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“Generalized Compensation Principle”

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Generalized Compensation Principle*

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Abstract

Economic disruptions (techonological change, trade liberalization, immigration flows) generally create winners and losers, i.e., wage gains for some individuals and wage losses for others. The compensation problem consists of designing a reform of the existing income tax system that offsets the wefare losses by redistributing the gains of the winners. We derive a closed-form formula for the compensating tax reform and its impact on the government budget when only distortionary tax instruments are available and wages are determined endogenously in general equilibrium. We apply this result to the compensation of automation in the U.S. and Germany.

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Introduction

Economic disruptions, for instance an inflow of immigration, a change in technology or an opening to international trade, generally create winners and losers, i.e., wage and welfare gains for some individuals and welfare losses for others. The welfare compensation problem consists of designing a reform of the tax-and-transfer system that offsets the losses by redistributing the gains of the winners. We solve this problem in an environment where only distortionary taxes are available and wages are determined endogenously in general equilibrium.

The traditional public finance literature (Kaldor [1939], Hicks [1939, 1940]) shows that in an economy where individualized lump-sum taxes are available, the tax reform that redistributes the welfare gains and losses caused by a disruption simply consists of raising (resp., lowering) the lump-sum tax liability of agents whose welfare increases (resp., decreases) from the shock, by an amount equal to their compensating variation. This standard Kaldor-Hicks approach is flawed, however. First, because of asymmetric information (Mirrlees [1971]), the only tax instrument at the disposal of the government, the labor income tax, is distortionary. Second, and most importantly, for many economic shocks it is crucial to model explicitly the endogeneity of wages.

For instance, consider an inflow of low-skilled immigrants – i.e., an exogenous (relative) increase in the total supply of low-skilled labor. In partial equilibrium, i.e. if wages were exogenous, this would not affect individual utilities. In general equilibrium, instead, this disruption lowers the wage of low-skilled workers whose marginal product of labor is decreasing, and raises the wage of high-skilled workers whose labor is complementary to the tasks performed by the immigrants (see, e.g., Card [2009]). Therefore, immigration flows have non-trivial welfare consequences only because the endogeneity of wages is explicitly taken into account. Similarly, the impact of automation on inequality can be understood as a race between education – the supply of high-skilled workers – and technology (see, e.g., Katz and Murphy [1992]). In both of these examples, movements in relative wages are fundamentally determined by movements in the relative *labor supplies* of different skills. As a result, standard public finance models in which labor supply is endogenous but wages are exogenous cannot properly account for the welfare implications of these disruptions.

Now suppose that in response to the disruption, the government implements a tax reform that aims at compensating the welfare of agents whose wage is adversely impacted. Since the only available policy tools are distortionary taxes, such a reform affects the agents' labor supply choices. By the exact same general equilibrium forces as we just described, these labor supply adjustments impact individuals' wages, and hence their utility. These welfare effects need to be themselves accounted for and compensated, using the distortionary tax code. Hence the combination of distortionary taxes and endogenous wages leads to an a priori complex fixed point problem for the compensating tax reform.

We start by analyzing the welfare compensation problem in a partial equilibrium environment where wages are exogenous. We show that the design of the compensating tax reform that brings every agent's utility back to its pre-disruption level is simple, even when distortionary income taxes are the only available instrument. The key insight here is that individual utility is only affected by the *average* tax rates of the reform – that is, the changes in *marginal* tax rates do not impact welfare. This follows from an envelope theorem argument: the marginal tax rate that the individual faces affects his indirect utility only through his optimal labor supply decision, so that the corresponding welfare effect is second-order. As a consequence, it is straightforward to show that a suitably designed adjustment in the average tax rate – namely, one that exactly cancels out the income gain or loss caused by the exogenous disruption, regardless of the marginal tax rate changes that it induces – is sufficient to achieve exact welfare compensation.

The analysis becomes significantly more complicated when distortionary taxes are coupled with general equilibrium forces. In this case, despite the envelope theorem, the endogenous changes in labor supply do matter for welfare, through their impact on wages resulting from the decreasing marginal productivities and the production complementarities. Therefore, in general equilibrium, because of the labor supply responses that they generate, the marginal rates of the tax reform affect directly the agent's utility, even conditional on the average tax rate change. As a result, to determine the compensating tax reform, we must solve for its average and its marginal rates simultaneously. This is the key difference with the partial equilibrium environment and the main technical challenge of our paper.

Our first main result is to derive a closed-form formula for the compensating tax reform in general equilibrium, in terms of elasticity variables that can be measured empirically. This formula is valid for arbitrary marginal wage disruptions; that is, our tax reform compensates the first-order effects on welfare caused by a shock. Our second main result is to derive a closed-form formula for the fiscal surplus, i.e., the impact on government budget of the disruption and its compensation. Thus, our analysis generalizes the traditional Kaldor-Hicks criterion and provides a simple test to determine whether economic shocks or policies generate aggregate gains, in the sense that offseting the individual welfare changes using only distortionary tax instruments is budget-feasible. More generally, the value (and not only the sign) of the fiscal surplus provides a policy-relevant monetary measure of the aggregate welfare gains or losses from the disruption.

We first show that the compensating tax reform features an element of progressivity that departs from the simple partial-equilibrium policy. This is because, when the marginal product of labor is decreasing, the compensation must be designed such that the (typically, negative) welfare effects caused by the higher average tax rates counteract the (typically, positive) effects caused endogenously by the higher marginal tax rates (in addition to those caused by the disruption itself). Thus, agents who face a higher average tax rate must also face a higher marginal tax rate. Ceteris paribus, this naturally leads taxes to grow with income at a rate (of progressivity) that is determined by the ratio between the labor supply and the labor demand elasticities, net of the rate of progressivity of the initial tax code. Second, skill complementarities in production generate additional indirect wage adjustments that also need to be compensated. But the marginal tax rates of this second round of compensation generate in turn further wage and welfare changes, and so on. We show that we can generally solve this fixed point problem – formally represented by an integro-differential equation – by defining inductively a sequence of functions that each capture a round of iterated compensation. Remarkably, if the production function is CES, we show that this series boils down to a uniform shift of the marginal tax rates in addition to the progressive reform derived above.

We finally propose a concrete application of our theory in the context of the robotization of the U.S. and the German economies between 1990 and 2007. We use Acemoglu and Restrepo [2017]'s data for the U.S., and Dauth et al. [2017] data for Germany, which give the estimated impact of an additional robot per one thousand workers on the wages of different skills – roughly the amount of automation observed in the U.S. between these dates. The closed-form solution that we derive allows us to easily evaluate the compensating reform quantitatively. We find that in the U.S., an additional robot per thousand workers requires a progressive tax reform, where the tax payment of agents at the 10^{th} (resp., 90^{th}) percentile of the wage distribution decreases (resp., increases) by 110% of their income loss (resp., 125% of their income gain) from the disruption. This represents a 2 percentage point decrease (resp., a 0.5 pp increase) in their average tax rate, and generates a positive $16 budget surplus for the government. In Germany, workers at the 10^{th} percentile should have their tax bill reduced by 310% of their income loss, while those at the 90^{th} percentile should have theirs reduced by 150% of their income loss.

Related literature. Our theoretical analysis of Section 2 builds on Kaplow [2004, 2012] and Hendren [2014], who extend the Kaldor-Hicks principle to the case of distortionary taxes in partial equilibrium. Our main contribution, however, is the analysis of the general equilibrium environment in which wages are endogenous. Most closely related to our general equilibrium framework, Guesnerie [1998], Itskhoki [2008] and Antras, de Gortari, and Itskhoki [2016] study compensating tax reforms and the welfare implications of trade liberalization within specific classes of distortionary taxes and tax reforms – linear for Guesnerie [1998] and CRP (as in Bénabou [2002], Heathcote, Storesletten, and Violante [2016], Heathcote, Storesletten, Violante, et al. [2017]) for Antras, de Gortari, and Itskhoki [2016], who moreover use a CES technology. While we do not consider a sophisticated model of trade, we solve the compensation problem allowing for both general nonlinear tax schedules and nonlinear tax reforms, as well as a general production function. More broadly, our model is within the class of Mirrleesean economies in general equilibrium. Stiglitz [1982a], Rothschild and Scheuer [2013], Sachs, Tsyvinski, and Werquin [2016] study optimal taxes in this environment for a given production function. Ales, Kurnaz, and Sleet [2015], Guerreiro, Rebelo, and Teles [2017], Thuemmel [2018], Costinot and Werning [2018], Hosseini and Shourideh [2018] characterize optimal income taxes, robot taxation, or trade policies in the presence of disruptions. None of these papers address the compensation problem, which is our main focus and leads to distinct economic insights. Sachs, Tsyvinski, and Werquin [2016] characterize the incidence of nonlinear tax reforms in general equilibrium – they do not try to find the compensating tax change, which is significantly more challenging as it requires solving not only for labor supply changes in response to a given tax reform, but also for the tax reform itself. Lastly, our application to automation relies on the empirical analyses of Acemoglu and Restrepo [2017] (for the U.S.) and Dauth et al. [2017] (for Germany), who estimate the impact of robots on the wage distribution.

Outline. In Section 1 we set up the model and define the welfare compensation problem. In Section 2 we solve for the compensating tax reform and the fiscal surplus in the standard partial-equilibrium Mirrlees framework, i.e., assuming that wages are exogenous. In Section 3 we analyze a simple version of our general-equilibrium environment, in which we make a number of functional form assumptions ensuring that all the relevant elasticity variables are constant. These allow us to derive in the simplest possible way the welfare compensating tax reform and analyze its economic implications. We calibrate the model and apply the resulting formula to the compensation of automation in Section 4. In Section 5 we relax all the functional form assumptions and solve the compensation problem in our most general environment. Section 6 concludes with a discussion of the benefits of the compensation approach over the standard optimal taxation approach. The proofs are gathered in the Appendix.

1 Welfare Compensation Problem

In this section we set up our general model and define the welfare compensation problem. Our goal is to design a tax reform that compensates the gains and losses of a given disruption of the initial equilibrium, and to evaluate its impact on government budget (fiscal surplus).

1.1 Initial equilibrium

There is a continuum of measure one of individuals indexed by their skill $i \in [0, 1]$. In the initial (undisrupted) economy, agents i earn a pre-tax wage $w_i \in \mathbb{R}_+$ that they take as given. Without loss of generality we order skills so that wages w_i are increasing in i. Hence the skill index $i \in [0, 1]$ can be interpreted as the agent's percentile in the wage distribution of the initial economy.

Agents with skill i have the utility function $u_i(c, l)$ over consumption c and labor supply l. They choose effort l_i, earn pre-tax income $y_i = w_i l_i$, and pay the tax $T(y_i)$, where the income tax schedule $T : \mathbb{R}_+ \to \mathbb{R}$ is twice continuously differentiable. Under standard assumptions on preferences, incomes $y_i = w_i l_i$ are strictly increasing in skills i, so that there are one-to-one maps between skills i, wages w_i and incomes y_i in the initial equilibrium.[1,2] Their welfare U_i is given by

$$U_i \quad = \quad u_i \left(w_i l_i - T \left(w_i l_i \right), l_i \right), \tag{1}$$

where labor supply l_i satisfies the first-order condition[3]

$$-\frac{u'_{i,l} \left(w_i l_i - T \left(w_i l_i \right), l_i \right)}{u'_{i,c} \left(w_i l_i - T \left(w_i l_i \right), l_i \right)} \quad = \quad \left[1 - T' \left(w_i l_i \right) \right] w_i. \tag{2}$$

There is a continuum of mass 1 of identical firms that produce output using as inputs the aggregate labor supply L_j of each type $j \in [0, 1]$. The aggregate production function is denoted by $\mathcal{F}(\{L_j\}_{j \in [0,1]})$. In equilibrium, firms earn no profits, and the wage w_i is equal to the marginal product of labor of skill i. Letting $\mathcal{F}'_i \equiv \partial \mathcal{F} / \partial L_i$, we have

$$w_i \quad = \quad \mathcal{F}'_i(\{L_j\}_{j \in [0,1]}). \tag{3}$$

We finally denote government revenue by

$$\mathcal{R} \quad = \quad \int_0^1 T \left(w_i l_i \right) \mathrm{d}i. \tag{4}$$

We define the local rate of progressivity $p\left(y_i\right) \equiv -\frac{y_i}{1 - T'(y_i)} \frac{\partial(1 - T'(y_i))}{\partial y_i}$ of the tax schedule at income y_i as (minus) the elasticity of the retention rate (one minus the marginal tax rate) $r_i = 1 - T'\left(y_i\right)$ with respect to gross income y_i.

1.2 Wage disruption

We now define a disruption of the initial equilibrium (1)-(4).

[1] This is the case if agents have a common utility function u that satisfies the Spence-Mirrlees condition. Importantly, because we focus on marginal perturbations, this ordering of wages need not be preserved by the disruption and the tax reform.

[2] We assume that incomes y_i belong to a compact interval $[\underline{y}, \bar{y}] \subset \mathbb{R}_+$ and have a continuous density $f_Y(\cdot)$. We denote by $\mathbb{E}[\cdot]$ the corresponding expectation.

[3] We assume that this equation has a unique solution.

Wage disruption. A disruption can be caused by various exogenous shocks: e.g., a perturbation of the production function $\mathcal{F}$ (due to, say, technological change) or of the distribution of aggregate labor supplies $\boldsymbol{L} = \{L_j\}_{j\in[0,1]}$ (due to, say, immigration flows). We denote by $\mu\hat{\boldsymbol{w}}^E \equiv \{\mu\hat{w}_i^E\}_{i\in[0,1]}$, where $\mu > 0$ is a constant, the percentage adjustment in the wage distribution $\boldsymbol{w} \equiv \{w_i\}_{i\in[0,1]}$ caused by these exogenous shocks, keeping individual labor supplies fixed. Without loss of generality, μ is pinned down by the normalization $\|\hat{\boldsymbol{w}}^E\| \equiv \sup_{i\in[0,1]} |\hat{w}_i^E| = 1$. Therefore the map $\hat{\boldsymbol{w}}^E = \{\hat{w}_i^E\}_{i\in[0,1]}$ is the (infinite-dimensional) *direction* of the disruption, and the scalar μ parametrizes its *size*.[4] Formally, a change in the production function from $\mathcal{F}$ to $\tilde{\mathcal{F}}$ and in labor supplies from $\boldsymbol{L} \equiv \{L_j\}_{j\in[0,1]}$ to $\tilde{\boldsymbol{L}}^E \equiv \{\tilde{L}_j^E\}_{j\in[0,1]}$ implies that the wage of agent i changes, on impact, from w_i to $w_i(1+\mu\hat{w}_i^E)$, where

$$\mu\hat{w}_i^E \equiv \frac{1}{w_i}[\tilde{\mathcal{F}}_i'(\tilde{\boldsymbol{L}}^E) - \mathcal{F}_i'(\boldsymbol{L})], \;\; \forall i.$$

Tax reform. In response to the disruption, the government can implement an arbitrarily non-linear tax reform $\mu\hat{T}(\cdot)$. Thus, the statutory tax payment at income y changes from $T(y)$ to $T(y) + \mu\hat{T}(y)$.[5]

Perturbed equilibrium. In response to the wage disruption $\mu\hat{\boldsymbol{w}}^E$ and the tax reform $\mu\hat{T}$, individuals optimally adjust their labor supply. In general equilibrium, this further impacts their wage, which in turn modifies their labor supply decisions, and so on. We denote by $\mu\hat{w}_i$ and $\mu\hat{l}_i$ the total endogenous percentage changes in individual i's wage and labor supply between the initial and the perturbed equilibria. Thus, the wages and labor supplies in the disrupted economy are respectively equal to $\tilde{w}_i = w_i(1+\mu\hat{w}_i^E + \mu\hat{w}_i)$ and $\tilde{l}_i = l_i(1+\mu\hat{l}_i)$.

Formally, the perturbed equilibrium is described by the following equations. The

[4]Since there are one-to-one maps between skills i, wages w_i, and incomes y_i, in the sequel we denote the wage disruption incurred by agent i interchangeably by $\hat{w}_i^E$ or $\hat{w}^E(y_i)$. Throughout the paper we focus on continuously differentiable functions $i \mapsto \hat{w}_i^E$ on $[0,1]$.

[5]In Section 3 we assume that the tax reforms $\hat{T}$ that the government can implement are continuously differentiable, bounded, with bounded first derivative. This defines a Banach space on which the norm of a function $\hat{T}$ is given by $\|\hat{T}\| = \sup_{y\in\mathbb{R}_+} |\hat{T}(y)| + \sup_{y\in\mathbb{R}_+} |\hat{T}'(y)|$. Note that the normalization of the tax reform (and, below, the endogenous wage and labor supply adjustments) by the same scalar $\mu > 0$ as the wage disruption is without loss of generality since we do not impose $\|\hat{T}\| = 1$.

perturbed welfare of agent i is given by

$$\tilde{U}_i = u_i[\tilde{w}_i\tilde{l}_i - T(\tilde{w}_i\tilde{l}_i) - \mu\hat{T}(\tilde{w}_i\tilde{l}_i), \tilde{l}_i], \tag{5}$$

where $(\tilde{w}_i, \tilde{l}_i)$ are defined by the perturbed first-order condition

$$-\frac{u'_{i,l}[\tilde{w}_i\tilde{l}_i - T(\tilde{w}_i\tilde{l}_i) - \mu\hat{T}(\tilde{w}_i\tilde{l}_i), \tilde{l}_i]}{u'_{i,c}[\tilde{w}_i\tilde{l}_i - T(\tilde{w}_i\tilde{l}_i) - \mu\hat{T}(\tilde{w}_i\tilde{l}_i), \tilde{l}_i]} = [1 - T'(\tilde{w}_i\tilde{l}_i) - \mu\hat{T}'(\tilde{w}_i\tilde{l}_i)]\tilde{w}_i, \tag{6}$$

and the perturbed wage equation

$$\tilde{w}_i = \tilde{\mathscr{F}}'_i(\{\tilde{L}_j\}_{j\in[0,1]}). \tag{7}$$

with $\tilde{L}_j \equiv \tilde{L}_j^E + \mu\hat{l}_j$. The perturbed government revenue is given by

$$\tilde{\mathcal{R}} = \int_0^1 [T(\tilde{w}_i\tilde{l}_i) + \mu\hat{T}(\tilde{w}_i\tilde{l}_i)]\mathrm{d}i. \tag{8}$$

1.3 Compensation and fiscal surplus

We can now formally set up the welfare compensation problem.

