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"Regulating Insurance Markets: Multiple Contracting and Adverse Selection"

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Regulating Insurance Markets: Multiple Contracting and Adverse Selection*

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Abstract

This paper studies an insurance market on which privately informed consumers can simultaneously trade with several firms operating under a regulation that prohibits cross-subsidies between contracts. The regulated game supports a single equilibrium allocation in which each layer of coverage is fairly priced given the consumer types who purchase it. This competitive allocation cannot be Pareto-improved by a social planner who observes neither consumers' types nor their trades with firms. Public intervention under multiple contracting and adverse selection should thus arguably target firms' pricing strategies, leaving consumers free to choose their preferred amount of coverage.

Keywords: Insurance Markets, Regulation, Multiple Contracting, Adverse Selection.
JEL Classification: D43, D82, D86.

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1 Introduction

Multiple contracting, whereby consumers purchase several policies from different public or private insurers to cover the same risk, is a widespread phenomenon in insurance markets. A case in point is the US life-insurance market, on which around 25% of consumers hold more than one term policy.[1] A similar phenomenon arises in annuity markets: for instance, the six million annuities in payment in the UK in 2013 were owned by about five millions individuals.[2] Most health-insurance markets also exhibit multiple contracting in ways that depend on the relative importance of the public and private insurance sectors:[3] in the US, the Medicare supplementary market enables 10 out of the 42 million consumers covered by Medicare to opt for Medigap plans issued by private firms; similarly, retirees can complement Medicare benefits with employer-sponsored retiree health plans.[4]

Since the early works of Arrow (1963), Akerlof (1970), Pauly (1974), and Rothschild and Stiglitz (1976), there has been a concern that these markets may be exposed to adverse selection, whereby high-risk consumers purchase more coverage than low-risk consumers, increasing the riskiness of the insured pool and thus firms' costs. Adverse selection can severely hinder private insurance provision, potentially causing a market breakdown.[5] As a result, the recent decades have witnessed many proposals for public intervention aimed at improving the efficiency of insurance markets under adverse selection. Yet few, if any, of these proposals explicitly take into account the implications of multiple contracting. The present paper addresses this issue.

To this end, we consider an insurance economy in which each consumer can purchase coverage from several firms to cover an idiosyncratic income loss. There are two types of consumers with different loss distributions and possibly different risk attitudes. The main restrictions are a single-crossing condition and a monotonicity condition on loss distributions

[1] A term life-insurance policy provides coverage for a limited period of time, which makes it a pure insurance product. Evidence about subscribers of such policies is provided by He (2009) on the basis of the Health and Retirement Study (HRS) panel.

[2] See the 2014 UK Insurance Key Facts Document issued by the Association of British Insurers, available at https://www.abi.org.uk/∼/media/Files/Documents/Publications/Public/2014/Key%20Facts/ABI%20Key%20Facts%202014.pdf.

[3] Private health insurance can be used as a source of basic coverage for consumers who choose not to obtain public health insurance. This is the case in Germany, the Netherlands, and Switzerland, where more than half of the population hold more than one policy (Paccagnella, Rebba, and Weber (2013)). Private insurance can also be used to fund healthcare needs that are already partially covered by public funds. This is the case in Australia, Denmark, and, in particular, France, where around 95% of the population complement public mandatory coverage with private coverage (Mossialos, Wenzl, Osborn, and Sarnak (2016)).

[4] The income from employment-based pension schemes is currently relevant for about half of the US retirees, see Poterba (2014).

[5] Consistent with this, Hendren (2013) finds that private information imposes large barriers to trade in life-insurance, disability, and long-term-care markets.

that together imply adverse selection, in the sense that consumers willing to make larger purchases of coverage are also more likely to incur large losses and are thus more costly to serve. Firms issue coinsurance contracts covering a fraction of the loss in exchange for an insurance premium. Unlike in Rothschild and Stiglitz (1976), these contracts are nonexclusive, so that consumers are free to combine contracts issued by different firms, and no firm can monitor the trades a consumer makes with its competitors.

In this context, the benchmark is provided by a more restrictive notion of constrained efficiency than the standard notion of informationally constrained efficient, or *second-best*, allocation, defined as an allocation that cannot be Pareto-improved within the set of budget-feasible and incentive-compatible allocations (Crocker and Snow (1985a)). Any second-best allocation can be implemented by a social planner as long as he perfectly monitors consumers' trades. By contrast, when the social planner, just like the firms in our economy, can neither prevent consumers from privately engaging in further trades nor monitor these trades, additional constraints reflecting this lack of information must be factored in to define an appropriate notion of *third-best* efficiency.

Although these constraints can be represented in different ways, recent contributions suggest that they severely restrict the set of allocations the social planner can implement. Attar, Mariotti, and Salanié (2020) argue that the third-best efficiency frontier reduces to a single allocation, precluding any redistribution of resources between the different types of consumers.[6] In this allocation, first described by Jaynes (1978), Hellwig (1988), and Glosten (1994), and thus hereafter referred to as the *JHG allocation*, low- and high-risk consumers purchase the same basic layer of coverage, which high-risk consumers complement by purchasing a complementary layer of coverage. The amount of coverage supplied on each layer matches the residual demand of the marginal type, and is fairly priced given the consumer types who purchase it. As a result, all incentive-compatibility constraints are slack, which implies that the JHG allocation does not belong to the second-best efficiency frontier (Crocker and Snow (1985a)).

Given the marginal form of Akerlof (1970) competitive pricing specific to the JHG allocation, it is natural to ask whether it can be implemented as the equilibrium outcome of a game in which strategic firms compete to serve the consumers' demand. A natural candidate is a competitive-screening game in which firms first simultaneously post arbitrary menus of contracts, from which privately informed consumers then choose in a nonexclusive way. However, Attar, Mariotti, and Salanié (2014) show that any equilibrium of this game

[6] Stiglitz, Yun, and Kosenko (2020) deem a larger set of allocations third-best efficient because they do not consider all the Pareto-improving trades that may be offered to the consumers.

must feature market breakdown, whereby low-risk consumers are entirely driven out of the market. As a result, an equilibrium exists, and implements the JHG allocation, if and only if the basic layer of coverage in the JHG allocation is degenerate, so that low-risk consumers are not willing to trade at a price equal to the consumers' average riskiness.

This paper proposes a simple regulation that overcomes this problem and enables one to implement the JHG allocation in a class of insurance economies. This regulation targets firms' pricing strategies, forbidding them to cross-subsidize between different contracts by engaging in below-cost pricing. In the regulated competitive-screening game we consider, firms can as above post menus of contracts that consumers are free to combine. The new feature is that a regulator now has the power to punish any profit-making firm that incurs a loss on a contract.

We provide two sets of results. Theorem 1 first shows that the JHG allocation is the only candidate equilibrium allocation of the regulated game. The intuition is that each firm aims at increasing its profit by complementing the aggregate coverage provided by its competitors. Under adverse selection, this gives rise to a form of Bertrand competition over each layer of coverage. This competitive behavior, which has no equivalent under exclusive competition, is not undermined by our regulation. An important byproduct of Theorem 1 is that every contract traded in equilibrium must break even on average given the consumer types who trade it, thus mimicking at the firm level the pricing of the aggregate layers of basic and complementary coverage. In particular, basic coverage is sold at average cost, to each consumer type. A related implication of Theorem 1 is that the aggregate amount of coverage supplied by firms at this price must exceed the demand of the low-risk type; otherwise, at least one firm would be indispensable for this type to obtain her equilibrium utility, and this firm could make a profit by raising the premium it charges. In this sense, any equilibrium must exhibit an excess supply of basic coverage.

We next implement the JHG allocation in an equilibrium of the regulated game. To clarify the logic of our equilibrium construction, it is useful to observe that, because low- and high-risk consumers are pooled on the same layer of basic coverage, the former subsidize the latter in the JHG allocation. This raises the issue of its fragility against cream-skimming deviations targeted at low-risk consumers. We show that such deviations can be blocked by firms including appropriate *latent contracts* in their equilibrium menus. These contracts are not meant to be traded in equilibrium; yet by posting them, firms offer additional coverage that high-risk consumers are willing to combine with that incorporated in any cream-skimming deviation. This makes it impossible for a firm to profitably deviate by separating low- from high-risk consumers.

In many circumstances that we characterize, equilibrium requires that latent contracts be issued at a price different from those of the contracts actually traded on the equilibrium path. Moreover, we identify an important class of consumer preferences, including quadratic and CARA preferences as special cases, for which a single latent contract is needed to block all cream-skimming deviations that, per se, attract low-risk consumers. This result is key for Theorem 2, which shows that, for consumer preferences in this class, and provided that there are sufficiently many firms, the JHG allocation can be implemented in a symmetric equilibrium in which all firms are active on the equilibrium path. The many-firm assumption ensures that the excess supply of basic coverage is sufficiently small to deter high-risk consumers from only purchasing basic coverage.

Theorem 3 complements Theorem 2 by providing necessary and sufficient conditions for free-entry equilibria, in which at least one firm is inactive on the equilibrium path—a standard requirement in models of competitive markets under adverse selection. Although Theorem 1 still holds under free entry, this refinement imposes additional restrictions on the relative sizes of the layers of basic and complementary coverage, because inactive firms must be deterred from exploiting the excess supply of basic coverage. As a result, this excess supply must now be large. Specifically, the excess supply of basic coverage is achieved by letting only a limited number of firms issue large basic-coverage contracts. In particular, such contracts offer more coverage than complementary-coverage contracts. We discuss the positive implications of this finding in Section 5.

Contributions to the Literature

The regulation we propose bears on the supply side rather than on the demand side of the economy: public intervention should target firms' pricing strategies, while consumers should be left free to choose their preferred amount of coverage. This contrasts with the policy recommendations that have been advocated in the literature. For instance, mandatory insurance is evoked in Akerlof (1970), and has been empirically investigated by Finkelstein (2004), Einav, Finkelstein, and Cullen (2010), and Einav and Finkelstein (2011). Similarly, Wilson (1977), Dahlby (1981), and Crocker and Snow (1985a) show that making basic coverage mandatory and simultaneously allowing private insurers to compete on an extended coverage permits to reach a second-best outcome.[7] Neither are taxes or subsidies needed, unlike in Crocker and Snow (1985b). Instead, we show that regulated markets are

[7] Villeneuve (2003) performs a similar analysis under nonexclusive competition, albeit in a framework that assumes linear pricing.

powerful enough to select a unique equilibrium allocation in which prices efficiently reflect costs—though this rule applies to successive layers of coverage and not to the aggregate coverage amounts purchased by each type of consumers.

This paper contributes to the recent literature on nonexclusive markets under adverse selection. Indeed, our regulation is motivated by the destabilizing role of cross-subsidies between contracts highlighted by Attar, Mariotti, and Salanié (2014). In the absence of regulation, they show that such cross-subsidies would allow any active firm to increase its profit by selling basic coverage to low-risk consumers only, while incurring a small loss by selling complementary coverage to high-risk consumers at slightly better terms than its competitors. The deviating firm would thereby minimize its losses by dumping on its competitors the burden of providing basic coverage to high-risk consumers (*lemon dropping*); this in turn would enable it to profitably attract low-risk consumers on a basic-coverage contract (*cherry picking*). This sophisticated deviation crucially exploits cross-subsidies and the nonexclusive nature of competition. As we indicated earlier, this implies that the unregulated nonexclusive competitive-screening game has no equilibrium in which low-risk consumers trade, and that an equilibrium can fail to exist altogether.[8] Free markets in which firms' pricing strategies are unchecked thus fail to be an effective device to share risk, and the regulation we propose is precisely designed to avoid such an undesirable outcome. In this sense, our work is directly connected to the normative analysis of Attar, Mariotti, and Salanié (2020), who single out the JHG allocation as the unique third-best efficient allocation. Our contribution is to show that, under suitable conditions on consumer preferences, the regulated game uniquely implements this allocation, thereby establishing instances of the First and Second Welfare Theorems for our economy.

Importantly, our approach does not rely on complex interfirm communication, unlike those of Jaynes (1978, 2011), Hellwig (1988), and Stiglitz, Yun, and Kosenko (2020). For these authors, an insurance contract issued by a firm can incorporate a potentially large number of restrictions contingent on the information disclosed by its customers and its competitors. The possibility to enforce exclusivity clauses and to withdraw a policy following a violation of contractual agreements makes their analyses of limited relevance to study competition on nonexclusive markets. In particular, their assumption that firms can choose not to disclose the contracts they issue is in tension with the assumption that consumers and, hence, in principle, firms should know about a firm selling insurance. Our regulated

[8]Though existence of a mixed-strategy equilibrium is guaranteed (Carmona and Fajardo (2009)), no explicit characterization is available; we may also be concerned about the descriptive relevance and the normative properties of such an equilibrium.

game eschews these conceptual problems by staying as close as possible to the nonexclusive-competition benchmark of Attar, Mariotti, and Salanié (2014).

Finally, our regulation can be related to the regulatory measures preventing retailers from engaging in below-cost pricing or loss-leading practices, which have been adopted in several countries since the early 90s.[9] Whereas these measures are typically motivated by the risk of predatory or anticompetitive pricing by multiproduct retailers (Chen and Rey (2012, 2019)), they have so far not been invoked as a device to help insurance markets function efficiently. More abstractly, we should observe that our regulatory requirement is in line with competitive-search and competitive-equilibrium models of markets with adverse selection. Indeed, the equilibrium concepts introduced in these approaches explicitly require that each contract break even, as for instance in Guerrieri, Shimer, and Wright (2010) and Azevedo and Gottlieb (2017). A key difference is that these authors focus on exclusive competition, while we allow for multiple contracting.

The paper is organized as follows. Section 2 describes the model. Section 3 shows that the JHG allocation is the only candidate equilibrium allocation of the regulated game. Section 4 shows how to implement the JHG allocation under oligopoly and free entry. Section 5 draws the lessons from our analysis. Section 6 concludes. The main appendix provides the proofs of Theorems 1–3. The online appendix collects supplementary material.

2 An Insurance Economy

We consider an economy in which each consumer can trade several coinsurance contracts with several firms. This captures markets where multiple policies pay for the same loss, such as life insurance. This also applies to cases in which the loss can be divided into units—such as drugs, care, and various indemnities for pain or loss of income—and consumers can cover different units with different insurers, the assumption being that all these units are fungible.

Consumers There is a continuum of consumers. Each consumer is privately informed of her type $i = 1, 2$ and the proportion of type i among consumers is $m_i > 0$. Every consumer of type i has initial wealth W_0 and faces the risk of a nonnegative idiosyncratic loss distributed according to a density f_i relative to a fixed measure $\boldsymbol{l}$; we assume that 0 belongs to the support of f_i. Each consumer aims at maximizing the expected utility of her final wealth. We denote by v_i the utility index of a consumer of type i, which is assumed to

[9]These practices are banned in several European countries (*The Economist* (2008) https://www.economist.com/node/10430246 describes the Belgian case), and restricted in a number of US states; in California, laws against loss leaders have been enforced as early as 1933.

be strictly increasing, strictly concave, twice continuously differentiable, and to satisfy the Inada condition $\lim_{W\to\infty} v_i'(W) = 0$.

Firms and Contracts There are $n \geq 3$ risk-neutral firms, indexed by k. Each firm observes the amount of coverage it sells to each consumer; thus it is without loss of generality to assume that each consumer can trade at most one contract with each firm. A *coinsurance contract* (q, t) between a firm and a consumer covers a fixed fraction $q \geq 0$ of her loss for a premium t, with premium rate $\frac{t}{q}$ if $q > 0$. A firm trading a contract (q, t) with a consumer of type i thus earns an expected profit $t - r_i q$, where r_i is type i's riskiness:

$$r_i \equiv \int l f_i(l)\, \boldsymbol{l}(\mathrm{d}l).$$

We assume that f_2 dominates f_1 in the monotone-likelihood-ratio order, which implies that type 2 has higher riskiness than type 1:

$$r_2 > r_1. \tag{1}$$

For instance, in the Rothschild and Stiglitz (1976) specification, there is a single loss level L, $\boldsymbol{l}$ is the counting measure on $\{0, L\}$, and $r_i \equiv f_i(L)L$, with $f_2(L) > f_1(L)$. We let $r \equiv m_1 r_1 + m_2 r_2$ be the average riskiness of a consumer.

Allocations Each consumer can combine coinsurance contracts issued by different firms. Her *aggregate trade* is then

$$(Q, T) \equiv \left(\sum_k q^k, \sum_k t^k \right), \tag{2}$$

where (q^k, t^k) is the contract she trades with firm k, with $(q^k, t^k) \equiv (0, 0)$ in case she chooses not to trade with firm k; the consumer thus overall bears a fraction $1 - Q$ of her loss. A symmetric aggregate allocation, hereafter simply *allocation*, is a pair of aggregate trades, one for each type.

It follows from our assumptions that type i's preferences over aggregate trades are represented by

$$u_i(Q, T) \equiv \int v_i(W_0 - (1 - Q)l - T) f_i(l)\, \boldsymbol{l}(\mathrm{d}l). \tag{3}$$

Observe that the function u_i is strictly concave and twice continuously differentiable, with $\frac{\partial u_i}{\partial T} < 0$. Hence type i's marginal rate of substitution of coverage for premia,

$$\tau_i(Q, T) \equiv -\frac{\frac{\partial u_i}{\partial Q}}{\frac{\partial u_i}{\partial T}}(Q, T),$$

is finite, except possibly when $Q = 0$, and is strictly and continuously decreasing along any of her indifference curves. The assumption that 0 belongs to the support of f_i, along with the Inada condition $\lim_{W \to \infty} v_i'(W) = 0$, ensures that, no matter her endowment point (Q, T), type i has a finite demand for coverage at any price $p > 0$,

$$\arg\max \{u_i(Q + Q', T + pQ') : Q' \geq 0\} < \infty, \tag{4}$$

reflecting that, along any indifference curve of type i, the marginal rate of substitution vanishes as aggregate coverage grows large.

The main assumption we impose on consumers' demand for coverage is the following single-crossing condition:

$$\text{for each } (Q, T),\ \tau_2(Q, T) > \tau_1(Q, T). \tag{5}$$

As a result, type 2 is more willing to increase her purchases of coverage than type 1. Together with (1), (5) generates adverse selection. When $v_1 = v_2$, (5) follows from the assumption that f_2 dominates f_1 in the monotone-likelihood-ratio order. More generally, (5) may hold even if type 1 is more risk-averse than type 2, as long as type 2 is sufficiently riskier than type 1; we refer to the online appendix for details.

The Regulated Game We consider a competitive-screening game in which firms compete by posting menus of contracts, subject to a regulation prohibiting cross-subsidies between contracts. The regulated game unfolds as follows.

1. Every firm k posts a compact menu of contracts C^k that contains at least the no-trade contract $(0, 0)$.

2. After privately learning her type, each consumer selects a contract from each of the menus C^k.

3. If a firm overall earns a nonnegative profit but incurs a loss on a contract it actually trades, then its profit is confiscated and the firm is fined.

The simultaneity of the menu offers in stage 1 captures the fact that a firm cannot react to the menus posted by its competitors. Moreover, because consumers in stage 2 make their purchase decisions only once all firms have posted their menus, a firm cannot make her menu offer contingent on the contracts selected by a consumer in the menus of its competitors; this captures the fact that a firm in a nonexclusive insurance market does not observe the amounts of coverage its customers actually purchase from each of its competitors.

