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"The Social Costs of Side Trading"

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The Social Costs of Side Trading*

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Abstract

We study resource allocation under private information when the planner cannot prevent bilateral side trading between consumers and firms. Adverse selection and side trading severely restrict feasible trades: each marginal quantity must be fairly priced given the consumer types who purchase it. The resulting social costs are twofold. First, second-best efficiency and robustness to side trading are in general irreconcilable requirements. Second, there actually exists a unique budget-feasible allocation robust to side trading, which deprives the planner from any capacity to redistribute resources between different types of consumers. We discuss the relevance of our results for insurance and financial markets.

Keywords: Adverse Selection, Side Trading, Second-Best Allocations.
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1 Introduction

The theory of incentives identifies the holding of private information by economic agents as a fundamental constraint on the allocation of resources (Hurwicz (1973)). Standard aggregate resource constraints must accordingly be supplemented by incentive-compatibility constraints that reflect the agents' ability to conceal their private information (Myerson (1979, 1982)). The problem of the optimal allocation of resources then reduces to that of characterizing informationally constrained efficient, or second-best, allocations (Harris and Townsend (1981)). The key finding is that a tradeoff arises between allocative efficiency and redistribution (Mirrlees (1971)).

A crucial assumption of theories of the second-best is that, although individual types are unobservable, individual trades can be perfectly monitored by the planning authority. Because few, if any, economic institutions have the required ability to monitor all individual trades, this calls for an explicit consideration of the role of side trading in the theory of resource allocation under private information, as first pointed out by Hammond (1979). The present paper contributes to analyzing this problem.

To this end, we consider a general trade environment in which firms can sell a divisible good to privately informed consumers who may be of two types. Consumers' preferences satisfy a single-crossing condition, and there is adverse selection in that consumers who are more eager to trade are also more costly to serve; private values arise as a limiting case when consumers' types are not payoff-relevant for the firms selling to them. This framework encompasses many applications, including the standard Rothschild and Stiglitz (1976) insurance economy as a prominent example.

In this setting, we characterize the allocations that can be achieved by a planner who observes neither consumers' types nor the trades they may conduct with firms. To do so, we refine the standard notion of incentive-feasibility by focusing on allocations that are robust to side trading. This reflects two additional constraints on resource allocation. First, the planner cannot force consumers to trade with him. Second, he cannot prevent them from engaging in mutually advantageous additional trades with a firm. We formalize these constraints by requiring the planner to offer a tariff such that no firm, acting as an entrant, can guarantee itself a positive profit by offering complementary side trades. This approach provides us with a modified criterion of incentive feasibility which is useful for evaluating the social costs of side trading.

Our main results show that these costs are twofold. First, second-best allocations, which can be characterized in our setting along the lines of Prescott and Townsend (1984) and

Crocker and Snow (1985), are typically not robust to side trading. This suggests that the planner's inability to monitor consumers' trades has significant welfare implications. Second, there actually exists only one budget-feasible allocation robust to side trading. That is, the threat of side trading effectively deprives the planner from any capacity to redistribute resources between different types of consumers. The allocation we characterize is thus the natural candidate for a competitive equilibrium, but, being the only feasible one under side trading, little, if anything, can be argued about its desirability.

A distinctive feature of our approach is that we model side trade as bilateral contracts between a consumer and a firm. This reflects our dissatisfaction with the standard way of representing unobservable side trades as transactions on Walrasian markets, which would call for a centralized market institution to monitor them. Bilateral trading plays a key role in our analysis. Our key Lemma 1, in particular, shows that budget-feasible allocations robust to side trading have a very peculiar price structure: each marginal quantity, or layer, is priced at the cost of serving the types who purchase it. This form of competitive pricing, reminiscent of Akerlof (1970), implies that there are no cross-subsidies between these layers, though there may be cross-subsidies between types. When the allocation is interior and separating, linear pricing can emerge only in the private-value limiting case.