Compensating tax reform. We define agent i's *compensating variation* $\mu\hat{U}_i$ by the difference in utilities between the initial and the perturbed equilibria, normalized by the (initial) marginal utility of consumption to obtain a monetary measure.[6] That is, $\mu\hat{U}_i \equiv (\tilde{U}_i - U_i)/u'_{i,c}$. The welfare compensation problem consists of designing a reform $\hat{T}$ of the existing tax code that offsets the welfare gains and losses of the wage disruption $\mu\hat{\boldsymbol{w}}^E$. Hence, the tax reform $\hat{T}$ must be designed such that each agent's compensating variation is equal to zero:

$$\hat{U}_i = 0, \ \forall i \in [0,1]. \tag{9}$$

[6]A positive (resp., negative) value implies that an individual i benefits (resp., loses) from the shocks. If the utility is quasilinear in consumption, it is the amount that agent i would be willing to pay, after the wage disruption $\mu\hat{\boldsymbol{w}}^E$ and the tax reform $\mu\hat{T}$, in order to be as well off as in the initial equilibrium.

Fiscal surplus. We define the *fiscal surplus* $\hat{\mathcal{R}}(\hat{\boldsymbol{w}}^E)$ as the change in government revenue induced by the disruption and the tax reform, i.e.,

$$\mu\hat{\mathcal{R}}(\hat{\boldsymbol{w}}^E) \;=\; \tilde{\mathcal{R}} - \mathcal{R}. \tag{10}$$

Marginal wage disruptions. Throughout the paper, we characterize analytically the solution to the welfare compensation problem for *marginal* wage disruptions, i.e., as $\mu \to 0$. Thus, our exercise consists of designing and evaluating the fiscal impact of a tax reform $\hat{T}$ that compensates the first-order welfare effects of a small wage disruption in the direction $\hat{\boldsymbol{w}}^E$.

Compensability and aggregate gains of a disruption. We say that a given economic shock $\{\hat{w}_i^E\}_{i\in[0,1]}$ is *compensable* if $\hat{\mathcal{R}}(\hat{\boldsymbol{w}}^E) \geq 0$. If this is the case, then it is possible to reform the initial tax code T to reach a Pareto improvement.[7] Conversely, it is possible that a disruption generates strictly positive aggregate gains, both in terms of gross incomes and government revenue, but that these gains are not compensable (i.e., the fiscal surplus $\hat{\mathcal{R}}(\hat{\boldsymbol{w}}^E)$ is negative), if the labor supply distortions that the disruption or the tax reform generate outweigh these gains. More generally, the value of the fiscal surplus $\hat{\mathcal{R}}(\hat{\boldsymbol{w}}^E)$, and not only its sign, carries important information: it provides a metric that allows to compare, in monetary units, the *aggregate welfare gains* (or losses) of different economic shocks. For example, suppose that a given disruption (say, automation) generates more revenue, after implementing the compensating tax reform, than another (say, an inflow of immigration). It follows that the government can achieve a strictly better Pareto improvement from the former shock.

Remark: a more general problem. It is natural to wonder what the compensating tax reform would be if the government's objective were to compensate every agent so that their welfare would be at least as large (rather than exactly as large) as in the initial economy, i.e., such that $\tilde{U}_i \geq U_i$ for all i in equation (5). To address this problem, we can directly specify the non-zero welfare improvements (or losses) $\hat{U}_i = h_i \in \mathbb{R}$ that one wants to achieve for each skill level. We then solve the compensation problem by replacing 0 with h_i in the right-hand side of (9). The corresponding

[7]For instance, the government can redistribute lump-sum the budget surplus.

tax reform and fiscal surplus can then be straightforwardly derived following identical steps as in the proofs of Propositions 1, 2 and 3 and Corollaries 1, 2, and 3 below.

2 Compensation in Partial Equilibrium

In this section, we show that the solution to the compensation problem takes a simple form in partial equilibrium, even when when taxes are distortionary. Suppose as in Mirrlees [1971] that wages are exogenous, i.e., the marginal product of labor is constant and skills are infinitely substitutable in production. Thus, the production function is given by

$$\mathcal{F}(\boldsymbol{L}) \;=\; \int_0^1 \theta_i L_i \mathrm{d}i, \tag{11}$$

so that the wage w_i is equal to the technological parameter θ_i in the initial equilibrium. In this case, the wage disruption $\mu\hat{\boldsymbol{w}}^E$ generates no further endogenous adjustment in the wage: $\hat{w}_i = 0$ for all $i \in [0,1]$, so that $\tilde{w}_i$ is simply equal to $w_i(1+\mu\hat{w}_i^E)$. We characterize in closed-form the solution to the welfare compensation problem, i.e., the compensating tax reform $\hat{T}$ and the fiscal surplus $\hat{\mathcal{R}}(\hat{\boldsymbol{w}}^E)$, for marginal wage disruptions. The proofs are gathered in Appendix A.

2.1 Labor supply elasticities

We start by introducing the relevant elasticity concepts. In Appendix A we give the (standard) analytical expressions of the Hicksian (compensated) elasticity $e_i^{S,r} > 0$ of labor supply of skill i with respect to the retention rate $r_i \equiv 1 - T'(w_i l_i)$, the income effect parameter $e_i^{S,n} < 0$ with respect to the non-labor (lump-sum) income n_i, and the elasticity $e_i^{S,w}$ of labor supply with respect to the wage w_i. These variables are respectively defined by:

$$e_i^{S,r} \;\equiv\; \frac{r_i}{l_i}\frac{\partial l_i}{\partial r_i}, \quad e_i^{S,n} \;\equiv\; r_i \frac{\partial l_i}{\partial n_i}, \quad e_i^{S,w} \;\equiv\; \left(1 - p\left(y_i\right)\right) e_i^{S,r} + e_i^{S,n}.$$

From these variables, we can then define the corresponding labor supply elasticities along the non-linear budget constraint,[8] as:

$$\{\varepsilon_i^{S,r}, \varepsilon_i^{S,n}, \varepsilon_i^{S,w}\} \equiv \frac{1}{1+p(y_i)\, e_i^{S,r}} \{e_i^{S,r}, e_i^{S,n}, e_i^{S,w}\}. \tag{12}$$

The labor supply elasticities $e_i^{S,w}$ and $\varepsilon_i^{S,r}$ differ from the standard elasticity with respect to the retention rate, $e_i^{S,r}$, because a wage change or the initial labor supply response to a tax change affect the marginal tax rate $T'(w_i l_i)$ faced by the agent, if the initial tax schedule is nonlinear, by an amount equal to the rate of progressivity $p(y_i)$ of the tax schedule; this in turn causes a further endogenous labor supply adjustment given by the elasticity $e_i^{S,r}$, leading to the correction terms $p(y_i)\, e_i^{S,r}$.

2.2 Incidence of disruptions and tax reforms

To characterize the compensating tax reform and the fiscal surplus, we derive first-order Taylor expansions around the initial equilibrium, as $\mu \to 0$, of the perturbed equilibrium conditions (5)-(6) and government revenue (8).

Welfare changes. A Taylor expansion of equation (5) implies that the change in the utility of agents i induced by the wage disruption and the tax reform is given by:

$$0 = \hat{U}_i = (1 - T'(y_i))\, y_i \hat{w}_i^E - \hat{T}(y_i), \tag{13}$$

where the first equality imposes that, once the new tax schedule is implemented, agents i keep the same level of welfare in the disrupted economy as in the initial equilibrium. This equation shows that, in partial equilibrium, the change in the utility of agents i is due to:

(i) their exogenous income gain or loss $y_i \hat{w}_i^E$ caused by the disruption,[9] weighted by the share $(1 - T'(y_i))$ that they keep after paying taxes (the first term of (13));

[8] These labor supply elasticity variables are standard in the literature, see e.g. Jacquet and Lehmann [2016]. They can be estimated empirically using, e.g., the methodology of Gruber and Saez [2002].

[9] Recall that $\hat{w}_i^E$ is the percentage wage change, so that $w_i \hat{w}_i^E$ is the absolute wage change, and $l_i \times (w_i \hat{w}_i^E)$ is the gross income change.

(ii) the change in their tax liability $\hat{T}(y_i)$ (the second term of (13)), which makes them poorer (resp., richer) if $\hat{T}(y_i) > 0$ (resp., < 0).

Labor supply changes. Next, a Taylor expansion of equation (5), which imposes that the labor supply of agent i remains optimal in the disrupted economy, can be expressed in terms of the elasticity notations introduced in Section 2.1 as follows. The disruption and tax reform lead to a change in labor supply equal to:

$$\hat{l}_i^{\text{pe}} = \varepsilon_i^{S,w} \hat{w}_i^E - \varepsilon_i^{S,r} \frac{\hat{T}'(y_i)}{1 - T'(y_i)} - \varepsilon_i^{S,n} \frac{\hat{T}(y_i)}{(1 - T'(y_i))\, y_i}. \tag{14}$$

This equation shows that agents i adjust their effort upwards if their wage increases by $\hat{w}_i^E$ (first term in (14)), their marginal tax rate decreases by $\hat{T}'(y_i)$ (second term), or their average tax rate increases by $\hat{T}(y_i)/y_i$. The magnitudes of these behavioral responses are respectively determined by the elasticities with respect to the wage ($\varepsilon_i^{S,w}$), the retention rate ($\varepsilon_i^{S,r}$), and the non-labor income ($\varepsilon_i^{S,n}$) that we defined in Section 2.1.

2.3 Compensating tax reform

Equation (13) immediately gives the tax reform $\hat{T}$ which ensures that, after reoptimizing their behavior, individuals remain as well off as before the wage disruption $\mu \hat{\boldsymbol{w}}^E$. Since there is a one-to-one map between skills i and incomes y_i, we let $\hat{w}^E(y_i) \equiv \hat{w}_i^E$ and $\varepsilon^{S,x}(y_i) \equiv \varepsilon_i^{S,x}$ for $x = r, n, w$. We obtain the following result.

Proposition 1. *Suppose that the production function is given by (11). The tax reform that compensates a marginal wage disruption in the direction $\hat{\boldsymbol{w}}^E$ is given by*

$$\hat{T}(y) = (1 - T'(y))\, y\, \hat{w}^E(y). \tag{15}$$

Equation (15) implies that if wages are exogenous, the compensating tax reform consists of increasing or decreasing the average tax rate (ATR) $\frac{\hat{T}(y_i)}{y_i}$ of each agent i by an amount equal to their net-of-tax wage gain or loss resulting from the disruption, $(1 - T'(y_i))\, \hat{w}_i^E$. This makes them just as well off as if the disruption had not occurred.

Taking stock. The crucial feature that allowed us to easily solve for the compensating tax reform $\hat{T}$ is that the changes in *marginal tax rates* (MTR), $\hat{T}'(y_i)$, do not enter equation (13) and therefore do not matter for welfare (conditional on the total tax bill change $\hat{T}(y_i)$). This follows from the envelope theorem: the MTR that an individual faces affect his utility only through his labor supply decision (equation (2)); but since labor supply is initially chosen optimally, these behavioral responses induce no first-order effect on welfare. As a result, it is sufficient to adjust every agent's total tax payment (or the ATR) to neutralize the income gain or loss due to the wage disruption $(1 - T'(y_i))\,\hat{w}_i^E$, regardless of the changes in MTR $\hat{T}'(y_i)$ that such a reform implies.

2.4 Fiscal surplus

We now derive the fiscal surplus $\hat{\mathcal{R}}(\hat{\boldsymbol{w}}^E)$, i.e., the change in government revenue (4) caused by the wage disruption and the compensating tax reform (15). It is decreasing in the deadweight loss induced by the compensating tax reform, which is determined by the individual labor supply adjustments (14) and hence by the marginal tax rate changes. Differentiating (15) implies that $\frac{\hat{T}'(y)}{1-T'(y)} = [1 - p(y) + \hat{\psi}^E(y)]\hat{w}^E(y)$, where the elasticity $\hat{\psi}^E(y) \equiv \frac{y}{\hat{w}^E(y)} \frac{d\hat{w}^E(y)}{dy}$ measures the local variation of the exogenous wage disruption along the income distribution.

Corollary 1. *Suppose that the production function is given by (11). The fiscal surplus generated by the wage disruption $\hat{\boldsymbol{w}}^E$ and the compensating tax reform (15) is given by*

$$\hat{\mathcal{R}}(\hat{\boldsymbol{w}}^E) \quad = \quad \mathbb{E}\left[\left\{1 - T'(y)\,\varepsilon^{S,r}(y)\,\hat{\psi}^E(y)\right\} y\,\hat{w}^E(y)\right]. \tag{16}$$

Corollary (1) provides a closed-form expression that allows us to determine whether a given economic shock $\{\hat{w}_i^E\}_{i\in[0,1]}$ is compensable, i.e., $\hat{\mathcal{R}}(\hat{\boldsymbol{w}}^E) \geq 0$. Note, in particular, that calculating the fiscal surplus (or, equivalently, the aggregate welfare gains or losses of the disruption $\hat{\boldsymbol{w}}^E$) does not require actually implementing or even computing the actual compensating tax reform (15). Indeed, the expression for $\hat{\mathcal{R}}(\hat{\boldsymbol{w}}^E)$ in (16) depends only on the exogenous disruption and the characteristics (tax rates, income distribution, labor supply elasticities) of the initial (undisrupted) economy.

Conclusion. Proposition 1 and Corollary (1) are the first step towards generalizing the standard Kaldor-Hicks criterion to the environment where type-specific lump-sum taxes are unavailable. The rest of the paper is devoted to the analysis of the case where general equilibrium effects are present.

3 Compensation in General Equilibrium with Constant Elasticities

We now characterize the compensating tax reform and the fiscal surplus when wages are endogenous. We start by presenting a very simple version of the general-equilibrium framework, which allows us to derive most transparently our main result – namely, a closed-form formula for the compensating tax reform and for the fiscal surplus of any marginal wage disruption. Specifically, we make several assumptions which ensure that the relevant behavioral and price elasticities are constant. The proofs and technical details are gathered in Appendix B. We relax these assumptions and solve the fully general model in Section 5.

3.1 Simplifying assumptions

We impose the following assumptions.[10]

Assumption 1 (**CEL**). *The utility function is quasilinear in consumption with an isoelastic disutility of labor effort: for some* $e > 0$ *and all* $i \in [0,1]$, $u_i(c,l) = c - \frac{l^{1+\frac{1}{e}}}{1+\frac{1}{e}}$.

Assumption 2 (**CRP**). *The labor income tax schedule has a constant rate of progressivity: for some* $p \in (-\infty, 1)$ *and* $\tau \in \mathbb{R}$, $T(y) = y - \frac{1-\tau}{1-p} y^{1-p}$.

Assumption 3 (**CES**). *The production function has a constant elasticity of substitution between skills: for some* $\varepsilon^D > 0$ *and* $\theta_j > 0$, $\mathcal{F}(\boldsymbol{L}) = [\int_0^1 \theta_j L_j^{1-1/\varepsilon^D} \mathrm{d}j]^{\varepsilon^D/(\varepsilon^D-1)}$.

Crucially, Assumptions 2 and 3 are only about the *initial* (undisrupted) economy. We do *not* impose that they remain satisfied after the disruption and the compensating tax reform. That is, we allow the production function and the tax schedule to be perturbed in arbitrary (non-CES and non-CRP) ways.

[10] Assumption 1 is standard in the taxation literature: see, e.g., Saez, Slemrod, and Giertz [2012] for supporting evidence. Assumptions 2 and 3 are those made by Heathcote, Storesletten, and Violante [2016], who show in particular that a CRP tax schedule closely approximates the U.S. tax-and-transfer system. The CES production function is equivalent to a setting as in Costinot and Vogel [2010] with a CES technology over tasks (e.g., manual, routine, abstract, etc.) to which worker skills are assigned, except that the assignment of skills to tasks remains fixed (e.g., there is a small switching cost) in response to a marginal disruption and tax reform.

3.2 Labor demand elasticities

We define the elasticity of the wage w_i of skill i with respect to the aggregate labor L_j of skill j. The cross-wage elasticities γ_{ij} and own-wage elasticities ε_j^D are defined by:

$$\frac{L_j}{w_i}\frac{\partial w_i}{\partial L_j} \quad = \quad \gamma_{ij} - \frac{1}{\varepsilon_j^D}\delta\left(i-j\right), \tag{17}$$

where $\delta\left(\cdot\right)$ is the Dirac delta function. To understand this definition, consider first the case where $i \neq j$, so that (17) reads $\gamma_{ij} = \frac{L_j}{w_i}\frac{\partial w_i}{\partial L_j}$. It is straightforward to show that, when the production function is CES, this cross-wage elasticity is equal to $\gamma_{ij} = \frac{w_j L_j}{\varepsilon^D \mathbb{E}y}$. This elasticity is positive, i.e., different skills are Edgeworth complements in production. Moreover, it does not depend on i, implying that an increase in the labor supply of type j raises the wages of all types $i \neq j$ by the same percentage amount – as a result, throughout Section 3 we simply denote it by γ_j. Second, consider the case where $i = j$. Equation (17) implies that the map $i \mapsto \frac{L_j}{w_i}\frac{\partial w_i}{\partial L_j}$ is discontinuous as $i \to j$.[11] That is, a change in the aggregate labor supply of type i affects the wage of skill i by a strictly smaller (in fact, negative) amount than the wages of other skills (even close neighbors) $j \neq i$, because the marginal product of labor of skill i is decreasing. With a CES technology, the own-wage elasticity $1/\varepsilon_j^D$, or equivalently the inverse elasticity of labor demand, is constant across the skill distribution and equal to $1/\varepsilon^D$.