Stage 3 embeds the key distinctive assumption of our model. Indeed, the truncated game in which stage 3 is omitted coincides with the nonexclusive-competition game analyzed in Attar, Mariotti, and Salanié (2014). While the equilibrium characterization developed by these authors crucially relies upon firms cross-subsidizing between different contracts, the proposed regulation prohibits such practices. It is important to observe that this prohibition bears on the profits realized on the contracts actually traded in stage 2.

A pure strategy for a consumer is a function that maps any menu profile $C \equiv (C^1, \ldots, C^n)$ into a selection of contracts $((q^1, t^1), \ldots, (q^n, t^n)) \in C^1 \times \ldots \times C^n$. Compactness of the menus C^k ensures that type i's utility-maximization problem

$$\max \left\{ u_i \left(\sum_k q^k, \sum_k t^k \right) : (q^k, t^k) \in C^k \text{ for all } k \right\}$$

always has a solution. Throughout the analysis, we focus on symmetric pure-strategy subgame-perfect equilibria, hereafter simply *equilibria*, in which all consumers of the same type i play the same pure strategy and thus select, given any menu profile C, the same contract $(q_i^k(C), t_i^k(C))$ in the menu C^k of firm k for all k. In line with stage 3 of the regulated game, we say that these contracts are *consistent with the regulation* provided firm k trades the same contract with each type, or, in case it does not and breaks even on average, $\sum_i m_i[t_i^k(C) - r_i q_i^k(C)] \geq 0$, provided each contract it trades breaks even, $t_i^k(C) - r_i q_i^k(C) \geq 0$ for all i. A *free-entry equilibrium* is an equilibrium in which at least one firm does not trade on the equilibrium path; we say in that case that this firm is *inactive*.

Remark The characterization of aggregate equilibrium allocations provided in Section 3 is more generally valid for any utility functions u_i over aggregate trades that are strictly quasiconcave and twice continuously differentiable, with $\frac{\partial u_i}{\partial T} < 0$, and that satisfy the finite-demand condition (4) and the single-crossing condition (5). What matters is that coverage is summarized by a one-dimensional index along which consumer preferences satisfy single crossing.[10] This allows for many alternative specifications of consumer preferences, such as multiplier preferences (Hansen and Sargent (2001)) or smooth ambiguity aversion (Klibanoff, Marinacci, and Mukerji (2005)).[11] The only point at which we rely on the expected-utility specification (3) is in Section 4, which discusses sufficient conditions for the existence of an equilibrium. We shall thus state our main results for general utility functions u_i over aggregate trades satisfying the above properties, indicating when appropriate the additional properties that are satisfied under expected utility.

[10] As a counterexample, contracts with deductibles do not nicely aggregate as in (2).

[11] See Attar, Mariotti, and Salanié (2021, online Appendix C) for a discussion.

3 Equilibrium Characterization

This section establishes the first half of our implementation result, namely, that the JHG allocation introduced by Jaynes (1978) and further studied by Hellwig (1988) and Glosten (1994) is the unique allocation that can be supported in an equilibrium of the regulated game. We first give a general characterization result. We then argue that equilibrium is consistent with free entry only under additional restrictions on the JHG allocation. We finally discuss the circumstances under which a necessary feature of equilibrium is that firms issue latent contracts to discipline their competitors.

3.1 The Uniqueness Result

The uniqueness part of our implementation result can be stated as follows.

Theorem 1 *The JHG allocation* $(Q_i^*, T_i^*)_{i=1,2}$ *defined by*

$$Q_1^* \equiv \arg\max\{u_1(Q, rQ) : Q \geq 0\}, \tag{6}$$

$$T_1^* \equiv rQ_1^*, \tag{7}$$

$$Q_2^* \equiv Q_1^* + \arg\max\{u_2(Q_1^* + Q, T_1^* + r_2 Q) : Q \geq 0\}, \tag{8}$$

$$T_2^* \equiv T_1^* + r_2(Q_2^* - Q_1^*), \tag{9}$$

is the unique candidate equilibrium allocation of the regulated game. Under expected utility, type 2 obtains full coverage, $Q_2^* = 1$, *while type 1 only obtain partial coverage,* $Q_1^* < 1$.

The JHG allocation features a marginal form of Akerlof (1970) competitive pricing. First, by (7) and (9), type 1 and type 2 purchase basic coverage Q_1^* at premium rate r and type 2 purchase complementary coverage $Q_2^* - Q_1^*$ at premium rate r_2; thus every *layer* Q_1^* and $Q_2^* - Q_1^*$ is fairly priced given the types who purchase it. Second, by (6) and (8), the size of each layer is optimal for type 1 and type 2, respectively, subject to this pricing constraint; thus type 1 purchases her demand at price r, and type 2 purchases her residual demand at price r_2. Fair pricing, by contrast, does not apply to aggregate coverage amounts, unlike in Rothschild and Stiglitz (1976): if $Q_1^* > 0$, then type 1 subsidizes type 2 in equilibrium as Q_1^* is sold at the average premium rate $r > r_1$.[12] Figure 1 depicts the candidate equilibrium allocation and the corresponding indifference curves $\mathcal{I}_1^*$ and $\mathcal{I}_2^*$ of type 1 and type 2 when both layers are strictly positive, so that the equilibrium marginal rates of substitution $\tau_1(Q_1^*, T_2^*)$ and $\tau_2(Q_2^*, T_2^*)$ of type 1 and type 2 are equal to r and r_2, respectively.

[12] Moreover, if $Q_2^* > Q_1^* > 0$, then Q_2^* is sold at a premium rate between r and r_2. Thus, compared to exclusive competition, nonexclusive competition reduces the convexity of the tariff for aggregate coverage.

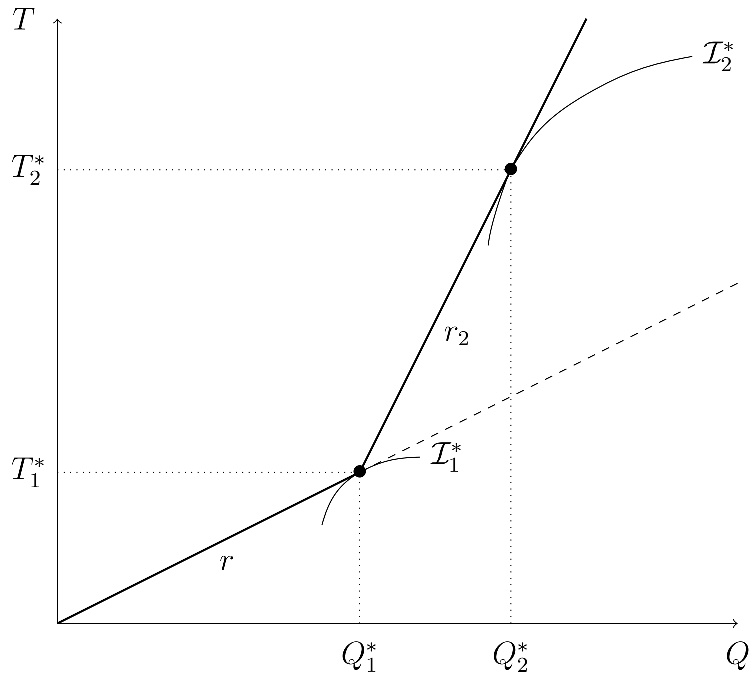

Figure 1: The JHG allocation.

It follows from Attar, Mariotti, and Salanié (2020, Theorem 2) that the JHG allocation is the only candidate equilibrium allocation under free entry. Indeed, in this case, the aggregate equilibrium trade (Q_1, T_1) of type 1 must at least provide her with utility $\max\{u_1(Q, rQ) : Q \geq 0\}$; otherwise, an inactive firm can issue a contract with a premium rate slightly higher than r that profitably attracts type 1 and remains profitable even if it attracts type 2. Similarly, the aggregate equilibrium trade (Q_2, T_2) of type 2 must at least provide her with utility $\max\{u_2(Q_1 + Q, T_1 + r_2 Q) : Q \geq 0\}$; otherwise, an inactive firm can issue a contract with a premium rate slightly higher than r_2 that profitably attracts type 2 in combination with the aggregate trade of type 1 and is even more profitable if it also attracts type 1. Budget-feasibility of the aggregate equilibrium allocation then implies that it must coincide with the JHG allocation. Thus, under free entry, the JHG allocation emerges as a simple implication of an inactive firm's inability to profitably attract both types 1 and 2, or only type 2, by issuing a single contract; in particular, the regulation has no bite.

By contrast, the difficulty in the proof of Theorem 1 is that we cannot a priori suppose that some firm is inactive in equilibrium. This means that we can no longer focus on deviations consisting of a single contract, and that we must now take into account the equilibrium profit a firm foregoes when deviating. If a firm could cross-subsidize between contracts, as in Attar, Mariotti, and Salanié (2014), then it could deviate by issuing a profitable contract targeted at one type while offering the coverage intended for the other type on the equilibrium path on more advantageous terms, incurring a loss on this second contract. By contrast, the inability of a firm to cross-subsidize between contracts in the regulated game requires more sophisticated arguments.

Because the JHG allocation is budget-balanced, each firm must earn zero profit in equilibrium. We show in the proof of Theorem 1 that this implies that any traded contract has premium rate r or r_2: that is, every active firm either sells basic coverage to both types, or complementary coverage to type 2 only. A related insight of our analysis is that no firm is indispensable in providing each type with her equilibrium utility; that is, each type could still obtain this level of utility, should a firm unilaterally withdraw its menu offer. As in the case of standard Bertrand competition, the argument is that, otherwise, this firm could make a profit by slightly increasing the premium it charges. Specifically, we show in the appendix that, if any firm withdraws its menu offer, type 1 can still trade (Q_1^*, T_1^*) in the aggregate, while type 2 can still obtain her equilibrium utility $u_2(Q_2^*, T_2^*)$ by purchasing an amount of coverage at least equal to Q_2^*. In particular, equilibrium requires that there be excess supply of coverage at price r; that is, the aggregate amount of coverage supplied at this price exceeds the demand Q_1^* of type 1, unless this demand is zero. This fact will play an important role in our equilibrium constructions, both under oligopoly (Section 4.2) and under free entry (Section 4.3).

Under expected utility, type 1 obtains partial coverage, $Q_1^* < 1$, while type 2 obtains full coverage, $Q_2^* = 1$; hence the complementary layer $Q_2^* - Q_1^*$ is strictly positive. By (6) and (7), type 1 purchases her demand at price r and is thus not attracted by further trades at price r_2. Similarly, by (8) and (9), type 2 purchases her residual demand at price r_2, and thereby strictly improves on her utility from purchasing basic coverage only. Thus, as illustrated in Figure 1, both types' incentive-compatibility constraints are slack. As a result, the candidate equilibrium allocation does not belong to the second-best efficiency frontier, which consists only of allocations in which at least one incentive-compatibility constraint is binding (Crocker and Snow (1985a)). Specifically, it would be possible to Pareto-improve on the candidate equilibrium allocation by providing type 1 with some small additional amount of coverage at premium rate r_1, thereby maintaining budget balance, while keeping both incentive-compatibility constraints slack.

However, the JHG allocation can be deemed efficient in a weaker, third-best sense. Indeed, this allocation can be characterized as the unique budget-feasible allocation a social planner can implement without inducing firms to secretly offer additional side trades to consumers (Attar, Mariotti, and Salanié (2020)). Thus a social planner who can neither observe consumer types nor monitor their trades with firms cannot Pareto-improve on the candidate equilibrium allocation. In this weak sense, Theorem 1 can be interpreted as a version of the First Welfare Theorem for our economy.

3.2 Free-Entry Equilibria and Size Restrictions

Standard approaches to the study of competitive insurance markets under adverse selection often postulate free entry. This premise is shared by strategic models à la Rothschild and Stiglitz (1976), as well as by the competitive-search and competitive-equilibrium models of Guerrieri, Shimer, and Wright (2010) and Azevedo and Gottlieb (2017), though these authors do not, as we do, consider the possibility of multiple contracting. As noted in our discussion of Theorem 1, the JHG allocation is a quite direct implication of free entry and budget feasibility, which can be drawn from the sole consideration of an inactive firm's optimal behavior. However, what we are looking for is a free-entry *equilibrium*, in which any firm's behavior is optimal given the menus posted by its competitors, regardless of whether it is active or inactive. While Theorem 1 is fully general and thus also holds in a free-entry equilibrium, this more demanding notion imposes additional restrictions on the candidate equilibrium allocation.

Specifically, these additional restrictions bear on the relative sizes of the basic and complementary layers of coverage. To see why, let us assume that the basic layer Q_1^* is strictly positive, that is, $\tau_1(0,0) > r$. The dispensability property then requires that the aggregate amount of coverage supplied at price r exceed Q_1^*. In any equilibrium, we must make sure that type 2 is not tempted to purchase basic coverage in excess of Q_1^*. But, in a free-entry equilibrium, we must in addition make sure that it is impossible for an inactive firm to exploit this excess supply of basic coverage to offer mutually advantageous trades to type 2. The resulting size restrictions can be formulated as follows.

Lemma 1 *Suppose $Q_1^* > 0$ and that the regulated game has a free-entry equilibrium. Then the JHG allocation must satisfy*

$$u_2(Q_2^*, T_2^*) \geq u_2(2Q_1^*, 2T_1^*), \tag{10}$$

$$Q_1^* > Q_2^* - Q_1^*. \tag{11}$$

Conditions (10)–(11) are easy to understand when only two firms issue contracts at the average premium rate r. First, because neither of them is indispensable, both must issue a contract equal to type 1's equilibrium aggregate trade (Q_1^*, T_1^*). Condition (10) then simply

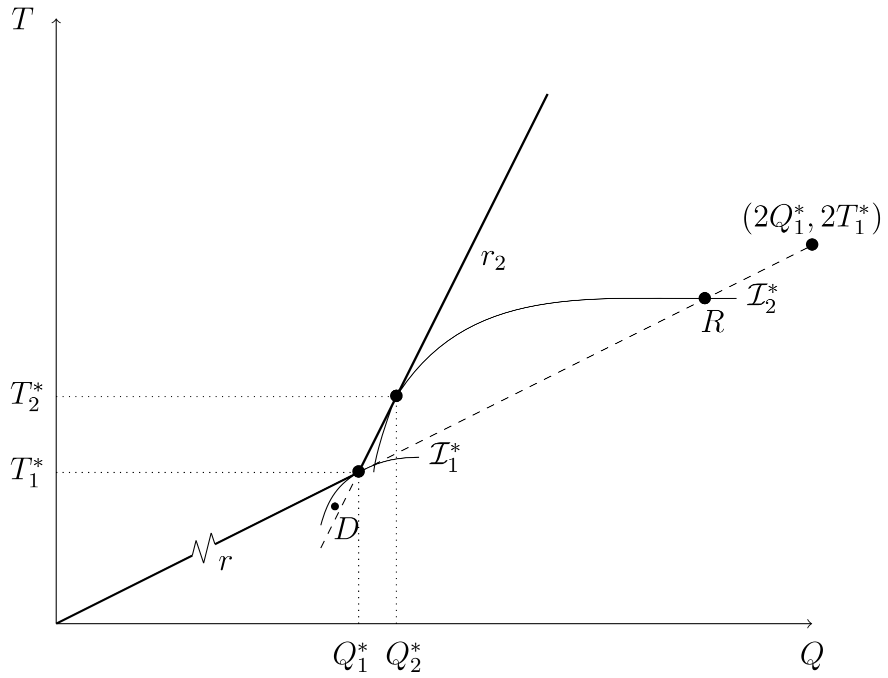

Figure 2: Size restrictions in a free-entry equilibrium.

expresses that type 2 is not strictly better off trading the contract (Q_1^*, T_1^*) twice on the equilibrium path. Next, if condition (11) were not satisfied, then an inactive firm could profitably attract type 2 by offering a coverage $Q_2^* - 2Q_1^*$ for a premium slightly lower than $T_2^* - 2T_1^*$, but at a premium rate higher than r_2; indeed, combined with the trade $(2Q_1^*, 2T_1^*)$, which, by assumption, is available on the equilibrium path, this offer would enable type 2 to pay less than T_2^* for her equilibrium coverage Q_2^*.[13] This logic easily extends when more than two firms issue contracts at the average premium rate r. Geometrically, conditions (10)–(11) are satisfied when the aggregate trade $(2Q_1^*, 2T_1^*)$ is located in the lower contour set of (Q_2^*, T_2^*) for type 2, to the right of (Q_2^*, T_2^*). This implies that the complementary layer is sufficiently small relative to the basic layer, as illustrated in Figure 2.

3.3 On the Necessity of Latent Contracts

Two types of contracts are traded in equilibrium: basic-coverage contracts with premium rate r, and complementary-coverage contracts with premium rate r_2. But other contracts may be issued by firms without being actually traded on the equilibrium path, and still be needed to sustain an equilibrium. Indeed, because cross-subsidies between types are present in equilibrium as soon as $Q_1^* > 0$, it is possible to design contracts—such as the contract

[13] Notice that this deviation is well-defined because $Q_2^* \neq 2Q_1^*$ in the equilibrium configuration under consideration; otherwise, type 2 would be strictly better off trading the contract (Q_1^*, T_1^*) twice instead of (Q_2^*, T_2^*) on the equilibrium path.

represented by D in Figure 2—that profitably attract type 1. The only way to block these cream-skimming deviations is to make them attract type 2 as well, who would trade the deviation in combination with other contracts issued by nondeviating firms. These contracts can take various forms, and we now discuss a few possibilities.

The first idea consists in relying on nonexclusivity, by trading the same contract several times. For example, consider the special case where type 2 is indifferent between her aggregate trade (Q_2^*, T_2^*) and purchasing twice the basic-coverage layer of the JHG allocation,

$$u_2(Q_2^*, T_2^*) = u_2(2Q_1^*, 2T_1^*).$$

Consider the following natural strategies for the firms: each of the $n \geq 3$ firms issues a basic-coverage contract (Q_1^*, T_1^*), and supplies any amount of complementary coverage at price r_2. Suppose in addition $2Q_1^* > Q_2^*$—otherwise, type 2 can obtain a utility higher than $u_2(Q_2^*, T_2^*)$ given these contracts. Now, any contract that attracts type 1, either per se or in combination with a basic-coverage contract, also attracts type 2, who can trade it in combination with one basic-coverage contract and/or complementary coverage. Because the aggregate trade (Q_1^*, T_1^*) is available at the deviation stage, as well as unlimited complementary coverage, this natural candidate forms an equilibrium.[14]

Nevertheless, this conclusion is not robust to a slight change in preferences. Let us keep the same natural strategies, but suppose now that type 2 strictly prefers her aggregate trade (Q_2^*, T_2^*) to purchasing twice the basic-coverage layer of the JHG allocation,

$$u_2(Q_2^*, T_2^*) > u_2(2Q_1^*, 2T_1^*),$$

as illustrated in Figure 2. Now, any firm k can deviate by issuing the contract (q, t) represented by D in Figure 2, which offers a coverage close to but lower than Q_1^* at a premium rate slightly lower than r. This contract profitably attracts type 1, and it does not attract type 2. Indeed, after the deviation, type 2 can still obtain her equilibrium utility $u_2(Q_2^*, T_2^*)$ by trading with two firms $l \neq k$. By contrast, combining (q, t) with the contract (Q_1^*, T_1^*) leads type 2 to excessive levels of coverage, thanks to the above inequalities; and combining (q, t) with one or several contracts $(Q_2^* - Q_1^*, T_2^* - T_1^*)$ leaves type 2 with a lower utility than trading (Q_2^*, T_2^*) in the aggregate. This shows that the deviation is profitable, and that this natural candidate fails to form an equilibrium.

