

The Social Costs of Side Trading*

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Abstract

We study resource allocation under private information when the planner cannot prevent bilateral side trading between consumers and firms. Adverse selection and side trading severely restrict feasible trades: each marginal quantity must be fairly priced given the consumer types who purchase it. The resulting social costs are twofold. First, second-best efficiency and robustness to side trading are in general irreconcilable requirements. Second, there actually exists only one budget-feasible allocation robust to side trading, which deprives the planner from any capacity to redistribute resources between different types of consumers. We discuss the relevance of our results for insurance and financial markets.

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1 Introduction

The theory of incentives identifies the holding of private information by economic agents as a fundamental constraint on the allocation of resources (Hurwicz (1973)). Standard aggregate resource constraints must accordingly be supplemented by incentive-compatibility constraints that reflect the agents' ability to conceal their private information (Myerson (1979, 1982)). The problem of the optimal allocation of resources then reduces to that of characterizing informationally constrained efficient, or second-best, allocations (Harris and Townsend (1981)). The key finding is that a tradeoff arises between redistribution and incentives (Mirrlees (1971)).

A crucial assumption of theories of the second-best is that, although individual types are unobservable, individual trades can be perfectly monitored by the planning authority. Because few, if any, economic institutions have the required ability to monitor all individual trades, this calls for an explicit consideration of the role of side trading in the theory of resource allocation under private information, as first pointed out by Hammond (1979).

To this end, we consider a general environment in which firms can provide a divisible good to privately informed consumers who may be of two types. Consumers' preferences satisfy a single-crossing condition, and there is adverse selection in that consumers who are more willing to trade are also more costly to serve; private values arise as a limiting case when consumers' types are not payoff-relevant for the firms selling to them. This framework encompasses many applications, including the standard Rothschild and Stiglitz (1976) insurance economy as a prominent example.

In this setting, we characterize the allocations that can be achieved by a planner who observes neither consumers' types nor the trades they may conduct with firms. To do so, we refine the standard notion of incentive-feasibility by focusing on allocations that are robust to side trading. This reflects two additional constraints on resource allocation. First, the planner cannot force consumers to trade with him; this is the case, for instance, when consumers can opt out of a publicly provided health-insurance plan, as in the current German system. Second, he cannot prevent them from engaging in mutually advantageous additional trades with a firm. We formalize these constraints by requiring the planner to offer a tariff such that no firm, acting as an entrant, can guarantee itself a positive profit by offering complementary side trades. This approach provides us with a modified criterion of incentive feasibility which is useful for evaluating the social costs of side trading. Focusing on two-type environments affords us a straightforward comparison with standard characterizations of the second-best efficiency frontier in both private-value (Bierbrauer and Boyer (2014)) and common-value (Prescott and Townsend (1984), Crocker and Snow (1985), Bisin and

Gottardi (2006)) environments.

We show that the social costs of side trading are twofold. First, second-best allocations are typically not robust to side trading, so that the planner's inability to monitor consumers' trades has significant welfare implications. Second, only one budget-feasible allocation is robust to side trading, so that the third-best efficiency frontier reduces to a point: the threat of side trading effectively deprives the planner from any capacity to redistribute resources between different types of consumers. The allocation we characterize is thus the natural candidate for a competitive equilibrium, but, being the only feasible one under side trading, little, if anything, can be argued about its desirability.

A distinctive feature of our approach is that we model side trade as bilateral contracts between a consumer and a firm. This reflects our dissatisfaction with the standard way of representing unobservable side trades as transactions on Walrasian markets, which would call for a centralized market institution to ensure that all these trades take place at the same price. Bilateral trading plays a key role in our analysis. Our key Lemma 1, in particular, shows that budget-feasible allocations robust to side trading have a very peculiar price structure: each marginal quantity, or layer, is priced at the cost of serving the types who purchase it. This form of competitive pricing, reminiscent of Akerlof (1970), implies that there are no cross-subsidies between these layers, though there may be cross-subsidies between types. When the allocation is interior and separating, linear pricing can emerge only in the private-value limiting case.

Lemma 1 has a simple but important implication that we state in Theorem 1: no second-best allocation in which only one incentive compatibility-constraint binds is robust to side trading. The reason is that, by the standard efficiency-at-the-top property, consumers for which this constraint binds must trade at the margin at the cost of serving them. As a result, the layer that connects the trades of the two types cannot be priced at the cost of serving the type who is the most willing to trade. But then, by Lemma 1, there always exists some side trade that a firm finds it profitable to conduct with at least one consumer type. For instance, in the Rothschild and Stiglitz (1976) insurance economy, any second-best allocation in which the high-risk type's incentive-compatibility constraint binds can be exploited by an entrant offering complementary coverage at a premium rate slightly higher than the high-risk fair premium rate, which this type is willing to trade along with the coverage provided by the allocation for the low-risk type.

