

WORKING PAPERS

N° TSE -1033

August 2019

“Regulating Insurance Markets: Multiple Contracting and Adverse Selection”

Andrea Attar, Thomas Mariotti, François Salanié

Regulating Insurance Markets: Multiple Contracting and Adverse Selection*

Andrea Attar[†] Thomas Mariotti[‡] François Salanié[§]

August 28, 2019

Abstract

We study insurance markets in which privately informed consumers can purchase coverage from several firms whose pricing strategies are subject to an anti-dumping regulation. The resulting regulated game supports a single allocation in which each layer of coverage is fairly priced given the consumer types who purchase it. This competitive allocation cannot be Pareto-improved by a social planner who can neither observe consumer types nor monitor their trades with firms. Accordingly, we argue that public intervention under multiple contracting and adverse selection should penalize firms that cross-subsidize between contracts, while leaving consumers free to choose their preferred amount of coverage.

Keywords: Insurance Markets, Regulation, Multiple Contracting, Adverse Selection.

JEL Classification: D43, D82, D86.

*This paper supersedes “Multiple Contracting in Insurance Markets” (TSE Working Paper No. 14-532) by the same authors. We thank Luca Anderlini, Felix Bierbrauer, Hector Chade, Pradeep Dubey, Christian Gollier, Piero Gottardi, Martin Hellwig, Wanda Mimra, Michael Peters, Maria Polyakova, Patrick Rey, Jean-Charles Rochet, Casey Rothschild, Bernard Salanié, Edward Schlee, and Jean Tirole for extremely valuable feedback. We also thank seminar audiences at Arizona State University, European University Institute, ETH Zürich, HEC Lausanne, Helsinki Center of Economic Research, Séminaire Roy, Toulouse School of Economics, Universität Bonn, University of Arizona, Tucson, as well as conference participants at the 2014 Université Paris Dauphine Workshop “Topics in Information Economics,” the 2015 CESifo Area Conference on Applied Microeconomics, the 2015 Conference of the Society for the Advancement of Economic Theory, the 2015 Southampton Winter Workshop in Economic Theory, the 2016 Conference of the American-European Health Economics Study Group, the 2016 CORE@50 Conference, the 2016 Econometric Society European Meeting, the 2016 Humboldt-Universität zu Berlin Workshop on Competition and Incentives, and the 2016 Society for Economic Dynamics Annual Meeting for many useful discussions. DaKang Huang provided excellent research assistance. Financial support from the ANR (Programme d’Investissement d’Avenir ANR-17-EURE-0010), the Chaire SCOR–TSE, the ERC (Starting Grant 203929-ACAP), and the Paul Woolley Research Initiative on Capital Market Dysfunctionalities is gratefully acknowledged.

[†]Toulouse School of Economics, CNRS, University of Toulouse Capitole, Toulouse, France, and Università degli Studi di Roma “Tor Vergata,” Roma, Italy. Email: andrea.attar@tse-fr.eu.

[‡]Toulouse School of Economics, CNRS, University of Toulouse Capitole, Toulouse, France. Email: thomas.mariotti@tse-fr.eu.

[§]Toulouse School of Economics, INRA, University of Toulouse Capitole, Toulouse, France. Email: francois.salanie@tse-fr.eu.

1 Introduction

Multiple contracting, whereby consumers purchase several policies from different public or private insurers to cover the same risk, is a widespread phenomenon in insurance markets. A case in point is the US life-insurance market, in which around 25% of consumers hold more than one term policy.¹ A similar phenomenon arises in annuity markets: for instance, the six million annuities in payment in the UK in 2013 were owned by about five millions individuals.² Most health-insurance markets also exhibit multiple contracting in ways that depend on the relative importance of the public and private insurance sectors:³ in the US, the Medicare supplementary market enables 10 out of the 42 million consumers covered by Medicare to opt for Medigap plans issued by private firms; similarly, retirees can complement Medicare benefits with employer-sponsored retiree health plans.⁴

Since the early works of Arrow (1963), Akerlof (1970), Pauly (1974), and Rothschild and Stiglitz (1976), there has been a presumption that these insurance markets are exposed to adverse selection. That is, firms' concern that high-risk consumers may be attracted by contracts with low premium rates can severely hinder private insurance provision, potentially leading to a market breakdown. As a result, many market interventions aimed at improving the efficiency of insurance provision under adverse selection have been proposed in recent decades. However, few, if any, of them explicitly take into account the implications of multiple contracting. The present paper contributes to analyzing this issue.

From the perspective of a single firm, the fact that its customers can also contract with its competitors makes it more difficult to screen them according to how much coverage they purchase. In the aggregate, when firms are free to compete through arbitrary menus of insurance contracts, the combination of adverse selection and multiple contracting leads to an extreme form of market breakdown: in equilibrium, high-risk consumers can obtain strictly positive coverage only if low-risk consumers are entirely driven out of the market (Attar,

¹A term life-insurance policy provides coverage for a limited period of time, which makes it a pure insurance product. Evidence about subscribers of such policies is provided by He (2009) on the basis of the Health and Retirement Study (HRS) panel.

²See the 2014 UK Insurance Key Facts Document issued by the Association of British Insurers, available at https://www.abi.org.uk/~/_/media/Files/Documents/Publications/Public/2014/Key%20Facts/ABI%20Key%20Facts%202014.pdf.

³Private health insurance can be used as a source of basic coverage for consumers who choose not to obtain public health insurance. This is the case in Germany, Netherlands, and Switzerland, where more than half of the population hold more than one policy (Paccagnella, Rebba, and Weber (2013)). Private insurance can also be used to fund healthcare needs that are already partially covered by public funds. This is the case in Australia, Denmark, and, in particular, France, where about 92% of the population complement public mandatory coverage with private coverage (Thomson, Osborne, Squires, and Jun (2013)).

⁴The income from employment-based pension schemes is currently relevant for about half of the US retirees, see Poterba (2014).

Mariotti, and Salanié (2014)). This calls for a determination of the combined welfare impact of adverse selection and multiple contracting.

In this respect, the benchmark is provided by the set of allocations that can be achieved by a social planner who observes neither consumers' riskiness nor their trades with private firms. Although the corresponding informational constraints can be represented in different ways, recent contributions single out a budget-balanced allocation in which low- and high-risk consumers purchase the same basic amount of coverage, which high-risk consumers complement by purchasing additional coverage. Each marginal amount of coverage, or layer, is fairly priced given the consumer types who purchase it, which corresponds to a marginal version of Akerlof (1970) pricing. Specifically, Stiglitz, Yun, and Kosenko (2018) and Attar, Mariotti, and Salanié (2019d) show that this allocation, first described by Jaynes (1978), Hellwig (1988), and Glosten (1994), cannot be Pareto-improved by a social planner who cannot monitor consumers' trades.⁵

This paper suggests a simple intervention which uniquely implements this Jaynes–Hellwig–Glosten (JHG) allocation in an equilibrium of a large class of insurance economies. The key feature of the regulation we propose is to prevent firms from destabilizing the market through dumping practices. Specifically, we consider a fully nonexclusive scenario in which firms post arbitrary menus of contracts that consumers are free to combine, and in which a regulatory agency has the power to punish any firm that makes a profit by cross-subsidizing between the different contracts it sells.

We provide two sets of results. Theorem 1 first shows that the JHG allocation is the only candidate equilibrium allocation of this regulated game. Thus our regulation does not undermine the power of competition under multiple contracting. The intuition is that each firm aims at increasing its profit by complementing the aggregate coverage provided by its competitors. Under adverse selection, this gives rise to a form of Bertrand competition over each layer of coverage. This competitive behavior, which has no counterpart under exclusive competition, is not hampered by our regulation. As the candidate equilibrium allocation cannot be Pareto-improved by a social planner who can neither observe consumer types nor monitor their trades with firms, it can be deemed third-best efficient.

An important by-product of Theorem 1 is that, in equilibrium, every active firm either provides basic coverage to both low- and high-risk consumers, or complementary coverage to high-risk consumers only. Each contract traded in equilibrium then breaks even on average given the consumers types who trade it, mimicking at the firm level the pricing of the

⁵As shown by Attar, Mariotti, and Salanié (2019d), this inability typically entails a Pareto loss relative to the second-best efficiency frontier, which is the relevant benchmark of efficiency under exclusive competition.

aggregate layers of basic and complementary coverage. A related implication of Theorem 1 is that no firm should be indispensable in providing basic coverage; otherwise it could make a profit by raising the premium it charges. Thus any equilibrium must exhibit an excess supply of basic coverage.

We next implement the JHG allocation in an equilibrium of the regulated game. To clarify the logic of our equilibrium analysis, it is useful to observe that the JHG allocation features cross-subsidization between different types, with low- and high-risk consumers being pooled on the same aggregate basic-coverage amount. This raises the issue of its fragility against cream-skimming deviations. In insurance economies, such deviations allow a firm to profitably serve only low-risk consumers, which suggests that, to sustain an equilibrium, firms should introduce some appropriate threats. Under multiple contracting, threats take the form of ad-hoc, latent contracts included in the firms' menus. These contracts are not meant to be traded in equilibrium but only to block cream-skimming deviations. We show that latent contracts are necessary to sustain an equilibrium.

Our equilibrium analysis is the first to provide a characterization of latent contracts in insurance markets subject to adverse selection. To this end, we identify an important class of consumer preferences, including quadratic and CARA preferences as special cases, for which a single latent contract is necessary and sufficient to deter all cream-skimming deviations that per se attract low-risk consumers. By posting this contract, firms make available additional coverage that low-risk consumers are willing to combine with that incorporated in any such deviation. This makes it impossible for a firm to profitably deviate by separating low-risk from high-risk consumers.

This result is key for Theorem 2, which shows that, provided that there are sufficiently many firms, the JHG allocation can be implemented in a symmetric equilibrium in which all firms are active on the equilibrium path. The many-firm assumption guarantees that the amount of basic coverage provided by each firm is sufficiently small so as to deter high-risk consumers from purchasing the excess supply of basic coverage. An interpretation of this equilibrium construction is that basic coverage is provided through a pool to which each firm contributes a small fraction, and from which each consumer is left free to choose whichever fraction she sees fit.

In contrast to Theorem 2, Theorem 3 provides necessary and sufficient conditions for free-entry equilibria in which at least one firm is inactive on the equilibrium path, a standard requirement in models of competitive markets under adverse selection. Although Theorem 1 still holds under free entry, this refinement imposes additional restrictions on the relative

sizes of basic and complementary coverage, because inactive firms must be deterred from exploiting the excess supply of basic coverage. As a result, this excess supply must now be large. Specifically, the excess supply of basic coverage is achieved by letting a limited number of firms offer large basic-coverage contracts. In particular, such contracts offer more coverage than complementary-coverage contracts.

Because, in equilibrium, complementary-coverage contracts have a higher premium rate than basic-coverage contracts, a striking implication of free entry is thus that the contracts offered by firms exhibit quantity discounts; intuitively, because multiple contracting a priori allows high-risk consumers to split their purchases between several small contracts, those should be charged a higher premium. Moreover, because only high-risk consumers purchase complementary coverage, we should observe a negative correlation between the coverage offered by a contract and its riskiness. Therefore, while the positive-correlation property emphasized by Chiappori and Salanié (2000) still holds when we consider the aggregate amount of coverage purchased by a consumer, it fails to hold at the firm level. This contrast between the demand- and the supply-side implications of equilibrium trades is a specific property of multiple-contracting environments. This result suggests that novel approaches exploiting firm-level data are needed to empirically identify adverse selection in insurance markets. We discuss this issue in Section 5.

The rationale for the regulation we propose lies in the destabilizing role played by cross-subsidies between contracts in our setting. To clarify this point, it is useful to consider the nonexclusive competition game in the absence of any intervention. In this setting, extensively analyzed in Attar, Mariotti, and Salanié (2014), a strictly positive amount of coverage for all consumer types cannot be supported in equilibrium. Indeed, if this were the case, any active firm could increase its profit by selling basic coverage to low-risk consumers only, while making a small loss by selling to high-risk consumers complementary coverage at slightly better terms than its competitors. The deviating firm would thereby minimize its losses by sharing with its competitors the cost of providing high coverage to high-risk consumers (“lemon dropping”); this in turn enables it to profitably attract low-risk consumers (“cherry picking”). This sophisticated deviation involves cross-subsidies and crucially exploits the nonexclusive nature of competition.⁶ Overall, free markets in which firms’ pricing strategies are unchecked fail to be an effective device to allocate resources. The regulation we propose is precisely designed to avoid such an undesirable outcome and allow market forces to efficiently

⁶Though existence of a mixed-strategy equilibrium is guaranteed (Carmona and Fajardo (2009)), no explicit characterization is available; we may also be concerned about the descriptive relevance and the normative properties of such an equilibrium.

play their role.

An attractive feature of our regulation is its parsimony in informational terms, as the regulator only needs to observe the aggregate profit that a firm earns on each contract. This is consistent with the recent evolution of financial reporting standards. Indeed, over the last decade, the International Accounting Standards Board has suggested several measures aimed at defining general principles that an entity should apply to report information in its financial statements about the nature of cash flows from insurance contracts (IASB (2013)). In particular, since 2011, insurance companies have been required to perform an onerous-contract test when circumstances indicate that the contract might be loss-making. As soon as a contract is so assessed, the company has to record a provision in its financial statements for the corresponding expected loss.

The regulation we propose can alternatively be seen as banning profits on basic-coverage contracts, a measure which is already in place in some insurance markets. For instance, in health insurance, Germany and Switzerland rely on a central fund to redistribute costs among firms according to a risk-equalization scheme.⁷ These cost-sharing mechanisms, by pooling and redistributing costs among sellers of a basic standardized coverage contract, prevent firms from earning abnormal profits on basic coverage.⁸

Finally, our implementation does not rely on firms exchanging information about their customers. Whereas regulation sometimes encourage this practice,⁹ communication between firms requires a rather sophisticated institutional setting to take place. Thus, while, under multiple contracting, each firm may in principle benefit from accessing information about all its customers' trades, the aggregation of this dispersed information would in practice involve complex information-sharing mechanisms, with possibly several rounds of communication. Our regulation bearing on realized profits seems in comparison easier to implement.

⁷See Thomson and Mossialos (2009, page 84). Besides Germany and Switzerland, other countries using such schemes include Australia, Ireland, Netherlands, and Slovenia.

⁸In Switzerland, the basic coverage contract is defined at the national level; then firms compete over prices to provide the corresponding amount of coverage. Yet, an additional rule specifies that costs are pooled and redistributed among firms. In Germany, the basic coverage contract is also defined at the national level and is offered by 134 not-for-profit, nongovernmental "sickness funds." Consumers contribute a fixed fraction of their wealth; these contributions are then centrally pooled and redistributed to sickness funds according to a rather precise risk-adjusted capitation formula. More generally, risk equalization involves transfer payments between firms so as to spread some of the claims cost of the high-risk, older, and less healthy consumers among all firms in the market, in proportion to their respective market shares.

⁹The Commission Regulation 267/2010 of March 24, 2010 states: "Collaboration between insurance undertakings or within associations of undertakings in the compilation of information (which may also involve some statistical calculations) allowing the calculation of the average cost of covering a specified risk in the past or, for life insurance, tables of mortality rates or of the frequency of illness, accident and invalidity, makes it possible to improve the knowledge of risks and facilitates the rating of risks for individual companies. This can in turn facilitate market entry and thus benefit consumers."

Contributions to the Literature

The regulation we propose is unique in the adverse-selection literature in that it bears on the supply rather than on the demand side of the economy: public intervention should target firms' pricing strategies, while consumers should be left free to choose their preferred amount of coverage. This contrasts with the policy recommendations that have been advocated in the exclusive-competition literature. For instance, mandatory insurance is evoked in Akerlof (1970), and has been empirically investigated by Finkelstein (2004), Einav, Finkelstein, and Cullen (2010), Einav and Finkelstein (2011). Similarly, Wilson (1977), Dahlby (1981), and Crocker and Snow (1985a) show that making basic coverage mandatory and simultaneously allowing private insurers to compete on an extended coverage permits to reach a second-best outcome.¹⁰ Neither are taxes or subsidies needed, unlike in Crocker and Snow (1985b). Instead, regulated markets are powerful enough to select a unique equilibrium in which prices efficiently reflect costs—though this rule applies to successive layers of coverage and not to the aggregate coverage amounts purchased by each type of consumers.

The penalization of cross-subsidies between contracts represents a minimal departure from Attar, Mariotti, and Salanié's (2014) canonical model of nonexclusive competition under adverse selection. It resolves an essential tension in this model by showing that market forces can achieve third-best efficiency whenever firms' pricing strategies are properly checked. This minimal departure implies that the extensive form of the regulated game remains remarkably simple. In particular, our equilibrium construction does not rely on complex interfirm communication, unlike those of Jaynes (1978, 2011), Hellwig (1988), and Stiglitz, Yun, and Kosenko (2018). In their formulations, an insurance contract issued by a firm can incorporate a potentially large number of restrictions contingent on the information disclosed by its customers and its competitors. The possibility to enforce exclusivity clauses and to withdraw a policy following a violation of contractual agreements make their analyses of limited relevance to study competition on nonexclusive markets. In particular, their assumption that firms can choose not to disclose their offer of insurance is in tension with the assumption that consumers and, hence, in principle, firms should know about a firm selling insurance. Our regulated game eschews these conceptual problems by staying as close as possible to the nonexclusive-competition benchmark.

