"Funding and financing infrastructure: the joint-use of public and private finance"

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FUNDING AND FINANCING INFRASTRUCTURE: THE JOINT-USE OF PUBLIC AND PRIVATE FINANCE \(^1\)

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ABSTRACT. The paper addresses the issue of the feasible level of private finance in a contracting model of infrastructure funding and financing. It characterizes the structure of financial contracts, deriving the conditions under which both public and private finance coexist. A key feature is that access to outside finance and the regulatory decision on pricing and the amount of public subsidy, hence the extent of price recovery, are jointly determined. Mobilizing private finance requires a combination of price for the service and subsidy to the service provider that is large enough, exacerbating the fundamental tension between financial viability through cost recovery and social inclusion. The paper then shows that the feasibility trade-off responds in non-trivial ways to changes in the economic and institutional environment likely to occur along the development path. While improvements along some of these dimensions, notably in the efficiency of bankruptcy procedures, appear to ease access to private finance, others, such as the cost of public funds, actually makes public finance more efficient. Using project data from the PPI database including information on the financial structure, we uncover an inverse U-shaped pattern in the share of private finance, peaking for countries in the upper-middle income range, which echoes our theoretical findings.

1. INTRODUCTION

The debate around the role of private partners in infrastructure ventures has dominated the infrastructure policy agenda at least since the revival of public-private partnerships (PPPs) in the 1980s. Currently, with many developing and emerging countries struggling with tight fiscal resources and mounting debts, and concessional funds from multilateral and bilateral agencies being limited, the question of whether and how to attract private resources to infrastructure projects once again is on the agenda.

Mobilizing resources to deliver infrastructure investments involves two related questions:

1. How infrastructure is funded, i.e., who eventually pays for the full cost of delivering the service. The range of possibilities goes from full cost recovery through users’ fees or other innovative mechanisms such as land value recovery, to full tax funding. Some of it comes down to the characteristics of the sector (lower levels of cost recovery are more likely in rural roads or sanitation than in electricity for example), and some to political and regulatory decisions about how to price services.

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2. How infrastructure is financed, i.e., who puts the money upfront to build and start operating it. Here the range goes from fully public to fully private finance, and involves a large variety of instruments (budgetary expenditures, bond markets, bank loans, equity issuance, etc.).

Funding is in essence a cash flow issue: Where does the money that will pay for the construction and operation of the service-providing piece of infrastructure ultimately come from? Typically, in the case of long-lived infrastructure investments such as roads or energy networks, such cash flows accrue gradually through time and are often significantly back-loaded, in the sense that very little is available in initial phases such as construction.

Financing, on the other hand, is the mechanism by which the equivalent of this potential accumulated cash flow is made available upfront to actually build the infrastructure and ensure it provides services to users. By extension, financing is often used in a second, related sense, to describe the complex arrangements used to perform this operation, including the agents involved in it, the instruments used and their specific characteristics such as maturity, cost, or repayment rules.

Several channels underpin the interdependence between funding and financing. The potential for cost recovery, pricing decisions and in particular regulatory ones, and the time profile of the cash flow, all impact the potential involvement of private financiers. Similarly, involving private financiers in the definition of the financing package over projects’ life cycles, from construction to operation, may be instrumental in ensuring a better monitoring of the projects, leading to lower costs and more timely delivery, and hence higher levels of cost recovery.

We address the issue of the feasible level of private finance in a simple model of infrastructure funding and financing, relevant to the different infrastructure sectors (transport, water, energy). In the context of a costly state verification contracting model involving three main agents -the government, the firm that delivers the service, and the financier- we characterize the structure of financial contracts, deriving the conditions under which public and private finance coexist. A central aspect of the model is that access to outside finance and the regulatory decision on pricing and the amount of public subsidy, hence the extent of price recovery, are jointly determined.

This framework makes clear the key trade-offs faced by policy makers when involving private finance in infrastructure projects. Specifically, it sheds light on how the feasibility and desirability of outside finance is affected by the nature of the projects and by the characteristics of the economic and institutional environment.

First and foremost, attracting private finance requires a combination of price for the service and (where needed) subsidy to the service provider that is large enough. Private financiers have a comparative advantage in auditing the service-providing firm under non-verifiability of costs. However, benefiting from this auditing capacity implies leaving a socially costly rent to the firm. To keep this rent in check, the optimal contract requires both increasing the audit probability and raising the price above the level under verifiability, at the risk of excluding the poorest consumers.

When demand is more elastic, for example in contexts where consumers are very poor
or credit constrained and react strongly to price increases, it is harder to move the price above the expected cost to raise revenues from user fees. Ceteris paribus, we therefore expect projects in those contexts to rely on greater shares of public subsidies. This tension between financial viability and inclusion, in the very places where the share of poor consumers to whom one would wish to extend the service is higher, is the first fundamental challenge of infrastructure finance in developing countries.

Our model then shows how different parameters of the economic and institutional environment affect this challenge. The viability condition for private finance implies that the cost to society of the service-providing firm’s rent increases with the cost of audit and bankruptcy. When resolving insolvency becomes cheaper and faster, i.e., the cost of audit decreases, a combination of higher private financing and lower price for the service becomes feasible, easing the trade-off highlighted above. Ensuring stronger creditors’ rights thus appears to be a crucial step in the quest to facilitate access to private finance.

Other key parameters have more mixed effects. Better institutional environments in the sense of reduced corruption, greater bureaucratic efficiency, or regulatory quality allow for higher levels of private finance, but this comes at the cost of higher prices for the service. Indeed, these improvements also mean that governments optimally (from an economic efficiency point of view) raise prices to obtain more financing but also to restrict demand and leave lower rents to the firms.

Finally, there are a number of characteristics for which improvements somewhat counterintuitively decrease the optimal level of private finance as well as the price of the service. These include the cost of public fund, the public deficit, as well as financial competitiveness. As budgetary pressures and the cost of taxation decrease, it becomes relatively more attractive for public authorities to rely on transfers rather than user fees, and both service prices and shares of private finance are reduced. Similarly, more efficient financial markets mean an enhanced sensitivity of finance to the regulated price. This results in both lower levels of private finance and lower prices.

Our results thus inform -and nuance- the conventional wisdom according to which the process of development, and the reforms it typically entails, should lead to greater reliance on private finance for infrastructure. We show that reforms alter the structure of infrastructure finance in several ways. First, by easing bankruptcy procedures, they allow for greater private finance shares without putting pressure on prices -a clear win-win policy. However, by also making public finance more efficient, some reforms symmetrically reduce the need for private finance, showing that strong public sectors with an ability to raise taxes and spend efficiently are clearly an effective solution to infrastructure finance.

**LITERATURE AND CONTRIBUTION.** Even though the topic is of considerable importance, very few papers have analyzed the interaction between financial constraints and the regulatory, or more generally the competitive, environment in which the firm evolves.

Regarding theoretical contributions, the literature can roughly be divided into two branches that correspond to different assumptions on the timing of contract signing, i.e., whether the financial contract is signed before or after the regulatory rules are defined, and on information.

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1See for example Lee, Miguel, and Wolfram (2016) on demand for rural electrification.
A first branch of the literature assumes that debt contracts strategically impact the relationship between the firm and its regulator or its competitors. In a seminal paper that triggered a more general literature on how industrial organization and corporate finance interact, Brander and Lewis (1986) show that debt contracts, because they make firms focus on the upside risk of their payoff, also render them more aggressive on the product market, the so called limited liability effect.

This idea was further adapted to regulatory contexts by Spiegel and Spulber (1994) and Dasgupta and Nanda (1993) who show how a regulated firm may choose to increase its debt to grasp more of the surplus when it bargains ex post on regulated prices with its regulator. This conclusion is consistent with empirical evidence to the extent that firms in regulated infrastructure sectors are often highly leveraged. This literature has stressed how the limited liability effect might also have consequences on incentives to make specific investments (Spiegel, 1996), how it helps predict the link between the power of regulatory incentives and the capital structure of firms in emerging markets (Moore et al., 2014), or how it might be exacerbated in some institutional context, noticeably when regulators are independent agencies (Bortolotti et al. 2011, Cambini and Spiegel, 2016).

How does asymmetric information affect these results? Increasing the share of investment financed through debt also means that the venture faces an excessive cost of bankruptcy from the point of view of the financier. As such the strategic value of debt contracts is fragile and might not be immune to the possibility of renegotiation once regulators have chosen the price of the service. Because of asymmetric information between lenders and borrowers, renegotiation might be costly which might restore some strategic value of debt in this context.

This impact of asymmetric information on renegotiation is well known from the general industrial organization literature on pre-commitment starting with Dewatripont (1988). In a more specific context of product market competition, Faure-Grimaud (2000) demonstrates that asymmetric information is a double-edged sword. On one hand and much in the spirit of what we are doing in this paper, his approach justifies using debt as an optimal contract as in the financial contracting literature (Townsend, 1978; Gale and Hellwig, 1985; Bolton and Scharfstein, 1990). On the other, the limited liability effect might be overturned. Highly leveraged firms may just be at a disadvantage on the product market, a theoretical finding that echoes some empirical evidence (Chevalier 1995a and 1995b).

Asymmetric information matters not only to analyze the firm’s interactions with financiers but also those with regulators, a fundamental premise of the New Regulatory Economics (Armstrong and Sappington, 2006; Baron and Myerson, 1982; Laffont and Tirole, 1993). Using a signaling model, Spiegel and Spulber (1993) argue that firms may be torn between two countervailing incentives. On the one hand, firms might want to downplay their costs to signal the high quality of their projects to financiers; on the other, they might also want to exaggerate their cost to get higher prices when bargaining with regulators. The result may be prices and financial contracts that are unresponsive to information. Still stressing asymmetric information in the relationship between firms

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2See Harris and Raviv (1992) for an introduction.
3Brander and Lewis (1988) investigate the robustness of these results when bankruptcy costs are no longer fixed costs but proportional to the firm’s output.
and financiers, but instead giving more commitment power to regulators, Lewis and Sappington (1995) show how those financial contracts should be designed.

A premise of the New Regulatory Economics—and one that we adopt here—is that public authorities set up the regulatory environment before firms approach financiers. However, our timing and information structure choices imply that there is no room for a limited liability effect in our analysis and that the issue of renegotiation does not arise. As in Iossa and Martimort (2012, 2015), the main usefulness of financiers is that they bring expertise in evaluating the firm’s performance. Yet the optimal debt contracts signed with financiers nevertheless change the preferences of the firm and thus modify its behavior with respect to the regulator. Roughly speaking, the cost structure of the firm must now account not only for the cost of bankruptcy but also for the agency cost of debt.

Our analysis also contrasts with Lewis and Sappington (1995) because public authorities are now interested by the economic regulation of the service and not only by its financial aspects. This allows to us to uncover a new set of interaction between regulation and financial contracting. On the one hand, public subsidies—where there are offered—act as a (public) equity stake that facilitates access to the financial market. On the other hand, the regulated prices for the services must account for the change in the firm’s cost structure that is induced by its relationship with financiers.

