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“Personalized Pricing and Distribution Strategies”

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Personalized Pricing and Distribution Strategies*

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Abstract

The availability of consumer data has led many firms to alter their pricing policy and move towards personalized pricing. This trend has implications for firms' strategies over which channels to use for reaching consumers. In this article, we develop a formal model to examine whether a brand manufacturer prefers to sell only through its own direct channel (mono distribution) or through an independent retailer as well (dual distribution). Compared with uniform pricing, personalized pricing allows for higher rent extraction but also leads to fiercer intra-brand competition in the latter case. We show that, if the manufacturer's and the retailer's channel are vertically differentiated and the manufacturer offers higher quality, mono distribution can be optimal under personalized pricing even if the retailer broadens the demand of the manufacturer's product. Instead, with uniform pricing, selling through both channels is always optimal. We also show that industry profits may be the largest in a hybrid pricing regime, in which the manufacturer forgoes the use of personalized pricing and only the retailer charges personalized prices. Instead, if the two channels are horizontally differentiated, or vertically differentiated with the independent retailer offering higher quality, dual distribution is the optimal strategy under both personalized and uniform pricing. Our results are able to explain the distribution strategies of manufacturers in different industries. They also imply that the insights about the effects of personalized pricing obtained in classic frameworks analyzing inter-brand competition between independent firms do not carry over to the case of intra-brand competition.

Keywords: personalized pricing, distribution strategies, vertical contracting, downstream competition.

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1 Introduction

The growing use of the Internet and advances in information technologies enable firms to gather unprecedented volumes of consumer data. This has led to important changes in their pricing policies by allowing them to practice price discrimination at finely-tuned levels. Firms tailor their prices according to consumers' purchase history, their physical location, the device they are using, their online search behavior, and so on.[1] For instance, in the apparel and fashion industry, most brand manufacturers, such as Desigual, Guess, and Marc O'Polo, and retailers, such as Zalando and Amazon, use coupons and specific promotions, based on their consumer data (e.g., through loyalty programs or information releases about sales), to implement different prices for different consumers. A similar practice can be observed in other industries, such as the telephone industry; for example, Chen and Iyer (2002) note that this industry was one of first where firms used customized pricing and specialized discounts to a large extent.[2]

The trend in data collection has consequences for pricing, but also for many long-term strategic decisions. In this article, we focus on the choice of distribution partners helping a manufacturer to reach out to consumers. This is a particularly important issue in the digital age, as technological advances have led to the emergence of new online retail companies, including general retailers such as Amazon Retail, which sells around 12 million products in the US ranging from clothing to grocery items,[3] and specialized ones such as Asos, which distributes fashion and cosmetic products of more than 850 brands. Whether to rely on these independent retailers or only on the direct channel is a key question for manufacturers.

Indeed, manufacturers do pursue different distribution strategies. For example, apparel and fashion brand manufacturers usually offer the full range of products on their own websites but only a limited range on Amazon Retail and Asos. Similarly, although several cosmetic and perfume producers are available at big retailers (e.g., Sephora), brands like The Body Shop or Glossier do not sell there. By contrast, in the market for consumer electronics, many brands offer almost their entire product portfolio through direct channels and through independent retailers, both general ones like Amazon Retail and specialized ones like John Lewis & Partners. For example, the

[1]For example, Tanner (2014) reports that buyers using a discount site, such as Nextag.com, receive prices as much as 23% lower than direct visitors. Large Internet retailers, such as Amazon and Staples, vary their prices according to customers' geographic location by up to 166%. Companies like Bloomberg and Axciom, which specialize in developing machine learning algorithms, act as data brokers and help firms to predict a consumer's willingness-to-pay (*The Economist*, 2014).

[2]In practice, firms may not know a consumer's valuation precisely. Although we will consider in this paper the benchmark of perfect information, the insights apply as well to fine-tuned price discrimination.

[3]See Nchannel (2020).

available variety of Sennheiser and Bang & Olufsen headphones is almost the same on the websites of these retailers as on the companies' direct websites.[4] The choice of distribution channels is also a relevant issue in offline markets. For instance, the advancement of wireless technology enabled mobile virtual network operators, such as Ting or Lycamobile, to enter the market for mobile phone services without rolling out their own networks. Established mobile operators had then to decide whether or not to grant these virtual operators access to their networks.

Motivated by these observations, the objective of this paper is to identify the implications of the availability of individual data for manufacturers' distribution strategies. How does personalized pricing change the incentives of a manufacturer to sell through an independent retailer? Is this decision influenced by the shape of consumers' demand for the two firms? Can a firm benefit from forgoing personalized pricing?

Our study differs from prior literature in two main ways. First, the relationship between personalized pricing and the distribution strategy has not been explored so far. The strategy literature (e.g., Lassar and Kerr, 1996; Hult et al. 2007) has recognized the importance of the distribution and supply chain choice for firm performance; however, the interaction with new pricing instruments that are possible through better data availability has not been studied. Second, the literature on personalized pricing (e.g., Thisse and Vives, 1988; Shaffer and Zhang, 1995, 2002; Chen *et al*, 2020) has focused on competition between independent firms—i.e., *inter-brand competition*. By contrast, we consider competition between a direct distribution channel and an independent channel—i.e., *intra-brand competition*. In this relationship, the wholesale contract between firms affects competition with personalized pricing and thereby also the optimal distribution strategy.

To study the strategic interaction between personalized pricing and distribution, we consider a setting with one brand manufacturer, selling directly to final consumers, and one independent retailer. The retailer adds value to the industry but also competes with the manufacturer in the downstream market. Our demand specification is sufficiently flexible to encompass standard models of product differentiation: vertical differentiation, with either the manufacturer or the retailer catering to high-valuation consumers, and horizontal differentiation.

For each demand pattern, we consider four different scenarios. In the first scenario,

[4]We conducted a search for Bang & Olufsen headphones on August 4, 2020. On the company's own website, 10 different models were available; 8 of these models were also available on Amazon Retail and on the website of John Lewis & Partners. A similar pattern holds for Sennheiser and Bose. By contrast, the newest collection of women's purses by Desigual consists of 20 models, which are available on Desigual's website; however, only one of these models is also available via Amazon Retail and none of them on the websites of Zalando and Asos. Again, a similar pattern can be observed for other Desigual products and for products of other fashion brands.

the manufacturer and the retailer offer uniform prices to final consumers. This represents a market in which consumer tracking is not possible. In the second scenario, both firms engage in personalized pricing. This reflects the situation in which the two firms have highly-frequented (e.g., online) stores allowing them to gather very precise consumer data. In the third scenario, only the manufacturer can set personalized prices. This represents for example a situation in which, thanks to past purchases, the manufacturer has better consumer data than the retailer. In the fourth scenario, only the retailer can set personalized prices. This reflects the situation in which a large retailer offers many products and is thereby able to collect more consumer data than a brand manufacturer. Our analysis therefore captures that new technologies allow firms to use personalized pricing as a trend but there are different capabilities of doing so, both on the market and at the firm level.

We also consider an extended setting in which the pricing regime is endogenous and negotiated by the firms—that is, personalized pricing is available to both firms, and they negotiate whether each of them adopts it or not.[5]

We start with the vertical differentiation pattern in which the manufacturer caters to high-valuation consumers—implying that the retailer broadens demand for low-valuation consumers. As long as the manufacturer offers a uniform retail price, dual distribution is optimal. This holds regardless of whether the retailer also sets a uniform price or charges personalized prices. In both cases, charging a high enough wholesale price suffices to attenuate the intensity of downstream competition. The retailer then adopts relatively high price(s), but nevertheless expands demand in the low-valuation segment—the manufacturer moreover increases its own (uniform) price and extracts more surplus from high-valuation consumers.

By contrast, if the manufacturer charges personalized prices, then mono distribution (i.e., to sell only through the direct channel) may become the optimal strategy—both when the retailer charges a uniform price and when it charges personalized ones. Specifically, relying exclusively on direct distribution is optimal when the retailer does not substantially broaden demand, as the effect of increased intra-brand competition then dominates the benefit of expanding demand. For example, when both firms can price discriminate, they can price aggressively in each other's strong segment without sacrificing margins in their own core business. As a result, it becomes more difficult to control intra-brand competition without impeding market expansion. Mono distribution is moreover more likely to be optimal when the retailer charges a uniform price, as this reduces its added value. These findings suggest that wholesale contracting and the possibility to charge personalized prices are crucial when determining the optimal

[5]The firms can contract on uniform pricing, for instance, by adopting privacy or fair treatment policies.

distribution strategy.

We then endogenize firms' pricing policies. Interestingly, we find that it can be profitable for the manufacturer not to use personalized pricing, even if it has the ability to do so. Restricting the manufacturer's pricing policy induces it to focus on its core market, thereby dampening the competitive pressure and allowing the retailer to extract more surplus. Hence, a hybrid pricing regime, in which only the retailer charges personalized prices, can achieve the right balance between allowing the retailer to add value and limiting intra-brand competition.

We next turn to the vertical differentiation pattern in which the manufacturer caters to low-valuation consumers—implying that the retailer adds value in the high-end segment. We show that dual distribution is then always optimal, regardless of the pricing regime. This is because the wholesale price is very effective at limiting intra-brand competition in this case. For instance, a wholesale price equal to consumers' maximal willingness-to-pay for the manufacturer's offering would fully protect it from intra-brand competition while still allowing the retailer to sell profitably to high-valuation consumers. Furthermore, as intra-brand competition can be kept under control, the most profitable regime is when both firms charge personalized prices to maximize rent-extraction.

Finally, we consider the horizontal differentiation pattern. We find that, there as well, the wholesale price is very effective at limiting intra-brand competition. For instance, a wholesale price equal to the willingness-to-pay of the consumer indifferent between the two firms' offerings enables them to segment the market—no firm can then profitably serve the other's core market. It follows that, again, dual distribution is always optimal, and the most profitable regime is when both firms charge personalized prices. These findings stress the importance of accounting for the role of the wholesale contract when assessing the impact of personalized pricing, as they are markedly different from those obtained in the case of inter-brand competition between independent firms. For instance, Thisse and Vives (1988) and Shaffer and Zhang (1995) show that firms are trapped in a prisoner's dilemma, in which personalized pricing reduces industry profits. By contrast, in the case of intra-brand competition between distribution channels, personalized pricing, *together* with an appropriate wholesale contract, maximizes industry profit.

Our insights can explain why brand manufacturers adopt different channel strategies across industries. For example, as already mentioned, apparel manufacturers such as Desigual or Marc O'Polo offer their entire product selection in their own online or brick-and-mortar stores, but only a small portion via online retail stores such as Asos and Zalando. Similarly, established mobile network operators have often been reluctant to grant mobile virtual network operators (MVNOs) access to their network—

prompting regulators to impose such access. In these markets, price discrimination is a common practice and, in addition, manufacturers' offers are usually more attractive for high-valuation consumers—in case of telephone service, this is due to the reputation of the established operators,[6] whereas in the apparel industry, this is because the image (and, in case of brick-and-mortar stores, also the atmosphere) of the manufacturer's channel is targeted towards the brand, which is not the case in independent stores. The fact that, in recent years, brands such as Adidas and Nike decided to sell fewer products through independent retailers[7] is also consistent with our prediction that the possibility of customizing pricing, which has strongly increased over the years, tends to favor mono distribution.

By contrast, in markets such as consumer electronics, price discrimination is less common. Furthermore, retailers often offer services such as next day delivery and/or free return, whereas manufacturers offer instead professional advice, which points towards horizontal rather than vertical differentiation. In line with our analysis, even when they have a direct channel, manufacturers also offer a large part of their product line via independent retailers.

The rest of the paper is organized as follows. The remainder of this section discusses the related literature. Section 2 presents the model. Section 3 considers the demand pattern in which the manufacturer offers the high-end product, endogenizes the choice of the pricing regime, and extends the analysis by allowing the manufacturer to shut down its direct channel. Section 4 analyzes the demand pattern of vertical differentiation in which the retailer offers the high-end product, and Section 5 studies the scenario of horizontal differentiation. Section 6 draws managerial implications and concludes.

Related literature. The literature on competition with price discrimination has almost exclusively focused on inter-brand competition between independent firms. In their seminal paper, Thisse and Vives (1988) analyze the effects of price discrimination for horizontally differentiated firms competing on a Hotelling line. They demonstrate that this leads to a prisoner's dilemma: firms adopt price discrimination but profits fall due to increased competition.[8] Shaffer and Zhang (1995) highlight a similar prisoner's dilemma when firms discriminate through coupon targeting and consumers differ in the cost of redeeming coupons. Chen and Iyer (2002) allow firms to choose the pro-

[6]The difference between the brand reputation of established operators and MVNOs has been emphasized by e.g. Banerjee and Dippon (2009) and Grand View Research (2020).

[7]See The Wall Street Journal (2019). For example, Adidas is reducing its product selection at Amazon's retail store and Nike is even no longer available there.

[8]Liu and Serfes (2013) extend the framework of Thisse and Vives (1988) by studying the effects of price discrimination in two-sided markets. Matsumura and Matsushima (2015) show instead that firms may choose not to price discriminate in order to limit rivals' incentives to engage in cost reduction.

portion of consumers for whom they acquire information. In this case, firms may benefit from consumer addressability and may refrain from acquiring full information.[9] Choudhary *et al.* (2005) consider instead competition between vertically differentiated firms, and find that pricing strategies can be non-monotonic in consumer valuations. Montes *et al.* (2019) and Chen *et al.* (2020) consider models in which consumers can prevent firms from exploiting information about their preferences. They show that this possibility can harm consumers and allow firms to benefit from price discrimination.[10]

Our paper contributes to this literature by studying the implications of personalized pricing on intra-brand competition, wholesale contracting, and on the choice of the optimal distribution strategy. To the best of our knowledge, the only two papers analyzing the effects of price discrimination on distribution channels are Liu and Zhang (2006) and Li (2018). Liu and Zhang (2006) consider a setting in which only the retailer has access to personalized pricing and the manufacturer can open a direct channel charging a uniform price. They show that the adoption of personalized pricing harms the retailer by inducing the manufacturer to charge a higher wholesale price, but can nevertheless be profitable by deterring the manufacturer from entering the downstream market. Li (2018) determines how behavior-based pricing shapes competition between two manufacturers which sell their products through exclusive retailers.[11] She shows that channel performance crucially depends on whether only retailers can adopt behavior-based pricing or manufacturers can do so as well. In contrast to these papers, we focus on an integrated manufacturer's decision to allow a retailer to enter the market, and study the implications of pricing strategies on this decision and the channel performance.

In the strategy literature, the importance of the distribution network and the supply chain on firm performance has been recognized in several papers—e.g., Lassar and Kerr (1996) and Hult *et al.* (2004, 2007). These studies provide empirical contributions, focusing on agency costs or the culture of competitiveness. We show that the pricing instruments, which have radically changed due to increased data availability, may be equally important for the performance of a distribution channel.

Finally, our paper also contributes to the literature on market foreclosure. Several papers show that a vertically integrated firm has the incentive to raise wholesale prices to a non-integrated downstream rival to dampen price competition (see e.g., Salinger,

[9]Shaffer and Zhang (2002) show that firms offering higher quality may benefit from personalized pricing, even though competition is fiercer. This is due to a gain in market share, which dominates the effect of lower prices.

[10]For empirical papers on estimates for the profitability of personalized pricing relative to uniform pricing in different set-ups, see e.g. Rossi *et al.* (1996), Dubé and Misra (2019), and Shiller (2020).

[11]Behavior-based price discrimination refers to the practice of charging consumers different prices dependent on their purchase history, see e.g. Acquisti and Varian (2005), Fudenberg and Tirole (2000), or Choe *et al.* (2018).

1988, Ordover *et al.*, 1990, Hart and Tirole, 1990, Chen, 2001, and Bourreau *et al.*, 2011).[12] However, if the rival adds value to the industry, for example, by offering a differentiated product, foreclosure takes place only partially, as the integrated firm benefits from entry through the wholesale revenue. Instead, our paper shows that an integrated firm may fully deny access to its products if price discrimination downstream is feasible.

2 The Model

Supply. A monopoly manufacturer, firm A, sells its good to final consumers through a direct distribution channel. In addition, it can also use an independent retailer, firm B, and choose a dual distribution strategy.[13] In order to highlight the strategic motive for mono or dual distribution, we assume away any fixed costs of opening a new distribution channel. For simplicity, we assume that variable costs are linear and normalize the production cost to zero; we denote firm i's distribution cost by c_i.

Demand. Consumers have heterogeneous preferences over the firms' offerings. Specifically, a consumer of type $x \in [0, X]$ derives from the offering of firm $i = A, B$ ("product i", thereafter) a utility given by $\theta_i - s_i x$. Without loss of generality, we suppose that consumers are ranked by decreasing order of preference for product A: $s_A > 0$; however, we allow s_B to be positive or negative.

We denote by $p_i \geq 0$ the price-cost margin of firm i and by $r_i = \theta_i - c_i$ the maximal value, net of the distribution cost, offered by firm i; the net value offered to a type-x consumer is therefore:

$$v_i(x) = r_i - s_i x.$$

Given total price $c_i + p_i$, a type-x consumer would consider buying firm i's product only if her utility $v_i(x) - p_i$ is non-negative. In what follows, we simply refer to p_i as the "price".

We assume that both products play an effective role. Specifically, letting:

$$\hat{x} \equiv \frac{r_A - r_B}{s_A - s_B}$$

denote the consumer type who receives the same value from both products, and:

$$\hat{v} \equiv \frac{s_A r_B - s_B r_A}{s_A - s_B}$$

[12] Rey and Tirole (2007) provide an overview of this literature.

[13] Later on, we also consider the possibility that the manufacturer shuts down its direct channel and distributes only through the independent retailer—i.e., follow a strategy of delegated distribution.

denote the corresponding net value, we maintain the following assumptions:

$$\hat{x} \in (0, X) \text{ and } \hat{v} > 0. \tag{1}$$

The first assumption ensures that no product "dominates" the other. The second assumption ensures that both products offer value to some consumers who have a positive willingness-to-pay (namely, those with a type close to $\hat{x}$). Together, these assumptions ensure that the two products compete for those consumers.

Our setting encompasses the classic models of vertical and horizontal differentiation:

- *Vertical differentiation:* $s_B > 0$. Consider for example the classic model of Shaked and Sutton (1982), in which consumers have heterogeneous tastes for quality; consumers' utilities are therefore *positively* correlated, as those who value quality are willing to pay more for either product. Assuming that quality is costly to offer, high-valuation consumers are then more profitable for the firm with the higher quality, whereas low-valuation consumers are more profitable for the low-quality firm.[14] Depending on which firm offers the higher quality, two cases arise:
  - firm A offers the higher quality: $s_A > s_B$. Firm A is then favored by high-valuation consumers (see Figure 1a)—the assumptions (1) then imply $r_A > r_B$.
  - firm B offers instead the higher quality: $s_B > s_A$. Firm A is then favored by low-valuation consumers (see Figure 1b) and $r_B > r_A$.
- *Horizontal differentiation:* $s_B < 0$. As in the classic Hotelling model, consumers' utilities are *negatively* correlated: consumers who are more attracted by one product are less attracted by the other one (see Figure 1c).[15]

The common feature of these demand patterns is the heterogeneity of consumers' valuations for the firms' offerings; vertical differentiation may be a better fit for some markets and horizontal differentiation a better fit for others.[16]

[14]In Shaked and Sutton (1982), a consumer with income t derives a utility $u_i \times t$ from consuming firm i's product. Indexing consumers by $x = X - t$, this corresponds to $s_i = u_i$ and $r_i = u_i \times X - c_i$: the net value generated by consumer x and firm i is indeed given by $u_i \times t - c_i = u_i \times (X - x) - c_i = r_i - s_i x$. Many papers focus on the case in which costs are the same for the two firms, which implies that one product dominates the other.

[15]In the standard Hotelling model, the firms are located at the two ends of a unit-length segment, whereas consumers are uniformly distributed along the segment and face a transportation cost t per unit of distance. Indexing consumers by their location x, and denoting their reservation value by V, this corresponds to $r_A = V - c_A$, $r_B = V - c_B - t$, $s_A = t$ and $s_B = -t$.

[16]We provide examples for each demand pattern when applying our insights to observed distribution structures.