Our regulation can also be related to the regulatory measures preventing retailers from engaging in below-cost pricing or loss-leading practices, which have been adopted in several

¹⁰Villeneuve (2003) performs a similar analysis under nonexclusive competition, albeit in a framework that assumes linear pricing.

countries since the early 90s.¹¹ Whereas these measures are typically motivated by the risk of predatory or anti-competitive pricing by multi-product retailers, as emphasized by Chen and Rey (2012, 2019), they have so far not been invoked as a device to help insurance markets function efficiently. More abstractly, we should observe that our regulatory requirement is in line with competitive-search and competitive-equilibrium models of markets with adverse selection. Indeed, the equilibrium concepts introduced in these approaches explicitly require that each contract break even, as for instance in Guerrieri, Shimer, and Wright (2010) and Azevedo and Gottlieb (2017). A key difference is that these authors focus on exclusive competition, while we allow for multiple contracting.

Our results are more generally relevant for the regulation of financial markets. In the aftermath of the recent crisis, the design of public intervention under the threat of adverse selection has become a central issue for the regulation of credit and interbank markets. In particular, it has been argued that liquidity-injection programs that provide a credible signal to uninformed lenders can help to avoid market breakdown by attracting the least profitable borrowers, either through direct lending (Philippon and Skreta (2012)) or by repurchasing low-quality assets (Tirole (2012)). In modern financial markets, however, borrowers' choices are not limited to opting out of a public program or exclusively participating to it, because they have the opportunity to complement such a program with additional funds raised on private markets.¹² In this context, our results suggest that public intervention is not needed to unfreeze the market but rather to improve its functioning. To achieve this goal, a program must successfully discipline lenders' strategic behavior by preventing them from engaging in dumping practices. Such an intervention would achieve budget balance, unlike those proposed by Philippon and Skreta (2012) and Tirole (2012), and induce all types of borrowers to participate.

The paper is organized as follows. Section 2 describes the model. Section 3 shows that the JHG allocation is the only candidate equilibrium allocation of the regulated game. Section 4 shows how to implement the JHG allocation under oligopoly and free entry. Section 5 draws the lessons from our analysis. Section 6 concludes. All proofs are in Appendices A and B.

¹¹These practices are banned in several European countries (*The Economist* (2008) <https://www.economist.com/node/10430246> describes the Belgian case), and restricted in a number of US states; in California, laws against loss leaders have been enforced as early as 1933.

¹²Focusing on the US interbank market over the 2007–2010 period, Armantier, Ghysels, Sarkar, and Shrader (2015) document how banks combine their loans at the Fed Discount Window with additional funding raised on ABCP and Repo markets, which exhibit similar lending terms with respect to eligibility, collateral and maturity. Similarly, Berger, Black, Bouwman, and Dlugosz (2016) show how, over the same period, the banks' reliance on both the Fed's Discount Window and Term Auction Facility liquidity programs significantly increased their aggregate lending.

2 An Insurance Economy

Consumers There is a continuum of consumers who can purchase coverage in exchange for an insurance premium. Each consumer is privately informed of her type $i = 1, 2$ and the proportion of type i among consumers is $m_i > 0$. Type i 's preferences over aggregate coverage-premia bundles $(Q, T) \in \mathbb{R}_+ \times \mathbb{R}$ are represented by a strictly quasiconcave and twice continuously differentiable utility function u_i , with $\partial_T u_i < 0$. Hence the marginal rate of substitution

$$\tau_i \equiv - \frac{\partial_Q u_i}{\partial_T u_i}$$

is well defined and strictly decreasing along her indifference curves. We impose the Inada condition that $\tau_i(Q, T)$ vanishes as Q grows large along any such curve. Hence, whatever her endowment point, type i has a finite demand for coverage at any premium rate $p > 0$. The following strict single-crossing condition is the key determinant of consumer demand:

$$\text{For each } (Q, T), \tau_2(Q, T) > \tau_1(Q, T). \quad (1)$$

Thus type 2 is more willing to increase her purchases of coverage than type 1.

Firms There are $n \geq 3$ identical firms. A firm providing a consumer of type i with coverage q for a premium t earns an expected profit $t - r_i q$, where r_i is type i 's riskiness. Type 2 has higher riskiness than type 1:

$$r_2 > r_1. \quad (2)$$

Together with (1), (2) generates adverse selection. We let $r \equiv m_1 r_1 + m_2 r_2$ be the average riskiness of a consumer. We assume that individual risks are independent and that an appropriate version of the law of large numbers applies (Sun (2006)).

Contracts Each firm monitors the amount of coverage it sells to each consumer, but not the amounts of coverage she purchases from its competitors. A *contract* is thus a pair (q, t) for some $q \geq 0$, with premium rate t/q if $q > 0$. A consumer's *aggregate trade* is

$$(Q, T) \equiv \left(\sum_k q^k, \sum_k t^k \right), \quad (3)$$

where (q^k, t^k) is the contract she trades with firm k , with $(q^k, t^k) = (0, 0)$ in case she chooses not to trade with firm k . A symmetric aggregate allocation, hereafter simply *allocation*, is a pair of aggregate trades, one for each type.

Coinsurance In our leading example, each type i has initial wealth W_0 and faces the risk of a loss \tilde{L} distributed according to a density f_i relative to a fixed measure λ . A contract (q, t) covers a fraction q of the loss for a premium t , and multiple contracts can be aggregated as in (3). Type i 's preferences have an expected-utility representation

$$u_i(Q, T) \equiv \int v(W_0 - (1 - Q)l - T) f_i(l) \lambda(dl), \quad (4)$$

where v is a twice continuously differentiable, strictly increasing, and strictly concave utility index. Type i 's riskiness is

$$r_i \equiv \int l f_i(l) \lambda(dl). \quad (5)$$

We assume that f_2 strictly dominates f_1 in the monotone-likelihood-ratio order, which implies (1)–(2).¹³ In the Rothschild and Stiglitz (1976) specification, there is a single loss level L , λ is the counting measure on $\{0, L\}$, and $r_i \equiv f_i(L)L$.

Our model encompasses many alternative specifications, such as rank-dependent expected utility (Quiggin (1982)), robust control (Hansen and Sargent (2007)), and smooth ambiguity aversion (Klibanoff, Marinacci, and Mukerji (2005)).¹⁴ It also allows for multiple loss levels as long as aggregation is feasible—as a counterexample, contracts with deductibles do not nicely aggregate. What matters is that coverage be summarized by a one-dimensional index and that preferences satisfy a single-crossing property along that dimension.

The Regulated Game We augment the nonexclusive-competition benchmark of Attar, Mariotti, and Salanié (2014) with an additional stage capturing an anti-dumping regulation. Specifically, we assume that a regulatory agency is able to credibly punish firms that end up with nonnegative profits while cross-subsidizing between contracts. The regulated game unfolds as follows.

1. Each firm k offers a compact menu of contracts C^k that contains at least the no-trade contract $(0, 0)$.
2. After privately learning her type, each consumer selects a contract from each of the menus C^k .
3. If a firm earns a nonnegative profit but makes a loss on a contract, then its profit is confiscated and the firm is fined.

¹³See Appendix B for a proof.

¹⁴See Attar, Mariotti, and Salanié (2019a, Appendix C) for a discussion.

For instance, the regulatory requirement is met if each contract offered by a firm is managed by a dedicated profit center, operating with its own budget, and cannot make a loss.

A pure strategy for a consumer is a function that maps any profile of menus (C^1, \dots, C^m) into a selection of contracts $((q^1, t^1), \dots, (q^n, t^n)) \in C^1 \times \dots \times C^m$. Compactness of the menus C^k ensures that type i 's utility-maximization problem

$$\max \left\{ u_i \left(\sum_k q^k, \sum_k t^k \right) : (q^k, t^k) \in C^k \text{ for all } k \right\}$$

always has a solution. Throughout the analysis, we focus on symmetric pure-strategy perfect Bayesian equilibria, hereafter simply *equilibria*, in which all consumers of the same type play the same pure strategy. A *free-entry equilibrium* is an equilibrium in which at least one firm is inactive, that is, does not trade on the equilibrium path.

3 Equilibrium Characterization

This section establishes the first half of our implementation result, namely, that the JHG allocation introduced by Jaynes (1978) and further studied by Hellwig (1988) and Glosten (1994) is the unique allocation that can be supported in an equilibrium of the regulated game. We first give a general characterization result. We then argue that equilibrium is consistent with free entry only under additional restrictions on the JHG allocation.

3.1 The Uniqueness Result

The uniqueness part of our implementation result can be stated as follows.

Theorem 1. *The JHG allocation defined by*

$$Q_1^* \equiv \arg \max \{u_1(Q, rQ) : Q \geq 0\}, \tag{6}$$

$$T_1^* \equiv rQ_1^*, \tag{7}$$

$$Q_2^* \equiv Q_1^* + \arg \max \{u_2(Q_1^* + Q, T_1^* + r_2Q) : Q \geq 0\}, \tag{8}$$

$$T_2^* \equiv T_1^* + r_2(Q_2^* - Q_1^*), \tag{9}$$

is the unique candidate equilibrium allocation of the regulated game.

In the JHG allocation, types 1 and 2 purchase basic coverage Q_1^* at premium rate r and type 2 purchase complementary coverage $Q_2^* - Q_1^*$ at premium rate r_2 . This is a marginal version of Akerlof (1970) competitive pricing: the *layers* Q_1^* and $Q_2^* - Q_1^*$ are fairly priced given the types who purchase them, and the size of each layer is optimal for type 1 and

type 2, respectively, subject to this constraint. Figure 1 depicts the candidate equilibrium allocation when both layers are strictly positive.

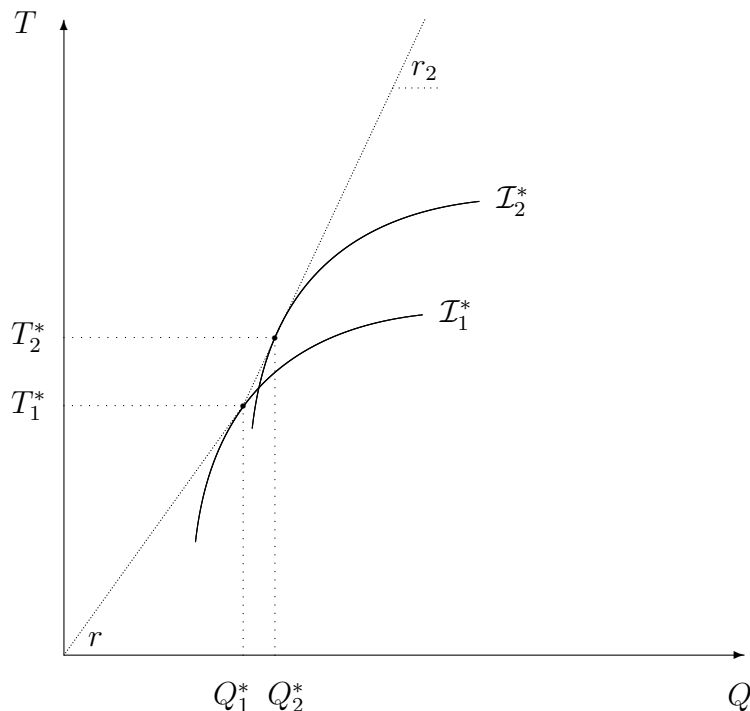


Figure 1 The candidate equilibrium allocation.

In the coinsurance example, type 1 obtains partial coverage, $Q_1^* < 1$, and type 2 obtains full coverage, $Q_2^* = 1$; hence the complementary layer is strictly positive. By (6)–(9), this implies that both types’ incentive-compatibility constraints are slack, so that the candidate equilibrium allocation does not belong to the second-best efficiency frontier (Crocker and Snow (1985a)). However, this allocation can be deemed efficient in a weaker, third-best sense. Indeed, the JHG allocation can be characterized as the unique budget-feasible allocation that is robust to side trading (Attar, Mariotti, and Salanié (2019d)). Thus a social planner who can neither observe consumer types nor monitor their trades with firms cannot Pareto-improve on the candidate equilibrium allocation. In this weak sense, Theorem 1 can be interpreted as a version of the First Welfare Theorem for our economy.

Because the JHG allocation is budget-balanced, firms must earn zero profit in equilibrium. We show in Appendix A that this implies that any traded contract has premium rate r or r_2 : that is, every active firm either provides basic coverage to both types, or complementary coverage to type 2 only. A related insight of our analysis is that, as in standard Bertrand competition, no firm is indispensable in providing each type with her equilibrium utility:

otherwise, it could earn a strictly positive profit by slightly increasing the premium it charges. Specifically, we show in Appendix A that, if any firm withdraws its menu offer, type 1 can still reach her equilibrium aggregate trade (Q_1^*, T_1^*) , while type 2 can still obtain her equilibrium utility $u_2(Q_2^*, T_2^*)$ by purchasing an amount of coverage at least equal to Q_2^* . In particular, equilibrium requires that there be excess supply of coverage at the average premium rate r . This fact will play an important role in our equilibrium constructions, both under oligopoly (Section 4.2) and under free entry (Section 4.3).

3.2 Free-Entry Equilibria and Size Restrictions

Standard approaches to the study of competitive insurance markets under adverse selection often postulate free entry. This premise is shared by strategic models à la Rothschild and Stiglitz (1976), as well as by the competitive-search and competitive-equilibrium models of Guerrieri, Shimer, and Wright (2010) and Azevedo and Gottlieb (2017). When evaluating the consequences of multiple contracting—a possibility disregarded by these authors—it is, therefore, important to show to which extent and under which conditions our characterization carries over to the case of free entry. In our strategic framework, this is captured by the notion of a free-entry equilibrium.

Because it characterizes the unique candidate equilibrium of the regulated game in full generality, independently of whether all firms are active on the equilibrium path, Theorem 1 also holds in a free-entry equilibrium.¹⁵ However, free entry imposes additional restrictions on the relative sizes of the basic and complementary layers. To see why, let us assume that the basic layer is strictly positive. The dispensability property then implies that the aggregate supply at premium rate r exceeds Q_1^* . We must thus make sure that type 2 be not tempted to purchase basic coverage in excess of Q_1^* . But, under free entry, we must also make sure that it be impossible for an inactive firm to exploit this excess supply of basic coverage. The resulting size restrictions can be formulated as follows.

Lemma 1. *If the regulated game has a free-entry equilibrium in which $Q_1^* > 0$, then*

$$u_2(Q_2^*, T_2^*) \geq u_2(2Q_1^*, 2T_1^*), \quad (10)$$

$$Q_1^* > Q_2^* - Q_1^* \quad (11)$$

hold at the JHG allocation.

¹⁵In fact, under free entry, we can invoke Attar, Mariotti, and Salanié (2019d, Theorem 2) to conclude that the JHG allocation remains the only candidate equilibrium allocation. The argument is simpler, as it only relies on inactive firms, so that we do not have to worry about the profit a firm must forego on existing contracts if it deviates from equilibrium.

Conditions (10)–(11) are easy to understand when only two firms issue contracts at the average premium rate r . First, because neither of them is indispensable, both must offer a contract equal to type 1’s equilibrium aggregate trade (Q_1^*, T_1^*) . Condition (10) then simply expresses that type 2 is not strictly better off trading (Q_1^*, T_1^*) twice on the equilibrium path. Next, if condition (11) were not satisfied, then an inactive firm could profitably attract type 2 by offering her a coverage $Q_2^* - 2Q_1^*$ for a premium slightly lower than $T_2^* - 2T_1^*$, but at a premium rate higher than r_2 ;¹⁶ indeed, combined with the trade $(2Q_1^*, 2T_1^*)$, which, by assumption, is available on the equilibrium path, this offer would allow type 2 to pay less than T_2^* for her equilibrium coverage Q_2^* . This logic easily extends when more than two firms issue contracts at the average premium rate r .

Geometrically, conditions (10)–(11) are satisfied when the aggregate trade $(2Q_1^*, 2T_1^*)$ is located in the lower contour set of (Q_2^*, T_2^*) for type 2, to the right of (Q_2^*, T_2^*) , as in Figure 1. This requires that the complementary layer be sufficiently small relative to the basic layer. In the coinsurance example, this is the case whenever the loss densities of type 1 and type 2 are not too different.

4 Equilibrium Existence

Theorem 1 singles out the JHG allocation as the unique candidate equilibrium allocation of the regulated game. We now provide conditions on consumer preferences under which an equilibrium exists, thereby establishing the second half of our implementation result. We consider two scenarios, the oligopoly case in which all firms are active on the equilibrium path, and the free-entry case in which at least one firm is inactive on the equilibrium path. To focus on the most relevant scenario for applications, which is also the most challenging for the theory, we throughout assume that both the basic and the complementary layers of the JHG allocation are strictly positive.¹⁷ As we will see, the structure of equilibrium menus is essentially determined by cream-skimming deviations, defined as contracts that only attract type 1, either per se or—and this is the specificity of multiple contracting—in combination with contracts offered by other firms. A necessary feature of our equilibrium constructions is that firms must issue latent contracts to discipline their competitors. These contracts are not traded in equilibrium but ensure that cream-skimming deviations are unprofitable.

¹⁶ Notice that, in this configuration, $Q_2^* \neq 2Q_1^*$ if an equilibrium exists. Otherwise, type 2 would be strictly better off trading (Q_1^*, T_1^*) twice instead of (Q_2^*, T_2^*) on the equilibrium path.

¹⁷This assumption is implicit in Jaynes (1978) and Hellwig (1988). When the JHG allocation is degenerate with $(Q_1^*, T_1^*) = (0, 0)$, it can be implemented by letting firms offer linear tariffs at the fair premium rate r_2 (Attar, Mariotti, and Salanié (2014)).

4.1 Cream Skimming and Latent Contracts

4.1.1 Why Latent Contracts Are Necessary

Two types of contracts are traded in equilibrium: basic-coverage contracts with premium rate r , and complementary-coverage contracts with premium rate r_2 . Is that enough to sustain an equilibrium? To answer this question, consider the configuration illustrated in Figure 2, where the JHG allocation satisfies (10)–(11). A natural candidate equilibrium would have each firm offering a basic-coverage contract (Q_1^*, T_1^*) and a complementary-coverage contract $(Q_2^* - Q_1^*, T_2^* - T_1^*)$. Notice that, in this construction, no firm is indispensable in providing each type i with her utility $u_i(Q_i^*, T_i^*)$, as required by equilibrium.