Regarding the more applied literature dealing specifically with developing countries issues, our paper also contributes to the understanding of how public and private finances may interact and shape the choice of projects, especially in the context of developing countries in urgent needs for infrastructure investment. Previous contributions include Estache et al. (2015), who analyze how the optimal combination of public finance, private debt, and private equity depends on the institutional environment and its limitations (capture, limited commitment, limited resources, etc.), and Engel et al. (2013), who provide conditions under which the Ricardian equivalence between public and private finance may fail.

By endogenizing the financing structure, our framework helps understand how different institutional reforms affect the optimal financing mix, much in the spirit of Laffont’s (2005n chapter 2) theory of the design of regulatory institutions at different stages of development. Importantly, our contribution departs from most of the literature on the relative merits of public versus private provision of infrastructure services in that it explicitly models the financial structure of the regulatory contract, and takes seriously the fact that private management is not necessarily synonymous with private finance, contrary to what is often the case in the policy literature.4

**Organization.** The paper is structured as follows. Section 2 develops the model and provides the main results on the link between regulatory prices and financial contracts. Section 3 develop several extensions of our main framework, with an eye on various comparative statics that lead to a number of empirical implications. There, we assess how our results change when the private and the social cost of bankruptcy differ, when consumption is subsidized, when financiers have market power, when public authorities have limited commitment, and when public deficit increase. Section 4 then provides stylized facts illustrating the model, and discusses policy implications using newly available data.

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4Cordella (2018) develops a model of project sequencing under alternative financing mechanisms
from the Private Participation in Infrastructure (PPI) database, which includes information on the financial structure of projects. Finally, Section 5 concludes. Proofs of the main results and the description of the data are relegated to appendices.

2. THE MODEL

A public authority wants to develop a public service which requires building an essential facility worth an outlay investment $I$. To do so, the public authority relies on the private sector while it still regulates the service, notably fixing its price. To finance such infrastructure, the public authority also relies on the private sector. In a first phase of contracting, a private firm, which is cashless, approaches outside financiers to obtain the required funds. In a second stage, the firm operates the service under the regulatory constraints. At this level of abstraction, this setting is consistent with the kind of private-public partnerships that are now fairly common throughout many sectors (transport, energy, water, environmental services, etc.) both in developed and developing countries. It may also conveniently represent the case of highly leveraged state-owned enterprises (SOEs) in regulated sectors (say a power plant or a water utility).

CONSUMERS. There is a continuum of potential consumers with mass one. Each of those consumers expresses demand for one unit of the service. Let denote by $p$ the price of the service. In all the above examples, this price remains regulated by the public authority and we will later assume that the public authority chooses its level. Consumers differ according to their valuation $\tilde{v}$ for this unit of service. We assume that $\tilde{v}$ is drawn on $\mathbb{R}_+$ according to an atomless cumulative distribution $H$. We denote by $h = H'$ the corresponding density function which is positive on $\mathbb{R}_+$.

A consumer with valuation $\tilde{v}$ thus buys this unit of service when $p < \tilde{v}$. We shall interpret the probability $1 - H(p)$ as the aggregate demand for the service. From this, it follows that consumers’ (overall) surplus can be defined as:

$$S(p) = \int_{p}^{\infty} (\tilde{v} - p) h(\tilde{v}) d\tilde{v} = \int_{p}^{\infty} (1 - H(\tilde{v})) d\tilde{v}.$$  

In this context, we might also define the elasticity of demand as $\varepsilon(p) = \frac{1 - H(p)}{h(p)}$. For future reference, we will suppose that the following monotonicity condition of the hazard rate, a quite familiar assumption in the screening literature,\(^5\) holds:

**Assumption 1**

$$\frac{1 - H(p)}{h(p)} \text{ non-increasing}.$$  

THE FIRM. Beyond the fixed-cost $I$ of setting up a key facility, the firm has a marginal cost $\tilde{c}$ of providing one unit of service. This random variable is drawn on $\mathbb{R}_+$ according to the atomless cumulative distribution $F$. We denote the corresponding positive density by $f = F'$. We denote by $E_{\tilde{c}}(\tilde{c})$ the average cost. Mimicking what we already did on the demand side, we will also impose (but somewhat refine) another standard monotonicity condition of the hazard rate on the cost distribution, namely:

\(^5\)See Bagnoli and Bergstrom (2005).
Assumption 2

\[
\frac{F(c)}{f(c)} \text{ increasing with } \lim_{c \to \infty} \frac{F(c)}{f(c)} = +\infty.
\]

For future reference, we also add a technical requirement. \(1 - F(c)\) should be integrable at \(+\infty\), i.e., \(\int_0^\infty (1 - F(\tilde{c}))d\tilde{c}\) converges towards zero as \(c\) goes to infinity, an assumption that always holds when \(\tilde{c}\) has finite support.

Let denote by \(T\) any lump-sum subsidy that the public authority may offer to the firm for providing the service. Subsidies are familiar tools for governments to ensure that firms break even in many regulated sectors which are relevant for our analysis. More generally, and we will come back on this interpretation later on, \(T\) could also be interpreted as any form of public investment into the project.\(^6\)

Taking into account the regulated profit and the addition of those subsidies, the firm at least breaks even (in expectations over possible realizations of its cost) when its profit is weakly positive:

\[
(2.2) \quad U(p, T) = (p - \mathbb{E}_\tilde{c}(\tilde{c}))(1 - H(p)) + T - I \geq 0.
\]

The Public Authority. Following a by now standard approach in New Regulatory Economics,\(^7\) the public authority maximizes a social welfare function which reflects consumers surplus, the firm’s profit but also the distortionary cost from taking the subsidy/public investment \(T\) from the public budget. Denoting by \(\lambda > 0\) the cost of public funds, this social welfare function writes as:

\[
W(p, T) = S(p) + U(p, T) - (1 + \lambda)T.
\]

Expressing \(T\) in terms of \(U = U(p, T)\) from (2.2) and using (2.1) and slightly abusing notations, we may obtain a more convenient expression of social welfare as:

\[
(2.3) \quad W(p, U) = \omega(p) - (1 + \lambda)I - \lambda U
\]

where

\[
\omega(p) = \int_{-\infty}^\infty (1 - H(\tilde{v}))(1 - H(p))d\tilde{v} + (1 + \lambda)(p - \mathbb{E}_\tilde{c}(\tilde{c}))(1 - H(p)).
\]

The public authority’s problem is to find \((p, U)\) so as to maximize the expression of social welfare \(W(p, U)\) in (2.3) subject to the break-even constraint (2.2).

The above expression of social welfare nicely illustrates how the contractual environment under scrutiny is akin to traditional forms of procurement. Indeed, everything happens as if the firm was not financing the investment on its own but instead the public authority was taking \(I\) from its own budget at a social cost \((1 + \lambda)I\). Doing so allows

\(^6\)A possible rationale for such public commitment, not explicitly modelled here, is that the public authority may help screening, among many potential projects, the ones with the highest social value.

\(^7\)Laffont and Tirole (1993) and Armstrong and Sappington (2006), among others.
to cut on the direct subsidy paid to the firm, which now becomes $T' = T - I$, and thus to save an equal amount $(1 + \lambda)I$. In this scenario of standard procurement, the public authority then chooses to delegate the management of the service to a firm which acts as a simple operator and is no longer concerned with the financial side of the project. As far as the overall social cost of the investment is concerned, there is a complete Ricardian equivalence between public and private funds.

This expression of welfare also shows that the cost of public funds plays a critical role on several fronts. First, increasing revenues by raising price above expected marginal cost allows the public authority to save on public subsidies and relieves the budgetary burden. Second, notice that the social cost of investment $(1 + \lambda)I$ differs from the private cost borne by the firm precisely because the cost of public funds $\lambda$ is positive. Again, it does not mean that private money is cheaper than public money. One dollar which is not directly invested by the public authority into the project ends up being nevertheless paid to the firm under the form of public subsidies. The cost of public funds in itself offers no justification for relying on private finance—a point already forcefully made by Engel, Fisher and Galetovic (2013). Third and as a consequence of the cost of public funds, transferring wealth to the firm is socially costly and the public authority wants to minimize the firm’s profit $U$, while satisfying the break-even constraint (2.2).

2.1. Self-Finance and/or Traditional Procurement

Consider a first scenario where the firm has initially enough cash to make the upfront investment $I$ by itself without relying on outside financiers. Alternatively, and following on a previous remark, this setting is formally equivalent to a scenario where the public authority invests $I$ by itself, i.e., a model of traditional procurement.

At the optimum, the break-even constraint (2.2) is necessarily binding ($U = 0$). The optimal regulated price $p^{sf}$ is thus chosen so as to maximize

$$\omega(p) - (1 + \lambda)I.$$ 

THE POSITIVE WEDGE BETWEEN PRICE AND EXPECTED COST. Observe that $\omega$ is quasi-concave thanks to Assumption 1. The optimal price $p^{sf}$ is thus immediately obtained from the following first-order condition:

$$\frac{p^{sf} - \mathbb{E}_c(\tilde{c})}{p^{sf}} = \frac{\lambda}{1 + \lambda \varepsilon(p^{sf})}.$$ 

The optimal price $p^{sf}$ follows a Ramsey pricing formula which is by now familiar from Regulatory Economics. The Lerner index is proportional to the inverse elasticity of demand, conveniently weighted by an increasing function of the cost of public funds. The logic for such positive price-cost margin is simple. Indeed, the public authority faces a trade-off between reducing budgetary pressures and mitigating allocative distortions. On the one hand, keeping the regulated price above marginal cost leaves more revenues to the firm which saves on public subsidies that would ensure that the firm breaks even. On the other hand, increasing price above (expected) marginal cost creates allocative distortions.

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8See Laffont and Tirole (1993, Chapter 1).
Comparative Statics. Simple comparative statics are immediately obtained from (2.4). As demand becomes more elastic, the optimal regulated price comes closer to the expected cost for the service. It is indeed harder to raise revenues from sales under those circumstances and public subsidies become a more attractive way of covering the upfront investment. When budgetary pressures are more stringent (i.e., for greater values of the cost of public funds $\lambda$), saving on public funds becomes more attractive. The wedge between price and (expected) cost thus also increases. Finally, when the expected cost itself increases, part of it is passed through to the regulated price.

Optimality of investment. Finally, the investment $I$ is socially valuable when:

\[(2.5) \quad \omega(p^{sf}) \geq (1 + \lambda)I.\]

Later, we shall be interested in how this condition becomes harder to satisfy when the firm has to rely on outside financiers and financial contracts are plagued with agency costs.

