Mergers and Demand-Enhancing Innovation*

Marc Bourreau[†] Bruno Jullien[‡] Yassine Lefouili[§]

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Abstract

We study the impact of horizontal mergers on merging firms' incentives to invest in demand-enhancing innovation. In our baseline model, we identify four effects of a symmetric merger on these incentives: the innovation diversion effect, the margin expansion effect, the demand expansion effect, and the per unit return to innovation effect. We offer sufficient conditions for a merger to reduce or raise merging firms' incentives to innovate in the absence of spillovers and efficiency gains in R&D, and find that a comparison between the innovation diversion and price diversion ratios is informative about the impact of a merger on innovation.

Keywords: Horizontal Mergers, Innovation, Competition.

JEL Classification: D43, L13, L40.

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[†]Telecom Paris, Institut Polytechnique de Paris, France; email: marc.bourreau@telecom-paris.fr

[‡]Toulouse School of Economics, CNRS, University of Toulouse Capitole, Toulouse, France; email: bruno.jullien@tse-fr.eu

[§]Toulouse School of Economics, University of Toulouse Capitole, Toulouse, France; email: yassine.lefouili@tse-fr.eu

1 Introduction

Competition authorities have been concerned with the effects of horizontal mergers on innovation for a long time.[1] However, they have paid greater attention to these effects in recent years. Gilbert and Greene (2015) find that the US Department of Justice and the Federal Trade Commission identified innovation concerns in approximately one-third of their merger challenges between 2004 and 2014. The European Commission has also taken action on grounds of adverse impacts on innovation in many merger cases over the last decade.[2]

Policy debates and the academic literature on the impact of horizontal mergers on innovation have highlighted several potentially conflicting effects.[3] They have also shown that the effects of mergers on demand-enhancing innovation are more complex than their effects on cost-reducing innovation.[4] As a result, the existing papers analyzing the impact of mergers on the former type of innovation have focused on specific demand functions.[5] In this paper, we use a different approach to study this issue. We consider a setting with a general demand function and decompose the impact of a merger on merging firms' incentives to invest in demand-enhancing innovation into several easily interpretable effects. Using this decomposition, we offer sufficient conditions under which the net impact of a merger on incentives to innovate is negative (or positive) and show that the comparison of two simple diversion ratios can help screen mergers in industries in which innovation plays a key role.

In our baseline model, we investigate the impact of a merger on merging firms' incentives to innovate, abstracting from nonmerging firms' reactions. Specifically, we study the

[1] For instance, the US Department of Justice challenged the proposed acquisition of General Motors' Allison Transmission Division by ZF Friedrichshafen AG in 1993 because the department considered not only that the merger could trigger traditional adverse price effects but also that it would harm innovation (Gilbert, 2020). In 1992, the European Commission identified potential innovation concerns when examining a merger between DuPont and ICI but decided to clear the merger.

[2] See, for instance, Novartis/GSK (case no. COMP/M.7276), GE/Alstom (case no. COMP/M.7278), Pfizer/Hospira (case no. COMP/M.7559), Dow/DuPont (case no. COMP/M. 7932), Bayer/Monsanto (case no. COMP/M.8084), and Bayer/BASF (case no. COMP/M.8851). The European Commission identified innovation concerns in all these mergers and cleared them only on the condition of implementation of remedies addressing these concerns.

[3] See, e.g., Baker (2007), Katz and Shelanski (2007), Shapiro (2012), Federico (2017), Federico et al. (2017, 2018), Motta and Tarantino (2018), Jullien and Lefouili (2018b), Denicolò and Polo (2019), Régibeau and Rockett (2019), Federico et al. (2020), and Gilbert (2020).

[4] See Motta and Tarantino (2018) and Jullien and Lefouili (2018b). Relatedly, Greenstein and Ramey (1998) and Chen and Schwartz (2013) have shown that the seminal result about the effect of competition on process innovation by Arrow (1962) does not always extend to the case of product innovation.

[5] See the discussion of Motta and Tarantino (2018) and Federico et al. (2018) in the related literature section.

impact of a merger between two symmetric duopolists on their incentives to innovate in an environment with no spillovers and no efficiency gains in R&D (but with potential efficiency gains in production). We show that the overall impact of the merger on innovation is the sum of four effects. First, the merger induces an *innovation diversion effect*: that is, it causes the impact of each merging firm's innovation investment on the other merging firm's demand to be internalized. We focus on the case in which this externality is negative, as this is the scenario about which competition authorities are most concerned.[6] Second, the merger affects the merging firms' output and, therefore, their incentives to innovate to increase their margins. This *margin expansion effect* is negative if the merger leads to lower output in the merging firms at a given innovation level, which is the case if there are no, or limited, efficiency gains in production. However, if efficiency gains in production are strong, the merger leads to an increase in the merging firms' output, and the margin expansion effect is then positive. Third, the merger affects the merging firms' margins and, therefore, their incentives to innovate to increase their demand. This *demand expansion effect* is positive because a merger leads to an increase in margins. Finally, the merger may generate a change in the return to investment per unit of output, which we call the *per unit return to innovation effect*. This effect can be either positive or negative.

Next, we investigate the (sign of) the overall impact of a merger on the merging firms' incentives to innovate. We first consider *P-neutral mergers*. These are mergers that do not affect prices when the innovation level of both firms is *fixed* at the equilibrium level that they choose when they are independent.[7] Studying the impact of such mergers on innovation is policy relevant because it allows us to determine the conditions under which one can use a *stand-alone* innovation theory of harm, i.e., a theory stipulating that even a merger that does not have any effects on prices (at given innovation levels) can have a negative impact on innovation (Denicolò and Polo, 2019).[8] The margin expansion effect and the per unit return to innovation effect vanish for P-neutral mergers, which implies that their overall impact on innovation is driven by the comparison of the demand expansion effect and the innovation diversion effect. We find that this impact is negative if and only if the price diversion ratio – commonly used by competitive authorities to assess the impact of mergers on prices– is less than the innovation diversion ratio – its counterpart for innovation analysis (Farrell and Shapiro, 2010; Salinger, 2019). We apply this result

[6] This implies that the innovation diversion effect is negative.

[7] The fact that a merger is P-neutral does *not* mean that it does not affect equilibrium prices but that any merger-induced change in equilibrium prices is driven by the effect of the merger on innovation incentives.

[8] In other words, a stand-alone innovation theory of harm posits that a merger can have a negative impact on innovation that is *not fully driven* by its impact on prices.

to several standard models and show that the overall impact of a P-neutral merger on innovation can be either negative, neutral, or positive, depending on the demand function.

We then consider *P-increasing mergers*, i.e., mergers that lead to higher prices when the innovation level of both firms is fixed at the equilibrium level chosen by the independent firms. Using our decomposition, we provide sufficient conditions for such mergers to reduce or raise incentives to innovate and apply our approach to several commonly used models. It turns out that the comparison of the innovation diversion ratio and the price diversion ratio still plays a key role in this case. Specifically, our results suggest that when the former is larger than the latter, the impact of a P-increasing merger on innovation is likely to be negative in the absence of spillovers and efficiency gains in R&D; however, when the latter is larger than the former, the impact can be either negative or positive.

We extend our baseline model to account for spillovers and efficiency gains in R&D and show that our decomposition can be adapted in a very natural way to incorporate them. Moreover, we find that the comparison between the innovation diversion ratio and the price diversion ratio remains relevant in environments with spillovers or efficiency gains as long as the diversion ratios are adjusted accordingly.

We also extend our analysis to an oligopolistic setting with merging and nonmerging firms and show how it is affected by the reactions of nonmerging firms to the merger. Importantly, we find that in this context as well, determining the impact of a P-neutral merger on merging firms' innovation level boils down to comparing the innovation diversion ratio and the price diversion ratio.

Finally, we discuss three other extensions of our baseline model. The first one allows for observable investments, which creates a strategic effect of innovation on prices. The second accounts for asymmetric demand and cost functions. The third shows that our approach can also be applied to cost-reducing innovation.

Related literature. While there is a vast and long-standing literature on the effect of competition on innovation,[9] the literature addressing the specific question of how mergers affect firms' incentives to innovate is more recent.

Motta and Tarantino (2018) primarily investigate the impact of horizontal mergers on process innovation and show that they reduce merging firms' incentives to engage in cost-reducing investment in the absence of spillovers and efficiency gains.[10] These authors also establish that this result extends to quality-improving investments for two specific demand

[9] See Gilbert (2006) for a recent survey and Schmutzler (2013) for a unified approach to this issue.

[10] See also Matsushima et al. (2013) for an analysis of the effects of a merger when heterogeneous oligopolists compete both in process innovation and on the product market.

functions under which a quality-improving investment is isomorphic to a cost-reducing investment. By contrast, our focus is on product rather than process innovation, and we study the impact of mergers on this type of innovation by using a general demand function. Our paper can therefore be seen as complementary to Motta and Tarantino (2018).

Federico et al. (2018) study the effect of a horizontal merger on firms' incentives to engage in incremental product innovation. Using simulations, they find that absent spillovers and efficiency gains, a merger is detrimental to innovation and consumer surplus for the three demand functions that they consider. Our approach is different in that we use a novel decomposition of the impact of a merger on innovation to provide sufficient conditions for the merger to reduce or raise the merging firms' incentives to innovate in a model with a general demand function. In particular, we show that the comparison of the innovation diversion ratio and the price diversion ratio is a key determinant of the net impact of a merger on merging firms' incentives to innovate. In this respect, our work is related to the paper by Gaudin (2021), who shows that these two ratios are also useful for the characterization of quality distortions under imperfect competition.

Federico et al. (2017) also analyze the effect of a merger on product innovation but focus on the case in which firms invest in R&D to develop new products. The authors find that the merger has a negative impact on innovation and consumer surplus. Considering a similar setting, Denicolò and Polo (2018) show that a merger between two firms can lead to an increase in their innovation incentives and consumer surplus if the merged entity does not find it optimal to spread its R&D expenditure evenly across the research units of the two merged firms.[11] Furthermore, Denicolò and Polo (2021) show that a merger may increase the merging firms' incentives to innovate because it allows them to share R&D knowledge and technologies.

For a setting where firms can undertake more than one research project, Letina (2016) and Gilbert (2019) show that a horizontal merger can decrease the variety of developed projects, and Moraga-González et al. (2019) find that a merger can either increase or decrease consumer welfare depending on whether the most profitable projects are also the most appropriable ones.[12] In the context of markets with buyer power, Loertscher and Marx (2019a, 2019b) show that a merger raises rivals' investment incentives and can raise the merging parties' investment incentives. Considering an environment with overlapping

[11]See also Jullien and Lefouili (2018a) for an extension of Federico et al. (2017) to the case of differentiated products.

[12]In the same vein, Letina et al. (2020) investigate how the possibility of acquiring entrants affects the R&D incentives of both incumbents and entrants in a model wherein firms are allowed to choose in which innovation projects to invest as well as how much to invest in those projects.

ownership, López and Vives (2019) show that increasing partial ownership interest in rivals decreases (increases) R&D if spillovers are sufficiently small (large).[13] Finally, Mermelstein et al. (2020) consider a dynamic model in which firms can reduce costs through either investment in building capital or mergers and show that merger policy can greatly affect firms' investment behavior and vice versa.

A related, yet different, strand of literature studies the impact of merger policy on firms' premerger (rather than postmerger) incentives to invest in settings where an incumbent may acquire an entrant.[14] Finally, there is a growing empirical literature on the effects of mergers on innovation; it shows that those effects are mixed.[15]

The paper proceeds as follows. We lay out our baseline model in Section 2. In Section 3, we present our main decomposition of the impact of a merger on innovation. We study the effects of P-neutral mergers in Section 4 and P-increasing mergers in Section 5. We incorporate spillovers into our setting in Section 6 and allow for the possibility of efficiency gains in R&D in Section 7. In Section 8, we consider a merger between two firms in an oligopoly setting. In Section 9, we extend our analysis to settings with observable investments, asymmetric demand and cost functions and show that our approach can also be applied to cost-reducing innovation. Section 10 concludes.

2 Baseline model

Consider two single-product firms, 1 and 2, producing differentiated goods. The firms compete in prices and can invest in innovation to increase demand for their products. Let $c \geq 0$ denote the firms' marginal cost of production and $C(\gamma_i)$ the investment cost that firm $i \in \{1, 2\}$ needs to incur to achieve an innovation level $\gamma_i \in [0, \bar{\gamma}]$. We assume that $C(.)$ is increasing and convex, with $C(0) = 0$. For conciseness, we impose that $C'(0) = 0$ and $C'(\bar{\gamma}) = +\infty$. In our baseline model, we suppose that firms set their prices and innovation levels simultaneously or, equivalently, that a firm does not observe its rival's innovation level before setting its price.[16]

[13] See also Vives (2020).

[14] See Jaunaux et al. (2017), Cabral (2020), Hollenbeck (2020), Katz (2020), Kamepalli et al. (2020), Letina et al. (2020), Motta and Peitz (2020), Motta et al. (2020), and Gilbert and Katz (2021a, 2021b).

[15] See, e.g., Grabowski and Kyle (2008), Ornaghi (2009), Guadalupe et al. (2012), Szücs (2014), Haucap et al. (2019), Bennato et al. (2019), and Igami and Uetake (2020).

[16] Oligopoly models with a simultaneous choice of price and R&D have been studied by Dasgupta and Stiglitz (1980), Levin and Reiss (1988), Ziss (1994), Leahy and Neary (1997), Cabral (2000), Vives (2008), and López and Vives (2019), among others. In Section 9.1, we extend our model to the case where innovation levels are observed before prices are set.

We assume that innovation affects the demand for both products but not their marginal cost of production.[17] Let $D_i\left(p_i, p_j, \gamma_i, \gamma_j\right)$ denote the demand addressed to firm $i \in \{1, 2\}$ when it sets its price and innovation level at p_i and γ_i and rival firm $j \neq i$ sets its price and innovation level at p_j and γ_j, and assume that the demand functions are symmetric – i.e., $D_i\left(p_i, p_j, \gamma_i, \gamma_j\right) = D_j\left(p_j, p_i, \gamma_j, \gamma_i\right)$ for any $\left(p_i, p_j, \gamma_i, \gamma_j\right)$ – and twice differentiable. A firm's demand is decreasing in its own price and increasing in its rival's price.[18] Moreover, we assume that an increase in a firm's innovation level leads to an increase in its own demand and a decrease in its rival's demand. Our analysis also applies to the case in which innovation by one firm has a positive impact on the rival's demand (see, e.g., Lin and Saggi, 2002) but we focus on the case where the impact is negative, as this is the scenario that is the most likely to raise anticompetitive concerns. Finally, we make the standard assumption that $\partial D_i/\partial p_i + \partial D_i/\partial p_j < 0$ (i.e., own effects dominate cross-effects) at symmetric prices and innovation levels $p_i = p_j$ and $\gamma_i = \gamma_j$. We also make a similar (reasonable) assumption regarding the effect of a uniform increase in innovation levels: $\partial D_i/\partial \gamma_i + \partial D_i/\partial \gamma_j > 0$ at symmetric prices and innovation levels $p_i = p_j$ and $\gamma_i = \gamma_j$.[19] We summarize these assumptions as follows:

Assumption 1: For any $i, j = 1, 2$, $j \neq i$, $(p_i, p_j) \in \mathbb{R}_+^2$, $\left(\gamma_i, \gamma_j\right) \in [0, \bar{\gamma}]^2$: (i) $\partial D_i/\partial p_i < 0 < \partial D_i/\partial \gamma_i$; (ii) $\partial D_i/\partial p_j > 0 \geq \partial D_j/\partial \gamma_i$; (iii) for any symmetric prices and innovation levels, $\partial D_i/\partial p_i + \partial D_i/\partial p_j < 0$ and $\partial D_i/\partial \gamma_i + \partial D_i/\partial \gamma_j > 0$.

