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# “Uncertain altruism and non-linear long-term care policies"

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# Uncertain altruism and non-linear long-term care policies*

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**Abstract**

We study the design of public long-term care (LTC) insurance when the altruism of informal caregivers is uncertain. We consider non-linear policies where the LTC benefit depends on the level of informal care, which is assumed to be observable while children's altruism is not. The traditional topping up and opting out policies are special cases of ours. Both total and informal care should increase with the children's level of altruism. This obtains under full and asymmetric information. Social LTC, on the other hand, may be non-monotonic. Under asymmetric information, social LTC is lower than its full information level for the lowest level of altruism, while it is distorted upward for the higher level of altruism. This is explained by the need to provide incentives to high-altruism children. The implementing contract is always such that social care *increases* with formal care.

**JEL classification**: H2, H5.

**Keywords**: Long term care, uncertain altruism, private insurance, public insurance, topping up, opting out.

# 1 Introduction

One particular lifetime risk that has recently garnered a lot of attention by economists is the one associated with old-age dependency and long-term care (LTC). This arises when the elderly depend on help to carry out daily activities. LTC is different from—albeit often complementary to—health care and particularly terminal care or hospice care. Dependent individuals do not only need medical care but also help with their everyday life routines. Providing this type of assistance is labor intensive and often quite costly, specially in severe cases of dependency that call for institutional care.

Currently, dependency presents the elderly with a significant financial risk of which social insurance covers only a small part.[1] At the same time, private insurance markets for LTC are thin and very expensive.[2] As a consequence, individuals often have to rely on their own private savings or on the informal care their family members provide (which estimates put at about two thirds of total LTC provision; see Norton (2000)). This is often insufficient as it leaves the elderly who cannot count on family solidarity without proper care.

More troubling are the various societal trends that point to an accelerating decline in family involvement. Family solidarity closely depends on the survival of a spouse and on the geographical proximity of children. Over the last decades, we have seen an increasing number of elderly living alone because of divorce and widowhood. As to the children, childless families are not infrequent and the mobility of children can make any nursing assistance impossible. Increased female labor force participation, population aging, and drastic changes in family values are other contributing factors. Additionally, long-run trends aside, informal care is subject to many random shocks.

[1]See for instance Lipszyc *et al.* 2012). In most European countries (with the exception of Germany) social health insurance does not pay for LTC. The specific program there are, like APA in France, covers only a small fraction of actual cost. Similarly, in the US, Medicare does not cover LTC; Medicaid, which is offered to families with minimal private sources, does.

[2]See for instance Brown and Finckelstein (2009, 2011).

There are pure demographic factors such as widowhood, absence or loss of children; divorce and migration too can be put in this category. Conflicts within the family or financial problems incurred by children also prevent them from helping their parents.

This paper studies the role and design of LTC policy when informal care is uncertain. This uncertainty is represented by a single parameter, $\beta$, referred to as the child's degree of altruism. One can also think about this as the (inverse of the) cost of providing care. This parameter is not publicly observable and while parents may eventually observe it this comes at a time when it is too late to make any decision. We concentrate on a single generation of parents over their life-cycle. When young, they work, consume, and save for their retirement. When old, they face the risk of becoming dependent. In case of dependence, the level of informal care is determined by the parameter $\beta$ which is randomly distributed over some interval.

The original feature of our analysis is that we consider nonlinear policies linking the public LTC transfer to the level of informal care, assuming that the latter is observable but children's altruism is not.

The issue of uncertain altruism and the role of social insurance in that context has previously been studied by Cremer, Gahvari and Pestieau (2014, 2017) and by Canta, Cremer and Gahvari (2017). The first two of these papers concentrate on the case where altruism is a binary variable. Children are either altruistic at some known degree or not altruistic at all. The third paper, like the current one considers a continuous distribution. All these papers consider a restricted set of policies and are agnostic about the information structure.

To be more precise, they consider two types of LTC policies. The first one, referred to as "topping up" ($TU$) provides a transfer to dependent elderly, which is non exclusive and can be supplemented by informal care. The second one, is an "opting out" scheme ($OO$); it provides care which is exclusive and cannot be topped up. One of the main messages that emerges from these studies is that none of these schemes appears to dominate in all circumstances. In particular the distribution of the degree of altruism is an important factor.

Both policies are special cases of the nonlinear schemes we consider. Under $TU$, the policy is flat and can in fact be implemented even when informal care is not observed. Under $OO$, we have a decreasing scheme but with rather extreme features: social LTC is positive is positive when informal care is zero and then jumps to zero as soon as some informal care is provided. We will also revisit this problem by having a fresh look at the $TU$ vs. $OO$ issue.

The distinction between $TU$ and $OO$ has been widely studied in the literature about in-kind vs cash transfers.[3] For instance, it has been shown to be relevant in the context of education and health both from a normative and a positive perspective.[4] In this paper we remain agnostic about the exact nature of the transfer and instead focus on its interaction with transfers within the family and specifically informal care provided by children. Furthermore this literature typically focuses on redistributive concerns while we concentrate on insurance issues. Individuals are identical *ex ante*, but they face two types of risk: that of becoming dependent and, when dependent, the risk of having children with a low degree of altruism.

A major concern raised by LTC policies is that of crowding out of informal care.[5] This may make public LTC insurance ineffective for some persons and overall more expensive. Within the context of informal care crowding out may occur both at the intensive and the extensive margins. Intensive margin refers to the reduction of informal care, possibly on a one by one basis, for parents who continue to receive aid from their children, even when social LTC is available. Crowding out at the extensive margin, on

[3]For a review of the literature, see Currie and Gahvari (2008).

[4]On the normative side, for instance, Blomquist and Christiansen (1998) show that both regimes can be optimal (to supplement an optimal income tax) depending on whether the demand for the publicly provided good increases or decreases with labor. From a positive perspective, $TU$ regimes may emerge from majority voting rules, as shown by Epple and Romano (1996).

[5]See for instance Cremer, Pestieau and Ponthière (2012) and and Grabowski *et al.* (2012).

the other hand, occurs when some children are dissuaded from providing any informal care. The specific policies considered in the earlier literature have different impacts on informal care. $TU$ involves crowding out both at the intensive and the extensive margins, whereas $OO$ crowds out informal care solely at the extensive margin.

Since we assume that informal care is observable and can thus to some extent be controlled, one might at first be tempted to think that crowding out is not an issue in our setting. However, this conjecture does not stand under closer scrutiny. Even under full information (about altruism) informal care is limited by the children's participation constraints. When the degree of altruism is not observable, children have to be given the appropriate incentives to provide informal care. The design of the social insurance scheme will then involve a tradeoff between power of incentives and informational rents.

