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Economics of stationary electricity storage with various charge and discharge durations

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Summary

Electricity storage encompasses a disparate list of technologies such as pumped-storage hydroelectricity, compressed-air energy storage, chemical batteries, and flywheels. These technologies can provide the electricity system with heterogeneous services of energy transfers across months, weeks, days or intra-days, power transfers for an hour, a few minutes or seconds, and can assist operators in load following, frequency control, and uninterrupted power supply.

The paper presents a unified economic analysis of these technologies and services. We underline the role of charge and discharge durations as a criterion for economic segmentation of technologies and services. We highlight the complementary value of storage in electricity systems with a high share of low variable cost and low carbon generation (nuclear, hydro, wind power, solar photovoltaic). We also underline the limited substitution value of storage for generation with high variable cost (gas combustion-turbines or gas-oil motor engines), given the cost of state-of-the-art storage technologies and the current relatively low cost of fossil fuels and low carbon pricing.

Keywords: storage, batteries, energy mix, shadow price, charge duration

JEL codes: Q4, C61, D47

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Introduction

Over the past decade, electricity storage has generated many economic studies on technologies and the services they can provide to the electricity system.

Per se, the subject is nothing new: the history of lead-acid batteries and PHS (Pumped Hydroelectric Storage) is nearly as long as that of the electricity sector. The basic economic rationale has also remained unchanged. But two facts have revived economists' attention to electricity storage for stationary use.

- Since the early 2000s, the array of technologies has widened with the use of lithium-ion batteries, first in portable devices (notably mobile phones, notebooks, and electric tools), then in electric cars, and more recently for stationary use. Other less mature technologies are considered at R&D level or in the mid- or long-term, such as sodium-sulfur batteries, flywheels, redox flow batteries, compressed-air storage, metal-air batteries, and super-capacitors. Hydrogen produced by electrolysis, stored and then reused for electricity (through fuel-cells or after methanation) is also a potential solution to electricity storage.[1]
- The recent deployment of intermittent wind power and solar PV has added new opportunities and requirements for storing electricity at a low cost, to the ones already provided by existing nuclear and hydro power.

As a result, the study of services provided by storage to the electricity system has naturally been growing. These include: energy transfers (also named arbitrage) across weeks, days, day-night, load-following; energy transfers for a few seconds or minutes (ancillary services) to provide frequency-control, quality improvement in electricity networks and UPS (Uninterrupted Power Supply) at locations of industrial consumers connected to the grid.[2] Interest is also increasing for electricity systems with a large share of variable renewables (wind power and solar PV), as storage can participate in the smoothing of intermittent power injections.[3]

The purpose of this paper is to provide a unified insight on cost-benefit analysis of electricity storage. To date, the published studies that include cost-benefit analysis have mostly given an incomplete view of the choice of storage technologies and of their sizing in relation with the sizing of generation capacities of renewable and other energy sources. This is not surprising: short-term scheduling and long-term investment analyses are both newer and more complex

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1 Strictly speaking, only superconductivity deserves the title of electricity storage. All other technologies store another form of energy: gravitational for PHS, mechanical and thermal for compressed-air, chemical for batteries, electrostatics for super-capacitors, hydrogen for "power to gas and gas to power".

2 For technological outlooks and a presentation of services, see for example Newbery (2018), EDF (2017), BNEF NEO (2017), IEA (2015), CESE (2015), DOE (2016), Wilson et al. 2018.

3 For analyses on electricity mix with variable renewables and storage, see for example Burtin, Silva (2015), Dai et al. (2017), Després et al. (2017), ADEME (2016), Steffen, Weber (2013), Esteban et al. (2012), DOE NREL (2012), Denholm, Hand (2011), De Jonghe et al. (2011), Gulagi et al. 2018, Segundo Sevilla et al. 2018.

for storage than for fossil-fuel plants. For the latter, scheduling through the merit-order of variable costs, and investing through the criterion of break-even points of the expected use duration, are classical and well understood.

Our first aim is to fill the gap at the analytical level. We also give quantified illustrations and comments on published studies in order to facilitate the understanding of the results.

The optimal running of a stored limited resource (oil, hydro in dams, CO_2 in the atmosphere) is well documented in the academic literature. It was applied to hydro in the 1950s and '60s, in operational research and dynamic programming.[4]

Our approach may be seen as a follow-up of this literature. First we determine the conditions for the optimal scheduling of a storage installation in a given power generation system, summarized by the chronicle of hourly wholesale market prices. The result is a short-term gross profit for the installation. Secondly we determine the conditions for optimally sizing the storage installation that maximizes long-term profit. Thirdly we examine the feedback of this sizing to the sizing of the power generation system, and in particular the savings in peak-load and half-baseload generation that storage might provide.[5]

Our main point is to make explicit the difference in value across technologies characterized by different charge and discharge durations, defined in the following way:

- Charge duration is the duration needed to fill up the reservoir initially empty at maximal inflow capacity.
- Symmetrically, discharge duration is the duration needed to empty the reservoir that was initially full at maximal outflow capacity.

The two durations differ because of the technology design, the inflow and outflow capacities, and energy losses in the process.

These durations are technical specifications. They differ from duration uses that are much longer. For instance an electric vehicle (EV) with a full battery can be used for a few hours, although its discharge duration is shorter than one hour.[6] Standard charge duration is also for a

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[4] Economists at the French utility EDF contributed to this, including PHS in the 1980s: see Lederer, Colleter (1981), Lederer, Torrion, Bouttes (1984), Note Bleue (1984), and Bauduin (1989). In the review of PHS development in France in the '80s, Bauduin 1989 stressed two economic features of storage: (1) a major share of baseload production plants with low variable costs (nuclear plants in the French context); and (2) a need for short-run modeling improvement to finely assess the periodicities of hourly marginal costs, beyond the classical analysis limited to the annual structure of hourly marginal costs sorted in a monotonic way, roughly sufficient for decisions to invest in thermal plants.

[5] This assumes a power system with investment needs in peak-load and half baseload generation plants. In cases of overcapacity like in continental Europe over the last few years, the benefits from storage, hence the relevance of installation development, is substantially reduced.

[6] This used to be the case for EVs with an autonomy of less than 150 km. New EVs with higher autonomy have longer discharge durations and much longer normal duration use.

few hours at home during the night, while technical charge duration is much shorter, as fast recharging stations demonstrate.

Storage technologies differ according to their charge and discharge durations. Durations of super-capacitors and flywheels are a matter of a few seconds or minutes. Charge and discharge durations of lithium-ion batteries are usually less than one hour, while some designed recently reach 4 hours. Sodium-Sulfur batteries made by company NGK have a 7-hour discharge duration. Compressed-Air Storage charge and discharge durations can exceed 10 hours. PHS charge and discharge durations are from a few hours to a few dozen hours.[7] In the future, hydrogen-based technologies could include storage with a few hundred hours of discharge duration. In fact, the storage segment of the full hydrogen chain value might have similarities with natural gas storage, for which discharge is seasonal.

Based on these duration differences, technologies can deliver different services to the electricity system. PHS are first designed for energy transfers across days and weeks (see Crampes and Moreaux, 2010), even though it can also deliver frequency control. Batteries have delivered UPS[8] and more recently frequency control services.[9] They have entered intra-day energy transfer markets even more recently, mainly on mini-grids in areas remote from interconnected grids. Flywheels have delivered UPS and more recently frequency control, but remain uneconomic for energy transfers.

Our unified economic approach with charge and discharge durations follows three steps.

In the first section, we analyze how to optimally schedule the charging and discharging of installed storage equipment. We determine both the primal variables – quantity of input during charging periods and output during discharging periods – and dual variables, in particular the reserve storage value. We show that the shorter the discharge and charge durations, the more frequently the reserve storage value will change.

In the second section, we determine the optimal sizing of storage installations by using the discharge duration as the decision variable. We first determine the optimal duration for an isolated storage plant by comparing the discounted value of profits computed in Section 1 and the investment annuity. In a second step, we address the problem of combining several plants characterized by different charge and discharge characteristics.

[7] In France, discharge durations of PHS in Montézic and Grand Maison are 40 hours and 30 hours respectively. Hydroelectric reservoirs also are usually classified by their discharge or charge durations: pondage reservoirs for a few hours, dams for a few dozens or hundreds of hours, and no reservoir in the case of run-of-river.

[8] Uninterrupted Power Supply usually includes an AC/DC converter and a storage equipment. Acid-lead batteries have been common in this service for decades. UPS with Li-ion batteries and flywheels is common now.

[9] Change in US federal regulations on wholesale power markets in 2011 (US FERC Order n°755) has been instrumental in favoring this function for batteries and flywheels. Frequency regulation by batteries was figured out in West Berlin in the 1980s.

The third section completes the long-term analysis of the whole power system production function, including investment in storage installations. We examine the feedback between well-sized storage installations and the sizing of the power generation system, e.g. the investment savings in peak-load and half-baseload generation plants that storage might provide.

Three results are highlighted.

1. Present state-of-art storage technologies provide potential economic savings in fuel costs for peak-load and half-baseload plants, consequently in CO_2 emissions, since most peak-load and half-baseload plants are fossil fuel plants.
2. All storage technologies are still so expensive that substitutability for peak-load and half baseload generation development is partial. This means that further development of baseload technologies (nuclear and renewables) coupled with storage to eradicate totally half baseload and peak-load technologies (gas plants) is unprofitable today. That may change with both a very high carbon tax for fossil-fuel plants and a breakthrough in storage technologies. Changes in consumers' behavior should also play a key role.
3. Studies on possible new electricity mix in 2050 illustrate the last point to a certain extent. Nearly 100% decarbonized electricity mixes with nuclear and renewable technologies show a growing role for storage development. Fossil-fueled plants that used to make the major baseload part of electricity worldwide, are kept in this new mix but relegated to a role of security of supply, used in half-baseload and peak-load.

1. Short run management of a storage installation

In this section, we determine the optimal scheduling for a storage equipment in a given power generation system summarized by data on hourly wholesale market prices. As we deal with heterogeneous storage technologies, our common generic vocabulary is either filling or storing or buying when the installation is being charged, and emptying or destoring or selling when the installation is being discharged.

Before setting the model, we need to explain what optimal scheduling means. It refers to the task of the pivotal agent in charge of piloting the operation of an existing installation for a given time span.

Upstream, this "guiding operator" can be seen as an agent reporting to the installation owner. Its task is to provide the owner with information on the expected optimal gross profit from the installation. An investor who has this information considers installation uprating or decommissioning.

Downstream, the pivotal agent is the principal of the operator in charge of the day-to-day storage process, i.e. immediate filling or emptying of the installation in view of spot market prices. What signal should be transmitted by the pivotal operator to the day-to-day operator, to enable the latter to take short-term decisions aligned with long-term profit maximization?[10] The answer is the "energy reserve value" that the day-to-day operator will compare with the market price to decide whether to store or destore energy. This task breakdown is not just theoretical. It is illustrated by the existence of separate departments within electric utilities that have numerous production plants to operate jointly with storage facilities, notably hydro dams and PHS (see for example Evans *et al.* 2013).

The entire Section 1 is devoted to tailoring the reserve value, starting from the technical parameters of the storage installation and given the electricity prices determined in energy markets.

1.1 Optimal management of a storage installation

We consider a discrete modeling of time where periods are labeled t=1,2,…,T. The total duration T is given (T may be for instance a whole year). Let us denote the parameters and variables included in the objective function of the decision-maker as follows:

p_t is the electricity price during period t in the system to which the storage installation is connected. It is the gain from a destored unit of energy sold to the system, and the cost of a stored unit of energy bought from the system.

q_{ot} (≥ 0) is the output at period t, i.e. energy destored and sold to the system.

q_{it} (≥ 0) is the input at period t, i.e. energy bought from the system and stored.

S_t (≥ 0) is the stock of energy contained in the installation at the beginning of period t.

$\boldsymbol{\Pi} = \sum_{t=1}^{T} \boldsymbol{p_t}(\boldsymbol{q_{ot}} - \boldsymbol{q_{it}})$ is the profit to be maximized on the time span {1;T}. For the sake of simplicity, there is no cash flow discounting within a cycle {1;T}. As explained above, the operations we describe last for a few months at the longest. We will introduce discounting in the investment discussion.

The storage installation is defined by four technical parameters:

K_o is the maximum capacity of discharging (output):[11] $q_{ot} \leq K_o,\ t = 1,2, \dots, T$

K_i is the maximum capacity of filling (input): $q_{it} \leq K_i,\ t = 1,2, \dots, T$

S_{max} is the maximum storage capacity of the equipment:[12] $S_t \leq S_{max},\ t = 1,2, \dots, T$.