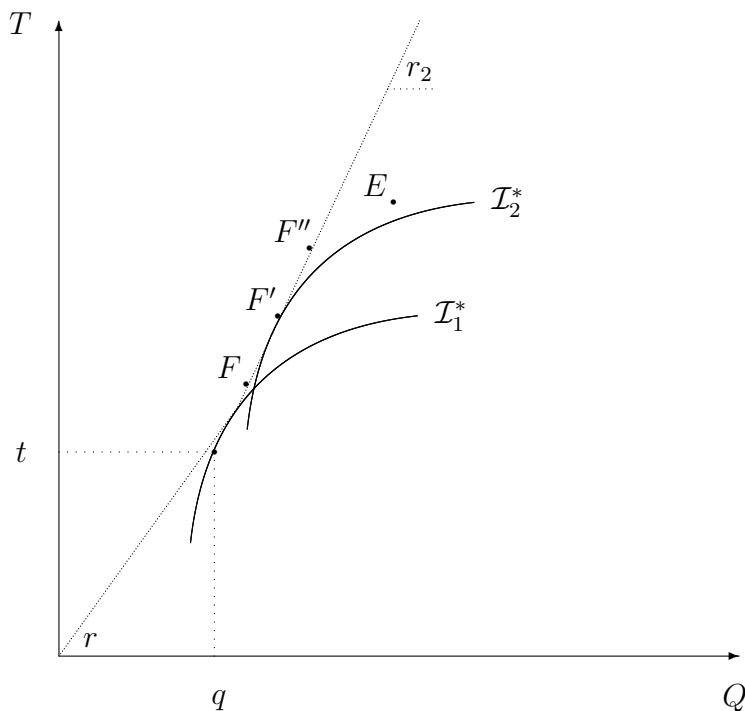


Figure 2 Cream skimming type 1.

Now, any firm k can deviate by offering the contract (q, t) , which offers an amount of coverage close to but lower than Q_1^* at a premium rate slightly lower than r , with $t - T_1^* > r_2(q - Q_1^*)$. This contract strictly attracts type 1 and is profitable if it does not attract type 2. This is indeed the case, because combining (q, t) with the contract (Q_1^*, T_1^*) (leading to aggregate trade E) or with one or several contracts $(Q_2^* - Q_1^*, T_2^* - T_1^*)$ (leading to aggregate trades F, F', F'', \dots) leaves type 2 with a lower utility than trading (Q_2^*, T_2^*) . As type 2 can obtain her equilibrium utility $u_2(Q_2^*, T_2^*)$ by trading with firms $l \neq k$, any firm k can thus

cream skim type 1 and make a profit.

The upshot of this discussion is that, to sustain an equilibrium, firms have to offer additional, latent contracts that are only meant to block cream-skimming deviations. We now turn to the study of how these contracts operate. Our goal is to provide a minimal implementation with as few latent contracts as possible.

4.1.2 Large Cream-Skimming Deviations

We first consider deviations that provide type 1 with utility at least $u_1(Q_1^*, T_1^*)$ even if she does not trade other contracts. These deviations are the standard ones considered in exclusive-competition models such as Rothschild and Stiglitz's (1976). We call them *large cream-skimming deviations* to emphasize the fact that a firm needs to provide a large amount of coverage to match $u_1(Q_1^*, T_1^*)$ and attract type 1. A latent contract (q^ℓ, t^ℓ) blocks large cream-skimming deviations if

$$\text{For each } (q, t), u_1(q, t) \geq u_1(Q_1^*, T_1^*) \text{ implies } u_2(q + q^\ell, t + t^\ell) \geq u_2(Q_2^*, T_2^*). \quad (12)$$

That is, if (q, t) per se attracts type 1, then it also attracts type 2 in combination with (q^ℓ, t^ℓ) . Then (q, t) cannot make a profit, because $u_1(q, t) \geq u_1(Q_1^*, T_1^*)$ implies by (6)–(7) that (q, t) has at most premium rate r .

Geometrically, (12) states that the translate of the upper contour set of (Q_1^*, T_1^*) for type 1 along the vector (q^ℓ, t^ℓ) lies in the upper contour set of (Q_2^*, T_2^*) for type 2. The following lemma shows that this property is satisfied by at most one latent contract.

Lemma 2. *The contract defined by*

$$u_2(Q_1^* + q_1^\ell, T_1^* + t_1^\ell) = u_2(Q_2^*, T_2^*), \quad (13)$$

$$\tau_2(Q_1^* + q_1^\ell, T_1^* + t_1^\ell) = r, \quad (14)$$

is the unique latent contract that possibly blocks large cream-skimming deviations.

Thus the requirement that large cream-skimming deviation be blocked by a single latent contract, in line with our minimal-implementation objective, singles out a unique candidate latent contract. We now provide a sufficient condition for (q_1^ℓ, t_1^ℓ) to satisfy (12), which is stated in terms of the Gaussian curvatures

$$\kappa_i \equiv \frac{1}{\|\partial U_i\|^3} \begin{vmatrix} -\partial^2 U_i & \partial U_i \\ -\partial U_i^\top & 0 \end{vmatrix}$$

of each type i 's indifference curves (Debreu (1972)).

Assumption C. For each i , $\kappa_i > 0$. Moreover, one of the following statements holds:

(i) For all Q_1, Q_2, T_1 , and T_2 , if $\tau_1(Q_1, T_1) = \tau_2(Q_2, T_2)$, then $\kappa_1(Q_1, T_1) > \kappa_2(Q_2, T_2)$.

(ii) For all Q_1, Q_2, T_1 , and T_2 , if $\tau_1(Q_1, T_1) = \tau_2(Q_2, T_2)$, then $\kappa_1(Q_1, T_1) = \kappa_2(Q_2, T_2)$.

That the curvatures of consumers' indifference curves nowhere vanish is a very weak requirement that we only impose for technical reasons.¹⁸

Assumption C(i) states that type 2's indifference curves are flatter than type 1's, once these curves are translated so as to make them tangent at the relevant aggregate trade. This differs from the standard single-crossing condition (1), which does not allow for translations. The latter are natural operations when consumers can combine several contracts, and we can view Assumption C as a second-order version of the single-crossing condition. Assumption C(ii) corresponds to the limiting case where any two pairs of indifference curves for types 1 and 2 are, over the relevant range, translates of each other. Assumption C can alternatively be phrased in terms of type 1's and type 2's Hicksian demand functions for coverage. We develop this interpretation in Appendix B.

The following result then holds.

Lemma 3. *If consumer preferences satisfy Assumption C, then the contract (q_1^ℓ, t_1^ℓ) blocks large cream-skimming deviations.*

Given the crucial role Assumption C plays in our equilibrium construction, it is obviously important to assess how restrictive it is. We examine this question in the context of the coinsurance example, only modified to allow for type-dependent utility indices v_i in (4).¹⁹ When there is a single loss level, Assumption C(i) is satisfied if $\min -v_1''/v_1' > \max -v_2''/v_2'$, that is, if type 1 is uniformly more risk-averse than type 2 in the sense of Aumann and Serrano (2008). When there are multiple loss levels, Assumption C(i) is satisfied if each type i 's density of losses belongs to a natural exponential family,

$$f_i(l) \equiv f(l|\theta_i) \equiv h(l) \exp(\theta_i l - A(\theta_i)), \quad (15)$$

and each type i has constant absolute risk aversion α_i , with $\alpha_1 > \alpha_2$.²⁰ This last requirement is in tension with the single-crossing condition (1), which now holds if and only if $\theta_2 - \theta_1 >$

¹⁸This amounts to assuming that preferences are nonlinear, even in a local sense. Such preferences are called *differentially strictly convex* by Mas-Colell (1985, Definition 2.6.1).

¹⁹See Appendix B for the proofs. More generally, the condition in Assumption C(i) is open, and thus defines a countable intersection of open sets in the space of smooth strictly convex preferences over (Q, T) that are strictly decreasing in T (Attar, Mariotti, and Salanié (2019b)).

²⁰Constant absolute risk aversion implies that Hicksian and Marshallian demands are the same.

$\alpha_1 - \alpha_2$, that is, if type 2 is sufficiently riskier than type 1. Assumption C(ii) is satisfied in the limiting case $\alpha_1 = \alpha_2$. Finally, (15) allows for a rich class of loss distributions, including Bernoulli, binomial, gamma, normal, and Poisson distributions.

These assumptions are common in the literature. The quadratic preferences in Biais, Martimort, and Rochet (2000) satisfy Assumption C(ii). Einav, Finkelstein, and Cullen (2010) and Einav, Finkelstein, Ryan, Schrimpf, and Cullen (2013) abstract from income effects in their estimations of the welfare cost of adverse selection and of the magnitude of selection on moral hazard in health insurance, see also Einav and Finkelstein (2011) and Chetty and Finkelstein (2013). As in Finkelstein and McGarry (2006), we allow preference-based ($\alpha_1 > \alpha_2$) and risk-based ($\theta_2 > \theta_1$) selection to act in offsetting directions, although not to the point of overturning adverse selection ($\theta_2 - \theta_1 > \alpha_1 - \alpha_2$).

4.2 Equilibrium under Oligopoly

We now turn to the construction of an equilibrium, starting with the oligopoly case.

4.2.1 Dispensability and Small Cream-Skimming Deviations

A necessary condition for equilibrium is that, if any firm withdraws its menu offer, type 1 can still trade (Q_1^*, T_1^*) . There are a priori various ways of ensuring this. In the oligopoly case, we shall simply assume that each of the n firms offers a fraction $1/(n-1)$ of the aggregate quantity Q_1^* at the average premium rate r , that is, the contract

$$(q_{1,n}^*, t_{1,n}^*) \equiv \frac{1}{n-1} (Q_1^*, T_1^*). \quad (16)$$

Thus the aggregate trade (Q_1^*, T_1^*) remains available following any firm's unilateral deviation. Notice that the excess supply of basic coverage is exactly equal to $q_{1,n}^*$, and thus by (16) is small when n is large.

A second necessary condition for equilibrium is that, if any firm withdraws its menu offer, type 2 can still obtain her equilibrium utility $u_2(Q_2^*, T_2^*)$. However, trading $n-1$ contracts $(q_{1,n}^*, t_{1,n}^*)$ does not allow type 2 to obtain her equilibrium utility, as $u_2(Q_2^*, T_2^*) > u_2(Q_1^*, T_1^*)$ if the complementary layer is strictly positive. Our equilibrium construction, therefore, requires a second latent contract, defined as

$$(q_{2,n}^\ell, t_{2,n}^\ell) \equiv (q_{1,n}^*, t_{1,n}^*) + (q_1^\ell, t_1^\ell). \quad (17)$$

In combination with $n-2$ contracts $(q_{1,n}^*, t_{1,n}^*)$, the contract $(q_{2,n}^\ell, t_{2,n}^\ell)$ allows type 2 to reach the aggregate trade $(Q_1^* + q_1^\ell, T_1^* + t_1^\ell)$ on her equilibrium indifference curve.

At this point, we should observe that the possibility of multiple contracting also allows a firm to exploit the offers of its competitors to attract a specific type. Indeed, a deviation may succeed in attracting only type 1 even if it only involves a small amount of coverage, a phenomenon that cannot arise in the standard exclusive-competition framework. Let us accordingly call *small cream-skimming deviations* the contracts that attract type 1, but only together with other contracts. We now informally argue that the contract $(q_{2,n}^\ell, t_{2,n}^\ell)$ is required to block any such deviation. Consider for instance a contract (q, t) with $t < rq$ that attracts type 1 in combination with $n - 1$ contracts $(q_{1,n}^*, t_{1,n}^*)$, but not with fewer such contracts. Then the latent contract (q_1^ℓ, t_1^ℓ) does not help blocking this deviation, because type 2 cannot trade it and at the same time mimic the trades of type 1, which already requires trading with all firms. Moreover, trading (q_1^ℓ, t_1^ℓ) on top of only $n - 2$ contracts $(q_{1,n}^*, t_{1,n}^*)$ need not be an attractive option for type 2, as, for instance, when the equilibrium indifference curves of type 1 and type 2 are translates of each other. Fortunately, the contract $(q_{2,n}^\ell, t_{2,n}^\ell)$, given its additive form (17), comes to help by ensuring that type 2 can effectively behave as if, at the deviation stage, $n - 1$ contracts $(q_{1,n}^*, t_{1,n}^*)$ were available for her to trade on top of (q, t) and (q_1^ℓ, t_1^ℓ) . Assumption C then ensures that she would thereby obtain a utility at least equal to $u_2(Q_2^*, T_2^*)$. She is thus willing to trade (q, t) and, because $t < rq$, the contract (q, t) makes a loss.

4.2.2 Existence of an Equilibrium

We are now ready to state the existence part of our implementation result.

Theorem 2. *Suppose that consumer preferences satisfy Assumption C. Then, if there are sufficiently many firms, the regulated game has an equilibrium.*

Together with Theorem 1, Theorem 2 can be interpreted as a weak version of the Second Welfare Theorem for our economy: under suitable conditions on consumer preferences, the regulated game has an equilibrium, and this equilibrium implements the unique third-best allocation.

Our equilibrium construction relies on four contracts made available by each firm: a basic-coverage contract $(q_{1,n}^*, t_{1,n}^*)$ with premium rate r , a complementary-coverage contract $(Q_2^* - Q_1^*, T_2^* - T_1^*)$ with premium rate r_2 , and the latent contracts (q_1^ℓ, t_1^ℓ) and $(q_{2,n}^\ell, t_{2,n}^\ell)$. The requirement that there be sufficiently many firms reflects two considerations.

First, because no firm is indispensable in offering basic coverage, there is excess supply of coverage at the average premium rate r . In particular, type 2 could trade n basic-coverage contracts $(q_{1,n}^*, t_{1,n}^*)$. For n large enough, the resulting aggregate trade $[n/(n - 1)](Q_1^*, T_1^*)$ is

arbitrarily close to (Q_1^*, T_1^*) . This ensures that type 2, for whom $u_2(Q_2^*, T_2^*) > u_2(Q_1^*, T_1^*)$, is not tempted by this option on the equilibrium path.

Second, the fact that the quantity Q_1^* is provided through a large number $n - 1$ of small contracts $(q_{1,n}^*, t_{1,n}^*)$ ensures that, if some firm k attempted to use a small cream-skimming deviation (q, t) with $t < rq$, then type 1 could obtain at least her equilibrium utility $u_1(Q_1^*, T_1^*)$ by trading (q, t) with firm k and at least one contract $(q_{1,n}^*, t_{1,n}^*)$ with some firm $l \neq k$.²¹ Then, under Assumption C, type 2 could mimic these trades, with the sole difference that she would trade $c_{2,n}^\ell = c_{1,n}^* + c_1^\ell$ instead of $c_{1,n}^*$ with firm l . By Lemma 3, she would thereby obtain at least her equilibrium utility $u_2(Q_2^*, T_2^*)$, thus blocking firm k 's attempt at cream skimming.

While our discussion has so far focused on cream-skimming deviations, the proof of Theorem 2 also has to deal with the case in which a firm attempts to screen type 1 and type 2 by offering a menu of contracts. We show that the regulation ensures that the latent contracts (q_1^ℓ, t_1^ℓ) and $(q_{2,n}^\ell, t_{2,n}^\ell)$ also discourage menu deviations.

One intuitive interpretation of this equilibrium construction is that basic coverage is provided through a *pool*, to which each firm contributes a small fraction $[1/(n - 1)](Q_1^*, T_1^*)$. These contracts are then aggregated, and consumers are left free to choose whichever fraction of the pool to purchase. When sufficiently many firms contribute to the pool, the latent contracts (q_1^ℓ, t_1^ℓ) and $(q_{2,n}^\ell, t_{2,n}^\ell)$ ensure that none of them has an incentive to deviate by offering coverage at a lower premium rate than the contracts offered by the other firms contributing to the pool, and the regulation ensures that no firm has an incentive to deviate by offering a menu of contracts.

Theorem 2 differs from related earlier contributions in three ways. First, the regulation we propose, by penalizing cross-subsidies between contracts, permits to support cross-subsidies between types in equilibrium. This contrasts with Attar, Mariotti, and Salanié (2014), whose unregulated nonexclusive-competition game has no equilibrium when the basic layer of the JHG allocation is strictly positive. Second, firms in the regulated game cannot exchange information about their customers' trades. This contrasts with Jaynes (1978, 2011), Hellwig (1988), and Stiglitz, Yun, and Kosenko (2018), whose equilibrium constructions explicitly rely on interfirm communication. Third, firms in the regulated game cannot react to the offers of their competitors. This contrasts with Hellwig (1988), who designs a specific sequential timing for the firms' offers requiring several steps of interfirm communication to implement the JHG allocation.

²¹Notice that we do not claim that this is necessarily the case at her best response.

4.3 Equilibrium under Free Entry

By construction, the equilibrium exhibited in Theorem 2 is not a free-entry equilibrium. Indeed, a putative inactive $n + 1^{\text{th}}$ firm would have an incentive to exploit the aggregate trade $[n/(n-1)](Q_1^*, T_1^*)$, available on the equilibrium path, to offer complementary coverage $Q_2^* - Q_1^* - q_{1,n}^*$ for a premium slightly lower than $T_2^* - T_1^* - t_{1,n}^*$: the corresponding contract would attract type 2 at a premium rate higher than r_2 and thus make a profit.²² We know from Lemma 1 that a free-entry equilibrium exists only under the size restrictions (10)–(11). The following result shows that these restrictions, along with Assumption C, are also sufficient for the existence of a free-entry equilibrium.