Our second main result, Theorem 2, states that, among the allocations that feature no cross-subsidies between layers, only one is robust to side trading, namely, the Pareto-efficient one that maximizes the utility of the consumer type who is the less willing to trade. This uniqueness result stands in stark contrast with the nondegenerate second-best efficiency

frontier that emerges from the standard tradeoff between redistribution and incentives. To complete our characterization, we evaluate whether this allocation can be second-best, hence considering the situations not covered by Theorem 1. Theorem 3 shows that a second-best allocation robust to side trading must either feature pooling of the two consumers' types, or each type purchasing her first-best quantity. We argue that these situations can only occur under very special assumptions on preferences and costs. Thus the unique third-best allocation typically does not belong to the second-best efficiency frontier.

Related Literature

While the constraints induced by private information on resource allocation are by now well understood, less is known about the impact of side trading on feasibility and redistribution. Starting with the early contributions of Hammond (1979, 1987), Allen (1985), and Jacklin (1987), several authors have attempted to identify the limits to risk sharing generated by consumers' side trading in financial markets. Cole and Kocherlakota (2001), Golosov and Tsyvinski (2007), Farhi, Golosov, and Tsyvinski (2009) have analyzed different private-value environments in which the planner is constrained by the existence of Walrasian markets on which privately informed consumers can complement their trades with the planner by trading linearly priced commodities. This raises two sets of questions, which we address in this paper. First, the existence of unobservable transactions is at odds with the idea that all trades take place in a centralized Walrasian market where a single market-clearing price is quoted—the issue is not so much that the quantities traded are unobservable, which is consistent with the functioning of a Walrasian market, but rather that side trades may take place at different prices and that bilateral contracting prevents the planner from monitoring the terms of these trades. Second, if one sticks to the Walrasian paradigm, extending the approach to incorporate common values and adverse selection presents severe conceptual difficulties (Prescott and Townsend (1984), Rustichini and Siconolfi (2008)); a possible way out is to rely on an Akerlof-like (1970) equilibrium, which however only obtains under linear utilities and/or indivisibilities (Attar, Mariotti, and Salanié (2011), Philippon and Skreta (2012), Tirole (2012)). We hence depart from the above literature in two ways. First, we offer an alternative representation of side trading, which we essentially regard as a bilateral rather than a centralized process. Second, we focus on trade under common values; hence, in a bilateral relationship, a firm's profit directly depends on the types of the consumers it trades with, which is arguably a prominent feature of insurance and financial markets.

The constraints induced by private contracting on redistribution between workers who are privately informed of their productivities have also been investigated by Stantcheva (2014) in the context of optimal taxation. In her framework, however, the State retains

full observability of workers' total incomes, as in Mirrlees (1971), which severely limits their ability to engage in side trading.

Our requirement that an allocation must be implementable by an entry-proof tariff to be robust to side trading is in line with the definition by Kahn and Mookherjee (1998) or Bisin and Guaitoli (2004) of third-best allocations in moral-hazard environments. In any such allocation, the planner's tariff must prevent consumers from complementing it with an additional profit-making contract provided by a firm. We extend this notion to private-information environments, and, in addition, we put no restriction on the side trades a firm can make available.

The unique third-best allocation that we characterize has been formerly derived by Glosten (1994) and Attar, Mariotti, and Salanié (2019a) as the outcome of competitive financial markets where market makers are restricted to post collections of limit orders and insiders thus face a convex aggregate tariff. In contrast with these authors, we adopt a fully normative perspective. This requires imposing no restriction on sides trades, which we assume to be fully bilateral. In line with this assumption, we allow firms to react to the planner's tariff by posting arbitrary tariffs. Jaynes (1978), Hellwig (1988), and Stiglitz, Yun, and Kosenko (2018) show that this allocation can also be obtained in an equilibrium of a modified Rothschild and Stiglitz (1976) insurance economy in which insurance companies can exchange information about their customers. Although the allocation is the same, the logic of our approach is entirely different. First, these authors allow firms to react to the information disclosed by their competitors by possibly enforcing exclusivity clauses, which is at odds with the very notion of side trading that we emphasize. Second, as noticed above, we are interested in the normative implications of side trading and not in characterizing the equilibrium of a given extensive-form game. We return to the issue of the decentralization of the third-best allocation in Section 4.

Our analysis shares with the common-agency literature (Martimort (2007)) the idea that what can be implemented by a principal—here, the planner—crucially depends on the trades made available by other principals—here, an entrant. Our normative approach, however, differs from the fully strategic approach adopted, in a similar environment, by Attar, Mariotti, and Salanié (2014). The idea is that when a planner contemplates implementing an allocation by a tariff, he always anticipates that an entrant, acting as a follower, may complement this tariff by providing consumers with further trading opportunities. The requirement that the planner's tariff be entry-proof then captures in a natural way the additional constraints that the possibility of side trades imposes on the planner.

The paper is organized as follows. Section 2 describes the model. Section 3 defines our

concept of robustness to side trading, fully characterizes the only feasible allocation satisfying this requirement, and shows that it is typically not second-best. Section 4 discusses the relevance of our results for insurance and financial markets. Proofs not given in the text can be found in the Appendix.

