Mergers and Demand-Enhancing Innovation*

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Abstract

This paper studies the impact of horizontal mergers on firms' incentives to invest in demand-enhancing innovation. In our symmetric baseline model, we identify four effects of a merger on innovation: the innovation diversion effect, the margin expansion effect, the demand expansion effect, and the per-unit return to innovation effect. The first two effects are negative, while the third one is positive, and the fourth one can be either positive or negative. We offer sufficient conditions for a merger to reduce or raise incentives to innovate in the absence of spillovers and synergies, and provide commonly used models in which they hold. Finally, we show that our approach can be extended to account for spillovers, synergies in R&D, synergies in production, and asymmetric demand and cost functions.

Keywords: Horizontal Mergers, Innovation, Competition.

JEL Classification: D43, L13, L40.

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1 Introduction

The debate on the impact of mergers on innovation has been recently revived by the European Commission's decision about the merger between Dow and Dupont.[1] This debate has highlighted several potentially conflicting effects of horizontal mergers on innovation,[2] and has shown the need for a general theoretical framework that would help to determine when a merger is likely to reduce or spur *product* innovation. This paper is a step towards such a framework.

We investigate the effect of a horizontal merger on demand-enhancing innovation. More specifically, we consider a merger between duopolists and compare their pre- and post-merger incentives to innovate. This is tantamount to focusing, in a model with more than two firms, on the *initial impetus* of the merger (Farrell and Shapiro, 2010; Federico et al., 2018), i.e., the way it affects merging firms' incentives to innovate for given rivals' behavior.

In our baseline model, we study the impact of a merger between two symmetric firms on their incentives to innovate in an environment with no spillovers, no synergies in R&D, and no – or limited – synergies. We derive a necessary and sufficient condition for the merger to reduce the equilibrium level of innovation and provide a decomposition of the merger's impact on innovation into four effects. First, the merger affects the merging firms' outputs and, therefore, their incentives to innovate in order to increase their margins. We call this the *margin expansion effect*. This effect is negative when the merger leads to lower output by the merging firms for a given innovation level, which is the case when synergies in production are absent or limited. Second, the merger affects the merging firms' margins and, therefore, their incentives to innovate in order to increase demand. This *demand expansion effect* is positive as a merger tends to increase margins. Third, the merger induces an *innovation diversion effect*: it leads to the internalization of the impact that each merging firm's innovation investment has on the other merging firm's demand. We focus on the case in which this externality is negative,[3] as this is the scenario that competition authorities are most concerned about. Finally, the merger may generate a change in the return to investment per unit of output, which we call the *per-unit return to innovation effect*. This effect can be either positive or negative. Importantly, we show that the sign of the sum of the demand expansion effect and the innovation diversion effect is the same as the sign of the difference between the price diversion ratio, commonly used by

[1] Case M.7932 – *Dow/DuPont.* The European Commission's March 2017 decision is available at http://ec.europa.eu/competition/mergers/cases/decisions/m7932_13668_3.pdf

[2] See, e.g., Baker (2007), Katz and Shelanski (2007), Shapiro (2012), Federico (2017), Denicolò and Polo (2018c), Jullien and Lefouili (2018b) and Régibeau and Rockett (2019).

[3] This implies that the innovation diversion effect is negative.

competition authorities to perform "upward pricing analysis" (Farrell and Shapiro, 2010), and the innovation diversion ratio, which is its counterpart for innovation analysis (Farrell and Shapiro, 2010; Salinger, 2016).

Using our decomposition, we provide sufficient conditions for a merger to reduce or raise incentives to innovate. More specifically, we show that a merger weakens firms' incentives to innovate if the innovation diversion ratio is greater than or equal to the price diversion ratio, and the per-unit return to investment in innovation does not increase too much after the merger. Conversely, a merger strengthens firms' incentives to innovate if the innovation diversion ratio is less than or equal to the price diversion ratio, and the per-unit return to investment in innovation increases sufficiently after the merger. We then apply our approach to several standard models, which we categorize according to whether the innovation diversion ratio is greater than, equal to, or less than the price diversion ratio. This reveals that in an environment in which there are no spillovers, no R&D synergies and no – or limited – production synergies, a horizontal merger reduces innovation incentives in several commonly used models, but that it can also lead to more innovation in other standard models.

Finally, we extend our baseline model to account for technological spillovers, synergies in R&D, (large) synergies in production, asymmetric demand and cost functions, observable investments, and cost-reducing innovation. In particular, we show that our decomposition can be adapted in a very natural way to incorporate the effects of spillovers and synergies. A key insight from our analysis is that synergies in production matter not only for the effect of a merger on prices but also for its effect on the incentives to innovate. More precisely, the fact that synergies in production may lead to an increase in output (for a given innovation level) implies that the margin expansion effect may become *positive*.

Related literature While there is a vast and long-standing literature on the effect of competition on innovation,[4] the literature addressing the specific question of how mergers affect firms' incentives to innovate is more recent and relatively small. Motta and Tarantino (2018) investigate primarily the impact of a horizontal merger on process innovation and show that a merger reduces merging firms' incentives to invest in cost-reducing R&D in the absence of spillovers and efficiency gains.[5] They also establish that this result extends to two special cases of quality-improving investments that are isomorphic to cost-reducing investments. By contrast, we focus on product innovation and consider a very general class of

[4]See Gilbert (2006) for a recent survey and Schmutzler (2013) for a unified approach to this issue.

[5]See also Matsushima et al. (2013) for an analysis of the effects of a merger when heterogeneous oligopolists compete both in process innovation and on the product market.

demand-enhancing R&D investments. Our paper can therefore be seen as complementary to Motta and Tarantino (2018).

Federico et al. (2017, 2018) analyze the effect of a merger on product innovation in a setting where investment in R&D affects the probability of success but *not* the value of the innovation, which is the focus of our analysis. They emphasize the negative innovation externality exerted by each firm on its competitors and show that the internalization of that externality by merging firms can lead to a decrease in innovation efforts under several standard demand specifications. Denicolò and Polo (2018a) and Jullien and Lefouili (2018a) clarify further the circumstances under which mergers result in less innovation when investments in R&D only affect the probability of innovation success, and establish that mergers can also lead to an increase in innovation incentives in that context. Moreover, Denicolò and Polo (2018b) show that a merger may increase the merging firms' incentives to innovate, because it allows them to share R&D knowledge and technologies.

In a setting where firms can undertake more than one research project, Letina (2016) and Gilbert (2019) show that a horizontal merger can decrease the variety of developed projects, and Moraga-González et al. (2019) find that a merger can either increase or decrease consumer welfare depending on whether the most profitable projects are also the most appropriable ones. In the context of markets with buyer power, Loertscher and Marx (2019a, 2019b) show that a merger raises rivals' investment incentives, and can raise merging parties' investment incentives. Considering an environment with minority shareholdings, López and Vives (2018) show that increasing partial ownership interest in rivals can increase incentives to innovate if R&D spillovers are sufficiently large. Finally, Mermelstein et al. (2018) consider a dynamic model in which firms can reduce costs through either investment in building capital or mergers and show that merger policy can greatly affect firms' investment behavior and vice-versa. There is also a growing empirical literature on the effects of mergers on innovation,[6] whose main message is that those effects are mixed.

The rest of the paper is organized as follows. We lay out our baseline model and state the equilibrium conditions defining price and innovation levels in Section 2. In Section 3 we present our main decomposition of the overall impact of a merger on innovation, which we apply to a number of specific models in Section 4. In Section 5 we extend our model to environments with technological spillovers, synergies in R&D, large synergies in production, asymmetric demand and cost functions, observable investments, and cost-reducing innovation. Section 6 concludes.

[6] See, e.g., Grabowski and Kyle (2008), Ornaghi (2009), Guadalupe et al (2012), Szücs (2014), Haucap et al. (2019), and Bennato et al. (2019).

2 Baseline model

Consider two single-product firms, 1 and 2, producing differentiated goods. The firms compete in prices and can invest in innovation to increase the demand for their products. Let $c \geq 0$ denote the firms' marginal cost of production and $C(\gamma_i)$ the investment cost firm $i \in \{1,2\}$ needs to incur to achieve an innovation level γ_i. We assume that $C(.)$ is increasing and convex, with $C(0) = 0$. In our baseline model, we suppose that firms set their prices and innovation levels simultaneously or, equivalently, that a firm does not observe its rival's innovation level before setting its price.[7]

We assume that innovation affects the demand for both products, but not their marginal cost of production.[8] Let $D_i(p_i, p_j, \gamma_i, \gamma_j)$ denote the demand addressed to firm $i \in \{1,2\}$ when it sets its price and innovation level at p_i and γ_i, and its rival sets its price and innovation level at p_j and γ_j, and assume that demand functions are symmetric, i.e., $D_i(p_i, p_j, \gamma_i, \gamma_j) = D_j(p_j, p_i, \gamma_j, \gamma_i)$ for any $(p_i, p_j, \gamma_i, \gamma_j)$. A firm's demand is decreasing in its own price and decreasing in its rival's price.[9] Moreover, we assume that an increase in a firm's innovation level leads to an increase in its own demand and a decrease in the rival's demand. Our analysis also applies to the case in which innovation by one firm has a positive impact on the rival's demand (see e.g. Lin and Saggi, 2002) but we will focus on the case in which the impact is negative as this is the scenario that raises anti-competitive concerns. Finally, we make the standard assumption that $\partial D_i/\partial p_i + \partial D_i/\partial p_j < 0$ (i.e., own effects dominate cross-effects) at symmetric prices and innovation levels $p_i = p_j$ and $\gamma_i = \gamma_j$. We also make a similar (reasonable) assumption regarding the effect of a uniform increase in innovation levels: $\partial D_i/\partial \gamma_i + \partial D_i/\partial \gamma_j > 0$ at symmetric prices and innovation levels $p_i = p_j$ and $\gamma_i = \gamma_j$.[10] We summarize these asumptions as follows:

Assumption 1: i) $\partial D_i/\partial p_i < 0 < \partial D_i/\partial p_j$ and $\partial D_i/\partial \gamma_i > 0 > \partial D_j/\partial \gamma_i$; ii) For any symmetric prices and innovation levels, $\partial D_i/\partial p_i + \partial D_i/\partial p_j < 0$ and $\partial D_i/\partial \gamma_i + \partial D_i/\partial \gamma_j > 0$.

[7] Oligopoly models with a simultaneous choice of price and R&D have been studied by Dasgupta and Stiglitz (1980), Levin and Reiss (1988), Ziss (1994), Leahy and Neary (1997), Cabral (2000), Vives (2008), and López and Vives (2018), among others. In Section 5.4, we extend our model to the case where innovation levels are observed before prices are set.