[10] Our concern here is limited to first best decentralization of decisions. We do not consider strategic issues of information asymmetry between the different layers of the organization.

[11] Rigorously, we should write $q_{ot} \leq K_o\ \Delta t$. To simplify notations, we fix $\Delta t=1$.

[12] When the installation uses several reservoirs as is the case in PHS, S_{max} is the smaller of the reservoirs' volumes. For electrochemical batteries and super-capacitors, reservoirs are the two electrodes. For air-compressed-air and flywheel, only the upper reservoirs have limited capacity.

r (<1) is a proxy for energy losses in the storage system. We assume that all losses occur at the filling stage.

The dynamic equation of the stock is: $S_{t+1} = S_t + rq_{it} - q_{ot} \quad t = 1,2,\ldots,T$

Stocks at the beginning and end of the time span under scrutiny are constrained by earlier inflows and later obligations: $S_1 \leq S_{initial}$ and $S_{final} \leq S_{T+1}$

To each constraint, we associate a dual (or shadow) variable, written between brackets in program **(P)** below; it is non-negative when the constraint is an inequality.

Given the technical data of the storage equipment and the energy prices forecast, the short-term problem to solve is the following program:

$$\textbf{(P)} \quad \Pi = \underset{\{q_{ot}\},\{q_{it}\}}{MAX} \sum_{t=1}^{T} p_t(q_{ot} - q_{it}) \text{ s.t.} \begin{cases} 0 \leq q_{ot} & (0 \leq \alpha_{ot}^{0}) \\ q_{ot} \leq K_o & (0 \leq \alpha_{ot}^{Ko}) \\ 0 \leq q_{it} & (0 \leq \alpha_{it}^{0}) \\ q_{it} \leq K_i & (0 \leq \alpha_{it}^{Ki}) \end{cases} \text{ and } \begin{cases} S_{t+1} - S_t = r\, q_{it} - q_{ot} & (\lambda_t) \\ 0 \leq S_t & (0 \leq \mu_t^0) \\ S_{final} \leq S_{T+1} & (0 \leq \mu_{T+1}^0) \\ S_t \leq S_{\max} & (0 \leq \mu_t^{S\max}) \\ S_1 \leq S_{initial} & (0 \leq \mu_1^{S\max}) \end{cases}$$

Given our assumptions on time scaling, physical variables and parameters (q_{ot}, q_{it}, S_t, S_{max}, $S_{initial}$, S_{final}) are measured in kWh, and parameters (K_o, K_{il}) are measured in kW. All dual variables α, μ and λ are measured in \$/kWh. The Lagrangian of the program is:

$$L = \sum_{t=1}^{T} \left\{ p_t(q_{ot} - q_{it}) + \lambda_t(r\, q_{it} - q_{ot} + S_t - S_{t+1}) + \alpha_{ot}^0 q_{ot} + \alpha_{it}^0 q_{it} + \alpha_{ot}^{Ko}(K_o - q_{ot}) + \alpha_{it}^{Ki}(K_i - q_{it}) \right\}$$
$$+ \sum_{t=2}^{T} \left\{ \mu_t^0 S_t + \mu_t^{S\max}(S_{\max} - S_t) \right\} \quad + \mu_{T+1}^0 (S_{T+1} - S_{final}) + \mu_1^{S\max}(S_{initial} - S_1)$$

(**P**) is a problem of linear optimization in a convex bounded set. The set of optima is a convex set defined by the following necessary and sufficient conditions:[13]

(1)	Optimal output (destoring)	q_{ot} such that :	$p_t + \alpha_{ot}^0 - \alpha_{ot}^{Ko} = \lambda_t$	for t=1, 2…,T
(2)	Optimal input (filling)	q_{it} such that :	$p_t - \alpha_{it}^0 + \alpha_{it}^{Ki} = r\lambda_t$	for t=1, 2…,T
(3)	Optimal stock	S_t such that :	$\lambda_t - \lambda_{t-1} = \mu_t^{Smax} - \mu_t^0$	for t=2, 3…,T

supplemented with terminal conditions ($\lambda_1 = \mu_1^{Smax}$ and $\lambda_T = \mu_{T+1}^0$) and qualification constraints.

1.2 The reserve value of energy

The reserve value of energy λ_t is the key economic indicator of optimal management. It is the shadow value of energy in stock at period t, i.e. the inside value of energy, to be compared with the outside value p_t. From the analysis of conditions (1)-(3), the charging/discharging

[13] Karush-Kuhn-Tucker conditions. See for example "KKT optimality conditions for convex problems" in Boyd, Vandenberghe, 2009, §5.5.3, p.244. Afterwards, we are exclusively concerned with the optimal scheduling of the installation. For the sake of simplicity, all primal and dual optimal variables use the same notation as the initial values of program P.

decisions and the dynamics of λ_t are characterized by the following lemma and propositions (proofs in the appendix).

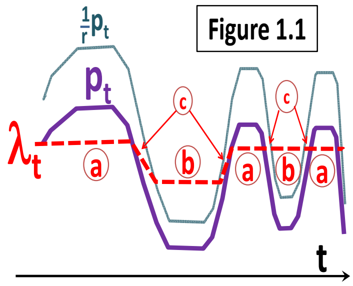
Lemma: It cannot be optimal to fill (q_{it}>0) and empty (q_{ot}>0) the reservoir at the same time.

This is due to charging losses r<1. Only r>1, which is technically impossible, would justify such a policy. Thus only three possibilities summarized in the following proposition remain.

Proposition 1

a) Destoring phases occur only when $p_t \geq \lambda_t$. When $p_t>\lambda_t$, the installation must be discharged at full capacity ($q_{ot} = K_o$). When $p_t=\lambda_t$, q_{ot} can take any value between 0 and K_o. In both cases, $q_{it} = 0$.

Moreover, if there are two successive periods where q_{ot}>0 and q_{ot+1}>0, then $\lambda_{t-1}=\lambda_t$. In other words, the reserve value is constant during consecutive destoring periods.



b) Storing phases occur only when $p_t \leq r\lambda_t$. When $p_t<r\lambda_t$, the installation must be charged at full capacity ($q_{it} = K_i$).[14] When $p_t=r\lambda_t$, q_{it} can take any value between 0 and K_i. In both cases, $q_{ot} = 0$.

Moreover, if there are two successive periods where q_{it}>0 and q_{it+1}>0, then $\lambda_{t-1}=\lambda_t$. In other words, the reserve value is constant during consecutive storing periods.

c) Idle phases. Periods with $p_t<\lambda_t<p_t/r$ correspond to inactive periods during which the stock is maintained at a constant level since $q_{ot} = q_{it}= 0$.

Figure 1.1 illustrates the three cases and the three following remarks:[15]

- **Remark 1.** The reserve value λ_t is a relevant indicator for using storage installation in a mix with power generation plants managed according to the cost merit order. At the optimum:
  i. The storage installation is scheduled for destoring (= producing) if its reserve value is less than the market price p_t that reflects the highest cost of running power generation plants sorted by their merit order (short-term marginal cost of the system); these phases are identified by **(a)** in the graph.

[14] Note that negative prices observed on some energy exchanges are an extreme case of $p_t < r\lambda_t$. In our proof in appendix, we exclude this case for the sake of simplicity.
[15] To facilitate the graphical depiction, time is considered as a continuous variable in the graphs.

ii. The installation is scheduled for storing (= consuming) if its reserve value weighted by r is greater than the short-term marginal cost of the system, measured by p_t. In a perfectly competitive market that reproduces the optimal hourly dispatching, the reserve value is the relevant indicator sent by the storage owners to the power exchange; these phases are identified by **(b)** in the graph.
iii. The installation remains idle when the market price is both too low for selling ($p_t \leq \lambda_t$) and too high for buying ($\lambda_t \leq p_t/r$) energy; these phases are identified by **(c)** in the graph.

- **Remark 2**. A common result of the three phases is that, as long as neither the cap nor the floor of the storage capacity is binding, the reserve value of stored energy λ_t remains unchanged. This actually is the Hotelling rule applied to our model, where the discounting rate is zero.[16] By contrast, the reserve value can change only during periods of empty or full storage capacity. In Figure 1, the reservoir is filling up during phases **(b)** and emptying during phases **(a)** but its shadow value remains constant because it is neither completely empty nor completely full. As a transition from **(a)** to **(b)** and back, phases **(c)** begin because either the reservoir is empty (then λ_t decreases) or full (then λ_t increases).
- **Remark 3.** Proposition 1 and Remarks 1 and 2 explain how λ_t and λ_{t+1} are related for t=1,…,T, i.e. they describe the trajectory of the shadow value of stored energy. To know the exact position of this trajectory, we have to use the internal "supply=demand" condition. Since terminal constraints are necessarily binding, this internal condition is $\sum_{t=1}^{T}(q_{ot} - rq_{it}) = S_{initial} - S_{final}$. Optimal solutions (KKT conditions) at periods 1 and T+1 are: $S_1=S_{initial}$, $S_T=S_{final}$, $\lambda_1=\mu_1^{Smax}$ and $\lambda_T=\mu_{T+1}^{0}$.

Let us consider sets of successive periods of storing or destoring. A destoring phase begins at date t when $q_{ot}>0$, with no possible date t_0 prior to t such that $q_{ot0}\geq 0$ during [t_0; t] and a strict inequality for at least one date in the interval.[17] A storing phase is defined in a symmetric way. Given a) and b) of Proposition 1, during each active storing or destoring period, λ_t remains constant. Thus, a phase is a series of successive periods where the internal energy value λ_t remains constant. Let u be the index of such a period and λ_{ou} (resp. λ_{iu}) the dual value of stored energy in phase u if it is a de-storing (resp. storing) set of periods. A cycle is defined as a pair of storing and de-storing phases, possibly separated by an idle phase. By convention, we suppose that the first phase is a destoring phase. We can state the following corollary, illustrated by cases **(c)** in Figure 1.1 (proof in appendix):

[16] For a proof in a more general framework, see Crampes and Moreaux (2018).
[17] In other words, there is no destoring prior to t not followed by storing.

Corollary 1

a) Each storing reserve value λ_{iu} of cycle u is necessarily no larger than the (previous) destoring reserve value λ_{ou} of the same cycle u.[18] The market price at the beginning of a storing phase is necessarily no larger than the market price of the latest preceding destoring period.

b) Each storing reserve value λ_{iu} of cycle u is necessarily no larger than the (succeeding) reserve value of destoring λ_{ou+1} of the following cycle u+1. The market price at the end of a destoring phase is necessarily no smaller than the market price of the earliest succeeding storing date.

To sum up, the reserve value of stored energy λ_t is:

- below local maxima of market prices during destoring phases;
- above local minima of market prices (discounted by r) during storing phases;
- between market values p_t and p_t/r during idle phases.

To provide further details on the trajectory of the reserve value and to extract its average level during [1;T] would require additional assumptions. For instance, in a steady-state regime ($S_{initial}$=S_{final} and $\lambda_1=\mu_1^{Smax}=\lambda_T=\mu_{T+1}^0$), if $K_o = rK_i$, r=1 (no loss efficiency) and $S_{max} = TK_o/2 = TK_i/2$, then λ_t would be constant all over [1;T] and equal to the median value of market prices.

1.3 The impact of discharge and charge durations on the value of the installation

We now examine the impact of the technical characteristics of the storage installation on its optimal management, its profit Π (this section) and on the reserve value (next section).

In program P, four parameters characterize the installation (K_o, K_i, S_{max} and r), and two parameters characterize initial and final conditions ($S_{initial}$ and S_{final}). Thus, profit Π associated with optimal management can be written as a function of these parameters: $\Pi(K_o,K_i,S_{max},S_{initial},S_{final})$.[19] As long as storage plants are small compared to installed production capacity, a change in their size has small or zero effect on electricity prices. We will discuss the endogeneity problem in Section 3.

Consequently, the dynamics of management is not drastically modified when all parameters (except r) are increased by the same homothetic transformation: when K_o, K_i, S_{max}, $S_{initial}$ and S_{final} are all multiplied by a positive scalar e, outputs and inputs at each period are all multiplied by e, profit is multiplied by e, and the reserve values at each period are left unchanged. More interesting is the impact of changes of one single parameter, in particular when the reservoir capacity S_{max} is increasing while other parameters are left unchanged.

[18] Consistent with our notation above.

[19] We omit parameter r, as we do not examine its impact on profit.

To go further in this direction, we define two key notions in storage problems.