2 The Economy

Consumers There is a continuum of consumers who can purchase a divisible good in exchange for monetary transfers. Each consumer is privately informed of her type $i = 1, 2$ and the proportion of type i among consumers is $m_i > 0$. Type i 's preferences over quantity-transfer bundles $(q, t) \in \mathbb{R}_+ \times \mathbb{R}$ are represented by a strictly quasiconcave and continuously differentiable utility function u_i , with $\partial_t u_i < 0$. Hence her marginal rate of substitution

$$\tau_i \equiv - \frac{\partial_q u_i}{\partial_t u_i}$$

is well defined and strictly decreasing along her indifference curves. We impose the Inada condition that $\tau_i(q, t)$ vanishes as q grows large along any such curve. Hence, whatever her endowment point, type i 's demand at any price $p > 0$ is finite. The following strict single-crossing condition is the key determinant of consumer demand:

$$\text{For all } q \text{ and } t, \tau_2(q, t) > \tau_1(q, t). \quad (1)$$

Thus type 2 is more willing to increase her purchases than type 1.

Firms The supply side of the economy is described by a constant-return-to-scale technology, with unit cost $c_i > 0$ of serving type i . Type 2 is weakly more costly to serve than type 1:

$$c_2 \geq c_1. \quad (2)$$

Together with (1), (2) typically generates adverse selection, whereas values are private in the limiting case $c_1 = c_2$. We let $c \equiv m_1 c_1 + m_2 c_2$ be the average cost of serving a consumer.

These assumptions hold in the Rothschild and Stiglitz (1976) insurance economy: c_i is type i 's riskiness, with $c_2 > c_1$, q is the amount of coverage she purchases, and t is the premium she pays in return. Our model encompasses many other specifications and is relevant for a broad spectrum of insurance, financial, and labor markets.

Incentive Feasibility and Efficiency A *contract* is a pair (q, t) for some $q \geq 0$, and with unit price t/q if $q > 0$. An *allocation* is a pair of contracts, one for each type. An allocation $(q_i, t_i)_{i=1,2}$ is *budget-feasible* if

$$m_1(t_1 - c_1 q_1) + m_2(t_2 - c_2 q_2) \geq 0.$$

To this aggregate resource constraint, we must add, following Myerson (1979, 1982) and Harris and Townsend (1981), constraints reflecting that the allocation of resources takes place under asymmetric information. An allocation $(q_i, t_i)_{i=1,2}$ is *incentive-compatible* if

$$u_1(q_1, t_1) \geq u_1(q_2, t_2) \text{ and } u_2(q_2, t_2) \geq u_2(q_1, t_1).$$

We denote these constraints by $IC_{1 \rightarrow 2}$ and $IC_{2 \rightarrow 1}$, respectively. An allocation is *incentive-feasible* if it is budget-feasible and incentive-compatible. A *second-best* allocation is Pareto-efficient among incentive-feasible allocations. This is the relevant notion of efficiency for a planner who perfectly monitors trades, but does not observe consumer types (Prescott and Townsend (1984)). In the Rothschild and Stiglitz (1976) insurance economy, the second-best efficiency frontier consists of a continuum of allocations (Crocker and Snow (1985), Bisin and Gottardi (2006)).

Tariffs A *tariff* T is a schedule specifying a transfer $T(q)$ to be paid in return for a quantity q , with $T(0) = 0$ in case a consumer chooses not to trade along the tariff and $T(q) = \infty$ in case the tariff does not allow consumers to purchase the quantity q . A tariff T *implements* the allocation $(q_i, t_i)_{i=1,2}$ if

$$\text{For each } i, q_i \in \arg \max \{u_i(q, T(q)) : q \geq 0\} \text{ and } t_i = T(q_i).$$

To ensure that the various maximization problems we will encounter have solutions, we impose the mild requirement that a tariff be lower semicontinuous, with $T(q)/q$ bounded away from 0 as q grows large; this notably holds true if T has a compact domain.

3 Second-Best Allocations and Side Trading

When side trading is feasible, the planner can no longer monitor trades between consumers and firms. This imposes two additional constraints on resource allocation. First, the planner cannot force consumers to trade with him, reflecting that they can opt out of any mechanism he could propose. To model this constraint, we require that the planner offer a tariff T^P , the key restriction being $T^P(0) = 0$. Second, the planner cannot prevent consumers from engaging in mutually advantageous additional trades with a firm. To model this constraint, we require that T^P be such that no firm, acting as an entrant, can guarantee itself a positive profit by offering complementary side trades.

Side trades are usually assumed to take place on Walrasian markets (Hammond (1979, 1987), Allen (1985), Jacklin (1987), Cole and Kocherlakota (2001), Golosov and Tsyvinski (2007), Farhi, Golosov, and Tsyvinski (2009)); in our context, this would amount to impose that the entrant must post a linear tariff. We find this at odds with the idea that side

trades cannot be monitored and instead allow the entrant to post an arbitrary tariff T^E ; the taxation principle (Hammond (1979), Guesnerie (1981), Rochet (1985)) ensures that this involves no loss of generality. Notice that we do not thereby prevent the entrant from offering a linear tariff; rather, the key idea is that the entrant can choose the terms of side trades and restrict the set of quantities consumers can choose from. This motivates the following definition.