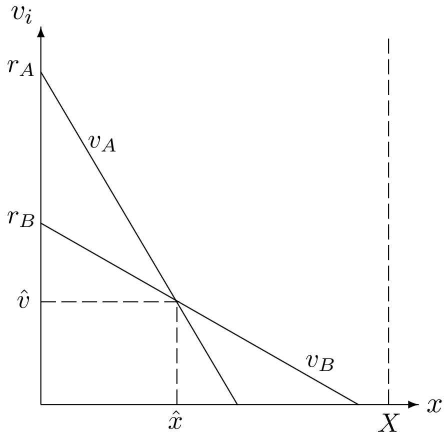

Figure 1a
Vertical differentiation:
A offers higher quality

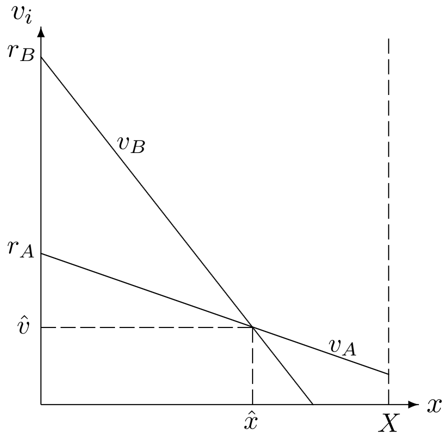

Figure 1b
Vertical differentiation:
B offers higher quality

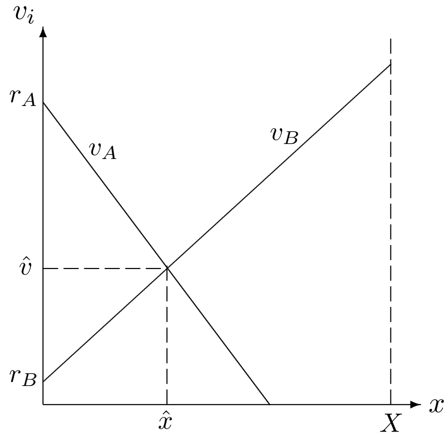

Figure 1c
Horizontal differentiation

Retail competition. A and B compete in prices for consumers; B maximizes its retail profit, whereas A maximizes its total profit, including the wholesale profit.

For each firm, we consider two types of pricing policies: uniform pricing (non-discrimination), in which the firm charges the same price to all consumers, and personalized pricing, in which the firm can perfectly discriminate consumers according to their types. Firm i's price is denoted by p_i under uniform pricing, and by $p_i(x)$ under personalized pricing. Combining the pricing policies of the firms, there are in total four different pricing regimes: two symmetric regimes (i.e., both firms charge a uniform price, and both firms charge personalized prices), and two hybrid regimes (i.e., A charges a uniform price and B personalized prices, and vice versa).

Wholesale contracting. We consider two-part tariffs of the form $T(q) = F + wq$, where F denotes the fixed fee and w the uniform wholesale price paid by B to A, and q is the quantity bought by B.[17] We suppose that the two firms adopt the Nash bargaining solution to negotiate about w and F, with bargaining power $\alpha \in [0, 1]$ for A and $1 - \alpha$ for B.

Timing. As discussed in the Introduction, the pricing regime is driven by technologies and data considerations: personalized pricing may be available in some industries, and not in others. This leads us to first treat the pricing regime as exogenous. However, if personalized pricing is available, firms have the opportunity not to use it. For this reason, we also consider for each demand configuration an extended version where we allow firms to contract not only on a wholesale tariff but also on their retail pricing policies.

The timing of the game is as follows:

- Stage 1: A and B negotiate the wholesale contract.

[17] In Online Appendix H, we show that our main results also hold with a linear wholesale contract.

- Stage 2: Active firms set their retail prices. Consumers then observe all retail prices and decide whether or not to buy, and from which firm to buy. If active, B then orders the quantity from A to satisfy its demand.

In the first stage, firms can share their joint profit through the fixed fee; hence, they seek to maximize the industry profit. Dual distribution is thus optimal if the resulting industry profit is larger than the one with mono distribution. For the sake of exposition, we will then say that dual distribution is the optimal strategy.

In the second stage, for symmetric pricing regimes, firms simultaneously set their prices. For asymmetric pricing regimes, we follow Thisse and Vives (1988), Liu and Zhang (2006), and Choe *et al*. (2018) in assuming that the firm charging a uniform price, say firm i, acts as a price leader: it sets p_i before the competitor sets its personalized prices $p_{-i}(x)$. This assumption ensures the existence of a pure-strategy Nash equilibrium. As pointed out by Thisse and Vives (1988), it is natural for asymmetric regimes, as i can announce and advertise its uniform price in advance, whereas this may be too complex or costly for the competitor. In addition, as noted by Choe *et al*. (2018) and Chen *et al*. (2020), the adjustment of a uniform price is a higher-level managerial decision, that is relatively slower in practice than the adjustment of personalized prices.

Solution concept. Our solution concept is subgame perfection.[18] In the case of price discrimination, asymmetric Bertrand competition for each consumer x is known to generate multiple equilibria. Following the literature, we focus on the equilibrium in which the firm offering the lower value prices at cost.[19]

Remark: wholesale personalized pricing. We focus on the case in which personalized pricing may be possible at the retail but not at the wholesale level. That is, the wholesale contract cannot be conditioned on consumers' types. While this would allow the firms to maximize the industry profit, it is usually infeasible. First, manufacturers are often unable to monitor which consumers their retailers are selling to; and even if they could obtain that information, it would be difficult to verify it in a court of law. Second, through direct interaction with its customers, the retailer may have access to data that is not available to the manufacturer.

[18]Formally, subgame perfection applies from stage 2 onwards. In stage 1, Nash bargaining could be also be achieved as the equilibrium of a non-cooperative random-proposer game in which each firm gets to make a take-it-or-leave-it offer with a probability reflecting its bargaining power. To obtain a deterministic outcome, it suffices to introduce a preliminary stage in which one firm (either one) makes an offer, with the random-proposer game acting as default option.

[19]This is the unique Coalition-Proof Nash equilibrium (in particular, it is the Pareto-dominant equilibrium from the firms' standpoint) and is also the unique trembling-hand perfect equilibrium.

3 Vertical differentiation with A offering higher quality

We first consider the scenario in which A offers the high-end product (i.e., $s_A > s_B > 0$). This is a prevalent scenario as consumers who highly value the manufacturer's product usually have a preference for the direct distribution channel (e.g., they regularly visit the manufacturer's store) whereas consumers of the retailer may discover the manufacturer's product only through browsing and have a lower valuation. As already noted, in this scenario the assumptions $\hat{x} > 0$ and $\hat{v} > 0$ imply $r_A > r_B > 0$ and:

$$\rho \equiv \frac{r_B}{r_A} > \frac{s_B}{s_A} \equiv \sigma.$$

To simplify the exposition, we restrict attention to the case in which no firm can serve all consumers. That is, letting:

$$\bar{x}_i \equiv \frac{r_i}{s_i},$$

denote firm i's marginal demand, we focus on the case $(\bar{x}_A <)\bar{x}_B \leq X$. None of our qualitative results hinges on this assumption, but it helps to convey our insights in a concise way.

3.1 Uniform Pricing by A

We first analyze the cases in which A cannot engage in personalized pricing. We obtain the following result:

Proposition 1: *If A charges a uniform price, then dual distribution is the unique optimal strategy, regardless of B's pricing regime.*

Proof: See Appendix A.

We now sketch the arguments underlying Proposition 1. If only A is active, it faces the monopoly demand $(r_A - p_A)/s_A$; it thus charges the monopoly price $p_A^m = r_A/2$, serves consumers $x \leq x_A^m = r_A/(2s_A)$, and obtains a profit of (the subscript U stands for *Uniform pricing*):

$$\Pi_U^m = \frac{r_A^2}{4s_A}.$$

Consider now the situation in which A and B are both active and charge a uniform price. They then charge retail prices p_A and p_B such that some consumers favor A whereas others favor B. Let $x_{AB} > 0$ denote the consumer indifferent between buying from A or B, and $x_B > 0$ denote the consumer indifferent between buying from B and not buying:

$$x_{AB}(p_A, p_B) = \frac{r_A - p_A - r_B + p_B}{s_A - s_B} \quad \text{and} \quad x_B(p_B) = \frac{r_B - p_B}{s_B}.$$

Any consumer $x < x_{AB}$ prefers A to B; hence, the demands for A and B are, respectively, x_{AB} and $x_B - x_{AB}$. The profit functions of the two firms (gross of the fixed fee) are then $\Pi_A = x_{AB}(p_A, p_B)p_A + [x_B(p_B) - x_{AB}(p_A, p_B)]\, w$ and $\Pi_B = [x_B(p_B) - x_{AB}(p_A, p_B)]\,(p_B - w)$.

In the first stage, A and B negotiate over w and F, following the Nash bargaining solution, taking into account that, in the second stage, each firm sets its retail price so as to maximize its own profit. This implies that the firms set w to maximize the industry profit and use F to share it according to their bargaining powers and outside options.[20] Denoting the equilibrium prices at the retail stage by $p_i(w)$, the maximization problem with respect to w is:

$$\max_{w}\ x_{AB}(p_A(w), p_B(w))p_A(w) + [x_B(p_B(w)) - x_{AB}(p_A(w), p_B(w))]\, p_B(w)$$

Dual distribution is optimal if the resulting industry profit is larger than A's profit from mono distribution, Π_U^m. As shown in Proposition 1, this is indeed always the case under uniform pricing.

We illustrate the intuition by Figure 2. To see that dual distribution leads to higher industry profit than mono distribution, note first that the firms can replicate the outcome of mono distribution by negotiating $w^m = v_B(x_A^m)$. This induces A to charge the monopoly price p_A^m and prevents B, which must charge at least w^m, from profitably attracting any consumer. Indeed, consumers with $x > x_A^m$ are not willing to pay w^m, and those with $x < x_A^m$ prefer A's product at price $p_A^m = v_A(x_A^m)$ to B's product at any price $p_B \geq w^m = v_B(x_A^m)$. Consider now a small reduction in the wholesale price, $w < w^m$, that generates a retail equilibrium in which B serves some consumers at price $p_B = w^m - dp$. In this retail equilibrium, B cannot obtain a negative profit and A cannot obtain less than what it would earn by charging $\hat{p}_A = p_A^m - dp$, so as to maintain its market share, $x_A = x_A^m$. Doing so would lead B to sell a quantity dx_B implicitly given by $dp \equiv -v_B'(x_A^m)\, dx_B$. Hence, the industry profit cannot be lower than $\pi_A + \pi_B \geq [(p_A^m - dp)\, x_A^m + w dx_B] + 0 \simeq \Pi_U^m + v_B(x_A^m)dx_B - x_A^m dp = \Pi_U^m + \frac{d}{dx}[v_B(x)\, x]\big|_{x=x_A^m} dx_B$, which exceeds Π_U^m: as B faces a more elastic monopolistic demand (that is, $|v_B'(x)|/v_B(x) < |v_A'(x)|/v_A(x)$),[21] its monopolistic output exceeds x_A^m (that is, $\frac{d}{dx}[v_B(x)\, x]\big|_{x=x_A^m} > 0$).

[20]Specifically, A's outside option is its profit from mono distribution whereas B's outside option is zero.

[21]This is due the fact that $\rho > \sigma$.

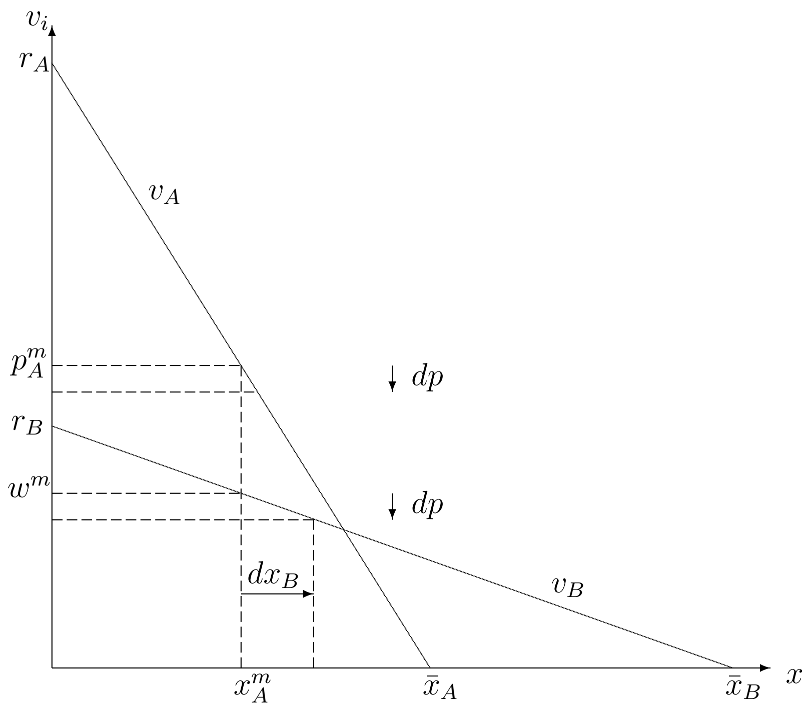

Figure 2: Uniform pricing

This shows that the industry profit is always larger if B is marginally active. This insight does not hinge on the demand being linear; it holds more generally as long as B's monopolistic output exceeds that of A.[22] Note however that the *equilibrium* wholesale price may be substantially lower than w^m, so that B's market share may also be substantial.

We now consider the scenario in which A still charges a uniform price but B charges personalized prices. The intuition why dual distribution is optimal in this case is illustrated by Figure 3, which depicts the equilibrium prices under uniform pricing, p_A^* and p_B^*, and the retail prices of B that would emerge if the two firms opted for dual distribution and set $w = p_B^*$ and $p_A = p_A^*$ (i.e., the equilibrium prices under uniform pricing).[23]

A then serves consumers $x < x_A^*$ (for whom $v_A(x) - v_B(x) = w$), whereas B serves consumers between x_A^* and x_B^* (for whom $v_B(x) = w$), and charges them prices equal to min $\{p_A^* + v_B(x) - v_A(x), v_B(x)\}$. The resulting industry profit is larger than under uniform pricing: in the segment served by A, the profit is the same because $p_A = p_A^*$; by contrast, in the segment served by B, B charges a strictly higher price than p_B^*. Because opting for dual distribution was already optimal with uniform pricing, and yields even more profits in the regime with personalized pricing by B, it also dominates mono

[22]We provide a proof of this statement in Online Appendix I.

[23]Remember that A acts as a price leader in this regime: w and p_A are chosen before B sets its retail prices.

distribution in the latter regime.

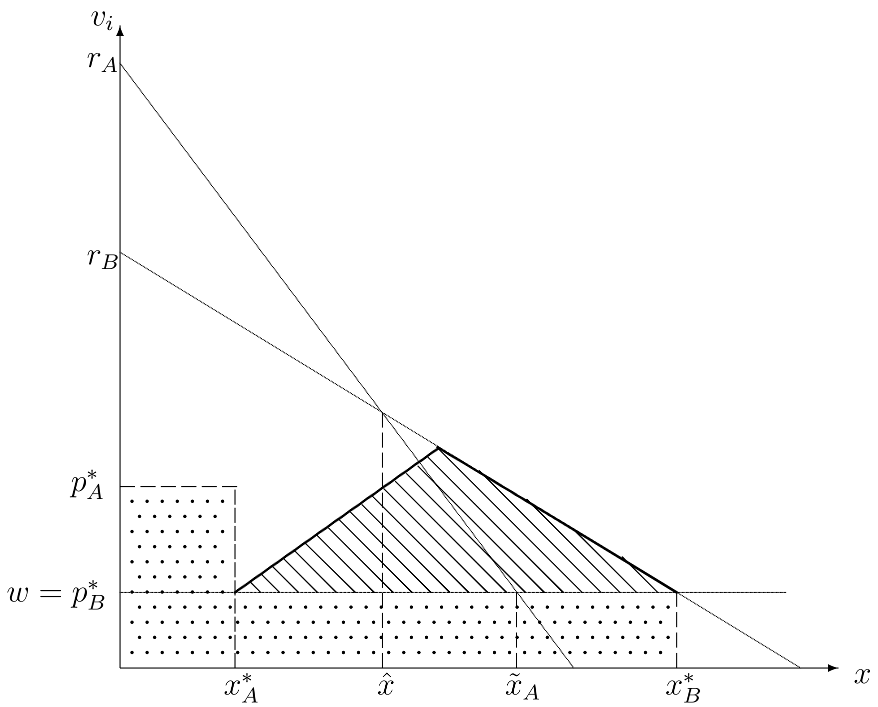

Figure 3: Profit with personalized pricing by B only if $w = p_B^*$

3.2 Personalized Pricing by A

We now turn to the cases in which A charges personalized prices. As in case of uniform pricing by A, we first state the main result, and then sketch the arguments behind it.

Proposition 2: *If A charges personalized prices, then mono distribution is the unique optimal strategy if and only if:*

$$\rho < \sigma \frac{2 + \delta(\sigma)\sqrt{1-\sigma}}{1+\sigma}, \tag{2}$$

where $\delta(\sigma) = 1$ if B charges personalized prices as well and $\delta(\sigma) = \sqrt{\frac{2+5\sigma+\sigma^2}{1+2\sigma}} > 1$ if B charges a uniform price.[24]

Proof: See Appendix B.

If B is not active, then A charges each consumer x a price $v_A(x)$, and thus obtains a

[24]The threshold given by (2) exceeds σ (hence, mono distribution is indeed optimal for some values of ρ) and even exceeds 1 if σ is large enough (in which case mono distribution is optimal for all relevant values of ρ).

profit of (the subscript P stands for *Personalized pricing*):

$$\Pi_P^m = \int_0^{\bar{x}_A} (r_A - s_A x)\, dx = \frac{r_A^2}{2s_A}. \tag{3}$$

We first consider the symmetric regime in which both firms charge personalized prices, starting with the retail stage (regime PP in what follows).

Retail competition. As firms now compete for each consumer x, three cases can arise.

If $v_B(x) < w$, then B cannot offer a positive value to consumers without incurring a loss; A then charges the monopoly price $p_A = v_A(x)$.

If instead $v_A(x) < w \leq v_B(x)$, A would have to price below w to win the consumer, and is therefore better off letting B serve this consumer; hence, B wins the competition by charging the monopoly price $v_B(x)$ (and A charges a price equal to its opportunity cost w, or any other price exceeding $v_A(x)$).

Finally, when $w \leq v_A(x), v_B(x)$. A's profit from such a consumer type is either $p_A(x)$, if A serves the consumer itself, or w, if instead B serves the consumer. As a consequence, w constitutes A's opportunity cost from serving the consumer. As w is B's real cost, a standard Bertrand argument applies: for consumers x with $v_i(x) > v_j(x)$, for $i \neq j \in \{A, B\}$, firm i wins the competition and sells to the consumer at price $p_i(x) = w + v_i(x) - v_j(x)$, whereas the other firm sets $p_j(x) = w$.[25]

Wholesale negotiation. We now turn to the determination of the wholesale contract. We first note that for any wholesale price w above $\hat{v}$, B is inactive in equilibrium: it is dominated by A in the consumer segment $x < \hat{x}$, and cannot offer a positive value at a profitable price in the segment $x > \hat{x}$. The profit thus cannot exceed Π_P^m.

If the firms negotiate a wholesale price $w \leq \hat{v}$, they are both active in the continuation equilibrium. Let:

$$\tilde{x}_i(w) \equiv \frac{r_i - w}{s_i}, \tag{4}$$

denote the marginal consumer willing to buy product i at price w. The profits of the two firms at the retail stage can be expressed as Π_A and Π_B, where:[26]

$$\Pi_A = \int_0^{\hat{x}} [w + v_A(x) - v_B(x)]\, dx + w\, [\tilde{x}_B(w) - \hat{x}],$$

and:

$$\Pi_B = \int_{\hat{x}}^{\tilde{x}_A(w)} [v_B(x) - v_A(x)]\, dx + \int_{\tilde{x}_A(w)}^{\tilde{x}_B(w)} [v_B(x) - w]\, dx.$$

[25] Both firms set a price of w to consumer $\hat{x}$.

[26] As we show in Appendix B, conditional on reaching an agreement (i.e., $w < \hat{v}$), the firms negotiate a wholesale price so that B expands potential demand; that is, B sells to consumers who would not be interested in buying from A at any positive price (i.e., w is sufficiently low that $\tilde{x}_B(w) > \bar{x}_A$).

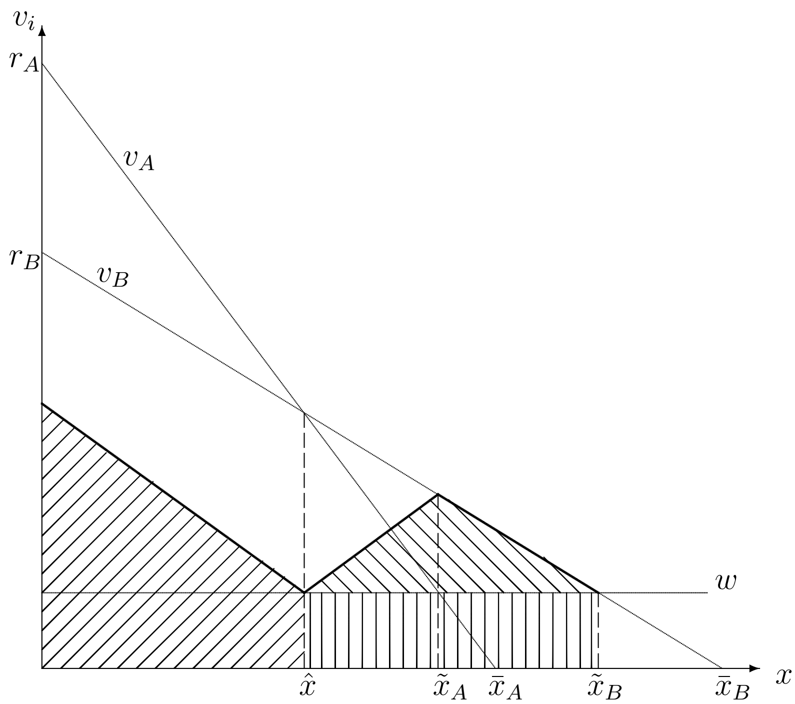

Figure 4: Industry profit in the pricing regime PP

These profit functions are illustrated by Figure 4, where the hatched area represents the industry profit. The first term in A's profit comes from consumers $x < \hat{x}$ (first region in Figure 4): A offers them a higher net value, and serves them at price $w + v_A(x) - v_B(x)$. The second term in A's profit reflects the wholesale revenue generated by consumers served by B (the other two rectangles in the figure). B's profit comes from consumers for whom it offers a higher net value, and can also be split in two parts. The first term corresponds to consumers $\hat{x} < x < \tilde{x}_A(w)$ (second triangle), for whom both firms compete, and so B only earns a margin $v_B(x) - v_A(x)$. The second term corresponds to consumers $\tilde{x}_A(w) < x < \tilde{x}_B(w)$ (third triangle), to whom A offers a lower net value than w, and so B can extract the full value and earn a margin $v_B(x) - w$.

