}$ (see eq. (3)) and, using (7), with information production proportional to $\frac{3}{16} = \frac{3}{4} \times (1 - \frac{3}{4})$. Suppose now this grade is refined by introducing a new cut-off in the middle, such that we would get a grade $m_2'$ for $s \in [\frac{1}{2}, \frac{3}{4}]$ and a grade $m_3'$ for $s \in (\frac{3}{4}, 1]$. Observing $m_2'$ would now induce belief $\mu_2' = \frac{5}{8}$ and information production proportional to $\frac{15}{64} > \frac{3}{16}$ while $m_3'$ induces a belief $\mu_3' = \frac{7}{8}$ and information production proportional to $\frac{7}{64} < \frac{3}{16}$. In other words, refining the test in this way improves information production incentives for resilience levels closer to $\frac{1}{2}$ and worsens them for resilience levels closer to 1.

This is intuitive as intermediate resilience levels are less conclusive about the bank's true capitalization, increasing a speculator's potential informational edge over the market maker. Which of these partitions is preferable depends on the type of bank for which the supervisor most values market information. The latter is most useful for banks that are "close calls," i.e., banks whose resilience levels are around $\widehat{s}$. Market information has little value for banks that have very high resilience levels because the test itself leaves little doubt as to the bank's true capitalization level. It may therefore be better to refine the initial partition. Although this example is a little extreme in terms of the chosen numbers, a robust intuition remains: eventually it is best not to dull information production incentives by including banks with very high resilience levels in a coarse test grade.

Note that in the region where a bank fails the test, the supervisor does not gain from further precision, just as in the benchmark. A coarse message $m_1$ is therefore weakly optimal.

In practice stress tests are carried out under a baseline scenario and at least one adverse scenario. Since banks are subject to continuous supervision, one can expect that all banks pass

[18]This is related to Crémer's (1995) idea of using an information-reducing arm's length relationship as a commitment device.

the baseline scenario (a supervisor would intervene before a bank reached the point of risking to fail the baseline scenario). One might think of $s_0 = 0$ as the baseline scenario. Under a lenient adverse scenario, a bank that fails the test would certainly be subject to an intervention. This has been the case, for example, for Monte dei Paschi di Siena who failed tests at several points in time, each resulting in a subsequent intervention. On the other hand, if a test is lenient, then passing it is not a guarantee of no intervention. For example, two Portuguese banks, Banco Comercial Português and Espirito Santo passed the 2011 stress tests, but were nonetheless forced to strengthen their balance sheets.[19]

In the US, stress tests have multiple adverse scenarios which can by and large be ranked by their severity. For example, under Dodd-Franks, the test does not allow banks' capital distributions to adjust downwards, making the scenario more severe than that is employed under CCAR. Similarly, adding counter-party defaults and / or a global market shock to the scenario applied to some banks provides additional information at the more severe end of the stress spectrum. From a descriptive point of view, our theory would predict that banks to whom even the most severe scenarios are applied, and who pass them, would have relatively little information in subsequent stock prices, compared to banks that pass a less severe adverse scenario. We would expect least information to be contained in the stock prices of those banks who fail outright the regular adverse scenario.

## 5 Microfoundation and Comparative Statics

In order to conduct comparative statics it is important to provide a microfoundation for the banks' overall value $V_\omega^a$ as well as its equity $E_\omega^a$ and debt values $D_\omega^a$. The debt value will be relevant for the analysis in Section 6. Without further microfoundation, one might, for example, consider a comparative static with respect to $\Delta V_l$. However, changing $\Delta V_l$ might also change something else (for example $\Delta E$), if one were to elaborate on the underlying reasons for the changes in parameter values.

Suppose the bank has assets in place $A_\omega$ whose value depends on the underlying state of nature and where $A_h > A_l = 0$. Moreover, the bank is financed with an exogenous amount of debt with face value $D$, to be repaid at date 4. We assume that the bank's CEO privately observes the value of assets in place.[20] The bank generates state-dependent cash flows at date

[19]The Banco de Portugal explained: "Following completion of the EU-wide stress test, the results determine that all four Portuguese banks meet the capital benchmark set out for the purpose of the stress test.However, two banks will devise, in agreement with Banco de Portugal, appropriate actions, such as increasing capital or putting in place asset disposals, to strengthen their balance sheets in the 3-month period immediately after the present publication." (see Banco de Portugal (2011))

[20]We exclude here the possibility of truthtelling contracts that might extract the banker's information. An older literature has considered how regulation, such as the pricing of deposit insurance, can induce truthtelling

4 if the supervisor allows it to continue without intervention at date 3. These cash flows can be high, $R$, medium, $r$, or low, 0 with $R > r > 0$. Moreover, at date 3, the banker can choose whether to act prudently or recklessly by choosing $\tilde{\varepsilon} \in \{0, \varepsilon\}$, where $\varepsilon \in (0, 1-p)$, to influence the distribution of cash flows $\Pr(R) = \frac{p+\tilde{\varepsilon}}{2}$, $\Pr(r) = 1 - (p + \tilde{\varepsilon})$ and $\Pr(0) = \frac{p+\tilde{\varepsilon}}{2}$.

We make several parametric assumptions, which are aimed at capturing the following: the banker, acting on behalf of shareholders, behaves prudently as long as the bank's balance sheet is in good health (assets are worth $A_h$), but engages in risk shifting when its balance sheet deteriorates (assets are worth $A_l = 0$).

First, assume that when cash flows are equal to $R$ the bank can repay its debt in full, irrespective of the value of assets, i.e.,

$$R > D. \tag{11}$$

Second, assume that when cash flows are medium ($r$), the bank cannot repay its debt when assets have a low value $A_l = 0$, i.e.,

$$D > r. \tag{12}$$

The two inequalities (11) and (12) imply that a poorly capitalized bank (i.e., in state $\omega = l$) behaves recklessly. To see this, compare the expected equity value from being prudent $\frac{p}{2}(R - D)$ to that from being reckless $\frac{p+\varepsilon}{2}(R - D)$, the latter obviously being higher.

Moreover, we make the following parametric assumption, which ensures that a well capitalized bank chooses to behave prudently:

$$2r + A_h - R \geq D. \tag{13}$$

Note that (13) implies

$$\frac{p}{2}(R + A_h - D) + (1-p)(r + A_h - D) \geq \frac{p+\varepsilon}{2}(R + A_h - D) + (1 - (p+\varepsilon))(r + A_h - D),$$

which means equity value is higher from being prudent (left-hand side of the inequality) than from being reckless (right-hand side of the inequality). Inequality (13) also implies $r + A_h - D \geq$

by the banker when the latter is also subject to an agency problem (e.g., Chan, Greenbaum and Thakor (1992), Giammarino, Lewis and Sappington (1993) or Freixas and Rochet (1998)). More recently, Philippon and Schnabl (2013) and Bhattacharya and Nyborg (2013) study the design of a bailout, when banks are privately informed about their funding needs and show that the optimal recapitalization seeks to reduce the bank's informational rent, among other things. The continued focus in research on the monitoring role of supervisors is arguably an implicit acknowledgement of the difficulty of implementing some of these schemes in practice. Moreover, having market information at her disposal, the supervisor could presumably reduce the informational rent left to the banker from a truthtelling mechanism. Hence, allowing for truthtelling contracts does not undermine the role of market information. However, taking these considerations fully on board would distract us from the paper's main message.

$R - r > 0$, so a well capitalized bank can repay its debt when the medium cash flow $r$ is realized.

Given the above parametric assumptions, when the supervisor does not intervene ($a = 1$), the bank's debt value equals the expected actual debt repaid:

$$\begin{aligned} D_h^1 &= \frac{p}{2} D + (1-p)\, D + \frac{p}{2} A_h, \\ D_l^1 &= \frac{p+\varepsilon}{2} D + (1-p-\varepsilon)\, r, \end{aligned} \tag{14}$$

while equity values are equal to the expected cash flows plus asset value net of the debt actually being repaid:

$$\begin{aligned} E_h^1 &= \frac{p}{2}(R + A_h - D) + (1-p)\,(r + A_h - D), \\ E_l^1 &= \frac{p+\varepsilon}{2}(R - D), \end{aligned} \tag{15}$$

The total bank value is just the sum of equity and debt:

$$\begin{aligned} V_h^1 &= D_h^1 + E_h^1 = \frac{p}{2} R + (1-p)\, r + A_h, \\ V_l^1 &= D_l^1 + E_l^1 = \frac{p+\varepsilon}{2} R + (1-p-\varepsilon)\, r. \end{aligned} \tag{16}$$

Intervention ($a = 0$) helps to limit the risk taking activities of the bank but also involves a loss in terms of expected cash flows.[21] Specifically, we assume that an intervention reduces the value of the bank's ongoing activities by a fraction $\delta < 1$. We assume, without loss of generality, that the value of assets in place, $A_\omega$ is not affected by the intervention. Assume that proceeds from selling the bank are not enough to repay debt $D$ in full even when a high-value bank is liquidated, i.e.,

$$D > \delta \left( \frac{p}{2} R + (1-p)\, r \right) + A_h. \tag{17}$$

It follows that bank equity value is zero following an intervention ($a = 0$),

$$E_h^1 = E_h^0 = 0,$$

and all remaining value goes to the creditors:

$$\begin{aligned} D_h^0 &= V_h^0 = \delta \left( \frac{p}{2} R + (1-p)\, r \right) + A_h, \\ D_l^0 &= V_l^0 = \delta \left( \frac{p}{2} R + (1-p)\, r \right). \end{aligned}$$

[21] One can think of intervention as taking the form of an acquisition of the bank's ongoing activities by a better capitalized bank who will not engage in risk shifting. The acquisition would be at a "fire sale" discount, reflecting, for example, the new bank's inferior ability to manage the on-going activities. Alternatively, one could assume that the bank is liquidated following an intervention, where liquidations incur a dead-weight loss.

Note that from (13) and (17), it follows that

$$2r > R, \tag{18}$$

which implies that expected cash flows are higher when the bank is prudent. Hence, the model captures a situation, where a bank that experiences an adverse shock to its balance sheet ($A_l = 0$) becomes undercapitalized (excessively levered), generating incentives to gamble. Since the supervisor's intervention can prevent excessive risk taking by the bank at the cost of impairing its ongoing activities, it is crucial for the supervisor to identify the bank's assets value $\omega$.[22]

Lastly, note that $\delta < 1$ implies $\Delta V_h > 0$. In order to ensure that $\Delta V_l > 0$, we need to assume that $\delta$ is not too low - otherwise intervention would never be optimal. Specifically, we assume

$$\delta > 1 - \frac{\frac{\varepsilon}{2}(2r - R)}{\frac{p}{2}R + (1-p)\,r}. \tag{19}$$

Equipped with our microfoundation we can relate the threshold, $\widehat{s}$, and the information sensitivity of equity, $\Delta E$, to parameters describing the banking system:

$$\begin{aligned} \widehat{s} &\triangleq \frac{\Delta V_l}{\Delta V_l + \Delta V_h} = 1 - (1-\delta)\frac{\frac{p}{2}R + (1-p)r}{\frac{\varepsilon}{2}(2r - R)} \in (0,1), \\ \Delta E &\triangleq E_h^1 - E_h^0 = \left(1 - \frac{p}{2}\right) A_h - (1-p)(D-r) - \frac{\varepsilon}{2}(R - D) > 0. \end{aligned} \tag{20}$$

The following comparative statics hold.

**Proposition 2** (Comparative Statics)**.** *The stress test is more lenient at the optimum, i.e.* $s_1$ *decreases, when:*

- *information acquisition is less expensive ($\tau$ decreases),*
- *the bank is less indebted ($D$ decreases),*
- *the risk shifting problem is less severe ($\varepsilon$ decreases),*
- *intervention is less efficient ($\delta$ decreases).*

*Proof.* See the Appendix. □

In general, the *pass* threshold $s_1$ is directly affected by changes in $\widehat{s}$ and indirectly by the

[22]Note that our model is rich enough to capture a forced recapitalization as an alternative way to think about a supervisor's intervention. A forced recapitalization would wipe out (or severely dilute) existing shareholders and require a potentially costly subsidy by the supervisor, providing her with an incentive to intervene only in state $\omega = l$. We can also include a cost of an ex-post bank failure (such as through negative spillovers on other banks) in the supervisor's objective function without this changing our results. In Appendix B, we show how this can be done.

optimal extent of distorting $s_1$ away from $\widehat{s}$. In developing the intuition for the comparative statics, we will make use of this distinction.

Consider a reduction in the cost of information acquisition, $\tau$. Firstly, note that $\tau$ has no direct effect on $\widehat{s}$. It does, however, have an effect on $s_1$: When financial markets can cheaply acquire information about the bank's fundamentals, private information becomes more precise. The benefit of distorting the stress test towards more leniency increases. A similar mechanism is at play following a reduction in the debt exposure of the bank, $D$, which again has no direct effect on $\widehat{s}$. A reduction in the debt level increases the information sensitivity of the equity claim and thereby the speculator's incentives to acquire more precise information.

By contrast, changes in $\varepsilon$ have both a direct effect on the benchmark resilience level $\widehat{s}$ and an indirect effect on the optimal extent of distorting $s_1$ away from $\widehat{s}$. As $\varepsilon$ increases, the risk shifting problem gets more severe. This makes it more costly for the supervisor to allow a poorly capitalized bank to continue without intervention. A bank therefore needs to exhibit a higher resilience level in order to be allowed to continue, i.e., $\widehat{s}$ increases. In addition there is an indirect effect. A more severe risk shifting problem (a higher $\varepsilon$) reduces the information sensitivity of the equity claim. Risk shifting creates value to equity holders precisely when the value of assets in place is low. Risk shifting therefore partially offsets the lower asset value implied by the low state and does so more strongly as $\varepsilon$ increases. This, however, dulls incentives for the speculator to acquire information, reducing the benefit of distorting the stress test towards more leniency. The direct effect of an increase in $\varepsilon$ on $\widehat{s}$ and the indirect effect thus work in the same direction towards less leniency.

Finally, a change in $\delta$ only has a direct effect on $\widehat{s}$ without changing the optimal distortion. As $\delta$ increases, interventions become less costly, lowering the intervention threshold $\widehat{s}$ and thus also $s_1$. A change in $\delta$ does not in itself affect information production incentives. Remember that the speculator, can only make a trading profit, if the bank is allowed to continue. The loss in bank value following an intervention, captured by $\delta$, therefore does not affect trading profits.