Throughout the paper we concentrate on intra-generational issues; the cost of the LTC program is borne by the generation who also benefits from it. In other words we consider a single generation of parents. The role of children is limited to their decision on the provision of informal care to their parents. The welfare of the grown-up children does not figure in social welfare, which accounts only for the expected lifecycle utility of parents. However, since children are subject to a participation constraint, their utility does also matter. In other words, we maximize parents' utilities for a given (minimum) utility level of children which amounts to characterizing a Pareto-efficient allocation. We first characterize the full information allocation, then turn to the case where neither the government nor the parents observe the children's level of altruism.

We show that, not surprisingly, both the consumption of dependent elderly and the level of informal care should increase with the children's level of altruism. This is the case under both full and asymmetric information. Social LTC, on the other hand, may be non-monotonic. On the one hand, children with a higher level of altruism provide more informal care, which reduces the need for social insurance. On the other hand, the more children care about their parents, the higher the optimal level of consumption of the latter. While the general expression is rather complicated our examples suggest that, under full information, while the optimal LTC transfer may first be increasing in $\beta$, it should decrease for larger levels of children's altruism and eventually drop to zero. This is in line with the idea that social LTC provides insurance against the risk of having children with a low degree of altruism.

Under asymmetric information, the consumption of the dependent elderly is distorted down for all levels of children's altruism, except for the highest one. This is the rather standard result of no distortion at the top, while the total care provided to all other parents is distorted downward to reduce informational rents. The level of informal care is also distorted down, except for the lowest level of altruism. Since informal care can be considered as a transfer from children to parents, this result is also intuitive and in line with traditional findings in contract theory. Finally, social LTC under asymmetric information is lower than its full information level for the lowest level of altruism. The opposite is true for the higher level of altruism, where $g$ is distorted upward. This may be surprising at first as it goes against the role of social care as insurance against children with low levels altruism. Intuitively, under asymmetric information, social LTC has to be distorted up at the top of the distribution in order to give incentives to high-altruism children to provide the appropriate level of informal care. This is more complicated under asymmetric information where incentive constraints have to be accounted for (rather than just the participation constraints under full information). Consequently, the most altruistic children have to be bribed into providing informal care by an increase in social care.

The paper is organized as follows. In Section 2 we present the model and derive the laissez faire allocation. In Section 3 we characterize the optimal allocation under full information. In Section 4 we turn to the case where the government does not observe the children's degree of altruism, and in Section 5 we provide some numerical illustration of the results. Finally, in Section 6 we summarize the main results and examine their implication for the $TU$ vs. $OO$ debate.

## 2 The model

Consider a single generation of parents facing the risk of becoming dependent when they are old and retired. If they become dependent, elderly parents may or may not receive informal care from their grown-up children. This in turn depends on how altruistic the children are. All parents are identical *ex ante* and each of them has one child to rely upon.

The sequence of events is as follows. Period 0 is when the government *designs and announces* its tax/transfer policy; this is the first stage of our game. In period 1, when young working parents each have one child, pay taxes, and save a *given* fraction of their income, $s$. No decision is made in this period. Finally, in period 2, parents have grown old, are retired, and may be dependent. For parents who remain healthy in old age, the game is over; they simply consume their savings. Dependent parents, on the other hand, move to stage 2, where their children, who have by now turned into working adults, decide if and how much informal care, $a$, they want to provide their parents with. Dependent parents also recevice from the government a LTC transfer $g$ conditional on $a$.

All parents face two sources of uncertainty. The first concerns their state of health in old age; they may either remain healthy or become dependent. The probability of dependence, $\pi$, is exogenously given and known. The second source of uncertainty relates to the degree of altruism of their children when they grow up, which is represented by $\beta \in [0, \overline{\beta}]$.[6] The higher is $\beta$ the more altruistic a child is. Children with $\beta = 0$ have no altruistic feelings towards their parents. The random variable $\beta$ is distributed according to the distribution function $F(\beta)$, with density $f(\beta)$. We assume that the hazard rate is increasing.

[6] We rule out $\beta < 0$, which represents a case where children are happier if their parents are worse off.

The public policy consists of a menu of contracts $(a(\beta), g(\beta))$ offered to children, where $a \geq 0$ denotes the informal care that children provide to their dependent elderly parents and $g$ is the public LTC benefit. In order to finance the LTC benefits, a tax $\tau$ is levied on young parents.

Parents provide a fixed labor supply when young, and they have no disutility associated with working. Their labor income is equal to $w\overline{L}$. Preferences are quasilinear in consumption when young; risk aversion is introduced through the concavity of second-period state dependent utilities. Denote the utility function for consumption when old and healthy by $U$ and when old and dependent by $H$. The parent's life-time expected utility is

$$EU = w\overline{L} - \tau - s + (1 - \pi) U(s) + \pi E[H(m)], \tag{1}$$

where $E$ is the expectation operator. Parents pay the tax $\tau$ in the first period. In case of dependence, they consume $m = s + a + g$. Assume that $U' > 0$, $U'' < 0$, $U > 0$, $U'(0) = \infty$, and that the same properties hold for $H$. Further, for any given $x$ we have $U'(x) < H'(x)$. We also assume that $H'(s) > 1$. This ensures that, if parents do not receive any informal care or LTC benefits, their marginal utility of consumption when dependent is greater than their first-period marginal utility in consumptions. In other words, parents would benefit from LTC insurance whose premium is paid in the first period.

Assume that grown-up children too have quasilinear preferences and that their altruism towards their parents comes into play only if the parents become dependent. The children's utility function is represented by

$$u = \begin{cases} y - a + \beta H(m) & \text{if the parent is dependent,} \\ y & \text{if the parent is non dependent,} \end{cases} \tag{2}$$

where $y$ denotes the working children's fixed income. No transfers are made to the healthy elderly parents regardless of the size of their savings, $s$.

## 2.1 Laissez faire

The altruistic children allocate an amount $a$ of their income $y$ to assist their dependent parents, given the parents' savings $s$. Its optimal level, $a^*$, is found through the maximization of equation (2). The first-order condition with respect to $a$ is, assuming an interior solution,

$$-1 + \beta H'(s+a) = 0. \tag{3}$$

Concavity of $H$ ensures that the second-order condition is satisfied and that $H'(s+a)$ is decreasing in $a$. Set $a = 0$ in the above equation and define $\beta_0(s)$ such that

$$\beta_0(s) \equiv \frac{1}{H'(s)}. \tag{4}$$

This function represents the minimum level of $\beta$, for a given $s$, at which a positive level of care is provided. We refer to a child with a $\beta$ equal to the threshold level as the "marginal child". Differentiating (4) yields

$$\frac{d\beta_0}{ds} = -\frac{H''}{(H')^2} > 0,$$

implying, not surprisingly, that an increase in parents' savings reduces the likelihood of children providing assistance in case of dependency.