Consider first the benchmark scenario in which firms act independently. In a symmetric equilibrium, the first-order condition for the pricing decision is:

$$(p - c)\frac{\partial D_i}{\partial p_i}(p, p, \gamma, \gamma) + D_i(p, p, \gamma, \gamma) = 0. \tag{1}$$

For the sake of exposition, we assume that this condition defines a unique function $\tilde{p}^*(\gamma)$.[20] Likewise, the first-order condition for the innovation decision is:

$$(p - c)\frac{\partial D_i}{\partial \gamma_i}(p, p, \gamma, \gamma) = C'(\gamma). \tag{2}$$

[17] We consider the case of cost-reducing innovations in Section 9.3.

[18] We discuss how our approach can be extended to the case of a merger between firms selling complementary products in the conclusion. See also Etro (2019) for an analysis of the impact of conglomerate mergers on innovation.

[19] Notice that the assumption that $\partial D_i/\partial \gamma_i + \partial D_i/\partial \gamma_j > 0$ at symmetric prices and innovation levels is equivalent to the assumption that an increase in one firm's innovation level (starting from a symmetric situation) has a positive effect on aggregate demand, i.e., $\partial D_i/\partial \gamma_i + \partial D_j/\partial \gamma_i > 0$ at symmetric prices and innovation levels.

[20] All our results hold without this uniqueness assumption.

We now make the following assumption regarding the price-innovation game.

Assumption 2: The duopoly price-innovation game has a unique symmetric equilibrium $(p^*, p^*, \gamma^*, \gamma^*)$ satisfying first-order conditions (1) and (2).[21]

The general idea behind the subsequent analysis is to use first-order conditions (1) and (2) to eliminate marginal costs and focus on equilibrium prices, innovation levels and demands. Let us first define the (symmetric) *marginal gain from innovation of the independent firm(s)* as

$$h^*(\gamma) \equiv h(\tilde{p}^*(\gamma), \gamma),$$

where

$$h(p, \gamma) \equiv D_i(p, p, \gamma, \gamma) \frac{\frac{\partial D_i}{\partial \gamma_i}(p, p, \gamma, \gamma)}{-\frac{\partial D_i}{\partial p_i}(p, p, \gamma, \gamma)}.$$

From (1), we can see that $h^*(\gamma)$ represents firm i's marginal gain from an increase in its innovation level γ_i when its price is set optimally, holding constant the innovation and price levels of firm j at γ and $\tilde{p}^*(\gamma)$, respectively. The symmetric equilibrium satisfies

$$h^*(\gamma^*) = C'(\gamma^*).$$

Consider now a merger between the two firms, and suppose that the merged entity keeps selling both products.[22] For now, we assume away any merger-induced efficiency gains in R&D[23] but allow for potential efficiency gains in production. More specifically, we suppose that the postmerger marginal costs of the merged entities are both given by $c - \sigma$, where $\sigma \geq 0$.

The merged entity's (monopoly) profit for levels of innovation γ_1 and γ_2 is given by

$$\Pi^M(\gamma_1, \gamma_2) \equiv \max_{p_1, p_2} (p_1 - c + \sigma) D_1(p_1, p_2, \gamma_1, \gamma_2) + (p_2 - c + \sigma) D_2(p_2, p_1, \gamma_2, \gamma_1) - C(\gamma_1) - C(\gamma_2).$$

We assume that this maximization problem behaves well in the following sense:

Assumption 3: The profit function $\Pi^M(\gamma_1, \gamma_2)$ is $\mathcal{C}^1$ and strictly quasiconcave in (γ_1, γ_2).

[21] The uniqueness of the equilibrium is not crucial for our analysis.

[22] See Johnson and Rhodes (2021) for an analysis of the effects of a horizontal merger in a setting where firms may reposition their product lines by adding or removing products of different qualities following the merger.

[23] We incorporate R&D synergies in our analysis in Section 7.

This assumption, combined with the symmetric nature of the demand system, implies that the merged entity's optimal innovation strategy is symmetric.[24] Therefore, we can restrict our attention to a single innovation level for both units of the merged entity, i.e., $\gamma_1 = \gamma_2 = \gamma$. For any given innovation level γ that applies to both products, the merged entity's optimal symmetric price $\tilde{p}^M(\gamma)$ is assumed to be positive and defined by the following first-order condition:

$$(p - c + \sigma)\left[\frac{\partial D_i}{\partial p_i}(p, p, \gamma, \gamma) + \frac{\partial D_j}{\partial p_i}(p, p, \gamma, \gamma)\right] + D_i(p, p, \gamma, \gamma) = 0. \tag{3}$$

Turning to the merged entity's innovation choice, the optimal innovation level for given symmetric prices is the solution to the following first-order condition:

$$(p - c + \sigma)\left[\frac{\partial D_i}{\partial \gamma_i}(p, p, \gamma, \gamma) + \frac{\partial D_j}{\partial \gamma_i}(p, p, \gamma, \gamma)\right] = C'(\gamma). \tag{4}$$

An optimal symmetric price-innovation pair $\left(p^M, \gamma^M\right)$ for the merged entity satisfies conditions (3) and (4).

Similarly to what we do in the case with independent firms, we define the *merged entity's marginal gain from innovation* as

$$l^M(\gamma) = l\left(\tilde{p}^M(\gamma), \gamma\right),$$

where

$$l(p, \gamma) \equiv -D_i(p, p, \gamma, \gamma)\, \frac{\frac{\partial D_i}{\partial \gamma_i}(p, p, \gamma, \gamma) + \frac{\partial D_j}{\partial \gamma_i}(p, p, \gamma, \gamma)}{\frac{\partial D_i}{\partial p_i}(p, p, \gamma, \gamma) + \frac{\partial D_j}{\partial p_i}(p, p, \gamma, \gamma)}.$$

From (3), we can see that $l^M(\gamma)$ corresponds to the slope of the merged entity's profit (gross of investment cost) with respect to γ_i (at $\gamma_i = \gamma$) when all prices are set optimally, holding constant the innovation level of the other unit (at $\gamma_j = \gamma$). Based on these definitions, the following proposition shows that the impact of the merger on innovation depends on the relative magnitude of the independent firms' and merged entity's marginal gain from innovation, evaluated at the independent firms' innovation level.

Proposition 1 *The impact of the merger on innovation, $\gamma^M - \gamma^*$, has the same sign as $l^M(\gamma^*) - h^*(\gamma^*)$.*

Proof. See Appendix. ■

[24] The assumption of continuous differentiability is meant only to simplify the exposition.

Proposition 1 thus shows that the merger increases (decreases) innovation if the merged entity's marginal gain from innovation is larger (smaller) than the independent firms' marginal gain from innovation. The comparison involves direct changes in incentives due to price and innovation externalities but also changes related to the difference between the merged entity's and independent firms' prices.

3 Decomposition of the effect of a merger on innovation

In this section, we show that the impact of a merger on innovation is a combination of four effects: the *innovation diversion effect*, the *margin expansion effect*, the *demand expansion effect*, and the *per unit return to innovation effect*.

To simplify the exposition, we adopt the following convention.

Convention: We denote with the superscript $*$ any function evaluated at symmetric innovation levels γ and independent firms' prices $\tilde{p}^*(\gamma)$ and with the superscript M any function evaluated at symmetric innovation levels γ and the merged entity's prices $\tilde{p}^M(\gamma)$. In particular,

$$D_i^*(\gamma) \equiv D_i(\tilde{p}^*(\gamma), \tilde{p}^*(\gamma), \gamma, \gamma) \text{ and } D_i^M(\gamma) \equiv D_i\left(\tilde{p}^M(\gamma), \tilde{p}^M(\gamma), \gamma, \gamma\right),$$

and for any $x \in \{p_i, \gamma_i, p_j, \gamma_j\}$,

$$\frac{\partial D_i^*(\gamma)}{\partial x} \equiv \frac{\partial D_i}{\partial x}(\tilde{p}^*(\gamma), \tilde{p}^*(\gamma), \gamma, \gamma), \quad \frac{\partial D_i^M(\gamma)}{\partial x} \equiv \frac{\partial D_i}{\partial x}\left(\tilde{p}^M(\gamma), \tilde{p}^M(\gamma), \gamma, \gamma\right).$$

3.1 Innovation diversion effect

To highlight the first effect, we isolate the terms in the merged entity's marginal gain from innovation $l^M(\gamma)$ that capture the impact of innovation in product i on the demand for that product. Eliminating the terms related to the impact of innovation on the demand for the other good, product j, we define

$$\hat{l}^M(\gamma) \equiv -D_i^M(\gamma) \frac{\frac{\partial D_i^M(\gamma)}{\partial \gamma_i}}{\frac{\partial D_i^M(\gamma)}{\partial p_i} + \frac{\partial D_j^M(\gamma)}{\partial p_i}}.$$

This term captures the marginal gain from innovation in the sales of product i for the merged entity. Using this term, we obtain the following decomposition of the impact of the merger on innovation:

$$l^M(\gamma) - h^*(\gamma) = \hat{l}^M(\gamma) - h^*(\gamma) + H_I(\gamma),$$
$$\text{where } H_I(\gamma) \equiv \hat{l}^M(\gamma) \frac{\frac{\partial D_j^M(\gamma)}{\partial \gamma_i}}{\frac{\partial D_i^M(\gamma)}{\partial \gamma_i}} < 0.$$

The term H_I captures the internalization by the merged entity of the *diversion* of sales that demand-enhancing innovation in one product induces for the other product. This term, which we refer to as the *innovation diversion effect*, is negative because the underlying innovation externality is negative.

3.2 Margin expansion and demand expansion effects

The *innovation diversion effect* would be the only effect at work if prices were exogenous and unaffected by the merger. However, in our setting, a merger may also affect prices, thus leading to changes in two key variables affecting a firm's incentives to invest in innovation: its demand and its margin.

To show how the price effects of a merger interact with the merging firms' incentives to innovate, we first discuss how a firm benefits from a demand-enhancing innovation based on the firm's postinnovation price. One extreme strategy is to set the latter at the preinnovation price level. In this case, the firm benefits from the innovation solely through an increase in demand. We call this a demand expansion strategy. The other extreme strategy is to set the postinnovation price at a level such that postinnovation demand is equal to preinnovation demand. In this case, the benefit from innovation is fully driven by the increase in the firm's margin. We call this a margin expansion strategy. In general, a firm's optimal strategy lies between the two polar cases described above. More formally, we can write the gain from changing the price and innovation levels from (p_i, γ_i) to (p_i', γ_i'), holding the price and innovation levels of the other firm constant at $(p_j, \gamma_j) = (p, \gamma)$, as:

$$\begin{aligned} & (p_i' - c) D_i(p_i', p, \gamma_i', \gamma) - (p_i - c) D_i(p_i, p, \gamma_i, \gamma) \\ = \; & (p_i' - c) \underbrace{[D_i(p_i', p, \gamma_i', \gamma) - D_i(p_i, p, \gamma_i, \gamma)]}_{\text{demand expansion}} + \underbrace{(p_i' - p_i)}_{\text{margin expansion}} D_i(p_i, p, \gamma_i, \gamma). \end{aligned}$$

The main channel through which the firm benefits from innovation is demand expansion if

$p_i' - p_i$ is relatively small, while the main channel is margin expansion if $D_i(p_i', p, \gamma_i', \gamma) - D_i(p_i, p, \gamma_i, \gamma)$ is relatively small. The extent to which the optimal postinnovation price is different from the preinnovation price ultimately depends on the comparison of the preinnovation and postinnovation price elasticities of demand. In particular, when direct and cross-elasticities of demand are little affected by innovation, demand expansion is the main driver of innovation efforts.

Related to the above discussion, a merger may affect firms' incentives to invest in demand-enhancing innovation in two ways. First, the merger affects firms' incentives to innovate with the aim of increasing margins. To see why, note that a firm's marginal benefit from a margin increase is equal to its demand. If the merger leads to higher prices and, therefore, lower demand, it lowers the merging firms' incentives to innovate to increase their margins. The reverse holds if the merger leads to lower prices. We call this the *margin expansion effect*. Second, the merger affects firms' incentives to innovate with the aim of increasing demand. This follows from the fact that the merger affects the marginal benefit from an increase in demand, which is equal to the margin. A higher margin following the merger raises the merging firms' incentives to innovate to increase demand. We call this the *demand expansion effect*.[25]

3.3 Final decomposition

To formally express the terms capturing the margin expansion and demand expansion effects in our model and isolate the last effect in our decomposition, we define the *per unit return to innovation* as

$$r_i\left(p_i, p_j, \gamma_i, \gamma_j\right) \equiv \frac{\frac{\partial D_i\left(p_i, p_j, \gamma_i, \gamma_j\right)}{\partial \gamma_i}}{-\frac{\partial D_i\left(p_i, p_j, \gamma_i, \gamma_j\right)}{\partial p_i}}.$$

We also define $r_i^*(\gamma)$ and $r_i^M(\gamma)$, following the same convention that we used above.

The ratio $r_i\left(p_i, p_j, \gamma_i, \gamma_j\right)$ measures the price increase that firm i can achieve when it increases innovation at the margin and raises prices to keep the volume of sales constant. Therefore, it can be interpreted as the return to innovation per unit of output. The *independent firm's marginal gain from innovation* can then be written as the product of the

[25] This effect was emphasized in the context of implementation of a new technology by Bourreau and Jullien (2018).

volume of output times the per unit return to innovation:

$$h^*(\gamma) = D_i^*(\gamma)\, r_i^*(\gamma).$$

The following proposition provides a decomposition of the impact of the merger on firms' investments in demand-enhancing innovation.

Proposition 2 *The change in innovation incentives induced by the merger can be decomposed as follows:*

$$l^M(\gamma^*) - h^*(\gamma^*) = H_M + H_R + H_D + H_I,$$

where

$$H_M \equiv \left[D_i^M(\gamma^*) - D_i^*(\gamma^*)\right] r_i^*(\gamma^*),$$

$$H_R \equiv D_i^M(\gamma^*)\left[r_i^M(\gamma^*) - r_i^*(\gamma^*)\right],$$

$$H_D \equiv \hat{l}^M(\gamma^*) \times \left(\frac{\frac{\partial D_j^M(\gamma^*)}{\partial p_i}}{-\frac{\partial D_i^M(\gamma^*)}{\partial p_i}}\right) > 0, \;\; H_I \equiv \hat{l}^M(\gamma^*) \times \left(\frac{\frac{\partial D_j^M(\gamma^*)}{\partial \gamma_i}}{\frac{\partial D_i^M(\gamma^*)}{\partial \gamma_i}}\right) < 0.$$

Proof. See Appendix. ■

The term H_M captures the *margin expansion effect*. It is negative if and only if the merger raises prices (at an innovation level fixed at the independent firms' value), i.e., $\tilde{p}^M(\gamma^*) > \tilde{p}^*(\gamma^*)$.[26] The term H_D is always positive and is larger the greater the magnitude of the derivative $\partial D_j/\partial p_i$, which drives the merged entity's incentives to increase prices (at a given innovation level) in comparison with those in the situation where firms set their prices independently. Since a higher price (and, therefore, a higher margin) provides the merged entity with stronger incentives to increase demand, we interpret the term H_D as capturing the *demand expansion effect*. As discussed above, the term H_I captures the *innovation diversion effect*. Finally, the term H_R captures a *per unit return to innovation effect* that measures the change in the per unit return to innovation due to the effect of the merger on prices. This effect can be either positive or negative.