[8] We consider the case of cost-reducing innovations in Section 5.5.

[9] See Etro (2019) for an analysis of the way a merger between firms selling complementary inputs affects innovation.

[10] Notice that the assumption that $\partial D_i/\partial \gamma_i + \partial D_i/\partial \gamma_j > 0$ at symmetric prices and innovation levels is equivalent to the assumption that an increase in one firm's innovation level (starting from a symmetric situation) has a positive effect on aggregate demand, i.e., $\partial D_i/\partial \gamma_i + \partial D_j/\partial \gamma_i > 0$ at symmetric prices and innovation levels.

Consider first the benchmark scenario in which firms act independently. In a symmetric equilibrium, the first-order condition for the pricing decision is:

$$(p-c)\frac{\partial D_i}{\partial p_i}(p,p,\gamma,\gamma)+D_i(p,p,\gamma,\gamma)=0. \tag{1}$$

For conciseness, we assume that this condition defines a unique function $\tilde{p}^*(\gamma)$.[11] Likewise, the first-order condition for the innovation decision is the following:

$$(p-c)\frac{\partial D_i}{\partial \gamma_i}(p,p,\gamma,\gamma)=C'(\gamma). \tag{2}$$

We now make the following assumption regarding the price-innovation game.

Assumption 2: The duopoly price-innovation game has a symmetric equilibrium $(p^*,p^*,\gamma^*,\gamma^*)$ satisfying first-order conditions (1) and (2).

Consider now a merger between the two firms, and suppose that the merged entity keeps selling the two products. For now, we assume away any merger-induced synergies in R&D, but allow for potential synergies in production. More specifically, we suppose that the post-merger marginal costs of the merging entities are both given by $c-\sigma$, where $\sigma \geq 0$. We will focus in the baseline model on the scenario in which synergies in production are either absent or "small," in a sense that we will define below.

The merged entity's (monopoly) profit for levels of investments γ_1 and γ_2 is given by

$$\Pi^M(\gamma_1,\gamma_2;\sigma)\equiv\max_{p_1,p_2}(p_1-c+\sigma)\left[D_1(p_1,p_2,\gamma_1,\gamma_2)+D_2(p_2,p_1,\gamma_2,\gamma_1)\right]-C(\gamma_1)-C(\gamma_2).$$

We assume that this monopoly maximization problem is well behaved in the following sense:

Assumption 3: The profit function $\Pi^M(\gamma_1,\gamma_2;\sigma)$ is strictly quasi-concave in (γ_1,γ_2).

Under this assumption, the merged entity's optimal innovation strategy is symmetric, and therefore, we can restrict our attention to a single innovation level for both components of the merged entity, i.e., $\gamma_1=\gamma_2=\gamma$. For any given innovation level γ that applies to both products, the merged entity's optimal symmetric price $\tilde{p}^M(\gamma)$ is defined by the following first-order condition:

$$(p-c+\sigma)\left[\frac{\partial D_i}{\partial p_i}(p,p,\gamma,\gamma)+\frac{\partial D_j}{\partial p_i}(p,p,\gamma,\gamma)\right]+D_i(p,p,\gamma,\gamma)=0. \tag{3}$$

[11] All our results carry over without the uniqueness assumption.

Let us finally make the assumption that, for a given innovation level, the merger leads to an increase in prices:

Assumption 4: For any given symmetric innovation level γ, the merged entity's optimal prices exist, are symmetric, and satisfy $\tilde{p}^M(\gamma^*) > p^*$.

The second part of this assumption means that synergies in production are sufficiently small for their effect on prices to be outweighed by the market power effect of the merger.[12] We will relax this assumption in Section 5.2.2.

Turning to the merged entity's innovation choice, the optimal innovation level for given symmetric prices is the solution to the following first-order condition:

$$(p - c + \sigma)\left[\frac{\partial D_i}{\partial \gamma_i}(p, p, \gamma, \gamma) + \frac{\partial D_j}{\partial \gamma_i}(p, p, \gamma, \gamma)\right] = C'(\gamma). \tag{4}$$

An optimal symmetric price-innovation pair $\left(p^M, \gamma^M\right)$ for the merged entity satisfies conditions (3) and (4).

The general idea behind the subsequent analysis is to use the first-order conditions to eliminate marginal costs and focus on equilibrium prices, innovation levels and demands. Let us first define an *independent firm's marginal gain from innovation* as

$$h^*(\gamma) \equiv -D_i(\tilde{p}^*(\gamma), \tilde{p}^*(\gamma), \gamma, \gamma)\frac{\frac{\partial D_i}{\partial \gamma_i}(\tilde{p}^*(\gamma), \tilde{p}^*(\gamma), \gamma, \gamma)}{\frac{\partial D_i}{\partial p_i}(\tilde{p}^*(\gamma), \tilde{p}^*(\gamma), \gamma, \gamma)}.$$

From (1), one can see that $h^*(\gamma)$ represents firm i's marginal gain from an increase in its innovation level γ_i, when its price is set optimally, holding constant the innovation and price levels of firm j at γ and $\tilde{p}^*(\gamma)$, respectively.

Similarly, we define the *merged entity's marginal gain from innovation* as

$$h^M(\gamma) \equiv -D_i\left(\tilde{p}^M(\gamma), \tilde{p}^M(\gamma), \gamma, \gamma\right)\frac{\frac{\partial D_i}{\partial \gamma_i}\left(\tilde{p}^M(\gamma), \tilde{p}^M(\gamma), \gamma, \gamma\right) + \frac{\partial D_j}{\partial \gamma_i}\left(\tilde{p}^M(\gamma), \tilde{p}^M(\gamma), \gamma, \gamma\right)}{\frac{\partial D_i}{\partial p_i}\left(\tilde{p}^M(\gamma), \tilde{p}^M(\gamma), \gamma, \gamma\right) + \frac{\partial D_j}{\partial p_i}\left(\tilde{p}^M(\gamma), \tilde{p}^M(\gamma), \gamma, \gamma\right)}.$$

From (3), this expression corresponds to the slope of the merged entity's profit (gross of investment cost) with respect to γ_i (at $\gamma_i = \gamma$), when all prices are set optimally, holding constant the innovation level of the other unit (at $\gamma_j = \gamma$). Based on these definitions, the following proposition shows that the impact of the merger on innovation depends on the

[12] This is true in particular in the special (benchmark) case where a merger does not induce any synergies in production.

relative magnitude of the pre- and post-merger marginal gain from innovation, evaluated at the pre-merger innovation level.

Proposition 1 *The impact of the merger on innovation, i.e.,* $\gamma^M - \gamma^*$*, has the same sign as* $h^M(\gamma^*) - h^*(\gamma^*)$*.*

Proof. See Appendix. ∎

Proposition 1 thus shows that the merger increases (resp., decreases) innovation if $h^M(\gamma^*) > h^*(\gamma^*)$ (resp., $h^M(\gamma^*) < h^*(\gamma^*)$).

3 Decomposition of the overall effect of a merger on innovation

In this section, we analyze the impact of a merger on innovation and show that this impact is a combination of four effects: the innovation diversion effect, the margin expansion effect, the demand expansion effect, and the per-unit return to innovation effect.

To highlight the first effect, we isolate the terms that capture the impact of innovation in product i on the demand for that product. For the sake of exposition, let us denote the monopoly output at innovation level γ as:

$$D_i^M(\gamma) \equiv D_i\left(\tilde{p}^M(\gamma), \tilde{p}^M(\gamma), \gamma, \gamma\right).$$

Eliminating the terms related to the impact of innovation on the demand for product j in the merged entity's marginal gain from innovation $h^M(\gamma)$, we define

$$\hat{h}^M(\gamma) \equiv -D_i^M(\gamma) \frac{\frac{\partial D_i}{\partial \gamma_i}\left(\tilde{p}^M(\gamma), \tilde{p}^M(\gamma), \gamma, \gamma\right)}{\frac{\partial D_i}{\partial p_i}\left(\tilde{p}^M(\gamma), \tilde{p}^M(\gamma), \gamma, \gamma\right) + \frac{\partial D_j}{\partial p_i}\left(\tilde{p}^M(\gamma), \tilde{p}^M(\gamma), \gamma, \gamma\right)}.$$

The term $\hat{h}^M(\gamma)$ represents the marginal gain from innovation on the sales of product i for the merged entity. This leads to the following decomposition:

$$h^M(\gamma) - h^*(\gamma) = \hat{h}^M(\gamma) - h^*(\gamma) + H_I(\gamma),$$
$$\text{where } H_I(\gamma) \equiv \hat{h}^M(\gamma) \frac{\frac{\partial D_j}{\partial \gamma_i}\left(\tilde{p}^M(\gamma), \tilde{p}^M(\gamma), \gamma, \gamma\right)}{\frac{\partial D_i}{\partial \gamma_i}\left(\tilde{p}^M(\gamma), \tilde{p}^M(\gamma), \gamma, \gamma\right)} < 0.$$

The term H_I captures the internalization by the merged entity of the *diversion* of sales that demand-enhancing innovation in one product induces for the other product. This term, which we refer to as the *innovation diversion effect*, is negative because the underlying innovation externality is negative.

To highlight the three other effects of the merger on innovation, we decompose the remaining term, $\hat{h}^M - h^*$, which compares the pre- and post-merger incentives to raise the demand for product i through innovation. Therefore, this term captures how an increase in *market power* due to the merger affects the incentives to innovate.

When a firm increases its investment in innovation from γ_i to $\gamma_i' = \gamma_i + d\gamma_i$, the benefit it derives depends on how prices are adjusted. If prices are only slightly adjusted, the main motivation for innovation is to expand the volume of sales. By contrast, if prices are adjusted such that the volume is barely affected by innovation, then the main motivation is to expand one's margin. In this case, a lower supply following the merger reduces the merging firms' incentives to innovate in order to increase their prices and, therefore, their margins. We refer to this effect as the *margin expansion effect.* In the case where prices are hardly affected by the increased innovation effort, a higher margin following the merger raises the merging firms' incentives to innovate in order to raise demand. We refer to this effect as the *demand expansion effect.*[13]

More formally, we can write the gain from changing the price and innovation levels from (p_i, γ_i) to (p_i', γ_i'), holding the price and innovation levels of the other firm constant at $\left(p_j, \gamma_j\right) = (p, \gamma)$, as:

$$\begin{aligned} & \left(p_i' - c\right) D_i\left(p_i', p, \gamma_i', \gamma\right) - \left(p_i - c\right) D_i\left(p_i, p, \gamma_i, \gamma\right) \\ = \; & \left(p_i' - c\right)\left(D_i\left(p_i', p, \gamma_i', \gamma\right) - D_i\left(p_i, p, \gamma_i, \gamma\right)\right) + \left(p_i' - p_i\right) D_i\left(p_i, p, \gamma_i, \gamma\right). \end{aligned}$$

The margin expansion effect outweighs the demand expansion effect if $D_i\left(p_i', p, \gamma_i', \gamma\right) - D_i\left(p_i, p, \gamma_i, \gamma\right)$ is relatively small, while the opposite is true if $p_i' - p_i$ is relatively small. Which effect dominates ultimately depends on the price-elasticity of demand at γ_i and γ_i'. In particular, when this elasticity is not affected by innovation, demand expansion is the main driver of innovation efforts.