Theorem 3. *Suppose that consumer preferences satisfy Assumption C. Then the regulated game has a free-entry equilibrium if and only if (10)–(11) hold at the JHG allocation.*

Our equilibrium construction relies on two active firms, each offering a menu recursively defined on the basis of three contracts: a basic-coverage contract (Q_1^*, T_1^*) with premium rate r , a complementary-coverage contract $(Q_2^* - Q_1^*, T_2^* - T_1^*)$ with premium rate r_2 , and the latent contract (q_1^ℓ, t_1^ℓ) . In addition, at least one inactive firm offers the same complementary-coverage contract as the active firms, so that, if an active firm withdraws its menu offer, type 2 can still obtain her equilibrium utility $u_2(Q_2^*, T_2^*)$ by purchasing basic coverage from the other active firm and complementary coverage from an inactive firm. Compared with Theorem 2, firms no longer play symmetric roles. In particular, active and inactive firms face different market configurations at the deviation stage; this issue is dealt with in the proof of Theorem 3 by showing that the latent contracts offered by active firms also deter entry by inactive ones. The regulation plays the same role as in the oligopoly case in making menu deviations unprofitable.

It is instructive to compare the equilibrium constructions in the oligopoly and the free-entry cases. In the oligopoly case, the excess supply of basic coverage that is needed to sustain an equilibrium is obtained by letting each of a large number of firms contribute a small fraction of the basic layer at the average premium rate r . The excess supply of basic coverage is thus small when there is a large number of firms. In the free-entry case, this excess supply is obtained by letting each of two firms offer the basic layer at the average premium rate r . The excess supply of basic coverage is thus large, even when only two firms

²²This should not be taken to say that this equilibrium is less likely to exist when there are more firms—on the contrary, Theorem 2 tells us that increased competition makes it easier to sustain. Instead, what this shows is that, although firms end up earning zero profit in equilibrium, their exact number matters for how much each of them is ready to contribute to the pool of basic coverage.

supply such coverage. However, the JHG allocation must then satisfy the size restrictions (10)–(11) for entry by an inactive firm to be unprofitable.

Theorem 3 differs from the earlier contributions of Glosten (1994) and Attar, Mariotti, and Salanié (2019a, 2019d) in that it is fully strategic. These authors characterize the JHG allocation as the unique budget-balanced allocation that can be implemented by an entry-proof tariff, but they make no attempt at generating this tariff as the outcome of a game played by competing firms. Indeed, Attar, Mariotti, and Salanié (2014, 2019c) show that this task cannot in general be accomplished in an unregulated nonexclusive-competition framework. By contrast, we construct a free-entry equilibrium of the regulated game that supports the JHG allocation under the additional size restrictions (10)–(11).

5 Discussion

In this section, we put our findings in perspective and relate them to the literature.

5.1 The Role of Regulation

The main insight of our analysis is that the penalization of cross-subsidies between contracts should play a key role in regulating insurance markets under multiple contracting and adverse selection. This reflects three types of considerations.

From a normative viewpoint, the regulation we propose allows a policy maker to reach third-best efficiency without shutting down competition. Indeed, competitive pricing obtains in equilibrium in spite of the restrictions imposed on firms' pricing strategies: as a result, our regulation does not hinder competition and does not prevent market forces from singling out a unique equilibrium allocation. Notice, in that respect, that fair pricing of layers is the relevant notion of competitiveness under multiple contracting because consumers are free to combine contracts issued by different firms. Fair pricing, by contrast, does not carry over to aggregate coverage *amounts*, unlike under exclusive competition: if $Q_1^* > 0$, type 1 subsidizes type 2 in equilibrium as Q_1^* is sold at the average premium rate $r > r_1$.²³

From a positive viewpoint, our regulation is instrumental in making menu deviations unprofitable. Indeed, in its absence, each firm could profitably deviate by offering two contracts when the basic layer is strictly positive. The first one would approximatively be the same as the one it trades with type 1 on the candidate equilibrium path, making a profit when

²³If, moreover, $Q_2^* - Q_1^* > 0$, then Q_2^* is sold at a premium rate between r and r_2 . Thus, compared to the separating equilibrium that arises under exclusive competition, in which each consumer pays the fair premium, r_1 or r_2 , associated to her type, multiple contracting reduces the convexity of the tariff for aggregate coverage.

traded by type 1. The second one would allow type 2 to purchase the complementary layer at a premium rate lower than r_2 , making a loss when traded by type 2. Because the deviating firm would now offer the complementary layer at better terms than its competitors, it would be optimal for type 2 to trade it on top of the basic layer these competitors collectively supply. By deviating in this way, the firm would make a small loss with type 2, which it would recoup by making a large profit with type 1. The regulation is precisely designed to make such a deviation prohibitively costly for the firm in question.

From a practical viewpoint, our regulation is not very demanding in informational terms. Indeed, if a firm attempts to deviate by cross-subsidizing between contracts, it attracts all consumers of type 2, effectively becoming the sole provider of complementary coverage. Thus, even if the loss on complementary coverage is small, the full force of the law of large numbers implies that it can be detected and punished by the regulator.²⁴ Moreover, in the light of our equilibrium construction, what matters for the regulator is the ability to detect firms that simultaneously sell basic and complementary coverage and that screen the two types using these two types of contracts. Hence an alternative regulation would be to assess the risk borne by each contract and to require that it be at least equal to the consumers' average riskiness.

5.2 Latent Contracts

A crucial feature of equilibrium is that latent contracts are necessary to block cream-skimming deviations. We have throughout aimed at characterizing the latent contracts that are required to this end. In particular, we have argued that the contract (q_1^ℓ, t_1^ℓ) uniquely performs the task of blocking large cream-skimming deviations. More generally, our goal has been to provide a minimal implementation with as few latent contracts as possible.

It should be noted that regulation and latent contracts play a complementary role in our analysis. This reflects that our regulation is specially designed for markets in which multiple contracting cannot be prohibited. By contrast, it would not help stabilizing the market under exclusive competition, where latent contracts by definition play no role as consumers can trade with at most one firm.

From a purely theoretical viewpoint, the role of latent contracts in competing-mechanism games has been well understood since the seminal work of Peters (2001) and Martimort and Stole (2002). In the context of insurance, their importance has been mainly emphasized in

²⁴We may also argue that, in practice, to attract all consumers of type 2, the deviating firm would have to charge a premium rate significantly lower than r_2 or pay significant advertising costs that the regulator could also observe.

moral-hazard environments. Hellwig (1983) and Arnott and Stiglitz (1991) argue that latent contracts deter entry in insurance markets where agents' effort decisions are not contractible, allowing active firms to earn strictly positive profits in equilibrium; the equilibrium structure of latent contracts and their welfare implications have been further examined by Bisin and Guaitoli (2004) and Attar and Chassagnon (2009).

By contrast, the importance of latent contracts in adverse-selection environments is much less appreciated in the literature. An exception is Attar, Mariotti, and Salanié (2011), but their analysis is restricted to the special case of linear preferences, which is not well suited for the study of insurance markets. The case of linear preferences is also special, in that latent contracts can take a simple linear form and can break even in all subgames. This is not the case in our minimal implementations. Indeed, because the slope of type 2's equilibrium indifference curve is higher at (Q_2^*, T_2^*) than at $(Q_1^* + q_1^\ell, T_1^* + t_1^\ell)$, we must have $q_1^\ell > Q_2^* - Q_1^*$. Therefore, the premium rate of (q_1^ℓ, t_1^ℓ) is higher than r and lower than r_2 . This implies that (q_1^ℓ, t_1^ℓ) makes strictly negative profits in all subgames in which it is traded by type 2 following a firm's attempt at cream skimming.

This property is not a special feature of the latent contracts we use in our equilibrium constructions. Rather, it is unavoidable when consumers have strictly convex preferences. It should be noted, however, that latent contracts can be included in active firms' menus; that is, no inactive firm issuing latent contracts is required to sustain an equilibrium. This makes their introduction less objectionable.

5.3 Free Entry, Quantity Discounts, and Negative Correlation

As we emphasized in Section 5.1, our analysis of the regulated game is the first to provide a positive equilibrium-existence result in a large class of insurance economies under multiple contracting and adverse selection. It may hence provide valuable insights on the implications of multiple contracting in insurance markets.

To illustrate this point, we focus on the size restrictions (10)–(11) under free entry. A direct implication of (7), (9), and (10)–(11) is that, in a free-entry equilibrium, traded contracts offering higher amounts of coverage have a lower premium rate:

$$Q_1^* > Q_2^* - Q_1^* \quad \text{and} \quad \frac{T_1^*}{Q_1^*} < \frac{T_2^* - T_1^*}{Q_2^* - Q_1^*}.$$

Therefore, while consumers pay a quantity premium for higher aggregate coverage, firms offer contracts that exhibit quantity discounts. In observable terms, this is a striking difference with the exclusive-competition case, in which whether data on traded contracts are collected

from consumer surveys or from the trade records of a single firm makes no difference as each consumer's demand must be met by a single contract sold by a single firm.

This result stands in stark contrast with the natural intuition that allowing for multiple contracting should push consumers towards splitting their demands between firms:²⁵ this intuition is misleading unless each firm is active, as in the oligopoly case. The reason why, in a free-entry equilibrium, firms end up proposing quantity discounts is that the basic layer must be larger than the complementary layer to prevent type 2 from purchasing basic coverage from different firms and to prevent inactive firms from entering the market.²⁶ This leads firms to only offer a few contracts that consumers can combine. In turn, consumers finds it in their interest to concentrate their trades on a minimum number of contracts: type 1 ends up trading a single contract, and type 2 two different contracts.

Since Chiappori and Salanié (2000), many empirical studies have tested the validity of the positive-correlation property, which states that, under adverse selection, there should be a positive correlation between the coverage purchased by a consumer and her riskiness. Due to the single-crossing assumption, this property still holds in our setting when we consider the aggregate coverage bought by a consumer: indeed, riskier consumers are also those who are more eager to purchase more coverage.²⁷

The above mentioned contrast, under multiple contracting, between the implications of equilibrium for the demand and supply sides of the market is also relevant for the positive-correlation property. Indeed, an implication of free-entry equilibrium is that, with data originating from a single firm, we should observe a negative correlation between risk and coverage, because the relatively small complementary layer is only purchased by type 2. Finally, a robust prediction of our analysis is that consumers holding more than one insurance policy should on average be more likely to experience a greater level of loss.

These observations are useful for assessing the empirical evidence, as exemplified by the work of Cawley and Philipson (1999) on life insurance, that of Cardon and Hendel (2001) on health insurance, and that of Finkelstein and Poterba (2004) on annuities. Because these papers take as a benchmark the exclusive-competition model, the above distinction between demand- and supply-side approaches is overlooked. As a result, the absence of quantity premia or the failure of the positive-correlation property are interpreted as rejecting the

²⁵See, for instance, Chiappori (2000) for an articulation of this view.

²⁶This explanation for quantity discounts differs from that proposed by Biais, Martimort, and Rochet (2000) and Chade and Schlee (2012) in the monopolistic case of Stiglitz (1977). In both papers, the shape of the hazard rate of the distribution of types plays an essential role.

²⁷Chiappori, Jullien, Salanié and Salanié (2006) show that this property, and similar ones, can also be derived in much more general settings from a simple inequality on equilibrium profits, even when single crossing is not postulated.

presence of adverse selection. Yet, multiple contracting is allowed and even prevalent on these markets, leading to a potential misspecification problem. At any rate, our results suggest that we should be careful when testing for the existence of quantity premia or for the positive-correlation property: in principle, we would need to observe each consumer's aggregate coverage and aggregate premium. In particular, checking only the contracts offered by firms is insufficient and even misleading.

6 Concluding Remarks

In modern economies, the insurance sector plays a key role by allowing agents to share risk. Because those risks are often private information, the properties of equilibrium allocations, and in fact the very existence of equilibrium, are still the subject of a lively debate among academics. The absence of consensus on the justifications and on the right design of public intervention may also be related to the fact that different countries display strikingly different regulatory systems for, in particular, health insurance.

This paper has put multiple contracting and adverse selection at the center stage of the analysis. Our main insight is that public intervention under these constraints should target firms' pricing strategies, while leaving consumers free to choose their preferred amount of coverage. The regulation we have proposed achieves this goal by penalizing cross-subsidies between contracts. As we have shown, this allows market forces to reach third-best efficiency, leading to an allocation in which each layer of coverage is fairly priced given the consumer types who purchase it. We have argued that this regulation is relatively light-handed and should be easy to implement.

Our analysis, by being the first to provide positive existence and efficiency results for insurance markets under multiple contracting and adverse selection, opens a new and rich avenue for empirical research. We also hope that it will renew the existing policy debates about health insurance, and more generally about the management of financial markets plagued by adverse selection.

Appendix A: Proofs of the Main Results

Proof of Theorem 1. An equilibrium allocation specifies individual trades (q_i^k, t_i^k) between each type i and each firm k , aggregate trades $(Q_i, T_i) \equiv (\sum_k q_i^k, \sum_k t_i^k)$ and utility levels $\mathbf{u}_i = u_i(Q_i, T_i)$ for each type i , and type-by-type and type-averaged profits $b_i^k \equiv t_i^k - r_i q_i^k$ and $b^k \equiv m_1 b_1^k + m_2 b_2^k$ for each firm k . Let

$$z_i^{-k}(q, t) \equiv \max \left\{ u_i \left(q + \sum_{l \neq k} q^l, t + \sum_{l \neq k} t^l \right) : (q^l, t^l) \in C^l \text{ for all } l \neq k \right\} \quad (\text{A.1})$$

be type i 's indirect utility from trading a contract (q, t) with firm k , which is continuous in (q, t) by Berge's maximum theorem (Aliprantis and Border (2006, Theorem 17.31)).

Our first result relates individual and aggregate profits on the complementary layers $q_2^k - q_1^k$ and $Q_2 - Q_1$,

$$\begin{aligned} s_2^k &\equiv t_2^k - t_1^k - r_2(q_2^k - q_1^k), \\ S_2 &\equiv T_2 - T_1 - r_2(Q_2 - Q_1). \end{aligned}$$

Observe that $s_2^k \neq 0$ implies $q_1^k \neq q_2^k$ in equilibrium.

Lemma A.1. *For each k ,*

$$S_2 > \max\{0, s_2^k\} \text{ implies } q_1^k = q_2^k \neq 0.$$

Proof. Suppose that $S_2 > \max\{0, s_2^k\}$ and, for each $\varepsilon_2 \geq 0$, define the contract

$$c_2^k(\varepsilon_2) \equiv (q_1^k + Q_2 - Q_1, t_1^k + T_2 - T_1 - \varepsilon_2).$$

This contract strictly attracts type 2 if $\varepsilon_2 > 0$ because, by trading it along with the contracts (q_1^l, t_1^l) offered by firms $l \neq k$, she ends up with the aggregate trade $(Q_2, T_2 - \varepsilon_2)$. The proof consists of three steps.

Step 1 We first claim that $z_1^{-k}(c_2^k(0)) < \mathbf{u}_1$. Otherwise, firm k can deviate by offering a contract $c_2^k(\varepsilon_2)$ with $\varepsilon_2 > 0$, which strictly attracts both types. Letting ε_2 go to zero, we obtain that, in equilibrium,

$$b^k \geq t_1^k - r q_1^k + T_2 - T_1 - r(Q_2 - Q_1).$$

Using the accounting identity

$$b^k = m_1(t_1^k - r_1 q_1^k) + m_2(t_2^k - r_2 q_2^k) = t_1^k - r q_1^k + m_2 s_2^k,$$

we can rewrite this inequality as

$$0 \geq S_2 - m_2 s_2^k + (r_2 - r)(Q_2 - Q_1).$$

But the first difference on the right-hand side of this inequality is strictly positive, because $S_2 > s_2^k > m_2 s_2^k$ if $s_2^k > 0$ and $S_2 > 0 \geq m_2 s_2^k$ otherwise, and the second difference is nonnegative, because $Q_2 \geq Q_1$ by single crossing. We have thus reached a contradiction. The claim follows.

Step 2 We next claim that $t_1^k > r_1 q_1^k$. Indeed, if firm k deviates by offering a contract $c_2^k(\varepsilon_2)$ with $\varepsilon_2 > 0$ small enough, it strictly attracts type 2 and does not attract type 1 by Step 1. Letting ε_2 go to zero, we obtain that, in equilibrium,

$$b^k \geq m_2 [t_1^k - r_1 q_1^k + T_2 - T_1 - r_2(Q_2 - Q_1)]$$

or, equivalently,

$$m_1(t_1^k - r_1 q_1^k) \geq m_2(S_2 - s_2^k) > 0.$$

The claim follows.