Admittedly, there are many other contracts we may include in the menus posted by firms, so as to ensure that cream-skimming deviations indeed attract type 2 as soon as they attract type 1. Fortunately, the following result generalizes the above intuitions to all equilibria.

[14]The proof follows from a direct geometrical argument and is omitted for the sake of brevity.

Lemma 2 *Suppose $Q_2^* > Q_1^* > 0$. Then, in any equilibrium of the regulated game, consumers are able to reach an aggregate trade (Q^P, T^P) such that*

$$u_2(Q^P, T^P) = u_2(Q_2^*, T_2^*) \quad \textit{and} \quad \tau_2(Q^P, T^P) \leq r.$$

Observe that this covers the nongeneric case discussed above, in which (Q^P, T^P) is simply $(2Q_1^*, 2T_1^*)$. A natural question is whether the aggregate trade (Q^P, T^P) displays specific qualitative features compared to the contracts actually traded in equilibrium. The following result provides a quite complete characterization of when this has to be the case.

Lemma 3 *Suppose $Q_2^* > Q_1^* > 0$. Then, the following holds:*

(i) If $u_2(2Q_1^*, 2T_1^*) > u_2(Q_2^*, T_2^*)$, then, in any equilibrium of the regulated game, some firm issues a contract with a premium rate different from r and r_2.

(ii) If $u_2(2Q_1^*, 2T_1^*) \leq u_2(Q_2^*, T_2^*)$, then, in any equilibrium of the regulated game, some firm issues a contract with a premium rate different from r and r_2, or consumers are able to reach the aggregate trade (Q^R, T^R) defined by

$$u_2(Q^R, T^R) = u_2(Q_2^*, T_2^*) \quad \textit{and} \quad T^R = rQ^R.$$

The aggregate trade (Q^R, T^R) is represented by R in Figure 2. The contracts whose necessity is proven in Lemma 3(i) have premium rates that differ from those of the contracts traded on the equilibrium path. Thus they are not actually traded in equilibrium, though they are necessary to sustain an equilibrium; hence the name *latent contracts*.

Overall, the upshot of this discussion is that, to sustain an equilibrium, firms have to issue additional, and sometimes latent contracts that are only meant to block cream-skimming deviations. We now turn to the study of how these contracts operate. Our goal is to provide a minimal implementation with as few latent contracts as possible.

4 Equilibrium Existence

Theorem 1 singles out the JHG allocation as the unique candidate equilibrium allocation of the regulated game. We now provide sufficient conditions for the existence of an equilibrium, thereby establishing the second half of our implementation result. We construct two types of equilibria, an oligopoly equilibrium in which all firms are active on the equilibrium path, and a free-entry equilibrium in which at least one firm is inactive on the equilibrium path. To focus on the most relevant scenario for applications, which is also theoretically the most challenging, we throughout assume that both the basic and complementary layers of the JHG allocation are strictly positive.[15] As we will see, the structure of equilibrium menus is essentially determined by cream-skimming deviations, defined as contracts that only attract type 1, either per se or—and this is the specificity of multiple contracting—in combination with contracts issued by nondeviating firms.

4.1 Large Cream-Skimming Deviations and Latent Contracts

We first consider deviations that attract type 1 even if she does not trade other contracts. These deviations are the standard ones considered by Rothschild and Stiglitz (1976) under exclusive competition. We call them *large cream-skimming deviations* to emphasize that a large amount of coverage is needed to match $u_1(Q_1^*, T_1^*)$ and attract type 1. Our goal in this section is to show that, under suitable conditions on consumer preferences, a single latent contract is needed to block all large cream-skimming deviations.

Specifically, suppose that each firm includes in its equilibrium menu a latent contract (q^ℓ, t^ℓ). We say that this contract *blocks large cream-skimming deviations* if

$$\text{for each } (q,t),\ u_1(q,t) \geq u_1(Q_1^*, T_1^*) \text{ implies } u_2(q+q^\ell, t+t^\ell) \geq u_2(Q_2^*, T_2^*). \qquad (12)$$

That is, if (q,t) per se attracts type 1, then it also attracts type 2 in combination with (q^ℓ, t^ℓ). As a result, (q,t) cannot make a profit, because $u_1(q,t) \geq u_1(Q_1^*, T_1^*)$ implies by (6)–(7) that (q,t) has at most premium rate r.

The geometrical interpretation of (12) is that the translate of the upper contour set of (Q_1^*, T_1^*) for type 1 along the vector (q^ℓ, t^ℓ) lies in the upper contour set of (Q_2^*, T_2^*) for type 2. However, type 2 cannot strictly prefer $(Q_1^* + q^\ell, T_1^* + t^\ell)$ to (Q_2^*, T_2^*), because these aggregate trades are available on the equilibrium path and type 2 trades (Q_2^*, T_2^*) in equilibrium. Thus she must be indifferent between $(Q_1^* + q^\ell, T_1^* + t^\ell)$ and (Q_2^*, T_2^*). By (12), this implies that the translate of the equilibrium indifference curve $\mathcal{I}_1^*$ of type 1 along the vector (q^ℓ, t^ℓ) is tangent at $(Q_1^* + q^\ell, T_1^* + t^\ell)$ to the equilibrium indifference curve $\mathcal{I}_2^*$ of type 2. Thus the marginal rate of substitution of type 2 at $(Q_1^* + q^\ell, T_1^* + t^\ell)$ must be equal to r. These properties single out a unique latent contract (q_1^ℓ, t_1^ℓ), which is illustrated in Figure 3. Here D represents a large cream-skimming deviation, which also attracts type 2 in combination with (q_1^ℓ, t_1^ℓ) because the point $D^\ell \equiv D + (q_1^\ell, t_1^\ell)$ lies below the translate $\mathcal{I}_1^\ell$ of $\mathcal{I}_1^*$, and, therefore, by (12), below $\mathcal{I}_2^*$. The following lemma summarizes this discussion.

[15] The complementary layer is always strictly positive under expected utility, as in this case $Q_1^* < 1$ and $Q_2^* = 1$. In general, when the basic layer is degenerate, $Q_1^* = 0$, the JHG allocation can be implemented by letting firms post linear tariffs at the fair premium rate r_2 (Attar, Mariotti, and Salanié (2014)).

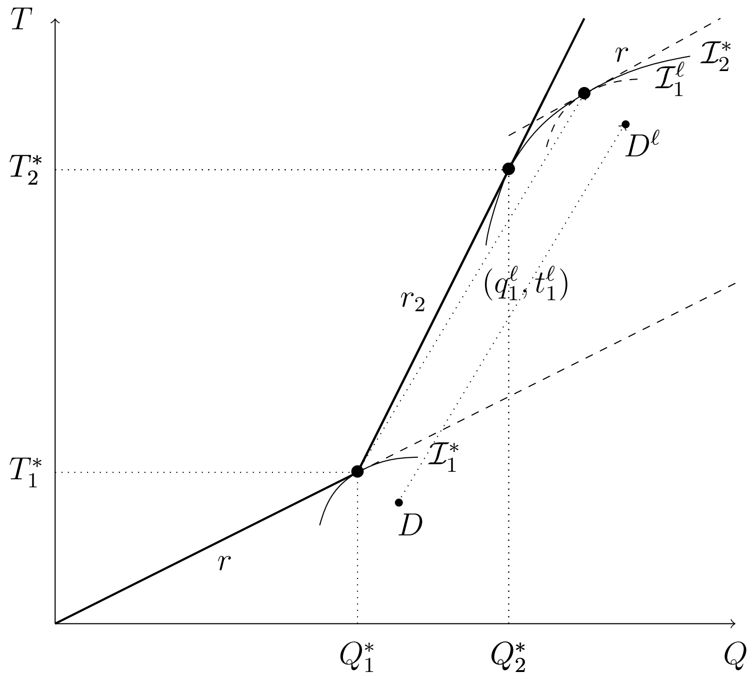

Figure 3: The latent contract (q_1^ℓ, t_1^ℓ).

Lemma 4 *The contract* (q_1^ℓ, t_1^ℓ) *defined by*

$$u_2(Q_1^* + q_1^\ell, T_1^* + t_1^\ell) = u_2(Q_2^*, T_2^*), \tag{13}$$

$$\tau_2(Q_1^* + q_1^\ell, T_1^* + t_1^\ell) = r \tag{14}$$

is the unique latent contract issued by all firms in equilibrium that possibly blocks large cream-skimming deviations.

Thus requiring that large cream-skimming deviation be blocked by a single latent contract pins down (q_1^ℓ, t_1^ℓ) as the unique candidate. However, because the reasoning underlying Lemma 4 is only local, we still need to make sure that (q_1^ℓ, t_1^ℓ) indeed satisfies the global condition (12). We now provide a sufficient condition for this stronger property, which is stated in terms of the Gaussian curvature $\kappa_i(Q_0, T_0)$ of every type i's indifference curve at any aggregate trade (Q_0, T_0). Intuitively, $\kappa_i(Q_0, T_0)$ measures how bent the indifference curve of type i at (Q_0, T_0) is at this point. Denoting by $T = \mathcal{I}_i(Q, u_i(Q_0, T_0))$ the functional expression of this indifference curve, we have

$$\kappa_i(Q_0, T_0) \equiv \frac{1}{\|\nabla u_i\|^3} \begin{vmatrix} -\nabla^2 u_i & \nabla u_i \\ -\nabla u_i^\top & 0 \end{vmatrix} (Q_0, T_0) = -\frac{\frac{\partial^2 \mathcal{I}_i}{\partial Q^2}(Q_0, u_i(Q_0, T_0))}{\left\{1 + \left[\frac{\partial \mathcal{I}_i}{\partial Q}(Q_0, u_i(Q_0, T_0))\right]^2\right\}^{\frac{3}{2}}}, \tag{15}$$

where of course $\frac{\partial \mathcal{I}_i}{\partial Q}(Q_0, u_i(Q_0, T_0)) = \tau_i(Q_0, T_0)$ (Debreu (1972)). Thus, for a given value of

$\tau_i(Q_0, T_0)$, the higher $\kappa_i(Q_0, T_0)$ is, the faster $\tau_i(Q, T)$ declines in a neighborhood of (Q_0, T_0) along the indifference curve of type i at (Q_0, T_0).

Now, a quick glance at Figure 3 reveals that, for the translation property (12) to be satisfied, the equilibrium indifference curve of type 2 must be flatter at $(Q_1^* + q_1^\ell, T_1^* + t_1^\ell)$ than the equilibrium indifference curve of type 1 at (Q_1^*, T_1^*); that is, at those points, the latter must exhibit at least as much curvature than the former. Generalizing this observation leads to the following assumption.

Assumption C *For all* i, $Q_0 > 0$, *and* T_0, $\kappa_i(Q_0, T_0) > 0$. *Moreover, one of the following properties is satisfied*:

(i) For all (Q_1, Q_2, T_1, T_2), *if* $\tau_1(Q_1, T_1) = \tau_2(Q_2, T_2)$, *then* $\kappa_1(Q_1, T_1) > \kappa_2(Q_2, T_2)$.

(ii) For all (Q_1, Q_2, T_1, T_2), *if* $\tau_1(Q_1, T_1) = \tau_2(Q_2, T_2)$, *then* $\kappa_1(Q_1, T_1) = \kappa_2(Q_2, T_2)$.

That the curvatures of consumers' indifference curves nowhere vanish is a very weak requirement that we only impose for technical reasons.[16] Property C(i) states that type 2's indifference curves are flatter than type 1's, once these curves are translated so as to make them tangent at the relevant aggregate trade. This differs from the standard single-crossing condition (5), which does not allow for translations. The latter are natural operations when consumers can combine contracts issued by different firms, and we can view Assumption C as a second-order version of the single-crossing condition. Property C(ii) is satisfied in the limiting case where any two pairs of indifference curves for types 1 and 2 are, over the relevant domains, translates of each other. Assumption C can alternatively be phrased in terms of type 1's and type 2's Hicksian demand functions for coverage; we develop this interpretation in the online appendix.

The following result then holds.

Lemma 5 *If consumer preferences satisfy Assumption C, then the contract* (q_1^ℓ, t_1^ℓ) *blocks large cream-skimming deviations.*

We defer the discussion of Assumption C until Section 4.4, where we show that it is consistent with the other assumptions of our model. In the next two sections, we provide two alternative equilibrium constructions, in which the firms' equilibrium menus include the latent contract (q_1^ℓ, t_1^ℓ).

[16] This amounts to assuming that preferences are nonlinear, even in a local sense. Such preferences are called *differentiably strictly convex* by Mas-Colell (1985, Definition 2.6.1).

4.2 Equilibrium under Oligopoly

We start with the oligopoly case. Our focus is on an equilibrium in which each firm posts the same menu, whose elements we first present in turn.

4.2.1 The Equilibrium Menu

The firms' equilibrium menu first includes two contracts that enable each type to reach her aggregate equilibrium trade.

The Basic-Coverage Contract We know from Theorem 1 that our candidate equilibrium must implement the JHG allocation. Thus both types must be able to purchase the basic layer Q_1^* in exchange for a transfer T_1^*. In addition, we also know that, if any firm withdraws its menu offer, then type 1 must still be able to trade (Q_1^*, T_1^*) in the aggregate. There are various ways of ensuring this; in the present oligopoly case, we shall simply assume that each of the n firms issues a basic-coverage contract, with premium rate r, consisting of a fraction $\frac{1}{n-1}$ of the aggregate trade (Q_1^*, T_1^*),

$$(q_{1,n}^*, t_{1,n}^*) \equiv \frac{1}{n-1}\,(Q_1^*, T_1^*). \tag{16}$$

Thus the aggregate trade (Q_1^*, T_1^*) remains available following any firm's unilateral deviation. Notice that the excess supply of basic coverage is exactly equal to $q_{1,n}^*$, which, by (16), is small when n is large. Because $u_2(Q_2^*, T_2^*) > u_2(Q_1^*, T_1^*)$, this implies that, for n large enough, type 2 will not be tempted to trade n basic-coverage contracts, one with each firm. The minimum number of firms required to support this construction need not be large in an absolute sense, and depends on how much the two types differ from each other; under expected utility, the further apart their loss distributions are, the smaller this number is.

The Complementary-Coverage Contract In the JHG allocation, type 2 must, on top of the basic layer Q_1^*, purchase the complementary layer $Q_2^* - Q_1^*$ in exchange for a transfer $T_2^* - T_1^*$. Because the basic layer is provided through $n-1$ basic-coverage contracts, the complementary layer must be provided through a single contract. We thus require that each firm issue the complementary-coverage contract $(Q_2^* - Q_1^*, T_2^* - T_1^*)$, with premium rate r_2. Trading one such contract with a firm and one basic-coverage contract with each of its competitors enables type 2 to reach her aggregate trade (Q_2^*, T_2^*) on the equilibrium path.

In line with the analysis in Sections 3.3 and 4.1, the firms' equilibrium menu also includes two latent contracts, which are not meant to be traded in equilibrium, and whose role is to block two types of cream-skimming deviations.

Blocking Large Cream-Skimming Deviations We first require that each firm issue the latent contract (q^ℓ_1, t^ℓ_1), which, under Assumption C, is known by Lemma 5 to block large cream-skimming deviations.

Blocking Small Cream-Skimming Deviations Because type 1 can combine contracts issued by different firms, a deviation may attract type 1 even if it only involves a small amount of coverage. Such deviations are specific to nonexclusive competition. We call them *small cream-skimming deviations* to emphasize that they attract type 1 only in combination with other contracts. To block such deviations, we additionally require that each firm issue the latent contract

$$(q^\ell_{2,n}, t^\ell_{2,n}) \equiv (q^*_{1,n}, t^*_{1,n}) + (q^\ell_1, t^\ell_1). \tag{17}$$

It should first be noted that, as requested by equilibrium, the contract $(q^\ell_{2,n}, t^\ell_{2,n})$ enables type 2 to still obtain her equilibrium utility $u_2(Q^*_2, T^*_2)$, should any firm withdraw its menu offer; indeed, she can reach the aggregate trade $(Q^*_1 + q^\ell_1, T^*_1 + t^\ell_1)$ on her equilibrium indifference curve by trading the contract $(q^\ell_{2,n}, t^\ell_{2,n})$ with a nondeviating firm in combination with the $n - 2$ basic-coverage contracts issued by the other nondeviating firms. We now informally argue that this contract blocks all small cream-skimming deviations. The key property, which is formally established in the proof of Theorem 2, is that, if n is large enough, then, for any small cream-skimming deviation (q, t), type 1 can at least obtain her equilibrium utility $u_1(Q^*_1, T^*_1)$ by trading (q, t) with the deviating firm in combination with contracts issued by nondeviating firms, *including at least one basic-coverage contract.* But then type 2 could mimic these trades, with the sole difference that she would substitute a latent contract $(q^\ell_{2,n}, t^\ell_{2,n})$ to this basic-coverage contract.[17] Under Assumption C, and following the logic of Lemma 5, doing so would by (17) enable her to at least obtain her equilibrium utility $u_2(Q^*_2, T^*_2)$, thus blocking the small cream-skimming deviation.

For each n, we let C^*_n be the menu consisting of the no-trade contract and these four contracts.

4.2.2 The Existence Result

We can now state our first existence result.

[17]The reason why we need n to be large enough, and hence the basic coverage $q^*_{1,n}$ offered by each nondeviating firm to be small enough, is that, otherwise, (q, t) may attract type 1 only in combination with complementary-coverage contracts or latent contracts issued by nondeviating firms, in which case the mimicking argument loses its bite. Intuitively, what is needed is that, at the deviation stage, type 1 gives priority to contracts with the lowest premium rate r when trading with nondeviating firms. This is not necessarily the case when there are few such firms, for her budget set is not convex.

Theorem 2 *Suppose that consumer preferences satisfy Assumption C. Then, provided there are sufficiently many firms, the regulated game has an equilibrium in which each of the n firms posts the menu C_n^*.*

Together with Theorem 1, Theorem 2 can be interpreted as a weak version of the Second Welfare Theorem for our economy: under suitable conditions on consumer preferences, the regulated game has an equilibrium, and this equilibrium implements the unique third-best allocation.

Whereas our discussion has so far focused on cream-skimming deviations, the proof of Theorem 2 also has to deal with the case in which a firm attempts to screen type 1 and type 2 by posting a menu of contracts. This is here, and only here, that the regulation has bite, as we now explain.

