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Horizontal Mergers Between Multi-Sided Platforms: Insights from Cournot Competition*

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Abstract

This paper discusses the literature on horizontal mergers between multi-sided platforms and argues that the Cournot model can provide useful insights into the welfare effects of such mergers. To illustrate those insights, we develop a simple model in which two-sided platforms offer a homogeneous service and compete *à la* Cournot, and derive the effects of "average-marginal-cost-preserving" mergers on consumers on both sides of the market. We conclude with a discussion of several research avenues that could be explored to understand better the impact of horizontal mergers between multi-sided platforms.

Keywords: Mergers, Multi-Sided Platforms, Cournot Competition
JEL Codes: L41, D43.

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1 Introduction

It is widely recognized that traditional merger analysis cannot be directly applied to multi-sided platform industries.[1] There are at least two reasons for that. First, a horizontal merger between multi-sided platforms not only affects the level of prices but also their structure. In particular, such a merger may lead to a lower price on one or more sides of the market even in the absence of cost-related efficiency gains.[2] Second, a merger between two or more platforms may generate network-related efficiencies that are absent in traditional markets. For instance, if the merger leads to the creation of a single platform or if the merging platforms are maintained but made interoperable, agents on each side of the market may end up interacting with more agents on the other side, which would affect positively their surplus if the relevant network effects are positive.

Several merger cases have involved multi-sided platforms.[3] Interestingly, the way competition authorities have addressed the competitive effects of mergers in these markets has been varying over time and across jurisdictions. While the multi-sided nature of the relevant market was overlooked in some cases, it was crucial to the decision in other cases.

For instance, in 2004, when analyzing the merger between the two weekly local newspapers Archant and Independent News and Media,[4] the UK Competition Commission focused on the impact of the merger on advertisers and ignored its effect on readers. In other media merger cases involving TV channels, such as BSkyb's acquisition of 24% of KirchpayTV[5] in 2000 and News Corporation's acquisition of 25% of Premiere[6] in 2008, the effect on the advertising side and the existence of network externalities seem to have been neglected to a large extent.[7]

In other cases, however, competition authorities have analyzed the effect on all the sides of the market when assessing mergers between platforms. An example is the decision by the US Department of Justice and the Federal Communications Commission to clear, in 2008, the merger between Sirius and XM, the only two US satellite digital radio services. As discussed by Belleflamme and Peitz (2015), the merger would probably have been blocked if the two-sided nature of the market had not been recognized since it would have been a '2-to-1' merger. However, by taking into account the advertisers' side, authorities widened the product market (by including other kinds of broadcast) and cleared the merger. The two-sidedness of the market was also critical in the decision of the European Commission to clear, in 2007, the

[1] See, for example, Wright (2004), Evans and Noel (2008), Evans and Schmalensee (2013) and Weyl and White (2014).

[2] See e.g. Chandra and Collard-Wexler (2009) and Leonello (2010).

[3] For a recent discussion of merger cases in multi-sided markets, see, for instance, Filistrucchi (2017).

[4] UK Competition Commission, "A report on the acquisition by Archant Limited of the London Newspapers of Independent News and Media Limited," 2004.

[5] Case No COMP/JV.37-BSKYB/KirchPayTV.

[6] Case No COMP/M.5121-News Corp/Premiere.

[7] See Filistrucchi et al. (2014) and Foros et al. (2015) for a discussion of these and other merger cases in which the multi-sidedness of the market was overlooked by competition authorities.

merger between Travelport and Worldspan.[8] The Commission claimed that this merger would not be detrimental to agents on one side of the market (travel agents) due to the need of the platforms to build a sufficiently large network to attract agents on the other side (travel service providers).[9]

Note that in the latter two cases competition authorities did not (at least explicitly) account for the potential efficiencies stemming from an increase in indirect network effects after the merger. These efficiencies played, however, a key role in the Netherlands Competition Authority's assessment of the merger between the (only) two Dutch yellow pages directories, European Directories and Truvo.[10] In particular, the merger was approved on the basis that advertisers and users would benefit from using a larger platform. This makes this case a "milestone" in the assessment of mergers between multi-sided platforms (Camesasca et al., 2009).

Despite the existence of many merger cases involving competing multi-sided platforms, the literature on such mergers remains scarce and competition authorities still lack clear guidance as to how they should be assessed. In this paper we discuss this literature and argue that the Cournot model can help address some of the challenges it faces.

To illustrate the insights that can be drawn from the Cournot model regarding mergers between multi-sided platforms, we develop a simple model in which K (≥ 2) two-sided platforms offer a homogeneous service and compete *à la* Cournot. We derive the effects of *average-marginal-cost-preserving (AMCP) mergers*, i.e., mergers that do not affect the industry-wide average marginal cost on any side of the market.[11] Considering first general demand functions, we show that the comparison between the pre-merger "externality-adjusted price" on each side of the market and the average marginal cost on that side plays a key role in determining the effect of a merger on consumers.[12] When both externality-adjusted prices are above (below) the corresponding average marginal costs, a merger harms (benefits) consumers on both sides. We then restrict attention to linear demand functions and establish that a merger harms consumers on both sides if (total) network effects are small, benefits consumers on one side and harms consumers on the other side if network effects are intermediate, and benefits consumers on both sides if network effects are large.

The remainder of the paper is organized as follows. In Section 2 we summarize the main results of the existing literature on horizontal mergers between multi-sided platforms. In Section 3 we discuss the Cournot model in multi-sided markets and explain why it can provide novel insights into the impact of mergers between multi-sided platforms. In Section 4 we lay out our model and derive predictions regarding the effect of a merger between two-sided platforms on

[8] Case No COMP/M.4523-Travelport/Worldspan.

[9] See paragraph 76 of the decision and Belleflamme and Peitz (2015, p. 644).

[10] Case 6246 European Directories-Truvo Nederland.

[11] Note that AMCP mergers include as a special case any merger in a symmetric industry in the absence of cost-related efficiency gains.

[12] The externality-adjusted price is defined as the price net of the value of the network externality received by the consumer.

consumers (on both sides of the market). In Section 5 we conclude with a discussion of avenues for future research.

2 A progress report on the existing literature

In a quite specific but provocative paper, Chandra and Collard-Wexler (2009) show that, in a two-sided market, even mergers to a monopoly may not lead to higher prices on either side of the market. Their model features two newspapers and two advertisers of differentiated products, and assumes that consumer preferences for newspapers and products are correlated. Marginal readers are less valuable to advertisers than captive consumers and some of them may be unprofitable to the newspaper because the advertising revenue they generate is insufficient to cover the subsidy they receive when buying the newspaper. However, a newspaper cannot exclude these readers because it is unable to engage in price discrimination. In a duopoly equilibrium, a small decrease in the price of a newspaper would attract the least valuable consumers away from the rival newspaper and, therefore, may *increase* the rival's profit. A merger makes platforms internalize this externality, which explains why it may lead to a decrease in newspaper prices.

In a similar vein, Leonello (2010) analyzes a merger to monopoly between two platforms located at the extreme points of a Hotelling line on both sides of the market. She finds that merging firms may have incentives to lower their prices on at least one side of the market if indirect network externalities are strong enough. Baranes et al. (2016) extend her analysis by considering four platforms equidistantly located on a Salop circle. Assuming linear externalities, they show that mergers between adjacent platforms may lead to lower prices if externalities are strong enough. By contrast, using a model with possibly non-linear externalities, and where platform differentiation is driven by a random utility that makes all platforms equidistant to each other, Tan and Zhou (2017) conclude that a merged entity always has incentives to raise prices (in the absence of cost-related efficiency gains). Both Baranes et al. (2016) and Tan and Zhou (2017) assume full market coverage, which is questionable for merger analysis since aggregate quantities are then fixed.

The importance of free media has motivated a line of research on multi-sided platforms that are solely financed by advertising.[13] Particular attention has been given to the impact of a merger on ad volumes and per viewer ad prices. The earlier literature concludes that, by reducing competition for viewers, a merger increases ad volumes and decreases per viewer ad prices (Anderson and Coate, 2005). Subsequent work by Anderson et al. (2012) and Anderson et al. (2018) shows that advertising congestion or multi-homing by viewers can reverse this finding. If viewers multi-home, platforms lose their monopoly position in delivering their audience to advertisers. As a result, a merger leads on the advertising side to the usual market power

[13] See e.g. Foros et al. (2015).

effect by which a monopolist reduces volume to increase price. In the presence of advertising congestion across platforms (for example, if viewers mix between platforms and have limited attention to ads), ad volume in a platform exerts a negative externality on other platforms that is internalized if a merger occurs. In this case, a merger may benefit consumers by reducing ad volume. Liu et al. (2004) reach a similar conclusion regarding the impact of a merger on product characteristics. They show that an industry with a larger number of competitors may provide content of lower quality, which may be detrimental to viewers.[14] Finally, Anderson and Peitz (2015) rely on an aggregative game approach to extend the analysis of competition in a media industry to an arbitrary number of platforms, and find that in the case of free media, a merger harms consumers but may or may not harm advertisers.

