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COSTLY DEFAULT AND ASYMMETRIC REAL BUSINESS CYCLES

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ABSTRACT. We augment a simple Real Business Cycle model with financial intermediaries that may default on their liabilities and a financial friction generating social costs of default. We provide a closed-form solution for the general equilibrium of the economy under specific assumptions, allowing for analytic results and straightforward simulations. Endogenous default generates asymmetric business cycles and our model replicates both the negative skew of GDP and the positive skew of credit spreads found in US data. Stronger financial frictions cause a rise in asymmetry and amplify the welfare costs of default. A Pigouvian tax on financial intermediation mitigates most of these negative effects at the cost of a steady-state distortion.

JEL Codes: E32, E44, G21

Keywords: Real Business Cycle model, default, financial frictions, asymmetry, skewness.

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Résumé Non Technique

Les cycles économiques présentent en général une asymétrie marquée : les expansions sont caractérisées par des hausses durables mais modérées de l'emploi et du PIB, tandis que les récessions sont plus brutales et accentuées. Cette asymétrie, documentée par une large littérature empirique pour les Etats-Unis, ainsi que pour d'autres économies avancées, semble s'accentuer depuis la fin des années 80. Certains auteurs ont également démontré que l'asymétrie des fluctuations du PIB peut être renforcée par une détérioration des conditions financières.

Ce papier contribue à expliquer l'asymétrie des cycles économiques. Nous modifions une version simplifiée du modèle canonique des cycles réels (RBC, Real Business Cycle) afin d'admettre la possibilité que certains intermédiaires financiers font défaut en cas de crise. Ce cadre nous permet d'étudier le lien entre les frictions financières dans l'économie et l'asymétrie du cycle en équilibre général. Nos résultats confirment que les facteurs financiers sont une importante source d'asymétrie. Nous discutons également des liens entre le degré d'asymétrie et le bien-être d'un ménage représentatif.

D'un point de vue technique, notre modélisation du secteur financier adopte une structure dite à générations imbriquées : à chaque période, une génération d'intermédiaires financiers quitte le marché tandis qu'une nouvelle entre. Cette hypothèse donne aux intermédiaires quittant le marché la possibilité de faire défaut sur leurs obligations, ce qu'ils choisissent de faire lorsque les conditions économiques se dégradent. Le modèle intègre également une friction financière, en imposant un coût d'audit aux nouveaux intermédiaires qui entrent sur le marché en période de crise. A cause de cette friction, le stress financier causé par un épisode de défaut se propage à l'économie réelle et génère un cycle économique asymétrique.

Certaines restrictions techniques nous permettent de calculer à la main la solution du modèle, de manière à obtenir une représentation exacte de l'équilibre général. Nous obtenons trois résultats principaux.

Premièrement, notre modèle est capable de reproduire deux propriétés importantes des données : l'asymétrie négative des variables agrégées (telles que le PIB) et l'asymétrie positive des écarts de taux d'intérêt (*credit spreads*). Dans les deux cas, l'asymétrie trouve sa source dans l'amplification par les frictions financières des effets d'épisodes de défaut occasionnels. Ainsi, nous pouvons démontrer que le degré d'asymétrie du cycle dans notre modèle augmente avec la sévérité des frictions financières.

Deuxièmement, nous démontrons l'existence d'un lien entre asymétrie et bien-être. L'intuition sous-jacente est simple : une asymétrie négative du PIB implique des récessions plus fréquentes et plus sévères, ce qui pénalise la consommation moyenne des ménages et donc leur bien-être.

Troisièmement, nous montrons qu'une politique simple, basée sur l'emploi d'une taxe "pigouvienne", permet de diminuer à la fois la probabilité et la sévérité des épisodes de défaut, au prix d'une distorsion des décisions d'investissement. Cet arbitrage signifie que la politique optimale doit égaliser le gain marginal résultant de la réduction des épisodes de défaut avec la perte marginale liée à la distorsion des décisions d'investissement. Il s'ensuit notamment que la taxe optimale augmente avec la sévérité des frictions financières dans l'économie, dans la mesure où celles-ci amplifient les coûts liés aux épisodes de défaut.

1. Introduction

Business cycles are asymmetric in the United States: expansions are characterized by long-lasting but moderate increases in aggregate variables such as GDP and employment, whereas recessions correspond to sudden but substantial drops in activity.[1] This pattern, which generates negative asymmetry (or negative skewness), has been documented by a number of authors, including Neftci (1984), Hamilton (1989), and Morley and Piger (2012). It also appears to be strengthening: recent work by Jensen, Petrella, Ravn, and Santoro (2019) finds that the skewness of US business cycles has become increasingly negative since the mid-1980s. These authors suggest that financial factors, in the form of rising private-sector leverage associated with occasionally binding borrowing constraints, can account for this surge in asymmetry. In addition, Adrian, Boyarchenko, and Giannone (2019) find that the lower tail in the distribution of GDP growth is associated with periods of deteriorating financial conditions, confirming the role of financial forces in driving macroeconomic skewness.

Building on this literature, our paper studies cyclical asymmetry in a Real Business Cycle (RBC) model augmented with financial intermediaries that may default on their liabilities, as in Gertler and Karadi (2011). We use this setup to yield insights about the relationship between the strength of financial frictions and asymmetry in general equilibrium. In doing so, we substantiate the idea that financial forces have the potential to explain cyclical asymmetry. We also study the relationship between asymmetry and welfare. In particular, the financial friction generating cyclical asymmetry in our model also causes welfare losses, and we study the effectiveness of a simple regulation in overcoming these properties.

More precisely, our framework builds on a RBC model with standard households and firms and a single technology shock, which we augment with a financial sector channeling funds from saving households to borrowing firms. We adopt a particular overlapping-generations structure for the financial sector: financial intermediaries live for two periods, with an old cohort exiting the market at each period and a new one entering. This setup generates an endogenous default decision in the financial sector: old intermediaries receive state-contingent earnings but face predetermined payments, so that they choose to default in bad states of the world. To propagate financial stress to

[1]Morley and Panovska (2019) document that business cycles are asymmetric in other industrialized economies as well.

the economy, we add a financial friction in the form of sunk accounting costs paid by new intermediaries entering in such a default state. This mechanism results in an endogenous amplification of bad technology shocks, which generates business cycle asymmetry in our model. To enrich the implications of the framework, we also introduce a tax on financial intermediation, which has a Pigouvian interpretation since it helps correct the externality arising from the financial sector. Bianchi (2011), Di Tella (2019), and Jeanne and Korinek (2019) use very similar instruments as shortcuts for macro-prudential regulation.

We solve the model under the specific assumptions of log utility, a Cobb-Douglas production function, and full capital depreciation. While these restrictions may not be realistic, they allow us to provide an exact non-linear representation of the general equilibrium of our economy and to obtain analytical results characterizing the behavior of the model. We obtain the following five results.

First, we show that the financial friction in our model penalizes capital accumulation in a way that mirrors the effects in DSGE models of negative shocks to investment efficiency (Fisher, 2006; Justiniano, Primiceri, and Tambalotti, 2010, 2011; Moura, 2018) or capital quality (Gertler and Karadi, 2011; Gourio, 2012; Brunnermeier and Sannikov, 2014). This finding provides a potential structural interpretation for these disturbances; it also corroborates Justiniano, Primiceri, and Tambalotti's (2011) view that investment shocks proxy for time-varying frictions to financial intermediation.

Second, the analytical solution highlights that asymmetry in our economy originates from a non-linearity due to the default decision: the distribution of default is truncated from above at zero (no-default periods) and features a right tail of positive realizations (default periods). Through the financial friction, this positive skew of default translates into negative skew for capital, output, and consumption. This is the mechanism through which our framework reproduces the stylized fact that US business cycles are negatively skewed.

Third, we obtain a closed-form expression for the equilibrium lending-deposit spread as a function of regulation, the financial friction, and current-period default. The spread inherits the positive skew of default, which makes our model qualitatively consistent with the empirical evidence provided in Ordonez (2013). Our framework also features the positive relationship between the size of financial frictions and the interest-rate asymmetry documented by Ordonez.

Fourth, we compute the probability that old financial intermediaries default in a given period as a function of regulation, showing that tighter regulation reduces the frequency of default events in the model.

Fifth, we analytically characterize the welfare loss caused by default using an accurate approximation. We decompose the loss in two terms: the first captures steady-state costs of regulation, which acts as a wedge on capital accumulation, and the second corresponds to cyclical costs linked to default events. We show that tighter regulation worsens steady-state costs but yields cyclical gains, and we prove that the cyclical gains of regulation increases with the size of the financial friction propagating default. This demonstrates that stronger financial frictions call for tighter regulation in our model.

Based on these analytical results, we then propose various quantitative experiments. Although very stylized, the model is able to reproduce the skewness measured in US data, and an additional advantage of the closed-form solution is that the simulations involve no approximation error. We report impulse-response functions (IRFs) showing that positive and negative technology shocks trigger asymmetric responses: the economy behaves like a RBC model after a positive shock, but negative shocks push intermediaries into default and generate financial stress, which amplifies the recession. We also analyze the relationship between structural parameters and cyclical skewness in our model. In particular, we show that asymmetry in quantities increases with the strength of the financial friction, in line with the findings of Jensen, Ravn, and Santoro (2018) and Jensen, Petrella, Ravn, and Santoro (2019). Finally, we numerically compute the welfare-maximizing regulation scheme and show that it strongly reduces the occurrence of default in the economy, as well as macroeconomic asymmetry.

Our paper lies at the intersection of several strands of literature. First, it belongs to the large collection of work focusing on business cycle asymmetry. In terms of documenting the skewness of US business cycles, we can mention Potter (1995) and Bloom, Guvenen, and Salgado (2016) in addition to the papers cited above. Many authors have also proposed theoretical explanations for this asymmetry, based on increasing returns (Acemoglu and Scott, 1997), capacity constraints (Hansen and Prescott, 2005), information constraints (Jovanovic, 2006; Van Nieuwerburgh and Veldkamp, 2006), or non-linear adjustment costs related to the labor market (Abbritti and Fahr, 2013). Compared to these papers, our main novelty is to focus on financial default as a source of non-linearity. Second, we contribute to the more recent literature linking business cycle asymmetry with financial factors. In particular, our model reproduces the association between negative skew and financial stress documented by Adrian, Boyarchenko, and Giannone (2019). Compared to Jensen, Petrella, Ravn, and Santoro (2019), we offer a simpler analytical framework and we emphasize default rather than leverage constraints as the potential source of asymmetry. Third and finally, our work relates to papers studying the stabilization properties of macro-prudential regulation in general equilibrium, for instance De Walque, Pierrard, and Rouabah (2010), Angeloni and Faia (2013), and Farhi and Werning (2016). However, we adopt a slightly different perspective: while most papers focus on volatility and its interplay with financial frictions and regulation, we put more weight on the welfare gains of reducing cyclical asymmetry by limiting the size and occurrence of default tail events. In that spirit, our work also echoes Mendoza and Yue (2012), who study the welfare consequences of default in small open economies.

The paper is organized as follows. Section 2 describes the model and provides the closed-form solution. It also discusses important equilibrium properties that can be characterized analytically. Section 3 provides the numerical experiments illustrating the asymmetric behavior of the economy, including IRFs and comparative statics for skewness statistics. Finally, Section 4 turns to welfare and regulation. Section 5 concludes. To increase readability, we relegate most mathematical proofs to Appendices.

2. Model

This section introduces our model of a real economy, which includes a representative household, a representative firm, and a financial sector channeling funds between the household and the firm. There is also a government raising taxes from the financial sector. The household owns all assets in the economy. The model has three key elements: (i) financial intermediaries bear all risk and may default on their liabilities, (ii) there is a social cost of default that intermediaries do not take into account, i.e. an externality, and (iii) there is a tax on financial intermediation akin to a regulation instrument. We impose conditions that guarantee an exact analytical solution and show how to solve the model.

