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“Group Size, Collective Action and Complementarities in Efforts"

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Group Size, Collective Action and Complementarities in Efforts

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Abstract: We revisit the group size paradox in a model where two groups of different sizes compete for a prize exhibiting a varying degree of rivalry and where group effort is given by a CES function of individual efforts. We show that the larger group can be more successful than the smaller group if the degree of complementarity is sufficiently high relative to the degree of rivalry of the prize.

Keywords: group size paradox; group contest; complementarity; (impure) public good

JEL classification: D72; D74

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1 Introduction

According to Olson (1965) larger groups are less effective than smaller groups because they face a greater free-rider or collective action problem.[1] Yet, even though individuals contribute less in larger groups, it does not necessarily imply that they produce lower levels of collective action since – by definition – a larger number of fellow members can contribute to group action. Esteban and Ray (2001) consider a group contest for a prize that can have mixed public-private characteristics and show that the group size paradox crucially depends on the degree of convexity of effort cost relative to the degree of rivalry of the prize. If group members face constant marginal costs and if the prize exhibits some degree of rivalry – implying that the per-capita value of the prize decreases with group size – then larger groups are less effective, as stated by Olson (1965). However, the group size paradox is fully reversed when the cost function has the elasticity of a quadratic function (or more) even though the prize is purely private and fully divided among a larger number of group members. The reason is that the higher level of individual effort contributed in smaller groups is not sufficient – due to the cost convexity – to counterbalance the lower number of contributors.

In this paper, we also consider a contest between two groups of different sizes for a prize exhibiting some degree of rivalry between a pure public good and a pure private good, and where group members incur constant, or increasing, marginal cost of contributing to collective action. However, in contrast to Esteban and Ray (2001) who assume, among many others, a 'summation technology' with perfect substitutability between individual efforts, we consider that the effective level of group action is given by a CES technology. This generalization allows us to take into account the possibility of complementarity between group members' efforts in collective action. Indeed, as first pointed out by Alchian and Demsetz (1972), we can argue that team or group production exists to the extent that it can exploit the complementarity of inputs, and this must be especially true when group activity aims at countering activities of competing groups.

The main result of our analysis is that the larger group has a higher probability of success than the smaller group if the degree of complementarity between group members' effort is sufficiently large relative to the degree of rivalry of the prize. For example, in the polar case of linear costs *and* of a purely private prize – which corresponds to the worst case scenario for the larger group in the standard model – we find that an elasticity of substitution across individual efforts smaller than 2 is sufficient for making the larger group more successful. This work thus complements that of Esteban and Ray (2001) by showing that larger groups can also be more effective in overcoming their free-rider problem through higher degrees of complementarity of individual efforts.

Few recent papers analyze contests between groups where group members' efforts are not perfect substitutes. Lee (2012) considers weakest-link contests for group-specific public prizes, while Chowdhury and al. (2013) take the other extreme by considering best-shot group contests. More closely related to the present analysis, Kolmar and Rommeswinkel (2013) assume that group effort is given by a CES function of group members' efforts and consider that group members are heterogeneous in their valuation of the pure public prize. As a result of this heterogeneity, groups with higher complementarity perform worse than similar groups with lower complementarity. Yet, they assume that individuals face linear costs and that groups compete for a pure public good. In the present paper, all agents have the same valuation of the prize and, thus, we focus primarily on the collective action problem as a function of group size when the prize is an impure public good.

[1] For a survey of the literature on the group-size paradox, see Pecorino (2015).

2 The model

Assume that two groups A and B with n_A and n_B identical members respectively, compete for an impure public prize Y. We denote by $e_j \equiv (e_{1j}, e_{2j}, ...e_{n_j j})$ the vector of individual efforts in group j for $j = A, B$. Group effort depends on group members' efforts according to a CES function, that is

$$F(\mathbf{e}_j) = \left(\sum_{j=1}^{n_j} e_{ij}^{\rho} \right)^{\frac{1}{\rho}} \text{ for } j = A, B, \tag{1}$$

where $\rho \in \{(-\infty, 0) \cup (0, 1]\}$ measures the degree of complementarity between individual efforts. The elasticity of substitution is $1/(1-\rho)$. For $\rho = 1$, we have perfect substitutability between individual efforts and Eq. (1) becomes the standard 'summation technology', i.e. $F(\mathbf{e}_j) = \sum_j e_{ij}$. For $\rho \longrightarrow -\infty$, we have perfect complementarity and in the limit we have $F(\mathbf{e}_j) = Min\{e_{ij}\}$ (referred as to the 'weakest-link' function).