As mentioned earlier, the tools of traditional merger analysis need to be adapted before being used in multi-sided markets. Affeldt et al. (2013) take a first step in this direction by proposing a way of adapting the Upward Pricing Pressure (UPP) measure (Farrell and Shapiro, 2010) to two-sided markets and apply it to a hypothetical merger in the Dutch newspaper market. Cosnita-Langlais et al. (2018) suggest an extended measure of UPP in two-sided markets that incorporates feedback effects[15] and show that it can lead to a merger assessment that is qualitatively different from the one obtained under Affeldt et al. (2013)'s measure.

Empirical evidence on the effects of media mergers suggests that ad prices increase while ad volumes may increase or decrease following a merger.[16] Jeziorski (2014) estimates a structural supply-and-demand model using data from the 1996-2006 merger wave in the US free radio industry, and concludes that the merger wave caused a 11% drop in ad volumes and a 6% increase in ad prices (per viewer).[17] Fan (2010) uses a structural model of the US daily newspaper market and simulates a merger in the Minneapolis market. She concludes that newspaper prices increase and circulation decreases, resulting in welfare losses for readers and advertisers. In addition, she finds significant welfare losses from adjustments in product characteristics (lower quality, lower local news ratio and lower variety). However, the empirical evidence on these adjustments is mixed. Sweeting (2010), for instance, concludes that radio stations become more differentiated after a merger. This conflicting result may reflect a tension between reducing the cost of content and reducing audience cannibalization.

[14] Their analysis is based on the Hotelling model with endogenous horizontal and vertical differentiation.

[15] More precisely, their measure takes into account how a price change on one side of the market may feed back on the optimal price on the other side of the market.

[16] There are several still unpublished papers on this topic. For instance, Brown and Williams (2002) study local free radio markets in the US and find that local concentration increases ad prices. By contrast, Chipty (2007) finds no effect of concentration on ad prices.

[17] This merger wave was triggered by the 1996 Telecommunications Act.

3 Cournot competition and mergers in multi-sided markets

The Cournot model has been a workhorse for traditional merger analysis (see e.g. Farrell and Shapiro, 1990; Nocke and Whinston, 2010, 2013). This model is primarily interpreted as imposing a specific form of firm's *conduct*. More precisely, its key assumption is that each firm anticipates that its rivals will maintain their quantities at a given level when setting its own strategy. An alternative interpretation is that the Cournot model is a reduced-form model for a capacity-then-price game (Kreps and Scheinkman, 1983).

Before discussing Cournot competition among multi-sided platforms, note that an important difference between an oligopoly model in a one-sided market and its counterpart in a multi-sided model is that the latter must not only set assumptions on firms' conduct but also on how demand on one side of the market is affected by what happens on the other side(s) of the market. In particular, the model needs to specify how the feedback effects related to indirect network effects are factored into competition.

In a number of multi-sided markets, a platform's capacity is a key strategic variable. This is the case for instance for nightclubs, shopping centers, and exhibition halls. In such industries, Cournot competition is a natural way of modeling platforms' behavior. Moreover, studying Cournot competition among platforms can be useful even when capacity constraints are less relevant. As mentioned earlier, one may interpret the assumption of Cournot competition on one side of the market as an assumption about platforms' conduct on that side: each platform anticipates that its competitors will respond to a change in its strategy by adjusting their strategies in a way that leaves their demands unchanged.

A more subtle issue is the treatment of feedback effects in multi-sided markets. Under the standard Bertrand oligopoly model - used in a large part of the literature on platforms - firms set publicly observable prices, and the demand on each side is assumed to adjust to any change on the other side(s) (induced by a change in prices). This implicitly amounts to assuming that quantities adjust faster than prices and, therefore, that some form of price rigidity exists.[18] This raises the question of whether alternative models may better fit situations where prices adjust faster. The existing literature features three approaches that can address this issue, one of them being the extension of the Cournot equilibrium concept to platforms. We now discuss the two other approaches and how they relate to the Cournot equilibrium.

White and Weyl (2016) argue that in practice platforms find ways around the users' coordination problem and propose a solution concept capturing this idea. More specifically, they assume that platforms set *insulating tariffs*, i.e., prices that are contingent on the market outcome on the other side in a way that makes sales on one side unrelated to the demand on the other side. In the case of homogeneous network effects - which we focus on in our model - this amounts to insuring users perfectly against any kind of coordination risk.

[18] Alternatively, one could assume that users are able to coordinate on a Nash equilibrium instantaneously, which is arguably a very strong form of rationality.

The concepts of insulated equilibrium and Cournot equilibrium are related as they both address the users' coordination problem. However, they differ in two dimensions. First, when network effects are heterogeneous, the insulating tariff insures only a "representative" consumer against the coordination risk related to the other side's participation, while the Cournot model entails such insurance for all users. Moreover, even when network effects are homogeneous, the two concepts differ in terms of assumed platforms' conduct. The insulated equilibrium uses sophisticated non-linear tariffs to resolve the users' coordination problem under a price-setting conduct assumption. By contrast, in the Cournot model *both* users' expectations and equilibrium prices are derived from the quantity-setting conduct assumption.[19]

A common advantage of both models is that they allow for the analysis of highly competitive platform markets.[20] Which model is best suited for applied analysis depends, at least partly, on which of the following two assumptions is considered as being the least restrictive: the sophistication of contracts that the insulated equilibrium relies on, or the quantity-setting assumption in the Cournot model.

A related solution concept was used by Katz and Shapiro (1985) in their seminal article on competition with (direct) network effects: in their baseline model users form expectations about network sizes and then firms take these expectations as given when they compete. The model is then solved under two assumptions: the first is a rational expectations assumption, i.e. users' expectations are fulfilled in equilibrium, and the second is an assumption of passive beliefs, i.e. users' expectations are not affected by off-equilibrium-path prices. This approach uncouples the formation of demand with network externalities from platforms' conduct and is therefore compatible with different conduct models. For instance, Katz and Shapiro (1985) and de Palma et al. (1999) assume that firms set quantities, while Hurkens and López (2014) consider price competition. While interesting, this approach suffers from a significant limitation when applied to multi-sided markets. As platforms take users' expectations about participation as given, they do not account for the value that a user creates for the other side(s) of the market when setting prices or quantities. Thus, this approach ignores a key driver of the differences between one-sided and multi-sided markets (Rochet and Tirole, 2003, 2006). Relatedly, note that the Cournot model may be viewed as a compromise between this approach and the standard price competition model for multi-sided markets. The reason is that it allows users' expectations about participation on other platforms to be simple (and in particular not affected by off-equilibrium-path prices) while assuming that those expectations are affected by the platforms, which makes the latter internalize the externalities between users on the two sides of the market.

To our knowledge, Katz and Shapiro (1985) are the first to consider the standard Cournot equilibrium concept in a model with network effects. In the appendix of their paper, they

[19] In particular, we may expect the Cournot equilibrium to be less competitive than the insulated equilibrium.