Definition 1 *The planner's tariff T^P is entry-proof if, for any entrant's tariff T^E , there exists a solution (q_i^P, q_i^E) to each type i 's problem*

$$\max\{U_i(q^P + q^E, T^P(q^P) + T^E(q^E)) : q^P \geq 0 \text{ and } q^E \geq 0\} \quad (3)$$

such that entry is not profitable:

$$m_1[T^E(q_1^E) - c_1q_1^E] + m_2[T^E(q_2^E) - c_2q_2^E] \leq 0. \quad (4)$$

An allocation is robust to side trading if it can be implemented by an entry-proof tariff.

It may at first be objected that we are tilting the odds in the planner's favor by de facto assuming that he is able, in case consumers are indifferent, to coordinate their behavior on an allocation in which the entrant does poorly. However, this assumption plays no role in the characterization results we offer in Theorems 1–3: indeed, Theorem 2 shows that this arguably weak entry-proofness concept already singles out a unique budget-feasible allocation robust to side trading; moreover, the necessary conditions we derive are obtained using contracts that are profitable for the entrant no matter the consumers' best responses. By contrast, strengthening our entry-proofness concept—for instance, by requiring that the planner's tariff be robust to entry no matter the consumers' best responses—would threaten the very existence of a budget-balanced allocation robust to side trading. This situation, however, is hardly specific to our model: indeed, when sustaining equilibria of competitive adverse-selection models, it is often necessary to assume that firms cannot sort out the least costly types at the deviation stage in case consumers are indifferent.¹

Any allocation robust to side trading is incentive-compatible. The question we ask is whether such an allocation can also be second-best. Our argument is twofold.

On the one hand, budget-feasible allocations robust to side trading have the following price structure.

Lemma 1 *In any budget-feasible allocation $(q_i, t_i)_{i=1,2}$ robust to side trading,*

$$t_1 = cq_1 \text{ and } t_2 - t_1 = c_2(q_2 - q_1). \quad (5)$$

¹This in particular the case when the equilibrium allocation involves some amount of pooling, as in Akerlof (1970), Miyazaki (1977), Wilson (1977), Spence (1978), and Attar, Mariotti, and Salanié (2011).

Proof. Because an allocation $(q_i, t_i)_{i=1,2}$ robust to side trading is incentive-compatible, it satisfies $q_2 \geq q_1$ by single crossing. Moreover,

$$t_1 \leq cq_1. \quad (6)$$

Otherwise, an entrant can supply q_1 at a price slightly above c : this profitably attracts type 1 as $T^P(0) = 0$, and remains profitable even if type 2 is attracted. Similarly,

$$t_2 - t_1 \leq c_2(q_2 - q_1). \quad (7)$$

Otherwise, an entrant can supply $q_2 - q_1$ at a price slightly above c_2 : this profitably attracts type 2 along with the contract (q_1, t_1) , and is even more profitable if type 1 is also attracted. Rewriting the resource constraint as

$$t_1 - cq_1 + m_2[t_2 - t_1 - c_2(q_2 - q_1)] \geq 0$$

and taking advantage of (6)–(7) yields (5). The result follows. ■

Hence pricing is competitive, in the sense that the prices of the *layers* q_1 and $q_2 - q_1$ reflect the costs of serving the types who purchases them. However, if $c_2 > c_1$ and $q_1 > 0$, then the *quantities* q_1 and q_2 are not priced competitively: as q_1 is sold at the average cost $c > c_1$, type 1 subsidizes type 2.

On the other hand, second-best allocations satisfy the following efficiency-at-the-top property.

Lemma 2 *In any second-best allocation $(q_i, t_i)_{i=1,2}$,*

(i) *If $IC_{2 \rightarrow 1}$ is slack, then $\tau_1(q_1, t_1) \leq c_1$, with equality if $q_1 > 0$.*

(ii) *If $IC_{1 \rightarrow 2}$ is slack, then $\tau_2(q_2, t_2) = c_2$.*

Proof. If $IC_{2 \rightarrow 1}$ or $IC_{1 \rightarrow 2}$ is slack, then $q_2 > q_1$ by incentive compatibility and single crossing. If $IC_{2 \rightarrow 1}$ is slack and $\tau_1(q_1, t_1) > c_1$, then $((q_1 + \varepsilon, t_1 + c_1\varepsilon), (q_2, t_2))$ is incentive-feasible for $\varepsilon > 0$ small enough and Pareto-dominates $(q_i, t_i)_{i=1,2}$, a contradiction. Thus $\tau_1(q_1, t_1) \leq c_1$. Moreover, if $q_1 > 0$ and $\tau_1(q_1, t_1) < c_1$, then $((q_1 - \varepsilon, t_1 - c_1\varepsilon), (q_2, t_2))$ is incentive-feasible for $\varepsilon > 0$ small enough and Pareto-dominates $(q_i, t_i)_{i=1,2}$, once again a contradiction. This proves (i). The proof of (ii) is similar, using $q_2 > 0$, and is therefore omitted. The result follows. ■

Combining Lemmas 1 and 2 yields our first main theorem.