This CES structure for the vector of individual contributions has been used by, among others, Cornes and Hartley (2007) and Ray et al. (2007) in a public good provision game; and by Münster (2009) and Kolmar and Rommeswinkel (2013) in the contest environment. However, when ρ changes from a positive value to a negative value, it fundamentally changes the nature of the model although it is never mentioned in the literature with the notable exception (to our knowledge) of Cubel and Sanchez-Pages (2014). Indeed, for $\rho < 0$ and a given level of individual effort, total production is lower when the group is composed of two (or more) members than when it has only one member.[2] Although it can find some support in situations where, for example, the managers of a company with similar abilities step on each other for consolidating their own power (see, e.g., Miles and Watkins, 2007), it makes less sense for addressing the collective problem within groups in a context of competition between those groups. In the following, we thus restrict the analysis to $\rho \in (0, 1]$.

The winning probability of group j, for $j = A, B$ and $j \neq k$, is given by the following contest-success function, i.e.

$$p_j(\mathbf{e}_j, \mathbf{e}_k) = \begin{cases} \dfrac{F(\mathbf{e}_j)}{F(\mathbf{e}_j) + F(\mathbf{e}_k)} & \text{if } F(\mathbf{e}_j) + F(\mathbf{e}_k) \neq 0, \\ \dfrac{1}{2} & \text{otherwise.} \end{cases} \tag{2}$$

All individuals in the two groups have the same valuation of the prize. The individual welfare of member i in group j is thus

$$\pi_{ij}(e_{ij}, \mathbf{e}_j, \mathbf{e}_k) = p_j(\mathbf{e}_j, \mathbf{e}_k) \frac{Y}{n_j^{\beta}} - c(e_{ij}). \tag{3}$$

$\beta \in [0, 1]$ measures the degree of rivalry of the prize Y. For $\beta = 0$, the prize is a pure public good while for $\beta = 1$, it is fully private and divisible among group members. The greater β, the larger is the rivalry of the prize. We will also assume that the cost of individual effort is isoleastic, i.e.,

$$c(e_{ij}) = \frac{1}{\gamma} e_{ij}^{\gamma} \text{ for } i = 1, 2, ..., n_j,\ j = A, B \text{ and } \gamma \geq 1. \tag{4}$$

[2] Let consider a certain level of individual effort $e > 0$, then the production function with n group members becomes $F(\mathbf{e}) = n^{1/\rho} e$, which reaches a maximum at $n = 1$ for $\rho < 0$.

We have the following Proposition.[3]

Proposition 1: *The group contest game admits a Nash equilibrium in pure strategies. In equilibrium, the level of effort of member i in group j, for $j = A, B$ and $j \neq k$, is given by the following first-order condition*

$$\frac{\partial F(\mathbf{e}_j)}{\partial e_{ij}} \frac{F(\mathbf{e}_k)}{[F(\mathbf{e}_j) + F(\mathbf{e}_k)]^2} \frac{Y}{n_j^{\beta}} - e_{ij}^{\gamma-1} \leq 0, \tag{5}$$

where $\partial F(\mathbf{e}_j)/\partial e_{ij} = \left[\sum_j e_{ij}^{\rho}\right]^{\frac{1}{\rho}-1} e_{ij}^{\rho-1}$.

We now focus on the symmetric interior equilibrium such that all members in a group exert the same level of effort that is $e_{ij} = e_j$ for any $i = 1, 2, ..., n_j$ and $j = A, B$. In this case, we have $F(\mathbf{e}_j) = n_j^{1/\rho} e_j$ and $\partial F(\mathbf{e}_j)/\partial e_{ij}\left|_{e_{ij}=e_j}\right. = n_j^{(1-\rho)/\rho}$. First-order conditions (5) can then be rewritten as

$$\frac{n_j^{(1-\rho)/\rho} n_k^{1/\rho} e_k}{[F(\mathbf{e}_j) + F(\mathbf{e}_k)]^2} \frac{Y}{n_j^{\beta}} = e_j^{\gamma-1}. \tag{6}$$