In the negotiation in the first stage, the firms maximize the industry profit:

$$\Pi(w) = \int_0^{\tilde{x}_A(w)} [w + |v_B(x) - v_A(x)|] \, dx + \int_{\tilde{x}_A(w)}^{\tilde{x}_B(w)} v_B(x) dx.$$

Taking the derivative with respect to w (and using $v_i(\tilde{x}_i(w)) = w$ for $i = A, B$) yields:

$$\Pi'(w) = \tilde{x}_A(w) + w\tilde{x}'_B(w). \tag{5}$$

When negotiating on the wholesale price w, firms face the following trade-off. By increasing w, the firms obtain a higher benefit from the inframarginal consumers in the range $x < \tilde{x}_A(w)$: as the two firms compete for these consumers, an increase in w increases the final consumer price by the same amount. However, increasing w has also a negative effect on the marginal consumer, $\tilde{x}_B(w)$, for whom B can charge the full value, $v_B(\tilde{x}_B(w)) = w$. By contrast, the revenue from consumers between $\tilde{x}_A(w)$ and $\tilde{x}_B(w)$ is unchanged, as these consumers continue buying from B and pay their reservation price.

Using (4), the first-order condition $\tilde{x}_A(w) + w\tilde{x}'_B(w) = 0$ yields (the subscript PP stands for *Personalized pricing by A and B*):[27]

$$w^*_{PP} = \frac{s_B r_A}{s_A + s_B}.$$

The associated industry profit is (recalling the notation $\rho \equiv r_B/r_A \in (0,1)$ and $\sigma \equiv s_B/s_A \in (0,\rho)$):

$$\Pi^*_{PP} = \frac{r_A^2}{2s_A} \frac{\sigma(1+3\sigma) - 4\sigma(1+\sigma)\rho + (1+\sigma)^2 \rho^2}{\sigma(1-\sigma^2)},$$

which is smaller than the monopoly profit Π^m_P given by (3) if and only if (2) holds.

In contrast to the case with uniform pricing by A, mono distribution may indeed occur when both downstream firms can price discriminate, which happens if ρ is sufficiently small. This result holds despite the fact that, with personalized pricing, the two firms share the market efficiently: consumers $x < \hat{x}$ (resp., $x > \hat{x}$) buy from A (resp., B). This was not true under uniform pricing, as B then sets a lower price and therefore also sells to consumers who have a relative preference for A's product. However, personalized pricing also allows a firm to lower the price charged to marginal consumers down to marginal production cost without sacrificing profit on inframarginal ones. This has two implications: first, B can serve additional consumers, and thereby expand the market, and second, B prices more aggressively in A's core market. This in turn makes A more aggressive. Competition is thus more intense, which dissipates profits; whenever this effect prevails, the firms do not reach an agreement, resulting in mono distribution. Instead, under uniform pricing, firms tend to focus on exploiting their market power over consumers in their respective core markets, which leads to relatively high prices, and dual distribution.

The equilibrium configurations under personalized pricing by A are depicted in Figure 5. Note first that under uniform pricing, dual distribution is optimal in the whole range $\rho > \sigma$ (i.e., the range in which B adds value to the industry).[28] By contrast, the optimal distribution choice under personalized pricing depends on the specific val-

[27] The profit function is concave as $\Pi''(w) = -(1/s_A + 1/s_B) < 0$.

[28] The figure also depicts the range $0 \le \rho < \sigma$, in which B does not add value, and, hence, mono distribution is always optimal.

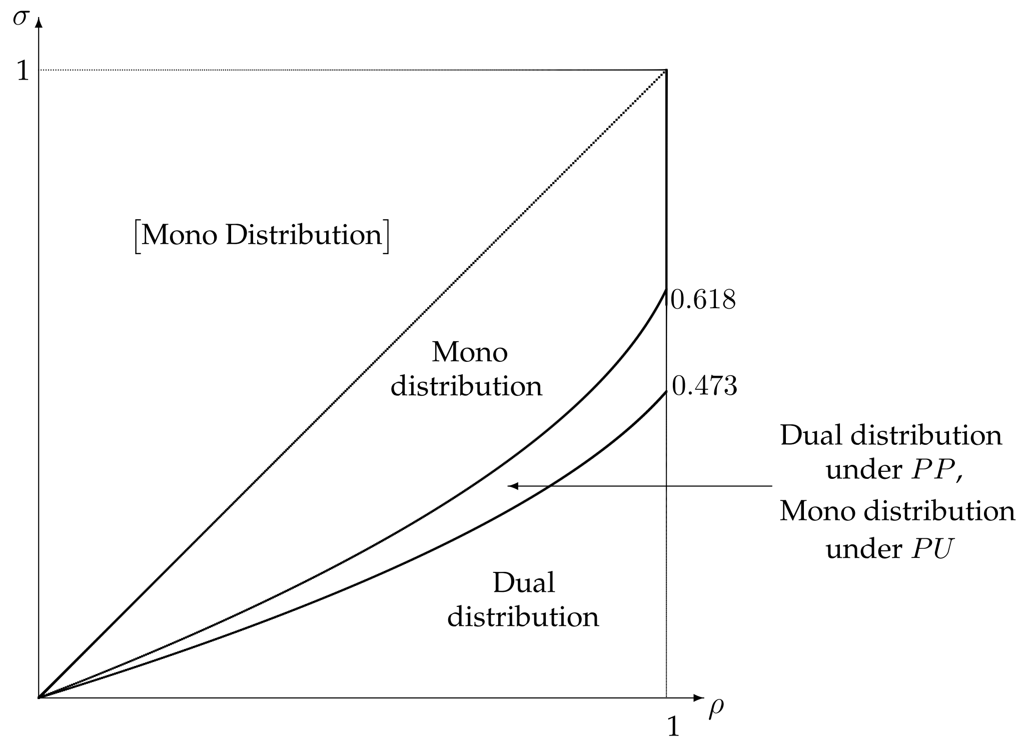

Figure 5: Equilibrium configurations in the pricing regimes PP and PU

ues of ρ and σ. The condition stated in Proposition 2 *(ii)* shows that mono distribution is more likely to be optimal, the lower the net additional value being brought by B, namely, the lower the relative intercept of B's demand function (as measured by ρ) and the steeper the relative slope (as measured by σ). In fact, for $\delta(\sigma) = 1$, condition (2) always holds if $\sigma \geq (\sqrt{5} - 1)/2 \approx 0.618$, as the right-hand side is then larger than 1. For $\sigma < (\sqrt{5} - 1)/2$, the right-hand side is strictly increasing in σ.

Finally, we turn to the situation in which only A charges personalized prices (regime PU in what follows). As stated in Proposition 2, mono distribution is again optimal if ρ is sufficiently small. The intuition is similar to the case with personalized pricing by both firms—i.e., dual distribution leads to competition and destroys profit from the high-valuation consumers to a sufficiently large extent that the industry profit is higher with mono distribution. However, as $\delta(\sigma)$ then exceeds 1, mono distribution is optimal for a larger range in the hybrid regime. In particular, as illustrated in Figure 5, mono distribution is then always optimal if $\sigma \geq 0.473$. This result emerges despite the fact that competition is less intense in the hybrid regime due to the fact that B charges only a uniform price. The intuition behind the result is as follows: with symmetric personalized pricing, the firms can benefit from B's ability to extract consumer rent, particularly so if σ is relatively small. By contrast, if B cannot charge personalized prices, this ability is reduced. In addition, B always demands a mark-up on the wholesale price, which implies that B serves fewer low-valuation consumers even if wholesale prices in both regimes were the same. Hence, the industry profit with dual distribution is lower in the hybrid regime, which leads to a larger range in which mono distribution is optimal.

An interesting insight emerging from our analysis concerns firms' market shares. Under uniform pricing, dual distribution is always optimal, regardless of firms' market shares. By contrast, under personalized pricing, dual distribution is not optimal under personalized pricing when B would have a small market share (e.g., ρ close to 0 or σ close to 1). The reason is that B would then add little value to the industry but still compete for high-valuation consumers. Table 1 illustrates this with a numerical example. We set $\sigma = 0.4$ and then report B's market shares for different values of ρ under uniform pricing by both firms and personalized pricing by both firms.[29]

ρ	Uniform Pricing	Personalized Pricing
0.5	0.27	0
0.6	0.45	0
0.7	0.59	0
0.8	0.69	0.74
0.9	0.77	0.89

Table 1: Market shares of firm B in the different regimes

It is worth noting that our insights are not driven by the common wisdom that personalized pricing intensifies competition and dissipates profit. First, this common wisdom, obtained in the context of horizontal differentiation, does not apply to the vertical differentiation pattern studied here. In Online Appendix G, we show that, for low values of σ, the industry profit under inter-brand competition,[30] is actually larger when both firms charge personalized rather than uniform prices. Second, whether personalized pricing increases or reduces profit under inter-brand competition provides little guidance for the optimal distribution strategy. In particular, mono distribution can be optimal even if personalized pricing would increase both firms' profits.

As mentioned in the introduction, our results are in line with the distribution strategies observed in different industries. For example, in the apparel and perfume industry, brand manufacturers often rely exclusively on direct distribution channels—as in case of the cosmetics brands The Body Shop, Glossier, or Pixi—or distribute only a limited part of their product portfolios through independent retailers. In the same vein, several recent antitrust cases were triggered by high-end brand suppliers—e.g., the perfume seller Coty or the sport shoe manufacturer Asics—forbidding sales through third-party websites. These markets are all characterized by (i) a demand pattern in which consumers who have a high willingness-to-pay for the brand also prefer the

[29] A similar comparison can be made when considering the hybrid regimes.

[30] In our setting, this amounts to setting the wholesale price equal to the marginal production cost.

direct distribution channel, and (ii) a widespread use of multiple forms of behavior-based price discrimination.

In addition, brands such as Nike or Adidas have started to limit the range of products sold through independent retailers. Nike, who already terminated a partnership with Amazon in December 2019, is now planning to reduce the number of its other partners and focus instead on direct-to-consumer avenues (e.g., the Nike app and Nike.com).[31] Similarly, Adidas lowered the selection of its products sold on Amazon in 2017 and plans to continue this strategy.[32] As the possibility of customizing prices or discounts has grown significantly in recent years,[33] these plans are consistent with our prediction that personalized pricing tends to favor mono distribution.

3.3 Endogenous Pricing Regime

In this section, we extend our model by considering the situation in which both pricing policies (i.e., personalized pricing and uniform pricing) are available to both firms, and firms endogenously decide on the pricing regime in their negotiation.

When being able to negotiate the pricing regime, the contract at the wholesale stage consists of four elements: the per-unit price w, the fixed fee F, A's pricing policy, and B's pricing policy. The firms choose these variables to maximize the industry profit, conditional on retail prices being chosen individually by each firm later. Throughout the section, we are particularly interested in the question whether or not firms benefit from not using personalized pricing.

The next proposition characterizes the equilibrium pricing regimes that occur for different parameter constellations:

Proposition 3: *If firms can contract on their pricing policies, then:*
(i) dual distribution together with personalized pricing by both firms is optimal if and only if:

$$\sigma\frac{2+\sqrt{1-\sigma}}{1+\sigma} \leq \rho \leq \frac{1+4\sigma}{3+2\sigma}; \tag{6}$$

(ii) dual distribution together with uniform pricing by A and personalized pricing by B is optimal if and only if:

$$\rho \geq \max\left\{\frac{1+4\sigma}{3+2\sigma}, \sigma+\sqrt{\frac{\sigma\left(1-\sigma^2\right)}{2+\sigma}}\right\};$$

(iii) for all other parameter combinations, mono distribution is optimal.

Proof: See Appendix C.

[31]See Business of Fashion (2019).
[32]See The Wall Street Journal (2019).
[33]Segment (2017), in an online survey, finds that more than 70% of customers expect firms to offer personalized discounts, which is a much larger percentage than a few years ago.

Figure 6 illustrates these insights.[34] If distributing via B does not expand demand significantly (σ large and/or ρ small), then mono distribution maximizes the industry profit, as it avoids downstream competition and allows to better exploit price discrimination. When instead the independent retailer brings enough value, the firms opt for dual distribution. In this case, the firms may benefit from restricting A to a uniform price, but prefer B to charge personalized prices. Specifically, restricting A's pricing policy is optimal when B is a relatively strong competitor for high-valuation consumers (i.e., ρ high enough); this is represented by the lower-right area in the figure. To see why this may occur, note that restricting A's pricing policy leads it to focus on its core market, which dampens the competitive pressure on B, allowing it to extract more surplus from medium-range consumers—i.e., those consumers at the margin between buying from A or from B. This however comes at a cost, as A's ability to extract rents from the high-valuation consumers is impeded.

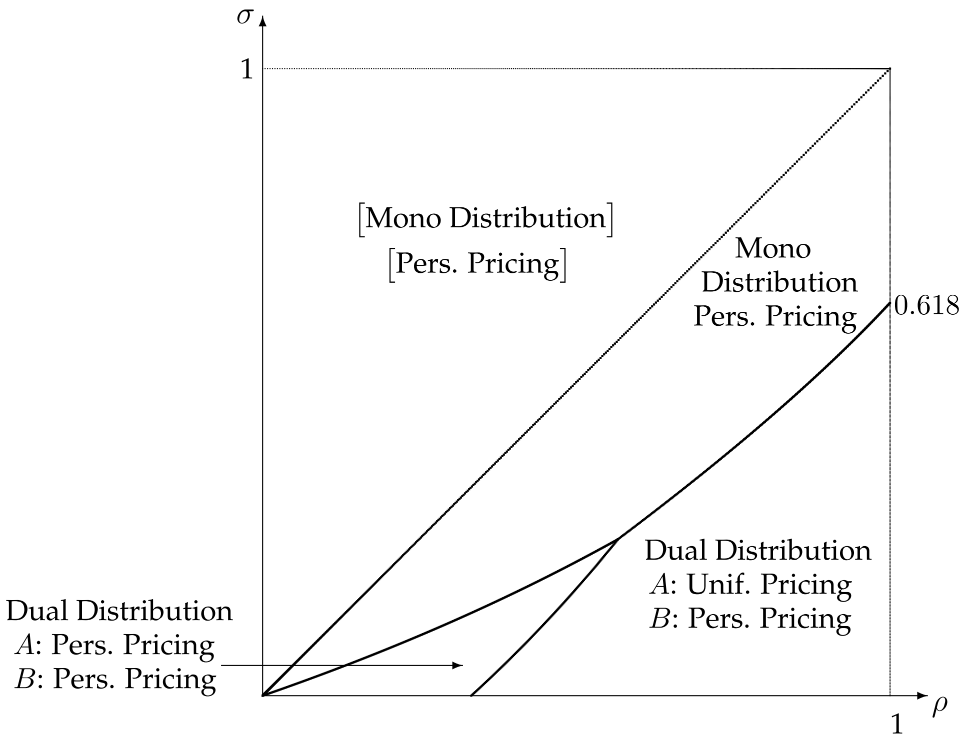

Figure 6: Endogenous pricing regime

3.4 Delegated Distribution

We assumed so far that A always uses its direct channel. In this section, we extend our model by allowing firms to opt for mono distribution by B—i.e., delegated dis-

[34]As the figure shows, the three thresholds of the proposition coincide at $\sigma = \tilde{\sigma} \approx 0.248$, where $\tilde{\sigma}$ is the unique solution to the equation $\sqrt{1-\sigma} = \sigma(3+2\sigma)$ in the range $(0, (\sqrt{5}-1)/2)$. Dual distribution together with personalized pricing by both firms can only be optimal if $\sigma \leq \tilde{\sigma}$, as otherwise (6) cannot hold. Similarly, dual distribution together with uniform pricing by A and personalized pricing by B can only be optimal if $\sigma < \left(\sqrt{5}-1\right)/2$, as otherwise $\sigma + \sqrt{\left(\sigma\left(1-\sigma^2\right)\right)/\left(2+\sigma\right)}$ would be larger than 1.

tribution. By the same logic as before, the firms will find this optimal when doing so maximizes the industry profit. The next proposition shows that delegated distribution can be optimal in each pricing regime:

Proposition 4: *In each pricing regime $XY \in \{UU, UP, PP, PU\}$, there exists a threshold $\rho_{XY}(\sigma)$ such that delegated distribution is optimal if $\rho > \rho_{XY}(\sigma)$.*[35] *Furthermore, these thresholds satisfy:*

$$\rho_{UP}(\sigma) < \rho_{PP}(\sigma) < \rho_{UU}(\sigma) < \rho_{PU}(\sigma).$$

Proof: See Appendix D.

In contrast to mono distribution by A, which can be optimal only if A charges personalized prices, delegated distribution can always be optimal. The intuition is as follows: in all four pricing regimes, delegated distribution is optimal if σ is relatively small, which implies that B increases demand substantially. Therefore, via using only B's channel, firms avoid competition, which allows B to profit most from its large demand. By contrast, A's relative advantage is not to broaden demand but to deliver a higher net value to high-valuation consumers. If A can only set a uniform price, its rent extraction possibility is limited, and mono distribution by A is never optimal.

In addition, the ranges in which delegated distribution is optimal in the different pricing regimes has a clear order. This range is larger if B charges personalized prices rather than a uniform price, and, given B's pricing policy, the range is larger if A can only set a uniform price. This is obvious when comparing mono distribution by A with delegated distribution, as the profit achieved by a firm under mono distribution is maximal when it can extract the entire surplus through personalized pricing. The proposition shows that the insight carries over when considering dual distribution as well.

4 Vertical Differentiation with B offering higher quality

We next consider the demand pattern in which B offers the high-end product (i.e., $s_B > s_A > 0$, or $\sigma > 1$). As mentioned, the assumptions $\hat{x} > 0$ and $\hat{v} > 0$ then imply $\rho \in (1, \sigma)$. The next proposition characterizes the optimal distribution strategy and pricing regime (part *(i)*), as well as the optimality of delegated distribution (part *(ii)*).

Proposition 5: *In the case of vertical differentiation with B offering the high-end product: (i) In the absence of delegated distribution: (a) dual distribution is the optimal strategy in all pricing regimes; (b) if firms can contract on their pricing policies, personalized pricing by both*

[35] $\rho_{PU}(\sigma) < 1$ for $\sigma < 1/2$ and the other thresholds always lie below 1; hence, delegated distribution is indeed optimal for some values of σ and ρ.

firms is optimal.

(ii) If delegated distribution is feasible, in each pricing regime $XY \in \{UU, UP, PP\}$ there exists a threshold $\rho_{XY}(\sigma)$ such that delegated distribution is optimal if $\rho > \rho_{XY}(\sigma)$;[36] *by contrast, it is never optimal in the regime PU. The thresholds can be ordered as follows:*

$$\rho_{UP}(\sigma) < \rho_{PP}(\sigma) < \rho_{UU}(\sigma) \quad \text{if} \quad \sigma < \check{\sigma},$$

$$\rho_{UP}(\sigma) < \rho_{UU}(\sigma) < \rho_{PP}(\sigma) \quad \text{if} \quad \sigma > \check{\sigma},$$

where $\check{\sigma} \equiv \left(3 + \sqrt{33}\right)/2 \approx 4.37$.

Proof: See Online Appendix E.

In contrast to the case studied in the previous section, dual distribution is always optimal when B is the one offering higher quality. To see why, consider first the situation where A charges a uniform price, and suppose that the firms negotiate a wholesale price equal to A's monopoly price under mono distribution, p_U^m. In the continuation equilibrium, A can then secure the monopoly profit Π_U^m by charging p_U^m: this can only increase sales—regardless of the price charged by B—and A obtains the same margin on every sale, regardless of which firm make it. Furthermore, as B can charge a higher price to high-valuation consumers, the industry profit increases compared to mono distribution. The argument carries over when A charges personalized prices, setting the wholesale price to the highest of A's monopoly prices, that is, $w = r_A$—in this case, A is strictly better off if B's makes a sale.

In addition, the industry profit is highest in the regime in which both firms charge personalized prices. Indeed, the optimal wholesale price then exceeds $\hat{v}$—i.e., the crossing point of the consumers' net value functions. This prevents B from competing for consumers in A's core segment, as consumers who have a higher net value for A's product have a net value for B's product that is lower than $\hat{v}$. A thus acts as a monopolist towards these consumers, and can extract more surplus with personalized pricing. A wholesale price above $\hat{v}$ also gives A a large revenue when B serves a consumer. This dampens competition in B's core segment, and there as well, charging personalized prices enables B to increases its profit.

