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# “The cost-efficiency carbon pricing puzzle”

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## Abstract

Any global temperature target must be translated into an intertemporal carbon budget and its associated cost-efficient carbon price schedule. Under the Hotelling's rule without uncertainty, the growth rate of this price should be equal to the interest rate. It is therefore a puzzle that many cost-efficiency IAM models yield carbon prices that increase at an average real growth rate above 7% per year, a very large return for traders of carbon assets. I explore whether uncertainties surrounding the development of green technologies could solve this puzzle. I show that future marginal abatement costs and aggregate consumption are positively correlated. This justifies doing less for climate change than in the safe case, implying a smaller initial carbon price, and an expected growth rate of carbon price that is larger than the interest rate. In the benchmark calibration of my model, I obtain an equilibrium interest rate around 1% and an expected growth rate of carbon price around 3.5%, yielding an optimal carbon price above 200 USD/tCO<sub>2</sub> within the next few years. I also show that the rigid carbon budget approach to cost-efficiency carbon pricing implies a large uncertainty surrounding the future carbon prices that support this constraint. I show that green investors should be compensated for this risk by a large risk premium embedded in the growth rate of expected carbon prices, rather than by a collar on carbon prices as often recommended.

*Keywords:* Carbon budget, risk-adjusted Hotelling's rule, climate finance, climate beta.

*JEL codes:* Q54, D81, G12.

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# 1 Introduction

How urgent is the necessity to decarbonize our economies? Can we wait another decade to get the anticipated low-cost low-carbon technologies before drastically reducing emissions? Should we again postpone the large increase in carbon price necessary to trigger this transition? In this paper, I address these key policy questions by recognizing that politicians have already fixed the climate objective of  $2^{\circ}\text{C}$  without knowing the cost of the green technologies that one will have to use in the next few decades to attain this ambitious objective. This ambition has been confirmed at the occasion of the COP-21 in Paris in 2015. As is well-known, it is associated to an intertemporal carbon budget constraint. Determining the optimal timing to consume this carbon budget is a problem isomorphic to the Hotelling's problem (Hotelling (1931)) of extracting a non-renewable resource (Nordhaus (1982), Chakravorty et al. (2006), Chakravorty et al. (2008), Schubert (2008), Lemoine and Rudik (2017), van der Ploeg (2018), Emmerling et al. (2019)). Under this cost-efficiency approach, abating one ton of  $\text{CO}_2$  today is a perfect substitute to abating one ton of  $\text{CO}_2$  in the future. Frontloading the abatement effort is an investment that has a single cost and a single benefit that are respectively equal to the present and future marginal abatement costs (MAC), i.e., to the present and future carbon prices. Along the optimal abatement path, this marginal investment should have a zero net present value. This is possible only if the growth rate of (expected) carbon price is equal to the (risk-adjusted) discount rate. This extended Hotelling's rule applied to climate change is simple and transparent.<sup>1</sup> The ambition of the climate target or the anticipation of future low-cost abatement technologies should influence the initial carbon price, but not its growth rate over time. In short, under an exogenous climate objective, the Hotelling's rule dictates the efficient timing of our climate efforts.

In most climate models, there is no uncertainty and green technological progresses together with the intertemporal carbon budget are known in advance. In that case, the growth rate of carbon prices should therefore be equal to the interest rate.<sup>2</sup> It is then a puzzle that most of these models generate carbon prices whose real growth rate is much larger than the interest rate. Figure 1 illustrates this observation. It describes the distribution of annualized real growth rates of world carbon prices from the database (<https://tntcat.iiasa.ac.at/AR5DB>) of models used in the 5th report of the IPCC. If one limits the analysis to the 767 calibrations of these models that estimate a world carbon prices for years 2020 and 2050, they yield an average annual growth rate of 7.04% for real carbon prices between these two dates, which is much larger than market interest rates. This suggests that the allocation of mitigation efforts is not intertemporally efficient. I refer to this observation as the "carbon pricing puzzle" of cost-efficient IAM models. It tells us that, compared to the recommendations extracted from

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<sup>1</sup>It is specific to the cost-efficient approach and does not need to hold in the cost-benefit approach used for example by Nordhaus (2018). For example, using a 3% discount rate, the U.S. administration published a scientific report (IAWG (2016)) based on a cost-benefit approach that recommends a price of 42 dollars (of 2007) per ton of  $\text{CO}_2$  in 2020, growing to 69 dollars (of 2007) in 2050. This yields a real growth rate of 1.65% per year. Because the carbon concentration in the atmosphere will continue to grow over time under the optimal mitigation strategy, carbon prices will grow in parallel, assuming a convex damage function.

<sup>2</sup>In the absence of any credibility problem, the decentralization of the allocation of the intertemporal carbon budget should be performed by allocating the corresponding permits in the economy, allowing for banking. Under certainty, these permits are risk-free, which implies that their value – the carbon price – should grow at the risk-free rate. Attempting to impose a larger growth rate will generate a disequilibrium (excess saving of permits).

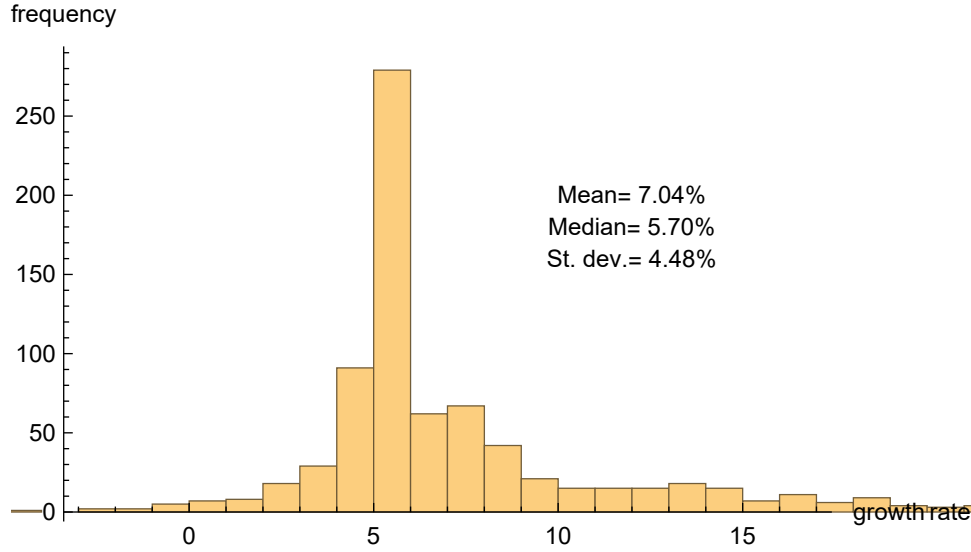


Figure 1: Histogram of the real growth rate (in % per year) of carbon prices between 2020 and 2050 from 767 calibrations of IAM models contained in the IPCC database (<https://tntcat.iiasa.ac.at/AR5DB>). The mean annual growth rate of 7.04%, with a median of 5.70% and a standard deviation of 4.48%.

these IPCC models, reallocating some climate efforts to the present would be socially desirable. In reality, these models explore second-best climate policies in which the intertemporal allocation of the carbon budget is not optimized, certainly because of the political unacceptability of a high carbon price. Rather, these models characterize carbon price schedules that are compatible with exogenously determined carbon emission targets at different dates. These "Representative Concentration Pathways" (RCP) are predetermined by the IPCC. The large growth rate of carbon prices suggests that the waiting game of international climate politics has infected the IPCC. It is a fair question to ask whether the IPCC should base its recommendations on the first-best allocation, or whether it should include the political acceptability constraint straight from the beginning into its analysis. My view on this is that second-best analyses are useful as long as they are clearly announced as such.

However, this initial puzzle is based on the premise that the evolution of abatement costs and carbon prices is certain. In this paper, I recognize that this key assumption is utterly unrealistic, and I explore the impact of uncertainty on the socially efficient growth rate of real carbon prices. Could it be possible that a low current carbon price (compensated by a larger growth rate of carbon prices) be socially desirable because of the uncertainty surrounding the cost of future green technologies? My analysis also predicts the growth rate of expected carbon prices if the intertemporal carbon budget is decentralized through a market for permits with full banking.

The abatement models using a cost-efficiency approach and a carbon budget rely on strong assumptions about the evolution of the abatement cost function during the next few decades (Pindyck (2013)). Obviously, technologically optimistic models allow for low prices and efforts in the short run by anticipation of the emergence of these low-cost mitigation technologies.

But in reality, technological changes are hard to predict. If they do not materialize, one will have to drastically increase carbon prices to satisfy the intertemporal carbon budget. Nobody really knows today what will be the mitigation cost associated to wind or solar energy in the future. Deep uncertainties also surround future electricity storage technologies and nuclear fusion for example. The extraordinary large uncertainty surrounding the emergence of economically viable renewable systems of energy is an inherent dimension of the energy transition. Similarly, IAM models are generally based on a deterministic growth of total factor productivity. Recognizing the uncertainty surrounding the growth of TFP in the long run should also be taken into account to determine the carbon price schedule. If economic growth is larger than expected, more abatement efforts will have to be implemented to compensate for the larger emissions and this will require a larger carbon price. As in the "quantity" approach proposed by Weitzman (1974) under uncertainty, I assume that the carbon budget is not sensitive to changes in the marginal abatement costs.

Uncertainty should affect the optimal timing of climate efforts and the carbon pricing system that support it.<sup>3</sup> The expected growth rate of carbon prices – which is also the expected return of abatement frontloading – should equal the discount rate adjusted for the riskiness of postponing or frontloading the abatement effort. The Consumption-based Capital Asset Pricing Model (CCAPM, Breeden (1979), Lucas (1978) and Rubinstein (1976)) tells us how to perform this adjustment. Suppose for example that, along the optimal path, marginal abatement costs are negatively correlated with aggregate consumption. Because the MAC is the future benefit of abatement frontloading, fighting climate change early has the extra benefit to hedge the macro risk in that context. Because of this negative CCAPM beta of early mitigation efforts, one should discount the future benefit of this early investment, i.e., the future MAC, at a rate lower than the risk-free rate to determine the current price of carbon. This means at the same time a larger current price of carbon, and a growth rate of the expected carbon price smaller than the risk-free rate. From a positive point of view, this carbon pricing system is compatible with an equilibrium, as investors in green technologies will have an expected rate of return smaller than the interest rate, just because such green investments hedge their global portfolio risk. On the contrary, if MAC and aggregate consumption correlate positively, i.e., if the climate beta is positive, the risk premium will be positive, the current price of carbon will be smaller, and the growth rate of expected carbon price will be larger than the interest rate. This policy provides the right price signal for private investors in renewables technologies to take account of the impact of their decisions on social welfare, as is the case on efficient financial markets for other investment projects.