2.1. **Setup.** Except for a small twist related to default, the household side of the model is fairly standard. At each period, the representative household consumes an amount c_t of the final good and saves d_t in deposits issued by financial intermediaries. Thus, the problem is to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln(c_t),$$

subject to the following budget constraint

$$c_t + d_t = \left(r_{t-1}^d - \Delta_t\right) d_{t-1} + \pi_t^c + \pi_t^f + t_t.$$

Here, E_0 is the expectation operator conditional on date-0 information, $\beta \in [0, 1[$ is the subjective discount factor, r_{t-1}^d is the (predetermined) gross return on deposits, π_t^c is corporate profits, π_t^f is financial profit, and t_t is a lump-sum transfer from the government. The only unusual term in the budget constraint is Δ_t, with $\Delta_t d_{t-1}$ capturing the financial loss incurred by the household when intermediaries default on their liabilities. We provide a microfounded expression for Δ_t in equation (4) below. For now, we just define the default rate on financial liabilities as

$$\frac{\Delta_t d_{t-1}}{r_{t-1}^d d_{t-1}} = \frac{\Delta_t}{r_{t-1}^d}. \tag{1}$$

The consumption-saving plan is characterized by the Euler equation

$$1 = \beta E_t \frac{\left(r_t^d - \Delta_{t+1}\right) c_t}{c_{t+1}}.$$

The production side is also standard. The representative firm uses the k_{t-1} units of capital available at date t to produce the final good in quantity

$$y_t = \epsilon_t k_{t-1}^{\alpha},$$

where $\alpha \in]0, 1[$. Productivity evolves according to

$$\epsilon_t = \epsilon_{t-1}^{\rho} \exp(u_t),$$

with $\rho \in [0, 1[$, $u_t \sim N(\mu, \sigma^2)$, and $\sigma \geq 0$. Corporate profits are given by

$$\pi_t^c = \epsilon_t k_{t-1}^{\alpha} - r_t^k k_{t-1}$$

and the production plan verifies

$$\alpha \epsilon_t k_{t-1}^{\alpha - 1} = r_t^k.$$

To engineer endogenous default events with macroeconomic effects while preserving an exact analytical solution, our modeling of financial intermediation is more involved. We postulate an overlapping-generations structure: intermediaries live for two periods so that, at each date, an old generation exits the market and a new cohort enters. Within this 2-period framework, we rely on three mechanisms:

- First, financial intermediaries bear all risk in the economy. Formally, we assume that intermediaries pay a predetermined return on their deposit liabilities and earn a return linked to the current state of technology on their assets.[2] As a result, bad technology shocks translate into unexpected lower profits in the financial sector, given predetermined costs.
- Second, it must be possible for financial intermediaries to default endogenously in bad states of the world. Our 2-period structure makes this straightforward: old intermediaries leave the economy at the end of each period and do not internalize future costs, so that they choose to default when bad shocks generate negative profits in the financial sector. We also assume that the government seizes any positive profit in case of default, which ensures that old intermediaries default only when their profits are negative.
- Third, new intermediaries must pay a cost when entering an economy in a state with default. This additional friction, which resembles sunk auditing or accounting costs, ensures that financial stress generates social costs for the economy. Similar mechanisms can be found in, among others, Carlstrom and Fuerst (1997), Bernanke, Gertler, and Gilchrist (1999), and Malherbe (2019). Without it, default would simply reallocate household income away from deposit earnings and toward profit earnings in a lump-sum fashion, with no effect on equilibrium allocations.

Digging into the details, young financial intermediaries entering the market at date t raise an amount d_t of deposits from the household, purchase k_t units of capital, and lend these to the firm. When the economy is in a default state, that is when $\Delta_t > 0$, young intermediaries must also pay an auditing cost equal to a fraction $\phi\Delta_t$ of their balance sheet, with $\phi \geq 0$. As a result, the aggregate balance sheet of financial intermediaries at the end of period t verifies

$$(1 + \phi\Delta_t)k_t = d_t. \tag{2}$$

In the following, we call ϕ the financial friction because it determines the size of the economic costs associated with default in our model. This friction generates an externality in our model because default is decided by old intermediaries that do not to take into

[2] Models with financial frictions typically postulate predetermined deposit rates; see for instance Bernanke, Gertler, and Gilchrist (1999), Iacoviello (2005), or Gertler and Karadi (2011).

account the feedback effects on other agents, in particular on the financing cost of new intermediaries.

In addition, new intermediaries have to pay a lump-sum tax τ_{t+1}^k to the government in the next period. This tax is the policy instrument in our model and it has a direct Pigouvian interpretation, since it helps correct the externality arising from the financial sector. It has also implications similar to standard capital requirements: we show below that a higher tax leads to a higher equilibrium spread between the lending and deposit rates and to a lower probability of default, so that the tax makes it possible to limit the riskiness of the financial sector. Bianchi (2011), Di Tella (2019), and Jeanne and Korinek (2019) use a similar shortcut to represent macro-prudential regulation. We assume that the tax is rebated lump sum to households within the period, so that the government budget constraint verifies

$$t_t = \tau_t^k.$$

At date $t+1$, old intermediaries earn $r_{t+1}^k k_t$ from their assets (we assume full capital depreciation) and have to pay $r_t^d d_t$ to the household and τ_{t+1}^k to the government. Old intermediaries may choose to default on their deposit liabilities, in which case they instead transfer their pre-tax income to the household. In contrast, they cannot default *vis-à-vis* the government: this assumption is consistent with the state being a senior creditor and ensures that the policy instrument remains effective in the model.[3] As a result, the profit of old intermediaries at date $t+1$ is given by

$$\pi_{t+1}^f = \max\left(r_{t+1}^k k_t - r_t^d d_t - \tau_{t+1}^k,\ -\tau_{t+1}^k\right), \tag{3}$$

where the first argument of the max operator corresponds to the no-default case and the second argument to the default case. It is immediate that intermediaries default only when asset income is below debt servicing costs:

$$r_{t+1}^k k_t < r_t^d d_t,$$

a situation we interpret as insolvency in the financial sector. Shifting time indexes backward, the size of default at date t verifies

$$\max\left(0,\ r_{t-1}^d d_{t-1} - r_t^k k_{t-1}\right).$$

[3]Usually, the highest priority claim in liquidation goes to fees and outstanding wages, which do not appear here. The state and tax collectors come next. Remaining creditors are then ranked in a descending order of seniority.

From the household budget constraint, we know that default is also equal to $\Delta_t d_{t-1}$, which implies

$$\Delta_t = \max\left(0,\, r^d_{t-1} - r^k_t \frac{k_{t-1}}{d_{t-1}}\right). \tag{4}$$

The representative household owns financial intermediaries, so that free entry in the market for intermediaries translates into the expected zero-profit condition:

$$\beta E_t \left(\frac{c_t}{c_{t+1}} \pi^f_{t+1}\right) = 0. \tag{5}$$

Finally, we note that our default model is reminiscent of the sovereign default literature (e.g. Aguiar and Gopinath, 2006; Arellano, 2008; Mendoza and Yue, 2012). In these papers, a country chooses to default when the short-term gains of not reimbursing debt are higher than the long-term costs, typically linked to exclusion from world financial markets for a number of periods. In our model, intermediaries default when there is an immediate advantage, since they exit immediately and do not internalize future costs. In addition, in the sovereign default literature, a country going into default experiences productivity losses reflecting inefficiencies linked to financial stress. In our economy, the financial friction ϕ creates a similar mechanism and governs the general equilibrium effects of default.

2.2. **Solution.** Gathering and rearranging the equations, the equilibrium of our model is characterized by the following system:

$$(6)\quad \begin{cases} \Delta_t = \max\left(r^d_{t-1} - \dfrac{r^k_t}{1+\phi\Delta_{t-1}},\, 0\right), & \text{(6a)} \\ c_t + (1+\phi\Delta_t)k_t = \epsilon_t k^\alpha_{t-1}, & \text{(6b)} \\ \beta E_t \left[\dfrac{\left(r^d_t - \Delta_{t+1}\right) c_t}{c_{t+1}}\right] = 1, & \text{(6c)} \\ r^k_t = \alpha \epsilon_t k^{\alpha-1}_{t-1}, & \text{(6d)} \\ E_t \max\left[\beta \dfrac{c_t}{c_{t+1}} \left(r^k_{t+1} - [1+\phi\Delta_t] r^d_t\right),\, 0\right] = E_t\, \beta \dfrac{c_t}{c_{t+1}} \dfrac{\tau^k_{t+1}}{k_t}, & \text{(6e)} \\ \epsilon_t = \epsilon^\rho_{t-1} \exp(u_t),\ u_t \sim N(\mu, \sigma^2). & \text{(6f)} \end{cases}$$

These equations highlight the three mechanisms we use to engineer default. First, the expression for Δ_t in (6a) makes it clear how financial intermediaries bear all aggregate risk: at each period, their cost is given in the form of predetermined deposit rates, but their earnings respond to current productivity developments *via* the return to capital.

Second, the same equation shows that intermediates default when the return to capital, appropriately weighted, is not sufficient to cover their liability cost. Third, the financial friction $\phi \Delta_t k_t$ in the resource constraint (6b) propagates default to aggregate variables.

Equation (6e), which combines equations (3) and (5), corresponds to the zero-profit condition in the market for intermediation and defines the equilibrium deposit rate r_t^d. It shows why the tax τ_{t+1}^k can be interpreted as a regulatory instrument, since an increase in τ_{t+1}^k implies, *ceteris paribus*, a higher lending-deposit spread and a lower probability of default. Moreover, the left-hand side of equation (6e) is the expectation of a random variable with support over positive values, so that it has to be strictly positive. As a result, system (6) is well defined only when $\tau_{t+1}^k > 0$ ensures that the right-hand side is also above zero. From an economic perspective, the lending-deposit spread becomes irrelevant for financial intermediaries as $\tau_{t+1}^k \to 0$, since in that case they may propose an infinitely high deposit rate, default at each period, and still earn a non-negative profit. However, this generates huge social costs and both capital and consumption converge to zero, so that the economy collapses. Below, we assume that $\tau_{t+1}^k > 0$ to avoid this pathological equilibrium.[4]

We also note how the max operator generates asymmetric effects of positive and negative productivity shocks in our model. A surprise increase in productivity raises the marginal product of capital and the income of old financial intermediaries, which are then solvable and pay back their debt. As a result, there is no default and new intermediaries face no entry cost. On the other hand, a surprise fall in productivity may push old intermediaries into default, increasing costs for new entrants and weighing on capital accumulation.

Below, we provide an analytical solution to system (6) that preserves this non-linearity. To build that solution, we impose a specific form on the policy instrument τ_t^k that provides a factorization of the free-entry condition in the market for financial intermediation:

Assumption 1. *The policy instrument is given by*

$$\tau_t^k = \tau \, r_t^k \, k_{t-1},$$

[4]This mechanism, which allows unregulated financial intermediaries to offer unsustainable returns on their liabilities (excessive risk taking) and end up defaulting (financial collapse) with negative spillovers to the whole economy, is close to the usual narrative of the 2008 financial crisis (see, among others, Hanson, Kashyap, and Stein, 2011).

with

$$\tau \equiv \tau(A,\sigma) = \Phi\left(\frac{A}{\sigma} - \frac{\sigma}{2}\right) - \Phi\left(\frac{A}{\sigma} - \frac{3\sigma}{2}\right)\exp(\sigma^2 - A) > 0,$$

where $A \in]0,\infty[$ *if* $\sigma = 0$ *and* $A \in]-\infty,\infty[$ *if* $\sigma > 0$ *and* $\Phi(.)$ *is the cdf of the standard normal distribution.*

Assumption 1 is sufficient to obtain an exact solution. It requires the policy instrument τ_t^k to be a tax on capital income, with constant rate $\tau > 0$. To keep an exact solution, τ has to depend on the volatility parameter σ, so we introduce an additional coefficient A to index the extent of regulation: given a value of σ, the policymaker can choose a value for τ by varying A. In light of this correspondence, from now on we refer to either τ and A as the policy instrument, depending on the context. The mapping from the desired τ to the implied A has no closed form. However, we prove in Appendix B that $\tau(A,0) = 1 - 1/\exp(A)$, $\lim_{A\to-\infty}\tau(A,\sigma) = 0$, $\lim_{A\to\infty}\tau(A,\sigma) = 1$, and $\partial\tau(A,\sigma)/\partial A > 0$ when $\sigma > 0$. Figure 9 in Appendix B provides a graphical illustration of the mapping between A and τ. Finally, Assumption 1 ensures that $\tau_t^k > 0$, so that the free-entry condition in system (6) is well defined.

An obvious caveat from Assumption 1 is that defining τ as a function of σ makes it difficult to isolate the economic effects of volatility in our model. In particular, comparative statics with respect to σ entail simultaneous movement in the tax rate, which mixes the consequences of a change in the variance of the technology shock with those of a change in regulation. As a result, we refrain from considering variations in σ in what follows; instead, we only consider independent movements in the other model parameters for a given level of volatility.