Lemma 1 has a simple but important implication that we state in Theorem 1: no second-best allocation in which only one incentive compatibility-constraint binds is robust to side trading. The reason is that, by the standard efficiency-at-the-top property, consumers for which this constraint binds must trade at the margin at the cost of serving them. As a result, the layer that connects the trades of the two types cannot be priced at the cost of serving the type who is the most eager to trade. But then, by Lemma 1, there always exists some side trade that a firm finds it profitable to conduct with at least one consumer type. For instance, in the Rothschild and Stiglitz (1976) insurance economy, any second-best allocation in which the high-risk type's incentive-compatibility constraint binds can be exploited by an entrant offering complementary coverage at a premium rate slightly higher than the high-risk fair premium rate, which this type is willing to trade along with the coverage provided by the allocation for the low-risk type.

Our second main result, Theorem 2, states that, among the allocations that feature no cross-subsidies between layers, only one is robust to side trading, namely, the Pareto-efficient one that maximizes the utility of the consumer type who is the less willing to trade. To complete our characterization, we evaluate whether this allocation can be second-best, hence considering the situations not covered by Theorem 1. Theorem 3 shows that a second-best allocation robust to side trading must either feature pooling of the two consumers' types, or each type purchasing her first-best quantity. We argue that these situations can only occur under very specific assumptions on preferences and costs.

Related Literature

While the constraints induced by private information on resource allocation are by now well understood, less is known about the impact of side trading on feasibility and redistribution. Starting with the early contributions of Hammond (1979, 1987), Allen (1985), and Jacklin (1987), several authors have attempted to identify the limits to risk sharing generated by consumers' side trading in financial markets. Cole and Kocherlakota (2001), Golosov and Tsyvinski (2007), Farhi, Golosov, and Tsyvinski (2009) have analyzed different private-value environments in which the planner is constrained by the existence of Walrasian markets on which privately informed consumers can complement their trades with the planner by trading linearly priced commodities. Our contribution to this literature is twofold. First, we offer an alternative representation of side trading, which we essentially regard as a bilateral, rather than a centralized, process. Second, we focus on trade under common values, so that, in a bilateral relationship, a firm's profit directly depends on the types of the consumers it trades with, an effect absent from the above literature, though arguably a prominent feature of insurance and financial markets.

The requirement that, to be robust to side trading, an allocation has to be implementable by an entry-proof tariff, is in line with the definition by Kahn and Mookherjee (1998) or Bisin and Guaitoli (2004) of third-best allocations in moral-hazard environments. In any such allocation, the planner's tariff must prevent consumers from complementing it with an additional profit-making contract provided by a firm. We extend this notion to private-information environments, and, in addition, we put no restriction on the side trades a firm can make available.

The allocation that we characterize as the only budget-feasible allocation robust to side trading was first introduced by Jaynes (1978), and further studied by Hellwig (1988) and Glosten (1994), in different contexts. Jaynes (1978) and Hellwig (1988) derive this allocation in strategic frameworks in which firms can exchange information about their customers. Glosten (1994) derives it in the context of financial markets where competitors are restricted to offer collections of limit orders. By contrast, we allow firms to offer arbitrary tariffs, in line with our assumption that side trades are fully bilateral.

The paper is organized as follows. Section 2 describes the model. Section 3 defines our concept of robustness to side trading, and shows that second-best allocations typically do not satisfy this requirement. Section 4 discusses the relevance of our results for insurance and financial markets. Proofs not given in the text can be found in the Appendix.

2 The Economy

Consumers There is a continuum of consumers who can purchase a divisible good in exchange for monetary transfers. Each consumer is privately informed of her type $i = 1, 2$ and the proportion of type i among consumers is $m_i > 0$. Type i's preferences over quantity-transfer bundles $(q, t) \in \mathbb{R}_+ \times \mathbb{R}$ are represented by a strictly quasiconcave and continuously differentiable utility function u_i, with $\partial_t u_i < 0$. Hence the marginal rate of substitution

$$\tau_i \equiv -\frac{\partial_q u_i}{\partial_t u_i}$$

is well defined and strictly decreasing along her indifference curves. We impose the Inada condition that $\tau_i(q, t)$ vanishes as q grows large along any such curve. Hence, whatever her endowment point, type i's demand at any price $p > 0$ is finite. The following strict single-crossing condition is the key determinant of consumer demand:

$$\text{For all } q \text{ and } t,\ \tau_2(q, t) > \tau_1(q, t). \tag{1}$$

Thus type 2 is more willing to increase her purchases than type 1.