Proposition 5*(i)* confirms that the insights from inter-brand competition provide little guidance in the case of intra-brand competition. As explained above, if A and B were independent firms with no wholesale contract, both personalized pricing and uniform pricing can be optimal. Instead, with an appropriate wholesale contract, dual distribution and personalized pricing is always optimal.

Part *(ii)* of the proposition shows that delegated distribution can be optimal in three

[36] Each threshold lies below σ; hence, delegated distribution is indeed optimal for any $\rho \in (\max\{1, \rho_{XY}(\sigma)\}, \sigma)$.

of the four pricing regimes. The only pricing regime in which this does not occur is PU. In this regime, A can extract the entire consumer surplus from low-valuation consumers whereas B cannot extract the entire surplus from any consumer. This implies that using A's direct channel is always profitable. Conversely, B adds most value relative to mono distribution by A in the regime UP; hence, delegated distribution occurs for the largest parameter range then. Comparing the two symmetric pricing regimes, delegated distribution is optimal for a smaller range in the regime PP than in UU if σ is high, that is, if A can serve a relatively large number of low-valuation consumers; shutting down the direct channel is then less profitable if A can extract their surplus through personalized pricing.

A vertical differentiation pattern in which the retailer caters to high-end consumers can for instance occur when the manufacturer is relatively small or unknown. For example, in the market for organic food, long-established retailers such as iHerb or Wholefoods offer a large variety, whereas manufacturers often specialize on a particular kind of product, and only low-valuation (price-sensitive) customers make the effort of visiting the manufacturers' websites. Consistent with the prediction of our model, in this market the manufacturers usually seek to distribute through independent retailers.

5 Horizontal Differentiation

Finally, we consider the demand pattern of horizontal differentiation (i.e., $s_A > 0 > s_B$, or $\sigma < 0$). The assumptions $\hat{x} > 0$ and $\hat{v} > 0$ then imply $\rho < 1$. The next proposition provides the results for this demand pattern:

Proposition 6: *In the case of horizontal differentiation:*
(i) In the absence of delegated distribution: (a) dual distribution is the optimal strategy in all pricing regimes; (b) if firms can contract on their pricing policies, personalized pricing by both firms is optimal.
(ii) If delegated distribution is feasible, in each pricing regime $XY \in \{UU, PU\}$ (i.e., if B charges a uniform price) there exists a threshold $\rho_{XY}(\sigma)$ such that delegated distribution is optimal if $\rho > \rho_{XY}(\sigma)$, with $\rho_{UU}(\sigma) < \rho_{PU}(\sigma)$;[37] by contrast, delegated distribution is never optimal in the regimes PP and PU (i.e., if B charges personalized prices).

Proof: See Online Appendix F.

In the case of horizontal differentiation, consumers $x \le \hat{x}$ constitute A's core market, whereas consumers $x \ge \hat{x}$ constitute B's core market. Firm can then avoid compe-

[37] Each threshold can lie below 1; hence, delegated distribution is indeed optimal for some values of $\rho < 1$.

tition by agreeing on, say, $w = \hat{v}$: B cannot profitably sell in A's core market, as these consumers have a willingness-to-pay for B's product which is below $\hat{v}$. Conversely, as w constitutes A's opportunity cost when competing against B,[38] A has no incentive to serve B's core market. However, B sells to consumers in its core market at a higher price than A can do with mono distribution. As a consequence, the industry profit with dual distribution is higher than with mono distribution. This holds regardless of the pricing regime.

In particular, if both firms charge personalized prices, setting $w = \hat{v}$ enables them to extract the entire consumer surplus: A sells to consumers $x \leq \hat{x}$ at personalized prices $v_A(x)$ and B sells to $x \geq \hat{x}$ at personalized prices $v_B(x)$. The equilibrium industry profit is thus higher whenever both firms charge personalized prices than in any other pricing regime, which implies that this regime is optimal if firms can negotiate on their pricing policies.[39]

This result is in sharp contrast to the one obtained in the classic papers on personalized pricing (e.g., Thisse and Vives, 1988, and Shaffer and Zhang, 1995), which consider inter-brand competition between independent firms. The possibility of personalized pricing then leads to a prisoner's dilemma in which firms choose personalized pricing but the industry profit is lower than with uniform pricing. Our analysis shows that in the case of intra-brand competition between distribution channels, the result reverses, and a well-designed wholesale contract allows firms to even extract the entire consumer surplus. Therefore, the result again shows that the common wisdom derived from inter-brand competition does not apply to intra-brand competition.

Finally, as stated in part *(ii)* of the proposition, delegated distribution can never be optimal if B charges personalized prices. The intuition is that setting $w = \hat{v}$ allows B to extract the entire consumer surplus in its core market even under dual distribution; hence, shutting down A's channel can not be optimal. By contrast, delegated distribution can be optimal if B charges a uniform price. This occurs if r_B is close to r_A, so that the net value functions intersect at a relatively low x. Setting $w = \hat{v}$ would then exacerbate double marginalization problems, whereas a lower wholesale price opens the door to competition; as a result, shutting down A's channel can become optimal.

Horizontal differentiation can arise when the brand manufacturer and the retailer provide different services, and customers with a similar willingness-to-pay for the manufacturer's good have heterogeneous preferences regarding these services. For example, brick-and-mortar stores and, sometimes, the websites of brand manufacturers

[38] A receives a margin of w whenever B sells to a consumer; w thus represents A's opportunity cost of serving the customer itself.

[39] Chen at al. (20120) also find in a different model that personalized pricing may allow firms to fully extract consumer surplus. In contrast to our model, their result is based on the effect that firms target different consumers.

may allow customers to obtain professional advice from trained salespeople, whereas online retailers may instead offer lower transaction costs (e.g., due to one-click shopping), free return policies or comparison services. Anecdotal evidence supporting our prediction that dual distribution is then likely to occur can be found in the market for consumer electronics, where a large fraction of the product line of many producers (e.g., Sennheiser or Samsung) with a direct channel is also available at independent retailers. Similarly, for jewelry in the medium segment, manufacturers, such as Pandora, offer almost their entire product portfolio not only directly but also through retailers such as Signet Jewelers (in the US) or Christ (in Germany, Austria, and Switzerland). The prevalence of personalized pricing may vary across these markets but this does not affect the distribution strategy.

6 Conclusion

This paper analyzes the effects of personalized pricing on the incentives of a brand manufacturer to opt for dual distribution. Adding an independent distribution channel enables the manufacturer to reach out to different consumer groups but triggers intra-brand competition with its own distribution channel. We show that the profitability of a dual distribution strategy crucially depends on the interplay between the possibility of charging personalized pricing and the demand pattern.

If the offers of manufacturer and retailer are vertically differentiated and the manufacturer caters better to high-valuation consumers—a scenario relatively common in e-commerce—choosing dual distribution is the optimal strategy if the manufacturer charges a uniform price to consumers. If instead the manufacturer charges personalized prices, then the two channels compete more intensely for each type of consumer; this dissipates profits to such an extent that the manufacturer opts for dual distribution only when the independent retailer expands demand significantly. These insights differ substantially from those obtained for competition between independent firms, but are in line with the channel structures observed in different industries. This shows that accounting for the possibility of influencing intra-brand competition through the wholesale contract is important for assessing the effects of personalized pricing.

We also find that a hybrid regime—in which only the retailer charges personalized prices—may lead to the highest industry profit. The manufacturer then extracts less surplus from high-valuation consumers, but benefits from reduced intra-brand competition. This implies that the manufacturer may optimally forgo charging personalized prices even if it has the possibility of doing so.

Finally, we show that if the two channels are horizontally differentiated, or if the independent retailer offers a higher-quality product, then dual distribution is optimal regardless of whether firms set personalized prices or only a uniform one.

An important managerial implication is that the extent to which price discrimination is feasible not only affects the pricing strategy but also the optimal distribution network. With prices becoming more and more finely tuned to consumer tastes, brand manufacturers risk fiercer competition with pure retailers, even if these retailers may appeal to different consumer groups. This calls for a cautious use of new distribution channels when price discrimination is possible at a finely-grained level. This holds particularly for products sold online, for which consumer data and purchase history is available, in case the manufacturers cater foremost to the high-end segment. It can then be more profitable to rely on mono distribution by the manufacturer or by the retailer, in order to avoid intra-brand competition. By contrast, dual distribution is beneficial if price discrimination is hard to achieve.

Another implication is that adopting a non-discriminatory pricing policy can be a profitable strategy for manufacturers. This is particularly true for companies facing the opportunity of distributing their products through a data-intensive retailer, which can perform price discrimination and broaden demand substantially. In that case, not using consumer data for its own distribution channel can achieve the right balance between rent extraction (by the retailer) and the avoidance of fierce intra-brand competition.

We conclude by briefly discussing two interesting avenues for future research emerging from our model. First, we considered a situation in which a firm can set personalized prices to all of its consumers, given that personalized pricing is possible. However, firms may have access to data only from consumers who have previously bought from the firm. A dynamic extension of our model in which firms set a uniform price in early periods but charge personalized prices to its previous customers in later periods can shed light on how price setting to learn about consumer preferences shapes channel design. Second, our model assumes that the brand manufacturer does not face competition from rival manufacturers. This is a reasonable assumption in markets in which brands are strongly differentiated and helps singling out the effects at work in a clear way. Analyzing whether new effects emerge with competition between manufacturers, and the resulting implications for wholesale contracts in such an extended scenario, constitutes a fruitful direction for future research.

Appendix

Appendix A: Proof of Proposition 1

A.1: Uniform pricing by B

We first analyze the situation of uniform pricing by both firms. To solve for the subgame-perfect equilibrium, we proceed by backward induction and first determine the reaction functions in the downstream stage. To simplify the exposition, we proceed under the assumption that both demands are positive in equilibrium and verify later that this is in fact true. The linearity of the demand functions ensures that firms' profit functions are strictly concave in their prices; hence, firms' reaction functions are characterized by the first-order conditions, which yield:

$$\begin{aligned} p_A(p_B; w) &= \frac{r_A - r_B + p_B + w}{2}, \\ p_B(p_A; w) &= \frac{r_B + w}{2} + \frac{s_B\left(p_A - r_A\right)}{2s_A}. \end{aligned}$$

Combining these reaction functions yields the equilibrium retail prices, as a function of the wholesale price w:

$$\begin{aligned} p_A\left(w\right) &= \frac{r_A(2s_A - s_B) + s_A(3w - r_B)}{4s_A - s_B}, \\ p_B\left(w\right) &= \frac{r_B(2s_A - s_B) + w(2s_A + s_B) - r_A s_B}{4s_A - s_B}. \end{aligned} \tag{7}$$

The associated demands are $D_A\left(w\right) = x_{AB}\left(p_A\left(w\right), p_B\left(w\right)\right)$ and $D_B\left(w\right) = x_B\left(p_B\left(w\right)\right) - x_{AB}\left(p_A\left(w\right), p_B\left(w\right)\right)$.

We now turn to the first stage. In the negotiation stage, firms choose w so as to maximize the industry profit, $\Pi\left(w\right) = p_A\left(w\right) D_A\left(w\right) + p_B\left(w\right) D_B\left(w\right)$. This profit is again a strictly concave function of w, as its second-order derivative is given by:

$$\Pi''\left(w\right) = -\frac{2s_A(4s_A + 5s_B)}{s_B(4s_A - s_B)^2} < 0.$$

Hence, the equilibrium wholesale price is characterized by the first-order condition, leading to (the subscript UU stands for *Uniform pricing by A and B*):

$$w^*_{UU} = \frac{s_B(4s_A(r_A + r_B) + r_A s_B)}{2s_A(4s_A + 5s_B)}.$$

Inserting the equilibrium prices into the demand functions D_A and D_B yields:

$$D_A^* = \frac{2s_A^2(2r_A - r_B) + s_B(3s_Ar_A - 4s_Ar_B - s_Br_A)}{2s_A(4s_A + 5s_B)(s_A - s_B)},$$
$$D_B^* = \frac{(2s_A + s_B)(r_Bs_A - r_As_B)}{s_B(4s_A + 5s_B)(s_A - s_B)}.$$

The assumption that the two demand functions intersect at a positive valuation (i.e., $r_A/s_A < r_B/s_B$) ensures that both equilibrium demands are positive. Indeed, D_A^* is strictly falling in r_B and is equal to $r_A(2s_A + s_B)/(2s_A(4s_A + 5s_B)) > 0$ at the highest possible value of r_B, which is $r_B = r_A$. Direct inspection of D_B shows that it is positive for $r_A/s_A < r_B/s_B$. As w^* constitutes a global maximum in the relevant range, and achieving $D_B = 0$ is feasible with a high enough w, it follows that in equilibrium it is optimal for the firms to generate positive sales for B. Indeed, the resulting profit, equal to:

$$\Pi_{UU}^* = \frac{r_A^2 s_B \left(5s_As_B + 4s_A^2 - s_B^2\right) + 4s_Ar_B \left(s_A + s_B\right)\left(s_Ar_B - 2s_Br_A\right)}{4s_As_B \left(s_A - s_B\right)\left(4s_A + 5s_B\right)}, \tag{8}$$

exceeds the monopoly profit that A can obtain with mono distribution, Π_U^m:

$$\Pi_{UU}^* - \Pi_U^m = \frac{(s_A + s_B)(s_Ar_B - r_As_B)^2}{s_As_B(4s_A + 5s_B)(s_A - s_B)} > 0.$$

A.2: Personalized pricing by B

We next analyze the case in which B charges personalized prices (and A still a uniform one). Given w and p_A, B's price response is such that consumers x with $v_A(x) - p_A > v_B(x) - w$, or:

$$x < \breve{x}\left(w, p_A\right) = \frac{r_A - p_A - r_B + w}{s_A - s_B},$$

end-up buying from A. Instead, consumers $\breve{x}\left(w, p_A\right) < x < \tilde{x}_B\left(w\right)$ end-up buying from B at price $p_B\left(x\right) = v_B\left(x\right) - \max\left\{v_A\left(x\right) - p_A, 0\right\}$. A's variable profit (gross of the fee F) is therefore given by:

$$p_A\breve{x}\left(w, p_A\right) + w\left[\tilde{x}_B\left(w\right) - \breve{x}\left(w, p_A\right)\right].$$

Optimizing this with respect to p_A yields:

$$p_A\left(w\right) = w + \frac{r_A - r_B}{2}.$$

We now turn to the wholesale stage. The two firms seek to maximize the industry profit given by:

$$\Pi = p_A(w)\check{x}(w) + \int_{\check{x}(w)}^{\hat{x}(w)} \left[p_A(w) + v_B(x) - v_A(x)\right] dx + \int_{\hat{x}(w)}^{\tilde{x}_B(w)} v_B(x)\, dx, \tag{9}$$

where $p_A(w) = w + (r_A - r_B)/2$, $\check{x}(w) = (r_A - p_A - r_B + w) / (s_A - s_B)$, $\hat{x}(w) = (r_A + r_B) / (2s_A) - w/s_A$, and $\tilde{x}_B(w) = (r_B - w)/s_B$. Maximizing this profit with respect to w yields (the subscript UP stands for the pricing regime in which A sets a *Uniform price* and B *Personalized prices*):[40]

$$w_{UP}^* = \frac{s_B(r_A + r_B)}{2(s_A + s_B)}.$$

Inserting $w = w_{UP}^*$ into (9), we obtain that the industry profit is given by:

$$\begin{aligned}\Pi_{UP}^* &= \frac{r_A^2 s_A s_B + 2r_A^2 s_B^2 - 4r_A r_B s_A s_B - 2r_A r_B s_B^2 + 2r_B^2 s_A^2 + r_B^2 s_A s_B}{4s_A^2 s_B - 4s_B^3} \\ &= \frac{r_A^2}{s_A} \frac{\sigma + 2\sigma^2 - 4\rho\sigma - 2\rho\sigma^2 + 2\rho^2 + \rho^2\sigma}{4\sigma\left(1 - \sigma^2\right)}.\end{aligned}$$

We next show that this profit exceeds the profit obtained under uniform pricing, which, from (8), can be written as:

$$\Pi_{UU}^* = \frac{r_A^2}{s_A} \frac{4\sigma + 5\sigma^2 - \sigma^3 - 8\rho\sigma - 8\rho\sigma^2 + 4\rho^2 + 4\rho^2\sigma}{4\sigma\left(1 - \sigma\right)\left(4 + 5\sigma\right)}.$$

We have:

$$\Pi_{UP}^* - \Pi_{UU}^* = \frac{\left(\sigma - \rho\right)^2 r_A^2}{4\sigma s_A} \frac{4 + 6\sigma + \sigma^2}{\left(4 + 5\sigma\right)\left(1 - \sigma^2\right)} > 0.$$

This establishes that the profit in the regime with uniform pricing by A and personalized pricing by B is larger than under uniform pricing by both firms.

We know from above that dual distribution is optimal in case both firms set uniform prices. Because the industry profit in the regime in which only B charges personalized prices (i.e., Π_{UP}^*) is larger than Π_{UU}^*, it must also be larger than the profit with mono distribution.

Appendix B: Proof of Proposition 2

B.1: Personalized pricing by B

We start with the situation of personalized pricing by both firms. As noted in the main text, if the wholesale price w is such that $w \geq \hat{v}$, B will be inactive;[41] hence, the industry profit cannot be larger than Π_P^m. Using the notation $\rho \equiv r_B/r_A \in (0, 1)$ and

[40] It is straightforward to check that the industry profit is a concave function of w.

[41] Recall that $\hat{v} = v_A(\hat{x}) = v_B(\hat{x})$.

$\sigma \equiv s_B/s_A \in (0, \rho)$, the threshold $\hat{v}$ is:

$$\hat{v} \equiv \frac{s_A r_B - s_B r_A}{s_A - s_B} = r_A \frac{\rho - \sigma}{1 - \sigma}.$$

We now focus on $w \leq \hat{v}$. We need to distinguish whether or not B finds it profitable to supply (some) consumers uninterested in A's product. From Figure 3, such consumers exist if and only if $\tilde{x}_B(w) > \bar{x}_A$. The latter inequality can only hold if w is sufficiently low, that is:

$$w < \underline{w} \equiv r_B - \frac{s_B}{s_A} r_A = r_A (\rho - \sigma).$$

Note that $\underline{w} = (1 - \sigma)\hat{v} < \hat{v}$.