In order to define formally the terms that capture the margin expansion effect and the demand expansion effect in our model, we define the (pre-merger) *per unit return to innovation* as

$$g^*(p, \gamma) \equiv -\frac{\frac{\partial D_i}{\partial \gamma_i}(p, p, \gamma, \gamma)}{\frac{\partial D_i}{\partial p_i}(p, p, \gamma, \gamma)}.$$

This ratio measures the price increase that the firm can achieve when it increases innovation

[13] This effect was emphasized in the context of coverage for a new technology by Bourreau and Jullien (2018).

at the margin and raises prices so as to maintain the volume of sales constant. Therefore, it can be interpreted as the return to innovation per unit of output. The *independent firm's marginal gain from innovation* can then be written as the product of the volume of output and the per unit return to innovation:

$$h^*(\gamma) = D_i\left(\tilde{p}^*(\gamma), \tilde{p}^*(\gamma), \gamma, \gamma\right) g^*\left(\tilde{p}^*(\gamma), \gamma\right).$$

Denoting $D_i^*(\gamma) \equiv D_i\left(\tilde{p}^*(\gamma), \tilde{p}^*(\gamma), \gamma, \gamma\right)$, the impact of the merger on firms' investments in demand-enhancing innovation net of the innovation diversion effect can be decomposed as follows:

$$\hat{h}^M(\gamma) - h^*(\gamma) = \underbrace{\left[D_i^M(\gamma) - D_i^*(\gamma)\right] g^*\left(\tilde{p}^*(\gamma), \gamma\right)}_{\equiv H_M} + \left[\left(-\frac{\frac{\partial D_i}{\partial \gamma_i}}{\frac{\partial D_i}{\partial p_i} + \frac{\partial D_j}{\partial p_i}}\right) - g^*\left(\tilde{p}^*(\gamma), \gamma\right)\right] D_i^M(\gamma),$$

where the derivatives are evaluated at $\left(\tilde{p}^M(\gamma), \tilde{p}^M(\gamma), \gamma, \gamma\right)$.

The first term H_M measures the effect of the change in output on innovation incentives. We decompose the remaining term into two terms, H_D and H_Δ, to derive our final decomposition:

Proposition 2 *The change in innovation incentives induced by the merger can be decomposed as follows:*

$$h^M(\gamma^*) - h^*(\gamma^*) = H_M + H_\Delta + H_D + H_I,$$

where

$$H_M \equiv \left[D_i^M(\gamma^*) - D_i^*(\gamma^*)\right] g^*(p^*, \gamma^*) < 0,$$

$$H_\Delta \equiv D_i^M(\gamma^*) \left[g^*\left(\tilde{p}^M(\gamma^*), \gamma^*\right) - g^*(p^*, \gamma^*)\right] \lesseqgtr 0,$$

$$H_D \equiv \hat{h}^M(\gamma^*) \times \left(\frac{\frac{\partial D_j}{\partial p_i}}{-\frac{\partial D_i}{\partial p_i}}\right) > 0, \;\; H_I \equiv \hat{h}^M(\gamma^*) \times \left(\frac{\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial \gamma_i}}\right) < 0,$$

and all the derivatives are evaluated at $\left(\tilde{p}^M(\gamma^*), \tilde{p}^M(\gamma^*), \gamma^*, \gamma^*\right)$.

Proof. See Appendix. ■

The term H_M captures the *margin expansion effect* discussed above. It is always negative under our assumption that the merger raises prices (for a given innovation level), i.e., $\tilde{p}^M(\gamma) > \tilde{p}^*(\gamma)$. The term H_D is always positive and is larger the greater the magnitude of the derivative $\partial D_j / \partial p_i$, which drives the merged entity's incentives to increase prices (for a given innovation level) with respect to the situation in which firms set their prices independently. Since a higher price (and, therefore, a higher margin) provides the merged entity with higher incentives to increase demand, we interpret term H_D as capturing the *demand expansion effect*. Finally, the term H_Δ captures a *per unit return to innovation effect* that measures the change in the per-unit return to innovation due to the merger; it can be either positive or negative depending on whether $g^*(p, \gamma^*)$ increases or decreases with p.

4 Applications

In this section, we show how our approach can be applied in standard settings. We first use our decomposition to provide sufficient conditions under which the overall effect of a merger on innovation is negative (resp., positive) when there are no spillovers, no R&D synergies, and no – or limited – production synergies. We then examine the impact of a merger on innovation under those circumstances in several commonly used models.

Let us first consider the term $H_D + H_I$, i.e., the combination of the demand expansion effect and the innovation diversion effect. It is easy to see that this term has the same sign as the difference between the price diversion ratio and the innovation diversion ratio, i.e.,

$$\underbrace{\frac{\frac{\partial D_j}{\partial p_i}}{-\frac{\partial D_i}{\partial p_i}}}_{\text{price diversion ratio}} - \underbrace{\frac{-\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial \gamma_i}}}_{\text{innovation diversion ratio}},$$

evaluated at $\left(\tilde{p}^M(\gamma^*), \tilde{p}^M(\gamma^*), \gamma^*, \gamma^*\right)$. Thus, the sign of $H_D + H_I$ captures whether the price externality firms exert on each other (for given innovation levels) is stronger or weaker than the innovation externality they exert on each other (for given prices). When the price externality is stronger, the merger induces a relatively large increase in margins, which leads to a demand expansion effect that is strong enough to outweigh the effect of sales cannibalization resulting from innovation on firms' incentives.

Similarly, combining the margin expansion effect and the per-unit return to innovation effect, we see that $H_M + H_\Delta$ has the same as the sign as

$$D_i^M(\gamma^*)\, g^*\left(\tilde{p}^M(\gamma^*), \gamma^*\right) - D_i^*(\gamma^*)\, g^*(p^*, \gamma^*).$$

This term captures the incentives to enhance demand for a given product that are not related to externalities exerted on the other product. Indeed, ignoring these externalities, the marginal gain from innovation can be defined as the extra profit on product i for a marginal increase of innovation γ_i when the price p_i is adjusted so that demand for product i remains constant:

$$D_i\left(p,p,\gamma,\gamma\right)\left.\frac{\partial p_i}{\partial \gamma_i}\right|_{\text{constant } D_i,}-C'\left(\gamma_i\right)=D_i\left(p,p,\gamma,\gamma\right)g^*\left(p,\gamma\right)-C'\left(\gamma\right).$$

The reasoning above shows that we can separate the overall effect of a merger on innovation incentives into a part that is related to externalities between products, and another one that is related to changes in the demand for each product induced by the internalization of these externalities. Based on this discussion, we now provide sufficient conditions for the merger to reduce (resp., raise) incentives to innovate.

Corollary 1 *The merger reduces incentives to innovate if the innovation diversion ratio is greater than or equal to the price diversion ratio (both evaluated at* $\left(\tilde{p}^M\left(\gamma^*\right),\tilde{p}^M\left(\gamma^*\right),\gamma^*,\gamma^*\right)$*) and* $D\left(p,p,\gamma^*,\gamma^*\right)g^*\left(p,\gamma^*\right)$ *is decreasing in* p*. It raises incentives to innovate if the innovation diversion ratio is less than or equal to the price diversion ratio and* $D\left(p,p,\gamma^*,\gamma^*\right)g^*\left(p,\gamma^*\right)$ *is increasing in* p*.*

Proof. Follows from the above discussion. ■

Consider the sufficient conditions under which the merger reduces incentives to innovate. The first condition is model-specific as there is no reason for diversion ratios to rank in any given way.[14] By contrast, the second condition is more likely to hold than not, as violating it requires not only g^* to increase but to increase sufficiently to compensate the decrease in demand resulting from the merger-induced increase in prices.

Corollary 1 shows that a key determinant of the effect of a merger on innovation is how the innovation diversion ratio compares to the price diversion ratio. Based on this, we now examine the impact of a merger on innovation in several standard models, which we categorize according to whether the innovation diversion ratio is greater than, equal to, or less than the price diversion ratio.

4.1 Models in which the innovation diversion ratio is equal to the price diversion ratio

In the class of models in which the innovation diversion ratio and the price diversion ratio are equal, a sufficient condition for a merger to reduce (resp. raise) incentives to innovate is that $D\left(p,p,\gamma^*,\gamma^*\right)g^*\left(p,\gamma^*\right)$ is decreasing (resp. increasing) in p.

[14] This will be confirmed by the specific models we consider below.

We focus on two commonly used types of models within this class. Consider first models with *hedonic prices*, i.e., for which there exists $Q(.,.)$ and $v(.)$ such that

$$D_i\left(p_i, p_j, \gamma_i, \gamma_j\right) = Q\left(p_i - v\left(\gamma_i\right), p_j - v\left(\gamma_j\right)\right), \text{ with } v'\left(\gamma\right) > 0. \tag{HED}$$

Let Q_1 and Q_2 denote the derivatives of Q with respect to its first and second argument. It is straightforward that the innovation diversion ratio and the price diversion ratio are both equal to $-\ Q_2/Q_1$, and that

$$D\left(p, p, \gamma, \gamma\right) g^*\left(p, \gamma\right) = v'\left(\gamma\right) Q\left(p - v\left(\gamma\right), p - v\left(\gamma\right)\right),$$

which is decreasing in p. Proposition 1 then implies that the merger reduces incentives to innovate under this specification.

Note that in these models with hedonic prices, the impact of a merger on innovation is driven solely by the margin expansion effect. The reason is that the demand expansion effect and the innovation diversion effect cancel out and the per-unit return to innovation effect is equal to zero. A key feature of these models is that their analysis is comparable to that of models with cost-reducing innovation (Motta and Tarantino, 2018). In Section 5.5, we confirm this isomorphism by extending our approach to cost-reducing innovation and showing that the margin expansion effect is the only effect at work for this type of innovation.