It remains to characterize the determinants of this carbon beta.<sup>4</sup> To do this, I develop

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<sup>3</sup>The theoretical question raised here is about how to adapt the Hotelling's rule to uncertainty. There has been a few attempts to answer this question in the late XXth century. For example, Pindyck (1978, 1980) explores the optimal extraction strategy of risk-neutral owners of a nonrenewable resource when exploration is possible or when the stock of this resource and the demand for it are unknown. This analysis is useful to examine a resource-rich country that is unable or unwilling to make this asset financially liquid, but it is not directly relevant in the context of the carbon budget problem. Indeed, households, investors and firms that will bear the mitigation risk will also bear all other statistically-linked risks in the economy. Our approach is closer to Gaudet and Howitt (1989), Gaudet and Khadr (1991) and Slade and Thille (1997) who examined the case of stochastic processes for economic growth and extraction costs in the context of a non-renewable resource.

<sup>4</sup>Dietz et al. (2018) examined the risk profile of carbon prices using the cost-benefit analysis of the DICE model. In this alternative approach, the key determinant of the climate beta is the income-elasticity of climate

a two-period model in which the dynamically optimal mitigation strategy is endogenously determined under uncertainty about the future abatement cost function, economic growth and carbon budget. I characterize the impact of these sources of uncertainty on the optimal growth rate of expected carbon price, and I realistically calibrate this model. In Section 4, I solve the classical asset pricing puzzles (Mehra and Prescott (1985), Weil (1989) and Kocherlakota (1996)) of the CCAPM by using two Barro’s approach based on extreme events (Barro (2006)). In this framework, I show that the beta of abatement frontloading is the income-elasticity of MACs. Multiplying this beta by the equilibrium aggregate risk premium tells us by how much the growth rate of expected carbon price should differ from the equilibrium interest rate. I show that the sign of this carbon beta is generally ambiguous, with different sources of uncertainty pushing the climate beta in opposite directions. However, a realistic calibration of the two-period model suggests a positive climate beta. This means that it is socially desirable to implement a climate strategy with a growth rate of expected carbon price that is larger than the interest rate, thereby allowing to start with a relatively low carbon price today. Thus, this analysis justifies using a discount rate for green technologies and planning for a growth rate of expected carbon prices that are larger than the interest rate. It could thus help solving the carbon pricing puzzle. However, the efficient growth rate of carbon prices is 3.5%, which is much smaller than the 7% observed on average in the database of models of the IPCC. The bottom line of my analysis remains that the RCPs of the IPCC inefficiently allocate abatement efforts over time. The same final concentration of GHG in the atmosphere could be obtained with a smaller impact on intergenerational welfare by abating more today, and abating less in the future.

Contrary to the norm in IAM modeling, I mostly use a two-period model in this paper because of the nature of the problem that I examine. The flow of climate impacts generated by our emissions today, which last for centuries and millennia, is ignored in this paper. It is replaced by an exogenous carbon budget. This considerably shortens the time span covered by the model, as we are expected to go to net-zero emissions by the middle of this century. In this temporal framework, it makes sense to split the time remaining to 2050 in two periods of 15 years, one period representing the current time during which one has to act under an uncertain technological future, and the other period representing the future, a time after 2035 where we will face much less technological uncertainty about the true abatement costs of the various green technologies. It is right to say that some uncertainty will remain unresolved in 2035 to choose the best technologies. But given the time to build green infrastructures, it is mostly the information available during the next decade that will determine the infrastructures available in 2050.

More recently, and independently of this paper, Olijslagers et al. (2023) have also examined an optimal dynamic carbon pricing model with an exogenous temperature target. In a continuous-time model with a stochastic consumption growth process and Epstein-Zin preferences, they show as I do in this paper that there is a positive carbon premium, which means that the growth rate of the expected carbon value must be larger than the interest rate. Although their paper has the important advantage to offer a richer dynamic framework, they limit the source of uncertainty to consumption growth. In this paper, I show that adding technological uncertainties has the potential to reverse the key result of a positive carbon risk premium. Another closely related paper is Edenhofer et al. (2024) in which the authors

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damages.

adapt my two-period model to take account of two sources of inefficiencies, namely the lack of commitment on the climate ambition by the regulator and technological externalities. That paper mostly confirms the main findings presented in this paper, with additional insights related to these two sources of inefficiencies.

A possible explanation of the carbon pricing puzzle is based on the existence of political constraints related to the social acceptability of climate policies around the world in the short run. Following Gollier and Tirole (2015) for example, these constraints are typically at play to postpone climate efforts to the future, a phenomenon of procrastination that could explain why the above-mentioned models support a low current carbon price and a large growth rate of this price. This raises the question of the credibility of long-term climate commitments. Laffont and Tirole (1996) take this question seriously by proposing a commitment device based on forward financial contracts. Harstad (2019) justifies strategic investments and investment subsidies in technologies that are strategic complements to future green investments when the social planner faces a time-consistency problem from hyperbolic discounting.

In the next section, I assume that the optimal abatement strategy under the carbon budget is known, and I characterize the properties of the carbon pricing system that supports this social optimum, assuming an exogenous statistical relation between MAC and aggregate consumption. Section 3 is devoted to a simple two-period model in which the price of carbon in the first period must be determined under uncertainty about economic growth, green innovation and carbon budget. The carbon beta is determined endogenously in this section. In Section 4, I calibrate this model. Before presenting the model, let me remind the readers that if my cost-efficiency approach focuses on the transition cost, it ignores the benefits of the transition. It is important to keep in mind that a carbon budget is imposed because it is socially desirable to limit the impacts of climate change.

## 2 CCAPM carbon pricing

In this section, I characterize the socially optimal expected growth rate of the carbon price based on the classical consumption-based CAPM model. In the spirit of this model, the optimality condition is translated into an asset pricing rule. This rule can be used as an optimality test for the underlying dynamic allocation. However, it provides only a partial characterization of the optimal allocation. Its full characterization is provided in the next section in a simplified framework.

Suppose that the economy has a representative agent whose rate of pure preference for the present is  $\rho$ . The von Neumann-Morgenstern utility function  $u$  of the representative agent is increasing and concave. Along the optimal path, the consumption per capita  $C_\tau|_{\tau \geq 0}$  evolves in a stochastic way. In the constellation of investment opportunities existing in the economy, consider a marginal incremental project that yields a cost  $I_0$  today and generates a single benefit  $B_t$  at date  $t$ , where  $B_t$  is potentially uncertain and statistically related to the stochastic process governing aggregate consumption. At the margin, investing in this project raises the discounted expected utility of the representative agent by

$$\Delta V = -I_0 u'(C_0) + e^{-\rho t} E[B_t u'(C_t)]. \quad (1)$$

The size of the investment in this project is optimal if and only if  $\Delta V = 0$ . If one reinterprets  $I_0$  as the current price of an asset yielding the single benefit  $B_t$  at date  $t$ , this optimality

condition is also an equilibrium condition and an asset pricing rule. The CCAPM makes use of this observation to price any asset in the economy. For example, the risk-free claim  $B_t = 1$  should be priced today as  $P_{ft} = e^{-\rho t} E[u'(C_t)]/u'(C_0)$ . This equilibrium condition gives us the interest rate  $r_{ft}$  in the economy once price  $P_{ft}$  is translated into a return  $r_{ft} = -t^{-1} \log(P_{ft})$ .

Let  $A'_\tau|_{\tau \geq 0}$  denote the dynamics of marginal abatement costs along the optimal allocation of mitigation efforts. It can also be interpreted as the dynamics of equilibrium carbon prices. If the climate policy is decentralized through a market for bankable emission permits for example, marginal abatement costs will be equalized across firms and individuals, and will be equal to the equilibrium carbon price. Let  $\delta$  denote the rate of natural decay of CO<sub>2</sub> in the atmosphere. Along the optimal mitigation path, contemplate a marginal reallocation of climate efforts consisting in reducing CO<sub>2</sub> emissions by 1 more ton today. This allows for a marginal increase in emissions by  $\exp(-\delta t)$  tons at date  $t$ , leaving the total carbon budget unaffected.<sup>5</sup> This investment yields an initial cost  $A'_0$  and generates a future benefit  $B_t = \exp(-\delta t)A'_t$ . Applying the optimality condition  $\Delta V = 0$  to this strategy, we must have that

$$A'_0 = \exp(-(\delta + \rho)t) \frac{E[A'_t u'(C_t)]}{u'(C_0)}. \quad (2)$$

Suppose that  $A'_t$  and  $C_t$  are comonotone<sup>6</sup>, a concept that is more demanding than "positive correlated", but is similar in spirit outside the gaussian world. Because of risk aversion, equation (2) implies that (see for example Gollier (2001), Proposition 15):

$$\frac{A'_0}{E[A'_t]} < \exp(-(\delta + \rho)t) \frac{E[u'(C_t)]}{u'(C_0)} = \exp(-(r_{ft} + \delta)t). \quad (3)$$

Let  $g_t = t^{-1} \log(E[A'_t]/A'_0)$  denote the growth rate of expected carbon price.<sup>7</sup> The above inequality directly implies that this growth rate is larger than  $r_{ft} + \delta$ . The opposite is true when  $A'_t$  and  $C_t$  are anti-comonotone. In the special case where  $A'_t$  is certain, the growth rate of carbon price should be equal to the sum of the interest rate and the rate of natural decay. This is the well-known Hotelling's rule adapted to carbon pricing under a fixed intertemporal carbon budget (Schubert (2008), Emmerling et al. (2019)). This proves the following proposition.

**Proposition 1.** *The growth rate of the expected carbon price that supports the optimal temporal allocation of abatement efforts is larger (smaller) than the sum of the interest rate and the rate of decay of carbon dioxide if the marginal abatement cost and aggregate consumption are (anti-)comonotone.*

From the social point of view, facing a positive correlation between marginal abatement costs and aggregate consumption is good news. It means that the worst-case scenarios in terms of abatement costs arise when aggregate consumption is large, i.e., when the marginal

<sup>5</sup>Dietz and Venmans (2019) claim that the relevant variable that affects the global temperature is not GHG concentration but rather cumulated emissions. In the calibration exercise, I set  $\delta$  to zero.

<sup>6</sup>Two random variables  $(X, Y)$  are said to be (anti-)comonotone iff for any pair  $(s, s')$  of states of nature,  $(X(s) - X(s'))(Y(s) - Y(s'))$  is non-negative (non-positive).

<sup>7</sup>In general, variables  $g_t$  and  $r_{ft}$  are maturity-dependent. Our findings should be understood as being applicable to any maturity.



abatement effort has a smaller utility impact. Abating more in the future reduces the macroeconomic risk. This hedging benefit raises the collective willingness to postpone abatement efforts. It reduces the efficient carbon price today, in exchange for a larger growth rate of the expected price. From the individual point of view, investors who abate early in exchange for saving their permits must be compensated for the fact that the benefit of doing so has a positive beta, in the sense that the return of this investment is smaller when other assets also perform poorly in the economy. Because the return of abatement frontloading is the growth rate of carbon price, this compensation takes the form of a growth rate of expected carbon price larger than the sum of the interest rate and the rate of natural decay.