Step 3 To safeguard the strictly positive profit it earns with type 1, firm k can deviate by offering two contracts, namely, a contract $c_2^k(\varepsilon_2)$ with $\varepsilon_2 > 0$, and a contract

$$c_1^k(\varepsilon_1) \equiv (q_1^k, t_1^k - \varepsilon_1)$$

with $\varepsilon_1 > 0$. Consider first type 1. By Step 1,

$$z_1^{-k}(c_2^k(0)) < \mathbf{u}_1 = z_1^{-k}(c_1^k(0)).$$

If ε_2 is small enough, this implies

$$z_1^{-k}(c_2^k(\varepsilon_2)) < \mathbf{u}_1 < z_1^{-k}(c_1^k(\varepsilon_1)).$$

Then type 1 trades $c_1^k(\varepsilon_1)$ following firm k 's deviation and firm k earns a nonnegative profit with type 1 by Step 2. Consider next type 2. By construction,

$$z_2^{-k}(c_1^k(0)) \leq \mathbf{u}_2 \leq z_2^{-k}(c_2^k(0)).$$

If ε_1 is small enough compared to ε_2 , this implies

$$\max\{z_2^{-k}(c_1^k(\varepsilon_1)), \mathbf{u}_2\} < z_2^{-k}(c_2^k(\varepsilon_2)).$$

Then type 2 trades $c_2^k(\varepsilon_2)$ following firm k 's deviation, and firm k earns a profit

$$t_1^k - r_2 q_1^k + S_2 - \varepsilon_2 = t_2^k - r_2 q_2^k + S_2 - s_2^k - \varepsilon_2$$

with type 2. If $t_2^k - r_2 q_2^k + S_2 - s_2^k > 0$ and ε_2 is small enough, then the deviation entails no cross-subsidies, and hence is consistent with the regulation. Letting ε_1 and ε_2 go to zero, we obtain that, in equilibrium,

$$b^k \geq m_1(t_1^k - r_1 q_1^k) + m_2(t_2^k - r_2 q_2^k + S_2 - s_2^k)$$

or, equivalently,

$$0 \geq m_2(S_2 - s_2^k),$$

contradicting the assumption $S_2 > s_2^k$. Hence $t_2^k - r_2 q_2^k + S_2 - s_2^k \leq 0$, which implies $t_2^k - r_2 q_2^k < 0$. The regulation then requires $q_1^k = q_2^k \neq 0$. The result follows. \blacksquare

A consequence of Lemma A.1 is that the aggregate profit on the complementary layer $Q_2 - Q_1$ is nonpositive.

Proposition A.1. $S_2 \leq 0$.

Proof. Suppose, by way of contradiction, that $S_2 > 0$. The proof consists of six steps.

Step 1 Our first observation is that $S_2 > 0$ implies that $s_2^k \geq 0$ for all k . Otherwise, $s_2^k < 0$ for some k and thus $q_1^k = q_2^k$ by Lemma A.1, a contradiction. Moreover, because $S_2 > 0$, there exists some l such that $s_2^l > 0$ and hence $q_1^l \neq q_2^l$. Lemma A.1 then implies $s_2^l \geq S_2$, and, from our first observation, we obtain $s_2^l = S_2 > 0$ and $s_2^k = 0$ for all $k \neq l$. Thus firm l trades different contracts (q_1^l, t_1^l) and (q_2^l, t_2^l) with types 1 and 2, respectively, and, by Lemma A.1 again, each firm $k \neq l$ trades the same contract $(q^k, t^k) \neq (0, 0)$ with each type. In particular, $Q_1 > 0$.

Step 2 Second, we claim that $b^k = 0$ for all $k \neq l$. Let $(Q^{-l}, T^{-l}) \equiv (\sum_{k \neq l} q^k, \sum_{k \neq l} t^k)$. Each firm $k \neq l$ can deviate by offering a contract $(Q^{-l}, T^{-l} - \varepsilon)$ with $\varepsilon > 0$, which strictly attracts both types. Letting ε go to zero, we obtain that, in equilibrium,

$$b^k \geq T^{-l} - rQ^{-l}$$

for all $k \neq l$. The claim then follows from the fact that

$$T^{-l} - rQ^{-l} = \sum_{k \neq l} b^k$$

and that there are at least two firms $k \neq l$ as $n \geq 3$. Notice that, because $r_2 > r_1$ and each firm $k \neq l$ is active on the equilibrium path, $b_2^k < 0$ for all $k \neq l$.

Step 3 Third, we claim that $\tau_1(Q_1, T_1) = r$. Otherwise, any firm $k \neq l$ can deviate by offering a contract $(q^k + \delta, t^k + \varepsilon)$ with δ and ε chosen so that

$$\tau_1(Q_1, T_1)\delta > \varepsilon > r\delta.$$

The first inequality ensures that this contract strictly attracts type 1 if δ and ε are small enough. If it attracts type 2, then firm k 's profit increases by $\varepsilon - r\delta$, a contradiction. If it does not attract type 2, then, because $b_2^k < 0$ by Step 2, firm k 's profit increases by $m_1(\varepsilon - r_1\delta) - m_2b_2^k > m_1(\varepsilon - r_1\delta)$, once again a contradiction. The claim follows.

Step 4 Fourth, we claim that $\mathbf{u}_1 = z_1^{-k}(0, 0)$ for all $k \neq l$. Otherwise, $\mathbf{u}_1 > z_1^{-k}(0, 0)$ for some $k \neq l$. Then firm k can deviate by offering a contract $(q^k, t^k + \varepsilon)$ with $\varepsilon > 0$, which strictly attracts type 1. If it attracts type 2, then firm k 's profit increases by ε , a contradiction. If it does not attract type 2, then, because $b_2^k < 0$ by Step 2, firm k 's profit increases by $m_1\varepsilon - m_2b_2^k > m_1\varepsilon$, once again a contradiction. The claim follows. As a result, for each $k \neq l$, there exists an aggregate trade (Q^{-k}, T^{-k}) made available by firms $m \neq k$ and such that $u_1(Q^{-k}, T^{-k}) = \mathbf{u}_1$.

Step 5 Fifth, we claim that $Q^{-k} < Q_1$. Notice that $Q^{-k} \leq Q_2$; otherwise, because $u_1(Q^{-k}, T^{-k}) = \mathbf{u}_1 \geq u_1(Q_2, T_2)$ by incentive compatibility, we have $u_2(Q^{-k}, T^{-k}) > \mathbf{u}_2$ by single crossing, a contradiction. We cannot have $Q^{-k} = Q_2$; otherwise, because $S_2 > 0$ and $\tau_1(Q_1, T_1) = r$ by Step 3, we again have $u_2(Q^{-k}, T^{-k}) > \mathbf{u}_2$. Finally, suppose, by way of contradiction, that $Q_1 \leq Q^{-k} < Q_2$. Then firm k can deviate by offering the contract $(Q_2 - Q^{-k}, T_2 - T^{-k} - \varepsilon)$ with $\varepsilon > 0$, which strictly attracts type 2 and possibly type 1 as well. However, because $u_1(Q^{-k}, T^{-k}) = \mathbf{u}_1$ and $S_2 > 0$, the premium rate of this contract is strictly greater than $(T_2 - T_1)/(Q_2 - Q_1) > r_2$ if ε is small enough, in contradiction with the zero-profit result in Step 2. The claim follows.

Step 6 By Steps 4–5, for any firm $k \neq l$, there exists an aggregate trade (Q^{-k}, T^{-k}) made available by firms $m \neq k$ and such that $u_1(Q^{-k}, T^{-k}) = \mathbf{u}_1$ and $Q^{-k} < Q_1$. Because $\tau_1(Q_1, T_1) = r$ by Step 3, $\tau_1(Q^{-k}, T^{-k}) > r$. Then firm k can deviate by offering a contract (δ, ε) with $\delta > 0$ and $\varepsilon > 0$ chosen so that

$$\tau_1(Q^{-k}, T^{-k})\delta > \varepsilon > r\delta.$$

The first inequality implies that this contract strictly attracts type 1 if δ and ε are small enough. If it attracts type 2, then firm k 's profit is $\varepsilon - r\delta > 0$, in contradiction with the zero-profit result in Step 2. If it does not attract type 2, then, because $b_2^k < 0$ by Step 2,

firm k 's profit is $m_1(\varepsilon - r_1\delta) - m_2b_2^k > m_1(\varepsilon - r_1\delta) > 0$, once again in contradiction with Step 2. Hence the result. \blacksquare

Our next goal is to show that each firm earns zero profit in equilibrium, that is, $B \equiv \sum_k b^k = 0$. The argument relies on the following intuitive lemma.

Lemma A.2. $T_1 \leq r_2Q_1$.

Proof. Suppose, by way of contradiction, that $T_1 > r_2Q_1$. In particular, $0 < Q_1 \leq Q_2$. Any firm k can then deviate by offering the contract $(Q_1, T_1 - \varepsilon_1)$ with $\varepsilon_1 > 0$, which strictly attracts type 1. Because $T_1 > r_2Q_1$, the worst case for firm k is that it does not attract type 2. Letting ε_1 go to zero, we obtain that, in equilibrium,

$$b^k \geq m_1(T_1 - r_1Q_1)$$

or, equivalently,

$$m_2(T_2 - r_2Q_2) \geq B - b^k.$$

Therefore, $T_2 \geq r_2Q_2$. Any firm k can then deviate by offering the contract $(Q_2, T_2 - \varepsilon_2)$ with $\varepsilon_2 > 0$, which strictly attracts type 2. Because $T_2 \geq r_2Q_2 > r_1Q_2$, the worst case for firm k is that it does not attract type 1. Letting ε_2 go to zero, we obtain that, in equilibrium,

$$b^k \geq m_2(T_2 - r_2Q_2) \geq B - b^k$$

for all k . Summing these inequalities over k yields $(n - 2)B \leq 0$ and hence, because $n \geq 3$, $B = b^k = T_2 - r_2Q_2 = 0$ for all k . This implies $T_1 = r_1Q_1$, in contradiction with $T_1 > r_2Q_1$. The result follows. \blacksquare

A consequence of Lemma A.2 is that the aggregate profits on both the basic and the complementary layers Q_1 and $Q_2 - Q_1$ are zero, and thus that each firm earns zero profit.

Proposition A.2. $B = T_1 - rQ_1 = S_2 = 0$.

Proof. Each firm k can deviate by offering the contract $(Q_1, T_1 - \varepsilon_1)$ with $\varepsilon_1 > 0$, which strictly attracts type 1. Because $T_1 \leq r_2Q_1$ by Lemma A.2, the worst case for firm k is that this contract attracts type 2. Letting ε_1 go to zero, we obtain that, in equilibrium,

$$b^k \geq T_1 - rQ_1.$$

Using the accounting identity

$$B = T_1 - rQ_1 + m_2S_2,$$

we can rewrite this inequality as

$$B - b^k \leq m_2 S_2$$

for all k . Because $S_2 \leq 0$ by Proposition A.1, we have $B = b^k = 0$ for all k , $S_2 = 0$, and $T_1 - rQ_1 = 0$. Hence the result. ■

We are now ready to complete the proof of Theorem 1.

Proposition A.3. $(Q_1, T_1) = (Q_1^*, T_1^*)$.

Proof. We know that each firm earns zero profit. If any firm k withdraws its menu offer, then each type i obtains at most her equilibrium utility,

$$\mathbf{u}_i \geq z_i^{-k}(0, 0). \quad (\text{A.2})$$

Now, observe that

$$z_1^{-k}(0, 0) \geq \max \{u_1(Q, rQ) : Q \geq 0\}. \quad (\text{A.3})$$

Otherwise, firm k can deviate by offering a contract at a premium rate slightly higher than r that strictly and profitably attracts type 1 and that remains profitable even if it attracts type 2. Chaining (A.2)–(A.3) and using the fact that $T_1 = rQ_1$ by Proposition A.3, we obtain that (Q_1, T_1) satisfies (6)–(7) and thus coincides with (Q_1^*, T_1^*) . Hence the result. ■

Remark A.1. The aggregate trade (Q_1^*, T_1^*) can only be reached by means of contracts with premium rate r . Otherwise, some contract (q, t) such that $q < Q_1^*$ and $t < rq$ is offered by some firm. Any other firm can then offer the contract $(Q_1^* - q, T_1^* - t - \varepsilon)$ with $\varepsilon > 0$. This contract strictly attracts type 1 along with the contract (q, t) and, because $T_1^* - t > r(Q_1^* - q)$, it is profitable for ε small enough even if it attracts type 2, a contradiction.

Lemma A.3. *The aggregate trade (Q_1^*, T_1^*) remains available if any firm k withdraws its menu offer.*

Proof. By Proposition A.3,

$$\mathbf{u}_1 = u_1(Q_1^*, T_1^*) = \max \{u_1(Q, rQ) : Q \geq 0\}.$$

Hence, by (A.2)–(A.3),

$$\mathbf{u}_1 = z_1^{-k}(0, 0)$$

for all k . As a result, for each k , there exists an aggregate trade (Q^{-k}, T^{-k}) made available by firms $l \neq k$ and such that $u_1(Q^{-k}, T^{-k}) = \mathbf{u}_1$. We show that $Q^{-k} = Q_1^*$ by eliminating the other cases. First, if $Q^{-k} < Q_1^*$, then, because $\tau_1(Q_1^*, T_1^*) = r$ and $T_1^* = rQ_1^*$ by Proposition A.3, the premium rate of (Q^{-k}, T^{-k}) is strictly less than r , in contradiction with Remark A.1. Second, if $Q^{-k} \geq Q_2^*$, then $u_2(Q^{-k}, T^{-k}) > \mathbf{u}_2$ by single crossing, a contradiction. Third, if $Q_1^* < Q^{-k} < Q_2^*$, then firm k can profitably deviate as in Step 5 of the proof of Proposition A.1, once again a contradiction. The result follows. ■

Proposition A.4. $(Q_2, T_2) = (Q_2^*, T_2^*)$.

Proof. For each k , we have, in analogy with (A.3),

$$z_2^{-k}(0, 0) \geq \max\{u_2(Q_1^* + Q, T_1^* + r_2Q) : Q \geq 0\}. \quad (\text{A.4})$$

Otherwise, firm k can offer a contract at a premium rate slightly higher than r_2 that strictly and profitably attracts type 2 along with the aggregate trade (Q_1^*, T_1^*) made available by firms $l \neq k$ by Lemma A.3, and that is even more profitable if it attracts type 1. Chaining (A.2) and (A.4) and using the fact that $T_2 - T_1 = r_2(Q_2 - Q_1)$ by Proposition A.3, we obtain that (Q_2, T_2) satisfies (8)–(9) and thus coincides with (Q_2^*, T_2^*) . Hence the result. ■

Remark A.2. Each traded contract is sold at premium rate r or r_2 . Indeed, consider a contract (q, t) with $q > 0$ that is traded in equilibrium. If it attracts both types, then it yields zero profit; thus its premium rate is r . If it attracts only one type, then it yields a nonnegative profit because of the regulation, and hence exactly zero profit; thus its premium rate is either r_1 or r_2 . Finally, Remark A.1 implies that a contract of the form (q, r_1q) with $q > 0$ cannot be traded only by type 1 in equilibrium.

Remark A.3. We know that no firm is indispensable to provide type 1 with her equilibrium aggregate trade. A slightly weaker property is satisfied for type 2, namely, that no firm is indispensable to provide her with her equilibrium utility: for each k , there exists an aggregate trade (Q^{-k}, T^{-k}) made available by firms $l \neq k$ and such that $u_2(Q^{-k}, T^{-k}) = \mathbf{u}_2$. We claim that $Q^{-k} \geq Q_2^*$. Otherwise, $T_2^* - T^{-k} > r_2(Q_2^* - Q^{-k})$ by (8)–(9) as u_2 is strictly quasiconcave. Firm k can then deviate by offering a contract $(Q_2^* - Q^{-k}, T_2^* - T^{-k} - \varepsilon)$ with $\varepsilon > 0$, which attracts type 2 along with the aggregate trade (Q^{-k}, T^{-k}) and is profitable for ε small enough, a contradiction.²⁸ ■

²⁸Unlike for type 1, the equilibrium aggregate trade of type 2 need not remain available if any firm who trades with her withdraws its menu offer. In Attar, Mariotti, and Salanié (2014), this property is satisfied because trades of negative quantities are allowed, which is not the case in the present setting.