# 6 Extensions

## 6.1 Debt Trading

As shown above, the equity claim becomes completely insensitive to private information when the supervisor intervenes. This, in turn, cancels the speculator's incentives to acquire information following a negative stress test result. In this extension, we allow the speculator to decide, after observing the stress test result $m_i$ and his private signal $z$, to trade either a risky debt claim or an equity claim.[23] In order to focus the analysis on the role of the information sensitivity of the traded claims, we assume that trade in the debt market works in exactly the same way as in the equity market, and that liquidity trades are drawn from the same distribution. This arguably exaggerates the potential positive role of debt markets in conveying information as the latter tend to be less liquid and, to the extent that trades occur over the counter, prices may be less readily available.

Following the notations previously used for equity and bank value, we denote by $D_\omega^a = V_\omega^a - E_\omega^a$ the bank's debt value in state $\omega$ following the supervisor's decision $a$. Relying on the microfoundations described in Section 5. one can show that

$$\begin{aligned} \Delta D^0 &\triangleq D_h^0 - D_l^0 = A_h > 0, \\ \Delta D^1 &\triangleq D_h^1 - D_l^1 = (1-p)(D-r) + \frac{p}{2}A_h + \frac{\varepsilon}{2}(2r - D) > 0. \end{aligned} \tag{21}$$

The first inequality implies that the speculator, contrary to the baseline model, can realize positive trading profits by trading bonds when the supervisor intervenes. On the other hand, suppose

$$\Delta E > \Delta D^1,$$

i.e., trading equity is more profitable than debt when the supervisor does not intervene. This amounts to assuming that the risk shifting problem is sufficiently mild

$$\varepsilon < \frac{2(1-p)(r - D + \frac{1}{2}A_h)}{r - D + \frac{1}{2}R}.$$

First, we determine the speculator's information acquisition choices.

**Lemma 4** (Speculator's Information Acquisition with Debt Trading)**.** *When the speculator can trade a debt or an equity claim, his optimal information acquisition is*

$$\sigma(\mu_i) = \begin{cases} \frac{1}{\tau}\Delta E \mu_i(1-\mu_i) & \text{if } \mu_i \geq \widehat{s} \\ \frac{1}{\tau}\Delta D^0 \mu_i(1-\mu_i) & \text{if } \mu_i < \widehat{s}. \end{cases} \tag{22}$$

*Proof.* See the Appendix. □

As in the baseline model, information acquisition is proportional to the residual uncertainty about the state, conditional on the stress test outcome. More interestingly, the speculator now acquires information even after a negative stress test result as he expects positive trading profits

[23] To keep things simple, we assume that the speculator can only trade in one claim and thus has to decide whether to trade in debt or equity, based on the expected trading profit. This assumption can be justified on the basis of position limits that might be imposed on the speculator, for example, due to financial constraints.

(when the order flow is uninformative) thanks to his superior information and the sensitivity of the debt claim to this information, $\Delta D^0 > 0$.

The supervisor benefits from the additional information in the following way. Receiving market information after negative stress tests ($\mu_i < \widehat{s}$) allows the supervisor to identify some well capitalized banks that mistakenly received a poor test result. Incidentally, bank equity holders also benefit from this additional information since it reduces the instances of supervisory intervention.

Determining the optimal stress test design is now significantly more difficult. Since different *fail* messages lead to different levels of information production we cannot restrict attention to a unique *fail* message as we did in the baseline model. This feature complicates the stress test design problem (reported in the Appendix as Lemma A1) so that we do not aim at providing a full characterization of the optimal stress test. Still, by simulating numerical examples we find that it is optimal to have some degree of coarseness in the interior of the $\mathcal{S}$ space and fully granular grades at the boundaries. More precisely, the optimal stress test features one or two coarse messages in the interior depending on parameter values. Intuitively, two coarse messages are optimal when $\widehat{s}$ is close to the center of the $\mathcal{S}$ space. In this instance, having a unique coarse message would lead to high information production (via either debt or equity trading) but also very poor decision making when the order flow turns out to be uninformative. Since information is now produced also for failed banks, the supervisor finds it optimal to have two distinct messages leading to relatively lower information production (one via debt trading, the other via equity trading) but more accurate intervention decisions when the order flow is uninformative. Ultimately, the possibility of trading debt helps to relax the speculator's participation constraint and reduces the extent of policy distortions.

Lastly, the optimal stress test with debt trading may generate a distortion towards conservatism for some parameters.

**Proposition 3.** *(Distortions towards Conservatism/Leniency with Debt Trading) Let* $m_j \triangleq \max_i \left\{ m_i \, s.t. \, \mu_i < \widehat{s} \right\}$ *be the highest stress test result that still induces the supervisor to intervene without market information. When the speculator can trade either debt or equity claims, the optimal stress test* $\mathcal{P}$ *exhibits*

- *motives towards conservatism,* $s_j > \widehat{s}$*, if* $\Delta D^0 / \Delta E \geq \kappa$*, for some* $\kappa > 1$*,*
- *motives towards leniency,* $s_j < \widehat{s}$*, otherwise.*

*Proof.* See the Appendix. □

A conservative stress test is one that is prone to give *fail* grades to banks that would have passed the test without information production by the market. The supervisor designs a conservative stress test whenever debt claims of failed banks are relatively more sensitive to private information than equity claims (high $\Delta D^0/\Delta E$). The result is intuitive. Since the speculator's private information may be publicly revealed by trading either securities, the supervisor tends to induce trading in the security for which the speculator is willing to acquire more information.

Increasing $D$ does not affect either $\widehat{s}$ or $\Delta D^0$ but reduces $\Delta E$ (see discussion after Proposition 2). It follows that the stress test would be conservative for banks with high debt exposures $D$. We would therefore expect more levered banks to benefit more from a conservative stress test design. Of course, this comparative static needs to be interpreted with caution, as we implicitly assume that a change in leverage does not by itself change liquidity (or anything else).

## 6.2 Weakly Correlated Information

In the baseline model, the stress test and the speculator aim at identifying the same information, namely the future value of a bank's assets in place. One may argue that in practice, a supervisor has a comparative advantage, for example, in monitoring the state of a bank's balance sheet, while financial markets may be more suited to identifying information about a bank's future cash flows from operations. Conceptually, one can capture this, by allowing for a more complex state space and distinguishing the states that the stress test can identify from those that a speculator can learn about. We now study a corresponding extension.

Specifically, we now consider a bi-dimensional state space with arbitrary positive correlation among states. Let $\omega \in \{l, h\}$ be the quality of the assets in place. Like before, $\omega$ determines whether the bank engages in risk shifting and hence whether an intervention is desirable. In addition there is a state $\theta \in \{b, g\}$ which affects the level of future cash flows of both the high and low risk projects. Let $V^a_{\omega,\theta}$ and $E^a_{\omega,\theta}$ denote the bank's total and equity values, respectively, conditional on the state $(\omega, \theta)$ and intervention decision $a$.

For reasons of tractability we make some simplifying assumptions. Since $\theta$ affects the level of expected cash flows for both projects, we assume that it does not affect the bank's decision to engage in risk shifting and hence it does not affect the desirability of an intervention. This allows us to write

$$\begin{aligned} V^1_{h,g} - V^0_{h,g} = V^1_{h,b} - V^0_{h,b} \triangleq \Delta V_h > 0, \\ V^0_{l,g} - V^1_{l,g} = V^0_{l,b} - V^1_{l,b} \triangleq \Delta V_l > 0. \end{aligned} \tag{23}$$

Moreover, we assume that the equity values relevant for purposes of trading claims depend only on the level of expected cash flows, i.e., on the state $\theta$, but not on $\omega$. This could be justified if $\omega$ is only publicly observed in the long term, beyond the trading horizon of the speculator. The speculator then cares indirectly about $\omega$ because it affects the intervention decision, but conditional on no intervention, the speculator's trading profits will not depend on $\omega$. Specifically, we assume that equity is wiped out following an intervention, i.e., $E^0_{\omega,\theta} = 0$, and

$$E^1_{\omega,\theta} = E^1_\theta \text{ for all } \omega \in \{l, h\}. \tag{24}$$

As in the baseline model, let

$$\Delta E \triangleq E^1_g - E^1_b > 0.$$

The joint prior beliefs over the states are

$$\begin{array}{lcc} \Pr(\omega,\theta) & g & b \\ h & \multicolumn{2}{c}{\multirow{2}{*}{$\begin{pmatrix} 1/4+\rho & 1/4-\rho \\ 1/4-\rho & 1/4+\rho \end{pmatrix}$}} \\ l & & \end{array} \tag{25}$$

where $\rho \in [0, 1/4)$ is a measure of how correlated the states $\omega$ and $\theta$ are. Note that, when $\rho = 1/4$, $\theta$ and $\omega$ are perfectly correlated, which brings us back to the baseline model in Section 3.

The stress test provides noisy information about the quality of the assets in place, just like in the baseline model and is defined as in Definition 1. The speculator now acquires information about the profitability of the bank. In particular, we assume that the speculator, by paying a cost $\frac{1}{2}\tau\sigma^2$, observes a private signal

$$z = \begin{cases} \theta & \text{with probability } \sigma \\ \varnothing & \text{with probability } 1-\sigma. \end{cases}$$

To sum up, the speculator cares about the result of the stress test because it provides information that correlates with profitability and because it is indicative of the supervisor's future intervention decision. Moreover, the supervisor cares about the stock price, because it may reveal future expected cash flows which correlate with the value of assets in place.

First, we describe how the supervisor's intervention decision takes into account that market information is only weakly informative about asset quality $\omega$. The supervisor's information set at date $t = 3$ is $\mathcal{I} \in \{(\theta, m_i), m_i\}$ depending on whether the trading activity revealed some information $\theta$ on top of the stress test result or not. As in the baseline model, when the supervisor learns nothing from the market, $\mathcal{I} = m_i$, she continues if $\mu_i \geq \widehat{s}$. Instead, when $\mathcal{I} = (\theta, m_i)$ she continues if

$$\Pr(\omega = h|\theta, m_i) = \frac{\mu_i \Pr(\theta|\omega = h)}{\mu_i \Pr(\theta|\omega = h) + (1-\mu_i)\Pr(\theta|\omega = l)} \geq \widehat{s},$$

so that we can determine two thresholds $\widehat{s}_\theta$ such that the supervisor continues if $\mu_i \geq \widehat{s}_\theta$. These thresholds are

$$\widehat{s}_g(\rho) \triangleq \frac{\left(\frac{1}{2} - 2\rho\right)\Delta V_l}{\left(\frac{1}{2} - 2\rho\right)\Delta V_l + \left(\frac{1}{2} + 2\rho\right)\Delta V_h},$$

$$\widehat{s}_b(\rho) \triangleq \frac{\left(\frac{1}{2} + 2\rho\right)\Delta V_l}{\left(\frac{1}{2} + 2\rho\right)\Delta V_l + \left(\frac{1}{2} - 2\rho\right)\Delta V_h}.$$

We emphasize that there is a set of stress test results $\mathcal{M}(\rho) \triangleq \{m_i \text{ s.t. } \mu_i \in [0, \widehat{s}_g(\rho)) \cup [\widehat{s}_b(\rho), 1]\}$ where the supervisor ignores the information produced by the market. In this region, the stress test result is so conclusive that it cannot be overcome by market information that only weakly correlates with asset quality. Note also that as $\rho$ decreases, market information is ignored more often as it is less indicative of the state the supervisor cares about. It follows that all the results $m_i$ s.t. $\mu_i \geq \widehat{s}_b(\rho)$ always induce the supervisor to continue and can be pooled in a unique result $m_n = s \in [s_{n-1}, 1]$. Similarly, all the $m_i$ s.t. $\mu_i < \widehat{s}_g(\rho)$ can be pooled in a unique result $m_1 = s \in [0, s_1]$.

The following lemma characterizes the optimal stress test when $\rho = 0$.

**Lemma 5** (Optimal Stress Test with Independent Information)**.** *When the stress test results $m_i$ are not informative about $\theta$, i.e. $\rho = 0$, the optimal stress test is a pass-fail experiment, that is it features*

- *a fail grade $m_1$ if $s \in [0, \widehat{s})$,*
- *a pass grade $m_2$ if $s \in [\widehat{s}, 1]$.*

*Proof.* The result directly follows from the fact that $\widehat{s}_g(0) = \widehat{s} = \widehat{s}_g(0)$. □

When market information is useless in predicting asset quality, the supervisor learns exclusively from the result of the test and ignores the market. Since her intervention decision is binary, it is sufficient to know whether $s$ is above or below the intervention threshold $\widehat{s}$.

We now determine the speculator's optimal information acquisition when $\rho > 0$.

**Lemma 6** (Speculator's Information Acquisition with Correlated Information)**.** *When the stress test results $m_i$ are weakly informative about $\theta$, i.e. $\rho \in (0, 1/4)$, the speculator's optimal information acquisition is*

$$\sigma(\mu_i) = \begin{cases} \frac{1}{\tau}\Delta E\left(\frac{1}{4} - 4\rho^2(1 - 2\mu_i)^2\right) & \text{if } \mu_i \geq \widehat{s} \\ 0 & \text{if } \mu_i < \widehat{s}. \end{cases} \tag{26}$$

*Proof.* See the Appendix. □

Similarly to the baseline model, the speculator's information acquisition is proportional to the residual uncertainty of his payoff-relevant state

$$
\begin{aligned}
\xi(\mu_i) &= \Pr(\theta = g|m_i)\Pr(\theta = b|m_i), \\
&= \Big(\mu_i\,(1/2 + 2\rho) + (1-\mu_i)\,(1/2 - 2\rho)\Big)\Big(\mu_i\,(1/2 - 2\rho) + (1-\mu_i)\,(1/2 + 2\rho)\Big), \\
&= \frac{1}{4} - 4\rho^2(1-2\mu_i)^2,
\end{aligned}
$$

as the higher the uncertainty, the higher the speculator's advantage with respect to the market maker and his incentives to acquire information. Moreover, when $\rho > 0$ the amount of information acquisition $\sigma(\mu_i)$ has the usual concave shape but now it becomes flatter when $\rho$ decreases. In other words, information acquisition is higher for any posterior beliefs $\mu_i$ when asset quality, and thus the stress test results, are less informative about profitability. This happens because, when $\rho$ decreases, the stress test becomes less reliable for the market maker to price the bank's share and thus the speculator's information advantage is preserved even when the stress test result is more conclusive. In the limit, when the stress test result is useless in predicting profitability ($\rho = 0$), information production is flat and does not depend on $\mu_i$.

The following proposition characterizes the optimal stress test when $\rho > 0$.