I illustrate Proposition 1 in two special cases. The benchmark case corresponds to the standard CCAPM. Suppose that relative risk aversion is a constant  $\gamma$ . Suppose also that aggregate consumption and marginal abatement costs evolve according to the following stochastic process:

$$dc_t = \mu_c dt + \sigma_c dz_t \quad (4)$$

$$da'_t = \mu_p dt + \phi \sigma_c dz_t + \sigma_w dw_t, \quad (5)$$

with  $c_t = \log C_t$  and  $a'_t = \log A'_t$ , and where  $z_t$  and  $w_t$  are two independent standard Wiener processes. This means that the logarithm of aggregate consumption and marginal costs are jointly normally distributed. Parameters  $\mu_c$  and  $\sigma_c$  are respectively the trend and the volatility of consumption growth. The trend of growth of the marginal abatement cost, and thus of the carbon price, is given by parameter  $\mu_p$ . The volatility of the marginal abatement cost has an independent component  $\sigma_w$  and a component coming from its correlation with economic growth. Notice that  $\phi$  can be interpreted as the elasticity of marginal abatement costs to unanticipated changes in aggregate consumption.

I provide a formal proof of the following proposition in the Appendix, together with the characterization of the risk-free rate and the aggregate risk premium. It is an application of the CCAPM for the pricing a non-renewable resource (Gaudet and Khadr (1991)).

**Proposition 2.** *Suppose that relative risk aversion is constant and that the logarithms of aggregate consumption and marginal abatement costs follow a bivariate Brownian process. Then, the growth rate  $g$  of the expected carbon price that supports the optimal temporal allocation of abatement efforts must be equal to the sum of three terms:*

- $\delta$ : the rate of natural decay of greenhouse gas in the atmosphere;
- $r_f$ : the interest rate in the economy;
- $\phi\pi$ : the abatement risk premium, which is the product of the income-elasticity ( $\phi$ ) of marginal abatement cost by the aggregate risk premium ( $\pi$ ) in the economy.

In short, we have that

$$g = \delta + r_f + \phi\pi, \quad (6)$$

where the interest rate  $r_f$  and the aggregate risk premium  $\pi$  are characterized in the Appendix. This result tells us that the CCAPM risk premium for carbon permits holds with a CCAPM "carbon beta" being equal to the income-elasticity  $\phi$  of the marginal abatement cost. An immediate consequence of Proposition 2 is that the growth rate of expected carbon

price is larger (smaller) than the sum of the interest rate and the rate of decay of carbon dioxide if the income-elasticity of marginal abatement costs is positive (negative). This is a special case of Proposition 1.

Under the stochastic process (4)-(5), the estimation of the key parameter  $\phi$  is rather simple. Indeed, this system implies that

$$\Delta \log(A'_t) = a + \phi \Delta \log(C_t) + \varepsilon_t, \quad (7)$$

where  $\Delta \log(A'_t)$  and  $\Delta \log(C_t)$  are respectively changes in log marginal cost and in log consumption, and  $\varepsilon_t$  is an independent noise that is normally distributed. This means that, under these assumptions, the OLS estimator of the slope of this linear equation is an unbiased estimator of the income-elasticity of the marginal abatement cost that must be used to determine the efficient growth rate of expected carbon price.

This CCAPM example is imperfect for at least two reasons. First, as is well-known, the CCAPM faces the standard puzzles of asset pricing, in particular the risk-free rate puzzle and the equity premium puzzle. Second, it is clear that the income-elasticity of the marginal abatement cost is endogenous and sensitive to the mitigation strategy that will be followed at equilibrium. This is why the remainder of this paper is devoted to the analysis of an alternative application of Proposition 1 that solves these two issues.

### 3 The determinants of the carbon beta under an exogenous carbon budget

In this section, I explore the determinants of the income-elasticity of the marginal abatement cost, i.e., the cost-efficient carbon beta. Because the current and future marginal abatement costs depend upon the intertemporal abatement strategy, its characterization requires solving the intertemporal carbon allocation problem. This cannot be easily done in a continuous-time framework. In this section, I solve this problem in a simple two-period framework. Suppose that the carbon budget constraint covers only two periods,  $t = 0$  and 1. The production of the consumption good is denoted  $Y_0$  and  $Y_1$  for periods 0 and 1 respectively, where  $Y_1$  is uncertain in period 0. The carbon intensity of the economy in the business-as-usual scenario in period  $t$  is denoted  $Q_t$ , so that  $Q_t Y_t$  tons of carbon dioxide are emitted in period  $t$  under this scenario. The country is committed not to exceed a total emission target  $T$  for the two periods, net of the natural carbon sinks. As stated for example in the Paris Agreement, the long-term carbon budget allocated to the countries could be modified depending upon new scientific information about the intensity of the climate change problem for example. In our model, this means that, in period 0, there may be some uncertainty about what the intertemporal carbon budget  $T$  will be in the future.

Compared to the business-as-usual scenario, the country must choose how much to abate in each period. Let  $X_t$  denote the number of tons of carbon dioxide abated due to actions implemented in period  $t$ , so that one can write the carbon budget constraint as follows:

$$e^{-\delta} (Q_0 Y_0 - X_0) + Q_1 Y_1 - X_1 \leq T, \quad (8)$$

where  $\delta$  is the rate of natural decay of carbon dioxide in the atmosphere. I hereafter assume that this ex-post carbon budget constraint is always binding, so that I can rewrite the

abatement in period 1 as a function of the other variables:

$$X_1 = X_1(X_0, Y_1, T) = e^{-\delta} (Q_0 Y_0 - X_0) + Q_1 Y_1 - T. \quad (9)$$

Because  $Y_1$  and  $T$  are uncertain, so is the abatement effort  $X_1$  in period 1 that will be necessary to satisfy the intertemporal carbon budget constraint.

Abating is costly. Let  $A_0(X_0)$  and  $A_1(X_1, \theta)$  denote the abatement cost function in periods 0 and 1 respectively. I assume that  $A_t$  is an increasing and convex function of  $X_t$ . In order to allow for technological uncertainty,  $A_1$  is a function of parameter  $\theta$ , which is unknown in period 0. Consumption in period  $t$  is the production net of the abatement cost in that period, i.e.,  $C_t = Y_t - A_t$ .

This framework allows us to include in the analysis long-term green investments made in period 0 that also reduce emissions in period 1 at zero marginal cost. Under this interpretation,  $X_0$  is the discounted value (at rate  $\delta$ ) of the flow of abated  $\text{CO}_2$  from the green action made in the first period, and  $X_1$  is the abatement in the second period net of the abatement generated by investments made in the previous period.

The problem of the social planner is thus to select the abatement strategy  $(X_0, X_1)$  that maximizes the intertemporal welfare function subject to the carbon budget constraint:

$$\max_{X_0, X_1} H(X_0, X_1) = u(Y_0 - A_0) + e^{-\rho} E[u(Y_1 - A_1)] \quad s.t. \quad (9). \quad (10)$$

The first-order condition of this problem is written as follows:

$$A'_0 u'(C_0) = e^{-\rho-\delta} E[A'_1 u'(C_1)], \quad (11)$$

where  $A'_t$  denote the partial derivative of the total abatement cost function with respect to abatement  $X_t$ .

We know from Proposition 1 that the growth rate of the expected carbon price is larger (smaller) than the interest rate plus the rate of natural decay when the marginal abatement cost and aggregate consumption are (anti-)comonotone. In the remainder of this section, I examine various special cases that highlight the factors that determine the nature of the statistical relation between these two random variables along the optimal path. To do this, let us fully differentiate  $A'_1$  and  $C_1$  with respect to the three sources of uncertainty  $(Y_1, \theta, T)$ :

$$dA'_1 = Q_1 A''_1 dY_1 - A'_1 dT + \frac{\partial A'_1}{\partial \theta} d\theta \quad (12)$$

$$dC_1 = (1 - Q_1 A'_1) dY_1 + A'_1 dT - \frac{\partial A_1}{\partial \theta} d\theta. \quad (13)$$

Suppose first that the only source of uncertainty in the economy is related to the exogenous growth of production  $Y_1$ , so that  $T$  and  $\theta$  are fixed. In that context, the only source of correlation between  $A'_1$  and  $C_1$  comes from the fact that both random variables covary positively with  $Y_1$ . From the first term in the right-hand side of equation (12), we see that  $A'_1$  is increasing in  $Y_1$ , since  $A''_1$  is positive. A positive productivity shock raises the marginal abatement cost. This is because it raises emissions under the business-as-usual together with the abatement effort to compensate it. Because the MAC is increasing in the effort, it covaries positively with  $Y_1$ . Suppose now that  $Q_1 A'_1$  is smaller than unity. From the first term in

the right-hand side of equation (13), this implies that a positive productivity shock increases consumption in spite of the fact that it also necessitates an additional abatement effort to compensate the excess emissions generated by the shock. Thus, under this condition, a positive productivity shock affects positively the MAC and aggregate consumption. Thus,  $A'_1$  and  $C_1$  are comonotone in this context. Using Proposition 1, this demonstrates the following proposition.

**Proposition 3.** *Suppose that the growth of aggregate production  $Y_1$  is the only source of uncertainty in the economy, and that  $Q_1 A'_1$  is smaller than unity. Then, it is socially desirable that the growth rate of expected carbon price be larger than the sum of the interest rate and the rate of decay of  $CO_2$ .*

A similar exercise based on the signs of the second terms in equations (12) and (13) can be done in a context where the only source of uncertainty is related to the intertemporal budget constraint  $T$ . In that case, a larger budget  $T$  implies a smaller abatement effort, and thus a larger share of production available for consumption rather than for abatement efforts. At the same time, because of the convexity of the cost function, the marginal abatement cost is smaller. Thus, aggregate consumption and marginal abatement cost are anti-comonotone. This yields the following result.

**Proposition 4.** *Suppose that the intertemporal carbon budget  $T$  is the only source of uncertainty in the economy. Then, it is socially desirable that the growth rate of expected carbon price be smaller than the sum of the interest rate and the rate of decay of  $CO_2$ .*

Suppose finally that the only source of uncertainty is about  $\theta$ , which is related to the speed of green technological progress. Suppose that an increase in  $\theta$  implies a reduction in both the total and the marginal abatement costs, i.e., that for all  $(X_1, \theta)$ ,

$$\frac{\partial A_1(X_1, \theta)}{\partial \theta} \leq 0 \quad \text{and} \quad \frac{\partial A'_1(X_1, \theta)}{\partial \theta} \leq 0. \quad (14)$$

A possible illustration is when marginal abatement cost is an uncertain constant, i.e., when  $A_1(X_1, \theta)$  is equal to  $\alpha + g(\theta)X_1$  with  $g' \leq 0$ , a case examined by Baumstark and Gollier (2010). In that context, a small  $\theta$  means at the same time a large marginal abatement cost and a large total abatement cost, and thus a low aggregate consumption. Thus,  $A'_1$  and  $C_1$  are anti-comonotone in that context, thereby demonstrating the following proposition.