Thus, from these two equilibrium conditions, we can obtain

$$e_j = \left(\frac{n_k}{n_j}\right)^{(1+\beta)/\gamma} e_k. \tag{7}$$

In a symmetric equilibrium, the winning probability of group j – for $j = A, B$ and $j \neq k$ – is thus given by

$$\begin{aligned} p_j(\mathbf{e}_j^*, \mathbf{e}_k^*) &= \frac{n_j^{\theta}}{n_j^{\theta} + n_k^{\theta}} \\ \text{where } \theta &= \frac{1}{\rho} - \frac{1+\beta}{\gamma}. \end{aligned} \tag{8}$$

We then have the following Proposition.

Proposition 2: *The larger group has a higher probability of success than the smaller group if and only if* $\rho \in (0, \min\{\gamma/(1+\beta), 1\}]$.

Using (6) and (7), the equilibrium level of individual effort in the symmetric equilibrium is given, for $j = A, B$ and $j \neq k$, by

$$e_j^* = \left[\frac{n_j^{\theta-1} n_k^{\theta}}{\left[n_j^{\theta} + n_k^{\theta}\right]^2} \frac{Y}{n_j^{\beta}}\right]^{\frac{1}{\gamma}}. \tag{9}$$

We have that $e_j^* > e_k^*$ if $n_k^{1+\beta} > n_j^{1+\beta}$. This is the illustration of the free-rider or collective action problem: the members of the larger group produce lower levels of efforts than

[3] The proof of Proposition 1 is given in the Appendix.

the members of the smaller group. Finally, substituting (8) and (9) into (3), individual welfare in the symmetric equilibrium is given, for $j = A, B$ and $j \neq k$, by

$$\pi_j^* \left(\mathbf{e}_j^*, \mathbf{e}_k^*\right) = \frac{n_j^{\theta-\beta}\left[\gamma\left(n_j^\theta + n_k^\theta\right) - n_j^{-1} n_k^\theta\right]}{\gamma\left[n_j^\theta + n_k^\theta\right]^2} Y. \tag{10}$$

3 The Group Size Paradox

In this Section, we determine when the group size paradox holds and when it does not depending on the various parameters. Let first consider that group members' efforts are perfectly substitutable, i.e. $\rho = 1$, so that each group impact function is given by the arithmetic sum of individual contributions. This is the standard assumption in much of the literature on group contests. We then have the following corollary.

Corollary 1: *Let $\rho = 1$. The larger group has a higher probability of success than the smaller group for $\gamma \geq 1 + \beta$, while the reverse holds for $\gamma < 1 + \beta$.*

Let further assume that the costs of contributing to collective action are linear, i.e. $\gamma = 1$ *and* that the prize is a pure public good, i.e. $\beta = 0$. This can correspond, for example, to the situation where 'green' and producer lobbies oppose each other on the strengthening of an environmental regulation. Group members always exert lower levels of efforts in the larger group because of the free-rider problem. But this is exactly compensated by the larger number of individuals contributing to group action. As a result, the two groups produce the same level of collective effort and have the same probability of success – that is 1/2 since $\theta = 0$ with $\rho = \gamma = 1$ and $\beta = 0$ – independently of the asymmetry in group size, as first shown by Katz and al. (1990). A key feature of this result is that the prize is a pure public good implying that per capita payoff is invariant with group size.

If, however, the prize exhibits some degree of rivalry, i.e. $\beta \neq 0$, then the per-capita benefit to each group member depends negatively on group size. This adds to the free-rider problem for reducing even more individual effort in the larger group. And if the marginal cost of effort remains constant, i.e. $\gamma = 1$, then a greater number participants to collective action cannot overcome both the greater free-rider problem and the lower individual stake in the larger group compared to the smaller group. As a result, the larger group has a lower probability of success, which corresponds to the Olson paradox. The extreme case where the prize is purely private – i.e. $\beta = 1$ – can correspond, for example, to a contest between two different departments of a company competing for a bonus to be equally distributed among the members of the wining department. In this case, the winning probability of a group is inversely related to its size – i.e. $p_j(\mathbf{e}_j^*, \mathbf{e}_k^*) = n_k/\left(n_j + n_k\right)$ – since $\theta = -1$ for $\rho = \gamma = \beta = 1$.