**Proposition 4** (Optimal Stress Test with Correlated Information)**.** *When the stress test results $m_i$ are weakly informative about $\theta$, i.e. $\rho \in (0, 1/4)$, there exist parameter values such that the optimal stress test $\mathcal{P}$ features*

- *a fail grade $m_1$ if $s \in [0, s_1)$ where $s_1 < \widehat{s}$,*
- *a pass grade $m_2$ if $s \in [s_1, s_2)$ where $2\widehat{s} - s_1 < s_2 < \widehat{s}_b(\rho)$,*
- *fully granular grades $s$ for $s \in [s_2, \widehat{s}_b(\rho)]$,*
- *a top grade $m_n$ if $s \in [\widehat{s}_b(\rho), 1]$.*

*Proof.* See the Appendix. □

The salient features of the optimal stress test in our baseline model ($\rho = 1/4$) remains optimal even when the test results are only weakly correlated with market information. In particular, the supervisor distorts the stress test towards leniency and introduce coarseness for intermediate resilience levels to stimulate information production. The key difference is that the stress test now presents a new coarse top grade $m_n$ for very high resilience levels which leads to continuation following any revealed market information. In this region price signals can be ignored since the test result is quite conclusive and price signals only weakly informative about asset quality. Hence, the supervisor's intervention decision is based exclusively on the stress test result. The highest resilience levels can thus be pooled in a unique grade as they all induce the supervisor to continue. Note that the region where the supervisor ignores market information grows larger when correlation decreases (since $\widehat{s}_b(\rho)$ is increasing), and thus also the size of the top grade $m_n$ increases.

# 7 Conclusion

Banking supervisors need to design stress tests in a way that best informs necessary interventions. Realistically, stress tests by themselves do not fully reveal in each instance whether an intervention is optimal. It is therefore important for supervisors to nurture other sources of information, such as those contained in the bank's share prices. This paper proposes and analyses an information design problem, taking into account the stress test's twofold objectives in providing direct information and in incentivizing the production of market information. It shows that making a stress test somewhat lenient can encourage the production of market information. It also shows that market information is most useful to the supervisor for banks whose stress resilience is at intermediate levels and the stress test can encourage information production for those banks by providing only a coarse message.

Although our model is set up to address the design of bank stress tests, we believe the underlying information design problem is pertinent in other contexts. For example, a credit rating agency needs to decide on a rating system, keeping in mind that this may have an impact on the information that speculators subsequently produce about the issuing firm. Similarly, there is a degree of freedom in setting up accounting rules so that a firm's financial health can appear better or worse (e.g., marking to market versus historical value rules, loan-loss accounting rules etc.). Little is understood about how such rules interact with other sources of information, in particular that contained in stock prices. Our paper proposes a tractable model that can be used in future research to address these questions.

## References


Banco de Portugal, (2011), EU-wide Stress Test: Main Results for the Portuguese Banks, *Press Release*, 15 July 2011.

Bhattacharya, Sudipto, (1982), Aspects of Monetary and Banking Theory and Moral Hazard, *Journal of Finance*, vol. 37(2), p. 371-384.

Bhattacharya, Sudipto, and Nyborg, Kjell (2013), Bank Bailout Menus, *Review of Corporate Finance Studies*, vol. 2(1), p. 29-61.

Bergemann, Dirk and Morris, Stephen, (2019), Information Design: A Unified Perspective, *Journal of Economic Literature*, 57, issue 1, p. 44-95.

Simon, Carl P. and Blume, Lawrence, (1994), Mathematics for economists, Norton, New York.

Board of Governors of the Federal Reserve System, (2022), 2022 Stress Test Scenarios.

Bond, Philip, Edmans, Alex and Goldstein, Itay, (2012), The Real Effects of Financial Markets, *Annual Review of Financial Economics*, 4, issue 1, p. 339-360.

Bond, Philip and Goldstein, Itay, (2015), Government Intervention and Information Aggregation by Prices, *Journal of Finance*, 70, issue 6, p. 2777-2812.

Bond, Philip, Goldstein, Itay and Prescott, Edward, (2010), Market-Based Corrective Actions, *Review of Financial Studies*, 23, issue 2, p. 781-820.

Bouvard, Matthieu, Chaigneau, Pierre and de Motta, Adolfo, (2015), Transparency in the Financial System: Rollover Risk and Crises, *Journal of Finance*, 70, issue 4, p. 1805-1837.

Calzolari, Giacomo and Lóránth, Gyöngyi, (2011), Regulation of multinational banks: A theoretical inquiry, *Journal of Financial Intermediation*, 20, issue 2, p. 178-198.

Calzolari, Giacomo, Colliard, Jean-Edouard and Lóránth, Gyöngyi, (2019), Multinational Banks and Supranational Supervision, *Review of Financial Studies*, 32, issue 8, p. 2997-3035.

Carletti, Elena, Dell'Ariccia, Giovanni and Marquez, Robert, (2021), Supervisory incentives in a banking union. *Management Science*, 67(1), pp.455-470.

Chan, Yuk-Shee, Greenbaum, Stuart I and Thakor, Anjan, (1992), Is Fairly Priced Deposit Insurance Possible?, *Journal of Finance*, 47, issue 1, p. 227-45.

Colliard, Jean-Edouard, (2019), Strategic Selection of Risk Models and Bank Capital Regulation, *Management Science*, 67, issue 6, p. 2591-2606.

Crémer, Jacques, (1995), Arms Length Relationships, *Quarterly Journal of Economics*, 140, p. 275-295.

Davis, Philip J. and Rabinowitz, Philip, (1984), Methods of Numerical Integration, Computer Science and Applied Mathematics, 2nd Edition, Academic Press, New York.

Davis, Jesse and Gondhi, Naveen, (forthcoming), Learning in Financial Markets: Implications for Debt Equity Conflicts, *Review of Financial Studies.*

Dow, James, Goldstein, Itay and Guembel, Alexander, (2017), Incentives for Information Production in Markets Where Prices Affect Real Investment, *Journal of the European Economic Association*, 15, issue 4, p. 877–909.

Fecht, Falko and Inderst, Roman and Pfeil, Sebastian, (2022), A Theory of the Boundaries of Banks with Implications for Financial Integration and Regulation, *CEPR Discussion Paper*, no. DP16880.

Flannery, Mark J. and Bliss, Robert R., (2019), Market Discipline in Regulation: Pre- and Post-Crisis , *Oxford Handbook of Banking*, ch. 23, p. 736–775.

Freixas, Xavier and Rochet, Jean-Charles, (1998), Fair pricing of deposit insurance. Is it possible? Yes. Is it desirable? No, *Research in Economics*, 52, issue 3, p. 217-232.

Giammarino, Ronald M., Lewis, Tracy and Sappington, David, (1993), An Incentive Approach to Banking Regulation, *Journal of Finance*, 48, issue 4, p. 1523-42.

Goldstein, Itay, (forthcoming), Information in Financial Markets and Its Real Effects, *Review of Finance.*

Goldstein, Itay and Huang, Chong, (2020), Credit Rating Inflation and Firms' Investments, *Journal of Finance*, 75, issue 6, p. 2929-2972.

Goldstein, Itay and Leitner, Yaron, (2018), Stress Tests and Information Disclosure, *Journal of Economic Theory*, 177, p. 34-69.

Goldstein, Itay and Sapra, Haresh, (2014), Should Banks' Stress Test Results be Disclosed? An Analysis of the Costs and Benefits, *Foundations and Trends in Finance*, 8, issue 1, p. 1-54.

Goldstein, Itay and Yang, Liyan, (2019), Good Disclosure, Bad Disclosure, *Journal of Financial Economics*, vol. 131, p. 118-138.

Goldstein, Itay and Yang, Liyan, (2017), Information Disclosure in Financial Markets, *Annual Review of Financial Economics*, vol. 9, p. 101-125.

Gorton, Gary and Huang, Lixin, (2004), Liquidity, Efficiency, and Bank Bailouts, *American Economic Review*, 94, issue 3, p. 455-483.

Hellmann, Thomas, Murdock, Kevin and Stiglitz, Joseph, (2000), Liberalization, Moral Hazard in Banking, and Prudential Regulation: Are Capital Requirements Enough?, *American Economic Review*, 90(1), p. 147-165.

Heitz, Amanda and Wheeler, Barrett, (2022), Bank Stress Test Disclosures, Private Information Production, and Price Informativeness, Working Paper, Available at SSRN: https://ssrn.com/abstract=3850954.

Kamenica, Emir and Gentzkow, Matthew, (2011), Bayesian Persuasion, *American Economic Review*, 101, issue 6, p. 2590-2615.

Inostroza, Nicolas (2019), Persuading Multiple Audiences: An Information Design Approach to Banking Regulation, Working Paper, Available at SSRN: https://ssrn.com/abstract=3450981.

Inostroza, Nicolas and Pavan, Alessandro (2021), Persuasion in Global Games with Application to Stress Testing, *CEPR Discussion Paper*, no. DP16484.

Kyle, Albert, (1985), Continuous Auctions and Insider Trading, *Econometrica*, 53, issue 6, p. 1315-35.

Leitner, Yaron and Bilge, Yilmaz, (2019), Regulating a Model, *Journal of Financial Economics*, 131, p.251-268.

Leitner, Yaron and Williams, Basil, (forthcoming), Model secrecy and stress tests, *Journal of Finance*.

Lenkey, Stephen and Song, Fenghua, (2017), Feedback, Investment, and Social Value of Financial Expertise, Working Paper, Available at SSRN: https://ssrn.com/abstract=3089981.

Morrison, Alan, D., and White, Lucy, (2005), Crises and Capital Requirements in Banking, *American Economic Review*, 95 (5), p. 1548-1572.

Orlov, Dmitry, Zryumov, Pavel and Skrzypacz, Andrzej, (2021), Design of Macro-Prudential Stress Tests, Stanford University Graduate School of Business Research Paper No. 17-41, Available at SSRN: https://ssrn.com/abstract=2977016.

Paul, Jonathan, (1992), On the Efficiency of Stock-Based Compensation, *Review of Financial Studies*, 5 (3), p. 471-502.

Philipon, Thomas and Schnabl, Philip, (2013), Efficient Recapitalization, *Journal of Finance*, 68 (1), p. 1-42.

Piccolo, Alessio and Shapiro, Joel, (2022), Credit Ratings and Market Information, *Review of Financial Studies*, 35 (10), p. 4425–4473.

Rochet, Jean-Charles, (1992), Capital requirements and the behaviour of commercial banks, *European Economic Review*, 36, issue 5, p. 1137-1170.

Shapiro, Joel and Zeng, Jing, (2020), Stress testing and bank lending, Working Paper, Available at SSRN: https://ssrn.com/abstract=3432291 .

Shen, Lin, (2021), Capital Flows in the Financial System and Supply of Credit, Working Paper, Available at SSRN: https://ssrn.com/abstract=3847983.

Siemroth, Christoph, (2019), The Informational Content of Prices When Policy Makers React to Financial Markets, *Journal of Economic Theory*, 179, p. 240-274.

Szydlowski, Martin (2020), Optimal financing and disclosure, *Management Science*, 67(1), p. 436-454.

Terovitis, Spyros, (2020), Information disclosure and the feedback effect in capital markets. *Journal of Financial Intermediation*, 49, article n. 100897.

Williams, Basil (2015), Stress Tests and Bank Portfolio Choice, Working Paper.

## Appendix

**Proof of Lemma 1**. Consider the trading game at date 2. The market maker observes anonymous orders and we denote the actual order as a pair, $X = (x_n, x_s)$ in which $x_n$ is the size of the order submitted by the liquidity trader and $x_s$ is the size of the order by the speculator. The order observed by the market maker $X$ can be thought of as a random reshuffling so that $X = (x_n, x_s)$ or $X = (x_s, x_n)$ with equal probability.

Consider the following speculator's equilibrium trading strategy:

$$x_s(z) = \begin{cases} 1 & \text{if } z = h \\ 0 & \text{if } z = \varnothing \\ -1 & \text{if } z = l. \end{cases}$$

First, we determine the price $p(X)$ chosen by the market maker as a function of the order $X$. When $X = (1, 1)$ the market maker infers that the informed speculator submitted a buy order and hence must have received a private signal $z = h$. It follows that the supervisor will allow the bank to continue ($a = 1$) at date 3, since $\mathcal{I} = \{\omega = h\}$. The market maker sets a price that reflects the speculator's private information and the supervisor's intervention decision, i.e. $p(1, 1) = E_h^1$. Similarly, when $X = (-1, -1)$ the low state is revealed and the market maker sets $p(-1, -1) = E_l^0 = 0$. When $X \in \{(1, -1), (-1, 1)\}$ the order flow allows no inference over the speculator's private information and the market maker's posterior belief therefore remains equal to the prior. When $X \in, \{(1, 0), (-1, 0)\}$ the market maker understands that the speculator received an uninformative signal $z = \varnothing$ and abstained from trading. Again, the market maker does not update from the prior. It follows that, for $X \in \{(1, -1), (-1, 1), (1, 0), (-1, 0)\} \triangleq X_\varnothing$, i) at date 3, the supervisor will take her intervention decision according to the outcome of the stress test ($\mathcal{I} = \{m_i\}$); and ii) the market maker sets a price $p(X) = \mu_i E_h^a + (1 - \mu_i) E_l^a$ with $a = 1$ if $\mu_i \geq \widehat{s}$ and $a = 0$ if $\mu_i < \widehat{s}$. Thus, the price schedule is

$$p(X) = \begin{cases} E_h^1 & \text{if } X = (1, 1) \\ \mu_i E_h^1 + (1 - \mu_i) E_l^1 & \text{if } X \in X_\varnothing \text{ and } \mu_i \geq \widehat{s} \\ \mu_i E_h^0 + (1 - \mu_i) E_l^0 = 0 & \text{if } X \in X_\varnothing \text{ and } \mu_i < \widehat{s} \\ E_l^0 = 0 & \text{if } X = (-1, -1). \end{cases}$$

Next, we compute the speculator's profits from the proposed trading strategy. If $z = h$, the speculator submits a buy order ($x_s = 1$). With probability 1/2 the liquidity trader trades in the same direction ($x_n = 1$ and $X = (1, 1)$) and the speculator's private information is revealed to the marker maker who then sets a price equal to $E_h^1$. The speculator's trading profits are nil, $E_h^1 - p(1,1) = 0$. With probability 1/2 the liquidity trader trades in the opposite direction ($x_n = -1$ and $X = (-1,1)$), the speculator retains his private information and makes profits equal to $E_h^1 - p(-1,1) = \Delta E(1-\mu_i)$ if $\mu_i \geq \widehat{s}$ and equal to 0 otherwise. It follows that, given $z = h$ the speculator's expected trading profits are $\frac{1}{2}\Delta E(1-\mu_i)$. If $z = l$, the speculator submits a sell order ($x_s = -1$) and, by the same reasoning trading profits are $\frac{1}{2}\Delta E\mu_i$ if $\mu_i \geq \widehat{s}$ and equal to 0 otherwise. If $z = \varnothing$ the speculator abstains from trading and makes profits equal to 0. In summary, the expected trading profits are

$$\pi(z,\mu_i) = \begin{cases} = \begin{cases} \frac{1}{2}\Delta E(1-\mu_i) & \text{if } z = h \\ 0 & \text{if } z = \varnothing \\ \frac{1}{2}\Delta E\mu_i & \text{if } z = l \end{cases} & \text{if } \mu_i \geq \widehat{s} \\ = 0 & \text{if } \mu_i < \widehat{s}. \end{cases}$$

Finally, we show that the proposed trading strategy is indeed optimal. Consider trading after a test inducing a belief $\mu_i \geq \widehat{s}$. Given $z = h$ it is optimal to buy: abstaining from trading yields profits equal to $0 < \frac{1}{2}\Delta E(1-\mu_i)$ and selling yields profits $-\frac{1}{2}\Delta E(1-\mu_i) < 0$. Given $z = l$ it is optimal to sell: abstaining from trading yields profits equal to $0 < \frac{1}{2}\Delta E\mu_i$. If the speculator buys instead, order flow will be either $X = (1,1)$ or $X = (-1,1)$. In either case the bank continues without intervention and expected trading profits are $\frac{1}{2}(E_l^1 - E_h^1) + \frac{1}{2}(E_l^1 - \mu_i E_h^1 - (1-\mu_i)E_l^1)$ which is negative. If $z = \varnothing$ and speculator buys, the bank will continue without intervention and expected equity value from the speculator's perspective is $\mu_i E_h^1 + (1-\mu_i)E_l^1$. The expected price he pays is $\frac{1}{2}(E_h^1 + \mu_i E_h^1 + (1-\mu_i)E_l^1)$ which is strictly greater, implying that the speculator loses in expectation. If the speculator sells instead, order flow can be $X = (-1,1)$, in which case the bank continues and has equity value $\mu_i E_h^1 + (1-\mu_i)E_l^1$. Since this is equal to the price paid in this state, profits are zero. Instead, order flow may be $X = (-1,-1)$. The price will now be zero, the bank will be subject to intervention and its equity value zero, yielding again zero trading profits.