**Proposition 5.** *Suppose that the speed of green technological progress  $\theta$  is the only source of uncertain. If total and marginal abatement costs are comonotone (condition (14)), it is socially desirable that the growth rate of expected carbon price be smaller than the sum of the interest rate and the rate of decay of  $CO_2$ .*

Up to this point, I only characterized the impact of uncertainty on the optimal growth rate of the carbon price. A more complete analysis would be to characterize its effect on the optimal abatement effort in the first period. This is a more difficult question. In order to address it, I simplify the problem by assuming that the marginal abatement cost in period 1 is constant but potentially uncertain:  $A_1(X_1, \theta) = \theta X_1$ . In that case, aggregate consumption in period 1 equals

$$C_1 = Y_1 - \theta \left( e^{-\delta} (Q_0 Y_0 - X_0) + Q_1 Y_1 - T \right).$$

Observe that in that case, the first period abatement  $X_0$  has a role similar to saving in the standard consumption-saving problem. Each ton of CO<sub>2</sub> "saved" in the first period generates an increase in consumption by  $R = \theta \exp(-\delta)$  in the second period, where  $R$  can be interpreted as the rate of return on savings. Suppose first that  $\theta$  is certain. It is well-known in that case that the uncertainty affecting future incomes raises optimal (precautionary) saving if and only if the individual is prudent (Drèze and Modigliani (1972), Leland (1968), Kimball (1990)). An individual is prudent if and only if the third derivative of  $u$  is positive. Applying this result to our context directly yields the following proposition. Notice that because the marginal abatement cost is certain, it must grow at the interest rate in this case.

**Proposition 6.** *Suppose that  $A_1(X_1, \theta) = \theta X_1$  and that the marginal abatement cost  $\theta$  is a known constant. Increasing risk on future production  $Y_1$  or on the intertemporal carbon budget  $T$  increases the initial abatement effort  $X_0$  if and only if the representative agent is prudent, i.e. iff  $u'$  is convex.*

When the marginal abatement cost is uncertain, the future return of abating more today becomes uncertain in that case. By risk aversion, this reinforces the willingness to abate in the first period because it also reduces the risk borne in the second period. Because of this second effect, prudence is sufficient but not necessary in this case.

**Proposition 7.** *Suppose that  $A_1(X_1, \theta) = \theta X_1$  and that the marginal abatement cost  $\theta$  is the only source of uncertainty. Increasing the risk affecting the marginal abatement cost  $\theta$  raises the initial abatement effort  $X_0$  if the representative agent is prudent.*

Proof: Consider two random variables,  $\theta_1$  and  $\theta_2$ , where  $\theta_2$  is riskier than  $\theta_1$  in the sense of Rothschild and Stiglitz (1970). Let  $G_i(X_0) = H_i(X_0, X_1(X_0, Y_1, T))$  denote the corresponding objective function, as described by (10). Let  $X_{0i}$  denote the optimal initial abatement under distribution  $\theta_i$  of the marginal abatement cost. The optimal abatement effort  $X_{01}$  under the initial uncertainty  $\theta_1$  satisfies the first-order condition

$$A'_0(X_{01})u'(Y_0 - A_0(X_{01})) = \beta E[\theta_1 u'(Y_1 - \theta_1 X_{11})], \quad (15)$$

where  $X_{11}$  is the optimal abatement effort in period 1 under the initial risk  $\theta_1$ , i.e.,  $X_{11} = X_1(X_{01}, Y_1, T)$ . Because  $G_2$  is concave in  $X_0$ , I obtain that  $X_{02}$  is larger than  $X_{01}$  if and only if  $G'_2(X_{01})$  is positive. Using condition (15), this condition can be written as follows:

$$E[\theta_2 u'(Y_1 - \theta_2 X_{11})] \geq E[\theta_1 u'(Y_1 - \theta_1 X_{11})]. \quad (16)$$

This is true for any Rothschild-Stiglitz risk increase if and only if function  $v$  is convex, where  $v(\theta)$  equals  $\theta u'(Y_1 - \theta X_{11})$  for all  $\theta$  in the joint support of  $\theta_1$  and  $\theta_2$ . It is easy to check that

$$v''(\theta) = -2X_{11}u''(Y_1 - \theta X_{11}) + \theta X_{11}^2 u'''(Y_1 - \theta X_{11}). \quad (17)$$

Because  $X_{11}$  is positive and  $u''$  is negative, we see that  $v$  is convex when  $u'''$  is positive. ■

## 4 Calibration

In this section, I calibrate the two-period model described in the previous section. A standard approach to climate policy in the western world is based on the hypothesis that the energy transition should be performed within the next 3 decades in order to remain below the 1.5°C objective with probability 1/2. I follow for example Metcalf (2023) to decompose the next 3 decades into two periods of 15 years, 2021-2035 and 2036-2050. I examine the case of the European Union (EU-28). I hereafter describe the calibration of this model. I assume a rate of pure preference for the present equaling  $\rho = 0.5\%$  per year, and a constant relative risk aversion  $\gamma = 3$ .

### 4.1 Economic growth

The current annual GDP of EU-28 is around 19,000 billions US dollars (GUSD). Assuming an annual growth rate of 1.4% per year over the period 2021-2035 yields a total production for this first period estimated at  $Y_0=315,000$  GUSD. The production  $Y_1$  of the second period is uncertain. A key element of this paper is that the recommended returns of green investments are compatible with the equilibrium returns of other assets in the economy, and with intertemporal social welfare. However, as is well-known, the CCAPM model that I use in this paper has been unable to predict observed asset prices when beliefs are normally distributed as assumed in Section 2. This model yields an interest rate that is too large and an aggregate risk premium that is too low.<sup>8</sup> In most of this paper, I use the resolution of these asset pricing puzzles that has been proposed by Barro (2006), who recognized the plausibility of infrequent large recessions that are not well represented in U.S. growth data. I follow the calibration proposed by Martin (2013). The change in log production during the second subperiod is equal to the sum of 15 independent draws of an annual growth rate  $x_i$  whose distribution compounds two normally distributed random variables:

$$\log\left(\frac{Y_1}{Y_0}\right) = \sum_{i=1}^{15} x_i \quad (18)$$

$$x_i \sim (h_{bau}, 1 - p; h_{cat}, p) \quad (19)$$

$$h_{bau} \sim N(\mu_{bau}, \sigma_{bau}^2) \quad (20)$$

$$h_{cat} \sim N(\mu_{cat}, \sigma_{cat}^2). \quad (21)$$

With probability  $1 - p$ , the annual growth rate is drawn from a "business-as-usual" normal distribution with mean  $\mu_{bau} = 2\%$  and volatility  $\sigma_{bau}^2 = 2\%$ . But with a small probability  $p = 1.7\%$ , the annual growth rate is drawn from a "catastrophic" normal distribution with a large negative  $\mu_{cat} = -35\%$  and a large volatility  $\sigma_{cat}^2 = 25\%$ . In Table 1, I describe the value of the parameters of the model that are used as a benchmark. The order of magnitude of the parameters of the production growth process is in the range of what has been considered by Barro (2006) and Martin (2013). It yields an annual trend of growth of 1.37% and an annual volatility of 6.12%.<sup>9</sup> It also generates an expected production of  $Y_1 = 387,000$  billions USD (GUSD) in the second period.

<sup>8</sup>See for example Kocherlakota (1996) and Cochrane (2017).

<sup>9</sup>It is interesting to compare the long run risk generated in this model to the one examined by Nordhaus (2018) and Christensen et al. (2018). They uses a survey of a panel of experts to characterize the uncertainty

## 4.2 Emissions, decarbonization and decay

The EU-28 emitted 3.647 GtCO<sub>2</sub>e in 2021. Under the Business-As-Usual (BAU), I assume that the 2021 annual emission level is maintained over each of the 15 years of the first period, implying  $E_0 = 55$  GtCO<sub>2</sub>e emitted in this scenario. When compared to the production  $Y_0$  estimated above, this yields a carbon intensity of  $Q_0 = 1.75 \times 10^{-4}$  GtCO<sub>2</sub>e/GUSD. Even without any mitigation policy, the world economy has benefitted from a natural reduction of the energy intensity of its global production over the recent decades. According to Clarke et al. (2014), the average rate of decline of the energy intensity has been approximately 0.8% per year between 1970 and 2010, before the implementation of stronger climate policies. This is why I assume in this calibration exercise that the carbon intensity in the second period under the BAU goes down to  $Q_1 = 1.55 \times 10^{-4}$  GtCO<sub>2</sub>e/GUSD in the BAU. This implies an expected total emission of around  $E_1 = 60$  GtCO<sub>2</sub>e in the second period under the BAU.

There exists an intense debate about the half-life of carbon dioxide in the atmosphere, and thus on its rate of natural decay. Dietz and Venmans (2019) convincingly argue that this rate should be set to zero on the basis of the hard sciences discovery (Leduc et al. (2016)) that the change in average temperature relative to preindustrial age is proportional to cumulative CO<sub>2</sub> emissions. This implies a total expected emission net of the natural decay for the European Union over the period 2021-2050 in the BAU around 115 GtCO<sub>2</sub>e.

## 4.3 Carbon budget

The European Union has recently decided to reduce emissions to 55% and 90% respectively in 2030 and 2040 with respect to 1990, when the EU-28 emitted 5.665 GtCO<sub>2</sub>e. Net-zero should be attained in 2050. Given that the Union emitted 3.647 GtCO<sub>2</sub>e in 2021, assuming linear interpolation within each decades yields a total carbon budget of 49 GtCO<sub>2</sub>e for period 2021-2050, compared to the 115 GtCO<sub>2</sub>e emitted in the BAU. This carbon budget is aligned on the share of the world carbon budget associated to the 2°C, assuming a constant per capita allocation.

Good and bad climate/scientific news are expected to be revealed within the first period which will affect this carbon budget. For example, the climate sensitivity parameter may be revised downwards or upwards in the future, as it has been in the past. This will affect the socially desirable level of the EU carbon budget. This is why I assume in the benchmark calibration that, seen from the decision date in 2021, the EU carbon budget  $T$  is normally distributed with mean  $\mu_T = 49$  and standard deviation  $\sigma_T = 5$ .