Region $w \leq \underline{w}$

In this region, in which $\tilde{x}_B(w) \geq \bar{x}_A$, as shown in the text, the industry profit is given by:

$$\Pi(w) = \int_0^{\tilde{x}_A(w)} [w + |v_B(x) - v_A(x)|]\, dx + \int_{\tilde{x}_A(w)}^{\tilde{x}_B(w)} v_B(x) dx.$$

It is strictly concave in w: using $v_A(\tilde{x}_A(w)) = v_B(\tilde{x}_B(w)) = w$, we have:

$$\Pi'(w) = \tilde{x}_A(w) + w \frac{d\tilde{x}_B}{dw}(w) = \frac{r_A - w}{s_A} - \frac{w}{s_B} = \frac{r_A}{s_A}\left(1 - \frac{1+\sigma}{\sigma}\frac{w}{r_A}\right),$$

and thus (as $\tilde{x}_A(w)$ and $\tilde{x}_B(w)$ are both linear and strictly decreasing in w):

$$\Pi''(w) = \frac{d\tilde{x}_A}{dw}(w) + \frac{d\tilde{x}_B}{dw}(w) < 0.$$

Region $\underline{w} < w \leq \hat{v}$

If instead $w > \underline{w}$, the industry profit includes an additional term, as illustrated by Figure A.1. This term corresponds to consumers in the region $\tilde{x}_B(w) < x \leq \bar{x}_A$: B does not find it profitable to supply these consumers (as $v_B(x) < w$), but they are still willing to buy from A, which can extract their full surplus. The industry profit can then be written as:

$$\Pi(w) = \int_0^{\tilde{x}_A(w)} [w + |v_B(x) - v_A(x)|]\, dx + \int_{\tilde{x}_A(w)}^{\tilde{x}_B(w)} v_B(x) dx + \int_{\tilde{x}_B(w)}^{\bar{x}_A} v_A(x) dx.$$

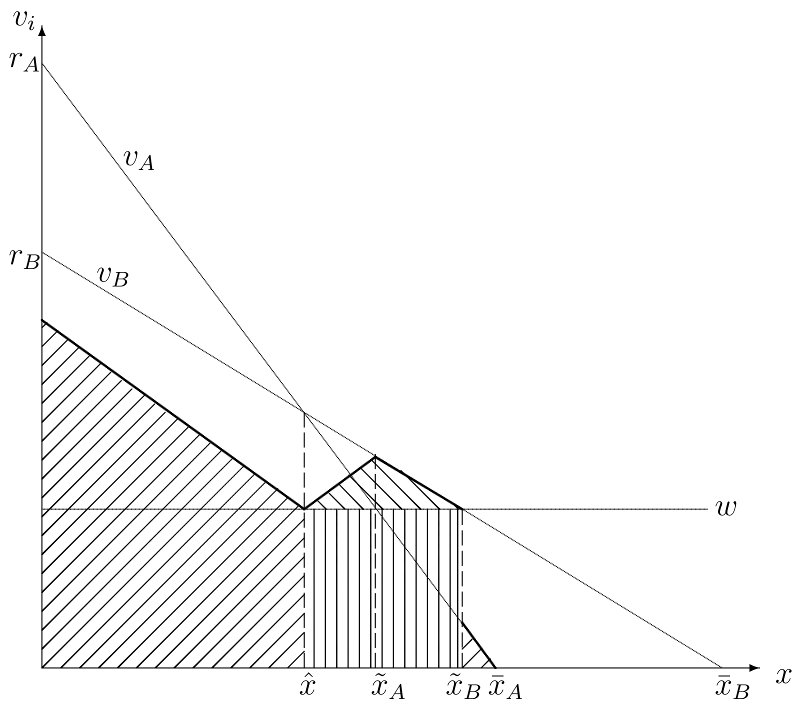

Figure A1: Profits if $w > \underline{w}$

The first-order derivative is equal to:

$$\begin{aligned}
\Pi'(w) &= \tilde{x}_A(w) + \left[w - v_A\left(\tilde{x}_B(w)\right)\right] \frac{d\tilde{x}_B}{dw}(w) \\
&= (r_A - w)\left(\frac{1}{s_A} + \frac{1}{s_B}\right) - (r_B - w)\frac{s_A}{s_B^2} \\
&= \frac{r_A}{s_A} \frac{\sigma^2 + \sigma - \rho + \left(1 - \sigma - \sigma^2\right)\frac{w}{r_A}}{\sigma^2}.
\end{aligned}$$

Hence:

$$\begin{aligned}
\Pi'_-\left(\hat{v}\right) &= \frac{r_A}{s_A}\frac{1-\rho}{1-\sigma}, \\
\Pi'_+\left(\underline{w}\right) &= \frac{r_A}{s_A}\frac{1+\sigma}{\sigma}\left(\sigma\frac{2+\sigma}{1+\sigma} - \rho\right), \\
\Pi''\left(w\right) &= \frac{1-\sigma-\sigma^2}{s_A\sigma^2}.
\end{aligned}$$

It follows that $\Pi\left(w\right)$ is strictly concave in w if:

$$\sigma > \hat{\sigma} = \frac{\sqrt{5}-1}{2} \simeq 0.618,$$

and is instead weakly convex if $\sigma \leq \hat{\sigma}$; in addition, $\Pi'\left(\hat{v}\right) > 0$ whereas $\Pi'\left(\underline{w}\right) \geq 0$ if and only if:

$$\rho \leq \hat{\rho}(\sigma) \equiv \sigma \frac{2+\sigma}{1+\sigma},$$

where $\hat{\rho}(\sigma)$ increases with σ and exceeds 1 for $\sigma \geq \hat{\sigma}$. Furthermore, not only is the profit function $\Pi(w)$ continuous at $w = \underline{w}$, its derivative $\Pi'(w)$ is also continuous:

$$\Pi'_{-}(\underline{w}) = \frac{r_A}{s_A}\left(1 - \frac{1+\sigma}{\sigma}\frac{w}{s_A}\right)\bigg|_{w=r_A(\rho-\sigma)} = \frac{r_A}{s_A}\frac{1+\sigma}{\sigma}\left(\sigma\frac{2+\sigma}{1+\sigma} - \rho\right) = \Pi'_{+}(\underline{w}).$$

Optimal distribution policy

As long as $w \geq \hat{v}$, B cannot attract any consumer at any profitable price: hence, doing so cannot be more profitable than mono distribution. Furthermore, if $\rho \leq \hat{\rho}(\sigma)$, then $\Pi'(\underline{w}) \geq 0$, implying that dual distribution cannot be more profitable than mono distribution:

- in the range $\underline{w} \leq w \leq \hat{v}$, the profit function $\Pi(w)$ is increasing, as it is quadratic and its derivative is non-negative at both ends of the range (namely, $\Pi'(\underline{w}) \geq 0$ and $\Pi'(\hat{v}) > 0$);
- in the range $w \leq \underline{w}$, the profit function $\Pi(w)$ is again increasing, as it is concave and its derivative is non-negative at the upper end of the range (namely, $\Pi'(\underline{w}) \geq 0$);
- it follows that the profit achieved under dual distribution cannot exceed $\Pi(\hat{v})$, which is less profitable than mono distribution.

As already noted, $\hat{\rho}(\sigma)$ is increasing in σ in the range $\sigma \in [0,1]$, and satisfies $\hat{\rho}(\sigma) \geq 1$ for $\sigma \geq \hat{\sigma}$. It follows that, if $\sigma \geq \hat{\sigma}$, then dual distribution cannot be more profitable than mono distribution, as we then have $\hat{\rho}(\sigma) \geq 1\ (> \rho)$.

If instead $\sigma < \hat{\sigma}$ and $\rho > \hat{\rho}(\sigma)$, then $\Pi'(\underline{w}) < 0$. From the analysis for the region $w \leq \underline{w}$ above, the first-order condition $\Pi'(w) = 0$ then determines the candidate optimal wholesale price, which is given by:

$$w = w^*_{PP} = \frac{s_B r_A}{s_A + s_B} \equiv \frac{\sigma r_A}{1+\sigma} \in (0, \underline{w}).$$

The corresponding profit is:

$$\Pi^*_{PP} = \frac{r_A^2}{2s_A}\frac{\sigma(1+3\sigma) - 4\sigma(1+\sigma)\rho + (1+\sigma)^2\rho^2}{\sigma(1-\sigma^2)}.$$

Compared with the profit from mono distribution, Π_P^m, dual distribution introduces a change in profit equal to:

$$\frac{r_A^2}{2s_A}\frac{\sigma^2(\sigma+3)-4\sigma(1+\sigma)\rho+(1+\sigma)^2\rho^2}{\sigma(1-\sigma^2)}.$$

The numerator of this expression is a convex quadratic polynomial of ρ and its roots are:

$$\sigma\frac{2-\sqrt{1-\sigma}}{1+\sigma} \text{ and } \sigma\frac{2+\sqrt{1-\sigma}}{1+\sigma}.$$

Furthermore, $\hat{\rho}(\sigma)$ lies between these two roots in the relevant range $\sigma<\hat{\sigma}$:

$$\begin{aligned}\frac{\sigma\frac{2-\sqrt{1-\sigma}}{1+\sigma}}{\hat{\rho}(\sigma)} &= \frac{\sigma\frac{2-\sqrt{1-\sigma}}{1+\sigma}}{\sigma\frac{2+\sigma}{1+\sigma}}=\frac{2-\sqrt{1-\sigma}}{2+\sigma}<1,\\ \frac{\sigma\frac{2+\sqrt{1-\sigma}}{1+\sigma}}{\hat{\rho}(\sigma)} &= \frac{\sigma\frac{2+\sqrt{1-\sigma}}{1+\sigma}}{\sigma\frac{2+\sigma}{1+\sigma}}=\frac{2+\sqrt{1-\sigma}}{2+\sigma}>1,\end{aligned}$$

where the last inequality stems from $\sqrt{1-\sigma}>\sigma$ in the relevant range $\sigma<\hat{\sigma}$. It follows that dual distribution is more profitable than mono distribution if and only if $\sigma<\hat{\sigma}$ and ρ exceeds the larger root, that is, if:

$$\rho>\tilde{\rho}(\sigma)\equiv\sigma\frac{2+\sqrt{1-\sigma}}{1+\sigma}$$

Note that $\tilde{\rho}(\sigma)$ is increasing in σ in the range $\sigma\leq\hat{\sigma}$, and exceeds 1 in the range $\sigma\geq\hat{\sigma}$. Hence, as $\rho<1$, the condition $\rho>\tilde{\rho}(\sigma)$ implies $\sigma<\hat{\sigma}$. Finally, $\tilde{\rho}(\sigma)$ is equivalent to the right-hand side of (2) with $\delta(\sigma)=1$.

B.2: Uniform pricing by B

We now move to the hybrid regime in which A charges personalized prices and B a uniform one. Again, we solve the game by backward induction. Consider first A's price response to a given w and p_B. Two market share configurations can occur, dependent on the value of p_B. When p_B is relatively small, the marginal consumer indifferent between buying from B and not buying, $x_B=(r_B-p_B)/s_B$, exceeds $\bar{x}_A$. A's best response is to serve consumers x with $v_A(x)-w>v_B(x)-p_B$, or:

$$x<x_{AB}(w)\equiv\frac{r_A-w-r_B+p_B}{s_A-s_B},$$

whereas consumers x with $x_{AB}(w)<x<x_B$ end-up buying from B.

Instead, when p_B is high enough so that $x_B<\bar{x}_A$, a third demand region exists between x_B and $\bar{x}_A$, in which consumers end-up buying from A. The thresholds for the first two demand regions are the same as in the market share configuration above.

We start with the first case. If B serves consumers x with $x_{AB}(w) < x < x_B$, its profit is:

$$(p_B - w)\left(\frac{r_B - p_B}{s_B} - \frac{r_A - w - r_B + p_B}{s_A - s_B}\right).$$

Maximizing with respect to p_B yields:

$$p_B\,(w) = \frac{s_A(r_B + w) - s_B(r_A - w)}{2s_A}. \tag{10}$$

We now turn to the negotiation at the wholesale stage. Because A charges to each consumer x a personalized price of $r_A - s_A x - r_B + s_B x + p_B(w)$, the industry profit is:

$$\int_0^{\frac{r_A - w - r_B + p_B(w)}{s_A - s_B}} [r_A - s_A x - r_B + s_B x + p_B(w)]\,dx + p_B(w)\left(\frac{r_B - p_B(w)}{s_B} - \frac{r_A - w - r_B + p_B(w)}{s_A - s_B}\right), \tag{11}$$

with $p_B(w)$ given by (10). Maximizing (11) with respect to w yields (the subscript PU stands for the pricing regime in which A sets *Personalized prices* and B a *Uniform price*)[42]

$$w^*_{PU} = \frac{s_B\left[r_A(2s_A + s_B) + r_B s_A\right]}{2s_A^2 + 5s_A s_B + s_B^2}.$$

This market configuration is only valid if $x_B \geq \bar{x}_A$. Comparing the two thresholds at the equilibrium values, we obtain that the inequality is fulfilled if and only if:

$$r_B \geq \frac{r_A s_B(2s_A^2 + 4s_A s_B + s_B^2)}{s_A^2(s_A + 2s_B)}$$

or, equivalently,

$$\rho \geq \frac{\sigma(2 + 4\sigma + \sigma^2)}{1 + 2\sigma}. \tag{12}$$

As ρ is bounded above by 1, this inequality can only be satisfied if $\sigma^2(4+\sigma) - 1 \leq 0$ (which is approximately equivalent to $\sigma \leq 0.473$). Inserting $w = w^*_{PU}$ in (11), the resulting industry profit is:

$$\begin{aligned}
\Pi^*_{PU} &= \frac{s_A^3 r_B^2 + s_A^2 s_B(2r_A^2 + 3r_B^2 - 4r_A r_B) + s_A s_B^2(5r_A^2 + 2r_B^2 - 8r_A r_B) - s_B^3 r_A^2}{2s_B(s_A - s_B)(2s_A^2 + 5s_A s_B + s_B^2)} \\
&= \frac{r_A^2}{s_A}\frac{\rho^2 + \sigma(2 - 4\rho + 3\rho^2) + \sigma^2(5 - 8\rho + 2\rho^2) - \sigma^3}{2\sigma(1-\sigma)(2 + 5\sigma + \sigma^2)}.
\end{aligned}$$

Compared with the profit from mono distribution, Π^m_P, the profit from dual distribu-

[42] The maximization problem is strictly concave.

tion is larger if and only if:[43]

$$\rho \geq \check{\rho}(\sigma) \equiv \sigma \frac{2+\sqrt{\frac{(1-\sigma)(2+5\sigma+\sigma^2)}{1+2\sigma}}}{1+\sigma}. \tag{13}$$

Note that $\check{\rho}(\sigma)$ is increasing in σ in the range $\sigma \in (0, 0.473)$ and exceeds 1 for $\sigma > 0.473$. As $\rho < 1$, the condition $\rho \geq \check{\rho}(\sigma)$ implies $\sigma < 0.473$. Moreover, $\check{\rho}(\sigma)$ is indeed larger than the right-hand side of (12) for $\sigma < 0.473$. Hence, if the first market configuration is valid, both firms are active if and only if (13) holds.

We now turn to the second market configuration. In this case, the industry profit is:

$$\int_0^{\frac{r_A-w-r_B+p_B(w)}{s_A-s_B}} \left[r_A - s_A x - r_B + s_B x + p_B(w)\right] dx \tag{14}$$

$$+p_B(w)\left(\frac{r_B-p_B}{s_B} - \frac{r_A-w-r_B+p_B(w)}{s_A-s_B}\right) + \int_{\frac{r_B-p_B}{s_B}}^{\frac{r_A}{s_A}} \left[r_A - s_A x\right] dx,$$

with $p_B(w)$ again given by (10). Maximizing with respect to w, we obtain that the second-order condition for an interior solution is fulfilled if and only if $\sigma^2(4+\sigma)-1 > 0$, resulting in a wholesale price of:

$$\frac{r_A s_B^3 + s_A s_B^2(3r_A + r_B) + s_A^2 s_B(r_A - r_B) - r_B s_A^3}{s_B^3 + 4s_B^2 s_A - s_A^3}.$$

However, at this wholesale price, B's demand is negative for $\sigma^2(4+\sigma)-1 > 0$. As a consequence, in case the maximization problem is concave, mono distribution is optimal. Instead, if $\sigma^2(4+\sigma)-1 \leq 0$, the maximization problem (14) is convex. It follows that w is optimally either set so high that B is not active, which results in mono distribution, or that w is set so low that $x_B \geq \bar{x}_A$.

In the latter case, the first market configuration is valid if (12) holds. Instead, if (12) is not fulfilled, the optimal w is set such that x_B exactly equals $\bar{x}_A$, or $(r_B - p_B(w))/s_B = r_A/s_A$, with $p_B(w)$ given by (10). Solving the last equation for w yields $w = (s_A r_B - s_B r_A)/(s_A + s_B)$. The resulting industry profit is:

$$\frac{r_A^2}{s_A} \frac{1+\rho^2 - 2\sigma\rho(1-\rho) - \sigma^2(1+4\rho) + 2\sigma^3 + \sigma^4}{2(1-\sigma)(1+\sigma)^2}.$$

Compared with Π_P^m, the profit with dual distribution is larger if and only if:

$$\rho \geq \sigma + (1+\sigma)\sqrt{\frac{\sigma(1-\sigma)}{1+2\sigma}}. \tag{15}$$

[43] Because Π_{PU}^* is a convex quadratic polynomial in ρ, the equation $\Pi_{PU}^* - \Pi_P^m = 0$ has two roots. The lower one is below zero, and the larger one is $\check{\rho}(\sigma)$.

However, the right-hand side of (15) is larger than the right-hand side of (12) for all values of σ, with $\sigma^2(4+\sigma)-1 \leq 0$. Since the profit function with $x_B = \bar{x}_A$ is only valid if (12) does not hold, (15) cannot be fulfilled for any admissible value of σ. Hence, mono distribution is optimal is this case.

As a consequence, dual distribution is more profitable than mono distribution if and only if $\rho \geq \check{\rho}(\sigma)$, which can only hold if $\sigma < 0.473$. Note that $\check{\rho}(\sigma)$ is equivalent to the right-hand side of (2) with $\delta(\sigma) = \sqrt{\frac{2+5\sigma+\sigma^2}{1+2\sigma}}$. Since:

$$\sqrt{\frac{2+5\sigma+\sigma^2}{1+2\sigma}} > 1,$$

mono distribution is optimal for a larger range in the hybrid regime—i.e., the regime in which only B charges a uniform price—compared to the symmetric regime in which both firms charge personalized prices.

Appendix C: Proof of Proposition 3

If firms can negotiate the pricing regime at the wholesale stage, they chose the one that maximizes the industry profit. As shown in the proof of Proposition 2, $\Pi^*_{UP} > \Pi^*_{UU}$, regardless of the values of ρ and σ. Therefore, firms will never choose the pricing regime of uniform pricing.

We next compare the two regimes in which A charges personalized prices—i.e., the regime in which B charges personalized prices as well and the regime in which B charges only a uniform price. If dual distribution is optimal in each of the two regimes, the respective profits are:

$$\Pi^*_{PP} = \frac{r_A^2}{2s_A} \frac{\sigma(1+3\sigma) - 4\sigma(1+\sigma)\rho + (1+\sigma)^2\rho^2}{\sigma(1-\sigma^2)}$$

and:

$$\Pi^*_{PU} = \frac{r_A^2}{s_A} \frac{\rho + \sigma(2-4\rho+3\rho^2) + \sigma^2(5-8\rho+2\rho^2) - \sigma^3}{2\sigma(1-\sigma)(2+5\sigma+\sigma^2)}.$$

Taking the difference yields:

$$\Pi^*_{PP} - \Pi^*_{PU} = \frac{r_A^2}{s_A} \frac{(1+3\sigma+\sigma^2)(\rho(1+\sigma)-2\sigma)^2}{2\sigma(1-\sigma^2)(2+5\sigma+\sigma^2)} > 0.$$

Moreover, as shown in Proposition 2, dual distribution is optimal for a larger range in the regime with personalized pricing of both firms than in the regime in which only A charges personalized prices. Because the profit from mono distribution is the same in both regimes and the profit from dual distribution is higher in case dual distribution is optimal, it follows that the regime with personalized pricing by both firms (weakly) dominates the hybrid regime. Therefore, firms do not choose the latter regime.

The preceding arguments imply that firms will either choose the regime in which they both charge personalized prices or the one in which only B charges personalized prices. In the latter regime, the industry profit is:

$$\Pi_{UP}^{*}=\frac{r_A^2}{s_A}\frac{\sigma+2\sigma^2-4\rho\sigma-2\rho\sigma^2+2\rho^2+\rho^2\sigma}{4\sigma\left(1-\sigma^2\right)}.$$

Instead, whenever both firms offer personalized prices, dual distribution is optimal if:

$$\rho>\tilde{\rho}(\sigma)\equiv\frac{\sigma\left(2+\sqrt{1-\sigma}\right)}{(1+\sigma)},$$

which, together with $\rho<1$, implies $\sigma<\hat{\sigma}=(\sqrt{5}-1)/2$. The industry profit from dual distribution is then equal to:

$$\Pi_{PP}^{*}=\frac{r_A^2}{s_A}\frac{\sigma(1+3\sigma)-4\rho\sigma(1+\sigma)+\rho^2(1+\sigma)^2}{2\sigma(1-\sigma^2)}.$$

Therefore:

$$\begin{aligned}\Pi_{PP}^{*}>\Pi_{UP}^{*}\quad&\Leftrightarrow\quad\frac{r_A^2}{s_A}\frac{\sigma(1+3\sigma)-4\rho\sigma(1+\sigma)+\rho^2(1+\sigma)^2}{2\sigma(1-\sigma^2)}>\frac{r_A^2}{s_A}\frac{\sigma+2\sigma^2-4\rho\sigma-2\rho\sigma^2+2\rho^2+\rho^2\sigma}{4\sigma\left(1-\sigma^2\right)}\\&\Leftrightarrow\quad\rho<g\left(\sigma\right)\equiv\frac{1+4\sigma}{3+2\sigma}.\end{aligned}$$

It follows that dual distribution together with personalized pricing by both firms is optimal if and only if:

$$\frac{\sigma\left(2+\sqrt{1-\sigma}\right)}{(1+\sigma)}\leq\rho\leq\frac{1+4\sigma}{3+2\sigma}$$

If instead $\rho\leq\tilde{\rho}(\sigma)$, then the industry profit with personalized pricing is the mono distribution profit $\Pi_{PP}^{m}=r_A^2/(2s_A)$, and thus:

$$\Pi_{UP}^{*}>\Pi_{P}^{m}\Longleftrightarrow\frac{r_A^2}{s_A}\frac{\sigma+2\sigma^2-4\rho\sigma-2\rho\sigma^2+2\rho^2+\rho^2\sigma}{4\sigma\left(1-\sigma^2\right)}>\frac{r_A^2}{2s_A}$$

$$\Longleftrightarrow\rho>\sigma+\sqrt{\frac{\sigma\left(1-\sigma^2\right)}{2+\sigma}}\quad\text{or}\quad\rho<\sigma-\sqrt{\frac{\sigma\left(1-\sigma^2\right)}{2+\sigma}}.$$

Because $\rho>\sigma$, the only relevant case is:

$$\rho>h\left(\sigma\right)\equiv\sigma+\sqrt{\frac{\sigma\left(1-\sigma^2\right)}{2+\sigma}}.$$

It is easy to check that $\sigma\left(2+\sqrt{1-\sigma}\right)/\left(1+\sigma\right)<h\left(\sigma\right)<g\left(\sigma\right)$ (resp., $\sigma\left(2+\sqrt{1-\sigma}\right)/\left(1+\sigma\right)>h\left(\sigma\right)>g\left(\sigma\right)$) for $\sigma<\tilde{\sigma}$ (resp., $\sigma>\tilde{\sigma}$), where $\tilde{\sigma}\simeq0.248$ is the unique solution in $(0,\hat{\sigma})$

to:

$$\sqrt{1-\tilde{\sigma}} = \tilde{\sigma}\left(3+2\tilde{\sigma}\right).$$

It follows that dual distribution together with uniform pricing by A and dual distribution by B is optimal if and only if:

$$\rho > \max\left\{g\left(\sigma\right), h\left(\sigma\right)\right\} = \begin{cases} g\left(\sigma\right) & \text{if } \sigma \leq \tilde{\sigma}, \\ h\left(\sigma\right) & \text{if } \tilde{\sigma} < \sigma < \hat{\sigma}. \end{cases}$$

It also follows from the preceding analysis that mono distribution is optmal if and only if:

$$\rho < \min\left\{\frac{\sigma\left(2+\sqrt{1-\sigma}\right)}{1+\sigma}, h\left(\sigma\right), 1\right\} = \begin{cases} \frac{\sigma\left(2+\sqrt{1-\sigma}\right)}{1+\sigma} & \text{if } \sigma \leq \tilde{\sigma}, \\ h\left(\sigma\right) & \text{if } \tilde{\sigma} < \sigma < \hat{\sigma}, \\ 1 & \text{if } \hat{\sigma} < \sigma < 1. \end{cases}$$

Appendix D: Proof of Proposition 4

We start the analysis with the regime in which both firms set uniform prices. If A's channel is shut down, B sets the monopoly price in the retail market, equal to $r_B/2$, and the industry profit is $r_B^2/(4s_B)$. Comparing this profit with the industry profit under dual distribution, which is given Π^*_{UU}, yields that delegated distribution gives a higher industry profit if and only if:

$$\rho \geq \rho_{UU} \equiv \frac{4(1+\sigma)-(1-\sigma)\sqrt{4+5\sigma}}{3+5\sigma}. \tag{16}$$

It is straightforward to check that this inequality always holds at $\rho = 1$ but is never fulfilled at $\rho = \sigma$. In addition, $\rho_{UU} \in (0,1)$ for all $\sigma \in (0,1)$, that is ρ_{UU} is in the interior of the admissible range.