4 P-neutral mergers

To understand how a merger affects firms' incentives to innovate, it is useful to abstract first from standard price effects. Specifically, we say that a merger is *P-neutral* if the

[26]This is for instance the case if there are no efficiency gains in production, i.e., $\sigma = 0$.

merger does not affect prices when the innovation level of both firms is *fixed* at the level chosen by independent firms, that is, if

$$\tilde{p}^M(\gamma^*) = p^*.$$

The fact that a merger is P-neutral does *not* mean that it does not affect equilibrium prices. Instead, it means that any merger-induced changes in equilibrium prices are driven by the effect of the merger on innovation incentives and the effect of innovation on prices.[27]

Our initial focus on the special case of P-neutral mergers is also motivated by their policy relevance. Studying the impact of such mergers helps to determine the conditions under which we can use a *stand-alone* innovation theory of harm stipulating that a merger that has no effect on prices (at given innovation levels) has a negative impact on innovation (Denicolò and Polo, 2019).

Consider a P-neutral merger. It is straightforward that the margin expansion effect (H_M) and the per unit return to innovation effect (H_R) vanish, as they both stem from changes in pricing behavior only. Therefore, the effect of a P-neutral merger on innovation is governed solely by the combination of the demand expansion effect and the innovation diversion effect – i.e., $H_D + H_I$.[28] Interestingly, the term $H_D + H_I$ has the same sign as the difference between the price diversion ratio and the innovation diversion ratio, that is,

$$\underbrace{\frac{\frac{\partial D_j^M(\gamma^*)}{\partial p_i}}{-\frac{\partial D_i^M(\gamma^*)}{\partial p_i}}}_{\text{price diversion ratio}} - \underbrace{\frac{-\frac{\partial D_j^M(\gamma^*)}{\partial \gamma_i}}{\frac{\partial D_i^M(\gamma^*)}{\partial \gamma_i}}}_{\text{innovation diversion ratio}}.$$

This leads us to the following result, which shows that the impact of a P-neutral merger on innovation can be derived from a mere comparison of the price diversion ratio and the innovation diversion ratio.

Corollary 1 *A P-neutral merger reduces (raises) incentives to innovate if the price diversion ratio is lower (greater) than the innovation diversion ratio, where both ratios are evaluated at* $(p^*, p^*, \gamma^*, \gamma^*)$.

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[27] A P-neutral merger would be CS-neutral in the terminology of Nocke and Whinston (2010) if the demand functions were not affected by the merger.

[28] Note that the demand expansion effect does not vanish for a P-neutral merger because it is driven by the effect of the merger on margins rather than prices. Since P-neutral mergers occur for a positive value of efficiency gains in production, they lead to a higher margin even though they do not affect prices (at given innovation levels).

The sign of $H_D + H_I$ captures whether the *price externality* that firms exert on each other is stronger or weaker than the *innovation externality* that they exercise on each other. If the price externality is stronger, the merger induces a relatively large increase in margins, which leads to a demand expansion effect substantial enough to outweigh the effect of sales cannibalization resulting from innovation on firms' incentives. As a result, the merged entity invests more in demand-enhancing innovation. By contrast, if the price diversion ratio is small in comparison to the innovation diversion ratio, the merged entity gains little from raising its demand but has a strong incentive to reduce cannibalization. In this case, the merger decreases investment in demand-enhancing innovation.

We now use Corollary 1 to determine the impact of a P-neutral merger on the merging firms' incentives to innovate for several commonly used demand functions. The following table summarizes our results – a more detailed presentation is provided in the Appendix.

Model	Demand function	Impact on innovation
Price-innovation index	$Q\left(\eta\left(p_i,\gamma_i\right),\eta\left(p_j,\gamma_j\right)\right)$	neutral
Quality-adjusted prices	$\frac{1}{\gamma_i}Q\left(\frac{p_i}{\gamma_i},\frac{p_j}{\gamma_j}\right)$	negative
Constant expenditures	$\frac{\eta(p_i,\gamma_i)}{p_i\eta(p_i,\gamma_i)+p_j\eta\left(p_j,\gamma_j\right)+K}$	negative
Quality-augmented linear demand	$\frac{\gamma_i\left[2\gamma_i(1-p_i)-\rho\gamma_j(1-p_j)\right]}{4-\rho^2}$	positive
Augmented Singh-Vives	$\frac{(\alpha+\tau\gamma_i)-\left(\alpha+\tau\gamma_j\right)\rho(\gamma_1,\gamma_2)-p_i+\rho(\gamma_1,\gamma_2)p_j}{1-\rho(\gamma_1,\gamma_2)^2}$	positive

Table 1: Impact of a P-neutral merger on innovation.

Our findings show that the impact of a P-neutral merger on firms' incentives to innovate may be either neutral, negative, or positive in standard models. Thus, a stand-alone innovation theory of harm stipulating that a merger with no price effects (at a fixed level of innovation) harms innovation is justified in some environments but not in others.

Note that the category of models with a price-innovation index includes the logit model (see Section 5.1). We also examine the impact of a P-neutral merger in the random coefficient logit model (Berry et al., 1995; Nevo, 2001), assuming a random coefficient for prices but a fixed coefficient for innovation. As we are not able to derive analytical results for this demand, we run numerical simulations of diversion ratios, building on Conlon and Mortimer (2020). Our results (available upon request) suggest that the innovation diversion ratio is larger than the price diversion ratio for this model, which implies that the impact of a P-neutral merger is negative in this case.

5 P-increasing mergers

We now consider *P-increasing* mergers, i.e., mergers leading to an increase in prices at an innovation level fixed at the equilibrium value chosen by the independent firms: $\tilde{p}^M(\gamma^*) > \tilde{p}^*(\gamma^*)$. In contrast to P-neutral mergers, the margin expansion effect is (strictly) negative and the per unit return to innovation effect can be different from zero. However, note that the term $H_M + H_R$, combining the margin expansion effect and the per unit return to innovation effect, has the same sign as

$$h^M(\gamma^*) - h^*(\gamma^*),$$

where $h^M(\gamma) \equiv D_i^M(\gamma) r_i^M(\gamma)$.

The term $h^M(\gamma^*) - h^*(\gamma^*)$ captures the incentives to enhance demand for a given product that are not related to the externalities exerted on the other product. Indeed, if we ignore these externalities, the marginal gain from innovation can be defined as the extra profit from product i for a marginal increase in innovation γ_i when the price p_i is adjusted so that demand for product i remains constant:

$$D_i(p, p, \gamma, \gamma) \left. \frac{\partial p_i}{\partial \gamma_i} \right|_{\text{constant } D_i,} - C'(\gamma) = h(p, \gamma) - C'(\gamma).$$

Thus, the sign of $H_M + H_R$ is positive (negative) if $h(p, \gamma^*)$ is increasing (decreasing) in price p.

The discussion above shows that we can separate the overall effect of a merger on innovation into one effect related to price and innovation externalities between products (captured by $H_D + H_I$) and another one related to changes in the demand for each product induced by the internalization of these externalities (captured by $H_M + H_R$). The sign of the former is positive (negative) if the price diversion ratio is greater (lower) than the innovation diversion ratio, while the sign of the latter depends on the monotonicity of $h(p, \gamma^*)$ with respect to p. In many, but not all, commonly used models, $h(p, \gamma^*)$ is decreasing in p.[29] In all those models, an increase in prices reduces firms' incentives to invest in demand-enhancing innovation, or equivalently, the combination of the margin expansion effect and the per unit return to innovation effect is negative.

We now determine the overall impact of a P-increasing merger on innovation in several standard models, which we categorize depending on whether the innovation diversion ratio is equal to, greater than, or lower than the price diversion ratio.

[29] This is confirmed by the specific models that we consider below.

5.1 Models in which the innovation diversion ratio is equal to the price diversion ratio

We first consider models in which the innovation diversion and price diversion ratios are equal. As shown in the Appendix, this holds for models that subsume the effect of prices and innovation through an index. For such models, the effect of a P-increasing merger on innovation is positive (negative) if $h(p, \gamma^*)$ is increasing (decreasing) in p.

We study two standard models within this class. Let us first consider the model with *hedonic prices*, in which the demand function is given by

$$D_i\left(p_i, p_j, \gamma_i, \gamma_j\right) = Q\left(p_i - \gamma_i, p_j - \gamma_j\right). \tag{HED}$$

where $Q(\cdot, \cdot)$ is decreasing in its first argument and increasing in its second argument. In this case, $h(p, \gamma) = Q(p - \gamma, p - \gamma)$ is decreasing in p, which implies that a P-increasing merger reduces incentives to innovate.[30]

Consider now the *multinomial logit* (MNL) model, in which demand is given by:[31]

$$D_i\left(p_i, p_j, \gamma_i, \gamma_j\right) = \frac{\exp u\left(\gamma_i, y - p_i\right)}{\exp u\left(\gamma_i, y - p_i\right) + \exp u\left(\gamma_j, y - p_j\right) + \exp u\left(0, y\right)}, \tag{MNL}$$

where u is increasing in both its arguments. In this model, y represents income, and $u(\gamma_i, y - p_i)$ is the mean utility from consuming one unit of product of quality γ_i paid at price p_i.

For this demand function, $h(p, \gamma)$ may be either increasing or decreasing in p. Denoting by u_1 and u_2 the derivatives of u with respect to its first and second arguments, respectively, and u_{12} and u_{22} the cross-derivative of u and the second derivative of u with respect to its second argument, respectively, we find the following sufficient condition for a P-increasing merger to reduce (raise) incentives to innovate.

[30] Note that in this model with hedonic prices, the impact of a merger on innovation is driven solely by the margin expansion effect. The reason is that the demand expansion effect and the innovation diversion effect cancel out, and the per unit return to innovation effect is equal to zero. A key feature of this model is that its mode of analysis is comparable to that of models with cost-reducing innovation (Motta and Tarantino, 2018). In Section 9.3, we confirm this isomorphism by extending our approach to cost-reducing innovation and showing that the margin expansion effect is the only effect at work for this type of innovation.

[31] See, e.g., Dubé (2019).

Corollary 2 *In the MNL model, a P-increasing merger reduces incentives to innovate if*

$$\frac{-u_{12}\left(\gamma^{*}, y-p\right)}{u_{1}\left(\gamma^{*}, y-p\right)}+\frac{u_{22}\left(\gamma^{*}, y-p\right)}{u_{2}\left(\gamma^{*}, y-p\right)}<0$$

for all $p \in\left[p^{*}, \tilde{p}^{M}\left(\gamma^{*}\right)\right]$. *A P-increasing merger raises incentives to innovate if*

$$\frac{-u_{12}\left(\gamma^{*}, y-p\right)}{u_{1}\left(\gamma^{*}, y-p\right)}+\frac{u_{22}\left(\gamma^{*}, y-p\right)}{u_{2}\left(\gamma^{*}, y-p\right)}>u_{2}\left(\gamma^{*}, y-p\right)$$

for all $p \in\left[p^{*}, \tilde{p}^{M}\left(\gamma^{*}\right)\right]$.

Proof. See Appendix. ■

Let us now provide specific utility functions satisfying the conditions stated in Corollary 2. First, it is easy to see that the sufficient condition under which a P-increasing merger reduces incentives to innovate holds in the case of a Cobb-Douglas utility function. Second, consider the case of a constant marginal utility of income, i.e., $u(\gamma, y-p)=v(\gamma)+f(\gamma)(y-p)$, with $v'(\gamma)>0$, $v'(\gamma)+f'(\gamma)y>0$ and $f(\gamma)>0$. We show in the Appendix that the sufficient condition for a P-increasing merger to raise incentives to innovate provided in Corollary 2 holds whenever a higher quality γ increases the marginal utility of income, i.e., f is increasing, and

$$\frac{f'\left(\gamma^{*}\right)}{f\left(\gamma^{*}\right)}>v'\left(\gamma^{*}\right)+y f'\left(\gamma^{*}\right).$$

Conversely, a P-increasing merger reduces incentives to innovate whenever f is decreasing.

5.2 Models in which the innovation diversion ratio is greater than the price diversion ratio

We now consider two classes of models in which the innovation diversion ratio is greater than the price diversion ratio: those with quality-adjusted prices and those with constant expenditures. In these models, a P-increasing merger reduces incentives to innovate if $h\left(p, \gamma^{*}\right)$ is nonincreasing in p.

5.2.1 Model with quality-adjusted prices

Let us first consider the model with quality-adjusted prices, where the demand of firm i is given by:[32]

$$D_i\left(p_i, p_j, \gamma_i, \gamma_j\right) = \frac{1}{\gamma_i} Q\left(\frac{p_i}{\gamma_i}, \frac{p_j}{\gamma_j}\right).$$

(See the Appendix for details.)

In this model, the overall effect of the merger on innovation is *a priori* ambiguous. On the one hand, the innovation diversion ratio is greater than the price diversion ratio, and therefore, the innovation diversion effect dominates the demand expansion effect. However, on the other hand, $h(p, \gamma)$ may be either increasing or decreasing in prices, and thus, the sum of the margin expansion effect and the per unit return to innovation effect has an ambiguous sign. Computing $h^*(\gamma^*)$ and $l^M(\gamma^*)$ and comparing them, we obtain the following result.

Corollary 3 *In the model with quality-adjusted prices, a P-increasing merger reduces incentives to innovate.*

Proof. See Appendix. ■

Motta and Tarantino (2018) show that absent any efficiency gains in production, the overall effect of the merger on innovation is negative under this specification. Corollary 3 shows that this result extends to any P-increasing merger.

5.2.2 Model with constant expenditures/CES

Let us now consider the class of models with constant expenditures, which includes CES demand as a special case (see, e.g., Vives, 1999). Firm i's demand can be written as:

$$D_i\left(p_i, p_j, \gamma_i, \gamma_j\right) = \frac{\eta\left(p_i, \gamma_i\right)}{p_i \eta\left(p_i, \gamma_i\right) + p_j \eta\left(p_j, \gamma_j\right) + K},$$

where K represents spending on other goods and the total spending $p_1 \eta(p_1, \gamma_1) + p_2 \eta(p_2, \gamma_2) + K$ is constant. (See the Appendix for details.)

As shown in the Appendix, the innovation diversion ratio is greater than the price diversion ratio. Therefore, a P-increasing merger reduces innovation if $h(p, \gamma^*)$ is decreasing in p, which leads to the following statement.

[32] This model has been considered, for instance, by Motta and Tarantino (2018).

Corollary 4 *In a model with constant expenditures, a P-increasing merger reduces incentives to innovate if* $\frac{\gamma}{\eta}\eta_2$ *is nonincreasing in* p *and* $-\frac{p}{\eta}\eta_1$ *is nondecreasing in* p*. This holds in particular for CES demand.*

Proof. See Appendix. ■

5.3 Models in which the innovation diversion ratio is lower than the price diversion ratio

Finally, we examine the impact of a merger on innovation in two models in which the innovation diversion ratio is lower than the price diversion ratio. In this class of models, a P-increasing merger raises incentives to innovate if $h(p, \gamma^*)$ is nondecreasing in p.

5.3.1 Model with quality-augmented linear demand

Let us first consider the quality-augmented linear model described in the Appendix, introduced by Sutton (1997, 1998) and used *inter alia* by Symeonidis (2000, 2003) and Federico et al. (2018), where firm i's demand is given by

$$D_i(p_i, p_j, \gamma_i, \gamma_j) = \frac{\gamma_i\left[2\gamma_i(1-p_i) - \rho\gamma_j(1-p_j)\right]}{4-\rho^2}.$$

In this model, the price diversion ratio is greater than the innovation diversion ratio, and $h(p, \gamma)$ is decreasing in p (over the relevant range $[0, 1]$). Thus, for P-increasing mergers, we have, on the one hand, $H_D + H_I > 0$ and, on the other hand, $H_M + H_R < 0$. Consequently, the effect of a merger on innovation is *a priori* ambiguous.

