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“Consumer Search and Retail Market Structure"

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Abstract

A puzzling feature of many retail markets is the coexistence of large multiproduct firms and smaller firms with narrow product ranges. This paper provides a possible explanation for this puzzle, by studying how consumer search frictions influence the structure of retail markets. In our model single-product firms which supply different products can merge to form a multiproduct firm. Consumers wish to buy multiple products, and due to search frictions value the one-stop shopping convenience associated with a multiproduct firm. We find that when search frictions are relatively large all firms are multiproduct in equilibrium. However when search frictions are smaller the equilibrium market structure is asymmetric, with different retail formats coexisting. This allows firms to better segment the market, and as such typically leads to the weakest price competition. When search frictions are low this asymmetric market structure is also the worst for consumers. Moreover due to the endogeneity of market structure, a reduction in the search friction can increase market prices and harm consumers.

Keywords: consumer search, multiproduct pricing, one-stop shopping, retail market structure, conglomerate merger
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1 Introduction

Many consumers place a high value on the convenience of one-stop shopping. They are often time-constrained, and so welcome the opportunity to buy a large basket of

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products in one place. Indeed in one survey by the UK Competition Commission in 2000, almost half the respondents said that the ability to buy everything in one location was the main factor influencing where they shopped.[1] Consequently product assortment is an important dimension along which retailers compete. Over time many retailers have attempted to become one-stop shops through aggressive increases in the size of their product assortments.[2] Nevertheless, and somewhat puzzlingly, in most retail markets large players like Wal-Mart or Amazon still coexist with many smaller retailers whose product ranges are much narrower.

Surprisingly, there is little research investigating why different retail formats can coexist despite consumers having a preference for one-stop shopping convenience. Our paper provides a framework to investigate this issue. As we explain in more detail below, we consider a model in which consumers find it costly and time-consuming to visit a retailer and learn about its prices and products. We consider incentives of firms selling different products to merge and sell them in one place, thus reducing consumers' search costs and providing them with one-stop shopping convenience. Our merger framework is partly motivated by the fact that many well-known retailers are increasingly growing their product ranges through mergers and acquisitions. For example Amazon's takeover of Whole Foods will facilitate its entry into the grocery sector, whilst its earlier acquisition of LoveFilm was designed to create a one-stop service for video streaming, DVD rental, and books. Along similar lines, the UK supermarket Sainsbury's recently bought the non-food retailer Argos, with the aim of eventually locating their products under the same roof. Meanwhile Starbucks acquired the La Boulange chain, closed its branches, but incorporated its bakery products into its own outlets. A recurring theme in these (and many other) examples is that the merger brings different but related products into one location, creating a one-stop shop for consumers. We therefore believe that our merger framework addresses an important issue in its own right. Of course retailers can also grow their product ranges organically, and so later in the paper we discuss how our main insights can also apply to this kind of growth.

[1] See https://goo.gl/MBnTyn. To put this in perspective, only 18% stated that price was the main factor. A more recent survey in the US confirms that many consumers no longer shop at traditional grocery stores, with 77% of respondents having bought groceries from big box stores like Wal-Mart or Target in 2013 (see https://goo.gl/Avui2B).

[2] For instance the Food Marketing Institute estimates that between 1975 and 2013, the number of products in an average US supermarket increased from around 9,000 to almost 44,000, at least some of which is due to new product categories. Messinger and Narasimhan (1997) provide empirical evidence that time-saving convenience is the most important driver of this growth in store size, whilst Seo (2015) estimates that the value of one-stop shopping convenience from grocery stores being able to sell liquor is about 8% of an average household's liquor expenditure.

In more detail, we consider a market in which there are two products (or product categories) and each of them is initially sold by two differentiated single-product firms. Each pair of single-product firms which supply different products choose whether to merge and form a multiproduct firm. This generates one of three possible market structures: either four single-product firms, or two multiproduct firms, or an asymmetric market with one multiproduct firm and two single-product firms. Consumers wish to buy one unit of each of the two products, but each consumer is initially uncertain about her personal valuation for a particular product as well as its price, and therefore must search a retailer in order to learn this information. We capture one-stop shopping convenience by assuming that it costs the same amount to search a single-product or a multiproduct retailer. The aim is then to understand how this search friction influences the final market structure.[3]

As a preliminary step, the paper first derives equilibrium prices and optimal consumer search rules in each of the three possible market structures. As an example consider the case of an asymmetric market, where one multiproduct firm competes with two single-product retailers. We show that here the multiproduct retailer charges lower prices, and yet earns higher profit because it is searched first by consumers and so ends up making many more sales. Intuitively consumers start by searching the large (generalist) retailer due to its one-stop shopping convenience. Many consumers stay there and purchase, due to the cost of searching again. As a result, a small (specialist) retailer is only searched by consumers who could not find a suitable product at the generalist, which it then exploits by charging relatively high prices. This prediction does not rely on differences in production costs, and is consistent with anecdotal evidence that larger firms are cheaper. This link between store size and pricing has also been confirmed empirically by, for example, Asplund and Friberg (2002) for Sweden and by Kaufman et al. (1997) for the US.

In our framework a merger influences retailers' profits in two distinct ways. Firstly there is a "search effect": when two single-product firms which supply different products merge, they provide one-stop shopping convenience and so are searched by more consumers. This increases the merged entity's sales of each product relative to the pre-merger situation. Secondly though, there is also a "price competition effect", because as the market structure changes so do retailers' optimal prices. In particular we show that the asymmetric market structure usually leads to the weakest price competition. Intuitively the small retailers are able to identify and exploit 'niche' consumers who do

[3]In our model consumers have both shopping frictions and imperfect information. This is a plausible assumption in many retail markets, where prices and product varieties may vary over time. An alternative modeling approach would be to have shopping frictions but perfect consumer information e.g. a framework like in Armstrong and Vickers (2010). We anticipate that our main result would still hold in such a setting, however the analysis (when there is at least one multiproduct firm) would not be simpler since the model would then resemble one of competitive mixed bundling.

not like the large retailer's products. As a result the two types of firm attract different types of clientele, and so are relatively well differentiated. One implication of this is that there is always at least one multiproduct firm in equilibrium, since the first pair of firms to merge benefit both from a higher demand and softer competition. More interestingly, we find that the size of the search friction determines whether or not a second merger occurs. In particular, if the second pair of firms merge, they too offer one-stop shopping convenience and so win back some demand, but also reduce market segmentation and so induce fiercer competition. We show that for a large search friction the first effect is more important and so a second merger occurs, however when the search friction is smaller the second effect dominates and the market ends up asymmetric. In the latter case, even the remaining small firms can do better than in the initial fragmented market.

To illustrate some of our results, first consider the merger between Amazon and Whole Foods. As one commentator put it "Adding groceries to its repertoire gets Amazon that much closer to being a one-stop destination for everything you buy... [and] reinforces the behavior by which customers search for things to buy on Amazon.com, rather than on a search engine like Google" i.e. the merger is designed to make Amazon *the* place to start shopping.[4] Similarly, and in line with our search effect whereby the merger should boost demand for each product, the commentator continues "... every Whole Foods customer [will have] to strongly consider signing up for Amazon Prime, which turns Amazon into their de facto source not only for online retail but for instant video and other media".[5] Second, our model suggests that firms may choose to remain small and target a particular customer niche, thereby avoiding tough competition with already-established large firms. In line with this, one writer noted that "Most of the mom-and-pop bookstores today have a little different clientele than Amazon" and so are relatively immune from further competition with it.[6] Along the same lines, and consistent with our model, Igami (2011) shows empirically that entry of very large supermarkets in Tokyo is bad for medium-sized incumbents but good for small stores. One interpretation given in the paper is precisely the idea that large supermarkets have a similar clientele to the former, but attract different consumers compared to the latter.[7] Third we also note that our model is consistent with, but gives an alternative perspective on, the emergence of the so-called long tail of niche providers on the internet. In particular our model suggests that when search costs are

[4]See https://goo.gl/25oF9e

[5]Quantitative evidence for our search effect comes from Sen et al. (2013), who find that when a supermarket adds a gas station it enjoys 14% more shopping trips and 7.7%-9.3% more spending on grocery items.

[6]See https://goo.gl/Ck3iTj

[7]Our model predicts that the effect on small firms is ambiguous and should depend on market conditions such as search frictions, and indeed evidence is mixed (see, e.g., Jia, 2008).

small, an asymmetric market structure with many niche firms is a natural consequence of firms' attempts to avoid fierce competition.

Finally our paper derives some novel predictions about how changes in search frictions affect competition and consumer welfare. Early predictions that the internet would herald a new era of 'frictionless commerce' have proved to be unfounded. We show that this is unsurprising once market structure is endogenized, since lower search frictions prompt a reorganization of the supply-side in such a way that prices may actually rise and welfare fall.

Related literature: Our search model with product differentiation builds on Wolinsky (1986) and Anderson and Renault (1999). These papers only study single-product search. We extend them to the multiproduct case where consumers need and firms (may) supply multiple products.

There is a growing literature on multiproduct consumer search. Lal and Matutes (1994) show that multiproduct search can lead to loss-leader pricing when some products are advertised. McAfee (1995) and Shelegia (2012) examine when and how multiproduct firms correlate their prices across products when consumers are heterogeneously informed.[8] Zhou (2014) investigates how multiproduct search generates a joint search effect, which creates complementarity between physically independent products such that multiproduct firms have a higher incentive to reduce their prices than single-product firms. Rhodes (2015) studies the relationship between the size of a retailer's product range, its pricing, and its advertising decision. He shows that a multiproduct retailer's low advertised prices can signal low prices on its unadvertised products. However all these papers assume an exogenously given market structure where each firm sells the same range of products. We depart from this literature by endogenizing market structure, and show that an asymmetric market structure can emerge as an equilibrium outcome.

There is also research on multiproduct firms and endogenous market structure when consumers have perfect information about firm offerings. Typically these papers consider a duopoly model where each firm can choose which varieties of a product to supply. The varieties are either horizontally differentiated (e.g. Shaked and Sutton, 1990), or vertically differentiated (e.g. Champsaur and Rochet, 1989), or both (e.g. Gilbert and Matutes, 1993). However in these papers there is no notion of one-stop shopping convenience, and moreover an asymmetric market with both large and small firms does not usually arise in equilibrium. (See Manez and Waterson, 2001 for a survey of this literature.) There are

[8]See also Baughman and Burdett (2015) and Kaplan et al. (2016) for more recent work in this direction. The former shows that assuming no consumer recall can greatly simplify the analysis of multiproduct search with price dispersion. The latter offers a search model with high and low valuation consumers which can explain relative price dispersion across retailers.

also papers on multiproduct competition which introduce shopping frictions whilst maintaining the assumption of perfectly informed consumers. However they typically assume an exogenous symmetric market where two firms supply the same range of products (e.g. Lal and Matutes, 1989, Klemperer, 1992, and Armstrong and Vickers, 2010).[9]

Also related is the literature on agglomeration. Baumol and Ide (1956) argue that larger retailers may attract more demand, because consumers are more willing to incur the time and transportation costs necessary to visit them. Stahl (1982) shows that due to a similar demand expansion effect, single-product firms have an incentive to co-locate (e.g. in a shopping mall) provided their products are not too substitutable. In a search environment firms may locate near each other either to offer consumers a higher chance of a good product match (Wolinsky, 1983), or as a way of guaranteeing consumers that they will face low prices (Dudey, 1990 and Non, 2010). Moraga-González and Petrikaitė (2013) show that when a subset of firms with differentiated versions of a product merge and sell all their products in a single shop, they become prominent and are searched first by consumers. However in all these papers consumers buy only one product, and so any one-stop shopping convenience does not arise from consumers' need to buy multiple products, even though this seems an important feature of many retail markets.

Finally, our paper is also related to the literature on conglomerate mergers (i.e., mergers between firms supplying different non-competitive products).[10] Since conglomerate mergers do not eliminate competitors and may generate cost synergies, economists and policymakers (especially in the US) often hold a benign view (see Church, 2008 for a survey). However our model shows that conglomerate mergers have a potential anti-competitive effect and can harm consumers. In independent and concurrent work, Chen and Rey (2015) examine conglomerate merger using a different framework. They find that conglomerate merger can also soften price competition, but that it benefits consumers (at least when bundling is infeasible). In addition, due to their modelling assumptions a second conglomerate merger is never profitable because it leads to Bertrand competition.[11]

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[9]See also Johnson (2017) for a multiproduct competition model where the market friction is that consumers are boundedly rational and make unplanned purchases. Section 3 of his paper considers an asymmetric market where one firm is exogenously able to carry more products than another.

[10]There are two types of conglomerate merger. One involves firms producing totally unrelated products (e.g. computers and tissues). The other involves firms producing complementary products, or products which belong to a range of products that are generally purchased by the same set of consumers. (See for example the EU guidelines on non-horizontal mergers.) The merger in our model is of the second type.

[11]Nalebuff (2000) and Thanassoulis (2011) come to a similar conclusion using different models where firms can use bundling. They argue that if all single-product firms merge and form multiproduct firms, the resulting bundle-against-bundle competition is so fierce that all firms are harmed. Hence they argue that an an asymmetric market will usually arise in equilibrium, though for different reasons compared to both us and Chen and Rey (2015).

The rest of the paper proceeds as follows. Section 2 outlines the main model, Section 3 characterizes the pricing equilibrium in various market structures, and Section 4 derives the equilibrium market structure. Section 5 discusses the robustness of our main results. Section 6 concludes with a discussion of some further extensions and managerial implications. All omitted proofs are available in the appendix.