- Discharge duration $D_o = S_{max}/K_o$ is the minimum time needed to empty the initially full reservoir, which means discharging at maximal outflow capacity.
- Charge duration $D_i = S_{max}/(rK_i)$ is the minimum time needed to fill the initially empty reservoir, which means charging at maximal inflow capacity.

Given their definitions, D_o and D_i are purely technical characteristics of the installation. They are not to be confused with the annual durations of destoring and storing that will be defined further. The following proposition states how profit evolves when technical characteristics are changed (proof in the appendix).

Proposition 2

Π is homogeneous of degree 1 with respect to the vector $\{K_o,K_i,S_{max},S_{initial},S_{final}\}$, and non-decreasing and concave with respect to the vector $\{K_o,K_i,S_{max},S_{initial}\}$. In particular, Π is a non-decreasing and concave function of S_{max}.

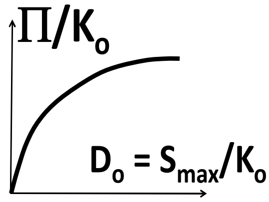
This means that for K_o and K_i fixed, each increase of Π per unit of increase of S_{max} is non-increasing with S_{max}. Equivalently, for K_o and K_i fixed, each increase of Π/K_o (or Π/rK_i) per unit of increase of discharge duration $D_o=S_{max}/K_o$ and charge duration $D_i=S_{max}/(rK_i)$ is non-increasing with simultaneous increases of S_{max}/K_o and S_{max}/K_i.

Corollary 2

Since Π is homogeneous of degree 1, an increase of S_{max} with K_o and K_i left unchanged is equivalent to an increase of $D_o=S_{max}/K_o$ and $D_i=S_{max}/(rK_i)$, i.e. an increase of both discharge and charge durations in the same relative proportions.[20]

The result of Corollary 2 is illustrated in Figure 1.2 by plotting varying discharge duration D_o and profit Π per unit of capacity of discharging K_o. It will be used in Section 2, when profit is compared to investment costs of technologies with different charge and discharge durations and with different discharging (or charging) capacities.

Figure 1.2



This purely analytical result is well known by practitioners. For instance, it has some resemblance with the new earning support scheme of capacity markets for battery storage with a duration

[20] To be rigorous, an increase of one parameter (such as S_{max}) has an influence on the relative position of initial and final conditions. This point is omitted here, inasmuch as we examine only regimes that have converged to a steady state, with $S_{initial}=S_{final}$ and $\lambda_1=\mu_1^{Smax}=\lambda_T=\mu_{T+1}^0$.

shorter than four hours, announced last year in the UK.[21]

But discussions with practitioners from various power systems also show that this kind of result is becoming common when comparing the value per kW (unit of capacity K_o) of equipment with such wide size differences (K_o) as Lithium-ion batteries for stationary use with discharge duration between 1 and 4 hours and Pumped Hydroelectric Storage with much longer discharge duration.

From now on, we limit the analysis to solutions that already converged to a steady state, i.e. such that $S_{T+1}=S_1=S_{initial}=S_{final}$.

Then energy destored is equal to energy stored during the whole period (net of energy losses). This is the case in a one-year period for a seasonal electricity system, with no exogenous shock (i.e. the market price p_t follows an annual periodicity) and for a storage installation having run for enough time.[22] In a steady state, dual variables satisfy $\lambda_1=\mu_1^{Smax}=\lambda_T=\mu_{T+1}^0$. From now on, we consider that T is the end of the year.

For convenience, we adopt the following notations:

H_o and H_i are the annual durations of destoring and storing respectively at full capacity equivalent during {1,…,T}. In steady states, H_o and H_i are related by the equality between total destored energy and total stored energy:

$$H_o = \frac{1}{K_o}\sum_{t=1}^{T} q_{ot} \qquad H_i = \frac{1}{K_i}\sum_{t=1}^{T} q_{it} \qquad H_o K_o = H_i r K_i$$

H_i and H_o are not to be confused with charge duration D_i and discharge duration D_o defined earlier. In fact, H_o/D_o can be interpreted as the number of equivalent full destoring phases and H_i/D_i can be interpreted as the number of equivalent full storing phases during a year.

(p_o, λ_o) and (p_i, λ_i) are the annual average prices and dual variables during destoring and storing phases, weighted by outputs and inputs respectively.

λ_{ou} and λ_{iu} are the time series of N dual values at dates of destoring/output beginning and storing/input beginning respectively, like in Section 1.2.

Using these notations, we can rewrite the maximized profit as follows (proof in appendix):

Proposition 3

The profit from the storage installation when optimally managed can be expressed as a function of price and capacity parameters. In a steady state:

---

21 See "De-rate hit for UK short-duration storage", Platts Power in Europe, December 18th , 2017.

22 In a periodic price-time series with periodicity lower than T, one can show that time to reach a steady state is less than T by using the Bellman Principle of Optimality.

$$\textbf{(4)}\ \Pi = \left(p_o - \tfrac{1}{r}p_i\right)H_oK_o = (p_o - \lambda_o)H_oK_o + (r\lambda_i - p_i)H_iK_i + S_{max}\sum_{u=1}^{N}(\lambda_{ou} - \lambda_{iu})$$

Equation (4) should not be misunderstood: Π (though homogeneous of degree one) is not a linear function of K_o, K_i and S_{max}, since the other parameters of the equation, be they visible (p_o, p_i, H_o, H_i, λ_{ou}, λ_{iu}) or invisible ($S_{initial}$ and S_{final}), also depend on K_o, K_i and S_{max}.

In the next section, we examine how the term $\sum_{u=1}^{N}(\lambda_{ou} - \lambda_{iu})$ varies with S_{max}.

1.4 Impact of charge and discharge durations on the reserve value

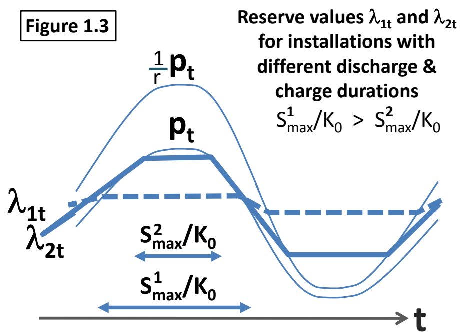
Longer charge and discharge durations contribute to lower differences in reserve values between destoring periods and storing periods, and to longer time of use of the installation across the whole period. This is made explicit in the following proposition (proof in appendix).

Proposition 4

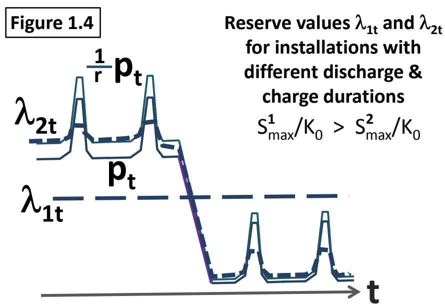
The sum $\sum_{u=1}^{N}(\lambda_{ou} - \lambda_{iu})$ of differences between destoring reserve values and storing reserve values is non-increasing when S_{max} is increasing.

As a result, the number of periods where λ_t is different from any λ_{ou} or λ_{iu}, i.e. where storage is inactive, is non-increasing with S_{max}. Inversely, the number of periods of active storing and destoring is non-decreasing: the average time of use of the installation is lengthened when S_{max} increases.

Figure 1.3 illustrates Proposition 4. Installation 1 (with energy value λ_{1t}) has discharge and charge durations longer than installation 2 (with energy value λ_{2t}). As the duration of use is the number of periods during which the reserve value is constant, the figure clearly shows a longer active period for installation 1.



As a consequence, the installation with longer charge and discharge duration is more often scheduled in the dispatching. This means that its reserve value is more often below the reserve value of the installation with the shorter duration during destoring periods, and more often above the reserve value of the installation with shorter duration during storing periods.



This does not mean that when S_{max} is increasing, <u>all</u> λ_t of destoring periods are decreasing and <u>all</u> λ_t of storing periods are increasing. Figure 1.4 illustrates a case where at some dates, installation 1 with higher

S_{max} is destoring while installation 2 with lower S_{max} is storing, and for some other dates installation 1 is storing while installation 2 is destoring.

Since the total difference between reserve values in destoring and storing periods is non-increasing with S_{max}, the maximum difference is for S_{max} starting from zero. The minimum difference is zero when the reserve value is constant for the whole period [0;T], as illustrated by λ_{1t} in Figure 1.4.

In the latter case, the cap and floor constraints on stock ($0 \leq S_t \leq S_{max}$) are never binding. This means that the stock constantly has a reserve value that is both too high (preventing full destoring), and too low (precluding complete fulfilling).

2. Investing in storage according to the charge and discharge durations

We now take into account the storage plant investment costs and analyze how charge and discharge durations affect these costs (§ 2.1). Using the information provided by the "planning operator" (see Section 1) or by market-priced long-term contracts associated with the future services of the installation, the investor compares the expected gross profit with the investment cost associated with different installation sizes, in order to obtain a non-negative net present value (§ 2.2).

2.1 The cost of storage installations

The different storage technologies to compare have different life durations. One way to compare their profitability is to compute the equivalent cost annuity of their investment and maintenance cost. Let A denote the annuity.[23] Let us explain its properties. A storage installation is made of different components. The manufacturer may have some leeway to design the maximum capacity of output K_o, the maximum capacity of input K_i and the maximum capacity of stored energy S_{max}. Let us assume there is no specific location scarcity (though it is crucial for hydro dams and PHS) and it is possible to replicate installations by industrialization. If the technology is mature and deployed at industry scale (with learning by doing effects already realized), we can agree that the total investment cost is a function homogeneous of degree 1 of the parameters $\{K_o, K_i, S_{max}\}$. The constant cost annuity of the installation, A, has the same homogeneity property. For a given "global size", the manufacturer

[23] Investment annuity per unit of capacity is an economic notion homogeneous to a marginal cost. In our setting, it is the cost of deciding to invest in the installation a unit time ahead (a year here). Justification of such an approach relies on the absence of technical progress and annual periodicity of prices (see details in appendix: "The dynamic problem of investment and annuities"). The approach can be extended to cases of technical progress, growing scarcity, and more generally non-steady state situations. In those cases, the investment annuity would be dependent on time i.e. it would stand for the "generalized long-run marginal cost").

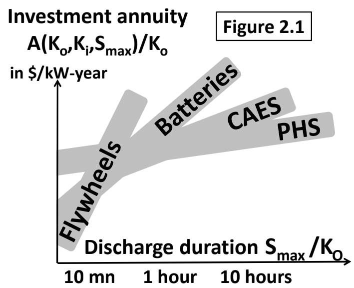
may have to choose among different triplets {K_o, K_i, S_{max}} matching with different discharge $D_o = S_{max}/K_o$ and charge $D_i = S_{max}/(rK_i)$ durations. For instance:

- PHS with alternators, turbines and pressure pipes of size K_o and K_i, and upstream and downstream dam reservoirs of size S_{max}.
- Electro-chemical batteries with Battery Management System of size K_o and K_i, exchange surfaces of electrodes of size K_o and K_i, and volumes of electrodes of size S_{max}.[24]

Some technologies such as sodium-sulfur batteries or flywheels may display few or no variety of discharge and charge durations on the market today. This reflects rational choices of the manufacturers and one of the conclusions of our paper: given the cost characteristics of each technology, their design must be adapted to the services they are most fitted for. For instance, one can imagine flywheels designed with discharge duration of a few hours but with a prohibitive cost that would prevent any practical use.

The result is that changing the design of the equipment and industrializing it to increase only one parameter (S_{max} for instance) should not lead to duplicating all components. Thus, average cost per unit of one parameter should be non-increasing with this parameter, ceteris paribus. For two alternative storage capacities, S^b_{max} and S^a_{max}, one should have:

$$S^b_{max} > S^a_{max} \quad \Rightarrow \quad \frac{A\left(K_o, K_i, S^b_{max}\right)}{S^b_{max}} \leq \frac{A(K_o, K_i, S^a_{max})}{S^a_{max}}$$



Batteries: lead-acid, li-ion, etc

CAES: Compressed Air Storage

It turns out that the cost of increasing S_{max} (or equivalently the charge and discharge durations $D_i = S_{max}/(rK_i)$ and $D_o = S_{max}/K_o$) may vary strongly among different families of technologies.

As Figure 2.1 illustrates (displaying for convenience A/K_0 with $D_o = S_{max}/K_o$ varying), this cost is extremely high for flywheels and super-capacitors, relatively low for PHS (but heavily dependent on site locations), and moderate for Compressed Air Energy Storage and electrochemical batteries, in spite of very significant advances in lithium-ion technologies.