[20] It is well known that equilibrium analysis in standard price competition models raises issues of multiplicity or non-existence when platforms are strongly substitutable from users' perspective (see e.g., Caillaud and Jullien, 2003 and Armstrong, 2006).

study an extension in which firms commit publicly to the quantities they set. The Cournot equilibrium concept has also been used in the literature on multi-sided markets but only in a limited number of papers. An early model of two-sided Cournot markets is provided by Schiff (2003) who considers a setting in which platforms set quantities on both sides of the market and the valuation of indirect network externalities is homogeneous within each side. He finds that a monopoly is socially preferable to a duopoly with incompatible platforms because of larger network effects under the former. Gabszewicz and Wauthy (2014) propose a similar model,[21] but with platforms that are located at the extremes of a Hotelling line, and show that heterogeneity in the valuation of externalities may lead to asymmetric market shares (despite platforms being symmetric *ex ante*).[22]

Finally, a number of papers in the media literature assume that media platforms set quantities on the advertiser side while setting (possibly zero) prices on the other side of the market.[23] For instance, Kind et al. (2007) build a model in which TV stations offer advertising space to advertisers and (free) content to viewers. They show that there is too little advertising when the channels' programs are close substitutes and that the more viewers dislike ads, the more likely it is that social welfare is increasing in the number of channels. Peitz and Valletti (2008) and Crampes et al. (2009) consider similar models but investigate also the case in which platforms charge viewers. This allows them to compare the free-to-air and pay-tv business models. A common feature of these papers is that consumers are assumed to single-home. Anderson et al. (2016) relax this assumption and show that a number of puzzles identified in the previous literature on media economics can be resolved by allowing for multi-homing consumers. A key difference between the approach adopted in those papers and the one we illustrate in the subsequent model is that the agents on the Cournot side of media platforms (i.e. advertisers) typically multi-home, while we consider a setting where agents single-home on both sides of the market. This implies that the nature of competition that we examine is different from the one considered in the media literature.

We argue that the Cournot model can provide useful insights regarding the effects of a merger between platforms on users (on both sides of the market) because of two appealing properties. First, unlike price competition models, the Cournot model generates predictions regarding externality-adjusted prices, which makes it possible to determine *directly* the welfare effects of a merger and, therefore, to sign those effects under a potentially larger set of circumstances than price competition. Second, the platform Cournot model is more tractable than price competition models when it comes to incorporate cost asymmetries in the model or when one wants to consider an industry with more than two platforms. The next section illustrates those advantages through a simple model in the spirit of Katz and Shapiro (1985).

[21] They consider both quantity competition and price competition.

[22] The logic is similar to the one behind the emergence of quality differentiation as a way of relaxing competition.

[23] See the surveys by Anderson and Jullien (2015) and Peitz and Reisinger (2015).

4 A simple model

4.1 Setup

Consider a two-sided market with a finite number of platforms indexed by $k \in \{1, ..., K\}$. The marginal cost of platform k on side $i \in \{1, 2\}$ is constant and denoted by $c_i^k \in \mathbb{R}_+$. On each side of the market, there is a unit mass of consumers. Platforms charge membership fees - which we will refer to as "prices" henceforth - on both sides of the markets. We assume further that platforms are homogeneous, which implies that consumers care only about the mass of consumers on the other side of the platform they join and the price they pay.[24] Consumers single-home and the utility of a consumer on side i of platform k paying a price p_i^k and having access to a mass n_j^k of consumers on the other side is:

$$u_i^k = \tilde{v}_i + \alpha_i n_j^k - p_i^k,$$

where $\alpha_i \in \mathbb{R}$ is known, with $\alpha_1 + \alpha_2 > 0$, and $\tilde{v}_i$ is a random individual stand-alone payoff distributed according to a strictly increasing and twice differentiable cdf $F_i : \mathbb{R} \to [0, 1]$.

Platforms simultaneously set the mass of consumers n_1^k and n_2^k they serve on each side of the market. Then, given the vector of quantities $\boldsymbol{n} \equiv \left(n_i^k\right)_{i=1,2}^{k=1,...,K}$, prices adjust to equate demand and supply on each platform. More precisely, a vector of prices $\boldsymbol{p} \equiv \left(p_i^k\right)_{i=1,2}^{k=1,...,K}$ is determined such that the allocation of consumers across platforms, $\boldsymbol{n}$, is consistent with individual utility maximization by consumers.

We refer to $z_i^k \equiv p_i^k - \alpha_i n_j^k$ as the externality-adjusted price on side i of platform k.[25] Since platforms are homogeneous, consumers on side i only join the platforms with the lowest externality-adjusted price on side i, denoted by $z_i \equiv \min_k\{z_i^k\}$. Therefore, all firms with positive participation have the same externality-adjusted price, z_i (as in Katz and Shapiro, 1985), and the total mass of consumers on side i who join a platform, $N_i \equiv \sum_{k=1}^K n_i^k$, is equal to $1 - F_i(z_i)$.

Let $Z_i : [0, 1] \to \mathbb{R} \cup \{-\infty, +\infty\}$ be the inverse demand function of side i consumers, i.e., let $Z_i(N_i)$ be the externality-adjusted price on side i for which N_i is the mass of consumers who join a platform. Since $Z_i(\cdot)$ is the inverse of $1 - F_i(\cdot)$, it is strictly decreasing and twice differentiable.

Given the quantities chosen by platforms, $\boldsymbol{n}$, prices are uniquely defined for platforms with positive participation and are given by:

$$p_i^k = Z_i(N_i) + \alpha_i n_j^k.$$

[24] This homogeneity assumption is similar to the one made by Katz and Shapiro (1985) in their model of direct network effects. A limitation of this approach is that it does not allow for products to be differentiated other than on the magnitude of network effects (that could be seen as a "quality" attribute).

[25] This is the counterpart in our multi-sided market setting of what Katz and Shapiro (1985) call the "hedonic price".

The resulting profit of platform k is:

$$\Pi^k = Z_1(N_1)\, n_1^k + Z_2(N_2)\, n_2^k + (\alpha_1+\alpha_2)\, n_1^k n_2^k - \left(c_1^k n_1^k + c_2^k n_2^k\right).$$

4.2 Equilibrium analysis

We focus on equilibria in which all platforms have positive participation on both sides. Each platform k maximizes its profit with respect to $\left(n_1^k, n_2^k\right)$. The corresponding first-order conditions are:

$$\begin{cases} Z_1(N_1) + Z_1'(N_1)\, n_1^k = c_1^k - (\alpha_1+\alpha_2)\, n_2^k \\ Z_2(N_2) + Z_2'(N_2)\, n_2^k = c_2^k - (\alpha_1+\alpha_2) n_1^k, \end{cases}$$

These conditions can be interpreted as follows. The marginal revenue on side 1 is given by $Z_1(N_1) + Z_1'(N_1)\, n_1^k + \alpha_1 n_2^k = p_1^k + \frac{\partial p_1^k}{\partial n_1^k} n_1^k$. In equilibrium, this is equated to the marginal "opportunity cost" on side 1, i.e. $c_1^k - \alpha_2 n_2^k$, which accounts for the fact that increasing n_1^k raises the cross-side externality and thus the price on the other side by a factor α_2. A similar reasoning applies to side 2.

We make the following assumption which ensures that the second-order conditions for optimality are satisfied (see Appendix).

Assumption 1. For every $(N_1, N_2) \in [0,1]^2$:

$$\begin{cases} 2Z_1'(N_1) + Z_1''(N_1)\, N_1 < 0 \text{ and } 2Z_2'(N_2) + Z_2''(N_2)\, N_2 < 0 \\ \left[2Z_1'(N_1) + \max\left\{Z_1''(N_1)\, N_1, 0\right\}\right]\left[2Z_2'(N_2) + \max\left\{Z_2''(N_2)\, N_2, 0\right\}\right] > (\alpha_1+\alpha_2)^2 \end{cases}$$

The first condition is standard and states that the one-sided monopoly profit is concave in quantity (see e.g. Novshek, 1985). The second condition imposes an upper bound on the magnitude of total network effects and is reminiscent of similar conditions in the literature on multi-sided markets (see e.g. Armstrong, 2006).

The aggregative nature of the Cournot game allows us to determine easily the aggregate demand. Summing the systems of first-order-conditions across platforms yields:

$$\begin{cases} KZ_1(N_1) + Z_1'(N_1)\, N_1 - \sum_{k=1}^{K} c_1^k + (\alpha_1+\alpha_2)\, N_2 = 0 \\ KZ_2(N_2) + Z_2'(N_2)\, N_2 - \sum_{k=1}^{K} c_2^k + (\alpha_1+\alpha_2)\, N_1 = 0 \end{cases} \tag{1}$$

We show in the Appendix that under Assumption 1, there exists a unique pair (N_1, N_2) that satisfies these conditions.[26]

[26] This also implies uniqueness of the individual platforms' quantities provided that $Z_1'(N_1)\, Z_2'(N_2)$ is not equal to $(\alpha_1+\alpha_2)^2$ at the equilibrium.