## 4.4 Abatement costs

The calibration of abatement costs is critically important in this model. Ideally, one would like to estimate the merit order of the myriad of actions of decarbonization in Europe. Over the last two decades, our economic profession failed to invest much in the micro-estimation

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in estimates of global output for the period 2010-2050. Experts were requested to estimate the average annual growth rate of the period. The resulting estimates were best fit using a normal distribution, with a mean of 2.59% and a standard deviation of 1.13%. This yields a standard deviation of  $\log(Y_{2050}/Y_{2010})$  equaling  $40 \times 1.13\% = 45.2\%$ . This should be compared to the standard deviation of  $\sqrt{40} \times 6.12\% = 38.7\%$  for this variable in my model. Thus, I assume long run output uncertainty whose intensity is similar to the sample in Nordhaus (2018).

of the MAC curve, with the consequence that we don't have much information about how to calibrate this function. I assume that the abatement cost function is quadratic:

$$A_t(X_t) = a_t X_t + \frac{1}{2} b_t X_t^2. \quad (22)$$

Parameter  $a_t$  measures the MAC in the BAU scenario ( $X_t = 0$ ). For the first period, I estimate it by the price of carbon permits observed in early 2021 on the EU-ETS market, around  $a_0 = 40$  GUSD/GtCO<sub>2</sub>e. The least-cost technology to decarbonize the EU economy lies in the energy sector and renewable electricity. In EPIC (2021), Michael Greenstone reports that between 2009 and 2021, the direct costs of producing electricity have fallen by nearly 86% for solar photovoltaic and 49% for wind. Taking the mean reduction of 67.5%, I assume that  $a_1$  is normally distributed with mean  $\mu_{a1} = 9$  and standard deviation  $\sigma_{a1} = 5$  (in GUSD/GtCO<sub>2</sub>e).

I calibrate the  $b$  parameter, which is the slope of the MAC curve by using estimations of the MAC of the backstop technology, which I assume to be Direct Air Capture with CO<sub>2</sub> transport and Storage (DACCS). DACCS offers a scalable, and permanent carbon removal technology. The current cost of DACCS is estimated at around 800 GUSD/GtCO<sub>2</sub>. I assume that this would be the MAC in the first period if one would decide to go to net-zero immediately. This means that  $a_0 + b_0 E_0 = 800$ , or  $b_0 = 13.8$ . Sievert et al. (2024) estimate the potential of reduction in this MAC of DACCS in the long term. Under their analysis, the best potential technology is associated to a high-temperature liquid solvent process. Their estimation provides a 90% interval of confidence of [226, 544] GUSD/GtCO<sub>2</sub> for the long-term MAC of that technology. This is compatible with assuming that  $b_1$  is normally distributed with mean  $\mu_{b1} = 6.4$  and standard deviation  $\sigma_{b1} = 1.35$  (in GUSD/GtCO<sub>2</sub>e<sup>2</sup>).

#### 4.5 Optimal abatement strategy in the benchmark calibration

I solved the first-order condition (11) numerically by using the Monte-Carlo method. I draw 300.000 random quadruplets  $(Y_1, a_1, b_1, T)$  to estimate the expectation of the right-hand side of this equality, expressed as a function of  $X_0$ . The optimal solution is described in Table 2. In the first period, it is optimal to abate 12.6 GtCO<sub>2</sub>. Taking account of the BAU annual emissions (early 2021 level) of 3.647 GtCO<sub>2</sub>, this corresponds to a reduction of 50.4% compared to the 1990 baseline. This looks quite aligned with the politically-determined EU target of 55% for 2030. In order to attain this objective, a carbon price of 213.5 USD/tCO<sub>2</sub> is necessary. It is in the range of the 190-2030 USD/tCO<sub>2</sub> recently estimated by the EPA for the social cost of carbon in 2020 and 2030 (EPA (2023)).

The optimal solution yields a specific stochastic consumption and mitigation path from which one can derive the equilibrium interest rate and the systematic risk premium:

$$r_f = \rho - \log \left( \frac{E[u'(C_1)]}{u'(C_0)} \right) \quad (23)$$

$$\pi = - \log \left( \frac{E[C_1 u'(C_1)]}{E[C_1] E[u'(C_1)]} \right). \quad (24)$$

As shown in Table 2, I obtain equilibrium asset prices that are in line with the real interest rate and the systematic risk premium that have been observed in the United States during



parameter	value	description
$\rho$	0.5%	annual rate of pure preference for the present
$\gamma$	3	concavity of utility function
$p$	1.7%	annual probability of a macroeconomic catastrophe
$\mu_{bau}$	2%	mean growth rate of production in a business-as-usual year
$\sigma_{bau}$	2%	volatility of the growth rate of production in a business-as-usual year
$\mu_{cat}$	-35%	mean growth rate of production in a catastrophic year
$\sigma_{cat}$	25%	volatility of the growth rate of production in a catastrophic year
$Y_0$	315,000	production in the first period (in GUSD)
$\delta$	0%	annual rate of natural decay of CO <sub>2</sub> in the atmosphere
$Q_0$	$1.75 \times 10^{-4}$	carbon intensity of production in period 0 (in GtCO <sub>2</sub> e/GUSD)
$Q_1$	$1.55 \times 10^{-4}$	carbon intensity of production in period 0 (in GtCO <sub>2</sub> e/GUSD)
$\mu_T$	49	expected carbon budget (in GtCO <sub>2</sub> e)
$\sigma_T$	5	standard deviation of the carbon budget (in GtCO <sub>2</sub> e)
$a_0$	40	marginal cost of abatement in the BAU, first period (in GUSD/GtCO <sub>2</sub> e)
$\mu_{a1}$	9	expected future marginal abatement cost in BAU (in GUSD/GtCO <sub>2</sub> e)
$\sigma_{a1}$	5	st.dev. of future marginal abatement cost in BAU (in GUSD/GtCO <sub>2</sub> e)
$b_0$	13.8	slope of the marginal abatement cost functions (in GUSD/GtCO <sub>2</sub> e <sup>2</sup> )
$\mu_{b1}$	6.4	expected slope of the marginal abatement cost functions (in GUSD/GtCO <sub>2</sub> e <sup>2</sup> )
$\sigma_{b1}$	1.35	st.dev. of slope of the marginal abatement cost functions (in GUSD/GtCO <sub>2</sub> e <sup>2</sup> )

Table 1: Benchmark calibration of the two-period model.

variable	value	description
$X_0$	12.56	optimal abatement in the first period (in GtCO <sub>2</sub> e)
$E[X_1]$	55.07	optimal expected abatement in the second period (in GtCO <sub>2</sub> e)
$p_0$	213.5	optimal carbon price in the first period (in USD/tCO <sub>2</sub> e)
$E[p_1]$	362.3	optimal expected carbon price in the second period (in USD/tCO <sub>2</sub> e)
$g$	3.52	annualized growth rate of expected carbon price (in %)
$r_f$	1.18	annualized interest rate (in %)
$\pi$	2.11	annualized systematic risk premium (in %)
$\phi$	1.16	OLS estimation of the income-elasticity of the marginal abatement cost
$\Delta V/V$	1.22	Relative welfare loss due to the transition (in %)

Table 2: Description of the optimal solution in the benchmark case. The welfare loss measures the reduction in the constant welfare-equivalent consumption level compared to the no-abatement strategy.

the last century (Kocherlakota (1996)). Notice also that the expected optimal abatement is much larger in the second period than in the first one. This is partly due to the anticipation of a larger expected price of carbon at 362.3 USD/tCO<sub>2</sub> in the second period. In expectation, the annualized growth rate of the carbon price equals 3.52%. This is much larger than the equilibrium interest rate of 1.18%. This is due to the fact that at the optimum, the marginal abatement cost is positively correlated with aggregate consumption, as shown in Figure 3. In fact, the OLS estimation of the income-elasticity of the marginal abatement cost is  $\phi \simeq 1.16$ .

It is noteworthy that equation (6) describing the CCAPM relationship existing between the  $g$ ,  $r_f$  and  $\pi$  through the climate beta  $\phi$  does not hold in this two-period framework with a non-normal distribution of log consumption. Future log consumption is not normal because of the additivity of the abatement cost, and (mostly) because of the risk of macro-catastrophes. This is a reminder that this CCAPM property of a linear relation between the excess expected return of an asset and its CCAPM beta holds only in the gaussian world (Gollier (2016)). Applying this rule in our framework would yield  $g = 1.18 + 1.16 \times 2.11 = 3.62\%$ , which is larger than the optimal  $g = 3.52\%$ .

As observed by Metcalf (2023), Aldy (2017) and Hafstead et al. (2017), carbon price predictability is the most important feature of a climate policy for the business community as it plans long-term investments in line with the energy transition. For example, Metcalf (2023) proposes to fix the annual growth rate of carbon price at 4% (plus inflation) as long as the path of emissions is in line with the objective. However, under uncertainty, the efficient growth rate of carbon price must be uncertain in this model because the resolution of the uncertainty affecting economic growth, green innovations and the carbon budget needs to be translated into a variable carbon price in the second period. In other words, it is not possible to be serious about the carbon budget constraint and, at the same time, to insure all economic agents against changes in the carbon price that is necessary to support the constraint. I represented the distribution of the carbon price  $p_1$  and its annualized growth rate respectively in Figures 4 and 5. The standard deviation of this annualized growth rate is equal to 2.5%. It reflects the uncertainties associated to the price of carbon necessary to satisfy the intertemporal carbon budget constraint. This constraint translates into an uncertain

abatement effort in the second period, as described in Figure 6. Investment decisions in energy transition should take account of these uncertainties. The attractiveness of green investments should come from their expected return rather than from their reduced riskiness, something that cannot be guaranteed under a rigid carbon budget.

The high uncertainty affecting the second period carbon price is also the consequence of the cost-efficiency approach used in this paper. In the alternative cost-benefit approach, the absence of green innovation would be partially compensated by allowing emissions to grow.<sup>10</sup> This is not possible if one takes the carbon budget constraint seriously. It implies that the carbon price has to grow faster under the cost-efficiency approach in this adverse scenario. Ex ante, this means that the carbon price uncertainty is larger.

Under this benchmark calibration, the cost of the transition is mostly postponed to the second period. Indeed, the optimal abatement effort in the first period yields a cost limited to 0.5% of the EU GDP. But in the second period, this abatement cost amounts to 2.6% of EU GDP in expectation, with a standard deviation of 0.9%. Although the representative agent is prudent the future incomes and costs are very uncertain, the expectation of much smaller abatement costs in the future justifies this relatively imprudent climate strategy.

What is the welfare cost of fighting climate change for the next three decades? To address this question, I measure welfare associated to a policy by the constant consumption level that generates the same discounted expected utility generated by that policy. Under the optimal carbon pricing rule, this "constant-equivalent consumption level" is equal to 328,776 GUSD. This should be compared to the constant-equivalent consumption level of 332,847 GUSD that is obtained with the zero ambition strategy, i.e., when  $X_0$  and  $X_1$  are set to zero. This means that fighting climate change has an effect on intertemporal welfare that is equivalent to a permanent reduction of consumption by 1.22%.<sup>11</sup>

#### 4.6 The welfare cost of delaying action

We have seen in the introduction that most calibrations of cost-efficiency IAM models in the literature yield a growth rate of carbon price that is much larger than the interest rate. Because these models assume no uncertainty, they imply a suboptimal allocation of the abatement effort over time, with a lack of effort in the short run, and too much effort in the long run. This may be due to the political command imposed to these calibrations. In this section, I am interested in measuring the welfare cost of this inefficiency. Our findings are summarized in Table 3.