We are now in position to state:

Proposition 1. *Under Assumption 1, system (6) has the closed-form solution*

$$(DEF)\quad \begin{cases} c_t = [1 - \alpha\beta(1-\tau)]\,\epsilon_t k_{t-1}^{\alpha}, & \text{(7a)} \\ k_t = \dfrac{\alpha\beta}{1+\phi\Delta_t}(1-\tau)\,\epsilon_t k_{t-1}^{\alpha}, & \text{(7b)} \\ \Delta_t = \dfrac{\alpha\epsilon_t k_{t-1}^{\alpha-1}}{1+\phi\Delta_{t-1}} \max\left[\exp\left(\mu + \dfrac{\sigma^2}{2} - u_t - A\right) - 1,\, 0\right], & \text{(7c)} \\ \epsilon_t = \epsilon_{t-1}^{\rho}\exp(u_t),\;\; u_t \sim N(\mu,\sigma^2). & \text{(7d)} \end{cases}$$

Proof. See Appendix C. □

2.3. **Equilibrium properties.** Finding a closed-form solution allows us to highlight analytically some important properties of the general equilibrium of our economy featuring default and financial frictions.

First, equation (7a) indicates that neither default nor the financial friction ϕ affect the equilibrium saving rate, given by $\alpha\beta(1-\tau)$. Indeed, we find a constant saving rate just as in the analytical RBC model. On the other hand, the saving rate is decreasing in τ, reflecting that tighter regulation weighs on capital accumulation.

Second, the law of motion of capital (7b) is changed in a way that makes our economy observationally equivalent to a RBC model with shocks to investment efficiency (Fisher, 2006; Justiniano, Primiceri, and Tambalotti, 2010, 2011; Moura, 2018) or capital quality (Gertler and Karadi, 2011; Gourio, 2012; Brunnermeier and Sannikov, 2014). Indeed, the financial friction arising from default lowers the amount of productive capital obtained from each unit of savings, which exactly mirrors the effect of negative investment efficiency shocks. We formally prove this equivalence in Appendix D, in which we also demonstrate the correspondence with capital quality shocks in our economy with full capital depreciation. Thus, our framework provides a potential micro-foundation for both investment efficiency and capital quality shocks in DSGE models. In particular, it rationalizes why these shocks proxy well for financial factors: in our setup, a negative productivity shock triggering default induces at the same time a fall in aggregate quantities, a rise in credit spreads, and a wedge that resembles investment efficiency and capital quality shocks (see Figure 1 below for impulse-response functions).

Third, equation (7c) shows that our analytical solution preserves the asymmetry of the model. The max operator truncates the equilibrium distribution of the default variable Δ_t to non-negative values, which implies a right tail and a positive skew when default is occasional. In that case, an important mass of the distribution lies at zero while the tail corresponds to positive values. In turn, the positive skew for Δ_t translates into a negative skew for capital: most of the time default is equal to zero and the transformation of investment into capital is unharmed, but bad shocks cause financial intermediaries to default and the associated social costs generate abnormally low capital realizations in the left tail of the distribution. Finally, given the log-linear production function and decision rule for consumption, both output and consumption (in logs) inherit the negative skew of capital. It follows that our model is able to reproduce the negative skewness of aggregate macroeconomic time series documented by the literature initiated by Neftci (1984), while maintaining the assumption of symmetric Gaussian productivity shocks.[5]

Fourth, the model also generates asymmetry in the spread between the lending and deposit rates. We show in Appendix C that the equilibrium spread between the expected return on credit and on deposits between t and $t+1$ verifies

$$s_t \equiv \frac{E_t r_{t+1}^k}{r_t^d} = \exp(A)(1+\phi\Delta_t). \tag{8}$$

The spread increases in the default variable Δ_t when there are financial frictions ($\phi > 0$), reflecting higher entry costs facing new intermediaries in bad states of the world. The relationship is linear, so that s_t inherits the asymmetry of Δ_t: credit spreads have a positive skew when defaults are rare events. Accordingly, our model is consistent with the positive skewness of spreads in both advanced and emerging economies found by Ordonez (2013). Since the strength of the link between s_t and Δ_t depends on ϕ, our framework is also consistent with the positive relationship between the extent of financial frictions and interest rate asymmetry that Ordonez finds in the data.

Fifth, the probability that default occurs at any given date can be computed as

$$\begin{aligned} \Pr[default] &= \Pr\left[r_{t+1}^k k_t - r_t^d d_t < 0\right] = \Pr\left[\exp(u_t) < \exp(\mu + \sigma^2/2 - A)\right] \\ &= \Pr\left[\exp(u_t) < E\exp(u_t - A)\right] = \Phi\left(\frac{\sigma}{2} - \frac{A}{\sigma}\right). \end{aligned} \tag{9}$$

This expression follows from the definition of the equilibrium spread (see Appendix C). Thanks to our 2-period overlapping-generations structure, the probability of default depends on neither current nor past economic conditions, which is key for analytical tractability. In addition, the probability of default decreases with A, that is with τ according to Assumption 1: tighter regulation lowers the occurrence of default events in our model. More precisely, when $\sigma > 0$ and $A \to -\infty$, then $\tau \to 0$ and $\Pr[default] \to 1$, meaning that intermediaries always default when they are not regulated. When $A = 0$, τ is slightly positive (for instance $\tau = 1.9\%$ when $\sigma = 0.05$) and $\Pr[default] \approx 50\%$. When $A \to \infty$, $\tau \to 1$ and default never happens. Finally, for $A > 0$ and in the limit case of $\sigma \to 0$, $\Pr[default] \to 0$, implying that intermediaries never default when the economy is deterministic.

[5] Altug, Ashley, and Patterson (1999) find no evidence of non-linearity in the Solow residual in the US economy.

Sixth, we can use equation (7c) and the balance sheet identity (2) to compute the amount defaulted by old intermediaries in each period as

$$\Delta_t d_{t-1} = \alpha \epsilon_t k_{t-1}^{\alpha} \max \left[\exp \left(\mu + \frac{\sigma^2}{2} - u_t - A \right) - 1, \, 0 \right]. \tag{10}$$

The terms on the right-hand side show that in our model the size of default depends on two features. The first is the scale of the economy, as measured by current production $\epsilon_t k_{t-1}^{\alpha}$, with larger economies being prone to potentially larger defaults. The second is the surprise in current productivity developments, as measured by the difference between the innovation u_t and its expected value augmented by A (which directly maps into the regulation instrument τ). Because of the max operator, only negative productivity surprises induce default; positive surprises instead trigger an unexpected rise in financial profits.

Finally, equations (8), (9), and (10) clarify how regulation affects the equilibrium of our model. In particular, tighter regulation through higher values of A (and τ) is associated with larger credit spreads that increase profits in the financial sector (equation (8)). As a result, the probability of default falls (equation (9)) and defaults are of limited value when they occur (equation (10)). These channels make the regulation tradeoff transparent in our model: on the one hand, regulation lowers the probability and the severity of default events, which is beneficial for the economy; on the other hand, this comes at the cost of larger credit spreads that distort capital accumulation and weigh on the economy. Obviously, the strength of the financial friction ϕ is crucial to determine whether tightening the regulation entails more advantages than drawbacks in a given economy.

Overall, our model combines two wedges that distort capital accumulation. The first reflects the social costs of default: it originates from the financial friction ϕ and generates business-cycle asymmetry. The second wedge arises from regulation: it captures the effect of the tax rate τ on the equilibrium consumption-saving tradeoff and does not generate asymmetry in itself. However, these two wedges are not independent because tighter regulation lowers the frequency and average size of default events in the model. Below, we analyze the interactions between these wedges in more detail.

2.4. **Central planner benchmark.** In the rest of the paper, we explore some equilibrium properties of our non-linear model (DEF), with a focus on asymmetry, welfare, and regulation. We need a benchmark for these analyses, and we take it to be the optimal allocation chosen by a benevolent central planner maximizing $E_0 \sum_{t=0}^{\infty} \beta^t \ln(c_t)$ subject to the resource constraint $c_t + k_t = \epsilon_t k_{t-1}^{\alpha}$. It is straightforward to show:

Proposition 2. *The central planner allocation verifies (McCallum, 1988)*

$$(CP) \quad \begin{cases} k_t = \alpha\beta\epsilon_t k_{t-1}^{\alpha}, \\ c_t = (1-\alpha\beta)\epsilon_t k_{t-1}^{\alpha}, \\ \epsilon_t = \epsilon_{t-1}^{\rho} \exp(u_t), \;\; u_t \sim N(\mu, \sigma^2). \end{cases}$$

Since we impose $\tau_t^k > 0$, Model (DEF) always features an inefficiency distorting capital accumulation. As a result, it is not possible to obtain the efficient allocation as an equilibrium outcome. However, the equilibrium allocation in Model (DEF) becomes arbitrarily close to the efficient outcome when there is no financial friction ($\phi = 0$) and when the regulation distortion vanishes ($\tau \to 0$).

3. Financial Frictions and Business Cycle Asymmetry

This section analyses how the interplay of endogenous default and financial frictions generates business cycle asymmetry in our model. We proceed in three steps. First, we parametrize the model. Second, we provide IRFs highlighting the asymmetric effects of positive and negative technology shocks in our setup, as well as the amplification arising from default and financial frictions. Third, we assess the role of key parameters — the financial friction ϕ and regulation τ — in shaping the model asymmetry.

3.1. **Parametrization.** Our assumption of full capital depreciation makes it difficult to come up with a proper calibration strategy, so instead we follow a different road and propose an illustrative parametrization.

We partition the model parameters in two sets. The first contains parameters for which it is relatively easy to pick reference values from the literature: these include the Cobb-Douglas exponent, the subjective discount factor, and the persistence of the technology process, which we set at $\alpha = 0.33$, $\beta = 0.97$, and $\rho = 0.90$.

The second set contains parameters either specific to our economy or that significantly affect the asymmetry. For this group of parameters, we choose values that generate relatively large effects in order to help identify the workings of the model. More precisely, we set the financial friction at $\phi = 4$ and we assume that the standard deviation of technology shocks is $\sigma = 0.05$, 5 times higher than the usual value of 0.01 in quarterly

TABLE 1. Parametrization

Parameter	Symbol	Value
Discount factor	β	0.97
Cobb-Douglas exponent	α	0.33
Technology shock: persistence	ρ	0.90
Technology shock: standard deviation	σ	0.05
Financial friction	ϕ	4.00
Regulation parameter	A	0.0013
Implied tax rate	τ	0.0194

models. Thus, our parametrization roughly targets an annual frequency, which is consistent with the value we choose for β. Finally, we adjust the regulation parameter A to obtain a 50% probability of default at each date, which yields $A = \sigma^2/2 = 0.0013$ (see equation (9)). The implied value of the tax rate is $\tau = 1.94\%$. One advantage of this particular choice is that negative technology shocks, however small, will push financial intermediaries into default.[6] Our parametrization also implies an average GDP cost of default of 2%, well below the 4.5% annual GDP cost of the 2007-2011 US financial crisis estimated by Laeven and Valencia (2018).[7] Table 1 reports the value assigned to each parameter.

This parametrization makes our model roughly consistent with the level of asymmetry measured in US data.[8] For instance, the skewness of log GDP was -0.24 between 1953 and 2018, and -0.44 between 1980 and 2018. These estimates suggest that the asymmetry of US GDP has been increasing in recent years, in line with the findings of Jensen, Petrella, Ravn, and Santoro (2019). When parametrized according to the entries in Table 1, our model implies the intermediary value of -0.35, well in the range supported

[6]Default would only occur for sufficiently negative shocks if the probability of default was below 50%, while default could occur even in presence of positive shocks if the probability was above 50%.

[7]In the model, the social cost depends on the *average size* of default $E(\Delta_t k_t)$ rather than just on the frequency of default. In that spirit, our setup is close to the "continuous default rate" model of Goodhart, Sunirand, and Tsomocos (2005) in which banks default at each period but in varying degrees.

[8]We use time series extracted from the FRED database. Output is annual real GDP in chained 2012 dollars (GDPC1), while the credit spread is the yearly average of Moody's seasoned Baa corporate bond yield relative to the yield on 10-year treasury bonds (BAA10YM). We remove the long-run trend of GDP using the HP filter with smoothing parameter 100, the standard value for annual series.