Proof of Lemma 1. Consider an equilibrium of the regulated game under free entry, assuming that such an equilibrium exists. Let K_r be the set of firms issuing contracts at premium rate r . Next, for each $k \in K_r$, let

$$C_r^k \equiv \{(q, t) \in C^k : q > 0 \text{ and } t = rq\}$$

be the set of such contracts offered by firm k . Finally, for all $k \in K_r$ and $(q, t) \in C_r^k$, let

$$\alpha(q, t) \equiv \frac{q}{Q_1^*}$$

be the fraction of the quantity Q_1^* covered by the contract (q, t) . Now, fix some $k \in K_r$ who trades a contract $(q^k, t^k) \in C_r^k$ on the equilibrium path, so that

$$0 < \alpha(q^k, t^k) \leq 1. \tag{A.5}$$

As shown in the proof of Theorem 1, type 1 can still trade (Q_1^*, T_1^*) if firm k withdraws its menu offer, and the aggregate trade (Q_1^*, T_1^*) can only be reached through contracts with premium rate r . Hence there exists a subset K_r^{-k} of $K_r \setminus \{k\}$ and contracts $(q^l, t^l) \in C_r^l$ offered by different firms $l \in K_r^{-k}$ such that

$$\sum_{l \in K_r^{-k}} \alpha(q^l, t^l) = 1. \tag{A.6}$$

Summing (A.5)–(A.6) yields

$$1 < \sum_{l \in K_r^{-k}} \alpha(q^l, t^l) + \alpha(q^k, t^k) \leq 2.$$

Because the aggregate trade $[1 + \alpha(q^k, t^k)](Q_1^*, T_1^*)$ is available on the equilibrium path, we must have

$$u_2(Q_2^*, T_2^*) \geq u_2([1 + \alpha(q^k, t^k)](Q_1^*, T_1^*)). \tag{A.7}$$

To conclude the proof, we only need to show that

$$[1 + \alpha(q^k, t^k)]Q_1^* > Q_2^*. \tag{A.8}$$

Indeed, (A.8) implies $2Q_1^* > Q_2^*$, which is (11). In turn, along with (A.7) and $2Q_1^* > Q_2^*$, (A.8) implies $u_2(Q_2^*, T_2^*) \geq u_2(2(Q_1^*, T_1^*))$, which is (10). To establish (A.8), observe first that, because $T_2^* > rQ_2^*$, it must be that

$$[1 + \alpha(q^k, t^k)]Q_1^* \neq Q_2^*.$$

Let us suppose, by way of contradiction, that $[1 + \alpha(q^k, t^k)]Q_1^* < Q_2^*$. Then an inactive firm could deviate by offering the contract $(Q_2^* - [1 + \alpha(q^k, t^k)]Q_1^*, T_2^* - r[1 + \alpha(q^k, t^k)]Q_1^* - \varepsilon)$ with $\varepsilon > 0$, which attracts type 2 along with the aggregate trade $[1 + \alpha(q^k, t^k)](Q_1^*, T_1^*)$. Because the JHG allocation satisfies

$$T_1^* - rQ_1^* = T_2^* - T_1^* - r_2(Q_2^* - Q_1^*) = 0,$$

we find that the corresponding profit on type 2,

$$T_2^* - r[1 + \alpha(q^k, t^k)]Q_1^* - r_2\{Q_2^* - [1 + \alpha(q^k, t^k)]Q_1^*\} - \varepsilon = (r_2 - r)\alpha(q^k, t^k)Q_1^* - \varepsilon,$$

is strictly positive for ε small enough. Hence entry is profitable even if type 1 is not attracted, a contradiction. The result follows. \blacksquare

Proof of Lemma 2. If a latent contract (q^ℓ, t^ℓ) blocks large cream-skimming deviations, then, by (12) applied to $(q, t) = (Q_1^*, T_1^*)$, we have

$$u_2(Q_1^* + q^\ell, T_1^* + t^\ell) \geq u_2(Q_2^*, T_2^*).$$

This inequality cannot be strict if the contract (q^ℓ, t^ℓ) is latent: otherwise, type 2 would be strictly better off trading it on top of type 1's aggregate trade (Q_1^*, T_1^*) , which, by Lemma A.3, she can make with any set of $n - 1$ firms. Hence (13). Next, by (12), the translate of the upper contour set of (Q_1^*, T_1^*) for type 1 along the vector (q^ℓ, t^ℓ) lies in the upper contour set of (Q_2^*, T_2^*) for type 2. As these two sets intersect at $(Q_1^* + q^\ell, T_1^* + t^\ell)$ by (13), we obtain along the lines of Benveniste and Scheinkman (1979, Lemma 1) that the slope of type 2's indifference curve at $(Q_1^* + q^\ell, T_1^* + t^\ell)$ must be equal to the slope of type 1's indifference curve at (Q_1^*, T_1^*) , that is, r . Hence (14). The result follows. \blacksquare

Proof of Lemma 3. We slightly abuse notation by identifying each type's equilibrium indifference curve with its functional expression $T = \mathcal{I}_i^*(Q)$. Recalling that

$$\mathcal{I}_2^*(Q_1^* + q_1^\ell) = \mathcal{I}_1^*(Q_1^*) + t_1^\ell, \tag{A.9}$$

we only need to prove that the translate of \mathcal{I}_1^* along the vector (q_1^ℓ, t_1^ℓ) does not cross \mathcal{I}_2^* at $(Q_1^* + q_1^\ell, \mathcal{I}_2^*(Q_1^* + q_1^\ell))$. We distinguish two cases, depending on whether Assumption C(i) or C(ii) is satisfied.

Assumption C(i) By (A.9), it is enough to show that

$$\mathcal{I}_1^{*'}(Q) \geq \mathcal{I}_2^{*'}(Q + q_1^\ell) \text{ if } Q \leq Q_1^*.$$

A sufficient condition for this is the following single-crossing property:

$$\mathcal{I}_1^{*'}(Q) = \mathcal{I}_2^{*'}(Q + q_1^\ell) \text{ implies } \mathcal{I}_1^{*''}(Q) < \mathcal{I}_2^{*''}(Q + q_1^\ell),$$

which, under Assumption C(i), is a direct implication of the identities

$$\begin{aligned} \tau_i^* &\equiv \mathcal{I}_i^{*'}, \\ \kappa_i^* &\equiv -\frac{\mathcal{I}_i^{*''}}{[1 + (\mathcal{I}_i^{*'})^2]^{3/2}}, \end{aligned}$$

along the equilibrium indifference curve \mathcal{I}_i^* of type i .

Assumption C(ii) We show that

$$\mathcal{I}_2^*(Q + q_1^\ell) = \mathcal{I}_1^*(Q) + t_1^\ell \tag{A.10}$$

for all Q in the relevant range, so that \mathcal{I}_2^* is the translate of \mathcal{I}_1^* along the vector (q_1^ℓ, t_1^ℓ) . Define implicitly a function ϕ by

$$\mathcal{I}_2^{*'}(\phi(Q) + q_1^\ell) = \mathcal{I}_1^{*'}(Q). \tag{A.11}$$

Notice that ϕ is strictly increasing as both \mathcal{I}_1^* and \mathcal{I}_2^* are strictly concave, and that

$$\phi(Q_1^*) = Q_1^* \tag{A.12}$$

by (14). As $\kappa_2^* > 0$ by Assumption C, $\mathcal{I}_2^{*''} < 0$. Hence, by the implicit function theorem, ϕ is differentiable with

$$\mathcal{I}_2^{*''}(\phi(Q) + q_1^\ell)\phi'(Q) = \mathcal{I}_1^{*''}(Q). \tag{A.13}$$

But, under Assumption C(ii), (A.11) implies

$$\mathcal{I}_2^{*''}(\phi(Q) + q_1^\ell) = \mathcal{I}_1^{*''}(Q),$$

so that, by (A.12)–(A.13), ϕ must be the identity function. Then (A.9) and (A.11) imply that (A.10) holds for all Q in the relevant range. The result follows. \blacksquare

Proof of Theorem 2. We show that, for n large enough, the regulated game has an equilibrium in which each firm offers the same menu C_n^* consisting of five contracts:

1. The no-trade contract $(0, 0)$;
2. The contract $c_{1,n}^* \equiv (q_{1,n}^*, t_{1,n}^*)$ defined by (16);

3. The contract $c_2^* \equiv (Q_2^* - Q_1^*, T_2^* - T_1^*)$;
4. The latent contract $c_1^\ell \equiv (q_1^\ell, t_1^\ell)$ defined by (13)–(14);
5. The latent contract $c_{2,n}^\ell \equiv (q_{2,n}^\ell, t_{2,n}^\ell)$ defined by (17).

Notice that, by construction, all offered contracts have premium rates at least equal to r . Moreover, because the complementary layer is strictly positive, we have $u_2(Q_2^*, T_2^*) > u_2(Q_1^*, T_1^*)$ and hence

$$u_2(Q_2^*, T_2^*) > u_2\left(\frac{n}{n-1}(Q_1^*, T_1^*)\right) = u_2(nc_{1,n}^*) \quad (\text{A.14})$$

for n large enough. The proof consists of four steps.

Step 1 We first claim that, on the equilibrium path, consumers have a best response such that they trade according to the JHG allocation, and each firm earns zero profit.

Consider first type 1. Because $(n-1)c_{1,n}^* = (Q_1^*, T_1^*)$ and $\tau_1(Q_1^*, T_1^*) = r$, and because all offered contracts have premium rates at least equal to r , trading $n-1$ contracts $c_{1,n}^*$ with, say, firms $k = 1, \dots, n-1$ is optimal for type 1, leading to the aggregate trade (Q_1^*, T_1^*) , as in the JHG allocation.

Consider next type 2. By trading $n-1$ contracts $c_{1,n}^*$ with firms $k = 1, \dots, n-1$ and one contract c_2^* with firm n , type 2 can reach the aggregate trade (Q_2^*, T_2^*) , as in the JHG allocation. Firms $k = 1, \dots, n-1$ then each trade $c_{1,n}^*$ with both types 1 and 2 at premium rate r and firm n trades c_2^* with type 2 at premium rate r_2 , so that each firm earns zero profit. We thus only have to check that type 2 has no profitable deviation. This follows from two observations. First, by (A.14), if n is large enough, type 2 is not tempted to trade the contract $c_{1,n}^*$ with each of the n firms. Second, by (13)–(14) and (17), trading some contracts c_1^ℓ or $c_{2,n}^\ell$, possibly along one or several contracts $c_{1,n}^*$ or c_2^* , brings type 2 at best on the line with slope r that supports her upper contour set of (Q_2^*, T_2^*) ; indeed, the best she can do is to trade $n-1$ contracts $c_{1,n}^*$ and one contract c_1^ℓ , or $n-2$ contracts $c_{1,n}^*$ and one contract $c_{2,n}^\ell$, reaching the aggregate trade $(Q_1^* + q_1^\ell, T_1^* + t_1^\ell)$ and thus obtaining, by (13), the same utility as at (Q_2^*, T_2^*) . The claim follows.

Step 2 We next prove that no firm k has a profitable deviation that only attracts type 2. With a slight abuse of terminology, we identify such a deviation C^k with the contract (q, t) that type 2 chooses to trade in C^k according to consumers' equilibrium strategy. The proof follows from three observations. First, type 2 can obtain her equilibrium utility $u_2(Q_2^*, T_2^*)$ by trading a contract $c_{1,n}^*$ with $n-2$ firms $l \neq k$ and a contract $c_{2,n}^\ell$ with the remaining firm

$l \neq k$. Second, if type 2 at most trades contracts $c_{1,n}^*$ or c_2^* with firms $l \neq k$ following firm k 's deviation, then the contract (q, t) can attract her only if $r_2q > t$, so that the deviation C^k is not profitable. Third, if type 2 trades some contracts c_1^ℓ or $c_{2,n}^\ell$ with firms $l \neq k$ following firm k 's deviation, possibly along one or several contracts $c_{1,n}^*$ or c_2^* , then this brings her at best on the line with slope r that supports her upper contour set of (Q_2^*, T_2^*) ; then the contract (q, t) can attract type 2 only if $rq > t$, and the deviation C^k is again not profitable.

Step 3 We then show that, for n large enough, no firm k has a profitable deviation that only attracts type 1 (hereafter *cream-skimming deviation*). As in Step 2, we identify such a deviation C^k with the contract (q, t) that type 1 chooses to trade in C^k according to consumers' equilibrium strategy. Observe that, because type 1 can purchase her demand Q_1^* at premium rate r from firms $l \neq k$ by trading a contract $c_{1,n}^*$ with each of them, a contract (q, t) can attract her only if $rq \geq t$; moreover, this contract is profitable only if $t \geq r_1q$. Hence any cream-skimming deviation belongs to the cone

$$X \equiv \{(q, t) : rq \geq t \geq r_1q\}.$$

We distinguish two cases.

First consider the case of a large cream-skimming deviation, that is, a contract $(q, t) \in X$ offered by some firm k such that

$$u_1(q, t) \geq u_1(Q_1^*, T_1^*). \quad (\text{A.15})$$

This case is easily dealt with under Assumption C, because, according to Lemma 3, (q, t) is blocked by the latent contract c_1^ℓ . Indeed, (A.15) implies

$$u_2((q, t) + c_1^\ell) \geq u_2(Q_2^*, T_2^*)$$

by (12); hence type 2 is also attracted by (q, t) in combination with a contract c_1^ℓ offered by some firm $l \neq k$. We can thus construct consumers' best response in such a way that both types trade (q, t) with firm k . But then, because $rq \geq t$ as $(q, t) \in X$, this deviation is not profitable, as desired.

Next consider the case of a small cream-skimming deviation, that is, a contract $(q, t) \in X$ offered by some firm k such that

$$z_{1,n}^{*-k}(q, t) \geq u_1(Q_1^*, T_1^*) > u_1(q, t), \quad (\text{A.16})$$

where, in analogy with (A.1),

$$z_{i,n}^{*-k}(q, t) \equiv \max \left\{ u_i \left(q + \sum_{l \neq k} q^l, t + \sum_{l \neq k} t^l \right) : (q^l, t^l) \in C_n^* \text{ for all } l \neq k \right\}$$

describes type i 's indirect utility from trading a contract (q, t) with firm k in the candidate equilibrium. Then (q, t) attracts type 1, but only in combination with other contracts offered by firms $l \neq k$. We can then no longer use the latent contract c_1^ℓ as in the case of a large cream-skimming deviation, because, for all we know, type 1 may have to trade with all firms $l \neq k$ to reach the utility $z_{1,n}^{*-k}(q, t)$.

However, notice that, if type 1 can obtain a utility at least equal to $u_1(Q_1^*, T_1^*)$ by trading (q, t) along with at least one contract $c_{1,n}^*$ and possibly other contracts, then, according to Lemma 3, type 2 can obtain a utility at least equal to $u_2(Q_2^*, T_2^*)$ by trading (q, t) along with at least one contract $c_{2,n}^\ell = c_{1,n}^* + c_1^\ell$ and these other contracts, which amounts to mimicking type 1's trades and trading an additional contract c_1^ℓ ; hence type 2 is also attracted by (q, t) and, as in the case of a large cream-skimming deviation, we can construct consumers' best response in such a way that this deviation is not profitable, as desired. We thus only need to show that this configuration arises for n large enough, uniformly in (q, t) . To do so, define, in analogy with (A.1),

$$z_{1,n}^{*-k}(q, t) \equiv \max \left\{ u_1 \left(q + \sum_{l \neq k} q^l, t + \sum_{l \neq k} t^l \right) : (q^l, t^l) \in C_n^* \text{ for all } l \neq k \right. \\ \left. \text{and } (q^l, t^l) = c_{1,n}^* \text{ for some } l \neq k \right\}$$

to be type 1's indirect utility from trading the contract (q, t) with firm k along with at least one contract $c_{1,n}^*$ with some firm $l \neq k$. By construction,

$$z_{1,n}^{*-k}(q, t) \geq \underline{z}_{1,n}^{*-k}(q, t).$$

The following lemma shows that, for n large enough, the additional constraint embedded in the definition of $\underline{z}_{1,n}^{*-k}(q, t)$ does not prevent type 1 from reaching her equilibrium utility, which completes the proof of Step 3.

Lemma A.4. *There exists $\underline{n} \in \mathbb{N}$ such that, for each $n \geq \underline{n}$,*

$$\underline{z}_{1,n}^{*-k}(q, t) \geq u_1(Q_1^*, T_1^*) \tag{A.17}$$

for all $(q, t) \in X$ that satisfy (A.16).

Proof. Extracting a subsequence if necessary, suppose, by way of contradiction, that there exists a sequence $((q_n, t_n))_{n \in \mathbb{N}}$ in X such that

$$z_{1,n}^{*-k}(q_n, t_n) \geq u_1(Q_1^*, T_1^*) > \max \{ u_1(q_n, t_n), \underline{z}_{1,n}^{*-k}(q_n, t_n) \}.$$

For each n , because $z_{1,n}^{*-k}(q_n, t_n) > u_1(q_n, t_n)$, type 1 must trade, on top of the contract (q_n, t_n) with firm k , some contracts $c_n^l \in C_n^*$ with firms $l \neq k$ to obtain the utility $z_{1,n}^{*-k}(q_n, t_n)$; moreover, because $z_{1,n}^{*-k}(q_n, t_n) > \underline{z}_{1,n}^{*-k}(q_n, t_n)$, all these contracts must be different from $c_{1,n}^*$. By definition, we have

$$u_1\left((q_n, t_n) + \sum_{l \neq k} c_n^l\right) = z_{1,n}^{*-k}(q_n, t_n) \geq u_1(Q_1^*, T_1^*). \quad (\text{A.18})$$

For each n , the contract (q_n, t_n) belongs to the compact set

$$Y \equiv X \cap \{(q, t) : q \leq Q_1^* \text{ and } u_1(q, t) \leq u_1(Q_1^*, T_1^*)\}$$

and hence the sequence $((q_n, t_n))_{n \in \mathbb{N}}$ can with no loss of generality be assumed to converge to some contract $(q_\infty, t_\infty) \in Y$.

Next, for each n , c_n^l belongs to the finite set $C_n^* \setminus \{c_{1,n}^*\}$ and at least one contract c_n^l is different from the no-trade contract $(0, 0)$; as a result, the sequence $(\sum_{l \neq k} c_n^l)_{n \in \mathbb{N}}$ is bounded away from $(0, 0)$. Noticing that

$$C_\infty^* \equiv \{(0, 0), c_2^*, c_1^l\}$$

is the closed limit of the sequence of sets $(C_n^* \setminus \{c_{1,n}^*\})_{n \in \mathbb{N}}$ (Aliprantis and Border (2006, Definition 3.80.1)), we can with no loss of generality assume that, for each $l \neq k$, the sequence $(c_n^l)_{n \in \mathbb{N}}$ converges to some $c_\infty^l \in C_\infty^*$ such that

$$\sum_{l \neq k} c_\infty^l \neq (0, 0).$$

We claim that

$$t_\infty < r q_\infty. \quad (\text{A.19})$$

Indeed, taking the limit in (A.18) yields, by continuity of u_1 ,

$$u_1\left((q_\infty, t_\infty) + \sum_{l \neq k} c_\infty^l\right) \geq u_1(Q_1^*, T_1^*).$$

However, the aggregate trade $\sum_{l \neq k} c_\infty^l$ has a premium rate higher than r as it is made up of contracts c_2^* or c_1^l . Because Q_1^* is the demand for coverage of type 1 at premium rate r and $T_1^* = rQ_1^*$, it must be that $t_\infty < r q_\infty$, as claimed.

Finally, because, for each n , $u_1(Q_1^*, T_1^*) > \underline{z}_{1,n}^{*-k}(q_n, t_n)$, we a fortiori have

$$u_1(Q_1^*, T_1^*) > \max\{u_1((q_n, t_n) + (Q^{-k}, T^{-k})) : (Q^{-k}, T^{-k}) \in C_{r,n}^{-k}\}, \quad (\text{A.20})$$

where

$$C_{r,n}^{-k} \equiv \{\nu c_{1,n}^* : \nu \in \mathbb{N} \text{ and } \nu \leq n-1\}$$

is the set of aggregate trades that consumers can make at premium rate r with firms $l \neq k$.