Consider trading after a test inducing a belief $\mu_i < \widehat{s}$. If the speculator sells, he always gets a price of zero, and there will always be an intervention so that equity value is also zero. Hence, selling yields zero profits. If the speculator buys, with probability $\frac{1}{2}$ order flow will be $X = (-1,1)$ in which case the price is zero, there will be an intervention and equity value will also be zero. With equal probability, order flow will be $X = (1,1)$, and the price equal to $E_h^1$. If the speculator deviated to purchasing information and learned $\omega = h$, he makes zero trading profits. Hence, the deviation generated a loss, net of the information acquisition cost. If the speculator deviated to buying without a positive signal, the expected value of equity is below $E_h^1$ so the speculator makes a loss.

Taking expectations over $z$, we get the expected equilibrium trading profits after the realization of the public signal $m_i$:

$$\begin{aligned}\pi(\mu_i) =& \Pr(\omega = h|m_i)\Pr(z = h|\omega = h)\left(\frac{1}{2}\Delta E(1-\mu_i)\right) + \\ &+ \Pr(\omega = l|m_i)\Pr(z = l|\omega = l)\left(\frac{1}{2}\Delta E\mu_i\right) \\ =& \mu_i\sigma\left(\frac{1}{2}\Delta E(1-\mu_i)\right) + (1-\mu_i)\sigma\left(\frac{1}{2}\Delta E\mu_i\right) \\ =& \sigma\Delta E\mu_i(1-\mu_i)\end{aligned}$$

for $\mu_i \geq \widehat{s}$ and $\pi(\mu_i) = 0$ otherwise. □

**Proof of Lemma 2**. Consider all the outcomes $m_i$ that induce beliefs $\mu_i < \widehat{s}$. All these $m_i$ result in no information production by the speculator ($\sigma(\mu_i) = 0$) and leads the supervisor to intervene ($a(\mu_i) = 0$). Hence, we can pool all these potential signals in a unique signal $m_1$ for all $s \in [0, s_1)$ with $\mu_1 < \widehat{s}$. The corresponding expected value for the supervisor generated by the outcome $m_1$ is

$$\begin{aligned}g(s_1) &= \frac{1}{2}\Big(\Pr(m_1|\omega = h)V_h^0 + \Pr(m_1|\omega = l)V_l^0\Big) \\ &= \frac{1}{2}\Big(s_1^2V_h^0 + (2s_1 - s_1^2)V_l^0\Big).\end{aligned}$$

Now, consider all the outcomes $m_i$ for $i \in \{2, 3, \ldots, n\}$. These outcomes induce posterior beliefs $\mu_i \geq \widehat{s}$ (otherwise we could have pooled the signal $m_i$ with $m_1$). If, at date 2, the order flow is uninformative the supervisor allows the bank to continue ($a(\mu_i) = 1$) at date 3. However, since the outcome induces a positive level $\sigma(\mu_i)$ of information production by the speculator, if the order flow reveals that $\omega = l$, the supervisor revises her intervention decision at date 3 by choosing $a = 0$. If the state is $\omega = l$, order flow reveals it with probability $\frac{1}{2}\sigma(\mu_i)$. The corresponding expected bank value generated by some outcome $m_i$ is

$$\begin{aligned}f(s_{i-1}, s_i) =& \frac{1}{2}\left(\Pr(m_i|\omega = h)V_h^1 + \Pr(m_i|\omega = l)\left(V_l^1 + \frac{1}{2}\sigma(\mu_i)\Delta V_l\right)\right) \\ =& \frac{1}{2}\left((s_i^2 - s_{i-1}^2)V_h^1 + \big(2(s_i - s_{i-1}) - (s_i^2 - s_{i-1}^2)\big)\left(V_l^1 + \frac{1}{2}\sigma(\mu_i)\Delta V_l\right)\right).\end{aligned}$$

The ex-ante expected value of the bank for a given stress test $\mathcal{P}$ can be written as

$$V(\mathcal{P}) = g(s_1) + \sum_{i=2}^{n} f(s_{i-1}, s_i). \tag{27}$$

Note that, the second term in (27) is a telescoping sum where

$$\sum_{i=2}^{n}(s_i^2 - s_{i-1}^2) = (1 - s_1^2),$$

$$\sum_{i=2}^{n}\big(2(s_i - s_{i-1}) - (s_i^2 - s_{i-1}^2)\big) = 2(1 - s_1) - (1 - s_1^2).$$

It follows that the objective function can be written as

$$\begin{aligned} V(\mathcal{P}) =& \frac{1}{2}\Big(s_1^2 V_h^0 + (2s_1 - s_1^2)V_l^0 + (1 - s_1^2)V_h^1 + \big(2(1 - s_1) - (1 - s_1^2)\big)V_l^1\Big) + \\ &+ \frac{1}{2}\sum_{i=2}^{n}\big(2(s_i - s_{i-1}) - (s_i^2 - s_{i-1}^2)\big)\frac{1}{2}\sigma(\mu_i)\Delta V_l \\ =& \chi(s_1) + \frac{1}{2}\sum_{i=2}^{n}(s_i - s_{i-1})\left(1 - \frac{s_{i-1} + s_i}{2}\right)\sigma(\mu_i)\Delta V_l, \end{aligned}$$

where

$$\begin{aligned} \chi(s_1) \triangleq& \frac{1}{2}\Big(s_1^2 V_h^0 + (2s_1 - s_1^2)V_l^0 + (1 - s_1^2)V_h^1 + \big(2(1 - s_1) - (1 - s_1^2)\big)V_l^1\Big) \\ =& \frac{1}{2}\Big(F_h(s_1)V_h^0 + F_l(s_1)V_l^0 + \big(1 - F_h(s_1)\big)V_h^1 + \big(1 - F_l(s_1)\big)V_l^1\Big). \end{aligned}$$

By applying the definitions in equations (3) and (7) we obtain the objective function in (10). □

**Proof of Lemma 3**. When $\frac{1}{\tau}\Delta E = 0$ the objective function in problem (9) is $V(\mathcal{P}) = \chi(s_1)$. The first order condition is:

$$\begin{aligned} \chi'(s_1) =& \frac{1}{2}\Big((2 - 2s_1)V_l^0 + (-2 + 2s_1)V_l^1 + 2s_1 V_h^0 - 2s_1 V_h^1\Big), \\ =& \Delta V_l - s_1(\Delta V_l + \Delta V_h) = 0. \end{aligned}$$

Solving for $s_1$ we get $s_1 = \Delta V_l/(\Delta V_l + \Delta V_h) = \widehat{s}$. □

**Proof of Proposition 1**. We first introduce some notation. Let $\mathcal{X} = \{x_0, x_1, \ldots, x_{n-1}, x_n\}$ be a partition of the interval $[a, b] \subset \mathbb{R}$ such that $a = x_0 < x_1 < x_2 \cdots < x_{n-1} < x_n = b$. Let $\Pi[a, b]$ be the set of all possible partitions $\mathcal{X}$ over the interval $[a, b]$. Let $\overline{\mathcal{X}}$ be the finest partition in $\Pi[a, b]$, i.e. such that $n \to \infty$; and let $\underline{\mathcal{X}}$ be the coarsest partition in $\Pi[a, b]$, i.e. such that $n = 1$.

Lastly, for some function $f : [a,b] \to \mathbb{R}$, we define

$$R(f, \mathcal{X}) \triangleq \sum_{i=1}^{n} (x_i - x_{i-1}) f\left(\frac{x_{i-1} + x_i}{2}\right)$$

as the midpoint Riemann sum of $f$ with respect to the partition $\mathcal{X}$. In what follows, we will use the following properties of the midpoint Riemann sum (see, e.g., Davis and Rabinowitz (1984) p. 54):

- if $f$ is convex over $[a,b]$ then $R(f, \overline{\mathcal{X}}) \geq R(f, \mathcal{X})$ for all $\mathcal{X}$ in $\Pi(a,b)$;
- if $f$ is concave over $[a,b]$ then $R(f, \underline{\mathcal{X}}) \geq R(f, \mathcal{X})$ for all $\mathcal{X}$ in $\Pi(a,b)$.

We proceed in four steps. Step 1 derives a key property of the optimal stress test: motives towards leniency, i.e. $s_1 < \widehat{s}$. Step 2 establishes the general structure of the stress test. Step 3 simplifies the objective function in problem (9) and writes it as a function of two thresholds $(s_1, s_2)$. Finally, Step 4 determines the optimal thresholds.

*Step 1* (Motives towards Leniency). The expression for $\Sigma(\mathcal{P})$ in equation (10) is equivalent to

$$\Sigma(\mathcal{P}) = \frac{1}{2\tau} \Delta E \Delta V_l \sum_{i=2}^{n} \frac{1}{2} (s_i^2 - s_{i-1}^2)(1 - \mu_i)^2.$$

Thus, the first order condition for the optimality of $s_1$ in problem (9) is

$$\frac{\partial V}{\partial s_1} = \Delta V_l - s_1(\Delta V_l + \Delta V_h) + \frac{1}{2\tau} \Delta E \Delta V_l \left(-s_1 (1 - \mu_2)^2 - \frac{1}{2}\left(s_2^2 - s_1^2\right)(1 - \mu_2)\right) = 0.$$

Isolating the $s_1$ in the second addend we get

$$s_1 = \frac{\Delta V_l}{\Delta V_l + \Delta V_h} - \frac{1}{2\tau} \Delta E \frac{\Delta V_l}{\Delta V_l + \Delta V_h} \left(s_1 (1 - \mu_2)^2 + \frac{1}{2}(s_2^2 - s_1^2)(1 - \mu_2)\right). \tag{28}$$

Since, by construction we have $0 < s_1 < s_2 < 1$ the term in brackets on the right-hand side is positive and for $\frac{1}{\tau} \Delta E > 0$ we have $s_1 < \Delta V_l / (\Delta V_l + \Delta V_h) = \widehat{s}$.

*Step 2.* (General Structure) Let $s_1$ solve the first order condition (28). Consider a subset $[s_1, 1] \subseteq \mathcal{S}$ and define the function $\widehat{\Sigma}(s) : [s_1, 1] \to \mathbb{R}$ as

$$\widehat{\Sigma}(s) = \frac{1}{2\tau} \Delta E \Delta V_l s (1 - s)^2.$$

Consider a partition $\mathcal{P}_1 \triangleq \{s_1, s_2, s_3, \ldots, s_{n-2}, s_{n-1}, 1\}$ of $[s_1, 1]$ and note that the stress test design problem (9) is equivalent to

$$\begin{aligned} \max_{\mathcal{P}_1} \; & R(\widehat{\Sigma}, \mathcal{P}_1) \\ \text{s.t.} \; & \frac{s_1 + s_2}{2} \geq \widehat{s} \\ & \frac{0 + s_1}{2} < \widehat{s}. \end{aligned} \tag{29}$$

Note also that the second constraint in (29) is always satisfied since from Step 1 we have $s_1 < \widehat{s}$. Moreover, the second derivative of $\widehat{\Sigma}(s)$ is

$$\widehat{\Sigma}''(s) = \frac{1}{2\tau} \Delta E \Delta V_l \left(6s - 4\right).$$

We distinguish 2 cases:

- Case 1: $s_1 < 2/3$: $\widehat{\Sigma}(s)$ is concave over $[s_1, 2/3]$ and convex over $[2/3, 1]$;
- Case 2: $s_1 \geq 2/3$: $\widehat{\Sigma}(s)$ is convex over its whole domain $[s_1, 1]$.

First, consider Case 1. Take a partition $\mathcal{P}_1' \triangleq \{s_1, s_2, s_3, \ldots, s_{j-1}, s_j, 2/3\}$ of $[s_1, 2/3]$ and a partition $\mathcal{P}_1'' \triangleq \{2/3, s_l, s_{l+1} \ldots, s_{n-2}, s_{n-1}, 1\}$ of $[2/3, 1]$ such that $\mathcal{P}_1' \cup \mathcal{P}_1'' = \mathcal{P}_1$ and $R(\widehat{\Sigma}, \mathcal{P}_1) = R(\widehat{\Sigma}, \mathcal{P}_1') + R(\widehat{\Sigma}, \mathcal{P}_1'')$. Since $\widehat{\Sigma}(s)$ is concave over $[s_1, 2/3]$ the coarsest partition $\underline{\mathcal{P}_1'}$ maximizes $R(\widehat{\Sigma}, \mathcal{P}_1')$. Conversely, since $\widehat{\Sigma}$ is convex over $[2/3, 1]$ the finest partition $\overline{\mathcal{P}_1''}$ maximizes $R(\widehat{\Sigma}, \mathcal{P}_1'')$. Thus, the partition $\mathcal{P}_1$ that solves problem (29) exhibits the same structure: 1 coarse subinterval $[s_1, s_2]$, for some optimally chosen $s_2$, and a set of infinitely fine subintervals over $[s_2, 1]$.
Next, consider Case 2. Since $\widehat{\Sigma}(s)$ is convex over its entire domain, the finest partition $\overline{\mathcal{P}_1}$ maximizes $R(\widehat{\Sigma}, \mathcal{P}_1)$. However, by Step 1 $s_1 < \widehat{s}$, so in order to have information acquisition following $m_2$, i.e. $(s_1 + s_2)/2 \geq \widehat{s}$ we need $s_2 > \widehat{s}$ otherwise $m_2$ could have been pooled with $m_1$ in Lemma 2. Thus, the partition $\mathcal{P}_1$ that solves problem (29) exhibits 1 coarse subinterval $[s_1, s_2]$, and a set of infinitely fine subintervals over $[s_2, 1]$.