Finally, as in Esteban and Ray (2001), let consider that group members face increasing marginal costs of contributing to collective action, i.e. $\gamma \geq 1$. They justify it by the fact that the input of the lobbying process can be time expended by group members. If this case, the larger group can be more successful provided the individual effort cost is sufficiently convex relative to the degree of rivalry of the prize. In particular, if the cost function is quadratic (or more) – i.e. $\gamma \geq 2$ – then the larger group has a greater probability of success than the smaller group even though the prize is fully private – i.e. $\beta = 1$ – since $\theta = 1 - (2/\gamma) \geq 0$ for $\rho = \beta = 1$. Due to the cost convexity, the higher level of individual effort in the smaller group is not sufficient to compensate its inherent disadvantage of having a smaller number of contributors.

If time is the input of collective action, we should consider the existence of complementarities in efforts as much as a convex cost structure. Thus, in order to focus on the complementarity effect – i.e. $\rho \neq 1$ – let $\gamma = 1$ and that the prize is either purely public or purely private. Recalling that the lower ρ, the greater is the complementarity between group members' efforts, we can establish the following Corollary.

Corollary 2: *Let* $\gamma = 1$. (i) *If the prize is a pure public good, i.e.* $\beta = 0$*, then the larger group has a higher probability of success than the smaller group for any* $\rho \in (0, 1]$*;* (ii) *If the prize is fully private and divisible among group members, i.e.* $\beta = 1$*, then the larger group has a higher probability of success than the smaller group for* $\rho \in (0, 1/2]$*, while the reverse holds for* $\rho \in (1/2, 1]$.

The CES structure with complementarity in efforts – i.e. $\rho \neq 1$ – implies that for a certain total amount of effort, a given group is more effective with a larger number of participants to collective action than with a smaller number of members exerting more individual effort. This can illustrate the Aristote's maxim that *"the whole is greater than the sum of its parts"* or that the size is an asset in group action. Think, for example, of demonstrations such as the "candlelight revolution" in South Korea in 2016 against President Park Geun-hyein, or the massive demonstrations in 2017 for and against the independence of Catalonia (from Spain). Clearly, the strength of these social movements was measured by their capacity to gather as many participants as possible at a specific time and place.

Point (ii) of Corollary 2 shows that in the special case of linear costs *and* of a purely private prize – which corresponds to the worst case scenario for the larger group in the standard model with $\rho = 1$ – we find that an elasticity of substitution across individual efforts – given by $1/(1-\rho)$ – smaller than 2 is sufficient for making the larger group more successful in the contest. Now, if the prize is a pure public good, then the larger group always has a higher probability of success than the smaller group if there is any complementarity at all between group members' efforts, even though the marginal cost is constant, as shown by point (i) of Corollary 2. The explanation is that the disadvantage of larger groups in terms of per-capita value of the prize – determining in turn group members' incentives and contributions – vanishes when the prize is purely public and nonexcludable. Thus our analysis complements that of Esteban and Ray (2001) by showing that complementarity in efforts works to the advantage of the larger group in the same way as does the convexity of the cost function.[4]

Finally, relax the assumption that $\gamma = 1$ and return to Proposition 2. If $\gamma > 1 + \beta$ the larger group always has a higher probability of success for $\rho = 1$ – as stated in Corollary 1 – and *a fortiori* for $\rho < 1$. If $\gamma \leq 1+\beta$, this is also the case provided that $\rho \in (0, \gamma/(1+\beta)]$. Thus, the higher the cost convexity and the lower the degree of rivalry of the prize, the larger is the size of the interval of admissible values of ρ for which the larger group has a higher probability of success.

4 Conclusion

As noted earlier, the very 'essence' of a group is the complementarity between the efforts of a certain number of agents pursuing the same objective. And it is this complementarity

[4] In fact, it can be easily shown that increasing the degree of complementarity raises the probability of success of the larger group, that is $\partial p_j/\partial \rho < 0$ when $n_j > n_k$, for any $\rho \in \{(-\infty, 0) \cup (0, 1]\}$ (and independently of β). Indeed, we have that $\partial p_j/\partial \rho = -n_j^\theta \left[\ln(n_j) - \ln(n_k)\right] / \left[\rho(n_j^\theta + n_k^\theta)\right]^2 < 0$, and thus increasing the degree of complementarity – i.e. decreasing ρ – raises p_j.