If the EU maintains the price of permits at its early 2021 level (40 USD/tCO<sub>2</sub>) for the next 15 years, it will be forced to increase it to 443 USD/tCO<sub>2</sub> in expectation during the second period, which corresponds to an annual growth rate of 16.03%. This vastly inefficient intertemporal allocation of efforts yields a welfare loss that is equivalent to a permanent reduction of consumption by 1.51%. Compared to the efficient policy, this represents an increase in welfare loss by 29 basis points, from 1.22%. In short, this means that postponing the effort by 15 years has an effect on welfare which is equivalent to a 24% increase in the welfare cost of fighting climate change compared to the efficient policy.

<sup>10</sup>This observation implies that marginal abatement cost and marginal abatement benefits are positively correlated, as explained by Stavins (2019).

<sup>11</sup>Of course, this measure does not take account of the benefits of reduced climate damages borne by future generations.

$p_0$	$E[p_1]$	$g$	$X_0$	$E[X_1]$	Welfare loss
40	442.8	16.03	0.00	67.63	1.51
100	415.0	9.49	4.34	63.29	1.35
150	391.8	6.40	7.96	59.67	1.26
200	368.6	4.08	11.58	56.05	1.23
213.5	362.3	3.52	12.56	55.07	1.22

Table 3: Cost of delaying the abatement effort. The initial price  $p_0$  is arbitrarily selected between the BAU level (40 USD/tCO<sub>2</sub>) and its efficient level (213.5 USD/tCO<sub>2</sub>). Units are as in Table 2.

It is noteworthy that the selection of an initial carbon price of 150 USD/tCO<sub>2</sub>, halfway between the BAU and the optimal carbon prices, yields a growth rate of expected carbon price of 6.4% per year, not far from what IAM models suggest. The welfare loss associated to this less inefficient policy is only 3% larger than when using the efficient policy.

#### 4.7 Risk sensitivity analysis

Table 4 provides some information about the sensitivity of the optimum to the intensity of the exogenous risk of the model. The most interesting comparison to the benchmark is obtained when all sources of risk are switched off. Suppose that all standard deviations are reduced to zero, together with the probability of catastrophe. To preserve the mean growth rate of output, I reduced the mean growth rate to  $\mu_{bau}$  to 1.37%. In this risk-free economy, we know that the efficient growth rate of carbon prices must be equal to the interest rate (since the rate of natural decay is zero). No risk premium should be included. However, the absence of uncertainty switches off the precautionary motive to reduce the interest rate,<sup>12</sup> which goes up to  $r_f = 4.16\% = g$  in this context. The large discount rate implies that very little effort is made in the first period, with a low initial carbon price. In the benchmark calibration, the reason for why much of the mitigation effort is postponed comes from the fact that the MAC is positively correlated with aggregate consumption. In this alternative context with no risk, there is an even stronger argument for delaying the effort, namely, the absence of any precautionary motive to invest. This "no risk" calibration is useful to relate my results to the IAM literature under certainty. In order to generate similar temporal abatement efforts and carbon prices in expectation, the sources of risk considered in my benchmark calibration can be approximately substituted by imposing a larger interest rate in standard IAM models, impatience replacing risk management as a justification to do little in the first period.

In the fourth column entitled "no catastrophe", I have solved the model by using the benchmark calibration except for the probability of catastrophe  $p$  that has been switched to zero, combined with a reduction of  $\mu_{bau}$  to 1.37% in order to leave  $E[Y_1]$  unchanged. In this calibration, the only source of risk on production growth comes from the gaussian noise  $\sigma_{bau}$ . This absence of macro catastrophe has the effect to raise the interest rate and to reduce the systematic risk premium to unrealistic levels. This observation justifies our choice of

<sup>12</sup>This precautionary motive to reduce the discount rate is best illustrated in the so-called "extended Ramsey rule" (equation (29) of the Appendix). For more details, see for example Gollier (2016).

variable	benchmark	no risk	no cata	no macro risk	no tech risk	no budget risk
$X_0$	12.6	11.1	11.3	11.2	12.6	12.5
$E[X_1]$	55.1	55.0	55.0	55.0	55.1	55.1
$p_0$	214	194	196	195	213	213
$E[p_1]$	362	362	362	361	362	362
$g$	3.52	4.16	4.08	4.12	3.53	3.53
$r_f$	1.18	4.16	3.99	4.15	1.09	1.01
$\pi$	2.11	–	0.11	0.00	2.19	2.25
$\phi$	1.22	–	0.81	-29.60	1.18	1.14

Table 4: Risk sensitivity analysis. The "no risk" context is obtained by equalizing all standard deviations to zero, by reducing the probability of catastrophe to zero, and by replacing  $\mu_{bau}$  by 1.37% to preserve the expected growth rate of production as in the benchmark. The "no catastrophe" context is obtained by shifting the probability of catastrophe  $p$  to zero, and by reducing the trend of growth to  $\mu_{bau}$  to 1.37%. The "no macro risk" context combines these changes with the shift of the volatility  $\sigma_{bau}$  to zero. In the "no tech risk" context, I switched  $\sigma_{a_1}$  and  $\sigma_{b_1}$  to zero compared to the benchmark. In the "no budget risk" case, I reduced  $\sigma_T$  to zero compared to the benchmark. Units are as in Table 2.

introducing macroeconomic catastrophes à la Barro in our calibration. In the next column, I fully eliminate the risk on  $Y_1$ . The consequence of eliminating the macro risk is to generate a negative climate beta ( $\phi = -29.6$ ), as suggested by our theoretical results. However, the technological uncertainty surrounding the energy transition is too small to generate a sizeable aggregate risk, and therefore a sizeable aggregate risk premium. In fact, we obtain  $\pi = 0.002\%$  in this case. This implies that, in spite of the large absolute value of the climate beta, the growth rate of carbon price is only marginally smaller than the equilibrium interest rate.

In the last two columns of Table 4, I document the results of simulations in which risks on technological progress or on the carbon budget  $T$  are switched off. Because these effects are relatively small, these results suggest that the main argument for a departure of the Hotelling's rule  $g = \delta + r_f$  comes from the macroeconomic uncertainty, not from technological risks or from carbon budget risks. These last two columns also tell us that the risk associated to climate change tends to reduce the equilibrium interest rate and the socially desirable risk-free discount rate. Indeed, recognizing that the emergence of mature green technologies and the level of the carbon budget are uncertain implies a lower interest rate, because of the enhanced precautionary motive to invest. It also implies a larger systematic risk premium. It can therefore contribute to the resolution of the classical asset pricing puzzles.

#### 4.8 Parameter sensitivity analysis

I now turn to the sensitivity analysis related to the non-risk parameters of the model. In Table 5, I first double the expected carbon budget from  $\mu_T = 49$  to 98 GtCO<sub>2</sub>e. This increases the

variable	benchmark	doubling carbon budget	better future least-cost tech	better future DACCS	Nordhaus	Stern
$X_0$	12.6	1.00	12.4	6.3	12.7	16.4
$E[X_1]$	55.1	18.7	55.2	61.3	54.9	51.2
$p_0$	214	54	211	127	215	266
$E[p_1]$	362	129	359	205	361	338
$g$	3.52	5.83	3.53	3.22	3.45	1.59
$r_f$	1.18	1.27	1.04	1.09	2.77	1.16
$\pi$	2.11	2.30	2.22	2.27	0.62	0.37
$\phi$	1.16	4.25	1.18	0.82	1.16	1.33

Table 5: Parameter sensitivity analysis. In the "doubling carbon budget" scenario, I increase the expected carbon budget from  $\mu_T = 49$  to 98 GtCO<sub>2</sub>e. In the "better future least-cost tech" scenario, I reduce by 50% the expected cost of the least-cost technologies in the second period, so that  $\mu_{a_1}$  is reduced from 9 to 4.5. In the "better future DACCS" scenario, I reduce by 50% the expected cost of direct air capture in the second period, so that  $\mu_{b_1}$  is reduced from 6.4 to 3.2. I double the curvature coefficient of the abatement cost function to  $b = 3.34$  in the scenario entitled "doubling cost curvature". In "Nordhaus", I increase the rate of pure preference for the present from  $\rho = 0.5\%$  to 1.5%, and I reduce relative risk aversion  $\gamma$  from 3 to 1.45. Finally, in "Stern", I reduce  $\rho$  to 0.1% and  $\gamma$  to 1. Units are as in Table 2.

income-elasticity of the MAC and the efficient growth rate of carbon price. This implies a reduction of the carbon price in the first period by 75%, and a large increase in the expected growth rate of the carbon price. Almost all efforts are postponed to the second period in that case. I also examined the effect of more optimistic green innovations. In the fourth column of Table 5, I document the optimal abatement strategy when the expected cost of renewable electricity is reduced by 50%. This has a very limited effect on the optimal strategy. On the contrary, doubling the expected cost reduction of DACCS has a sizeable effect on carbon prices and on the allocation of the efforts over time. This optimistic belief reduces the optimal effort in the first period by 50%.

The last two columns of Table 5 are related to the Nordhaus-Stern controversy on the discount rate. The benchmark calibration was made compatible with observed asset prices. Following Barro (2006), I introduced macro catastrophes and I assumed a constant relative risk aversion equaling  $\gamma = 3$ . I also used a rate of pure preference for the present equaling  $\rho = 0.5\%$ . These two coefficients are subject to an intense debate in our profession. Nordhaus (2018) uses a larger  $\rho = 1.5\%$ , and a smaller  $\gamma = 1.45$ . It is a surprise that this calibration yields an optimal abatement strategy that is almost equivalent to our benchmark calibration. The interest rate is too large, but the reduced risk premium compensates this effect, so that the expected growth rate of carbon price at  $g = 3.45\%$  is almost equal to its benchmark level. Stern (2007) uses a smaller  $\rho = 0.1\%$  and a smaller  $\gamma = 1$ . It yields an equilibrium interest rate very similar than in my benchmark calibration, but also a very small aggregate risk premium  $\pi = 0.37\%$ . This implies an optimal growth rate of carbon price that is more than 50% smaller than in the benchmark calibration, implying a larger first-period carbon

price and a larger first-period abatement effort. We recover here the traditional conflict between Nordhaus and Stern recommendations about the speed at which our economies should decarbonize.<sup>13</sup>

## 5 Conclusion

The future social and private benefits of most investments in renewable energy are uncertain by nature. One of their crucial social benefits is the reduction in emissions of CO<sub>2</sub>, whose anticipated future value should be a key driver to induce market players to invest. Under a fixed intertemporal carbon budget constraint, the carbon price should send the right signal about the evolution of both the scarcity of emission permits and the cost of abatement efforts. For the sake of efficiency, it needs to be sensitive to macroeconomic and technological shocks. I have shown in this paper that, along the optimal mitigation path, the marginal abatement cost is positively correlated with aggregate consumption. To be more precise, I have shown that the MAC has a CCAPM-beta close to 1. Abating early generates a social benefit – the future MAC saved – whose risk profile is not different from a claim on aggregate consumption. It should be priced accordingly, with a discount rate equaling the interest rate plus the aggregate risk premium. This means that, along the optimal mitigation path, the expected MAC and carbon price should grow at that rate. This provides the right compensation of early green technology adopters for the risk they take.