Theorem 1 *A second-best allocation in which only one incentive-compatibility constraint binds is not robust to side trading.*

Proof. Suppose first that only $IC_{1 \rightarrow 2}$ binds. Then $q_2 > q_1$ by incentive compatibility and single crossing, and $\tau_1(q_1, t_1) \leq c_1$ by Lemma 2(i). Moreover, because type 1's preferences are strictly convex and $IC_{1 \rightarrow 2}$ binds, we have $t_2 - t_1 < c_1(q_2 - q_1) \leq c_2(q_2 - q_1)$. By Lemma 1, $(q_i, t_i)_{i=1,2}$ is not robust to side trading.

Suppose next that only $IC_{2 \rightarrow 1}$ binds. Then $q_2 > q_1$ by incentive compatibility and single crossing, and $\tau_2(q_2, t_2) = c_2$ by Lemma 2(ii). Moreover, because type 2's preferences are strictly convex and $IC_{2 \rightarrow 1}$ binds, we have $t_2 - t_1 > c_2(q_2 - q_1)$. By Lemma 1, $(q_i, t_i)_{i=1,2}$ is not robust to side trading. Hence the result. ■

Theorem 1 covers most cases emphasized in the literature. For instance, in the Rothschild and Stiglitz (1976) insurance economy, either $IC_{1 \rightarrow 2}$ or $IC_{2 \rightarrow 1}$ bind in all but the pooling second-best allocation (Crocker and Snow (1985)), and Theorem 1 implies that none of these allocations is robust to side trading.

This leaves only two cases in which a second-best allocation may be robust to side trading: when both $IC_{1 \rightarrow 2}$ and $IC_{2 \rightarrow 1}$ bind, which corresponds to a pooling allocation, or when both $IC_{1 \rightarrow 2}$ and $IC_{2 \rightarrow 1}$ are slack. Both cases can arise, as we show below, but only under very special assumptions on preferences and costs.

To study these cases, we strengthen Lemma 1 by establishing that a unique allocation is budget-feasible and robust to side trading. In this allocation, the first layer is optimal for type 1 at price c , while the second layer is optimal for type 2 at price c_2 , conditional on her purchasing the first layer. This allocation is thus Pareto-efficient—maximizing type 1's utility—among those satisfying (5).

Theorem 2 *The third-best allocation defined by*

$$q_1^* \equiv \arg \max \{u_1(q, cq) : q \geq 0\}, \quad (8)$$

$$t_1^* \equiv cq_1^*, \quad (9)$$

$$q_2^* \equiv q_1^* + \arg \max \{u_2(q_1^* + q, t_1^* + c_2q) : q \geq 0\}, \quad (10)$$

$$t_2^* \equiv t_1^* + c_2(q_2^* - q_1^*), \quad (11)$$

is the only budget-feasible allocation robust to side trading.

Proof. (Uniqueness) Because an allocation $(q_i, t_i)_{i=1,2}$ robust to side trading is incentive-compatible, it satisfies $q_2 \geq q_1$ by single crossing. Moreover,

$$u_1(q_1, t_1) \geq \max \{u_1(q, cq) : q \geq 0\}. \quad (12)$$

Otherwise, an entrant can offer a contract with unit price slightly above c that profitably attracts type 1 as $T^P(0) = 0$, and remains profitable even if type 2 is attracted. Similarly,

$$u_2(q_2, t_2) \geq \max \{u_2(q_1 + q, t_1 + c_2q) : q \geq 0\}. \quad (13)$$

Otherwise, an entrant can offer a contract with unit price slightly above c_2 that profitably attracts type 2 along with the contract (q_1, t_1) , and is even more profitable if type 1 is also attracted. Finally, if $(q_i, t_i)_{i=1,2}$ is budget-feasible, then (5) holds, so that (12)–(13) are equalities. Thus $(q_i, t_i)_{i=1,2}$ is the third-best allocation defined by (8)–(11).

(Existence) By (8)–(11), the piecewise-linear convex tariff

$$T^P(q) \equiv 1_{\{q \leq q_1^*\}}cq + 1_{\{q > q_1^*\}}[cq_1^* + c_2(q - q_1^*)] \quad (14)$$

implements the third-best allocation. Now, suppose that an entrant posts a tariff T^E . The following monotonicity property is established in the Appendix.

Lemma 3 *There exists a solution $((q_i^P, q_i^E))_{i=1,2}$ to (3) such that $q_2^E \geq q_1^E$.*

Let us fix such a solution in what follows. As T^P allows type 1 to purchase her optimal quantity q_1^* at price c , we must have

$$T^E(q_1^E) \leq cq_1^E. \quad (15)$$

Moreover, because $q_2^E \geq q_1^E$, type 2 could alternatively obtain the same aggregate quantity $q_2^P + q_2^E$ as in her best response by purchasing q_1^E from the entrant and $q_2^P + q_2^E - q_1^E$ from the planner, paying overall $T^P(q_2^P + q_2^E - q_1^E) + T^E(q_1^E)$. As she chooses to pay $T^P(q_2^P) + T^E(q_2^E)$ instead, we must have

$$T^E(q_2^E) - T^E(q_1^E) \leq T^P(q_2^P + q_2^E - q_1^E) - T^P(q_2^P). \quad (16)$$