The complementarity between revenues and public subsidies. From the binding break-even constraint (2.2), we obtain the following expression of the optimal public subsidy $T^{sf}$:

\[(2.6) \quad T^{sf} = I - (p^{sf} - \mathbb{E}_c(\tilde{c}))(1 - H(p^{sf})) = I - \frac{\lambda}{1 + \lambda} \frac{(1 - H(p^{sf}))^2}{h(p^{sf})}.\]

Therefore the optimal price remains above expected cost, and:

\[T^{sf} < I.\]

The reason the investment should not be entirely financed through public subsidies is that public funds are socially costly. Indeed, part of this investment can already be recouped by what consumers directly pay for the service, which is socially cheaper. There exists a complementarity between two modes of financing the investment, either through revenues from the service or through public subsidies.

2.2. The Benefits of Outside Finance

There are two main reasons why relying on outside financiers could a priori be found attractive. First, the operator may not have retained enough earnings from previous operations elsewhere to bear the initial cost of investment by itself. From a previous remark, the model of Section 2.1 can nevertheless be also interpreted as picturing the case where the public authority directly pays for the investment. The fact that the firm has no cash to start with is thus not a real concern and public subsidies could be used to relieve such constraint anyway. However, nonlinearities in the cost of public funds could render such solution difficult to implement, especially at times of raising budgetary discipline.10

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9One ad hoc modeling of such nonlinearity would consists in supposing that the cost of public funds is increasing and convex, say $\Lambda(T)$ (with $\Lambda' \geq 0$ and $\Lambda'' \geq 0$). Section 3.6 below provides some modeling for such nonlinearities.

10Schmitz (2013) analyzes how budgetary pressures may affect the traditional theory of PPP, which following Hart (2001), Bennet and Iossa (2006), Iossa and Martimort (2015) and Martimort and Pouyet (2008) suppose that task bundling under PPP saves on various agency and transaction costs. Under such circumstances, while public subsidies might be preferred for investments of small size, relying on outside financiers might be a least costly solution to build the infrastructure if the upfront outlay is of significant magnitude.
Second, outside financiers may bring expertise that is not available within the public sphere. Given the underlying uncertainty on costs and thus on profits, outside financiers might use their comparative advantage in monitoring the project and offer better assessments of its prospects. In the sequel, we shall assume that private financiers bring their unique auditing expertise.

To understand why such knowledge on costs can be useful, observe that the solution \((p^{sf}, T^{sf})\) obtained in Section 2.1 is only feasible if the firm can run losses. Indeed, when the realized cost \(\tilde{c}\) is above its expectation \(E(\tilde{c})\), the subsidy \(T^{sf}\) is no longer sufficient to ensure a positive profit. If the firm is protected by limited liability, the subsidy and/or the price to deliver the service must be raised to ensure that, when such adverse events occur, the firm does not go bankrupt. The first solution one may think of to avoid such a problem would be to offer a cost-contingent subsidy \(T^{sf}(\tilde{c})\) of the form:

\[
T^{sf}(\tilde{c}) = I - (p^{sf} - \tilde{c})(1 - H(p^{sf})).
\]

Even when costs are verifiable, increasing subsidies from \(T^r\) to a greater value \(T^{sf}(\tilde{c})\) opens the door to costly renegotiation and to all sorts of transaction costs, including political ones, which may be of significant magnitude. As such, the auditing ability of outside financiers underpins the efficiency gains from bringing in private finance.

In the sequel, we shall assume that costs are non-verifiable. In those circumstances, the cost-contingent subsidy \(T^{sf}(\tilde{c})\) is no longer immune to manipulations. The firm may indeed inflate its cost to grab more subsidies. Given a large upper bound of the firm’s cost support, the project may be abandoned altogether. Audit becomes useful to check the adequacy of the firm’s reported costs with their exact realizations and discipline such behavior.

**Debt contracts.** Following the literature on costly state verification (Townsend, 1979; Gale and Hellwig, 1985; Border and Sobel, 1987), we thus assume that outside financiers bring funds \(I\). These financiers can audit the firm’s claim on its realized cost. Auditing the firm is itself costly but, on the other hand, the public authority faces an infinite cost of doing so. Under non-verifiability of costs, such an audit is necessary to enforce repayments from the firm for the loan it received.

For the sake of simplicity, we shall normalize the cost of audit and evaluate this cost per unit of demand. This normalization is useful to avoid any scale effect that would arise if audit becomes comparatively more attractive as the firm’s scale of activity expands. If audit takes place on a subset \(A \subseteq \sim \Omega_+\) of claims \(\tilde{c}\) regarding realized cost, the expected total cost of audit can thus be written as

\[
\mu(1 - H(p)) \text{Proba}\{\tilde{c} \in A\}
\]

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11This point was already suggested in the **PPPs** context by Iossa and Martimort (2012, 2015) in moral hazard models where the public authority does not control the price of the service, and demand for the service is nonelastic.


13Our results could of course be extended to a context where the public authority could also audit cost, but at a cost disadvantage. We simplify the presentation by making the assumption that the public authority has no such expertise.
where $\mu$ is the marginal cost of audit. We follow the corporate finance literature and interpret the probability of an audit as the probability of bankruptcy and the corresponding auditing cost as a bankruptcy cost. Then, those costs should be viewed as costs of reorganizing the ventures, terminating contracts with clients and input providers, severing employment relationships, saling dedicated assets at less than their market value, etc. From now on we will refer to $\mu$ as the cost of bankruptcy.

The costly state verification literature (Townsend, 1979; Gale and Hellwig, 1985; Border and Sobel, 1987) has demonstrated that an optimal financial contract in such an environment is a debt contract. The set of claims that trigger an audit is thus of the form $A = \{\hat{c} \geq \check{c}\}$ whose probability measure is $1 - F(\check{c})$. Denoting by $D$ the face value of debt, an audit therefore takes place when the firm’s claimed cost $\check{c}$ lies above the threshold $\hat{c}$ which is defined as:

$$D = (p - \check{c})(1 - H(p)) + T. \tag{2.7}$$

We assume that outside financiers are competitive so that the following break-even condition must hold:

$$I = DF(\check{c}) + \int_{\check{c}}^{\infty} ((p - c)(1 - H(p)) + T) dF(c) - \mu(1 - H(p))(1 - F(\check{c})). \tag{2.8}$$

The right-hand side is the benefit obtained by lenders net of the cost of audit. When the firm claims that its cost exceeds the threshold $\hat{c}$, lenders audit the firm, verify that the claim is correct at a truthful revelation equilibrium (thanks to the Revelation Principle that applies to that context\textsuperscript{14}) but seize the corresponding profit $(p - c)(1 - H(p)) + T$. When the firm claims instead that its cost is below the threshold $\check{c}$, audit does not take place. Lenders then receive a fixed repayment $D$.

Observe that the regulatory environment affects the face value of debt $D$ and the audit threshold $\hat{c}$ through the choice of the public instruments $p$ and $T$. We will analyze this link in more details below.

**The feasibility of outside finance.** Manipulating (2.7) and (2.8) and denoting by $I_{pr} = I - T$ the share of the investment that ends up being financed by the private sector, we may define the threshold value $\check{c}$ (or alternatively the face value of debt through (2.7)) as the solution to the following equation

$$I_{pr} = (1 - H(p)) \left( p - \check{c} - \int_{\check{c}}^{\infty} (1 - F(c)) dc - \mu(1 - F(\check{c})) \right). \tag{2.9}$$

The right-hand side is the revenue that investors may recoup. Because lenders are competitive, this revenue just covers private investment. This revenue can be decomposed into two different terms. On the one hand, for each unit of demand, financiers have of course to pay a cost of audit over the verification zone $[\check{c}, +\infty)$. On the other hand, everything happens as if, for each unit of demand, a competitive lender could only recoup a price-cost margin $p - \hat{c}$. The quantity

$$\hat{c} = \check{c} + \int_{\check{c}}^{\infty} (1 - F(c)) dc = \check{c}F(\check{c}) + \int_{\check{c}}^{\infty} cdF(c)$$

\textsuperscript{14}See Border and Sobel (1987) for a formal proof.
is the expected (marginal) cost perceived by the lender. This perceived cost actually amounts to the average cost conditional on information which has been collected through the audit. For all cost realizations below $\hat{c}$, there is no cost verification and lenders only know that the firm’s cost is below $\hat{c}$, an event that occurs with probability $F(\hat{c})$. For all cost realizations above $\hat{c}$, cost verification takes place and lenders perfectly learn the firm’s cost which makes a contribution $\int_{\hat{c}}^{\infty} c dF(c)$ to the perceived cost $\hat{c}$.

It is useful to rewrite (2.9) as:

$$(2.10) \quad \varphi(\hat{c}) = p - \frac{I - T}{1 - H(p)},$$

where

$$\varphi(\hat{c}) = \hat{c} + \int_{\hat{c}}^{\infty} (1 - F(\hat{c}))d\hat{c} + \mu(1 - F(\hat{c})).$$

Observe that $\varphi(\hat{c})$ is the sum of the expected (marginal) cost perceived by the lender (namely $\hat{c} + \int_{\hat{c}}^{\infty} (1 - F(\hat{c}))d\hat{c}$) and the cost of bankruptcy $\mu(1 - F(\hat{c}))$ per unit of demand.

Next Lemma highlights some important properties satisfied by the function $\varphi$.

**Lemma 1** $\varphi$ is quasi-convex and achieves a minimum at $c_m$ defined by:

$$(2.11) \quad \frac{F(c_m)}{f(c_m)} = \mu.$$  

To better understand the meaning of $\varphi$, suppose that there is a single potential lender who thus has monopoly power in fixing the terms of a debt contract. The net payoff to such monopolistic lender could be written as:

$$(2.12) \quad (1 - H(p))(p - \varphi(\hat{c})) + T - I.$$  

In other words, a minimum of $\varphi$ also maximizes the profit of a monopolistic lender. This financier chooses an audit zone that optimally trades off the benefit of reducing the cost of audit against the benefit that it allows in asking the firm for a greater repayment; a familiar intuition from the work of Townsend (1978) and Gale and Hellwig (1985). Increasing the audit zone when reducing $c_m$ by a small amount $dc$ marginally increases the cost of audit by $(1 - H(p)) \times \mu f(c_m)dc$. At the same time, the repayment can be increased by $(1 - H(p)) \times dc$ for all cost realizations $c \leq c_m$ which yields a gain $(1 - H(p))F(c_m)dc$. A monopolistic lender chooses a threshold $c_m$ for the audit zone so as to equate those marginal gains and costs.

From Lemma 1, it follows that (2.10) has a unique solution $\hat{c} \geq c_m$ only if the following feasibility condition is satisfied:\footnote{There is also another solution to (2.10) that lies on the decreasing part of $\varphi$ (i.e., such that $\hat{c}_- \leq c_m$). Because the audit zone is greater with $\hat{c}_- \leq c_m$ than with the solution $\hat{c}$ on the increasing part of $\varphi$, such solution does not minimize the cost of audit and cannot be selected in any welfare maximizing policy.}

$$p - \frac{I - T}{1 - H(p)} \geq \varphi(c_m).$$
Through (2.7), this solution defines the corresponding face value of debt $D$. As the face value of debt $D$ increases, the audit zone also increases ($\hat{c}$ decreases).