Let $\bar{c}_i \equiv \sum_k c_i^k / K$ denote the average marginal cost on side i across platforms, $\bar{n}_i \equiv N_i/K$ the average mass of side i consumers across platforms, and $\varepsilon_i(z_i) \equiv -\frac{z_i}{Z_i^{-1}(z_i)} \frac{dZ_i^{-1}(z_i)}{dz_i}$ the (absolute value of the) elasticity of demand on side i with respect to the externality-adjusted price z_i. In a Cournot equilibrium in which all platforms have positive participation on both sides, all externality-adjusted prices on a given side are the same:

$$p_i^k - \alpha_i n_j^k = z_i.$$

Rearranging condition (1) shows that the externality-adjusted price on a given side is determined by a condition similar to the Lerner formula under Cournot competition, adjusted for the externalities:[27]

$$\frac{z_i - \bar{c}_i + (\alpha_1 + \alpha_2)\,\bar{n}_j}{z_i} = \frac{1}{K\varepsilon_i\,(z_i)}.$$

Observe that the distribution of marginal costs across platforms is irrelevant: only the average marginal cost on each side matters (as long as all platforms are active). Notice also that only total network effects $\alpha_1 + \alpha_2$ matter for the externality-adjusted prices and, therefore, for equilibrium aggregate quantities on each side.[28]

One advantage of the Cournot model in one-sided markets is that it relates the aggregate Lerner index to the Herfindahl-Hirschmann Index (HHI)[29] and the price-elasticity of demand. A natural question that arises is how the standard Lerner index should be modified to account for network effects and whether (and how) the HHI on each side should be adjusted to be a good measure of this index. One possible approach is to modify the Lerner index for a given side of the market by incorporating the network externalities received by that side. This amounts to replacing prices in the standard definition of the Lerner index by externality-adjusted prices.[30] We show in the Appendix that the HHI needs to be adjusted downward with respect to its standard definition, and that this adjustment should be greater the larger the network effects and the larger the "correlation" between firms' market shares on the two sides of the market.

[27] The externality-adjusted price is the natural counterpart for the Cournot model of the externality-adjusted cost for the Bertrand model with differentiated platforms. In the Cournot model, each firm perceives a residual demand curve $D_i\left(p_i^k - \alpha_i n_j^k\right) - \sum_{l \neq k} n_i^l$, where $D_i(p) = 1 - F_i(p)$. Following Rochet and Tirole (2006), the optimal price for platform k solves

$$\frac{p_i^k - \left(c_i^k - \alpha_j n_j^k\right)}{p_i^k} = \frac{1}{\eta_i}$$

where η_i is the elasticity of the residual demand curve, which is given by $\eta_i = \frac{p_i^k D_i'\left(p_i^k - \alpha_i n_j^k\right)}{D_i\left(p_i^k - \alpha_i n_j^k\right) - \sum_{l \neq k} n_i^l} = \frac{p_i^k}{Z_i'(N_i) n_i^k}$ in the Cournot model. Thus, the difference with price competition is only about the nature of the residual demand curve. While the notion of externality-adjusted cost highlights the opportunity cost of selling on each side, the notion of externality-adjusted price is more intuitive for Cournot competition because the latter is equated across platforms in equilibrium.

[28] This is driven by the linearity of externalities in our setup, and implies in particular that the effect of a merger on consumers in our model does not depend on the sign of network externalities on each side but only on the magnitude of total network externalities. This is a limitation of our model.

[29] The HHI is the sum of the squared market shares of all firms in the industry.

[30] This approach is in line with the view that we should care about welfare on *each* side of the market.

4.3 The effect of a merger between two platforms

4.3.1 General demand

Notice first that in our homogeneous Cournot model, the effect of a merger on side-i consumer surplus is fully captured by its effect on side-i total participation. Indeed, side-i consumer surplus is given by $\int_0^{N_i} [\tilde{v}_i - Z_i(N_i)]\, dF_i(\tilde{v}_i)$,which depends on participation only through N_i. Therefore, a merger raises consumer surplus on a given side of the market if and only if it raises participation on that side (or, equivalently, if it lowers the externality-adjusted price on that side).

A merger between two (or more) platforms may affect total participation on each side of the market through two channels. First, it can change the average marginal cost of the industry, either upward or downward depending on which platforms merge.[31] In the Appendix, we show that a decrease (increase) in the average marginal cost of the industry on any side of the market benefits (harms) consumers on both sides. Second, a merger affects total participation on each side through the change in the number of platforms in the market. This effect is more complex. To investigate it we introduce the concept of *average-marginal-cost-preserving (AMCP) mergers*, i.e., mergers that do not affect the average marginal costs of the industry $\bar{c}_1$ and $\bar{c}_2$. The following proposition provides sufficient conditions for such a merger to benefit or harm consumers on both sides of the market.

Proposition 1 *Consider an AMCP merger between two platforms and assume that all platforms are active both before and after the merger.*

- If the pre-merger externality-adjusted prices are above average marginal costs on both sides of the market, then the merger harms consumers on both sides.

- If the pre-merger externality-adjusted prices are below average marginal costs on both sides of the market, then the merger benefits consumers on both sides.

Proof. See Appendix. ■

This proposition shows that "non-standard" welfare effects of mergers can only arise if at least one externality-adjusted price is below average marginal cost in the corresponding side. When both prices are large enough for the externality-adjusted price to be above average marginal cost on both sides, the market power effect of mergers dominates potential efficiency gains stemming from larger participation on each platform. When both externality-adjusted prices are below average marginal cost, the reverse holds.

In the case when only one externality-adjusted price is below cost, it is possible that one side benefits and the other is harmed by the merger. To address this issue and provide sufficient

[31] Assuming that the merged platform's marginal cost is the lowest cost of the merging platforms, a merger between two platforms reduces average marginal cost (on a given side) if and only if the marginal cost of the less efficient one is above the industry average marginal cost.

conditions (for the merger to harm or benefit consumers) that are formulated in terms of the primitives of the model, we now consider the special case of linear demand.

4.3.2 Linear demand

Assume that the demand function on each side of the market is linear. More precisely, suppose that $Z_i(N_i) = 1 - N_i$ for $i = 1, 2$. We assume further that $c_i^k < 1$ for any $i \in \{1, 2\}$ and $k \in \{1, ..., K\}$.

In this scenario, Assumption 1 reduces to $\alpha_1 + \alpha_2 < 2$. Moreover, one can rewrite the sufficient conditions provided in Proposition 1 using the primitives of the model: both externality-adjusted prices are above (below) the corresponding average marginal costs if and only if total network effects $\alpha_1 + \alpha_2$ are sufficiently small (large).[32]

We now provide provide a more precise characterization of the impact of an AMCP merger on consumers.

Proposition 2 *Suppose that demand is linear, and w.l.o.g. that $\bar{c}_2 \leq \bar{c}_1 < 1$. Assuming that all platforms are active,[33] there exist two thresholds $f(K, \bar{c}_1, \bar{c}_2) \in (0, 1)$ and $g(K, \bar{c}_1, \bar{c}_2) \in (1, 2)$ such that a marginal AMCP reduction of K:*

- harms consumers on both sides of the market if $0 \leq \alpha_1 + \alpha_2 < f(K, \bar{c}_1, \bar{c}_2)$,

- benefits consumers on side 1 and harms consumers on side 2 if $f(K, \bar{c}_1, \bar{c}_2) < \alpha_1 + \alpha_2 < g(K, \bar{c}_1, \bar{c}_2)$,

- benefits consumers on both sides of the market if $g(K, \bar{c}_1, \bar{c}_2) < \alpha_1 + \alpha_2 < 2$.

Proof. See Appendix. ■

While the above results are derived only for a marginal decrease in K, they extend to a discrete change from K to $K - 1$ as long as the corresponding condition holds everywhere between $K - 1$ and K. We show in the Appendix that f is increasing in K while g is decreasing in K. This implies that: (i) a merger harms consumers on both sides if $\alpha_1 + \alpha_2 < f(K-1, \bar{c}_1, \bar{c}_2)$, (ii) a merger benefits consumers on side 1 and harms consumers on side 2 if $f(K, \bar{c}_1, \bar{c}_2) < \alpha_1 + \alpha_2 < g(K, \bar{c}_1, \bar{c}_2)$, and (iii) a merger benefits consumers on both sides if $\alpha_1 + \alpha_2 > g(K-1, \bar{c}_1, \bar{c}_2)$.

The findings for the small and large network effect scenarios confirm the results of Proposition 1. More interestingly, we show that a *see-saw effect* arises for network effects of intermediate size: in that case, a reduction of K benefits consumers on the higher-cost side while it harms consumers on the lower-cost side. Interestingly, which side benefits from the merger does not depend on which side receives the highest network benefits (as only total network effects $\alpha_1 + \alpha_2$ matter).