The renewable industry has often lobbied to obtain guarantees about future carbon prices, with the claim that it is a necessary condition for a rapid energy transition. This request is not substantiated. Rather than offering guarantees about future prices – a policy which would limit the quality of future price signals, one should offer them a larger expected rate of return for their investments in renewable energies, as a compensation for the risk that these investments yields. Again, this takes the form of planning a larger growth rate of expected carbon prices. Very risk-averse green investors should look for financial products that could hedge the carbon price volatility at market price. Of course, these recommendations rely on a credible institution able to implement a carbon pricing mechanism that is dual to our collective climate ambition.

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<sup>13</sup>Under a cost-benefit approach, Nordhaus would also assume a larger carbon budget than Stern. This is not taken into account in the discussion based on an exogenous carbon budget.

## References

- ALDY, J. (2017): “Designing and updating a U.S. carbon tax in an uncertain world,” *Harvard Environmental Law Review*, 41, 28–40.
- BARRO, R. (2006): “Rare Disasters and Asset Markets in the Twentieth Century,” *Journal of Finance*, 121, 823–866.
- BAUMSTARK, L. AND C. GOLLIER (2010): “La dynamique de la valeur carbone dans un cadre incertain,” TSE Working paper.
- BREEDEN, D. (1979): “An intertemporal asset pricing model with stochastic consumption and investment opportunities,” *Journal of Financial Economics*, 7, 265–296.
- CHAKRAVORTY, U., B. MAGNÉ, AND M. MOREAUX (2006): “A Hotelling model with a ceiling on the stock of pollution,” *Journal of Economic Dynamics and Control*, 30, 2875–2904.
- CHAKRAVORTY, U., M. MOREAUX, AND M. TIDBALL (2008): “Ordering the extraction of polluting non renewable resources,” *American Economic Review*, 98, 1128–1144.
- CHRISTENSEN, P., K. GILLINGHAM, AND W. D. NORDHAUS (2018): “Uncertainty in forecasts of long-run economic growth,” *Proceedings of the National Academy of Science*, 115, 5409–5414.
- CLARKE, L., K. JIANG, K. AKIMOTO, B. M., G. BLANFORD, K. FISHER-VANDEN, J.-C. HOURCADE, V. KREY, E. KRIEGLER, A. LOSCHEL, D. MCCOLLUM, S. PALTSEV, S. ROSE, P. R. SHUKLA, M. TAVONI, B. C. C. VAN DER ZWAAN, AND D. VAN VUUREN (2014): “Assessing transformation pathways,” in *Climate Change 2014: Mitigation of Climate Change. Contribution of Working Group III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change*, ed. by O. Edenhofer, R. Pichs-Madruga, Y. Sokona, E. Farahani, S. Kadner, K. Seyboth, A. Adler, I. Baum, S. Brunner, P. Eickemeier, B. Kriemann, J. Savolainen, S. Schlomer, C. von Stechow, T. Zwickel, and J. Minx, Cambridge, UK and New York, NY, USA: Cambridge University Press.
- COCHRANE, J. (2017): “Macro-finance,” *Review of Finance*, 945–985.
- DIETZ, S., C. GOLLIER, AND L. KESSLER (2018): “The climate beta,” *Journal of Environmental Economics and Management*, 87, 258–274.
- DIETZ, S. AND F. VENMANS (2019): “Cumulative carbon emissions and economic policy: In search of general principles,” *Journal of Environmental Economics and Management*, 96, 108–129.
- DRÈZE, J. AND F. MODIGLIANI (1972): “Consumption decisions under uncertainty,” *Journal of Economic Theory*, 5, 308–335.
- EDENHOFER, O., K. LESSMAN, AND I. TAHRI (2024): “Asset pricing and the carbon beta of externalities,” *Journal of Environmental Economics and Management*, 125.



- EMMERLING, J., L. DROUET, K.-I. VAN DER WIJST, D. VAN VUUREN, V. BOSETTI, AND M. TAVONI (2019): “The role of the discount rate for emission pathways and negative emissions,” *Environmental Research Letters*, 14.
- EPA (2023): “Report on the Social Cost of Greenhouse Gases: Estimates Incorporating Recent Scientific Advances,” Tech. rep., U.S. Environmental Protection Agency.
- EPIC (2021): “U.S. energy and climate roadmap. Evidence-based policies for effective action,” Tech. rep., University of Chicago.
- GAUDET, G. AND P. HOWITT (1989): “A note on uncertainty and the Hotelling rule,” *Journal of Environmental Economics and Management*, 16, 80–86.
- GAUDET, G. AND A. KHADR (1991): “The evolution of natural resource prices under stochastic investment opportunities: An intertemporal asset-pricing approach,” *International Economic Review*, 32, 441–455.
- GOLLIER, C. (2001): *The economics of risk and time*, MIT Press.
- (2016): “Evaluation of long-dated assets : The role of parameter uncertainty,” *Journal of Monetary Economics*, 84, 66–83.
- GOLLIER, C. AND J. TIROLE (2015): “Negotiating effective institutions against climate change,” *Economics of Energy and Environmental Policy*, 4, 5–27.
- HAFSTEAD, M., G. METCALF, AND C. ROBERTON (2017): “Adding Quantity Certainty to a Carbon Tax: The Role of a Tax Adjustment Mechanism for Policy Pre-Commitment,” *Harvard Environmental Law Review*, 41, 41–57.
- HARSTAD, B. (2019): “Technology and time inconsistency,” *Journal of Political Economy*, forthcoming.
- HOTELLING, H. (1931): “The economics of exhaustible resources,” *Journal of Political Economy*, 39, 137–175.
- IAWG (2016): “Interagency Working Group on the Social Cost of Greenhouse Gases: Technical update of the social cost of carbon for regulatory impact analysis under executive order 12866,” Tech. rep., Washington: Executive Office of the President, Available: <https://obamawhitehouse.archives.gov/sites/default/files/omb/assets/inforeg/technical-update-social-cost-of-carbon-for-regulator-impact-analysis.pdf>.
- KIMBALL, M. (1990): “Precautionary saving in the small and in the large,” *Econometrica*, 58, 53–73.
- KOCHERLAKOTA, N. (1996): “The equity premium: It’s still a puzzle,” *Journal of Economic Literature*, 34, 42–71.
- LAFFONT, J. AND J. TIROLE (1996): “Pollution Permits and Compliance Strategies,” *Journal of Public Economics*, 62, 85–125.

- LEDUC, M., H. MATTHEWS, AND R. ELIA (2016): “Regional estimates of transient climate response to cumulative CO2 emissions,” *Nature Climate Change*, 6, 10.1038.
- LELAND, H. (1968): “Savings and uncertainty: The precautionary demand for savings,” *Quarterly Journal of Economics*, 45, 621–636.
- LEMOINE, D. AND I. RUDIK (2017): “Steering the Climate System: Using Inertia to Lower the Cost of Policy,” *American Economic Review*, 107, 2947–57.
- LUCAS, R. (1978): “Asset prices in an exchange economy,” *Econometrica*, 46, 1429–46.
- MARTIN, I. (2013): “Consumption-based asset pricing with higher cumulants,” *Review of Economic Studies*, 80, 745–773.
- MEHRA, R. AND E. C. PRESCOTT (1985): “The Equity Premium: A Puzzle,” *Journal of Monetary Economics*, 15, 145–161.
- METCALF, G. (2023): “An emissions assurance mechanism: Adding environmental certainty to a carbon tax,” *Review of Environmental Economics and Policy*, 14, 114–130.
- NORDHAUS, W. (1982): “How Fast Should We Graze the Global Commons?” *The American Economic Review*, 72, 242–246.
- NORDHAUS, W. D. (2018): “Productions and uncertainties about climate change in an era of minimal climate policies,” *American Economic Journal: Economic Policy*, 10, 333–360.
- OLIJSLAGERS, S., F. VAN DER PLOEG, AND S. VAN WIJNBERGEN (2023): “On current and future carbon prices in a risky world,” *Journal of Economic Dynamics and Control*, 146, 104569.
- PINDYCK, R. (1978): “The optimal exploration and production of nonrenewable resources,” *Journal of Political Economy*, 86, 841–861.
- (1980): “Uncertainty and exhaustible resource markets,” *Journal of Political Economy*, 88, 1203–1225.
- (2013): “Climate change policy: What do the models tell us?” *Journal of Economic Literature*, 51, 860–872.
- ROTHSCHILD, M. AND J. STIGLITZ (1970): “Increasing Risk I: A definition,” *Journal of Economic Theory*, 2, 225–243.
- RUBINSTEIN, M. (1976): “The valuation of uncertain income streams and the pricing of options,” *The Bell Journal of Economics*, 7, 407–425.
- SCHUBERT, K. (2008): “La valeur carbone: Niveau initial et profil temporel optimaux,” Paris School of Economics.
- SIEVERT, K., T. SCHMIDT, AND B. STEFFEN (2024): “Considering technology characteristics to project future costs of direct air capture,” *Joule*, 8, 979–999.

- SLADE, M. AND H. THILLE (1997): “Hotelling confronts CAPM: A test of the theory of exhaustible resources,” *The Canadian Journal of Economics*, 30, 685–708.
- STAVINS, R. (2019): “The future of U.S. carbon-pricing policy,” NBER WP 25912.
- STERN, N. (2007): *The Economics of Climate Change: The Stern Review*, Cambridge University Press.
- VAN DER PLOEG, F. (2018): “The safe carbon budget,” *Climatic Change*, 147, 47–59.
- WEIL, P. (1989): “The equity premium puzzle and the risk-free rate puzzle,” *Journal of Monetary Economics*, 24, 401–421.
- WEITZMAN, M. L. (1974): “Prices vs. quantities,” *Review of Economic Studies*, 41, 477–491.