The above feasibility condition amounts to requiring a positive payoff to the monopolistic financier and can also be written as:

\[(2.13) \quad (p - \varphi(c_m))(1 - H(p)) + T - I \geq 0.\]

This feasibility condition describes all public policies $(p, T)$ that render outside finance feasible. Were the public authority able to choose a pair $(p, T)$ that instead violates (2.13), by picking either a subsidy level or imposing a regulated price that are too small, relying on outside financiers would no longer be feasible. We would then be back to the case where the expertise of financiers to assess costs cannot be used. Given that the firm is protected by limited liability, the subsidy $T$ or the regulated price would have to be raised sufficiently to induce participation from the firm even for the worst realization of its costs (assuming a bounded distribution of cost realizations).

By means of (2.7), the threshold $c_m$ thus corresponds to the greatest audit zone and the maximal amount of debt $D_m$ that makes outside finance feasible, namely:

\[D_m = (p - c_m)(1 - H(p)) + T.\]

A pair $(p, T)$ that would strictly satisfy (2.13) corresponds to a solution to (2.10) (on the increasing branch of $\varphi$) with a lower cut-off $\hat{c} > c_m$ (a smaller audit zone) and a lower face value of debt $D < D_m$.

Next Lemma thus allows us to write the feasibility condition (2.13) in a simpler form.

**Lemma 2** For a feasible debt contract, the threshold $\hat{c}$ beyond which audit takes place necessarily satisfies:

\[(2.14) \quad \hat{c} \geq c_m.\]

**Feasibility condition in terms of the firm’s profit.** Because lenders are competitive and make zero profit with a debt contract inducing a cut-off $\hat{c}$, the firm’s net profit also stands for the overall profit of the implicit coalition it forms with its lenders including the cost of audit. It leads us to write the firm’s profit as:

\[(2.15) \quad U(p,T) = (p - \mathbb{E}_c(\hat{c}))(1 - H(p)) + T - I - \mu(1 - H(p))(1 - F(\hat{c})).\]

Slightly abusing notations and now making the dependence of $U$ on $(p, \hat{c})$ explicit, this remark allows us to give in the next lemma an alternative and very compact expression of the firm’s profit.

**Lemma 3** The firm’s profit under a debt contract with audit for $c \geq \hat{c}$ satisfies:

\[(2.16) \quad U(p, \hat{c}) = (1 - H(p)) \int_{0}^{\hat{c}} F(c)dc.\]
This expression is remarkable. While under self-finance or traditional procurement, the firm is making zero profit, it now always enjoys some positive rent from having signed a debt contract with outside financiers while being privately informed on its costs. The firm’s rent is proportional to the demand for the service, namely $1 - H(p)$. That it depends positively on its scale of activities is a rather intuitive result since the proportionality follows from the fact that audit costs are counted per unit of demand. In addition, the rent is also proportional to a second term $\int_0^\hat{c} F(c) dc$, which comes from the fact that, with a debt contract, the firm repays financiers less than what it gets whenever its cost realization falls below the threshold $\hat{c}$. For all cost realizations $c \leq \hat{c}$ (so no audit) and for each unit of demand, the firm enjoys some gain $\hat{c} - c$ from such low repayment. Integrating those benefits over all infra-marginal costs then gives a payoff $\int_0^\hat{c} (\hat{c} - c) dF(c) = \int_0^\hat{c} F(c) dc$ where the equality follows from a simple integration by parts.

The firm’s rent is of course socially costly and both the pricing of the service and the debt contract will be modified accordingly to account for these extra costs. Equation (2.16) already shows how this may happen. First, by raising the regulated price and decreasing demand, the public authority is able to reduce this rent. Second, increasing the audit zone and choosing a lower threshold also helps.

On more technical grounds, we may notice that Condition (2.16) also establishes a one-to-one relationship between the firm’s payoff $U(p, \hat{c})$ and the face value of debt, or alternatively the threshold $\hat{c}$ above which outside financiers audit the project. In other words, controlling the firm’s profit through prices and subsidies allows the public authority to indirectly command the firm’s level of debt. Although the public authority has no direct control of the debt contract in practice, it may make the venture more attractive for outside financiers and change the cost of outside finance by simply increasing the public subsidy or raising the regulated prices so as to boost revenue from the service. This simple observation will later allow us to write the social welfare maximization in terms of $p$ and $\hat{c}$; understanding that the public authority indirectly commands access to outside finance. For the time being, it is important to stress that a lower likelihood of audit (i.e., a greater value of $\hat{c}$) and a lower level of debt also means that the firm obtains a greater information rent.

### 2.3. The Optimal Regulatory Charter

The optimal regulatory charter consists of a regulated price $p$, a public subsidy $T$, together with a financial contract which is optimally designed by competitive lenders given this regulatory contract. Using the expression of $U(p, \hat{c})$, social welfare becomes:

$$W(p, \hat{c}) = \omega(p) - (1 + \lambda)I - \lambda U(p, \hat{c}) - (1 + \lambda)(1 - H(p))\mu(1 - F(\hat{c}))$$

where we again make explicit the dependence on $(p, \hat{c})$. This expression highlights how the social cost of audit is counted in the public authority’s objectives. Since financiers are competitive, the private cost of audit ends up being borne by the firm itself and it can be recouped through extra public subsidies, whose social cost is $(1 + \lambda)(1 - H(p))\mu(1 - F(\hat{c}))$.

The properties of the optimal regulatory charter are summarized in the next proposition.

**Proposition 1** Suppose that financiers are competitive. The optimal regulatory charter $(p^*, \hat{c}^*)$ entails the following properties.
1. Price is above its value under traditional procurement, \( p^* > p^{sf} \), with:

\[
\frac{p^* - c^*}{p^*} = \frac{\lambda}{1 + \lambda \varepsilon(p^*)}
\]

where

\[
c^* = \mathbb{E}_c(\hat{c}) + \mu(1 - F(\hat{c}^*)) + \frac{\lambda}{1 + \lambda} \int_{0}^{\hat{c}^*} F(c) dc > \mathbb{E}_c(\hat{c}).
\]

2. The firm’s profit is always positive:

\[
U(p^*, \hat{c}^*) = (1 - H(p^*)) \int_{0}^{\hat{c}^*} F(c) dc > 0.
\]

3. Bankruptcy is more likely than with traditional procurement. We have \( \hat{c}^* > c_m \) with:

\[
\frac{F(\hat{c}^*)}{f(\hat{c}^*)} = \mu \left( 1 + \frac{1}{\lambda} \right).
\]

**Higher prices and more audit.** Leaving a positive rent to the firm is now socially costly. From our discussion of Lemma 3, reducing the firm’s information rent requires jointly acting on the regulated price and the probability of bankruptcy.

As far as price distortions are concerned, observe that increasing the price of the service depresses demand. It has thus a direct impact in reducing the overall cost of audit but also the cost of the firm’s information rent since both costs are counted per unit of demand. The pricing formula (2.17) ends up being very close to that obtained with traditional procurement except for a change in the value of the costs. Everything indeed happens as if the firm’s average cost \( \mathbb{E}_c(\hat{c}) \) was now replaced by a virtual cost \( c^* \) which is strictly greater. This virtual cost incorporates not only the cost of audit (the second term on the right-hand side of (2.18)) but also the extra cost of leaving information rent to the firm (the third term on the right-hand side of (2.18)) per unit of demand.

The presence of the cost of audit in this formula comes from the fact that the cost of bankruptcy borne by financiers finally ends up being passed onto the public authority since it must be covered through subsidies to ensure that financiers break even and that the firm operates.

Since the public authority cares about the firm’s payoff *per se*, the social cost of the firm’s rent is less than the private cost that is perceived by financiers. More formally, the virtual costs considered by the regulator differ from the costs, perceived by the lenders, that serve to compute (2.10).

Turning now to the probability of audit, the public authority can also reduce the firm’s information rent by inducing more audit by its financiers. This objective is mainly achieved by now reducing the direct subsidy so as to force financiers to take a greater share of the project. The level of debt increases and, by the same token, the likelihood of bankruptcy also does so.
Observe that $p^*$ increases with the cost of public funds. As budgetary pressures increase, it becomes more attractive to use the firm’s revenue to facilitate its access to outside finance and less attractive to rely on direct subsidies.

Finally, interpreting (2.17) as a pass-through formula also shows that, with private finance, the regulated price increases with the cost of bankruptcy in (higher values of $\mu$) or with the cost of public funds (higher values of $\lambda$) because their impacts on the firm’s virtual cost are similar. This leads us to the following testable implications.

**Testable Implications 1**  
*Regulated prices are higher when the cost of bankruptcy increases.*

**Optimal Level of Debt.** The public authority designs a regulatory environment so as to induce outside financiers to ask for a debt which remains below what a monopolistic lender would chose, and which thus induces less audit $\hat{c}^* > c_m$. The intuition for this result is straightforward. A monopolistic lender cares about extracting the firm’s rent and to do so relies on a very likely audit ($\hat{c} = c_m$) and the highest possible level of debt $D_m$. Instead, the public authority is less concerned by rent extraction and wants to induce less audit and a lower level of debt. As the cost of public funds increases, the public authority and outside financiers become more alike in evaluating the firm’s rent. As a result, we obtain the following implications.

**Testable Implications 2**  
*Firms have more debt and bankruptcy is more likely when the cost of public funds increases or when the cost of bankruptcy decreases.*

**Optimality of Investment.** The investment $I$ is now socially valuable when:

\[
\omega(p^*) \geq (1 + \lambda)I + (1 - H(p^*)) \left( (1 + \lambda)\mu(1 - F(\hat{c}^*)) + \lambda \int_{c^*}^{\hat{c}^*} F(c)dc \right).
\]

This condition is clearly harder to satisfy than its counterpart (2.5) in the traditional procurement scenario because now both the social cost of audit and the social cost of the firm’s rent must also be covered by social surplus. As a result, fewer projects will be undertaken with outside finance than if traditional procurement was possible. It is important to notice that this comparison is driven by the assumption that, under traditional procurement, costs are supposed to be verifiable while a costly audit is necessary under the private finance scenario. Alternatively, with non-verifiable costs and under the assumption that the public authority faces a (much) higher cost of audit than private financiers, traditional procurement would allow fewer projects to be undertaken.

**The Mix Between Public Subsidies and Outside Finance.** Using (2.19) and (2.18) now yields an expression of the optimal public subsidy as:

\[
T^* = I - (1 - H(p^*)) \left( p^* - c_e^* - \frac{1}{1 + \lambda} \int_{c_e^*}^{\hat{c}^*} F(c)dc \right),
\]

or simplifying further

\[
T^* = I - \frac{\lambda}{1 + \lambda} \frac{(1 - H(p^*))^2}{h(p^*)} + \frac{U(p^*, \hat{c}^*)}{1 + \lambda}.
\]
This condition again bears some resemblance with what was found under direct finance, namely Equation (2.6). The main difference comes from the fact that part of the public subsidy is now dissipated and goes to the firm under the form of information rent.