It follows that the optimal $\mathcal{P}$, exhibits 2 coarse signals $m_1$ if $s \in [0, s_1)$, $m_2$ if $s \in [s_1, s_2)$ and fully granular grades $s$ for $s \in [s_2, 1]$.

*Step 3.* (Simplifying the Objective Function) Step 2 allows us to write the objective $V(\mathcal{P})$ as a function of the thresholds $(s_1, s_2)$ only. Note that if the stress test $\mathcal{P}$ has fully granular grades for $s \in [s_2, 1]$ we have $\nu_i = ds$ and $\mu_i = s$ for $i \in \{3, \ldots, n\}$ where $n \to \infty$. It follows that the objective function reduces to

$$V(s_1, s_2) = \chi(s_1) + \frac{1}{2\tau}\Delta E \Delta V_l \left( \nu_2 \mu_2 (1-\mu_2)^2 + \int_{s_2}^{1} s(1-s)^2\, ds \right).$$

Adding and subtracting $\frac{1}{2\tau}\Delta E \Delta V_l \int_{s_1}^{s_2} s(1-s)^2\, ds$ on the right-hand side we get

$$V(s_1, s_2) = \chi(s_1) + \frac{1}{2\tau}\Delta E \Delta V_l \left( \nu_2 \mu_2 (1-\mu_2)^2 - \int_{s_1}^{s_2} s(1-s)^2 ds + \int_{s_1}^{1} s(1-s)^2\, ds \right).$$

Let the first two addends in the bracket on the right hand side be denoted by

$$A(s_1, s_2) \triangleq (s_2 - s_1)\mu_2(1-\mu_2)^2 - \int_{s_1}^{s_2} s(1-s)^2 ds.$$

Notice that

$$(s_2 - s_1)\mu_2(1-\mu_2)^2 = \frac{1}{12}(s_2 - s_1)(12\mu_2 + 12\mu_2^3 - 24\mu_2^2),$$

and

$$\begin{aligned}\int_{s_1}^{s_2} s(1-s)^2 ds &= \frac{1}{2}(s_2^2 - s_1^2) + \frac{1}{4}(s_2^4 - s_1^4) - \frac{2}{3}(s_2^3 - s_1^3), \\ &= \frac{1}{12}(s_2 - s_1)\Big(12\mu_2 + 6\mu_2(s_1^2 + s_2^2) - 8(s_1^2 + s_2^2 + s_1 s_2)\Big).\end{aligned}$$

It follows that

$$\begin{aligned}A(s_1, s_2) &= \frac{1}{12}(s_2 - s_1)\Big(12\mu_2^3 - 24\mu_2^2 - 6\mu_2(s_1^2 + s_2^2) + 8(s_1^2 + s_2^2 + s_1 s_2)\Big), \\ &= \frac{1}{12}(s_2 - s_1)\Big(3\mu_2(s_1 + s_2)^2 - 6(s_1 + s_2)^2 - 6\mu_2(s_1^2 + s_2^2) + 8(s_1^2 + s_2^2 + s_1 s_2)\Big).\end{aligned}$$

Exploiting the following identities:

$$\begin{aligned}(s_1 + s_2)^2 &= (s_2 - s_1)^2 + 4 s_1 s_2, \\ (s_1^2 + s_2^2) &= (s_2 - s_1)^2 + 2 s_1 s_2, \\ (s_1^2 + s_2^2 + s_1 s_2) &= (s_2 - s_1)^2 + 3 s_1 s_2,\end{aligned}$$

we get

$$A(s_1, s_2) = \frac{1}{12}(s_2 - s_1)^3(2 - 3\mu_2)$$

so that the objective function becomes

$$V(s_1, s_2) = \chi(s_1) + \frac{1}{2\tau}\Delta E \Delta V_l \left( \frac{1}{12}\nu_2^3(2 - 3\mu_2) + \int_{s_1}^{1} s(1-s)^2 ds \right).$$

*Step 4* (Optimal Thresholds). By Step 3, the supervisor solves the following stress test design problem

$$\begin{aligned} &\max_{s_1,s_2} V(s_1,s_2) \\ &\text{s.t. } \frac{s_1+s_2}{2} \geq \widehat{s}. \end{aligned} \tag{30}$$

Let $\lambda$ be the Lagrange multiplier associated to the first constraint. The Lagrangian of problem (30) is

$$\mathcal{L}(s_1,s_2,\lambda) = V(s_1,s_2) + \lambda\left(\frac{s_1+s_2}{2} - \widehat{s}\right).$$

The Karush-Kuhn-Tucker conditions (see, e.g., Simon and Blume (1994)) are

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial s_1} &= \Delta V_l - s_1(\Delta V_l + \Delta V_h) + \\ &\quad - \frac{1}{2\tau}\Delta E \Delta V_l \left(\frac{3}{12}(\nu_2)^2(2-3\mu_2) + \frac{3}{24}(\nu_2)^3 + s_1(1-s_1)^2\right) + \frac{1}{2}\lambda = 0, \\ \frac{\partial \mathcal{L}}{\partial s_2} &= \frac{1}{2\tau}\Delta E \Delta V_l \left(\frac{3}{12}(\nu_2)^2(2-3\mu_2) - \frac{3}{24}(\nu_2)^3\right) + \frac{1}{2}\lambda = 0, \\ \lambda\frac{\partial \mathcal{L}}{\partial \lambda} &= \lambda\left(\frac{s_1+s_2}{2} - \widehat{s}\right) = 0, \\ \frac{\partial \mathcal{L}}{\partial \lambda} &= \frac{s_1+s_2}{2} - \widehat{s} \geq 0, \\ \lambda &\geq 0. \end{aligned} \tag{31}$$

First, consider the case of a non-binding constraint, that is $\lambda = 0$. The second equation in (31) simplifies to $s_2 = 1 - \frac{1}{2}s_1$. Plugging $s_2$ in the first equation in (31), after simple algebra we get a system of equations that implicitly defines the optimal thresholds

$$\begin{aligned} 0 &= s_1 - \widehat{s} + \frac{1}{2\tau}\Delta E\, \widehat{s}\left(\frac{1}{4}\left(1-\frac{3}{2}s_1\right)^3 + s_1(1-s_1)^2\right) \triangleq k(s_1), \\ s_2 &= 1 - \frac{1}{2}s_1. \end{aligned} \tag{32}$$

Note that the constraint is non-binding as long as $\widehat{s} < 2/3$. In fact, when $\lambda = 0$ we have $s_2 = 1 - \frac{1}{2}s_1 > s_1$, that is $s_1 < 2/3$. Moreover, $(s_1+s_2)/2 = \big(s_1 + (1-\frac{1}{2}s_1)\big)/2 \geq \widehat{s}$ so that we have $s_1 \geq 4\widehat{s} - 2$. It follows that $2/3 > s_1 \geq 4\widehat{s} - 2$, or equivalently $\widehat{s} < 2/3$.
The $s_1$ that solves the first equation in (32) lies in the interval $(0, \widehat{s})$. In fact, note that $\lim_{s_1 \to 0} k(s_1) = -\widehat{s} + \frac{1}{2\tau}\Delta E \widehat{s}\frac{1}{4} < 0$ (since $\frac{1}{\tau}\Delta E < 4$ by assumption (4)) and $\lim_{s_1 \to \widehat{s}} k(s_1) = \frac{1}{2\tau}\Delta E \widehat{s}\left(\frac{1}{4}\left(1-\frac{3}{2}\widehat{s}\right)^3 + \widehat{s}(1-\widehat{s})^2\right) > 0$ (since $1 - \frac{3}{2}\widehat{s} > 0$). Since $k(s_1)$ is continuous and strictly increasing over the interval $(0, \widehat{s})$ it crosses the horizontal axis only once over the interval $(0, \widehat{s})$.

Moreover, $s_2 < 1$ for $s_1 > 0$, and $s_2 > s_1$ for $s_1 < 2/3$. In summary, when $\lambda = 0$, we have $0 < s_1 < \widehat{s} < s_2 < 1$.

Now, consider the case of a binding constraint, that is $\lambda > 0$ (i.e. $\widehat{s} \geq 2/3$). The complementary slackness condition implies $s_2 = 2\widehat{s} - 1$. Equating the left-hand sides of the first two equations in (31) we get rid of the Lagrange multiplier. Plugging $s_2$ in the resulting equation, after simple algebra we get a system of equations that implicitly defines the optimal thresholds

$$\begin{aligned} 0 &= s_1 - \widehat{s} + \frac{1}{2\tau}\Delta E\,\widehat{s}\left(2(2-3\widehat{s})(\widehat{s}-s_1)^2 + s_1(1-s_1)^2\right) \triangleq z(s_1), \\ s_2 &= 2\widehat{s} - s_1. \end{aligned} \tag{33}$$

The $s_1$ that solves the first equation in (33) lies in the interval $(0, \widehat{s})$. In fact, note that $\lim_{s_1 \to 0} z(s_1) = -\widehat{s} + \frac{1}{2\tau}\Delta E \widehat{s}\big(2(2-3\widehat{s})\widehat{s}^2\big) < 0$ (since $2-3\widehat{s} \leq 0$) and $\lim_{s_1 \to \widehat{s}} z(s_1) = \frac{1}{2\tau}\Delta E \widehat{s}\big(\widehat{s}(1-\widehat{s})^2\big) > 0$ (since $\frac{1}{2\tau}\Delta E > 0$ and $\widehat{s} > 0$). Since $z(s_1)$ is continuous and strictly increasing over the interval $(0, \widehat{s})$ it crosses the horizontal axis only once over the interval $(0, \widehat{s})$. Moreover, $s_2 = 2\widehat{s} - s_1 < 1$ since $s_1 < \widehat{s} < 1$, and $s_2 > s_1$ since $s_1 < \widehat{s}$. In summary, when $\lambda > 0$, we have $0 < s_1 < \widehat{s} < s_2 < 1$. □

**Proof of Proposition 2**. We replicate equation (20) here for ease of exposition:

$$\begin{aligned} \widehat{s} &= 1 - (1-\delta)\frac{\frac{p}{2}R + (1-p)r}{\frac{\varepsilon}{2}(2r - R)}, \\ \Delta E &= \left(1 - \frac{p}{2}\right) A_h - (1-p)(D - r) - \frac{\varepsilon}{2}(R - D). \end{aligned}$$

Given our parametric assumptions in Section 5, one can easily show that

$$\begin{aligned} \frac{\partial \widehat{s}}{\partial D} &= 0, & \frac{\partial \Delta E}{\partial D} &= -(1-p) + \tfrac{\varepsilon}{2} < 0, \\ \frac{\partial \widehat{s}}{\partial \varepsilon} &= 2(1-\delta)\frac{\frac{p}{2}R + (1-p)r}{\varepsilon^2(2r - R)} > 0, & \frac{\partial \Delta E}{\partial \varepsilon} &= -\tfrac{1}{2}(R - D) < 0, \\ \frac{\partial \widehat{s}}{\partial \delta} &= \frac{\frac{p}{2}R + (1-p)r}{\frac{\varepsilon}{2}(2r - R)} > 0, & \frac{\partial \Delta E}{\partial \delta} &= 0. \end{aligned}$$

First, consider the case where the constraint in problem (30) is non-binding. From Step 4 of Proposition 1 we know that $\widehat{s} < 2/3$ and that the first-order conditions are given in equation (32). We define

$$\begin{aligned} G(s_1) &\triangleq s_1(1-s_1)^2 + \frac{1}{4}\left(1 - \frac{3}{2}s_1\right)^3, \\ g(s_1) &\triangleq G'(s_1) = (1-s_1)(1-3s_1) - \frac{9}{8}\left(1 - \frac{3}{2}s_1\right)^2, \end{aligned}$$

so that the first-order condition for the optimality of $s_1$ can be written as

$$s_1 - \widehat{s} + \frac{1}{2\tau}\Delta E\,\widehat{s}\,G(s_1) = 0. \tag{34}$$

To perform comparative statics about the degree of leniency $s_1$, relying on the implicit function theorem, we compute the total differential of (34) with respect to the variable of interest $x \in \{\tau, D, \varepsilon, \delta\}$, we solve for $\partial s_1/\partial x$ and study the sign of this partial derivative. This procedure yields

$$\begin{aligned}
\frac{\partial s_1}{\partial \tau} &= \frac{\frac{1}{2\tau^2}\Delta E\widehat{s}G(s_1)}{1+\frac{1}{2\tau}\Delta E\widehat{s}g(s_1)} > 0,\\
\frac{\partial s_1}{\partial D} &= -\frac{\frac{\partial \Delta E}{\partial D}\frac{1}{2\tau}\widehat{s}\,G(s_1)}{1+\frac{1}{2\tau}\Delta E\widehat{s}g(s_1)} > 0,\\
\frac{\partial s_1}{\partial \varepsilon} &= \frac{\frac{\partial \widehat{s}}{\partial \varepsilon}\left(1-\frac{1}{2\tau}\Delta E G(s_1)\right) - \frac{\partial \Delta E}{\partial \varepsilon}\frac{1}{2\tau}\widehat{s}\,G(s_1)}{1+\frac{1}{2\tau}\Delta E\widehat{s}\,g(s_1)} > 0,\\
\frac{\partial s_1}{\partial \delta} &= \frac{\frac{\partial \widehat{s}}{\partial \delta}\left(1-\frac{1}{2\tau}\Delta E G(s_1)\right)}{1+\frac{1}{2\tau}\Delta E\widehat{s}g(s_1)} > 0.
\end{aligned}$$

The above inequalities follow from the following facts. Remember that by Proposition 1 we have $s_1 < \widehat{s}$, this implies that the $s_1$ that solves equation (34) is such that $G(s_1) > 0$. Moreover, $G(s_1) < 1/4$ for $s_1 > 0$ and $g(s_1) > -1/3$ for any $s_1 < 2/3$. Since $\frac{1}{\tau}\Delta E < 4$ we have that $1 - \frac{1}{2\tau}\Delta E G(s_1) > 0$ and $1 + \frac{1}{2\tau}\Delta E\widehat{s}g(s_1) > 0$.