Firms The supply side of the economy is described by a constant-return-to-scale technology, with unit cost $c_i > 0$ of serving type i. Type 2 is weakly more costly to serve than type 1:

$$c_2 \geq c_1. \tag{2}$$

Together with (1), (2) typically generates adverse selection, whereas values are private in the limiting case $c_1 = c_2$. We let $c \equiv m_1 c_1 + m_2 c_2$ be the average cost of serving a consumer.

These assumptions hold in the Rothschild and Stiglitz (1976) insurance economy: c_i is type i's riskiness, with $c_2 > c_1$, q is the amount of coverage she purchases, and t is the premium she pays in return. Our model encompasses many other specifications and is relevant for a broad spectrum of insurance, financial, and labor markets.

Incentive Feasibility and Efficiency A *contract* is a pair (q, t) for some $q \geq 0$, and with unit price t/q if $q > 0$. An *allocation* is a pair of contracts, one for each type. An allocation $(q_i, t_i)_{i=1,2}$ is *budget-feasible* if

$$m_1(t_1 - c_1 q_1) + m_2(t_2 - c_2 q_2) \geq 0.$$

To this aggregate resource constraint, we must add, following Myerson (1979, 1982) and Harris and Townsend (1981), constraints reflecting that the allocation of resources takes place under asymmetric information. An allocation $(q_i, t_i)_{i=1,2}$ is *incentive-compatible* if

$$u_1(q_1, t_1) \geq u_1(q_2, t_2) \text{ and } u_2(q_2, t_2) \geq u_2(q_1, t_1).$$

We denote these constraints by $IC_{1\to 2}$ and $IC_{2\to 1}$, respectively. An allocation is *incentive-feasible* if it is budget-feasible and incentive-compatible. A *second-best* allocation is Pareto-efficient among incentive-feasible allocations. This is the relevant notion of efficiency for a planner who perfectly monitors trades, though not consumer types (Prescott and Townsend (1984), Crocker and Snow (1985)).

Tariffs A *tariff* T is a schedule specifying a transfer $T(q)$ to be paid in return for a quantity q, with $T(0) = 0$ in case a consumer chooses not to trade along the tariff and $T(q) = \infty$ in case the tariff does not allow consumers to purchase the quantity q. A tariff T *implements* the allocation $(q_i, t_i)_{i=1,2}$ if

$$\text{For each } i, \; q_i \in \arg\max \{u_i(q, T(q)) : q \geq 0\} \text{ and } t_i = T(q_i).$$

To ensure that the various maximization problems we will encounter have solutions, we impose the mild requirement that a tariff be lower semicontinuous, with $T(q)/q$ bounded away from 0 as q grows large; this holds true, notably, if T has a compact domain.

3 Second-Best Allocations and Side Trading

When side trading is feasible, the planner can no longer monitor trades. This imposes two additional constraints on resource allocation. First, the planner cannot force consumers to trade with him. To model this constraint, we require that the planner offer a tariff T^P, the key restriction being $T^P(0) = 0$. Second, the planner cannot prevent consumers from engaging in mutually advantageous additional trades with a firm. To model this constraint, we require that T^P be such that no firm, acting as an entrant, can guarantee itself a positive profit by offering complementary side trades.

Side trades are usually assumed to take place on Walrasian markets (Hammond (1979, 1987), Allen (1985), Jacklin (1987), Cole and Kocherlakota (2001), Golosov and Tsyvinski (2007), Farhi, Golosov, and Tsyvinski (2009)); in our context, this would amount to impose that the entrant must post a linear tariff. We find this at odds with the idea that side trades cannot be monitored and instead allow the entrant to post an arbitrary tariff T^E; the taxation principle (Hammond (1979), Guesnerie (1981), Rochet (1985)) ensures that this involves no loss of generality. Hence the following definition.