**Testable Implications 3**  
With outside finance, public subsidies are of a greater magnitude.

### 3. Alternative Assumptions

We now assess the robustness of our findings to various alternative assumptions and derive related testable implications.

#### 3.1. Social Cost of Bankruptcy

Firms in infrastructure sectors are often big players involved in projects of prime importance for the economy. When their project get bankrupt, it generates a social cost that differs from the simple private cost of bankruptcy borne by financiers. To illustrate the consequences of that wedge between the private and the social costs of bankruptcy in our framework, let us suppose that in case of bankruptcy, the public authority must also account for a loss which, expressed in terms of compensatory payments and still per unit of demand, is worth:

\[(1 + \lambda)\kappa(1 - H(p))(1 - F(\hat{c}))\]

where \(\kappa > 0\).

It is straightforward to check that Proposition 1 carries over mutatis mutandis with the only modification coming from replacing \(\mu\) by \(\mu + \kappa\). We obtain immediately.

**Testable Implications 4**  
Firms have less debt and bankruptcy is less likely when the social cost of bankruptcy increases.

#### 3.2. Consumption Subsidy

In many circumstances, the public authority does not provide lump-sum subsidies to the firm but instead directly subsidizes consumption, thereby boosting demand and indirectly the firm’s revenues much like what a lump-sum subsidy \(T\) would do. To see the impact of a per unit of consumption subsidy \(s\) on demand and profits, observe that the aggregate demand becomes \(1 - H(\tilde{p})\) where \(\tilde{p} = p - s\) is now the perceived price paid by consumers while \(p\) remains the price charged by the firm. With these notations, we may rewrite the firm’s profit as:

\[(\tilde{p} - c^e)(1 - H(\tilde{p})) + s(1 - H(\tilde{p})) - I - \mu(1 - H(\tilde{p}))(1 - F(\hat{c}))\]

This expression shows that everything happens as if the price charged by the firm was now \(\tilde{p}\) itself while the firm also receives an overall subsidy \(s(1 - H(\tilde{p}))\) which plays the same role as the lump-sum subsidy \(T\) used previously. Of course, with such an implementation, there is no direct transfer given to the firm and proceeding that way certainly saves on the cost of public funds. On the other hand, this consumption subsidy is still taken from taxpayers elsewhere in the economy and it still has the very same social cost worth
\((1 + \lambda)s(1 - H(\hat{p}))\), another instance of Ricardian equivalence. It follows that welfare is now maximized when the perceived price and the consumption subsidy satisfy

\[
\hat{p}^{cs} = p^* \quad \text{and} \quad s^{cs} = \frac{T^*}{1 - H(p^*)}.
\]

Anticipating that consumption is subsidized, the firm charges a higher price that undoes the impact of such subsidy on demand.

An immediate corollary of our Testable Implications 3 is thus:

**Testable Implications 5** With outside finance, and in the absence of direct transfers to the firm both consumption subsidies and prices are of a greater magnitude.

**Remark.** Suppose now that there is an hidden extra social cost of using lump-sum transfers to directly subsidize the firm; i.e., raising \(T\) now costs \((1 + \lambda')T\) with \(\lambda' > \lambda\). Motivations for such extra cost might come from the need to prevent corrupted political decision-makers and bureaucrats from embezzling public funds and to increase transparency of the process.\(^{16}\) Clearly, the above equivalence fails and using a consumption subsidy dominates.

### 3.3. Congruence Between Public Authority and the Industry

Our baseline model can be readily extended to allow for the possibility that the public authority and the firm are more congruent. We will be somewhat agnostic on the source of such alignment. The model is crude enough to account for the possibility of such congruence coming from direct corruption of public officials,\(^ {17}\) or from the fact that, for electoral reasons,\(^ {18}\) the public authority may want to please a constituency, which has a stake in the firm, as when employment in those regulated firms is an issue of critical importance at elections time.

Expressing again \(T\) in terms of \(U\) from (2.2) and using (2.1), we finally obtain:

\[
(3.1) \quad W(p, T) = \omega(p) - (1 + \lambda)I - (\lambda - \alpha)U.
\]

where, following Baron and Myerson (1982), \(\alpha \geq 0\) denotes the degree of congruence between the public authority and the firm. This expression highlights that the firm’s information rent remains costly as long as the congruence parameter is not too large, namely \(\alpha < \lambda\), an assumption made from now on. Yet, the incentives to distort prices and increase the likelihood of bankruptcy for rent extraction reasons diminish in this context. Indeed, the firm’s virtual cost is lower. As a result, the price decreases.

Mimicking our earlier findings yields the following proposition, which proof is immediate:

**Proposition 2** Suppose that financiers are competitive but that the public authority’s and the firm’s objectives are congruent. The optimal regulatory charter \((p^\alpha, \hat{c}^\alpha)\) entails the following properties.

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\(^ {16}\)On this issue, see Martimort and Straub (2009).

\(^ {17}\)Dixit (2010).

\(^ {18}\)Baron (1988).
1. Price is above its value under traditional procurement, \( p^\alpha > p^{sf} \) but lower as \( \alpha \) increases:

\[
\frac{p^\alpha - c^\alpha}{p^\alpha} = \frac{1}{1 + \lambda \varepsilon(p^\alpha)}
\]

where

\[
c^\alpha = \mathbb{E}_c(\check{c}) + \mu(1 - F(\check{c}^\alpha)) + \frac{\lambda - \alpha}{1 + \lambda} \int_{\check{c}^\alpha}^{\hat{c}^\alpha} F(c)dc > \mathbb{E}_c(\check{c}).
\]

2. Bankruptcy becomes less likely as \( \alpha \) increases:

\[
\frac{F(\check{c}^\alpha)}{f(\check{c}^\alpha)} = \mu \left(1 + \frac{1}{\lambda - \alpha}\right).
\]

Testable implications immediately follow.

**Testable Implications 6** Regulated prices are lower, firms have less debt and bankruptcy is less likely when firms and public authorities are more congruent.

If we keep in mind that more congruence might reflect the degree of corruption that prevails in weak institutional environments, we might also obtain:

**Testable Implications 7** Regulated prices are lower, firms have less debt and bankruptcy is less likely in weaker institutional environments.

### 3.4. Financiers With Monopoly Power

Our assumption that financiers are competitive might certainly be easier to justify in the case of well-developed financial markets than what might prevail in most developing countries, where various forms of transaction costs and entry barriers might give market power to financial institutions. We now investigate how our previous findings are modified in that case.

We already know from Lemma 1 that a monopolistic lender always fixes the highest possible probability of bankruptcy since it wants to extract from the firm as much information rent as possible. Such lender offers a debt contract which defines a threshold for the audit zone at \( \hat{c} = c_m \). Importantly, this choice is independent of the regulatory instruments \((p, T)\) used by the public authority. While with competitive financiers, the public authority was able to indirectly control for the optimal probability of bankruptcy, this control is now lost in the case of a monopolistic lender. Anticipating the added restriction that the probability of bankruptcy is now responsive to the monopolistic lender’s sole objectives, the public authority must still choose his regulatory instruments \((p, T)\) with a view on participation of this lender. In other words, the feasibility requirement (2.13) still applies. Since public subsidies are socially costly, this constraint is again binding at the optimum and even a monopolistic lender ends up making zero profit. From this it follows that the firm’s rent is still given by (2.16) with the additional constraint that \( \hat{c} = c_m \). Of course, social welfare is always lower in that scenario.

*Mutatis mutandis*, we can derive the main features of the optimal regulatory charter as follows.
**Proposition 3** Suppose that financiers have monopoly power. The optimal regulatory charter \((p^m, c^m)\) entails the following properties.

1. Price is above its value with competitive financiers, \(p^m > p^*\), with:

\[
(3.5) \quad \frac{p^m - c^m}{p^m} = \frac{\lambda}{1 + \lambda \varepsilon(p^m)}
\]

where

\[
(3.6) \quad c^m = E\tilde{c}(\tilde{c}) + \mu(1 - F(c_m)) + \frac{\lambda}{1 + \lambda} \int_0^{c_m} F(c) dc > c^*.
\]

2. The firm’s profit is lower than with competitive financiers:

\[
(3.7) \quad U(p^m, \hat{c}^m) = (1 - H(p^m)) \int_0^{c_m} F(c) dc < U(p^*, \hat{c}^*).
\]

3. Bankruptcy is more likely than with competitive financiers:

\[
(3.8) \quad \hat{c}^m = c_m < c^*.
\]

Pushing a bit further the interpretation of our results, we conjecture that, when financial markets are not well developed, maybe because of existing entry barriers, and market power is a concern even in a less crude way than modeled here, the firm has more debt and bankruptcy becomes more likely. The virtual cost of the firm increases, which further raises prices and depresses demand. The firm’s information rent also diminishes as a result of both a demand reduction and a more likely bankruptcy.

We can state sum up those comparative statics as the following testable implications:

**Testable Implications 8** Regulated firms have less debt, bankruptcy is less likely, and regulated prices are lower as financial markets are more developed.

### 3.5. Limited Commitment

An important assumption of our analysis is that the public authority has the ability to commit to the regulatory instruments, i.e., namely \((T, p)\), and that this stability of the regulatory environment is viewed as attractive enough by financiers. In practice, the public authority’s commitment ability is certainly more limited, which in turn makes it harder to attract financiers. Outside financiers will request a premium for the risk of changes in the rules of the game.

To view how this premium translates into distortions of prices and levels of indebtedness, we now analyze how the regulatory contract is modified under limited commitment. Our starting assumption is that, once a debt contract with outside financiers has already been signed, the public authority may gain, maybe in response to short-term electoral incentives, from decreasing the price \(p\) below what financiers expected. Let us denote by \((p^e, T^e)\) the regulatory contract that is expected at equilibrium. The public authority enjoys an extra gain \(B(p^e - p)\) from decreasing the price \textit{ex post}. We assume that \(B\) is increasing, concave and satisfies \(B(0) = 0\). The parameter \(B'(0)\) will capture the magnitude of incentives to decrease the price for electoral reasons. Of course, reducing
the price in response to those short-term incentives while, at the same time, the face-value of debt remains the same would increase the probability of bankruptcy. It means that the public authority must also increase the public subsidy $T$ in order to guarantee that outside financiers still break even.\footnote{Observe that this increase in the public subsidy is needed as long as the debt contract itself cannot be perfectly renegotiated. Our implicit assumption here is that such renegotiation is just impossible. In practice, reorganizing the firm’s activity in times of financial distress, selling some of the firm’s assets on short notice, and more generally restructuring claims among various claimholders is certainly a costly process as it has been argued at length in the recent corporate finance literature.} Formally, a change in regulatory instruments $(p, T)$ maintains a non-negative profit for financiers if and only if

$$\varphi \left( p - \frac{D - T}{1 - H(p)} \right) \leq p - \frac{I - T}{1 - H(p)}. \quad (3.9)$$

This condition (3.9), which immediately follows from (2.7) and (2.10), thus determines all the possible deviations available to the public authority. Of course, this condition also holds at the conjectured regulatory contract $(p^e, T^e)$ but it is then an equality since conjectures must be correct at equilibrium. This implicitly defines the value of $D$. Yet, the public authority’s incentives to cut the regulatory price off path obviously do matter which in turn determines the equilibrium values of the regulatory contract.