To determine the overall impact of the merger, note that the independent firms' and merged entity's prices for a given (symmetric) level of innovation γ are[33]

$$\widetilde{p}^*(\gamma) = c + (1-c)\frac{2-\rho}{4-\rho} < 1 \text{ and } \widetilde{p}^M(\gamma) = \frac{1+c-\sigma}{2}, \tag{5}$$

respectively. Interestingly, $\widetilde{p}^*(\gamma) = p^*$ and $\widetilde{p}^M(\gamma) = p^M$ do not depend on the innovation level γ under this specification. Therefore, innovation is monetized only through an increase in demand by the merged entity, which suggests that the demand expansion effect plays an important role in this model. We obtain the following result.

[33] See Symeonidis (2003) for the derivation of equilibrium prices.

Corollary 5 *In the presence of quality-augmented linear demand, a merger raises incentives to innovate if and only if*

$$p^M < 1 - \sqrt{1 - \frac{\rho}{4}} + \sqrt{1 - \frac{\rho}{4}} p^*,$$

which happens if and only if the level of efficiency gains in production σ is sufficiently large. Conversely, a P-increasing merger reduces incentives to innovate if and only if it leads to a sufficiently high increase in prices.

Proof. See Appendix. ■

Thus, a P-increasing merger strengthens the merging firm's incentives to innovate if the price increase is moderate due to efficiency gains in production but weakens those incentives if the price increase is large enough. The demand expansion effect is strong enough to outweigh the other effects in the former scenario but not in the latter.

5.3.2 Augmented Singh-Vives model

Consider now the augmented Singh and Vives model (described in the Appendix), where the demand for firm i is given by

$$D_i\left(p_i, p_j, \gamma_i, \gamma_j\right) = \frac{(\alpha + \tau\gamma_i) - \left(\alpha + \tau\gamma_j\right)\rho(\gamma_1, \gamma_2) - p_i + \rho(\gamma_1, \gamma_2)p_j}{1 - \rho(\gamma_1, \gamma_2)^2}.$$

Under this specification, the innovation diversion ratio is lower than the price diversion ratio. Moreover, $h\left(p, \gamma^*\right)$ is decreasing in p, and therefore, the overall effect of a P-increasing merger is *a priori* ambiguous. However, combining all the effects, we find that a merger enhances innovation incentives if the price increase following the merger is moderate enough.

Corollary 6 *In the augmented Singh and Vives model (with $H_I < 0$), a merger raises incentives to innovate if and only if*

$$\frac{\left(a\left(\gamma^*\right) - \tilde{p}^M\left(\gamma^*\right)\right)^2 - \left(a\left(\gamma^*\right) - p^*\right)^2 \gamma^*\delta}{1 - \gamma^*\delta} > \left(\tilde{p}^M\left(\gamma^*\right) - p^*\right)\frac{\tau}{\delta}, \tag{6}$$

which happens if the price increase following the merger is sufficiently small. Condition (6) *holds for a merger with no efficiency gains in production (i.e., $\sigma = 0$) if*

$$\left(a\left(\gamma^{*}\right)-c\right)\left[\frac{1+4\gamma^{*2}\delta^{2}}{2\left(1-\gamma^{*}\delta\right)\left(1-4\gamma^{*2}\delta^{2}\right)}\right]>\frac{\tau}{\delta}, \tag{7}$$

which is satisfied if γ^* is large or (holding γ^* constant) τ is small or δ is large.

Proof. See Appendix. ■

In line with our general model, we have focused on the case in which the innovation diversion effect is negative ($H_I < 0$). It can be shown that the latter condition and condition (7) can hold simultaneously.[34] This implies that there are parameters under which a merger with no efficiency gains and a negative innovation diversion effect has a positive impact on innovation.

6 Technological spillovers

It is well known that a firm's R&D may benefit other firms, including its rivals, through technological spillovers (d'Aspremont and Jacquemin, 1988; Bloom et al., 2013; López and Vives, 2019). In this section, we show how our approach should be adapted to account for such spillovers.

Let us assume that there exists a degree of spillovers $\lambda \in [0,1]$ such that the demand addressed to firm i is given by $D_i\left(p_i, p_j, \gamma_i + \lambda\gamma_j, \gamma_j + \lambda\gamma_i\right)$. In other words, a share λ of the demand-enhancing innovation efforts of firm i spills over to firm j (and vice versa).

Let $\hat{\gamma}_i \equiv \gamma_i + \lambda\gamma_j$ for $i = 1, 2$ and $\hat{\gamma} \equiv (1+\lambda)\gamma$, and denote by $(\hat{p}^*, \hat{\gamma}^*)$ the (symmetric) independent firms' equilibrium level of innovation. We show in the Appendix that Proposition 1 extends in a very natural way to the scenario in which there are spillovers. Specifically, we establish that in the presence of spillovers, the impact of the merger on innovation has the same sign as $l_\lambda^M\left(\hat{\gamma}^*\right) - h_\lambda^*\left(\hat{\gamma}^*\right)$, where $l_\lambda^M(.)$ and $h_\lambda^*(.)$ are obtained from $l^M(.)$ and $h^*(.)$ by replacing $\frac{\partial D_i}{\partial \gamma_i}$ and $\frac{\partial D_j}{\partial \gamma_i}$ with $\frac{\partial D_i}{\partial \hat{\gamma}_i} + \lambda\frac{\partial D_i}{\partial \hat{\gamma}_j}$ and $\frac{\partial D_j}{\partial \hat{\gamma}_i} + \lambda\frac{\partial D_j}{\partial \hat{\gamma}_j}$, respectively, and replacing the arguments $\left(\tilde{p}^*(\gamma), \tilde{p}^*(\gamma), \gamma, \gamma\right)$ and $\left(\tilde{p}^M(\gamma), \tilde{p}^M(\gamma), \gamma, \gamma\right)$ with $\left(\tilde{p}^*(\hat{\gamma}), \tilde{p}^*(\hat{\gamma}), \hat{\gamma}, \hat{\gamma}\right)$ and $\left(\tilde{p}^M(\hat{\gamma}), \tilde{p}^M(\hat{\gamma}), \hat{\gamma}, \hat{\gamma}\right)$, respectively.

We can again provide a decomposition of the overall impact of the merger on incentives to innovate into several effects:

$$l_\lambda^M\left(\hat{\gamma}^*\right) - h_\lambda^*\left(\hat{\gamma}^*\right) = H_{M\lambda} + H_{D\lambda} + H_{I\lambda} + H_{R\lambda} + E_\lambda,$$

[34] This holds because $\frac{a(\gamma^*)(1+4\delta^2\gamma^{*2})}{2(1-\gamma^*\delta)(1-4\delta^2\gamma^{*2})} > \frac{a(\gamma^*)\delta\gamma^*}{2(1-\delta\gamma^*)(1-2\delta\gamma^*)}$.

where $H_{M\lambda}$, $H_{D\lambda}$, $H_{I\lambda}$, and $H_{R\lambda}$ are obtained from H_M, H_D, H_I, and H_R, respectively, by making the replacements specified above and

$$E_\lambda \equiv \lambda \hat{l}^M(\hat{\gamma}^*) \left[\frac{\frac{\partial D_j^M(\hat{\gamma})}{\partial \hat{\gamma}_i}}{\frac{\partial D_i^M(\hat{\gamma})}{\partial \hat{\gamma}_i}} \times \frac{\frac{\partial D_j^M(\hat{\gamma})}{\partial p_i}}{-\frac{\partial D_i^M(\hat{\gamma})}{\partial p_i}} + 1 \right] > 0.$$

The terms $H_{M\lambda}$, $H_{D\lambda}$, $H_{I\lambda}$, $H_{R\lambda}$ capture the margin expansion effect, the demand expansion effect, the innovation diversion effect, and the per unit return to innovation effect, respectively, as in the baseline model. The additional term E_λ captures a new *spillover effect* and is positive.[35]

Furthermore, we find that the sum of the innovation diversion effect, the demand expansion effect and the spillover effect, $H_{D\lambda} + H_{I\lambda} + E_\lambda$, which are the only effects at work for a P-neutral merger, has the same sign as the difference between the price diversion ratio and a spillover-adjusted innovation diversion ratio. Specifically, we have the following result.

Proposition 3 *Assume that there are R&D spillovers and denote the spillover rate as λ. The effect of a P-neutral merger on the incentives to innovate has the same sign as:*

$$\underbrace{\frac{\frac{\partial D_j^M(\hat{\gamma})}{\partial p_i}}{-\frac{\partial D_i^M(\hat{\gamma})}{\partial p_i}}}_{\textit{price diversion ratio}} - \underbrace{\frac{-\frac{\frac{\partial D_j^M(\hat{\gamma})}{\partial \hat{\gamma}_i}}{\frac{\partial D_i^M(\hat{\gamma})}{\partial \hat{\gamma}_i}} - \lambda}{1 + \lambda \frac{\frac{\partial D_j^M(\hat{\gamma})}{\partial \hat{\gamma}_i}}{\frac{\partial D_i^M(\hat{\gamma})}{\partial \hat{\gamma}_i}}}}_{\textit{spillover-adjusted innovation diversion ratio}}.$$

This proposition shows that our result that the impact of a P-neutral merger on firms' incentives to innovate is (fully) determined by the comparison of the price diversion and innovation diversion ratios still holds in a setting with spillovers as long as the innovation diversion ratio is adjusted to account for spillovers.

Note that the denominator of the spillover-adjusted innovation diversion ratio is always positive. This follows from the assumption that $\lambda \in [0, 1]$ and $\partial D_i / \partial \hat{\gamma}_i + \partial D_i / \partial \hat{\gamma}_j > 0$. Thus, the sign of the spillover-adjusted innovation diversion ratio is given by the sign of

[35] To see why this term is positive, note that $\frac{\partial D_j}{\partial \hat{\gamma}_i} \times \frac{\frac{\partial D_j}{\partial p_i}}{-\frac{\partial D_i}{\partial p_i}} + \frac{\partial D_i}{\partial \hat{\gamma}_i} > \min\left[\frac{\partial D_i}{\partial \hat{\gamma}_i}, \frac{\partial D_j}{\partial \hat{\gamma}_i} + \frac{\partial D_i}{\partial \hat{\gamma}_i}\right]$ because $\partial D_i / \partial p_i + \partial D_i / \partial p_j$ is negative and $\partial D_i / \partial \hat{\gamma}_i + \partial D_i / \partial \hat{\gamma}_j$ is positive.

the difference between the innovation diversion ratio and the spillover rate:

$$\left(\frac{-\frac{\partial D_j^M(\hat{\gamma})}{\partial \hat{\gamma}_i}}{\frac{\partial D_i^M(\hat{\gamma})}{\partial \hat{\gamma}_i}}\right) - \lambda.$$

This sign can be related to the magnitude of the net innovation pressure (NIP) defined by Salinger (2019). Considering an environment with no price competition, Salinger (2019) shows that a merger reduces innovation if and only if $NIP > 1$, where

$$NIP = \frac{\left(\frac{\partial D_i}{\partial \gamma_i} + \frac{\partial D_j}{\partial \gamma_i}\right)(1+\lambda)}{\frac{\partial D_i}{\partial \gamma_i} + \lambda \frac{\partial D_j}{\partial \gamma_i}}.$$

It is straightforward to see that this condition holds if and only if the spillover-adjusted innovation diversion ratio is positive, i.e., $\lambda < -\frac{\partial D_j/\partial \gamma_i}{\partial D_i/\partial \gamma_i}$.

7 Efficiency gains in R&D

Suppose that the merger leads to a reduction in the cost of R&D investments.[36] More specifically, assume that the postmerger cost of R&D is given by $C(\gamma)/(1+\mu)$, where $\mu \geq 0$ is a measure of the size of efficiency gains in R&D. To simplify the exposition, we abstract from any efficiency gains in production and set σ to zero. The only first-order condition that is affected by efficiency gains in R&D is associated with the merged entity's innovation level, i.e., equation (4), which becomes

$$(1+\mu)(p-c)\left[\frac{\partial D_i}{\partial \gamma_i}(p,p,\gamma,\gamma) + \frac{\partial D_j}{\partial \gamma_i}(p,p,\gamma,\gamma)\right] = C'(\gamma).$$

The independent firms' equilibrium price and the merged entity's price for a given (symmetric) innovation level are still given by $\tilde{p}^*(\gamma)$ and $\tilde{p}^M(\gamma)$, respectively. Therefore, the result in Proposition 1 can be extended to the case of efficiency gains in R&D of size μ as follows: the impact of the merger on innovation has the same sign as $(1+\mu)\, l^M(\gamma^*) - h^*(\gamma^*)$.

We can decompose the impact of the merger on innovation into the same four effects that are at play in the baseline model:

$$(1+\mu)\, l^M(\gamma^*) - h^*(\gamma^*) = H_M + H_{D\mu} + H_{I\mu} + H_R.$$

[36] Davidson and Ferrett (2007) emphasize the importance of R&D synergies in shaping the profitability of a merger. In contrast, we focus on how they affect innovation efforts.

The terms H_M and H_R, which capture the margin expansion effect and the return to innovation effect, respectively, remain the same as in the baseline model, while the two other terms must be adjusted as follows:

$$H_{D\mu} \equiv \hat{l}^M(\gamma^*)\left(\mu + \frac{\frac{\partial D_j^M(\gamma^*)}{\partial p_i}}{-\frac{\partial D_i^M(\gamma^*)}{\partial p_i}}\right) > 0,$$

$$H_{I\mu} \equiv (1+\mu) H_I = (1+\mu)\hat{l}^M(\gamma^*)\left(\frac{\frac{\partial D_j^M(\gamma^*)}{\partial \gamma_i}}{-\frac{\partial D_i^M(\gamma^*)}{\partial \gamma_i}}\right) < 0.$$

Consider now a P-neutral merger. In this case, the overall impact of the merger on innovation is given by the sum of the adjusted demand expansion and innovation diversion effects, $H_{D\mu} + H_{I\mu}$. It is straightforward to show that this sum has the same sign as the difference between a synergy-adjusted price diversion ratio and the innovation diversion ratio. Specifically, we have the following result:

Proposition 4 *Assume that the merged entity's R&D cost function is given by $C(\gamma)/(1+\mu)$, where μ measures efficiency gains in R&D. The effect of a P-neutral merger on the incentives to innovate has the same sign as:*

$$\underbrace{\frac{\frac{\frac{\partial D_j^M(\gamma^*)}{\partial p_i}}{-\frac{\partial D_i^M(\gamma^*)}{\partial p_i}} + \mu}{1+\mu}}_{\textit{synergy-adjusted price diversion ratio}} - \underbrace{\frac{\frac{\partial D_j^M(\gamma^*)}{\partial \gamma_i}}{-\frac{\partial D_i^M(\gamma^*)}{\partial \gamma_i}}}_{\textit{innovation diversion ratio}}.$$

This shows that the comparison of the price diversion ratio and the innovation diversion ratio remains the only determinant of the impact of a P-neutral merger on innovation in the presence of efficiency gains in R&D as long as the price diversion ratio is adjusted to account for these efficiency gains.

8 Oligopoly

In this section, we extend our analysis to a merger between two firms in an oligopoly. For the sake of conciseness, we assume that there are three firms, indexed by $i \in \{1, 2, 3\}$.[37]

[37] With more than one outsider, the analysis can be extended by aggregating outsiders' reaction into a joint reaction to the merged entity's strategy (see Deneckere and Davidson, 1985).