3.3 Incidence of disruptions and tax reforms

In general equilibrium, the initial wage disruption $\mu\hat{\boldsymbol{w}}^E$ generates additional endogenous wage adjustments $\mu\hat{w}_i$, which in turn affect every agent's utility and choice of labor supply. As in Section 2, we start by deriving first-order Taylor expansions around the initial equilibrium (i.e., as $\mu \to 0$) of the perturbed equilibrium conditions (5) to (8).

Endogenous wage changes. A Taylor expansion of equation (7) implies that the endogenous wage changes $\hat{w}_i$, expressed in terms of the elasticities introduced in

[11] The Dirac notation ensures that the Euler theorem holds: $\int_0^1 w_i L_i \times \frac{L_j}{w_i}\frac{\partial w_i}{\partial L_j} \mathrm{d}i = 0$.

Section 3.2, are given by

$$\hat{w}_i \quad = \quad -\frac{1}{\varepsilon^D}\hat{l}_i + \int_0^1 \gamma_j \hat{l}_j \mathrm{d}j. \tag{18}$$

This equation has the following economic interpretation. A one percent increase in the labor supply of individuals with skill i leads to a $-1/\varepsilon^D$ percent change in their own wage, because the marginal product of labor is decreasing. A one percent increase in the labor supply of agents with skill $j \in [0,1]$ leads to a γ_j percent change in the wage of type i through complementarities between skills in production.

Labor supply changes. A Taylor expansion of equation (5), which imposes that the labor supply of agent i remains optimal in the disrupted economy, implies that the disruption and tax reform lead to the following change in labor supply, expressed in terms of the elasticities introduced in Section 2.1:

$$\hat{l}_i \quad = \quad \varepsilon^{S,w}[\hat{w}_i^E + \hat{w}_i] - \varepsilon^{S,r}\frac{\hat{T}'(y_i)}{1 - T'(y_i)}. \tag{19}$$

This expression is analogous to equation (14) obtained in partial equilibrium, except that the relevant wage change is now the sum of the exogenous disruption $\hat{w}_i^E$ and the general-equilibrium adjustments $\hat{w}_i$. Substituting for $\hat{w}_i$ using equation (18) implies that, in addition to the direct effects already present in partial equilibrium, the adjustment $\hat{l}_i$ in the labor supply of agent i is now also affected by those of all other agents j, $\{\hat{l}_j\}_{j\in[0,1]}$, because of the cross-wage complementarities. We obtain:

$$\hat{l}_i \quad = \quad \phi\hat{l}_i^{\mathrm{pe}} + \phi\varepsilon^{S,w}\int_0^1 \gamma_j \hat{l}_j \mathrm{d}j, \tag{20}$$

where $\phi \equiv (1 + \frac{\varepsilon^{S,w}}{\varepsilon^D})^{-1}$ and $\hat{l}_i^{\mathrm{pe}}$ is given by (14) with $\varepsilon_i^{S,n} = 0$. Equation (20) is an integral equation in the labor supply changes of all agents. We show in Appendix B that its solution $\{\hat{l}_i\}_{i\in[0,1]}$ as a function of the exogenous wage disruption function $\hat{\boldsymbol{w}}^E$ and the tax reform $\hat{T}$ is given by:

$$\hat{l}_i \quad = \quad \phi\hat{l}_i^{\mathrm{pe}} + \phi\varepsilon^{S,w}\int_0^1 \gamma_j \hat{l}_j^{\mathrm{pe}} \mathrm{d}j. \tag{21}$$

It has the same structure and interpretation as (20), except that the unknown labor supply changes $\hat{l}_j$ in the integral are replaced by their (known) partial equilibrium expressions $\hat{l}_j^{\text{pe}}$. Therefore, the labor supply of agent i is directly affected by the marginal tax rate changes of agents i (through $\hat{l}_i^{\text{pe}}$) and j (through $\hat{l}_j^{\text{pe}}$ and the cross-wage complementarity γ_j) for all $j \in [0,1]$.

Welfare changes. Finally, a Taylor expansion of equation (5) around the initial equilibrium implies that the (normalized) change $\hat{U}_i$ in the utility of agent i induced by the wage disruption $\mu\hat{\boldsymbol{w}}^E$ and the tax reform $\mu\hat{T}$ is given by:

$$0 \;=\; \hat{U}_i \;=\; (1-T'(y_i))\, y_i[\hat{w}_i^E + \hat{w}_i] - \hat{T}(y_i)\,. \tag{22}$$

where the first equality imposes that agent i keeps the same level of welfare in the disrupted economy as in the initial equilibrium. This expression generalizes equation (13), again replacing $\hat{w}_i^E$ with $\hat{w}_i^E + \hat{w}_i$. Substituting for this term using equation (18), we obtain that, in addition to the two partial-equilibrium forces described in Section 2, a third channel now impacts the compensating variation of the agent:

(iii) the endogenous changes $\hat{l}_i$ and $\{\hat{l}_j\}_{j\in[0,1]}$ in the labor supplies of type-i and type-j agents, by impacting the wage of skill i (equation (18)), have a first-order impact on the indirect utility of agent i.

As we will see, this additional effect is crucial for the design of the compensating tax reform.

Taking stock. We gather and discuss the results obtained so far. In contrast to equation (13) obtained in partial equilibrium, (22) does not directly yield the solution for the compensating tax change $\hat{T}(y_i)$ as a function of the exogenous disruption $\hat{w}_i^E$. This is because the agent's utility is also affected by the endogenous wage adjustment $\hat{w}_i$, which in turn is determined by the labor supply responses $\{\hat{l}_j\}_{j\in[0,1]}$ (equation (18)). Therefore, despite the envelope theorem, the endogenous changes in labor supply now have first-order effects on welfare (via their impact on wages). But $\hat{l}_j$ depends on the marginal tax rate changes $\hat{T}'(y_j)$, through standard substitution effects (equation (21)). Therefore, in general equilibrium and when only distortionary tax instruments are available, both the average *and* the marginal rates of the tax reform have first-order impacts on welfare. Specifically, a higher *average* tax rate at a given income y^*, $\hat{T}(y^*) > 0$, implies a reduction in the welfare of agent y^*, by directly making him poorer, just like in partial equilibrium (last term in equation (22)). Moreover, in general equilibrium, a higher *marginal* tax rate at income y^*, $\hat{T}'(y^*) > 0$, leads to the following fiscal externalities:

(a') a higher average tax rate for all incomes $y > y^*$, since $\hat{T}(y) = \int_0^y \hat{T}'(x)\,dx$, which reduces the welfare of these agents;

(b') an *increase* in the welfare of agent y^*, who works less, as in partial equilibrium (substitution effect, second term in equation (14)) and hence earns a higher wage (decreasing marginal product, first term in equation (18));[12]

(c') a *decrease* in the welfare of all agents $y \neq y^*$, whose wage decreases due to the lower labor supply of agent y^* (production complementarities, second term in equation (18)).

Thus, the consequences of a given tax reform are much richer, and the design of the compensating policy significantly more complex, than in partial equilibrium. Suppose that the planner implements the tax reform (15) that would compensate every agent's welfare in partial equilibrium. Through standard substitution effects, this tax reform affects individual labor supplies and hence, through decreasing returns and complementarities in production, the wage distribution. These lead to additional first-order welfare effects that need to be themselves compensated. One can only do so by further reforming the tax-and-transfer system, which implies further changes in marginal tax rates, etc. Therefore, the combination of distortionary tax instruments with elastic labor supply (whereby marginal tax rates affect labor supply behavior) and general equilibrium (whereby labor supply decisions determine wages) leads to a fixed point problem for the compensating tax reform. Formally, the tax reform $\hat{T}$ is the solution to an integro-differential equation that we derive in Lemma 2 in the Appendix. We derive and analyze the solution to this equation in Section 3.4.

[12] The fact that an agent is made better off from a higher marginal tax rate (conditional on a total tax payment) follows from the same logic as the "trickle-down" result of Stiglitz [1982b] that implies lower optimal high-income tax rates than in partial equilibrium. See Corollary 4 in Appendix B for a formal result.

3.4 Compensating tax reform

We now give a complete analytical characterization of the compensating tax reform in response to any wage disruption in general equilibrium. For ease of notation, we define the total wage disruption faced by agent i by

$$\hat{\Omega}_i^E = \phi \hat{w}_i^E + \phi \varepsilon^{S,w} \int_0^1 \gamma_j \hat{w}_j^E \mathrm{d}j. \tag{23}$$

In partial equilibrium, we have $\hat{\Omega}_i^E = \hat{w}_i^E$. In general equilibrium, $\hat{\Omega}_i^E$ accounts for the full incidence of the initial shock (absent the tax reform) on the wage of agent i, i.e., the direct impact $\hat{w}_i^E$ plus all of the indirect effects caused by other agents' wage adjustments $\{\hat{w}_j^E\}_{j\in[0,1]}$ via skill complementarities γ_j. Importantly, it is possible that empirical studies that evaluate the impact of a disruption on the wage distribution capture not only the direct effect of the disruption, $\{\hat{w}_i^E\}_{i\in[0,1]}$, but also all of the indirect effects due to the labor demand spillovers in general equilibrium; this is the case, for instance, in our empirical application in Section 4. In this case, the compensation formula we derive in Proposition 2 can be applied directly using $\{\hat{\Omega}_i^E\}_{i\in[0,1]}$ as a primitive. Since there is a one-to-one map between skills i and incomes y_i, we denote by $\hat{\Omega}^E(y_i) \equiv \hat{\Omega}_i^E$.

Proposition 2. *Suppose that Assumptions 1, 3 and 2 hold. The tax reform that compensates a marginal wage disruption in the direction* $\hat{\boldsymbol{w}}^E$ *is given by*

$$\hat{T}(y) = (1 - T'(y))\, y \left[\int_y^{\bar{y}} \Pi(y,z)\, \hat{\Omega}^E(z)\, \mathrm{d}z - \lambda \right], \tag{24}$$

where we let

$$\Pi(y,z) = \frac{\varepsilon^D}{\phi \varepsilon^{S,r} z} \left(\frac{y}{z} \right)^{\varepsilon^D / \varepsilon^{S,r}}, \tag{25}$$

and λ *is a constant equal to*

$$\lambda = \frac{1}{\mathbb{E}y} \left\{ \mathbb{E}\left[\int_y^{\bar{y}} y \Pi(y,z)\, \hat{\Omega}^E(z)\, \mathrm{d}z \right] - \mathbb{E}[y \hat{\Omega}^E(y)] \right\}.$$

Formula (24) is a closed-form expression, as it depends only on the exogenous wage disruption $\hat{\boldsymbol{w}}^E$ (or $\hat{\Omega}^E$) and on variables that are observed in the pre-disruption economy: statutory marginal tax rates, elasticity of labor supply, and elasticity of substitution between skills. Therefore it is straightforward to implement such a tax reform in practice (see Section 4 for an application).

3.5 Economic analysis of Proposition 2

We start by giving a heuristic derivation of Proposition 2 and then provide its economic meaning and implications.

Heuristic derivation of (24). To understand formula (24), we can easily show (see Appendix B) that equations (18), (19) and (22) imply

$$\frac{\hat{T}(y_i)}{y_i} = (1 - T'(y_i))\,\hat{\Omega}_i^E + \frac{\phi\varepsilon^{S,r}}{\varepsilon^D}\hat{T}'(y_i) - (1 - T'(y_i))\,\phi\lambda, \tag{26}$$

where $\lambda \equiv \varepsilon^{S,r}\int_0^1 \gamma_j \frac{\hat{T}'(y_j)}{1-T'(y_j)}\mathrm{d}j$ is a constant independent of i. As in Section 2, the change in the ATR, $\frac{\hat{T}(y_i)}{y_i}$, must compensate the wage, and hence welfare, gains or losses incurred by agent i. The first term in the right-hand side of (26), $(1 - T'(y_i))\,\hat{\Omega}_i^E$, is the net-of-tax wage change caused by the exogenous disruption, already present in partial equilibrium (equation (15)). The second term accounts for the fact that, in general equilibrium, an increase in the MTR of agents i by $\hat{T}'(y_i)$ reduces their labor supply by $\phi\varepsilon^{S,r}\hat{T}'(y_i)$, and hence raises their own wage by $\frac{\phi\varepsilon^{S,r}}{\varepsilon^D}\hat{T}'(y_i)$.[13] The third term accounts for the additional fiscal externalities caused by increases in the MTR of agents $j \neq i$ by $\hat{T}'(y_j)$, which lead to reductions in their labor supplies and hence in the wage of agent i through the complementarities γ_j.[14] Thus, the change in the average tax rate must now compensate not only the wage disruption $\hat{\Omega}_i^E$, but also the wage adjustments generated endogenously by the marginal tax rates of the reform. As a consequence, the reform $\hat{T}$ satisfies the first-order ODE (26). We derive its solution in closed-form using standard techniques to obtain equation (24).

[13] The scaling factor ϕ accounts for the fact that the marginal product of labor is decreasing, so that the agent's initial labor supply adjustment (say, increase) $\varepsilon^{S,r}\hat{T}'(y_i)$ lowers his wage by a factor $1/\varepsilon^D$, which in turn leads him to reduce his labor supply by a factor $\varepsilon^{S,w}/\varepsilon^D$, therefore dampening his initial response by $\phi \equiv 1/[1 + \frac{\varepsilon^{S,w}}{\varepsilon^D}]$.

[14] Note that if the production function is not CES, the cross-wage elasticities depend directly on i, so that λ is no longer be a constant. This makes the analysis more difficult, since equation (26) is then an integro-differential equation with non-separable kernel, rather than an ODE. We solve this general case in Section 5.

Balancing the ATR and the MTR. Equation (26) formalizes the key insight that, in general equilibrium, both the ATR *and* the MTR of the tax reform affect welfare and therefore have to be determined simultaneously: an increase in the former (resp., the latter) lowers (resp., raises) the agent's utility, and the welfare compensating tax reform must be such that the total effect of these two instruments exactly cancels out that of the exogenous disruption. Suppose in particular that the government implements the partial equilibrium compensation (15). This tax reform is constructed so that its average tax rates exactly compensate the wage gains or losses of the disruption. While the implied adjustments in marginal tax rates can be ignored in partial equilibrium (because of the envelope theorem), in general equilibrium they lead to additional, and hence unintended, welfare consequences that need to be themselves compensated (second term in the right hand side of (26)). Namely, the welfare gain created by a higher marginal tax rate $\hat{T}'(y) > 0$ must be counteracted by a welfare loss of equal magnitude via an increase in the average tax rate $\frac{\hat{T}(y)}{y} > 0$. These joint adjustments in marginal and average tax rates tend to make the compensating tax reform progressive, as we now describe.

Progressivity of the tax reform. To fix ideas, consider a disruption that does not affect the wage of agents with skill $i < i^*$, and ignore for now the cross-wage complementarities (so that $\lambda = 0$ in (26)). Suppose first that $\frac{\varepsilon^D}{\phi\varepsilon^{S,r}} = 1$, i.e., $\frac{\varepsilon^D}{\varepsilon^{S,r}} = p$.[15] Equation (26) then reads $\frac{\hat{T}(y_i)}{y_i} = \hat{T}'(y_i)$ for all $i < i^*$. This requires that the average and the marginal tax rates of the reform must exactly coincide: an increase in the former (resp., in the latter) generates welfare losses (resp., welfare gains) of the same magnitude. It follows immediately that the compensating tax schedule $\hat{T}$ must be linear on $[\underline{y}, y_{i^*})$. More generally, the relationship $\frac{\hat{T}(y_i)}{y_i} = \frac{\varepsilon^D}{\phi\varepsilon^{S,r}}\hat{T}'(y_i)$ implies that the ratio between the average and the marginal tax rates must be equal to the constant $\frac{\varepsilon^D}{\phi\varepsilon^{S,r}} = 1 + \frac{\varepsilon^D}{\varepsilon^{S,r}} - p$. This implies that the compensating tax reform satisfies $\frac{\hat{T}(y_i)}{y_i} \propto y_i^{\varepsilon^D/\varepsilon^{S,r} - p}$ for all $i < i^*$. Therefore, the tax reform is *progressive* on $[\underline{y}, y_{i^*})$ if and only if $\frac{\varepsilon^D}{\varepsilon^{S,r}} > p$. Intuitively, a higher MTR at income $y_i \in [\underline{y}, y_{i^*})$ mechanically raises the ATR of every agent with income $y_j > y_i$. This creates a utility loss that must be compensated, in turn, by a further increase in the MTR at income y_j, and so on. This "race" between the MTR and the ATR leads to exponentially increasing average

[15] This follows from the definition of $\phi = 1/[1 + \frac{(1-p)\varepsilon^{S,r}}{\varepsilon^D}]$ which implies $\frac{\varepsilon^D}{\phi\varepsilon^{S,r}} = 1 + \frac{\varepsilon^D}{\varepsilon^{S,r}} - p$.

and marginal tax rates on $[\underline{y}, y_{i^*})$. The key parameter driving the progressivity of the compensating reform is the ratio between the elasticities of labor demand and labor supply, net of the rate of progressivity p of the pre-existing tax code. This is because (the inverse of) this ratio determines the extent to which an increase in the marginal tax rate raises the agent's welfare – it lowers his labor supply proportionally to $\varepsilon^{S,r}$, which in turn raises his wage proportionally to $1/\varepsilon^{D}$. Empirically, the inequality $\frac{\varepsilon^D}{\varepsilon^{S,r}} > p$ is clearly satisfied, since we have $p \approx 0.15$, $\varepsilon^{S,r} \approx 0.3$, and $\varepsilon^D \geq 0.5$.