Now, consider the case where the constraint in problem (30) is binding. From Step 4 of Proposition 1 we know that $\widehat{s} \geq 2/3$ and that the first-order conditions are given in equation (33). We define

$$\begin{aligned}
G(s_1, \widehat{s}\,) &\triangleq 2(2-3\widehat{s})(\widehat{s}-s_1)^2 + s_1(1-s_1)^2,\\
g(s_1, \widehat{s}) &\triangleq \frac{\partial G}{\partial s_1} = -4(2-3\widehat{s})(\widehat{s}-s_1) + (1-s_1)^2 - 2s_1(1-s_1),\\
h(s_1, \widehat{s}) &\triangleq \frac{\partial G}{\partial \widehat{s}} = -6(\widehat{s}-s_1)^2 + 4(2-3\widehat{s})(\widehat{s}-s_1),
\end{aligned}$$

so that the first-order condition for the optimality of $s_1$ can be written as

$$s_1 - \widehat{s} + \frac{1}{2\tau}\Delta E\,\widehat{s}\,G(s_1, \widehat{s}) = 0, \tag{35}$$

Relying again on the implicit function theorem we get

$$\frac{\partial s_1}{\partial \tau} = \frac{\frac{1}{2\tau^2}\Delta E \widehat{s} G(s_1,\widehat{s})}{1+\frac{1}{2\tau}\Delta E \widehat{s} g(s_1,\widehat{s})} > 0,$$

$$\frac{\partial s_1}{\partial D} = -\frac{\frac{\partial \Delta E}{\partial D}\frac{1}{2\tau}\widehat{s}\, G(s_1,\widehat{s})}{1+\frac{1}{2\tau}\Delta E \widehat{s} g(s_1,\widehat{s})} > 0,$$

$$\frac{\partial s_1}{\partial \varepsilon} = \frac{\frac{\partial \widehat{s}}{\partial \varepsilon}\left(1-\frac{1}{2\tau}\Delta E\Big(G(s_1,\widehat{s})+\widehat{s}\, h(s_1,\widehat{s})\Big)\right) - \frac{\partial \Delta E}{\partial \varepsilon}\frac{1}{2\tau}\widehat{s}\, G(s_1,\widehat{s})}{1+\frac{1}{2\tau}\Delta E \widehat{s}\, g(s_1,\widehat{s})} > 0,$$

$$\frac{\partial s_1}{\partial \delta} = \frac{\frac{\partial \widehat{s}}{\partial \delta}\left(1-\frac{1}{2\tau}\Delta E\Big(G(s_1,\widehat{s})+\widehat{s}\, h(s_1,\widehat{s})\Big)\right)}{1+\frac{1}{2\tau}\Delta E \widehat{s} g(s_1,\widehat{s})} > 0.$$

The above inequalities follow from the following facts. We have $G(s_1,\widehat{s}) > 0$. Moreover, $G(s_1,\widehat{s}) + \widehat{s}\, h(s_1,\widehat{s}) < 1/4$ and $g(s_1,\widehat{s}) > -1/3$ for $s_1 \in (0,\widehat{s})$ and $\widehat{s} \geq 2/3$. Since $\frac{1}{\tau}\Delta E < 4$ we have that $1 - \frac{1}{2\tau}\Delta E\big(G(s_1,\widehat{s}) + \widehat{s}\, h(s_1,\widehat{s})\big) > 0$ and $1 + \frac{1}{2\tau}\Delta E\widehat{s} g(s_1,\widehat{s}) > 0$. □

**Proof of Lemma 4**. Consider the trading game at date $t = 2$. Let $x_s^E(z,\mu_i)$ and $x_s^D(z,\mu_i)$ be the speculator's market orders in the equity and debt market, respectively. Similarly, let $x_n^E$ and $x_n^D$ be the liquidity trader's market orders. Let $X^E = (x_n^E, x_s^E)$ and $X^D = (x_n^D, x_s^D)$ be the aggregate anonymous order in the equity and debt market, respectively. Consider the following speculator's equilibrium trading strategy:

$$x_s^E(z,\mu_i) = \begin{cases} = \begin{cases} 1 & \text{if } z = h \\ 0 & \text{if } z = \varnothing \\ -1 & \text{if } z = l \end{cases} & \text{if } \mu_i \geq \widehat{s} \\ \quad 0 & \text{if } \mu_i < \widehat{s}, \end{cases}$$

$$x_s^D(z,\mu_i) = \begin{cases} \quad 0 & \text{if } \mu_i \geq \widehat{s} \\ = \begin{cases} 1 & \text{if } z = h \\ 0 & \text{if } z = \varnothing \\ -1 & \text{if } z = l \end{cases} & \text{if } \mu_i < \widehat{s}. \end{cases}$$

In other words, whenever the stress test result is positive, i.e. such that $\mu_i \geq \widehat{s}$, the speculator trades only equity: buying on positive private information ($z = h$), selling on negative information ($z = l$) and abstaining when uninformed ($z = \varnothing$). Instead, whenever the stress test result is negative, the speculator only trades debt, using a similar trading strategy.

Given the above trading strategy, we can compute the prices $p^E(X^E)$ and $p^D(X^D)$ chosen by the market maker as a function of the aggregate order $X^E$ and $X^D$ as we did in Lemma 1

$$p^E(X^E) = \begin{cases} E_h^1 & \text{if } X^E = (1,1) \\ \mu_i E_h^1 + (1-\mu_i)E_l^1 & \text{if } X^E \in X_\varnothing^E \text{ and } \mu_i \geq \widehat{s} \\ \mu_i E_h^0 + (1-\mu_i)E_l^0 = 0 & \text{if } X^E \in X_\varnothing^E \text{ and } \mu_i < \widehat{s} \\ E_l^0 = 0 & \text{if } X^E = (-1,-1), \end{cases}$$

$$p^D(X^D) = \begin{cases} D_h^1 & \text{if } X^D = (1,1) \\ \mu_i D_h^1 + (1-\mu_i)D_l^1 & \text{if } X^D \in X_\varnothing^D \text{ and } \mu_i \geq \widehat{s} \\ \mu_i D_h^0 + (1-\mu_i)D_l^0 & \text{if } X^D \in X_\varnothing^D \text{ and } \mu_i < \widehat{s} \\ D_l^0 & \text{if } X^D = (-1,-1), \end{cases}$$

where $X_\varnothing^E = X_\varnothing^D = \{(1,-1),(-1,1),(1,0),(-1,0)\}$. The speculator's equilibrium trading profits can be computed as in Lemma 1 and shown to be equal to:

$$\pi(z,\mu_i) = \begin{cases} = \begin{cases} \frac{1}{2}\Delta E(1-\mu_i) & \text{if } z = h \\ 0 & \text{if } z = \varnothing \\ \frac{1}{2}\Delta E\mu_i & \text{if } z = l \end{cases} & \text{if } \mu_i \geq \widehat{s}, \\ = \begin{cases} \frac{1}{2}\Delta D^0(1-\mu_i) & \text{if } z = h \\ 0 & \text{if } z = \varnothing \\ \frac{1}{2}\Delta D^0\mu_i & \text{if } z = l \end{cases} & \text{if } \mu_i < \widehat{s}. \end{cases}$$

We now show that the proposed trading strategy is indeed optimal. We already shown in Lemma 1 that trading a security in any other direction yields lower trading profits to the speculator. Here, we also need to show that trading another security (in any direction) yields lower profits. Consider trading after a test inducing $\mu_i \geq \widehat{s}$ (so that the supervisor continues when the order flow is uninformative, i.e. $X \in X_\varnothing^E$ or $X \in X_\varnothing^D$). Given $z = h$ it is optimal to buy equity: buying debt yields profits equal to $\frac{1}{2}(D_h^1 - p^D(1,1)) + \frac{1}{2}(D_h^1 - p^D(1,-1)) = \frac{1}{2}\Delta D^1(1-\mu_i) < \frac{1}{2}\Delta E(1-\mu_i)$ (since by assumption we have $\Delta D^1 < \Delta E$); selling debt yields profits $\frac{1}{2}(p^D(-1,-1) - D_h^0) + \frac{1}{2}(p^D(-1,1) - D_h^1) = -\frac{1}{2}(\Delta D^0 + (1-\mu_i)\Delta D^1) < 0 < \frac{1}{2}\Delta E(1-\mu_i)$; while abstaining yields profits equal to $0 < \frac{1}{2}(1-\mu_i)\Delta E$. Given $z = l$ it is optimal to sell equity: buying debt yields profits equal to $\frac{1}{2}(D_l^1 - p^D(1,1)) + \frac{1}{2}(D_l^1 - p^D(1,-1)) = -\frac{1}{2}(\Delta D^1 + \mu_i\Delta D^1) < 0 < \frac{1}{2}\Delta E\mu_i$; selling debt yields profits equal to $\frac{1}{2}(p^D(-1,-1) - D_l^0) + \frac{1}{2}(p^D(-1,1) - D_l^1) = \frac{1}{2}\mu_i\Delta D^1 < \frac{1}{2}\Delta E\mu_i$; while abstaining yields profits equal to $0 < \frac{1}{2}\Delta E\mu_i$. Given $z = \varnothing$ it is optimal to abstain from trading equity: buying debt yields profits equal to $\frac{1}{2}(\mu_i D_h^1 + (1-\mu_i)D_l^1 - p^D(1,1)) + \frac{1}{2}(\mu_i D_h^1 + (1-\mu_i)D_l^1 - p^D(1,-1)) = -\frac{1}{2}(1-\mu_i)\Delta^1 < 0$; while selling debt yields profits equal to $\frac{1}{2}(p^D(-1,-1) - \mu_i D_h^0 - (1-\mu_i)D_l^0) + \frac{1}{2}(p^D(-1,1) - \mu_i D_h^1 - (1-\mu_i)D_l^1) = -\frac{1}{2}\Delta D^0\mu_i < 0$.

It can be shown, in a similar manner, that it is optimal to trade debt after a test inducing $\mu_i < \widehat{s}$. Taking the expectation over $z$, we get the gross expected trading profits after the realization of the public signal $m_i$

$$\pi(\mu_i) = \begin{cases} \sigma \Delta E \mu_i (1-\mu_i) & \text{if } \mu_i \geq \widehat{s} \\ \sigma \Delta D^0 \mu_i (1-\mu_i) & \text{if } \mu_i < \widehat{s}, \end{cases}$$

and maximizing the net trading profits $\pi(\mu_i) - \frac{1}{2}\tau\sigma^2$ with respect to $\sigma$ gives the speculator's optimal information acquisition in (22). □

**Lemma A1** (Stress Test Design Problem with Debt Trading)**.** *When the speculator can trade a debt or an equity claim, the supervisor solves the following stress test design problem:*

$$\begin{aligned} \max_{\mathcal{P}} \; & V(\mathcal{P}) = \chi(s_j) + \Sigma(\mathcal{P}) \\ \text{s.t. } & \mu_{j+1} \geq \widehat{s} \\ & \mu_j < \widehat{s}, \end{aligned} \tag{36}$$

*where*

$$\begin{aligned} j &\triangleq \max\left\{ i \in \{1,2,\ldots,n\} : \mu_i < \widehat{s} \right\}, \\ \chi(s_j) &= \frac{1}{2}\Big( F_h(s_j) V_h^0 + \big(1 - F_h(s_j)\big) V_h^1 + F_l(s_j) V_l^0 + \big(1 - F_l(s_j)\big) V_l^1 \Big), \\ \Sigma(\mathcal{P}) &= \frac{1}{2\tau} \left( \Delta D^0 \Delta V_h \sum_{i=1}^{j} \nu_i \mu_i^2 (1-\mu_i) + \Delta E \Delta V_l \sum_{i=j+1}^{n} \nu_i \mu_i (1-\mu_i)^2 \right). \end{aligned} \tag{37}$$

**Proof of Lemma A1**. To derive the objective function we proceed as in Lemma 2. Unlike before, we cannot pool all the lower subintervals in a unique message since they induce different levels of information acquisition. The value accrued to the supervisor from some $m_i$ s.t. $\mu_i < \widehat{s}$ is:

$$g(s_{i-1}, s_i) = \frac{1}{2} \left( (s_i^2 - s_{i-1}^2) \left( V_h^0 + \frac{1}{2}\sigma(\mu_i)\Delta V_h \right) + \big( 2(s_i - s_{i-1}) - (s_i^2 - s_{i-1}^2) \big) V_l^0 \right),$$

while the value for some $m_i$ s.t. $\mu_i \geq \widehat{s}$ is (as in Lemma 2):

$$f(s_{i-1}, s_i) = \frac{1}{2} \left( (s_i^2 - s_{i-1}^2) V_h^1 + \big( 2(s_i - s_{i-1}) - (s_i^2 - s_{i-1}^2) \big) \left( V_l^1 + \frac{1}{2}\sigma(\mu_i)\Delta V_l \right) \right).$$

Let $j \triangleq \max\left\{i \in \{1, 2, \ldots, n\} : \mu_i < \widehat{s}\right\}$. The objective function can be written as:

$$V(\mathcal{P}) = \sum_{i=1}^{j} g(s_{i-1}, s_i) + \sum_{i=j+1}^{n} f(s_{i-1}, s_i),$$

and exploiting the fact that the objective is a telescoping sum, after some algebra we get the expressions in equation (37). □

**Proof of Proposition 3**. The first order condition for the optimality of $s_j$ in problem (36) is

$$\begin{aligned}\frac{\partial V(\mathcal{P})}{\partial s_j} &= \Delta V_l - s_j(\Delta V_l + \Delta V_h) + \\ &+ \frac{1}{2\tau}\Delta D^0 \Delta V_h \left(\nu_j \mu_j (1-\mu_j) + \mu_j^2 \left(1 - \frac{1}{2}s_j\right)\right) + \\ &- \frac{1}{2\tau}\Delta E \Delta V_l \left(\nu_{j+1}\mu_{j+1}(1-\mu_{j+1}) + (1-\mu_{j+1})^2 s_j\right) = 0.\end{aligned}$$

Dividing both sides by $\Delta V_l + \Delta V_h$ and isolating the $s_j$ in the first line we get:

$$\begin{aligned}s_j = \frac{\Delta V_l}{\Delta V_l + \Delta V_h} &+ \frac{1}{2\tau}\Delta D^0 \frac{\Delta V_h}{\Delta V_l + \Delta V_h}\left(\nu_j \mu_j (1-\mu_j) + \mu_j^2\left(1 - \frac{1}{2}s_j\right)\right) + \\ &- \frac{1}{2\tau}\Delta E \frac{\Delta V_l}{\Delta V_l + \Delta V_h}\left(\nu_{j+1}\mu_{j+1}(1-\mu_{j+1}) + (1-\mu_{j+1})^2 s_j\right).\end{aligned}$$