Noticing that

$$C_{r,\infty}^{-k} \equiv \{(Q^{-k}, T^{-k}) : Q^{-k} \in [0, Q_1^*] \text{ and } T^{-k} = rQ^{-k}\}$$

is the closed limit of the sequence of sets $(C_{r,n}^{-k})_{n \in \mathbb{N}}$, taking the limit superior in (A.20) yields, by continuity of u_1 ,

$$\begin{aligned} u_1(Q_1^*, T_1^*) &\geq \limsup_{n \rightarrow \infty} \max \{u_1((q_n, t_n) + (Q^{-k}, T^{-k})) : (Q^{-k}, T^{-k}) \in C_{r,n}^{-k}\} \\ &\geq \max \{u_1((q_\infty, t_\infty) + (Q^{-k}, T^{-k})) : (Q^{-k}, T^{-k}) \in C_{r,\infty}^{-k}\} \\ &\geq u_1(Q_1^*, T_1^* + t_\infty - rq_\infty), \end{aligned}$$

where the third inequality follows from letting type 1 trade $(q_\infty, t_\infty) \in Y$ with firm k and $(Q_1^* - q_\infty, r(Q_1^* - q_\infty)) \in C_{r,\infty}^{-k}$ with firms $l \neq k$. Because $t_\infty < rq_\infty$ by (A.19), we have reached a contradiction. The result follows. \blacksquare

Step 4 Finally, we check that no firm k has a profitable menu deviation

$$C^k \equiv \{(0, 0), (q_1^k, t_1^k), (q_2^k, t_2^k)\},$$

where (q_i^k, t_i^k) is the contract selected by type i . To be consistent with the regulation, this menu must be such that $t_i^k \geq r_i q_i^k$ for all i if $(q_1^k, t_1^k) \neq (q_2^k, t_2^k)$. By Step 3, we can focus on the case $(q_2^k, t_2^k) \neq (0, 0)$. We construct consumers' equilibrium strategy in such a way that each type i is attracted by the contracts (q_i^k, t_i^k) at the deviation stage only if she is thereby strictly better off, that is,

$$z_{i,n}^{*-k}(q_i^k, t_i^k) > z_{i,n}^{*-k}(0, 0).$$

Indeed, otherwise, she may as well not trade with firm k and obtain her equilibrium utility $u_i(Q_i^*, T_i^*) = z_{i,n}^{*-k}(0, 0)$ made available by firms $l \neq k$. We distinguish two cases.

Either $(q_1^k, t_1^k) \neq (q_2^k, t_2^k)$. Then the regulation requires $t_2^k \geq r_2 q_2^k$. However, given the menus C_n^* offered by firms $l \neq k$, type 2 is strictly better off trading the contract (q_2^k, t_2^k) with firm k only if its premium rate is lower than r_2 . That is,

$$z_{2,n}^{*-k}(q_2^k, t_2^k) > z_{2,n}^{*-k}(0, 0) \text{ implies } t_2^k < r_2 q_2^k.$$

Therefore, this case is ruled out by the regulation and our specification of the buyer's best response at the deviation stage.

Or $(q_1^k, t_1^k) = (q_2^k, t_2^k) \equiv (q, t)$. Because type 1 can purchase her demand Q_1^* for coverage at premium rate r from firms $l \neq k$, she is strictly better off trading the contract (q, t) with firm k only if its premium rate is lower than r . That is,

$$z_{1,n}^{*-k}(q, t) > z_{1,n}^{*-k}(0, 0) \text{ implies } t < rq.$$

But then the deviation C^k is not profitable. Hence the result. ■

Proof of Theorem 3. The necessity of conditions (10)–(11) follows from Lemma 1. We show that, under conditions (10)–(11), the regulated game has a free-entry equilibrium in which there are only two firms, say, firms 1 and 2, who are active on the equilibrium path and who offer the same menu C^* , constructed as follows. First, C^* includes three basic contracts:

1. The no-trade contract $(0, 0)$;
2. The contract $c_1^* \equiv (Q_1^*, T_1^*)$;
3. The contract $c_2^* \equiv (Q_2^* - Q_1^*, T_2^* - T_1^*)$.

Let then C_∞ be the smallest set that contains these three contracts and that is closed under addition with the latent contract $c_1^\ell \equiv (q_1^\ell, t_1^\ell)$ defined by (13)–(14); that is,

$$c \in C_\infty \text{ implies } c + c_1^\ell \in C_\infty.$$

Notice that C_∞ is unbounded and hence not compact. To construct from C_∞ a compact menu C^* , consider the line D with slope r_1 that supports type 1's upper contour set of (Q_1^*, T_1^*) , which is well defined by the Inada condition, and let H be the corresponding lower closed half space. Because contracts in C_∞ have premium rates at least equal to r , whereas the line D has slope r_1 , the set $C_\infty \cap H$ is finite. We define C^* as follows:

$$C^* \equiv (C_\infty \cap H) + \{(0, 0), c_1^\ell\}. \tag{A.21}$$

Finally, each inactive firm $k \neq 1, 2$ offers the contract c_2^* . The remainder of the proof consists of four steps.

Step 1 The proof that consumers have a best response such that, on the equilibrium path, they trade according to the JHG allocation, is similar to that of Step 1 of the proof of Theorem 2, except that both types now trade the contract c_1^* with, say, firm 1, and type 2 in addition trades c_2^* with firm 2. In particular, by (10), type 2 is not tempted to trade two contracts c_1^* with firms 1 and 2. All firms earn zero profit.

Step 2 The proof that no firm k has a profitable deviation that only attracts type 2 is similar to that of Step 2 of the proof of Theorem 2, observing in addition that type 2 can obtain her equilibrium utility $u_2(Q_2^*, T_2^*)$ by trading c_1^* with one active firm and c_2^* with an inactive firm.

Step 3 We next show that no firm k has a cream-skimming deviation. As in Step 3 of the proof of Theorem 2, let X be the cone to which such potential deviations must belong. We distinguish two cases.

First consider the case of an active firm, say, firm 1. Let $(q, t) \in X$ be some contract, offered by firm 1, that attracts type 1 in combination with a contract $c \in C^*$ offered by firm 2 and ν contracts c_2^* offered by firms $k \neq 1, 2$. Because firm 2 offers the contract c_1^* , the equilibrium utility of type 1 remains available following firm 1's deviation. Hence

$$u_1((q, t) + c + \nu c_2^*) \geq u_1(Q_1^*, T_1^*). \quad (\text{A.22})$$

Now, $(q, t) \in X$ implies $t \geq r_1 q$. As a result, it must be that $c \in C_\infty \cap H$, where C_∞ and H are defined as above; otherwise, type 1 would not be willing to combine (q, t) with c . Therefore, by (A.21), $c + c_1^\ell \in C^*$. In particular, because Assumption C is satisfied, we can apply Lemma 2, so that (A.22) implies

$$u_2((q, t) + c + c_1^\ell + \nu c_2^*) \geq u_2(Q_2^*, T_2^*)$$

by (12); hence type 2 is also attracted by (q, t) in combination with the contract $c + c_1^\ell$ offered by firm 2 and the ν contracts c_2^* offered by firms $k \neq 1, 2$. We can thus construct consumers' best response in such a way that both types trade (q, t) with firm 1. But then, because $r q \geq t$ as $(q, t) \in X$, this deviation is not profitable, as desired.

Next consider the case of an inactive firm $k \neq 1, 2$. Let $(q, t) \in X$ be some contract, offered by firm k , that attracts type 1 in combination with contracts $c, c' \in C^*$ offered by firms 1 and 2 and ν contracts offered by firms $l \neq 1, 2, k$. In analogy with (A.22), we have

$$u_1((q, t) + c + c' + \nu c_2^*) \geq u_1(Q_1^*, T_1^*). \quad (\text{A.23})$$

By the same reasoning as above, we have $c + c' \in (C_\infty + C_\infty) \cap H$ and hence $c, c' \in C_\infty \cap H$. Therefore, for instance, $c' + c_1^\ell \in C^*$. Applying again Lemma 2, (A.23) implies

$$u_2((q, t) + c + c' + c_1^\ell + \nu c_2^*) \geq u_2(Q_2^*, T_2^*)$$

by (12); hence type 2 is also attracted by (q, t) in combination with the contract c and $c' + c_1^\ell$ offered by firms 1 and 2, respectively, and the ν contracts offered by firms $l \neq 1, 2, k$, and we can conclude as in the previous case.

Step 4 There remains to check that no firm k has a profitable menu deviation. For active firms, the proof proceeds as in Step 3 of the proof of Theorem 2. For inactive firms, the key observation is that, under conditions (10)–(11), no such firm can exploit the trade $(2Q_1^*, 2T_1^*)$ made available by active firms on the equilibrium path to attract type 2 in a profitable way. The proof then proceeds as for active firms. Hence the result. ■

Appendix B: Omitted Calculations

B.1 On Single Crossing and Coinsurance

We show that, if f_2 dominates f_1 in the monotone-likelihood-ratio order, then (1)–(2) hold. As for (2), we have

$$r_2 \equiv \int l f_2(l) \boldsymbol{\lambda}(dl) > \int l f_1(l) \boldsymbol{\lambda}(dl) \equiv r_1.$$

Consider next (1). For all (Q, T) and i ,

$$\tau_i(Q, T) = \frac{\int l v'(W_0 - (1 - Q)l - T) f_i(l) \boldsymbol{\lambda}(dl)}{\int v'(W_0 - (1 - Q)l - T) f_i(l) \boldsymbol{\lambda}(dl)} = \int l g_i(l | Q, T) \boldsymbol{\lambda}(dl), \quad (\text{B.1})$$

where $g_i(\cdot | Q, T)$ is the risk-neutral density

$$g_i(l | Q, T) \equiv \frac{v'(W_0 - (1 - Q)l - T) f_i(l)}{\int v'(W_0 - (1 - Q)l - T) f_i(l) \boldsymbol{\lambda}(dl)}. \quad (\text{B.2})$$

If f_2 dominates f_1 in the monotone-likelihood-ratio order, then, by (B.2), $g_2(\cdot | Q, T)$ also dominates $g_1(\cdot | Q, T)$ in the monotone-likelihood-ratio order. It then follows from (B.1) that $\tau_2(Q, T) > \tau_1(Q, T)$, which is precisely (1). ■

B.2 On Assumption C

B.2.1 Hicksian Demands

We first show that Assumption C is equivalent to the property that type 2's Hicksian demand for coverage be more sensitive than type 1's to changes in the premium rate, whatever utility levels are used as references. Specifically, letting $H_i(p, \mathbf{u})$ be type i 's Hicksian demand function for coverage, the following result holds.²⁹

Lemma B.1. *Assumptions C(i) and C(ii) are equivalent to the mappings*

$$p \mapsto H_2(p, \mathbf{u}_2) - H_1(p, \mathbf{u}_1)$$

being strictly decreasing and constant for all \mathbf{u}_1 and \mathbf{u}_2 , respectively.

²⁹That is, $H_i(p, \mathbf{u})$ is the first component of $\arg \min \{pQ - T : u_i(Q, T) \geq \mathbf{u}\}$.

Proof. We slightly abuse notation by identifying each type's indifference curve associated to utility level \mathbf{u} with its functional expression $T = \mathcal{I}_i(Q, \mathbf{u})$. The strict quasiconcavity of u_i implies that, for each \mathbf{u} , $\mathcal{I}_i(Q, \mathbf{u})$ is strictly concave with respect to Q . By construction, the slope of $\mathcal{I}_i(\cdot, \mathbf{u})$ is type i 's marginal rate of substitution,

$$\partial_Q \mathcal{I}_i(Q, \mathbf{u}) = \tau_i(Q, \mathcal{I}_i(Q, \mathbf{u})),$$

and $\partial_Q \mathcal{I}_i(\cdot, \mathbf{u})$ is the inverse of the Hicksian demand function,

$$\partial_Q \mathcal{I}_i(Q, \mathbf{u}) = p \text{ if and only if } Q = H_i(p, \mathbf{u}). \quad (\text{B.3})$$

Assumption C(i) states that, if

$$\partial_Q \mathcal{I}_1(Q_1, \mathbf{u}_1) = \partial_Q \mathcal{I}_2(Q_2, \mathbf{u}_2),$$

then

$$-\frac{\partial_{QQ}^2 \mathcal{I}_1(Q_1, \mathbf{u}_1)}{\{1 + [\partial_Q \mathcal{I}_1(Q_1, \mathbf{u}_1)]^2\}^{\frac{3}{2}}} > -\frac{\partial_{QQ}^2 \mathcal{I}_2(Q_2, \mathbf{u}_2)}{\{1 + [\partial_Q \mathcal{I}_2(Q_2, \mathbf{u}_2)]^2\}^{\frac{3}{2}}},$$

so that

$$\partial_{QQ}^2 \mathcal{I}_1(Q_1, \mathbf{u}_1) < \partial_{QQ}^2 \mathcal{I}_2(Q_2, \mathbf{u}_2).$$

Call $p \equiv \partial_Q \mathcal{I}_1(Q_1, \mathbf{u}_1) = \partial_Q \mathcal{I}_2(Q_2, \mathbf{u}_2)$. Overall, Assumption C(i) reduces to the following condition:

$$\partial_{QQ}^2 \mathcal{I}_1(\partial_Q \mathcal{I}_1(\cdot, \mathbf{u}_1)^{-1}(p), \mathbf{u}_1) < \partial_{QQ}^2 \mathcal{I}_2(\partial_Q \mathcal{I}_2(\cdot, \mathbf{u}_2)^{-1}(p), \mathbf{u}_2).$$

That is, the mapping

$$p \mapsto \partial_Q \mathcal{I}_2(\cdot, \mathbf{u}_2)^{-1}(p) - \partial_Q \mathcal{I}_1(\cdot, \mathbf{u}_1)^{-1}(p)$$

is strictly decreasing in p , which, by (B.3), is the desired property of Hicksian demand functions. The proof for Assumption C(ii) is similar, replacing all inequalities by equalities. The result follows. \blacksquare

B.2.2 A Covariance Formula

We now show how to express the second derivative of a consumer's indifference curve with respect to quantities as a covariance. That is, in analogy with (B.1), we differentiate

$$\partial_Q \mathcal{I}_i(Q, \mathbf{u}) = \tau_i(Q, \mathcal{I}_i(Q, \mathbf{u})) = \int l \frac{v'_i(W_0 - (1 - Q)l - \mathcal{I}_i(Q, \mathbf{u}))f_i(l)}{\int v'_i(W_0 - (1 - Q)l - \mathcal{I}_i(Q, \mathbf{u}))f_i(l)} \boldsymbol{\lambda}(dl)$$

with respect to Q . We hereafter omit the index i and the arguments of the functions for the sake of clarity. With this convention,

$$\partial_Q \mathcal{I} = \int l \frac{v' f}{\int v' f d\boldsymbol{\lambda}} d\boldsymbol{\lambda},$$

and differentiating yields

$$\begin{aligned} \partial_{QQ}^2 \mathcal{I} &= \int l \frac{[v''(l - \partial_Q \mathcal{I}) \int v' f d\boldsymbol{\lambda} - v' \int v''(l - \partial_Q \mathcal{I}) f d\boldsymbol{\lambda}] f}{(\int v' f d\boldsymbol{\lambda})^2} d\boldsymbol{\lambda} \\ &= - \int l \left[\alpha(l - \partial_Q \mathcal{I}) - \int \alpha(l - \partial_Q \mathcal{I}) \frac{v' f}{\int v' f d\boldsymbol{\lambda}} d\boldsymbol{\lambda} \right] \frac{v' f}{\int v' f d\boldsymbol{\lambda}} d\boldsymbol{\lambda}, \end{aligned} \quad (\text{B.4})$$

where $\alpha \equiv -v''/v'$. Notice that the term in brackets in (B.4) has zero mean under the risk-neutral density $g \equiv v' f / \int v' f d\boldsymbol{\lambda}$. Hence, inverting the convention, we obtain

$$\partial_{QQ}^2 \mathcal{I}_i(Q, \mathbf{u}) = -\mathbf{Cov}_{g_i(\cdot|Q, \mathcal{I}_i(Q, \mathbf{u}))}[\tilde{L}, \alpha_i(W_0 - (1 - Q)\tilde{L} + \mathcal{I}_i(Q, \mathbf{u}))][\tilde{L} - \partial_Q \mathcal{I}_i(Q, \mathbf{u})]. \quad (\text{B.5})$$

In the next two sections, we study the implications of (B.5) in two applications.

B.2.3 A Single Loss Level

Suppose that there is a single loss level L , so that type i 's preferences are represented by

$$u_i(Q, T) = r_i v_i(W_0 - (1 - Q)L - T) + (1 - r_i)v_i(W_0 - T).$$

We show that Assumption C(i) is satisfied if type 1 is uniformly more risk-averse than type 2, that is, letting $\alpha_i(w) \equiv -v_i''(w)/v_i'(w)$ be type i 's coefficient of absolute risk aversion at wealth w , if $\min \alpha_1 > \max \alpha_2$. For each i , we have, in analogy with (B.1), and using the notation of Section B.1,

$$g_i(L | Q_i, \mathcal{I}_i(Q_i, \mathbf{u}_i)) = \frac{1}{L} \tau_i(Q_i, \mathcal{I}_i(Q_i, \mathbf{u}_i)) = \frac{1}{L} \partial_Q \mathcal{I}_i(Q_i, \mathbf{u}_i).$$

Hence

$$\partial_Q \mathcal{I}_1(Q_1, \mathbf{u}_1) = \partial_Q \mathcal{I}_2(Q_2, \mathbf{u}_2) \equiv \partial_Q \mathcal{I}$$

implies

$$g_1(L | Q_1, \mathcal{I}_1(Q_1, \mathbf{u}_1)) = g_2(L | Q_2, \mathcal{I}_2(Q_2, \mathbf{u}_2)) \equiv \frac{1}{L} \partial_Q \mathcal{I}.$$

That is, the two risk-neutral densities are the same under the premise of Assumption C. Now, for each i , we can apply our covariance formula (B.5), yielding

$$\partial_{QQ}^2 \mathcal{I}_i(Q_i, \mathbf{u}_i) = -\partial_Q \mathcal{I} (L - \partial_Q \mathcal{I}) \bar{\alpha}_i(Q_i, \mathbf{u}_i), \quad (\text{B.6})$$

where

$$\bar{\alpha}_i(Q_i, \mathbf{u}_i) \equiv \frac{1}{L} \partial_Q \mathcal{I} \alpha_i(W_0 - \mathcal{I}_i(Q_i, \mathbf{u}_i)) + \left(1 - \frac{1}{L} \partial_Q \mathcal{I}\right) \alpha_i(W_0 - (1 - Q)L - \mathcal{I}_i(Q_i, \mathbf{u}_i)).$$

Because $\min \alpha_1 > \max \alpha_2$, we have $\bar{\alpha}_1(Q_1, \mathbf{u}_1) > \bar{\alpha}_2(Q_2, \mathbf{u}_2)$ and thus, by (B.6),

$$\partial_Q \mathcal{I}_1(Q_1, \mathbf{u}_1) = \partial_Q \mathcal{I}_2(Q_2, \mathbf{u}_2) \text{ implies } \partial_{QQ}^2 \mathcal{I}_1(Q_1, \mathbf{u}_1) < \partial_{QQ}^2 \mathcal{I}_2(Q_2, \mathbf{u}_2),$$

which is precisely Assumption C(i).