Definition 1. *The planner's tariff* T^P *is* entry-proof *if, for any entrant's tariff* T^E*, there exists a solution* (q_i^P, q_i^E) *to every type* i*'s problem*

$$\max\{U_i(q^P + q^E, T^P(q^P) + T^E(q^E)) : q^P \geq 0 \text{ and } q^E \geq 0\} \tag{3}$$

such that entry is not profitable:

$$m_1[T^E(q_1^E) - c_1 q_1^E] + m_2[T^E(q_2^E) - c_2 q_2^E] \leq 0. \tag{4}$$

An allocation is robust to side trading *if it can be implemented by an entry-proof tariff.*

Any allocation robust to side trading is incentive-compatible. The question we ask is whether such an allocation can also be second-best. Our argument is twofold.

On the one hand, budget-feasible allocations robust to side trading have the following price structure.

Lemma 1. *In any budget-feasible allocation* $(q_i, t_i)_{i=1,2}$ *robust to side trading,*

$$t_1 = cq_1 \text{ and } t_2 - t_1 = c_2(q_2 - q_1). \tag{5}$$

Proof. Because an allocation $(q_i, t_i)_{i=1,2}$ robust to side trading is incentive-compatible, it satisfies $q_2 \geq q_1$ by single crossing. Moreover,

$$t_1 \leq cq_1. \tag{6}$$

Otherwise, an entrant can supply q_1 at a price slightly above c: this profitably attracts type 1 as $T^P(0) = 0$, and remains profitable even if type 2 is attracted. Similarly,

$$t_2 - t_1 \leq c_2(q_2 - q_1). \tag{7}$$

Otherwise, an entrant can supply $q_2 - q_1$ at a price slightly above c_2: this profitably attracts type 2 along with the contract (q_1, t_1), and is even more profitable if type 1 is also attracted. Rewriting the resource constraint as

$$t_1 - cq_1 + m_2[t_2 - t_1 - c_2(q_2 - q_1)] \geq 0$$

and taking advantage of (6)–(7) yields (5). The result follows. ■

Hence pricing is competitive, in the sense that the prices of the *layers* q_1 and $q_2 - q_1$ reflect the costs of serving the types who purchases them. However, if $c_2 > c_1$ and $q_1 > 0$, then the *quantities* q_1 and q_2 are not priced competitively: as q_1 is sold at the average cost $c > c_1$, type 1 subsidizes type 2.

On the other hand, second-best allocations satisfy the following efficiency-at-the-top property.

Lemma 2. *In any second-best allocation* $(q_i, t_i)_{i=1,2}$,

(i) If $IC_{2\to1}$ *is slack, then* $\tau_1(q_1, t_1) \leq c_1$, *with equality if* $q_1 > 0$.

(ii) If $IC_{1\to2}$ *is slack, then* $\tau_2(q_2, t_2) = c_2$.

Proof. If $IC_{2\to1}$ or $IC_{1\to2}$ is slack, then $q_2 > q_1$ by incentive compatibility and single crossing. If $IC_{2\to1}$ is slack and $\tau_1(q_1, t_1) > c_1$, then $((q_1 + \varepsilon, t_1 + c_1\varepsilon), (q_2, t_2))$ is incentive-feasible for $\varepsilon > 0$ small enough and Pareto-dominates $(q_i, t_i)_{i=1,2}$, a contradiction. Thus $\tau_1(q_1, t_1) \leq c_1$. Moreover, if $q_1 > 0$ and $\tau_1(q_1, t_1) < c_1$, then $((q_1 - \varepsilon, t_1 - c_1\varepsilon), (q_2, t_2))$ is incentive-feasible for $\varepsilon > 0$ small enough and Pareto-dominates $(q_i, t_i)_{i=1,2}$, once again a contradiction. This proves (i). The proof of (ii) is similar, using $q_2 > 0$, and is therefore omitted. The result follows. ■

Combining Lemmas 1 and 2 yields our first main theorem.