2.2 Impact of charge and discharge durations choice on investment

Section 1.3 examined the impact of a limited number of technical parameters on the gross economic value of a storage installation $\Pi(K_o,K_i,S_{max})$[25], without identification of the technology

[24] Usually illustrated in chemistry by the "Ragonne diagrams" for each technological family of batteries. The difference at the cell level is noticeable between full battery electric cars (BEV) and plug-in hybrid electric cars (PHEV). Actually, BEV have higher autonomy than PHEV first because the parameter S_{max} of individual cells is relatively higher ceteris paribus (i.e. with K_o and K_i boiled down to similar size), and secondly because the number of cells per battery is larger.

used (PHS, battery or other). Section 2.1 examined analytics of investment annuity $A(K_o,K_i,S_{max})$ with respect to these parameters, highlighting some stylized facts common to various technologies. Now we briefly deal with a full cost-benefit analysis, uniform across storage technologies, and limited to small or marginal investment relative to the entire electric system, i.e. with no impact on market prices, which will be dealt with in Section 3.

As we focus on steady-state values and costs, the inter-temporal net present value analysis can be reduced to the analysis of the net equivalent annual profit $\Pi(K_o,K_i,S_{max})$ - $A(K_o,K_i,S_{max})$.[26]

An investment is to be made if the annuity of its Net Present Value is positive

$$\textbf{(5)} \qquad \Pi(K_o,K_i,S_{max}) \geq A(K_o,K_i,S_{max})$$

Additionally, as we focus on equipment with values and costs homogeneous of degree 1 wrt $\{K_o,K_i,S_{max}\}$, our problem can be limited to an analysis of the relative variations of the parameters, in particular to variations in S_{max} (or $D_o=S_{max}/K_o$ and $D_i=S_{max}/rK_i$) that provide positive Π-A ceteris paribus.[27]

As $\Pi(K_o,K_i,S_{max})$ is concave wrt S_{max} and $A(K_o,K_i,S_{max})$ is affine wrt S_{max} for a given technology, the NPV annuity $\Pi(K_o,K_i,S_{max})$ - $A(K_o,K_i,S_{max})$ is concave wrt S_{max}. Thus, each increase of the annuity Π-A per unit of increase of S_{max} is non-increasing with S_{max} (property of concave functions). In practice, given market prices and technologies, there is always a bounded value S^m of S_{max} beyond which Π-A is decreasing (increment of Π-A with S_{max} are negative) and below which Π-A is increasing (increment of Π-A with S_{max} are positive). If the annuity is positive for S^m, it is positive (but decreasing) for higher values of S_{max}, up to a value S^*, beyond which it is negative, and it is positive for lower values of S_{max}, down to a value S^{**}, below which it can be negative. Investment is then profitable for all values of S_{max} that belong to the interval $[S^{**};S^*]$.

The non-existence of a positive NPV occurs in two cases:

- The chosen storage technology is not yet mature or competitive. This is the case, for instance, of lithium-air batteries currently in preliminary R&D phases.
- Profit Π can be too low due to depressed electricity market prices, notably because of excess power generation capacity. This has been the case in many parts of Europe over the last few years, as a consequence of lower economic growth, lower fossil-fuel prices, and accelerated development of capacities in renewables. Before 2008, the magnitude of market price differences was high and development of PHS was still significant in Europe.

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[25] Here we set aside technical efficiency r (given exogenously, with no sensibility analysis), and initial and final conditions $S_{initial}$ and S_{final}.

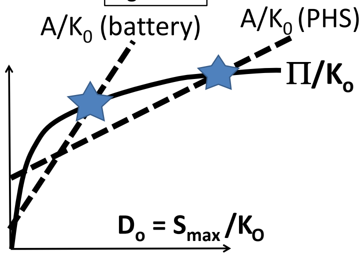
[26] See "The dynamic problem of investment and annuities" in the appendix.

[27] The social net benefit of the investment is positive as long as the net profit is positive, and is to be realized if there is competitive access to the technology. In case of a non-regulated private monopoly on the technology, the investor may choose S_{max} to maximize Π-A.

Hereafter we limit the analysis to cases where the net profit is positive with two relatively mature technologies: PHS and lithium-ion batteries. Their respective fixed costs[28] wrt S_{max} or S_{max}/K_o is illustrated in Figure 2.1. If we assume identical values of the efficiency coefficient r and of the ratios $D_o= S_{max}/K_0$ and $D_i=S_{max}/K_i$ (even though absolute size of PHS may be 100 times the size of batteries), they have the same economic value Π/K_o wrt S_{max}/K_o, as illustrated in Figure 1.2.

Figure 2.2 superimposes Figures 1.2 and 2.1 and illustrates the full cost and benefit analysis. The symbols ★ indicate the threshold values S* of S_{max} for PHS and batteries, beyond which net respective profits Π-A are negative. Given the cost curve difference between both technologies, their threshold values are different. Note that for very short discharge and charge durations, the cost of batteries can be lower than the cost of PHS. Thus for such durations, net profits are higher for batteries.

Figure 2.2



In the current state-of-the art of storage technologies, in all cases Π-A becomes negative beyond some duration threshold, at most a few weeks. This means that seasonal electricity storage cannot be competitive.[29]

Figure 2.2 also illustrates the rationale for simultaneous development of several technologies with different durations D_o associated to distinct service provisions $\Pi(D_o)$.

- For services requiring duration less than a couple of minutes or a few seconds with high added value per kWh (per kW and per hour of duration), technologies with low annuity for low discharge and charge duration are appropriate. Large diffusion of lead-acid batteries for UPS[30] among industrial consumers requiring enhanced quality of electricity compared to the one provided by the grid illustrates our case.
- For services requiring a duration of less than a few tens of minutes, such as frequency-control, lithium-ion and lead-acid batteries may be the most suitable.
- Energy transfers between day and night require durations of a few hours, for which batteries are much less competitive, except for special cases such as out-of-grid load pockets. Decreasing the cost of lithium-ion batteries might also change the situation in the future.

[28] A represents the full fixed cost of the storage installation: fixed operation & maintenance costs are implicitly included.

[29] Hydro dams can have seasonal storage, but are not included in our analysis (hydro dams can be viewed as storage technologies where inputs are provided by nature "for free").

[30] UPS (Un-interruptible Power Supply) that includes an inverter and a storage battery.

- For services requiring energy transfers beyond 10 or 20 hours, PHS with adequate discharge and charge durations has until now been the only competitive technology. Incidentally, PHS with such durations can also provide the shorter services mentioned previously.[31]

3. Feedback of storage on the generation mix

In the former sections, we took market prices and the power generation mix as given. Actually, one can expect some changes due to the development of efficient storage equipment. Note that the large-scale entry of storage in the electricity industry will have consequences on energy prices because: i) when charging, the operators of storage plants create an additional demand, then push prices up, and ii) when discharging they create an additional supply, then they pull prices down. To remain in line with our hypothesis of exogenous prices (Section 1), we assume that either each operator is too small to be a price maker, or that large operators abide by antitrust rules so that they do not exert their market power, neither as buyers (during charging phases) nor as suppliers (during discharging phases).

We first explain and then illustrate that the possibility for storage to replace a given production technology is impaired by its limited discharge and charge durations.

3.1 Feedback of storage non-marginal investment on market prices

In Figure 2.2 in Section 2.2, below the threshold values of S_{max} of each storage technology, investment opportunities provide strictly positive net present values. Thus, in a perfectly competitive market with free entry, many investors enter the storage activity. Prices are then pushed up during charging phases and pulled down during de-charging phases. Profit Π, based on price differences (equation 4), is reduced. At the long-run equilibrium for storage investment there should be no remaining marginal investment opportunity providing for strictly positive net present values for any technology.

This equilibrium is illustrated by Figures 3.1 a, b and c, where only two storage technologies are supposed to be competitive (batteries and PHS). In Figure 3.1.a, symbols ★ indicate only two optimal D_o in the long-run equilibrium. In Figure 3.1.b, symbols indicate two continuums of optimal D_o, one for each storage technology. Remembering equation (4) in Section 1.3 and the dynamics of the reserve value analyzed in Section 1.2, Figure 3.1.b indicates that there would be multiple periods during which reserve values would be equal to

[31] We can make a comparison with generation technologies: appropriate plants for baseload can and do deliver power during peaking hours, along with peak-load units, without fully replacing them.

prices (storage would be the marginal technology on the electricity market), reflecting higher competitiveness of storage compared to Figure 3.1.a.

Figure 3.1.a

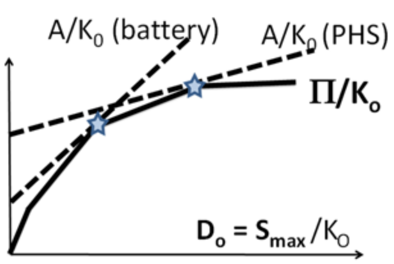


Figure 3.1.b

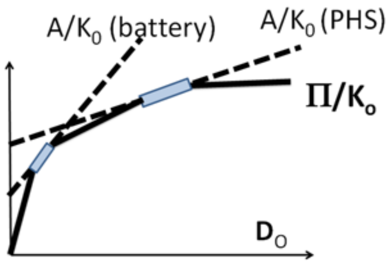
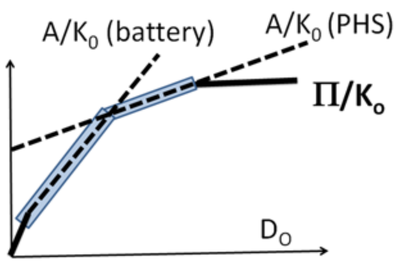


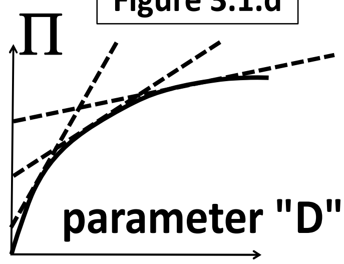
Figure 3.1.c



In Figure 3.1.c, symbol indicates one continuum of optimal Do, including all the shortest possible usage of storage technologies. In this situation, all the marginal technologies associated with the highest market prices would be storage technologies in de-charging phases. This would reflect low investment costs and high competitiveness of storage technologies: without any storage possibility, high market prices usually reflect the cost of peak-load generation plants, as the next section explains. Thus, Figure 3.1.c illustrates a scenario where storage technologies have become sufficiently competitive to be able to fully substitute peak-load generation plants.

In practice, as we will see in next sections, Figures 3.1.a and 3.1.b are presently more relevant than Figure 3.1.c, given the current cost of state-of-the-art storage technologies and the current relatively low cost of fossil fuels and low carbon prices.

Figure 3.1.d



Figures 3.1.a, b and c are an application to storage technologies of the general case pictured in 3.1.d, where the marginal service provided for by a family of given comparable technologies with a varying parameter D (gross profit Π, in continuous line) is compared to the complete unit costs of the family of technologies (dashed line). This figure is standard to represent the family of dispatchable generation technologies distinguishing peak-load, half baseload and baseload plants according to the annual duration D of capacity use (see details in the next section).

Tangency of profit and cost curves illustrates a long-run equilibrium: when investments are optimized, short-run and long-run marginal costs (or forward market prices and expected spot market prices) are equal by the envelope. Note that a figure is relevant for a family of "homogeneous" technologies: both profit and cost functions of the given parameter are different for storage technologies, for dispatchable generation technologies, or else (renewable non-dispatchable technologies for instance).

3.2 The limited ability of storage to save investment in generation

We first recall the basic economics of power generation (Section 3.1.1) and then explain the potential impact of storage technologies on the production mix. [32]

3.2.1 Power generation without storage

Assume that the power generation system has been developed according to economic criteria. In such a system, we have roughly four types of generation technologies:[33]

- Baseload technologies have the lowest short-term variable costs. They are called first in the economic dispatching. In this category we find wind power, solar PV, run-of-the-river hydro plants, all with zero variable costs when the primary source is available, and nuclear[34] with variable cost below $ 10/MWh. In some countries, baseload is provided by brown coal plants, or black coal plants, or even gas plants, with higher variable cost.
- Half-baseload technologies are called when baseload generation capacity is fully used. These are CCGT (Combined-Cycle Gas Turbine) with variable cost between $ 20 and $ 50 /MWh depending on gas and carbon prices.[35] Most coal plants have a lower variable cost (in particular when carbon prices are low), but in some countries they are used as half-baseload.
- Peak-load technologies are called into operation when baseload and half baseload capacities are fully used. It is the case of combustion gas turbines (CTs) with variable cost roughly twice the cost of CCGTs.[36] In areas with no access to natural gas, oil combustion engines are used with variable cost between $ 150 and 300/MWh.
- Extreme peak-load periods may entail loss-of-load with the highest variable cost. This means for instance implicit costs of around $ 20,000/MWh for situations required to occur less than 24 hours every 10 years[37] and the corresponding development of gas turbines (annuity around $ 50/kW/year) for expected marginal use of 24 hours every 10 years.