Let us now consider the *multinomial logit* (MNL) model, in which the demand is given by (see, e.g., Dubé, 2019):

$$D_i\left(p_i, p_j, \gamma_i, \gamma_j\right) = \frac{\exp u\left(\gamma_i, y - p_i\right)}{\exp u\left(\gamma_i, y - p_i\right) + \exp u\left(\gamma_j, y - p_j\right) + \exp u\left(0, y\right)}, \tag{MNL}$$

where u is increasing in both its arguments, γ_i is the quality of product i, y is income and $u\left(\gamma_i, y - p_i\right)$ is the mean utility from consuming one unit of quality γ_i paid at price p_i.

Denoting by u_1 and u_2 the derivatives of u with respect to its first and second argument, we find that

$$\frac{\frac{\partial D_j}{\partial p_i}}{-\frac{\partial D_i}{\partial p_i}} = \frac{-\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial \gamma_i}} = \frac{\exp u\left(\gamma, y - p\right)}{\exp u\left(\gamma, y - p\right) + \exp u\left(0, y\right)}$$

at $p_i = p_j = p$ and $\gamma_i = \gamma_j = \gamma$.

The following proposition provides a sufficient condition for the merger to reduce (resp., raise) incentives to innovate in this case.

Proposition 3 *In the MNL model, the merger reduces incentives to innovate if*

$$\frac{-u_{12}\left(\gamma^{*}, y-p\right)}{u_{1}\left(\gamma^{*}, y-p\right)}+\frac{u_{22}\left(\gamma^{*}, y-p\right)}{u_{2}\left(\gamma^{*}, y-p\right)}<0$$

for all $p \in \left[p^{*}, \tilde{p}^{M}\left(\gamma^{*}\right)\right]$. *The merger raises incentives to innovate if*

$$\frac{-u_{12}\left(\gamma^{*}, y-p\right)}{u_{1}\left(\gamma^{*}, y-p\right) u_{2}\left(\gamma^{*}, y-p\right)}+\frac{u_{22}\left(\gamma^{*}, y-p\right)}{\left(u_{2}\left(\gamma^{*}, y-p\right)\right)^{2}}>1$$

for all $p \in \left[p^{*}, \tilde{p}^{M}\left(\gamma^{*}\right)\right]$.

Proof. See Appendix. ■

Let us now provide specific utility functions satisfying the above conditions. First, it is easy to see that the sufficient condition under which a merger reduces incentives to innovate holds in the case of a Cobb-Douglas utility function.

Second, consider the case of a constant marginal utility of income, i.e., $u(\gamma, y-p)=v(\gamma)+f(\gamma)(y-p)$, with $v'(\gamma)>0$, $v'(\gamma)+f'(\gamma)y>0$ and $f(\gamma)>0$. On the one hand, we show in the Appendix that the sufficient condition for a merger to raise incentives to innovate provided in Proposition 3 holds whenever

$$\frac{-f'\left(\gamma^{*}\right)}{f\left(\gamma^{*}\right)}>v'\left(\gamma^{*}\right),$$

which requires that higher quality γ reduces the marginal utility of income, i.e., f is decreasing. On the other hand, it is straightforward that the sufficient condition for a merger to reduce incentives to innovate holds whenever f is increasing.

4.2 Models in which the innovation diversion ratio is less than the price diversion ratio

We now consider two classes of models in which the innovation diversion ratio is greater than the price diversion ratio: those with quality-adjusted prices, and those with constant expenditures.

4.2.1 Models with quality-adjusted prices

Let us first consider demand functions for which there exists a function $Q(.,.)$ such that

$$D_{i}\left(p_{i}, p_{j}, \gamma_{i}, \gamma_{j}\right)=\frac{1}{\gamma_{i}} Q\left(\frac{p_{i}}{\gamma_{i}}, \frac{p_{j}}{\gamma_{j}}\right). \tag{QAP}$$

Denoting by Q_1 and Q_2 the derivatives of Q with respect to its first and second argument, we assume that $Q_1 < Q_1 + Q_2 < 0$ and that $Q + \frac{p}{\gamma} Q_1 < 0$, which ensures that innovation raises own demand. In this setting, p_i/γ_i can be interpreted as the quality-adjusted price. This class of models has been considered, for instance, by Motta and Tarantino (2018).

The price diversion ratio and the innovation diversion ratio at symmetric prices and innovation levels are given by

$$\frac{\frac{\partial D_j}{\partial p_i}}{-\frac{\partial D_i}{\partial p_i}} = -\frac{Q_2}{Q_1} \text{ and } \frac{-\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial \gamma_i}} = -\frac{pfQ_2}{Q + pfQ_1} > -\frac{Q_2}{Q_1}. \tag{5}$$

Thus, the innovation diversion ratio is greater than the price diversion ratio. Moreover, we have

$$D\left(p, p, \gamma, \gamma\right) g^*\left(p, \gamma\right) = \frac{1}{\gamma} Q\left(\frac{p}{\gamma}, \frac{p}{\gamma}\right)\left(\frac{Q}{Q_1}\left(\frac{p}{\gamma}, \frac{p}{\gamma}\right) + \frac{p}{\gamma}\right),$$

which may be either increasing or decreasing in prices. Thus, the overall effect of the merger on innovation is *a priori* ambiguous: the demand expansion effect is dominated by the innovation diversion effect (since the innovation diversion ratio is greater than the innovation diversion ratio) but the sum of the margin expansion effect and the per-unit return to innovation effect has an ambiguous sign. To evaluate further the overall impact of the merger on innovations in this model, let us notice that the pre- and post-merger prices for $\gamma = \gamma^*$ satisfy

$$\begin{aligned} \gamma^*\left(s^* + \frac{Q\left(s^*, s^*\right)}{Q_1\left(s^*, s^*\right)}\right) &= c, \\ \gamma^*\left(s^{M*} + \frac{Q\left(s^{M*}, s^{M*}\right)}{Q_1\left(s^{M*}, s^{M*}\right) + Q_2\left(s^{M*}, s^{M*}\right)}\right) &= c - \sigma, \end{aligned}$$

where $s^* = \tilde{p}^*\left(\gamma^*\right)/\gamma^*$ and $s^{M*} = \tilde{p}^M\left(\gamma^*\right)/\gamma^*$. The pre- and post-merger marginal gains from innovation (evaluated at $\gamma = \gamma^*$) can then be written as

$$h^*\left(\gamma^*\right) = \frac{Q\left(s^*, s^*\right)}{\gamma^*}\left(s^* + \frac{Q\left(s^*, s^*\right)}{Q_1\left(s^*, s^*\right)}\right) = \frac{cQ\left(s^*, s^*\right)}{\left(\gamma^*\right)^2}$$

and

$$h^M\left(\gamma^*\right) = \left(s^{M*} + \frac{Q\left(s^{M*}, s^{M*}\right)}{Q_1\left(s^{M*}, s^{M*}\right) + Q_2\left(s^{M*}, s^{M*}\right)}\right)\frac{1}{\gamma^*} Q\left(s^M, s^M\right) = \frac{\left(c - \sigma\right) Q\left(s^{M*}, s^{M*}\right)}{\left(\gamma^*\right)^2},$$

respectively. Thus, the innovation incentives (at $\gamma = \gamma^*$) are related to the volume of sales, and, more precisely, to total variable production costs. Therefore, under this specification, any merger that would reduce output – or total variable costs – for a fixed innovation level reduces incentives to innovate.[15]

4.2.2 Models with constant expenditures/CES

Let us now consider the class of models with constant expenditures (see Vives, 1999). In these models, there exist $K \geq 0$ and a function $\eta(.,.)$, such that firm i's demand can be written as

$$\begin{aligned} D_i\left(p_i, p_j, \gamma_i, \gamma_j\right) &= \frac{\eta\left(p_i, \gamma_i\right)}{p_i \eta\left(p_i, \gamma_i\right) + p_j \eta\left(p_j, \gamma_j\right) + K}, \\ \text{with } \frac{\partial \eta}{\partial p} &< 0 < \frac{\partial \eta}{\partial \gamma} \text{ and } p\frac{\partial \eta}{\partial p} + \eta < 0; \end{aligned} \tag{CE}$$

where the last condition ensures that goods are substitutes. Here, K represents spending on other goods, so that total spending $p_1 \eta\left(p_1, \gamma_1\right) + p_2 \eta\left(p_2, \gamma_2\right) + K$ is constant. In the special case of a CES demand function with substitutable goods, we have

$$\eta\left(p, \gamma\right) = \left(\frac{p}{\gamma}\right)^{-\sigma} \text{ with } \sigma > 1. \tag{CES}$$

In this class of models, the diversion ratios at symmetric prices and innovation levels are given by:

$$\frac{\frac{\partial D_j}{\partial p_i}}{-\frac{\partial D_i}{\partial p_i}} = \frac{p\eta \frac{\partial \eta}{\partial p} + \eta^2}{\left(p\eta + K\right)\frac{\partial \eta}{\partial p} - \eta^2} < \frac{-\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial \gamma_i}} = \frac{p\eta}{p\eta + K},$$

where the inequality follows from $\frac{\partial \eta}{\partial p} < 0$. Thus, using Corollary 1, we conclude that the merger reduces innovation if $D\left(p, p, \gamma, \gamma\right) g^*\left(p, \gamma\right)$ is decreasing in p. This leads to the following statement.

Corollary 2 *In a model with constant expenditures (CE), the merger reduces incentives to innovate if* $\frac{\gamma}{\eta}\frac{\partial \eta}{\partial \gamma}$ *is non-increasing in* p *and* $-\frac{p}{\eta}\frac{\partial \eta}{\partial p}$ *is non-decreasing in* p*. This holds in particular for the CES demand.*

Proof. See Appendix. ■

[15] Motta and Tarantino's analysis shows through a direct argument that absent any production synergies, the overall effect of the merger on innovation is negative under this specification.

4.3 Models in which the innovation diversion ratio is less than the price diversion ratio

We now examine the impact of a merger on innovation in two models in which the innovation diversion ratio is lower than the price diversion ratio.

4.3.1 Model with quality-augmented linear demand

Let us consider the following linear demand system, considered by Sutton (1997, 1998), Symeonidis (2000, 2003) and Federico et al. (2018):

$$D_i\left(p_i, p_j, \gamma_i, \gamma_j\right) = \frac{\gamma_i\left[2\gamma_i\left(1-p_i\right)-\rho\gamma_j\left(1-p_j\right)\right]}{4-\sigma^2},$$

where $\rho \in (0,2)$ is an inverse measure of the degree of horizontal differentiation, and γ_i is the quality of product i.