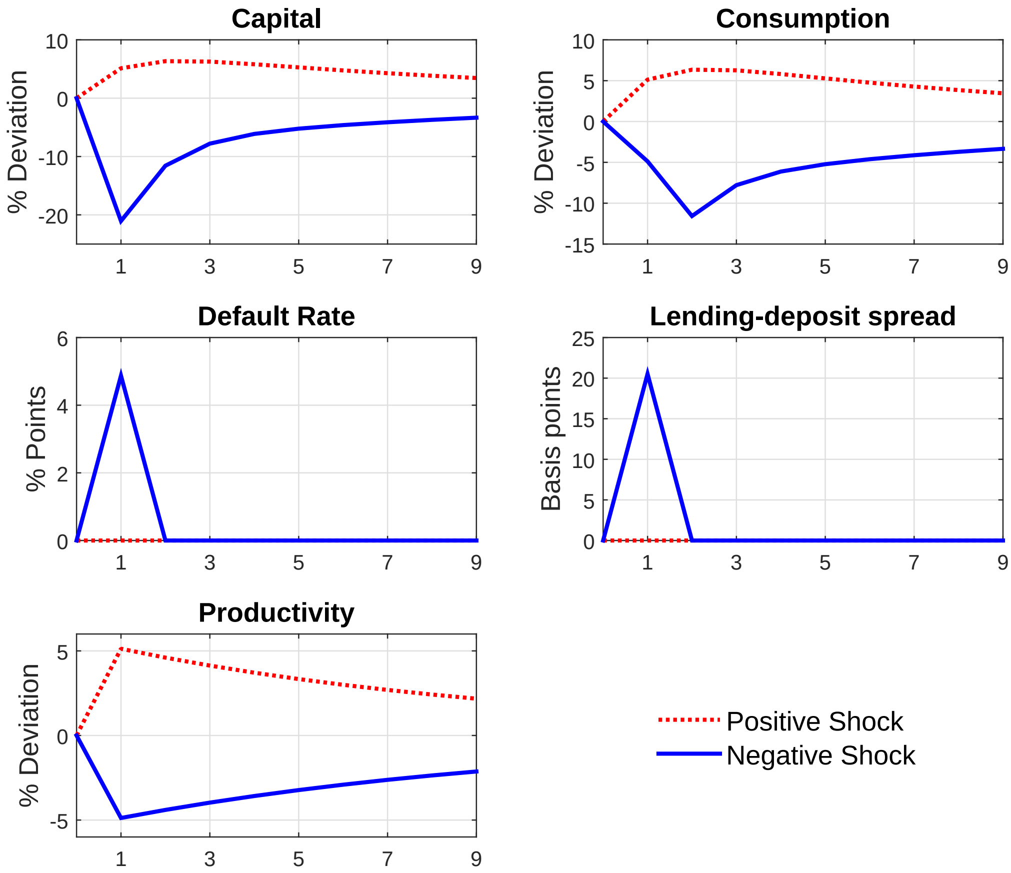
by the data. As for the credit spread, it has a positive skewness of 0.38 between 1953 and 2018, and of 0.82 in the more recent 1980-2018 period. Our parametrization overshoots these values, as it generates a skew of 1.59 for the spread. However, it is qualitatively consistent with spreads having a positive skew in the data.

3.2. **Asymmetric effects of technology shocks.** Based on this parametrization, we compute the equilibrium path of the model following positive and negative technology shocks. We choose the deterministic steady state as the initial condition, so that the level of default was always zero in the past, and we hit the economy with a one-time, one-standard-deviation technology shock, either positive or negative. We report the resulting IRFs in Figure 1. Equation (7a) demonstrates that consumption and output have identical dynamics in our model, and we report the response of consumption only.

The dashed red lines represent the dynamic effects induced by the positive shock. These are pretty standard. Productivity increases by 5% on impact and then gradually returns to its long-run level. Higher productivity leads to an immediate rise in production of similar magnitude, which is absorbed by increased levels of consumption and investment. The additional units of capital available for future production, together with the persistence of the technology shock, slightly amplifies the economy's response in the short run and generates hump-shaped dynamics in capital, consumption, and output. There is no default and the lending-deposit spread is constant. As a result, our model displays exactly the same movements as the central planner benchmark.

The solid blue lines represent the economy's response to the negative technology shock. There is a clear asymmetry compared to the effects of the positive shock, as well as significant amplification. The unexpected fall in productivity lowers the return to capital and the income of financial intermediaries. As a result, intermediaries go into default: the default rate on financial liabilities almost reaches 5%. Default occurs only when the shock hits the economy, since there is no surprise afterward. However, the effects are long lasting. Through the financial friction, default entails a large social cost that weighs on investment, which drops by 20% on impact — four times more than productivity. Consumption also falls immediately, though this only reflects lower productivity (recall that the equilibrium saving rate is constant). Finally, the financial friction also causes a 20-point increase in the lending-deposit spread. At future dates, there are negative spillovers due to the lower capital levels and consumption reaches a trough of -10% two periods after the shock. The economy then gradually returns to its steady state.

FIGURE 1. Asymmetric effects of technology shocks.



Notes. IRFs to one-standard-deviation positive and negative technology shocks, starting from the deterministic steady state of the model. The default rate is defined as $\Delta_t d_{t-1}/(r^d_{t-1} d_{t-1}) = \Delta_t / r^d_{t-1}$ and the spread is in deviation from its deterministic steady-state value $\exp(A)$. See Table 1 for the parametrization.

These IRFs highlight two key properties of our model. First, positive and negative technology shocks generate asymmetric responses from the endogenous variables because of the non-linearity of default. Second, the financial friction provides an amplification mechanism for negative shocks, both on impact and in later periods. All amplification originates from the social cost associated with the financial friction: as discussed in Section 2.1, setting $\phi = 0$ would eliminate both the asymmetry and the amplification related to default.

3.3. **Forces shaping asymmetry.** We now explore in more detail how selected parameters contribute to the asymmetric behavior of our the model. We focus on the financial friction ϕ and regulation τ: the first determines the strength of the negative spillovers from default, while the second governs the probability of default (recall equation (9)).[9] We conduct various experiments simulating the model, changing one parameter at a time and evaluating asymmetry using skewness statistics based on artificial samples of $500,000$ periods. Throughout, we study the log of the capital stock, which maps directly into output given the production structure while avoiding issues related to the treatment of default costs that would arise if we studied GDP instead. We also consider the equilibrium lending-deposit spread. Finally, we note that the central planner benchmark yields symmetric distributions for all variables, with zero skewness for all parameter values.

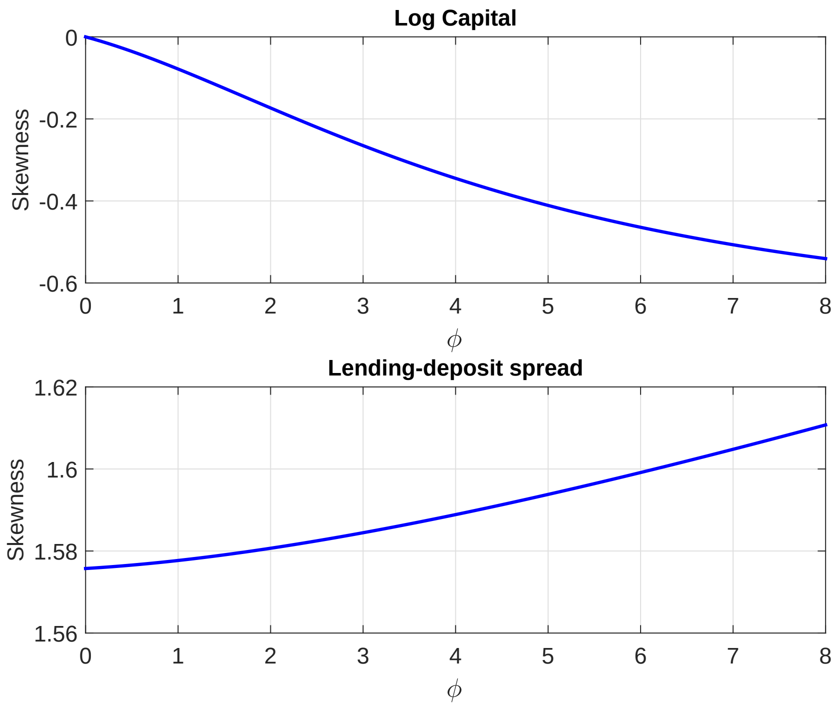
3.3.1. *Financial friction ϕ.* We start by varying the strength of the financial friction in the model, i.e. parameter ϕ. Greater friction leaves the probability of default unchanged but associates default events with larger social costs that weigh more on capital accumulation. As a result, we expect higher values of ϕ to correspond to a more negatively skewed distribution for equilibrium log capital, and a more positively skewed distribution for the lending-deposit spread.

Figure 2 corroborates these insights. The top panel shows a decreasing relationship between ϕ and the skewness of capital in our model. When $\phi = 0$, default arises once every two periods on average but it does not distort the equilibrium distribution of log capital, which is symmetric just as in the central planner benchmark. In that case, the model has no friction able to propagate financial stress to aggregate quantities and default is just a lump-sum transfer. As ϕ rises, stronger friction makes default more costly and the distribution of capital has an increasingly negative skewness. The intuition is straightforward: higher values of ϕ amplify the economy's response to bad technology shocks without affecting the response to good shocks, which induces a longer left tail in the distribution of aggregate quantities.

The bottom panel indicates that the skewness of the lending-deposit spread increases slightly with ϕ. This is consistent with equation (8), which shows that larger social costs translate into higher equilibrium credit spreads. The positive relationship between the skew of credit spreads and the strength of financial frictions is also in line with the empirical findings reported in Ordonez (2013). There is an interesting discontinuity at $\phi = 0$. Without financial frictions, the spread is not affected by default and remains

[9] As explained above, we do not consider experiments varying volatility σ because of its simultaneous effect on regulation τ.

FIGURE 2. Relationship between financial friction ϕ and asymmetry.



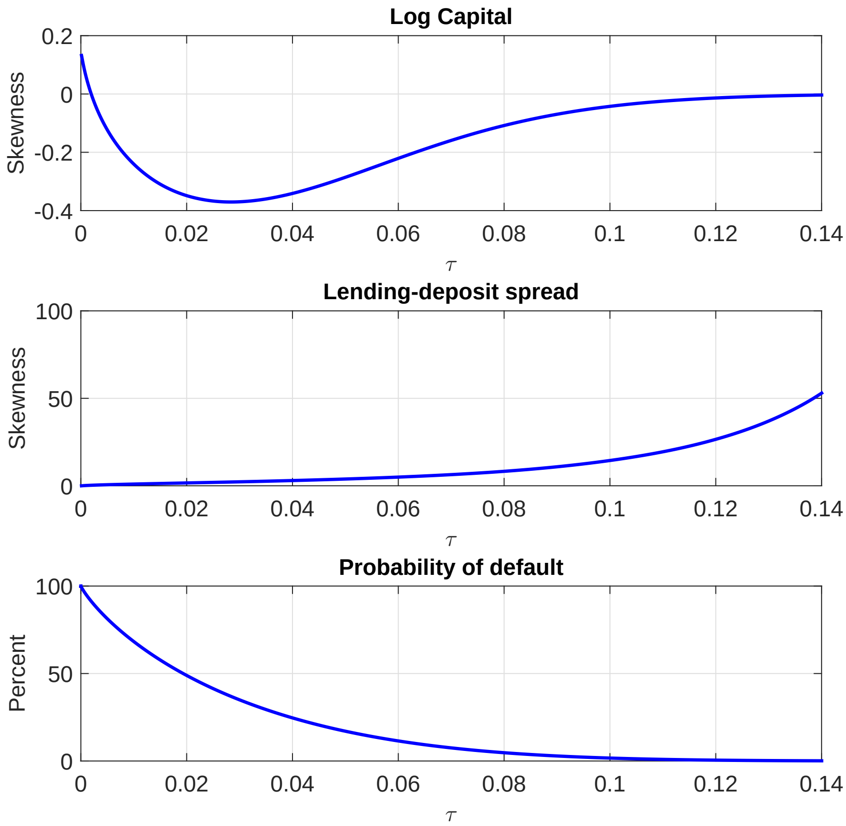
Notes. The figure shows the skewness of log capital and the credit spread as a function of the financial friction ϕ, with other model parameters fixed at the values described in Table 1. Statistics computed on samples with $500,000$ observations.

constant; as a result, its skew is not defined even though default Δ_t has a positive skew. For any $\phi > 0$, however small, the spread inherits the asymmetry of Δ_t and the financial friction just acts as an amplifying factor.

3.3.2. *Regulation* τ. We now consider the effect of regulation on asymmetry. From a practical perspective, we vary parameter A in Assumption 1 but we report the results as functions of the tax-like composite parameter τ to ease interpretation. Equation (9) shows that tighter regulation reduces the equilibrium probability of default at any volatility level. As a result, one could expect a negative effect of τ on asymmetry: as regulation increases, defaults occur less often and particularly low realizations of log capital are scarcer, as are high realizations of the lending-deposit spread.

Figure 3 shows that the relationship between regulation and asymmetry is more complicated in our model. The top panel indicates a U-like relationship between τ and the skew of log capital: asymmetry is close to zero when regulation is tight, increases

FIGURE 3. Relationship between regulation τ and asymmetry.