Effect of the cross-wage complementarities. Finally, the skill complementarities lead to a constant change (in percentage terms) in the retention rate of the tax schedule: $\frac{\hat{T}'(y_i)}{1-T'(y_i)} = \lambda (1-p)$. We can easily show that this is equivalent to an increase in the parameter τ of the baseline tax schedule $T(y) = y - \frac{1-\tau}{1-p} y^{1-p}$ by an amount $\hat{\tau}$ equal to $\frac{\hat{\tau}}{1-\tau} = \lambda (1-p)$. The last term in (26) therefore requires a uniform percentage shift in the tax rates of the compensation in addition to the progressive reform described in the previous paragraph.

3.6 Fiscal surplus

We finally derive the fiscal surplus $\hat{\mathcal{R}}$ implied by the disruption and the tax reform.

Corollary 2. *Suppose that Assumptions 1, 3 and 2 hold. The fiscal surplus generated by the wage disruption $\hat{\boldsymbol{w}}^E$ and the compensating tax reform (24) is given by*

$$\hat{\mathcal{R}}(\hat{\boldsymbol{w}}^E) = \mathbb{E}\left[y \hat{\Omega}^E(y)\right] + \frac{\varepsilon^D}{\phi} \mathbb{E}\left[T'(y)\left(y\hat{\Omega}^E(y) - \phi \int_{\underline{y}}^{\bar{y}} y \Pi(y,z) \hat{\Omega}^E(z) \, \mathrm{d}z\right)\right] \tag{27}$$

Remarks analogous to those in Section 2 apply. In particular, a disruption $\hat{\boldsymbol{w}}^E$ is compensable, and a Pareto improvement can be achieved, if $\hat{\mathcal{R}} \geq 0$. Note also that formula (27) is given only as a function of the exogenous disruption and hence does not require calculating explicitly the tax reform (24).

4 Application: Compensating Automation

In this section, we show how our theoretical results can be implemented in an empirical application: compensating the welfare consequences of robotization in the

U.S. and the German economies.[16] This exercise also allows us to evaluate the importance of the general equilibrium effects that motivate this paper. Throughout the analysis we assume that the economy is described by the model of Section 3. The initial tax schedule is CRP with $p = 0.156$ and $\tau = -3$ in the U.S. (Heathcote, Storesletten, and Violante [2016]), and $p = 0.128$ in Germany (Kindermann, Mayr, and Sachs [2017]). The production function is CES with $\varepsilon^D = \infty$ (partial equilibrium), $\varepsilon^D = 0.6$ (Dustmann, Frattini, and Preston [2013]), or $\varepsilon^D = 1.5$. We estimate the labor supply elasticity using the data of Acemoglu and Restrepo [2017] and find $\varepsilon^{S,r} = 0.47$, which is in the ballpark of empirical estimates.

Automation in the U.S. Using the 1990 and 2007 U.S. Census data, Acemoglu and Restrepo [2017] have estimated the impact of one additional robot per thousand workers[17] on wages, employment, and hours worked. These estimates are obtained by comparing people in the same skill cell but who reside in commuting zones with different exposure to industrial automation. They include both the direct effects of robots on employment and wages and any indirect spillover effects that might arise because of a resulting decline in local demand; in other words, they estimate the total disruption $\hat{\Omega}^E$ rather than the direct impact $\hat{\boldsymbol{w}}^E$.

The left panel of Figure 1 plots the wage disruption $100 \times \hat{\Omega}^E(y)$ (i.e., the percentage change in the wage) along the baseline (1990) earnings distribution, as well as the standard errors. Consistent with our theoretical analysis, we group agents by wage deciles, so that the values of the wage disruption in the y-axis are those reported in Figure 13 of Acemoglu and Restrepo [2017]. This figure shows that the change in the wage due to automation is increasing with the agent's position in the income distribution. The lowest wages in 1990 are reduced by 1.84%, while the 80th and 90th percentiles experience an estimated increase in their wage of 0.31% and 0.34%. The solid blue curves in the right panel of Figure 1 and the left panel of Figure 2 give the corresponding percentage changes in gross incomes $y\hat{\Omega}^E(y)$ for each decile of the baseline (1990) earnings distribution.

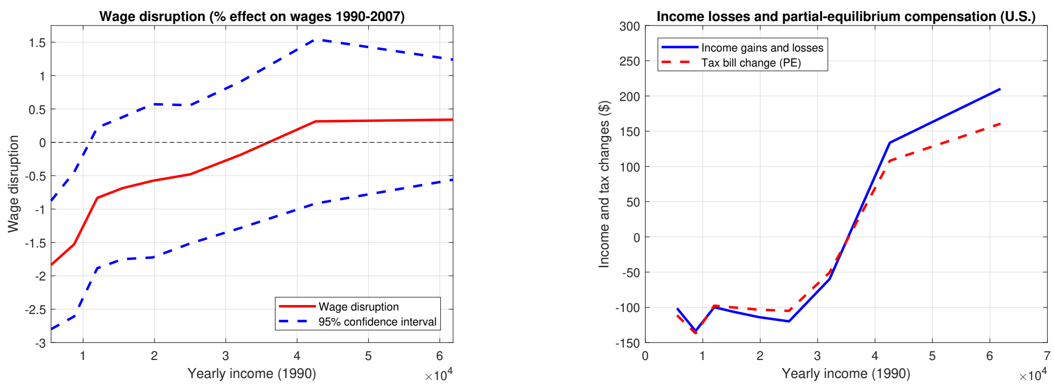
In the right panel of Figure 1, we plot the compensating tax reform $\hat{T}$ (dashed red curve) obtained in the partial equilibrium environment (formula (15)). The partial-equilibrium compensation tracks one-for-one the shape of the income gains and losses

[16] Costinot and Werning [2018] use the same data to compute the optimal robot tax, rather than the labor income tax reform that offsets the welfare gains and losses of the disruption.

[17] This corresponds to the increase in robots observed in the U.S. between 1990 and 2007.

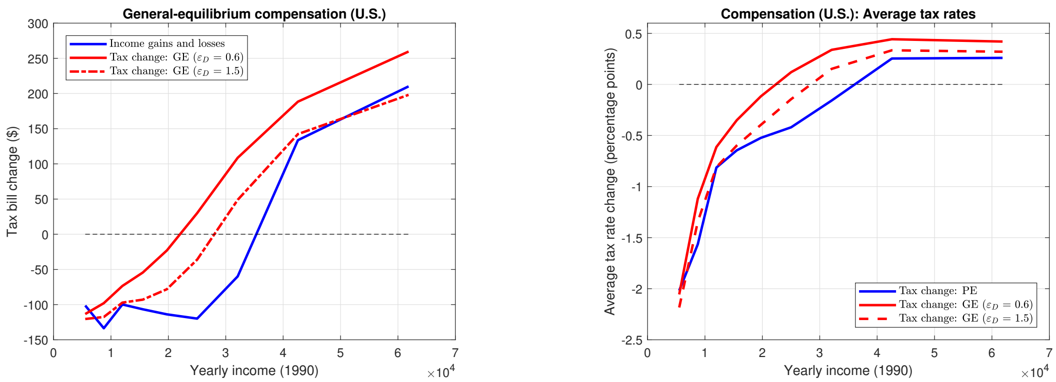
(solid curve), correcting only for the fact that the initial tax schedule is progressive so that gross income changes differ from net income changes. The 10th income percentile ($5,500 per year) have their tax bill reduced by $100 (i.e., 110% of their income loss), while the 90th income percentile ($62,000 per year) face a tax increase of $160 per year (i.e., 76% of their income gain).

Figure 1: Wage disruption (left) and Partial-equilibrium compensation (right)



In general equilibrium, however, the large changes in MTR that the previous tax reform generates would lead to sizable unintended first-order welfare consequences. The left panel of Figure 2 plots the compensation in general equilibrium. The solid (resp., dashed-dotted) red curves give the tax changes $\hat{T}(y)$ for $\varepsilon^D = 0.6$ (resp., $\varepsilon^D = 1.5$) in response to the disruption (solid blue curve). Since the tax change at a given income y in formula (24) depends on the disruption affecting agents with incomes larger than y and up to the top of the income distribution $\bar{y}$, we need to make an assumption about the disruption on incomes higher than the largest in our dataset (about $60,000): we assume that they incur the same wage disruption as those who earn $60,000, i.e., their wage increases by 0.34%. To compensate their income loss, low-income agents get a tax rebate equal to $113 if $\varepsilon^D = 0.6$ (i.e., 111.9% of their income loss) or $120 if $\varepsilon^D = 1.5$ (i.e., 118.9% of their income loss). To redistribute their income gain, high-income agents face an increase in their tax payment equal to $260 if $\varepsilon^D = 0.6$ (i.e., 124% of their income gain) or $198 if $\varepsilon^D = 1.5$ (i.e., 94% of their income gain). The right panel of Figure 2 plots the corresponding changes in the average tax rates induced by the reform, i.e. $\hat{T}(y)/y$. The average tax rate on low incomes is reduced by 2.1 percentage points (resp., 2.18 pp) if $\varepsilon^D = 0.6$ (resp., $\varepsilon^D = 1.5$), while that on high incomes is increased by 0.42 pp (resp., 0.32 pp). Finally, applying the formula of Corollary 2, we obtain that the disruption generates a fiscal deficit per capita of $-\$37.3$ in partial equilibrium and $-\$11.5$ if $\varepsilon^D = 1.5$, and a fiscal surplus of \$16 if $\varepsilon^D = 0.6$.[18]

Figure 2: General-equilibrium compensation (left) and Average tax changes (right)



Note that the compensation of a disruption that primarily benefits high-income agents is such that the tax increases must be front-loaded – e.g., in the left panel of Figure 2, individuals between the first and the fourth deciles face a higher tax payment increase (resp., a smaller decrease) than their income gain (resp., loss) caused by the disruption. The remainder of the compensation is ensured by the fact that their marginal tax rate also rises. This front-loading avoids the steep increase in marginal tax rates between the 3rd and 4rd deciles (i.e., between \$25,000 and \$45,000) that the partial-equilibrium compensation would create (right panel of Figure 1). Moreover, the increase in tax payment at the top of the income distribution is larger than the increase in income caused by the disruption, and larger than the partial-equilibrium compensation. Again, this is because these agents also face an increase in their marginal tax rate, which raises their welfare and compensates for the difference between their larger tax bill and their benefit from automation. Therefore, while optimal taxation analyses typically suggest that "trickle-down" forces imply lower marginal tax rates at the top in general equilibrium (Stiglitz [1982b], Rothschild and Scheuer [2013]), the compensation exercise by contrast requires higher marginal and average tax increases on high incomes than in partial equilibrium in response to an increasing wage disruption: the compensation at the 90^{th} percentile is 1.6 times higher once the general-equilibrium forces are taken into account.

[18] Recall that these numbers are for one additional robot per thousand workers; when more robots are introduced, the compensation should be scaled accordingly.

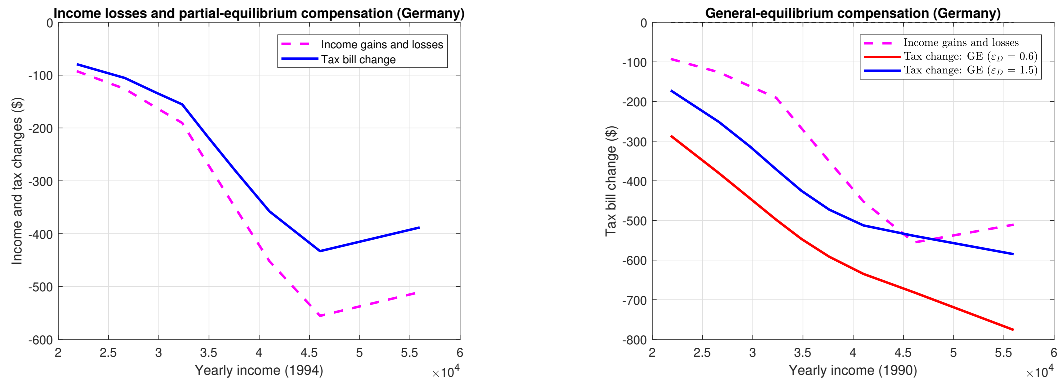
Automation in Germany. Next, we compute the tax reform that compensates the effects of automation on the wages of manufacturing workers in Germany between 1994 and 2014 using the empirical estimates of Dauth, Findeisen, Südekum, and Woessner [2017]. Contrary to Acemoglu and Restrepo [2017], they find that robots caused wage losses for the whole population, and that these losses were larger for higher-income agents. This difference between the two papers is partly due to the fact that they use a different methodology: while Acemoglu and Restrepo [2017] estimate a decile-specific effect of the population-wide exposure to robots (i.e., one per thousand workers), Dauth, Findeisen, Südekum, and Woessner [2017] estimate instead a single overall impact of exposure to robots that they then multiply by the average change in robot exposure over the twenty-year period at each decile of the distribution. They find in Column 6 of Table 7 that an increase of one additional robot per worker reduces earnings by 1.0822%. Moreover, they estimate the exposure to robots along the earnings distribution up to 500,000€ and find that higher incomes faced stronger exposure – and hence incurred larger income losses. The earnings losses of each decile are represented by the dashed magenta curves in both panels of Figure 3.

The solid blue curve in the left panel of Figure 3 plots the compensation in partial equilibrium, which mirrors the income loss induced by automation. In the right panel of Figure 3, we plot the compensating tax reform in general equilibrium for $\varepsilon^D = 0.6$ and $\varepsilon^D = 1.5$. The tax rebate is larger than in partial equilibrium, and almost everywhere larger than the income loss due to the disruption because marginal tax rate changes are negative. If $\varepsilon^D = 0.6$, the bottom decile of incomes should have their tax payment reduced by \$286 per year (i.e., 310% of their income loss), while the top decile should have theirs reduced by \$776 per year (i.e., 152% of their income loss). Finally, these figures imply reductions in the tax rates equal to 1.3 percentage points at the bottom and 1.4 pp at the top. If $\varepsilon^D = 1.5$, the tax rebates are \$172 (186% of the income loss) and \$585 (115% of the income loss) at the bottom and the top, respectively.

5 Compensation in the General Model

In this section we relax many of the assumptions we made in Section 3 and derive a closed-form generalization of formula (24) for the compensating tax reform.

Figure 3: General-equilibrium compensation: Germany



5.1 Initial equilibrium

Agents differ along two dimensions: their skill $i \in [0, 1]$, as in the previous sections, and their fixed cost of participating in the labor force $\kappa \in \mathbb{R}_+$. These two characteristics can be arbitrarily correlated in the population. An agent with types (i, κ) has idiosyncratic preferences over consumption c and labor supply l described by $u_i(c, l) - \kappa \mathbb{I}_{\{l>0\}}$, where the utility function u_i is general (i.e., non-quasilinear) twice continuously differentiable function that satisfies $u'_{i,c} > 0$, $u''_{i,cc} \leq 0$, $u'_{i,l}, u''_{i,ll} < 0$, and where $\mathbb{I}_{\{l>0\}}$ is an indicator function equal to 1 if the agent is employed. If the agent decides to work, he earns a wage w_i, chooses labor supply (hours) l_i, earns pre-tax labor income $y_i = w_i l_i$, and pays a labor income tax $T(y_i)$.[19] If he decides to stay unemployed, his labor supply is equal to zero and he earns the government-provided transfer $-T(0)$. Finally, he also owns an exogenous quantity k_i of the economy's total capital stock, which earns a pre-tax return r.[20] Capital income is taxed at the constant rate τ.

[19]As in Section 1, we order skills so that there is a one-to-one map between skills i and wages w_i in the initial equilibrium with tax schedule T.

[20]We impose that all agents with a given skill i, i.e. a given wage w_i, own the same amount of capital, which ensures that they all choose the same level of labor supply (conditional on working) l_i, independent of their fixed cost of working. We can easily relax this restriction by assuming that agents i who are employed in the initial equilibrium own a different amount of capital than agents with the same skill i but who are not employed. However, if we allowed the level of capital (and hence labor supply) to vary more generally with the fixed cost of working κ, a tax system that consists of a labor income tax schedule and a constant capital tax rate would not be sufficient to compensate the impact of arbitrary wage disruptions, unless individual preferences have no income effects on labor supply.

The maximization problem of agent (i,κ) reads

$$U_{i,\kappa} \equiv \max\left\{\max_{l>0}\ u_i\left(c_i\left(l\right),l\right)-\kappa\,;\, u_i\left(c_i\left(0\right),0\right)\right\}. \tag{28}$$

where $c_i\left(l\right)$ is defined by the budget constraint: $c_i\left(l\right)=w_i l-T\left(w_i l\right)+\left(1-\tau\right)rk_i$ for any $l\geq 0$. Conditional on working, agent (i,κ) chooses labor supply l_i that satisfies the first-order condition

$$-\frac{u'_{i,l}\left(c_i\left(l_i\right),l_i\right)}{u'_{i,c}\left(c_i\left(l_i\right),l_i\right)} = \left[1-T'\left(w_i l_i\right)\right]w_i. \tag{29}$$

We assume that there is a unique solution l_i to this problem. Moreover, the agent decides to participate if and only if his fixed cost of work κ is smaller than a threshold κ_i, given by

$$\kappa_i = u_i\left[w_i l_i-T\left(w_i l_i\right)+\left(1-\tau\right)rk_i,l_i\right]-u_i\left[-T\left(0\right)+\left(1-\tau\right)rk_i,0\right]. \tag{30}$$

Denote by $h_i\left(\kappa\right)$ the density of κ conditional on skill i and by $L_i=l_i\int_0^{\kappa_i}h_i\left(\kappa\right)d\kappa$ the total amount of labor supplied by workers of skill i.

Firms produce output using the aggregate labor supply L_i of each type $i\in[0,1]$ and the aggregate capital stock K, which we assume to be in fixed supply. The aggregate production function is denoted by $\mathscr{F}(\left\{L_i\right\}_{i\in[0,1]},K)$. We assume that $\mathscr{F}$ has constant returns to scale. In equilibrium, firms earn no profits and the wage w_i is equal to the marginal product of type-i labor, i.e.,

$$w_i = \mathscr{F}'_i(\left\{L_j\right\}_{j\in[0,1]},K). \tag{31}$$

The equilibrium interest rate is equal to the marginal product of capital, i.e., $r=\mathscr{F}'_K(\left\{L_j\right\}_{j\in[0,1]},K)$.