that can make size an asset in group contests. We indeed show that larger groups can be more successful than smaller groups if the degree of complementarity is sufficiently large even though the prize is purely private and must be divided among a larger number of group members. A question remains as to whether the members of larger groups also obtain higher levels of per-capita utility. If the prize is purely public, then a greater probability of success necessarily implies a greater per capita payoff simply because this last is independent of group size. If, however, the prize is purely private then it requires a higher degree of complementarity than that for which the larger group has a higher probability of success.[5]

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Appendix

From the first-order condition, the first derivative of $\pi_{ij}\left(e_{ij},\mathbf{e}_j,\mathbf{e}_{-j}\right)$ with respect to e_{ij} can be written as follows

$$\frac{p_j(.)(1-p_j(.))}{F\left(\mathbf{e}_j\right)}\frac{\partial F\left(\mathbf{e}_j\right)}{\partial e_{ij}}\frac{Y}{n_j^{\beta}}-e_{ij}^{\gamma-1}, \tag{A1}$$

where $p_j(.)$ stands for $p_j\left(\mathbf{e}_j,\mathbf{e}_{-j}\right)$.

Now let $\Psi_j(.)\equiv p_j(.)(1-p_j(.))/F\left(\mathbf{e}_j\right)$. This derivative can thus be re-written as

$$\Psi_j(.)\frac{\partial F\left(\mathbf{e}_j\right)}{\partial e_{ij}}\frac{Y}{n_j^{\beta}}-e_{ij}^{\gamma-1}. \tag{A2}$$

We now characterize the second derivative of $\pi_{ij}\left(e_{ij},\mathbf{e}_j,\mathbf{e}_k\right)$ with respect to e_{ij}. We have

$$\left[\frac{\partial\Psi_j(.)}{\partial e_{ij}}\frac{\partial F\left(\mathbf{e}_j\right)}{\partial e_{ij}}+\Psi_j(.)\frac{\partial^2 F\left(\mathbf{e}_j\right)}{\partial e_{ij}^2}\right]\frac{Y}{n_j^{\beta}}-(\gamma-1)\,e_{ij}^{\gamma-2}. \tag{A3}$$

We have

$$\frac{\partial\Psi_j(.)}{\partial e_{ij}}=\frac{1}{\left[F\left(\mathbf{e}_j\right)\right]^2}\left\{\frac{\partial p_j(.)}{\partial e_{ij}}(1-2p_j(.))F\left(\mathbf{e}_j\right)-p_j(.)(1-p_j(.))\frac{\partial F\left(\mathbf{e}_j\right)}{\partial e_{ij}}\right\}. \tag{A4}$$

We also have

$$\frac{\partial p_j(.)}{\partial e_{ij}}=\frac{p_j(.)(1-p_j(.))}{F\left(\mathbf{e}_j\right)}\frac{\partial F\left(\mathbf{e}_j\right)}{\partial e_{ij}}. \tag{A5}$$

[5] The details of the analysis of individual welfare are given in the working paper version available upon request. Specifically, when the prize is purely private, the elasticity of substitution among group members' efforts must be lower than 1.5 – i.e. $\rho\leq 1/3$ – for the members of the larger group to obtain higher levels of per-capita utility, while an elasticity smaller than 2 is sufficient for the larger group to be more successful in terms of probability of success. This result actually contrasts with that of Esteban and Ray (2001), who found that the members of the larger group always get lower per-capita payoffs when the prize is purely private.

Substituting into (A4), we thus have

$$\frac{\partial \Psi_j(.)}{\partial e_{ij}} = -2p_j(.)\frac{p_j(.)(1-p_j(.))}{\left[F\left(\mathbf{e}_j\right)\right]^2}\frac{\partial F\left(\mathbf{e}_j\right)}{\partial e_{ij}}, \tag{A6}$$

which can be rewritten as

$$\frac{\partial \Psi_j(.)}{\partial e_{ij}} = -2p_j(.)\frac{\Psi_j(.)}{F\left(\mathbf{e}_j\right)}\frac{\partial F\left(\mathbf{e}_j\right)}{\partial e_{ij}}. \tag{A7}$$