Second, we analyze the pricing regime UP. Without the presence of A's channel, B extracts the entire surplus in the retail market, which implies that the industry profit is $r_B^2/(2s_B)$. Instead, with dual distribution, the industry profit is Π^*_{UP}. Comparing the two profits yields:

$$\frac{r_B^2}{2s_B} \geq \Pi^*_{UP} \quad \Leftrightarrow \quad \rho \geq \rho_{UP} \equiv \frac{2+\sigma-\sqrt{3(1+\sigma)(1-\sigma)}}{1+2\sigma}. \tag{17}$$

This inequality always holds at the upper bound $\rho = 1$. At the lower bound $\rho = \sigma$, the inequality is also fulfilled if $\sigma \geq 1/2$. Hence, delegated distribution is optimal in this regime for all admissible values of ρ if $\sigma \geq 1/2$, and, for $\rho \geq \rho_{UP}$ if $\sigma < 1/2$.

Third, we turn to the regime PU. Without A's channel, the industry profit is

$r_B^2/(4s_B)$ because B charges only a uniform price. Instead, if A's channel is open, dual distribution is optimal if and only if:

$$\rho \geq \sigma \frac{2 + \sqrt{\frac{(1-\sigma)(2+5\sigma+\sigma^2)}{1+2\sigma}}}{1+\sigma} \tag{18}$$

and leads to a profit of Π_{PU}^*; otherwise, mono distribution by A is optimal with a profit of $\Pi_P^m = r_A^2/(2s_A)$. Comparing Π_P^m with the profit from delegated distribution (i.e., $r_B^2/(4s_B)$) yields that the latter is larger if and only if $\rho \geq \sqrt{2\sigma}$. As the upper bound of ρ equals 1, this inequality can only be fulfilled if $\sigma \leq 0.5$. Because this comparison is only relevant if (18) does not hold, we need to check if $\sqrt{2\sigma}$ is smaller than the right-hand side of (18). Thus is true if and only if $0.357 \lessapprox \sigma$. It follows that for $0.357 \lessapprox \sigma \leq 0.5$, delegated distribution is optimal if $\rho \geq \sqrt{2\sigma}$, whereas for $\sigma > 0.5$, delegated distribution can never be optimal.

Instead, if dual distribution is optimal in case A's channel is open, we obtain:

$$\frac{r_B^2}{4s_B} \geq \Pi_{PU}^* \quad \Leftrightarrow \quad \rho \geq \frac{4(1+2\sigma) - (1-\sigma)\sqrt{2(2+5\sigma+\sigma^2)}}{3+8\sigma+\sigma^2}. \tag{19}$$

This threshold is larger than the one of the right-hand side of (19) if and only if $0 \leq \sigma < 0.357$, approximately. Hence, delegated distribution is optimal if $\rho \geq \rho_{PU}$, with:

$$\rho_{PU} \equiv \begin{cases} \frac{4(1+2\sigma)-(1-\sigma)\sqrt{2(2+5\sigma+\sigma^2)}}{3+8\sigma+\sigma^2} & \text{if} \quad 0 < \sigma \lessapprox 0.357; \\ \sqrt{2\sigma} & \text{if} \quad 0.357 \gtrapprox \sigma < 1; \end{cases} \tag{20}$$

Finally, proceeding in the same way for the regime PP, we obtain that delegated distribution is optimal if:

$$\rho_{PP} \equiv \begin{cases} 1 - \sqrt{\frac{1-\sigma}{2(1+\sigma)}} & \text{if} \quad 0 < \sigma \lessapprox 0.157; \\ \sqrt{\sigma} & \text{if} \quad 0.157 \gtrapprox \sigma < 1; \end{cases} \tag{21}$$

We now compare the ranges for which delegated distribution is optimal in the four pricing regimes. We start with a comparison of the regimes PU with UU. In the latter, delegated distribution is optimal if $\rho \geq \rho_{UU}$ holds, whereas in the former delegated distribution is optimal if $\rho \geq \rho_{PU}$. We start with the case $0 \leq \sigma < 0.357$. The difference:

$$\rho_{PU} - \rho_{UU} \quad \Leftrightarrow \quad \frac{4(1+2\sigma) - (1-\sigma)\sqrt{2(2+5\sigma+\sigma^2)}}{3+8\sigma+\sigma^2} - \frac{4(1+\sigma) - (1-\sigma)\sqrt{4+5\sigma}}{3+5\sigma}$$

equals 0 at the lower bound $\sigma = 0$. Instead, at the upper bound it is approximately equal to 0.034. The difference is also increasing in σ, which implies that it is positive for all σ between 0 and 0.357. In the range, $0.357 < \sigma \leq 1$, the relevant comparison is:

$$\sqrt{2\sigma} - \frac{4(1+\sigma) - (1-\sigma)\sqrt{4+5\sigma}}{3+5\sigma}.$$

It is easy to check that this difference is again equal to 0.034 at the lower bound. At the upper bound, it is equal to $\sqrt{2} - 1 > 0$. It is also increasing for all values of σ in the range between 0.357 and 1. It follows that $\rho_{PU} > \rho_{UU}$, which implies that the range in which delegated distribution is optimal in the regime PU is a subset of the one in the regime UU.

Next, we compare ρ_{UU} with ρ_{PP}. In the latter regime, we need to distinguish whether σ is below or above approximately 0.157. We start again with the former case. The difference:

$$\rho_{UU} - \rho_{PP} \quad \Leftrightarrow \quad \frac{4(1+\sigma) - (1-\sigma)\sqrt{4+5\sigma}}{3+5\sigma} - 1 - \sqrt{\frac{1-\sigma}{2(1+\sigma)}}$$

equals $1/\sqrt{2} - 1/3 > 0$ at $\sigma = 0$, it is approximately equal to 0.339 at $\sigma = 0.157$, and increasing for all values of σ between 0 and 0.157. Turning to the range $0.157 < \sigma \leq 1$, the relevant comparison is:

$$\frac{4(1+\sigma) - (1-\sigma)\sqrt{4+5\sigma}}{3+5\sigma} - \sqrt{\sigma},$$

which is approximately equal to 0.339 at $\sigma = 0.157$, decreasing in σ for $\sigma \in (0.157, 1]$, and equal to 0 at $\sigma = 1$. It follows that $\rho_{UU} > \rho_{PP}$.

Finally, we compare ρ_{PP} with ρ_{UP}, noting that in the regime UP, delegated distribution maximizes the industry profit for all $\sigma \geq 1/2$. We start with the region $0 \leq \sigma < 0.157$. At the lower bound $\sigma = 0$, the difference $\rho_{PP} - \rho_{UP}$ equals $\sqrt{3} - 1 - 1/\sqrt{2} = 0.025 > 0$, and the upper bound $\sigma = 0.157$, this difference is approximately 0.056. The difference is also increasing in $\sigma \in [0, 0.157)$, which implies that it is positive in the entire range. Turning to the range $0.157 < \sigma \leq 0.5$, the difference is approximately equal to 0.056 at $\sigma = 0.157$, is decreasing in σ for $\sigma < 1$, and equals 0 at $\sigma = 1$; hence the difference is positive for $\sigma \in [0, 0.157)$. It follows that $\rho_{PP} > \rho_{UP}$.

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Online Appendix
— not submitted for publication —

Online Appendix E: Proof of Proposition 5

We start with part *(i)*, statement *(a)*—i.e., we show that dual distribution is the optimal strategy in all pricing regimes (in the absence of delegated distribution). Following the same structure as in the case in which A offers the high-quality product, we analyze first the situation in which A sets a uniform price and then the one in which A sets personalized prices.

E.1: Uniform pricing by A

Suppose that the firms agreed on the wholesale tariff $F = 0$ and $w = p_A^m$.

Assume first that B also charges a uniform price. In the continuation equilibrium, B then charges a price $p_B > w = p_A^m$ on all consumers served and obtains a positive share the market,[44] whereas A charges some price p_A^+.[45] Suppose now that A deviates and charges the mono-distribution price p_A^m. Following this deviation, total demand (weakly) exceeds that of the mono-distribution outcome (as consumers have more choice) and A obtains a margin p_A^m (either directly through p_A or indirectly through w). It follows that B's profit is strictly positive after the deviation, and A's profit is weakly larger. Therefore, in the continuation equilibrium in which A best responds to p_B, the same result must hold. As a consequence, the tariff $F = 0$ and $w = p_A^m$ yields a continuation equilibrium that strictly increases B's profit and weakly increases A's profit.

The same reasoning applies to the case in which B charges personalized prices. In the continuation equilibrium, after firms agreed on a tariff $F = 0$ and $w = p_A^m$, B's demand is positive, and A charges some price p_A^{++}. A deviation to $p_A = p_A^m$ then gives A a profit that is weakly larger than Π_A^m due to the fact that it obtains the same price p_A^m on all consumers served and demand weakly exceeds that with mono distribution.

[44] To see this, suppose instead that B does not attract any consumer. In that case, B prices at cost (i.e., $p_B = w = p_A^m$) and A prices so as to attract consumers with type $x = 0$ (i.e., $p_A \leq p_A^m - (r_B - r_A) < p_A^m$), and obtains a profit lower than Π_A^m (as it charges $p_A \neq p_A^m$, and B attracts no additional consumer). But then, A would profitably deviate by charging $p'_A = p_A^m$: compared with the mono distribution outcome, this would (weakly) expand demand (as consumers can now buy from both firms) and A would obtain the same margin on each customer (either directly through p'_A, or indirectly through w); hence, the deviation brings a profit of at least Π_A^m.

[45] Compared with mono distribution, B now charges a "lower" price (the mono distribution outcome can be interpreted as B charging $p_B \geq r_B$), and in response A also lowers its own price.

E.2: Personalized pricing by A

Under mono distribution, A charges each consumer a price $v_A(x)$. Suppose now that the firms agree on the two-part tariff $F = 0$ and $w = r_A$. In the continuation equilibrium, B obtains a positive demand from consumers close enough to $x = 0$: these consumers have a net willingness-to-pay for B's product equal to $v_B(x)$, which is larger than B's wholesale price $w = r_A$, and A earns a higher margin by letting B serve these consumers, as $w = r_A > v_A(x)$. Therefore, A charges prices larger than or equal to w to the consumers served by B, and obtains a higher margin compared to mono distribution. This holds regardless of whether B charges personalized prices or a uniform price. In addition, B does not offer a positive net utility to consumers it does not serve: it it did, B would serve these consumers because A is better off by letting B serve these consumers and get a margin $w > v_A(x)$ instead of serving the consumers itself. It follows that A can still charge a price of $p_A(x) = v_A(x)$ to these consumers. Hence, A obtains the same profit as under mono distribution from consumers that it serves but a strictly larger profit from consumers served by B. Therefore, the two-part tariff $F = 0$ and $w = r_A$ increases the profits of both firms.

E.3: Endogenous Pricing Policy

We next prove statement *(b)* of part *(i)*, that is, we show that if firms can contract on their pricing policies, they choose the regime of personalized pricing by both firms. To simplify the exposition, we focus on the case in which no firm can serve all consumers, that is $\bar{x}_A < X$. All our results also hold if this inequality was not fulfilled.

We start with the regime in which both firms set a uniform price. Following the analysis in Section 3 and the proof of Proposition 1, we denote the consumer indifferent between buying from B or A by $x_{BA} > 0$, whereby:

$$x_{BA}(p_A, p_B) = \frac{r_B - p_B - r_A + p_A}{s_B - s_A},$$

and the consumer indifferent between buying from A and not buying by $x_A > 0$, whereby:

$$x_B(p_A) = \frac{r_A - p_A}{s_A}.$$

The profit functions (gross of the fixed fee) are then $\Pi_A = x_{BA}(p_A, p_B)w + [x_A(p_A) - x_{BA}(p_A, p_B)]\, p_A$ and $\Pi_B = x_{BA}(p_A, p_B)(p_B - w)$. Solving for the equilibrium retail prices, as a function of the wholesale price w, yields:

$$\begin{aligned} p_A(w) &= \frac{r_A(2s_B - s_A) + s_A(3w - r_B)}{4s_B - s_A}, \\ p_B(w) &= \frac{r_B(2s_B - s_A) + w(2s_B + s_A) - r_A s_B}{4s_B - s_A}. \end{aligned}$$

The associated demands are $D_A(w) = x_A(p_A(w)) - x_{BA}(p_A(w), p_B(w))$ and $D_B(w) = x_{BA}(p_A(w), p_B(w))$. In the first stage, firms choose w so as to maximize the industry profit, $\Pi(w) = p_A(w) D_A(w) + p_B(w) D_B(w)$. Inserting the respective demands and prices, we obtain that equilibrium wholesale price is:

$$w^*_{UU} = \frac{s_B(s_A(r_A + 4r_B) + 4r_A s_B)}{2(5s_A + 4s_B)}.$$

Inserting the equilibrium prices into the industry profit yields:

$$\Pi^*_{UU} = \frac{r_A^2(5s_A s_B + 4s_B^2 - s_A^2) + 4s_A r_B(s_A + s_B)(2r_A - r_B)}{4s_A(s_B - s_A)(5s_A + 4s_B)}$$

We next turn to the case in which only B charges personalized prices. Given w and p_A, B's price response is such that consumers x with $v_B(x) - w > v_A(x) - p_A$, or:

$$x < \frac{r_B - w - r_A + p_A}{s_B - s_A},$$

buy from B. A's maximization problem with respect to p_A is therefore given by:

$$w\left(\frac{r_B - w - r_A + p_A}{s_B - s_A}\right) + p_A\left(\frac{r_A - p_A}{s_A} - \frac{r_B - w - r_A + p_A}{s_B - s_A}\right),$$

leading to an optimal p_A of:

$$p_A(w) = \frac{w s_A}{s_B}\frac{r_A s_B - r_B s_A}{2s_B}.$$

We now turn to the wholesale stage. The two firms seek to maximize the industry profit given by:

$$\Pi = p_A(w)\left(\frac{r_A - p_A(w)}{s_A} - \frac{r_B - w - r_A + p_A(w)}{s_B - s_A}\right) + \int_0^{\frac{r_B - w - r_A + p_A(w)}{s_B - s_A}} [p_A(w) + v_B(x) - v_A(x)]\, dx.$$

Maximizing this profit with respect to w yields $w^*_{UP} = (r_A s_B + r_B s_A) / (2(s_A + s_B))$. Inserting $w = w^*_{UP}$ into the industry profit yields:

$$\Pi^*_{UP} = \frac{r_A^2 s_B(2s_A + s_B) - r_B s_A(s_A + 2s_B)(2r_A - r_B)}{4s_A(s_A + s_B)(s_B - s_A)}.$$

Comparing Π^*_{UP} and Π^*_{UU}, we obtain:

$$\Pi^*_{UP} - \Pi^*_{UU} = \frac{(r_B - r_A)^2 \left(s_A^2 + 4s_B^2 + 6s_A s_B\right)}{4\left(5s_A + 4s_B\right)\left(s_A + s_B\right)\left(s_B - s_A\right)} > 0.$$

Third, we determine the industry profit in case both firms charge personalized prices. Following the analysis in Section 3 and the proof of Proposition 2, taking into account that the optimal wholesale price will be between $\hat{v}$ and r_A, the industry profit in this case can be written as:

$$\Pi = \int_0^{\frac{r_A - w}{s_A}} \left[w + v_B(x) - v_A(x)\right] dx + \int_{\frac{r_A - w}{s_A}}^{\frac{r_B - w}{s_B}} \left[v_B(x)(x)\right] dx + \int_{\frac{r_B - w}{s_B}}^{\frac{r_A}{s_A}} \left[v_A(x)(x)\right] dx.$$

The first term is the profit from consumers close $x = 0$, who have a higher valuation than w for the product of each firm, which implies that the two firms compete for these consumers. The second term is the profit from consumers in the middle, whose valuation for B's product is above w but their valuation for A's product is below w. As w is A's opportunity cost, A is better off when B serves these consumers, which implies that B extracts their entire consumer surplus. Finally, the third term is the profit from consumer whose valuation for B's product is below w, which implies that A can extract their consumer surplus. Maximizing with respect to w, we obtain that the optimal wholesale price is:

$$w^*_{PP} = \frac{r_A s_A \left(s_A + s_B\right) - r_B s_A^2}{s_B^2 - s_A^2 + s_A s_B},$$

leading to an industry profit of:

$$\Pi^*_{PP} = \frac{r_A^2 \left(s_A + s_B\right)^2 - r_B s_A \left(2s_A + s_B\right)\left(2r_A - r_B\right)}{2s_A\left(s_B^2 - s_A^2 + s_A s_B\right)}.$$

We next turn to the regime in which only A sets personalized prices. Following the same steps as in the proof of Proposition 2, A's best response to p_B is to serve all consumers x with $v_A(x) - w > v_B(x) - p_B$. Since, as shown above, B is always active in equilibrium, this implies that A serves consumers x, such that $\left(r_B - p_B\right)/s_B < x \leq r_A/s_A$. Therefore, B's profit at the retail stage is:

$$\left(p_B - w\right)\left(\frac{r_B - p_B}{s_B}\right),$$

leading to an optimal retail price of $p_B(w) = \left(r_B + w\right)/w$. Turning to the wholesale stage, firms maximize the industry profit, given by:

$$p_B(w)\left(\frac{r_B - p_B(w)}{s_B}\right) + \int_{\frac{r_B - p_B(w)}{s_B}}^{\frac{r_A}{s_A}} v_A(x)dx,$$

with respect to w. This yields $w^*_{PU} = (2r_A s_B - r_B s_A) / (2s_B - s_A)$. Inserting this into the industry profit yields:

$$\Pi^*_{PU} = \frac{2r_A^2 s_B + r_B s_A (r_B - 2r_A)}{2s_A (2s_B - s_A)}.$$

Comparing Π^*_{PP} and Π^*_{PU}, we obtain:

$$\Pi^*_{PP} - \Pi^*_{PU} = \frac{(r_B - r_A)^2 (s_B^2 + 2s_A s_B - s_A^2)}{2(s_B^2 - s_A^2 + s_A s_B)(2s_B - s_A)} > 0.$$

Finally, to determine the optimal pricing regime in the negotiation, we need to compare Π^*_{PP} with Π^*_{UP}. The sign of the difference between the two profits is given by:

$$\text{sign}\{\Pi^*_{PP} - \Pi^*_{PU}\} = \text{sign}\left\{r_A^2 (s_B^2 - 2s_A^2)(s_A^2 + s_B^2 + s_A s_B) + r_B s_A^2 (2r_A - r_B)(3s_A^2 - s_B^2 + s_A s_B)\right\}. \tag{22}$$

At the lower bound of s_A—i.e., $s_A = 0$—the right-hand side of (22) is $r_A^2 s_B^4$, which is strictly positive. At the upper bound of s_A—i.e., $s_A = r_A s_B / r_B$—the right-hand side of (22) is $2r_A^2 s_B^4 (r_B - r_A)(r_A^3 + r_B(r_B^2 - r_A^2)) / r_B^4$, which is again strictly positive due to the fact that $r_B > r_A$. Finally, taking the derivative of $\Pi^*_{PP} - \Pi^*_{PU}$ with respect to s_A, we obtain that it is a polynomial of third order, which is increasing at the lower bound and is either increasing or decreasing at the upper bound. In case it is decreasing at the upper bound, the sign of $\Pi^*_{PP} - \Pi^*_{PU}$ must be positive in the admissible range, as otherwise the derivative changes its sign more than three times, which is impossible for a third-order polynomial. In case it is increasing at the upper bound, we can show that the derivative never changes its sign in the admissible range, which again implies that $\Pi^*_{PP} > \Pi^*_{PU}$ for all s_A in admissible range. Therefore, firms choose the regime PP in case they can contract on their pricing policies.