The linear model also sheds light on merger profitability in a multi-sided Cournot setting. Absent cost-related efficiency gains, a merger between two platforms is profitable in our symmetric linear model if there are no more than four firms, marginal costs are not too high and

[32] We provide the exact thresholds on total network effects in the Appendix.

[33] Condition (4) in the proof of the proposition is a necessary and sufficient condition for this to hold.

network effects are large enough.[34] In that case, the network efficiency gains due to higher concentration outweigh the negative output contraction effect that undermines the profitability of mergers in Cournot models (see e.g. Salant et al. 1983).

5 Conclusion and future research

This paper argues that the Cournot model can be a useful addition to the toolkit for merger analysis in multi-sided markets. We illustrate this by deriving the effect of a merger between platforms on its users in a simple two-sided Cournot model. We believe that extensions of this model may help address some of the issues raised by the analysis of mergers in multi-sided markets both in theory and in practice.

The Cournot model can be a valuable complement for the canonical Bertrand model of competition between platforms. This is particularly clear for the analysis of mergers involving competition among homogeneous platforms, for which standard Bertrand models raise major tractability issues. Moreover, the Cournot model allows to relate the HHI (or an adapted version of it) to measures of market power such as the adjusted versions of the Lerner index we discussed. This can help define rules for the preliminary screening of mergers in two-sided markets.

The existing models of horizontal mergers in multi-sided markets usually impose strong restrictions regarding the nature of competition between platforms and the behavior of their users. This contrasts with the diversity of both the business models used by platforms and the consumption patterns of their users. It is therefore not surprising that there are still many issues that need to be addressed by future research.

A first obvious, yet important, issue is the extension of models of mergers in multi-sided markets to situations where agents multi-home. With few exceptions (such as Anderson et al., 2012 and Anderson et al., 2018), the existing models, including ours, assume that agents single-home on both sides of the market. Multi-homing may however occur on one or both sides of the market and changes dramatically the nature of competition between platforms. Indeed, while single-homing consumers choose between one or the other platform, multi-homing consumers choose how many platforms to join. The value of a platform for a multi-homing consumer is then reduced to its *incremental* value so that direct price comparison may not be relevant for her. In this context, the literature on media platforms has shown that the effects of a merger differ from the case where marginal consumers are single-homing. A general analysis of the effect of mergers in a model with endogenous decisions to single-home or multi-home remains to be done.[35]

[34] When marginal costs are equal to zero, a merger between two out of three platforms is profitable if $\alpha_1 + \alpha_2 > 0.5858$, and a merger between two out of four platforms is profitable if $\alpha_1 + \alpha_2 > 1.5858$.

[35] See Ambrus et al. (2016) and Anderson et al. (2018) for models of two-sided media market with endogenous multi-homing on the consumer side.

A second issue that still needs to be addressed is the interaction between business models and mergers. Indeed, a merger may have a very different impact if the platforms are free for users on one side or charge users on all sides. In particular, the price may increase or decrease on a given side depending on whether the other side is charged a positive price or not. Moreover, the effect may depend on whether the platforms choose quantities on the paying side, as assumed in media models, or prices, as assumed in matching models. A different, but related, question is whether a merger may induce a change of business model. For instance, platforms that are free on one side due to intense competition on any of the two sides may choose to charge a positive price once a merger relaxes competitive pressure.

It would also be interesting to examine the incentives of the merged entity to maintain both platforms after a merger. In our homogeneous Cournot model, a merger is tantamount to a reduction in the number of platforms. In the case of differentiated platforms, this may not be the case and the merged entity may choose to maintain both platforms. This raises the question of a merged entity's incentive to make the platforms it owns interoperable when it keeps them separate.[36] Interoperability typically reduces platforms' differentiation. Therefore, when deciding on interoperability, the merged entity needs to seek the right balance between the benefits of network effects induced by interoperability and the benefits of platforms' differentiation.

Finally, note that all the existing theoretical papers analyzing the effects of mergers among multi-sided platforms assume static competition. Although some equilibrium concepts - such as the insulated equilibrium or the Cournot equilibrium - are guided by implicit dynamic considerations, it would be useful to move towards a more dynamic analysis of mergers in multi-sided markets. This is even more important for platform markets than for traditional markets because of two features, emphasized by Arthur (1996), that are particularly prevalent in technology markets. First, the presence of network effects combined with rapid technological progress makes the market highly volatile with rapid change of leadership. Second, some platform markets tend to exhibit a "winner-take-all" feature so that there is competition *for* the market but little competition *on* the market. Competition in this context is then shaped by history dependence (Arthur, 1989), incumbency advantage (Biglaiser et al., 2013; Biglaiser and Crémer, 2016) and consumers' expectations (Jullien and Pavan, 2017; Halaburda et al., 2017). In this context, a static view of mergers between platforms may be misleading either by over-emphasizing current strong market position in a dynamically competitive environment or by underestimating the effect of a merger between future potential competitors.

[36] For instance, after acquiring Waze, Google allowed direct navigation between Google Maps and Waze.

6 Appendix

Sufficiency of Assumption 1 for the second-order condition

The second-order conditions for optimality of the best reply are:

$$\begin{cases} 2Z_1'(N_1) + Z_1''(N_1)\, n_1^k < 0 \\ \left[2Z_1'(N_1) + Z_1''(N_1)\, n_1^k\right]\left[2Z_2'(N_2) + Z_2''(N_2)\, n_2^k\right] > (\alpha_1+\alpha_2)^2 . \end{cases} \tag{2}$$

To check that these conditions are satisfied under Assumption 1, notice that

$$2Z_i'(N_i) + Z_i''(N_i)\, n_i^k < 2Z_i'(N_i) + \max\left\{Z_i''(N_i)\, N_i, 0\right\} < 0.$$

This implies that

$$\begin{aligned} \left[2Z_1'(N_1) + Z_1''(N_1)\, n_1^k\right]\left[2Z_2'(N_2) + Z_2''(N_2)\, n_2^k\right] &> \\ \left[2Z_1'(N_1) + \max\left\{Z_1''(N_1)\, N_1, 0\right\}\right]\left[2Z_2'(N_2) + \max\left\{Z_2''(N_2)\, N_2, 0\right\}\right] &> (\alpha_1+\alpha_2)^2 . \end{aligned}$$

Uniqueness of the solution to the system of conditions (1)

Under Assumption 1, $KZ_1(N_1) + Z_1'(N_1)\, N_1$ is a continuous and strictly decreasing function of N_1, which ranges from $+\infty$ (when $N_1 = 0$) to $-\infty$ (when $N_1 = 1$). Therefore, there is a unique and interior value of N_1 for which the first equation of (1) holds, as a function of N_2, which we denote by $R_1(N_2)$. It is clear that $R_1 : [0,1] \to [0,1]$ is continuous and strictly increasing, with derivative given by $R_1'(N_2) = \frac{-(\alpha_1+\alpha_2)}{(K+1)Z_1'(N_1)+Z_1''(N_1)N_1}$.

Define $R_2 :\ [0,1] \to [0,1]$ in a similar way, and observe that R_1 and R_2 surely intersect. If $R_1'(N_2)R_2'(N_1) < 1$, the "reaction" functions R_1 and R_2, which exhibit strategic complementarity, intersect only once. Hence, uniqueness of a pair (N_1, N_2) that satisfies (1) is guaranteed if:

$$\left[(K+1)Z_1'(N_1) + Z_1''(N_1)N_1\right]\left[(K+1)Z_2'(N_2) + Z_2''(N_2)N_2\right] > (\alpha_1+\alpha_2)^2,$$

which is implied by Assumption 1. Indeed,

$$(K+1)Z_i'(N_i) + Z_i''(N_i)N_i < 2Z_i'(N_i) + \max\left\{Z_i''(N_i)\, N_i, 0\right\} < 0$$

which leads to

$$\begin{aligned} &\left[(K+1)Z_1'(N_1) + Z_1''(N_1)N_1\right]\left[(K+1)Z_2'(N_2) + Z_2''(N_2)N_2\right] \\ > \quad &\left[2Z_1'(N_1) + \max\left\{Z_1''(N_1)\, N_1, 0\right\}\right]\left[2Z_2'(N_2) + \max\left\{Z_2''(N_2)\, N_2, 0\right\}\right] > (\alpha_1+\alpha_2)^2 \end{aligned}$$