The government levies taxes on labor and capital incomes. The initial labor income tax schedule is twice continuously differentiable but is allowed to be arbitrarily nonlinear. We restrict the initial tax schedule and tax reforms on capital income to be linear.

5.2 The welfare compensation problem

A wage disruption, defined as in Section 1.2, may be due to exogenous shocks to the production function, the distribution of labor supplies, and the aggregate capital stock. We denote by $\mu\hat{r}^E$ the corresponding disruption to the interest rate – i.e., the difference between the marginal productivities of capital before and after the shock keeping individual labor supplies fixed at their pre-disruption level. The government can implement an arbitrarily nonlinear reform $\mu\hat{T}$ of the labor income tax schedule, and a reform $\mu\hat{\tau}$ of the capital income tax rate. In response to a disruption $(\mu\hat{\boldsymbol{w}}^E, \mu\hat{r}^E)$ and a tax reform $(\mu\hat{T}, \mu\hat{\tau})$, individuals optimally adjust their labor supply and participation decisions. In general equilibrium, this further impacts their wage and the interest rate, which in turn affects again their labor supply choices, and so on. We denote by $\mu\hat{w}_i$, $\mu\hat{r}$, $\mu\hat{l}_i$ and $\mu\hat{\kappa}_i$ the total endogenous changes in individual i's wage, interest rate, labor supply (conditional on working), and participation threshold, respectively, following the disruption and tax reform. That is, in the disrupted economy we have $\tilde{w}_i = w_i(1 + \mu\hat{w}_i^E + \mu\hat{w}_i)$, $\tilde{r} = r(1 + \mu\hat{r}^E + \mu\hat{r})$, $\tilde{l}_i = l_i(1 + \mu\hat{l}_i)$ and $\tilde{\kappa}_i = \kappa_i + \mu\hat{\kappa}_i$. We finally denote by $\tilde{U}_{i,\kappa} = U_{i,\kappa} + \mu u'_{i,c}\hat{U}_{i,\kappa}$ the resulting indirect utility of agents with type (i, κ) in the final equilibrium. The welfare compensation problem consists of designing the reform $(\mu\hat{T}, \mu\hat{\tau})$ of the tax system such that the welfare of every agent is the same as it was before the wage disruption; that is, $\tilde{U}_{i,\kappa} = U_{i,\kappa}$ for all $(i, \kappa) \in [0, 1] \times \mathbb{R}_+$.

We start by proving that if the government implements the welfare compensating policy, then it must be the case that no agent switches participation status, i.e., $\hat{\kappa}_i = 0$ for all i. Indeed, first note that we can choose to adjust the capital income tax rate by $\frac{\hat{\tau}}{1-\tau} = \hat{r}$, so that the net of tax return $(1-\tau)\, r$, and hence the capital income of each agent, remains constant. Thus, we can leave unchanged the welfare $u_i[-T\,(0) + (1-\tau)\, rk_i, 0]$ of agents who are unemployed both before and after the perturbation by keeping the unemployment transfer $-T\,(0)$ unaffected. Moreover, in order to leave unchanged the welfare of agents who are employed both before and after the perturbation, the combination of the wage disruption and the tax reform must make the utility $\tilde{U}_i \equiv u_i[\tilde{w}_i\tilde{l}_i - T(\tilde{w}_i\tilde{l}_i) - \mu\hat{T}(\tilde{w}_i\tilde{l}_i) + (1-\tau)\, rk_i, \tilde{l}_i]$ equal to its initial value U_i for all i. Now, since the participation decision (30) of an individual with skill i depends only on the difference between the utilities conditional on employment and on unemployment, we obtain that the participation threshold κ_i must also remain constant for all i. That is, in order to leave everyone's welfare unchanged, the compensating tax reform must ensure that the individuals who were employed (resp., unemployed) before the disruption remain so in the new equilibrium.[21]

The welfare compensation problem consists of constructing a labor income tax reform $\hat{T}$ such that the welfare of each employed agent in the disrupted economy is equal to their welfare in the initial equilibrium. As before, our goal is to characterize analytically the solution to the welfare compensation problem for marginal wage disruptions, i.e., as $\mu \to 0$. The problem is a straightforward extension of (5)-(8). Its formal statement and the proofs are gathered in Appendix C.

5.3 Compensation in general equilibrium

We now turn to the general-equilibrium model and follow the same steps as in Section 3. We define the cross-wage elasticities γ_{ij} and labor demand elasticities ε_j^D by (17). The endogenous wage changes are given by:

$$\hat{w}_i \;=\; -\frac{1}{\varepsilon_i^D}\frac{\hat{l}_i}{l_i} + \int_0^1 \gamma_{ij}\hat{l}_j \mathrm{d}j. \tag{32}$$

This equation generalizes (to the case of non-constant own- and cross-wage elasticities) equation (18) in the simpler model of Section 3 and has the same economic interpretation.

The change in the indirect utility of agent i induced by the wage disruption and the tax reform (weighted by the marginal utility of consumption), $\hat{U}_i$, is given by:

$$0 \;=\; \hat{U}_i \;=\; \left(1 - T'\left(y_i\right)\right) y_i \left[\hat{w}_i^E + \hat{w}_i\right] - \hat{T}\left(y_i\right). \tag{33}$$

This equation generalizes (to the case of non-quasilinear preferences) equation (22) in the simpler model of Section 3 and has the same economic interpretation.

The change in the labor supply of agent i is given by:

$$\hat{l}_i \;=\; \varepsilon_i^{S,w}\left[\hat{w}_i^E + \hat{w}_i\right] - \varepsilon_i^{S,r}\frac{\hat{T}'\left(y_i\right)}{1 - T'\left(y_i\right)} + \varepsilon_i^{S,n}\frac{\hat{T}\left(y_i\right)}{\left(1 - T'\left(y_i\right)\right) y_i}. \tag{34}$$

This equation generalizes (to the case of non-constant elasticities and non-quasilinear

[21] This implies in particular that the values of the elasticities of participation with respect to the tax rates (which otherwise would matter to determine the endogenous wage adjustments $\hat{w}_i$) are irrelevant for the construction of the compensating tax reform.

preferences) equation (19) in the simpler model of Section 3 and has a similar economic interpretation, except that labor supply now also adjusts in response to a change in the average tax rate $\hat{T}(y_i)/y_i$ by an amount given by the income effect parameter $\varepsilon_i^{S,n}$.

Using (32) to substitute for the endogenous wage adjustment $\hat{w}_i$ in equation (34) leads to an integral equation for the labor supply changes of all agents, $\{\hat{l}_j\}_{j\in[0,1]}$. This equation is analogous to (20) except that the variables $\phi_i, \varepsilon_i^{S,w}, \varepsilon_i^{S,r}, \gamma_{ij}$ now depend explicitly on i and that there are income effects. The following lemma, which follows from Proposition 1 in Sachs, Tsyvinski, and Werquin [2016] and is proved in the Appendix, gives the closed-form solution to this equation.

Lemma 1. *Assume that* $\int_{[0,1]^2} \mid \delta_i \varepsilon_i^{S,w} \gamma_{ij} \mid^2 \mathrm{d}i\mathrm{d}j < 1$.[22] *The solution to (34) is given by: for all* $i \in [0,1]$,

$$\hat{l}_i \quad = \quad \phi_i \hat{l}_i^{\mathrm{pe}} + \phi_i \varepsilon_i^{S,w} \int_0^1 \Gamma_{ij} \phi_j \hat{l}_j^{\mathrm{pe}} \mathrm{d}j, \tag{35}$$

where $\hat{l}_i^{\mathrm{pe}}$ *is defined by (14),* $\phi_i \equiv 1/[1 + \frac{\varepsilon_i^{S,w}}{\varepsilon_i^D}]$*, and* $\Gamma_{ij} \equiv \sum_{n=0}^{\infty} \Gamma_{ij}^{(n)}$ *with* $\Gamma_{ij}^{(0)} = \gamma_{ij}$ *and for all* $n \geq 1$*,* $\Gamma_{ij}^{(n)} = \int_0^1 \Gamma_{ik}^{(n-1)} \phi_k \varepsilon_k^{S,w} \gamma_{kj} \mathrm{d}k$.

Equation (35) shows that the percentage change in the labor supply of type i, $\hat{l}_i$, is the sum of two terms. The first, $\hat{l}_i^{\mathrm{pe}}$, is the partial-equilibrium expression (14), weighted by ϕ_i. This scaling factor accounts for the fact that the marginal product of labor is decreasing, so that the agent's initial labor supply adjustment (say, increase) $\hat{l}_i^{\mathrm{pe}}$ lowers his wage by a factor $1/\varepsilon_i^D$, which in turn leads him to reduce his labor supply by a factor $\varepsilon_i^{S,w}/\varepsilon_i^D$, therefore dampening his initial response by ϕ_i. The second term in (14) accounts for the fact that the wage disruption and the tax reform also lead to percentage increases $\phi_j \hat{l}_j^{\mathrm{pe}}$ in the labor supplies of agents of type $j \neq i$. These responses impact the wage of agent i by $\Gamma_{ij} \phi_j \hat{l}_j^{\mathrm{pe}}$, where Γ_{ij} captures the total elasticity of the wage of skill i with respect to the labor supply of type j. This total cross-wage elasticity, defined by a series $\sum_{n=0}^{\infty} \Gamma_{ij}^{(n)}$, contains the direct effect $\Gamma_{ij}^{(0)} = \gamma_{ij}$, as well as the infinite sequence of feedback cross-wage effects between skills j and i that occur in general equilibrium: for each $n \geq 1$, $\Gamma_{ij}^{(n)}$ accounts for the

[22] This condition ensures that the series defining Γ_{ij} converges. The assumptions made in Section 3 provide sufficient conditions on primitives such that it is satisfied.

impact of l_j on w_i via the wage and hence labor supply adjustments of n intermediate types – e.g., for $n = 1$, $l_j \xrightarrow{\gamma_{kj}} w_k \xrightarrow{\varepsilon_k^{S,w}} l_k \xrightarrow{\gamma_{ik}} w_i$.[23]

Taking stock. As in the simpler model of Section 3, individual welfare is now affected both by the average tax rates *and* the marginal tax rates – as a result, equation (33) does not directly lead to a formula for the compensating tax reform: we need to solve for the fixed point between the average and marginal tax rates of the compensation. Suppose, to simplify the discussion, that the cross-wage elasticities γ_{ij} are positive for all $i \neq j$. A higher average tax rate at income y^*, $\hat{T}(y^*) > 0$, implies:[24]

(a) a reduction in welfare of agent y^*, by directly making him poorer, as in partial equilibrium (third term in equation (33));

(b) a reduction in welfare of agent y^*, by making him work more (income effect, third term in (14)) and hence earn a lower wage (decreasing marginal product, first term in (32));[25]

(c) an increase in welfare of all agents $y \neq y^*$, whose wage increases due to the higher labor supply of agent y^* (production complementarities, second term in (32)).

Moreover, a higher marginal tax rate at income y^*, $\hat{T}'(y^*) > 0$, has the same consequences (a'), (b'), and (c') as described in Section 3.3. The compensation must take into account all of these effects of taxes on individual welfare that arise in general equilibrium. The non-constant elasticities and the presence of income effects make the construction of this tax reform more difficult than in the environment of Section 3.

[23] See Sachs, Tsyvinski, and Werquin [2016] for details.

[24] We ignore the effects of the changes in the average and marginal tax rates on wages and welfare through agents' participation decisions, since we argued above that no agent switches participation status if the government implements the correct compensating reform.

[25] Because the cross-wage elasticities γ_{ij}, and hence Γ_{ij}, are positive, the wage and welfare of agent y^* are still reduced after taking into account the second, third, etc. rounds of general equilibrium spillovers. This follows from equation (47) in the Appendix. The same reasoning applies for the next bullet point. See Corollary 4 in Appendix B for a formal proof.

Main result. The next result, which generalizes Proposition 2, gives an analytical characterization of the compensating tax reform in response to any wage disruption in general equilibrium.[26] The total wage disruption faced by agent i is now defined as

$$\hat{\Omega}_i^E \quad = \quad \phi_i \hat{w}_i^E + \phi_i \int_0^1 \Gamma_{ij} \phi_j \varepsilon_j^{S,w} \hat{w}_j^E \mathrm{d}j. \tag{36}$$

It accounts for the full incidence of the initial shock on wages and has the same interpretation as (23) in the simpler environment (in which we have $\Gamma_{ij} = \phi_j^{-1}\gamma_j$).

Proposition 3. *Consider a marginal disruption of the wage distribution $\boldsymbol{w}$ in the direction $\hat{\boldsymbol{w}}^E = \{\hat{w}_i^E\}_{i\in[0,1]}$. The following tax reform $\hat{T}$ solves the welfare compensation problem: for all i,*

$$\hat{T}(y) \quad = \quad (1 - T'(y))\, y \int_y^{\bar{y}} \Pi(y,z) \left[\hat{\Omega}_z^E + \int_{\underline{y}}^{\bar{y}} \Lambda(z,x)\, \hat{\Omega}^E(x)\, \mathrm{d}x\right] \mathrm{d}z, \tag{37}$$

where we let

$$\Pi(y,z) \quad \equiv \quad \frac{\varepsilon^D(z)}{\phi(z)\, \varepsilon^{S,r}(z)\, z} e^{-\int_y^z \frac{1}{\varepsilon^{S,r}(x)}\left[\varepsilon^D(x) + 2\varepsilon^{S,n}(x)\right]\frac{\mathrm{d}x}{x}}, \tag{38}$$

and where $\Lambda(y,z) \equiv \sum_{n=0}^{\infty} \Lambda^{(n)}(y,z)$ with

$$\Lambda^{(0)}(y,z) \quad = \quad \phi(y)\, \Gamma(y,z)\, \varepsilon^D(z) - \int_{\underline{y}}^{z} \phi(y)\, \Gamma(y,x)\, \varepsilon^D(x)\, \Pi(x,z)\, \mathrm{d}x, \tag{39}$$

and for all $n \geq 1$,

$$\Lambda^{(n)}(y,z) = \int_{\underline{y}}^{\bar{y}} \Lambda^{(n-1)}(y,x)\, \phi(x)\, \Lambda^{(0)}(x,z)\, \mathrm{d}x.$$

Analogously to equation (24), formula (37) features two main departures from the partial-equilibrium compensation (15). First, the progressivity variable Π is a direct generalization of the corresponding term in (25), and has the same interpre-

[26] As usual, since there is a one-to-one map between types i and incomes y_i, we can change variables and index by income the wages $w(y_i) \equiv w_i$, wage disruptions $\hat{w}^E(y_i) \equiv \hat{w}_i^E$ and $\hat{\Omega}^E(y_i) \equiv \hat{w}_i^E$, and elasticities $\varepsilon^{S,x}(y_i) \equiv \varepsilon_i^{S,x}$ for $x \in \{r, w, n\}$, $\varepsilon^D(y_i) \equiv \varepsilon_i^D$, $\phi(y_i) \equiv \phi_i$, and $\gamma(y_i, y_j) \equiv \gamma_{ij}/y'_j$, $\Gamma(y_i, y_j) \equiv \Gamma_{ij}/y'_j$, where $y'_j \equiv \frac{\partial y_j}{\partial j}$.

tation. Second, the integral in the square brackets of (37) accounts for the cross-wage effects originating from the skill complementarities in production. It is more complex than in (24). Indeed, the functional equation (26) is more involved when the labor supply of type k does not have the same impact on the wage of different skills, so that Γ_{jk} can depend arbitrarily on j. Our proof shows that for each k, the welfare impact of these indirect wage adjustments is determined by the first term $\Lambda^{(0)}(y,z)$ in (39), so that the total effect on type j is given by $\int_{\underline{y}}^{\bar{y}} \Lambda^{(0)}(y,z)\, \hat{\Omega}^E(z)\, \mathrm{d}z$. This welfare change needs to be itself compensated using the tax schedule, thus leading to the term $(1-T'(y))\, y \int_{\underline{y}}^{\bar{y}} \Pi(y,z)\, [\int_{\underline{y}}^{\bar{y}} \Lambda^{(0)}(z,x)\, \hat{\Omega}^E(x)\, \mathrm{d}x] \mathrm{d}z$ in (37). In turn, the marginal tax rates of this second round of compensation generate further wage and welfare changes for all of the agents. These again must be compensated (third round of "compensating the compensation"), leading to the term $(1-T'(y))\, y \int_{\underline{y}}^{\bar{y}} \Pi(y,z)\, [\int_{\underline{y}}^{\bar{y}} \Lambda^{(1)}(z,x)\, \hat{\Omega}^E(x)\, \mathrm{d}x] \mathrm{d}z$ in (37). The full sequence of tax reforms that achieves the fixed point of the compensation problem is constructed by defining inductively the sequence of variables $\Lambda^{(n)}(y,z)$ for all $n \geq 0$, where each $\Lambda^{(n)}(y,z)$ captures one round of iterated compensation.

Corollary 3. *The fiscal surplus generated by the disruption and the compensating tax reform is given by*

$$\hat{\mathcal{R}} = \int_{\underline{y}}^{\bar{y}} \boldsymbol{\rho}(y) \left[\hat{\Omega}^E(y) + \int_{\underline{y}}^{\bar{y}} \Lambda(y,z)\, \hat{\Omega}^E(z)\, \mathrm{d}z \right] \mathrm{d}y \tag{40}$$

where we denote

$$\boldsymbol{\rho}(y) = \left(\varepsilon^{S,w}(y) + \varepsilon^D(y)\right) T'(y)\, y f_Y(y) + \int_{\underline{y}}^{y} \Pi(z,y) \left(1 - \varepsilon^D(z)\, T'(z)\right) z f_Y(z)\, \mathrm{d}z.$$

6 Conclusion: Compensation Principle vs. Optimal Taxation

The classic policy question of compensating winners and losers from an economic disruption becomes quite involved when the environment features both distortionary taxes and general equilibrium. At the same time, both of these considerations are important in many applied and policy questions (e.g., to compensate the adverse effects of technical change or immigration). We provide a general closed-form formula for the design of the welfare-compensating tax reform in general equilibrium. This equation has a clear economic meaning and is easy to implement in practical applications.