2 The Model

There are a large number of consumers in the market with measure normalized to one. Each consumer is interested in buying two different products 1 and 2. Initially there are four single-product firms in the market: two of them, denoted by 1_A and 1_B, sell product 1, and the other two, denoted by 2_A and 2_B, sell product 2. The marginal cost of supplying each product is normalized to zero.

Each product is horizontally differentiated across its two sellers. For example, the sellers supply two different brands of the product with different styles. If a consumer buys product i_k, $i = 1, 2$ and $k = A, B$, she obtains utility $v_i + u_{ik}$. Here v_i is the basic valuation for product i and is the same for all consumers, and u_{ik} is a consumer specific match utility of product i_k and is a random draw from the distribution $G_i(u)$ with support $[\underline{u}_i, \overline{u}_i]$ and density $g_i(u)$. The realization of u_{ik} is i.i.d. across consumers, which reflects for example consumers' idiosyncratic tastes, and is also i.i.d. across k, which implies no systematic quality differences across the two variants of each product. This random utility approach for product differentiation is developed in Perloff and Salop (1985).[12] For simplicity, we also suppose the two products are symmetric so that $v_1 = v_2 = v$ and $G_1 = G_2 = G$, and their match utilities are independent of each other. Consumers have unit demand for each product, i.e. they will only buy a unit of one version of each product. If a consumer buys two products with match utilities u_1 and u_2 respectively and makes a total payment P, she obtains a surplus $2v + u_1 + u_2 - P$.

As we describe in more detail below, consumers would benefit from the one-stop shopping convenience from visiting a multiproduct firm which supplies both products. We consider a two-stage game. At the first stage, each pair of firms (1_k,2_k), $k = A, B$, which supply different products, has the opportunity to merge and form a multiproduct firm.[13]

[12] Product differentiation could alternatively be modeled using a spatial model, for example a Hotelling line. However in spatial models match utilities are correlated across firms, which makes search analysis less tractable. This is why the search literature with differentiated products (e.g. Wolinsky, 1986, and Anderson and Renault, 1999) uses the random utility framework with independent match utilities.

[13] Equivalently one firm has the opportunity to acquire the other. We assume that horizontal merger between two firms selling the same product is not permitted (or is too costly), for instance due to antitrust policy.

Their merger decisions can be simultaneous (in which case we focus on pure-strategy equilibria) or sequential. We assume that merger is costless and does not affect consumer preferences for each product or the marginal cost of supplying each product.[14] At the second stage, after observing the market structure firms simultaneously choose their prices and consumers search and make their purchases. We assume that multiproduct firms charge separate prices for each product and do not use more advanced pricing strategies such as bundling,[15] and consumers can multi-stop shop.

Consumers know whether a firm is single-product or multiproduct, but they initially have imperfect information about the (actual) prices firms are charging and the match utilities of all products. They only know the match utility distribution $G(u)$ and hold rational expectations about each firm's pricing strategy. However they can learn a firm's prices and match utilities by incurring a search cost $s > 0$. To capture the idea of one-stop shopping convenience, we will assume that visiting a multiproduct firm costs the same as visiting a single-product one. The search process is sequential, and as is standard in the literature consumers are assumed to have costless recall, i.e., they can to go back to a previously visited firm to buy its products without paying any extra costs.[16] This setup is based on the classic single-product search framework developed in Wolinsky (1986) and Anderson and Renault (1999).

Following Perloff and Salop (1985) and Anderson and Renault (1999), we assume that the basic valuation for each product v is sufficiently large so that all consumers buy both products, i.e. each product market is fully covered. As we often see in the literature on oligopolistic competition, this assumption simplifies our analysis but does not affect the basic insights. Given full market coverage, v does not affect our analysis and so we will henceforth ignore it. Full market coverage also implies that consumers visit at least one firm for each product. To have effective competition in each possible market structure, it must be the case that some consumers visit a second firm to compare products. To ensure this happens we will assume that the search cost is not too high:

$$s < \int_{\underline{u}}^{\overline{u}} (u - \underline{u}) dG(u) \equiv \overline{s} \ . \tag{1}$$

That is, even if a consumer is only looking for one product, if she finds the lowest possible match utility at the first firm, she has an incentive to search the second firm when they

[14] In practice mergers may be costly to propose, but could also generate economies of scope and therefore long-term cost savings. We assume this away to highlight the effect of one-stop shopping convenience.

[15] Indeed in most retail markets we do not observe store-wide bundling.

[16] The assumption of costless recall makes sense when, for example, the search cost is a product inspection cost, or it is a travel cost but consumers can call back a previously visited store to place an order. Introducing a small return cost would significantly complicate the analysis, but would not affect our main insights.

both have the same (expected) price.

Both consumers and firms are assumed to be risk neutral. We use the concept of Perfect Bayesian Equilibrium to analyze the second stage of the game where the market structure is given. Each firm sets its prices to maximize profits, given its expectation of consumers' search behavior and other firms' pricing strategies. Consumers search optimally to maximize their expected surplus, given their rational beliefs about firms' pricing strategies (which of course depend on how many firms are single-product or multiproduct). In addition, even if a consumer searches a firm and observes off-equilibrium price(s), she still holds the equilibrium belief about the unsampled firms' prices.[17]

Finally, we assume that the density function $g(u)$ is strictly log-concave. (This implies that $1-G(u)$ is strictly log-concave as well, or equivalently G has an increasing hazard rate $\frac{g(u)}{1-G(u)}$.) This regularity condition is standard in the random utility oligopoly literature. It is satisfied by, for example, the power distribution with $G(u) = u^\beta$ and $\beta \geq 1$ (which includes the uniform distribution), and many other commonly used distributions such as normal, logistic and extreme value, as well as any truncated versions of them. (See Bagnoli and Bergstrom, 2005 for a comprehensive list of log-concave distributions).

3 Pricing Under Different Market Structures

To examine the equilibrium market structure, we first derive the pricing equilibrium in each possible market structure. There are three market structures to consider: (i) if no merger has occurred, a fragmented market with four independent single-product firms, (ii) if only one pair of firms has merged, an asymmetric market with one multiproduct firm and two single-product firms, and (iii) if both pairs of firms have merged, a symmetric market with two multiproduct firms. We compare prices across market structures in the end of this section.

3.1 A fragmented market with four single-product firms

With four single-product firms, a consumer's search process is completely separable across the two product markets. In each market we have a duopoly version of the sequential search model in Anderson and Renault (1999). Consider the market for product i. Given the two firms are symmetric, following the tradition in the search literature we look for a symmetric equilibrium where both firms charge the same price p_0 and consumers search in a random order (i.e. half of the consumers visit firm i_A first and the other half visit

[17] Notice that in our model there are no economic shocks which are correlated across firms (such as aggregate cost shocks), and so their pricing decisions are independent of each other.

firm i_B first).[18]

To derive the equilibrium price for product i, suppose firm i_A unilaterally deviates and charges a price p_0' while firm i_B charges the equilibrium price. Notice that consumers do not observe this price deviation before they search, so this does not affect their search order. Firm i_A's demand has three sources: consumers who visit i_A first and buy immediately, consumers who visit i_A first but continue to search i_B and then return to buy at i_A, and consumers who visit i_B first and continue to search and buy at i_A.

Consider a consumer who visits i_A first. Suppose she finds match utility u_{iA}. Given she holds the belief that firm i_B charges the equilibrium price p_0, she will stop searching and buy immediately at i_A if and only if

$$\int_{u_{iA}-p_0'+p_0}^{\overline{u}} [u_{iB} - p_0 - (u_{iA} - p_0')]dG(u_{iB}) \leq s \ ,$$

where the left-hand side is the incremental benefit from sampling the second firm i_B, since if the consumer continues to search she will buy there if $u_{iB} - p_0 \geq u_{iA} - p_0'$ and will otherwise return and buy from i_A. Define a as the solution to

$$\int_{a}^{\overline{u}} (u - a)dG(u) = s \ . \tag{2}$$

(The left-hand side is decreasing in a, so this equation has a unique solution $a \in (\underline{u}, \overline{u})$ given $s \in (0, \overline{s})$, and a is decreasing in s.) Then the consumer will stop searching and buy at i_A immediately if $u_{iA} - p_0' \geq a - p_0$. (Here $a - p_0$ is interpreted as the reservation surplus above which a consumer will stop searching. If firms charge the same price, a is the reservation match utility above which a consumer will stop searching.) This generates the first portion of firm i_A's demand

$$\frac{1}{2}[1 - G(a - p_0 + p_0')] \ . \tag{3}$$

If the consumer finds $u_{iA} - p_0' < a - p_0$ at i_A, she will continue to search i_B but will return and buy at firm i_A if $u_{iB} - p_0 < u_{iA} - p_0'$. This generates the second portion of firm i_A's demand

$$\frac{1}{2}\Pr[u_{iB} - p_0 < u_{iA} - p_0' < a - p_0] = \frac{1}{2}\int_{\underline{u}}^{a-p_0+p_0'} G(u_{iA} - p_0' + p_0)dG(u_{iA}) \ . \tag{4}$$

Now consider a consumer who visits firm i_B first. Since firm i_B is charging the equilibrium price p_0, and since the consumer holds an equilibrium belief about firm i_A's price,

[18] We will discuss asymmetric equilibrium in a symmetric market (with either four single-product firms or two multiproduct firms) and its implication for the equilibrium market structure in Section 5.

she will come to visit i_A if $u_{iB} < a$. She will be surprised by i_A's deviation price but will still buy at i_A if $u_{iA} - p_0' > u_{iB} - p_0$. This generates the third portion of firm i_A's demand

$$\frac{1}{2}\Pr[u_{iB} < a \text{ and } u_{iB} - p_0 < u_{iA} - p_0'] = \frac{1}{2}\int_{\underline{u}}^{a} [1 - G(u_{iB} - p_0 + p_0')]dG(u_{iB}) \ . \quad (5)$$

Let $Q_0(p_0')$ be firm i_A's demand at price p_0' when i_B is charging the equilibrium price. It equals the sum of (3)–(5). Notice that $Q_0(p_0) = \frac{1}{2}$ due to firm symmetry and full market coverage. Then one can check that the first-order condition for p_0 to be the equilibrium price is[19]

$$\frac{1}{p_0} = -\frac{Q_0'(p_0)}{Q_0(p_0)} = g(a)[1 - G(a)] + 2\int_{\underline{u}}^{a} g(u)^2 du \ . \quad (6)$$

In equilibrium each firm earns a profit $\pi_0 = \frac{1}{2}p_0$.

To illustrate, consider the uniform distribution example with $G(u) = u$. Then we have $s \in (0, \frac{1}{2})$ from (1), and $a = 1 - \sqrt{2s} \in (0, 1)$ from (2). Using (6) we derive

$$p_0 = \frac{1}{2 - \sqrt{2s}} \ . \quad (7)$$

It increases from $\frac{1}{2}$ to 1 when s increases from 0 to $\frac{1}{2}$.

3.2 An asymmetric market

Now suppose 1_A and 2_A merge into a multiproduct firm A, while 1_B and 2_B remain independent. Since the multiproduct firm A offers one-stop shopping convenience, we look for an equilibrium where all consumers visit it first. In this case we sometimes say firm A is the prominent firm. (We will discuss the possibility of other equilibria later on.) Let p_A be the multiproduct firm's price for each of its products and p_B be each single-product firm's price. Notice that we need to verify that it is indeed rational for consumers to visit firm A first after taking into account the price difference between the firms.

We first notice that given the cost of visiting each single-product firm is separable, a consumer's search decision when she is at the multiproduct firm is also separable between the two products. Formally, if the multiproduct firm A charges p_A' for product i, a consumer who finds match utilities (u_{1A}, u_{2A}) will continue to visit the single-product firm i_B if and only if the expected benefit of doing so $\int_{u_{iA}-p_A'+p_B}^{\overline{u}} [u_{iB} - p_B - (u_{iA} - p_A')]dG(u_{iB})$ is greater than the search cost s, regardless of u_{jA} ($j \neq i$) and the multiproduct firm's price

[19] If $p[1 - G(p)]$ is concave, the first-order condition is also sufficient for defining the equilibrium price. (See Appendix B in Anderson and Renault (1999) for other conditions which ensure the existence of a symmetric pure-strategy pricing equilibrium.) Simple calculation shows that p_0 decreases in a (so increases in s) if $1 - G$ is log-concave.

for the other product. This is exactly the same as in the single-product search discussed before. Therefore, a consumer will continue to search i_B if and only if $u_{iA} - p'_A < a - p_B$, where a is defined in (2). The only difference compared to the fragmented market is that now consumers do not search randomly.[20]

Consider the market for product i. The demand for the multiproduct firm's product, if it charges p'_A while its single-product rival i_B sets the equilibrium price p_B, is

$$[1 - G(a - p_B + p'_A)] + \int_{\underline{u}}^{a-p_B+p'_A} G(u_{iA} - p'_A + p_B) dG(u_{iA}) \ . \tag{8}$$

Given all consumers visit firm A first, the first term is from the consumers who find $u_{iA} - p'_A \geq a - p_B$ and so buy immediately. This is similar to (3) in the previous case. The second term is from the consumers who find $u_{iA} - p'_A < a - p_B$ and so continue to search firm i_B but eventually return to firm A. This is similar to (4) in the previous case.