The price diversion ratio and the innovation diversion ratio at symmetric prices and innovation levels are given by

$$\frac{\frac{\partial D_j}{\partial p_i}}{-\frac{\partial D_i}{\partial p_i}} = \frac{\rho}{2} \text{ and } \frac{-\frac{\partial D_j}{\partial \gamma_i}\left(p,p,\gamma,\gamma\right)}{\frac{\partial D_i}{\partial \gamma_i}\left(p,p,\gamma,\gamma\right)} = \frac{\rho}{4-\rho},$$

respectively, which implies that the price diversion ratio is greater than the innovation diversion ratio. Moreover, we find in this case that

$$D\left(p,p,\gamma,\gamma\right) g^*\left(p,\gamma\right) = \frac{\gamma}{2+\sigma}\frac{\left(4-\rho\right)\left(1-p\right)^2}{2},$$

which is decreasing in p. Thus, we have on the one hand, $H_D + H_I > 0$, and on the other hand, $H_M + H_\Delta < 0$. Therefore, the overall effect of a merger on innovation is *a priori* ambiguous.

To determine the overall impact of the merger, note that the pre- and post-merger prices for a given (symmetric) level of innovation γ are[16]

$$\widetilde{p}^*\left(\gamma\right) = c + \left(1-c\right)\frac{2-\rho}{4-\rho} \text{ and } \widetilde{p}^M(\gamma) = \frac{1+c}{2},$$

respectively.[17] Interestingly, $p^*\left(\gamma\right)$ and $p^M(\gamma)$ do not depend on the innovation level γ. This

[16] See Symeonidis (2003) for the derivation of equilibrium prices.

[17] We ignore any post-merger production synergies post-merger, i.e., we set $\sigma = 0$.

implies in particular that innovation is monetized only through an increase in demand by the merged entity,[18] and that the demand expansion effect plays a key role this model. However, a direct computation shows that

$$h^*(\gamma) = \frac{2}{4-\rho}\frac{\gamma(1-c)^2}{2+\rho} > h^M(\gamma) = \frac{1}{2}\frac{\gamma(1-c)^2}{2+\rho}.$$

Thus, the demand expansion effect is not strong enough to outweigh the other effects, and the merger always reduces incentives to innovate in this model.

4.3.2 Singh-Vives model with endogenous horizontal and vertical differentiation

We investigate the effect of a merger on innovation in the linear demand model of Singh and Vives (1984) in a context where innovation has both a horizontal and a vertical dimension. Assume that the utility of the representative consumer is given by

$$U(q_1, q_2, m) = a_1 q_1 + a_2 q_2 - (q_1^2 + q_2^2)/2 - \rho q_1 q_2 + m,$$

where (q_1, q_2) is the vector of quantities, m is the numeraire good, and $\rho \in [0, 1]$ represents the degree of substitutability between the product of firm 1 and the product of firm 2. Products are independent if $\rho = 0$ and perfect substitutes if $\rho = 1$.

Suppose that R&D raises product quality as in the model with hedonic prices, which we capture by assuming that $a_i = a(\gamma_i) = \alpha + \tau\gamma_i$. We also assume that R&D allows firms to increase the differentiation between their products as in Lin and Saggi (2002). Formally, the degree of substitutability is given by $\rho(\gamma_1, \gamma_2) = 1 - \delta(\gamma_1 + \gamma_2)$, and we suppose that R&D costs are sufficiently high so that $\delta(\gamma_1 + \gamma_2) < 1$ in equilibrium (i.e., $2\delta\gamma < 1$ in the symmetric equilibrium).

The demand for firm i is then given by

$$D_i(p_i, p_j, \gamma_i, \gamma_j) = \frac{(\alpha + \tau\gamma_i) - (\alpha + \tau\gamma_j)\rho - p_i + \rho p_j}{1 - \rho^2}.$$

To ensure that the innovation diversion effect H_I is negative we assume that the parameters

[18] Note also that, while the pre-merger equilibrium prices for potentially asymmetric quality levels $p_i^*(\gamma_i, \gamma_j)$ is increasing in γ_i (see Symeonidis, 2003), the equilibrium demand of a firm is increasing in its quality level.

of the model are such that

$$\frac{a\left(\gamma^{*}\right)\delta\gamma^{*}}{2(1-\delta\gamma^{*})(1-2\delta\gamma^{*})}\delta<\tau. \tag{6}$$

Under this specification, straightforward computations show that the innovation diversion ratio is less than the price diversion ratio:

$$H_D+H_I=\frac{\delta a\left(\gamma^{*}\right)^{2}}{8(1-\delta\gamma^{*})}>0.$$

Moreover, $g^{*}\left(p,\gamma\right)=\frac{(a(\gamma)-p)\delta^{2}\gamma}{1-\delta\gamma}+\tau$. As $g^{*}\left(p,\gamma\right)$ is decreasing in p, we have $H_{\Delta}+H_M<0$. Hence, the overall effect of the merger is *a priori* ambiguous. However, combining all the effects, we find that

$$H_M+H_D+H_I+H_{\Delta}=\frac{a\left(\gamma^{*}\right)\left(1-2\delta\gamma^{*}\right)}{4(1-\delta\gamma^{*})(1+2\delta\gamma^{*})}\left(\frac{a\left(\gamma^{*}\right)\left(1+4\delta^{2}\gamma^{*2}\right)}{2(1-\gamma^{*}\delta)(1-4\delta^{2}\gamma^{*2})}\delta-\tau\right), \tag{7}$$

which implies that the merger leads to an increase in incentives to innovate if δ/τ is not too small.

Proposition 4 *In the augmented Singh and Vives model with a negative innovation diversion effect, the merger raises incentives to innovate if the vertical effect of R&D investments is not too strong (i.e., τ is not too large, given ρ^{*}, α and τ), if there is little differentiation between products (ρ^{*} is small, given τ, δ and α), or if demand is large enough (α is large, given ρ^{*}, τ and δ).*

Proof. See Appendix. ■

We should emphasize that condition (6), which ensures that the innovation diversion effect is negative, is compatible with expression (7) being positive.[19] In other words, the set of parameters for which the merger has a positive impact on innovation is not empty.

5 Extensions

In this section, we show how the approach we developed in our baseline model can be extended to account for technological spillovers, synergies in R&D, (large) synergies in production, asymmetric demand and cost functions, and observable investments. We also

[19] This holds because $\frac{a(\gamma^{*})(1+4\delta^{2}\gamma^{*2})}{2(1-\gamma^{*}\delta)(1-4\delta^{2}\gamma^{*2})}>\frac{a(\gamma^{*})\delta\gamma^{*}}{2(1-\delta\gamma^{*})(1-2\delta\gamma^{*})}$.

provide an extension in which we investigate the impact of a merger on investments in cost-reducing innovation.

5.1 Technological spillovers

It is well known that a firm's R&D may benefit its rivals through technological spillovers (d'Aspremont and Jacquemin, 1988; Bloom et al., 2013; López and Vives, 2018). To capture this, let us now assume that there exists a degree of spillovers $\lambda \in [0,1]$ such that the demand addressed to firm i is given by $D_i\left(p_i, p_j, \gamma_i + \lambda\gamma_j, \gamma_j + \lambda\gamma_i\right)$. In other words, a share λ of the demand-enhancing innovation efforts of firm i spills over to firm j (and vice versa).

Let $\hat{\gamma}_i \equiv \gamma_i + \lambda\gamma_j$ for $i = 1,2$ and $\hat{\gamma} \equiv (1+\lambda)\gamma$, and denote $(\hat{p}^*, \hat{\gamma}^*)$ the (symmetric) pre-merger equilibrium level of innovation. The following statement shows that Proposition 1 extends in a very natural way to the scenario in which there are spillovers.

Proposition 5 *In the presence of spillovers, the impact of the merger on innovation has the same sign as* $h_\lambda^M(\hat{\gamma}^*) - h_\lambda^*(\hat{\gamma}^*)$*, where* $h_\lambda^M(.)$ *and* $h_\lambda^*(.)$ *are obtained from* $h^M(.)$ *and* $h^*(.)$*, respectively, by:*

- replacing $\frac{\partial D_i}{\partial \gamma_i}$ *and* $\frac{\partial D_j}{\partial \gamma_i}$ *with* $\frac{\partial D_i}{\partial \hat{\gamma}_i} + \lambda \frac{\partial D_i}{\partial \hat{\gamma}_j}$ *and* $\frac{\partial D_j}{\partial \hat{\gamma}_i} + \lambda \frac{\partial D_j}{\partial \hat{\gamma}_j}$*, respectively,*

- and replacing the arguments $(\tilde{p}^*(\gamma), \tilde{p}^*(\gamma), \gamma, \gamma)$ *and* $\left(\tilde{p}^M(\gamma), \tilde{p}^M(\gamma), \gamma, \gamma\right)$ *with* $(\tilde{p}^*(\hat{\gamma}), \tilde{p}^*(\hat{\gamma}), \hat{\gamma}, \hat{\gamma})$ *and* $\left(\tilde{p}^M(\hat{\gamma}), \tilde{p}^M(\hat{\gamma}), \hat{\gamma}, \hat{\gamma}\right)$*, respectively.*

Proof. See Appendix. ■

We can again provide a decomposition of the overall impact of the merger on incentives to innovate into several effects:

$$h_\lambda^M(\hat{\gamma}^*) - h_\lambda^*(\hat{\gamma}^*) = H_{M\lambda} + H_{D\lambda} + H_{I\lambda} + H_{\Delta\lambda} + E_\lambda$$

where $H_{M\lambda}$, $H_{D\lambda}$, $H_{I\lambda}$, and $H_{\Delta\lambda}$ are obtained from H_M, H_D, H_I, and H_Δ, respectively, by making the replacements specified in Proposition 5, and

$$E_\lambda \equiv \lambda \hat{h}^M(\hat{\gamma}^*)\left[\frac{\frac{\partial D_j}{\partial \hat{\gamma}_i}}{\frac{\partial D_i}{\partial \hat{\gamma}_i}} \times \frac{\frac{\partial D_j}{\partial p_i}}{-\frac{\partial D_i}{\partial p_i}} + 1\right] > 0.$$

The terms $H_{M\lambda}$, $H_{D\lambda}$, $H_{I\lambda}$, $H_{\Delta\lambda}$ capture the margin expansion effect, the demand expansion effect, the innovation diversion effect, and the per-unit return to innovation effect, respectively, as in the baseline model. The additional term E_λ captures a new *spillover* *effect*, and is positive.[20]