To do so, it is useful to recall that, in the absence of regulation—that is, in a truncated version of our game in which stage 3 is omitted—no equilibrium exists when the basic layer of the JHG allocation is strictly positive (Attar, Mariotti, and Salanié (2014)). Indeed, while the JHG allocation remains the unique candidate equilibrium allocation, a firm can now profitably deviate by issuing two contracts, exploiting the fact that the aggregate trade (Q_1^*, T_1^*) is made available by its competitors. The first contract is approximatively the same as the one it trades with type 1 on the candidate equilibrium path, and makes a profit when traded by type 1 only. The second contract enables type 2 to purchase the complementary layer at a premium rate slightly lower than r_2. Because the deviating firm now offers the complementary layer at slightly more advantageous terms, it is optimal for type 2 to trade it on top of the basic layer collectively supplied by its competitors. By deviating in this way, the firm manages to only incur a small loss with type 2, which it more than recoups by making a large profit with type 1.

Our regulation is precisely designed to outlaw such a deviation. Specifically, we show in the proof of Theorem 2 that, given the menus C_n^* posted by its competitors, a deviating firm cannot strictly screen type 1 and type 2 without incurring a loss with type 2, thereby exposing itself to being punished if it overall earns a nonnegative profit. The only remaining possibility is to attract both types with the same contract, but this cannot be profitable as type 1 can purchase her demand at price r from the nondeviating firms. Overall, the regulation we propose, by penalizing cross-subsidies between contracts, permits to support cross-subsidies between types in equilibrium.

Finally, Theorem 2 differs from earlier contributions in two other ways. First, firms in the regulated game cannot exchange information about their customers' trades. This contrasts with Jaynes (1978, 2011), Hellwig (1988), and Stiglitz, Yun, and Kosenko (2020), whose equilibrium constructions explicitly rely on interfirm communication. Second, firms in the regulated game cannot react to the offers of their competitors. This contrasts with Hellwig (1988), whose sequential timing requires several steps of interfirm communication to implement the JHG allocation, and with Attar, Mariotti, and Salanié (2021), whose discriminatory ascending-auction mechanism enables each firm to instantaneously react to the past supply decisions of its competitors.

4.3 Equilibrium under Free Entry

The equilibrium exhibited in Theorem 2 is not a free-entry equilibrium. Indeed, an inactive $n+1^{\text{th}}$ firm could offer complementary coverage $Q_2^* - Q_1^* - q_{1,n}^*$ for a premium slightly lower than $T_2^* - T_1^* - t_{1,n}^*$: in combination with the aggregate trade $\frac{n}{n-1}(Q_1^*, T_1^*)$ made available by the active firms, this would attract type 2 at a premium rate higher than r_2 and thus make a profit.[18] We know from Lemma 1 that a free-entry equilibrium exists only under the size restrictions (10)–(11). The following result shows that, under Assumption C, these restrictions are also sufficient for the existence of a free-entry equilibrium.

Theorem 3 *Suppose that consumer preferences satisfy Assumption C. Then the regulated game has a free-entry equilibrium if and only if the JHG allocation satisfies (10)–(11).*

Our equilibrium construction relies on two active firms, each posting a menu recursively defined on the basis of the basic-coverage contract (Q_1^*, T_1^*), the complementary-coverage contract $(Q_2^* - Q_1^*, T_2^* - T_1^*)$, and the latent contract (q_1^ℓ, t_1^ℓ). In addition, the inactive firms issue the same complementary-coverage contract as the active firms. Thus, if an active firm withdraws its menu offer, type 2 can still obtain her equilibrium utility $u_2(Q_2^*, T_2^*)$ by purchasing basic coverage from the other active firm and complementary coverage from an inactive firm. Compared with Theorem 2, firms no longer play symmetric roles. In particular, active and inactive firms face different market configurations at the deviation stage; this issue is dealt with in the proof of Theorem 3 by showing that the latent contracts issued by active firms also deter entry by inactive ones. The regulation plays the same role as in the oligopoly case by making menu deviations unprofitable.

It is instructive to compare our equilibrium constructions in the oligopoly and free-entry cases. In the oligopoly case, the excess supply of basic coverage needed to sustain an

[18] This does not mean that this equilibrium is less likely to exist when there are more firms—on the contrary, Theorem 2 tells us that more competition makes it easier to sustain. Instead, what this shows is that, although firms end up earning zero profit in equilibrium, the number of firms matters for how much each of them is ready to contribute to the provision of basic coverage.

equilibrium is obtained by letting each of a large number of firms contribute a small fraction of the basic layer Q_1^*. The excess supply of basic coverage is thus small when there is a large number of firms. In the free-entry case, this excess supply is obtained by letting each of two firms offer the basic layer Q_1^*. The excess supply of basic coverage is thus large, even when only two firms supply such coverage. However, the JHG allocation must then satisfy the size restrictions (10)–(11) for entry by an inactive firm to be unprofitable.

Theorem 3 differs from the earlier contributions of Glosten (1994) and Attar, Mariotti, and Salanié (2020) in that these authors characterize the JHG allocation as the unique budget-balanced allocation supported by an entry-proof market tariff, but do not implement it as the outcome of a game. Indeed, Attar, Mariotti, and Salanié (2014, 2019b) show that this task cannot in general be accomplished through an unregulated competitive-screening game. By contrast, we construct a free-entry equilibrium of the regulated game that uniquely supports the JHG allocation under the additional size restrictions (10)–(11).

4.4 On Assumption C

Given the crucial role Assumption C plays in our equilibrium construction, it is important to assess how restrictive it is. The following examples show that Assumption C is consistent with the other assumptions of our model.[19]

Example 1 When there is a single loss level L, as in Rothschild and Stiglitz (1976), property C(ii) is satisfied if type 1 and type 2 have the same CARA utility index, while property C(i) is satisfied if $\inf_W -\frac{v_1''}{v_1'}(W) > \max_W -\frac{v_2''}{v_2'}(W)$, that is, if type 1 is uniformly more risk-averse than type 2 in the sense of Aumann and Serrano (2008). The latter condition is clearly in tension with the single-crossing condition (5); Lemma S.2 in the online appendix shows that these conditions are consistent provided type 2 is sufficiently riskier than type 1, so that she is more willing to increase her purchases of coverage despite being less risk-averse.

Example 2 When there are multiple loss levels and every type i's density of losses belongs to a natural exponential family

$$f_i(l) \equiv h(l) \exp(\theta_i l - A(\theta_i)), \tag{18}$$

property C(ii) is satisfied if type 1 and type 2 have the same CARA utility index, while property C(i) is satisfied if every type i has a CARA utility index with absolute risk aversion

[19]Though these two examples rely on the expected-utility representation (3), other specifications are possible. For instance, preferences represented by quadratic utility functions $u_i(Q,T) = \theta_i Q - \frac{\alpha}{2} Q^2 - T$ as in Biais, Martimort, and Rochet (2000) also satisfy property C(ii).

α_i and type 1 is more risk-averse than type 2, $\alpha_1 > \alpha_2$. Again, the latter condition is clearly in tension with the single-crossing condition (5); Lemma S.4 in the online appendix shows that these conditions are consistent provided type 2 is sufficiently riskier than type 1 in the sense that $\theta_2 - \theta_1 > \alpha_1 - \alpha_2$.

While these two examples show that Theorems 2–3 are not vacuous, they admittedly rest on strong assumptions, namely, that both types have the same CARA utility index or that type 1 be uniformly more risk-averse than type 2.[20] In particular, our analysis falls short of providing a general existence result in the Rothschild and Stiglitz (1976) specification beyond the CARA case, reflecting that the possibility of combining contracts can interact with income effects in a complex way.

This does not mean, however, that an equilibrium necessarily fails to exist beyond the cases delineated in Examples 1–2. First, the condition in property C(i) is open, and thus defines a countable intersection of open sets in the space of smooth strictly convex preferences over (Q, T) that are strictly decreasing in T (Attar, Mariotti, and Salanié (2019a)); thus Examples 1–2 are robust. Second, the strong assumptions on consumers' utility indices made in these example are the price to pay to guarantee that Assumption C is satisfied, and thus that Lemma 5 holds, irrespective of the loss probabilities or the distribution of types. Once these are fixed, the translation property (12), which is all we need to establish Theorems 2–3, may be satisfied even though Assumption C is not.

5 Discussion

In this section, we put our findings in perspective and relate them to the literature.

5.1 On Latent Contracts

A key feature of our equilibrium constructions is that we explicitly characterize the latent

[20]Though restrictive, the CARA assumption, implying that Hicksian and Marshallian demands for coverage are the same, is common in the applied literature; see, for instance, Einav, Finkelstein, and Cullen (2010), Einav and Finkelstein (2011), Chetty and Finkelstein (2013), and Einav, Finkelstein, Ryan, Schrimpf, and Cullen (2013). The negative correlation between risk aversion and riskiness postulated in Examples 1–2 is ultimately an empirical matter. In the annuity market, for instance, it may be argued that individuals who are more likely to live longer—and thus are more risky from the perspective of insurance companies—may also be less risk-averse, especially when it comes to wealth after death. In their study of the long-term-care market, Finkelstein and McGarry (2006) develop a proxy for risk aversion, using information on how cautiously respondents behave in terms of preventive health activities. They find that people who are more risk-averse by this measure are more likely to purchase coverage but less likely to use long-term care, consistent with multidimensional private information and advantageous selection based on risk aversion. Example 2 allows preference-based ($\alpha_1 > \alpha_2$) and risk-based ($\theta_2 > \theta_1$) selection to act in offsetting directions, although not to the point of overturning adverse selection ($\theta_2 - \theta_1 > \alpha_1 - \alpha_2$).

contracts needed to block cream-skimming deviations. Specifically, Lemmas 4–5 strengthen Lemmas 2–3 by showing that, under Assumption C, the contract (q^ℓ_1, t^ℓ_1) by itself blocks large cream-skimming deviations, and is the single contract to do so.[21] This has allowed us to provide a minimal implementation with as few latent contracts as possible.

The role of latent contracts in competing-mechanism games has been well understood since the seminal work of Peters (2001) and Martimort and Stole (2002). In the context of insurance, their importance has been mainly emphasized in moral-hazard environments. Hellwig (1983) and Arnott and Stiglitz (1991) argue that latent contracts deter entry on insurance markets when agents' effort decisions are not contractible, enabling active firms to earn strictly positive profits in equilibrium. The equilibrium structure of latent contracts and their welfare implications have been further examined by Bisin and Guaitoli (2004) and Attar and Chassagnon (2009).

By contrast, the importance of latent contracts in adverse-selection environments is much less appreciated in the literature. An exception is Attar, Mariotti, and Salanié (2011), but their analysis is restricted to the special case of linear preferences, which is not well suited for the study of insurance markets. This case is also special in that latent contracts can be issued at the same price as the contracts traded on the equilibrium path and can break even in all subgames. This is not the case in our implementation of the JHG allocation, because the premium rate of (q^ℓ_1, t^ℓ_1) is higher than r and lower than r_2; thus (q^ℓ_1, t^ℓ_1) incurs a loss in any subgame in which it is traded by type 2 following a deviation. This property is not a special feature of the latent contracts we use in our equilibrium constructions, and is unavoidable when consumers have strictly convex preferences.

5.2 A Proposal for Actual Regulation

The main insight of our analysis is that penalizing cross-subsidies between contracts may play a key role in regulating insurance markets under multiple contracting and adverse selection. This reflects three types of considerations.

From a normative viewpoint, our regulation allows a policy maker to reach third-best efficiency without shutting down competition. Indeed, equilibrium leads to competitive pricing in spite of the restrictions imposed on firms' pricing strategies: thus our regulation does not hinder competition and does not prevent market forces from singling out a unique equilibrium allocation. Notice, in that respect, that fair pricing of layers, and not of aggregate

[21] By contrast, the equilibrium construction sketched in Section 3.3 in the special case where $u_2(Q^*_2, T^*_2) = u_2(2Q^*_1, 2T^*_1)$ relies on each firm also supplying any amount of complementary coverage at price r_2.

coverage amounts, is the relevant notion of competitiveness under multiple contracting because consumers are free to combine contracts issued by different firms.

From a practical viewpoint, our regulation is not overly demanding in informational terms. Indeed, if a firm attempts to deviate by cross-subsidizing between contracts, it attracts all consumers of type 2. Thus, even if the loss the firm incurs on each complementary-coverage contract it trades is small, the law of large numbers implies that it can be detected and punished by the regulator.[22] Moreover, in the light of our equilibrium construction, what matters for the regulator is the ability to detect firms that simultaneously sell basic and complementary coverage and that screen the two types using these two types of contracts. Hence an alternative regulation would be to assess the risk borne by each contract and to require that it be at least equal to the consumers' average riskiness.[23] Overall, the regulator only needs to observe the total profits a firm earns on each contract.[24]

Finally, our proposal does not rely on firms exchanging information about their customers. Actual regulation sometimes encourages this practice;[25] yet communication between firms requires a rather sophisticated institutional setting to take place. Thus, while firms under multiple contracting may in principle benefit from accessing information about all their customers' trades, the aggregation of this dispersed information would in practice involve complex information-sharing mechanisms, possibly with several rounds of communication. Our regulation bearing on realized profits seems in comparison easier to implement.

5.3 Positive Implications

Our analysis so far has been essentially normative. However, instead of penalizing cross-subsidies between contracts, the regulation we propose can also be seen as banning profits

[22]We may also argue that, in practice, to attract all consumers of type 2, the deviating firm would have to charge a premium rate significantly lower than r_2 or pay significant advertising costs that the regulator could also observe.

[23]With more than two types of consumers, the regulator would have to assess the risk borne by each contract traded by a firm and compare it to the terms of trade.

[24]This is consistent with the recent evolution of financial reporting standards. Indeed, over the last decade, the International Accounting Standards Board has suggested several measures aimed at defining general principles that an entity should apply to report information in its financial statements about the nature of cash flows from insurance contracts (IASB (2013)). In particular, since 2011, insurance companies have been required to perform an onerous-contract test when circumstances indicate that the contract might be loss-making. As soon as a contract is so assessed, the company has to record a provision in its financial statements for the corresponding expected loss.

[25]The Commission Regulation 267/2010 of March 24, 2010 states: "Collaboration between insurance undertakings or within associations of undertakings in the compilation of information (which may also involve some statistical calculations) allowing the calculation of the average cost of covering a specified risk in the past or, for life insurance, tables of mortality rates or of the frequency of illness, accident and invalidity, makes it possible to improve the knowledge of risks and facilitates the rating of risks for individual companies. This can in turn facilitate market entry and thus benefit consumers."

on basic-coverage contracts, a measure that is already in place in several insurance markets. For instance, the health-insurance systems in Germany and Switzerland rely on a central fund to redistribute costs among firms according to a risk-equalization scheme.[26] These cost-sharing mechanisms, by pooling and redistributing costs among sellers of a standardized basic-coverage contract, prevent firms from earning abnormal profits on basic coverage.[27] Because our analysis of the regulated game provides an equilibrium-existence result, it may thus provide valuable insights on the positive implications of multiple contracting, in particular for these insurance markets where a similar regulation is used.

5.3.1 Quantity Discounts

To formulate these insights, we focus on conditions (10)–(11) for a free-entry equilibrium, which imply that traded contracts offering higher coverage have lower premium rates,

$$Q_1^* > Q_2^* - Q_1^* \quad \text{and} \quad \frac{T_1^*}{Q_1^*} = r < r_2 = \frac{T_2^* - T_1^*}{Q_2^* - Q_1^*}.$$

Thus, while consumers pay a quantity premium for higher aggregate coverage, firms issue contracts that exhibit quantity discounts. In observable terms, this is a striking difference with the exclusive-competition case, in which whether data on traded contracts are collected from consumer surveys or from the trade records of a single firm is irrelevant as each consumer's demand must be met by a single contract issued by a single firm.

This result stands in stark contrast with the natural intuition that allowing for multiple contracting should push consumers towards splitting their demands between firms:[28] indeed, this intuition is misleading unless each firm is active, as in the oligopoly case. The reason why, in a free-entry equilibrium, firms end up proposing quantity discounts is that the basic layer must be larger than the complementary layer to prevent type 2 from purchasing basic coverage from different firms and to prevent inactive firms from entering the market.[29] This

[26] See Thomson and Mossialos (2009, page 84). Besides Germany and Switzerland, other countries using such schemes include Australia, Ireland, the Netherlands, and Slovenia.

[27] In Switzerland, the basic-coverage contract is defined at the national level; then firms compete over prices to provide the corresponding amount of coverage. Yet an additional rule specifies that costs are pooled and redistributed among firms. In Germany, the basic-coverage contract is also defined at the national level and was offered in 2009 by 134 not-for-profit, nongovernmental "sickness funds." Consumers contribute a fixed fraction of their wealth; these contributions are then centrally pooled and redistributed to sickness funds according to a rather precise risk-adjusted capitation formula. More generally, risk equalization involves transfer payments between firms so as to spread some of the claims cost of the high-risk, older, and less healthy consumers among all firms in the market, in proportion to their respective market shares.

[28] See, for instance, Chiappori (2000) for an articulation of this view.

[29] This explanation for quantity discounts differs from that proposed by Biais, Martimort, and Rochet (2000) and Chade and Schlee (2012) in the monopolistic case of Stiglitz (1977). In both papers, the shape of the hazard rate of the distribution of types plays an essential role.

leads firms to only issue a few contracts that consumers can combine. In turn, consumers find it in their interest to concentrate their trades on a minimum number of contracts: type 1 ends up trading a single contract, and type 2 two different contracts.

5.3.2 A Negative-Correlation Property

Since Chiappori and Salanié (2000), many empirical studies have tested the validity of the positive-correlation property, which states that, under adverse selection, there should be a positive correlation between the coverage purchased by a consumer and her riskiness. Due to the single-crossing condition, this property still holds in our setting when we consider the aggregate coverage bought by a consumer: indeed, riskier consumers are also those who are more eager to purchase more coverage.[30]

Yet the above mentioned contrast, under multiple contracting, between the implications of equilibrium for the demand and supply sides of the market is also relevant for the positive-correlation property. Indeed, an implication of free-entry equilibrium is that, with data originating from a single firm, we should observe a negative correlation between risk and coverage, because the relatively small complementary layer is only purchased by type 2. Finally, a robust prediction of our analysis is that consumers holding more than one insurance policy should on average be more likely to experience greater losses.

These general observations are useful for assessing the empirical evidence, as exemplified by the work of Cawley and Philipson (1999) on life insurance, that of Cardon and Hendel (2001) on health insurance, and that of Finkelstein and Poterba (2004) on annuities. Because these papers take as a benchmark the exclusive-competition model, the above distinction between demand- and supply-side approaches is overlooked. As a result, the absence of quantity premia or the failure of the positive-correlation property are interpreted as rejecting the presence of adverse selection; yet multiple contracting is allowed and even prevalent on these markets, leading to a potential misspecification problem. Admittedly, these markets are not regulated in a manner similar to the one we propose. However, when it comes to markets that are subject to such a regulation, such as the German and Swiss health-insurance markets, our results suggest that we should be careful when testing for the existence of quantity premia or for the positive-correlation property: in principle, we would need to observe each consumer's aggregate coverage and aggregate premia. In particular, checking only the contracts traded by firms would be insufficient and even misleading.

[30] Chiappori, Jullien, Salanié, and Salanié (2006) show that this property, and similar ones, can be derived in much more general settings from a simple inequality on equilibrium profits even when single crossing is not postulated.