Since both terms in brackets are positive, the optimal stress test exhibits a distortion towards leniency ($s_j < \Delta V_h/(\Delta V_l + \Delta V_h) = \widehat{s}$) or towards conservatism ($s_j > \widehat{s}$) depending on the parameter values. In particular, there exist a $\kappa > 1$ such that $\Delta D^0/\Delta E \geq \kappa$ and $s_j > \widehat{s}$, otherwise we have $s_j < \widehat{s}$. □

**Proof of Lemma 6**. Consider the trading game at date $t = 2$. Let $x_s(z)$ and $x_n$ be the market order of the speculator and the liquidity trader, respectively. Consider the following speculator's equilibrium trading strategy:

$$x_s(z) = \begin{cases} 1 & \text{if } z = g \\ 0 & \text{if } z = \varnothing \\ -1 & \text{if } z = b. \end{cases}$$

Given the above trading strategy, we can compute the price $p(X)$ chosen by the market maker as a function of the anonymous aggregate order flow $X = (x_n, x_s)$ for some intervention decision

$a$:

$$p(X,a)=\begin{cases} E_g^a & \text{if } X=(1,1) \\ \Pr(\theta=g|m_i)E_g^a+\big(1-\Pr(\theta=g|m_i)\big)E_b^a & \text{if } X\in X_\varnothing \\ E_b^a & \text{if } X=(-1,-1) \end{cases}$$
$$=\begin{cases} E_g^a & \text{if } X=(1,1) \\ E_b^a+\Pr(\theta=g|m_i)\Delta E & \text{if } X\in X_\varnothing \\ E_b^a & \text{if } X=(-1,-1) \end{cases}$$

where $X_\varnothing=\{(1,-1),(-1,1),(1,0),(-1,0)\}$. The speculator's equilibrium trading profits are

$$\pi(z,\mu_i,a)=\begin{cases} =\begin{cases} \frac{1}{2}\Delta E\big(1-\Pr(\theta=g|m_i)\big) & \text{if } z=g \\ 0 & \text{if } z=\varnothing \\ \frac{1}{2}\Delta E\Pr(\theta=g|m_i) & \text{if } z=b \end{cases} & \text{if } a=1 \\ =0 & \text{if } a=0 \end{cases}$$

where

$$\Pr(\theta=g|m_i)=\mu_i\left(\frac{1}{2}+2\rho\right)+(1-\mu_i)\left(\frac{1}{2}-2\rho\right),$$
$$=\frac{1}{2}-2\rho(1-2\mu_i).$$

Now, notice that the speculator makes positive trading profits only when the supervisor is expected to continue *even when the order flow is uninformative*, i.e. when $\mu_i\geq\widehat{s}$, in fact:

- for $\mu_i\in\big[0,\widehat{s}_g(\rho)\big)$, the supervisor always intervenes so the speculator makes zero profits;
- for $\mu_i\in\big[\widehat{s}_g(\rho),\widehat{s}\big)$, the supervisor continues only when the order flow is $X=(1,1)$ and reveals that the speculator received $z=g$, but since the order flow is revealing the speculator makes zero profits;
- for $\mu_i\in\big[\widehat{s},\widehat{s}_b(\rho)\big)$, the supervisor continues when the order flow is $X=(1,1)$ or $X\in X_\varnothing$ and reveals that the speculator received $z=g$ or reveals nothing, so the speculator makes positive expected profits when $X\in X_\varnothing$;
- for $\mu_i\in\big[\widehat{s}_b(\rho),1\big]$, the supervisor always continues, so the speculator makes positive expected profits when $X\in X_\varnothing$.

Thus, the speculator's equilibrium trading profits are equal to:

$$\pi(z,\mu_i) = \begin{cases} = \begin{cases} \frac{1}{2}\Delta E\big(1-\Pr(\theta=g|m_i)\big) & \text{if } \ z=g \\ 0 & \text{if } \ z=\varnothing \\ \frac{1}{2}\Delta E \Pr(\theta=g|m_i) & \text{if } \ z=b \end{cases} & \text{if } \ \mu_i \geq \widehat{s} \\ = 0 & \text{if } \ \mu_i < \widehat{s}. \end{cases}$$

Proceeding as in Lemma 1, it is possible to show that the proposed trading is indeed optimal, hence we omit the proof here.

Taking the expectation over $z$, we get the gross expected trading profits after the realization of the public signal $m_i$

$$\begin{aligned} \pi(\mu_i) &= \begin{cases} \sigma\Delta E \Pr(\theta=g|m_i)\big(1-\Pr(\theta=g|m_i)\big) & \text{if } \ \mu_i \geq \widehat{s} \\ 0 & \text{if } \ \mu_i < \widehat{s}, \end{cases} \\ &= \begin{cases} \sigma\Delta E\left(\frac{1}{4}-4\rho^2(1-2\mu_i)^2\right) & \text{if } \ \mu_i \geq \widehat{s} \\ 0 & \text{if } \ \mu_i < \widehat{s}, \end{cases} \end{aligned}$$

and maximizing the net trading profits $\pi(\mu_i) - \frac{1}{2}\tau\sigma^2$ with respect to $\sigma$ gives the speculator's optimal information acquisition in equation (26). □

**Lemma A2** (Stress Test Design Problem with Correlated Information)**.** *When the stress test results $m_i$ are weakly informative about $\theta$, i.e. $\rho \in (0, 1/4)$, the supervisor solves the following stress test design problem:*

$$\begin{aligned} &\max_{\mathcal{P}} V(\mathcal{P}) = \chi(s_1) + \Sigma(\mathcal{P}) \\ &\text{s.t. } \mu_1 < \widehat{s} \\ &\qquad \mu_2 \geq \widehat{s} \\ &\qquad \mu_{n-1} < \widehat{s}_b(\rho) \\ &\qquad \mu_n \geq \widehat{s}_b(\rho), \end{aligned} \tag{38}$$

*where*

$$\begin{aligned} \chi(s_1) &= \frac{1}{2}\Big(F_h(s_1)V_h^0 + \big(1-F_h(s_1)\big)V_h^1 + F_l(s_1)V_l^0 + \big(1-F_l(s_1)\big)V_l^1\Big), \\ \Sigma(\mathcal{P}) &= \frac{1}{2\tau}\Delta E \sum_{i=2}^{n-1} \nu_i \left(\frac{1}{4} - 4\rho^2(1-2\mu_i)^2\right)\left(\Delta V_l\left(\frac{1}{2}+2\rho\right)(1-\mu_i) - \Delta V_h\left(\frac{1}{2}-2\rho\right)\mu_i\right), \\ V_\omega^a &\triangleq \Pr(\theta=g|\omega)V_{\omega,g}^a + \Pr(\theta=b|\omega)V_{\omega,b}^a\,. \end{aligned} \tag{39}$$

**Proof of Lemma A2**. Let $V^a_\omega$ denote the bank's expected value conditional on the state being $\omega$ after intervention decision $a$, i.e.

$$V^a_\omega \triangleq \Pr(\theta = g|\omega)V^a_{\omega,g} + \Pr(\theta = b|\omega)V^a_{\omega,b}.$$

Consider all the $m_i$ such that $\mu_i \in \big[0, \widehat{s}_g(\rho)\big)$. The speculator acquires no information and the supervisor always intervenes. Hence, we can pool all these messages $m_i$ in a unique message $m_1 = s \in [0, s_1]$. Next, consider all the results $m_i$ such that $\mu_i \in \big[\widehat{s}_g(\rho), \widehat{s}\big)$. The speculator acquires no information, hence the supervisor's decision is based exclusively on $m_i$ and this leads to intervention. Hence, all these $m_i$ induce the same players' behaviour as $m_1$ and we can pool them together with $m_1$. The stress test result $m_1$ brings value to the supervisor equal to:

$$\begin{aligned} g(s_1) =& \Pr(\omega = h)\Pr(m_1|\omega = h)V^0_h + \Pr(\omega = l)\Pr(m_1|\omega = l)V^0_l \\ =& \frac{1}{2}\big(s_1^2 V^0_h + (2s_1 - s_1^2)V^0_l\big). \end{aligned}$$

Next, consider all the $m_i$ such that $\mu_i \in [\widehat{s}, \widehat{s}_b(\rho))$. The speculator acquires information $\sigma(\mu_i)$ and the supervisor continues unless the order flow reveals $Y = b$. Hence, all these $m_i$ bring value to the supervisor equal to:

$$\begin{aligned} f(s_{i-1}, s_i) =& \Pr(\omega = h)\Pr(m_i|\omega = h)\left(\Pr(g|\omega = h)V^1_{h,g} + \Pr(b|\omega = h)\left(V^1_{h,b} - \frac{1}{2}\sigma(\mu_i)\Delta V_h\right)\right) + \\ &+ \Pr(\omega = l)\Pr(m_i|\omega = l)\left(\Pr(g|\omega = l)V^1_{l,g} + \Pr(b|\omega = l)\left(V^1_{l,b} + \frac{1}{2}\sigma(\mu_i)\Delta V_l\right)\right) \\ =& \frac{1}{2}(s_i^2 - s_{i-1}^2)\left(V^1_h - \left(\frac{1}{2} - 2\rho\right)\frac{1}{2}\sigma(\mu_i)\Delta V_h\right) + \\ &+ \frac{1}{2}\big(2(s_i - s_{i-1}) - (s_i^2 - s_{i-1}^2)\big)\left(V^1_l + \left(\frac{1}{2} + 2\rho\right)\frac{1}{2}\sigma(\mu_i)\Delta V_l\right). \end{aligned}$$

Lastly, consider all the $m_i$ such that $\mu_i \in [\widehat{s}_b(\rho), 1]$. The speculator acquires information $\sigma(\mu_i)$ but the supervisor continues in any case. Hence, we can pool all these results in a unique message $m_n$, that brings value to the supervisor equal to:

$$\begin{aligned} h(s_{n-1}) =& \Pr(\omega = h)\Pr(m_n|\omega = h)V^1_h + \Pr(\omega = l)\Pr(m_n|\omega = l)V^1_l \\ =& \frac{1}{2}\big((1 - s_{n-1}^2)V^1_h + \big(2(1 - s_{n-1}) - (1 - s_{n-1}^2)\big)V^1_l\big). \end{aligned}$$

Thus, the expected bank value can be written as:

$$V(\mathcal{P}) = g(s_1) + \sum_{i=2}^{n-1} f(s_{i-1}, s_i) + h(s_{n-1}),$$

and exploiting the fact that $V(\mathcal{P})$ is a telescoping sum we get the formula in equation (39). □

**Proof of Proposition 4**. We refer to the notation introduced at the beginning of Proposition 1. We proceed in two steps. Step 1 establishes that the optimal stress test exhibits motives towards leniency, $s_1 < \widehat{s}$. Step 2 characterizes the general structure of the optimal stress test for some parameter values.

*Step 1* (Motives towards Leniency). The expression for $\Sigma(\mathcal{P})$ in equation (39) is equivalent to

$$\Sigma(\mathcal{P}) = \frac{1}{2\tau} \Delta E \sum_{i=2}^{n-1} \Gamma(s_{i-1}, s_i)\Phi(s_{i-1}, s_i),$$

where

$$\Gamma(s_{i-1}, s_i) \triangleq \nu_i \left(\frac{1}{4} - 4\rho^2\right) + 8\rho^2(s_i^2 - s_{i-1}^2)(1 - \mu_i),$$
$$\Phi(s_{i-1}, s_i) \triangleq \left(\Delta V_l \left(\frac{1}{2} + 2\rho\right)(1 - \mu_i) - \Delta V_h \left(\frac{1}{2} - 2\rho\right)\mu_i\right).$$

The first order condition for the optimality of $s_1$ in problem (38) is

$$\frac{\partial V}{\partial s_1} = \Delta V_l - s_1(\Delta V_l + \Delta V_h) + \frac{1}{2\tau}\Delta E \left(\frac{\partial \Gamma}{\partial s_1}\Phi(s_1, s_2) + \Gamma(s_1, s_2)\frac{\partial \Phi}{\partial s_1}\right) = 0, \tag{40}$$

where

$$\frac{\partial \Gamma}{\partial s_1} = -\left(\left(\frac{1}{4} - 4\rho^2\right) + 8\rho^2\left(2s_1(1 - \mu_i) + (s_2^2 - s_1^2)\frac{1}{2}\right)\right) < 0,$$
$$\frac{\partial \Phi}{\partial s_1} = -\frac{1}{2}\left(\Delta V_l\left(\frac{1}{2} + 2\rho\right) + \Delta V_h\left(\frac{1}{2} - 2\rho\right)\right) < 0.$$

Isolating the $s_1$ in the second term of equation (40) we get

$$s_1 = \frac{\Delta V_l}{\Delta V_l + \Delta V_h} + \frac{1}{2\tau}\Delta E \frac{1}{\Delta V_l + \Delta V_h}\left(\frac{\partial \Gamma}{\partial s_1}\Phi(s_1, s_2) + \Gamma(s_1, s_2)\frac{\partial \Phi}{\partial s_1}\right)$$

By Lemma A2 $\mu_{n-1} < \widehat{s}_b(\rho)$ so $\Phi(s_{n-2}, s_{n-1}) > 0$, and since by construction $s_1 < s_{n-1}$, we also have $\Phi(s_1, s_2) > 0$. Moreover, $\Gamma(s_1, s_2) > 0$. It follows that the term in brackets on the right-hand side is negative and for $\frac{1}{\tau}\Delta E > 0$ we have $s_1 < \Delta V_l/(\Delta V_l + \Delta V_h) = \widehat{s}$.