Remark B.1. The assumption $\min \alpha_1 > \max \alpha_2$ is in tension with the single-crossing condition (1), which requires that type 2 be more willing to increase her purchases of coverage than type 1. A sufficient condition for (1) to hold in spite of this assumption is that the difference $\max \alpha_1 - \min \alpha_2$ be not too large. This can be seen as follows. By (B.1), type i 's marginal rate of substitution is

$$\tau_i(Q, T) = \frac{f_i(L)v'_i(W_0 - (1 - Q)L - T)L}{f_i(L)v'_i(W_0 - (1 - Q)L - T) + [1 - f_i(L)]v'_i(W_0 - T)}.$$

Thus (1) holds if, for each (Q, T) ,

$$\left[\frac{1 - f_2(L)}{f_2(L)} \right] \left[\frac{v'_2(W_0 - T)}{v'_2(W_0 - (1 - Q)L - T)} \right] < \left[\frac{1 - f_1(L)}{f_1(L)} \right] \left[\frac{v'_1(W_0 - T)}{v'_1(W_0 - (1 - Q)L - T)} \right]. \quad (\text{B.7})$$

We have $v''_2/v'_2 \leq -\min \alpha_2$ and hence

$$\frac{v'_2(W_0 - T)}{v'_2(W_0 - (1 - Q)L - T)} \leq \exp(-\min \alpha_2 (1 - Q)L).$$

Similarly, we have $v''_1/v'_1 \geq -\max \alpha_1$ and hence

$$\frac{v'_1(W_0 - T)}{v'_1(W_0 - (1 - Q)L - T)} \geq \exp(-\max \alpha_1 (1 - Q)L).$$

Thus, by (B.7), a sufficient condition for (1) is

$$\ln \left(\frac{f_2(L)}{1 - f_2(L)} \right) - \ln \left(\frac{f_1(L)}{1 - f_1(L)} \right) > (\max \alpha_1 - \min \alpha_2)L.$$

That is, type 2 is not too less risk-averse and sufficiently riskier than type 1.

B.2.4 The CARA–Exponential Case

We now generalize the results of the previous section to the case of CARA utility functions v_1 and v_2 , with absolute risk aversions $\alpha_1 > \alpha_2$, and losses distributions f_1 and f_2 in the natural exponential family (15), with parameters $\theta_2 > \theta_1$. Taking advantage of quasilinearity, we

can drop the transfer and utility variables for the sake of clarity. By (B.1), type i 's marginal rate of substitution is

$$\tau_i(Q) = \mathcal{I}'_i(Q) = \psi(\alpha_i(1 - Q) + \theta_i), \quad (\text{B.8})$$

where, for each x in the relevant range,

$$\psi(x) \equiv \int l \frac{\exp(xl)h(l)}{\int \exp(xl)h(l) \boldsymbol{\lambda}(dl)} \boldsymbol{\lambda}(dl).$$

Now, observe that

$$\psi'(x) \propto \int l^2 \frac{\exp(xl)h(l)}{\int \exp(xl)h(l) \boldsymbol{\lambda}(dl)} \boldsymbol{\lambda}(dl) - \left(\int l \frac{\exp(xl)h(l)}{\int \exp(xl)h(l) \boldsymbol{\lambda}(dl)} \boldsymbol{\lambda}(dl) \right)^2,$$

which is strictly positive by Jensen's inequality. Hence $\psi(x)$ is strictly increasing in x . Thus

$$\tau_1(Q_1) = \tau_2(Q_2) \text{ implies } \alpha_1(1 - Q_1) + \theta_1 = \alpha_2(1 - Q_2) + \theta_2.$$

As a result, the risk-neutral densities $g_1(\cdot | Q_1)$ and $g_2(\cdot | Q_2)$ are the same. Now, for each i , we can apply our covariance formula (B.5), yielding, as α_i is constant,

$$\mathcal{I}''_i(Q_i) = -\alpha_i \mathbf{Var}_{g_i(\cdot | Q_i)}[\tilde{L}]. \quad (\text{B.9})$$

Because $\alpha_1 > \alpha_2$ and the variances are the same, we have, by (B.9),

$$\mathcal{I}'_1(Q_1) = \mathcal{I}'_2(Q_2) \text{ implies } \mathcal{I}''_1(Q_1) < \mathcal{I}''_2(Q_2),$$

which is precisely Assumption C(i). Assumption C(ii) is satisfied in the limiting case $\alpha_1 = \alpha_2$.

Remark B.2. It follows from (B.8) and the strict monotonicity of ψ that (1) holds if and only if $\theta_2 - \theta_1 > \alpha_1 - \alpha_2$, in line with the single-loss model of Section B.2.3.

References

- [1] Akerlof, G.A. (1970): “The Market for “Lemons”: Quality Uncertainty and the Market Mechanism,” *Quarterly Journal of Economics*, 84(3), 488–500.
- [2] Aliprantis, C.D., and K.C. Border (2006): *Infinite Dimensional Analysis: A Hitchhiker’s Guide*, Berlin, Heidelberg, New York: Springer.
- [3] Armantier, O., E. Ghysels, A. Sarkar, and J. Shrader (2015): “Discount Window Stigma during the 2007-2008 Financial Crisis,” *Journal of Financial Economics*, 118(2), 317–335.
- [4] Arnott, R.J. and J.E. Stiglitz (1993), “Equilibrium in Competitive Insurance Markets with Moral Hazard,” National Bureau of Economic Research Working Paper No. 3588.
- [5] Arrow, K.J. (1963): “Uncertainty and the Welfare Economics of Medical Care,” *American Economic Review*, 53(5), 941–973.
- [6] Attar, A., and A. Chassagnon (2009): “On Moral Hazard and Nonexclusive Contracts,” *Journal of Mathematical Economics*, 45(9–10), 511–525.
- [7] Attar, A., T. Mariotti, and F. Salanié (2011): “Nonexclusive Competition in the Market for Lemons,” *Econometrica*, 79(6), 1869–1918.
- [8] Attar, A., T. Mariotti, and F. Salanié (2014): “Nonexclusive Competition under Adverse Selection,” *Theoretical Economics*, 9(1), 1–40.
- [9] Attar, A., T. Mariotti, and F. Salanié (2019a): “Entry-Proofness and Market Breakdown under Adverse Selection,” Toulouse School of Economics Working Paper No. 17-788.
- [10] Attar, A., T. Mariotti, and F. Salanié (2019b): “On a Class of Smooth Preferences,” *Economic Theory Bulletin*, 7(1), 37–57.
- [11] Attar, A., T. Mariotti, and F. Salanié (2019c): “On Competitive Nonlinear Pricing,” *Theoretical Economics*, 14(1), 297–343.
- [12] Attar, A., T. Mariotti, and F. Salanié (2019d): “The Social Costs of Side Trading,” Toulouse School of Economics Working Paper No. 19-1017.
- [13] Aumann, R.J., and R. Serrano (2008): “An Economic Index of Riskiness,” *Journal of Political Economy*, 116(5), 810–836.
- [14] Azevedo, E.M., and D. Gottlieb (2017): “Perfect Competition in Markets with Adverse

- Selection,” *Econometrica*, 85(1), 67–105.
- [15] Benveniste, L.M., and J.A. Scheinkman (1979): “On the Differentiability of the Value Function in Dynamic Models of Economics,” *Econometrica*, 47(3), 727–732.
- [16] Berger, A.N., L.K. Black, C.H.S. Bouwman, and J. Dlugosz (2015): “The Federal Reserve’s Discount Window and TAF Programs: “Pushing on a String?”” Unpublished Manuscript, Wharton School.
- [17] Biais, B., D. Martimort, and J.-C. Rochet (2000): “Competing Mechanisms in a Common Value Environment,” *Econometrica*, 68(4), 799–837.
- [18] Bisin, A., and D. Guaitoli (2004): “Moral Hazard and Nonexclusive Contracts,” *RAND Journal of Economics*, 35(2), 306–328.
- [19] Cardon, J.H., and I. Hendel (2001): “Asymmetric Information in Health Insurance: Evidence from the National Medical Expenditure Survey,” *RAND Journal of Economics*, 32(3), 408–427.
- [20] Carmona, G., and J. Fajardo (2009): “Existence of Equilibrium in Common Agency Games with Adverse Selection,” *Games and Economic Behavior*, 66(2), 749–760.
- [21] Cawley, J., and T. Philipson (1999): “An Empirical Examination of Information Barriers to Trade in Insurance,” *American Economic Review*, 89(4), 827–846.
- [22] Chade, H., and E. Schlee (2012): “Optimal Insurance with Adverse Selection,” *Theoretical Economics*, 7(3), 571–607.
- [23] Chen, Z., and P. Rey (2012): “Loss Leading as an Exploitative Practice,” *American Economic Review*, 102(7), 3462–3482.
- [24] Chen, Z., and P. Rey (2019): “Competitive Cross-Subsidization,” *RAND Journal of Economics*, 50(3), 645–665.
- [25] Chetty, R., and A. Finkelstein (2013) “Social Insurance: Connecting Theory to Data,” in *Handbook of Public Economics*, Volume 5, ed. by A. Auerbach, R. Chetty, M. Feldstein, and E. Saez. Amsterdam: Elsevier, 111–193.
- [26] Chiappori, P.-A. (2000): “Econometric Models of Insurance under Asymmetric Information,” in *Handbook of Insurance*, ed. by G. Dionne. Boston, Dordrecht, London: Kluwer Academic Publishers, 365–394.

- [27] Chiappori, P.-A., B. Jullien, B. Salanié, and F. Salanié (2006): “Asymmetric Information in Insurance : Some Testable Implications,” *RAND Journal of Economics*, 37(4), 783–798.
- [28] Chiappori P.-A., and B. Salanié (2000): “Testing for Asymmetric Information in Insurance Markets,” *Journal of Political Economy*, 108(1), 56–78.
- [29] Crocker, K.J., and A. Snow (1985a): “The Efficiency of Competitive Equilibria in Insurance Markets with Asymmetric Information,” *Journal of Public Economics*, 26(2), 207–219.
- [30] Crocker, K.J., and A. Snow (1985b): “A Simple Tax Structure for Competitive Equilibrium and Redistribution in Insurance Markets with Asymmetric Information,” *Southern Economic Journal*, 51(4), 1142–1150.
- [31] Dahlby, B.G. (1981): “Adverse Selection and Pareto Improvements through Compulsory Insurance,” *Public Choice*, 37(3), 547–558.
- [32] Debreu, G. (1972): “Smooth Preferences,” *Econometrica*, 40(4), 603–615.
- [33] Einav, L., and A. Finkelstein (2011): “Selection in Insurance Markets: Theory and Empirics in Pictures,” *Journal of Economic Perspectives*, 25(1), 115–138.
- [34] Einav, L., A. Finkelstein, and M.R. Cullen (2010): “Estimating Welfare in Insurance Markets Using Variation in Prices,” *Quarterly Journal of Economics*, 125(2), 877–921.
- [35] Einav, L., A. Finkelstein, S.P. Ryan, P. Schrimpf, and M.R. Cullen (2013): “Selection on Moral Hazard in Health Insurance,” *American Economic Review*, 103(1), 178–219.
- [36] Finkelstein, A. (2004): “Minimum Standards, Insurance Regulation and Adverse Selection: Evidence from the Medigap Market,” *Journal of Public Economics*, 88(12), 2515–2547.
- [37] Finkelstein, A., and K. McGarry (2006): “Multiple Dimensions of Private Information: Evidence from the Long-Term Care Insurance Market,” *American Economic Review*, 96(4), 938–958.
- [38] Finkelstein, A., and J.M. Poterba (2004): “Adverse Selection in Insurance Markets: Policyholder Evidence from the U.K. Annuity Market,” *Journal of Political Economy*, 112(1), 183–208.

- [39] Glosten, L.R. (1994): “Is the Electronic Open Limit Order Book Inevitable?” *Journal of Finance*, 49(4), 1127–1161.
- [40] Guerrieri, V., R. Shimer, and R. Wright (2010): “Adverse Selection in Competitive Search Equilibrium,” *Econometrica*, 78(6), 1823–1862.
- [41] Hansen, L.P., and T.J. Sargent (2007): *Robustness*, Princeton: Princeton University Press.
- [42] He, D. (2009): “The Life Insurance Market: Asymmetric Information Revisited,” *Journal of Public Economics*, 93(9–10), 1090–1097.
- [43] Hellwig, M.F. (1983): “On Moral Hazard and Non-Price Equilibria in Competitive Insurance Markets,” Discussion Paper No. 109, Institut für Gesellschafts- und Wirtschaftswissenschaften, Sonderforschungsbereich 21, Universität Bonn.
- [44] Hellwig, M.F. (1988): “A Note on the Specification of Interfirm Communication in Insurance Markets with Adverse Selection,” *Journal of Economic Theory*, 46(1), 154–163.
- [45] IASB (2013): *Insurance Contracts*, London: IFRS Foundation Publications Department.
- [46] Jaynes, G.D. (1978): “Equilibria in Monopolistically Competitive Insurance Markets,” *Journal of Economic Theory*, 19(2), 394–422.
- [47] Jaynes, G.D. (2011): “Equilibrium and Strategic Communication in the Adverse Selection Insurance Model,” Yale University Economics Department Working Paper No. 91.
- [48] Klibanoff, P., M. Marinacci, and S. Mukerji (2005): “A Smooth Model of Decision Making Under Ambiguity,” *Econometrica*, 73(6), 1849–1892.
- [49] Martimort, D., and L. Stole (2002): “The Revelation and Delegation Principles in Common Agency Games,” *Econometrica*, 70(4), 1659–1673.
- [50] Mas-Colell, A. (1985): *The Theory of General Economic Equilibrium: A Differentiable Approach*, Cambridge, UK: Cambridge University Press.
- [51] Paccagnella, O., V. Rebba, and G. Weber (2013): “Voluntary Private Health Care Insurance Among the Over 50s in Europe,” *Health Economics*, 22(3), 289–315.

- [52] Pauly, M.V. (1974): “Overinsurance and Public Provision of Insurance: The Roles of Moral Hazard and Adverse Selection,” *Quarterly Journal of Economics*, 88(1), 44–62.
- [53] Peters, M. (2001): “Common Agency and the Revelation Principle,” *Econometrica*, 69(5), 1349–1372.
- [54] Philippon T., and V. Skreta (2012): “Optimal Interventions in Markets with Adverse Selection,” *American Economic Review*, 102(1), 1–28.
- [55] Poterba, J.M (2014): “Retirement Security in an Aging Society,” *American Economic Review*, 104(5), 1–30.
- [56] Quiggin, J. (1982): “A Theory of Anticipated Utility,” *Journal of Economic Behavior and Organization*, 3(4), 323–343.
- [57] Rothschild, M., and J.E. Stiglitz (1976): “Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information,” *Quarterly Journal of Economics*, 90(4), 629–649.
- [58] Stiglitz, J.E. (1977): “Monopoly, Non-Linear Pricing and Imperfect Information: The Insurance Market,” *Review of Economic Studies*, 44(3), 407–430.
- [59] Stiglitz, J.E., J. Yun, and A. Kosenko (2018): “Characterization, Existence, and Pareto Optimality in Insurance Markets with Asymmetric Information with Endogenous and Asymmetric Disclosures: Revisiting Rothschild–Stiglitz,” National Bureau of Economic Research Working Paper No. 24711.
- [60] Sun, Y.N. (2006): “The Exact Law of Large Numbers via Fubini Extension and Characterization of Insurable Risks,” *Journal of Economic Theory*, 126(1), 31–69.
- [61] Thomson, S., and E. Mossialos (2009): “Private Health Insurance in the European Union,” Studies and Reports on Health and Long-Term Care, European Commission, Brussels.
- [62] Thomson, S., R. Osborne, D. Squires, and M. Jun (2013): “International Profiles of Health Care Systems, 2013,” Commonwealth Fund Publication No. 1717, The Commonwealth Fund, New York.
- [63] Tirole, J. (2012): “Overcoming Adverse Selection: How Public Intervention Can Restore Market Functioning,” *American Economic Review*, 102(1), 29–59.

- [64] Villeneuve, B. (2003): “Mandatory Pensions and the Intensity of Adverse Selection in Life Insurance Markets,” *Journal of Risk and Insurance*, 70(3), 527–548.
- [65] Wilson, C. (1977): “A Model of Insurance Markets with Incomplete Information,” *Journal of Economic Theory*, 16(2), 167–207.