The HHI in a two-sided Cournot model

Denote $s_i^k \equiv n_i^k/N_i$ the market share of platform k on side i and $HHI_i \equiv \sum_{k=1}^K \left(s_i^k\right)^2$ the Herfindahl-Hirschmann index on side i. Let us consider the following adjusted Lerner index for platform k on side i obtained by replacing prices with externality-adjusted prices:

$$L_i^k \equiv \frac{z_i - c_i^k}{z_i}.$$

The corresponding aggregate Lerner index for side i is given by

$$L_i \equiv \sum_{k=1}^{K} s_i^k L_i^k.$$

The first-order condition for the maximization of platform k's profit with respect to n_i^k is

$$z_i + Z_i'(N_i)\, n_i^k = c_i^k - (\alpha_1 + \alpha_2)\, n_j^k$$

where $j \neq i$, and can be rewritten as

$$L_i^k = \frac{s_i^k}{\varepsilon_i} - \frac{\alpha_1 + \alpha_2}{z_i} N_j s_j^k.$$

Therefore,

$$L_i = \frac{HHI_i}{\varepsilon_i} - \frac{\alpha_1 + \alpha_2}{z_i} S N_j = \frac{HHI_i - \overbrace{\frac{\alpha_1 + \alpha_2}{z_i} S \varepsilon_i N_j}^{\text{adjustment term}}}{\varepsilon_i}, \tag{3}$$

where

$$S \equiv \sum_{k=1}^{K} s_1^k s_2^k$$

can be interpreted as a measure of the "correlation" between the platforms' market shares on the two sides of the market. Expression (3) shows that for the HHI to be a good measure of the aggregate Lerner index on a given side, it needs to be adjusted downward with respect to its standard definition for a one-sided market. This adjustment should be greater the larger the network effects and the larger the correlation between market shares on the two sides of the market.

Effect of a change in the average marginal cost of the industry on consumers

Denote by N_i^* the total mass of side-i consumers that are served in equilibrium. Differentiating the system of conditions (1) with respect to $\bar{c}_1$ yields

$$\begin{cases} \left[(K+1) Z_1'(N_1^*) + Z_1''(N_1^*)\, N_1^*\right] \frac{\partial N_1^*}{\partial \bar{c}_1} + (\alpha_1 + \alpha_2) \frac{\partial N_2^*}{\partial \bar{c}_1} = K \\ (\alpha_1 + \alpha_2) \frac{\partial N_1^*}{\partial \bar{c}_1} + \left[(K+1) Z_2'(N_2^*) + Z_2''(N_2^*)\, N_2^*\right] \frac{\partial N_2^*}{\partial \bar{c}_1} = 0. \end{cases}$$

Solving this system leads to

$$\begin{cases} \frac{\partial N_1^*}{\partial \bar{c}_1} = \frac{K\left[(K+1)Z_2'\left(N_2^*\right)+Z_2''\left(N_2^*\right)N_2^*\right]}{\left[(K+1)Z_1'\left(N_1^*\right)+Z_1''\left(N_1^*\right)N_1^*\right]\left[(K+1)Z_2'\left(N_2^*\right)+Z_2''\left(N_2^*\right)N_2^*\right]-(\alpha_1+\alpha_2)^2} \\ \frac{\partial N_2^*}{\partial \bar{c}_1} = \frac{-K(\alpha_1+\alpha_2)}{\left[(K+1)Z_1'\left(N_1^*\right)+Z_1''\left(N_1^*\right)N_1^*\right]\left[(K+1)Z_2'\left(N_2^*\right)+Z_2''\left(N_2^*\right)N_2^*\right]-(\alpha_1+\alpha_2)^2}. \end{cases}$$

Assumption 1 implies that that the numerator of $\frac{\partial N_1^*}{\partial \bar{c}_1}$ is negative and that the (common) denominator of $\frac{\partial N_1^*}{\partial \bar{c}_1}$ and $\frac{\partial N_2^*}{\partial \bar{c}_1}$ is positive. It follows that $\frac{\partial N_1^*}{\partial \bar{c}_1}$ and $\frac{\partial N_2^*}{\partial \bar{c}_1}$ are negative, which implies that a decrease (increase) in the average marginal cost of the industry on a given side of the market benefits (harms) consumers on both sides of the market.

Proof of Proposition 1

Denote by N_i^* the total mass of side-i consumers that are served in equilibrium and z_i^* the equilibrium externality-adjusted price on side i. Differentiating the system of conditions (1) with respect to K (it is instrumental to treat K as a continuous variable) yields

$$\begin{cases} \left[(K+1)Z_1'\left(N_1^*\right)+Z_1''\left(N_1^*\right)N_1^*\right]\frac{\partial N_1^*}{\partial K}+(\alpha_1+\alpha_2)\frac{\partial N_2^*}{\partial K} = \bar{c}_1 - Z_1\left(N_1^*\right) \\ (\alpha_1+\alpha_2)\frac{\partial N_1^*}{\partial K}+\left[(K+1)Z_2'\left(N_2^*\right)+Z_2''\left(N_2^*\right)N_2^*\right]\frac{\partial N_2^*}{\partial K} = \bar{c}_2 - Z_2\left(N_2^*\right). \end{cases}$$

This leads to

$$\begin{cases} \frac{\partial N_1^*}{\partial K} = \frac{\Delta_1^*}{\left[(K+1)Z_1'\left(N_1^*\right)+Z_1''\left(N_1^*\right)N_1^*\right]\left[(K+1)Z_2'\left(N_2^*\right)+Z_2''\left(N_2^*\right)N_2^*\right]-(\alpha_1+\alpha_2)^2} \\ \frac{\partial N_2^*}{\partial K} = \frac{\Delta_2^*}{\left[(K+1)Z_1'\left(N_1^*\right)+Z_1''\left(N_1^*\right)N_1^*\right]\left[(K+1)Z_2'\left(N_2^*\right)+Z_2''\left(N_2^*\right)N_2^*\right]-(\alpha_1+\alpha_2)^2} \end{cases}$$

where

$$\Delta_i^* \equiv \left[\bar{c}_i - z_i^*\right]\left[(K+1)Z_{-i}'\left(N_{-i}^*\right)+Z_{-i}''\left(N_{-i}^*\right)N_{-i}^*\right] - \left[\bar{c}_{-i} - z_{-i}^*\right](\alpha_1+\alpha_2).$$

Assumption 1 implies that the denominator of $\frac{\partial N_1^*}{\partial K}$ and $\frac{\partial N_2^*}{\partial K}$ is positive. Therefore, the sign of $\partial N_i^*/\partial K$ is the same as the sign of Δ_i^*.

Since $\alpha_1+\alpha_2 > 0$ and $(K+1)Z_i'(N_i)+Z_i''(N_i)N_i < 0$ (under Assumption 1), the effect of a merger on the mass of users on each side depends on the signs of the externality-adjusted price-cost margins $z_1^* - \bar{c}_1$ and $z_2^* - \bar{c}_2$. More precisely:

- If the externality-adjusted price is above cost on both sides, then a (marginal) decrease in K leads to a decrease in the mass of users served on both sides of the market.

- If the externality-adjusted price is below cost on both sides, then a (marginal) decrease in K leads to an increase in the mass of users served on both sides of the market.

- Otherwise, the effect of a (marginal) decrease in K on consumers is generally ambiguous.

Finally, to verify that the effects on consumers of a discrete reduction in K have the same sign as those of a marginal decrease in K, observe that:

- If $z_i^* > \bar{c}_i$ for both $i = 1, 2$, then $\partial N_i^*/\partial K > 0$, which means that $\partial z_i^*/\partial K < 0$. This implies that margins remain positive as K diminishes and thus the effect on z_i^* of a discrete reduction in K is the integral of the effect of a marginal reduction in K, whose sign remains constant.

- If $z_i^* < \bar{c}_i$ for both $i = 1, 2$, then $\partial N_i^*/\partial K < 0$, which means that $\partial z_i^*/\partial K > 0$. This implies that margins remain negative as K diminishes and thus the effect on z_i^* of a discrete reduction in K is the integral of the effect of a marginal reduction in K, whose sign remains constant.