E.4: Delegated Distribution

We last prove the statement in part *(ii)* of the proposition. To so so, we proceed in the same way as in the proof of Proposition 4 by comparing the respective profits from dual distribution (i.e., Π^*_{UU}, Π^*_{UP}, Π^*_{PP}, and Π^*_{PU}) with the profits from delegated distribution. The latter are $r_B^2/(4s_B)$, in case B sets only a uniform price, and $r_B^2/(2s_B)$, in case B sets personalized prices.

Starting with the regime in which both firms set a uniform price, the comparison between Π^*_{UU} and $r_B^2/(4s_B)$ yields that $\Pi^*_{UU} - r_B^2/(4s_B) = r_A^2(s_B - s_A)/(4s_A s_B) > 0$ at the lower bound of r_B (i.e., $r_B = r_A$) and $\Pi^*_{UU} - r_B^2/(4s_B) = -\left(r_A^2(s_B - s_A)\right)/\left(4s_A(5s_A + 4s_B)\right) < 0$ at the upper bound of r_B (i.e., $r_B = r_A s_B/s_A$). In addition, the difference is strictly decreasing in r_B, and the threshold value of r_B at which both profits are equal to each other is:

$$\rho_{UU} \equiv \frac{4\sigma(1+\sigma) - (\sigma - 1)\sqrt{\sigma(5+4\sigma)}}{5+3\sigma},$$

where we again used the definitions $\rho \equiv r_B/r_A$ and $\sigma \equiv s_B/s_A$. It is easy to check that the threshold is below σ for all $\sigma > 1$. Therefore, delegated distribution is optimal if and only if $\rho > \rho_{UU}$.

Turning to the regime in which only B sets personalized prices, we obtain that $\Pi^*_{UP} - r_B^2/(2s_B)$ is positive at the lower bound of r_B if and only if $s_B > 2s_A$ but strictly negative at the upper bound of r_B. The difference is again strictly decreasing in r_B. If a solution to $\Pi^*_{UP} = r_B^2/(4s_B)$ in the admissible range exists, which is the case if $s_B > 2s_A$, or $\sigma > 2$, the threshold value at which both profits are equal to each other is:

$$\rho_{UP} \equiv \frac{\sigma\left(2\sigma + 1 - \sqrt{3(\sigma+1)(\sigma-1)}\right)}{2+\sigma}.$$

This threshold is again below σ for $\sigma > 1$, which implies that delegated distribution is optimal if and only if $\rho > \rho_{UP}$.

Proceeding in the same way in the regime in which both firms set personalized prices yields that $\Pi^*_{PP} - r_B^2/(2s_B)$ is positive at the lower bound of r_B, negative at the upper bound of r_B, and strictly decreasing in r_B. The threshold value at which the two profits are equal to each other is:[46]

$$\rho_{PP} \equiv \frac{\sigma(2+\sigma) - \sqrt{\sigma(\sigma^2 + \sigma - 1)}}{1+\sigma};$$

hence, delegated distribution is optimal if and only if $\rho > \rho_{PP}$.

Finally, in the regime in which only A sets personalized prices, the difference between $\Pi^*_{PU} - r_B^2/(4s_B)$ is:

$$\frac{\left(2r_A s_B - r_B s_A\right)^2}{4s_A s_B\left(2s_B - s_A\right)},$$

which is strictly positive for all parameters in the admissible range.

Therefore, we have shown that delegated distribution is optimal in the regimes UU, UP, and PP for some values of ρ and σ but that it is never optimal in the regime PU.

[46]This threshold is also below σ for $\sigma > 1$.

We next compare the threshold values in the regimes UU, UP, and PP. We start with a comparison between ρ_{UU} and ρ_{UP}. Taking the difference between ρ_{UU} and ρ_{UP} yields:

$$\frac{\sigma\left(5+3\sigma\right)\sqrt{3\left(\sigma+1\right)\left(\sigma-1\right)}-\left(\sigma-1\right)\left[\sigma\left(3+2\sigma\right)-\left(2+\sigma\right)\sqrt{\sigma\left(5+4\sigma\right)}\right]}{\left(2+\sigma\right)\left(5+3\sigma\right)},$$

which is equal to 0 at the lower bound of σ (i.e., $\sigma = 1$)[47] but strictly increasing in σ for all $\sigma > 1$. It follows that $\rho_{UU} > \rho_{UP}$.

Next, we compare ρ_{PP} and ρ_{UP}. Taking the difference between ρ_{PP} and ρ_{UP} yields:

$$\frac{\sigma\left(1+\sigma\right)\sqrt{3\left(\sigma+1\right)\left(\sigma-1\right)}-\sigma\left(\sigma^2-\sigma-3\right)-\left(2+\sigma\right)\sqrt{\sigma\left(\sigma^2+\sigma-1\right)}}{\left(2+\sigma\right)\left(1+\sigma\right)},$$

which is again 0 at $\sigma = 1$ but strictly increasing in σ for all $\sigma > 1$; hence, $\rho_{PP} > \rho_{UP}$.

Lastly, the difference between ρ_{UU} and ρ_{PP} is given by:

$$\frac{\sigma\left(\sigma^2-3\sigma-6\right)+\sigma\left(5+3\sigma\right)\sqrt{\sigma\left(\sigma^2+\sigma-1\right)}-\left(1+\sigma\right)\left(1-\sigma\right)\sqrt{\sigma\left(5+4\sigma\right)}}{\left(1+\sigma\right)\left(5+3\sigma\right)}. \tag{23}$$

This difference is again 0 at $\sigma = 1$. It is increasing in σ for $\sigma \searrow 1$ but negative as $\sigma \to \infty$. Setting (23) equal to 0 and solving for σ yields $\left(3+\sqrt{33}\right)/2 \equiv \check{\sigma}$ as the unique solution for $\sigma > 1$. It follows that $\rho_{UU} \geq \rho_{PP}$ if $\sigma \leq \check{\sigma}$ and $\rho_{UU} < \rho_{PP}$ if $\sigma > \check{\sigma}$.

Online Appendix F: Proof of Proposition 6

We first show that dual distribution is the optimal strategy in all pricing regimes, that is, statement *(a)* of part *(i)*.

F.1: Uniform pricing by A

Let p_A^m define firm A's optimal (uniform) price under mono distribution, and Π_A^m the associated profit. To show that dual distribution is optimal, it suffices to show that an appropriate two-part tariff leads to a continuation equilibrium in which both firms earn greater profit (and one firm does strictly so) than under mono distribution. To see this, suppose that the firms agreed instead on the wholesale tariff $F = 0$ and $w = \hat{v}$.

Assume first that B also charges a uniform price. In the continuation equilibrium, B then charges a price $p_B > w = \hat{v}$, such that it obtains a positive share the market: as consumers close to X have a higher valuation for B's product than for A's product, there exists an $\hat{x}_B \in (\hat{x}, X)$ such that consumers $x > \hat{x}_B$ buy from B in the continuation

[47]Recall that $s_B \geq s_A$ if B offers the high-quality product.

equilibrium. Suppose now that A charges the mono-distribution price p_A^m. If $p_A^m \geq \hat{v}$, then B still attracts all consumers $x > \hat{x}_B$; hence, firm A's profit becomes $\Pi_A^m + \hat{v}(X - \hat{x}_B) > \Pi_A^m$. If instead $p_A^m < \hat{v}$, then B still attracts those consumers that are close enough to X;[48] hence, compared to mono distribution, total sales can only increase, and A earns a greater margin ($\hat{v} > p_A^m$) on the consumers served by B. We thus have that, by charging p_A^m, A obtains a greater profit than Π_A^m. It follows that, in the continuation equilibrium (where it best responds to p_B), A a fortiori obtains a profit higher than Π_A^m. In other words, the tariff $F = 0$ and $w = \hat{v}$ yields a continuation equilibrium that strictly increases each firm's profit.

The same reasoning applies to the case in which B charges personalized prices. In the continuation equilibrium, B attracts some consumers, and by charging p_A^m in the retail stage, A obtains a higher profit than Π_A^m, either by expanding the market served or by earning a higher margin on existing customers.

F.2: Personalized pricing by A

Under mono distribution, firm A charges each consumer a price $v_A(x)$. Suppose now that the firms agree on the two-part tariff $F = 0$ and $w = \hat{v}$.

If both firms charge personalized prices, A still charges $v_A(x)$ to consumers $x < \hat{x}$ (as B cannot offer them any positive value at any price $p_B(x) \geq w = \hat{v}$), and now obtains $w > v_A(x)$ on consumers $x > \hat{x}$ (which are served by B at price $p_B(x) = v_B(x)$).[49]

Consider now the case in which B charges a uniform price. In the continuation equilibrium, A still charges $p_A(x) = v_A(x)$ to consumers $x < \hat{x}$, for the same reason as above. For consumers $\hat{x} < x \leq X$, B charges the price p_B that maximizes its monopoly profit, given the wholesale price $w = \hat{v}$, and attracts some consumers $x > \hat{x}_B$, where $\hat{x}_B \in (\hat{x}, X)$. To see this, note first that A charges $p_A(x) = v_A(x)$ to all consumers x, with $v_B(x) < p_B$, because these consumers obtain a negative utility when buying from B, which implies that A can extract the entire consumer surplus. Instead, for consumers who obtain a positive utility when buying from B, A can earn a higher margin—i.e., $\hat{v} = v_A(\hat{x}) > v_A(x)$—when B serves these consumers than when serving these consumers itself. Therefore, in the continuation equilibrium, A charges these consumers any price larger than or equal to $v_A(x)$, and B serves these consumers. It follows that, again, A earns a higher margin on the consumers served by B, and obtains a higher profit compared to mono distribution.

[48] In the case of mono distribution, either A covers part of the market, or it covers all the market but leaves no net utility to the marginal consumer $x = X$. In both cases, B attracts consumers close enough to X with the price p_B.

[49] Recall that, as usual, we focus on the Pareto efficient equilibrium, in which A does not sell below its opportunity cost (given here by $w = \hat{v}$).

F.3: Endogenous Pricing Regime

The arguments in F.1 and F.2 established that dual distribution is optimal in all pricing regimes. Because firms can extract the entire consumer surplus in the regime in which they both charge personalized prices (by negotiating a wholesale price $w = \hat{v}$) whereas this is not possible in any other pricing regime, the regime in with personalized pricing by both firms yields the highest industry profit. Hence, if firms can contract on their pricing policies, they will choose this regime. This establishes statement *(b)* of part *(i)*.

F.4: Delegated Distribution

Finally, turning to part *(ii)* of the proposition, it is easy to see from the arguments above, that delegated distribution is never optimal if B charges personalized prices: a wholesale price of $w = \hat{v}$ then ensures that B extracts the entire surplus from consumers $x \in (\hat{x}, X]$ but does not compete with A for consumers $x \in [0, \hat{x}]$. As A provides a higher net value to the latter consumers, the profit with dual distribution is higher than with delegated distribution.

If B can only charge a uniform price, we can proceed in the same way as in the proof of the last proposition to determine the equilibrium industry profit with dual distribution.[50] We start with the regime of uniform pricing by both firms. Solving for the equilibrium in the retail market, plugging the equilibrium retail prices in the industry profit, and maximizing the resulting industry profit with respect to w, we obtain that the optimal w is set in such a way that the marginal consumer indifferent between both firms receives a utility of zero. Two cases can then arise: first, A and B do not compete in the retail market, even if both set their respective monopoly prices—i.e., $p_A^m = r_A/2$ and $p_B^m = (r_B - s_B X)/2$.[51] In this case, the optimal wholesale price equals zero, as this avoids double marginalization, and achieves the highest profit possible with uniform pricing. Second, competition in the retail market occurs if both firms set their respective monopoly price. Solving for the optimal wholesale price in that case yields:

$$w_{UU}^* = \frac{s_A r_B - s_B (r_A - s_A X)}{s_A}. \tag{24}$$

This wholesale price ensures that the utility of the consumer indifferent between both firms is zero. Determining the boundaries between the cases yields the former case occurs if $r_B \leq -(s_B (s_A X - r_A))/s_A$. The equilibrium industry profit with dual distri-

[50] Without loss of generality, we focus on the case in which A cannot serve all consumers, that is, $X > r_A/s_A$.

[51] Recall that $s_B < 0$ with horizontal differentiation.

bution is then:

$$\Pi_{UU}^{*}=\begin{cases}\frac{r_A^2}{4s_A}-\frac{(r_B-s_BX)^2}{4s_B} & \text{if} \quad r_B\leq-\frac{s_B(s_AX-r_A)}{s_A},\\ \frac{r_A^2}{4s_A}-\frac{(2s_AX-r_A)(s_Br_A-2s_Ar_B)}{4s_A^2} & \text{if} \quad r_B>-\frac{s_B(s_AX-r_A)}{s_A}.\end{cases} \tag{25}$$

The profit with delegated distribution (i.e., only B is active) can be determined to get $-(r_B-s_BX)^2/(4s_B)$. Comparing the two profits with each other yields that in case $r_B\leq-(s_B(s_AX-r_A))/s_A$ or, equivalently, $\rho\leq-\sigma(s_AX/r_A-1)$,[52] delegated distribution is dominated by dual distribution. This is intuitive: in both cases, B achieves the monopoly profit. In addition, with dual distribution, also A achieves its monopoly profit whereas A's channel is shut down in case of delegated distribution. Instead, if $\rho>-(\sigma(s_AX/r_A-1))$, a comparison of the profits yields that the one from delegated distribution is larger than the one from dual distribution if and only if:

$$\rho>\rho_{UU}\equiv-\sigma(s_AX/r_A-1)+\sqrt{-\sigma}. \tag{26}$$

The right-hand side of (26) is strictly larger than $-\sigma(s_AX/r_A-1)$—i.e., the threshold value for which the comparison is valid. It is also below the upper bound of ρ, which is 1, if $r_A>-(\sigma s_AX)/\left(1-\sigma-\sqrt{-\sigma}\right)$. This inequality can indeed be satisfied if X is close to its lower bound r_A/s_A. It follows that there exist parameter values such that ρ_{UU} is in the admissible range.

For the pricing regime in which A charges personalized prices and B a uniform one, we can follow the same steps. There are again two cases, one in which firms do not compete in the retail market, even at a wholesale price of zero, and one, in which this is not the case. Determining the boundaries between the two cases, we obtain that the former one occurs if and only if $r_B>-(s_B(s_AX-2r_A))/s_A$. In this case, the optimal wholesale price is indeed zero. Solving for the optimal wholesale price in the second case, we obtain:

$$w_{PU}^{*}=\frac{s_Ar_B-s_B(2r_A-s_AX)}{s_A-2s_B}. \tag{27}$$

At this wholesale, the consumer indifferent between both firms again receives a utility of zero. In addition, as A charges personalized prices and does not compete with B, also all consumers to the left of the indifferent consumer receive no utility. Determining the equilibrium industry profit with dual distribution then yields:

$$\Pi_{PU}^{*}=\begin{cases}-\frac{(s_BX+r_B)(s_B(s_AX-4r_A)+r_Bs_A)}{8s_B^2}-\frac{(r_B-s_BX)^2}{4s_B} & \text{if} \quad r_B\leq-\frac{s_B(s_AX-2r_A)}{s_A},\\ \frac{s_B^2X^2+2X(s_Ar_B-s_B(r_A+r_B))+(r_A-r_B)^2}{2(s_A-2s_B)} & \text{if} \quad r_B>-\frac{s_B(s_AX-2r_A)}{s_A}.\end{cases} \tag{28}$$

[52] Here, we used again the definition $\sigma\equiv s_B/s_A<0$ and $\rho\equiv r_B/r_A\in(0,1)$.

Comparing Π^*_{PU} with the profit from delegated distribution, $-\left(r_B - s_B X\right)^2 / \left(4 s_B\right)$, yields again that for $r_B \leq -\left(s_B\left(s_A X - 2r_A\right)\right)/s_A$ or, equivalently, $\rho \leq -\sigma\left(s_A X/r_A - 2\right)$, the former is larger than the latter. Instead, if $\rho > -\sigma\left(s_A X/r_A - 2\right)$, the profit from delegated distribution is larger than the one from dual distribution if and only if:

$$\rho > \rho_{PU} \equiv -\sigma\left(s_A X/r_A - 2\right) + \sqrt{-2\sigma\left(1-2\sigma\right)}. \tag{29}$$

The right-hand side of (29) is strictly larger than $-\sigma\left(s_A X/r_A - 2\right)$ and is below the upper bound of ρ if $r_A > -1 + \left(\sigma s_A X\sqrt{2\sigma(1-2\sigma)}\right)/\left(1-2\sigma\right)$. This inequality is again satisfied if X is close to its lower bound r_A/s_A. It follows that there exist parameter values such that ρ_{PU} is in the admissible range.

Finally, it is easy to check, by comparing the values in (25) and (28) for the different ranges, that Π^*_{PU} is larger than Π^*_{UU} for all parameter combinations, that is, the profit from dual distribution is larger in the regime PU than in the regime UU. It follows that $\rho_{PU} < \rho_{UU}$.

Online Appendix G: Comparison of inter-brand with intra-brand competition

In this appendix, we analyze a situation of inter-brand competition between two independent firms in the scenario of vertical differentiation in which A is more attractive to high-valuation consumers. As in the last two appendices, we compare the profits of the two symmetric regimes (i.e., uniform pricing by both firms and personalized pricing by both firms) with each other. We show that the insights from such an analysis are misleading if applied to a situation of intra-brand competition between two channels, which is partly governed by the negotiated wholesale contract.

The case of inter-brand competition between independent firms is equivalent to a situation in which no wholesale contract exists, that is, the wholesale price w equals zero. We can then apply the same techniques as in the proof of Proposition 2 to solve for the aggregate profits of the firms in the two pricing regimes via inserting $w = 0$ in the respective equations. This yields:

$$\Pi^A_{UU}(w=0) = \frac{r_A^2}{s_A}\frac{\left(2-\rho-\sigma\right)^2}{(4-\sigma)^2(1-\sigma)} \quad \text{and} \quad \Pi^B_{UU}(w=0) = \frac{r_A^2}{s_A}\frac{\left(2\rho-\sigma(1+\rho)\right)^2}{\sigma(4-\sigma)^2(1-\sigma)}$$

for the uniform pricing regime, and:

$$\Pi^A_{PP}(w=0) = \frac{r_A^2}{s_A}\frac{\left(1-\rho\right)^2}{2(1-\sigma)} \quad \text{and} \quad \Pi^B_{PP}(w=0) = \frac{r_A^2}{s_A}\frac{\left(\rho-\sigma\right)^2}{2\sigma(1-\sigma)}.$$

for the personalized pricing regime. Comparing $\Pi^i_{PP}(w=0)$ with $\Pi^i_{UU}(w=0)$, $i = A, B$, we obtain that A benefits from personalized pricing if and only if:

$$\rho \leq \rho_A(\sigma) \equiv \frac{\left(\sqrt{2}-1\right)\left(2\sqrt{2}-\sigma\right)}{4-\sqrt{2}-\sigma}, \tag{30}$$

and B benefits from personalized pricing if and only if:

$$\rho \geq \rho_B(\sigma) \equiv \frac{\sigma\left(\sqrt{2}+1\right)\left(4-\sqrt{2}-\sigma\right)}{2\sqrt{2}+\sigma}. \tag{31}$$

Turning to the joint profits, the difference between the firms' aggregate profits with personalized pricing and their profits with uniform pricing (i.e., $\Pi^A_{PP}(w=0)+\Pi^B_{PP}(w=0)-\Pi^A_{UU}(w=0)-\Pi^B_{UU}(w=0)$) is:

$$\frac{r_A^2}{2s_A}\frac{\left(8+14\sigma-9\sigma^2+\sigma^3\right)\left(\rho^2+\sigma\right)-4\rho\sigma\left(12-6\sigma+\sigma^2\right)}{(1-\sigma)\sigma(4-\sigma)^2}. \tag{32}$$

The numerator is a convex quadratic polynomial of ρ with the two roots:

$$\underline{\rho}(\sigma) \equiv \frac{2\sigma\left(12-6\sigma+\sigma^2\right)-(1-\sigma)(4-\sigma)\sqrt{\sigma(12\sigma-4-\sigma^2)}}{8+14\sigma-9\sigma^2+\sigma^3}$$

and

$$\overline{\rho}(\sigma) \equiv \frac{2\sigma\left(12-6\sigma+\sigma^2\right)+(1-\sigma)(4-\sigma)\sqrt{\sigma(12\sigma-4-\sigma^2)}}{8+14\sigma-9\sigma^2+\sigma^3}.$$

It is straightforward to check that for $\sigma < 2\left(3-2\sqrt{2}\right) \approx 0.343$, the numerator of (32) is always positive (i.e., no root exists in this case), which implies that firms' aggregate profits are larger with personalized prices. In addition, for these values of σ, $\rho_A(\sigma) > \rho_B(\sigma)$, which implies that both firms benefit from personalized pricing if and only if $\rho_B(\sigma) \leq \rho \leq \rho_A(\sigma)$. Instead, for $\sigma \geq 2\left(3-2\sqrt{2}\right)$, the joint profit with personalized pricing is larger if $\rho \leq \underline{\rho}(\sigma)$ or $\rho \geq \overline{\rho}(\sigma)$. The threshold $\overline{\rho}(\sigma)$ is increasing in σ and reaches the upper bound for ρ, which is 1, at $\sigma = 5-\sqrt{17} \approx 0.877$. Similarly, $\underline{\rho}(\sigma)$ is decreasing in σ and reaches the lower bound for ρ, which is σ, also at $\sigma = 5-\sqrt{17} \approx 0.877$. It follows that for $\sigma > 5-\sqrt{17}$, the joint profit from uniform pricing is higher in the admissible range for ρ.