Condition (3) implies that whenever $\beta \geq \beta_0 > 0$, $a^*$ satisfies [7]

$$s + a^* = (H')^{-1}\left(\frac{1}{\beta}\right) \equiv m(\beta). \tag{5}$$

Conversely, when $\beta < \beta_0$, we have $a^* = 0$. The consumption of dependent parents is then equal to

$$m = \begin{cases} s & \text{if } \beta < \beta_0, \\ m(\beta) & \text{if } \beta \geq \beta_0. \end{cases} \tag{6}$$

[7] A corner solution at $a = y$, cannot be ruled out. To avoid a tedious and not very insightful multiplication of cases we assume throughout the paper that the constraint $a \leq y$ is not binding in equilibrium.

It also follows, from equation (5) and $d\beta_0/ds > 0$, that savings crowd out informal care in two ways. First, by increasing $\beta_0$, an increase in savings makes it less likely that a positive level of care is provided (crowding out at the extensive margin). Second, when informal care is provided, it is crowded out on a one to one basis by savings as long as the solution remains interior (crowding out at the intensive margin). Differentiating (6) yields

$$\frac{dm}{d\beta} = \begin{cases} 0 & \text{if } \beta < \beta_0, \\ \frac{-1}{\beta^2 H''(m)} > 0 & \text{if } \beta \geq \beta_0, \end{cases}$$

where the second line is positive because $H$ is concave.[8] As expected, a dependent parent's total consumption increases with the degree of altruism of their child. In other words, parents are left without insurance against the risk of having a child with a low degree of altruism. Figure 1 represents equation (6). It illustrates the consumption level of dependent parents as a function of the child's degree of altruism.

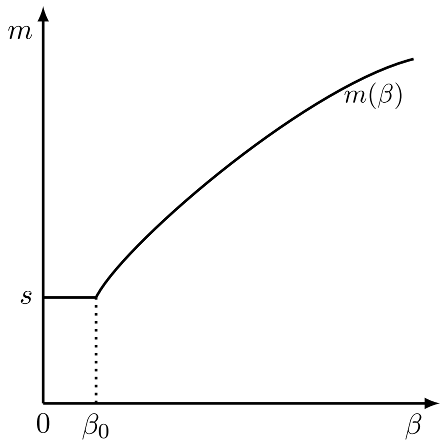


Figure 1: *Laissez faire allocation: consumption of dependent parents as a function of the children's degree of altruism.*

[8]The function $m$ is not differentiable at $\beta = \beta_0$. To avoid cumbersome notation we use $dm/d\beta$ for the right derivative at this point.

# 3 Full information

The government observes the type $\beta$ of each child and sets a menu $(a(\beta), g(\beta))$ maximizing the *ex ante* utility of parents. The problem of the government is

$$\begin{aligned}
\max_{a(\beta),g(\beta)} \quad & EU = w\overline{L} - s - \tau + (1-\pi)u(s) + \pi \int_0^{\overline{\beta}} H(s + a(\beta) + g(\beta))f(\beta)d\beta \\
\text{s.t.} \quad & \tau \geq \pi \int_{\beta} g(\beta)f(\beta)d\beta \\
& y - a(\beta) + \beta H\left(s + a(\beta) + g(\beta)\right) \geq y + \beta H(s) \quad \forall \beta \\
& a(\beta) \geq 0, \quad g(\beta) \geq 0 \quad \forall \beta,
\end{aligned}$$

where the first constraint is the government's resource constraint and the second one is the children's participation constraint which stipulates that their utility must be at least equal to a minimal level. We set this equal to their utility when they do not participate in the provision of care, but setting this at any other arbitrary level would not change the results. Under full information we simply calculate a Pareto-efficient allocation and specifying a different utility level for children would yield another allocation on the utility possibility curve, with otherwise similar properties; see the comments following equation (10) for further discussion. As an alternative, one could have set the children's minimum utility at their $LF$ level. Though, intuitively appealing this would be not be consistent from a contract theory perpective. Specifically, once a goverment program is in place, the $LF$ may no longer be an option for them.[9]

The first constraint will hold with equality. Substituting it into the objective func-

[9]Such an assumption would not affect the qualititative results under full information. It would also not change any result under asymmetric information because for $\beta = 0$ the non participation solution corresponds to the $LF$ and, as shown below, solely the participation constraint of this type binds.

tion, the problem of the government can be rewritten as

$$\begin{aligned}
\max_{a(\beta),g(\beta)} \quad & EU = w\overline{L} - s - \pi \int_0^{\overline{\beta}} g(\beta)f(\beta)d\beta + (1-\pi)u(s) - \pi y \\
& + \pi \int_0^{\overline{\beta}} H(s + a(\beta) + g(\beta))f(\beta)d\beta \\
\text{s.t.} \quad & - a(\beta) + \beta H\left(s + a(\beta) + g(\beta)\right) - \beta H(s) \geq 0 \quad \forall\beta \qquad (7) \\
& a(\beta) \geq 0, \quad g(\beta) \geq 0 \quad \forall\beta.
\end{aligned}$$

The first order conditions with respect to $a(\beta)$ and $g(\beta)$ are, respectively

$$\frac{\partial EU}{\partial a} = \pi H'(s + a(\beta) + g(\beta))f(\beta) + \lambda_\beta[\beta H'(s + a(\beta) + g(\beta)) - 1] + \mu_\beta = 0, \qquad (8)$$

and

$$\frac{\partial EU}{\partial g} = -\pi f(\beta) + \pi H'(s + a(\beta) + g(\beta))f(\beta) + \lambda_\beta \beta H'(s + a(\beta) + g(\beta)) + \nu_\beta = 0, \quad (9)$$

where $\lambda_\beta$, $\mu_\beta$, and $\nu_\beta$ are the Lagrange multipliers associated with the participation constraint and the non-negativity constraints for the $\beta$-type child, respectively. Equation (8) implies that $\lambda_\beta > 0$ for all $\beta$: under full information the participation constraint is binding for all children. Intuitively, if this was the case it would be possible to increase $EU$ by increasing $a(\beta)$ until the constraint becomes binding.

We shall now use these conditions to study the properties of the solutions. In particular we examine the possibility of both interior and corner solutions as well as the shape of $a(\beta)$ and $g(\beta)$.

## 3.1 Interior solutions

When the solution is interior, combining (8) and (9) yields

$$(1 + \beta)H'(s + a(\beta) + g(\beta)) = 1, \qquad (10)$$

Interestingly, this conditions implies that parents receive *more* than full insurance. This is because increasing insurance transfers ($a$ or $g$) also makes them less expensive, since children partially internalize them.