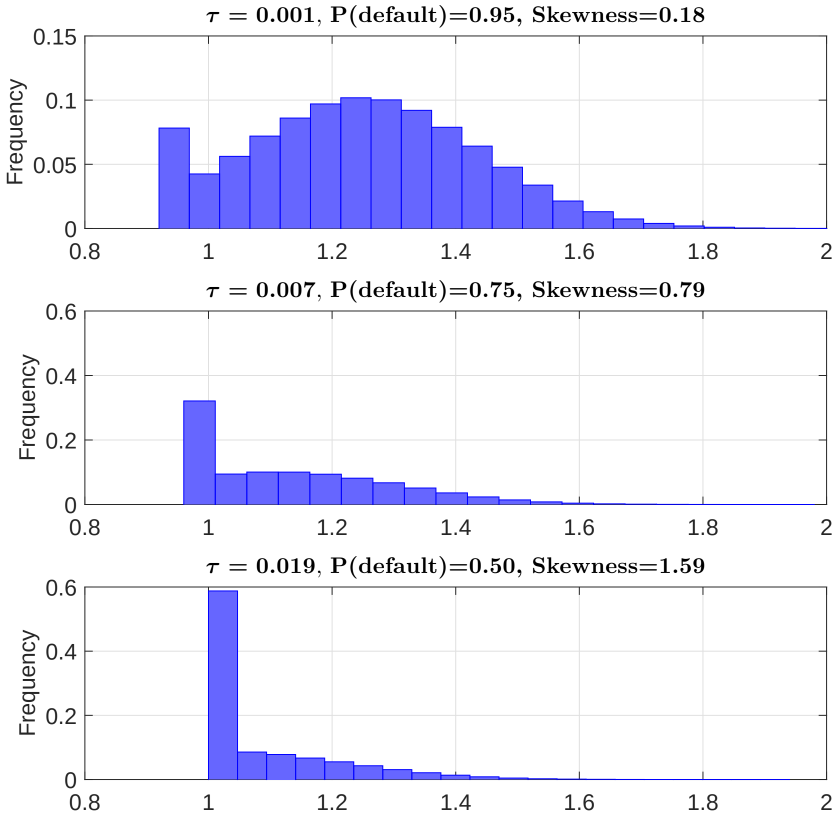


Notes. The figure shows the skewness of log capital and the credit spread as a function of regulation τ, with other model parameters fixed at the values described in Table 1. Statistics computed on samples with $500,000$ observations.

progressively as τ decreases, before flattening out and eventually falling as regulation becomes very lenient. On the contrary, the central panel displays an increasing relationship between the skew of the spread and the tightness of regulation.

The behavior of capital asymmetry is best understood by reading the chart from right to left, that is starting from tight regulation and moving toward less regulated economies. When τ is high, regulation is strong enough to prevent default in equilibrium. As a result, the probability of default is essentially zero and log capital has a symmetric distribution. Relaxing regulation by lowering τ entails an increase in the frequency of default, so that social costs weigh on intermediation more often and generate lower realizations of capital. The skewness of log capital falls toward more negative values, signaling that asymmetry increases. As regulation becomes lenient, the probability of default increases: social costs are incurred more often, so that the corresponding low values of capital become more

FIGURE 4. Distribution of credit spreads for different regulation τ.



Notes. The figure shows the distribution of credit spreads for different values of regulation τ. Statistics computed on samples with $500,000$ observations.

of a norm and less of a tail realization. This explains why skewness statistics recover when τ falls enough: there is initially a reduction in negative skew as the left tail of the distribution is compressed, which eventually turns into positive skew as default becomes the norm and rare non-default periods generate a right tail instead.

The behavior of credit spreads follows from equations (8) and (9), which show that the probability of default declines as regulation increases and that the lending-deposit spread is constant when financial intermediaries do not default. Hence, tightening regulation concentrates the distribution of credit spreads around the no-default constant value, leaving a thinner right tail of larger spreads. We illustrate this concentration pattern in Figure 4, which reports the distribution of credit spreads for increasing values of τ. In the top panel, regulation is lax and default occurs almost every period, so that the distribution of spreads has a wide support. In the lower panels, regulation becomes tighter and the no-default threshold absorbs more of the distribution as the right tail flattens. As Figure 3 shows, this specific form of asymmetry leads to large skewness statistics for credit spreads.

4. Welfare and Regulation

So far, we have proposed a positive analysis of how frictions give rise to cyclical asymmetry in our model. In this final section, we take a different perspective: frictions generate inefficiencies and yield a suboptimal equilibrium allocation, which justifies a normative analysis. Building on our analytical solution, we provide an exact expression for welfare and use it to study the costs of default. To push the computations further, we introduce an approximation that preserves the non-linearity of the max operator. Finally, we resort to numerical simulations to characterize the welfare-maximizing regulation policy and its effects on welfare and asymmetry.[10]

4.1. **Analytical results.** As in Lester, Pries, and Sims (2014), we focus on unconditional welfare as measured by the average value function of the representative household. Using the results from Propositions 1 and 2, straightforward algebra demonstrates that, in Models (CP) and (DEF), welfare verifies

$$\begin{aligned}(1-\alpha)(1-\beta)\mathcal{W}_{CP} &= (1-\alpha)\ln(1-\alpha\beta)+\alpha\ln(\alpha\beta)+\frac{\mu}{1-\rho}\\(1-\alpha)(1-\beta)\mathcal{W}_{DEF} &= (1-\alpha)(1-\beta)\mathcal{W}_{CP}\\&\quad+(1-\alpha)\ln\left[\frac{1-\alpha\beta(1-\tau)}{1-\alpha\beta}\right]+\alpha\ln(1-\tau)-\alpha E\ln(1+\phi\Delta_t),\end{aligned}$$

where E is the unconditional expectation operator. It follows that the welfare difference between Models (DEF) and (CP) is given by

$$(1-\alpha)(1-\beta)\left(\mathcal{W}_{DEF}-\mathcal{W}_{CP}\right)=-\Big[\underbrace{g(\tau)}_{\text{steady-state cost}}+\underbrace{\alpha E\ln(1+\phi\Delta_t)}_{\text{cyclical cost}}\Big], \tag{11}$$

where

$$g(\tau)=(1-\alpha)\ln\left[\frac{1-\alpha\beta}{1-\alpha\beta(1-\tau)}\right]-\alpha\ln(1-\tau).$$

We show in Appendix E that $g(\tau)>0$ for $\tau\in\ [0,1[$.

The next proposition summarizes the welfare ranking between the central planner benchmark and the economy with default:

[10] In the following, we refer to this welfare-maximizing policy as the optimal policy, keeping implicit that it is only constrained-optimal and does not restore the efficient central-planner allocation.

Proposition 3. *For all $\sigma \geq 0$ and $\tau(A, \sigma) \in]0, 1[$, we have*

$$\mathcal{W}_{DEF} < \mathcal{W}_{CP}.$$

In addition, $\mathcal{W}_{DEF} \to \mathcal{W}_{CP}$ when $\sigma = 0$ and $\tau \to 0$, and $\mathcal{W}_{DEF} \to \mathcal{W}_{CP}$ when $\phi = 0$ and $\tau \to 0$.

Proof. See Appendix E. □

Proposition 3 shows that welfare is always lower in the model with default. Equation (11) decomposes this welfare loss into two sources. First, there is a cost $g(\tau)$ representing the distortion to capital accumulation induced by regulation. We refer to it as the steady-state cost because tighter regulation reduces the average levels of GDP, consumption, and capital even in a deterministic economy without default. Second, there is a cost $\alpha E \ln(1 + \phi\Delta_t)$ linked to default events. It is strictly positive when uncertainty generates occasional default events ($\sigma > 0$) and when default is amplified by financial frictions ($\phi > 0$). In particular, this cost encapsulates the negative welfare consequences of second- and third-order moments induced by uncertainty, which explains why we refer to it as the cyclical cost. In the limiting cases in which default does not occur ($\sigma = 0$) or does not propagate ($\phi = 0$), only the regulation cost matters and the economy converges to the central planner allocation when $\tau \to 0$. Obviously, this discussion neglects the key point that tighter regulation also lowers the probability and size of default, as well as the related social costs. We discuss this point below.

The literature about financial regulation typically finds that tighter regulation yields long-term gains by lowering default risks, at the expense of short-run losses representing the economic costs of, e.g., increasing capital requirements (Van den Heuvel, 2008; Clerc, Derviz, Mendicino, Moyen, Nikolov, Stracca, Suarez, and Vardoulakis, 2015; Mendicino, Nikolov, Suarez, and Supera, 2018). Our model, which displays no transitional dynamics after a change in τ, follows a different logic. In particular, both the costs of tighter regulation and the welfare gains materialize immediately when τ increases: on the one hand, financial intermediation becomes at once more costly; on the other hand, it also becomes safer as the probability of default falls. Thus, the trade-off is not between a short-term cost and a long-term gain, but between a cost and a gain that will be experienced today and at all future dates.

It is difficult to obtain further analytical results about welfare because the cost $E \ln(1+\phi\Delta_t)$ in equation (11) cannot be explicitly written in terms of structural parameters.

Still, we can make some progress at the cost of a slight approximation described in the following:

Assumption 2. *The equilibrium is such that*

(A1) $\ln(1+\phi\Delta_t) \approx \phi\Delta_t$*;*

(A2) $\Delta_t \Delta_{t-1} \approx 0$*;*

(A3) $\epsilon_{t-1}^{\rho} k_{t-1}^{\alpha-1}$ *is accurately approximated by a first-order Taylor expansion around the deterministic steady state;*

(A4) $\phi < 1/\theta$*, where*

$$\theta = \frac{\exp\left(\sigma^2/2\right)\left[\exp(-A)\Phi\left(\sigma/2 - A/\sigma\right) - \Phi\left(-\sigma/2 - A/\sigma\right)\right]}{\beta(1-\tau)} > 0.$$

Assumption 2 essentially requires that the economy is not too volatile around its deterministic steady state, so that the linearizations involved in ($A1$) and ($A3$) remain accurate; that defaults are not too large, so that ($A2$) holds; and that the financial friction is not too strong, so that ($A4$) is verified. At the same time, the assumption preserves the non-linearity of the max operator and thus the asymmetry due to default. We show in Appendix F that the resulting approximation error is small for a wide range of parameter values.

Also in Appendix F, we demonstrate that under Assumption 2 the cyclical cost in equation (11) simplifies to

$$\alpha E \ln(1+\phi\Delta_t) \approx \frac{\alpha\phi\theta}{1-\phi\theta},$$

where ($A4$) from Assumption 2 ensures that the right-hand side is positive. We can then show:

Proposition 4. *Under Assumptions 1 and 2,*

$$\frac{\partial E \ln(1+\phi\Delta_t)}{\partial\phi} > 0, \quad \frac{\partial E \ln(1+\phi\Delta_t)}{\partial\tau} < 0, \quad \frac{\partial^2 E \ln(1+\phi\Delta_t)}{\partial\tau\partial\phi} < 0.$$

Proof. See Appendix G. □

Given equation (11), Proposition 4 makes three statements valid in the vicinity of the deterministic steady state of our model.

First, higher financial frictions ϕ deteriorate welfare in Model (DEF), since they amplify the cyclical cost. This is not surprising, as stronger frictions make default events more costly to the economy.

Second, there is a clear tradeoff related to regulation. On the one hand, tighter regulation impairs welfare through $g(\tau)$, since $\partial g(\tau)/\partial \tau > 0$. This is a steady-state cost that reflects the distortion to capital accumulation induced by regulation. On the other hand, tighter regulation improves welfare by mitigating the social costs of default, as can be seen from the negative response of $E \ln(1 + \phi\Delta_t)$ to an increase in τ. This is a cyclical effect that captures the lower frequency and smaller size of default events in a regulated economy.

Third, the cyclical effect is more important when financial frictions are high, as shown by the cross-partial derivative of $E \ln(1 + \phi\Delta_t)$ with respect to ϕ and τ. Defining the optimal policy as the one balancing positive and negative effects on welfare, our analytic argument makes it clear that higher financial frictions justify tighter regulation: the cyclical cost reduction from increasing τ is larger, while the steady-state cost is unchanged.