Assuming that the political authority’s objective remains quasi-concave in that limited commitment scenario so that necessary conditions for optimality are also sufficient, we obtain the following characterization.

**PROPOSITION 4** Suppose that the public authority has limited commitment. Necessary conditions for a regulatory charter $(p^{lc}, \hat{c}^{lc})$ to be an equilibrium are as follows.

1. The regulated price decreases with the magnitude of ex post incentives $B'(0)$:

$$\frac{p^{lc} - \hat{c}^{lc}}{p^{lc}} = \frac{\lambda}{1 + \lambda \varepsilon(p^{lc})} - \frac{B'(0)}{(1 + \lambda)p^{lc}h(p^{lc})} \quad (3.10)$$

where

$$\hat{c}^{lc} = \mathbb{E}_{\hat{c}}(\hat{c}) + \mu(1 - F(\hat{c}^{lc})) + \frac{\lambda}{1 + \lambda} \int_{0}^{\hat{c}^{lc}} F(c) dc. \quad (3.11)$$

2. Bankruptcy is less likely than under full commitment:

$$\hat{c}^{lc} > c^e. \quad (3.12)$$

There exists in fact a multiplicity of equilibria to the game, thanks to the rational expectations that financiers hold. Those equilibria differ in terms of the price and debt level of the firm. Yet, all those equilibria are characterized by downward price distortions and less debt. The intuition for why the public authority has some incentives to reduce the price for the service is straightforward. It is a direct consequence from the fact that the marginal electoral gains of doing so is positive. Less intuitive is the fact that bankruptcy is less likely and that debt decreases. Indeed, the set of possible deviations from any putative equilibrium contract is constrained by the requirement of keeping financiers on board. This means that any cut on price has also to come with an increase in the direct subsidy to the firm so as to pay for its debt. This puts an extra burden on the public budget and such constraint is of course released by inducing lower levels of debt. We can thus conclude this section with the following implications.
Testable Implications 9  Firms have less debt, bankruptcy is less likely, and regulated prices are lower when public authorities have more limited commitment.

3.6. Endogenizing the Cost of Public Funds

In this section, we endogenize the value of the cost of public funds and show the robustness of our previous results. Doing so will also allow us to relate our findings to exogenous parameters like the level of deficit or the distribution of income. To see how, suppose that society is made of two groups: owners of the firm and consumers of its service. Owners, who are in mass one, enjoy the firm’s rent. Owners pay a tax $T_0$ to finance the infrastructure and, more generally, any public deficit $E$. They also enjoy a wealth endowment $\Omega_o$ (which should be viewed as net of their expenditures on other goods and services). For simplicity, we also assume that profits are taxed away. Overall, owners get a net utility worth:

$$U_0 = \Omega_o + (1 - H(p)) \int_0^{\hat{c}} F(c) dc - T_o.$$

Consumers, who are also in mass one, receive a wealth endowment $\Omega_c$ to start with, enjoy the consumption of the service at price $p$ but on top also pay a tax $T_c$ to finance the infrastructure. Their net utility thus writes as:

$$U_c = \Omega_c + \int_p^{+\infty} (1 - H(\tilde{v})) d\tilde{v} - T_c.$$

The contributions $T_o$ and $T_c$ should be set so as to cover not only the investment, the firm’s rent but also a deficit $E$ if needed. Given that the firm’s profit, net of the auditing costs, is completely taxed away, the public authority’s budget constraint can be written as:

$$(3.13) \quad T_o + T_c + (p - \mathbb{E}_c(\hat{c}))(1 - H(p)) - \mu(1 - H(p))(1 - F(\hat{c})) = I + E + (1 - H(p)) \int_0^{\hat{c}} F(c) dc.$$

The public authority’s problem is now to maximize a social welfare function, namely $\mathcal{V}(U_0) + \mathcal{V}(U_c)$, where $\mathcal{V}$ is increasing and strictly concave, subject to the budget balance condition (3.13). Strict concavity of $\mathcal{V}$ means that the public authority has some redistributive concerns. Since both groups receive the same weight in his objective, the public authority would ideally equalize their payoffs. When taxation is frictionless, perfect redistribution is achieved while, at the same time, the overall surplus

$$\int_p^{+\infty} (1 - H(\tilde{v})) d\tilde{v} + (p - \mathbb{E}_c(\hat{c}))(1 - H(p)) - \mu(1 - H(p))(1 - F(\hat{c}))$$

is maximized. At the optimum, the regulated price is now just equal to the expected cost of the service:

$$(3.14) \quad p^{end} = \mathbb{E}_c(\hat{c}).$$

When there are no frictions in redistribution, the fact that owners enjoy the firm’s information rent has no impact. Indeed, that rent can be passed onto consumers at no social cost. Efficiency thus follows.
As a by-product of the fact that the firm’s information rent no longer needs to be minimized, audit has no social value. Since audit is costly, it should never take place in this context and:

\[(3.15) \quad \hat{c}_{\text{end}} = +\infty.\]

Of course, efficiency is no longer achieved when redistribution is costly. One possibility to model such friction is to assume that consumers and owners cannot be distinguished. Taxes have thus to be identical for the two groups, i.e.:

\[(3.16) \quad T_o = T_c \equiv T.\]

This constraint is enough to generate frictions and offer an endogenization of the cost of public funds.

**Proposition 5** Suppose that (3.16) holds. The optimal regulatory charter \((p^{\text{end}}, \hat{c}^{\text{end}})\) again satisfies condition (2.17), (2.18) and (2.20) with \(\lambda\), the non-negative Lagrange multiplier for (3.13), satisfying

\[(3.17) \quad \lambda = \frac{\mathcal{V}' \left( \Omega_o + (1 - H(p^{\text{end}})) \int_{0}^{\hat{c}^{\text{end}}} F(c) dc - T^{\text{end}} \right)}{\mathcal{V}' \left( \Omega_c + \int_{p^{\text{end}}}^{+\infty} (1 - H(\bar{v})) d\bar{v} - T^{\text{end}} \right)} \]

where

\[(3.18) \quad T^{\text{end}} = I - (p^{\text{end}} - E_c(\hat{c}))(1 - H(p^{\text{end}})) + \mu(1 - H(p^{\text{end}}))(1 - F(\hat{c}^{\text{end}})) + (1 - H(p^{\text{end}})) \int_{0}^{\hat{c}^{\text{end}}} F(c) dc + E.\]

Those formula are suggestive of the following comparative statics. Suppose that \(\Omega_o\) is much larger than \(\Omega_c\) (the rich class owns the firm) and \(\mathcal{V}\) exhibits decreasing inequality aversion (i.e., \(\frac{d}{dx} \left( -\frac{\mathcal{V}'(x)}{\mathcal{V}(x)} \right) \leq 0 \)).\(^{20}\) Then a greater deficit \(E\) has a direct effect in increasing the cost of public funds (while of course an indirect effect impacts the optimal variables \(p^{\text{end}}\) and \(\hat{c}^{\text{end}}\)). This suggests in turn that, when communities face more stringent budget constraint, price distortions are more pronounced while the probability of bankruptcy increases as well. In that case, everything happens as if a poorer political authority was implicitly increasing the bargaining power of financiers in their relationship with the firm; moving closer to the solution found with a monopolistic lender. It implies that the level of debt increases and bankruptcy becomes more likely.

**Testable Implications 10** Regulated prices are higher, firms have more debt, and bankruptcy is more likely when public authorities are constrained by greater public deficits.

\(^{20}\)Martimort (2001) presents a political economy model of redistribution based on this assumption.
4. STYLIZED FACTS AND POLICY IMPLICATIONS

Our model helps put in perspective the current renewed interest around private financing for infrastructure, which appears in different guises. The first one is the strong recurrent narrative in policy circles on the need to generate a growing stream of PPPs to boost infrastructure investment levels. The line of argument relies both on the need to mobilize additional resources, and on the belief that the involvement of private counterparts improves project efficiency.

However, by recent estimates PPPs still represent at most 10% of the approximately $1.3 trillion invested yearly in infrastructure by developing countries' government, not even considering the fact that most of these projects involve an important share of public funds, probably close to half, either directly or indirectly through guarantees (Fay et al., 2017). This means that doubling PPPs would add at around 5% to infrastructure investment in developing countries. Similarly, the jury is still out on the efficiency argument.

The second one is the idea that trillions of dollars held by institutional investors such as pension funds, sovereign wealth funds, and insurance companies, could somehow be channeled towards infrastructure projects (Arezki et al., 2017). Again, there are strong indications that the scope for this should not be overestimated. Indeed, Inderst and Stewart (2014) rely on several available database on institutional investors to estimate an amount of around $11 trillion of assets under management worldwide, of which under optimistic assumptions, at most $60 billion could flow to infrastructure investment in emerging and developing countries.

Finally, the third argument refers to the idea that public finance may under certain conditions crowd-in private finance towards infrastructure projects through so-called blended finance arrangements. A refined version of this argument is found in the World Bank’s ‘Maximizing Finance for Development’ approach, which establishes a step-by-step approach to prioritize and attract non-government guaranteed, commercial financing, by assessing its viability and addressing if needed specific regulatory and institutional reforms (World Bank, 2017a).

These different narratives are clearly related, and call for a better understanding of the specific aspects that determine the desirability and availability of private financing for infrastructure. Our framework helps clarify this debate. It first shows why relatively little private money flows into infrastructure, namely that this requires a combination of regulated prices and transfers that are high enough. In cases where bankruptcy costs are very high, as is indeed the case in most developing countries, the feasibility constraints are very high, as is indeed the case in most developing countries, the feasibility constraints

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21 If anything the Figure for 2016 was even lower, at around 5%. See World Bank (2017b).
23 Note that the discussion often confuses the issues of private management and/or ownership of assets with that of private financing. While these might be related, there need not be a one-to-one relationship. For example, State Owned Enterprises (SOEs) can also be commercially financed, through the emission of bonds or bank loans, providing an example of public ownership together with private finance. Also, SOEs often play the role of corporate "private" partners in PPPs. See the recent Fitch report on Chinese PPPs, Reuters February 19, 2017, http://www.reuters.com/article/fitch-ppps-to-drive-china-infrastructure-idUSFit989432.
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may simply be impossible to satisfy, precluding access to private finance. Clearly, providing more efficient mechanisms to resolve bankruptcy cases tops the list of desirable policy reforms in that respect.