[32] For the sake of simplicity, transmission constraints are omitted, though transmission economics should obviously be introduced in a more full-fledged analysis: substitutability or complementarity between storage and transmission is an important issue. Also note that the methodology to analyze energy storage has close links with the methodology to analyze energy transmission. None is a production activity, both transfer energy from nodes to nodes. In the case of transmission, nodes are spatial while they are temporal in the case of storage. On transmission, see for example Joskow, Tirole 2005.

[33] We discard imports and demand response.

[34] Nuclear can also be thought as a storage capacity to be refueled every year or less. In such a situation, like any storage facility, nuclear has a reserve value to be added to its variable cost.

[35] For instance, with the wholesale gas price equal to $ 3/MBtu ($ 10.2 /MWhg) and a carbon price equal to $ 6/ton ($ 1.2 /MWhg), we obtain $ 11.4/MWhg, hence $ 19 per electric MWh (thermal efficiency: 60%). For a CO_2 price ten times higher (under tight climate policy) and a gas price twice as high (in the long term, even with tight climate policy), the variable cost of energy from CCGTs reaches $ 54 per electric MWh.

[36] Fuel costs are similar to CCGT's, but with thermal efficiency roughly halved, hence a variable cost that is doubled.

[37] Present criteria used by the Federal Energy Regulatory Commission in the US. In France, it is 3 hours per year.

A power system developed according to economic criteria without storage has the following properties (see Boiteux 1964a and 1964b, and Joskow 2011 for the particularities of intermittent renewable technologies):

- When demand is above the sum of all baseload, half-baseload and peak-load available power generation capacities, and thus entails some demand response or rationing mechanism (loss-of-load), average wholesale market prices[38] are above variable costs of infra-marginal peak-load technologies. Pertained revenue, noted R_a, is sufficient to pay for fixed-cost annuities of peak-load units; it also partially contributes to pay for annuities of infra-marginal half-baseload and baseload power generation capacities.
- When demand is above the sum of baseload and half-baseload available power generation capacities and below the sum of baseload, half-baseload and peak-load available power generation capacities, average wholesale market prices are above variable costs of infra-marginal half-baseload technologies. Pertained revenue, noted R_b, summed up with R_a, is sufficient to pay for fixed cost annuities of half-baseload units. Both revenues also contribute to the financing of baseload capacity.
- When demand is above baseload available power generation capacity and below the sum of baseload and half-baseload available power generation capacities, average wholesale market prices are above variable costs of infra-marginal baseload technologies. Pertained revenue, denoted R_c, summed up with R_a and R_b, is sufficient to pay for fixed cost annuities of baseload units.

Nuclear, CCGT and CT are examples of dispatchable power generation capacities, with significant guaranteed power (we assume that they are constant for the whole year, for legibility of figures below). Intermittent or variable renewables (wind power or solar PV) can be considered as baseload power generation capacities varying with dates and states of nature, used in priority to nuclear power[39].

Let H_a, H_{a+b} and H_{a+b+c} denote the cumulated annual number of periods where market prices are above variable costs of respectively peak-load, half-baseload and baseload (nuclear) power generation technologies. $H_a<H_{a+b}<H_{a+b+c}$ is straightforward. Given the relative investment and variable costs of available technologies, this ranking provides "break-even annual durations" between each family of technologies. Even though the notion of break-even annual duration must be adjusted to include variable renewables, let us suppose here that variable renewables can be economically developed up to a capacity level where it is never sufficient to supply all demand at any moment. Thus the "remaining baseload" is satisfied by other means (nuclear in our example).

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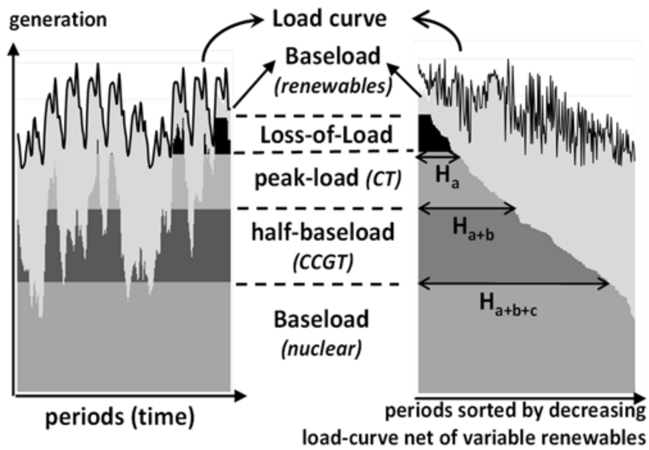
[38] Prices on energy only markets completed with Capacity Requirement Mechanisms, irrespective of what they are.
[39] We omit here kinetic constraints on nuclear and fossil fuel thermal plants.

Our stylized power system without storage is illustrated by Figure 3.2.a where baseload, half-baseload and peak load are sorted according to their variable cost merit order to supply some demand load curve. Variable renewables are displayed at the top of the generation stack for readability reasons, whereas it is clear that their output is used first before nuclear, CCGT and CT.

Figure 3.2.b illustrates the same power system with each period (hours) of the year sorted by decreasing load-curve net of variable renewables generation. Figure 3.1.b allows one to visualize the durations H_a, H_{a+b} and H_{a+b+c}.

Supply of demand load curve with power generation capacities without storage equipment

Figure 3.2.a **Figure 3.2.b**



Let us replace quantities by prices on the Y axis, with prices equal to nuclear fuel cost levels when only renewables and nuclear are called, to CCGT fuel cost when renewables, nuclear and CCGT are called, to CT fuel cost when all generation technologies are called, and to Value of Loss of Load when power generation capacities are not sufficient to match demand. Then, with periods sorted according to the criteria of Figure 3.2.b on the X axis, we would have a decreasing price curve.

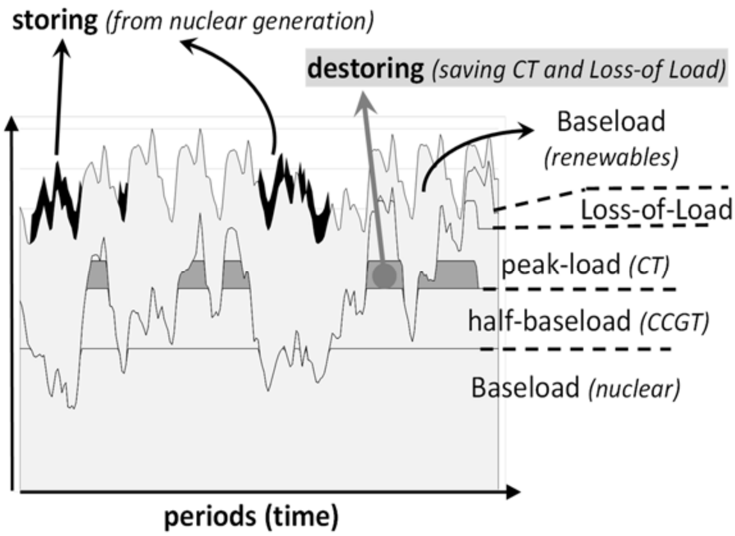
To give some rough numerical illustration, the whole year (24h/day x 365 days=8760 hours) may be divided into periods of 1000-2000 hours when demand is supplied by baseload and market prices (variable costs of baseload technologies) are low, 5000-6000 hours when market prices are based on variables costs of half-baseload technologies, 10-500 hours when market prices are based on peak-load unit variables costs, and a few hours for loss-of-load.

3.2.2 Economics of the power generation system with storage

In this sub-section we introduce a (non-identified) storage technology at a significant scale, so that it can change the generation mix in a non-marginal way. Figure 3.3 uses Figure 3.2.a. to illustrate the impact of storage with limited discharge and charge durations on our stylized electricity mix.

Periods of excess renewables and nuclear power generation capacities are used for storing at

Figure 3.3



low cost. Periods of loss-of-load and peak-load dispatching are used for destoring (saving of high costs).

But due to limited charge durations, not all excess renewables and nuclear generation capacities can be used. Limited discharge duration prevents storage installation from totally avoiding loss-of-load and dispatching peak-load at full capacity (storage is empty at the right side of Figure 3.3).

With a larger storage installation, in particular longer discharge and charge durations, we would be on the right side of Figure 3.1 where the investment cost increase would be higher than the gross profit.

Storage can reduce the installed capacity of peak-load units only if the periods of loss-of-load are all shorter than the discharge duration of the installed storage equipment (which is not the case in Figure 3.3). But this implies neither the complete disappearance of loss-of-load situations nor of peak-load units, which can occur in situations rare enough for not justifying a substitution with the storage installation. Equations (4) and (5) tell us that the total annual duration use must be sufficient to justify investment in storage installations. Our results are in line with those of Newbery 2018 who "*argues against the simplistic assumption that batteries, and indeed building more storage generally, offer the natural solution to balancing an increasingly renewables-dominated electricity system*". Other flexibility solutions (imports, back up by CCGT, demand response), are still cheaper than storage installations to solve the intermittency of renewables.

Another result is worth mentioning here: as storage possibility gives higher valuation to baseload generation power plants, it can be efficient to further develop the latter in partial substitution with half-baseload generation power plants. In the new situation, periods initially with lower market prices would be replaced by periods with higher market prices, in fact reflecting reserve value of storage equipment in storing phase.[40]

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40 The proof of these results is not provided here. The approach would be similar to the one adopted for proofs in appendix (convex analysis) but would require a full-fledged model of both generation and storage equipment, and would make the present paper much longer.

3.3 Numerical illustrations with present state-of-the art storage technologies

The case illustrated in Figure 3.3 with no investment savings in generation capacity may be typical of large interconnected power networks, where the reference generation technologies are one-cycle combustion gas turbines (CT) for peak-load, combined cycle gas turbines (CCGT) for half-baseload, and nuclear, coal or CCGT for baseload.

The marginal value from investing in one kW of CT results from avoiding a few dozen hours of loss-of-load that could occur once every 5 or 10 years.[41] It does not exclude possibilities of more frequent shorter periods of power cuts. As an illustration, let us consider a loss-of-load of 24 hours every 10 years, with a CT fixed cost of $ 50/kW/year and variable fuel cost of $ 100/MWh.[42] The result is a total cost of $ 50.24 /kW/year.[43] This figure is the expected gross profit that a storage installation able to provide the same service would earn to avoid CT investment. Today, the fixed cost of PHS with discharge durations of 25 hours is still twice or three times higher than this value. Costs of lithium-ion batteries remain still higher, even after the announcements in 2017 by Tesla and Bloomberg for 2025.[44]

This situation has another illustration in the NREL report for the Department of Energy (NREL 2012), with a scenario of a US electricity system in 2050 where 80% of annual generation is expected to come from renewables. The study shows a significant development of storage plants for intra-day transfers (batteries) and intra-week transfers (PHS and CAES). Nevertheless, total capacity from gas plants (together with remaining nuclear, coal and hydro plants) is as important as total peak-load demand. That means that storage installations are not expected to save investment in gas peaking units.

Alternative specific situations may be figured out, for instance for a small electricity system with low electricity consumption variations between seasons (tropical areas for example), where peaking unit usage is more frequent and more expensive, and using gas-oil fueled internal combustion engines because of lack of access to natural gas. Let us assume that gas-oil ICE (Internal Combustion Engine) might be used 3 hours a day 300 days a year (900 hours a year), at night when solar PV cannot produce anything. For a fixed cost (investment and maintenance) around $ 30/kW/year and electricity variable cost around $ 200/MWh (refined and distributed oil from crude oil at $ 50/bbl), we obtain a value of $ 210/kW/year.[45] This value

[41] See Footnote 37 above for France and the USA.
[42] See Footnotes 35 and 36.
[43] € 50 /kW/year + € 100/1000 /kWh x (24/10) hours/year = € 50.24 /kW/year.
[44] See BNEF NEO 2017.
[45] € 30 /kW/year + € 200/1000 /kWh x 900 hours/year = € 210 /kW/year.

is larger than the present cost of lithium-ion batteries with a discharge duration of 3 hours.[46] Then, for out-of-grid places in tropical areas, PV + batteries should be preferred to gas-oil ICE.