Furthermore, we find that the sum of the innovation diversion effect, the demand expansion effect and the spillover effect, $H_{D\lambda} + H_{I\lambda} + E_\lambda$, has the same sign as:

$$\underbrace{\frac{\frac{\partial D_j}{\partial p_i}}{-\frac{\partial D_i}{\partial p_i}}}_{\text{price diversion ratio}} \quad - \quad \underbrace{\frac{-\frac{\frac{\partial D_j}{\partial \hat{\gamma}_i}}{\frac{\partial D_i}{\partial \hat{\gamma}_i}} - \lambda}{1 + \lambda \frac{\frac{\partial D_j}{\partial \hat{\gamma}_i}}{\frac{\partial D_i}{\partial \hat{\gamma}_i}}}}_{\text{spillover-adjusted innovation diversion ratio}}$$

The denominator of the spillover-adjusted innovation diversion ratio is always positive. This follows from the assumptions that $\lambda \in [0, 1]$ and $\partial D_i/\partial \hat{\gamma}_i + \partial D_i/\partial \hat{\gamma}_j > 0$. Thus, the sign of the spillover-adjusted innovation diversion ratio is given by the sign of the difference between the innovation diversion ratio and the spillover rate:

$$\left(\frac{-\frac{\partial D_j}{\partial \hat{\gamma}_i}}{\frac{\partial D_i}{\partial \hat{\gamma}_i}} \right) - \lambda.$$

This sign can be related to the magnitude of the "net innovation pressure" (NIP) defined in Salinger (2016):

$$NIP = \frac{\left(1 + \frac{\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial \gamma_i}}\right)(1 + \lambda)}{1 + \lambda \frac{\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial \gamma_i}}}.$$

Considering an environment with no price competition, Salinger (2016) shows that a merger reduces innovation if and only if $NIP > 1$. It is straightforward to see that this condition holds if and only if the spillover-adjusted innovation diversion ratio is positive, i.e., $\lambda < -\frac{\partial D_j/\partial \gamma_i}{\partial D_i/\partial \gamma_i}$.

5.2 Synergies

5.2.1 Synergies in R&D

We first consider the case where the merger induces synergies that reduce the cost of R&D investments.[21] Let us assume that the post-merger cost of R&D is given by $\frac{C(\gamma)}{1+\mu}$, where

[20] To see why this term is positive, note that $\frac{\partial D_j}{\partial \hat{\gamma}_i} \times \frac{\frac{\partial D_j}{\partial p_i}}{-\frac{\partial D_i}{\partial p_i}} + \frac{\partial D_i}{\partial \hat{\gamma}_i} > \min\left[\frac{\partial D_i}{\partial \hat{\gamma}_i}, \frac{\partial D_j}{\partial \hat{\gamma}_i} + \frac{\partial D_i}{\partial \hat{\gamma}_i}\right]$ because $\partial D_i/\partial p_i + \partial D_i/\partial p_j$ is negative and $\partial D_i/\partial \hat{\gamma}_i + \partial D_i/\partial \hat{\gamma}_j$ is positive.

[21] Davidson and Ferrett (2007) emphasize the importance of R&D synergies in shaping the profitability of a merger. In contrast, we focus on how they affect innovation efforts.

$\mu \geq 0$ is a measure of the size of R&D synergies. The only first-order condition that is affected by R&D synergies is the one associated with the merged entity's innovation level, i.e., Equation (4), which becomes[22]

$$(1+\mu)(p-c)\left[\frac{\partial D_i}{\partial \gamma_i}(p,p,\gamma,\gamma)+\frac{\partial D_j}{\partial \gamma_i}(p,p,\gamma,\gamma)\right]=C'(\gamma).$$

The equilibrium price and the monopoly price for a given (symmetric) innovation level are still given by $\tilde{p}^*(\gamma)$ and $\tilde{p}^M(\gamma)$, respectively. The result in Proposition 1 can then be extended as follows.

Proposition 6 *In the presence of R&D synergies of size μ, the impact of the merger on innovation has the same sign as $(1+\mu)h^M(\gamma^*)-h^*(\gamma^*)$.*

We can decompose the impact of the merger on innovation into the same four effects that are at play in the baseline model. We have:

$$(1+\mu)h^M(\gamma^*)-h^*(\gamma^*)=H_M+H_{D\mu}+H_{I\mu}+H_\Delta.$$

The terms H_M and H_Δ, which represent the margin expansion effect and the return to innovation effect, respectively, remain the same as in the baseline model, while the two other terms must be adjusted as follows:

$$H_{D\mu}\equiv\hat{h}^M(\gamma^*)\left(\mu+\frac{\frac{\partial D_j}{\partial p_i}}{-\frac{\partial D_i}{\partial p_i}}\right)>0,$$

$$H_{I\mu}\equiv(1+\mu)H_I=(1+\mu)\hat{h}^M(\gamma^*)\left(\frac{\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial \gamma_i}}\right)<0.$$

The sum of the adjusted demand expansion and innovation diversion effects, $H_{D\mu}+H_{I\mu}$, has the same sign as the difference between a synergy-adjusted price diversion ratio and the innovation diversion ratio:

$$\underbrace{\frac{\frac{\frac{\partial D_j}{\partial p_i}}{-\frac{\partial D_i}{\partial p_i}}+\mu}{1+\mu}}_{\text{synergy-adjusted price diversion ratio}}-\underbrace{\frac{-\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial \gamma_i}}}_{\text{innovation diversion ratio}}.$$

[22] To simplify the exposition, we abstract from any production synergies and set σ to zero.

Note that, since $\partial D_i/\partial p_i + \partial D_i/\partial p_j < 0$ from our assumptions, the synergy-adjusted price diversion ratio is increasing in μ.

5.2.2 Synergies in production

We now show that when there are strong synergies in production, they influence not only the effect of the merger on prices but also its effect on innovation. Recall that we have assumed that the merger generates synergies that allow the merging firms to decrease their marginal cost by $\sigma \in [0, c]$. Using Equation (3), we obtain an optimal price for the merged entity, $\tilde{p}^M(\gamma, \sigma)$, where we now make the dependence on σ explicit. While we have focused so far on the case where $\tilde{p}^M(\gamma, \sigma) > \tilde{p}^*(\gamma)$ (Assumption 4), the opposite may be true if synergies are strong enough.

Let us make the natural assumption that $\tilde{p}^M(\gamma, \sigma)$ is decreasing in σ. Suppose furthermore that there exists $\tilde{\sigma}(\gamma) \in (0, c)$ such that $\tilde{p}^M(\gamma, \tilde{\sigma}(\gamma)) = p^*(\gamma)$. Then, if production synergies are sufficiently strong (i.e., $\sigma > \tilde{\sigma}(\gamma)$), the post-merger price $\tilde{p}^M(\gamma, \sigma)$ is smaller than the pre-merger price $\tilde{p}^*(\gamma)$. In other words, efficiencies in production outweigh the market power effect of the merger. Given (3) and (4), and the fact that the pre-merger outcome is not affected by merger-induced synergies, Proposition 1 still holds for $\tilde{p}^M(\gamma, \sigma) < \tilde{p}^*(\gamma)$ and our decomposition remains valid as well. A key difference with the baseline model, however, is that the margin expansion effect, H_M, is *positive* if production synergies are strong enough so that the monopoly price is smaller than the competitive price at the pre-merger innovation level, i.e., if $\sigma > \tilde{\sigma}(\gamma^*)$. If this condition holds, and g^* is monotonic in p, the term H_Δ representing the per-unit return to innovation effect, is also of the opposite sign compared to the baseline model.

In the special case where $\sigma = \tilde{\sigma}(\gamma^*)$, i.e., when the pre- and post-merger prices are identical if the innovation level is set at its pre-merger level γ^*, we have $H_M = H_\Delta = 0$. This leads us to the following result, which shows that when the merger has a neutral effect on prices (for an innovation level fixed at its pre-merger value), its impact on innovation can be derived from the mere comparison of the price diversion ratio and the innovation diversion ratio.

Proposition 7 *Suppose that $\sigma = \tilde{\sigma}(\gamma^*)$. Then, the merger reduces (resp., raises) incentives to innovate if the price diversion ratio is greater (resp., less) than the innovation diversion ratio, where both ratios are evaluated at $\left(\tilde{p}^M(\gamma^*), \tilde{p}^M(\gamma^*), \gamma^*, \gamma^*\right)$.*

5.3 Asymmetric demand and cost functions

We now extend our analysis to a setting in which the demand functions D_i, the marginal costs c_i and the innovation cost functions C_i are potentially asymmetric.

Consider first the pre-merger situation. Assume that the pricing game derived from the price-innovation game by fixing the innovation levels of firms 1 and 2 to γ_1 and γ_2, respectively, has a unique equilibrium. The corresponding equilibrium price pair $(\tilde{p}_1^*(\gamma_1,\gamma_2), \tilde{p}_2^*(\gamma_1,\gamma_2))$ is the solution to the following system of first-order conditions:

$$\begin{cases} (p_1 - c_1)\frac{\partial D_1}{\partial p_1}(p_1,p_2,\gamma_1,\gamma_2) + D_1(p_1,p_2,\gamma_1,\gamma_2) = 0 \\ (p_2 - c_2)\frac{\partial D_2}{\partial p_2}(p_1,p_2,\gamma_1,\gamma_2) + D_2(p_1,p_2,\gamma_1,\gamma_2) = 0. \end{cases} \tag{8}$$

Likewise, assume that the innovation game derived from the price-innovation game by fixing the prices of firms 1 and 2 to p_1 and p_2, respectively, has a unique symmetric equilibrium. The corresponding equilibrium pair of innovation levels $(\tilde{\gamma}_1^*(p_1,p_2), \tilde{\gamma}_2^*(p_1,p_2))$ is the solution to the following system of first-order conditions:

$$\begin{cases} (p_1 - c_1)\frac{\partial D_1}{\partial \gamma_1}(p_1,p_2,\gamma_1,\gamma_2) = C_1'(\gamma_1) \\ (p_2 - c_2)\frac{\partial D_2}{\partial \gamma_2}(p_1,p_2,\gamma_1,\gamma_2) = C_2'(\gamma_2). \end{cases} \tag{9}$$

Consider now the post-merger situation. As in the baseline model, we assume away any merger-induced synergies in R&D but allow for potential merger-induced synergies in production. More specifically, we suppose that the post-merger production costs of the two merging entities are given by $c_1 - \sigma_1$ and $c_2 - \sigma_2$, respectively, where σ_1 and σ_2 are "small" in a sense that will be made clear later.