In addition, even if this feasibility constraint is satisfied, the implications in term of higher prices and lower access are likely to restrict the social acceptability of private arrangements and fragilize the social contract between the state and the population, including non-poor segments.\(^{24}\)

The other important aspect highlighted by our model is that other institutional and efficiency improvements, notably those regarding the cost of raising taxation and the competitiveness of the financial sector, have the opposite effect of enhancing the comparative advantage of public finance and should therefore reduce the likelihood to rely on private debt. Again, this explains why countries do not switch to extensive private financing as their public sectors become more efficient.

Overall, these effects paint a complex picture, in which reforms and institutional improvements have heterogeneous effects that may increase the feasibility of both private and public finance along the development path. We conjecture that this may lead to a non-linear relationship between the prevalence of private finance for infrastructure and the level of development, as different effects dominate at different level of per capita GDP.

Next, we use a sample of infrastructure projects from the World Bank Private Participation in Infrastructure (PPI) database to illustrate these different effects.

Starting 2015, the PPI team collected, whenever possible, information on the financing structure of the projects. The PPI database includes any infrastructure project (energy, telecommunications, transport, water and sanitation) in low- and middle-income countries for which information is available in publicly available sources, which have at least 20% private participation in the project contract, and in which private parties assume operating risk, by either operating the project alone or jointly with a public counterpart, or owning an equity share in the project. As such, this data base covers a large variety of projects and is relevant to illustrate our model.

The PPI data base contains detailed financial information for 361 projects, across four infrastructure sectors (energy, transport, water and sanitation, and ICT backbone) and three years (2015-2017; see Table I). The projects span 55 countries across 6 regions of the world.\(^{25}\)

For each of these projects, the data reports the total financing amount, disaggregated between public and private financing. These two categories are further disaggregated in the following way:

\[
\text{PublicFinancing} = \text{PublicDebt} + \text{PublicEquity} + \text{DirectGovernmentSupport}
\]

with Public Debt = Multilateral Debt + Bilateral Debt + Institutional Debt + Public National Debt, and

\[
\text{PrivateFinancing} = \text{CommercialDebt} + \text{PrivateEquity}.
\]

\(^{24}\)See Martimort and Straub (2009), and Bonnet et al. (2012) for such an argument in the context of infrastructure privatization.

\(^{25}\)See the list of countries in the Appendix.
Based on this, the financing indicators used in the paper are computed as follows:

\[(4.3) \quad \text{Share of Private Financing} = \frac{\text{Private Financing}}{\text{Total Financing}},\]

where Total Financing = Public Financing + Private Financing, and

\[(4.4) \quad \text{Share of Private Debt} = \frac{\text{Private Debt}}{\text{Total Debt}},\]

where Total Debt = Public Debt + Commercial Debt.

Following the model, the institutional and financial market characteristics that are relevant to the financial structure of projects fall in several categories.\(^{26}\)

First, the availability of private project financing is likely to be dependent on features of the financial sector, including the perceived cost of bankruptcy and the level of competitiveness. We consider the following proxies:

- The Doing Business Resolving Insolvency indicator, as a measure of the cost of audit facing financial institutions that lend to the projects.
- The ratio of the assets of the five largest banks to total commercial banking assets, as a measure of the competitiveness of local financial markets, from the World Bank Global Financial Development Database.

Second, we consider the cost of public funds, due to distortionary taxation:

- The marginal cost of public funds (MCF) estimates computed by Ensor (2016) for 106 countries, following Auriol and Warlters’ (2012) model.

Third, we consider two measures of institutional quality from the World Bank Worldwide Governance Indicators (WGI): Regulatory Quality, and Control of Corruption.

Figure 1 illustrates the correlations between the project-level ratio of private to total debt, as defined in (4.4), and the level of development, measured with per capita GDP.\(^{27}\) The non-linear fit reveals an inverse U-shaped relationship, with the share of private finance being maximized at an intermediate level of pcGDP. The left panel uses the whole set of projects with financial information. To adjust for the fact that countries with a lot of projects such as Brazil, India, or Mexico, are potentially biasing the pattern, the right panel shows country averages pooled over the three years.

In the Appendix, Figure A1 provides equivalent correlations using instead the ratio of private to total financing defined in (4.3). The conclusions are unchanged.

As argued above, the concave pattern linking private finance use and pcGDP is likely to be the result of complex relationships between different features of the institutional environment and the optimal finance mix. In Table II, we regress the project-level share of private debt on each of the characteristics defined above, controlling for subsector and region fixed effects.

Column 1 shows the regression equivalent of the plots in Figure 1. The coefficient for the per capita GDP and its square are strongly significant and imply that the relationship

\(^{26}\)See the Appendix for a detailed description of all the variables and their sources.

\(^{27}\)The sample covers 327 projects, excluding 34 projects for which there is no debt financing.
has an inverse U-shaped form and becomes negative above US$9320. In column 2, the coefficient for resolving insolvency is positive and significant, indicating that projects in countries with more efficient insolvency frameworks attract higher shares of private debt as predicted by our model. A similar correlation is obtained in columns 3 and 4 for less corrupt environments and those characterized by better regulatory quality.

In column 5, on the other hand, projects in countries with a higher marginal cost of public funds also boast higher shares of private debt, and so do, in column 6, those in countries with more concentrated banking sectors although the coefficient is not significant.

These correlations illustrate and confirm the main relationships posited by our model. While they should of course not be taken as causal claims, they suggest that the optimal level of private debt is the result of the complex interplay between features of the environment that determine the efficiency of the private financing mechanisms and others that have to do with the cost of public finance.

5. CONCLUDING REMARKS

We have developed a model of infrastructure funding and financing, in which access to outside finance, the extent of price recovery resulting from the regulatory decision on pricing, and the amount of public subsidy, are jointly determined. This model highlights a number of key trade-offs faced by policymakers when involving private finance in infrastructure projects. A number of lessons stand out.

First, mobilizing private finance for infrastructure projects in developing countries requires setting a combination of price for the service and subsidy to the service provider that are large enough. This exacerbates the fundamental tension between financial viability based on cost recovery and social inclusion, and might be politically difficult to handle in countries with large fractions of poor consumers.

Second, this feasibility trade-off is affected by the nature of the projects and by characteristics of the economic and institutional environment, such as the efficiency of bankruptcy procedures, the cost of public funds, regulatory capacity, and corruption among others.

Interestingly, while improvements along some of these dimensions appear to ease access to private finance, for others it actually makes public finance more efficient. The latter is particularly true for reductions in the cost of public funds, which tilt the balance towards more public subsidies.

As a result, the evolution of the desirability of private finance for infrastructure as countries transit the development path appears to be less obvious than posited by the different narratives reviewed above. In fact, we uncover an inverse U-shaped pattern, peaking up for countries in the upper-middle income range.

Countries with infrastructure deficit should therefore not consider the expansion of private finance as the sole way to increase investment. In fact, policy reforms meant to improve the business environment are likely to have competing effects. Some will make

\[28\text{It is likely, however, that local financial conditions are not strictly relevant for the financing of projects that often involve international operators with access to international financial markets.}\]
infrastructure ventures more attractive for private financiers, while others will improve public sectors’ ability to raise taxes and spend efficiently. Our analysis suggest that both paths should be pursued.

REFERENCES


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Figure 1.— Projects’ Share of Private Debt as a Function of per capita GDP
### TABLE I
**Infrastructure Projects with Financial Information**

<table>
<thead>
<tr>
<th>Year</th>
<th>Energy</th>
<th>ICT</th>
<th>Transport</th>
<th>Water &amp; sanitation</th>
<th>All</th>
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<tr>
<td>2015</td>
<td>93</td>
<td>0</td>
<td>14</td>
<td>9</td>
<td>116</td>
</tr>
<tr>
<td>2016</td>
<td>77</td>
<td>1</td>
<td>14</td>
<td>1</td>
<td>93</td>
</tr>
<tr>
<td>2017</td>
<td>117</td>
<td>1</td>
<td>29</td>
<td>5</td>
<td>152</td>
</tr>
<tr>
<td>Total</td>
<td>287</td>
<td>2</td>
<td>57</td>
<td>15</td>
<td>361</td>
</tr>
</tbody>
</table>

### TABLE II
**Share of Private Debt: Conditional Correlations**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>pcGDP</td>
<td>0.149***</td>
<td>(0.0333)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>pcGDP squared</td>
<td>-0.00802***</td>
<td>(0.00251)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Resolving Insolvency</td>
<td>0.00684**</td>
<td>(0.00264)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Control of corruption</td>
<td>0.161**</td>
<td>(0.0722)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regulatory quality</td>
<td>0.280***</td>
<td>(0.0681)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost of Public Funds</td>
<td>1.538***</td>
<td>(0.293)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Banking Concentration</td>
<td>0.00249</td>
<td>(0.00219)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Yes</th>
<th>Yes</th>
<th>Yes</th>
<th>Yes</th>
<th>Yes</th>
<th>Yes</th>
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</thead>
<tbody>
<tr>
<td>Subsector F.E.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Region F.E.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Observations</td>
<td>326</td>
<td>278</td>
<td>326</td>
<td>326</td>
<td>141</td>
<td>323</td>
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<tr>
<td>R-squared</td>
<td>0.206</td>
<td>0.131</td>
<td>0.144</td>
<td>0.175</td>
<td>0.403</td>
<td>0.131</td>
</tr>
</tbody>
</table>
**PROOF OF LEMMA 1:** Differentiating $\varphi$ w.r.t. $c$ yields:

$$\frac{\varphi'(c)}{f(c)} = -\mu + \frac{F(c)}{f(c)}.$$  

From Assumption 1, the right-hand side above is increasing and thus has a unique zero, whatever the value of $\nu$. Hence, $\varphi$ is quasi-convex with a minimum at $c_m$ defined in (2.11).  

**Q.E.D.**

**PROOF OF LEMMA 2:** We first rewrite the feasibility condition (2.13) in terms of $U(p, \hat{c})$, once the private investment $I - T$ has been eliminated using (2.15) to get:

$$U(p, \hat{c}) \geq (1 - H(p)) (\varphi(c_m) - E\hat{c}(\hat{c}) - \mu(1 - F(\hat{c})).$$

Using (2.16) to express the left-hand side and simplifying, this condition becomes:

$$(A1) \quad \int_{c_m}^{\hat{c}} F(c) dc - \mu F(\hat{c}) \geq \int_{0}^{c_m} F(c) dc - \mu F(c_m).$$

Observe that for $c \geq c_m$ we have:

$$\frac{d}{dc} \left( \int_{0}^{c} F(c) dc - \mu F(c) \right) = F(c) - \mu f(c) \geq 0.$$

Thus, the feasibility condition (A1) holds when (2.14) is satisfied.  

**Q.E.D.**

**PROOF OF LEMMA 3:** From the fact that lenders are competitive, the firm’s profit can be expressed as in (2.15). Using (2.9), we can now rewrite:

$$\frac{U(p, \hat{c})}{1 - H(p)} = \hat{c} - \bar{c}(\hat{c}) - \int_{\hat{c}}^{\infty} (1 - F(c)) dc.$$

Integrating by parts the right-hand side, we finally obtain (2.16).  