Our analysis does not nest general cases of multidimensional worker heterogeneity. In particular, the disruptions we consider do not have heterogeneous effects within income levels. In general, such multi-dimensional shocks could not be compensated with a one-dimensional income tax instrument. We can easily add "tags" and implement tax reforms that target some, say, sectors or occupations rather than others, as long as there is no endogenous switching between sectors. In its full generality, however, the multi-dimensional compensation problem would require richer policy instruments than a simple labor income tax schedule and is left for future research.

We conclude this paper by highlighting the advantages of the compensation approach, taken in this paper, over the more traditional optimal taxation approach (see, in general equilibrium environments, Ales, Kurnaz, and Sleet [2015], Rothschild and Scheuer [2013], Sachs, Tsyvinski, and Werquin [2016]).[27] First, the compensation approach is more tractable. We are able to derive a closed-form solution in very general environments, while the optimal tax formula is much more complex and must be solved numerically even in simple general-equilibrium settings. Second, our formula depends only on the evaluation of sufficient statistics (elasticities, income distribution) in the *current*, pre-disruption, economy rather than in a fictional economy where the optimal tax schedule would already be implemented; it can thus be directly applied using current data. Moreover, the policy response to a given economic disruption is given by a reform of the *actual* (e.g., U.S.) tax schedule, rather than of the optimal one, which was not implemented in the first place – this makes our insights more directly policy-relevant. Finally, the compensation approach allows Pareto comparisons, so no position on a social welfare function must be taken – our formula depends only on variables that are measurable empirically and does not rely on interpersonal comparisons of welfare. We believe that these benefits make the compensation problem a fruitful alternative to optimal taxation.

[27] Of course, by discussing the benefits of the "compensation" approach, we do not mean to claim that it is always superior to the "optimum" approach. Fundamentally, they answer different questions.

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A Proofs of Sections 2

Proof of equation (13). The change in utility of agent i in response to the disruption and tax reform is given by:

$$\tilde{U}_i - U_i = u_i[\tilde{w}_i\tilde{l}_i - T(\tilde{w}_i\tilde{l}_i) - \mu\hat{T}(\tilde{w}_i\tilde{l}_i), \tilde{l}_i] - u_i\left[w_i l_i - T\left(w_i l_i\right), l_i\right]$$

where $\tilde{w}_i = w_i(1+\mu\hat{w}_i^E)$ and $\tilde{l}_i = l_i(1+\mu\hat{l}_i)$. A first-order Taylor expansion of this equation around the initial equilibrium, i.e., as $\mu \to 0$, yields:

$$\tilde{U}_i - U_i = \mu\left[\left(1-T'\left(w_i l_i\right)\right)\left(w_i l_i\hat{l}_i + w_i l_i\hat{w}_i^E\right) - \hat{T}\left(w_i l_i\right)\right]u'_{i,c} + \mu l_i\hat{l}_i u'_{i,l} + o\left(\mu\right), \tag{41}$$

Using the fact that $\left(1-T'\left(w_i l_i\right)\right)w_i l_i\hat{l}_i u'_{i,c} + l_i\hat{l}_i u'_{i,l} = 0$, which follows from the first-order condition (2) or from the envelope theorem, leads to (13).

□

Proof of equation (14). The perturbed first-order condition of agent i in response to the disruption and tax reform is given by:

$$\begin{aligned} 0 &= [1-T'(\tilde{w}_i\tilde{l}_i) - \mu\hat{T}'(\tilde{w}_i\tilde{l}_i)]\tilde{w}_i u'_{i,c}[\tilde{w}_i\tilde{l}_i - T(\tilde{w}_i\tilde{l}_i) - \mu\hat{T}(\tilde{w}_i\tilde{l}_i) + Rk_i, \tilde{l}_i] \\ &\quad + u'_{i,l}[\tilde{w}_i\tilde{l}_i - T(\tilde{w}_i\tilde{l}_i) - \mu\hat{T}(\tilde{w}_i\tilde{l}_i) + Rk_i, \tilde{l}_i]. \end{aligned}$$

A first-order Taylor expansion of this equation around the initial equilibrium, i.e., as $\mu \to 0$, gives:

$$\begin{aligned} 0 &= \left[\left(1-T'\left(y_i\right)\right)^2 w_i l_i u''_{i,cc} + \left(1-T'\left(y_i\right)\right) l_i u''_{i,cl} + \left(1-T'\left(y_i\right)\right)u'_{i,c} - w_i l_i T''\left(y_i\right)u'_{i,c}\right] w_i\hat{w}_i^E \\ &\quad + \left[\left(1-T'\left(y_i\right)\right)^2 w_i^2 u''_{i,cc} + 2\left(1-T'\left(y_i\right)\right) w_i u''_{i,cl} + u''_{i,ll} - w_i^2 T''\left(y_i\right)u'_{i,c}\right] l_i\hat{l}_i \\ &\quad - w_i u'_{i,c}\hat{T}'\left(y_i\right) - \left[\left(1-T'\left(y_i\right)\right) w_i u''_{i,cc} + u''_{i,cl}\right]\hat{T}\left(y_i\right). \end{aligned}$$

The Hicksian (compensated) labor supply elasticity e_i^c and the income effect parameter e_i^n are respectively equal to (see, e.g., Saez [2001] p. 227):

$$e_i^c = \frac{\frac{u'_{i,l}}{l_i}}{\left(\frac{u'_{i,l}}{u'_{i,c}}\right)^2 u''_{i,cc} - 2\left(\frac{u'_{i,l}}{u'_{i,c}}\right)u''_{i,cl} + u''_{i,ll}}, \quad e_i^n = \frac{-\left(\frac{u'_{i,l}}{u'_{i,c}}\right)^2 u''_{i,cc} + \left(\frac{u'_{i,l}}{u'_{i,c}}\right)u''_{i,cl}}{\left(\frac{u'_{i,l}}{u'_{i,c}}\right)^2 u''_{i,cc} - 2\left(\frac{u'_{i,l}}{u'_{i,c}}\right)u''_{i,cl} + u''_{i,ll}}. \tag{42}$$

Solving the previous equation for $\hat{l}_i$ then implies

$$\hat{l}_i = \frac{\left(1-p_i\right)e_i^c + e_i^n}{1+p\left(y_i\right)e_i^c}\hat{w}_i^E - \frac{e_i^c}{1+p\left(y_i\right)e_i^c}\frac{\hat{T}'\left(y_i\right)}{1-T'\left(y_i\right)} - \frac{e_i^n}{1+p\left(y_i\right)e_i^c}\frac{\hat{T}\left(y_i\right)}{\left(1-T'\left(y_i\right)\right)y_i}.$$

Using the definitions of the elasticities along the nonlinear budget constraint $\varepsilon_i^{S,r}, \varepsilon_i^{S,n}, \varepsilon_i^{S,w}$ leads to equation (14).

□

Proof of Proposition 1. Equation (15) follows immediately from (13) and a change of variables from skills i to incomes y_i.

□

Proof of Corollary 1. The effect of the wage disruption and the corresponding compensating tax reform on government budget is given by

$$\hat{\mathcal{R}}(\hat{\boldsymbol{w}}^E) = \lim_{\mu \to 0} \frac{1}{\mu} \left\{ \int_0^1 \left[T(\tilde{w}_i \tilde{l}_i) + \mu \hat{T}(\tilde{w}_i \tilde{l}_i) \right] \mathrm{d}i - \int_0^1 T(w_i l_i)\, \mathrm{d}i \right\},$$

A first-order Taylor expansion around the initial equilibrium implies that this expression is equal to:

$$\hat{\mathcal{R}}(\hat{\boldsymbol{w}}^E) = \int_0^1 \hat{T}(w_i l_i)\, \mathrm{d}i + \int_0^1 [\hat{w}_i^E + \hat{l}_i] w_i l_i T'(w_i l_i)\, \mathrm{d}i,$$

Using equation (15), we can rewrite this expression as

$$\hat{\mathcal{R}}(\hat{\boldsymbol{w}}^E) = \int_0^1 [\hat{w}_i^E + T'(y_i)\, \hat{l}_i] y_i \mathrm{d}i. \tag{43}$$

Now differentiate $\hat{T}(y)$ with respect to y in (15) to obtain the marginal tax rates of the reform. Denoting by $y_i' \equiv \frac{dy_i}{di}$, we obtain:

$$\begin{aligned} \hat{T}'(y_i) &= \frac{1}{y_i'} \left[-y_i' T''(y_i)\, y_i \hat{w}_i^E + (1 - T'(y_i))\, y_i' \hat{w}_i^E + (1 - T'(y_i))\, y_i \frac{d\hat{w}_i^E}{di} \right] \\ &= (1 - T'(y)) \left[(1 - p(y))\, \hat{w}^E(y) + y \frac{d\hat{w}^E(y)}{dy} \right]. \end{aligned}$$

Therefore, equation (14) can be rewritten as:

$$\hat{l}_i = \varepsilon_i^{S,w} \hat{w}_i^E - \varepsilon_i^{S,r} \left[(1 - p(y_i))\, \hat{w}_i^E + \frac{y_i}{y_i'} \frac{d\hat{w}_i^E}{di} \right] - \varepsilon_i^{S,n} \hat{w}_i^E = -\varepsilon_i^{S,r} \frac{y_i}{y_i'} \frac{d\hat{w}_i^E}{di},$$

where we used the fact that $\varepsilon_i^{S,w} = (1 - p(y_i))\, \varepsilon_i^{S,r} + \varepsilon_i^{S,n}$. Substituting into equation (43) and changing variables from skills to incomes therefore leads to (16).

□

B Proofs of Section 3

Proof of equation (18). Consider an exogenous disruption $\mu \hat{\mathcal{F}}^E$ of the initial production function and a tax reform $\mu \hat{T}$, with $\mu > 0$ (the proof can be extended immediately to a disruption of the aggregate labor supply distribution). The corresponding wage disruption is defined by

$$\hat{w}_i^E = \frac{\partial \hat{\mathcal{F}}^E}{\partial L_i}(\{L_j\}_{j \in [0,1]}).$$

Denote by $\mu\hat{w}_i$ and $\mu\hat{l}_i$ the first-order endogenous percentage changes as $\mu \to 0$ in the wage and labor supply of type i, and let $\tilde{w}_i = w_i(1+\mu\hat{w}_i^E + \mu\hat{w}_i)$ and $\tilde{l}_i = l_i(1+\mu\hat{l}_i)$. In the perturbed equilibrium, the wage is equal to the marginal product of the labor of the corresponding type:

$$\tilde{w}_i \quad = \quad \frac{\partial[\mathcal{F}+\mu\hat{\mathcal{F}}^E]}{\partial L_i}(\{L_j(1+\mu\hat{l}_j)\}_{j\in[0,1]}).$$

A first-order Taylor expansion in $\mu \to 0$ of this equation around the initial equilibrium yields the following expression for the Gateaux derivative of the wage functional:\hat{w}_{i}

$$\begin{aligned}
\hat{w}_i \quad &\equiv \quad \lim_{\mu\to 0} \frac{1}{\mu w_i}[\tilde{w}_i - w_i - \mu\hat{w}_i^E] \\
&= \quad \lim_{\mu\to 0} \frac{1}{\mu w_i}\left\{\frac{\partial[\mathcal{F}+\mu\hat{\mathcal{F}}^E]}{\partial L_i}(\{L_j(1+\mu\hat{l}_j)\}_{j\in[0,1]}) - \frac{\partial\mathcal{F}}{\partial L_i}(\{L_j\}_{j\in[0,1]}) - \mu\frac{\partial\hat{\mathcal{F}}^E}{\partial L_i}(\{L_j\}_{j\in[0,1]})\right\}.
\end{aligned}$$

This expression is equal to

$$\hat{w}_i \quad = \quad \frac{1}{w_i}\int_0^1 \hat{l}_j L_j \frac{\partial^2\mathcal{F}(\boldsymbol{L})}{\partial L_i \partial L_j}dj.$$

The cross-wage elasticities are given, for $i \neq j$, by

$$\begin{aligned}
\frac{L_j}{w_i}\frac{\partial^2\mathcal{F}(\boldsymbol{L})}{\partial L_i\partial L_j} \quad &= \quad \frac{L_j}{w_i}\frac{\partial^2}{\partial L_j}\left\{\theta_i L_i^{-1/\varepsilon^D}\left[\int_0^1 \theta_j L_j^{1-1/\varepsilon^D}dj\right]^{\frac{1}{\varepsilon^D-1}}\right\} \\
&= \quad \frac{1}{\varepsilon^D}\frac{\theta_j L_j^{1-1/\varepsilon^D}}{\int_0^1 \theta_k L_k^{1-1/\varepsilon^D}dk} \quad = \quad \frac{1}{\varepsilon^D}\frac{w_j L_j}{\mathcal{F}(\boldsymbol{L})} \quad \equiv \quad \gamma_j,
\end{aligned}$$

and the own-wage elasticities by

$$\begin{aligned}
\frac{L_i}{w_i}\frac{\partial^2\mathcal{F}(\boldsymbol{L})}{\partial L_i^2} \quad &= \quad \frac{L_i}{w_i}\frac{\partial^2}{\partial L_i}\left\{\theta_i L_i^{-1/\varepsilon^D}\left[\int_0^1 \theta_j L_j^{1-1/\varepsilon^D}dj\right]^{\frac{1}{\varepsilon^D-1}}\right\} \\
&= \quad \gamma_i - \frac{1}{\varepsilon^D}\frac{1}{w_i}\theta_i L_i^{-1/\varepsilon^D}\left[\int_0^1 \theta_j L_j^{1-1/\varepsilon^D}dj\right]^{\frac{1}{\varepsilon^D-1}}\delta(0) \quad = \quad \gamma_i - \frac{1}{\varepsilon^D}\delta(0).
\end{aligned}$$

Hence we obtain

$$\hat{w}_i \quad = \quad \int_0^1 \hat{l}_j\left\{\gamma_j - \frac{1}{\varepsilon^D}\delta(i-j)\right\}dj,$$

which leads to equation (18).

□

Proof of equations (19, 20, 22). The proofs of equations (19) and (22) are identical to those of equations (14) and (13) in Section A, except that we now have $\varepsilon^{S,n} = 0$ (by Assumption 1) and $\tilde{w}_i = w_i(1+\mu\hat{w}_i^E+\mu\hat{w}_i)$ rather than $\tilde{w}_i = w_i(1+\mu\hat{w}_i^E)$. Equation (20) is easily obtained by substituting for $\hat{w}_i$ into (19) using (18) and solving for $\hat{l}_i$.

□

Proof of equation (26). Equation (20) is an integral equation in $\{\hat{l}_i\}_{i\in[0,1]}$. To solve this equation, multiply both sides by γ_i and integrate from 0 to 1 to get:

$$\begin{aligned}\int_0^1 \gamma_i \hat{l}_i di &= \phi\varepsilon^{S,w}\int_0^1 \gamma_i \hat{w}_i^E di - \phi\varepsilon^{S,r}\int_0^1 \gamma_i \frac{\hat{T}'(y_i)}{1-T'(y_i)}di + \phi\varepsilon^{S,w}\left(\int_0^1 \gamma_i di\right)\left(\int_0^1 \gamma_j \hat{l}_j dj\right)\\ &= \frac{1}{1-\phi\varepsilon^{S,w}\int_0^1 \gamma_i di}\left\{\phi\varepsilon^{S,w}\int_0^1 \gamma_i \hat{w}_i^E di - \phi\varepsilon^{S,r}\int_0^1 \gamma_i \frac{\hat{T}'(y_i)}{1-T'(y_i)}di\right\}.\end{aligned}$$

Using the facts that $\int_0^1 \gamma_i di = \frac{1}{\varepsilon^D}$ and $\phi = 1/(1+\frac{\varepsilon^{S,w}}{\varepsilon^D})$ to simplify this expression, and then substituting for $\int_0^1 \gamma_j \hat{l}_j di$ in (20), leads to

$$\hat{l}_i = \phi\varepsilon^{S,w}\left\{\hat{w}_i^E + \varepsilon^{S,w}\int_0^1 \gamma_j \hat{w}_j^E dj\right\} - \phi\varepsilon^{S,r}\left\{\frac{\hat{T}'(y_i)}{1-T'(y_i)} + \varepsilon^{S,w}\int_0^1 \gamma_j \frac{\hat{T}'(y_j)}{1-T'(y_j)}dj\right\}.$$

This leads to equation (21). Note in particular that in the absence of a tax reform, the total labor supply adjustment $\hat{l}_i$ would be given by $\varepsilon^{S,w}\hat{\Omega}_i^E$. Next, substitute for $\hat{w}_i^E + \hat{w}_i$ in (22) using (19) to get

$$\hat{T}(y_i) = \frac{1}{\varepsilon^{S,w}}(1-T'(y_i))\, y_i \hat{l}_i + \frac{\varepsilon^{S,r}}{\varepsilon^{S,w}} y_i \hat{T}'(y_i).$$

Using the expression we derived above for $\hat{l}_i$, we get:

$$\hat{T}(y_i) = (1-T'(y_i))\, y_i \hat{\Omega}_i^E + \phi\frac{\varepsilon^{S,r}}{\varepsilon^D} y_i \hat{T}'(y_i) - (1-T'(y_i))\, y_i \phi\varepsilon^{S,r}\int_0^1 \gamma_j \frac{\hat{T}'(y_j)}{1-T'(y_j)}dj.$$

Letting $\lambda \equiv \varepsilon^{S,r}\int_0^1 \gamma_j \frac{\hat{T}'(y_j)}{1-T'(y_j)}dj$ and changing variables from skills i to incomes y_i in this equation leads to (26).