For any given innovation levels γ_1 and γ_2, the merged entity's optimal price pair $\left(\tilde{p}_1^M(\gamma_1,\gamma_2), \tilde{p}_2^M(\gamma_1,\gamma_2)\right)$ is defined by the following system of first-order conditions:

$$\begin{cases} (p_1 - c_1 + \sigma_1)\frac{\partial D_1}{\partial p_1}(p_1,p_2,\gamma_1,\gamma_2) + (p_2 - c_2 + \sigma_2)\frac{\partial D_2}{\partial p_1}(p_1,p_2,\gamma_1,\gamma_2) + D_1(p_1,p_2,\gamma_1,\gamma_2) = 0 \\ (p_1 - c_1 + \sigma_1)\frac{\partial D_1}{\partial p_2}(p_1,p_2,\gamma_1,\gamma_2) + (p_2 - c_2 + \sigma_2)\frac{\partial D_2}{\partial p_2}(p_1,p_2,\gamma_1,\gamma_2) + D_2(p_1,p_2,\gamma_1,\gamma_2) = 0. \end{cases}$$

Combining these two equations leads to

$$\begin{cases} p_1 - c_1 + \sigma_1 = \frac{D_2\frac{\partial D_2}{\partial p_1} - D_1\frac{\partial D_2}{\partial p_2}}{\frac{\partial D_1}{\partial p_1}\frac{\partial D_2}{\partial p_2} - \frac{\partial D_2}{\partial p_1}\frac{\partial D_1}{\partial p_2}} \\ p_2 - c_2 + \sigma_2 = \frac{D_1\frac{\partial D_1}{\partial p_2} - D_2\frac{\partial D_1}{\partial p_1}}{\frac{\partial D_1}{\partial p_1}\frac{\partial D_2}{\partial p_2} - \frac{\partial D_2}{\partial p_1}\frac{\partial D_1}{\partial p_2}}. \end{cases}$$

We can now state the counterparts to Assumptions 2-4 in the current setting.

Assumption 2': The duopoly price-innovation game has an equilibrium $(p_1^*, p_2^*, \gamma_1^*, \gamma_2^*)$ satisfying first-order conditions (8) and (9).

Assumption 3': The profit function $\Pi^M(\gamma_1, \gamma_2; \sigma_1, \sigma_2)$ is strictly quasi-concave in (γ_1, γ_2) ,where $\Pi^M(\gamma_1, \gamma_2; \sigma_1, \sigma_2)$ is the merged entity's (monopoly profit) for levels of investments γ_1 and γ_2:

$$\begin{aligned}\Pi^M(\gamma_1, \gamma_2; \sigma_1, \sigma_2) \equiv \max_{p_1, p_2} \{ & (p_1 - c_1 + \sigma_1) D_1(p_1, p_2, \gamma_1, \gamma_2) + \\ & (p_2 - c_2 + \sigma_2) D_2(p_2, p_1, \gamma_2, \gamma_1) - C(\gamma_1) - C(\gamma_2)\}.\end{aligned}$$

Assumption 4': For any given innovation levels γ_1 and γ_2, the merged entity's optimal prices exist and are such that $\tilde{p}_1^M(\gamma_1, \gamma_2) > \tilde{p}_1^*(\gamma_1, \gamma_2)$ and $\tilde{p}_2^M(\gamma_1, \gamma_2) > \tilde{p}_2^*(\gamma_1, \gamma_2)$.

The *independent firm's marginal gain from innovation* is now given by

$$h_i^*(\gamma_1, \gamma_2) \equiv -D_i \frac{\frac{\partial D_i}{\partial \gamma_i}}{\frac{\partial D_i}{\partial p_i}}$$

for firm $i = 1, 2$, where all functions are evaluated $(\tilde{p}_1^*(\gamma_1, \gamma_2), \tilde{p}_2^*(\gamma_1, \gamma_2), \gamma_1, \gamma_2)$, while the *merged entity's marginal gain from innovation* in product $i = 1, 2$ is

$$h_i^M(\gamma_1, \gamma_2) \equiv \frac{\left(D_j \frac{\partial D_j}{\partial p_i} - D_i \frac{\partial D_j}{\partial p_j}\right) \frac{\partial D_i}{\partial \gamma_i} + \left(D_i \frac{\partial D_i}{\partial p_j} - D_j \frac{\partial D_i}{\partial p_i}\right) \frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial p_i} \frac{\partial D_j}{\partial p_j} - \frac{\partial D_j}{\partial p_i} \frac{\partial D_i}{\partial p_j}}.$$

where all functions are evaluated at $\left(\tilde{p}_1^M(\gamma_1, \gamma_2), \tilde{p}_2^M(\gamma_1, \gamma_2), \gamma_1, \gamma_2\right)$.

Finally, we assume that the pre-merger and post-merger net benefits from innovation in product i, i.e., $h_i^*(\gamma_1, \gamma_2) - C'(\gamma_i)$ and $h_i^M(\gamma_1, \gamma_2) - C'(\gamma_i)$, are decreasing in γ_i. Moreover, we say that the pre-merger (resp. post-merger) innovation efforts are *strategic complements* if $h_1^*(\gamma_1, \gamma_2)$ (resp., $h_1^M(\gamma_1, \gamma_2)$) is increasing in γ_2 and $h_2^*(\gamma_1, \gamma_2)$ (resp., $h_2^M(\gamma_1, \gamma_2)$) is increasing in γ_1.

The next proposition shows that when innovation efforts are strategic complements, the comparison of an independent firm's marginal gain from innovation and the merged entity's marginal gain from innovation (as defined above) is still informative regarding the impact of the merger on innovation.

Proposition 8 *(i) If the pre-merger innovation efforts are strategic complements and $h_i^M(\gamma_1, \gamma_2) < h_i^*(\gamma_1, \gamma_2)$ for $i = 1, 2$ and any (γ_1, γ_2), then the merger leads to less innovation in both products, i.e., $\gamma_i^M < \gamma_i^*$ for $i = 1, 2$.*

(ii) If the post-merger innovation efforts are strategic complements and $h_i^M(\gamma_1, \gamma_2) > h_i^*(\gamma_1, \gamma_2)$ for $i = 1, 2$ and any (γ_1, γ_2), then the merger leads to more innovation in both products, i.e., $\gamma_i^M > \gamma_i^*$ for $i = 1, 2$.

Proof. See Appendix. ■

As an illustration, we provide in the Appendix a sufficient condition under which the post-merger innovation efforts are strategic complements in the Singh-Vives model with endogenous horizontal and vertical differentiation.

5.4 Observable investments

In the baseline model, we assumed that the price and innovation decisions were taken simultaneously by the firms, or equivalently, that a firm could not observe its rival's investment before setting its price. We now assume that a firm's investment in R&D is observed by its rival before prices are set. For given investment levels, the profit-maximizing price for an independent firm i, $\tilde{p}_i^*(\gamma_i, \gamma_j)$, is the solution to the following first-order condition:

$$(p_i - c)\frac{\partial D_i}{\partial p_i}(p_i, p_j, \gamma_i, \gamma_j) + D_i(p_i, p_j, \gamma_i, \gamma_j) = 0.$$

With observable investments, the first-order condition with respect to γ_i becomes

$$(p_i - c)\left(\frac{\partial D_i}{\partial \gamma_i} + \frac{\partial \tilde{p}_j^*}{\partial \gamma_i}\frac{\partial D_i}{\partial p_j}\right) = C'(\gamma_i), \tag{10}$$

where $\partial D_i/\partial p_j$ is evaluated at $\left(\tilde{p}_i^*(\gamma_i, \gamma_j), \tilde{p}_j^*(\gamma_i, \gamma_j), \gamma_i, \gamma_j\right)$ and $\partial \tilde{p}_j^*/\partial \gamma_i$ is evaluated at (γ_i, γ_j). Therefore, firm i takes into account not only the direct effect of its investment on its profit, but also the strategic effect that goes through firm j's pricing reaction. The first-order conditions associated to the merged entity's maximization program remain the same as before. Therefore, the decomposition in our baseline setting remains valid as long as we replace the partial derivative $\partial D_i/\partial \gamma_i$ by $\partial D_i/\partial \gamma_i + \partial \tilde{p}_j^*/\partial \gamma_i \times \partial D_i/\partial p_j$ in the independent firm's marginal gain from innovation. This leads us to the following decomposition:

$$h^M(\gamma^*) - h^*(\gamma^*) = H_M + H_D + H_I + H_\Delta + H_O,$$

where

$$H_O = -D_i(\gamma)\, g^*(p, \gamma)\frac{\partial \tilde{p}_j^*}{\partial \gamma_i}.$$

The sign of the additional term H_O is the opposite of the sign of the strategic effect on the rival's price, $\partial\tilde{p}_j^*/\partial\gamma_i$. It seems natural to assume that when firm i invests more in innovation, firm j reacts by setting a lower price. In the Appendix, we provide sufficient conditions on firms' demand functions which ensure that $\partial\tilde{p}_j^*/\partial\gamma_i \leq 0$. In this case, the last term of the decomposition, H_O, is positive. Compared to the baseline model, when investment is observable, a merger allows firms to internalize the negative strategic effect of their investments on profits, which tends to stimulate innovation.

5.5 Cost-reducing innovation

In this extension, we depart from the baseline framework by applying our approach to *cost-reducing* innovation. Let us denote by $D_i(p_i, p_j)$ the demand addressed to firm i, and assume that firm i can reduce its marginal cost from an initial level c to $c - \gamma_i$, by investing $C(\gamma_i)$. As in the baseline model with demand-enhancing innovation, we assume that the duopoly price-innovation game has a unique, symmetric, equilibrium and that the equilibrium innovation level γ^* is positive. Also, denoting by $\sigma \geq 0$ the synergies in production induced by the merger, suppose that the profit function of the merged entity with optimized prices, i.e.,

$$\Pi^M(\gamma_1, \gamma_2; \sigma) = \max_{p_1, p_2} (p_1 - c + \sigma + \gamma_1) D_1(p_1, p_2) + (p_2 - c + \sigma + \gamma_2) D_2(p_1, p_2) - C(\gamma_1) - C(\gamma_2),$$

is strictly quasi-concave in (γ_1, γ_2), and that for a given symmetric innovation level γ, the merger leads to higher prices, i.e., $\tilde{p}^M(\gamma) > \tilde{p}^*(\gamma)$ (with similar notations as in our baseline model).