6 Concluding Remarks

In modern economies, the insurance sector plays a key role by allowing agents to share risk. Because those risks are often private information, the properties of equilibrium allocations, and in fact the very existence of equilibrium, are still the subject of a lively debate among academics. The absence of consensus on the justifications and on the right design of public intervention may also be related to the fact that different countries display strikingly different regulatory systems for, in particular, health insurance.

This paper has put multiple contracting and adverse selection at the center stage of the analysis. Our main insight is that public intervention under these constraints should target firms' pricing strategies, while leaving consumers free to choose their preferred amount of coverage. The regulation we have proposed achieves this goal by penalizing cross-subsidies between contracts. As we have shown, this allows market forces to reach third-best efficiency, leading to an allocation in which each layer of coverage is fairly priced given the consumer types who purchase it. We have argued that this regulation is relatively light-handed and should be easy to implement.

We have followed Rothschild and Stiglitz (1976) and Attar, Mariotti, and Salanié (2014) in assuming that there are only two types of consumers, which imposed some discipline on the analysis. Though the details of the equilibrium characterization and of the equilibrium construction are likely to depend on this assumption, the general message that one of the goals of regulation should be to proscribe cross-subsidies between contracts is of independent interest. The key point is that the risk borne by each contract traded by a firm should reflect an upper-tail conditional expectation of riskiness in the population. Whether this can be achieved by market forces curbed by a light-handed regulation or necessitates a more direct intervention is an interesting avenue for future research.

Finally, our analysis, by being the first to provide positive existence and efficiency results for insurance markets under multiple contracting and adverse selection, opens a new and rich avenue for empirical research. We also hope that it will renew the existing policy debates about health insurance, and more generally about the management of financial markets plagued by adverse selection.

Appendix

Proof of Theorem 1. Suppose there exists an equilibrium, and fix an equilibrium in which every firm k posts the menu C^k. Then

$$z_i^{-k}(q,t) \equiv \max\left\{u_i\left(q + \sum_{l\neq k} q^l, t + \sum_{l\neq k} t^l\right) : (q^l,t^l) \in C^l \text{ for all } l \neq k\right\} \tag{A.1}$$

is type i's indirect utility from trading a contract (q,t) with firm k given the menus posted by firms $l \neq k$, which is continuous in (q,t) by Berge's maximum theorem (Aliprantis and Border (2006, Theorem 17.31)). The equilibrium outcome is specified by individual trades (q_i^k, t_i^k) between every type i and every firm k, leading to aggregate trades $(Q_i,T_i) \equiv (\sum_k q_i^k, \sum_k t_i^k)$ and utility levels $\mathbf{u}_i \equiv u_i(Q_i,T_i)$ for every type i, and to type-by-type profits $b_i^k \equiv t_i^k - r_i q_i^k$ and type-averaged profits $b^k \equiv m_1 b_1^k + m_2 b_2^k$ for every firm k.

Our first result relates individual and aggregate profits on the complementary layers $q_2^k - q_1^k$ and $Q_2 - Q_1$, which are given by

$$s_2^k \equiv t_2^k - t_1^k - r_2(q_2^k - q_1^k) \quad \text{and} \quad S_2 \equiv T_2 - T_1 - r_2(Q_2 - Q_1),$$

respectively. Observe that $s_2^k \neq 0$ implies $q_1^k \neq q_2^k$ in equilibrium.

Lemma A.1 *For each k,*

$$S_2 > \max\{0, s_2^k\} \textit{ implies } q_1^k = q_2^k \neq 0.$$

Proof. Suppose $S_2 > \max\{0, s_2^k\}$ and, for each $\varepsilon_2 \geq 0$, define the contract

$$c_2^k(\varepsilon_2) \equiv (q_1^k + Q_2 - Q_1, t_1^k + T_2 - T_1 - \varepsilon_2).$$

This contract strictly attracts type 2 if $\varepsilon_2 > 0$ because, by combining it with the contracts (q_1^l, t_1^l) issued by firms $l \neq k$, she ends up with the aggregate trade $(Q_2, T_2 - \varepsilon_2)$. The proof consists of three steps.

Step 1 We first claim that $z_1^{-k}(c_2^k(0)) < \mathbf{u}_1$. Otherwise, firm k can deviate by issuing a contract $c_2^k(\varepsilon_2)$ with $\varepsilon_2 > 0$, which strictly attracts both types. Letting ε_2 go to zero, we obtain that, in equilibrium,

$$b^k \geq t_1^k - rq_1^k + T_2 - T_1 - r(Q_2 - Q_1).$$

Using the accounting identity

$$b^k = m_1(t_1^k - r_1 q_1^k) + m_2(t_2^k - r_2 q_2^k) = t_1^k - rq_1^k + m_2 s_2^k,$$

we can rewrite this inequality as

$$0 \geq S_2 - m_2 s_2^k + (r_2 - r)(Q_2 - Q_1).$$

But the first difference on the right-hand side of this inequality is strictly positive, because $S_2 > s_2^k > m_2 s_2^k$ if $s_2^k > 0$ and $S_2 > 0 \geq m_2 s_2^k$ otherwise, and the second difference is nonnegative, because $Q_2 \geq Q_1$ by single crossing. We have thus reached a contradiction. The claim follows.

Step 2 We next claim that $t_1^k > r_1 q_1^k$. Indeed, if firm k deviates by issuing a contract $c_2^k(\varepsilon_2)$ with $\varepsilon_2 > 0$ small enough, it strictly attracts type 2 and does not attract type 1 by Step 1. Letting ε_2 go to zero, we obtain that, in equilibrium,

$$b^k \geq m_2[t_1^k - r_2 q_1^k + T_2 - T_1 - r_2(Q_2 - Q_1)]$$

or, equivalently,

$$m_1(t_1^k - r_1 q_1^k) \geq m_2(S_2 - s_2^k) > 0.$$

The claim follows.

Step 3 To safeguard the profit it makes with type 1, firm k can deviate by issuing two contracts, namely, a contract $c_2^k(\varepsilon_2)$ with $\varepsilon_2 > 0$, and a contract

$$c_1^k(\varepsilon_1) \equiv (q_1^k, t_1^k - \varepsilon_1)$$

with $\varepsilon_1 > 0$. Consider first type 1. By Step 1,

$$z_1^{-k}(c_2^k(0)) < \mathbf{u}_1 = z_1^{-k}(c_1^k(0)).$$

If ε_2 is small enough, this implies

$$z_1^{-k}(c_2^k(\varepsilon_2)) < \mathbf{u}_1 < z_1^{-k}(c_1^k(\varepsilon_1)).$$

Then type 1 trades $c_1^k(\varepsilon_1)$ following firm k's deviation and, if ε_1 is small enough, firm k makes a profit with type 1 by Step 2. Consider next type 2. By construction,

$$z_2^{-k}(c_1^k(0)) \leq \mathbf{u}_2 \leq z_2^{-k}(c_2^k(0)).$$

If ε_1 is small enough compared to ε_2, this implies

$$\max\{z_2^{-k}(c_1^k(\varepsilon_1)), \mathbf{u}_2\} < z_2^{-k}(c_2^k(\varepsilon_2)).$$

Then type 2 trades $c_2^k(\varepsilon_2)$ following firm k's deviation, and firm k earns a profit

$$t_1^k - r_2 q_1^k + S_2 - \varepsilon_2 = t_2^k - r_2 q_2^k + S_2 - s_2^k - \varepsilon_2$$

with type 2. If $t_2^k - r_2 q_2^k + S_2 - s_2^k > 0$ and ε_2 is small enough, then the deviation entails no cross-subsidies, and hence is consistent with the regulation. Letting ε_1 and ε_2 go to zero, we obtain that, in equilibrium,

$$b^k \geq m_1(t_1^k - r_1 q_1^k) + m_2(t_2^k - r_2 q_2^k + S_2 - s_2^k)$$

or, equivalently,

$$0 \geq m_2(S_2 - s_2^k),$$

a contradiction. Hence $t_2^k - r_2 q_2^k + S_2 - s_2^k \leq 0$, which implies $t_2^k - r_2 q_2^k < 0$ and thus $q_2^k \neq 0$. This is consistent with the regulation only if $q_1^k = q_2^k$. The result follows. ■

A consequence of Lemma A.1 is that the aggregate profit on the complementary layer $Q_2 - Q_1$ is nonpositive.

Proposition A.1 $S_2 \leq 0$.

Proof. Suppose, by way of contradiction, that $S_2 > 0$. The proof consists of six steps.

Step 1 Our first observation is that $S_2 > 0$ implies that $s_2^k \geq 0$ for all k. Otherwise, $s_2^k < 0$ for some k and thus $q_1^k = q_2^k$ by Lemma A.1, a contradiction. Moreover, because $S_2 > 0$, there exists some l such that $s_2^l > 0$ and hence $q_1^l \neq q_2^l$. Lemma A.1 then implies $s_2^l \geq S_2$, and, from our first observation, we obtain that $s_2^l = S_2 > 0$ and $s_2^k = 0$ for all $k \neq l$. Thus firm l trades different contracts (q_1^l, t_1^l) and (q_2^l, t_2^l) with types 1 and 2, respectively, and, by Lemma A.1 again, every firm $k \neq l$ trades the same contract $(q^k, t^k) \neq (0, 0)$ with each type. In particular, $Q_1 > 0$.

Step 2 Second, we claim that $b^k = 0$ for all $k \neq l$. Let $(Q^{-l}, T^{-l}) \equiv (\sum_{k \neq l} q^k, \sum_{k \neq l} t^k)$. Any firm $k \neq l$ can deviate by issuing a contract $(Q^{-l}, T^{-l} - \varepsilon)$ with $\varepsilon > 0$, which strictly attracts both types. Letting ε go to zero, we obtain that, in equilibrium,

$$b^k \geq T^{-l} - r Q^{-l}$$

for all $k \neq l$. The claim then follows from the fact that

$$T^{-l} - r Q^{-l} = \sum_{k \neq l} b^k$$

and that there are at least two firms $k \neq l$ as $n \geq 3$. Notice that, because $r_2 > r_1$ and every firm $k \neq l$ is active on the equilibrium path, $b_2^k < 0$ for all $k \neq l$.

Step 3 Third, we claim that $\tau_1(Q_1, T_1) = r$. Otherwise, any firm $k \neq l$ can deviate by issuing a contract $(q^k + \delta, t^k + \varepsilon)$ with δ and ε chosen so that

$$\tau_1(Q_1, T_1)\delta > \varepsilon > r\delta.$$

The first inequality ensures that this contract strictly attracts type 1 if δ and ε are small enough. If it attracts type 2, then firm k's profit increases by $\varepsilon - r\delta$, a contradiction. If it does not attract type 2, then, because $b_2^k < 0$ by Step 2, firm k's profit increases by $m_1(\varepsilon - r_1\delta) - m_2 b_2^k > 0$, once again a contradiction. The claim follows.

Step 4 Fourth, we claim that $\mathbf{u}_1 = z_1^{-k}(0,0)$ for all $k \neq l$. Otherwise, $\mathbf{u}_1 > z_1^{-k}(0,0)$ for some $k \neq l$. Then firm k can deviate by issuing a contract $(q^k, t^k + \varepsilon)$ with $\varepsilon > 0$, which strictly attracts type 1. If it attracts type 2, then firm k's profit increases by ε, a contradiction. If it does not attract type 2, then, because $b_2^k < 0$ by Step 2, firm k's profit increases by $m_1\varepsilon - m_2 b_2^k > m_1\varepsilon$, once again a contradiction. The claim follows. As a result, for each $k \neq l$, there exists an aggregate trade (Q^{-k}, T^{-k}) made available by firms $m \neq k$ and such that $u_1(Q^{-k}, T^{-k}) = \mathbf{u}_1$.

Step 5 Fifth, we claim that $Q^{-k} < Q_1$. Notice that $Q^{-k} \leq Q_2$; otherwise, because $u_1(Q^{-k}, T^{-k}) = \mathbf{u}_1 \geq u_1(Q_2, T_2)$ by incentive compatibility, we have $u_2(Q^{-k}, T^{-k}) > \mathbf{u}_2$ by single crossing, a contradiction. We cannot have $Q^{-k} = Q_2$; otherwise, because $S_2 > 0$ and $\tau_1(Q_1, T_1) = r$ by Step 3, we again have $u_2(Q^{-k}, T^{-k}) > \mathbf{u}_2$. Finally, suppose, by way of contradiction, that $Q_1 \leq Q^{-k} < Q_2$. Then firm k can deviate by issuing the contract $(Q_2 - Q^{-k}, T_2 - T^{-k} - \varepsilon)$ with $\varepsilon > 0$, which strictly attracts type 2 and possibly type 1 as well. However, because $u_1(Q^{-k}, T^{-k}) = \mathbf{u}_1$ and $S_2 > 0$, the premium rate of this contract is strictly greater than $\frac{T_2 - T_1}{Q_2 - Q_1} > r_2$ if ε is small enough, in contradiction with the zero-profit result in Step 2. The claim follows.

Step 6 By Steps 4–5, for every firm $k \neq l$, there exists an aggregate trade (Q^{-k}, T^{-k}) made available by firms $m \neq k$ and such that $u_1(Q^{-k}, T^{-k}) = \mathbf{u}_1$ and $Q^{-k} < Q_1$. Because $\tau_1(Q_1, T_1) = r$ by Step 3, $\tau_1(Q^{-k}, T^{-k}) > r$. Then firm k can deviate by issuing a contract (δ, ε) with $\delta > 0$ and $\varepsilon > 0$ chosen so that

$$\tau_1(Q^{-k}, T^{-k})\delta > \varepsilon > r\delta.$$

The first inequality implies that this contract strictly attracts type 1 if δ and ε are small enough. If it attracts type 2, then firm k's profit is $\varepsilon - r\delta > 0$, in contradiction with the zero-profit result in Step 2. If it does not attract type 2, then, because $b_2^k < 0$ by Step 2, firm k's profit is $m_1(\varepsilon - r_1\delta) - m_2 b_2^k > m_1(\varepsilon - r_1\delta) > 0$, once again in contradiction with Step 2. Hence the result. ■

Our next goal is to show that each firm earns zero profit in equilibrium, that is, $B \equiv \sum_k b^k = 0$. The argument relies on the following intuitive lemma.

Lemma A.2 $T_1 \leq r_2 Q_1$.

Proof. Suppose, by way of contradiction, that $T_1 > r_2 Q_1$. In particular, $0 < Q_1 \leq Q_2$. Any firm k can then deviate by issuing the contract $(Q_1, T_1 - \varepsilon_1)$ with $\varepsilon_1 > 0$, which strictly attracts type 1. Because $T_1 > r_2 Q_1$, the worst case for firm k is that it does not attract type 2. Letting ε_1 go to zero, we obtain that, in equilibrium,

$$b^k \geq m_1(T_1 - r_1 Q_1)$$

or, equivalently,

$$m_2(T_2 - r_2 Q_2) \geq B - b^k.$$

Therefore, $T_2 \geq r_2 Q_2$. Any firm k can then deviate by issuing the contract $(Q_2, T_2 - \varepsilon_2)$ with $\varepsilon_2 > 0$, which strictly attracts type 2. Because $T_2 \geq r_2 Q_2 > r_1 Q_2$, the worst case for firm k is that it does not attract type 1. Letting ε_2 go to zero, we obtain that, in equilibrium,

$$b^k \geq m_2(T_2 - r_2 Q_2) \geq B - b^k$$

for all k. Summing these inequalities over k yields $(n-2)B \leq 0$ and hence, because $n \geq 3$, $B = b^k = T_2 - r_2 Q_2 = 0$ for all k. This implies $T_1 = r_1 Q_1$, in contradiction with $T_1 > r_2 Q_1$. The result follows. ■

A consequence of Proposition A.1 and Lemma A.2 is that the aggregate profits on both the basic and the complementary layers Q_1 and $Q_2 - Q_1$ are zero, and thus that each firm earns zero profit.

Proposition A.2 $B = T_1 - rQ_1 = S_2 = 0$.

Proof. Any firm k can deviate by issuing the contract $(Q_1, T_1 - \varepsilon_1)$ with $\varepsilon_1 > 0$, which strictly attracts type 1. Because $T_1 \leq r_2 Q_1$ by Lemma A.2, the worst case for firm k is that this contract attracts type 2. Letting ε_1 go to zero, we obtain that, in equilibrium,

$$b^k \geq T_1 - rQ_1.$$

Using the accounting identity

$$B = T_1 - rQ_1 + m_2 S_2,$$

we can rewrite this inequality as

$$B - b^k \leq m_2 S_2$$

for all k. Because $S_2 \leq 0$ by Proposition A.1, we obtain that $B = b^k = 0$ for all k, $S_2 = 0$, and $T_1 - rQ_1 = 0$. Hence the result. ■

We are now ready to complete the proof of Theorem 1.

Proposition A.3 $(Q_1, T_1) = (Q_1^*, T_1^*)$.

Proof. We know that each firm earns zero profit in equilibrium. If any firm k withdraws its menu offer, then every type i obtains at most her equilibrium utility, that is,

$$\mathbf{u}_i \geq z_i^{-k}(0,0). \tag{A.2}$$

Now, observe that

$$z_1^{-k}(0,0) \geq \max\{u_1(Q, rQ) : Q \geq 0\}. \tag{A.3}$$

Otherwise, firm k can deviate by issuing a contract at a premium rate slightly higher than r that strictly and profitably attracts type 1 and that remains profitable even if it attracts type 2. Chaining (A.2)–(A.3) and using the fact that $T_1 = rQ_1$ by Proposition A.2, we obtain that (Q_1, T_1) satisfies (6)–(7) and thus coincides with (Q_1^*, T_1^*). Hence the result. ■

Remark A.1 The aggregate trade (Q_1^*, T_1^*) can only be reached by means of contracts with premium rate r. Otherwise, some contract (q, t) such that $q < Q_1^*$ and $t < rq$ is issued by some firm. Any other firm can then issue the contract $(Q_1^* - q, T_1^* - t - \varepsilon)$ with $\varepsilon > 0$. This contract strictly attracts type 1 in combination with (q, t) and, because $T_1^* - t > r(Q_1^* - q)$, it is profitable for ε small enough even if it attracts type 2, a contradiction.

Lemma A.3 *The aggregate trade (Q_1^*, T_1^*) remains available if any firm k withdraws its menu offer.*

Proof. By Proposition A.3,

$$\mathbf{u}_1 = u_1(Q_1^*, T_1^*) = \max\{u_1(Q, rQ) : Q \geq 0\}.$$

Hence, by (A.2)–(A.3),

$$\mathbf{u}_1 = z_1^{-k}(0,0)$$

for all k. As a result, for each k, there exists an aggregate trade (Q^{-k}, T^{-k}) made available by firms $l \neq k$ and such that $u_1(Q^{-k}, T^{-k}) = \mathbf{u}_1$. We show that $Q^{-k} = Q_1^*$ by eliminating the other cases. First, if $Q^{-k} < Q_1^*$, then, because $\tau_1(Q_1^*, T_1^*) = r$ and $T_1^* = rQ_1^*$ by Proposition A.3, the premium rate of (Q^{-k}, T^{-k}) is lower than r, in contradiction with the reasoning of Remark A.1. Second, if $Q^{-k} \geq Q_2^*$, then $u_2(Q^{-k}, T^{-k}) > \mathbf{u}_2$ by single crossing, a contradiction. Third, if $Q_1^* < Q^{-k} < Q_2^*$, then firm k can profitably deviate as in Step 5 of the proof of Proposition A.1, once again a contradiction. The result follows. ■

Proposition A.4 $(Q_2, T_2) = (Q_2^*, T_2^*)$.