Because T^P is convex with slope at most c_2 and $q_2^E \geq q_1^E$,

$$T^P(q_2^P + q_2^E - q_1^E) - T^P(q_2^P) \leq c_2(q_2^E - q_1^E). \quad (17)$$

Collecting (15) and (16)–(17) yields

$$T^E(q_1^E) - cq_1^E + m_2[T^E(q_2^E) - T^E(q_1^E) - c_2(q_2^E - q_1^E)] \leq 0,$$

which is (4). This shows that T^P is entry-proof. Hence the result. ■

The uniqueness of the third-best allocation contrasts with the multiplicity of second-best allocations, which form a nondegenerate frontier. The planner is thus severely constrained by

his inability to monitor trades, which effectively prevents any kind of redistribution between different types of consumers.

The existence of such an allocation for any binary distribution of types is also noteworthy. Nonexclusivity, or consumers' ability to combine the contracts offered by an entrant with those offered by the planner, is key to this result. While this enlarges the set of contracts an entrant can use to attract consumers, this also gives the planner more instruments to deter entry. These take the form of *latent contracts*, which are not meant to be traded but only to make entry unprofitable. Of course, the planner must make sure that, by offering latent contracts, he does not create new profitable entry opportunities. The *third-best tariff* (14) strikes a balance between these two requirements.

In the adverse-selection case $c_2 > c_1$, type 1's and type 2's marginal rates of substitution at the third-best allocation are strictly ordered, $\tau_1(q_1^*, t_1^*) < \tau_2(q_2^*, t_2^*)$. In particular, we have $\tau_1(q_1^*, t_1^*) = c < c_2 = \tau_2(q_2^*, t_2^*)$ if the third-best allocation is interior and separating. This contrasts with private-value models where side trades take place on Walrasian markets, which calls for an equalization of marginal rates of substitution (Hammond (1979, 1987)). Yet incentive-compatible gains from trade between types 1 and 2 are exhausted at the third-best allocation, subject to the side-trading constraint. Indeed, supposing that consumers have access to the same constant-return-to-scale technology as firms, the minimum price at which type 1 would be willing to sell a small additional quantity to type 2 is c_2 , and at this price type 2 is not willing to buy. In that sense, the third-best allocation is the only candidate for a competitive equilibrium.

Regarding the proof of Theorem 2, an interesting duality is that the third-best allocation is the only candidate for a budget-feasible allocation robust to side trading even if the entrant can only offer a single contract, while the third-best tariff is entry-proof even if the entrant can post an arbitrary tariff. This differs from Glosten (1994), who in his analysis of limit-order markets requires the entrant's tariff to satisfy a property he dubs *single crossing* and that generalizes convexity. Another important difference is that Theorem 2 does not require consumers' preferences to be quasilinear, which makes it relevant for standard insurance economies.

We are now ready to address the remaining cases not covered by Theorem 1.

Theorem 3 *If a second-best allocation is robust to side trading, then it coincides with the third-best allocation and one of the following conditions holds:*

- (i) *The third-best allocation is pooling, that is, $\tau_2(q_1^*, t_1^*) \leq c_2$.*
- (ii) *The third-best allocation is separating and first-best, that is, $c_1 = c_2$ or $\tau_1(0, 0) \leq c_1$.*

Proof. By Theorem 2, if a second-best allocation is robust to side trading, then it coincides with the third-best allocation. By Theorem 1, we only need to consider two cases.

(i) If $IC_{1 \rightarrow 2}$ and $IC_{2 \rightarrow 1}$ bind, then $q_2^* = q_1^*$ by incentive compatibility and single crossing. Hence the third-best allocation is pooling, which amounts to $\tau_2(q_1^*, t_1^*) \leq c_2$ by (10)–(11).

(ii) If $IC_{1 \rightarrow 2}$ and $IC_{2 \rightarrow 1}$ are slack, then $q_2^* > q_1^*$ by incentive compatibility and single crossing. Hence the third-best allocation is separating. Two cases can arise. If $q_1^* > 0$, then $\tau_1(q_1^*, t_1^*) = c_1$ by Lemma 2(i) and $\tau_1(q_1^*, t_1^*) = c$ by (8)–(9), so that $c_1 = c_2$. If $q_1^* = 0$, then $\tau_1(0, 0) \leq c_1$ by Lemma 2(i). In either case, each type i trades efficiently at cost c_i , so that the third-best allocation is first-best. Hence the result. ■

Condition (i) is clearly extreme. It cannot hold in a Rothschild and Stiglitz (1976) insurance economy, because the optimal coverage of type 1 at the average premium rate c is only partial, while type 2 is willing to purchase additional coverage at the fair premium rate c_2 until she reaches full insurance. In the case of quasilinear preferences, condition (i) together with the condition $\tau_1(q_1^*, t_1^*) \leq c$ implied by (8)–(9) entails that type 1's first-best quantity is at least as large as type 2's, and strictly larger if $q_1^* > 0$, a case of nonresponsiveness (Caillaud, Guesnerie, Rey, and Tirole (1988)).