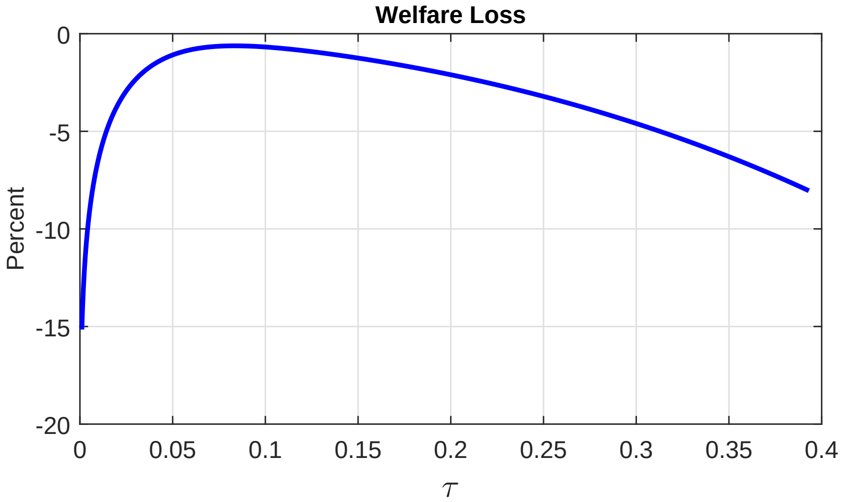
4.2. **Numerical results.** Finally, we document the properties of optimal regulation in our model using numerical analyses. As before, we base the simulations on artificial samples with $500,000$ periods.

In our benchmark parametrization, we find that the welfare function is concave in the regulation instrument τ.[11] This is clear from Figure 5, which reports the welfare losses resulting from varying τ (more precisely A) while keeping other model parameters constant at the values in Table 1. Concavity of welfare essentially follows from the two costs apparent in equation (11). For low values of τ, regulation is lenient: as a result, the steady-state distortion is small while the cyclical costs associated with default are important, so that there are welfare gains from tightening regulation. In contrast, when τ is high regulation is tight so default occurs rarely: in this case, the distortions on capital accumulation are stronger and deregulation improves welfare.

Because of concavity, there exists an interior value of τ that maximizes welfare in Model (DEF). In our baseline parametrization, this optimal regulation instrument corresponds to an (annual) tax rate of $\tau^\star = 8.28\%$. More generally, optimal regulation balances the welfare effects of marginally raising or lowering the tax rate: at the optimal level $\tau^\star$, the cyclical welfare benefit of limiting the negative consequences of default by raising the tax rate is equal to the steady-state welfare cost of larger capital distortions.

[11] We checked numerically that welfare being concave in τ is a robust implication of our model by varying the parameters ϕ and σ.

FIGURE 5. Welfare as a function of the tax instrument τ.



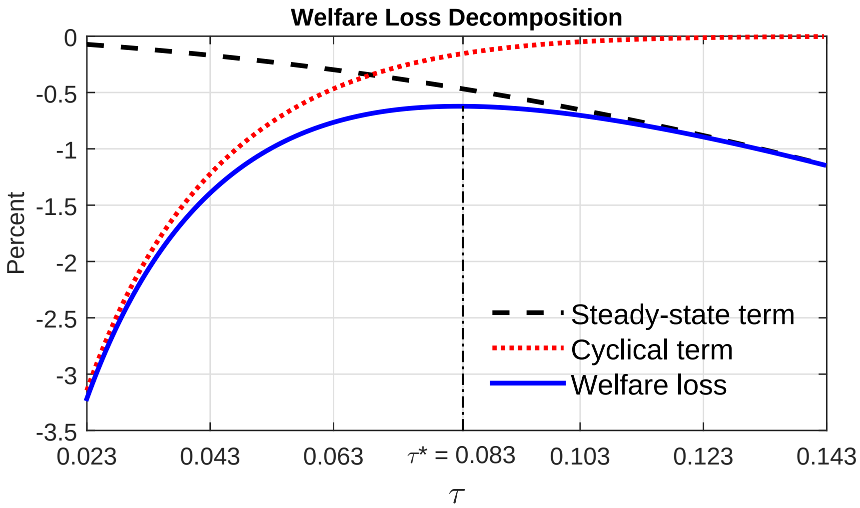
Notes. The figure shows the welfare loss resulting from varying the regulation instrument τ, with other model parameters fixed at the values described in Table 1. Welfare losses are expressed as percent deviations from the central planner benchmark W^{CP}. Statistics computed on samples with $500,000$ observations.

This indifference condition is apparent in Figure 6, which decomposes the welfare function around $\tau^{\star}$: the slopes of the steady-state and cyclical terms are equal at the optimal regulation. The chart also confirms the (local) analytical insight that optimal regulation is tighter when financial frictions are larger: a rise in ϕ amplifies cyclical costs and leaves steady-state costs unchanged, shifting the dotted red line downward and calling for a higher tax rate to restore the slope equality between the two cost functions.

Figure 7 provides more insights about how optimal regulation responds to stronger financial frictions in our model. On each chart, the solid blue line indicates the equilibrium outcome under the optimal regulation given the value of the friction parameter ϕ, whereas the dashed red line corresponds to the outcome observed under the benchmark parametrization described in Table 1. Hence, comparison between the two lines shed light on the effects of regulation in our model.

The top-left panel shows the optimal regulation tax $\tau^{\star}$ as a function of the friction ϕ, keeping all other parameters fixed. The chart confirms the existence of an increasing relationship: optimal regulation is close to zero when financial frictions are very small (i.e. when default entails virtually no cyclical cost), and then increases smoothly with ϕ. The relationship is also concave, implying that it is optimal to react less than proportionately

FIGURE 6. Forces shaping optimal regulation.



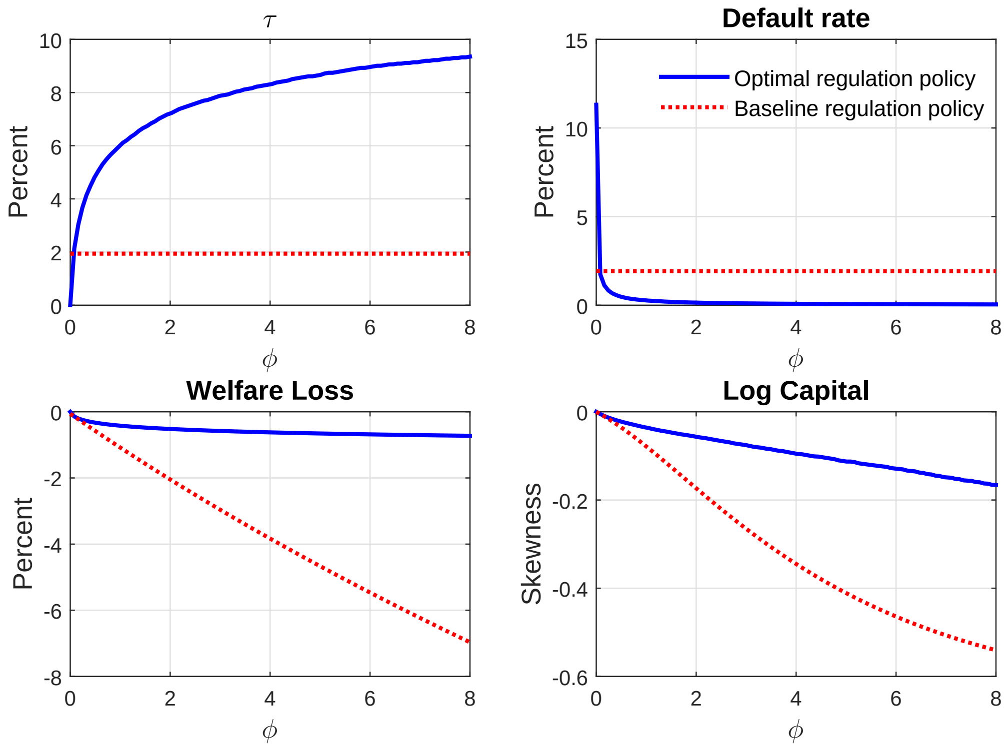
Notes. The figure shows the welfare loss resulting from varying the regulation instrument τ around its optimal value $\tau^\star$, with other model parameters fixed at the values described in Table 1. The dashed and dotted lines provide the decomposition of welfare into the steady-state and cyclical costs defined in equation (11). Welfare losses are expressed as percent deviations from the central planner benchmark W^{CP}. Statistics computed on samples with $500,000$ observations.

to increases in financial frictions. This property is explained by the behavior of the average equilibrium default rate shown in the top-right panel of the figure: average default is very responsive to the initial tightening in regulation, but then stabilizes slightly above zero. Since tightening regulation limits cyclical costs by reducing the average size of default, the gains from additional increases in τ are smaller and smaller.

The remaining two charts in Figure 7 indicate the gains from optimal regulation in terms of both welfare and cyclical asymmetry. Two results stand out. First, the optimal policy is able to contain the welfare losses from increasing financial frictions. For instance, equilibrium welfare is 4% below the central planner benchmark when $\phi = 4$ and the tax is calibrated as in Table 1, while the loss is well below 1% under optimal regulation. When $\phi = 8$, the loss is still below 1% when regulation is optimal, compared to 7% under the baseline policy. Second, the optimal policy limits the equilibrium asymmetry in capital, and thus also in GDP and consumption.

To better understand the relationship between asymmetry and welfare, Figure 8 plots the welfare loss as a function of the skewness of log capital for different values of ϕ, under

FIGURE 7. Gains from optimal regulation.

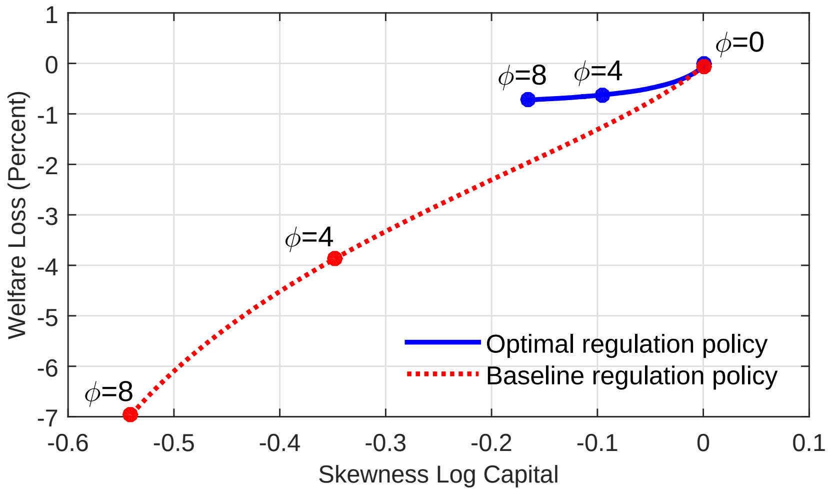


Notes. The blue lines show the optimal tax, the default rate, the welfare loss, and the skewness of log capital as functions of the financial friction ϕ when regulation is set optimally. The red lines correspond to the case in which regulation is fixed at the level calibrated in Table 1. Welfare losses are expressed as percent deviations from the central planner benchmark W^{CP} and the default rate is Δ_t/r^d_{t-1}. Statistics computed on samples with $500,000$ observations.

both the optimal regulation policy (blue line) and the baseline policy (red line). In both cases, we observe that a rise in asymmetry (more negative skewness) is associated with greater welfare losses. In addition, for any given level of financial frictions, moving from the baseline policy to the optimal policy limits asymmetry and improves welfare.

These results indicate that the optimal regulation goes a long way toward limiting the aggregate effects of default and financial frictions in our model. They also suggest a direct link between welfare and asymmetry, as limiting welfare losses requires compressing the left tail of the equilibrium distribution of log capital, GDP, and consumption. In our framework, this link arises through default, which generates both negative skew in quantities and cyclical welfare costs.

FIGURE 8. Reduced-form relationship between asymmetry and welfare.



Notes. The blue line show the relationship between the skewness of log capital and the welfare loss when ϕ changes under the optimal regulation, while the red line corresponds to the case in which regulation is fixed at the level calibrated in Table 1. Welfare losses are expressed as percent deviations from the central planner benchmark W^{CP}. Statistics computed on samples with $500,000$ observations.

5. Conclusion

This paper develops a Real Business Cycle model with endogenous default and financial regulation. We prove analytically that: (i) financial frictions mirror the effect of a negative shock to capital accumulation; (ii) endogenous default generates asymmetric business cycles; (iii) tighter regulation decreases steady-state consumption but lowers the probability of default, which may generate welfare gains. We illustrate these theoretical results through various quantitative experiments. In particular, we show that the size of financial frictions amplifies business-cycle asymmetry and that skewed business cycles are associated with welfare losses.

We see at least four interesting extensions of our stylized framework. First, considering partial capital depreciation would allow the equilibrium saving rate to vary over time and potentially depend on the level of financial frictions. Second, and in the same vein, an endogenous labor supply would highlight interactions between financial frictions and the labor market, especially during crises. Third, we could take an extended version of the model with partial depreciation and endogenous labor to the data to check whether it is able to reproduce observed asymmetries. Fourth and finally, the variance of productivity shocks deserves more attention, since volatility affects the decision rule for capital accumulation through the cost of default.