Proof. For each k, we have, in analogy with (A.3),

$$z_2^{-k}(0,0) \geq \max\{u_2(Q_1^* + Q, T_1^* + r_2 Q) : Q \geq 0\}. \tag{A.4}$$

Otherwise, firm k can issue a contract at a premium rate slightly higher than r_2 that strictly and profitably attracts type 2 in combination with the aggregate trade (Q_1^*, T_1^*) made available by firms $l \neq k$ by Lemma A.3, and that is even more profitable if it also attracts type 1. Chaining (A.2) and (A.4) and using the fact that $T_2 - T_1 = r_2(Q_2 - Q_1)$ by Proposition A.3, we obtain that (Q_2, T_2) satisfies (8)–(9) and thus coincides with (Q_2^*, T_2^*). Hence the result. ■

Remark A.2 Any traded contract is sold at premium rate r or r_2. Indeed, consider a contract (q, t) with $q > 0$ that is traded in equilibrium. If it attracts both types, then it yields zero profit; thus its premium rate is r. If it attracts only one type, then it must yields a nonnegative profit to be consistent with the regulation, and hence exactly zero profit; thus its premium rate is either r_1 or r_2. Finally, because Remark A.1 implies that a contract with premium rate r_1 cannot be traded by type 1 in equilibrium, this premium rate must be r_2.

Remark A.3 We know that no firm is indispensable to provide type 1 with her equilibrium aggregate trade. A slightly weaker property is satisfied for type 2, namely, that no firm is indispensable to provide her with her equilibrium utility: for each k, there exists an aggregate trade (Q^{-k}, T^{-k}) made available by firms $l \neq k$ and such that $u_2(Q^{-k}, T^{-k}) = \mathbf{u}_2$. We claim that $Q^{-k} \geq Q_2^*$. Otherwise, $T_2^* - T^{-k} > r_2(Q_2^* - Q^{-k})$ by (8)–(9) because u_2 is strictly quasiconcave. Firm k can then deviate by issuing a contract $(Q_2^* - Q^{-k}, T_2^* - T^{-k} - \varepsilon)$ with

$\varepsilon > 0$, which attracts type 2 in combination with the aggregate trade (Q^{-k}, T^{-k}) and is profitable for ε small enough, a contradiction.[31]

The proof of Theorem 1 is now complete. Hence the result. ■

Proof of Lemma 1. Suppose $Q_1^* > 0$ and that there exists a free-entry equilibrium, and fix such an equilibrium in which every firm k posts the menu C^k. Let K_r be the set of firms issuing contracts at premium rate r and, for each $k \in K_r$, let

$$C_r^k \equiv \{(q,t) \in C^k : q > 0 \text{ and } t = rq\}$$

be the set of such contracts issued by firm k. Finally, for all $k \in K_r$ and $(q,t) \in C_r^k$, let

$$\alpha(q,t) \equiv \frac{q}{Q_1^*}$$

be the fraction of Q_1^* covered by the contract (q,t). Now, consider some firm $k \in K_r$ that trades a contract $(q^k, t^k) \in C_r^k$ on the equilibrium path, so that

$$0 < \alpha(q^k, t^k) \leq 1. \tag{A.5}$$

By Lemma A.3, type 1 can still trade (Q_1^*, T_1^*) if firm k withdraws its menu offer, and, by Remark A.1, the aggregate trade (Q_1^*, T_1^*) can only be reached through contracts with premium rate r. Hence there exists a subset K_r^{-k} of $K_r \setminus \{k\}$ and contracts $(q^l, t^l) \in C_r^l$ issued by firms $l \in K_r^{-k}$ such that

$$\sum_{l \in K_r^{-k}} \alpha(q^l, t^l) = 1. \tag{A.6}$$

Summing (A.5)–(A.6) yields

$$1 < \sum_{l \in K_r^{-k}} \alpha(q^l, t^l) + \alpha(q^k, t^k) \leq 2.$$

Because the aggregate trade $[1 + \alpha(q^k, t^k)](Q_1^*, T_1^*)$ is available on the equilibrium path, we must have

$$u_2(Q_2^*, T_2^*) \geq u_2([1 + \alpha(q^k, t^k)](Q_1^*, T_1^*)). \tag{A.7}$$

To conclude the proof, we only need to show that

$$[1 + \alpha(q^k, t^k)]Q_1^* > Q_2^*. \tag{A.8}$$

[31] Unlike for type 1, the equilibrium aggregate trade of type 2 need not remain available if any firm who trades with her withdraws its menu offer. In Attar, Mariotti, and Salanié (2014), this property is satisfied because trades of negative quantities are allowed, which is not the case in the present setting.

Indeed, (A.8) implies $2Q_1^* > Q_2^*$, which is (11). In turn, along with (A.7) and $2Q_1^* > Q_2^*$, (A.8) implies $u_2(Q_2^*, T_2^*) \geq u_2(2Q_1^*, 2T_1^*)$, which is (10). To establish (A.8), observe first that, because $T_2^* > rQ_2^*$, it must be that

$$[1 + \alpha(q^k, t^k)]Q_1^* \neq Q_2^*.$$

Let us suppose, by way of contradiction, that $[1 + \alpha(q^k, t^k)]Q_1^* < Q_2^*$. Then an inactive firm can issue the contract $(Q_2^* - [1 + \alpha(q^k, t^k)]Q_1^*, T_2^* - r[1 + \alpha(q^k, t^k)]Q_1^* - \varepsilon)$ with $\varepsilon > 0$, which attracts type 2 in combination with the aggregate trade $[1 + \alpha(q^k, t^k)](Q_1^*, T_1^*)$. Because the JHG allocation satisfies

$$T_1^* - rQ_1^* = T_2^* - T_1^* - r_2(Q_2^* - Q_1^*) = 0,$$

we find that the corresponding profit on type 2,

$$T_2^* - r[1 + \alpha(q^k, t^k)]Q_1^* - r_2\{Q_2^* - [1 + \alpha(q^k, t^k)]Q_1^*\} - \varepsilon = (r_2 - r)\alpha(q^k, t^k)Q_1^* - \varepsilon,$$

is strictly positive for ε small enough. Hence this contract is profitable even if it does not attract type 1, a contradiction. The result follows. ■

Proof of Lemma 2. Suppose $Q_2^* > Q_1^* > 0$ and that there exists an equilibrium, and fix an equilibrium in which every firm k posts the menu C^k. Because $Q_1^* > 0$, some firm k sells $q_1^k > 0$ to type 1 on the equilibrium path. Because all the contracts traded by type 1 on the equilibrium path have premium rate r, q_1^k is sold for a transfer rq_1^k.

For all i and Q, let $\mathcal{I}_i^*(Q)$ be the transfer for the aggregate coverage Q along the equilibrium indifference curve of type i, that is, $u_i(Q, \mathcal{I}_i^*(Q)) = u_i(Q_i^*, T_i^*)$. By assumption, $\mathcal{I}_i^*$ is strictly concave and continuously differentiable, except possibly at zero. We also define A^{-k} as the set of aggregate trades (Q, T) that can be reached with firms $l \neq k$, that is, $A^{-k} = \sum_{l \neq k} C^l$. Because A^{-k} is compact, the function

$$y_i^{-k}(q) \equiv \max\{\mathcal{I}_i^*(q + Q) - T : (Q, T) \in A^{-k}\} \tag{A.9}$$

is well-defined. Now, suppose that firm k deviates by issuing the contract (q, t). This contract attracts type i if and only if there exists $(Q, T) \in A^{-k}$ such that

$$u_i(q + Q, t + T) \geq u_i(Q_i^*, T_i^*)$$

or, equivalently, $t \leq y_i^{-k}(q)$. Therefore, if $r_1 q < t < y_1^{-k}(q)$, the contract (q, t) strictly and profitably attracts type 1. Hence, it must also attract type 2, and this must make the deviation unprofitable. This shows that $r_1 q < t < y_1^{-k}(q)$ implies that $t \leq y_2^{-k}(q)$ and $t \leq rq$, or, equivalently, that $r_1 q < y_1^{-k}(q)$ implies that $y_1^{-k}(q) \leq y_2^{-k}(q)$ and $y_1^{-k}(q) \leq rq$.

Now, because type 1 purchases aggregate coverage Q_1^* at price r in equilibrium, it must be that $(Q_1^* - q_1^k, r(Q_1^* - q_1^k))$ belongs to A^{-k}. Therefore,

$$y_1^{-k}(q) \geq Y^{-k}(q) \equiv \mathcal{I}_1^*(q + Q_1^* - q_1^k) - r(Q_1^* - q_1^k).$$

The function $Y^{-k}(q)$ on the right-hand side of this inequality is strictly concave in q, with value rq_1^k and derivative r at $q = q_1^k$. This shows that the condition $r_1 q < y_1^{-k}(q)$ is satisfied in a neighborhood $\mathcal{Q}$ of q_1^k. From our previous result, it follows that $Y^{-k}(q) \leq y_1^{-k}(q) \leq y_2^{-k}(q)$ for all $q \in \mathcal{Q}$. Moreover, because the contract (q_1^k, rq_1^k) is issued by firm k on the equilibrium path, it must be that type 2 can at most obtain her equilibrium utility if she selects this contract; that is, it must be that $y_2^{-k}(q_1^k) \leq rq_1^k$. These two facts together imply $Y^{-k}(q_1^k) = y_1^{-k}(q_1^k) = y_2^{-k}(q_1^k) = rq_1^k$.

Consider next a strictly increasing sequence $(q_N)_{N \in \mathbb{N}}$ in $\mathcal{Q}$ converging to q_1^k, and, for each N, fix some solution (Q_N, T_N) to problem (A.9) for $i = 2$ and $q = q_N$, so that

$$\text{for each } N \in \mathbb{N},\ y_2^{-k}(q_N) = \mathcal{I}_2^*(q_N + Q_N) - T_N. \tag{A.10}$$

Because $(Y^{-k})'(q_1^k) = r$, there exists a function $\eta : \mathcal{Q} \to \mathbb{R}$ such that $\lim_{q \to q_1^k} \eta(q) = 0$ and $Y^{-k}(q) = Y^{-k}(q_1^k) + r(q - q_1^k) + \eta(q)(q - q_1^k)$ for all $q \in \mathcal{Q}$. Hence

$$\begin{aligned}
\text{for each } N \in \mathbb{N},\ r + \eta(q_N) &= \frac{Y^{-k}(q_1^k) - Y^{-k}(q_N)}{q_1^k - q_N} \\
&\geq \frac{y_2^{-k}(q_1^k) - y_2^{-k}(q_N)}{q_1^k - q_N} \\
&\geq \frac{\mathcal{I}_2^*(q_1^k + Q_N) - \mathcal{I}_2^*(q_N + Q_N)}{q_1^k - q_N},
\end{aligned} \tag{A.11}$$

where the first inequality follows from $Y^{-k}(q_1^k) = y_2^{-k}(q_1^k)$ and $Y^{-k}(q_N) \leq y_2^{-k}(q_N)$, and the second inequality follows from (A.10) and the definition of $y_2^{-k}(q_1^k)$.

Finally, for each N, (Q_N, T_N) belongs to the compact set A^{-k}, so that the sequence $\{(Q_N, T_N)\}_{N \in \mathbb{N}}$ admits a convergent subsequence, with limit $(\hat{Q}^{-k}, \hat{T}^{-k}) \in A^{-k}$. Moreover, by Berge's maximum theorem (Aliprantis and Border (2006, Theorem 17.31)), the sequence $\{y_2^{-k}(q_N)\}_{n \in \mathbb{N}}$ converges to $y_2^{-k}(q_1^k) = rq_1^k$. Taking limits in (A.10)–(A.11), we obtain

$$rq_1^k = \mathcal{I}_2^*(q_1^k + \hat{Q}^{-k}) - \hat{T}^{-k} \quad \text{and} \quad (\mathcal{I}_2^*)'(q_1^k + \hat{Q}^{-k}) \leq r.$$

This shows that the aggregate trade $(Q^P, T^P) \equiv (q_1^k + \hat{Q}^{-k}, rq_1^k + \hat{T}^{-k})$ belongs to the equilibrium indifference curve of type 2, with $\tau_2(Q^P, T^P) \leq r$, and is thus strictly at the right of (Q_2^*, T_2^*). Finally, consumers can reach (Q^P, T^P) in equilibrium because it is the sum of a contact (q_1^k, rq_1^k) issued by k and of an element of A^{-k}. The result follows. ■

Proof of Lemma 3. Suppose that the aggregate trade $(\hat{Q}^{-k}, \hat{T}^{-k})$ constructed in the proof of Lemma 2 is only built up from contracts with premium rates r or r_2. Then this also holds true for (Q^P, T^P), so that one can write

$$Q^P = Q + Q' \quad \text{and} \quad T^P = rQ + r_2 Q'.$$

Because $Q^P > Q_2^*$, it must be that $Q > Q_1^*$. If $Q' > 0$, then this construction involves at least one contract with premium rate r_2, which must be issued by some firm $l \neq k$. However, because $Q > Q_1^*$ and $u_2(Q^P, T^P) = u_2(Q_2^*, T_2^*)$, firm l can deviate by issuing a contract with a premium rate higher than r_2 that strictly and profitably attracts type 2 in combination with the aggregate trade (Q, rQ), and that is even more profitable if it also attracts type 1. We have thus reached a contradiction.

Therefore, as the aggregate trade $(\hat{Q}^{-k}, \hat{T}^{-k})$ is only built up from contracts with premium rates r or r_2, the only possibility is that it has premium rate r. It follows that $(Q^P, T^P) = (Q^R, T^R)$, where $u_2(Q^R, T^R) = u_2(Q_2^*, T_2^*)$ and $T^R = rQ^R$. Notice finally that $\hat{Q}^{-k} \leq Q_1^*$; otherwise, and by the same reasoning as above, firm k can strictly and profitably attract type 2. Because we also have $q_1^k \leq Q_1^*$, it must thus be that

$$Q^R = q_1^k + \hat{Q}^{-k} \leq 2Q_1^*,$$

which implies $u_2(2Q_1^*, 2T_1^*) \leq u_2(Q_2^*, T_2^*)$ by construction of (Q^R, T^R). By contraposition, we obtain that, if $u_2(2Q_1^*, 2T_1^*) > u_2(Q_2^*, T_2^*)$, then some firm $l \neq k$ must issue a contract with a premium rate different from r and r_2. The result follows. ■

Proof of Lemma 4. If a latent contract (q^ℓ, t^ℓ) issued by all firms in equilibrium blocks large cream-skimming deviations, then, by (12) applied to $(q, t) = (Q_1^*, T_1^*)$, we have

$$u_2(Q_1^* + q^\ell, T_1^* + t^\ell) \geq u_2(Q_2^*, T_2^*).$$

This inequality cannot be strict. Otherwise, on the equilibrium path, type 2 would be strictly better off trading (q^ℓ, t^ℓ) with one firm in combination with the aggregate trade (Q_1^*, T_1^*), which, by Lemma A.3, is made available by the other firms. Hence (13). Next, by (12), the translate of the upper contour set of (Q_1^*, T_1^*) for type 1 along the vector (q^ℓ, t^ℓ) lies in the upper contour set of (Q_2^*, T_2^*) for type 2. As these two sets intersect at $(Q_1^* + q^\ell, T_1^* + t^\ell)$

by (13), we obtain along the lines of Benveniste and Scheinkman (1979, Lemma 1) that the slope of type 2's indifference curve at $(Q_1^* + q^\ell, T_1^* + t^\ell)$ must be equal to the slope of type 1's indifference curve at (Q_1^*, T_1^*), that is, r. Hence (14). Finally, that (q_1^ℓ, t_1^ℓ) is well-defined, with $Q_1^* + q_1^\ell > Q_2^*$, follows from $\tau_2(Q_2^*, T_2^*) = r_2$ along with the fact that type 2's marginal rate of substitution strictly and continuously decreases along her equilibrium indifference curve and vanishes as aggregate coverage grows large. The result follows. ■

Proof of Lemma 5. Defining $\mathcal{I}_i^*$ as in the proof of Lemma 2, observe that, by construction,

$$\mathcal{I}_2^*(Q_1^* + q_1^\ell) = \mathcal{I}_1^*(Q_1^*) + t_1^\ell, \tag{A.12}$$

Suppose first that property C(i) is satisfied. By (A.12), a sufficient condition for the translation property (12) is

$$\text{for each } Q \in \operatorname{dom} \mathcal{I}_1^*,\ \mathcal{I}_1^{*\prime}(Q) \gtreqless \mathcal{I}_2^{*\prime}(Q + q_1^\ell) \text{ if } Q \lesseqgtr Q_1^*.$$

In turn, a sufficient condition for this is the following single-crossing condition:

$$\mathcal{I}_1^{*\prime}(Q) = \mathcal{I}_2^{*\prime}(Q + q_1^\ell) \text{ implies } \mathcal{I}_1^{*\prime\prime}(Q) < \mathcal{I}_2^{*\prime\prime}(Q + q_1^\ell),$$

which, under property C(i), is by (15) a direct implication of the identities

$$\tau_i(Q, \mathcal{I}_i^*(Q)) = \mathcal{I}_i^{*\prime}(Q) \quad \text{and} \quad \kappa_i(Q, \mathcal{I}_i^*(Q)) = -\frac{\mathcal{I}_i^{*\prime\prime}(Q)}{\{1 + [\mathcal{I}_i^{*\prime}(Q)]^2\}^{\frac{3}{2}}}$$

along the equilibrium indifference curve of type i.

Suppose next that property C(ii) is satisfied. We show that

$$\text{for each } Q \in \operatorname{dom} \mathcal{I}_1^*,\ \mathcal{I}_2^*(Q + q_1^\ell) = \mathcal{I}_1^*(Q) + t_1^\ell, \tag{A.13}$$

so that, over the relevant domains, $\mathcal{I}_2^*$ is obtained from $\mathcal{I}_1^*$ by translation along the vector (q_1^ℓ, t_1^ℓ), which again implies (12). Define implicitly a function ϕ by

$$\mathcal{I}_2^{*\prime}(\phi(Q) + q_1^\ell) = \mathcal{I}_1^{*\prime}(Q). \tag{A.14}$$

Notice that ϕ is strictly increasing as both $\mathcal{I}_1^*$ and $\mathcal{I}_2^*$ are strictly concave, and that

$$\phi(Q_1^*) = Q_1^* \tag{A.15}$$

by (14). For each $Q > 0$, we have $\mathcal{I}_2^{*\prime\prime}(\phi(Q) + q_1^\ell) > 0$ as $\kappa_2^*(\phi(Q) + q_1^\ell, \mathcal{I}_2^*(\phi(Q) + q_1^\ell)) > 0$ by Assumption C. Hence, by the implicit function theorem, ϕ is differentiable and

$$\text{for each } Q \in \operatorname{dom} \mathcal{I}_1^*,\ \mathcal{I}_2^{*\prime\prime}(\phi(Q) + q_1^\ell)\phi'(Q) = \mathcal{I}_1^{*\prime\prime}(Q). \tag{A.16}$$

Now, under property C(ii), (A.14) implies

$$\mathcal{I}_2^{*\prime\prime}(\phi(Q) + q_1^\ell) = \mathcal{I}_1^{*\prime\prime}(Q).$$

Hence, by (A.15)–(A.16), ϕ must be the identity function. This in turn implies, by (A.12) and (A.14), that (A.13) holds for all $Q > 0$. The result follows. ■

Proof of Theorem 2. We must show that, for n large enough, the regulated game has an equilibrium in which each firm posts the same menu C_n^* consisting of five contracts:

1. The no-trade contract $(0, 0)$;
2. The basic-coverage contract $c_{1,n}^* \equiv (q_{1,n}^*, t_{1,n}^*)$ defined by (16);
3. The complementary-coverage contract $c_2^* \equiv (Q_2^* - Q_1^*, T_2^* - T_1^*)$;
4. The latent contract $c_1^\ell \equiv (q_1^\ell, t_1^\ell)$ defined by (13)–(14);
5. The latent contract $c_{2,n}^\ell \equiv (q_{2,n}^\ell, t_{2,n}^\ell)$ defined by (17).