Observe that while the policy is determined to maximize solely the parents welfare, children's utility effectively matters. Under full information the participation constraint is binding for *all* children. Consequently the problem amounts to maximizing the parents' utility for a given level of the children's welfare so that we effectively determine a Pareto-efficient allocation.

Totally differentiating (10) yields

$$\frac{\partial(g^*(\beta)+a^*(\beta))}{\partial\beta} = -\frac{H'}{(1+\beta)H''} > 0. \tag{11}$$

Consequently, under symmetric information, the *total transfer* to dependent parents $a+g$ increases as the level of children altruism increases.

When the solution is interior the optimal policy under symmetric information is such that (10) holds and the participation constraints of the children hold with equality. Using (10), for each $\beta$ the public transfer $g$ can be expressed as a function of $a$ of the form

$$g^*(\beta) = m^*(\beta) - s - a^*(\beta), \tag{12}$$

where $m(\beta) = H'^{-1}(1/(1+\beta))$. Substituting for $g$ in the participation constraints we obtain

$$a^*(\beta) = \beta H(m(\beta)) - \beta H(s). \tag{13}$$

Differentiating this expression with respect to $\beta$ yields, using (11)

$$\frac{\partial a^*(\beta)}{\partial\beta} = H(m(\beta))-H(s)+\beta H'(m(\beta))m'(\beta) = H(m(\beta))-H(s)-\frac{\beta}{(1+\beta)^3H''(m(\beta))}, \tag{14}$$

which is positive as long as $m(\beta) \geq s$. Not surprisingly informal care increases with the level of children's altruism.

The derivative of $g(\beta)$ with respect to $\beta$ is

$$\frac{\partial g^*(\beta)}{\partial \beta} = -H(m(\beta))+H(s)+[1-\beta H'(m(\beta))]m'(\beta) = -H(m(\beta))+H(s)-\frac{1}{(1+\beta)^3 H''(m(\beta))}. \tag{15}$$

The sign of this expression is ambiguous. On the one hand, as $\beta$ increases, $a(\beta)$ increases. This reduces the government's transfer. On the other hand, an increase in $\beta$ entails an increase of the optimal level of consumption when dependent, $m(\beta)$, since altruistic children partially internalize it. This increase $g$. The sign of the derivative then depends on whether the optimal $a(\beta)$ grows faster than the optimal $m(\beta)$.

### 3.2 Corner solutions

We now turn to the issue of possible corner solutions. First, note that there exist no $\beta$ such that the solution implies neither informal nor social care: $a^*(\beta) = g^*(\beta) = 0$. Suppose this was the case. Then, the participation constraint of type $\beta$ would be binding. But this constraint would continue to by satisfied when setting $g(\beta) = \varepsilon$, with $\varepsilon$ arbitrarily small, a policy which provides parents with a higher welfare as long as $H'(s) > 1$, so that parents enjoy insurance. This contradicts the fact that $(a^*, g^*)$ is a solution to the government problem.

Second, for $\beta = 0$, the solution implies $a^*(0) = 0$ and $g^*(0) > 0$. Irrespective on the value of $g$, the participation constraint is satisfied only for $a^* = 0$, which is then the optimal value of informal care for the non-altruistic child. The first-order condition with respect to $g$ for $\beta = 0$ is then given by

$$\frac{\partial EU}{\partial g} = -\pi f(0) + \pi H'(s+g) f(0) + \nu_0 = 0,$$

Under the assumption that $H' > 1$, the transfer $g$ is always interior, so that $H'(s+g) = 1$. When the child is not altruistic, no informal care should (or could) be provided and parents are insured through $g$.

Third, when $\beta > 0$, we never have $a^*(\beta) = 0$. To see this, assume that for some $\beta > 0$ the solution to the government problem is $(a^*, g^*)$ with $a^* = 0$. But then, the participation constraint is satisfied with strict inequality, and it would continue to be satisfied for $a = \varepsilon$, with $\varepsilon$ arbitrarily small, a solution which implies a higher welfare for the parents. This contradicts the fact that $(a^*, g^*)$ is a solution to the government problem.

Fourth, for any $\beta \in (0, \beta_0)$, where $\beta_0$ is the marginal child in the laissez faire defined by (4), $a^* > 0$ and $g^* = 0$ cannot be a solution. Suppose that for some $\beta$ the solution $(a^*, g^*)$ is such that $a^* > 0$ and $g^* = 0$. This implies that there exist an $a^* > 0$ such that

$$a^* = \beta H\left(s + a^*\right) - \beta H(s). \tag{16}$$

However, such a level $a^*$ exists if and only if $\beta > \beta_0$, the threshold above which children provide informal care in the laissez faire. To see this, notice that the LHS of (16) is a linear function of $a$ with slope 1 and intercept equal to zero. The RHS is increasing and concave in $a$, and it is equal to zero if $a = 0$. Then, (16) is always satisfied at $a = 0$, and there exist another positive solution if and only if the slope of the RHS evaluated at $a = 0$ is greater than one, that is if and only if $\beta H'(s) > 1$. But this in turn is the case only when $\beta > \beta_0$, with $\beta_0$ defined by (4)

Finally, when $\beta > \beta_0$, we may have either an interior solution or a corner solution such that $a > 0$ and $g = 0$. A solution with $a^* > 0$ and $g^* = 0$ is characterized by

$$\frac{\partial EU}{\partial a} = \pi H'(s + a^*) f(\beta) + \lambda_\beta [\beta H'(s + a^*) - 1] = 0 \iff \lambda_\beta = \frac{\pi H'(s + a^*) f(\beta)}{1 - \beta H'(s + a^*)}, \tag{17}$$

$$\frac{\partial EU}{\partial g} = -\pi f(\beta) + \pi H'(s + a^*) f(\beta) + \lambda_\beta \beta H'(s + a^*) < 0, \tag{18}$$

and (16). Condition (17) implies that $1 - \beta H'(s + a^*) > 0$. Intuitively, when $g$ is equal to zero, the level of informal care implied by a binding participation constraint is larger than its laissez faire level. Substituting (17) into (18) yields

$$-1 + (1 + \beta) H'(s + a^*) < 0,$$

so that an increase in $g$ starting from 0 would indeed decrease welfare.

The results of this section are summarized in the following proposition.