Comparison of the externality-adjusted prices with the corresponding average marginal costs under a linear demand

Both externality-adjusted prices are above the corresponding average marginal costs if and only if

$$\alpha_1 + \alpha_2 < \sqrt{\left(\frac{K}{2}\frac{1-\min(\bar{c}_1,\bar{c}_2)}{1-\max(\bar{c}_1,\bar{c}_2)}\right)^2 + K + 1} - \frac{K}{2}\frac{1-\min(\bar{c}_1,\bar{c}_2)}{1-\max(\bar{c}_1,\bar{c}_2)},$$

and both externality-adjusted prices are above the corresponding average marginal costs if and only if

$$\alpha_1 + \alpha_2 > \sqrt{\left(\frac{K}{2}\frac{1-\max(\bar{c}_1,\bar{c}_2)}{1-\min(\bar{c}_1,\bar{c}_2)}\right)^2 + K + 1} - \frac{K}{2}\frac{1-\max(\bar{c}_1,\bar{c}_2)}{1-\min(\bar{c}_1,\bar{c}_2)}.$$

Proof of Proposition 2

The first-order conditions for the maximization of Π^k with respect to n_1^k and n_2^k are given by:

$$\begin{cases} 1 - N_1 - n_1^k - c_1^k + (\alpha_1+\alpha_2)n_2^k = 0 \\ 1 - N_2 - n_2^k - c_2^k + (\alpha_1+\alpha_2)n_1^k = 0. \end{cases}$$

Summing the first-order-conditions across platforms and solving the corresponding system leads to the following equilibrium total participations on the two sides of the market (assuming all platforms are active in equilibrium):

$$\begin{cases} N_1^* = \frac{\frac{K}{K+1}}{1-\left(\frac{\alpha_1+\alpha_2}{K+1}\right)^2}\left[(1-\bar{c}_1) + \frac{\alpha_1+\alpha_2}{K+1}(1-\bar{c}_2)\right] \\ N_2^* = \frac{\frac{K}{K+1}}{1-\left(\frac{\alpha_1+\alpha_2}{K+1}\right)^2}\left[(1-\bar{c}_2) + \frac{\alpha_1+\alpha_2}{K+1}(1-\bar{c}_1)\right]. \end{cases}$$

Denote $H = \frac{1}{K+1}$, $\alpha = \alpha_1 + \alpha_2$, $d_i = 1 - \bar{c}_i$ (recall that $\alpha \in [0, 2)$ and $d_2 \geq d_1 > 0$). It is straightforward to show that all platforms are active if and only if the following condition holds for any platform k:

$$\frac{H\left(1-\alpha^2\right)}{1-\alpha^2 H^2}\left[1-\bar{c}_i + \alpha H\left(1-\bar{c}_j\right)\right] + \bar{c}_i + \alpha\bar{c}_j > c_i^k + \alpha c_j^k \text{ for } i = 1, 2,\ j \neq i \tag{4}$$

The equilibrium mass of users served in equilibrium on side 1 can be rewritten as

$$N_1^* = \frac{1-H}{1-\alpha^2 H^2}\left(d_1 + \alpha H d_2\right).$$

Therefore,

$$\begin{aligned}
\frac{\partial N_1^*}{\partial H} &= \frac{-\left(1-\alpha^2 H^2\right) + 2\alpha^2 H\left(1-H\right)}{\left(1-\alpha^2 H^2\right)^2}\left(d_1 + \alpha H d_2\right) + \frac{1-H}{1-\alpha^2 H^2}\alpha d_2 \\
&= \frac{J\left(\alpha, H, d_1, d_2\right)}{\left(1-\alpha^2 H^2\right)^2}
\end{aligned}$$

where

$$J\left(\alpha, H, d_1, d_2\right) \equiv -d_1 + \underbrace{d_2\left[1-2H\right]}_{>0}\alpha + \underbrace{H\left(2-H\right)d_1}_{>0}\alpha^2 + \underbrace{H^2 d_2}_{>0}\alpha^3$$

which implies that

$$\frac{\partial J}{\partial \alpha}\left(\alpha, H, d_1, d_2\right) > 0$$

for any $\alpha \geq 0$. This, combined with the fact that $J\left(0, H, d_1, d_2\right) = -d_1 < 0$ and the continuity of $J\left(\alpha, H, d_1, d_2\right)$ with respect to α implies that there exists a unique threshold $\tilde{\alpha}\left(H, d_1, d_2\right) \in (0, 2]$ such that $J\left(\tilde{\alpha}\left(H, d_1, d_2\right), H, d_1, d_2\right) = 0$, and that $\frac{\partial N_1^*}{\partial H} < 0$ for $\alpha \in [0, \tilde{\alpha}\left(H, d_1, d_2\right))$ and $\frac{\partial N_1^*}{\partial H} > 0$ for $\alpha \in \left(\tilde{\alpha}\left(H, d_1, d_2\right), 2\right)$.

Similarly,

$$\frac{\partial N_2^*}{\partial H} = \frac{J\left(\alpha, H, d_2, d_1\right)}{\left(1-\alpha^2 H^2\right)^2}$$

By analogy, the threshold $\tilde{\alpha}\left(H, d_2, d_1\right) \in (0, 2]$ is such that $\frac{\partial N_2^*}{\partial H} < 0$ for $\alpha \in [0, \tilde{\alpha}\left(H, d_2, d_1\right))$ while $\frac{\partial N_2^*}{\partial H} > 0$ for $\alpha \in \left(\tilde{\alpha}\left(H, d_2, d_1\right), 2\right)$.

Notice that $J\left(0, H, d_1, d_2\right)$ and $J\left(0, H, d_2, d_1\right)$ are both negative, that $J\left(1, H, d_1, d_2\right) > 0 > J\left(1, H, d_2, d_1\right)$, and that $J\left(2, H, d_2, d_1\right) > \left(-1 + 4H(2-H)\right)d_1 > 0$. This implies that $0 < \tilde{\alpha}\left(H, d_1, d_2\right) < 1 < \tilde{\alpha}\left(H, d_2, d_1\right) < 2$. Denoting $f(K, \bar{c}_1, \bar{c}_2) \equiv \tilde{\alpha}\left(H, d_1, d_2\right)$ and $g(K, \bar{c}_1, \bar{c}_2) \equiv \tilde{\alpha}\left(H, d_2, d_1\right)$, we get the result.

Monotonicity of f and g

From $J\left(\tilde{\alpha}\left(H, d_1, d_2\right), H, d_1, d_2\right) = 0$ it follows that

$$\left(-d_2 + d_1\alpha\right)2H\alpha = \left(-d_1 + d_2\alpha\right)\left(1 - H^2\alpha^2\right)$$

at $\alpha = \tilde{\alpha}\left(H, d_1, d_2\right)$, which implies that $-d_1 + d_2\tilde{\alpha}\left(H, d_1, d_2\right) < 0$.

Therefore,

$$\frac{\partial J}{\partial H}\left(\tilde{\alpha}\left(H, d_1, d_2\right), H, d_1, d_2\right) = \frac{\left(-d_1 + d_2\tilde{\alpha}\left(H, d_1, d_2\right)\right)\left[3H_2^2\left(\tilde{\alpha}\left(H, d_1, d_2\right)\right)^2 - 1\right]}{H} > 0$$

Differentiating $J\left(\tilde{\alpha}\left(H,d_1,d_2\right),H,d_1,d_2\right)=0$ with respect to H and using the fact that $\partial J/\partial\alpha>0$ we get:

$$\frac{\partial\tilde{\alpha}\left(H,d_1,d_2\right)}{\partial H}=\frac{-\frac{\partial J}{\partial H}\left(\tilde{\alpha}\left(H,d_1,d_2\right),H,d_1,d_2\right)}{\frac{\partial J}{\partial\alpha}\left(\tilde{\alpha}\left(H,d_1,d_2\right),H,d_1,d_2\right)}<0$$

Hence, $\partial f(K,\bar{c}_1,\bar{c}_2)/\partial K>0$. Similarly, we can show that $-d_2+d_1\tilde{\alpha}\left(H,d_2,d_1\right)>0$ and use the same reasoning as above to prove that $\partial g(K,\bar{c}_1,\bar{c}_2)/\partial K<0$.

References

[1] Affeldt, P., L. Filistrucchi, and T.J. Klein, 2013, "Upward pricing pressure in two-sided markets," *The Economic Journal*, 123, 505-523.

[2] Ambrus, A., Calvano, E., and M. Reisinger, 2016, "Either or both competition: A "two-sided" theory of advertising with overlapping viewerships," *American Economic Journal: Microeconomics*, 8(3), 189-222.

[3] Anderson, S.P. and S. Coate, 2005, "Market provision of broadcasting: A welfare analysis," *Review of Economic Studies,* 72, 947-972.

[4] Anderson, S.P., Ø. Foros, Kind, H.J., 2016, "Competition for advertisers and for viewers in media markets," *The Economic Journal*, 128(608), 34-54.