□

Our analysis relies crucially on properly characterizing and accounting for the incidence of taxes (both average and marginal) on individual utilities. Before deriving the compensating tax reform, we characterize this incidence in an important special case that nests the model of Section 3.1.

Corollary 4. *Assume that there are no income effects and that the cross-wage elasticities satisfy $\gamma_{ji} \geq 0$ for all i, j. Then, for a given total (average) tax change $\hat{T}(y_i)$ at income y_i, a higher marginal tax rate $\hat{T}'(y_i) > 0$ raises the utility of agents with skill i and lowers that of all other agents. That is, $\hat{U}_i > 0$, and $\hat{U}_j < 0$ for all $j \neq i$.*

Proof of Corollary 4. Suppose that $\gamma_{ji} > 0$ for all i, j, which implies that $\Gamma_{ji} > 0$ for all i, j. (In the model of Section 3.1, we simply have $\Gamma_{ij} = \phi^{-1}\gamma_j$.) We then have, by equation (47), for any

$j \in [0,1]$,

$$\hat{w}_j = \frac{\phi_j \varepsilon_j^{S,r}}{\varepsilon_j^D} \frac{\hat{T}'(y_j)}{1-T'(y_j)} - \phi_j \int_0^1 \Gamma_{ji} \phi_i \varepsilon_i^{S,r} \frac{\hat{T}'(y_i)}{1-T'(y_i)} di.$$

Since $\phi_j \Gamma_{ji} \phi_i \varepsilon_i^{S,r} > 0$, a higher marginal tax rate $\hat{T}'(y_i) > 0$ at income y_i lowers the wage, and hence lowers the utility (conditional on the total tax change $\hat{T}(y_j)$ at income y_j), of type $j \neq i$. This is because the higher tax rate lowers the labor supply of type i and the labor of type j is complementary to that of type i in production. Moreover, since $\frac{\phi_j \varepsilon_j^{S,r}}{\varepsilon_j^D} > 0$, a higher marginal tax rate $\hat{T}'(y_j) > 0$ at income y_j raises the wage, and hence raises the utility (conditional on the total tax change $\hat{T}(y_j)$ at income y_j), of type j. The easiest way to show this is to consider a tax reform at income y_j only, i.e., $\hat{T}'(y) = \delta(y-y_j)$. We then have

$$\hat{w}_i = \frac{\phi_j \varepsilon_j^{S,r}}{\varepsilon_j^D (1-T'(y_j))} \delta(0) - \frac{\phi_j \Gamma_{jj} \phi_j \varepsilon_j^{S,r}}{1-T'(y_j)} > 0.$$

□

Proof of Proposition 2. We now solve the ODE (26). Since there is a one-to-one map between skills i and incomes y_i, we denote by $\hat{\Omega}^E(y_i) \equiv \hat{\Omega}_i^E$ and change variables to express (26) in terms of incomes. The change of variables for the cross wage elasticities γ_j is given by:

$$\gamma(y_j) \equiv \frac{\gamma_j}{y_j'} = \frac{1}{\varepsilon^D} \frac{y_j f_Y(y_j)}{\mathbb{E}y},$$

where f_Y is the pdf of incomes. Now, noting that $1/(\phi \frac{\varepsilon^{S,r}}{\varepsilon^D}) = 1-p+\frac{\varepsilon^D}{\varepsilon^{S,r}}$, the homogeneous equation reads:

$$\hat{T}'(y) - \left(1-p+\frac{\varepsilon^D}{\varepsilon^{S,r}}\right) \frac{1}{y} \hat{T}(y) = 0.$$

Its general solution is given by

$$\hat{T}_H(y) = C y^{1-p+\frac{\varepsilon^D}{\varepsilon^{S,r}}},$$

where C is a constant. Using the method of variation of the parameter, we find a particular solution of the form

$$\hat{T}_P(y) = C(y) y^{1-p+\frac{\varepsilon^D}{\varepsilon^{S,r}}},$$

where the function $C(y)$ is given by:

$$C(y) = \left(1-p+\frac{\varepsilon^D}{\varepsilon^{S,r}}\right) \int_y^{\bar{y}} (1-T'(x)) x^{-(1-p+\frac{\varepsilon^D}{\varepsilon^{S,r}})} \left[\hat{\Omega}^E(x) - \phi\lambda\right] dx.$$

The general solution to (26) is then equal to:

$$\hat{T}(y) = \hat{T}_H(y) + \hat{T}_P(y) = \left(1 - p + \frac{\varepsilon^D}{\varepsilon^{S,r}}\right)(1 - T'(y)) \int_y^{\bar{y}} \left(\frac{y}{x}\right)^{1+\frac{\varepsilon^D}{\varepsilon^{S,r}}} \hat{\Omega}^E(x)\,dx$$
$$- \left[1 - \left(\frac{y}{\bar{y}}\right)^{\frac{\varepsilon^D}{\varepsilon^{S,r}}}\right](1 - T'(y))\,y\lambda + Cy^{1-p+\frac{\varepsilon^D}{\varepsilon^{S,r}}},$$

where we used $(1 - T'(x)) = (1-\tau)\,x^{-p} = (1 - T'(y))\left(\frac{y}{x}\right)^p$. Denoting the constant $D \equiv \frac{C}{1-\tau} + \lambda\bar{y}^{-\varepsilon^D/\varepsilon^{S,r}}$ we get

$$\hat{T}(y) = \left(1 - p + \frac{\varepsilon^D}{\varepsilon^{S,r}}\right)(1 - T'(y)) \int_y^{\bar{y}} \left(\frac{y}{x}\right)^{1+\frac{\varepsilon^D}{\varepsilon^{S,r}}} \hat{\Omega}^E(x)\,dx$$
$$-(1 - T'(y))\,y\lambda + D(1 - T'(y))\,y^{1+\frac{\varepsilon^D}{\varepsilon^{S,r}}}.$$

Finally, to find the constant λ, note that

$$\frac{y\hat{T}'(y)}{1 - T'(y)} = \left(1 - p + \frac{\varepsilon^D}{\varepsilon^{S,r}}\right)\left[\frac{\hat{T}(y)}{1 - T'(y)} - y\hat{\Omega}^E(y) + \phi\lambda y\right]$$
$$= \left(1 - p + \frac{\varepsilon^D}{\varepsilon^{S,r}}\right)\left\{\left(1 - p + \frac{\varepsilon^D}{\varepsilon^{S,r}}\right)\int_y^{\bar{y}} \left(\frac{y}{x}\right)^{1+\frac{\varepsilon^D}{\varepsilon^{S,r}}} \hat{\Omega}^E(x)\,dx - y\hat{\Omega}^E(y)\right.$$
$$\left. + (\phi - 1)\,\lambda y + Dy^{1+\frac{\varepsilon^D}{\varepsilon^{S,r}}}\right\}.$$

Using the expression for γ_j and the change of variables from skills to incomes,

$$\lambda \equiv \frac{\varepsilon^{S,r}}{\varepsilon^D \mathbb{E}y} \int_{\underline{y}}^{\bar{y}} \frac{y\hat{T}'(y)}{1 - T'(y)} f_Y(y)\,dy$$
$$= \frac{1}{\phi\mathbb{E}y}\left\{\left(1 - p + \frac{\varepsilon^D}{\varepsilon^{S,r}}\right)\mathbb{E}\left[\int_y^{\bar{y}} \left(\frac{y}{x}\right)^{1+\frac{\varepsilon^D}{\varepsilon^{S,r}}} \hat{\Omega}^E(x)\,dx\right] - \mathbb{E}\left[y\hat{\Omega}^E(y)\right] + D\mathbb{E}\left[y^{1+\frac{\varepsilon^D}{\varepsilon^{S,r}}}\right]\right\} + \left(1 - \frac{1}{\phi}\right)\lambda$$
$$= \frac{1}{\mathbb{E}y}\left\{\left(1 - p + \frac{\varepsilon^D}{\varepsilon^{S,r}}\right)\mathbb{E}\left[\int_y^{\bar{y}} \left(\frac{y}{x}\right)^{1+\frac{\varepsilon^D}{\varepsilon^{S,r}}} \hat{\Omega}^E(x)\,dx\right] - \mathbb{E}\left[y\hat{\Omega}^E(y)\right] + D\mathbb{E}\left[y^{1+\frac{\varepsilon^D}{\varepsilon^{S,r}}}\right]\right\}.$$

Now suppose that the initial tax schedule T is Pareto optimal and that there is no disruption, i.e., $\hat{\Omega}^E(y) = 0$. Imposing that the tax reform should be $\hat{T} = 0$ in this case requires $D = 0$. This concludes the proof.

□

Proof of Corollary 2. Recall that the firscal surplus is given by

$$\hat{\mathcal{R}} = \int_0^1 \hat{T}(y_i)\,di + \int_0^1 T'(y_i)\left[\hat{w}_i^E + \hat{w}_i + \hat{l}_i\right] y_i di.$$

Substituting for $\hat{l}_i$ using (19) and for $\hat{w}_i^E + \hat{w}_i$ using (22), we can write

$$\hat{\mathcal{R}} \;=\; \int_0^1 \hat{T}(y_i)\,di + \int_0^1 T'(y_i)\left[\left(1+\varepsilon^{S,w}\right)\frac{\hat{T}(y_i)}{\left(1-T'(y_i)\right)y_i} - \varepsilon^{S,r}\frac{\hat{T}'(y_i)}{1-T'(y_i)}\right]y_i di.$$

Using the ODE (26) to substitute for $\hat{T}'(y_i)$ in this expression, we get

$$\hat{\mathcal{R}} \;=\; \int_0^1 \left[1+\left(1-\varepsilon^D\right)\frac{T'(y_i)}{1-T'(y_i)}\right]\hat{T}(y_i)\,di - \lambda\varepsilon^D\int_0^1 T'(y_i)\,y_i di + \frac{\varepsilon^D}{\phi}\int_0^1 T'(y_i)\,y_i\hat{\Omega}^E(y_i)\,di.$$

Now, the definition of λ and the ODE (26) require

$$\lambda \;=\; \varepsilon_r^S\int_0^1 \gamma_j\frac{\hat{T}'(y_j)}{1-T'(y_j)}dj \;=\; \frac{1}{\phi\mathbb{E}y}\int_0^1 y_j\left(\frac{\hat{T}(y_j)}{\left(1-T'(y_i)\right)y_i} - \hat{\Omega}^E(y_j) + \phi\lambda\right)dj,$$

so that

$$\int_0^1 \frac{\hat{T}(y_j)}{1-T'(y_j)}dj \;=\; \int_0^1 y_j\hat{\Omega}^E(y_j)\,dj.$$

Inserting this term into the formula for $\hat{\mathcal{R}}$ above leads to

$$\begin{aligned}\hat{\mathcal{R}} \;=\; & \int_0^1 y_i\hat{\Omega}^E(y_i)\,di - \varepsilon^D\int_0^1 \frac{T'(y_i)}{1-T'(y_i)}\hat{T}(y_i)\,di \\ & -\lambda\varepsilon^D\int_0^1 T'(y_i)\,y_i di + \frac{\varepsilon^D}{\phi}\int_0^1 T'(y_i)\,y_i\hat{\Omega}^E(y_i)\,di.\end{aligned}$$

Next, using the solution (24) to the ODE for $\hat{T}$ leads to

$$\begin{aligned}\hat{\mathcal{R}} \;=\; & \int_0^1 y_i\hat{\Omega}^E(y_i)\,di + \frac{\varepsilon^D}{\phi}\int_0^1 T'(y_i)\,y_i\hat{\Omega}^E(y_i)\,di \\ & -\frac{\varepsilon^D}{\phi\frac{\varepsilon^{S,r}}{\varepsilon^D}}\int_0^1 T'(y_i)\left\{\int_{y_i}^{\bar{y}}\left(\frac{y_i}{x}\right)^{1+\frac{\varepsilon^D}{\varepsilon^{S,r}}}\hat{\Omega}^E(x)\,dx\right\}di.\end{aligned}$$

This concludes the proof.

□

C Proofs of Section 5

The welfare compensation problem is characterized by the following equations:

$$U_i \;=\; \tilde{U}_i \;\equiv\; u_i[\tilde{w}_i\tilde{l}_i - T(\tilde{w}_i\tilde{l}_i) - \mu\hat{T}(\tilde{w}_i\tilde{l}_i) + (1-\tau)\,rk_i, \tilde{l}_i], \tag{44}$$

where $(\tilde{w}_i, \tilde{l}_i)$ are defined by the perturbed first-order condition

$$-\frac{u'_{i,l}[\tilde{w}_i\tilde{l}_i - T(\tilde{w}_i\tilde{l}_i) - \mu\hat{T}(\tilde{w}_i\tilde{l}_i) + (1-\tau)\,rk_i, \tilde{l}_i]}{u'_{i,c}[\tilde{w}_i\tilde{l}_i - T(\tilde{w}_i\tilde{l}_i) - \mu\hat{T}(\tilde{w}_i\tilde{l}_i) + (1-\tau)\,rk_i, \tilde{l}_i]} = [1 - T'(\tilde{w}_i\tilde{l}_i) - \mu\hat{T}'(\tilde{w}_i\tilde{l}_i)]\tilde{w}_i, \tag{45}$$

and the perturbed wage equation

$$\tilde{w}_i = \tilde{\mathscr{F}}'_i(\{L_j + \hat{L}^E_j + \mu\hat{l}_j\}_{j\in[0,1]}, \tilde{K}). \tag{46}$$

Proof of equations (32), (33), (34). The derivations of these equations are analogous to those of equations (18), (22), (19) above.

□

Proof of Lemma 1. This lemma follows from Sachs, Tsyvinski, and Werquin [2016]; for completeness, we give its proof here. Using equations (18) and (19), we obtain that the labor supply adjustments $\{\hat{l}_i\}_{i\in[0,1]}$ satisfy the following linear Fredholm integral equation:

$$\begin{aligned}\hat{l}_i &= \varepsilon^{S,w}_i\left[\hat{w}^E_i - \frac{1}{\varepsilon^D_i}\hat{l}_i + \int_0^1 \gamma_{ij}\hat{l}_j dj\right] - \varepsilon^{S,r}_i\frac{\hat{T}'(y_i)}{1-T'(y_i)} + \varepsilon^{S,n}_i\frac{\hat{T}(y_i)}{(1-T'(y_i))\,y_i} \\ &= \phi_i\left[\varepsilon^{S,w}_i\hat{w}^E_i - \varepsilon^{S,r}_i\frac{\hat{T}'(y_i)}{1-T'(y_i)} + \varepsilon^{S,n}_i\frac{\hat{T}(y_i)}{(1-T'(y_i))\,y_i}\right] + \phi_i\varepsilon^{S,w}_i\int_0^1 \gamma_{ij}\hat{l}_j dj.\end{aligned}$$

Denoting the expression in square brackets by $\hat{l}^{\text{pe}}_i$, and substituting for $\hat{l}_j$ in the integral leads to

$$\begin{aligned}\hat{l}_i &= \frac{\phi_i\hat{l}^{\text{pe}}_i}{l_i} + \phi_i\varepsilon^{S,w}_i\int_0^1 \gamma_{ij}\left[\phi_j\hat{l}^{\text{pe}}_j + \phi_j\varepsilon^{S,w}_j\int_0^1 \gamma_{jk}\hat{l}_k dk\right]dj \\ &= \left[\phi_i\hat{l}^{\text{pe}}_i + \phi_i\varepsilon^{S,w}_i\int_0^1 \gamma_{ij}\phi_j\hat{l}^{\text{pe}}_j dj\right] + \phi_i\varepsilon^{S,w}_i\int_0^1\left[\int_0^1 \gamma_{ik}\phi_k\varepsilon^{S,w}_k\gamma_{kj}dk\right]\hat{l}_j dj \\ &\equiv \left[\phi_i\hat{l}^{\text{pe}}_i + \phi_i\varepsilon^{S,w}_i\int_0^1 \gamma_{ij}\phi_j\hat{l}^{\text{pe}}_j dj\right] + \phi_i\varepsilon^{S,w}_i\int_0^1 \Gamma^{(1)}_{ij}\hat{l}_j dj,\end{aligned}$$

where $\Gamma^{(0)}_{ij} = \gamma_{ij}$ and $\Gamma^{(1)}_{ij} = \int_0^1 \Gamma^{(0)}_{ik}\phi_k\varepsilon^{S,w}_k\gamma_{kj}dk$. By induction, it is easy to show that for all $N \geq 0$,

$$\hat{l}_i = \left[\phi_i\hat{l}^{\text{pe}}_i + \phi_i\varepsilon^{S,w}_i\int_0^1\left\{\sum_{n=0}^N \Gamma^{(n)}_{ij}\right\}\phi_j\hat{l}^{\text{pe}}_j dj\right] + \phi_i\varepsilon^{S,w}_i\int_0^1 \Gamma^{(N+1)}_{ij}\hat{l}_j dj$$

where for all $n \geq 0$, $\Gamma^{(n+1)}_{ij} = \int_0^1 \Gamma^{(n)}_{ik}\phi_k\varepsilon^{S,w}_k\gamma_{kj}dk$. The condition $\int_0^1\int_0^1 \mid \phi_i\varepsilon^{S,w}_i\gamma_{ij} \mid^2 didj < 1$ ensures that the series $\sum_{n=0}^N \Gamma^{(n)}_{ij}$ converges as $N \to \infty$. This implies equation (35). Finally, note that we can write the endogenous wage changes as

$$\begin{aligned}\hat{w}_i &= -\frac{\phi_i\varepsilon^{S,w}_i}{\varepsilon^D_i}\hat{w}^E_i + \frac{\phi_i\varepsilon^{S,r}_i}{\varepsilon^D_i}\frac{\hat{T}'(y_i)}{1-T'(y_i)} - \frac{\phi_i\varepsilon^{S,n}_i}{\varepsilon^D_i}\frac{\hat{T}(y_i)}{(1-T'(y_i))\,y_i} \\ &\quad + \phi_i\int_0^1 \Gamma_{ij}\phi_j\left[\varepsilon^{S,w}_j\hat{w}^E_j - \varepsilon^{S,r}_j\frac{\hat{T}'(y_j)}{1-T'(y_j)} + \varepsilon^{S,n}_j\frac{\hat{T}(y_j)}{(1-T'(y_j))\,y_j}\right]dj,\end{aligned} \tag{47}$$

which follows from equations (14), (19) and (35).