**Proposition 1** *Assume that private savings are such that $H'(s) > 1$. If the government perfectly observes the level of altruism of the children, $\beta$, and if the level of informal care, $a$, and the level of public LTC transfer, $g$, are contractible, the optimal policy $(a^*(\beta), g^*(\beta))$ has the following properties:*

*(i) There exist no level of altruism $\beta \geq 0$ such that $(a^*(\beta), g^*(\beta)) = (0, 0)$, which would imply that both types of care are equal to zero.*

*(ii) $a^*(0) = 0$ and $g^*(0) > 0$ is implicitly defined by $H'(s + g^*(0)) = 1$.*

*(iii) For all $\beta \in (0, \beta_0]$, with $\beta_0$ defined by (4), $(a^*(\beta), g^*(\beta))$ is interior, and given by (10) and (7).*

*(iv) For $\beta \in (\beta_0, \overline{\beta}]$, corner solutions such that $a(\beta) > 0$ and $g(\beta) = 0$ are possible. Otherwise, $(a^*(\beta), g^*(\beta))$ is given by (10) and (7).*

*(v) Parents' consumption when old and dependent increases in $\beta$, and so does informal care.*

### 3.3 Implementation

Implementing the symmetric information policy requires (assuming that the solution of the government's proble is interior for all $\beta$)

$$g'_a(a, \beta) = 1/\beta > 0 \tag{19}$$

and

$$g(a^*(\beta), \beta) = g^*(\beta). \tag{20}$$

To derive condition (19), note that the first order condition of a child facing a schedule $g(a)$ is

$$-1 + \beta H'(m)[1 + g'_a(a, \beta)] = 0.$$

This expression is satisfied by the full information solution as defined by (10) when $g'_a(a,\beta) = 1/\beta > 0$. In words, social LTC must *increase* with the level of informal care.

Combining (19) and (20) it then follows that, under symmetric information, the first-best allocation can be decentralized by assigning to each type $\beta$ a function

$$g(a,\beta) = \frac{1}{\beta}a + [g^*(\beta) - \frac{1}{\beta}a^*(\beta)].$$

The implementing function for any given $\beta$ is thus an affine function. Its slope decreases in $\beta$ as well as its vertical intercept because

$$\frac{\partial[g^*(\beta) - \frac{1}{\beta}a^*(\beta)]}{\partial\beta} = m'(\beta)[1 - (1+\beta)H'(m(\beta))] - \frac{1+\beta}{\beta}[H(m(\beta)) - H(s)] < 0,$$

where the sign follows from (10) and from $m(\beta) \geq s$.

# 4 Asymmetric information

Under asymmetric information, the utility of a child with altruism parameter $\beta$ reporting $\tilde{\beta}$ is

$$u_c(\beta, \widetilde{\beta}) = y + \beta H(s + a(\widetilde{\beta}) + g(\widetilde{\beta})) - a(\widetilde{\beta}). \tag{21}$$

Incentive compatibility implies that

$$\frac{\partial u_c(\beta, \widetilde{\beta})}{\partial \widetilde{\beta}}|_{\widetilde{\beta}=\beta} = 0. \tag{22}$$

Then, an incentive compatible policy has to satisfy the local incentive compatibility constraint

$$\frac{\partial u_c(\beta)}{\partial\beta} = H(s + a + g) > 0. \tag{23}$$

In words, under asymmetric information children should get a rent that is increasing with their level of altruism.

The problem of the government is now

$$\begin{aligned}
\max_{a(\beta),g(\beta)} \quad & EU = w\overline{L} - s - \tau + (1-\pi)u(s) + \pi \int_\beta H(s + a(\beta) + g(\beta))f(\beta)d\beta \\
s.t. \quad & \tau \geq \pi \int_\beta g(\beta)f(\beta)d\beta \\
& u_c(\beta) = y + \beta H(s + a(\beta) + g(\beta)) - a(\beta) \quad \forall\beta && (24)\\
& u_c(\beta) \geq y + \beta H(s) \quad \forall\beta && (25)\\
& \dot{u}_c = H(s + a(\beta) + g(\beta)). && (26)
\end{aligned}$$

In addition to the participation constraint of the children, the government has now to fulfill the local incentive compatibility constraint.[10] Since $u_c$ is increasing, and $u_c$ increases faster than $y + \beta H(s)$ as long as $a + g \geq 0$, only the participation constraint of the lowest altruism type will be binding. When $a + g = 0$, on the other hand, we have bunching over the relevant interval. We neglect this issue in the analytical part, but this possibility is illustrated in the second and third example provided in Section 5.

$$\begin{aligned}
\max_{a(\beta),g(\beta)} \quad & \pi \int_\beta [H(s + a(\beta) + g(\beta)) - g(\beta)]f(\beta)d\beta \\
s.t. \quad & u_c = y + \beta H(s + a(\beta) + g(\beta)) - a(\beta) \quad \forall\beta \\
& \dot{u}_c = H(s + a(\beta) + g(\beta)).
\end{aligned}$$

The corresponding Hamiltonian is

$$\mathcal{H} = [H(s + a + g) - g]f(\beta) + \eta_\beta(u_c - y - \beta H(s + a + g) + a) + \mu_\beta H(s + a + g) \quad (27)$$

where $u_c$ is the state variable, $a$ and $g$ are the control variables, $\eta_\beta$ is the Lagrange multiplier associated with the first constraint, and $\mu_\beta$ is the co-state.

The first order conditions are

$$\frac{\partial \mathcal{H}}{\partial a} = \pi H'(s + a + g)f(\beta) + \eta_\beta(1 - \beta H'(s + a + g)) + \mu_\beta H'(s + a + g) = 0, \quad (28)$$

[10] The second order condition of the individual revelation problem is that $m(\beta)$ is non-decreasing. We assume for the moment that this condition is satisfied, and show that this is indeed the case for the aymmetric information solution.

$$\frac{\partial \mathcal{H}}{\partial g} = -\pi f(\beta) + \pi H'(s+a+g)f(\beta) - \eta_\beta \beta H'(s+a+g)) + \mu_\beta H'(s+a+g) = 0, \quad (29)$$

$$\frac{\partial \mathcal{H}}{\partial u_c} = \eta_\beta = -\dot{\mu}_\beta, \quad (30)$$

and

$$\frac{\partial \mathcal{H}}{\partial \mu} = H(s+a+g) = \dot{u}_c \quad (31)$$

Substituting (28) into (29) yields $-\pi f(\beta) = \eta_\beta$. Since $\dot{\mu}_\beta = -\eta_\beta = \pi f(\beta)$ and $\mu_{\overline{\beta}} = 0$ from the transversality condition, the co-state can be rewritten as

$$\mu_\beta = -\int_\beta^{\overline{\beta}} \pi f(\beta) d\beta = -\pi(1 - F(\beta)). \quad (32)$$