Demand for firm i_B's product, if it charges price p'_B while firm A sets its equilibrium price p_A, is

$$\int_{\underline{u}}^{a-p_B+p_A} [1 - G(u_{iA} - p_A + p'_B)] dG(u_{iA}) \ . \tag{9}$$

Consumers will come to visit i_B only if $u_{iA} - p_A < a - p_B$ given they hold the equilibrium belief about firm i_B's price, and will then buy from i_B if $u_{iB} - p'_B > u_{iA} - p_A$. This is similar to (5) in the previous case.

Define $\Delta \equiv p_B - p_A$ and

$$Q(\Delta) \equiv 1 - \int_{\underline{u}}^{a-\Delta} [1 - G(u + \Delta)] dG(u) \ . \tag{10}$$

Notice that $Q(\Delta)$ is the equilibrium demand for firm A's product i (i.e. (8) evaluated at $p'_A = p_A$), and $1 - Q(\Delta)$ is the equilibrium demand for firm i_B (i.e. (9) evaluated at $p'_B = p_B$). Due to full market coverage, the equilibrium demands depend only on the price difference Δ. Notice that $Q(0) = \frac{1}{2} + \frac{1}{2}[1 - G(a)]^2 > \frac{1}{2}$. This confirms that when the two firms charge the same price, firm A has a higher demand since it is prominent. Since $Q(\Delta)$ increases in Δ, firm A's demand will be even higher if it charges a lower price than i_B.

Using the introduced notation, one can verify that the first-order conditions for the equilibrium prices (p_A, p_B) are[21]

$$p_A = \frac{Q(\Delta)}{Q'(\Delta)} \tag{11}$$

[20] In this sense the situation is similar to the non-random search studied in Armstrong, Vickers and Zhou (2009). They do not assume full market coverage but focus on the uniform distribution case, while we deal with a general distribution under the assumption of full market coverage.

[21] As in the case with four single-product firms, the first-order conditions are also sufficient for defining the equilibrium prices if $p[1 - G(p)]$ is concave.

and

$$p_B = \frac{1 - Q(\Delta)}{Q'(\Delta) - g(a-\Delta)[1 - G(a)]} \ . \tag{12}$$

In equilibrium firm A's per product profit is $\pi_A = p_A Q(\Delta)$ and each single-product firm's profit is $\pi_B = p_B(1 - Q(\Delta))$.

The analysis so far implicitly assumes that $a - \Delta > \underline{u}$ and that it is optimal for consumers to visit firm A first. The following result shows that this is indeed the case under our log-concavity assumption. (All omitted proofs can be found in the appendix.)

Lemma 1 *For any $s \in (0, \overline{s})$, the system of (11) and (12) has a solution $\Delta \in (0, a - \underline{u})$, and $\lim_{s \to 0} \Delta = \lim_{s \to \overline{s}} \Delta = 0$.*

When $\Delta > 0$ (so $p_A < p_B$), the consumer search order is indeed optimal, since the multiproduct firm both provides one-stop shopping convenience and offers lower prices. This proves existence of the proposed equilibrium where all consumers visit the multiproduct firm first. The intuition for $p_A < p_B$ is as follows. Consumers are more likely to buy at the first firm they visit because search is costly. Therefore only those who are rather unsatisfied with the multiproduct firm's products will choose to visit the single-product firms. As a result, consumers who visit a single-product firm must on average prefer its product over the multiproduct firm's. In this sense we can interpret the multiproduct firm as a generalist, and the single-product firms as specialists who only serve consumers with strong preferences for their products. This implicit market segmentation gives single-product firms some extra market power to charge a higher price. This prediction fits the casual observation that large retailers tend to be cheaper than small ones. Notice that this holds in our model even if the multiproduct firm does not have any cost advantage, and so it complements the usual cost-based explanation. The price difference Δ disappears in the two limit cases because both the multiproduct firm and the two single-product firms will act either as in the perfect information case, or as in the monopoly case respectively.

It is worth mentioning that although the multiproduct firm is cheaper than its smaller competitors, it earns a higher profit from each product (i.e. $\pi_A > \pi_B$) due to its higher demand. This can be seen from a revealed preference argument. Suppose for product i the multiproduct firm A privately deviates and charges the same price p_B as the single-product firm i_B. Then its profit from product i will drop to $p_B Q(0)$, but it is still greater than $\pi_B = p_B(1 - Q(\Delta))$ since $Q(0) > \frac{1}{2} > 1 - Q(\Delta)$.

To illustrate the equilibrium prices, consider again the uniform distribution example. Equations (11) and (12) simplify to

$$p_A = \frac{1}{1-\Delta}[1 - a + \Delta + \frac{1}{2}(a^2 - \Delta^2)], \quad p_B = 1 - \frac{1}{2}(a + \Delta) \ ,$$

where $a = 1 - \sqrt{2s}$. The system has a unique solution:

$$p_A = \frac{1}{16}(3K - 5a - 5), \quad p_B = \frac{1}{16}(K - 7a + 9) \ , \tag{13}$$

where $K \equiv \sqrt{17a^2 - 30a + 49}$. Both prices increase from $\frac{1}{2}$ to 1 when s increases from 0 to $\frac{1}{2}$.

Discussion of other possible equilibria: In the asymmetric market, there is no equilibrium where the multiproduct firms charge the same price as its single-product rivals. Otherwise, all consumers would visit the multiproduct firm first because of its one-stop shopping convenience, and this search order would not support symmetric pricing across firms. On the other hand, following a similar argument for $p_A < p_B$, in principle it is possible to have an alternative equilibrium in which consumers visit the two single-product firms first and they therefore charge lower prices than the multiproduct firm. However for this to be an equilibrium, the price difference has to be large enough to compensate consumers for the extra search cost incurred by visiting the two single-product firms first. It is hard to rule out this alternative equilibrium in general, but we can do it numerically in the uniform distribution case.

3.3 A symmetric market with two multiproduct firms

Now suppose both pairs of firms merge and we have two symmetric multiproduct firms A and B. This is a multiproduct search model studied in Zhou (2014). We look for a symmetric equilibrium where both firms charge the same price p_m for each product and consumers search in a random order (i.e. half of the consumers visit firm A first and the other half visit firm B first).

We first explain the optimal stopping rule in multiproduct search. Consider a consumer who visits firm A first. After finding out firm A's prices and match utilities, she faces the following three options: stop searching and buy both products immediately, or buy one product first and keep searching for the other, or keep searching for both products. However, in our model the second option is always dominated by the third: if the consumer decides to search firm B, given the search cost is always s regardless of how many products she is still searching for, and given that she will have costless recall thereafter and so can freely mix and match among the two firms, she should keep searching for both products. Therefore, if the two firms charge the same prices, the consumer will stop searching and buy both products immediately at firm A if its match utilities (u_{1A}, u_{2A}) satisfy

$$\int_{u_{1A}}^{\overline{u}} (u_{1B} - u_{1A}) dG(u_{1B}) + \int_{u_{2A}}^{\overline{u}} (u_{2B} - u_{2A}) dG(u_{2B}) \leq s \ , \tag{14}$$

where the left-hand side is the expected benefit from sampling firm B. Otherwise, she should buy none of the products and continue to search. The equality of (14) defines a reservation frontier $u_{2A} = \phi(u_{1A})$ in the match utility space $[\underline{u}, \overline{u}]^2$ as depicted in Figure 1 below: if the match utilities (u_{1A}, u_{2A}) at firm A are in the stopping region above this frontier, the consumer should buy both products immediately, and otherwise she should continue to visit firm B. It is clear that the reservation frontier $\phi(\cdot)$ should be decreasing and satisfy $\phi(a) = \overline{u}$ where a is defined in (2). (It can also be shown that $\phi(\cdot)$ must be convex.)

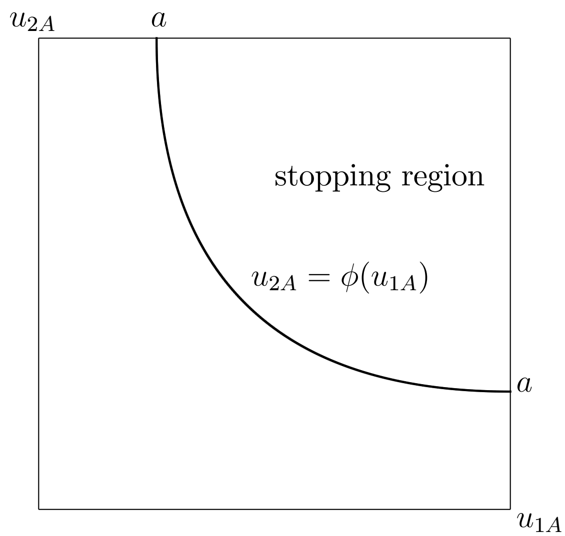

Figure 1: The reservation frontier in multiproduct search

Though logically similar to single-product search, the analysis in multiproduct search is technically more involved. We refer the reader to Zhou (2014) for the details of demand analysis and how to derive the equilibrium price. The first-order condition for p_m to be the equilibrium price is[22]

$$\frac{1}{p_m} = \int_a^{\overline{u}} [2 - G(u_1) - G(\phi(u_1))] g(\phi(u_1)) g(u_1) du_1 + 2 \int_{\underline{u}}^{\overline{u}} \int_{\underline{u}}^{\phi(u_1)} g(u_2)^2 g(u_1) du_2 du_1 \ , \quad (15)$$

where $\phi(u_1) = \overline{u}$ for $u_1 \in [\underline{u}, a]$. In equilibrium firms share the market equally, so each firm's per product profit is $\pi_m = \frac{1}{2} p_m$.

In the uniform distribution example, the reservation frontier solves

$$(1 - u_1)^2 + (1 - \phi(u_1))^2 = 2s \ ,$$

so it is the arc of a quarter-circle with a radius $\sqrt{2s}$. Then the double integral in (15) is just 1 minus the area of the stopping region which is $\frac{1}{4}\pi(\sqrt{2s})^2 = \frac{1}{2}\pi s$, where $\pi \approx 3.14$ is

[22] As explained in Zhou (2014), for many common distributions (including the uniform distribution) the first-order condition is also sufficient for defining the equilibrium price.

the mathematical constant. So the second term in (15) equals $2-\pi s$. The first term can be decomposed into two parts: $\int_a^1[1-\phi(u)]du + \int_a^1(1-u)du$. The first part is the area of the stopping region so equals $\frac{1}{2}\pi s$, and the second part equals s by the definition of a. Therefore, the equilibrium price is

$$p_m = \frac{1}{2-(\frac{\pi}{2}-1)s} \ . \tag{16}$$

It increases from $\frac{1}{2}$ to about 0.583 when s increases from 0 to $\frac{1}{2}$.

A main feature in multiproduct search is a "joint-search effect". When a firm reduces one product's price, more consumers who visit it first will stop searching. Once they stop searching, as we explained before they will buy both products. In other words, reducing one product's price increases the demand for the other product as well. This makes the two products like complements even if they are physically independent, inducing each firm to price more aggressively than in single-product search.[23] We formally compare price between single-product and multiproduct search in next subsection.

3.4 Price comparison

We now compare prices across the three market structures.

Proposition 1 *(i)* $p_m < p_0 < p_B$ *for any* $s \in (0, \overline{s})$.
(ii) Suppose $g(\underline{u}), g(\overline{u}) > 0$. *There exist* $0 < s_1 < s_2 < \overline{s}$ *such that (a) for* $s < s_1$, $p_m < p_0 < p_A < p_B$ *if*

$$g(\underline{u})^2 + g(\overline{u})^2 > 2\left(\int_{\underline{u}}^{\overline{u}} g(u)^2 du\right)^2 \ , \tag{17}$$

and $p_m < p_A < p_0 < p_B$ *if the opposite strict inequality holds; (b)* $p_m < p_0 < p_A < p_B$ *for* $s > s_2$.
(iii) $p_m < p_0 < p_A < p_B$ *for any* $s \in (0, \overline{s})$ *in the uniform distribution case with* $G(u) = u$.

The first result in Proposition 1(i) is that price competition between two multiproduct firms is fiercer than between two single-product firms. For instance, in the uniform example with $s = 0.1$, we have $p_m \approx 0.51$ which is lower than $p_0 \approx 0.64$ by about 20%. Two factors drive this result. First, since searching a multiproduct firm is as costly as searching a single-product firm, more consumers are willing to sample both firms in the multiproduct firm case. This intensifies price competition. Second, the joint-search effect

[23] As discussed in Zhou (2014), this joint-search effect can increase in s such that the equilibrium price in multiproduct search can decrease in s even under the log-concavity condition.

in the multiproduct firm case further drives price down.[24] The second result in Proposition 1(i) is that p_B is the highest price across all three market structures since we already know $p_A < p_B$.

However, it is hard to compare p_A with p_0 and p_m in general. More progress can be made when s is small or large as shown in Proposition 1(ii). In both cases, the price in the symmetric case with two multiproduct firms is always the lowest, but when s is small the comparison between p_A and p_0 depends on the details of the match utility distribution. One can check that (17) holds, for example, for any linear density function defined on $[0, 1]$ except for the uniform one,[25] but it can also be easily voilated, for example, when both $g(\underline{u})$ and $g(\overline{u})$ are close to 0. Proposition 1(iii) rank prices in the uniform distribution example, and they are depicted in Figure 2 below (where from top to bottom the curves are p_B, p_A, p_0 and p_m).