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Appendix A. Equivalence between Tax and Capital Requirements

In this appendix, we show that regulatory policies implemented through taxes or capital requirements on financial intermediaries produce equivalent effects. Our argument largely mirrors that of Bianchi (2011).

Consider a partial-equilibrium version of the model from Section 2.1. Furthermore, assume that there is no financial friction ($\phi = 0$). The financial intermediary borrows d and lends k, subject to the balance-sheet constraint $k = d$. It earns $r^k k$ from assets and pays $r^d d$ on liabilities. Default happens when $r^k k < r^d d$, that is with probability

$$\Pr[default] = \Pr\left[\frac{r^k}{r^d} < 1\right],$$

where we used the balance-sheet constraint to simplify quantities. Independently of default, the financial intermediary must pay a tax $\tau r^k k$ proportional to capital income, where $\tau > 0$ is a tax rate. The free-entry condition for financial intermediation is therefore

$$E \max[r^k k - r^d d,\, 0] = \tau r^k k,$$

which can be simplified to

$$E \max\left[1 - \frac{r^d}{r^k},\, 0\right] = \tau.$$

It follows immediately that raising the tax rate τ increases the equilibrium lending-deposit spread r^k/r^d, which in turn reduces the probability of default $\Pr[r^k/r^d < 1]$.

Now, consider a similar economy in which capital requirements replace taxes: the financial intermediary must finance at least a fraction $\gamma \in [0,1[$ of the loans it issues with equity e, i.e. $e \geq \gamma k$. Assume that raising equity is more costly than raising deposits, for instance because deposits yield a liquidity service to the household. Other possible justifications include the outcome of moral-hazard problems or tax disadvantages on equity (Bianchi, 2011). Since the return on equity is higher than the return on deposits, the equity constraint is always binding and $e = \gamma k$. The intermediary's balance-sheet becomes $k = d + \gamma k$, equivalently $(1-\gamma)k = d$. Default still occurs whenever $r^k k < r^d d$, that is with probability

$$\Pr[default] = \Pr\left[\frac{r^k}{(1-\gamma) r^d} < 1\right].$$

For simplicity, we take the cost of equity to be given by $(1+\eta) r^d \gamma k$, with $\eta > 0$ being the additional cost of equity relative to deposits. The free-entry condition in the market for intermediation is then

$$E \max[r^k k - r^d d,\, 0] = (1+\eta) r^d \gamma k,$$

which can be simplified to

$$E \max\left[\frac{r^k}{(1-\gamma) r^d} - 1,\, 0\right] = (1+\eta)\frac{\gamma}{1-\gamma}.$$

Hence, raising the capital adequacy ratio γ increases the credit spread r^k/r^d, which ends up reducing the probability of default $\Pr[r^k/((1-\gamma)r^d) < 1]$. It follows that a regulator could use either of a tax policy or a capital requirement to reduce the probability of financial default in this model.

Appendix B. Properties of τ

Consider first the case of $\sigma = 0$. Then $A \leq 0$ implies $\tau(A, 0) = 0$, which we exclude since we need $\tau_t^k > 0$. We therefore impose $A \in]0, \infty[$ when $\sigma = 0$. Under this constraint, we have $\tau(A, 0) = 1 - 1/\exp(A)$, which is increasing in A. Moreover, $\lim_{A\to 0} \tau(A, 0) = 0$ and $\lim_{A\to\infty} \tau(A, 0) = 1$.

In the more general case of $\sigma > 0$, we do not need to restrict the support of A, which belongs to $] - \infty, \infty[$. The derivative of the tax rate with respect to A is

$$\frac{\partial \tau(A, \sigma)}{\partial A} = \exp(\sigma^2 - A)\Phi\left(\frac{A}{\sigma} - \frac{3\sigma}{2}\right) > 0.$$

Using l'Hospital rule to deal with an indeterminate form, we find that $\lim_{A\to-\infty} \tau(A, \sigma) = 0$. Furthermore, $\lim_{A\to\infty} \tau(A, \sigma) = 1$ is evident.

Figure 9 provides a graphical illustration of the mapping between τ and A for both $\sigma > 0$ and $\sigma = 0$.

Appendix C. Proof of Proposition 1

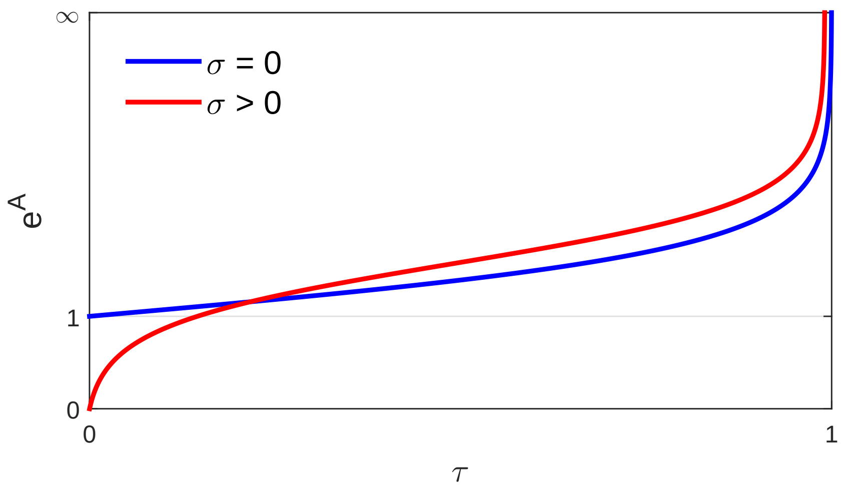
We use a guess-and-verify approach. Suppose that the policy function for consumption verifies

$$c_t = \Gamma \epsilon_t k_{t-1}^{\alpha},$$

where $\Gamma \geq 0$ is an unknown coefficient. Using this guess, equation (6d) and Assumption 1, the free-entry equation (6e) can be written as

$$E_t \max\left[\alpha \epsilon_t^{\rho} k_t^{\alpha-1} - \frac{r_t^d (1 + \phi\Delta_t)}{\exp(u_{t+1})},\, 0\right] = \tau \alpha \epsilon_t^{\rho} k_t^{\alpha-1}.$$

FIGURE 9. Mapping between the desired tax rate τ and A given σ.



Notes. These curves directly result from Assumption 1.

To simplify the notation, define $\mu_{1,t} = \alpha\epsilon_t^{\rho}k_t^{\alpha-1}$ and $\mu_{2,t} = r_t^d(1+\phi\Delta_t)$. Remark that both $\mu_{1,t}$ and $\mu_{2,t}$ are known as of date t, so that the only source of uncertainty is u_{t+1}. Knowing that $u \sim N(\mu, \sigma^2)$ and using $f(\cdot)$ to denote its pdf., the above equation is equivalent to

$$\int_{\ln\frac{\mu_{2,t}}{\mu_{1,t}}}^{\infty} f(u)du - \frac{\mu_{2,t}}{\mu_{1,t}}\int_{\ln\frac{\mu_{2,t}}{\mu_{1,t}}}^{\infty} \exp(-u)f(u)du = \tau.$$

After some algebra, this can be expressed as

$$\Phi\left(\frac{\mu - \ln\frac{\mu_{2,t}}{\mu_{1,t}}}{\sigma}\right) - \frac{\mu_{2,t}}{\mu_{1,t}}\exp\left(-\mu + \frac{\sigma^2}{2}\right)\Phi\left(\frac{\mu - \sigma^2 - \ln\frac{\mu_{2,t}}{\mu_{1,t}}}{\sigma}\right) = \tau.$$

A solution to this equation is

$$\mu_{1,t} = \kappa\,\mu_{2,t}, \tag{12}$$

where $\kappa > 0$ must verify

$$\Phi\left(\frac{\mu + \ln\kappa}{\sigma}\right) - \frac{1}{\kappa}\exp\left(-\mu + \frac{\sigma^2}{2}\right)\Phi\left(\frac{\mu - \sigma^2 + \ln\kappa}{\sigma}\right) = \tau.$$

It is impossible to find a closed-form expression $\kappa = \kappa(\mu, \sigma, \tau)$ in the general case. However, we can impose Assumption 1 requiring that

$$\tau = \Phi\left(\frac{A}{\sigma} - \frac{\sigma}{2}\right) - \Phi\left(\frac{A}{\sigma} - \frac{3\sigma}{2}\right)\exp(\sigma^2 - A),$$

with $A \in]0, \infty[$ if $\sigma = 0$ and $A \in]-\infty, \infty[$ if $\sigma > 0$. Then it turns out that

$$\kappa = \exp\left(-\mu - \frac{\sigma^2}{2} + A\right),$$

solves the equation.

Plugging κ, $\mu_{1,t}$ and $\mu_{2,t}$ into equation (12), we obtain

$$E_t r_{t+1}^k = \exp\left(\mu + \frac{\sigma^2}{2}\right) \alpha \epsilon_t^\rho k_t^{\alpha-1} = \exp(A) r_t^d (1 + \phi\Delta_t).$$

We use this relationship between the marginal product of capital and the deposit rate to simplify equations (6a) and (6c) into

$$\begin{aligned} \Delta_t &= \frac{\alpha \epsilon_{t-1}^\rho k_{t-1}^{\alpha-1}}{1 + \phi\Delta_{t-1}} \max\left[0, \exp\left(\mu + \frac{\sigma^2}{2} - u_t - A\right) - 1\right], \\ \frac{1}{\epsilon_t k_{t-1}^\alpha} &= \frac{\alpha\beta}{k_t(1 + \phi\Delta_t)} E_t \min\left[\exp\left(\mu + \frac{\sigma^2}{2} - u_{t+1} - A\right), 1\right]. \end{aligned} \tag{13}$$

Define $l_t = E_t \min[\exp(\mu + \sigma^2/2 - u_{t+1} - A), 1]$. Then,

$$\begin{aligned} l_t &= \Phi\left(\frac{\sigma}{2} - \frac{A}{\sigma}\right) + \exp\left(\mu + \frac{\sigma^2}{2} - A\right) \int_{\mu + \frac{\sigma^2}{2} - A}^{\infty} \exp(-u) f(u) du \\ &= \Phi\left(\frac{\sigma}{2} - \frac{A}{\sigma}\right) + \exp\left(\sigma^2 - A\right) \left[1 - \Phi\left(\frac{3\sigma}{2} - \frac{A}{\sigma}\right)\right] \\ &= 1 - \tau. \end{aligned}$$

Inserting this expression into the Euler equation (13) yields

$$k_t = \frac{\alpha\beta}{1 + \phi\Delta_t} (1 - \tau) \epsilon_t k_{t-1}^\alpha.$$

Merging this equation with the resource constraint (6b), we obtain

$$c_t = [1 - \alpha\beta(1 - \tau)] \, \epsilon_t k_{t-1}^\alpha.$$

This validates our initial guess for the consumption policy function, whose unknown coefficient verifies

$$\Gamma = [1 - \alpha\beta(1 - \tau)].$$

Appendix D. Equivalence with Investment Efficiency Shocks and Capital Quality Shocks

This appendix shows the observational equivalence between our set-up with default and financial frictions and a model with shocks to the efficiency of investment. We also demonstrate the correspondence with capital quality shock when capital fully depreciates. Throughout, we abstract from the tax rate τ without loss of generality.

An influential strand of the literature argues that investment shocks, which affect the transformation of private savings into productive capital, play a prominent role in US business cycles (see, among others, Greenwood, Hercowitz, and Krusell, 2000; Fisher, 2006; Justiniano, Primiceri, and Tambalotti, 2010, 2011; Moura, 2018). In addition, Justiniano, Primiceri, and Tambalotti (2011) show that shocks to investment efficiency proxy for financial disturbances in DSGE models, an insight that our framework corroborate.

We introduce an investment efficiency shock into the central planner model from Section 2.4; see Fernandez-Villaverde and Rubio-Ramirez (2007) for a very similar set-up. Household preferences and firm technology remain unchanged, but the aggregate resource constraint becomes

$$k_t = z_t(y_t - c_t), \tag{14}$$

where $z_t \in]0, 1]$ is the investment shock. If $z_t = 1$ at all periods, we recover the central planner economy in which savings are fully transformed into productive capital. Here, we instead assume that, although z_t equals 1 at the deterministic steady state, it may occasionally be below than 1. In that case, a contraction in the efficiency of investment lowers the amount of productive capital obtained out of savings, with negative consequences on aggregate production.