Theorem 1. *A second-best allocation in which only one incentive-compatibility constraint binds is not robust to side trading.*

Proof. Suppose first that only $IC_{1\to2}$ binds. Then $q_2 > q_1$ by incentive compatibility and single crossing, and $\tau_1(q_1, t_1) \leq c_1$ by Lemma 2(i). Moreover, because type 1's preferences are strictly convex and $IC_{1\to2}$ binds, we have $t_2 - t_1 < c_1(q_2 - q_1) \leq c_2(q_2 - q_1)$. By Lemma 1, $(q_i, t_i)_{i=1,2}$ is not robust to side trading.

Suppose next that only $IC_{2\to1}$ binds. Then $q_2 > q_1$ by incentive compatibility and single crossing, and $\tau_2(q_2, t_2) = c_2$ by Lemma 2(ii). Moreover, because type 2's preferences are strictly convex and $IC_{2\to1}$ binds, we have $t_2 - t_1 > c_2(q_2 - q_1)$. By Lemma 1, $(q_i, t_i)_{i=1,2}$ is not robust to side trading. Hence the result. ■

Theorem 1 covers most cases emphasized in the literature. For instance, in the Rothschild and Stiglitz (1976) insurance economy, either $IC_{1\to2}$ or $IC_{2\to1}$ bind in all but the pooling second-best allocation (Crocker and Snow (1985)), and Theorem 1 implies that none of these allocations is robust to side trading.

This leaves only two cases in which a second-best allocation may be robust to side trading: when both $IC_{1\to2}$ and $IC_{2\to1}$ bind, which corresponds to a pooling allocation, or when both

$IC_{1\to 2}$ and $IC_{2\to 1}$ are slack. Both cases can arise, as we show below, but only under very specific assumptions on preferences and costs.

To study these cases, we strengthen Lemma 1 by establishing that a unique allocation is budget-feasible and robust to side trading. In this allocation, the first layer is optimal for type 1 at price c, while the second layer is optimal for type 2 at price c_2, conditional on her purchasing the first layer. This allocation is thus Pareto-efficient—maximizing type 1's utility—among those satisfying (5).

Theorem 2. *The* JHG allocation *defined by*

$$q_1^* \equiv \arg\max\{u_1(q, cq) : q \geq 0\}, \tag{8}$$

$$t_1^* \equiv cq_1^*, \tag{9}$$

$$q_2^* \equiv q_1^* + \arg\max\{u_2(q_1^* + q, t_1^* + c_2 q) : q \geq 0\}, \tag{10}$$

$$t_2^* \equiv t_1^* + c_2(q_2^* - q_1^*), \tag{11}$$

is the only budget-feasible allocation robust to side trading.

Proof. (Uniqueness) Because an allocation $(q_i, t_i)_{i=1,2}$ robust to side trading is incentive-compatible, it satisfies $q_2 \geq q_1$ by single crossing. Moreover,

$$u_1(q_1, t_1) \geq \max\{u_1(q, cq) : q \geq 0\}. \tag{12}$$

Otherwise, an entrant can offer a contract with unit price slightly above c that profitably attracts type 1 as $T^P(0) = 0$, and remains profitable even if type 2 is attracted. Similarly,

$$u_2(q_2, t_2) \geq \max\{u_2(q_1 + q, t_1 + c_2 q) : q \geq 0\}. \tag{13}$$

Otherwise, an entrant can offer a contract with unit price slightly above c_2 that profitably attracts type 2 along with the contract (q_1, t_1), and is even more profitable if type 1 is also attracted. Finally, if $(q_i, t_i)_{i=1,2}$ is budget-feasible, then (5) holds, so that (12)–(13) are equalities. Thus $(q_i, t_i)_{i=1,2}$ is the JHG allocation defined by (8)–(11).

(Existence) By (8)–(11), the piecewise-linear convex tariff

$$T^P(q) \equiv 1_{\{q \leq Q_1^*\}} cq + 1_{\{q > Q_1^*\}}[cq_1^* + c_2(q - q_1^*)] \tag{14}$$

implements the JHG allocation. Now, suppose that an entrant posts a tariff T^E. The following monotonicity property is established in the Appendix.