Condition (ii) is also extreme. Indeed, the third-best allocation is then first-best, and the third-best tariff is linear with slope c_2 . In the private-value case $c_1 = c_2$, each type trades efficiently at marginal cost. In the adverse-selection case $c_2 > c_1$, a separating second-best allocation is robust to side trading only if type 1 is not willing to trade at cost c_1 and hence is in some sense irrelevant.

This answers the question we raised in this section: second-best efficiency and robustness to side trading are irreconcilable requirements, except in very special cases. Overall, our results suggest that the threat of side trading constitutes a serious obstacle to efficiency and redistribution in private-information economies. In the limiting case of private values, side trading poses no threat to efficiency, as it leads to a first-best allocation; yet the requirement that there be no cross-subsidies between layers precludes the planner from redistributing resources between different consumer types. By contrast, under adverse selection, the social costs of side trading are twofold: first, the threat of side trading moves the economy away from the second-best efficiency frontier; second, it precludes redistribution.

4 Decentralization and Market Intervention

The Decentralization Problem Determining whether constrained-efficient allocations can be supported in an equilibrium of an adverse-selection economy is a central issue for welfare economics. Bisin and Gottardi (2006) provide a positive answer to this question in

the case of fully observable trades: for any second-best allocation, there exists a system of transfers ensuring that this allocation obtains in an equilibrium of a decentralized economy in which firms compete by offering exclusive contracts. However, in the polar case where firms can only observe their own trades with consumers, decentralizing a third-best allocation is a more delicate task. To perform it, one needs to explicitly model firms' behavior when no information on consumers' aggregate trades is available to them.

In perfectly competitive markets, it is standard to assume, following Bisin and Gottardi (1999), that nonexclusive competition forces prices to be linear. Yet, as Theorem 3 shows, this is consistent with third-best efficiency only in the special private-value case. A natural alternative is to consider strategic models in which firms compete through menus of bilateral contracts. Side trading is then captured by letting consumers free to trade with any subset of firms, in contrast with the exclusivity assumption made in standard competitive screening models following Rothschild and Stiglitz (1976).

Attar, Mariotti, and Salanié (2014) provide a general analysis of this nonexclusive-competition scenario. Their main result is that a positive level of trade for type 2 can be supported in a pure-strategy equilibrium only if type 1 is left out of the market. Indeed, equilibrium requires that type 1 not be willing to consume at price c , and type 2 purchasing her optimal quantity at price c_2 . Given (8)–(11), the resulting equilibrium allocation is a degenerate third-best one in which $(q_1^*, t_1^*) = (0, 0)$. A necessary and sufficient condition for decentralization is $\tau_1(0, 0) \leq c$, that is, Akerlof's (1970) condition for a market breakdown in which only the worse-quality goods are traded. In all other cases, a pure-strategy equilibrium fails to exist altogether.

An alternative strategy that has been pursued in the literature consists in designing extensive-form games that incorporate inter-firm communication. This approach, initiated by Jaynes (1978) and Hellwig (1988), and further developed by Stiglitz, Yun, and Kosenko (2018), allows each firm to dynamically react to the information about its customers' trades disclosed by its competitors. This may induce a firm to withdraw its initial offers or to enforce exclusivity clauses, which suggests that these models are better interpreted as models of endogenous exclusivity than of side trading. In addition, enforcing any such mode of communication would require a very sophisticated institutional setting to properly take into account firms' incentives to reveal information over time.

By contrast, we suggest below some simple market interventions that decentralize the third-best allocation. The novelty of our proposal consists in the explicit consideration of public programs that the agents can complement by resorting to the private sector, therefore endogenously determining the aggregate level of trade. Their simplicity lies in the fact that little information is required for public authorities. We develop our discussion in the context

of insurance and financial markets.

Insurance Markets In modern health-insurance systems, public insurance schemes for the provision of basic coverage do not prevent an active role for the private sector. In Germany, consumers can opt out from the public-insurance scheme to buy basic coverage designed and priced by private insurance companies. Consumers also often have the option to complement basic coverage with additional privately provided coverage, such as *mutuelles* in France. Finally, different forms of mandatory health insurance, whereby consumers are not allowed to remain uninsured, are in place in several systems, as in France, Germany, Japan, Netherlands, and Switzerland.²

Our analysis suggests a simple intervention that achieves a mix of public and private insurance, with no need for observability requirements. It consists in letting the State offer any amount of basic coverage up to q_1^* at the average premium rate c . As private insurance companies are willing to provide any amount of complementary coverage at the high premium rate c_2 , the State together with any insurance company make the third-best tariff available. Because this tariff is entry-proof, no insurance company has an incentive to deviate and entry is impossible. Implementing the third-best allocation is therefore compatible with letting consumers free to choose their preferred level of coverage. This is reminiscent of the universal health-care vouchers advocated by Emanuel and Fuchs (2005, 2007), whereby universal coverage is provided while letting consumers free to purchase additional services or amenities on private insurance markets.