## Appendix: Proof of Proposition 2

Let  $r_t = t^{-1} \log(E[B_t]/I_0)$  denote the expected return of an asset whose current price is  $I_0$  and future benefit at date  $t$  is  $B_t$ . Using equation (1), the optimality condition can be written as follows:

$$e^{-r_t t} = e^{-\rho t} \frac{E[B_t u'(C_t)]}{u'(C_0) E[B_t]}. \quad (25)$$

Using equation (25), the risk-adjusted discount rate  $r_{ct}$  to discount a claim on aggregate consumption must satisfy the following efficiency condition:

$$\exp(-r_{ct} t) = \exp(-\rho t) \frac{E[C_t u'(C_t)]}{u'(C_0) E[C_t]}. \quad (26)$$

The systematic risk premium  $\pi_t$  is the extra expected rate of return of a claim on aggregate consumption over the interest rate that must compensate agents who accept to bear the macroeconomic risk:

$$\pi_t = r_{ct} - r_{ft}. \quad (27)$$

Under the two assumptions of the proposition, equation (2) implies that

$$1 = e^{-(\rho+\delta)t} E \left[ \frac{A'_t u'(C_t)}{A'_0 u'(C_0)} \right] = e^{-(\rho+\delta)t} E [\exp(a'_t - \gamma c_t)].$$

Notice that our assumptions implies that  $a'_t - \gamma c_t$  is normally distributed with mean  $\mu_x - \gamma \mu_c$  and variance  $(1 - \gamma \phi)^2 \sigma_c^2 + \sigma_w^2$ . By Stein's Lemma, the above condition can then be rewritten as follows:

$$1 = \exp \left( \left( -\rho - \delta + \mu_p - \gamma \mu_c + 0.5(\phi - \gamma)^2 \sigma_c^2 + 0.5 \sigma_w^2 \right) t \right),$$

or, equivalently,

$$\mu_p + 0.5 \phi^2 \sigma_c^2 + 0.5 \sigma_w^2 = \delta + \rho + \gamma \mu_c - 0.5 \gamma^2 \sigma_c^2 + \phi \gamma \sigma_c^2. \quad (28)$$

In this economy, the following standard CCAPM formula for the risk-free interest rate can be derived from its implicit definition  $\exp(-r_{ft} t) = \exp(-\rho t) E(C_t/C_0)^{-\gamma}$ , yielding:

$$r_{ft} = r_f = \rho + \gamma \mu_c - 0.5 \gamma^2 \sigma_c^2. \quad (29)$$

The systematic risk premium  $\pi_t$  is given by equation (27). Using Stein's Lemma twice to estimate  $r_{ct}$  given by equation (26) yields the following result:

$$\pi_t = \pi = \gamma \sigma_c^2. \quad (30)$$

Notice also that, using Stein's Lemma again, we have that the expected marginal abatement cost satisfies the following condition:

$$E \frac{A'_t}{A'_0} = E \exp(a'_t) = \exp \left( \left( \mu_p + 0.5 \phi^2 \sigma_c^2 + 0.5 \sigma_w^2 \right) t \right).$$

This implies that the growth rate  $g$  of expected marginal abatement cost is a constant given by

$$g = \frac{dEA'_t/dt}{EA'_t} = \mu_p + 0.5\phi^2\sigma_c^2 + 0.5\sigma_w^2.$$

Because in a decentralized economy, the marginal abatement cost is equal to the price of carbon in all states of nature and at all dates,  $g$  can also be interpreted as the growth rate of expected carbon price. Combining these properties implies that one can rewrite condition (28) as follows:

$$g = \delta + r_f + \phi\pi. \tag{31}$$

This concludes the proof of Proposition 2. ■

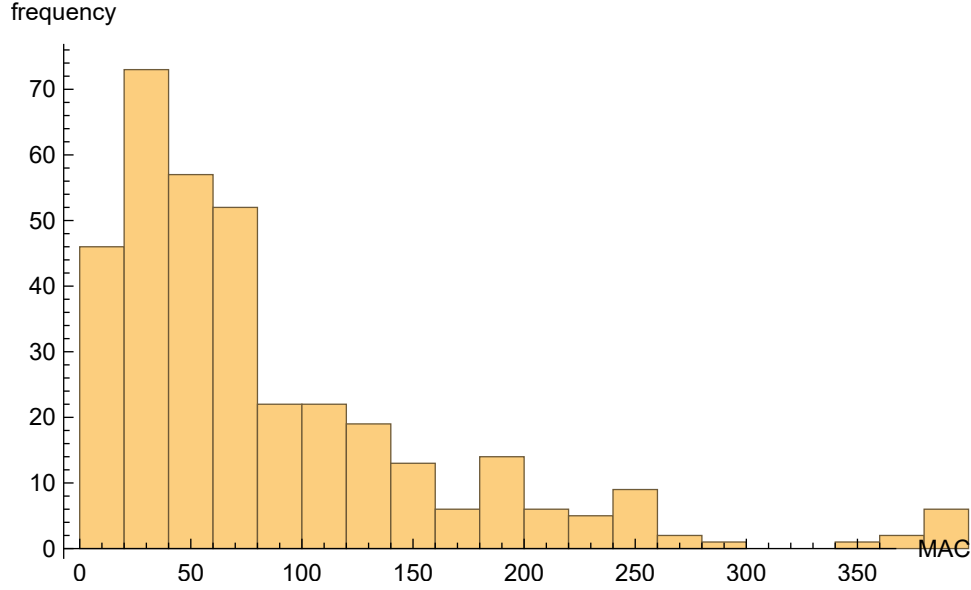


Figure 2: Histogram of the world marginal abatement costs for 2030 extracted from the IPCC database (<https://tntcat.iiasa.ac.at/AR5DB>). I have selected the 374 estimates of carbon prices (in US\$2005/tCO<sub>2</sub>) in 2030 from the IAM models of the database compatible with a target concentration of 450ppm.

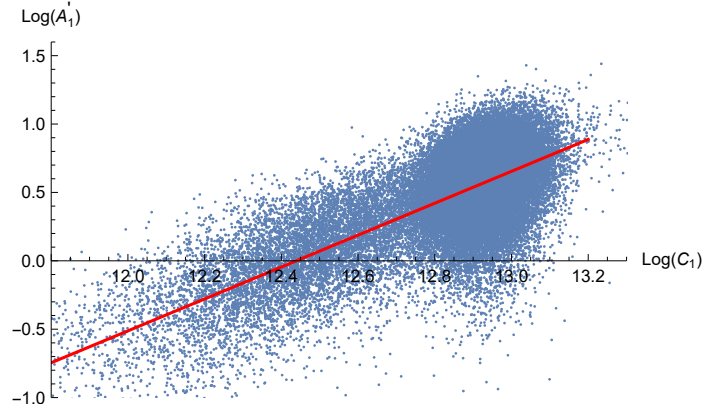


Figure 3: Monte-Carlo simulation under the benchmark case. For the sake of readability of the figure, I limited the simulation to 50.000 draws of the quadruplets  $(Y_1, a_a, b_1, T)$  to estimate the optimal abatement strategy. The figure illustrates the positive statistical relation between log consumption growth and the log marginal abatement costs (and thus log carbon price) in the second period. The red curve depicts the OLS estimation in log-log.

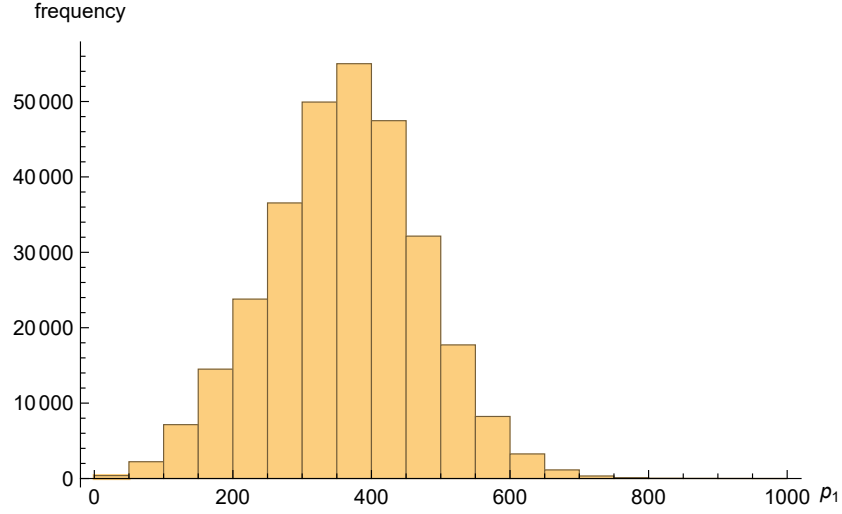


Figure 4: Empirical probability distribution of the carbon price  $p_1$  (in US\$/tCO<sub>2</sub>e) under the optimal abatement strategy in the benchmark calibration of the two-period model. The Monte-Carlo simulation uses a sample of 300.000 draws of the quadruplet  $(Y_1, a_1, b_1, T)$ .

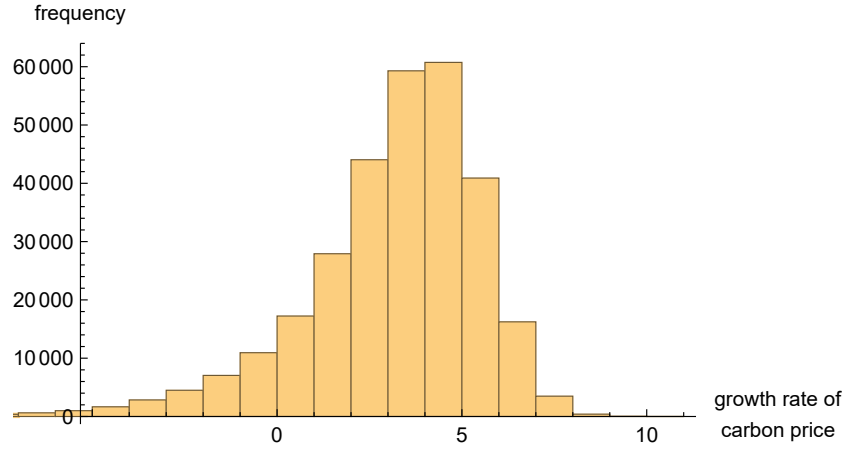


Figure 5: Empirical probability distribution of the annualized growth rate of carbon price under the optimal abatement strategy in the benchmark calibration of the two-period model. The growth rate is in percent per year. The mean growth rate is 3.52% and the standard deviation is equal to 2.5%.

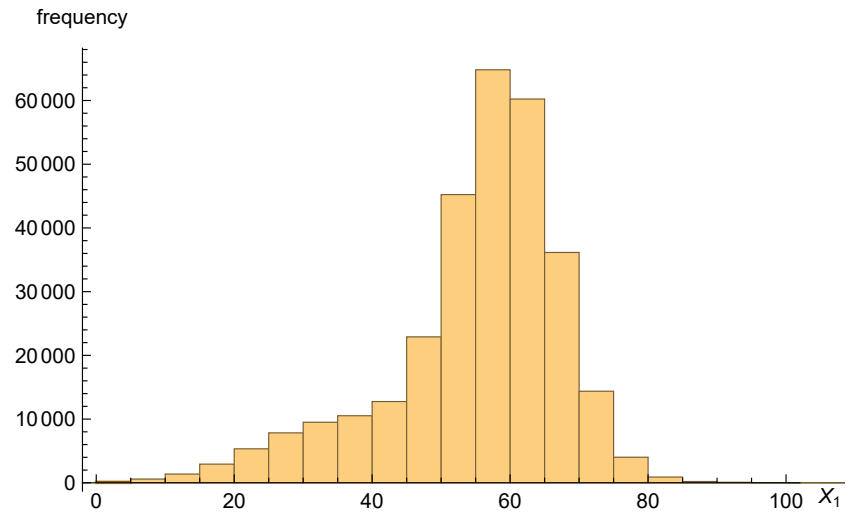


Figure 6: Empirical probability distribution of the abatement effort  $X_1$  (in GtCO<sub>2</sub>e) under the optimal abatement strategy in the benchmark calibration of the two-period model.