Using the expression above and proceeding by substitution, conditions (28) and (29) yield

$$\begin{aligned} & -1 + (1+\beta)H'(s+a+g) + \frac{\mu_\beta}{\pi f(\beta)} H'(s+a+g) = 0 \\ \iff & \left[ (1+\beta) - \frac{1-F(\beta)}{f(\beta)} \right] H'(s+a+g) = 1. \end{aligned} \quad (33)$$

With respect to the case with symmetric information, the sum $a+g$ is now distorted down in order to relax incentive compatibility constraints. Denoting by $AI$ asymmetric information and $FI$ asymmetric information, $m^{AI}(\overline{\beta}) = m^{FI}(\overline{\beta})$, and $m^{AI}(\beta) < m^{FI}(\beta)$ for all $\beta < \overline{\beta}$. Under the increasing hazard rate property, $1+\beta-(1-F(\beta))/f(\beta)$ increases in $\beta$ and is positive if $\beta = \overline{\beta}$. Then two cases are possible. If $(1 - 1/f(0))H'(s) \geq 1$, then the solution of (33) is always interior with $g(\beta) + a(\beta) > 0$ and increasing in $\beta$. If $(1 - 1/f(0))H'(s) < 1$, then there exist a threshold $\widetilde{\beta} \in (0, \overline{\beta})$ such that $g(\beta) + a(\beta) = 0$ for all $\beta \leq \widetilde{\beta}$, and $g(\beta) + a(\beta) > 0$ and increasing for $\beta > \widetilde{\beta}$.

Assuming an interior solution for all $\beta \geq 0$, using the children incentive compatibility and participation constraint of the lowest type, we obtain[11]

$$u_c(\beta) = y + \int_0^{\beta} H(m(z))dz, \tag{34}$$

and substituting $u_c(\beta)$ with its value in (24) yields

$$a(\beta) = \beta H(m(\beta)) - \int_0^{\beta} H(m(z))dz. \tag{35}$$

The solution for the government problem is given by (33) and (35). Using the participation constraints and (34) we have

$$\int_0^{\beta} H(m(z))dz \geq \beta H(s).$$

Then,

$$a(\beta) \leq \beta H(m(\beta)) - \beta H(s),$$

with the condition holding with strict inequality for all $\beta > 0$. This condition implies that, for all $\beta$, $a^{AI}(\beta) < a^{*}(\beta)$. Since $m^{AI}(\overline{\beta}) = m^{*}(\overline{\beta})$, one has that, for the highest altruism type, $\overline{\beta}$, $g^{AI}(\overline{\beta}) > g^{*}(\overline{\beta})$. Conversely, for the lowest altruism type $m^{AI}(0) < m^{*}(0)$ and $a^{AI}(0) = a^{*}(0) = 0$ so that $g^{AI}(0) < g^{*}(0)$. The consumption of dependent parents, $m^{AI}(\beta)$, is non-decreasing, and strictly increases whenever the solution is interior. Then, $a^{FI}$ is also non-decreasing and strictly increases if the solution is interior.

Our results are summarized in the following proposition

[11] If If $(1 - 1/f(0))H'(s) < 1$, then

$$u_c(\beta) = \begin{cases} y + \beta H(s) & \text{if } \beta \leq \widetilde{\beta}, \\ y + \tilde{\beta} H(s) + \int_{\widetilde{\beta}}^{\beta} H(m(z))dz & \text{if } \beta > \widetilde{\beta}, \end{cases}$$

and

$$a(\beta) = \begin{cases} 0 & \text{if } \beta \leq \widetilde{\beta}, \\ \beta H(m(\beta)) - \widetilde{\beta} H(s) - \int_{\widetilde{\beta}}^{\beta} H(m(z))dz. & \text{if } \beta > \widetilde{\beta}, \end{cases}$$

**Proposition 2** *Assume that private savings are such that $H'(s) > 1$. When the level of children's altruism, $\beta$, is not publicly observable, while the level of informal care, $a$, and the level of LTC insurance, $g$, are contractible, the optimal policy $(a^{AI}(\beta), g^{AI}(\beta))$ has the following properties:*

*(i) If $(1-1/f(0))H'(s) < 1$, then there exist a threshold $\tilde{\beta} \in (0, \overline{\beta})$ such that $g^{AI}(\beta)+a^{AI}(\beta) = 0$ for all $\beta \leq \tilde{\beta}$, and $g^{AI}(\beta) + a^{AI}(\beta) > 0$ and increasing for $\beta > \tilde{\beta}$.*

*(ii) If $(1-1/f(0))H'(s) > 1$, then for all $\beta$ $g^{AI}(\beta) + a^{AI}(\beta) \geq 0$ and increasing.*

*(ii) For all $\beta < \overline{\beta}$, we have $m^{AI}(\beta) < m^{*}(\beta)$ (downward distortion of total care) while $m^{AI}(\overline{\beta}) = m^{*}(\overline{\beta})$ (no distortion at the top).*

*(ii) For all $\beta > 0$, we have $a^{AI}(\beta) < a^{*}(\beta)$, while $a^{AI}(0) = a^{*}(0) = 0$.*

*(iv) Social LTC satisfies $g^{AI}(0) < g^{*}(0)$ and $g^{AI}(\overline{\beta}) > g^{*}(\overline{\beta})$.*

*(v) Parents' consumption when old and dependent, $m^{AI}(\beta)$, is non-decreasing in $\beta$, and so is informal care, $a^{AI}(\beta)$.*

## 4.1 Implementation

The implementation of the above allocation (assuming that the solution of the government's problem is interior for all $\beta$) implies a function $g(a)$ such that the solution to the children's problem is equal to $a^{AI}(\beta)$. Unlike in the full information case, $g$ cannot be conditioned on $\beta$; the same function $g(a)$ applies to all types. Combining (33) with the children's first order condition we obtain

$$\beta(1+g'(a)) = (1+\beta) - \frac{1-F(\beta)}{f(\beta)} \iff g'(a) = \frac{1}{\beta}\left\{1 - \frac{1-F(\beta)}{f(\beta)}\right\}, \tag{36}$$

where $\beta = (a^{AI})^{-1}(a)$. According to (33), $1-1/f(0) \geq 0$ is a necessary condition for the solution to be interior for all $\beta$. If this is the case, the increasing hazard rate property implies that $1-(1-F(\beta))/f(\beta) > 0$ for all $\beta$, which in turns implies that $g'(a) > 0$ for all levels of $a$. In words, the consumption of the parents in the laissez faire would be too low with respect to the asymmetric information optimal allocation. Then, the LTC policy provides extra incentives to children to provide informal care. It is also easy to verify that, $g'(a^{AI}(\overline{\beta})) = 1/\overline{\beta}$.

Under asymmetric information, the slope $g'(a)$ as defined in (36) is always smaller than the slope of the schedule $g(a)$ under full information, $g'_a(a, \beta) = 1/\beta$, except for the highest altruism type. This is due to the fact that $a$ is distorted down under asymmetric information.