Each firm chooses a price p_i and a level of innovation γ_i, and we assume again that these choices are simultaneous (hence, a firm does not observe the other firms' innovation level before choosing its price). Firms 1 and 2 are the merging firms, and firm 3 is the outsider. We denote by $\phi_3 = (p_3, \gamma_3)$ the strategy of the outsider and by $D_i\left(p_i, p_j, \gamma_i, \gamma_j, \phi_3\right)$ the demand for $i, j \in \{1, 2\}, i \neq j$. Building on our baseline model, we assume that the merging firms have symmetric demand and the same production and innovation cost functions and that Assumption 1 holds for any given strategy ϕ_3 of the third firm. The outsider may, however, have a different marginal cost c_3, investment cost C_3 or demand D_3. The best response of firm 3 to a symmetric strategy (p, γ) of firms 1 and 2 is denoted $R_3(p, \gamma) : \mathbb{R}_+^2 \to \mathbb{R}_+^2$.

In the benchmark scenario in which firms act independently, the symmetric first-order conditions for prices and innovation levels of firms 1 and 2, given the outsider's strategy ϕ_3, can now be written as

$$(p - c)\frac{\partial D_i}{\partial p_i}(p, p, \gamma, \gamma, \phi_3) + D_i(p, p, \gamma, \gamma, \phi_3) = 0 \tag{8}$$

for the price and

$$(p - c)\frac{\partial D_i}{\partial \gamma_i}(p, p, \gamma, \gamma, \phi_3) = C'(\gamma) \tag{9}$$

for the innovation level. We extend Assumption 2 to the oligopoly setting as follows:

Assumption 2': The oligopoly price-innovation game has a unique equilibrium, in which firms 1 and 2 play symmetric strategies $(p^*, p^*, \gamma^*, \gamma^*)$ satisfying first-order conditions (8) and (9) and firm 3 plays strategy $\phi_3^* = R_3(p^*, \gamma^*)$.

Holding constant the strategy ϕ_3^*, Section 2 characterizes the behavior of firms 1 and 2. In particular, equation (8) defines the equilibrium price of firms 1 and 2 as a function $\tilde{p}^*(\gamma, \phi_3)$ of their innovation level γ and firm 3's strategy. As in the baseline model, we can define the independent firm's marginal gain from innovation $h(p, \gamma, \phi_3)$ and conclude that the equilibrium innovation level of firms 1 and 2 satisfies

$$h(\tilde{p}^*(\gamma^*, \phi_3^*), \gamma^*, \phi_3^*) = C'(\gamma^*).$$

Considering the situation in which firms 1 and 2 merge, we can similarly define the merged entity's marginal gain from innovation $l(p, \gamma, \phi_3)$. If the postmerger equilibrium strategy of firm 3 is denoted $\phi_3^M = R_3\left(p^M, \gamma^M\right)$, the symmetric equilibrium innovation level of each of the merging firms after the merger satisfies

$$l\left(p^M,\gamma^M,\phi_3^M\right)=C'\left(\gamma^M\right). \tag{10}$$

For the analysis of the postmerger equilibrium, it is convenient to decompose the equilibrium conditions into two sets of conditions: equation (10) for the merging firms' innovation level, on the one hand, and the other equilibrium conditions (for prices and the third firm's innovation), on the other hand. For this purpose, we define the *postmerger accessory game* for any given γ as the game where the innovation level of the merged firm is fixed at $\gamma_1=\gamma_2=\gamma$ so that the merged entity chooses only the prices $p_i,\ i=1,2$ while the third firm chooses both its price and innovation level. For the oligopolistic setting that we consider in this section, we cannot rely on the global optimality of the choices of the merged entity to compare the postmerger and premerger situations because the outsider's strategy changes. We therefore replace Assumption 3 with the following one:

Assumption 3': (i) The postmerger accessory game has an equilibrium $\left(\hat{p}^M(\gamma),R_3\left(\hat{p}^M(\gamma),\gamma\right)\right)$, which is unique and continuous in γ. (ii) The postmerger game equilibrium $\left(p^M,\gamma^M,\phi_3^M\right)$ is symmetric in products 1 and 2 and uniquely characterized by equilibrium conditions: $p^M=\hat{p}^M\left(\gamma^M\right)$, $\phi_3^M=R_3\left(p^M,\gamma^M\right)$ and $l\left(p^M,\gamma^M,\phi_3^M\right)=C'\left(\gamma^M\right)$.

In this environment, we say that a merger is P-neutral if efficiency gains in production are such that if we hold the merged entity's innovation level fixed at the premerger equilibrium level γ^*, the merger would not affect the merged entity's price. In other words, a merger is P-neutral if

$$\hat{p}^M(\gamma^*)=p^*.$$

Note that if a merger is P-neutral, then at a constant innovation level γ^* of the merged firms, the merger would not affect the price nor the innovation level of the outsider. Indeed, firm 3 would still choose $\phi_3^*=R_3(p^*,\gamma^*)$ in this case. It follows that as in the baseline model, a P-neutral merger affects equilibrium prices (and quantities) if and only if it affects the equilibrium innovation level of the merged entity.

The next proposition shows that our main result about the overall impact of a P-neutral merger on the merging firms' incentives to innovate extends to an oligopoly environment.

Proposition 5 *(i) The impact of a P-neutral merger on the merging firms' innovation level, $\gamma^M-\gamma^*$, has the same sign as $l(p^*,\gamma^*,\phi_3^*)-h(p^*,\gamma^*,\phi_3^*)$.*
(ii) A P-neutral merger reduces (raises) the merging firms' innovation level if the innova-

tion diversion ratio is greater (lower) than the price diversion ratio, where both ratios are evaluated at $(p^*, p^*, \gamma^*, \gamma^*, \phi_3^*)$.

Proof. See Appendix. ■

Therefore, the mere comparison of the diversion ratios allows us to sign the effect of a P-neutral merger on the merging firms' innovation level, even in the presence of an outsider.[38] As the comparison is made based on a constant strategy for the outsider, all our conclusions relating to P-neutral mergers extend to the oligopolistic environment considered here.

An argument similar to that in the proof of Proposition 5 shows that a P-increasing merger – such that $\hat{p}^M(\gamma^*) > p^*$ – has a negative effect on the merging firms' innovation level if

$$l\left(\hat{p}^M(\gamma^*), \gamma^*, R_3\left(\hat{p}^M(\gamma^*), \gamma^*\right)\right) < h\left(p^*, \gamma^*, \phi_3^*\right).$$

A difficulty in this case is that the comparison between the LHS and the RHS in the inequality above depends on the change of the outsider's strategy from ϕ_3^* to $R_3\left(\hat{p}^M(\gamma^*), \gamma^*\right)$.

9 Other extensions

In this section, we extend our analysis to settings with observable investments in R&D and asymmetric demand and cost functions. We also show that our approach can be adapted to assess the impact of a merger on investment in cost-reducing innovation.

9.1 Observable investments

In the baseline model, we assume that the price and innovation decisions are taken simultaneously by the firms or, equivalently, that a firm cannot observe its rival's investment before setting its price. We now assume that a firm's investment in R&D is observed by its rival before prices are set. At given investment levels, the profit-maximizing price for an independent firm i, $\tilde{p}_i^*\left(\gamma_i, \gamma_j\right)$, is the solution to the following first-order condition:

$$(p_i - c)\frac{\partial D_i}{\partial p_i}\left(p_i, p_j, \gamma_i, \gamma_j\right) + D_i\left(p_i, p_j, \gamma_i, \gamma_j\right) = 0.$$

With observable investments, the first-order condition with respect to γ_i becomes

$$(p_i - c)\left(\frac{\partial D_i}{\partial \gamma_i} + \frac{\partial \tilde{p}_j^*}{\partial \gamma_i}\frac{\partial D_i}{\partial p_j}\right) = C'(\gamma_i), \tag{11}$$

[38] This extends to the case of several outsiders.

where $\partial D_i/\partial p_j$ is evaluated at $\left(\tilde{p}_i^*\left(\gamma_i,\gamma_j\right),\tilde{p}_j^*\left(\gamma_i,\gamma_j\right),\gamma_i,\gamma_j\right)$ and $\partial\tilde{p}_j^*/\partial\gamma_i$ is evaluated at $\left(\gamma_i,\gamma_j\right)$. Therefore, firm i takes into account not only the direct effect of its investment on its profit but also the strategic effect that operates through firm j's pricing reaction. The first-order conditions associated with the merged entity's maximization program remain the same as before. Therefore, the decomposition in our baseline setting remains valid as long as we replace the partial derivative $\partial D_i/\partial\gamma_i$ with $\partial D_i/\partial\gamma_i+\partial\tilde{p}_j^*/\partial\gamma_i\times\partial D_i/\partial p_j$ in the independent firm's marginal gain from innovation. This leads us to the following decomposition:

$$l^M\left(\gamma^*\right)-h^*\left(\gamma^*\right)=H_M+H_D+H_I+H_R+H_O,$$

where

$$H_O=-D_i\left(\gamma\right)g^*\left(p,\gamma\right)\frac{\partial\tilde{p}_j^*}{\partial\gamma_i}.$$

The sign of the additional term H_O is the opposite of the sign of the strategic effect on the rival's price, $\partial\tilde{p}_j^*/\partial\gamma_i$. It seems natural to assume that when firm i invests more in innovation, firm j reacts by setting a lower price. In the Appendix, we provide sufficient conditions on firms' demand functions, which ensure that $\partial\tilde{p}_j^*/\partial\gamma_i\leq 0$. In this case, the last term of the decomposition, H_O, is positive. When investment is observable, unlike in the baseline model, a merger allows firms to internalize the negative strategic effect of their investments on profits, which tends to stimulate innovation.

9.2 Asymmetric demand and cost functions

We now extend our analysis to a setting in which the demand functions D_i, the marginal costs c_i and the innovation cost functions C_i are potentially asymmetric. We maintain the assumptions of the baseline model on the cost of innovation C_i, $i=1,2$ (denoting as $\bar{\gamma}_i$ the upper bound of firm i's innovation level) and Assumption 1 on demand.

Consider first the scenario in which the two firms are independent. Assume that the pricing game derived from the price-innovation game by fixing the innovation levels of firms 1 and 2 to γ_1 and γ_2, respectively, has a unique equilibrium. The corresponding equilibrium price pair $\left(\tilde{p}_1^*\left(\gamma_1,\gamma_2\right),\tilde{p}_2^*\left(\gamma_1,\gamma_2\right)\right)$ is the solution to the following system of first-order conditions:

$$\begin{cases}\left(p_1-c_1\right)\frac{\partial D_1}{\partial p_1}+D_1=0\\ \left(p_2-c_2\right)\frac{\partial D_2}{\partial p_2}+D_2=0.\end{cases}\tag{12}$$

Likewise, the system of first-order conditions for the equilibrium pair of innovation levels of firms 1 and 2 in the price-innovation game is:

$$\begin{cases} (p_1 - c_1) \frac{\partial D_1}{\partial \gamma_1} = C_1' (\gamma_1) \\ (p_2 - c_2) \frac{\partial D_2}{\partial \gamma_2} = C_2' (\gamma_2) . \end{cases} \tag{13}$$

Consider now the postmerger situation. As in the baseline model, we assume away any merger-induced efficiency gains in R&D but allow for potential merger-induced efficiency gains in production. More specifically, we suppose that the postmerger production costs of the two merged entities are given by $c_1 - \sigma_1$ and $c_2 - \sigma_2$, respectively, where $\sigma_1, \sigma_2 \geq 0$.

For any given innovation levels γ_1 and γ_2, the merged entity's optimal price pair $\left(\tilde{p}_1^M (\gamma_1, \gamma_2), \tilde{p}_2^M (\gamma_1, \gamma_2)\right)$, assumed to be positive, is defined by the following system of first-order conditions:

$$\begin{cases} (p_1 - c_1 + \sigma_1) \frac{\partial D_1}{\partial p_1} + (p_2 - c_2 + \sigma_2) \frac{\partial D_2}{\partial p_1} + D_1 = 0 \\ (p_1 - c_1 + \sigma_1) \frac{\partial D_1}{\partial p_2} + (p_2 - c_2 + \sigma_2) \frac{\partial D_2}{\partial p_1} + D_2 = 0. \end{cases}$$

Combining these two equations leads to

$$\begin{cases} p_1 - c_1 + \sigma_1 = \frac{D_2 \frac{\partial D_2}{\partial p_1} - D_1 \frac{\partial D_2}{\partial p_2}}{\frac{\partial D_1}{\partial p_1} \frac{\partial D_2}{\partial p_2} - \frac{\partial D_2}{\partial p_1} \frac{\partial D_1}{\partial p_2}} \\ p_2 - c_2 + \sigma_2 = \frac{D_1 \frac{\partial D_1}{\partial p_2} - D_2 \frac{\partial D_1}{\partial p_1}}{\frac{\partial D_1}{\partial p_1} \frac{\partial D_2}{\partial p_2} - \frac{\partial D_2}{\partial p_1} \frac{\partial D_1}{\partial p_2}} . \end{cases}$$

We can now state the counterparts to Assumptions 2-3 for the current setting.

Assumption 2": The duopoly price-innovation game has an equilibrium $(p_1^*, p_2^*, \gamma_1^*, \gamma_2^*)$ satisfying first-order conditions (12) and (13).

Assumption 3": The profit function $\Pi^M (\gamma_1, \gamma_2)$ is $\mathcal{C}^1$ and strictly quasiconcave in (γ_1, γ_2), where $\Pi^M (\gamma_1, \gamma_2)$ is the merged entity's profit for levels of investments γ_1 and γ_2:

$$\begin{aligned} \Pi^M (\gamma_1, \gamma_2) \equiv & \max_{p_1, p_2} \{ (p_1 - c_1 + \sigma_1) D_1 (p_1, p_2, \gamma_1, \gamma_2) + \\ & (p_2 - c_2 + \sigma_2) D_2 (p_2, p_1, \gamma_2, \gamma_1) - C (\gamma_1) - C (\gamma_2) \}. \end{aligned}$$

The *independent firm's marginal gain from innovation* is now given by

$$h_i^* (\gamma_1, \gamma_2) \equiv -D_i \frac{\frac{\partial D_i}{\partial \gamma_i}}{\frac{\partial D_i}{\partial p_i}}$$

for firm $i = 1, 2$, where all functions are evaluated at $(\tilde{p}_1^* (\gamma_1, \gamma_2), \tilde{p}_2^* (\gamma_1, \gamma_2), \gamma_1, \gamma_2)$, while the *merged entity's marginal gain from innovation* in product $i = 1, 2$ is

$$l_i^M (\gamma_1, \gamma_2) \equiv \frac{\left(D_j \frac{\partial D_j}{\partial p_i} - D_i \frac{\partial D_j}{\partial p_j} \right) \frac{\partial D_i}{\partial \gamma_i} + \left(D_i \frac{\partial D_i}{\partial p_j} - D_j \frac{\partial D_i}{\partial p_i} \right) \frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial p_i} \frac{\partial D_j}{\partial p_j} - \frac{\partial D_j}{\partial p_i} \frac{\partial D_i}{\partial p_j}},$$

where all functions are evaluated at $\left(\tilde{p}_1^M (\gamma_1, \gamma_2), \tilde{p}_2^M (\gamma_1, \gamma_2), \gamma_1, \gamma_2\right)$. Under Assumption 3", the merged entity's innovation levels are the unique solution of

$$l_i^M \left(\gamma_1^M, \gamma_2^M\right) = C_i' \left(\gamma_i^M\right), \quad i = 1, 2.$$

We say that the merged entity's innovation efforts are *strategic complements* if $l_1^M (\gamma_1, \gamma_2)$ is increasing in γ_2 and $l_2^M (\gamma_1, \gamma_2)$ is increasing in γ_1.

The next proposition shows that when the merged entity's innovation efforts are strategic complements, the comparison of an independent firm's marginal gain from innovation and the merged entity's marginal gain from innovation (as defined above) still determines the impact of the merger on the merging firms' incentives to innovate.