Again, even in this situation, peak-load units are not fully substituted by batteries if the VoLL is high (e.g. in hospitals and maternity clinics). It remains the possibility of infrequent events, such as several days of very low sun power and very low generation from solar PV, where recourse to peak-load units remains cheaper than investment in additional batteries that would be used in rare circumstances.

Conclusion

We have presented a unified cost-benefit analysis for storage technologies providing heterogeneous services to the electricity system. In particular, we have highlighted the role of discharge and charge durations as a key metric for segmenting storage technologies and services.

The deterministic analytical framework, although simplistic, provides for key intuitions. Complementary analysis with demand and renewables uncertainty would show the essential quality of storage equipment as an insurance device on top of a mere buffering function.[47] Access to storage equipment gives a premium (the convenience yield highlighted by Pindyck 1993) since it allows unexpected demand to be met without changing the production process.

We also have shown how an electric system with efficient mix and size might integrate several types of storage installations to provide fossil fuel savings in peak-load and half base-load plants, and possibly capacity savings in such plants, without reaching full substitutability.

This is the result of both the current cost of state-of-the-art storage technologies and the current relatively low cost of fossil fuels and low carbon prices. These conditions might change during the next decades if significant technological progress impacts storage technologies and environmental policy becomes more stringent. Determining the thresholds for these economic parameters to trigger full substitutability between carbonized fossil fuels and renewables (or nuclear) with storage is an open topic for further research.

Finally, as flexibility requirements on demand side are growing, our analytical analysis might be relevant to common final electricity uses that display storage characteristics such as water heating from direct electricity or heat pumps, air-conditioning, air heating or cooling processes with "inertia", cloth washing, and electric car charging and discharging.

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[46] The announced cost of the battery in West Burton (UK) with a discharge duration of 4 hours is $ 1600/kWh, hence an annuity of $ 207/kW/year for a lifetime of 10 years and a cost of capital of 5%.

[47] Similarly, uncertainty on natural water inflows has always been key for the optimal management and cost-benefit analysis of hydro dams.

References

ADEME 2016, “Mix électrique 100% renouvelable. Analyse et optimisations”. www.ademe.fr.

Bauduin Ph, 1989, “Réflexion sur la valeur actuelle des méthodes de choix d’investissement hydraulique à Electricité de France, Revue de l’Energie n°410, mars-avril 1989.

BNEF NEO 2017, « New Energy Outlook 2017, Bloomberg New Energy Finance.

Boiteux M., 1964a, “The Choice of Plant and Equipment for the Production of Electricity.” In J.R. Nelson ed. Marginal Cost Pricing in Practice, Prentice-Hall, Englewood, N.J. Reprint from previous publication in French.

Boiteux M., 1964b, “Marginal Cost Pricing.” In J.R. Nelson ed. Marginal Cost Pricing in Practice, Prentice-Hall, Englewood, N.J. Reprint from previous publication in French (1949).

Boyd S, Vandenberghe L, 2009, "Convex Optimization", Cambridge University Press 7th printing 2009, https://web.stanford.edu/~boyd/cvxbook/bv_cvxbook.pdf.

Burtin A., Silva V., 2015, "Technical and Economic Analysis of the European Electricity System with 60% RES, EDF R&D, 17 Jun 2015

http://www.energypost.eu/wp-content/uploads/2015/06/EDF-study-for-download-on-EP.pdf

CESE, 2015, "Le stockage de l’énergie électrique : une dimension incontournable de la transition énergétique", Les avis du Conseil Economique, Social et Environnemental, A. Obadia. Juin 2015. www.lecese.fr/sites/default/files/pdf/Avis/2015/2015_16_stockage_electricite_transition_nrj.pdf

Crampes C., Moreaux M., 2010, "Pumped storage and cost saving", Energy Economics 32 (2010) 325–333

Crampes C. et Moreaux M., 2018, "Apports naturels en eau dans les barrages-réservoirs et règle de Hotelling", TSE Working paper.

Dai H, Fujimori S, Silva Herran D, Shiraki H, Masui T, Matsuoka Y, 2017, “The impacts on climate mitigation costs of considering curtailment and storage of variable renewable energy in a general equilibrium model, Energy Economics 64, 2017

De Jonghe C., Delarue E, Belmans R., D’haeseleer W., 2011, “Determining optimal electricity technology mix with high level of wind power penetration”, Applied Energy 88, 2011.

Denholm P, Hand M, 2011, “Grid flexibility and storage required to achieve very high penetration of variable renewable electricity; Energy Policy 39, 2011

Després J., Mima S., Kitous A., Criqui P., Hadjsaid N., Noirot I., 2017, “Storage as a flexibility option in power systems with high shares of variable renewable energy sources: a POLES-based analysis”, Energy Economics, 64, 2017.

DOE EIA 2016, "Capital Cost Estimates for Utility Scale Electricity Generating Plants", US Department of Energy, Energy Information Administration. Parts on PHP and batteries http://www.eia.gov/analysis/studies/powerplants/capitalcost/pdf/capcost_assumption.pdf

EDF 2017, "Le stockage de l'électricité : un défi pour la transition énergétique", ouvrage collectif (Bart J.B., Bénéfice E., Brincourt T., Brisse A., Cagnac A., Delille G., Hinchliffe T., Jeandel E., Lancel G., Lefebvre T., Loevenbruck P., Nekrasov A., Pastor E., Penneau J.F., Radvanyi E., Soler R., Stevens P., Torcheux L.). Ed Lavoisier.

Esteban M, Zhang Q, Utama A, 2012, "Estimation of the energy storage requirement of a future 100% renewable energy system in Japan", Energy Policy 47, 2012q

Evans L, Guthrie G, Lu A, 2013, "The role of storage in a competitive electricity market and the effects of climate change", Energy Economics 36, 405–418

Gulagi A., Bogdanov D., Breyer C., The role of storage technologies in energy transition pathways towards achieving a fully sustainable energy system for India, Journal of Energy Storage 17 (2018) 525–539

IEA, 2015, "Energy Technology Perspectives", Part 1, Chap .2, Energy Storage, International Energy Agency.

Joskow P., 2011, "Comparing the Costs of Intermittent and Dispatchable Electricity Generating Technologies", American Economic Review: Papers & Proceedings 2011, 100:3, 238–241

https://economics.mit.edu/files/6317

Joskow P., Tirole J., 2005, "Merchant Transmission Investment", The Journal of Industrial Economics, vol. 53, n° 2, June, p. 233–264

Lederer P., Torrion P., Bouttes J.P. 1984, "A global feedback for the French System Generation Planning", 8th, August 1984

Lederer P., Colleter P. 1981, "Optimal Operation Feedback for the French Hydropower System", CORS-TIMS-ORSA Joint National Meeting, Toronto, May 1981

Newbery D. 2018 "Shifting demand and supply over time and space to manage intermittent generation: The economics of electrical storage", Energy Policy, Volume 113, February, Pages 711-720.

NREL 2012, "Renewable Electricity Futures Study: Exploration of High-Penetration Renewable Electricity Futures", National Renewable Energy Laboratory www.nrel.gov/docs/fy12osti/52409-1.pdf and 52409-2.pdf.

Pindyck R. 1993, "The Present Value Model of Rational Commodity Pricing", The Economic Journal, Vol. 103, May, n° 418, pp. 511-530.

Segundo Sevilla F., Parra D., Wyrsch N., Patel M., Kienzle F., Korba P., Techno-economic analysis of battery storage and curtailment in a distribution grid with high PV penetration, Journal of Energy Storage 17 (2018) 73–83

Steffen B., Weber C., 2013, "Efficient storage capacity in power systems with thermal and renewable generation", Energy Economics 36, 2013

Wilson I.A.G., Barbour E., Ketelaer T., Kuckshinrichs W., An analysis of storage revenues from the time-shifting of electrical energy in Germany and Great Britain from 2010 to 2016, Journal of Energy Storage 17 (2018) 446–456

Appendix. Proofs of propositions

A1. Proof of the Lemma

If it were true that $q_{it}>0$ and $q_{ot}>0$ for the same t, we would have $\alpha_{ot}^0 = \alpha_{it}^0 = 0$. From condition (1), $p_t - \alpha_{ot}^{Ko} = \lambda_t$ hence $p_t \geq \lambda_t$. From condition (2), $p_t + \alpha_{it}^{Ki} = r\lambda_t$ hence $p_t \leq r\lambda_t < \lambda_t$ which is not compatible with $p_t \geq \lambda_t$, except for the particular case $p_t = \lambda_t = 0$ (which gives no benefit whatever $q_{ot}>0$ and $q_{it}>0$).

A.2. Proof of proposition 1

At the beginning of each period t, the operator knows the market price p_t and the quantity stored S_t.

1. Whenever $0 < S_t < S_{max}$, multipliers associated with these inequalities have a null value, $\mu_t^0 = \mu_t^{Smax} = 0$. Then, by (3) $\lambda_t - \lambda_{t-1} = 0$. Whenever $S_t = S_{max}$, $\mu_t^{Smax} \geq \mu_t^0 = 0$, so that $\lambda_{t-1} \leq \lambda_t$. Whenever $S_t = 0$, $\mu_t^0 \geq \mu_t^{Smax} = 0$, so that $\lambda_{t-1} \geq \lambda_t$.
2. Destoring periods. Let us first consider periods where $p_t \geq \lambda_t$ and show that they characterize destoring periods.

   If $p_t > \lambda_t$, then $\alpha_{ot}^{Ko} > \alpha_{ot}^0 \geq 0$ by (1), so that $q_{ot} = K_o$ by the complementary slackness conditions and $q_{it}=0$ from the Lemma. The result is $S_{t+1} < S_t$.

   If $0 < p_t = \lambda_t$, from (1), $\alpha_{ot}^{Ko} = \alpha_{ot}^0 = 0$ since $\alpha_{ot}^{Ko} = \alpha_{ot}^0 > 0$ would entail $q_{ot}=K_o$ and $q_{ot}=0$, a contradiction. Moreover, $p_t = \lambda_t > r\lambda_t$,[48] then by (2) $\alpha_{it}^{Ki} > 0$. We conclude that $q_{it}=0$ and $0 \leq q_{ot} \leq K_o$ including the possibility of $0 < q_{ot} < K_o$.

   If $p_t < \lambda_t$, $\alpha_{ot}^0 > 0$ by (1). Then $q_{ot}=0$.

[48] For the sake of simplicity, we exclude the possibility of negative wholesale prices ($p_t<0$) although they occur at some periods of excess supply in power exchanges. Consequently, the reserve value cannot be negative and $\lambda_t \geq r\lambda_t$.

In other words, destoring can occur only when $p_t \geq \lambda_t$. Moreover, destoring with full exhaustion of the stock without saturating the discharging capacity ($0<q_{ot}=S_t<K_0$) can occur only when $p_t=\lambda_t$ ($p_t>\lambda_t$ would imply $\alpha_{ot}^{Ko} > 0$ and $q_{ot}=K_0$).

Moreover, if there are two successive periods where $q_{ot}>0$ and $q_{ot+1}>0$, then $0 < S_t < S_{t-1} \leq S_{max}$, then $0 < S_t < S_{max}$. It follows from Remark 2 in the text that $\lambda_{t-1}=\lambda_t$. In other words, the reserve value is constant during successive destoring periods.

3. Storing periods. We use the same arguments in a symmetrical way to prove the following assertions: periods where $p_t \leq r\lambda_t$ characterize storing periods, and periods where $p_t < r\lambda_t$ characterize storing periods in which charging capacity is saturated ($q_{it}=K_{it}$); storing reaching full stock without saturating charging capacity can occur only when $p_t=r\lambda_t$; the reserve value is constant during successive storing periods.

4. If $p_t < \lambda_t < p_t/r$, then $\alpha_{ot}^0 > \alpha_{ot}^{Ko} \geq 0$ by (1) and $\alpha_{it}^0 > \alpha_{it}^{Ki} \geq 0$ by (2) so that $q_{ot} = 0$ and $q_{it} = 0$ by the complementary slackness conditions. As a result, $S_{t+1} = S_t$.