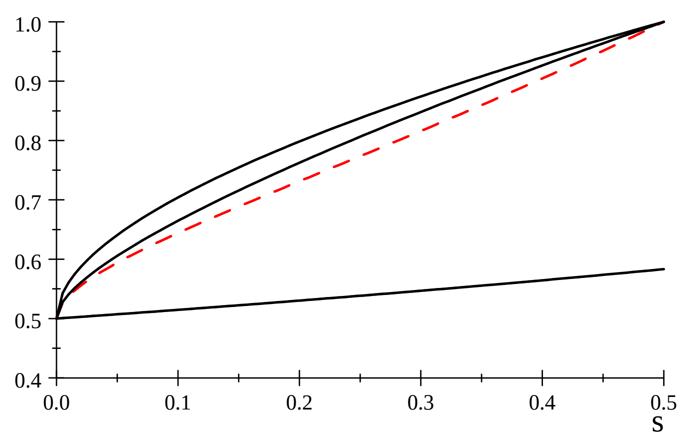

Figure 2: Price comparison across market structures (uniform distribution)

One main observation from this price comparison is that the price competition can be the weakest in the asymmetric market. Intuitively, this is because the small firms in the asymmetric market charge high prices due to the market segmentation logic explained before, and by strategic complementarity this can even induce the multiproduct firm to

[24]This joint-search effect is so strong that p_m can be lower than p_0 even if visiting a multiproduct firm is twice as costly as visiting a single-product firm. Consider the uniform example, and suppose that visiting a single-product firm only costs $\frac{s}{2}$ (in which case the search cost condition (1) becomes $s < 1$). It is easy to check that $p_m < p_0$ for any $s \in (0, 1)$. For example when $s = 0.1$, $p_m \approx 0.51$ is still lower than $p_0 \approx 0.59$ by about 13.5%.

[25]For the uniform distribution, the equality of (17) holds and so the first-order approximations of p_0 and p_A are equal when s is small such that they cannot guide the comparison of p_0 and p_A. But result (iii) shows that $p_0 < p_A$ in the uniform case.

set high prices as well. As we will show in next section, due to this price competition effect even the small firms in the asymmetric market can earn more than in the initial fragmented market. In other words, the first merger can benefit all firms.

4 Equilibrium Market Structure

We are now ready to investigate the equilibrium market structure. Notice that the equilibrium structure will be the same regardless of whether the firms make their merger decisions simultaneously (in pure strategies) or sequentially. We start with the following useful observations:

Lemma 2 *(i) $\pi_0 < \pi_A$ for any $s \in (0, \overline{s})$, and so the fragmented market structure with four single-product firms is never an equilibrium outcome.*
(ii) Then if $\pi_B \geq \pi_m$, the equilibrium market structure is asymmetric with a multiproduct firm and two single-product firms, while if $\pi_B < \pi_m$, the equilibrium market structure is symmetric with two multiproduct firms.

Result (i) is immediately implied by $p_B > p_0$ from Proposition 1(i). Notice that in the asymmetric market structure, firm A could always unilaterally deviate by charging p_B and get a profit $p_B Q(0)$, which is greater than $\pi_0 = \frac{p_0}{2}$ given $p_B > p_0$ and $Q(0) > \frac{1}{2}$. Hence by revealed preference, when firm A charges its equilibrium price, its profit strictly exceeds π_0. Result (ii) is obvious.

It is hard to fully characterize the equilibrium market structure for a general distribution. Nevertheless progress can be made under mild conditions if the search cost s is sufficiently small or large, and we can also fully solve for the equilibrium market structure in the uniform distribution case.

Proposition 2 *(i) Suppose $g(\underline{u}), g(\overline{u}) > 0$. There exist $0 < s_3 < s_4 < \overline{s}$ such that the equilibrium market structure is asymmetric if $s < s_3$ and symmetric with two multiproduct firms if $s > s_4$.*
(ii) In the uniform distribution case with $G(u) = u$, there exists $\hat{s} \in (0, \overline{s})$ such that the equilibrium market structure is asymmetric when $s < \hat{s}$ and otherwise symmetric with two multiproduct firms.

Proposition 2 can be explained as follows. Intuitively a merger between a pair of firms leads to two different effects. Firstly, there is a "search effect": the merged entity offers one-stop shopping convenience and so receives a larger demand for its products. Secondly though, there is a "price competition effect": the merger changes the market structure and hence the intensity of competition. At least in the cases covered in the above proposition, starting from the initial situation with four single-product firms, if one pair deviates and merges, both of these effects work in their favor: they secure higher demand and price competition is also softened in the asymmetric market structure. As a result, $\pi_A > \pi_0$ and there will be no equilibrium with the fragmented market structure. However once there is already a multiproduct firm in the market, the second pair faces a trade-off when they contemplate merging. On the one hand, a merger restores symmetry and random search, and therefore wins back some demand. However on the other hand, the merger intensifies competition because as discussed earlier price competition is fiercest when there are two multiproduct firms. As one might expect, the search effect is stronger when the search cost is higher, and so the second pair of single-product firms choose to merge only if s is high enough. Consequently our model predicts that when the search friction is relatively small, the industry settles on an asymmetric market structure. Intuitively the large (generalist) and small (specialist) firms target different consumer segments as a way to soften competition.

Surprisingly, if the search effect is relatively weak, it is possible that the first merger benefits *all* firms in the industry - including those not involved at all in the merger. The following result provides a sufficient condition for that to happen.

Corollary 1 *Suppose $g(\overline{u}) > 0$. There exists $s_5 \in (0, \overline{s})$ such that $\pi_B > \pi_0$ (i.e. the remaining single-product firms benefit from a merger) if $s < s_5$ and (17) holds.*

So far we have assumed that merger is costless. If we introduce a fixed cost of merging, we might expect the asymmetric market structure to arise more often. To illustrate this, in the uniform distribution case we can show that $\pi_A - \pi_0 > \pi_m - \pi_B$ i.e. because the symmetric market with two multiproduct firms is very competitive, a second merger is always less profitable than the first. Consequently if the cost of merging is between $\pi_m - \pi_B$ and $\pi_A - \pi_0$, the market will end up being asymmetric even if $\pi_m > \pi_B$.[26]

4.1 Consumer surplus and total welfare

Finally we compare consumer surplus and total welfare across market structures to see (i) whether they are maximized under the equilibrium market structure, and (ii) how they change with the search friction once we account for endogeneity of market structure.

As a preliminary step, we first develop a method to calculate consumer surplus. This is useful because brute-force calculation of consumer surplus in search models with product differentiation tends to be messy.

[26] Of course if the merger cost is sufficiently high, even the first merger will not happen and the market will end up being fragmented.

Lemma 3 *Under any of the three market structures, the expected consumer surplus in equilibrium equals*

$$V - \int_0^s T(x)dx \ ,$$

where V is consumer surplus when consumers can freely search and compare all options given the equilibrium prices associated with the actual search cost s, and $T(x)$ is the expected number of searches a consumer conducts when the search cost is x given the equilibrium prices associated with the actual search cost s.

Our method of calculating consumer surplus is economically intuitive. It decomposes consumer surplus into two separate parts: one which is only related to match utilities and prices, and the other which is only related to the search cost. Moreover as can be seen from the proof, our method applies for a general case with more firms, and so is interesting in its own right. Total welfare is then the sum of consumer surplus and industry profit.

We use the case of a fragmented market to illustrate how the above method works. Since search is totally separable between the two products, let us consider the search market for product i. Then $V = \mathbb{E}[\max\{u_{iA}, u_{iB}\}] - p_0$ since the consumer will choose the best product i when search is free, and $T(x) = 1 + G(a(x))$, where $a(x)$ is the reservation match utility defined in (2) when the search cost is x, since the consumer will search the second firm if and only if the first firm's match utility is below $a(x)$. Therefore per-product consumer surplus in the fragmented market is

$$CS_0 = \mathbb{E}[\max\{u_{iA}, u_{iB}\}] - p_0 - \int_0^s [1 + G(a(x))]dx \ ,$$

and per-product total welfare is $TW_0 = CS_0 + p_0$. (Since the basic valuation v for each product does not affect the comparison across market structures, we do not include it in the calculation.) In the uniform distribution example, $\mathbb{E}[\max\{u_{iA}, u_{iB}\}] = \frac{2}{3}$ and $a(x) = 1 - \sqrt{2x}$, so one can check that

$$CS_0 = \frac{2}{3} - p_0 - 2s\left(1 - \frac{\sqrt{2s}}{3}\right) \ . \tag{18}$$

Denote by CS_A and TW_A the per-product consumer surplus and total welfare in the asymmetric market, and by CS_m and TW_m the per-product consumer surplus and total welfare in the symmetric market with two multiproduct firms. We then have the following welfare comparison results:

Proposition 3 *(i) $CS_m > CS_A, CS_0$ if the symmetric market with two multiproduct firms has the lowest market prices, and $TW_m > TW_A, TW_0$.*

(ii) In the uniform distribution case with $G(u) = u$, there exists an $\tilde{s} \in (0, \overline{s})$ such that $CS_m > CS_0 > CS_A$ (i.e. the asymmetric market structure is worst for consumers) when $s < \tilde{s}$, and otherwise $CS_m > CS_A \geq CS_0$ (i.e. the fragmented market structure is worst for consumers), while $TW_m > TW_A > TW_0$ for any $s \in (0, \overline{s})$.

It is not surprising that the symmetric market with two multiproduct firms has the highest total welfare. Since the market is fully covered, payments from consumers to firms are just transfers which do not directly affect welfare. Instead, welfare is determined by the efficiency of the match between consumers and products, and also how much consumers end up searching. The symmetric market with two multiproduct firms is therefore better because one-stop shopping convenience induces consumers to spend less on search. Moreover compared to the asymmetric market, it is also better because prices are the same across firms and so consumers make socially efficient search and purchase decisions. If the symmetric market also leads to the lowest industry profit, then it must be the best for consumers. The total welfare comparison between the fragmented market and the asymmetric market is trickier: the former has the advantage of no price dispersion, but the latter has search cost savings from visiting the multiproduct firm. It turns out that at least in the uniform distribution case the search cost effect dominates, and so the asymmetric market is more efficient. For consumer surplus, the asymmetric market has an additional disadvantage since it is most expensive. Consequently, when the search cost effect is relatively weak, the asymmetric market becomes the worst for consumers among all possible market structures.

Two observations immediately follow from Propositions 2 and 3. Firstly, the equilibrium market structure is not necessarily optimal for consumers and total efficiency because the asymmetric market can arise in equilibrium. Secondly, due to the endogeneity of the market structure, reducing the search cost can harm both consumers surplus and social efficiency. This happens when the market structure switches from the symmetric one with two multiproduct firms to the asymmetric one.

5 Robustness Discussion

Asymmetric equilibrium in symmetric markets. In the pricing analysis of the two symmetric market structures, we have focused on a symmetric equilibrium where firms charge the same price in each product market and consumers search in a random order. This is the tradition in the search literature when firms are *ex ante* symmetric. However, as suggested by the analysis of the asymmetric market structure in Section 3.2, there also exist asymmetric equilibria in a symmetric market: if all consumers expect a particular firm to be cheaper, they visit it first such that it optimally charges less.[27] In the following, we discuss this issue and argue that our prediction concerning equilibrium market structure remains qualitatively unchanged even if we consider asymmetric equilibrium in symmetric markets.

In the fragmented market with four single-product firms, the asymmetric equilibrium in each product market is the same as that characterized in the asymmetric market. Recall that the prominent multiproduct firm A earns more on each product than its single-product rivals i.e. $\pi_A > \pi_B$. We therefore have that $\pi_A + \pi_B < 2\pi_A$, which implies that starting with a fragmented market, a prominent firm in product market i and a non-prominent firm in product market $j \neq i$ have an incentive to merge. Consequently the fragmented market structure cannot arise in equilibrium.

Suppose now that a multiproduct firm already exists, and consider whether the remaining pair of single-product firms want to merge. Assuming that following the merger the two multiproduct firms will play an asymmetric equilibrium, we need to distinguish between whether the second pair of firms that merge become prominent or (stay) non-prominent. Firstly, suppose the second pair that merge are non-prominent. Intuitively they are likely to be worse off compared to the symmetric equilibrium which we studied earlier: price competition between two multiproduct firms is still fierce due to the joint search effect, but now the second merger does not restore random search. This suggests that an asymmetric market structure is more likely to arise compared to our earlier analysis with symmetric equilibrium. Secondly though, it is possible that when the second pair of firms merge they become prominent. (However this seems less plausible, because it requires that the merger causes consumers to completely reverse their search order.) In this case, one would expect the asymmetric market structure to arise less often.

The asymmetric equilibrium with two multiproduct firms has not been explored in the literature.[28] The analysis is more complicated than the equilibrium analysis of an asymmetric market that we did before. The details are reported in a separate online appendix. There we derive the first-order conditions for the equilibrium prices and also show a similar result to Proposition 2(i). In particular when $g(\underline{u}), g(\overline{u}) > 0$, the equilibrium market structure is asymmetric when the search cost is sufficiently small, and it is symmetric with two multiproduct firms when the search cost is sufficiently high, regardless of whether the second pair of single-product firms will become prominent or non-prominent after their merger. In the uniform distribution example, there is no simple analytical solution for the asymmetric equilibrium prices. However, numerical simulations suggest a cut-off result like Proposition 2(ii). Specifically, if the firms involved in a sec-

[27] Notice however that this asymmetric equilibrium requires strong coordination among consumers.

[28] Zhou (2014) focuses on the symmetric equilibrium.

ond merger remain non-prominent, the market structure is asymmetric for s smaller than about 0.17, and otherwise has two multiproduct firms. If instead the firms involved in a second merger become prominent afterwards, equilibrium market structure is the same except that the threshold falls to around 0.045. This confirms our intuitive discussion above, because in our earlier analysis where firms played symmetric equilibrium following the second merger, the threshold was about 0.092.