Proposition 6 *Assume that the merged entity's innovation efforts are strategic complements (i.e., $l_1^M (\gamma_1, \gamma_2)$ is increasing in γ_2 and $l_2^M (\gamma_1, \gamma_2)$ is increasing in γ_1). Then, a merger reduces innovation in both products if $l_i^M (\gamma_1^*, \gamma_2^*) < h_i^* (\gamma_1^*, \gamma_2^*)$ for $i = 1, 2$, and a merger boosts innovation in both products if $l_i^M (\gamma_1^*, \gamma_2^*) > h_i^* (\gamma_1^*, \gamma_2^*)$ for $i = 1, 2$.*

Proof. See Appendix. ■

As an illustration, we provide in the Appendix a sufficient condition under which the merged entity's innovation efforts are strategic complements in the augmented Singh-Vives model.

Following the same logic as that for symmetric mergers, we can define a P-neutral merger as a merger with synergies σ_1 and σ_2 such that at constant innovation levels γ_1^* and γ_2^*, the merger does not affect prices. In other words, a merger is P-neutral if

$$\left(\tilde{p}_1^M (\gamma_1^*, \gamma_2^*), \tilde{p}_2^M (\gamma_1^*, \gamma_2^*)\right) = (p_1^*, p_2^*).$$

We define price and innovation diversion ratios for each product $i = 1, 2$ as in the baseline model. Our main result about the impact of P-neutral mergers on innovation extends to the asymmetric setting considered here as follows:

Corollary 7 *Assume that the merged entity's innovation efforts are strategic complements. A P-neutral merger reduces (raises) innovation in both products if the price diversion ratio is lower (higher) than the innovation diversion ratio for both products, where both ratios are evaluated at* $(p^*, p^*, \gamma^*, \gamma^*)$.

Proof. See Appendix. ■

Thus, the comparison of the innovation diversion ratios with the corresponding price diversion ratios still determines the impact of a P-neutral merger on innovation (in both products) as long as the outcome of the comparison is the same for both products.

9.3 Cost-reducing innovation

In this extension, we depart from the baseline framework by applying our approach to *cost-reducing* innovation. Let us denote by $D_i(p_i, p_j)$ the demand addressed to firm i, and assume that firm i can reduce its marginal cost from an initial level c to $c - \gamma_i$ by investing $C(\gamma_i)$. As in the baseline model with demand-enhancing innovation, we assume that the duopoly price-innovation game has a unique, symmetric equilibrium (and that the equilibrium innovation level γ^* is positive). Also, denoting by $\sigma \geq 0$ the efficiency gains in production induced by the merger, suppose that the profit function of the merged entity with optimized prices, i.e.,

$$\Pi^M(\gamma_1, \gamma_2) = \max_{p_1, p_2} (p_1 - c + \sigma + \gamma_1) D_1(p_1, p_2) + (p_2 - c + \sigma + \gamma_2) D_2(p_1, p_2) - C(\gamma_1) - C(\gamma_2),$$

is strictly quasiconcave in (γ_1, γ_2).

The key difference from the baseline model is that the first-order condition that gives the equilibrium innovation level as a function of the symmetric equilibrium price is the same whether firms act independently or as a merged entity. This condition is given by

$$D_i(p, p) = C'(\gamma). \tag{14}$$

Therefore, the marginal gain from innovation is $D_i^*(\gamma) = D_i(\tilde{p}^*(\gamma), \tilde{p}^*(\gamma))$ if firms act independently and $D_i^M(\gamma) = D_i\left(\tilde{p}^M(\gamma), \tilde{p}^M(\gamma)\right)$ if they merge. The strict quasiconcavity of $\Pi^M(.,.)$ then ensures that a necessary and sufficient condition for the merger to reduce innovation is that $D_i^M(\gamma^*) - D_i^*(\gamma^*)$ is negative. This term captures the *margin expansion effect* in the present setting.[39] It is negative if and only if $\tilde{p}^M(\gamma^*) > \tilde{p}^*(\gamma^*)$, that is, if

[39] The only difference between the margin expansion effect in the cost-reducing innovation setting and our baseline demand-enhancing innovation setting is that firms increase their margins by decreasing their

and only if the merger is P-increasing. Thus, unlike in the case of demand-enhancing innovation, the margin expansion effect is the only effect driving the impact of a merger on cost-reducing innovation. This finding confirms Motta and Tarantino's clear-cut result that a merger to monopoly reduces the incentives to conduct cost-reducing investments in the absence of spillovers and efficiency gains and extends it to any P-increasing merger. Moreover, it implies that a P-neutral merger always has a neutral effect on merging firms' incentives to invest in cost-reducing innovation.

10 Conclusion

In this paper, we provide a novel decomposition of the impact of a merger on merging firms' incentives to innovate and use it to provide sufficient conditions under which this impact is negative (or positive) in the absence of spillovers and efficiency gains in R&D. It turns out that the impact of a horizontal merger on innovation crucially depends on the comparison between the price diversion ratio and the innovation diversion ratio. This is particularly the case for P-neutral mergers, that is, mergers that would not affect prices if the merging firms' innovation levels were fixed at their premerger equilibrium levels. More specifically, we find that P-neutral mergers have a negative (positive) impact on the merging firms' incentives to invest in demand-enhancing innovation if the innovation diversion ratio is greater (less) than the price diversion ratio. Moreover, our analysis of several standard models suggests that the impact of a P-increasing merger on merging firms' incentives to innovate is likely to be negative if the innovation diversion ratio is greater than the price diversion ratio but can be either positive or negative if the innovation diversion ratio is less than the price diversion ratio.

One advantage of our approach is that it can be extended to account for spillovers and efficiency gains and can also be adapted to investigate the effect of horizontal mergers on cost-reducing innovation. In particular, the diversion ratios can be easily adjusted to incorporate spillovers and efficiency gains in R&D. Note also that our model can be extended in a very simple way to study the impact of a merger between two firms producing complementary products on their incentives to invest in demand-enhancing innovation.[40] Our main decomposition still holds in this case, but the signs of some of the effects in the decomposition change. First, the margin expansion effect becomes positive even in the absence of any efficiency gains in production because the merger leads to lower prices. Sec-

marginal costs instead of increasing their prices.

[40] We simply need to assume that $\partial D_i/\partial p_j < 0$ for $i \neq j$ instead of $\partial D_i/\partial p_j > 0$.

ond, the demand expansion effect becomes negative because the price externality becomes positive. Third, the sign of the innovation diversion effect can be either positive or negative depending on whether the innovation of one firm has a positive or negative effect on the demand of the other firm.[41]

Another implication of our analysis is that empirically assessing how firms tend to monetize their investments in product innovations in a given industry can shed light on the expected effect of a merger on innovation in this industry. More specifically, everything else held equal, a merger is more likely to have a negative (positive) effect on the merging firms' incentives to innovate in industries in which firms derive profits from their innovations primarily by increasing their margins (their sales).

Finally, our results suggest that mergers that increase merging firms' incentives to invest in demand-enhancing innovation in the absence of spillovers and efficiency gains in R&D may also lead to an increase in prices, which shows that there may be a trade-off between the impact of a merger on innovation and its effect on prices. We leave this trade-off and its implications for consumer surplus for future research.

Appendix

Proof of Proposition 1

From the positivity of γ^*, and from (1) and (2), it follows that $h^*(\gamma^*) = C'(\gamma^*)$. Moreover, from (3) and (4), it follows that $\frac{d\Pi^M}{d\gamma}(\gamma,\gamma) = 2\left[l^M(\gamma) - C'(\gamma)\right]$. Assumption 2 implies that $\gamma^M > \gamma^*$ if and only if $\frac{d\Pi^M}{d\gamma}(\gamma^*,\gamma^*) > 0$, which yields the result.

Proof of Proposition 2

We have

$$l^M(\gamma^*) - h^*(\gamma^*) = H_M + \left[\left(-\frac{\frac{\partial D_i}{\partial \gamma_i}}{\frac{\partial D_i}{\partial p_i} + \frac{\partial D_j}{\partial p_i}}\right) - r^*(\gamma^*)\right] D_i^M(\gamma^*) + H_I,$$

[41] Note that both scenarios are plausible. For instance, an innovation that would allow customers to use the same product for a longer time would have a positive effect on demand for the complementary product. However, an innovation that would add new features to a product that make the value of the complementary product lower would have a negative effect on demand for the latter.

and

$$
\begin{aligned}
\left[\left(-\frac{\frac{\partial D_i}{\partial \gamma_i}}{\frac{\partial D_i}{\partial p_i}+\frac{\partial D_j}{\partial p_i}}\right)-r^*\left(\gamma^*\right)\right] D_i^M\left(\gamma^*\right) &= \left[\left(-\frac{\frac{\partial D_i^M(\gamma^*)}{\partial \gamma_i}}{\frac{\partial D_i^M(\gamma^*)}{\partial p_i}+\frac{\partial D_j^M(\gamma^*)}{\partial p_i}}\right)-\left(-\frac{\frac{\partial D_i^M(\gamma^*)}{\partial \gamma_i}}{\frac{\partial D_i^M(\gamma^*)}{\partial p_i}}\right)\right] D_i^M\left(\gamma\right) \\
&\quad +\left[r^M\left(\gamma^*\right)-r^*\left(\gamma^*\right)\right] D_i^M\left(\gamma^*\right) \\
&= \left[1-\frac{1}{\frac{\partial D_i^M(\gamma^*)}{\partial p_i}}\left(\frac{\partial D_i^M\left(\gamma^*\right)}{\partial p_i}+\frac{\partial D_j^M\left(\gamma^*\right)}{\partial p_i}\right)\right] \hat{l}^M\left(\gamma^*\right)+H_R \\
&= -\frac{\frac{\partial D_j^M(\gamma^*)}{\partial p_i}}{\frac{\partial D_i^M(\gamma^*)}{\partial p_i}} \hat{l}^M\left(\gamma^*\right)+H_R=H_D+H_R,
\end{aligned}
$$

which completes the proof.

Effect of a P-neutral merger on innovation

To determine the impact on innovation of a P-neutral merger, we compare the price diversion ratio and the innovation ratio for different commonly used demand functions.

Price-innovation index. Consider first models based on a price-innovation index $\eta(p_i, \gamma_i)$, with $\partial\eta/\partial p_i > 0$ and $\partial\eta/\partial\gamma_i < 0$, for which demand is given by

$$
D_i\left(p_i, p_j, \gamma_i, \gamma_j\right)=Q\left(\eta(p_i, \gamma_i), \eta(p_i, \gamma_i)\right). \tag{PII}
$$

We denote by Q_1 and Q_2 the derivatives of Q with respect to its first and second argument, respectively, and assume that $Q_1 < 0$ and $Q_2 > 0$. The innovation diversion ratio and the price diversion ratio are both equal to $-Q_2/Q_1$ at symmetric prices and innovation levels.

Quality-adjusted prices. Now, let us consider demand functions for which there exists a function $Q(.,.)$ such that

$$
D_i\left(p_i, p_j, \gamma_i, \gamma_j\right)=\frac{1}{\gamma_i} Q\left(\frac{p_i}{\gamma_i}, \frac{p_j}{\gamma_j}\right). \tag{QAP}
$$

We assume that $Q_1 < Q_1 + Q_2 < 0$ and that $Q + \frac{p}{\gamma}Q_1 < 0$, which ensures that innovation raises own demand. In this setting, p_i/γ_i can be interpreted as the quality-adjusted price of firm i. The price diversion ratio and the innovation diversion ratio at symmetric prices and innovation levels are such that

$$\frac{\frac{\partial D_j}{\partial p_i}}{-\frac{\partial D_i}{\partial p_i}} = -\frac{Q_2}{Q_1} < -\frac{\frac{p}{\gamma}Q_2}{Q + \frac{p}{\gamma}Q_1} = \frac{-\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial \gamma_i}} .$$

Constant expenditures. Consider now the class of models with constant expenditures (see, e.g., Vives, 1999). In these models, there exist $K \geq 0$ and a function $\eta(.,.)$, such that firm i's demand can be written as

$$D_i\left(p_i, p_j, \gamma_i, \gamma_j\right) = \frac{\eta\left(p_i, \gamma_i\right)}{p_i \eta\left(p_i, \gamma_i\right) + p_j \eta\left(p_j, \gamma_j\right) + K}, \tag{CEX}$$

with $\partial\eta/\partial p < 0 < \partial\eta/\partial\gamma$ and $p\partial\eta/\partial p + \eta < 0$, which ensures that goods are substitutes. In this demand setting, K represents spending on other goods, so that the total spending $p_1\eta\left(p_1, \gamma_1\right) + p_2\eta\left(p_2, \gamma_2\right) + K$ is constant. In this class of models, the price and innovation diversion ratios at symmetric prices and innovation levels are such that

$$\frac{\frac{\partial D_j}{\partial p_i}}{-\frac{\partial D_i}{\partial p_i}} = \frac{p\eta\frac{\partial \eta}{\partial p} + \eta^2}{\left(p\eta + K\right)\frac{\partial \eta}{\partial p} - \eta^2} < \frac{p\eta}{p\eta + K} = \frac{-\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial \gamma_i}},$$

where the inequality follows from the fact that $\partial\eta/\partial p < 0$.

Quality-augmented linear demand. Now, consider the following linear demand system, considered by Sutton (1997, 1998) and Symeonidis (2000, 2003):

$$D_i\left(p_i, p_j, \gamma_i, \gamma_j\right) = \frac{\gamma_i\left[2\gamma_i\left(1 - p_i\right) - \rho\gamma_j\left(1 - p_j\right)\right]}{4 - \rho^2}, \tag{QAD}$$

where $\rho \in (0, 2)$ is an inverse measure of the degree of horizontal differentiation, and γ_i is the quality of product i. The price and innovation diversion ratios at symmetric prices and innovation levels are such that

$$\frac{\frac{\partial D_j}{\partial p_i}}{-\frac{\partial D_i}{\partial p_i}} = \frac{\rho}{2} > \frac{\rho}{4 - \rho} = \frac{-\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial \gamma_i}}.$$

Augmented Singh-Vives. Finally, we consider the linear demand model of Singh and Vives (1984) in a context where innovation has both a horizontal and a vertical dimension. Assume that the utility of the representative consumer is given by

$$U(q_1, q_2, m) = a_1 q_1 + a_2 q_2 - (q_1^2 + q_2^2)/2 - \rho q_1 q_2 + m,$$

where (q_1, q_2) is the vector of quantities, m is the numeraire good, and $\rho \in [0,1]$ represents the degree of substitutability between the product of firm 1 and the product of firm 2. Products are independent if $\rho = 0$ and perfect substitutes if $\rho = 1$.

We assume that innovation has two effects. First, it raises product quality, which we capture by assuming that $a_i = a(\gamma_i) = \alpha + \tau\gamma_i$. Second, it allows firms to increase the differentiation between their products as in Lin and Saggi (2002). Formally, the degree of substitutability is given by $\rho(\gamma_1, \gamma_2) = 1 - \delta(\gamma_1 + \gamma_2)$, and we suppose that R&D costs are sufficiently high so that $\delta(\gamma_1 + \gamma_2) < 1$ in equilibrium (i.e., $2\delta\gamma < 1$ in the symmetric equilibrium).