A.3. Proof of corollary 1

Let us prove part a). Let λ_{iu} denote the reserve value associated to a date t_{iu} opening a storing phase. By definition, the date τ of activity prior and closest to t_{iu} is necessarily a destoring date. Let λ_τ denote the associated reserve value. During the interval of activity $[\tau+1;t_{iu}]$, the stock is necessarily not full ($S_t<S_{max}$) since it is possible to do some storing at t_{iu}. Then $\mu_t^{Smax} = 0$ for all $t \in [\tau+1;t_{iu}]$. From (3) $\lambda_t - \lambda_{t-1} = -\mu_t^0 \leq 0$, $t= \tau+1, \tau+2, \ldots, t_{iu}$.. Hence, $\lambda_{iu} - \lambda_\tau \leq 0$.

Now, let t_{ou} denote the date of destoring beginning prior to τ and closest to τ. By definition of t_{ou}, there cannot be any storing activity for $t \in [t_{ou}+1; \tau]$, and the stock is not empty since it is still possible to do some destoring at date τ. As a result, $\mu_t^0 = 0$ for all $t \in [t_{ou}+1; \tau]$. It is also true that the storage installation is not full since it will be possible to do some storing at date t_{iu} without any activity between τ and t_{iu}. Thus $\mu_t^{Smax} = 0$ for all $t \in [t_{ou}+1; \tau]$. From (3) $\lambda_t - \lambda_{t-1} = 0$, $t= t_{ou}+1, t_{ou}+2, \ldots, \tau$, hence $\lambda_{ou} - \lambda_\tau = 0$.

Combining: $\lambda_{iu} - \lambda_\tau \leq 0$ and $\lambda_{ou} - \lambda_\tau = 0$, we obtain $\lambda_{iu} \leq \lambda_{ou}$. QED.

Moreover, $\lambda_\tau \leq p_\tau$ from (1) since τ is a destoring period and $p_{tiu} \leq r\lambda_{iu}$ from (2) since t_{iu} is a storing period. Since $\lambda_{iu} \leq \lambda_\tau$, we have $p_{tiu} \leq r\lambda_{iu} \leq r\lambda_\tau \leq rp_\tau < p_\tau$. Given the definition of τ and t_{iu}, this shows that the market price at the beginning of a storing phase is necessarily no larger than the market price at the end of the previous discharging phase.

The proof of b) follows the same steps.

A.4. Proof of proposition 2

Let q denote the column vector $\{q_{o1},\ldots,q_{oT},q_{i1},\ldots q_{iT},S_1,\ldots,S_{T+1}\}$ of the 3xT+1 flow and stock variables, p the line vector $\{p_1,\ldots,p_T,-p_1,\ldots,-p_T,0,\ldots,0\}$ of the 3xT+1 prices for all corresponding variables, p.q the scalar product of prices and quantities, Λ the vector $\{\lambda_1,\ldots,\lambda_T\}$ of the dual variables of the T equations of stock dynamics (the components of Λ may be negative or positive), A the Tx(3xT+1) matrix of the stock dynamics (depending on the parameter r), K the column vector of the parameters of the 3xT+1 inequality constraints associated with (K_o, K_i , S_{max}, $S_{initial}$, S_{final}), α^K the vector of the 3xT+1 non-negative dual variables of the 3xT+1 inequality constraints associated with (K_o, K_i , S_{max}, $S_{initial}$, S_{final}), α^0 the vector of the 3xT+1 no negative dual variables associated with the 3xT+1 inequality constraints of variables positivity:

$$K = \left\{ \underbrace{K_o, K_o, \ldots, K_o}_{T\ parameters}, \underbrace{K_i, K_i, \ldots, K_i}_{T\ parameters}, \underbrace{S_{inital}, S_{\max}, \ldots, S_{\max}}_{T+1\ parameters} \right\}$$

$$\alpha^K = \left\{ \underbrace{\alpha_{o1}^{K_o}, \ldots, \alpha_{oT}^{K_o}}_{T\ parameters}, \underbrace{\alpha_{i1}^{K_i}, \ldots, \alpha_{iT}^{K_i}}_{T\ parameters}, \underbrace{\mu_1^{S_{\max}}, \ldots, \mu_{T+1}^{S_{\max}}}_{T+1\ parameters} \right\} \qquad \alpha^0 = \left\{ \underbrace{\alpha_{o1}^0, \ldots, \alpha_{oT}^0}_{T\ parameters}, \underbrace{\alpha_{i1}^0, \ldots, \alpha_{iT}^0}_{T\ parameters}, \underbrace{\mu_1^0, \ldots, \mu_{T+1}^0}_{T+1\ parameters} \right\}$$

K is a linear mapping of the vector $\{K_o, K_i, S_{max}, S_{initial}, S_{final}\}$.

With these notations, the original optimization problem can be written in a condensed way:

$$\text{(P)} \qquad \underset{q}{Max}\ p.q \quad s.t. \quad \begin{cases} q \geq 0 & (\alpha^0 \geq 0) \\ q \leq K & (\alpha^K \geq 0) \\ A.q = 0 & (\Lambda) \end{cases}$$

In the following, we suppose that the domain of possible K guarantees that a feasible q (i.e. such as $q\geq0$, $q\leq K$ and $A.q=0$) always exists.[49]

Since (P) is a problem of linear optimization in a convex non-empty and bounded set, there is a bounded solution. Let $\Pi(K)$ denote the optimal profit for each vector K of the parameters, and $\Pi(K_o,K_i,S_{max},S_{initial},S_{final})$ the same profit highlighting the only varying parameters.

The following two points are straightforward:

- $\Pi(K)$ is homogeneous of degree 1 wrt K. Since the program is linear, if we multiply K by any scalar e>0, optimal q and Π are multiplied by e. Since K is a linear mapping of $\{K_o,K_i,S_{max},S_{initial},S_{final}\}$, **$\Pi(K_o,K_i,S_{max},S_{initial},S_{final})$ is homogeneous of degree 1 wrt $\{K_o,K_i,S_{max},S_{initial},S_{final}\}$.**
- $\Pi(K)$ is non-decreasing wrt K when S_{final} is kept unchanged, as increasing K with S_{final} kept unchanged means increasing the feasible domain of solutions. As K is a linear mapping of

[49] If $K_i>0$, $K_o>0$, $S_{max}>0$, $S_{max}\geq S_{initial}\geq0$ and $S_{max}\geq S_{final}\geq0$, the set of feasible q is non empty.

{K_o,K_i,S_{max},S_{final},S_{final}} with positive coefficients, Π **is non-decreasing wrt {K_o,K_i,S_{max},$S_{initial}$}. In particular, Π is non-decreasing wrt S_{max}.**

In order to prove concavity of Π(K) with respect to K, we rely on duality theory (including the analysis of the so called "perturbed problem", i.e. comparative statics with respect to K).

Let L denote the Lagrangian of (P): $L(q,\alpha^K,\alpha^0,\Lambda,K) = p.q + \alpha^K.(K-q) + \alpha^0.q + \Lambda.A.q$

Since (P) is a problem of linear optimization in a convex, non-empty and bounded set, the solution of (P) for each K exists and is a saddle-point for variables q and (α^K,α^0,Λ)[50]. That means that profit Π(K) can be expressed either as a maximum for the problem (P) with respect to "primal" variables q, or as a minimum for the "Lagrange dual function" wrt to dual variables α and Λ in the following way:

$$\Pi(K) = \underset{\alpha,\Lambda}{Inf}\ \underset{q\in\Re^{3T+1}}{Sup}\ L(q,\alpha^K,\alpha^0,\Lambda,K) \quad = \quad \underset{\alpha,\Lambda}{Inf}\ \underset{q\in\Re^{3T+1}}{Sup}\left[p.q+\alpha^K.(K-q)+\alpha^0.q+\Lambda.A.q\right]$$

The solution q of Sup [p.q+α^K.(K-q)+α^0.q+Λ.A.q] depends on α and Λ [51], not on K.

$$\text{Thus we can write} \qquad \Pi(K) = \underset{\alpha,\Lambda}{Inf}\left[\alpha^K.K + \underset{q\in\Re^{3T+1}}{Sup}\left[p-\alpha^K+\alpha^0+\Lambda.A.\right]q\right]$$

Note that since the optimal q is dependent on optimal α and Λ, which in turn depend on K, the optimal q also depends on K. Then for any parameter with values K^a and K^b we can state the following:

$$\begin{cases} \forall\alpha,\Lambda \quad \Pi(K^a) \le \alpha.K^a + \underset{q}{Sup}\left[p.q-\alpha^K.q+\alpha^0.q+\Lambda.A.q\right] \\ \forall\alpha,\Lambda \quad \Pi(K^b) \le \alpha.K^b + \underset{q}{Sup}\left[p.q-\alpha^K.q+\alpha^0.q+\Lambda.A.q\right] \end{cases}$$

$$\Rightarrow\ \forall\alpha,\Lambda,\forall e\in[0;1] \quad e\Pi(K^a)+(1-e)\Pi(K^b) \le \alpha.\left(eK^a+(1-e)K^b\right)+\underset{q}{Sup}\left[p-\alpha^K.\alpha^0+\Lambda.A\right]q$$

$$\Rightarrow\ \forall e\in[0;1] \quad e\Pi(K^a)+(1-e)\Pi(K^b) \le \underset{\Lambda,\alpha}{Inf}\left[\alpha.\left(eK^a+(1-e)K^b\right)+\underset{q}{Sup}\left[p-\alpha^K+\alpha^0+\Lambda.A\right]q\right]$$

$$\Rightarrow\ \forall e\in[0;1] \quad e\Pi(K^a)+(1-e)\Pi(K^b) \le \Pi\left(eK^a+(1-e)K^b\right)$$

By definition, the last equation means that Π(K) is concave with respect to K. Since variations of K with {K_o,K_i,S_{max},$S_{initial}$ } are linear with positive coefficients, and since the composed function of a concave function and of a linear mapping with positive coefficients is concave, **Π(K_o,K_i,S_{max},$S_{initial}$) is concave wrt {K_o,K_i,S_{max},$S_{initial}$ }**. In particular, Π **is concave wrt S_{max}**. QED.

The last inequality above is equivalent to:

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[50] Strong duality theorem. For the proof and the saddle-point interpretation, see for instance Boyd, Vandenberghe, 2009, §5.3.2 pp.234-236 and §5.4.2 pp.238-239.

[51] In particular, unless p-α+Λ.A =0 (KKT conditions of Proposition 1), the solution for q is unbounded and Π=+∞.

$$\forall e\epsilon[0;1]\ \Pi[K^b]-\Pi[K^a]\leq\frac{\Pi[eK^a+(1-e)K^b]-\Pi[K^a]}{1-e}\qquad\text{(A1)}$$

The equation is true for any pair of values K^a and K^b of the same parameter, thus true when variations of K result from variations of S_{max} only. Let Π^a, Π^c and Π^b be the profits associated with three scalar values of S_{max}: $S^a<S^c<S^b$. The other parameters K_o and K_i remain unchanged. Using inequality (A1) with $e = (S^b-S^c)/(S^b-S^a)$ and $1-e= (S^c-S^a)/(S^b-S^a)$, we have:

$$\frac{\Pi^b-\Pi^a}{S^b-S^a}\leq\frac{\Pi^c-\Pi^a}{S^c-S^a}\quad\Leftrightarrow\quad\frac{\Pi^b-\Pi^c}{S^b-S^c}\leq\frac{\Pi^c-\Pi^a}{S^c-S^a}$$

which shows that increments of Π per unit of S_{max} variations are non-increasing with S_{max}. QED.

A.5. Proof of Proposition 3

Let us examine the expression of Π with "primal" variables (outputs and inputs). H_0 and H_i are defined as the total duration respectively of destoring and storing at full capacity equivalent during {1,…,T}. In a steady state, H_0 and H_i are related by the equality between total destored energy and total stored energy:

$$H_o=\frac{1}{K_o}\sum_{t=1}^{T}q_{ot}\qquad H_i=\frac{1}{K_i}\sum_{t=1}^{T}q_{it}\qquad H_oK_o=H_ir\,K_i$$

The average prices p_o and p_i are defined by

$$\sum_{t=1}^{T}p_tq_{ot}=\sum_{q_{ot}>0}p_tq_{ot}=p_o\sum_{q_{ot}>0}q_{ot}=p_oH_oK_o\ \text{and}\ \sum_{t=1}^{T}p_tq_{it}=\sum_{q_{it}>0}p_tq_{it}=p_i\sum_{q_{it}>0}q_{it}=p_iH_iK_i=\tfrac{1}{r}p_iH_oK_o$$

The expression of Π is straightforward: $\Pi=\sum_{t=1}^{T}p_t(q_{ot}-q_{it})=\left(p_o-\frac{1}{r}p_i\right)H_oK_o$

We now examine the expression of Π in terms of the dual variables.