**Q.E.D.**

**PROOF OF PROPOSITION 1:** The public authority’s maximization problem can now be written as:

$$(P) : \max_{(p, \hat{c})} W(p, \hat{c}) \text{ subject to (2.14) and (2.16).}$$

First, observe that (2.16) is necessarily binding at the optimum of $(P)$. The objective function can thus be written as:

$$W(p, \hat{c}) = \omega(p) - (1 + \lambda)I - (1 + \lambda)\mu(1 - H(p))(1 - F(\hat{c})) - \lambda(1 - H(p)) \int_{0}^{\hat{c}} F(c) dc.$$  

We compute

$$\frac{\partial W}{\partial \hat{c}}(p, \hat{c}) = (1 - H(p))(1 + \lambda)\mu f(\hat{c}) - \lambda F(\hat{c}).$$
By an argument similar to that made in the Proof of Lemma 1, we can easily show that \( W(p, \hat{c}) \) is quasi-concave in \( \hat{c} \) and maximum for \( \hat{c}^* \) defined in (2.20). From this, and using the monotonicity condition in Assumption 2, it follows that \( \hat{c}^* > c_m \). Thus, Condition (2.14) is slack. We immediately obtain (2.19). Finally, the first-order condition w.r.t. \( p \) yields (2.17).

**Proof of Proposition 3:** The formula (3.5) to (3.7) are directly obtained by taking into account that necessarily \( \hat{c} = c_m \) with a monopolistic lender. Turning now to some comparative statics, observe that \( c^* \) achieves the minimum of the quasi-convex function:

\[
\mu(1 - F(c)) + \frac{\lambda}{1 + \lambda} \int_0^c F(c) dc.
\]

Hence, the inequality in (3.6) follows. From which it also follows that \( p^m > p^* \). Using (3.8), (3.7) is readily obtained.

**Proof of Proposition 4:** Let us still denote by \( \hat{c} \) the threshold value beyond which bankruptcy arises if the debt level is fixed at \( D \) and the public authority deviates to a new regulatory contract \( (p, T) \):

(A2) \[ \hat{c} = p - \frac{D - T}{1 - H(p)}. \]

To ensure that financiers break even even after a deviation to \( (p, T) \), it must also be that

(A3) \[ \varphi(\hat{c}) \leq p - \frac{I - T}{1 - H(p)}. \]

Taken together (A2) and (A3) give us Condition (3.9). Of course, (A2) and (A3) also hold at the equilibrium value \( (p_e, T_e) \) with (A3) being an equality. We may accordingly define \( \hat{c}^e \) as the equilibrium value of the bankruptcy threshold:

(A4) \[ \hat{c}^e = p - \frac{D - T_e}{1 - H(p^e)}. \]

(A5) \[ \varphi(\hat{c}^e) = p^e - \frac{I - T_e}{1 - H(p^e)}. \]

For any possible deviation \( (p, T) \), we may also express the firm’s payoff in terms of \( \hat{c} \) and the putative deviation \( (p, T) \) as

\[
U(p, T) = (p - \mathbb{E}_\hat{c}(\hat{c}))(1 - H(p)) + T - I - \mu(1 - H(p)) \left(1 - F \left(p - \frac{D - T}{1 - H(p)}\right)\right).
\]

Proceeding as in Lemma 3, we also obtain

(A6) \[ U(p, T) \geq (1 - H(p)) \int_0^{\hat{c}} F(c) dc. \]
Finally, we observe that eliminating $D$ from (A2), (A3), (A4) and (A5) gives us an equivalent expression for Condition (3.9) as:

\[(A7) \quad (\phi(\hat{c}e) - \hat{c}e)(1 - H(p^e)) \geq (\phi(\hat{c}) - \hat{c})(1 - H(p)).\]

Taking into account its ex post incentives to decrease the price, the public authority’s maximization problem can now be written as:

\[(P) : \max_{(p, \hat{c})} \mathcal{W}(p, \hat{c}) + B(p^e - p) \text{ subject to (A6) and (A7)}.\]

Of course, at equilibrium, the solution to (P) must be $(p^e, \hat{c}e)$ because conjectures are correct. Notice also that (A6) must be binding. Inserting the value of $U(p, \hat{c})$ so obtained into the maximand, we may form the Lagrangean for this problem. We denote by $\zeta$ the non-negative Lagrange multiplier for (A7). Assuming that this Lagrangean is quasi-concave in $(p, \hat{c})$, the first-order condition for optimality w.r.t. $p$ is then given by:

\[-(1+\lambda)(p - E\hat{c}(\hat{c}))h(p) + \lambda(1-H(p)) + \lambda h(p) \int_0^{\hat{c}} F(c)dc - \lambda F(\hat{c}) + B'(p^e - p) + h(p)\zeta (\phi(\hat{c}) - \hat{c} - (\phi(\hat{c}e) - \hat{c}e)) = 0.\]

Taking into account that conjectures are correct at equilibrium so that $\hat{c}lc = \hat{c}e$ and $p lc = p e$ and simplifying yields (3.10).

The first-order condition for optimality w.r.t. $\hat{c}$ writes as

\[(A8) \quad (1 + \lambda)f(\hat{c}) - \lambda F(\hat{c}) = \zeta(F(\hat{c}) - 1 - \mu f(\hat{c})).\]

Or, after simplifications,

\[\frac{\lambda F(\hat{c}lc) + \zeta(1 - F(\hat{c}lc))}{f(\hat{c}lc)} = \mu(1 + \lambda - \zeta).\]

Taking into account that $\phi'(\hat{c}) - 1 = F(\hat{c}) - 1 - \mu f(\hat{c}) < 0$, (A8) and that conjectures are correct at equilibrium implies that the inequality (3.12) holds since $\zeta \geq 0$. Finally, observe that any positive $\zeta$ is consistent with an equilibrium. \(Q.E.D.\)

**Proof of Proposition 5:** We assume that taxes for consumers and owners cannot be distinguished. The public authority’s maximization problem can now be written as:

\[(P^{end}) : \max_{(p, \hat{c}, T_c = T)} \mathcal{V} \left( \Omega_0 + (1 - H(p)) \int_0^{\hat{c}} F(c)dc - T \right) + \mathcal{V} \left( \Omega_c + \int_p^{+\infty} (1 - H(\tilde{v}))d\tilde{v} - T \right) \]

subject to (2.14)

and the following budget constraint (taking into account that there is a unit mass of both consumers and owners)

\[(A9) \quad 2T + (p - E\hat{c}(\hat{c}))(1 - H(p)) - \mu(1 - H(p))(1 - F(\hat{c})) = I + E + (1 - H(p)) \int_0^{\hat{c}} F(c)dc.\]
Let $\lambda$ be the non-negative Lagrange multiplier for (A9). We may write the Lagrangean for ($\mathcal{P}^{\text{end}}$) as:

$$\mathcal{V}\left(\Omega_0 + (1 - H(p)) \int_0^{\hat{c}} F(c) dc - T\right) + \mathcal{V}\left(\Omega_c + \int_p^{+\infty} (1 - H(\tilde{v})) d\tilde{v} - T\right)$$

$$+ \lambda \left(2T - I + (p - \mathbb{E}_c(\hat{c}))(1 - H(p)) - \mu(1 - H(p))(1 - F(\hat{c})) - (1 - H(p)) \int_0^{\hat{c}} F(c) dc - E\right).$$

• Optimizing w.r.t. $T$ yields:

$$\lambda = \frac{1}{2} \mathcal{V}'\left(\Omega_0 + (1 - H(p)) \int_0^{\hat{c}} F(c) dc - T\right) + \frac{1}{2} \mathcal{V}'\left(\Omega_c + \int_p^{+\infty} (1 - H(\tilde{v})) d\tilde{v} - T\right)$$

• Optimizing w.r.t. $p$ yields:

$$-h(p) \mathcal{V}'\left(\Omega_0 + (1 - H(p)) \int_0^{\hat{c}} F(c) dc - T\right) \int_0^{\hat{c}} F(c) dc - (1 - H(p)) \mathcal{V}'\left(\Omega_c + \int_p^{+\infty} (1 - H(\tilde{v})) d\tilde{v} - T\right)$$

$$+ \lambda \left(- (p - \mathbb{E}_c(\hat{c})) h(p) + 1 - H(p) + \mu h(p)(1 - F(\hat{c})) + h(p) \int_0^{\hat{c}} F(c) dc\right) = 0.$$

Simplifying yields (2.17) and (2.18) with the suggested definition of $\lambda$ in (3.17).

• Optimizing w.r.t. $\hat{c}$ yields:

$$(1 - H(p)) F(\hat{c}) \mathcal{V}'\left(\Omega_0 + (1 - H(p)) \int_0^{\hat{c}} F(c) dc - T\right) - \lambda \mu (1 - H(p)) f(\hat{c}) = 0.$$ 

Simplifying by using again the definition of $\lambda$ in (3.17) yields (2.20).

Q.E.D.
APPENDIX B

Data Description

Cost of audit / bankruptcy: Doing Business Resolving Insolvency indicator. It is a country-level proxy for the time, cost and outcome of insolvency proceedings involving domestic entities as well as the strength of the legal framework applicable to judicial liquidation and reorganization proceedings. We use the Resolving Insolvency DTF (distance to frontier) variable, which measures an economy's distance to frontier is indicated on a scale from 0 to 100, where 0 represents the lowest performance and 100 the frontier. See http://www.doingbusiness.org/Methodology/resolving-insolvency for more details (accessed 03/21/18).

Cost of Public Funds: We use the marginal cost of public funds (MCF) estimates computed by Ensor (2016) for 106 countries, following the methodology of Auriol and Warlters (2012). The variable measures the change in social welfare associated with raising an additional unit of tax revenue using a particular tax instrument. The paper report estimates corresponding to the average marginal cost of public funds for five key tax instruments: domestic sales taxes, import and export taxes, and corporate and personal income taxes.

Institutional quality: We use World Bank Worldwide Governance Indicators (WGI) country level governance indicators for 2015 and 2016. We focus on two dimensions of governance: Regulatory Quality, and Control of Corruption. Indicators range from approximately -2.5 to 2.5, with higher values corresponding to better outcomes. See http://info.worldbank.org/governance/wgi/ for more details (accessed 03/21/18).

Figure A1.— Projects’ Share of Private Finance as a Function of per capita GDP
<table>
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<th>Country</th>
<th>Number of projects</th>
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<tbody>
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<td>Argentina</td>
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<tr>
<td>Bangladesh</td>
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<td>China</td>
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<td>Colombia</td>
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<td>Dominican Republic</td>
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<td>Ecuador</td>
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<tr>
<td>Egypt, Arab Rep.</td>
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<tr>
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<td>Ghana</td>
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<td>Vietnam</td>
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<td>Zambia</td>
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