The demand for firm i in this model is then given by

$$D_i\left(p_i, p_j, \gamma_i, \gamma_j\right) = \frac{(\alpha + \tau\gamma_i) - \left(\alpha + \tau\gamma_j\right)\rho(\gamma_1, \gamma_2) - p_i + \rho(\gamma_1, \gamma_2) p_j}{1 - \rho(\gamma_1, \gamma_2)^2}. \tag{ASV}$$

To ensure that the innovation diversion effect H_I is negative, we assume that the parameters of the model are such that

$$\frac{\left(\alpha - \tilde{p}^M(\gamma^*)\right)\delta^2\gamma^*}{1 - \delta\gamma^*(3 - \delta\gamma^*)} < \tau.$$

Under this specification, the innovation diversion ratio is lower than the price diversion ratio as:

$$-\frac{\frac{\partial D_j^M}{\partial \gamma_i}}{\frac{\partial D_i^M}{\partial \gamma_i}} = \frac{\tau\rho(1+\rho) + (1-\rho)(a-p)\frac{\partial\rho}{\partial\gamma}}{\tau(1+\rho) - (1-\rho)(a-p)\frac{\partial\rho}{\partial\gamma}} < -\frac{\frac{\partial D_j^M}{\partial p_i}}{\frac{\partial D_i^M}{\partial p_i}} = \rho.$$

Proof of Corollary 2

In a model with an MNL demand, we have

$$r(p, p, \gamma, \gamma) = \frac{u_1(\gamma, y-p)}{u_2(\gamma, y-p)}$$

and, therefore,

$$h(p, \gamma) = \frac{u_1(\gamma, y-p)\exp u(\gamma, y-p)}{2u_2(\gamma, y-p)\exp u(\gamma, y-p) + u_2(\gamma, y-p)\exp u(0, y)}.$$

The derivative of $h(p,\gamma^*)$ with respect to p has the same sign as

$$[2(u_1u_{22}-u_{12}u_2)(\gamma^*,y-p)]\exp u(\gamma^*,y-p)+\left[(u_1u_{22}-u_{12}u_2-u_1u_2^2)(\gamma^*,y-p)\right]\exp u(0,y).$$

If $u_1u_{22}-u_{12}u_2<0$ or, equivalently, $-u_{12}/u_1+u_{22}/u_2<0$, then both the first term into brackets and the second term into brackets are negative and $h(p,\gamma^*)$ decreases with p. Therefore, the merger reduces incentives to innovate. If $-u_{12}u_2-u_1u_2^2+u_1u_{22}>0$ or, equivalently, $-u_{12}/u_1+u_{22}/u_2>u_2$, then both the first term into brackets and the second term into brackets are positive and $h(p,\gamma^*)$ increases with p. Therefore, the merger raises incentives to innovate.

Conditions for the merger to reduce or raise innovation incentives with a constant marginal utility of income

In the special case where utility is given by $u(\gamma,y-p)=v(\gamma)+f(\gamma)(y-p)$, the sufficient condition under which the merger reduces incentives to innovate is equivalent to

$$\frac{f'(\gamma^*)}{v'(\gamma^*)+f'(\gamma^*)(y-p)}<0,$$

and, therefore, it holds whenever $f(\gamma)$ is decreasing.

The sufficient condition under which the merger raises incentives to innovate is equivalent to

$$\frac{-f(\gamma^*)[v'(\gamma^*)+f'(\gamma^*)(y-p)]+f'(\gamma^*)}{v'(\gamma^*)+f'(\gamma^*)(y-p)}>0,$$

for all $p\in\left[p^*,\tilde{p}^M(\gamma^*)\right]$, which holds if and only if

$$-f(\gamma^*)[v'(\gamma^*)+f'(\gamma^*)(y-p)]+f'(\gamma^*)>0$$

for all $p\in\left[p^*,\tilde{p}^M(\gamma^*)\right]$. For this inequality to hold, it is necessary that $f'(\gamma^*)>0$. Using this, a sufficient condition for the inequality to hold is that $-f(\gamma^*)[v'(\gamma^*)+f'(\gamma^*)y]+f'(\gamma^*)<0$, which we can write as

$$\frac{f'(\gamma^*)}{f(\gamma^*)}>v'(\gamma^*)+yf'(\gamma^*).$$

Proof of Corollary 3

To evaluate the impact of the merger on innovation in this model, let us notice that the pre- and post-merger prices for $\gamma = \gamma^*$ satisfy

$$\begin{aligned}
\gamma^* \left(s^* + \frac{Q(s^*, s^*)}{Q_1(s^*, s^*)} \right) &= c, \\
\gamma^* \left(s^{M*} + \frac{Q\left(s^{M*}, s^{M*}\right)}{Q_1\left(s^{M*}, s^{M*}\right) + Q_2\left(s^{M*}, s^{M*}\right)} \right) &= c - \sigma,
\end{aligned}$$

where $s^* = \tilde{p}^*(\gamma^*)/\gamma^*$ and $s^{M*} = \tilde{p}^M(\gamma^*)/\gamma^*$. The pre- and post-merger marginal gains from innovation (evaluated at $\gamma = \gamma^*$) can then be written as

$$h^*(\gamma^*) = \frac{Q(s^*, s^*)}{\gamma^*} \left(s^* + \frac{Q(s^*, s^*)}{Q_1(s^*, s^*)} \right) = \frac{cQ(s^*, s^*)}{(\gamma^*)^2}$$

and

$$l^M(\gamma^*) = \frac{Q\left(s^M, s^M\right)}{\gamma^*} \left(s^{M*} + \frac{Q\left(s^{M*}, s^{M*}\right)}{Q_1\left(s^{M*}, s^{M*}\right) + Q_2\left(s^{M*}, s^{M*}\right)} \right) = \frac{(c - \sigma) Q\left(s^{M*}, s^{M*}\right)}{(\gamma^*)^2},$$

respectively. Thus, the innovation incentives (at $\gamma = \gamma^*$) are related to the volume of sales, and, more precisely, to total variable production costs. Therefore, under this specification, any P-increasing merger reduces incentives to innovate, as it reduces output.

Proof of Corollary 4

We have already shown that the innovation diversion ratio is larger than the price diversion ratio under this specification. Therefore, a merger reduces innovation if $h(p, \gamma)$ is decreasing in prices. We find that

$$\begin{aligned}
h(p, \gamma) &= \frac{1}{2p\eta + K} \frac{(p\eta + K)\eta\eta_2}{-(p\eta + K)\eta_1 + \eta^2} \\
&= \frac{1}{2p\eta + K} \frac{(p\eta + K)p\eta}{(p\eta + K)\left(-\frac{p\eta_1}{\eta}\right) + p\eta} \left(\frac{\eta_2}{\eta}\right).
\end{aligned}$$

Then, we have for any $\beta = -\frac{p\eta_1}{\eta}$:

$$\frac{d}{dx} \frac{1}{2x + K} \frac{(x + K)x}{(x + K)\beta + x} = \frac{K\left(K^2\beta + x^2(\beta - 1) + 2Kx\beta\right)}{(K + 2x)^2 (x + K\beta + x\beta)^2} > 0.$$

As $p\eta$ decreases in p when goods are substitutes, this implies that $h(p, \gamma)$ is decreasing in prices if $\frac{\eta_2}{\eta}$ is non-increasing in p and $-\frac{p\eta_1}{\eta}$ is non-decreasing in p.

For a CES demand, we have $\frac{\gamma}{\eta}\eta_2 = -\frac{p}{\eta}\eta_1 = \beta$, which yields the result.

Proof of Corollary 5

Direct computations show that

$$h^{*}(\gamma)=\frac{4-\rho}{2}\frac{\gamma(1-p^{*})^{2}}{2+\rho} \text{ and } l^{M}(\gamma)=\frac{1}{2}\frac{4\gamma\left(1-p^{M}\right)^{2}}{2+\rho}.$$

The merger raises incentives to innovate if $h^{*}(\gamma^{*})<h^{M}(\gamma^{*})$, which yields the result. It does not hold for $\sigma=0$ because in that case

$$h^{*}(\gamma)=\frac{2}{4-\rho}\frac{\gamma(1-c)^{2}}{2+\rho}>l^{M}(\gamma)=\frac{1}{2}\frac{\gamma(1-c)^{2}}{2+\rho}.$$

However, the condition holds if p^{M} is sufficiently close to p^{*}. Hence, the condition holds if and only if the increase in prices induced by the merger is not too large. Using the fact that

$$h^{*}(\gamma)=\frac{2}{4-\rho}\frac{\gamma(1-c)^{2}}{2+\rho} \quad \text{and} \quad h^{M}(\gamma)=\frac{1}{2}\frac{\gamma(1-c+\sigma)^{2}}{2+\rho},$$

we can see that $h^{*}(\gamma^{*})<l^{M}(\gamma^{*})$ holds if the level of efficiency gains is sufficiently large,

$$\sigma>(1-c)\left[\sqrt{\frac{1}{1-\rho/4}}-1\right].$$

Proof of Corollary 6

On the one hand, the demand expansion effect dominates the innovation diversion effect, since

$$H_{D}+H_{I}=\frac{\delta\left[a(\gamma^{*})-\widetilde{p}^{M}(\gamma^{*})\right]}{2(1-\delta\gamma^{*})}>0.$$

On the other hand, for a P-increasing merger, we have

$$H_{M}+H_{R}=-\frac{\widetilde{p}^{M}(\gamma^{*})-\widetilde{p}^{*}(\gamma^{*})}{2(1-\delta\gamma^{*})^{2}}\left[\tau(1-\delta\gamma^{*})+2\delta^{2}\gamma^{*}\left(a(\gamma^{*})-\frac{\widetilde{p}^{M}(\gamma^{*})+\widetilde{p}^{*}(\gamma^{*})}{2}\right)\right]<0.$$

Therefore, the overall effect of the merger on innovation is *a priori* ambiguous. We find that

$$H_{D}+H_{I}+H_{M}+H_{R}=\frac{\delta}{2(1-\delta\gamma^{*})^{2}}\left[\left(a(\gamma^{*})-\widetilde{p}^{M}(\gamma^{*})\right)^{2}-\delta\gamma^{*}\left(a(\gamma^{*})-\widetilde{p}^{*}(\gamma^{*})\right)^{2}\right]-\frac{\tau\left[\widetilde{p}^{M}(\gamma^{*})-\widetilde{p}^{*}(\gamma^{*})\right]}{2(1-\delta\gamma^{*})},$$

which is positive if and only if

$$\frac{\left(a\left(\gamma^*\right)-\widetilde{p}^M\left(\gamma^*\right)\right)^2}{\left(1-\delta\gamma^*\right)}-\delta\gamma^*\frac{\left(a\left(\gamma^*\right)-\widetilde{p}^*\left(\gamma^*\right)\right)^2}{\left(1-\delta\gamma^*\right)}>\frac{\tau}{\delta}\left[\widetilde{p}^M\left(\gamma^*\right)-\widetilde{p}^*\left(\gamma^*\right)\right], \tag{15}$$

where $\delta\gamma^* < 1/2$ from our assumptions. The second part of the proposition follows directly by computing $\widetilde{p}^M\left(\gamma^*\right)$ and $\widetilde{p}^*\left(\gamma^*\right)$ in the special case $\sigma = 0$.

Extension of Proposition 1 to an environment with spillovers

In the presence of spillovers, the counterparts to the first-order conditions (1) and (2) for the duopoly game are given by

$$(p-c)\frac{\partial D_i}{\partial p_i}\left(p,p,\hat{\gamma},\hat{\gamma}\right)+D_i\left(p,p,\hat{\gamma},\hat{\gamma}\right)=0$$

and

$$(p-c)\left(\frac{\partial D_i}{\partial\hat{\gamma}_i}+\lambda\frac{\partial D_i}{\partial\hat{\gamma}_j}\right)\left(p,p,\hat{\gamma},\hat{\gamma}\right)=C'\left(\gamma\right),$$

respectively.

The counterparts to (3) and (4) for the multi-product merged entity are:[42]

$$(p-c+\sigma)\left(\frac{\partial D_i}{\partial p_i}+\frac{\partial D_j}{\partial p_i}\right)\left(p,p,\hat{\gamma},\hat{\gamma}\right)+D_i\left(p,p,\hat{\gamma},\hat{\gamma}\right)=0,$$

and

$$(p-c+\sigma)\left(1+\lambda\right)\left(\frac{\partial D_i}{\partial\hat{\gamma}_i}+\frac{\partial D_j}{\partial\hat{\gamma}_i}\right)\left(p,p,\hat{\gamma},\hat{\gamma}\right)=C'\left(\gamma\right),$$

respectively.

These first-order conditions show that the analysis leading to Proposition 1 in the baseline model extends to an environment with spillovers as long as $\frac{\partial D_i}{\partial\gamma_i}$ and $\frac{\partial D_j}{\partial\gamma_i}$ are replaced with $\frac{\partial D_i}{\partial\hat{\gamma}_i}+\lambda\frac{\partial D_i}{\partial\hat{\gamma}_j}$ and $\frac{\partial D_j}{\partial\hat{\gamma}_i}+\lambda\frac{\partial D_j}{\partial\hat{\gamma}_j}$, respectively, and the arguments $\left(\tilde{p}^*\left(\gamma\right),\tilde{p}^*\left(\gamma\right),\gamma,\gamma\right)$ and $\left(\tilde{p}^M\left(\gamma\right),\tilde{p}^M\left(\gamma\right),\gamma,\gamma\right)$ are replaced with $\left(\tilde{p}^*\left(\hat{\gamma}\right),\tilde{p}^*\left(\hat{\gamma}\right),\hat{\gamma},\hat{\gamma}\right)$ and $\left(\tilde{p}^M\left(\hat{\gamma}\right),\tilde{p}^M\left(\hat{\gamma}\right),\hat{\gamma},\hat{\gamma}\right)$, respectively.

Proof of Proposition 5

By assumptions 2' and 3', γ^M is the unique solution of

$$l\left(\hat{p}^M\left(\gamma\right),\gamma,R_3\left(\hat{p}^M\left(\gamma\right),\gamma\right)\right)=C'\left(\gamma\right).$$

[42] To simplify the exposition, we assume away any synergies in production, i.e., $\sigma = 0$.

Consider a P-neutral merger. Then $\hat{p}^M(\gamma^*) = p^*$ and $R_3(\hat{p}^M(\gamma^*), \gamma^*) = R_3(p^*, \gamma^*) = \phi_3^*$. The function $l(\hat{p}^M(\gamma), \gamma, R_3(\hat{p}^M(\gamma), \gamma)) - C'(\gamma)$ is continuous by assumption 2', positive at $\gamma = 0$ and negative at $\gamma = \bar{\gamma}$ by assumption 2". This, combined with the uniqueness of γ^M, implies that the curve of function $l(\hat{p}^M(\gamma), \gamma, R_3(\hat{p}^M(\gamma), \gamma)) - C'(\gamma)$ crosses the horizontal axis before γ^* if and only if it is negative at γ^*, that is if and only if $l(p^*, \gamma^*, \phi_3^*) - h(p^*, \gamma^*, \phi_3^*) < 0$. The latter holds if and only if the innovation diversion ratio is less than the price diversion ratio (with both ratios evaluated at $(p^*, p^*, \gamma^*, \gamma^*, \phi_3^*)$).