Financial Markets In the aftermath of the recent crisis, the opportunity for agents to opt out of a public program and trade in private markets has been acknowledged as a key constraint for the design of financial institutions in the presence of adverse selection. In this respect, recent works have suggested a rationale for liquidity-injection programs that provide a credible signal to uninformed lenders by rejuvenating the relevant markets. An optimal intervention then typically consists in attracting only the least profitable borrowers, either through direct lending (Philippon and Skreta (2012)), or by repurchasing low-quality assets (Tirole (2012)). By participating in a bailout program, a borrower may however end up signalling her financial weakness to the market, creating a stigma effect with potentially perverse implications (Gorton (2015)).

While bailout policies are derived under the assumption that public and private liquidity are mutually exclusive, our approach offers a general theoretical framework for evaluating public interventions in situations where privately informed borrowers may complement a

²We refer to the surveys of Thomson and Mossialos (2009) and Thomson, Osborne, Squires, and Jun (2013) for institutional details and cross-country evidence.

public program with additional funds raised on private markets. A possible intervention would require public liquidity provision to involve a price sufficiently low, c , so as to attract all borrowers, and a borrowing limit q_1^* such that no overborrowing by the least profitable ones is possible. Further borrowing may then take place on private markets at price c_2 . Overall, such an intervention would implement the third-best allocation, thereby achieving budget balance, unlike those proposed by Philippon and Skreta (2012) and Tirole (2012), and inducing all types of borrowers to participate. This in turn would make it harder to infer their individual financial conditions, mitigating the impact of the stigma effect. Finally, the corresponding allocation of funds is the only one that can be reached under a budget-balanced program under the threat of side trading.

5 Concluding Remarks

We would like to conclude by mentioning two avenues for future research.

First, we have followed Rothschild and Stiglitz (1976) and many authors since in assuming that there are only two types of consumers. This, in particular, enabled us to directly compare our results to the characterization of the second-best efficiency frontier in Crocker and Snow (1985). Yet the question naturally arises to which extent our results are robust to this assumption. Attar, Mariotti, and Salanié (2019a) show that, in general, there exists a unique *convex* tariff for the planner that is entry-proof and implements a budget-feasible allocation. However, it is not straightforward to extend this uniqueness result to more than two types when the planner's tariff is nonconvex.³ Preliminary work suggests that imposing more structure on the consumers' preferences, as arises for instance in a Rothschild and Stiglitz (1976) economy, might be instrumental for such an extension.

Second, we have throughout the analysis postulated that, given the planner's tariff, an entrant can make a take-it-or-leave-it tariff offer to consumers. In that sense, bilateral trading is frictionless in our model. An alternative to this assumption, and to the standard Walrasian approach to side trading, would be to assume that bilateral trades are the outcome of a dynamic matching and bargaining game between firms and consumers searching for trade partners, in the spirit of Gale (2000). The planner may then be able to manipulate the outcome of this game by making search more costly, possibly enabling him to enlarge the set of implementable allocations and to achieve redistribution. More generally, the optimal manipulation of the market for side trades, which we have by assumption ruled out from our analysis, is an important topic for future investigations.

³Technically, this is because such a tariff may cause the consumers' indirect utility functions, as defined in the proof of Lemma 3, to fail to satisfy a single-crossing condition. Notice in any case that the *existence* of a budget-balanced allocation robust to side trading is always guaranteed.

Appendix

Proof of Lemma 3. Each type i evaluates any contract (q^E, t^E) she can trade with the entrant through the indirect utility function

$$z_i^{-P}(q^E, t^E) \equiv \max\{u_i(q^P + q^E, T^P(q^P) + t^E) : q^P \geq 0\}. \quad (\text{A.1})$$

Because u_i is strictly quasiconcave and T^P is convex, the maximum in (A.1) is attained at a unique $\widehat{q}_i^P(q^E)$, and $(\widehat{q}_i^P(q^E), q_i^E)$ is a solution to (3) if and only if

$$q_i^E \in \arg \max\{z_i^{-P}(q^E, T^E(q^E)) : q^E \geq 0\}. \quad (\text{A.2})$$

According to Attar, Mariotti, and Salanié (2019b, Lemma 1), the convexity of the tariff T^P and the strict single-crossing condition for the functions u_i imply the following single-crossing condition for the functions z_i^{-P} :

For all $\underline{q}^E < \bar{q}^E$, \underline{t}^E , and \bar{t}^E , $z_1^{-P}(\underline{q}^E, \underline{t}^E) < z_1^{-P}(\bar{q}^E, \bar{t}^E)$ implies $z_2^{-P}(\underline{q}^E, \underline{t}^E) < z_2^{-P}(\bar{q}^E, \bar{t}^E)$.

To conclude, suppose that $q_2^E < q_1^E$ at some solution $((q_i^P, q_i^E))_{i=1,2}$ to (3). By (A.2),

$$z_2^{-P}(q_2^E, T^E(q_2^E)) \geq z_2^{-P}(q_1^E, T^E(q_1^E)).$$

Because $q_2^E < q_1^E$, the above single-crossing condition then implies

$$z_1^{-P}(q_2^E, T^E(q_2^E)) \geq z_1^{-P}(q_1^E, T^E(q_1^E)).$$

Thus $(\widehat{q}_1^P(q_2^E), q_2^E)$ is also a solution to (3) for type 1. The result follows. ■

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