# 5 Illustration

In this section we illustrate the results of Proposition 1 and 2 using explicit functional forms for $H$ and $F$.

First, we assume $H(m) = \ln(m) + C$, and that $F(\beta) = 2\beta$, so that $\beta$ is uniformly distributed between 0 and 1/2. We also assume that $s = 0.5$, so that $H' > 1$ for all $\beta$, and that $C = -\ln(0.4)$, so that $H > 0$ for all $\beta$. These assumptions ensure that the solution of the full and asymmetric information problems are both interior. Figures 2 presents the full and asymmetric information levels of dependent parents' consumption (net of savings), informal care, and LTC transfer as functions of children's altruism. Parents consumption and informal care are both increasing in $\beta$, as it was shown analytically. Furthermore, both $m$ and $g$ are distorted down under asymmetric information. The optimal LTC transfer displays a different pattern. Under full information, it is first increasing and then increasing in $\beta$. Under asymmetric information this is not the case any more, as $g^{AI}$ increases in $\beta$. In this case, the need to provide incentives to informal care givers prevails over the insurance motive of the LTC policy.

We then turn to the case where $H(m) = \ln(m) - \ln(0.4)$, $F(\beta) = \beta$, and $s = 0.5$. In words, we consider now a uniform distribution of $\beta$ between zero and one. The optimal policy is illustrated in Figure 3. In this case $\widetilde{\beta} = 0.25$. Below this threshold $a^{AI} + g^{AI} = 0$. Except for these corner solutions, the pattern is similar to the one depicted in Figure 2.

Finally, we consider a different functional form for the utility function of the dependent, $H(m) = m^{\varepsilon}/\varepsilon$, with $\varepsilon = 0.5$, keeping $s = 0.5$ and $F(\beta) = \beta$. The optimal policy, illustrated in Figure 4, is qualitatively similar to the ones in Figures 2 and 3.

# 6 Summary and concluding comments

This papers has studied the design of public LTC insurance when children's altruism and thus informal care is uncertain. We have considered non-linear policies where the LTC benefit depends on the amount of informal care received, assuming that the latter is observable but children's altruism is not. The traditional $TU$ and $OO$ policies are special cases of ours.

We have shown that, both total care and informal care received by parents should increase with the children's level of altruism. This is the case both under full and asymmetric information. Social LTC, on the other hand, may be non-monotonic. Our examples suggest that while it may first be increasing in $\beta$, it should decrease for larger levels of child altruism and eventually drop to zero. This is in line with the idea that social LTC provides insurance against the risk of having children with a low degree of altruism.

Under asymmetric information, the consumption of dependent elderly is distorted down for all levels of children's altruism, except for the highest one. The level of informal care is also distorted down, except for the lowest level of altruism. These are rather classical contract theory results. More interestingly, social LTC under asymmetric information is lower than its full information level for the lowest level of altruism. The opposite is true for the higher level of altruism, where it is distorted upward. While this goes against the role of social care as insurance against children with low levels altruism, it is explained by the need to provide incentives. Intuitively, under asymmetric information, social LTC has to be distorted up at the top of the distribution in order to give incentives to high-altruism children to provide the appropriate level of informal

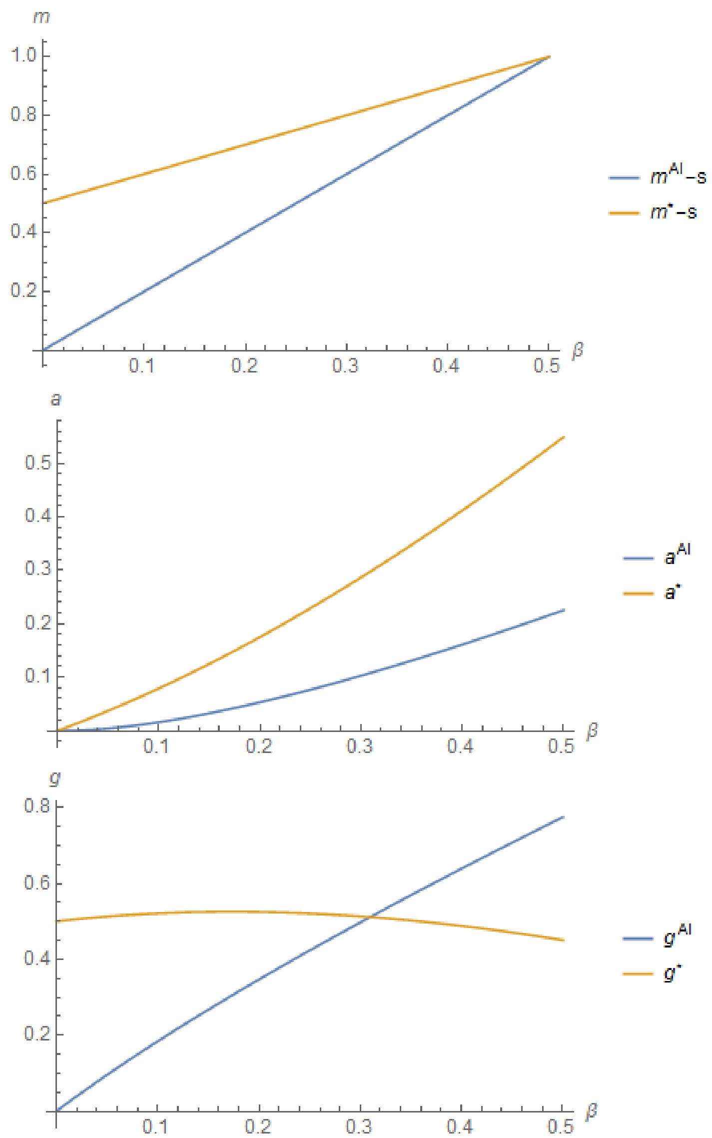


Figure 2: *Optimal consumption of dependent parents (net of private savings), informal care, and LTC insurance as a function of the children' s altruism. Case with* $H(m) = \ln(m) - \ln(0.4)$, $F(\beta) = 2\beta$, *and* $s = 0.5$. *Superscript AI denotes the asymmetric information case, * denotes the full information case)*