Sign of the strategic effect on rival's price $\partial\tilde{p}_j^*/\partial\gamma_i$

Differentiating the first-order condition

$$(p_i^* - c)\frac{\partial D_i}{\partial p_i}(p_i^*, p_j^*, \gamma_i, \gamma_j) + D_i(p_i^*, p_j^*, \gamma_i, \gamma_j) = 0$$

with respect to γ_i and rearranging terms leads to

$$\left[2\frac{\partial D_i}{\partial p_i} + (\tilde{p}_i^* - c)\frac{\partial^2 D_i}{\partial p_i^2}\right]\frac{\partial \tilde{p}_i^*}{\partial \gamma_i} + \left[\frac{\partial D_i}{\partial p_j} + (\tilde{p}_i^* - c)\frac{\partial^2 D_i}{\partial p_i \partial p_j}\right]\frac{\partial \tilde{p}_j^*}{\partial \gamma_i} = -(\tilde{p}_i^* - c)\frac{\partial^2 D_i}{\partial p_i \partial \gamma_i} - \frac{\partial D_i}{\partial \gamma_i}.$$

Similarly, differentiating the same first-order condition with respect to γ_j and rearranging terms, we obtain

$$\left[\frac{\partial D_i}{\partial p_j} + (\tilde{p}_i^* - c)\frac{\partial^2 D_i}{\partial p_i \partial p_j}\right]\frac{\partial \tilde{p}_i^*}{\partial \gamma_j} + \left[2\frac{\partial D_i}{\partial p_i} + (\tilde{p}_i^* - c)\frac{\partial^2 D_i}{\partial p_i^2}\right]\frac{\partial \tilde{p}_j^*}{\partial \gamma_j} = -(\tilde{p}_i^* - c)\frac{\partial^2 D_i}{\partial p_i \partial \gamma_j} - \frac{\partial D_i}{\partial \gamma_j}.$$

Therefore, under the assumption of symmetric demand functions, the following system of linear equations in $\left(\partial\tilde{p}_i^*/\partial\gamma_i, \partial\tilde{p}_j^*/\partial\gamma_i\right)$ is satisfied in a symmetric equilibrium:

$$\begin{cases} A\frac{\partial \tilde{p}_i^*}{\partial \gamma_i} + B\frac{\partial \tilde{p}_j^*}{\partial \gamma_i} = E \\ B\frac{\partial \tilde{p}_i^*}{\partial \gamma_i} + A\frac{\partial \tilde{p}_j^*}{\partial \gamma_i} = F \end{cases} \tag{16}$$

where $A \equiv 2\frac{\partial D_i}{\partial p_i} + (\tilde{p}^* - c)\frac{\partial^2 D_i}{\partial p_i^2}$, $B \equiv \frac{\partial D_i}{\partial p_j} + (\tilde{p}^* - c)\frac{\partial^2 D_i}{\partial p_i \partial p_j}$, $E \equiv -(\tilde{p}^* - c)\frac{\partial^2 D_i}{\partial p_i \partial \gamma_i} - \frac{\partial D_i}{\partial \gamma_i}$ and $F \equiv -(\tilde{p}^* - c)\frac{\partial^2 D_i}{\partial p_i \partial \gamma_j} - \frac{\partial D_i}{\partial \gamma_j}$.

The solution to (16) is given by

$$\begin{cases} \frac{\partial \tilde{p}_i^*}{\partial \gamma_i} = \frac{AE - BF}{A^2 - B^2} \\ \frac{\partial \tilde{p}_j^*}{\partial \gamma_i} = \frac{AF - BE}{A^2 - B^2} \end{cases}$$

Assume that $\frac{\partial^2 D_i}{\partial p_i^2} \leq 0$, $\frac{\partial^2 D_i}{\partial p_i^2} + \frac{\partial^2 D_i}{\partial p_i \partial p_j} \geq 0$, and $\frac{\partial^2 D_i}{\partial p_i \partial \gamma_i} + \frac{\partial^2 D_i}{\partial p_i \partial \gamma_j} \geq 0$. This implies in particular that $A < 0 < B$, $E < 0 < F$, $A + B < 0$ and $E + F < 0$. It is straightforward to show that this set of inequalities implies that $\frac{\partial \tilde{p}_j^*}{\partial \gamma_i} < 0$.

Proof of Proposition 6

First, note that (γ_1^*, γ_2^*) is solution of the following system of equations

$$\left\{ \begin{array}{l} h_1^*(\gamma_1, \gamma_2) = C_1'(\gamma_1) \\ h_2^*(\gamma_1, \gamma_2) = C_2'(\gamma_2) \end{array} \right. , \tag{17}$$

and (whenever interior) $\left(\gamma_1^M, \gamma_2^M\right)$ is the unique solution of:

$$\left\{ \begin{array}{l} l_1^M(\gamma_1, \gamma_2) = C_1'(\gamma_1) \\ l_2^M(\gamma_1, \gamma_2) = C_2'(\gamma_2) \end{array} \right. . \tag{18}$$

Notice that

$$\frac{\partial \Pi^M(\gamma_1, \gamma_2)}{\partial \gamma_i} = l_i^M(\gamma_1, \gamma_2) - C_i'(\gamma_i).$$

Denote $R_1^M(\gamma_2) = \arg\max_\gamma \Pi^M(\gamma, \gamma_2)$. Whenever positive it is the unique solution of $l_1^M(\gamma_1, \gamma_2) = C_1'(\gamma_1)$ in γ_1 (the existence and uniqueness are guaranteed by assumptions 3" and $C'(\bar{\gamma}) = +\infty$). Denote similarly $R_2^M(\gamma_1) = \arg\max_\gamma \Pi^M(\gamma_1, \gamma)$.Again it is the unique solution of $l_2^M(\gamma_1, \gamma_2) = C_2'(\gamma_2)$ in γ_2.

Thus, we have

$$\gamma_1^* = R_1^*(\gamma_2^*) \quad ; \quad \gamma_2^* = R_2^*(\gamma_1^*)$$

and

$$\gamma_1^M = R_1^M\left(\gamma_2^M\right) \quad ; \quad \gamma_2^M = R_2^M\left(\gamma_1^M\right).$$

Differentiating $h_1^*(R_1^*(\gamma_2), \gamma_2) = C_1'(R_1^*(\gamma_2))$ with respect to γ_2 yields

$$\frac{dR_1^*}{d\gamma_2} = \frac{-\frac{\partial h_1^*}{\partial \gamma_2}(R_1^*(\gamma_2), \gamma_2)}{\frac{\partial h_1^*}{\partial \gamma_2}(R_1^*(\gamma_2), \gamma_2) - C_1''(R_1^*(\gamma_2))},$$

which has the same sign as $\frac{\partial h_1^*}{\partial \gamma_2}(R_1^*(\gamma_2), \gamma_2)$ (since the denominator is negative by assumption). Likewise $\frac{dR_2^*}{d\gamma_1}$, $\frac{dR_1^M}{d\gamma_2}$, $\frac{dR_2^M}{d\gamma_1}$ have the same signs as $\frac{\partial h_2^*}{\partial \gamma_1}(\gamma_1, R_2^*(\gamma_1))$, $\frac{\partial l_1^M}{\partial \gamma_2}\left(R_1^M(\gamma_2), \gamma_2\right)$, and $\frac{\partial l_2^M}{\partial \gamma_1}\left(\gamma_1, R_2^M(\gamma_1)\right)$, respectively.

Assume now that $l_1^M(\gamma_1, \gamma_2)$ is increasing in γ_2 and $l_2^M(\gamma_1, \gamma_2)$ is increasing in γ_1. This implies that $R_1^M(.)$ and $R_2^M(.)$ are non-decreasing. Consider first the scenario in which $l_i^M(\gamma_1^*, \gamma_2^*) < h_i^*(\gamma_1^*, \gamma_2^*)$ for $i = 1, 2$. In this case, $\gamma_1^* > R_1^M(\gamma_2^*)$ and $\gamma_2^* > R_2^M(\gamma_1^*)$. To see why the latter inequalities hold, notice that

$$\frac{\partial \Pi^M(\gamma_1^*,\gamma_2^*)}{\partial \gamma_i} = l_i^M(\gamma_1^*,\gamma_2^*) - C_i'(\gamma_1^*) < h_i^*(\gamma_1^*,\gamma_2^*) - C_i'(\gamma_1^*) = 0,$$

which implies by strict quasi-concavity in γ_i that $\gamma_i^* > R_i^M(\gamma_j^*)$.

We have then

$$\begin{aligned}
\gamma_2 &\leq R_2^M(\gamma_1^*) \Rightarrow \gamma_2 < \gamma_2^* \Longrightarrow R_1^M(\gamma_2) \leq R_1^M(\gamma_2^*) \\
\gamma_1 &\leq R_1^M(\gamma_2^*) \Rightarrow \gamma_1 < \gamma_1^* \Longrightarrow R_2^M(\gamma_1) \leq R_2^M(\gamma_1^*)
\end{aligned}$$

Consider the mapping

$$\begin{aligned}
\left[0, R_1^M(\gamma_2^*)\right] \times \left[0, R_2^M(\gamma_1^*)\right] &\longmapsto \left[0, R_1^M(\gamma_2^*)\right] \times \left[0, R_2^M(\gamma_1^*)\right] \\
(\gamma_1,\gamma_2) &\longmapsto \left(R_1^M(\gamma_2), R_2^M(\gamma_1)\right)
\end{aligned}$$

This mapping is continuous on a compact support, hence it has a fixed point. Then, assumption 3" ensures that it represents the merged entity's optimal innovation levels. This implies that $\gamma_1^M \leq R_1^M(\gamma_2^*) < R_1^*(\gamma_2^*) = \gamma_1^*$ and $\gamma_2^M \leq R_2^M(\gamma_1^*) < R_2^*(\gamma_1^*) = \gamma_1^*$.

Consider now the scenario in which $l_i^M(\gamma_1^*,\gamma_2^*) > h_i^*(\gamma_1^*,\gamma_2^*)$ for $i = 1, 2$. In this case, $\gamma_1^* < R_1^M(\gamma_2^*)$ and $\gamma_2^* < R_2^M(\gamma_1^*)$.

We have then

$$\begin{aligned}
\gamma_2 &\geq R_2^M(\gamma_1^*) \Rightarrow R_1^M(\gamma_2) \geq R_1^M(\gamma_2^*) \\
\gamma_1 &\leq R_1^M(\gamma_2^*) \Rightarrow R_2^M(\gamma_1) \geq R_2^M(\gamma_1^*)
\end{aligned}$$

Consider the mapping

$$\begin{aligned}
\left[R_1^M(\gamma_2^*), \bar{\gamma}_1\right] \times \left[R_2^M(\gamma_1^*), \bar{\gamma}_2\right] &\longmapsto \left[R_1^M(\gamma_2^*), \bar{\gamma}_1\right] \times \left[R_2^M(\gamma_1^*), \bar{\gamma}_2\right] \\
(\gamma_1,\gamma_2) &\longmapsto \left(R_1^M(\gamma_2), R_2^M(\gamma_1)\right)
\end{aligned}$$

This is continuous on a compact support, hence it has a fixed point. Then Assumption 3" ensures that it represents the merged entity's optimal innovation levels and assumption 1 with $C'(\bar{\gamma}) = +\infty$ implies that it less than $\bar{\gamma}$. This implies that $\bar{\gamma} > \gamma_1^M \geq R_1^M(\gamma_2^*) > \gamma_1^*$ and $\bar{\gamma} > \gamma_2^M \geq R_2^M(\gamma_1^*) > \gamma_1^*$.

Proof of Corollary 7

We have

$$\begin{aligned} l_i^M(\gamma_1^*,\gamma_2^*) - h_i^*(\gamma_1^*,\gamma_2^*) &= \frac{\left(D_j\frac{\partial D_j}{\partial p_i} - D_i\frac{\partial D_j}{\partial p_j}\right)\frac{\partial D_i}{\partial \gamma_i} + \left(D_i\frac{\partial D_i}{\partial p_j} - D_j\frac{\partial D_i}{\partial p_i}\right)\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial p_i}\frac{\partial D_j}{\partial p_j} - \frac{\partial D_j}{\partial p_i}\frac{\partial D_i}{\partial p_j}} + D_i\frac{\frac{\partial D_i}{\partial \gamma_i}}{\frac{\partial D_i}{\partial p_i}} \\ &= \frac{\left(D_i\frac{\partial D_i}{\partial p_j} - D_j\frac{\partial D_i}{\partial p_i}\right)\frac{\partial D_i}{\partial \gamma_i}\left(\frac{\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial \gamma_i}} - \frac{\frac{\partial D_j}{\partial p_i}}{\frac{\partial D_i}{\partial p_i}}\right)}{\frac{\partial D_i}{\partial p_i}\frac{\partial D_j}{\partial p_j} - \frac{\partial D_j}{\partial p_i}\frac{\partial D_i}{\partial p_j}} \end{aligned}$$

This is negative for both products if

$$\frac{\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial \gamma_i}} - \frac{\frac{\partial D_j}{\partial p_i}}{\frac{\partial D_i}{\partial p_i}} < 0 \text{ for } i,j=1,2,\ i\neq j,$$

and is positive for both products if the reverse holds.

Condition under which the post-merger innovation efforts are strategic complements in the augmented Singh-Vives model

Consider the demand function

$$D_i\left(p_i,p_j,\gamma_i,\gamma_j\right) = \frac{\alpha+\tau\gamma_i - \left(\alpha+\tau\gamma_j\right)\rho - p_i + \rho p_j}{1-\rho^2}.$$

Denoting $a_i = \alpha+\tau\gamma_i$ and $a_j = \alpha+\tau\gamma_j$, the post-merger first-order condition with respect to p_i is

$$a_i - a_j\rho - p_i + \rho p_j - p_i + c_i + \rho\left(p_j - c_j\right) = 0,$$

which leads to the following post-merger prices

$$p_i = \frac{1}{2}\left(a_i + c_i\right);\ p_j = \frac{1}{2}\left(a_j + c_j\right)$$

Denoting $A_i = \alpha + \tau\gamma_i - c_i$, the profit with optimal prices is

$$\begin{aligned} \Pi\left(a_i,a_j,\rho\right) &= \frac{\frac{1}{4}A_i^2 + \frac{1}{4}A_j^2 - \frac{1}{2}A_iA_j\rho}{1-\rho^2} \\ &= \frac{1}{4}\frac{\left(A_i - A_j\right)^2}{1-\rho^2} + \frac{1}{2}\frac{A_iA_j}{1+\rho}. \end{aligned}$$

Hence

$$\frac{\partial \Pi}{\partial \rho} = \frac{2\rho}{\left(1-\rho^2\right)^2}\frac{1}{4}\left(A_i - A_j\right)^2 - \frac{1}{2}\frac{A_i A_j}{\left(1+\rho\right)^2}$$

and

$$\frac{\partial^2 \Pi}{\partial \rho^2} = \frac{1+3\rho^2}{\left(1-\rho^2\right)^3}\frac{1}{2}\left(A_i - A_j\right)^2 + \frac{A_i A_j}{\left(1+\rho\right)^3}.$$

Considering now the derivative with respect to γ_i, we have

$$\frac{\partial \Pi}{\partial \gamma_i} = -\delta\frac{\partial \Pi}{\partial \rho} + \frac{\tau}{2}\left(\frac{A_i - A_j}{1-\rho^2} + \frac{A_j}{1+\rho}\right).$$

Differentiating this with respect to ρ yields

$$\frac{\partial^2 \Pi}{\partial \rho \partial \gamma_j} = -\frac{2\rho}{\left(1-\rho^2\right)^2}\frac{\tau}{2}\left(A_i - A_j\right) - \frac{\tau}{2}\frac{A_i}{\left(1+\rho\right)^2}$$

Thus, a sufficient condition for post-merger strategic complementarity, i.e., $\frac{\partial^2 \Pi}{\partial \gamma_i \partial \gamma_j} > 0$ is

$$\delta\frac{A_i + A_j}{1+\rho} > \tau\frac{\rho}{1-\rho},$$

where $\rho = 1 - \delta\left(\gamma_1 + \gamma_2\right)$. Using the expressions of A_i and A_j, this condition can be written as

$$2\alpha - c_1 - c_2 > \frac{\tau}{\delta}\left(\frac{3\rho - 1}{1-\rho}\right). \tag{19}$$

It is easy to see that Condition (19) is more likely to hold the smaller ρ and the larger α. Note that it is compatible with the condition $H_I < 0$, that is, $2\alpha < 2\frac{\tau}{\delta}\left(\frac{-1+4\rho+\rho^2}{1-\rho}\right)$.

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