The model has a simple solution. At each period, the consumption-saving plan is characterized by the Euler equation

$$\frac{1}{z_t c_t} = \alpha\beta E_t \frac{y_{t+1}}{k_t c_{t+1}}.$$

Using the aggregate resource constraint, this is also

$$\frac{y_t}{c_t} = 1 + \alpha\beta E_t \frac{y_{t+1}}{c_{t+1}}.$$

Since $\alpha\beta < 1$, substituting forward and imposing the transversality condition yields

$$c_t = (1 - \alpha\beta)y_t, \tag{15}$$

so that the equilibrium saving rate does not depend on the investment shock z_t. This is not the case of capital accumulation, which is given by

$$k_t = z_t \alpha \beta \epsilon_t k_{t-1}^{\alpha}. \tag{16}$$

It is immediate that equations (15) and (16) are equivalent to equations (7a) and (7b) from Proposition 1 when

$$z_t = \frac{1}{1+\phi\Delta_t} \in]0,1].$$

It follows that our model with default and financial frictions provides a micro-foundation for investment efficiency shocks. More precisely, a negative investment shock in the above model is observationally equivalent to the negative externality arising from endogenous default in Model (DEF).

Several papers mimic the aggregate effects of financial crisis using disturbances to capital quality (see for instance Gertler and Karadi, 2011; Gertler, Kiyotaki, and Queralto, 2012, and many others). To model these shocks, we slightly modify the central planner economy from Section 2.4 to introduce incomplete capital depreciation at rate $\delta \in]0,1]$ and a capital quality shock $\psi_t \in]0,1]$. Define in-process capital s_t as the sum of after-depreciation productive capital $(1-\delta)k_{t-1}$ and investment i_t:

$$s_t = (1-\delta)k_{t-1} + i_t,$$

and assume that in-process capital is transformed into next-period productive capital after the realization of a multiplicative capital quality shock:

$$k_t = \psi_t s_t.$$

Merging these equations, the aggregate resource constraint becomes

$$y_t = c_t + i_t = c_t + s_t - (1-\delta)k_{t-1} = c_t + \frac{k_t}{\psi_t} - (1-\delta)k_{t-1}.$$

In the special case of $\delta = 1$, this simplifies into

$$k_t = \psi_t(y_t - c_t),$$

which is equivalent to the resource constraint (14) from the model with investment efficiency shocks. It follows that the model solution is given by equations (15) and (16), in which the capital quality shock ψ_t simply replaces the investment efficiency shock z_t. Hence, in an economy with full capital depreciation a capital quality shock is also observationally equivalent to the negative externality arising from endogenous default in Model (DEF).

Appendix E. Proof of Proposition 3

Define

$$g(\tau) = (1-\alpha)\ln\left[\frac{1-\alpha\beta}{1-\alpha\beta(1-\tau)}\right] - \alpha\ln(1-\tau),$$

with $\tau \in]0,1[$ according to Assumption 1. Since $\lim_{\tau\to 0} g(\tau) = 0$, $\lim_{\tau\to 1} g(\tau) = \infty$, and $\partial g(\tau)/\partial\tau > 0$, we have $g(\tau) > 0$. Moreover, $\Delta_t \geq 0$ by definition, so that $\phi \geq 0$ implies $E\ln(1+\phi\Delta_t) \geq 0$. Together, these restrictions prove the first part of the proposition.

When $\sigma = 0$, $\Delta_t = 0$ from equation (7c) and $\mathcal{W}_{DEF} \to \mathcal{W}_{CP}$ when $\tau \to 0$. This proves the second part of the proposition.

When $\phi = 0$, $\ln(1+\phi\Delta_t) = 0$ and $\mathcal{W}_{DEF} \to \mathcal{W}_{CP}$ when $\tau \to 0$. This proves the last part of the proposition.

Appendix F. Welfare Approximation

This appendix proves the welfare approximations from Section 4.1.

Replacing ϵ_t by its expression (7d) in equation (7c) gives

$$\Delta_t(1+\phi\Delta_{t-1}) = \alpha\epsilon_{t-1}^{\rho} k_{t-1}^{\alpha-1} \max\left[\exp\left(\mu + \frac{\sigma^2}{2} - A\right) - \exp(u_t),\ 0\right].$$

Under simplification ($A2$) from Assumption 2, taking the unconditional expectation of both sides of the equality and applying the Law of Iterated Expectations gives

$$E\Delta_t \approx E\left\{\alpha\epsilon_{t-1}^{\rho} k_{t-1}^{\alpha-1} E_{t-1}\max\left[\exp\left(\mu + \frac{\sigma^2}{2} - A\right) - \exp(u_t),\ 0\right]\right\}.$$

The conditional expectation is

$$\int_{-\infty}^{\mu+\sigma^2/2-A}\left[\exp\left(\mu + \frac{\sigma^2}{2} - A\right) - \exp(u)\right] f(u)du = \exp\left(\mu + \frac{\sigma^2}{2}\right) h(\sigma, A),$$

where we define

$$h(\sigma, A) = \exp(-A)\Phi\left(\frac{\sigma}{2} - \frac{A}{\sigma}\right) - \Phi\left(-\frac{\sigma}{2} - \frac{A}{\sigma}\right).$$

It easy to show that $\lim_{A\to-\infty} h(\sigma, A) = \infty$ and $\lim_{A\to\infty} h(\sigma, A) = 0$. Moreover, the partial derivative verifies

$$\frac{\partial h(\sigma, A)}{\partial A} = -\exp(-A)\Phi\left(\frac{\sigma}{2} - \frac{A}{\sigma}\right) < 0.$$

Together with the limits as $A \to \pm\infty$, this implies $h(\sigma, A) > 0$. Overall, the expected value of the default term is thus

$$E\Delta_t = \alpha \exp\left(\mu + \frac{\sigma^2}{2}\right) h(\sigma, A) E\left(\epsilon_{t-1}^{\rho} k_{t-1}^{\alpha-1}\right). \tag{17}$$

To obtain an analytical expression for the last term, we use simplification $(A3)$ from Assumption 2 and take a log-linear approximation around the non-stochastic steady state of the model. This gives

$$E\left(\epsilon_{t-1}^{\rho} k_{t-1}^{\alpha-1}\right) \approx \bar{\epsilon}^{\rho} \bar{k}^{\alpha-1} E\left[1 + \rho\left(\ln \epsilon_{t-1} - \ln \bar{\epsilon}\right) + (\alpha - 1)\left(\ln k_{t-1} - \ln \bar{k}\right)\right],$$

where upper bars denote non-stochastic steady-state levels. From equations (7b) and (7d), we obtain

$$\begin{aligned}
\bar{\epsilon} &= \exp(\mu)^{\frac{1}{1-\rho}}, \\
\bar{k} &= \left[\alpha\beta(1-\tau)\bar{\epsilon}\right]^{\frac{1}{1-\alpha}}, \\
E \ln \epsilon_t &= \frac{\mu}{1-\rho}, \\
(1-\alpha) E \ln k_t &= \ln[\alpha\beta(1-\tau)] + \frac{\mu}{1-\rho} - E \ln(1 + \phi\Delta_t).
\end{aligned}$$

Finally, simplification $(A1)$ from Assumption 2 allows to write the last equation as

$$(1-\alpha) E \ln k_t \approx \ln[\alpha\beta(1-\tau)] + \frac{\mu}{1-\rho} - \phi E\Delta_t.$$

It follows that

$$E\left[\epsilon_{t-1}^{\rho} k_{t-1}^{\alpha-1}\right] = \frac{1 + \phi E\Delta_t}{\alpha\beta(1-\tau)\exp(\mu)}. \tag{18}$$

Consolidating equations (18) and (17) then yields

$$E\Delta_t = \frac{\theta}{1 - \phi\theta}, \quad \text{with } \theta = \exp\left(\frac{\sigma^2}{2}\right) \frac{h(\sigma, A)}{\beta(1-\tau)}.$$

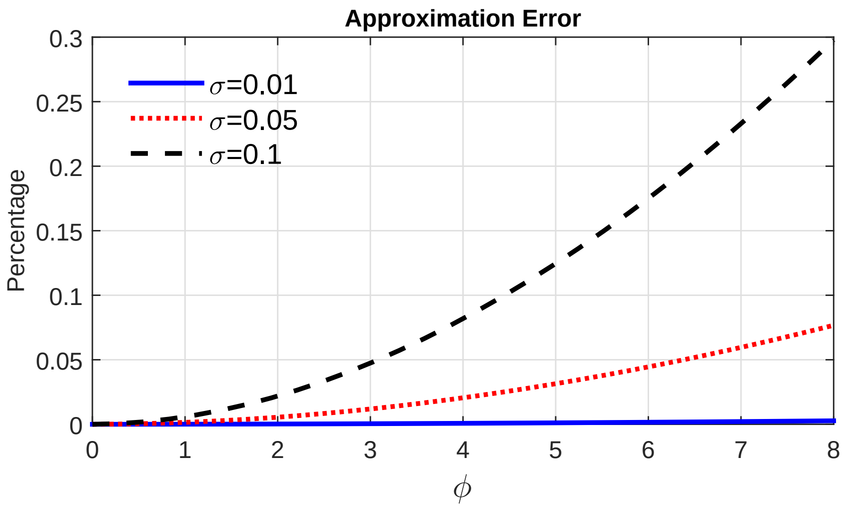
It is clear that $h(\sigma, A) > 0$ implies $\theta > 0$. Equation $(A4)$ from Assumption 2 then implies that $E\Delta_t > 0$, which is consistent with default having a non-negative support.

Finally, these computations yield an analytical expression for the last term in welfare $\mathcal{W}_{DEF}$: relying once more on simplification $(A1)$ from Assumption 2, we have

$$E \ln(1 + \phi\Delta_t) \approx \phi E\Delta_t = \frac{\phi\theta}{1 - \phi\theta} \geq 0.$$

Figure 10 shows that the approximation error resulting from Assumption 2 is small for a wide range of parameter. We have also verified that $(A4)$ holds for all parameter configurations used in this figure.

FIGURE 10. Approximation error due to Assumption 2.



Notes. The figure shows the approximation error in welfare computations induced by Assumption 2. It reports, for different (ϕ, σ) combinations, the absolute value of the ratio $(\widetilde{\mathcal{W}}_{DEF} - \mathcal{W}_{DEF})/\mathcal{W}_{DEF}$, where $\widetilde{\mathcal{W}}_{DEF}$ is the analytical welfare approximation and $\mathcal{W}_{DEF}$ is the exact model welfare. Statistics computed on samples with $500,000$ observations. τ is kept constant at the value reported in Table1 in all simulations.

APPENDIX G. PROOF OF PROPOSITION 4

The sign of the first partial derivative is immediate since $\partial E \ln(1+\phi\Delta_t)/\partial\phi = \theta/(1-\phi\theta)^2 > 0$.

To prove the sign of the second partial derivative, we know from Appendix B that $\partial\tau/\partial A > 0$. Therefore, it is equivalent to prove $\partial E \ln(1+\phi\Delta_t)/\partial\tau < 0$ or $\partial E \ln(1+\phi\Delta_t)/\partial A < 0$. Using the properties of τ and $h(\sigma, A)$ derived in Appendices B and F, we obtain

$$\frac{\partial\theta}{\partial A} = -\exp\left(\frac{\sigma^2}{2}\right)\frac{(1-\tau)\Phi\left(-\sigma/2 - A/\sigma\right) + h(A,\sigma)\Phi\left(\sigma/2 - A/\sigma\right)}{\beta(1-\tau)^2} < 0,$$

which implies in turn

$$\frac{\partial E \ln(1+\phi\Delta_t)}{\partial A} = \frac{\partial\theta/\partial A}{(1-\phi\theta)^2} < 0.$$

This proves the second result.

Finally, it is equivalent to prove the sign of the last partial derivative with respect to τ or A:

$$\text{sign}\left(\frac{\partial^2 E \ln(1+\phi\Delta_t)}{\partial\tau\partial\phi}\right) = \text{sign}\left(\frac{\partial^2 E \ln(1+\phi\Delta_t)}{\partial A\partial\phi}\right).$$

Then, the above results as well as (A4) in Assumption 2 imply

$$\frac{\partial^2 E \ln(1+\phi\Delta_t)}{\partial A \partial \phi} = \frac{\partial}{\partial A}\left[\frac{\theta}{(1-\phi\theta)^2}\right] = \frac{(1-\phi\theta)(1+\phi\theta)}{(1-\phi\theta)^4}\,\frac{\partial \theta}{\partial A} < 0.$$

Since all partial derivatives are themselves differentiable, Schwarz's theorem (see, e.g., Rudin, 1976, for details) implies

$$\frac{\partial^2 E \ln(1+\phi\Delta_t)}{\partial \phi \partial \tau} < 0.$$