Let α_j and Λ_j denote a solution of the dual problem for a value of the 3T+1 column vector K:[52]

$$\alpha_j^K,\alpha_j^0,\Lambda_j\ \ st\quad\Pi(K)=\underset{\alpha,\Lambda}{Inf}\left[\alpha^K.K+\underset{q}{Sup}\left[p-\alpha^K+\alpha^0+\Lambda.A\right]q\right]=\alpha_j^K.K+\underset{q}{Sup}\left[p-\alpha_j^K+\alpha_j^0+\Lambda_j.A.\right]q$$

Since solutions and Π are bounded (strong duality results), we can use the KKT conditions mentioned in Proposition 1: $p-\alpha_j^K+\alpha_j^0+\Lambda_jA=0$ and $\underset{q}{\mathrm{Sup}}\left[p-\alpha_j^K+\alpha_j^0+\Lambda_jA\right]q=0$.

The result is:

$$\Pi(K)=\alpha_j^KK=\ K_o\sum_{t=1}^{T}\alpha_{ot}^{Ko}+K_i\sum_{t=1}^{T}\alpha_{it}^{Ki}+S_{\max}\sum_{t=2}^{T}\mu_t^{S\max}+\mu_1^{S\max}S_{initial}-\mu_{T+1}^{0}S_{final}$$

In steady state, $S_{initial}=S_{final}$ and $\mu_{T+1}^0=\mu_1^{Smax}$, so that:

[52] While the optimal profit is unique, the (convex) set of optimal variables is not necessarily a singleton.

$$\Pi(K) = K_o \sum_{t=1}^{T} \alpha_{ot}^{Ko} + K_i \sum_{t=1}^{T} \alpha_{it}^{Ki} + S_{\max} \sum_{t=2}^{T} \mu_t^{S\max}$$

From equations (1), (2) and (3) in Section 1.2, at date t: α_{ot}^{Ko}=Max(0,p_t-λ_t), α_{it}^{Ki}=Max(0,rλ_t-p_t) and μ_t^{Smax}=Max(0,λ_t-λ_{t-1}). Then we have:

$$\Pi(K) \quad = \quad K_o \sum_{t=1}^{T} Max(0, p_t - \lambda_t) + K_i \sum_{t=1}^{T} Max(0, r\lambda_t - p_t) + S_{\max} \sum_{t=2}^{T} Max(0, \lambda_t - \lambda_{t-1})$$

The terms associated with K_0 (respectively K_i) are strictly positive only during destoring (respectively storing) periods. In Section 1.2, we denoted λ_{ou} and λ_{iu} the time series of N dual values at dates of destoring/output beginning and storing/input beginning respectively. Given the definitions of p_o, p_i, λ_o and λ_i:

$$\sum_{q_{ot}>0} \lambda_t q_{ot} = \lambda_o \sum_{q_{ot}>0} q_{ot} = \lambda_o \; H_o K_o \quad \text{and} \quad \sum_{q_{it}>0} \lambda_t q_{it} = \lambda_i \sum_{q_{it}>0} q_{it} = \lambda_i \; H_i K_i = \tfrac{1}{r} \lambda_i \; H_o K_o$$

We can write: [53]

$$\Pi = K_o (p_o - \lambda_o) H_o \quad + K_i \, (r\lambda_i - p_i) H_i \quad + S_{\max} \sum_{u=1}^{N} (\lambda_{ou} - \lambda_{iu})$$

which proves equation (4). QED.

A.6. Proof of Proposition 4

Consider two different vectors of parameters K^j (j=a,b) and the associated solutions of the dual problem: $\alpha_j^K, \alpha_j^0, \Lambda_j$. In the proof of Proposition 3, we have seen that $p - \alpha_j^K + \alpha_j^0 + \Lambda_j . A = 0$, $\operatorname{Sup}_q \; [p - \alpha_j^K + \alpha_j^0 + \Lambda_j . A] . q = 0$ and $\Pi(K^j) = \alpha_j^K . K^j$. Thus

$$\begin{cases} \Pi(K^a) = \alpha_a^K . K^a \leq \; \alpha_b^K . K^a + \underset{q \in \Re^{3T+1}}{Sup} \left[p.q - \alpha_b^K .q + \alpha_b^0 .q + \Lambda_b .A.q \right] = \alpha_b^K . K^a \\ \Pi(K^b) = \alpha_b^K . K^b \leq \alpha_a^K . K^b + \underset{q \in \Re^{3T+1}}{Sup} \left[p.q - \alpha_a^K .q + \alpha_a^0 .q + \Lambda_a .A.q \right] = \alpha_a^K . K^b \end{cases}$$

$$\Rightarrow \begin{cases} (\alpha_b^K - \alpha_a^K) . K^a \geq 0 \\ (\alpha_b^K - \alpha_a^K) . K^b \leq 0 \end{cases}$$

$$\Rightarrow \quad (\alpha_b^K - \alpha_a^K) . (K^b - K^a) \leq 0$$

In particular, if the difference between K^a and K^b is given only by distinct values of the parameter S_{max}, say S_{max}^a and S_{max}^b, we have:

$$\left(\sum_{t=2}^{T} \mu_t^{S_{\max}^b} - \sum_{t=2}^{T} \mu_t^{S_{\max}^b} \right) (S_{\max}^b - S_{\max}^a) \leq 0 \qquad \text{Thus} \quad S_{\max}^b - S_{\max}^a > 0 \Rightarrow \sum_{t=2}^{T} \mu_t^{S_{\max}^b} \leq \sum_{t=2}^{T} \mu_t^{S_{\max}^a}$$

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[53] The term in S_{max} may initially include λ_1=μ_1^{Smax} and λ_T= μ_{T+1}^0. But they vanish in steady-state.

$$\text{We conclude that } \sum_{t=2}^{T} \mu_t^{S_{max}} \text{ is non} - \text{increasing when } S_{max} \text{ is increasing.}$$

$$\text{From the proof of Proposition 3, we have } \sum_{t=2}^{T} \mu_t^{S_{max}} = \sum_{t=2}^{T} Max(0, \lambda_t - \lambda_{t-1}) = \sum_{u=1}^{N} (\lambda_{ou} - \lambda_{iu})$$

$$\text{Thus } \sum_{u=1}^{N} (\lambda_{ou} - \lambda_{iu}) \text{ is non increasing when } S_{max} \text{is increasing. QED.}$$

A.7. The dynamic problem of investment and annuities

We recall how to tackle investment issues across several years, and the circumstances under which one can rely on an analysis in terms of annuities.

We have seen in Section 2 that, when optimally operated, storage installation generates a value increasing at a decreasing rate with the size of the installation S_{max},, then with the charging and discharging durations for given K_i and K_0. We now address the threefold "long run problem": choosing the size, the beginning and the end of the investment operation.

*** Isolated equipment**

Let us denote

m = 1, 2, … periods measured in multiples of T. For instance T is one year, and m is a number of years.

M the lifetime of a storage installation, measured in numbers of T.

I(S_{max},m,M) the investment cost of a storage installation of size S_{max} (and K_0, K_i), put on line at the beginning of period m for a duration M. It is increasing and convex in S_{max} and M (the latter because of equipment wear) and decreasing with m if there is technological progress. Also, I(0_+,m,M)>0.

Π(S_{max},m) the operating profit over a phase of length T beginning at date m. It is increasing and weakly concave in S_{max}. The way it changes with m depends on the evolution of technologies and regulations in the electricity industry.

ρ the discount rate for a T period.[54]

The Net Present Value of a facility of size S_{max} installed at the start of year m for a duration M is:

$$\Pi(S_{max}, m, M) = -I(S_{max}, m, M)\frac{1}{(1+\rho)^{m-1}} + \sum_{j=m}^{m+M-1} \frac{\pi(S_{max}, j)}{(1+\rho)^j}$$

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[54] There is no discounting within periods {1;2;…;T}, {T+1;T+2;…;2T}, etc. The discounting rate ρ applies between T and T+1, 2T and 2T+1, etc. This reflects a common accounting practice (with a discontinuity from December 31 to January 1), not an economic one. In a fully rigorous model, we should apply constant discounting over each period. That would slightly change equations (1), (2) and (3) in Section 1.2, but not their interpretation.

Maximization gives the optimal values S^*_{max}, m* and M* as solutions of the system of simultaneous relations

$$\Pi(S^*_{max}, m^*, M^*) \geq \Pi(S_{max}, m^*, M^*) \quad \forall S_{max} \neq S^*_{max}$$
$$\Pi(S^*_{max}, m^*, M^*) \geq \Pi(S^*_{max}, m \ \ , M^*) \quad \forall m \neq m^*$$
$$\Pi(S^*_{max}, m^*, M^*) \geq \Pi(S^*_{max}, m^*, M) \quad \forall M \neq M^*$$

To have a profitable installation, these local conditions are to be complemented with the global condition $\Pi(S^*_{max}, m^*, M^*) \geq 0$.

There is currently no anticipated disruption in each type of storage technology for the next decade. Then, $I(S_{max}, m, M)$ will not vary with *m*. Additionally, if we maintain our hypothesis of steady-state in the operation of the installation, $\Pi(S_{max}, m)$ will not change with *m* either. There result is that the beginning date is either right now if $\Pi(S^*_{max}, 1, M^*) \geq 0$ or otherwise never. Under stationary conditions and no anticipated technological progress that would make the current equipment obsolete, the finishing date is determined by the technical characteristics of the equipment, and then it is exogenous. Let us denote it by $\bar{M}$. It results that the optimal size
is the solution to $\Pi(S^*_{max}, 1, \bar{M}) > \Pi(S_{max}, 1, \bar{M}) \ \forall S_{max} \neq S^*_{max} \ if \ \Pi(S^*_{max}, 1, \bar{M}) \geq 0$.

Let $A(S^*_{max})$ be the constant equivalent annuity of the investment $I(S^*_{max}, 1, \bar{M})$. It is formally defined by $\sum_{j=1}^{\bar{M}} A\frac{1}{(1+\rho)^j} = I(S^*_{max}, 1, \bar{M})$. Clearly, $A(S^*_{max})$ has the same functional properties as $I(S^*_{max}, 1, \bar{M})$ regarding S^*_{max}.

Let $\pi(S^*_{max})$ denote the constant yearly operating profit. Using these notations, we can write the profitability condition as $\pi(S^*_{max}) \geq A(S^*_{max})$.

If there is free entry in the storage industry, newcomers will push the electricity buying prices (resp. selling prices) up (resp. down). Then the price differential will progressively decrease down to the point where the operating profit of storage operators just covers the investment cost. At that point of free entry equilibrium, $\Pi(S^*_{max}, 1, \bar{M}) = 0$ or $\pi(S^*_{max}) = A(S^*_{max})$.

* Competition between technologies

How to invest in several technologies of storage when they have different charge and discharge durations? Let *k*= 1, 2 be the index of two different technologies. If the two technologies do not have the same lifetime, a comparison of their net present values on equal grounds necessitates a hypothesis of successive renewals until a common duration is reached.

Knowing this common number of renewals (denoted by *n*), one can compute the constant equivalent annuity of technology *k*, $A_k(S^*_{max,k})$ now defined by

$$\sum_{j=1}^{(n+1)\bar{M}_k} \frac{A_k(S^*_{max,k})}{(1+\rho)^j} = \sum_{j=0}^{n} \frac{I(S^*_{max}, j\bar{M}_k+1, \bar{M}_k)}{(1+\rho)^{j\bar{M}_k}}$$

To reach a given discharged duration D_o the efficient technology is the one with the lower cost per discharged kWh $A_k(S^*_{max,k})/K_{ok}$. Assuming that $A_1(S_{max,1} = 0_+) > A_2(S_{max,2} = 0_+)$, $A'_1(S^*_{max,1}) < A'_2(S^*_{max,2})$ and $K_{o1}=K_{o2}=K_o$, we can determine the threshold $\widehat{D}_0$ such that $A_1(\widehat{D}_0 K_o) = A_2(\widehat{D}_0 K_o)$. Then:

For $D_o > \widehat{D}_0$, 1 is the least-cost technology because its high fixed cost can be stretched over a long duration of use.

For $D_o < \widehat{D}_0$, 2 is the least-cost technology because the high cost of investment varying with the size $S^*_{max,k}$ is compensated by its low fixed cost.

If the system operator needs only one type of storage service (i.e. one single discharge duration), only one technology will be used, on condition that it is profitable, i.e. its yearly operating profit is larger than the annuity. Actually, several types of storage services are necessary. Thus, several technologies with different discharge durations may be required, which can necessitate specific reward rules (e.g. flexibility premium, capacity payment).