Substituting into (A3), we have

$$\left[-2p_j(.)\frac{\Psi_j(.)}{F\left(\mathbf{e}_j\right)}\left(\frac{\partial F\left(\mathbf{e}_j\right)}{\partial e_{ij}}\right)^2 + \Psi_j(.)\frac{\partial^2 F\left(\mathbf{e}_j\right)}{\partial e_{ij}^2}\right]\frac{Y}{n_j^{\beta}} - (\gamma-1)\, e_{ij}^{\gamma-2}. \tag{A8}$$

We also have $\partial F\left(\mathbf{e}_j\right)/\partial e_{ij} = \left[\sum_j e_{ij}^{\rho}\right]^{\frac{1}{\rho}-1} e_{ij}^{\rho-1}$ and hence

$$\frac{\partial^2 F\left(\mathbf{e}_j\right)}{\partial e_{ij}^2} = \left(\frac{1}{\rho}-1\right)\left(\sum\nolimits_j e_{ij}^{\rho}\right)^{\frac{1}{\rho}-2}\rho e_{ij}^{\rho-1}e_{ij}^{\rho-1} + (\rho-1)\left(\sum\nolimits_j e_{ij}^{\rho}\right)^{\frac{1}{\rho}-1} e_{ij}^{\rho-2}. \tag{A9}$$

Simplifying we have that

$$\frac{\partial^2 F\left(\mathbf{e}_j\right)}{\partial e_{ij}^2} = \left(\sum\nolimits_j e_{ij}^{\rho}\right)^{\frac{1}{\rho}-2} e_{ij}^{\rho-2}\left[(1-\rho)\, e_{ij}^{\rho} + (\rho-1)\sum\nolimits_j e_{ij}^{\rho}\right]. \tag{A10}$$

Simplifying again, it can be rewritten as

$$\frac{\partial^2 F\left(\mathbf{e}_j\right)}{\partial e_{ij}^2} = (\rho-1)\left(\sum\nolimits_j e_{ij}^{\rho}\right)^{\frac{1}{\rho}-2}\left(\sum\nolimits_{k\neq i} e_{kj}^{\rho}\right) e_{ij}^{\rho-2}. \tag{A11}$$

which is always negative since $\rho \leq 1$. It follows that the second derivative of $\pi_{ij}\left(e_{ij}, \mathbf{e}_j, \mathbf{e}_k\right)$ with respect to e_{ij} given by (A3) is always negative. Therefore, the game admits a Nash equilibrium in pure strategies■

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Group Size, Collective Action and Complementarities in Efforts: Another Notion of Group Effectiveness

(Not for publication)

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Abstract: We explore a second notion of group effectiveness that is that which relates group size to per-capita payoffs.

Per-Capita Payoffs

We now determine whether individuals are better-off in the larger (say group j) or in the smaller group (say group k). For the sake of simplicity, we assume $\gamma = 1$ and that the prize is either a pure public good, i.e. $\beta = 0$, or a fully divisible prize, i.e. $\beta = 1$. From (10) we can write the difference between the welfare of a member of the larger group j and that of a member of the smaller group k as:

$$\Pi_j - \Pi_k = \frac{n_j^{\theta-\beta}\left[n_j^{\theta} + n_k^{\theta} - n_j^{-1}n_k^{\theta}\right] - n_k^{\theta-\beta}\left[n_k^{\theta} + n_j^{\theta} - n_k^{-1}n_j^{\theta}\right]}{\left[n_j^{\theta} + n_k^{\theta}\right]^2} Y.$$

Recall that $\theta = (1/\rho) - (1 + \beta)$ and let consider that $\Psi_j = n_j^{\theta} + n_k^{\theta} - n_j^{-1}n_k^{\theta}$ and $\Psi_k = n_k^{\theta} + n_j^{\theta} - n_k^{-1}n_j^{\theta}$. We can see that $\Psi_j > \Psi_k$ for $\rho \in (0, 1]$, independently of β. Indeed $\Psi_j > \Psi_k$ can be reduced to $n_j^{\theta+1} > n_k^{\theta+1}$, where $\theta + 1 = \frac{1}{\rho} - \beta > 0$ for any $\rho \in (0, 1]$. For $\beta = 0$, we also have $n_j^{\theta-\beta} > n_k^{\theta-\beta}$ and thus $\Pi_j - \Pi_k > 0$. Now, for $\beta = 1$, we still have $\Psi_j > \Psi_k$ but $n_j^{\theta-\beta}$ can be larger or smaller that $n_k^{\theta-\beta}$ depending on the value of ρ. We have $\theta - 1 = \frac{1}{\rho} - 3$ so that $n_j^{\theta-1} > n_k^{\theta-1}$ for $\rho \in (0, 1/3]$, implying that $\Pi_j - \Pi_k > 0$. But for $\rho \in (1/3, 1]$ we have that $n_j^{\theta-1} < n_k^{\theta-1}$ and $\Psi_j > \Psi_k$. Therefore, we rewrite the above equation for $\beta = 1$ as:

$$\Pi_j - \Pi_k = \frac{n_j^{2\theta-1} - n_k^{2\theta-1} + (n_j n_k)^{\theta}\left[\left(n_j^{-1} - n_k^{-1}\right) - \left(n_j^{-2} - n_k^{-2}\right)\right]}{\left[n_j^{\theta} + n_k^{\theta}\right]^2} Y. \tag{11}$$

The expression $\left(n_j^{-1} - n_k^{-1}\right) - \left(n_j^{-2} - n_k^{-2}\right)$ is always negative for $n_j > n_k$ and the expression $n_j^{2\theta-1} - n_k^{2\theta-1}$ is also negative for $\rho \in (0.4, 1]$ since $2\theta - 1 = (2/\rho) - 5$ when $\beta = 1$. Therefore, we can state:

Proposition 3: (i) *When the prize is a pure public good, i.e. $\beta = 0$, then a member of the larger group gets higher welfare than a member of the smaller group for $\rho \in (0, 1]$.* (ii) *When the prize is a fully divisible good, i.e. $\beta = 1$, a member of the larger group gets a higher welfare than a member of the smaller group for $\rho \in (0, 1/3]$, while the reverse holds for $\rho \in (0.4/1]$. However, the result is ambiguous for $\rho \in (1/3, 0.4]$.*

When the prize is a pure public good – i.e. $\beta = 0$ – so that the per capita value of the prize is independent of group size, the larger group always has a larger probability of success and in addition group members produce lower levels of efforts because of a greater free-rider problem. Thus, in this case, a higher probability of success necessarily implies higher individual welfare and thus members are always better off in a larger group.

When the prize is purely private – i.e. when $\beta = 1$ – the per-capita payoff is not directly related to the probability of success since an increase in group size reduces the availability of the prize to individual members (because of the rivalry). The cost of individual effort is still lower in a larger group but the effect is ambiguous on the probability of success and it depends on the elasticity of substitution between group members' efforts. Again, if this one is lower than two – that is if ρ lower than one half – then the larger group has a higher probability of success but it requires a smaller ρ – that is lower than $1/3$ – for individual members to be better off in a larger group than in a smaller group. This is nevertheless a strong difference with the analysis of Esteban and Ray (2001). They show that individuals are always worse off in a larger group independently of whether it has a lower or greater probability of success.

The crucial difference is that the positive group size bias in our analysis comes from the existence of complementarities rather than from the convexity of the individual cost function. And this complementarity can compensate, for a larger group, the greater free-rider problem, the lower individual stake and thus the lower incentive to contribute, but also the direct negative impact on the per-capita value of the prize.

Finally, the result is ambiguous for $\rho \in (1/3, 0.4]$. We thus present in the following some numerical examples. We set the value of the fully divisible prize at $Y = 100$.

Table 1: Numerical examples for $\rho \in (1/3, 0.4]$

ρ	n_j	n_k	$\Pi_j - \Pi_k$
0.37	4	3	−0.026
0.37	5	2	0.635
0.37	4	2	0.876
0.37	100	50	−0.135
0.34	4	3	0.951
0.34	5	2	3.604
0.34	4	2	3.538
0.34	100	50	−0.003

Table 1 shows that the sign of $\Pi_j - \Pi_k$ depends on both ρ and on the asymmetry in group size, but also on the total number of players involved in the contest. For example, whether $\rho = 0.34$ or $\rho = 0.37$ and for the same ratio of group size $n_j/n_k = 2$, the sign of $\Pi_j - \Pi_k$ changes as the total numbers of players increases from $(4 + 2) = 6$ to $(100 + 50) = 150$.