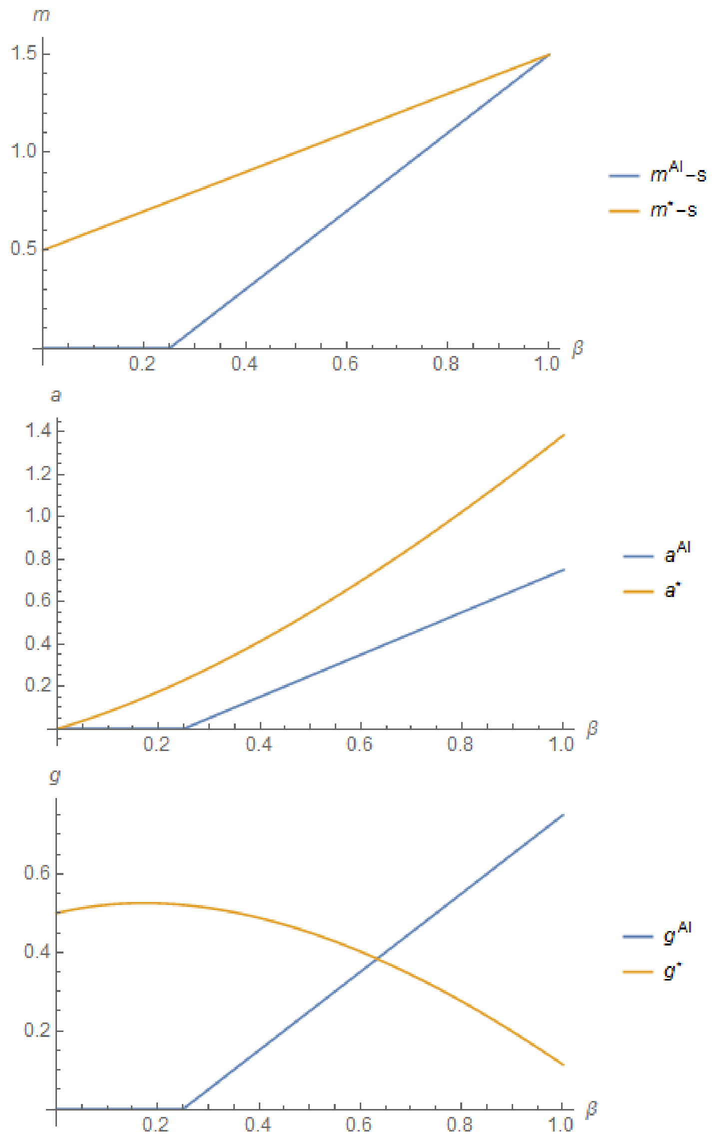


Figure 3: *Optimal consumption of dependent parents (net of private savings), informal care, and LTC insurance as a function of the children' s altruism. Case with* $H(m) = \ln(m) - \ln(0.4)$, $F(\beta) = \beta$, *and* $s = 0.5$. *Superscript AI denotes the asymmetric information case, * denotes the full information case)*

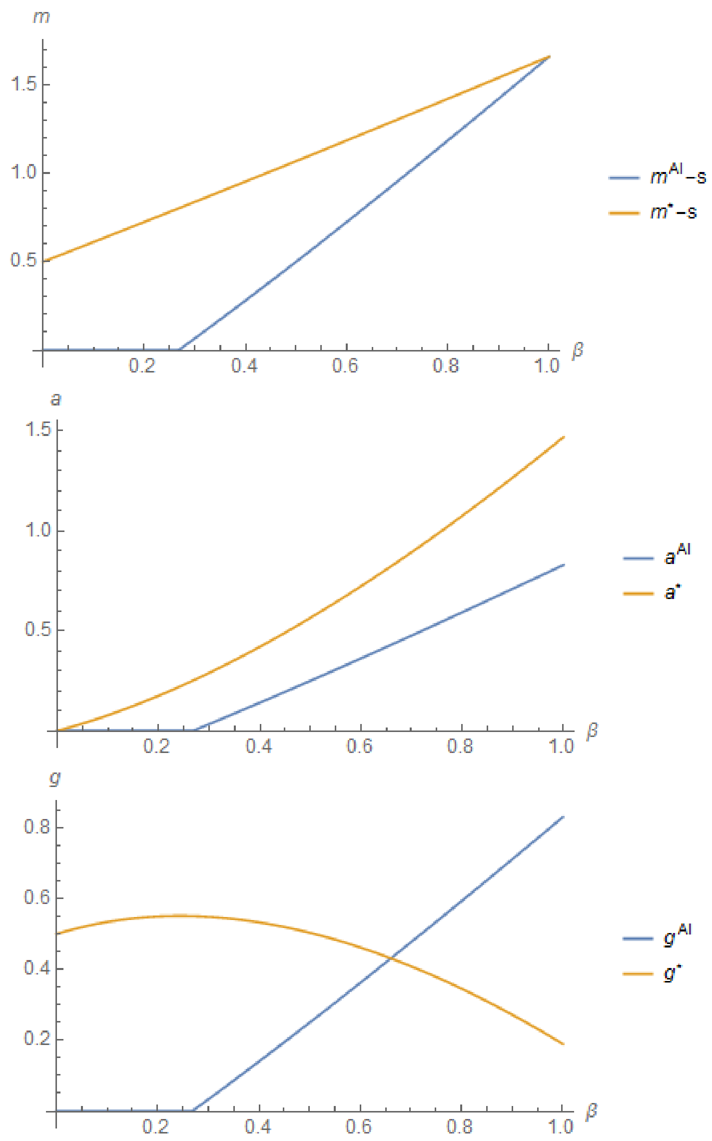


Figure 4: *Optimal consumption of dependent parents (net of private savings), informal care, and LTC insurance as a function of the children' s altruism. Case with* $H(m) = m^{\epsilon}/\epsilon$, $F(\beta) = \beta$, $\epsilon = 0.5$ *and* $s = 0.5$. *Superscript AI denotes the asymmetric information case, * denotes the full information case)*

care.

Returning to the $TU$ vs. $OO$ issue mentioned in the Introduction the optimal policy obviously corresponds to neither of these special case. It is not flat and while under full information public LTC may eventually drop to zero for sufficiently large levels of $a$ (and $\beta$) the schedule does not resemble and $OO$ policy either, which would have a discontinuity at zero. Under asymmetric information, on the other hand, $g$ will be larger than under full information for large $\beta$'s and our illustration suggests that it may even be increasing all along. This is because altruistic children have to be provided with incentives not to mimick children with lower levels of altruism. This draws us even further away from $TU$ or $OO$.

From a practical perspective the main lesson is that what we need is not a social system that can be topped up by informal care but provide social LTC to supplement or top up private care. In other words, ther should be no penalty for informal care through a reduction of social care. Quite the opposite, social care has to increase sufficiently with informal care to provide the appropriate incentives to caregivers. While informal care is in reality difficult to observe so that the implementation of such a policy is not trivial, it does not appear to be impossible. When informal care is "rewarded" by extra social care, families have no incentive to hide it and some degree of observability, albeit imperfect, is not unrealistic.

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