Homogeneous products. Our main model has assumed that each product has two differentiated versions, and so consumers search for both higher product suitability and lower prices. However in some retail markets firms supply very similar or even identical versions of a product, and so consumers care mainly about prices. In an earlier version of this paper, we also study an alternative model with homogeneous products and price dispersion. There we consider the same two-stage game as in the main model, but adopt a different approach to model shopping frictions (in the spirit of Varian (1980)): a fraction $\alpha \in (0, 1)$ of consumers are "shoppers", who can search and multi-stop shop freely, while the remaining fraction $1-\alpha$ of consumers are "non-shoppers", who can visit only one firm (e.g. due to time constraints) and so value the one-stop shopping convenience from having multiproduct firms the market. We show a similar result concerning equilibrium market structure: asymmetric market arises when $1 - \alpha$ (which measures the search friction) is relatively small.

6 Conclusion

This paper offers a tractable framework to study equilibrium retail market structure when consumers buy multiple products and value one-stop shopping convenience. We have shown that the size of the search friction plays an important role in determining the equilibrium market structure. When search frictions are relatively high the market has all large firms. However when search frictions are relatively low, the market is asymmetric with a mix of large and small firms, since this allows firms to target different consumer niches and thereby soften competition. As such, our model provides a simple explanation for the puzzling observation that large and small retailers usually coexist. It is also consistent with anecdotal evidence that online markets - where search costs should typically be lower - are even less symmetric than offline ones.[29] Of course we do not wish to claim that ours is the only explanation for why some retailers may choose to remain small. Other considerations such as financial constraints are also likely to be important. Nevertheless our paper provides a novel strategic explanation which is complementary to those already in the literature.

[29] For example in 2012 Amazon sold more than its top 12 online competitors combined.

Throughout the paper we have chosen to focus on a merger framework. This matches the observation (as detailed earlier in the introduction) that retailers are increasingly growing their assortments through mergers and acquisitions. Nevertheless in an earlier version of the paper, we showed that our insights are robust when firms can choose their product range directly. In particular we considered a homogenous-product model with two products, three firms and a mixture of shoppers and non-shoppers. Firms simultaneously chose which product(s) to stock, with the first one being free but the second incurring a stocking cost. We showed that when a firm contemplated stocking a second product, it faced the same trade-off between search and price competition effects as in our merger framework. We further showed that the number of multiproduct firms was increasing in the search friction, and that for low search costs and an intermediate stocking cost an asymmetric market emerged with one multiproduct firm and two single-products firms selling a different product. Hence the main insights from our merger framework carried over, albeit in a less parsimonious way.

Our paper has several managerial implications. Firstly, when a market already has some large retailers, other firms may find it optimal to remain small so as to soften competition. This is especially true in markets where it is relatively easy for consumers to learn product and price information, for example in markets with high internet penetration. Secondly, large and small retailers should target different types of consumer, and tailor their pricing strategies accordingly. Finally, our model also suggests that the negative effect of lower search costs on industry profits can be mitigated, and even overturned, provided that managers are flexible and able to quickly adjust their product offerings.

Appendix: Omitted Proofs and Details

Proof of Lemma 1. (11) and (12) imply

$$\Delta = \frac{1 - Q(\Delta)}{Q'(\Delta) - g(a-\Delta)[1-G(a)]} - \frac{Q(\Delta)}{Q'(\Delta)} \equiv \Phi(\Delta) \ . \tag{19}$$

Since $Q(0) = \frac{1}{2} + \frac{1}{2}[1-G(a)]^2$, we have $1 - Q(0) = Q(0) - [1-G(a)]^2$. Then

$$\Phi(0) > 0 \Leftrightarrow \frac{Q(0) - [1-G(a)]^2}{Q'(0) - g(a)[1-G(a)]} > \frac{Q(0)}{Q'(0)} \Leftrightarrow \frac{Q(0)}{Q'(0)} > \frac{1-G(a)}{g(a)} \ .$$

This is true because

$$
\begin{aligned}
Q'(0) &= g(a)[1-G(a)] + \int_{\underline{u}}^{a} g(u)dG(u) \\
&= g(a)[1-G(a)] + \int_{\underline{u}}^{a} \frac{g(u)}{1-G(u)}[1-G(u)]dG(u) \\
&< g(a)[1-G(a)] + \frac{g(a)}{1-G(a)}\ [G(a) - \frac{1}{2}G(a)^2] \\
&= \frac{g(a)}{1-G(a)}Q(0)\ ,
\end{aligned}
$$

where the inequality uses the strict log-concavity of $1-G$ (or equivalently that $\frac{g}{1-G}$ is increasing), which is implied by our assumption that g is stictly log-concave.

On the other hand, using L'Hôpital's rule we have that

$$
\Phi(a-\underline{u}) = \frac{1-G(a)}{g(a)} - \frac{1}{g(\underline{u})[1-G(a)]} < \frac{1}{g(\underline{u})}[1 - \frac{1}{1-G(a)}] < 0 < a - \underline{u}\ ,
$$

where the first inequality again uses the strict log-concavity of $1-G$. Therefore by continuity $\Phi(\Delta) = \Delta$ has a solution between 0 and $a - \underline{u}$.

When $s \to \overline{s}$, we have $a \to \underline{u}$ and so $\Delta \to 0$. When $s \to 0$, we have $a \to \overline{u}$ and so (19) becomes

$$
\Delta = \frac{1-2Q(\Delta)}{Q'(\Delta)}\ .
$$

It does not hold for any $\Delta > 0$ since $Q(\Delta) > \frac{1}{2}$ for $\Delta > 0$, but it holds for $\Delta = 0$ given $Q(0) = \frac{1}{2}$ at $a = \overline{u}$. ■

Proof of Proposition 1. (i) We first show $p_m < p_0$ when $1-G$ is strictly log-concave (which is implied by the strict log-concavity of g). From (15), we have

$$
\begin{aligned}
\frac{1}{p_m} &> \int_a^{\overline{u}} [1-G(\phi(u_1))]g(\phi(u_1))g(u_1)du_1 + 2\int_{\underline{u}}^{\overline{u}}\int_{\underline{u}}^{\phi(u_1)} g(u_2)^2 g(u_1)du_2du_1 \\
&= \int_{\underline{u}}^{\overline{u}} \left\{ [1-G(\phi(u_1))]g(\phi(u_1)) + 2\int_{\underline{u}}^{\phi(u_1)} g(u_2)^2du_2 \right\} dG(u_1)\ .
\end{aligned}
$$

(The inequality is from discarding $\int_a^{\overline{u}} [1-G(u_1)]\, g(\phi(u_1))\, g(u_1)\, du_1$ in (16). The equality used the fact that $\phi(u_1) = \overline{u}$ and so $1-G(\phi(u_1)) = 0$ for $u_1 \le a$.) When $1-G$ is strictly log-concave, the curly-bracket term is a strictly increasing function of $\phi(u_1)$. Since $\phi(u_1) \ge a$, we have

$$
\frac{1}{p_m} > \int_{\underline{u}}^{\overline{u}} \left\{ [1-G(a)]g(a) + 2\int_{\underline{u}}^{a} g(u_2)^2du_2 \right\} dG(u_1) = [1-G(a)]g(a) + 2\int_{\underline{u}}^{a} g(u)^2du = \frac{1}{p_0}\ .
$$

We then prove $p_0 < p_B$. We need the following result which will be proved later:

Claim 1 *When g is log-concave, $g(a-\Delta)[1-G(a)] + 2\int_{\underline{u}}^{a-\Delta} g(u+\Delta)\,dG(u)$ is a decreasing function of Δ.*

The claim implies that

$$g(a-\Delta)[1-G(a)]+2\int_{\underline{u}}^{a-\Delta} g(u+\Delta)\,dG(u) \leq g(a)[1-G(a)]+2\int_{\underline{u}}^{a} g(u)\,dG(u) = \frac{1}{p_0} \; .$$

Using the fact that $Q'(\Delta) = g(a-\Delta)[1-G(a)] + \int_{\underline{u}}^{a-\Delta} g(u+\Delta)\,dG(u)$, we then have

$$\frac{Q(\Delta)}{p_A} + \frac{1-Q(\Delta)}{p_B} \leq \frac{1}{p_0} \; .$$

Since we already know $p_A < p_B$, we can then conclude $p_B > p_0$.

Proof of Claim 1. The derivative of the objective function with respect to Δ is

$$-g'(a-\Delta)[1-G(a)] - 2g(a)\,g(a-\Delta) + 2\int_{\underline{u}}^{a-\Delta} g'(u+\Delta)\,dG(u) \; . \tag{20}$$

We aim to show this is negative. First, notice that the log-concavity of $1-G$ implies

$$[1-G(a)]\frac{g'(a)}{g(a)} + g(a) \geq 0 \; .$$

Meanwhile, we have

$$\frac{g'(a)}{g(a)} \leq \frac{g'(a-\Delta)}{g(a-\Delta)}$$

from the log-concavity of g. Therefore,

$$[1-G(a)]\frac{g'(a-\Delta)}{g(a-\Delta)} + g(a) \geq 0 \Rightarrow -g'(a-\Delta)[1-G(a)] - g(a)\,g(a-\Delta) \leq 0 \; . \tag{21}$$

Second, we have

$$\begin{aligned}
\int_{\underline{u}}^{a-\Delta} g'(u+\Delta)\,g(u)\,du &= \int_{\underline{u}}^{a-\Delta} \frac{g'(u+\Delta)}{g(u+\Delta)} g(u) g(u+\Delta) du \\
&\leq \int_{\underline{u}}^{a-\Delta} \frac{g'(u)}{g(u)} g(u) g(u+\Delta) du \\
&= \int_{\underline{u}}^{a-\Delta} g'(u)\,g(u+\Delta) du \\
&= g(a-\Delta)g(a) - g(\underline{u})g(\underline{u}+\Delta) - \int_{\underline{u}}^{a-\Delta} g(u)\,g'(u+\Delta) du \; .
\end{aligned}$$

(The inequality used the log-concavity of g, and the last step is from integration by parts.)
Then

$$2\int_{\underline{u}}^{a-\Delta} g'(u+\Delta)\,g(u)\,du \leq g(a-\Delta)g(a) - g(\underline{u})g(\underline{u}+\Delta) \; ,$$

and so

$$-g(a)\,g(a-\Delta)+2\int_{\underline{u}}^{a-\Delta} g'(u+\Delta)\,dG(u)\leq -g(\underline{u})g(\underline{u}+\Delta)\leq 0\ . \tag{22}$$

It is then clear that (21) and (22) imply that (20) is negative. ■

(ii-a) We aim to show the result when s is close to 0, or equivalently when a is close to $\overline{u}$ since a is strictly decreasing in s. Suppose $a=\overline{u}-\epsilon$, where $\epsilon>0$ but very small. We first approximate equilibrium prices in various market structures using a Taylor expansion. We use the notation

$$p\equiv\frac{1}{2\int_{\underline{u}}^{\overline{u}} g(u)^2du}\ , \tag{23}$$

which is actually the equilibrium price under any market structure when information is perfect i.e. when $s=0$.

Under the fragmented market structure, suppose the (first-order) linear approximation of the equilibrium price is $p_0\approx p+k_0\epsilon$, where k_0 is to be determined. Notice that

$$g(a)[1-G(a)]=g(\overline{u}-\epsilon)[1-G(\overline{u}-\epsilon)]\approx g(\overline{u})^2\epsilon\ ,$$

and

$$\int_{\underline{u}}^{a} g(u)^2du=\int_{\underline{u}}^{\overline{u}-\epsilon} g(u)^2du\approx\frac{1}{2p}-g(\overline{u})^2\epsilon\ ,$$

where p is defined in (23). Then (6) requires

$$1\approx(p+k_0\epsilon)\left(\frac{1}{p}-g(\overline{u})^2\epsilon\right)\ ,$$

which after discarding all higher-order terms allows us to solve

$$k_0=p^2g(\overline{u})^2\ . \tag{24}$$

Under the symmetric market structure with two multiproduct firms, suppose the (first-order) linear approximation of the equilibrium price is $p_m\approx p+k_m\epsilon$, where k_m is to be determined. The first term in (15) equals

$$\begin{aligned}
&\int_{\overline{u}-\epsilon}^{\overline{u}}[2-G(u)-G(\phi(u))]g(\phi(u))g(u)du\\
\approx\ &[2-G(\overline{u}-\epsilon)-G(\phi(\overline{u}-\epsilon))]g(\phi(\overline{u}-\epsilon))g(\overline{u}-\epsilon)\epsilon\\
=\ &[1-G(\overline{u}-\epsilon)]g(\overline{u})g(\overline{u}-\epsilon)\epsilon\ .
\end{aligned}$$

(The equality uses $\phi(\overline{u}-\epsilon)=\phi(a)=\overline{u}$.) This clearly has no first-order effects. Half of the second term in (15) equals

$$\int_{\underline{u}}^{a}\int_{\underline{u}}^{\overline{u}} g(u_2)^2g(u_1)du_2du_1+\int_{a}^{\overline{u}}\int_{\underline{u}}^{\phi(u_1)} g(u_2)^2g(u_1)du_2du_1$$

It is easy to verify that

$$\int_{\underline{u}}^{\overline{u}-\epsilon}\int_{\underline{u}}^{\overline{u}} g(u_2)^2 g(u_1)du_2du_1 \approx \frac{1}{2p}(1-g(\overline{u})\epsilon)$$

and

$$\int_{\overline{u}-\epsilon}^{\overline{u}}\int_{\underline{u}}^{\phi(u_1)} g(u_2)^2 g(u_1)du_2du_1 \approx \frac{1}{2p}g(\overline{u})\epsilon \ ,$$

where again we use $\phi(\overline{u}-\epsilon)=\phi(a)=\overline{u}$. Since the ϵ terms just cancel each other out, the second term in (15) has no first-order effects either. We can thus conclude that $k_m=0$. That is, in multiproduct search when a decreases slightly from $\overline{u}$, it has no first-order impact on the equilibrium price.