Lemma 3. *There exists a solution $((q_i^P, q_i^E))_{i=1,2}$ to (3) such that $q_2^E \geq q_1^E$.*

Let us fix such a solution in what follows. As T^P allows type 1 to purchase her optimal quantity q_1^* at price c, we must have

$$T^E(q_1^E) \leq c q_1^E. \tag{15}$$

Moreover, because $q_2^E \geq q_1^E$, type 2 could alternatively obtain the same aggregate quantity $q_2^P + q_2^E$ as in her best response by purchasing q_1^E from the entrant and $q_2^P + q_2^E - q_1^E$ from the planner, paying overall $T^P(q_2^P + q_2^E - q_1^E) + T^E(q_1^E)$. As she chooses to pay $T^P(q_2^P) + T^E(q_2^E)$ instead, we must have

$$T^E(q_2^E) - T^E(q_1^E) \leq T^P(q_2^P + q_2^E - q_1^E) - T^P(q_2^P). \tag{16}$$

Because T^P is convex with slope at most c_2 and $q_2^E \geq q_1^E$,

$$T^P(q_2^P + q_2^E - q_1^E) - T^P(q_2^P) \leq c_2(q_2^E - q_1^E). \tag{17}$$

Collecting (15) and (16)–(17) yields

$$T^E(q_1^E) - c q_1^E + m_2[T^E(q_2^E) - T^E(q_1^E) - c_2(q_2^E - q_1^E)] \leq 0,$$

which is (4). This shows that T^P is entry-proof. Hence the result. ■

The uniqueness of a budget-feasible allocation robust to side trading contrasts with the multiplicity of second-best allocations, which form a nondegenerate frontier. The planner is thus severely constrained by his inability to monitor trades, which effectively prevents any kind of redistribution between different types of consumers.

The existence of such an allocation for any distribution of types is also noteworthy. Nonexclusivity, or consumers' ability to combine the contracts offered by an entrant with those offered by the planner, is key to this result. While this enlarges the set of contracts an entrant can use to attract consumers, this also gives the planner more instruments to deter entry. These take the form of *latent contracts*, which are not meant to be traded but only to make entry unprofitable. Of course, the planner must make sure that, by offering latent contracts, he does not create new profitable entry opportunities. The *JHG tariff* (14) strikes a balance between these two requirements.

In the adverse-selection case $c_2 > c_1$, type 1's and type 2's marginal rates of substitution at the JHG allocation are ordered, $\tau_1(q_1^*, t_1^*) < \tau_2(q_2^*, t_2^*)$. In particular, $\tau_1(q_1^*, t_1^*) = c < c_2 = \tau_2(q_2^*, t_2^*)$ if the JHG allocation is interior and separating. This contrasts with private-value models where side trades take place on Walrasian markets, which calls for an equalization of marginal rates of substitution (Hammond (1979, 1987)). Yet incentive-compatible gains from trade between types 1 and 2 are exhausted at the JHG allocation, subject to the side-trading constraint. Indeed, supposing that consumers have access to the same constant-return-to-scale technology as firms, the minimum price at which type 1 would be willing to sell a small additional quantity to type 2 is c_2, and at this price type 2 is not willing to buy. In that sense, the JHG allocation is the only candidate for a competitive equilibrium.

Regarding the proof of Theorem 2, an interesting duality is that the JHG allocation is the only candidate for a budget-feasible allocation robust to side trading even if the entrant can only offer a single contract, while the JHG tariff is entry-proof even if the entrant can post an arbitrary tariff. This differs from Glosten (1994), who in his analysis of limit-order markets requires the entrant's tariff to satisfy a property he dubs *single crossing* and that generalizes convexity. Another difference is that Theorem 2 does not require consumers' preferences to be quasilinear.

We are now ready to address the remaining cases not covered by Theorem 1.

Theorem 3. *If a second-best allocation is robust to side trading, then it coincides with the JHG allocation and one of the following conditions holds:*

(i) The JHG allocation is pooling, that is, $\tau_2(q_1^*, t_1^*) \leq c_2$.