*Step 2.* (General Structure) Let $s_1$ solve the first order condition (40). Consider a subset $[s_1, s_{n-1}] \subseteq \mathcal{S}$ and define the function $\widehat{\Sigma}(s) : [s_1, s_{n-1}] \to \mathbb{R}$ as

$$\widehat{\Sigma}(s) = \frac{1}{2\tau}\Delta E \left(\frac{1}{4} - 4\rho^2(1-2s)^2\right)\left(\Delta V_l \left(\frac{1}{2} + 2\rho\right)(1-s) - \Delta V_h \left(\frac{1}{2} + 2\rho\right) s\right). \tag{41}$$

Consider a partition $\mathcal{P}_1 \triangleq \{s_1, s_2, s_3, \ldots, s_{n-1}\}$ of $[s_1, s_{n-1}]$ and note that the stress test design problem (38) is equivalent to

$$\begin{aligned} \max_{\mathcal{P}_1} \; & R(\widehat{\Sigma}, \mathcal{P}_1) \\ \text{s.t. } & \mu_1 < \widehat{s} \\ & \mu_2 \geq \widehat{s} \\ & \mu_{n-1} < \widehat{s}_b(\rho) \\ & \mu_n \geq \widehat{s}_b(\rho). \end{aligned} \tag{42}$$

Note that, since the term in the first bracket in equation (41) is positive for $\rho < 1/4$, we have $\widehat{\Sigma}(s) \geq 0$ for $s \in [s_1, \widehat{s}_b(\rho)]$ and $\widehat{\Sigma}(s) < 0$ for $s \in (\widehat{s}_b(\rho), s_{n-1}]$. Moreover, the second order derivative of $\widehat{\Sigma}(s)$ is

$$\widehat{\Sigma}''(s) = \frac{1}{2\tau}\Delta E\, 16\rho^2 \left(\Delta V_l \left(\frac{1}{2} + 2\rho\right)(6s-4) - \Delta V_h \left(\frac{1}{2} - 2\rho\right)(2-6s)\right),$$

so that, for $\rho > 0$, we have $\widehat{\Sigma}''(s) \leq 0$ for $s \in [s_1, (1+\widehat{s}_b(\rho))/3]$ and $\widehat{\Sigma}''(s) > 0$ for $s \in [(1+\widehat{s}_b(\rho))/3, s_{n-1}]$.

Consider parameter values such that i) $\widehat{s} \in \big(0; (1+\widehat{s}_b(\rho))/3\big)$, i.e. $\widehat{\Sigma}(s)$ is concave over $[s_1, (1+\widehat{s}_b(\rho))/3)$ and convex over $[(1+\widehat{s}_b(\rho))/3, s_{n-1}]$; and ii) and $\widehat{s}_b(\rho) \in \big((1+\widehat{s}_b(\rho))/3; 1\big)$, i.e. $\widehat{\Sigma}(s)$ crosses the horizontal axis when it is convex.
Consider, a partition $\mathcal{P}_1' \triangleq \{s_1, s_2, \ldots, s_{j-1}, s_j\}$ of $[s_1, (1+\widehat{s}_b(\rho))/3]$ and a partition $\mathcal{P}_1'' \triangleq \{s_j, s_{j+1}, \ldots, s_{n-2}, s_{n-1}\}$ of $[(1+\widehat{s}_b(\rho))/3, s_{n-1}]$ such that $\mathcal{P}_1' \cup \mathcal{P}_1'' = \mathcal{P}_1$ and $R(\widehat{\Sigma}, \mathcal{P}_1) = R(\widehat{\Sigma}, \mathcal{P}_1') + R(\widehat{\Sigma}, \mathcal{P}_1'')$. Since $\widehat{\Sigma}(s)$ is concave over $[s_1, (1+\widehat{s}_b(\rho))/3]$ the coarsest partition $\underline{\mathcal{P}_1'}$ maximizes $R(\widehat{\Sigma}, \mathcal{P}_1')$. Conversely, since $\widehat{\Sigma}(s)$ is convex over $[(1+\widehat{s}_b(\rho))/3, s_{n-1}]$ the finest partition $\overline{\mathcal{P}_1''}$ maximizes $R(\widehat{\Sigma}, \mathcal{P}_1'')$ over the domain of $s$ where $\widehat{\Sigma}(s)$ is non-negative (i.e. for $s \in [(1+\widehat{s}_b(\rho))/3, \widehat{s}_b(\rho)]$), and setting $s_{n-1} = \widehat{s}_b(\rho)$ restricts $\widehat{\Sigma}(s)$ to be non-negative over its domain. Thus, the partition $\mathcal{P}_1$ that solves problem (42) exhibits 1 coarse subinterval $[s_1, s_2]$,

for some optimally chosen $s_2$, and a set of infinitely fine subintervals over $[s_2, \widehat{s}_b(\rho)]$.
It follows that, for the parameter values considered, the optimal $\mathcal{P}$ exhibits 2 coarse messages $m_1$ if $s \in [0, s_1)$, $m_2$ if $s \in [s_1, s_2)$, fully granular grades $s$ for $s \in [s_2, \widehat{s}_b(\rho))$ and a coarse grade $m_n$ if $s \in [\widehat{s}_b(\rho), 1]$. $\square$

# Appendix B

### Alternative Microfoundation of Bank Value $V_\omega^a$ and Equity $E_\omega^a$

In this section we show how the supervisor's intervention can be interpreted as a bail-out, whereby a capital shortfall is made up by an equity issue plus government money. Let us also suppose that the government finds it costly to have the bank default on its creditors *ex post*, maybe because such a bank failure exerts negative externalities on the rest of the banking system. This will provide an additional motive for the supervisor to inject government money into a poorly capitalized bank.

The bank's incentive to behave prudently when its balance sheet is impaired can be restored by injecting fresh equity capital $A_h$. The value of the bank's equity, after a bank has raised capital $A_h$ in state $\omega = l$ is

$$\frac{p}{2}\left(R + A_h - D\right) + (1-p)\left(r + A_h - D\right).$$

Given our prior parametric assumptions, even if the new equity holders are given 100% of the ownership of the bank, the debt overhang is so severe that they cannot break even, that is

$$\frac{p}{2}\left(R + A_h - D\right) + (1-p)\left(r + A_h - D\right) < A_h.$$

Recapitalizing the bank therefore requires (i) wiping out the old equity holders entirely, and (ii) an injection of government funds of magnitude

$$\begin{aligned} b &\equiv A_h - \frac{p}{2}\left(R + A_h - D\right) + (1-p)\left(r + A_h - D\right) \\ &= \frac{p}{2}\left(2r + A_h - R - D\right) + D - r > 0. \end{aligned}$$

Note that in this specification, there is a cost to the supervisor of bailing out the bank. One could imagine that some of the cost is recovered because the supervisor holds claims against the bank's future cash flows. Suppose for simplicity, that the bail-out money is a pure subsidy.

Suppose that a bank default exerts an externality of magnitude $k$. The supervisor maximizes the expected value of the bank minus the expected cost of a bank failure and minus the subsidy. Note that the new money raised by fresh equity washes out of the planner's objective function since this money was in the economy regardless. As before, a well capitalized bank will not engage in risk shifting and will only default if its cash flow is 0.[24] Hence, the supervisor's payoff

[24] Assume that $2A_h < D$ so that even a good bank that was recapitalized erroneously will default in this case.

from allowing a bank to continue without intervention in the high state $\omega = h$ is:

$$V_h^1 = \frac{p}{2}R + (1-p)r + A_h - \frac{p}{2}k,$$

while her payoff from re-capitalizing a bank in the good state is

$$V_h^0 = \frac{p}{2}R + (1-p)r + A_h - \frac{p}{2}k - b,$$

i.e., the supervisor loses from providing a subsidy to a well capitalized bank and $V_h^1 > V_h^0$.

The supervisor's payoff when failing to recapitalize a poorly capitalized bank is

$$V_l^1 = \frac{p+\varepsilon}{2}R + (1-p-\varepsilon)r - \left(1 - p - \varepsilon + \frac{p+\varepsilon}{2}\right)k,$$

while the payoff from recapitalizing is

$$V_l^0 = \frac{p}{2}R + (1-p)r - \frac{p}{2}k - b.$$

Hence, it is beneficial to recapitalize in the low state if $V_l^0 > V_l^1$, i.e., when

$$\frac{p}{2}R + (1-p)r > b - \left(1 - p - \varepsilon + \frac{\varepsilon}{2}\right)k + \frac{p+\varepsilon}{2}R + (1-p-\varepsilon)r.$$

In other words, the benefit from a recapitalization is now two-fold. First, it increases bank value because a recapitalized bank refrains from risk shifting, and second, it reduces the failure externality. The cost of recapitalizing comes in the form of the social cost of the subsidy to the bank. One can now redo the previous analysis with the new expressions for $V_\omega^a$.

## Baseline Model with a Parametric Prior

In this section, we show that our main result (Proposition 1) is robust to the introduction of parametric prior beliefs over the state $\omega$.

Let $\Pr(\omega = h) = \alpha$. The posterior beliefs given a message $m_i = s \in [s_{i-1}, s_i]$ are

$$\begin{aligned}
\Pr(\omega = h | m_i) &= \frac{(s_i^2 - s_{i-1}^2)\alpha}{(s_i^2 - s_{i-1}^2)\alpha + \big(2(s_i - s_{i-1}) - (s_i^2 - s_{i-1}^2)\big)(1-\alpha)} \\
&= \frac{(s_{i-1} + s_i)\alpha}{(s_{i-1} + s_i)\alpha + (2 - (s_{i-1} + s_i))(1-\alpha)} \\
&= \frac{\mu_i \alpha}{\mu_i \alpha + (1-\mu_i)(1-\alpha)}
\end{aligned}$$

where $\mu_i = (s_{i-1} + s_i)/2$. Let $\xi(\mu_i) = \Pr(\omega = h|m_i)\big(1 - \Pr(\omega = h|m_i)\big)$.

The intervention policy of the supervisor and the information production by the speculator are unaffected, but obviously $\alpha$ impacts the player's behavior trough its effect on the posterior beliefs. In particular, (for a given stress test) the more optimistic is the prior, the more optimistic is the posterior, so i) the supervisor will intervene less often, and ii) the speculator produces less information.

$$a(\mathcal{I}) = \begin{cases} 1 & \text{if} \quad \Pr(\omega = h|\mathcal{I}) \geq \widehat{s} \\ 0 & \text{if} \quad \Pr(\omega = h|\mathcal{I}) < \widehat{s}, \end{cases}$$

$$\sigma(m_i) = \begin{cases} \frac{1}{\tau}\Delta E\, \xi(\mu_i) & \text{if} \quad \Pr(\omega = h|m_i) \geq \widehat{s} \\ 0 & \text{if} \quad \Pr(\omega = h|m_i) < \widehat{s}. \end{cases}$$

The stress test design problem of the supervisor can be derived as in Lemma 2. All the messages such that $\Pr(\omega = h|m_i) < \widehat{s}$ can be pooled in a unique message $m_1$ - inducing no information production and intervention - and yielding value to the supervisor equal to:

$$\begin{aligned} g(s_1) &= \Pr(\omega = h)\Pr(m_1|\omega = h)V_h^0 + \Pr(\omega = l)\Pr(m_1|\omega = l)V_l^0 \\ &= \alpha s_1^2 V_h^0 + (1-\alpha)(2s_1 - s_1^2)V_l^0. \end{aligned}$$

The value accrued to the supervisor from subsequent messages, inducing positive information production, is instead:

$$\begin{aligned} f(s_{i-1}, s_i) =& \Pr(\omega = h)\Pr(m_i|\omega = h)V_h^1 + \Pr(\omega = l)\Pr(m_i|\omega = l)\left(V_l^1 + \frac{1}{2}\sigma(s_{i-1}, s_i)\Delta V_l\right) \\ &= \alpha(s_i^2 - s_{i-1}^2)V_h^1 + (1-\alpha)\big(2(s_i - s_{i-1}) - (s_i^2 - s_{i-1}^2)\big)\left(V_l^1 + \frac{1}{2}\sigma(s_{i-1}, s_i)\Delta V_l\right). \end{aligned}$$

Since $V(\mathcal{P}) = g(s_1) + \sum_{i=2}^n f(s_{i-1}, s_i)$ is a telescoping sum, after simple algebra we get the following objective function:

$$\begin{aligned} V(\mathcal{P}) &= \chi(s_1) + \Pr(\omega = l)\frac{1}{2\tau}\Delta E \Delta V_l \sum_{i=2}^n \Pr(m_i|\omega = l)\xi(\mu_i) \\ &= \chi(s_1) + (1-\alpha)\frac{1}{2\tau}\Delta E \Delta V_l \sum_{i=2}^n \big(2(s_i - s_{i-1}) - (s_i^2 - s_{i-1}^2)\big)\xi(\mu_i) \qquad (43) \\ &= \chi(s_1) + (1-\alpha)\frac{1}{\tau}\Delta E \Delta V_l \sum_{i=2}^n \nu_i\,(1-\mu_i)\,\xi(\mu_i), \end{aligned}$$

where:

$$\chi(s_1) = \Pr(\omega = l)\Big(F(s_1|l)V_l^0 + \big(1 - F(s_1|l)\big)V_l^1\Big) + \Pr(\omega = h)\Big(F(s_1|h)V_h^0 + \big(1 - F(s_1|h)\big)V_h^1\Big)$$
$$= (1-\alpha)\Big((2s_1 - s_1^2)V_l^0 + \big(2(1-s_1) - (1-s_1^2)\big)V_l^1\Big) + \alpha\Big(s_1^2 V_h^0 + (1-s_1^2)V_h^1\Big).$$

Thus, the stress test design problem is

$$\max_{\mathcal{P}} V(\mathcal{P})$$
$$\text{s.t. } \mu_2 \geq \widehat{s}$$
$$\mu_1 < \widehat{s}.$$

This problem can be solved as in Lemma 3 and Proposition 1. When there is no information production by financial markets ($\frac{1}{\tau}\Delta E \to 0$) the optimal stress test is a pass-fail experiment with passing threshold

$$s_1 = \frac{(1-\alpha)\Delta V_l}{(1-\alpha)\Delta V_l + \alpha \Delta V_h} \triangleq \widehat{s}(\alpha).$$

Instead, when there is information production by financial markets ($\frac{1}{\tau}\Delta E > 0$) it can be shown that the optimal stress test exhibits motives towards leniency, i.e. $s_1 < \widehat{s}(\alpha)$. Moreover, the stress test features the same structure as in Proposition 1. In fact, notice that the second term in $V(\mathcal{P})$ is a midpoint Riemann sum of the function

$$\widehat{\Sigma}(s) = (1-\alpha)\frac{1}{\tau}\Delta E \Delta V_l\ (1-s)\frac{s\alpha}{s\alpha + (1-s)(1-\alpha)}\frac{(1-s)(1-\alpha)}{s\alpha + (1-s)(1-\alpha)}$$
$$= \frac{1}{\tau}\Delta E \Delta V_l\ s\alpha \left(\frac{(1-s)(1-\alpha)}{s\alpha + (1-s)(1-\alpha)}\right)^2$$

whose second order derivative $\widehat{\Sigma}''(s)$ is either i) positive for all $s \in [s_1, 1]$, or ii) positive for $s \in [s_1, \widetilde{s}]$ and negative for $s \in [\widetilde{s}, 1]$ for some $\widetilde{s} < 1$, depending on the values of $\Delta V_l$ and $\Delta V_h$. Hence, the same reasoning in the proof of Proposition 1 applies.