□

Lemma 2. *Let* $\tau_{ij} \equiv \frac{(1-T'(y_i))y_i}{(1-T'(y_j))y_j}$. *The compensating tax reform* $\hat{T}$ *satisfies the following functional equation: for all* $i \in [0,1]$,

$$\begin{aligned}(1-T'(y_i))\,y_i\hat{\Omega}_i^E \;=\;& -\phi_i \frac{\varepsilon_i^{S,r}}{\varepsilon_i^D} y_i \hat{T}'(y_i) + \phi_i \left(1 + \frac{\varepsilon_i^{S,w}}{\varepsilon_i^D} + \frac{\varepsilon_i^{S,n}}{\varepsilon_i^D}\right)\hat{T}(y_i) \\ &+ \phi_i \int_0^1 \Gamma_{ij}\tau_{ij}\phi_j \left[\varepsilon_j^{S,r} y_j \hat{T}'(y_j) - \varepsilon_j^{S,n}\hat{T}(y_j)\right] dj.\end{aligned} \tag{48}$$

Proof of Lemma 2. Equations (34) and (35) imply that the wage adjustments $\{\hat{w}_i\}_{i\in[0,1]}$ are given by

$$\begin{aligned}\hat{w}_i^E + \hat{w}_i \;=\;& \frac{1}{\varepsilon_i^{S,w}}\hat{l}_i + \frac{\varepsilon_i^{S,r}}{\varepsilon_i^{S,w}} \frac{\hat{T}'(y_i)}{1-T'(y_i)} - \frac{\varepsilon_i^{S,n}}{\varepsilon_i^{S,w}} \frac{\hat{T}(y_i)}{(1-T'(y_i))\,y_i} \\ =\;& \phi_i \hat{w}_i^E - \frac{(\phi_i - 1)\,\varepsilon_i^{S,r}}{\varepsilon_i^{S,w}} \frac{\hat{T}'(y_i)}{1-T'(y_i)} + \frac{(\phi_i - 1)\,\varepsilon_i^{S,n}}{\varepsilon_i^{S,w}} \frac{\hat{T}(y_i)}{(1-T'(y_i))\,y_i} \\ &+ \phi_i \int_0^1 \Gamma_{ij}\left[\phi_j \varepsilon_j^{S,w}\hat{w}_j^E - \phi_j \varepsilon_j^{S,r} \frac{\hat{T}'(y_j)}{1-T'(y_j)} + \phi_j \varepsilon_j^{S,n} \frac{\hat{T}(y_j)}{(1-T'(y_j))\,y_j}\right] dj.\end{aligned}$$

Using this equation with $\frac{\phi_i - 1}{\varepsilon_i^{S,w}} = -\frac{\phi_i}{\varepsilon_i^D}$, we can substitute for $\hat{w}_i^E + \hat{w}_i$ in the constraint (33) to rewrite it as

$$\begin{aligned}0 \;=\;& (1-T'(y_i))\,y_i\phi_i \left[\hat{w}_i^E + \frac{\varepsilon_i^{S,r}}{\varepsilon_i^D} \frac{\hat{T}'(y_i)}{1-T'(y_i)} - \frac{\varepsilon_i^{S,n}}{\varepsilon_i^D} \frac{\hat{T}(y_i)}{(1-T'(y_i))\,y_i}\right] \\ &+ (1-T'(y_i))\,y_i\phi_i \int_0^1 \Gamma_{ij}\phi_j \left[\varepsilon_j^{S,w}\hat{w}_j^E - \varepsilon_j^{S,r} \frac{\hat{T}'(y_j)}{1-T'(y_j)} + \varepsilon_j^{S,n} \frac{\hat{T}(y_j)}{(1-T'(y_j))\,y_j}\right] dj - \hat{T}(y_i),\end{aligned}$$

which leads to (48).

□

Proof of Proposition 3. Changing variables from i to y_i in equation (48) leads to

$$\hat{T}'(y_i) - \left(\frac{\varepsilon^{S,w}(y_i) + \varepsilon^{S,n}(y_i) + \varepsilon^D(y_i)}{\varepsilon^{S,r}(y_i)\,y_i}\right)\hat{T}(y_i) \;=\; -\frac{\varepsilon^D(y_i)}{\phi(y_i)\,\varepsilon^{S,r}(y_i)\,y_i}(1-T'(y_i))\,y_i\mathcal{A}(y_i), \tag{49}$$

where we denote

$$\mathcal{A}(y_i) \;\equiv\; \hat{\Omega}^E(y_i) - \phi(y_i)\int_{\underline{y}}^{\bar{y}} \frac{\Gamma(y_i,y_j)\,\phi(y_j)}{(1-T'(y_j))\,y_j}\left[\varepsilon^{S,r}(y_j)\,y_j\hat{T}'(y_j) - \varepsilon^{S,n}(y_j)\,\hat{T}(y_j)\right] dy_j \tag{50}$$

and the changes of variables imply

$$\hat{\Omega}^E(y_i) \quad = \quad \phi(y_i)\,\hat{w}^E(y_i) + \phi(y_i)\int_{\underline{y}}^{\bar{y}} \Gamma(y_i,y_j)\,\phi(y_j)\,\varepsilon^{S,w}(y_j)\,\hat{w}^E(y_j)\,dy_j.$$

(In the sequel, we denote the arguments y_i, etc. as indices for conciseness.) Equation (49) is a first-order ordinary differential equation. Using standard techniques and the definition $\varepsilon_{y_i}^{S,w} = (1-p(y_i))\,\varepsilon_{y_i}^{S,r} + \varepsilon_{y_i}^{S,n}$, we can express its general solution (up to a constant c_0, equal to 0 if the initial tax schedule is Pareto efficient) as

$$\begin{aligned}
\hat{T}(y_i) &= \int_{y_i}^{\bar{y}} \frac{\varepsilon_{y_j}^D}{\phi_{y_j}\varepsilon_{y_j}^{S,r} y_j} e^{-\int_{y_i}^{y_j}\left(1-p(y_k)+\frac{\varepsilon_{y_k}^D+2\varepsilon_{y_k}^{S,n}}{\varepsilon_{y_k}^{S,r}}\right)\frac{dy_k}{y_k}} (1-T'(y_j))\,y_j\mathcal{A}(y_j)\,dy_j \\
&= \int_{y_i}^{\bar{y}} \frac{\varepsilon_{y_j}^D}{\phi_{y_j}\varepsilon_{y_j}^{S,r} y_j} e^{-\int_{y_i}^{y_j}\frac{\varepsilon_{y_k}^D+2\varepsilon_{y_k}^{S,n}}{\varepsilon_{y_k}^{S,r}}\frac{dy_k}{y_k}} \frac{(1-T'(y_i))\,y_i}{(1-T'(y_j))\,y_j}(1-T'(y_j))\,y_j\mathcal{A}(y_j)\,dy_j \\
&= (1-T'(y_i))\,y_i\int_{y_i}^{\bar{y}} \Pi(y_i,y_j)\,\mathcal{A}(y_j)\,dy_j. \qquad (51)
\end{aligned}$$

where the second equality uses the definition $\frac{p(y_k)}{y_k} = \frac{T''(y_k)}{1-T'(y_k)}$ and integrates this expression. Using (49) and (51), we can rewrite that auxiliary function $\mathcal{A}(\cdot)$ as

$$\begin{aligned}
\mathcal{A}(y_i) &= \hat{\Omega}_{y_i}^E - \phi_{y_i}\int_{\underline{y}}^{\bar{y}} \frac{\Gamma_{y_i,y_j}\phi_{y_j}}{(1-T'(y_j))\,y_j}\left[-\frac{\varepsilon_{y_j}^D}{\phi_{y_j}}(1-T'(y_j))\,y_j\mathcal{A}(y_j) + \left(\varepsilon_{y_j}^{S,w}+\varepsilon_{y_j}^D\right)\hat{T}(y_j)\right]dy_j \\
&= \hat{\Omega}_i^E + \phi_{y_i}\int_{\underline{y}}^{\bar{y}} \Gamma_{y_i,y_j}\varepsilon_{y_j}^D\mathcal{A}(y_j)\,dy_j - \phi_{y_i}\int_{\underline{y}}^{\bar{y}} \Gamma_{y_i,y_j}\varepsilon_{y_j}^D\frac{\hat{T}(y_j)}{(1-T'(y_j))\,y_j}dy_j \\
&= \hat{\Omega}_i^E + \phi_{y_i}\int_{\underline{y}}^{\bar{y}} \Gamma_{y_i,y_j}\varepsilon_{y_j}^D\mathcal{A}(y_j)\,dy_j - \phi_{y_i}\int_{y_j=\underline{y}}^{\bar{y}}\int_{y_k=y_j}^{\bar{y}} \Gamma_{y_i,y_j}\varepsilon_{y_j}^D\Pi_{y_j,y_k}\mathcal{A}(y_k)\,dy_k dy_j
\end{aligned}$$

where the second equality uses the fact that $\phi_{y_j}(\varepsilon_{y_j}^{S,w}+\varepsilon_{y_j}^D) = \varepsilon_{y_j}^D$. Inverting the order of the two integrals in the last line implies that this expression can be rewritten as

$$\begin{aligned}
\mathcal{A}(y_i) &= \hat{\Omega}_i^E + \phi_{y_i}\int_{\underline{y}}^{\bar{y}} \Gamma_{y_i,y_j}\varepsilon_{y_j}^D\mathcal{A}(y_j)\,dy_j - \phi_{y_i}\int_{y_k=\underline{y}}^{\bar{y}}\left\{\int_{y_j=\underline{y}}^{y_k}\Gamma_{y_i,y_j}\varepsilon_{y_j}^D\Pi_{y_j,y_k}dy_j\right\}\mathcal{A}(y_k)\,dy_k \\
&= \hat{\Omega}_i^E + \phi_{y_i}\int_{\underline{y}}^{\bar{y}}\left\{\Gamma_{y_i,y_j}\varepsilon_{y_j}^D - \int_{\underline{y}}^{y_j}\Gamma_{y_i,y_k}\varepsilon_{y_k}^D\Pi_{y_k,y_j}dy_k\right\}\mathcal{A}(y_j)\,dy_j.
\end{aligned}$$

But this is a standard linear Fredholm integral equation, with kernel given by

$$\Lambda_{y_i,y_j}^{(0)} \equiv \phi_{y_i}\left[\Gamma_{y_i,y_j}\varepsilon_{y_j}^D - \int_{\underline{y}}^{y_j}\Gamma_{y_i,y_k}\varepsilon_{y_k}^D\Pi_{y_k,y_j}dy_k\right].$$

Its solution is therefore known in closed form (see, e.g., Zemyan [2012]). Assume that

$$\int_{[0,1]^2} \left|\Lambda^{(0)}_{y_i,y_j}\right|^2 di dj < 1,$$

which ensures the convergence of the series $\sum_{n=0}^{\infty} \Lambda^{(n)}_{y_i,y_j}$ defined in Proposition 3. This condition is satisfied in the case under the assumptions of Section 3.1. Following analogous steps as in the proof of Lemma 1, we get

$$\mathcal{A}(y_i) \quad = \quad \hat{\Omega}^E_{y_i} + \int_{\underline{y}}^{\bar{y}} \left\{ \sum_{n=0}^{\infty} \Lambda^{(n)}_{y_i,y_j} \right\} \hat{\Omega}^E_{y_j} dy_j. \tag{52}$$

From equations (51) and (52), we obtain the solution to the compensating tax reform problem $\hat{T}(y_i) = (1 - T'(y_i))\, y_i \int_{y_i}^{\bar{y}} \Pi_{y_i y_j} \mathcal{A}(y_j)\, dy_j$, leading to formula (37).

□

Proof of Corollary 3. The effect of the wage disruption and the corresponding compensating tax reform on government budget is given by

$$\hat{\mathcal{R}} \quad = \quad \int_0^1 \hat{T}(y_i)\, f(i)\, di + \int_0^1 \left[\hat{w}^E_i + \hat{w}_i + \hat{l}_i\right] w_i l_i T'(w_i l_i)\, di.$$

Using equations (19) and (35), the second integral in the right hand side can be rewritten as

$$\begin{aligned}
&\int_0^1 T'(y_i)\, y_i \left(1 + \varepsilon^{S,w}_i\right) \phi_i \left[\hat{w}^E_i + \int_0^1 \Gamma_{ij}\phi_j \varepsilon^{S,w}_j \hat{w}^E_j dj\right] di \\
&- \int_0^1 T'(y_i)\, y_i \left\{ E_i \varepsilon^{S,r}_i \frac{\hat{T}'(y_i)}{1 - T'(y_i)} + \left(1 + \varepsilon^{S,w}_i\right) \phi_i \int_0^1 \Gamma_{ij}\phi_j \varepsilon^{S,r}_j \frac{\hat{T}'(y_j)}{1 - T'(y_j)} dj \right\} di \\
&+ \int_0^1 T'(y_i)\, y_i \left\{ E_i \varepsilon^{S,n}_i \frac{\hat{T}(y_i)}{(1 - T'(y_i))\, y_i} + \left(1 + \varepsilon^{S,w}_i\right) \phi_i \int_0^1 \Gamma_{ij}\phi_j \varepsilon^{S,n}_j \frac{\hat{T}(y_j)}{(1 - T'(y_j))\, y_j} dj \right\} di,
\end{aligned}$$

with $E_i \equiv (\frac{1}{\varepsilon^{S,w}_i} + 1)\phi_i - 1/\varepsilon^{S,w}_i = 1 - 1/\varepsilon^D_i$. Equation (48) implies that

$$\begin{aligned}
& -\phi_i \int_{\underline{y}}^{\bar{y}} \Gamma_{ij}\phi_j \varepsilon^{S,r}_j \frac{\hat{T}'(y_j)}{1 - T'(y_j)} dj + \phi_i \int_{\underline{y}}^{\bar{y}} \Gamma_{ij}\phi_j \varepsilon^{S,n}_j \frac{\hat{T}(y_j)}{(1 - T'(y_j))\, y_j} dj \\
= \quad & -\hat{\Omega}^E_i - \phi_i \frac{\varepsilon^{S,r}_i}{\varepsilon^D_i} \frac{\hat{T}'(y_i)}{1 - T'(y_i)} + \phi_i \left(1 + \frac{\varepsilon^{S,w}_i}{\varepsilon^D_i} + \frac{\varepsilon^{S,n}_i}{\varepsilon^D_i}\right) \frac{\hat{T}(y_i)}{(1 - T'(y_i))\, y_i}.
\end{aligned}$$

Tedious but straightforward algebra implies that the previous expression can thus be rewritten as

$$\begin{aligned}
& - \int_0^1 T'(y_i)\, y_i \left[\varepsilon^{S,r}_i \frac{\hat{T}'(y_i)}{1 - T'(y_i)} - \left(1 + \varepsilon^{S,w}_i + \varepsilon^{S,n}_i\right) \frac{\hat{T}(y_i)}{(1 - T'(y_i))\, y_i}\right] di \\
= & \int_0^1 T'(y_i)\, y_i \left[\frac{\varepsilon^D_i}{\phi_i} \mathcal{A}(y_i) + \left(1 - \varepsilon^D_i\right) \frac{\hat{T}(y_i)}{(1 - T'(y_i))\, y_i}\right] di,
\end{aligned}$$

where the second equality uses equation (49). Using the solution for $\hat{T}$ derived in (51) as a function of the auxiliary function $\mathcal{A}$, and changing variables from skills to incomes, allows us to rewrite this expression as

$$\begin{aligned}
&\int_{\underline{y}}^{\bar{y}} T'(y_i)\, y_i \left[\frac{\varepsilon_{y_i}^D}{\phi_{y_i}}\mathcal{A}(y_i) + \left(1-\varepsilon_{y_i}^D\right)\int_{y_i}^{\bar{y}} \Pi_{y_i y_j}\mathcal{A}(y_j)\, dy_j\right] f_Y(y_i)\, dy_i \\
= &\int_{\underline{y}}^{\bar{y}} \left[T'(y_i)\,\frac{\varepsilon_{y_i}^D}{\phi_{y_i}} y_i f_Y(y_i)\right]\mathcal{A}(y_i)\, dy_i + \int_{\underline{y}}^{\bar{y}}\left[\int_{\underline{y}}^{y_i} T'(y_j)\left(1-\varepsilon_{y_j}^D\right)\Pi_{y_j y_i} y_j f_Y(y_j)\, dy_j\right]\mathcal{A}(y_i)\, dy_i
\end{aligned}$$

where the second equality inverts the order of the two integrals. Finally, using (37), we can rewrite the mechanical effect of the tax reform on government revenue as

$$\begin{aligned}
\int_{\underline{y}}^{\bar{y}} \hat{T}(y_i)\, f_Y(y_i)\, dy_i &= \int_{\underline{y}}^{\bar{y}} \left(1-T'(y_i)\right) y_i \int_{y_i}^{\bar{y}} \Pi_{y_i,y_j}\mathcal{A}(y_j)\, dy_j\, f_Y(y_i)\, dy_i \\
= &\int_{\underline{y}}^{\bar{y}}\left[\int_{\underline{y}}^{y_i}\left(1-T'(y_j)\right) y_j \Pi_{y_j,y_i} f_Y(y_j)\, dy_j\right]\left\{\hat{\Omega}_{y_i}^E + \int_{\underline{y}}^{\bar{y}} \Lambda_{y_i,y_k}\hat{\Omega}_{y_k}^E\, dy_k\right\} dy_i,
\end{aligned}$$

where in the last equality used (52) and inverted the order of integrals. Collecting the terms leads to equation (40).

□