(ii) The JHG allocation is separating and first-best, that is, $c_1 = c_2$ *or* $\tau_1(0, 0) \leq c_1$.

Proof. By Theorem 2, if a second-best allocation is robust to side trading, then it coincides with the JHG allocation. By Theorem 1, we only need to consider two cases.

(i) If $IC_{1\to2}$ and $IC_{2\to1}$ bind, then $q_2^* = q_1^*$ by incentive compatibility and single crossing. Hence the JHG allocation is pooling, which amounts to $\tau_2(q_1^*, t_1^*) \leq c_2$ by (10)–(11).

(ii) If $IC_{1\to2}$ and $IC_{2\to1}$ are slack, then $q_2^* > q_1^*$ by incentive compatibility and single crossing. Hence the JHG allocation is separating. Two cases can arise. If $q_1^* > 0$, then $\tau_1(q_1^*, t_1^*) = c_1$ by Lemma 2(i) and $\tau_1(q_1^*, t_1^*) = c$ by (8)–(9), so that $c_1 = c_2$. If $q_1^* = 0$, then $\tau_1(0, 0) \leq c_1$ by Lemma 2(i). In either case, each type i trades efficiently at cost c_i, so that the JHG allocation is first-best. Hence the result. ■

Condition (i) is clearly extreme. It cannot hold in a Rothschild and Stiglitz (1976) insurance economy, because the optimal coverage of type 1 at the average premium rate c is only partial, while type 2 is willing to purchase additional coverage at the fair premium rate c_2 until she reaches full insurance. In the case of quasilinear preferences, condition (i) together with the condition $\tau_1(q_1^*, t_1^*) \leq c$ implied by (8)–(9) entails that type 1's first-best quantity is at least as large as type 2's, and strictly larger if $q_1^* > 0$, a case of nonresponsiveness (Caillaud, Guesnerie, Rey, and Tirole (1988)).

Condition (ii) is also extreme. Indeed, the JHG allocation is then first-best, and the JHG tariff is linear with slope c_2. In the knife-edged private-value case, each type trades efficiently at cost $c_1 = c_2$. In the adverse-selection case $c_2 > c_1$, a separating second-best allocation is robust to side trading only if type 1 is not willing to trade at cost c_1 and hence is in some sense irrelevant.

This answers the question we raised in this section: second-best efficiency and robustness to side trading are irreconcilable requirements, except in very special cases. Overall, our results suggest that the threat of side trading constitutes a serious obstacle to efficiency and redistribution in private-information economies. In the limiting case of private values, side trading poses no threat to efficiency, as it leads to a first-best allocation; yet the requirement that there be no cross-subsidies between layers precludes the planner from redistributing resources between different consumer types. By contrast, under adverse selection, the social costs of side trading are twofold: first, the threat of side trading moves the economy away from the second-best efficiency frontier; second, it precludes redistribution.

4 Policy Implications

On a more positive note, our analysis also yields novel insights for the design of public programs when agents can complement them by resorting to the private sector. We discuss this issue in the context of insurance and financial markets.

Insurance Markets In modern health-insurance systems, public insurance schemes for the provision of basic coverage do not prevent an active role for the private sector. Indeed, consumers may have the choice to opt out from the public-insurance scheme to buy basic coverage designed and priced by private insurance companies, as in Germany. Alternatively, they may have the option to complement basic coverage with additional privately provided coverage, such as *mutuelles* in France. At the same time, different forms of mandatory health insurance, whereby consumers are not allowed to remain uninsured, are in place in several systems, as in France, Germany, Japan, Netherlands, and Switzerland.[1] From a theoretical viewpoint, the design and the enforcement of a public-insurance scheme in the presence of such constraints appears to be a delicate task.

[1] We refer to the surveys of Thomson and Mossialos (2009) and Thomson, Osborne, Squires, and Jun (2013) for institutional details and cross-country evidence.