Under the asymmetric market structure, suppose the (first-order) linear approximation of the equilibrium prices is $p_A \approx p+k_A\epsilon$ and $p_B \approx p+k_B\epsilon$, where k_A and k_B are to be determined. Then $\Delta \approx \delta\epsilon$, where $\delta = k_B - k_A$. One can verify that

$$\int_{\underline{u}}^{a-\Delta}[1-G(u+\Delta)]dG(u) \approx \int_{\underline{u}}^{\overline{u}-(1+\delta)\epsilon}[1-G(u+\delta\epsilon)]dG(u) \approx \frac{1}{2}-\frac{\delta}{2p}\epsilon \ .$$

Hence

$$Q(\Delta) \approx \frac{1}{2}+\frac{\delta}{2p}\epsilon \ . \tag{25}$$

One can also verify that $g(a-\Delta)[1-G(a)] \approx g(\overline{u})^2\epsilon$, and

$$\int_{\underline{u}}^{a-\Delta} g(u+\Delta)dG(u) \approx \int_{\underline{u}}^{\overline{u}-(1+\delta)\epsilon} g(u+\delta\epsilon)dG(u) \approx \int_{\underline{u}}^{\overline{u}} g(u)^2du - \left((1+\frac{\delta}{2})g(\overline{u})^2+\frac{\delta}{2}g(\underline{u})^2\right)\epsilon \ ,$$

where we use $\int_{\underline{u}}^{\overline{u}} g'(u)g(u)du = \frac{1}{2}(g(\overline{u})^2-g(\underline{u})^2)$. Hence

$$Q'(\Delta) = g(a-\Delta)[1-G(a)] + \int_{\underline{u}}^{a-\Delta} g(u+\Delta)dG(u) \approx \frac{1}{2p}-\frac{\delta}{2}\left(g(\overline{u})^2+g(\underline{u})^2\right)\epsilon \ .$$

Therefore (11) requires

$$(p+k_A\epsilon)\left(\frac{1}{2p}-\frac{\delta}{2}\left(g(\overline{u})^2+g(\underline{u})^2\right)\epsilon\right) \approx \frac{1}{2}+\frac{\delta}{2p}\epsilon \ ,$$

from which we can solve

$$k_A = [1+p^2(g(\overline{u})^2+g(\underline{u})^2)]\delta \ . \tag{26}$$

Similarly (12) requires

$$(p+k_B\epsilon)\left(\frac{1}{2p}-\left((1+\frac{\delta}{2})g(\overline{u})^2+\frac{\delta}{2}g(\underline{u})^2\right)\epsilon\right) = \frac{1}{2}-\frac{\delta}{2p}\epsilon \ ,$$

from which we can solve

$$k_B = 2p^2 g(\overline{u})^2 + [p^2(g(\overline{u})^2 + g(\underline{u})^2) - 1]\delta \ . \tag{27}$$

From (26), (27) and $\delta = k_B - k_A$, it is easy to derive

$$\delta = \frac{2}{3} p^2 g(\overline{u})^2 \ , \tag{28}$$

and it then follows that

$$k_A = \frac{2}{3} p^2 g(\overline{u})^2 [1 + p^2(g(\overline{u})^2 + g(\underline{u})^2)] \ , \tag{29}$$

and

$$k_B = \frac{2}{3} p^2 g(\overline{u})^2 \left[2 + p^2(g(\overline{u})^2 + g(\underline{u})^2)\right] \ . \tag{30}$$

Notice that k_0, k_A and k_B are strictly positive given $g(\overline{u}) > 0$. Then we can compare prices in a meaningful way when s is close to 0. p_m is the smallest since $k_m = 0$, and one can also easily verify that p_B is the biggest since $k_B > k_A, k_0$. It remains to compare p_0 and p_A. $p_0 < p_A$ if and only if $k_0 < k_A$. Using (24) and (29), one can check that $k_0 < k_A$ if and only if $\frac{1}{2} < p^2(g(\overline{u})^2 + g(\underline{u})^2)$, which is equivalent to condition (17) by using (23).[30]

(ii-b) We then show the result when s is close to $\overline{s}$, or equivalently when a is close to $\underline{u}$. The first observation is that as $s \to \overline{s}$ we have $\Delta \to 0$ and $p_0, p_A, p_B \to \frac{1}{g(\underline{u})}$. Meanwhile we also know from the proof of $p_m < p_0$ that

$$\frac{1}{p_m} > \frac{1}{p_0} + \int_a^{\overline{u}} [1 - G(u)] \, g(\phi(u)) \, g(u) \, du \ .$$

Since the final integral term is bounded away from 0 as $a \to \underline{u}$, we conclude that in the limit $p_m < p_0, p_A, p_B$.

For the rest of the proof, we need to approximate p_0, p_A, and p_B when s is close to $\overline{s}$. The procedure is similar to the proof of (ii-a). Suppose $a = \underline{u} + \epsilon$, where $\epsilon > 0$ but very small. We first approximate $p_0 \approx \frac{1}{g(\underline{u})} + \hat{k}_0 \epsilon$, where $\hat{k}_0$ is to be determined. Notice that

$$g(a)[1 - G(a)] + 2\int_{\underline{u}}^{a} g(u)^2 \, du \approx g(\underline{u}) + [g'(\underline{u}) + g(\underline{u})^2]\epsilon$$

by discarding all higher order terms. Then one can check that

$$\hat{k}_0 = -\frac{g'(\underline{u}) + g(\underline{u})^2}{g(\underline{u})^2} \ .$$

[30] Notice that if $k_0 = k_A$ (which is true in the uniform distribution example), the (first-order) approximation does not help the comparison of p_0 and p_A.

Given $1-G$ is strictly log-concave, we have $g'[1-G]+g^2>0$. This, together with $1-G(\underline{u})=1$, implies $\hat{k}_0<0$.

Then let us approximate $p_A \approx \frac{1}{g(\underline{u})}+\hat{k}_A\epsilon$ and $p_B \approx \frac{1}{g(\underline{u})}+\hat{k}_B\epsilon$, where k_A and k_B are to be determined. Then $\Delta \approx \hat{\delta}\epsilon$, where $\hat{\delta}=\hat{k}_B-\hat{k}_A$. One can check that

$$Q(\Delta) \approx 1-(1-\hat{\delta})g(\underline{u})\,\epsilon \ ,$$

and

$$Q'(\Delta) \approx g(\underline{u})+\left[(1-\hat{\delta})g'(\underline{u})-\hat{\delta}g(\underline{u})^2\right]\epsilon \ .$$

Then from $p_A Q'(\Delta)=Q(\Delta)$, we can derive

$$\hat{k}_A = 2\hat{\delta}-1-(1-\hat{\delta})\frac{g'(\underline{u})}{g(\underline{u})^2} \ .$$

Similarly, using the first-order condition for p_B we can derive $\hat{k}_B=0$. Then from $\hat{\delta}=\hat{k}_B-\hat{k}_A$, we solve

$$\hat{k}_A = -\frac{g'(\underline{u})+g(\underline{u})^2}{g'(\underline{u})+3g(\underline{u})^2}$$

Finally, given $g'(\underline{u})+g(\underline{u})^2>0$ under log-concavity, we have $\hat{k}_A\in(\hat{k}_0,0)$. Together with $\hat{k}_B=0$, this implies that $p_0<p_A<p_B$ when s is close to $\overline{s}$.

(iii) The uniform distribution is strictly log-concave, so we already have $p_m<p_0$ and $p_A<p_B$. It remains to prove $p_0<p_A$. Using (7) and (13), one can check that $p_0<p_A$ if and only if $a(1-a)^2(3+2a)>0$, which must be true given $a\in(0,1)$. ■

Proof of Proposition 2. Recall that we have shown in Lemma 2(i) that $\pi_A>\pi_0$ and so at least one multiproduct firm will emerge in the market. Hence, to determine the equilibrium market structure we only need to examine whether the second pair of single-product firms want to merge or not (i.e., to compare π_B with π_m).

(i) We first deal with the relatively simple case where s is high. (In this case we do not need the price approximations derived before.) Suppose $s\to\overline{s}$ (or $a\to\underline{u}$). Under the asymmetric market structure, $\Delta\to 0$ and $Q(\Delta)\to 1$, so (11) and (12) imply that $p_A,p_B\to\frac{1}{g(\underline{u})}$. (L'Hôpital's rule is needed when taking the limit in (12). Intuitively in either case each firm acts as a monopolist.) Then $\pi_B=p_B(1-Q(\Delta))\to 0<\pi_m=\frac{1}{2}p_m$. Therefore two multiproduct firms emerge in equilibrium when s is sufficiently close to $\overline{s}$.

We then turn to the case where s is small. In this case we need the price approximations when s is close to 0 in the proof of Proposition 1. Using (25), (28) and (30), we have

$$\pi_B = p_B(1-Q(\Delta)) \approx (p+k_B\epsilon)\left(\frac{1}{2}-\frac{\delta}{2p}\epsilon\right) \approx \frac{p}{2}+\frac{1}{3}p^2 g(\overline{u})^2[1+p^2(g(\overline{u})^2+g(\underline{u})^2)]\epsilon \ .$$

Then it is clear that $\pi_B > \pi_m$ given $\pi_m \approx \frac{p}{2}$ and $g(\overline{u}) > 0$. Therefore, we conclude that the asymmetric market arises in equilibrium when s is sufficiently small.

(ii) We now consider the uniform distribution case. Recall that $\pi_B = p_B[1 - Q(\Delta)]$. In the uniform distribution case,

$$1 - Q(\Delta) = \int_0^{a-\Delta} (1 - u - \Delta)du = \frac{1}{2}((1-\Delta)^2 - (1-a)^2) ,$$

and $\Delta = p_B - p_A = \frac{1}{8}(7 - a - K)$, where $K = \sqrt{17a^2 - 30a + 49}$. Hence

$$\pi_B(a) = \frac{1}{16}(K-7a+9)\times\frac{1}{2}\left[(1-\Delta)^2 - (1-a)^2\right] = \frac{1}{32}(K-7a+9)[\frac{1}{64}(1+a+K)^2-(1-a)^2] .$$

It is easy to check that $\pi_B(0) = 0$ and $\pi_B(1) = \frac{1}{4}$. Lengthy calculations also show that $\pi_B(a)$ is concave in a, and that $\pi'_B(1) = -\frac{1}{8}$.

On the other hand,

$$\pi_m(a) = \frac{1}{2}p_m = \frac{1}{2[2 - (\frac{1}{2}\pi - 1)\frac{(1-a)^2}{2}]} .$$

It is easy to see that $\pi_m(a)$ is decreasing and convex in a, and that $\pi_m(1) = \frac{1}{4}$. One can also check that $\pi'_m(1) = 0$.

Given the above properties of $\pi_B(a)$ and $\pi_m(a)$, we can conclude that there exists $\hat{a} \in (0, 1)$ such that $\pi_B(a) > \pi_m(a)$ if and only if $a > \hat{a}$. Since a is decreasing in s, this implies the desired result.[31] ■

Proof of Corollary 1. Suppose s is close to zero so that $a = \overline{u} - \epsilon$. Using the approximation results in the proof of Proposition 1, we have

$$\pi_0 = \frac{p_0}{2} \approx \frac{p}{2} + \frac{1}{2}p^2 g(\overline{u})^2\epsilon$$

and

$$\pi_B \approx \frac{p}{2} + \frac{1}{3}p^2 g(\overline{u})^2[1 + p^2(g(\overline{u})^2 + g(\underline{u})^2)]\epsilon .$$

Given $g(\overline{u}) > 0$, $\pi_B > \pi_0$ if and only if $p^2(g(\overline{u})^2 + g(\underline{u})^2) > \frac{1}{2}$. Using the expression for p in (23), this simplifies to (17). ■

Proof of Lemma 3. In the equilibrium of all of our search models, each consumer maximizes her expected surplus, given the equilibrium prices, by choosing an optimal search rule (which specifies how to search and which option to select eventually). Let Θ be the set of all possible search rules. If a consumer adopts search rule $\theta \in \Theta$, her expected surplus can be written as $U(\theta) - sT(\theta)$, where $U(\theta)$ is the expected utility

[31] Numerical calculation shows that the threshold is $\hat{s} \approx 0.092$.

from the purchased option under θ and $T(\theta)$ is the expected number of searches. (Note that any monetary payment has been included in U, and neither U nor T involves the search cost s directly.) Let $V(s) \equiv \max_{\theta\in\Theta} U(\theta) - sT(\theta)$, and $T(s) \equiv T(\theta(s))$ where $\theta(s) \equiv \arg\max_{\theta\in\Theta} U(\theta) - sT(\theta)$ is the optimal search rule given s. (As standard in the search literature, both Θ and the optimization problem are well behaved provided that probabilistic search rules are allowed.) Notice that $V(s)$ must be convex in s since the objective function in the maximization problem is linear in s. As a result, $V(s)$ is differentiable almost everywhere, and $V'(s) = -T(s)$ by an envelope argument. Then

$$V(s) = V(0) + \int_0^s V'(x)dx = V(0) - \int_0^s T(x)dx.$$

Here $V(0)$ is the consumer's expected surplus, given the equilibrium prices, if the search cost were zero (in which case the consumer would freely compare all options), and $T(x)$ is the expected number of searches, given the equilibrium prices, if the search cost were x. Notice that this argument works regardless of whether we are considering random or non-random search (as long as consumers choose the optimal search rule), and single-product or multiproduct search. ■

Proof of Proposition 3. (i) We first compare total welfare in the general case. Notice that with full market coverage, consumer payment is a pure transfer between consumers and firms, so total welfare only measures the match efficiency between consumers and products (i.e., the match quality minus the search cost). In the case with two multiproduct firms, after visiting the first firm, say, A, a consumer can search as in the fragmented market: if $u_{1A} < a$, she continues to visit firm B and then buys the better product 1; after this is done for product 1, if $u_{1B} < a$, she visits firm B again by paying the search cost again and then buys the better product 2. This search process is clearly suboptimal. But even so, the consumer still does better than in the fragmented market in terms of the match efficiency because the first search only costs $\frac{s}{2}$ for each product. This proves $TW_m > TW_0$. After visiting firm A, the consumer can also adopt a similar suboptimal search rule by replacing the above a by $a - \Delta$. That will generate the same match efficiency as in the asymmetric case. This proves $TW_m > TW_A$.[32]

When $p_m < p_0, p_A, p_B$, the symmetric market with two multiproduct firms has the lowest industry profit, so the consumer surplus result follows immediately. (Recall that $p_m < p_0 < p_B$ under the log-concavity condition, but we have not been able to show $p_m < p_A$.)