Notice that, by construction, all the nonnull contracts in C_n^* have premium rates at least equal to r. Moreover, because, by assumption, the complementary layer is strictly positive, we have $u_2(Q_2^*, T_2^*) > u_2(Q_1^*, T_1^*)$ and hence

$$u_2(Q_2^*, T_2^*) > u_2\left(\frac{n}{n-1}\,(Q_1^*, T_1^*)\right) = u_2(nc_{1,n}^*) \tag{A.17}$$

for n large enough. The proof consists of four steps.

Step 1 We first claim that, on the equilibrium path, consumers have a best response such that they trade according to the JHG allocation, so that each firm earns zero profit.

Consider first type 1. Because $(n-1)c_{1,n}^* = (Q_1^*, T_1^*)$ and $\tau_1(Q_1^*, T_1^*) = r$, and because all the nonnull contracts in C_n^* have premium rates at least equal to r, trading $n-1$ contracts $c_{1,n}^*$ with, say, firms $k = 1, \ldots, n-1$ is optimal for type 1, leading her to the aggregate trade (Q_1^*, T_1^*), as in the JHG allocation.

Consider next type 2. By trading $n-1$ contracts $c_{1,n}^*$ with firms $k = 1, \ldots, n-1$ and one contract c_2^* with firm n, type 2 can reach the aggregate trade (Q_2^*, T_2^*), as in the JHG allocation. Firms $k = 1, \ldots, n-1$ then each trade $c_{1,n}^*$ with both types 1 and 2 at premium rate r and firm n trades c_2^* with type 2 at premium rate r_2, so that each firm earns zero profit. We thus only have to check that type 2 has no profitable deviation. This follows from two observations. First, by (A.17), if n is large enough, type 2 is not tempted to trade the contract $c^*_{1,n}$ with each of the n firms. Second, by (13)–(14) and (17), trading some contracts c^ℓ_1 or $c^\ell_{2,n}$, possibly along one or several contracts $c^*_{1,n}$ or c^*_2, brings type 2 at best on the line with slope r that supports her upper contour set of (Q^*_2, T^*_2); indeed, the best she can do is to trade $n-1$ contracts $c^*_{1,n}$ and one contract c^ℓ_1, or $n-2$ contracts $c^*_{1,n}$ and one contract $c^\ell_{2,n}$, reaching the aggregate trade $(Q^*_1 + q^\ell_1, T^*_1 + t^\ell_1)$ and thus obtaining, by (13), the same utility as at (Q^*_2, T^*_2). The claim follows.

Step 2 We next prove that no firm k has a profitable deviation that only attracts type 2. With a slight abuse of terminology, we identify such a deviation C^k with the contract (q, t) type 2 chooses to trade in C^k according to her best response. The proof follows from three observations. First, type 2 can obtain her equilibrium utility $u_2(Q^*_2, T^*_2)$ by trading a contract $c^*_{1,n}$ with $n-2$ firms $l \neq k$ and a contract $c^\ell_{2,n}$ with the remaining firm $l \neq k$. Second, if type 2 at most trades contracts $c^*_{1,n}$ or c^*_2 with firms $l \neq k$ following firm k's deviation, then the contract (q, t) attracts her only if $r_2 q \geq t$, and thus the deviation C^k is not profitable. Third, if type 2 trades some contracts c^ℓ_1 or $c^\ell_{2,n}$ with firms $l \neq k$ following firm k's deviation, possibly along one or several contracts $c^*_{1,n}$ or c^*_2, then this brings her at best on the line with slope r that supports her upper contour set of (Q^*_2, T^*_2); hence the contract (q, t) attracts type 2 only if $rq \geq t$, and again the deviation C^k is not profitable.

Step 3 We then prove that, for n large enough, no firm k has a profitable deviation that only attracts type 1 (hereafter *cream-skimming deviation*). As in Step 2, we identify such a deviation C^k with the contract (q, t) type 1 chooses to trade in C^k according to her best response. Observe that, because type 1 can purchase her demand Q^*_1 at price r from firms $l \neq k$ by trading a contract $c^*_{1,n}$ with each of them, the contract (q, t) attracts her only if $rq \geq t$; moreover, this contract is profitable only if $t > r_1 q$. Hence any cream-skimming deviation must belong to the cone

$$X \equiv \{(q, t) : rq \geq t \geq r_1 q\}.$$

We distinguish two cases.

Consider first the case of a large cream-skimming deviation, that is, a contract $(q, t) \in X$ issued by some firm k such that

$$u_1(q, t) \geq u_1(Q^*_1, T^*_1). \tag{A.18}$$

This case is easily dealt with under Assumption C, because, according to Lemma 5, (q, t) is blocked by the latent contract c^ℓ_1. Indeed, (A.18) implies

$$u_2((q, t) + c^\ell_1) \geq u_2(Q^*_2, T^*_2)$$

by (12); hence (q,t) also attracts type 2 in combination with a contract c_1^ℓ issued by some firm $l \neq k$. We can thus construct the consumers' best response in such a way that both types trade the contract (q,t) with firm k. But then, because $rq \geq t$ as $(q,t) \in X$, this deviation is not profitable, as desired.

Consider next the case of a small cream-skimming deviation, that is, a contract $(q,t) \in X$ issued by some firm k such that

$$z_{1,n}^{*-k}(q,t) \geq u_1(Q_1^*, T_1^*) > u_1(q,t), \tag{A.19}$$

where, in analogy with (A.1),

$$z_{i,n}^{*-k}(q,t) \equiv \max\left\{u_i\left(q + \sum_{l\neq k} q^l, t + \sum_{l \neq k} t^l\right) : (q^l, t^l) \in C_n^* \text{ for all } l \neq k\right\}$$

is type i's indirect utility from trading the contract (q,t) with firm k given the menus posted by firms $l \neq k$. Thus (q,t) attracts type 1, but only in combination with other contracts issued by firms $l \neq k$. We can then no longer use the latent contract c_1^ℓ as in the case of a large cream-skimming deviation, because, for all we know, type 1 may have to trade with all firms $l \neq k$ to obtain the utility $z_{1,n}^{*-k}(q,t)$.

However, notice that, in case type 1 can at least obtain her equilibrium utility $u_1(Q_1^*, T_1^*)$ by trading (q,t) in combination with one contract $c_{1,n}^*$ and possibly other contracts, then, according to Lemma 5, type 2 can at least obtain her equilibrium utility $u_2(Q_2^*, T_2^*)$ by trading (q,t) in combination with one contract $c_{2,n}^\ell = c_{1,n}^* + c_1^\ell$ and these other contracts, which amounts to mimicking type 1's trades and trading an additional contract c_1^ℓ; hence, in this case, (q,t) also attracts type 2 and, by the same reasoning as for a large cream-skimming deviation, we can thus construct the consumers' best response in such a way that this deviation is not profitable, as desired. We thus only need to show that this case arises for n large enough, uniformly in (q,t). To do so, let, in analogy with (A.1),

$$\underline{z}_{1,n}^{*-k}(q,t) \equiv \max\left\{u_1\left(q + \sum_{l\neq k} q^l, t + \sum_{l \neq k} t^l\right) : (q^l, t^l) \in C_n^* \text{ for all } l \neq k \right.$$
$$\left. \text{and } (q^l, t^l) = c_{1,n}^* \text{ for some } l \neq k\right\}$$

be type 1's indirect utility from trading the contract (q,t) with firm k in combination with at least one contract $c_{1,n}^*$ issued by some firm $l \neq k$. By construction,

$$z_{1,n}^{*-k}(q,t) \geq \underline{z}_{1,n}^{*-k}(q,t).$$

The following lemma shows that, for n large enough, the additional constraint embedded in the definition of $\underline{z}^{*-k}_{1,n}(q,t)$ does not prevent type 1 from at least obtaining her equilibrium utility, which completes the proof of Step 3.

Lemma A.4 *There exists $\underline{n} \in \mathbb{N}$ such that, for each $n \geq \underline{n}$,*

$$\underline{z}^{*-k}_{1,n}(q,t) \geq u_1(Q_1^*, T_1^*) \tag{A.20}$$

for all $(q,t) \in X$ that satisfy (A.19).

Proof. Suppose, by way of contradiction, that there exists a sequence $((q_n, t_n))_{n \in \mathbb{N}}$ in X such that

$$z^{*-k}_{1,n}(q_n, t_n) \geq u_1(Q_1^*, T_1^*) > \max\{u_1(q_n, t_n), \underline{z}^{*-k}_{1,n}(q_n, t_n)\}.$$

For each n, because $u_1(Q_1^*, T_1^*) > u_1(q_n, t_n)$, type 1 must trade, on top of the contract (q_n, t_n) issued by firm k, some contracts $c_n^l \in C_n^*$ issued by firms $l \neq k$ to obtain the utility $z^{*-k}_{1,n}(q_n, t_n)$; moreover, because $u_1(Q_1^*, T_1^*) > \underline{z}^{*-k}_{1,n}(q_n, t_n)$, all these contracts must be different from $c^*_{1,n}$. By definition, we have

$$u_1\left((q_n, t_n) + \sum_{l \neq k} c_n^l\right) = z^{*-k}_{1,n}(q_n, t_n) \geq u_1(Q_1^*, T_1^*). \tag{A.21}$$

For each n, the contract (q_n, t_n) belongs to the compact set

$$Y \equiv X \cap \{(q,t) : q \leq Q_1^* \text{ and } u_1(q,t) \leq u_1(Q_1^*, T_1^*)\}.$$

Thus we can with no loss of generality assume that the sequence $((q_n, t_n))_{n \in \mathbb{N}}$ converges to some contract $(q_\infty, t_\infty) \in Y$. Notice that $q_\infty \leq Q_1^*$ by definition of Y.

Next, for each n, c_n^l belongs to the finite set $C_n^* \setminus \{c^*_{1,n}\}$ and at least one contract c_n^l is different from the no-trade contract; hence, by construction of the contracts c_2^*, c_1^ℓ, and $c_{2,n}^\ell$, the sequence $(\sum_{l \neq k} c_n^l)_{n \in \mathbb{N}}$ is bounded away from $(0,0)$. Noticing that

$$C_\infty^* \equiv \{(0,0), c_2^*, c_1^\ell\}$$

is the closed limit of the sequence of sets $(C_n^* \setminus \{c^*_{1,n}\})_{n \in \mathbb{N}}$ (Aliprantis and Border (2006, Definition 3.80.1)), we can with no loss of generality assume that, for each $l \neq k$, the sequence $(c_n^l)_{n \in \mathbb{N}}$ converges to some contract $c_\infty^l \in C_\infty^*$ such that

$$\sum_{l \neq k} c_\infty^l \neq (0,0).$$

We claim that

$$t_\infty < rq_\infty. \tag{A.22}$$

Indeed, taking the limit in (A.21) yields, by continuity of u_1,

$$u_1\left((q_\infty, t_\infty) + \sum_{l \neq k} c^l_\infty\right) \geq u_1(Q^*_1, T^*_1).$$

However, the aggregate trade $\sum_{l \neq k} c^l_\infty$ has a premium rate higher than r as it is only built up from contracts c^*_2 or c^ℓ_1. Because Q^*_1 is the demand for coverage of type 1 at price r and $T^*_1 = rQ^*_1$, it must be that $t_\infty < rq_\infty$, as claimed.

Finally, because, for each n, $u_1(Q^*_1, T^*_1) > \underline{z}^{*-k}_{1,n}(q_n, t_n)$, we a fortiori have

$$u_1(Q^*_1, T^*_1) > \max\{u_1((q_n, t_n) + (Q^{-k}, T^{-k})) : (Q^{-k}, T^{-k}) \in C^{*-k}_{r,n}\}, \tag{A.23}$$

where

$$C^{*-k}_{r,n} \equiv \{\nu c^*_{1,n} : \nu \in \mathbb{N} \text{ and } \nu \leq n - 1\}$$

is the set of aggregate trades that consumers can make at premium rate r with firms $l \neq k$. Noticing that

$$C^{*-k}_{r,\infty} \equiv \{(Q^{-k}, T^{-k}) : Q^{-k} \in [0, Q^*_1] \text{ and } T^{-k} = rQ^{-k}\}$$

is the closed limit of the sequence of sets $(C^{*-k}_{r,n})_{n \in \mathbb{N}}$, taking the limit superior in (A.23) yields, by continuity of u_1,

$$\begin{aligned}
u_1(Q^*_1, T^*_1) &\geq \limsup_{n \to \infty} \max\{u_1((q_n, t_n) + (Q^{-k}, T^{-k})) : (Q^{-k}, T^{-k}) \in C^{*-k}_{r,n}\} \\
&\geq \max\{u_1((q_\infty, t_\infty) + (Q^{-k}, T^{-k})) : (Q^{-k}, T^{-k}) \in C^{*-k}_{r,\infty}\} \\
&\geq u_1(Q^*_1, T^*_1 + t_\infty - rq_\infty),
\end{aligned}$$

where the third inequality follows from letting type 1 trade $(q_\infty, t_\infty) \in Y$ with firm k and $(Q^*_1 - q_\infty, r(Q^*_1 - q_\infty)) \in C^{*-k}_{r,\infty}$ with firms $l \neq k$, which is feasible as $q_\infty \leq Q^*_1$. Because $t_\infty < rq_\infty$ by (A.22), and because Q^*_1 is the demand for coverage of type 1 at price r and $T^*_1 = rQ^*_1$, we have thus reached a contradiction. The result follows. ■

Step 4 There remains to prove that no firm k has a profitable menu deviation

$$C^k \equiv \{(0, 0), (q_1, t_1), (q_2, t_2)\},$$

where $(q_i, t_i) \neq (0,0)$ is the contract type i chooses to trade in C^k according to her best response. If $(q_1, t_1) \neq (q_2, t_2)$, then this deviation is consistent with the regulation only if $t_2 \geq r_2 q_2$. However, we can construct type 2's best response in such a way that she trades the contract (q_2, t_2) only if it strictly attracts her; otherwise, type 2 may as well obtain her equilibrium utility $u_2(Q_2^*, T_2^*)$ by only trading with firms $l \neq k$. Arguing as in Step 2, we then obtain $t_2 < r_2 q_2$, a contradiction. If $(q_1, t_1) = (q_2, t_2) \equiv (q, t)$, then, because type 1 can purchase her demand Q_1^* for coverage at price r from firms $l \neq k$, it must be that $t \leq rq$ and the deviation is not profitable. Hence the result. ■

Proof of Theorem 3. The necessity of conditions (10)–(11) follows from Lemma 1. We show that, under conditions (10)–(11), the regulated game has a free-entry equilibrium in which only two firms, say, firms 1 and 2, are active on the equilibrium path and post the same menu C^*, which is recursively defined on the basis of the following three contracts:

1. The no-trade contract $(0,0)$;
2. The basic-coverage contract $c_1^* \equiv (Q_1^*, T_1^*)$;
3. The complementary-coverage contract $c_2^* \equiv (Q_2^* - Q_1^*, T_2^* - T_1^*)$.

Specifically, let C_∞ be the smallest set that contains these three contracts and that is closed under addition with the latent contract $c_1^\ell \equiv (q_1^\ell, t_1^\ell)$ defined by (13)–(14); that is,

$$c \in C_\infty \text{ implies } c + c_1^\ell \in C_\infty.$$

Notice that C_∞ is unbounded and hence not compact. To construct from C_∞ a compact menu C^*, consider the line M with slope r_1 that supports type 1's upper contour set of (Q_1^*, T_1^*), which is well-defined as $\tau_1(Q_1^*, T_1^*) = r$ and type 1's marginal rate of substitution strictly and continuously decreases along her equilibrium indifference curve and vanishes as aggregate coverage grows large. Let H be the lower closed half-space defined by M. Because nonnull contracts in C_∞ have premium rates at least equal to r, whereas the line M has slope r_1, the set $C_\infty \cap H$ is finite. We define C^* as follows:

$$C^* \equiv (C_\infty \cap H) + \{(0,0), c_1^\ell\}. \tag{A.24}$$

Finally, every inactive firm $k \neq 1, 2$ issues the contract c_2^*. The remainder of the proof consists of four steps.

Step 1 The proof that consumers have a best response such that, on the equilibrium path, they trade according to the JHG allocation, is similar to that of Step 1 of the proof of Theorem 2, except that both types now trade the contract c_1^* with, say, firm 1, and type 2 in addition trades c_2^* with firm 2. In particular, by (10), type 2 is not tempted to trade two contracts c_1^* with firms 1 and 2. All firms earn zero profit.

Step 2 The proof that no firm has a profitable deviation that only attracts type 2 is similar to that of Step 2 of the proof of Theorem 2, observing in addition that type 2 can obtain her equilibrium utility $u_2(Q_2^*, T_2^*)$ by trading c_1^* with one active firm and c_2^* with an inactive firm.

Step 3 We next show that no firm has a cream-skimming deviation. As in Step 3 of the proof of Theorem 2, let X be the cone of such deviations. We distinguish two cases.

Consider first the case of an active firm, say, firm 1. Let $(q,t) \in X$ be some contract, issued by firm 1, that attracts type 1 in combination with a contract $c \in C^*$ issued by firm 2 and $\nu \in \{0, \dots, n-2\}$ contracts c_2^* issued by firms $k \neq 1, 2$. Because firm 2 issues the contract c_1^*, the equilibrium utility of type 1 remains available following firm 1's deviation. Hence we have

$$u_1((q,t) + c + \nu c_2^*) \geq u_1(Q_1^*, T_1^*). \tag{A.25}$$

Now, $(q,t) \in X$ implies $t \geq r_1 q$. As a result, it must be that $c \in C_\infty \cap H$, where C_∞ and H are defined as above; otherwise, type 1 would not be willing to combine (q,t) with c. Therefore, by (A.24), $c + c_1^\ell \in C^*$. In particular, because Assumption C is satisfied, we can apply Lemma 2, so that (A.25) implies

$$u_2((q,t) + c + c_1^\ell + \nu c_2^*) \geq u_2(Q_2^*, T_2^*)$$

by (12); hence (q,t) also attracts type 2 in combination with the contract $c + c_1^\ell$ issued by firm 2 and the ν contracts c_2^* issued by firms $k \neq 1, 2$. We can thus construct the consumers' best response in such a way that both types trade the contract (q,t) with firm 1. But then, because $rq \geq t$ as $(q,t) \in X$, this deviation is not profitable, as desired.