In this respect, our analysis suggests a simple intervention that achieves a mix of public and private insurance, with no need for observability requirements. It consists in letting the State offer any amount of basic coverage up to q_1^* at the average premium rate c. As private insurance companies are willing to provide any amount of complementary coverage at the high premium rate c_2, the State together with any insurance company make the JHG tariff available. Because this tariff is entry-proof, no insurance company has an incentive to deviate and entry is impossible. Implementing the JHG allocation is therefore compatible with letting consumers free to choose their preferred level of coverage. This is reminiscent of the universal health-care vouchers advocated by Emanuel and Fuchs (2005, 2007), whereby universal coverage is provided while letting consumers free to purchase additional services or amenities on private insurance markets.

Financial Markets In the aftermath of the recent crisis, the opportunity for agents to opt out of a public program and trade in private markets has been acknowledged as a key constraint for the design of financial institutions in the presence of adverse selection. In this respect, recent works have suggested a rationale for liquidity-injection programs that provide a credible signal to uninformed lenders by rejuvenating the relevant markets. An optimal intervention then typically consists in attracting only the least profitable borrowers, either through direct lending (Philippon and Skreta (2012)), or by repurchasing low-quality assets (Tirole (2012)). By participating in a bailout program, a borrower may however end up signalling her financial weakness to the market, creating a stigma effect with potentially perverse implications (Gorton (2015)).

While bailout policies are derived under the assumption that public and private liquidity are mutually exclusive, our approach offers a general theoretical framework for evaluating public interventions in situations where privately informed borrowers may complement a public program with additional funds raised on private markets. A possible intervention would require public liquidity provision to involve a price sufficiently low, c, so as to attract all borrowers, and a borrowing limit q_1^* such that no overborrowing by the least profitable ones is possible. Further borrowing may then take place on private markets at price c_2. Overall, such an intervention would implement the JHG allocation, thereby achieving budget balance, unlike those proposed by Philippon and Skreta (2012) and Tirole (2012), and inducing all types of borrowers to participate. This in turn would make it harder to infer their individual financial conditions, mitigating the impact of the stigma effect. Finally, the corresponding allocation of funds is the only one that can be reached under a budget-balanced program under the threat of side trading.

Appendix

Proof of Lemma 3. Each type i evaluates any contract (q^E, t^E) she can trade with the entrant through the indirect utility function

$$z_i^{-P}(q^E, t^E) \equiv \max\{u_i(q^P + q^E, T^P(q^P) + t^E) : q^P \geq 0\}. \tag{A.1}$$

Because u_i is strictly quasiconcave and T^P is convex, the maximum in (A.1) is attained at a unique $\widehat{q}_i^P(q^E)$, and $(\widehat{q}_i^P(q^E), q_i^E)$ is a solution to (3) if and only if

$$q_i^E \in \arg\max\{z_i^{-P}(q^E, T^E(q^E)) : q^E \geq 0\}. \tag{A.2}$$

According to Attar, Mariotti, and Salanié (2019, Lemma 1), the convexity of the tariff T^P and the strict single-crossing condition for the functions u_i imply the following single-crossing condition for the functions z_i^{-P}:

For all $\underline{q}^E < \overline{q}^E$, $\underline{t}^E$, and $\overline{t}^E$, $z_1^{-P}(\underline{q}^E, \underline{t}^E) < z_1^{-P}(\overline{q}^E, \overline{t}^E)$ implies $z_2^{-P}(\underline{q}^E, \underline{t}^E) < z_2^{-P}(\overline{q}^E, \overline{t}^E)$.

To conclude, suppose that $q_2^E < q_1^E$ at some solution $((q_i^P, q_i^E))_{i=1,2}$ to (3). By (A.2),

$$z_2^{-P}(q_2^E, T^E(q_2^E)) \geq z_2^{-P}(q_1^E, T^E(q_1^E)).$$

Because $q_2^E < q_1^E$, the above single-crossing condition then implies

$$z_1^{-P}(q_2^E, T^E(q_2^E)) \geq z_1^{-P}(q_1^E, T^E(q_1^E)).$$

Thus $(\widehat{q}_1^P(q_2^E), q_2^E)$ is also a solution to (3) for type 1. The result follows. ■

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