The result shows that inter-brand competition with independent firms is less fierce in a situation of vertical differentiation than in one of horizontal differentiation. As mentioned in the main text, Thisse and Vives (1988) and Shaffer and Zhang (1995) show that in a Hotelling model aggregate profits with personalized pricing are unambiguously lower than those with uniform pricing. Instead, our analysis shows that with vertical differentiation, this is not necessarily true and there is indeed a sizable range in which the opposite result occurs.

The comparison between the profits from personalized pricing and from uniform pricing in case of inter-brand competition provides little guidance for the question whether or not dual distribution is optimal in case of intra-brand competition. First, as shown in Proposition 1, if the manufacturer offers the low-end product, then dual distribution is always optimal in case of intra-brand competition, regardless of the pricing regime. Therefore, the optimal distribution regime in this case is unrelated to the ranges in which personalized increases or decreases profits in a situation of inter-brand competition. Second, if the manufacturer offers the high-end product, as shown in Proposition 2, mono distribution is optimal under intra-brand competition if ρ is close to σ—i.e., if $\rho < \sigma\left(2+\sqrt{1-\sigma}\right)/(1+\sigma)$. Instead, as shown above, with inter-brand competition, for all values of $\sigma \leq 0.877$ personalized pricing leads to higher aggregate profits than uniform pricing if ρ is close to σ. Moreover, mono distribution can be optimal even if personalized pricing increases the profits of *both* firms in the case of inter-brand competition. The latter occurs if $\rho_B(\sigma) \leq \rho \leq \rho_A(\sigma)$. From the proof of Proposition 2, mono distribution under intra-brand competition is optimal if and only if $\rho \geq \tilde{\rho}(\sigma)$. It is straightforward to check that $\tilde{\rho}(\sigma) > \rho_B(\sigma)$ for $\sigma \in (0,1)$ and $\tilde{\rho}(\sigma) < \rho_A(\sigma)$ for $\sigma \lessapprox 0.230$; hence, the situation occurs for $\sigma \in (0, 0.230)$ if $\rho_B(\sigma) \leq \rho \leq \tilde{\rho}(\sigma)$.

Online Appendix H: Linear wholesale tariff

In this appendix, we show that our main results carry over to the case with a linear wholesale tariff. In contrast to the two-part tariff analyzed in the main model, linear tariffs create double marginalization problems and tend to generate inefficiently high prices. Yet, because of their simplicity or for fairness reasons,[53] linear tariffs are sometimes used in practice.[54] We restrict our attention to the two situations in which both firms charge only a uniform price and in which both firms charge personalized prices. We also simplify the exposition by allocating all bargaining power at the wholesale stage to A (i.e., we assume that A makes the wholesale contract offer).[55] We then obtain the following proposition:

Proposition: *Suppose that wholesale contracts are restricted to a wholesale price offered by A. Dual distribution is optimal in case both firms charge only a uniform retail price. By*

[53] Cui *et al.* (2007) show that a linear wholesale price contract can be efficient if the retailer is inequity averse when comparing its profit with that of the manufacturer.

[54] This is, for example, the case of the U.S. pay-TV industry; see Crawford and Yurukoglu (2012) and Crawford *et al.* (2018).

[55] The qualitative result are similar if instead B made the offer.

contrast, when both firms offer personalized prices, mono distribution is optimal if and only if:

$$\rho \leq \sigma \frac{2+\sqrt{2(1-\sigma)}}{1+\sigma}.$$

The proposition shows that our main insights carry over when considering linear wholesale prices instead of two-part tariffs. The intuition is the same as before. Dual distribution expands demand in the low-end segment but triggers competition with the manufacturer's own distribution channel. As long as that channel charges a uniform price, this competition is not too fierce and can be sufficiently mitigated through an appropriate wholesale price. Dual distribution is therefore optimal. When instead both firms offer personalized prices, competition is tougher; mono distribution is then optimal if B does not add enough value to the industry. Compared to the setting in which the wholesale contract consists of a two-part tariff, the range in which mono distribution is optimal is now even larger, as a linear wholesale price contract does not allow firms to share their joint profits in any way they wish to, which implies that A obtains a smaller part of B's profit than with two-part tariffs.

Proof of the proposition:

With dual distribution, the second stage of the game leads, as before, to downstream prices given by (7). We now consider the first stage for the two pricing regimes.

Under uniform pricing, the profit function of A is now $\Pi_A = D_A p_A + D_B w$. Inserting the corresponding demand functions, p_A and p_B from (7), and maximizing with respect to w, we obtain that the equilibrium wholesale price is (using "**" to distinguish from the equilibrium that arises with two-part tariffs):[56]

$$w_{UU}^{**} = \frac{r_A s_B^2 + 8 r_B s_A^2}{2 s_A (8 s_A + s_B)} = \frac{r_A}{2} \frac{8\rho + \sigma^2}{8+\sigma}.$$

Inserting w_{UU}^{**} into the profit yields the equilibrium profit with dual distribution:

$$\begin{aligned} \Pi_{UU}^{**} &= \frac{4 s_A^3 r_B^2 + 8 s_A^2 s_B r_A (r_A - r_B) - r_A^2 s_B^2 (3 s_A + s_B)}{4 s_A s_B (8 s_A + s_B)(s_A - s_B)} \\ &= \frac{r_A^2}{4 s_A} \frac{4\rho^2 + 8\sigma(1-\rho) - \sigma^2 (3+\sigma)}{\sigma(1-\sigma)(8+\sigma)}. \end{aligned} \tag{33}$$

As in Section 4, it can be checked that demands D_A and D_B are both positive at $w = w_{UU}^{**}$, implying that dual distribution is optimal. Indeed, comparing Π_{UU}^{**} with $\Pi_U^m =$

[56]The second-order condition is $-2s_A(8s_A+s_B)/(s_B(4s_A-s_B)^2) < 0$, implying that the profit function is concave.

$r_A^2/4s_A$, yields:

$$\Pi_{UU}^{**} - \Pi_U^m = \Pi_U^m \frac{4\left(\sigma-\rho\right)^2}{\sigma\left(1-\sigma\right)\left(8+\sigma\right)} > 0.$$

We now turn to personalized pricing. As in the case of two-part tariffs, in the range $w \geq \hat{v}$, B is inactive and so A cannot obtain more than the mono distribution profit Π_P^m. We thus focus on $w \leq \hat{v}$, distinguishing again between $w \leq \underline{w} = r_A\left(\rho-\sigma\right)$ and $w > \underline{w}$. We start with the former case. With linear tariffs, the profit function is:

$$\Pi_A(w) = \int_0^{\hat{x}} \left[w + v_A(x) - v_B(x)\right] dx + \int_{\hat{x}}^{\tilde{x}_B(w)} w dx,$$

which is strictly concave in w:

$$\Pi_A'\left(w\right) = \tilde{x}_B\left(w\right) + w\frac{d\tilde{x}_B}{dw}\left(w\right) = \frac{r_B - w}{s_B} - \frac{w}{s_B} = \frac{r_B - 2w}{s_B}, \tag{34}$$

and thus $\Pi_A'' = -2/s_B < 0$. When instead $w > \underline{w}$, A's profit can be written as:

$$\Pi_A\left(w\right) = \int_0^{\hat{x}} w + v_A(x) - v_B(x) dx + \int_{\hat{x}}^{\tilde{x}_B(w)} w dx + \int_{\tilde{x}_B(w)}^{\bar{x}_A} v_A(x) dx.$$

The first derivative is equal to:

$$\begin{aligned}
\Pi_A'\left(w\right) &= \tilde{x}_B(w) + \left[w - v_A\left(\tilde{x}_B\left(w\right)\right)\right]\frac{d\tilde{x}_B}{dw}\left(w\right) \\
&= \frac{r_B - 2w + r_A}{s_B} - \frac{s_A}{s_B}\frac{r_B - w}{s_B} \\
&= \frac{r_A}{s_A\sigma^2}\left[\sigma - \rho\left(1-\sigma\right) + \left(1-2\sigma\right)\frac{w}{r_A}\right].
\end{aligned}$$

Hence:

$$\begin{aligned}
\Pi_{A-}'\left(\hat{v}\right) &= \frac{r_A}{s_A}\frac{1-\rho}{1-\sigma}, \\
\Pi_{A+}'\left(\underline{w}\right) &= \Pi_-'\left(\underline{w}\right) = \frac{r_A}{s_A\sigma}\left(2\sigma-\rho\right), \\
\Pi_A''\left(w\right) &= \frac{1-2\sigma}{s_A\sigma^2}.
\end{aligned}$$

It follows that $\Pi\left(w\right)$ is strictly concave in w if $\sigma > \hat{\sigma}^{**} = 1/2$ and is weakly convex otherwise; in addition, $\Pi_A'\left(\hat{v}\right) > 0$ whereas $\Pi_A'\left(\underline{w}\right) \geq 0$ if and only if:

$$\rho \leq \hat{\rho}^{**}\left(\sigma\right) \equiv 2\sigma,$$

where $\hat{\rho}^{**}\left(\sigma\right)$ increases with σ and exceeds 1 for $\sigma \geq \hat{\sigma}^{**}$. Furthermore, the profit function $\Pi_A\left(w\right)$ and its derivative $\Pi_A'\left(w\right)$ are both continuous at $w = \underline{w}$.

As mentioned above, as long as $w \geq \hat{v}$, A cannot obtain a higher profit than with mono distribution. Furthermore, if $\rho \leq \hat{\rho}^{**}(\sigma)$, then $\Pi_A'(\underline{w}) \geq 0$, implying that dual distribution cannot be more profitable than mono distribution:

- in the range $\underline{w} \leq w \leq \hat{v}$, the profit function $\Pi_A(w)$ is increasing, as it is quadratic and its derivative is non-negative at both ends of the range (namely, $\Pi_A'(\underline{w}) \geq 0$ and $\Pi_A'(\hat{v}) > 0$);
- in the range $w \leq \underline{w}$, the profit function $\Pi_A(w)$ is again increasing, as it is concave and its derivative is non-negative at the upper end of the range (namely, $\Pi_A'(\underline{w}) \geq 0$);
- it follows that the profit achieved under dual distribution cannot exceed $\Pi_A(\hat{v})$, which is less profitable than mono distribution.

As already noted, $\hat{\rho}^{**}(\sigma)$ is increasing in σ, and satisfies $\hat{\rho}^{**}(\sigma) \geq 1$ for $\sigma \geq \hat{\sigma}^{**}$. It follows that, if $\sigma \geq \hat{\sigma}^{**}$, then dual distribution cannot be more profitable than mono distribution, as we then have $\hat{\rho}^{**}(\sigma) \geq 1\,(> \rho)$.

If instead $\sigma < \hat{\sigma}^{**}$ and $\rho > \hat{\rho}^{**}(\sigma)$, then $\Pi_A'(\underline{w}) < 0$ and, in the range $w \leq \underline{w}$, from (34), $\Pi_A(w)$ is maximal for $w_{PP}^{**} = r_B/2$, which lies below $\underline{w}$ and yields a profit equal to:

$$\Pi_{PP}^{**} = \frac{2r_A^2 s_B + r_B^2(s_A + s_B) - 4r_A r_B s_B}{4s_B(s_A - s_B)} = \frac{r_A^2}{4s_A}\frac{2\sigma + \rho^2(1+\sigma) - 4\rho\sigma}{\sigma(1-\sigma)}.$$

Compared with the profit from mono distribution, $\Pi_P^m = r_A^2/2s_A$, dual distribution introduces a change in profit equal to:

$$\Pi_{PP}^{**} - \Pi_P^m = \Pi_P^m\left(\frac{2\sigma - 4\rho\sigma + \rho^2(1+\sigma)}{2\sigma(1-\sigma)} - 1\right) = \Pi_P^m \frac{2\sigma^2 - 4\sigma\rho + (1+\sigma)\rho^2}{2\sigma(1-\sigma)}.$$

The numerator of this expression is a convex quadratic polynomial of ρ and its roots are:

$$\sigma\frac{2 - \sqrt{2(1-\sigma)}}{1+\sigma} \text{ and } \sigma\frac{2 + \sqrt{2(1-\sigma)}}{1+\sigma}.$$

Furthermore, $\hat{\rho}^{**}(\sigma)$ lies between these two roots in the relevant range $\sigma < \hat{\sigma}^{**}$:

$$\begin{aligned} \frac{\sigma\frac{2-\sqrt{2(1-\sigma)}}{1+\sigma}}{\hat{\rho}^{**}(\sigma)} &= \frac{2 - \sqrt{2(1-\sigma)}}{2(1+\sigma)} < 1, \\ \frac{\sigma\frac{2+\sqrt{2(1-\sigma)}}{1+\sigma}}{\hat{\rho}^{**}(\sigma)} &= \frac{\frac{2+\sqrt{2(1-\sigma)}}{1+\sigma}}{2} = \frac{2 + \sqrt{2(1-\sigma)}}{2(1+\sigma)} > 1, \end{aligned}$$

where the last inequality stems from $\sqrt{2(1-\sigma)} > 2\sigma$ in the relevant range $\sigma < \hat{\sigma}^{**}$. It follows that dual-distribution is more profitable than mono-distribution if and only if

$\sigma < \hat{\sigma}^{**}$ and ρ exceeds the larger root, that is, if:

$$\rho > \tilde{\rho}^{**}(\sigma) \equiv \sigma \frac{2 + \sqrt{2(1-\sigma)}}{1+\sigma}.$$

Note that $\tilde{\rho}^{**}(\sigma)$ is increasing in σ in the range $\sigma \leq \hat{\sigma}^{**}$, and exceeds 1 in the range $\sigma \geq \hat{\sigma}^{**}$. Hence, as $\rho < 1$, the condition $\rho > \tilde{\rho}^{**}(\sigma)$ implies $\sigma < \hat{\sigma}^{**}$.

Online Appendix I: Generalization of Proposition 2, Part *(i)*

In this appendix, we generalize part *(i)* of Proposition 2 to an extended setting in which consumers with unit demand obtain values, net of distribution costs, of $v_A(x)$ and $v_B(x)$ for the products of the two firms, where $v_A(\cdot)$ and $v_B(\cdot)$ are both twice continuously differentiable, x is distributed according to a twice continuously differentiable c.d.f. $G(x)$ over $\mathbb{R}_+$ and:

- $\forall x \in \mathbb{R}_+$, $v'_A(x) < v'_B(x) < 0$;
- $v_i(\bar{x}_i) = 0$ for some $\bar{x}_i > 0$; and
- $v_A(\hat{x}) = v_B(\hat{x}) > 0$ for some $\hat{x} > 0$.

This implies that, as in our baseline model, the curves $v_A(\hat{x})$ and $v_B(\hat{x})$ intersect exactly once, and this intersection occurs in the positive quadrant.

Let:

$$D_i^m(p_i) \equiv G\left(v_i^{-1}(p_i)\right),$$

denote the monopolistic demand for firm i's product:

$$p_i^m \equiv \arg\max_{p_i} p_i D_i^m(p_i),$$

denote firm i's monopoly price:

$$x_i^m \equiv v_i^{-1}(p_i^m),$$

denote the location of the associated marginal consumer, and:

$$q_i^m \equiv D_i^m(p_i^m) = G(x_i^m) \quad \text{and} \quad \pi_i^m \equiv p_i^m q_i^m,$$

denote the monopoly output and profit, respectively. Our working assumption is that B would seek to serve more consumers than A in these monopoly situations:

Assumption A: B's monopoly profit function is strictly quasi-concave and $q_B^m > q_A^m$.

Let $w^m \equiv v_B(x_A^m)$. For $w \geq w^m$, there exists a continuation equilibrium in which A charges its monopoly price, p_A^m, and B does not serve any consumer (e.g., by charging $p_B = w^m$). If instead $w < w^m$, both firms can obtain a positive market share: A then faces a demand:

$$D_A(p_A, p_B) \equiv G\left(\Delta^{-1}(p_A - p_B)\right),$$

where:

$$\Delta(x) \equiv v_A(x) - v_B(x),$$

whereas B faces a demand given by:

$$D_B(p_A, p_B) \equiv D_B^m(p_B) - D_A(p_A, p_B).$$

For the sake of exposition, we will assume that there then exists an equilibrium where both firms obtain a positive market share, which is moreover "well-behaved":

Assumptions B: For any $w \leq w^m$, there exists a unique downstream equilibrium, $(p_A^e(w), p_B^e(w))$, where $p_A^e(w)$ and $p_B^e(w)$ are continuous and increasing in w, and such that $p_A^e(w^m) = p_A^m$ and $p_B^e(w^m) = w^m$.

We have:

Proposition: *Under Assumptions A and B, dual distribution is the unique optimal distribution strategy under uniform pricing.*

Proof: We first consider the regime in which both firms charge a uniform price. Starting from a situation in which the firms negotiate $w = w^m$, and thus A obtains Π_A^m, consider a small reduction in the wholesale price from w^m to $w < w^m$, together with a fixed fee, $F(w)$, designed to appropriate B's profit (or almost all of it, to ensure acceptance). A then obtains (almost all of) the industry profit, which can be expressed as:

$$\Pi(w) = \Pi_A(w) + \Pi_B(w),$$

where:

$$\begin{aligned}
\Pi_A(w) &= p_A^e(w) D_A(p_A^e(w), p_B^e(w)) + w D_B(p_A^e(w), p_B^e(w)) + F(w), \\
\Pi_B(w) &= [p_B^e(w) - w] D_B(p_A^e(w), p_B^e(w)) - F(w).
\end{aligned}$$

By deviating from the downstream equilibrium and charging:

$$\hat{p}_A(w) = p_B^e(w) - v_B(x_A^m) + v_A(x_A^m) = p_A^m + p_B^e(w) - w^m,$$

A would maintain its output of q_A^m, and generate an output $\hat{q}_B = D_B^m(p_B^e(w)) - q_A^m$ for B. Therefore:

$$\begin{aligned}
\Pi_A(w) &\geq \hat{p}_A(w) D_A(\hat{p}_A(w), p_B^e(w)) + wD_B(\hat{p}_A(w), p_B^e(w)) + F(w) \\
&= [p_A^m + p_B^e(w) - w^m] q_A^m + w[D_B^m(p_B^e(w)) - q_A^m] + F(w) \\
&= \pi_A^m + [p_B^e(w) - w - w^m] q_A^m + wD_B^m(p_B^e(w)) + F(w).
\end{aligned}$$

Likewise, noting that B could always choose to deviate from the downstream equilibrium and charge $p_B = w$, we have:

$$\Pi_B(w) \geq -F(w).$$

Adding these two inequalities yields (recalling that $\Pi(w) = \Pi_A(w) + \Pi_B(w)$):

$$\Pi(w) - \pi_A^m \geq \phi(w) \equiv [p_B^e(w) - w - w^m] q_A^m + wD_B^m(p_B^e(w)).$$

Note that $\phi(w^m) = 0$ because $p_B^e(w^m) = w^m$ and $D_B^m(w^m) = G(x_A^m) = q_A^m$. Taking the derivative of $\phi(w)$ and evaluating it at $w = w^m$, we obtain (again using $p_B^e(w^m) = w^m$ and $q_A^m = D_B^m(w^m)$):

$$\begin{aligned}
\phi'(w^m) &= \left[\frac{dp_B^e}{dw}(w) - 1\right] q_A^m + D_B^m(w^m) + w\frac{dD_B^m}{dp_B^e}(p_B^e(w))\frac{dp_B^e}{dw}(w) \\
&= \frac{dp_B^e}{dw}(w)\left[D_B^m(w^m) + w^m\frac{dD_B^m}{dp_B^e}(w^m)\right],
\end{aligned}$$

where the expression within bracket is negative from Assumption A.[57] It follows that a reduction of w below w^m is strictly profitable, implying that dual distribution is the unique optimal mode of distribution.

Turning to the hybrid regime in which B charges personalized prices, the same logic as in the main text can be applied. In particular, setting $p_A = p_A^*$ and $w = p_B^*$, where p_A^* and p_B^* are the equilibrium retail prices under uniform pricing, delivers a higher industry profit than dual distribution with uniform pricing, and therefore also a higher profit than Π_U^m.

[57]Indeed, Assumption A implies that firm B's optimal monopoly demand is strictly larger than $q_A^m = D_B^m(w^m)$; hence, firm B's monopoly price is below w^m, which implies that the first-order condition evaluated at w^m is negative.

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