[32]We do not have a general comparison between TW_A and TW_0, because the former saves on search costs for the first pair of products, but has a less efficient search for the second pair of products due to the price dispersion across firms.

(ii) In the uniform distribution case, since p_m is the lowest price among all three possible market structures, the results in (i) apply. Then it remains to compare the asymmetric market with the fragmented market. Using (18) and $a = 1 - \sqrt{2s}$, we rewrite the consumer surplus and total welfare in the fragmented market as:

$$CS_0 = a - \frac{1}{3}a^3 - \frac{1}{1+a} \quad \text{and} \quad TW_0 = a - \frac{1}{3}a^3 \ .$$

In the asymmetric market, per-product consumer surplus is

$$CS_A = \mathbb{E}[\max\{u_{iA} - p_A, u_{iB} - p_B\}] - \frac{1}{2}\int_0^s [1 + 2G(a(x) - \Delta)] \ dx \ ,$$

where the first expectation term is $V/2$ in Lemma 3, and $1+2G(a(x)-\Delta)$ is the expected number of searches when the search cost is x. (The latter is because a consumer will visit the multiproduct firm for sure, and after that visit each single-product firm with probability $G(a(x)-\Delta)$.) Per-product total welfare in this case is $TW_A = CS_A + \pi_A + \pi_B$.

In the uniform distribution case, one can check

$$CS_A = \frac{1}{2} + \frac{(1-\Delta)^3}{6} - p_A - s\left(\frac{3}{2} - \Delta - \frac{2\sqrt{2s}}{3}\right) \ .$$

This can be rewritten as

$$CS_A = \frac{1}{16}\left[\frac{(1+a+K)^3}{192} - (3K - 5a - 13) - \frac{(1-a)^2(19a + 3K - 1)}{3}\right] \ ,$$

where $K = \sqrt{17a^2 - 30a + 49}$. Using the profit expressions, one can also check that

$$TW_A = \frac{1}{1536}\left[139 + 789a + 393a^2 - 513a^3 + 3(7-a)(1+a)K + 3(3-a)K^2 - K^3\right] \ .$$

Define $\Delta CS(a) = CS_0 - CS_A$. Simple calculations show that $\Delta CS(0) = -\frac{1}{4}$, $\Delta CS(1) = 0$ and $\Delta CS'(1) = -\frac{1}{12}$. Lengthier calculations show that $\Delta CS(a)$ is concave in $a \in [0,1]$. Hence there exists a critical $\tilde{a}$ such that $\Delta CS(a) < 0$ if and only if $a < \tilde{a}$. Given a decreases in s, this proves the consumer surplus result.

Define $\Delta TW(a) = TW_0 - TW_A$. Simple calculations show that $\Delta TW(0) = -\frac{1}{4}$, $\Delta TW(1) = 0$ and $\Delta TW'(1) = 0$. Lengthier calculations also show that $\Delta TW(a)$ is strictly concave in $a \in (0,1)$, and hence $\Delta TW(a) < 0$ for $a < 1$ and the total welfare result follows. ■

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Details of the Asymmetric Equilibrium With Two Multiproduct Firms
[Online Appendix: Not For Publication]

In this supplementary document, we characterize the asymmetric equilibrium in the symmetric market with two multiproduct firms A and B. Suppose all consumers visit firm A first in equilibrium. Let p^i_{mk} be firm k's price for its product i. Let $\mathbf{\Delta} = (\Delta_1, \Delta_2)$, where $\Delta_i \equiv p^i_{mB} - p^i_{mA}$ is the price difference of product i across firms. Denote by $S(\mathbf{\Delta})$ the stopping region in firm A's match utility space, and let $NS(\mathbf{\Delta})$ be the complement. The stopping region is characterized by a reservation frontier $\phi_{\mathbf{\Delta}}(u_1) \equiv \phi(u_1 + \Delta_1) - \Delta_2$, where $\phi(\cdot)$ is the reservation frontier in the symmetric case with $\mathbf{\Delta} = \mathbf{0}$ and it solves

$$\int_{u_1}^{\overline{u}} [1 - G(x)]dx + \int_{\phi(u_1)}^{\overline{u}} [1 - G(x)]dx = s \ .$$

A consumer will stop searching and buy both products immediately at firm A if the match utilities discovered there are such that $u_2 > \phi_{\mathbf{\Delta}}(u_1)$.

Since each firm's two products are symmetric, we look for an equilibrium where $p^1_{mk} = p^2_{mk} = p_{mk}$ and $\Delta_1 = \Delta_2 = \Delta = p_{mB} - p_{mA}$. Suppose an equilibrium with $\Delta \in (0, \overline{u} - \underline{u})$ exists for any $s \in (0, \overline{s})$ and in equilibrium all consumers buy (i.e., the market is fully covered).[1] Let π_{mk} be firm k's equilibrium profit from each product. Then the asymmetric market structure arises in equilibrium if the second pair of single-product firms become the non-prominent firm after merger and $\pi_{mB} < \pi_B$, or if the second pair become the prominent one after merger and $\pi_{mA} < \pi_B$. While if these inequalities are violated the equilibrium market structure has two multiproduct firms. The following two graphs depict the reservation frontier in equilibrium:

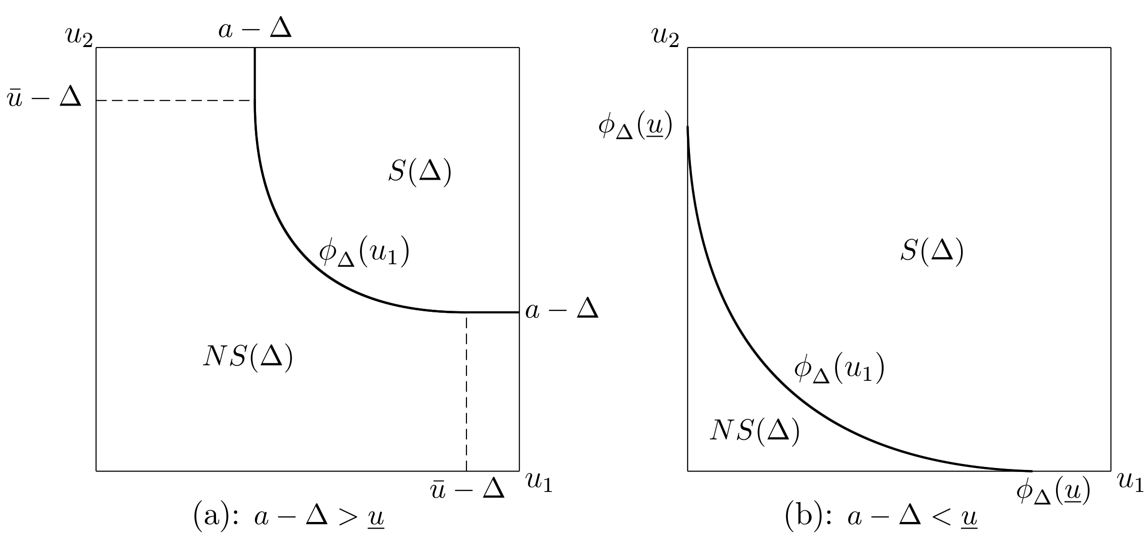


Figure 3: The reservation frontier in asymmetric equilibrium

[1] If $\Delta \geq \overline{u} - \underline{u}$, no consumers will search beyond the first firm even if search is almost costless.

Figure 3(a) is the case for $a - \Delta > \underline{u}$, where $\phi_{\mathbf{\Delta}}(u_1)$ has a vertical segment with $\phi_{\mathbf{\Delta}}(u_1) \in [\overline{u}-\Delta, \overline{u}]$ at $u_1 = a-\Delta$, and a horizontal segment with $\phi_{\mathbf{\Delta}}(u_1) = a-\Delta$ for $u_1 \in [\overline{u}-\Delta, \overline{u}]$; Figure 3(b) is the case for $a - \Delta < \underline{u}$, where $\phi_{\mathbf{\Delta}}(u_1)$ hits the vertical axis at $u_2 = \phi_{\mathbf{\Delta}}(\underline{u})$ and hits the horizontal axis at $u_1 = \phi_{\mathbf{\Delta}}(\underline{u})$ (where we have used the fact $\phi^{-1}(\cdot) = \phi(\cdot)$ since the two products are symmetric). Notice that $a = \overline{u}$ at $s = 0$, so the first case applies when s is small; while $a = \underline{u}$ at $s = \overline{s}$, so the second case applies when s is large.

We assume that the equilibrium prices p_{mA} and p_{mB} are determined by the first-order conditions (up to some possible corner solution adjustment when $a - \Delta = \underline{u}$ as we will discuss later). We first consider the case with $a - \Delta > \underline{u}$ (so that Figure 3(a) applies). Suppose firm A unilaterally deviates and charges $p_{mA}-\varepsilon$ for its product 2 so that $\Delta_2 = \Delta + \varepsilon$. This shifts the reservation frontier downward by ε everywhere. Let $\mathbf{\Delta}(\varepsilon) = (\Delta, \Delta+\varepsilon)$. Then firm A's deviation profit is

$$(2p_{mA} - \varepsilon) \int_{S(\mathbf{\Delta}(\varepsilon))} dG(\mathbf{u}) + \int_{NS(\mathbf{\Delta}(\varepsilon))} [p_{mA} G(u_1 + \Delta) + (p_{mA} - \varepsilon) G(u_2 + \Delta + \varepsilon)] dG(\mathbf{u}) \ .$$

Here the first term is the profit from consumers who buy immediately at firm A, and the second term is the profit from consumers who choose to search on and visit firm B but eventually come back to buy something from firm A (where $G(u_1 + \Delta)$ is the chance that firm A's product 1 is better than firm B's product 1 and $G(u_2 + \Delta + \varepsilon)$ is the chance that firm A's product 2 is better than firm B's product 2). Noticing that the price deviation affects both $S(\mathbf{\Delta}(\varepsilon))$ and $NS(\mathbf{\Delta}(\varepsilon))$, one can check that the first-order condition implies

$$p_{mA} = \frac{\int_{S(\mathbf{\Delta})} dG(\mathbf{u}) + \int_{NS(\mathbf{\Delta})} G(u_2 + \Delta) dG(\mathbf{u})}{\int_{NS(\mathbf{\Delta})} g(u_2 + \Delta) dG(\mathbf{u}) + \int_{a-\Delta}^{\overline{u}} [2 - G(u_1 + \Delta) - G(\phi(u_1 + \Delta))] g(\phi_{\mathbf{\Delta}}(u_1)) dG(u_1)} \ . \tag{1}$$

Here the numerator is the equilibrium demand for firm A's product 2.

Suppose now firm B unilaterally deviates and charges $p_{mB} - \varepsilon$ for its product 2. Then firm B's deviation profit is

$$\int_{NS(\mathbf{\Delta})} \{p_{mB}[1 - G(u_1 + \Delta)] + (p_{mB} - \varepsilon)[1 - G(u_2 + \Delta - \varepsilon)]\} dG(\mathbf{u}) \ .$$

When a consumer who has discovered match utilities (u_1, u_2) at firm A comes to visit firm B, she will buy firm B's product 1 with probability $1 - G(u_1 + \Delta)$ and buy firm B's product 2 with probability $1 - G(u_2 + \Delta - \varepsilon)$. Notice that here the price deviation does not appear in $NS(\Delta)$, since whether a consumer will come to visit firm B or not depends on the expected equilibrium prices of firm B (instead of the actual deviation price). This also implies that firm B's pricing problem is totally separable between the two products. The first-order condition is then

$$p_{mB} = \frac{\int_{NS(\mathbf{\Delta})} [1 - G(u_2 + \Delta)] dG(\mathbf{u})}{\int_{NS(\mathbf{\Delta})} g(u_2 + \Delta) dG(\mathbf{u})} \ . \tag{2}$$

Here the numerator is the equilibrium demand for firm B's product 2. (Given full market coverage, the sum of the two numerators in (1) and (2) equals 1.)