[5] Anderson, S.P., Ø. Foros, Kind, H.J., 2018, "The importance of consumer multi-homing (joint purchases) for market performance: mergers and entry in media markets,", working paper.

[6] Anderson, S.P., Ø. Foros, Kind, H.J., and M. Peitz, 2012, "Media market concentration, advertising levels, and ad prices," *International Journal of Industrial Organization*, 30, 321-325.

[7] Anderson, S.P. and B. Jullien, 2015, "The advertising-financed business model in two-sided media markets," in S. P. Anderson, D. Strömberg and J. Waldfogel (Eds.): *Handbook of Media Economics,* Elsevier, 41-90.

[8] Anderson, S.P. and M. Peitz, 2015, "Media see-saws: Winners and losers on media platforms," mimeo.

[9] Arthur, W.B., 1989, "Competing technologies, increasing returns, and lock-in by historical events," *The Economic Journal*, 99(394), 116-131.

[10] Arthur, W.B., 1996, "Increasing returns and the two worlds of business," *Harvard Business Review*, 74(4), 100-109.

[11] Baranes, E., Cortade, T. and A. Cosnita-Langlais, 2016, “Merger control on two-sided markets: Is there need for an efficiency defense”, mimeo.

[12] Belleflamme, P. and M. Peitz, 2015, *Industrial organization: Markets and strategies*, Second Edition, Cambridge University Press.

[13] Biglaiser, G. and J. Crémer, 2016, “The value of incumbency in heterogeneous platforms”, CESifo Working Paper, No. 5829.

[14] Biglaiser, G., Crémer, J. and G. Dobos, 2013, “The value of switching costs”. *Journal of Economic Theory*, 148(3), 935-952.

[15] Brown, K. and G. Williams, 2002, “Consolidation and advertising prices in local radio markets,” Media Ownership Working Group, FCC (2002).

[16] Caillaud, B. and B. Jullien, 2003, “Chicken & egg: Competition among intermediation service providers,” *The RAND Journal of Economics*, 34, 309-328.

[17] Camesasca, P.D., M. Meulenbelt, M., Chellingsworth, T., Daems, I., and J. Vandenbussche, 2009, “The Dutch yellow pages merger case: 2-1 will go!,” *European Competition Law Review* 1, 4-13.

[18] Chandra, A. and A. Collard-Wexler, 2009, “Mergers in two-sided markets: An application to the Canadian newspaper industry,” *Journal of Economics and Management Strategy*, 18, 1045-1070.

[19] Chipty, T., 2007, “FCC Media ownership Study#5: Station ownership and programming in radio.” Charles River Associates (CRA) International. Boston.

[20] Cosnita-Langlais, A., Johansen, B.O., and L. Sørgard, 2018, “Upward price pressure in two-sided markets: Incorporating feedback effects,” Economix working paper n° 2018-3.

[21] Crampes, C., Haritchabalet, C, and B. Jullien, “Adversting, competition and entry in media industries,” *The Journal of Industrial Economics*, 57(1), 7-31.

[22] Evans, D.S., 2003, “The antitrust economics of multi-sided platform markets,” *Yale Journal on Regulation*, 20, 325.

[23] Evans, D.S. and R. Schmalensee, 2013, “The antitrust analysis of multi-sided platform businesses,” NBER working paper No. w18783.

[24] Evans, D.S. and M.D. Noel, 2008, “The analysis of mergers that involve multisided platform businesses,” *Journal of Competition Law and Economics*, 4(3), 663-695.

[25] Fan, Y., 2013,“Ownership consolidation and product characteristics: A Study of the US Daily Newspaper Market,” *The American Economic Review*, 103(5), 1598-1628.

[26] Farrell, J. and C. Shapiro, 1990, "Horizontal mergers: An equilibrium analysis," *The American Economic Review*, 80, 107-126.

[27] Filistrucchi, L., 2017, "Network effects and the efficiency defense in mergers among two-sided platforms," mimeo.

[28] Filistrucchi, L., Klein, T.J., and T.O. Michielsen, 2012, "Assessing unilateral merger effects in a two-sided market: An application to the Dutch daily newspaper market," *Journal of Competition Law & Economics,* 8(2), 297-329.

[29] Filistrucchi, L., D. Geradin, E. Van Damme, and P. Affeldt, 2014, "Market definition in two-sided markets: Theory and practice," *Journal of Competition Law & Economics* 10, 293-339.

[30] Foros, Ø, Kind, H.J., and L. Sørgard, 2015, "Merger policy and regulation in media industries", in S. P. Anderson, D. Strömberg and J. Waldfogel (Eds.): *Handbook of Media Economics,* Elsevier, 226-261.

[31] Gabszewicz, J. and X. Wauthy, 2014, "Vertical product differentiation and two-sided markets," *Economics Letters,* 123, 58-61.

[32] Halaburda, H., Jullien, B., and Y. Yehezkel, 2016, "Dynamic competition with network externalities: Why history matters", TSE Working Paper 16-636.

[33] Hurkens, S. and A. López, 2014, "Mobile termination, network externalities and consumer expectations", *Economic Journal*, 124, 1005-1039.

[34] Jeziorski, P., 2014, "Effects of mergers in two-sided markets: The US radio industry," *American Economic Journal: Microeconomics*, 6(4), 35-73.

[35] Jullien, B. and A. Pavan, 2017, "Information management and pricing in platform markets," TSE Working Paper, n. 13-429.

[36] Katz, M.L. and C. Shapiro, 1985, "Network externalities, competition, and compatibility," *The American Economic review*, 75(3), 424-440.

[37] Kind, H.J., T. Nilssen, and L. Sørgard, 2007, "Competition for viewers and advertisers in a TV Oligopoly," *Journal of Media Economics*, 20(3), 211-233.

[38] Kreps, D. M. and J. A. Scheinkman, 1983, "Quantity precommitment and Bertrand competition yield Cournot outcomes," *The Bell Journal of Economics*, 14(2), 326-337.

[39] Leonello, A., 2010, "Horizontal mergers in two-sided markets," European University Institute, mimeo.

[40] Liu, Y., Putler, D.S., and C.B. Weinberg, 2004, "Is having more channels really better? A model of competition among commercial television broadcasters," *Marketing Science*, 23, 120-133.

[41] Nocke, V. and M.D. Whinston, 2010, "Dynamic merger review," *Journal of Political Economy*, 118(6), 1200-1251.

[42] Nocke, V. and M.D. Whinston, 2013, "Merger policy with merger review," *American Economic Review*, 103(2), 1006-1033.

[43] Novshek, W., 1985, "On the existence of Cournot equilibrium," *The Review of Economic Studies*, 52(1), 85-98.

[44] de Palma, A., Leruth, L., and P. Régibeau, 1999, "Partial compatibility with network externalities and double purchase," *Information Economics and Policy*, 11, 209–227.

[45] Peitz, M. and M. Reisinger, 2015, "The economics of Internet media," in S. P. Anderson, D. Strömberg and J. Waldfogel (Eds.): *Handbook of Media Economics,* Elsevier, 445-530.

[46] Peitz, M. and T. Valletti, 2008, "Content and advertising in the media: Pay-tv versus free-to-air," *International Journal of Industrial Organization*, 26(4), 949-965.

[47] Rochet, J-C. and J. Tirole, 2003, "Platform competition in two-sided markets," *Journal of the European Economic Association*, 1(4), 990-1029.

[48] Rochet, J-C. and J. Tirole, 2006, "Two-sided markets: A progress report," *The RAND Journal of Economics*, 37(3), 645-667.

[49] Salant, S.W., Switzer, S., and R.J. Reynolds, 1983, "Losses from horizontal merger: The effect of an exogenous change in industry structure on Cournot-Nash equilibrium," *The Quarterly Journal of Economics,* 98(2),185-199.

[50] Schiff, A., 2003, "Open and closed systems of two-sided networks," *Information Economics and Policy*, 15(4), 425-442.

[51] Sweeting, A., 2010, "The effects of mergers on product positioning: evidence from the music radio industry," *The RAND Journal of Economics*, 41, 372-397.

[52] Tan, G., and J. Zhou, 2017, "Price competition in multi-sided markets," mimeo.

[53] Weyl, E.G. and A. White, 2014, "Let the right 'one' win: Policy lessons from the new economics of platforms," Working paper version: December 8, 2014

[54] White, A. and E.G. Weyl, 2016, "Insulated platform competition," working paper.

[55] Wright, J., 2004, "One-sided logic in two-sided markets," *Review of Network Economics*, 3, 44-64.