Consider next the case of an inactive firm $k \neq 1, 2$. Let $(q,t) \in X$ be some contract, issued by firm k, that attracts type 1 in combination with contracts $c, c' \in C^*$ issued by firms 1 and 2 and $\nu \in \{0, \dots, n-3\}$ contracts c_2^* issued by firms $l \neq 1, 2, k$, provided such firms exist. In analogy with (A.25), we have

$$u_1((q,t) + c + c' + \nu c_2^*) \geq u_1(Q_1^*, T_1^*). \tag{A.26}$$

By the same reasoning as in the previous case, it must be that $c + c' \in (C_\infty + C_\infty) \cap H$ and hence $c, c' \in C_\infty \cap H$. Therefore, for instance, $c' + c_1^\ell \in C^*$. Applying again Lemma 2, we obtain that (A.26) implies

$$u_2((q,t) + c + c' + c_1^\ell + \nu c_2^*) \geq u_2(Q_2^*, T_2^*)$$

by (12); hence (q,t) also attracts type 2 in combination with the contracts c and $c' + c_1^\ell$ issued by firms 1 and 2, respectively, and the ν contracts issued by firms $l \neq 1, 2, k$. We can then conclude as in the previous case.

Step 4 There remains to prove that no firm k has a profitable menu deviation. For active firms, the proof proceeds as in Step 3 of the proof of Theorem 2. For inactive firms, the key observation is that, under conditions (10)–(11), no such firm can exploit the trade $(2Q_1^*, 2T_1^*)$ made available by active firms on the equilibrium path to attract type 2 in a profitable way. The proof then proceeds as for active firms. Hence the result. ■

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Online Appendix

S.1 A Reformulation of Assumption C

The following result shows that Assumption C is equivalent to the property that type 2's Hicksian demand for coverage be more sensitive, or as sensitive, to changes in the premium rate than type 1's, whatever utility levels are used as references.

Lemma S.1 *Let $H_i(p, \mathbf{u})$ be type i's Hicksian demand function. Then property C(i) (C(ii)) is satisfied if and only if* $\left|\frac{\partial H_2}{\partial p}(p, \mathbf{u}_2)\right| > (=) \left|\frac{\partial H_1}{\partial p}(p, \mathbf{u}_1)\right|$ *for all* $(p, \mathbf{u}_1, \mathbf{u}_2)$.

Proof. Let $T = \mathcal{I}_i(Q, \mathbf{u})$ be the functional expression of type i's indifference curve at utility level $\mathbf{u}$. Then $H_i(\cdot, \mathbf{u})$ is the inverse of $\frac{\partial \mathcal{I}_i}{\partial Q}(\cdot, \mathbf{u})$, so that $\frac{\partial H_i}{\partial p}(p, \mathbf{u}) = \left[\frac{\partial^2 \mathcal{I}_i}{\partial Q^2}(H_i(p, \mathbf{u}), \mathbf{u})\right]^{-1}$ for all $(p, \mathbf{u})$. By (15), property C(i) is satisfied if and only if

$$\text{for all } (Q_1, Q_2, \mathbf{u}_1, \mathbf{u}_2),\ \frac{\partial \mathcal{I}_1}{\partial Q}(Q_1, \mathbf{u}_1) = \frac{\partial \mathcal{I}_2}{\partial Q}(Q_2, \mathbf{u}_2) \text{ implies } \frac{\partial^2 \mathcal{I}_1}{\partial Q^2}(Q_1, \mathbf{u}_1) < \frac{\partial^2 \mathcal{I}_2}{\partial Q^2}(Q_2, \mathbf{u}_2),$$

that is, because $p = \frac{\partial \mathcal{I}_i}{\partial Q}(Q_i, \mathbf{u}_i)$ if and only if $Q_i = H_i(p, \mathbf{u}_i)$, if and only if

$$\text{for all } (p, \mathbf{u}_1, \mathbf{u}_2),\ \frac{\partial^2 \mathcal{I}_1}{\partial Q^2}(H_1(p, \mathbf{u}_1), \mathbf{u}_1) < \frac{\partial^2 \mathcal{I}_2}{\partial Q^2}(H_2(p, \mathbf{u}_2), \mathbf{u}_2),$$

which is the desired property of Hicksian demand functions. The proof for property C(ii) is similar, replacing all inequalities by equalities. The result follows. ■

S.2 Omitted Calculations

If every type i's preferences have the expected-utility representation (3), then type i's marginal rate of substitution of coverage for premia is

$$\tau_i(Q, T) = \int l g_i(l \,|\, Q, T)\, \boldsymbol{l}(\mathrm{d}l), \tag{S.1}$$

where $g_i(\cdot \,|\, Q, T)$ is the risk-neutral density

$$g_i(l \,|\, Q, T) \equiv \frac{v_i'(W_0 - (1-Q)l - T) f_i(l)}{\int v_i'(W_0 - (1-Q)\ell - T) f_i(\ell)\, \boldsymbol{l}(\mathrm{d}\ell)}. \tag{S.2}$$

We first provide a convenient expression for $\frac{\partial^2 \mathcal{I}_i}{\partial Q^2}$.

Lemma S.2 *Let $\alpha_i(W) \equiv -\frac{v_i''}{v_i'}(W)$ be type i's coefficient of absolute risk-aversion at wealth W, and let $\tilde{L}$ be a random variable with density f_i with respect to the measure $\boldsymbol{l}$. Then*

$$\frac{\partial^2 \mathcal{I}_i}{\partial Q^2}(Q, \mathbf{u}) = -\mathbf{Cov}_{g_i(\cdot|Q, \mathcal{I}_i(Q,\mathbf{u}))}\left[\tilde{L}, \alpha_i(W_0 - (1-Q)\tilde{L} + \mathcal{I}_i(Q, \mathbf{u}))\left[\tilde{L} - \frac{\partial \mathcal{I}_i}{\partial Q}(Q, \mathbf{u})\right]\right]. \tag{S.3}$$

Proof. By (S.1)–(S.2), we only need to differentiate

$$\frac{\partial \mathcal{I}_i}{\partial Q}(Q, \mathbf{u}) = \tau_i(Q, \mathcal{I}_i(Q, \mathbf{u})) = \int l \, \frac{v_i'(W_0 - (1-Q)l - \mathcal{I}_i(Q, \mathbf{u}))f_i(l)}{\int v_i'(W_0 - (1-Q)\ell - \mathcal{I}_i(Q, \mathbf{u}))f_i(\ell)\, \boldsymbol{l}(\mathrm{d}\ell)} \, \boldsymbol{l}(\mathrm{d}l)$$

with respect to Q. Omitting the index i and the arguments of the functions for the sake of clarity, this yields

$$\begin{aligned}\frac{\partial^2 \mathcal{I}}{\partial Q^2} &= \int l \, \frac{\left[v''\left(l - \frac{\partial \mathcal{I}}{\partial Q}\right)\int v' f \, \mathrm{d}\boldsymbol{l} - v' \int v''\left(l - \frac{\partial \mathcal{I}}{\partial Q}\right) f \, \mathrm{d}\boldsymbol{l}\right] f}{\left(\int v' f \, \mathrm{d}\boldsymbol{l}\right)^2} \, \mathrm{d}\boldsymbol{l} \\ &= -\int l \left[\alpha\left(l - \frac{\partial \mathcal{I}}{\partial Q}\right) - \int \alpha\left(l - \frac{\partial \mathcal{I}}{\partial Q}\right)\frac{v' f}{\int v' f \, \mathrm{d}\boldsymbol{l}} \, \mathrm{d}\boldsymbol{l}\right] \frac{v' f}{\int v' f \, \mathrm{d}\boldsymbol{l}} \, \mathrm{d}\boldsymbol{l},\end{aligned}$$

which implies (S.3) upon noticing that the bracketed term on the right-hand side of this equality has zero mean under the risk-neutral density $g = \frac{v' f}{\int v' f \, \mathrm{d}\boldsymbol{l}}$. The result follows. ■

We now use Lemma S.2 to show that Assumption C is consistent with the other assumptions of our model. We consider two examples to this end.

Example 1 Suppose first that there is a single loss level L, so that

$$u_i(Q, T) = f_i(L)v_i(W_0 - (1-Q)L - T) + [1 - f_i(L)]v_i(W_0 - T). \tag{S.4}$$

The following result then holds.

Lemma S.3 *The preferences represented by (S.4) satisfy both property C(i) and the single-crossing condition if type 1 is uniformly more risk-averse than type 2,*

$$\inf_W \alpha_1(W) > \sup_W \alpha_2(W), \tag{S.5}$$

and type 2 is sufficiently riskier than type 1,

$$\ln\left(\frac{f_2(L)}{1 - f_2(L)}\right) - \ln\left(\frac{f_1(L)}{1 - f_1(L)}\right) > \left[\sup_W \alpha_1(W) - \inf_W \alpha_2(W)\right] L. \tag{S.6}$$

Moreover, the preferences represented by (S.4) with $f_2(L) > f_1(L)$ satisfy both property C(ii) and the single-crossing condition if type 1 and type 2 have the same CARA utility index.

Proof. Consider first Assumption C. By (S.1)–(S.2) and (S.4),

$$g_i(L \,|\, Q_i, \mathcal{I}_i(Q_i, \mathbf{u}_i)) = \frac{\tau_i(Q_i, \mathcal{I}_i(Q_i, \mathbf{u}_i))}{L} = \frac{\frac{\partial \mathcal{I}_i}{\partial Q}(Q_i, \mathbf{u}_i)}{L}.$$

Hence

$$\frac{\partial \mathcal{I}_1}{\partial Q}(Q_1, \mathbf{u}_1) = \frac{\partial \mathcal{I}_2}{\partial Q}(Q_2, \mathbf{u}_2) \equiv \frac{\partial \mathcal{I}}{\partial Q}$$

implies

$$g_1(L\,|\,Q_1, \mathcal{I}_1(Q_1, \mathbf{u}_1)) = g_2(L\,|\,Q_2, \mathcal{I}_2(Q_2, \mathbf{u}_2)) = \frac{\frac{\partial \mathcal{I}}{\partial Q}}{L}.$$

That is, the two risk-neutral densities are the same under the premise of Assumption C. We can now apply the covariance formula (S.3) to obtain

$$\frac{\partial^2 \mathcal{I}_i}{\partial Q^2}(Q_i, \mathbf{u}_i) = -\frac{\partial \mathcal{I}}{\partial Q}\left(L - \frac{\partial \mathcal{I}}{\partial Q}\right)\overline{\alpha}_i(Q_i, \mathbf{u}_i),$$

where

$$\overline{\alpha}_i(Q_i, \mathbf{u}_i) \equiv \frac{\frac{\partial \mathcal{I}}{\partial Q}}{L}\,\alpha_i(W_0 - \mathcal{I}_i(Q_i, \mathbf{u}_i)) + \left(1 - \frac{\frac{\partial \mathcal{I}}{\partial Q}}{L}\right)\alpha_i(W_0 - (1 - Q_i)L - \mathcal{I}_i(Q_i, \mathbf{u}_i)).$$

If (S.5) holds, then $\overline{\alpha}_1(Q_1, \mathbf{u}_1) > \overline{\alpha}_2(Q_2, \mathbf{u}_2)$, so that

$$\frac{\partial \mathcal{I}_1}{\partial Q}(Q_1, \mathbf{u}_1) = \frac{\partial \mathcal{I}_2}{\partial Q}(Q_2, \mathbf{u}_2) \text{ implies } \frac{\partial^2 \mathcal{I}_1}{\partial Q^2}(Q_1, \mathbf{u}_1) < \frac{\partial^2 \mathcal{I}_2}{\partial Q^2}(Q_2, \mathbf{u}_2),$$

which is property C(i) by (15). If type 1 and type 2 have the same CARA utility index, then

$$\frac{\partial \mathcal{I}_1}{\partial Q}(Q_1, \mathbf{u}_1) = \frac{\partial \mathcal{I}_2}{\partial Q}(Q_2, \mathbf{u}_2) \text{ implies } \frac{\partial^2 \mathcal{I}_1}{\partial Q^2}(Q_1, \mathbf{u}_1) = \frac{\partial^2 \mathcal{I}_2}{\partial Q^2}(Q_2, \mathbf{u}_2),$$

which is property C(ii).

Consider next the single-crossing condition. By (S.1)–(S.2), (5) holds if and only if

$$\left[\frac{1 - f_2(L)}{f_2(L)}\right]\left[\frac{v_2'(W_0 - T)}{v_2'(W_0 - (1 - Q)L - T)}\right] < \left[\frac{1 - f_1(L)}{f_1(L)}\right]\left[\frac{v_1'(W_0 - T)}{v_1'(W_0 - (1 - Q)L - T)}\right]. \quad \text{(S.7)}$$

This is clearly the case if $v_1 = v_2$ and $f_2(L) > f_1(L)$. Assume from now on that $v_1 \neq v_2$ and that (S.5) holds. Suppose first $Q \leq 1$. As $\frac{v_2''}{v_2'}(W) \leq -\inf_W \alpha_2(W)$, we have

$$\frac{v_2'(W_0 - T)}{v_2'(W_0 - (1 - Q)L - T)} \leq \exp\Big(-\inf_W \alpha_2(W)\,(1 - Q)L\Big).$$

Similarly, because $\frac{v_1''}{v_1'}(W) \geq -\sup_W \alpha_1(W)$,

$$\frac{v_1'(W_0 - T)}{v_1'(W_0 - (1 - Q)L - T)} \geq \exp\left(-\sup_W \alpha_1(W)\,(1 - Q)L\right).$$

Thus a sufficient condition for (S.7) to hold for a fixed $Q \leq 1$ is

$$\ln\left(\frac{f_2(L)}{1 - f_2(L)}\right) - \ln\left(\frac{f_1(L)}{1 - f_1(L)}\right) > \left[\sup_W \alpha_1(W) - \inf_W \alpha_2(W)\right](1 - Q)L.$$

In turn, (S.6) is a sufficient condition for this to hold for all $Q \leq 1$ under (S.5). Suppose next $Q > 1$. As $-\frac{v_2''}{v_2'}(W) \leq \sup_W \alpha_2(W)$, we have

$$\frac{v_2'(W_0 - T)}{v_2'(W_0 - (1-Q)L - T)} \leq \exp\Big(-\sup_W \alpha_2\,(1-Q)L\Big).$$

Similarly, because $-\frac{v_1''}{v_1'}(W) \geq \inf_W \alpha_1(W)$,

$$\frac{v_1'(W_0 - T)}{v_1'(W_0 - (1-Q)L - T)} \geq \exp\Big(-\inf_W \alpha_1(W)\,(1-Q)L\Big).$$

As $f_2(L) > f_1(L)$ under (S.5)–(S.6) and $\inf_W \alpha_1(W) > \sup_W \alpha_2(W)$ under (S.5), this implies that (S.7) holds for all $Q > 1$. The result follows. ■

Example 2 Suppose next that every type i has a CARA utility index with absolute risk-aversion coefficient α_i and a loss density f_i that belongs to the natural exponential family (18), which includes, beyond the Bernoulli case of Example 1, binomial, gamma, and Poisson loss distributions. Hence, up to a multiplicative constant,

$$u_i(Q,T) = -\exp(\alpha_i T)\int \exp([\alpha_i(1-Q)+\theta_i]l)h(l)\,\boldsymbol{l}(\mathrm{d}l) \tag{S.8}$$

and type i's preferences are quasilinear. The following result then holds.

Lemma S.4 *The preferences represented by (S.8) satisfy both property C(i) and the single-crossing condition if type 1 is uniformly more risk-averse than type 2,*

$$\alpha_1 > \alpha_2, \tag{S.9}$$

and type 2 is sufficiently riskier than type 1,

$$\theta_2 - \theta_1 > \alpha_1 - \alpha_2. \tag{S.10}$$

Moreover, the preferences represented by (S.8) with $\theta_2 > \theta_1$ satisfy both property C(ii) and the single-crossing condition if type 1 and type 2 have the same CARA utility index.

Proof. Consider first Assumption C. Thanks to quasilinearity, we can simplify notation by omitting transfers and utility levels. By (S.1)–(S.2) and (S.8),

$$\tau_i(Q) = \mathcal{I}_i'(Q) = \psi(\alpha_i(1-Q)+\theta_i), \tag{S.11}$$

where, for each x in the relevant range,

$$\psi(x) \equiv \int l\,\frac{\exp(xl)h(l)}{\int \exp(x\ell)h(\ell)\,\boldsymbol{l}(\mathrm{d}\ell)}\,\boldsymbol{l}(\mathrm{d}l).$$

Observe that

$$\psi'(x) \propto \int l^2 \frac{\exp(xl)h(l)}{\int \exp(x\ell)h(\ell)\,\boldsymbol{l}(\mathrm{d}\ell)}\,\boldsymbol{l}(\mathrm{d}l) - \left[\int l \frac{\exp(xl)h(l)}{\int \exp(x\ell)h(\ell)\,\boldsymbol{l}(\mathrm{d}\ell)}\,\boldsymbol{l}(\mathrm{d}l)\right]^2 > 0$$

by Jensen's inequality, so that $\psi(x)$ is strictly increasing in x. Hence

$$\tau_1(Q_1) = \tau_2(Q_2)$$

implies

$$\alpha_1(1 - Q_1) + \theta_1 = \alpha_2(1 - Q_2) + \theta_2 \equiv x$$

and thus

$$g_1(l\,|\,Q_1) = g_2(l\,|\,Q_2) = g(l) \equiv \frac{\exp(xl)h(l)}{\int \exp(x\ell)h(\ell)\,\boldsymbol{l}(\mathrm{d}\ell)}.$$

That is, the two risk-neutral densities are the same under the premise of Assumption C. We can now apply the covariance formula (S.3) to obtain

$$\mathcal{I}_i''(Q_i) = -\alpha_i \mathbf{Var}_g[\tilde{L}].$$

If (S.9) holds, then

$$\mathcal{I}_1'(Q_1) = \mathcal{I}_2'(Q_2) \text{ implies } \mathcal{I}_1''(Q_1) < \mathcal{I}_2''(Q_2),$$

which is property C(i) by (15). If type 1 and type 2 have the same CARA utility index, then

$$\mathcal{I}_1'(Q_1) = \mathcal{I}_2'(Q_2) \text{ implies } \mathcal{I}_1''(Q_1) = \mathcal{I}_2''(Q_2),$$

which is property C(ii).

Consider next the single-crossing condition. By (S.11), (5) holds if and only if

$$\alpha_2(1 - Q) + \theta_2 > \alpha_1(1 - Q) + \theta_1. \tag{S.12}$$

This is clearly the case if $\alpha_1 = \alpha_2$ and $\theta_2 > \theta_1$. If $\alpha_1 > \alpha_2$ as in (S.9), a necessary and sufficient condition for (S.12) to hold for all Q is (S.10). The result follows. ■