When $a-\Delta<\underline{u}$ (so that Figure 3(b) applies), the first-order conditions are the same except that $\int_{a-\Delta}^{\overline{u}}$ in the denominator of (1) is replaced by $\int_{\underline{u}}^{\phi_{\mathbf{\Delta}}(\underline{u})}$. An analytical investigation of the system of the first-order conditions is harder than in the case of asymmetric market structure. However, an approximation analysis when s is close to 0 or equivalently when a is close to $\overline{u}$ (in which case Figure 3(a) applies) can be done. As a result, we can prove a result parallel to result (i) in Proposition 2 in the main paper.

Claim 1 *Suppose $g(\underline{u}), g(\overline{u})>0$ and that two multiproduct firms play an asymmetric equilibrium. There exist $0<\hat{s}_1<\hat{s}_2<\overline{s}$ such that the equilibrium market structure is asymmetric if $s<\hat{s}_1$ and symmetric with two multiproduct firms if $s>\hat{s}_2$.*

Proof. As in the proof of result (ii) in Proposition 1 of the main paper, we approximate prices when a is close to $\bar{u}$ (or equivalently, s is close to 0). Hence the relevant prices to consider are those in equations (1) and (2). Consider $a=\bar{u}-\epsilon$ where $\epsilon>0$ but very small. Suppose the (first-order) linear approximations of the equilibrium prices are $p_{mA}\approx p+k_{mA}\epsilon$ and $p_{mB}\approx p+k_{mB}\epsilon$, where p is the price that prevails under full information and solves $1/p=2\int_{\underline{u}}^{\overline{u}} g(u)^2\,du$. We now solve for k_{mA} and k_{mB}, and let $\delta=k_{mB}-k_{mA}$.

First consider the expression (2) for p_{mB}. The numerator can be written more explicitly as

$$\int_{\underline{u}}^{a-\Delta}\left(\int_{\underline{u}}^{\overline{u}-\Delta}[1-G(u_2+\Delta)]\,dG(u_2)\right)dG(u_1) + \int_{a-\Delta}^{\overline{u}-\Delta}\left(\int_{\underline{u}}^{\phi(u_1+\Delta)-\Delta}[1-G(u_2+\Delta)]\,dG(u_2)\right)dG(u_1) + \int_{\overline{u}-\Delta}^{\overline{u}}\left(\int_{\underline{u}}^{a-\Delta}[1-G(u_2+\Delta)]\,dG(u_2)\right)dG(u_1)\ .$$

(Recall that $\phi_{\mathbf{\Delta}}(u_1)=\phi(u_1+\Delta)-\Delta$.) Substituting in $a=\overline{u}-\epsilon$ and $\Delta=\delta\epsilon$, we can then write the numerator of (2) in terms of ϵ:

$$\int_{\underline{u}}^{\overline{u}-\epsilon-\delta\epsilon}\left(\int_{\underline{u}}^{\overline{u}-\delta\epsilon}[1-G(u_2+\delta\epsilon)]\,dG(u_2)\right)dG(u_1) + \int_{\overline{u}-\epsilon-\delta\epsilon}^{\overline{u}-\delta\epsilon}\left(\int_{\underline{u}}^{\phi(u_1+\delta\epsilon)-\delta\epsilon}[1-G(u_2+\delta\epsilon)]\,dG(u_2)\right)dG(u_1) + \int_{\overline{u}-\delta\epsilon}^{\overline{u}}\left(\int_{\underline{u}}^{\overline{u}-\epsilon-\delta\epsilon}[1-G(u_2+\delta\epsilon)]\,dG(u_2)\right)dG(u_1)\ .$$

Using the first-order Taylor approximation around the point $\epsilon=0$, the first term in this expression is approximated by

$$\frac{1}{2}-\left[\frac{(1+\delta)\,g(\overline{u})}{2}+\frac{\delta}{2p}\right]\epsilon\ ,$$

whilst the second term is approximately equal to $g(\overline{u})\,\epsilon/2$ and the third term is approximately equal to $\delta g(\overline{u})\,\epsilon/2$. Hence we conclude that

$$\int_{NS(\Delta)} [1 - G(u_2 + \Delta)]\, dG(u) \approx \frac{1}{2} - \frac{\delta}{2p}\epsilon \ . \tag{3}$$

Following the same procedure, it is also straightforward to derive that

$$\int_{NS(\Delta)} g(u_2 + \Delta)\, dG(u) \approx \frac{1}{2p} - \delta \frac{g(\underline{u})^2 + g(\bar{u})^2}{2}\epsilon \ . \tag{4}$$

Consequently using equation (2) and dropping higher order terms, we obtain the following equation which determines k_{mA} and k_{mB}:

$$k_{mB} + \delta - \delta \left[g(\overline{u})^2 + g(\underline{u})^2 \right] p^2 = 0 \ . \tag{5}$$

Second consider the expression (1) for p_{mA}. Since the numerator is firm A's demand, which consists of all consumers who do not purchase from firm B, we can immediately use equation (3) to infer that

$$\int_{S(\Delta)} dG(u) + \int_{NS(\Delta)} G(u_2 + \Delta)\, dG(u) \approx \frac{1}{2} + \frac{\delta}{2p}\epsilon \ .$$

Moreover the first term in the denominator of equation (1) has already been approximated above in equation (4). In addition it is straightforward to see that the second term in the denominator is not first order. Combining this information with equation (1), and again dropping higher order terms, we obtain another equation which determines k_{mA} and k_{mB}:

$$k_{mA} - \delta - \delta \left[g(\overline{u})^2 + g(\underline{u})^2 \right] p^2 = 0 \ . \tag{6}$$

Notice that equations (5) and (6) have a unique solution given by $k_{mA} = k_{mB} = 0$ (and so $\delta = 0$), and thus we conclude that $p_{mA} \approx p_{mB} \approx p$, that $\int_{NS(\Delta)} [1 - G(u_2 + \Delta)]\, dG(u) \approx 1/2$, and hence $\pi_{mA} \approx \pi_{mB} \approx p/2$. We have shown in the proof of result (i) in Proposition 2 in the main paper that π_B in the asymmetric market is greater than $p/2$ when $a = \overline{u} - \epsilon$. Therefore, we can conclude that $\pi_{mA}, \pi_{mB} < \pi_B$ when a is sufficiently close to $\overline{u}$, or equivalently when the search cost is sufficiently small. This implies an asymmetric market structure for small s.

(ii) Suppose now that s is close to $\overline{s}$. Clearly since firm A is searched first it must earn a strictly positive profit (i.e. $\pi_{mA} > 0$). We now argue that firm B must also earn strictly positive profit (i.e. $\pi_{mB} > 0$). Suppose to the contrary that it does not (i.e. all consumers buy immediately at firm A). (i) Suppose that the consumer who finds $(\underline{u}, \underline{u})$ at firm A strictly prefers to buy immediately without searching. Then firm A could slightly increase both prices without losing any demand, which would contradict the assumption that its price is determined by the first-order condition. (ii) Suppose instead that the consumer who finds $(\underline{u}, \underline{u})$ at firm A is just indifferent between searching and not. Then if firm A

slightly increases both prices, consumers around the corner of $(\underline{u}, \underline{u})$ in the match utility space will start searching firm B. In other words, the non-stopping region of NS will appear around that corner. But this only has a second-order effect on firm A's demand, so firm A's deviation must be profitable. This again yields a contradiction.

In the proof of result (i) of Proposition 2 in the main paper, we have shown that $\pi_B \to 0$ as $s \to \bar{s}$. Therefore, we have $\pi_{mA}, \pi_{mB} > \pi_B$ when $s \to \bar{s}$. This implies that the second pair of single-product firms will choose to merge, and so a symmetric market structure with two multiproduct firms arises in equilibrium for large s. ■

We now proceed to study the uniform-distribution example. When $a > \Delta$, one can check that the first-order conditions simplify to

$$p_{mA} = \frac{Q(\Delta)}{1 + s - \Delta} \tag{7}$$

and

$$p_{mB} = \frac{1 - Q(\Delta)}{1 - \frac{1}{2}\pi s - (1 + \sqrt{2s})\Delta}\ , \tag{8}$$

where $Q(\Delta) = \frac{1}{2} + \frac{2}{3}s\sqrt{2s} + (1+s)\Delta - \frac{1}{2}\Delta^2$ is the demand for a product of firm A. When $a < \Delta$, the first-order conditions simplify to

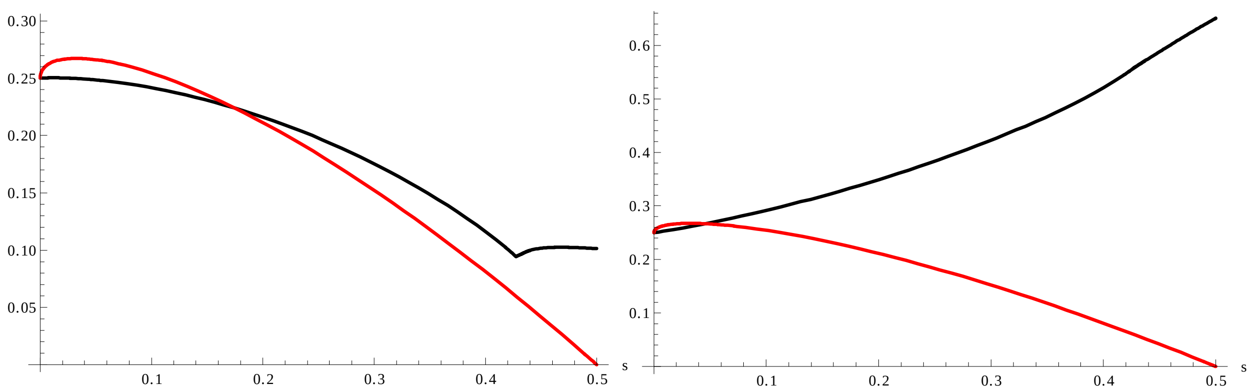
$$p_{mA} = \frac{\hat{Q}(\Delta)}{3s - 2A(\Delta) - (1-\Delta)\sqrt{A(\Delta)}} \tag{9}$$

and

$$p_{mB} = \frac{1 - \hat{Q}(\Delta)}{\int_0^{\phi(\Delta)-\Delta} \int_0^{\phi(u_1+\Delta)-\Delta} du_2 du_1} \tag{10}$$

where $A(\Delta) = 2s - (1-\Delta)^2$, $\phi(\Delta) = 1 - \sqrt{A(\Delta)}$, and $\hat{Q}(\Delta) = 1 - \frac{1-\Delta}{3}(s - 2A(\Delta)) - \frac{A(\Delta)}{3}\sqrt{A(\Delta)}$ is the demand for a product of firm A. (The denominator in the p_{mB} equation does not have a simple elementary expression.) Unfortunately, neither of the two systems has a simple analytical solution. But numerical calculation is easy to do. In the following, we report the details.

The exact nature of the equilibrium depends on how s compares with two thresholds s' and s'', where $s' \approx 0.427$ and $s'' \approx 0.436$. When $s < s'$ the equilibrium prices satisfy $a > \Delta$ and jointly solve equations (7) and (8). On the other hand, when $s > s''$ the equilibrium prices satisfy $a < \Delta$ and jointly solve equations (9) and (10). Interestingly we find that when $s \in (s', s'')$, the equilibrium prices satisfy $a = \Delta$ and firm B's price is pinned down by equation (8). In other words, in this case firm A's problem has a corner solution. To understand why, notice that for s relatively small the reservation frontier is as depicted in Figure 3(a). Therefore when firm A reduces its price for, say, product 2 the whole frontier shifts down, and demand is relatively price elastic. However as s increases, the reservation frontier moves south-west, and touches the axes when $s = s'$. At this point firm A faces a kinked demand curve and wants to price in such a way that $a = \Delta$. To see why, notice that if firm A increases price, the reservation frontier moves north-east and demand is relatively sensitive as before. However if firm A slightly decreases price, the situation resembles that depicted in Figure 3(b), where suddenly the horizontal segment on the reservation frontier with $u_1 \in [\overline{u} - \Delta, \overline{u}]$ disappears and so the length of the reservation frontier is decreased by a discrete amount; equivalently, demand is much less price sensitive. Finally though, as s increases from s' to s'', firm A becomes relatively more expensive because its price satisfies $a = \Delta$. Consequently at $s = s''$ it becomes worthwhile for firm A to reduce price in relative terms to attract more marginal consumers, and so the equilibrium solution again satisfies the interior first-order conditions.



(a): Non-prominent case

(b) Prominent case

Figure 4: Profit comparison for second merger

Figures 4(a) and 4(b) above plot equilibrium profits in this uniform example. The red lines depict π_B, the profit earned by a non-prominent firm in the asymmetric market structure. The black lines depict (per-product) profit earned by the second pair of single-product firms if they proceed to merge: Figure 4(a) assumes they remain non-prominent after merger and so the black line is π_{mB}, whilst Figure 4(b) assumes that they become prominent after merger and so the black line is π_{mA}. In either case, the post-merger profit is less than π_B for a sufficiently small s. As reported in Section 5 in the main paper, the asymmetric market structure arises for s below approximately 0.17 and 0.045 respectively, and otherwise the equilibrium market structure consists of two multiproduct firms.[2]

[2] One interesting observation is that whilst π_{mA} monotonically increases in s, π_{mB} is non-monotonic. Intuitively the latter arises because as discussed above, once s reaches s' the prominent multiproduct firm has less incentive to reduce price. Consequently the prominent firm becomes a weaker competitor, which by strategic complementarity benefits the non-prominent multiproduct firm as well. However once s is sufficiently above s'', the prominent firm starts to become more aggressive again, and steals demand away from the non-prominent firm causing its profit to fall again.