Under these assumptions, the merged entity's innovation strategy is symmetric. The key difference with the baseline model is that the first-order condition which gives the equilibrium innovation level as a function of the symmetric equilibrium price is the same whether firms act independently or as a merged entity. This condition is given by

$$D_i(p, p) = C'(\gamma). \tag{11}$$

Therefore, the marginal gain from innovation is $D_i^*(\gamma) = D_i(\tilde{p}^*(\gamma), \tilde{p}^*(\gamma))$ if firms act independently and $D_i^M(\gamma) = D_i\left(\tilde{p}^M(\gamma), \tilde{p}^M(\gamma)\right)$ if they merge. The strict quasi-concavity of $\Pi^M(.,.)$ then ensures that a necessary and sufficient condition for the merger to reduce innovation is that $D_i^M(\gamma^*) - D_i^*(\gamma^*)$ is negative. This term captures the *margin expansion effect* in the present setting.[23] It is indeed negative, as the merger leads to higher prices,

[23] The only difference between the margin expansion effect in the cost-reducing innovation setting and

i.e., $\tilde{p}^M(\gamma) > \tilde{p}^*(\gamma)$ (by assumption), which reduces the demand addressed to each firm (since $\partial D_i/\partial p_i + \partial D_i/\partial p_j < 0$ at symmetric prices). Thus, contrary to the case of demand-enhancing innovation, the margin expansion effect is the only effect driving the impact of a merger on cost-reducing innovation. This confirms Motta and Tarantino's clear-cut result that a merger to monopoly reduces the incentives to conduct cost-reducing investments when they are not observable to rivals.

6 Conclusion

Our analysis sheds light on the impact of a merger on investment in demand-enhancing innovation. In our baseline model with no spillovers, no synergies in R&D and no – or limited – synergies in production, we identified the following four effects of a merger between symmetric duopolists on their incentives to innovate: the *margin expansion effect* which is particularly relevant when innovation raises margins but has little effect on the volume of sales, the *demand expansion effect* which is key when innovation raises demand but has little effect on margins, the *innovation diversion effect* that arises whenever innovation affects sales of other products, and the *per-unit return to innovation effect* which appears whenever the merger affects the gain from innovation per unit of output. Using this decomposition, we provided sufficient conditions under which the overall impact on innovation incentives is negative. It turns out that the impact of a horizontal merger on innovation depends crucially, albeit not solely, on the comparison between the price diversion ratio and innovation diversion ratio, which calls for a distinction between environments in which the innovation diversion ratio is greater than the price diversion ratio and environments in which the reverse holds. Our analysis of several standard models suggests that in the former class of environments, the impact of a merger on innovation is likely to be negative in the absence of spillovers and synergies, while in the latter the impact can be either positive or negative.

One advantage of our approach is that it can be easily extended to account for spillovers, synergies in R&D, and synergies in production and can also be adapted to cost-reducing innovation. We leave for future research the extension of our analysis to a setting with $N \geq 3$ firms in which a subset of firms merge. That would allow us to investigate the effect of a merger on merging and non-merging firms' incentives to innovate (incorporating equilibrium effects). Finally, while the effect of mergers on innovation is interesting in itself, the ultimate question that most competition authorities are seeking to answer when

our baseline demand-enhancing innovation setting is that firms increase their margins by decreasing their marginal costs instead of increasing their prices.

evaluating a merger is its effect on consumer surplus. Therefore, it would be interesting to embed our analysis in a full-fledged assessment of the impact of a merger on consumers.

Appendix

Proof of Proposition 1

From the positivity of γ^*, and from (1) and (2), it follows that $h^*(\gamma^*) = C'(\gamma^*)$. Moreover, from (3) and (4), it follows that $\frac{d\Pi^M}{d\gamma}(\gamma,\gamma) = 2\left[h^M(\gamma) - C'(\gamma)\right]$. Assumption 2 implies that $\gamma^M > \gamma^*$ if and only if $\frac{d\Pi^M}{d\gamma}(\gamma^*,\gamma^*) > 0$, which yields the result.

Proof of Proposition 2

We have

$$h^M(\gamma^*) - h^*(\gamma^*) = H_M + \left[\left(-\frac{\frac{\partial D_i}{\partial \gamma_i}}{\frac{\partial D_i}{\partial p_i} + \frac{\partial D_j}{\partial p_i}}\right) - g^*(p^*,\gamma^*)\right] D_i^M(\gamma^*) + H_I,$$

and

$$\begin{aligned}\left[\left(-\frac{\frac{\partial D_i}{\partial \gamma_i}}{\frac{\partial D_i}{\partial p_i} + \frac{\partial D_j}{\partial p_i}}\right) - g^*(p^*,\gamma^*)\right] D_i^M(\gamma^*) &= \left[\left(-\frac{\frac{\partial D_i}{\partial \gamma_i}}{\frac{\partial D_i}{\partial p_i} + \frac{\partial D_j}{\partial p_i}}\right) - \left(-\frac{\frac{\partial D_i}{\partial \gamma_i}}{\frac{\partial D_i}{\partial p_i}}\right)\right] D_i^M(\gamma) \\ &\quad + \left[g^*\left(\tilde{p}^M(\gamma^*),\gamma^*\right) - g^*\left(\tilde{p}^*(\gamma),\gamma\right)\right] D_i^M(\gamma^*) \\ &= \left[1 - \frac{1}{\frac{\partial D_i}{\partial p_i}}\left(\frac{\partial D_i}{\partial p_i} + \frac{\partial D_j}{\partial p_i}\right)\right]\hat{h}^M(\gamma^*) + H_\Delta \\ &= -\frac{\frac{\partial D_j}{\partial p_i}}{\frac{\partial D_i}{\partial p_i}}\hat{h}^M(\gamma^*) + H_\Delta = H_D + H_\Delta,\end{aligned}$$

which completes the proof.

Proof of Proposition 3

In a model with an MNL demand, we have

$$g^*(p,\gamma) = \frac{u_1(\gamma, y-p)}{u_2(\gamma, y-p)}$$

and, therefore,

$$g^*(p,\gamma) D(p,p,\gamma,\gamma) = \frac{u_1(\gamma,y-p)\exp u(\gamma,y-p)}{2u_2(\gamma,y-p)\exp u(\gamma,y-p) + u_2(\gamma,y-p)\exp u(0,y)}.$$

Hence, the derivative of $g^*(p,\gamma^*) D(p,p,\gamma^*,\gamma^*)$ with respect to p has the same sign as
$[-(u_{12}+u_1u_2)(\gamma^*,y-p)]\exp u(\gamma^*,y-p)[2u_2(\gamma^*,y-p)\exp u(\gamma^*,y-p)+u_2(\gamma^*,y-p)\exp u(0,y)]$
$-u_1(\gamma^*,y-p)\exp u(\gamma^*,y-p)\{[-2(u_{22}+u_2^2)(\gamma^*,y-p)]\exp u(\gamma^*,y-p)-u_{22}(\gamma^*,y-p)\exp u(0,y)\}$,
which has the same sign as

$$[2(-u_{12}u_2+u_1u_{22})(\gamma^*,y-p)]\exp u(\gamma^*,y-p)+\left[(-u_{12}u_2-u_1u_2^2+u_1u_{22})(\gamma^*,y-p)\right]\exp u(0,y).$$

If $u_1u_{22}-u_{12}u_2<0$ or, equivalently, $-u_{12}/u_1+u_{22}/u_2<0$, then both the first term between brackets and the second term between brackets are negative and, therefore, $g^*(p,\gamma^*)D(p,p,\gamma^*,\gamma^*)$ decreases with p. Corollary 1 then implies that the merger reduces incentives to innovate. If $-u_{12}u_2-u_1u_2^2+u_1u_{22}>0$ or, equivalently, $-u_{12}/(u_1u_2)+u_{22}/(u_2)^2>1$, then both the first term between brackets and the second term between brackets are positive and, therefore, $g^*(p,\gamma^*)D(p,p,\gamma^*,\gamma^*)$ increases with p. Corollary 1 then implies that the merger reduces incentives to innovate.

Condition for the merger to raise incentives to innovate in the case of a utility with constant marginal utility of income

In the special case $u(\gamma,y-p)=v(\gamma)+f(\gamma)(y-p)$, the sufficient condition under which the merger raises incentives to innovate writes

$$-f(\gamma^*)\{[v'(\gamma^*)+f'(\gamma^*)(y-p)]f(\gamma^*)+f'(\gamma^*)\}>0,$$

for all $p\in\left[p^*,\tilde{p}^M(\gamma^*)\right]$, which holds if and only if

$$[v'(\gamma^*)+f'(\gamma^*)(y-p)]f(\gamma^*)+f'(\gamma^*)<0$$

for all $p\in\left[p^*,\tilde{p}^M(\gamma^*)\right]$. For this inequality to hold, it is necessary that $f'(\gamma^*)<0$. Using this, it is easy to see that a sufficient condition for the inequality to hold is that $v'(\gamma^*)f(\gamma^*)+f'(\gamma^*)<0$, which we can write as

$$\frac{-f'(\gamma^*)}{f(\gamma^*)}>v'(\gamma^*).$$

Proof of Corollary 2

From Proposition 1 and the fact that the innovation diversion ratio is larger than the price diversion ratio, a merger reduces innovation if $D(p,p,\gamma,\gamma)g^*(p,\gamma)$ is decreasing in prices. We have

$$
\begin{aligned}
D(p,p,\gamma,\gamma)\, g^*(p,\gamma) &= \frac{1}{2p\eta+K} \frac{(p\eta+K)\,\eta\frac{\partial\eta}{\partial\gamma}}{-(p\eta+K)\frac{\partial\eta}{\partial p}+\eta^2} \\
&= \frac{1}{2p\eta+K} \frac{(p\eta+K)\,p\eta}{(p\eta+K)\left(-\frac{p}{\eta}\frac{\partial\eta}{\partial p}\right)+p\eta} \left(\frac{1}{\eta}\frac{\partial\eta}{\partial\gamma}\right)
\end{aligned}
$$

Then we have for any $\sigma = -\frac{p}{\eta}\frac{\partial\eta}{\partial p}$:

$$
\frac{d}{dx}\frac{1}{2x+K}\frac{(x+K)\,x}{(x+K)\,\sigma+x} = \frac{K\left(K^2\sigma+x^2(\sigma-1)+2Kx\sigma\right)}{(K+2x)^2\,(x+K\sigma+x\sigma)^2} > 0
$$

As $p\eta$ decreases in p when goods are substitutes, this implies that Dg is decreasing in prices if $\frac{1}{\eta}\frac{\partial\eta}{\partial\gamma}$ is non-increasing and $-\frac{p}{\eta}\frac{\partial\eta}{\partial p}$ is non-decreasing.

For a CES demand, we have $\frac{\gamma}{\eta}\frac{\partial\eta}{\partial\gamma} = -\frac{p}{\eta}\frac{\partial\eta}{\partial p} = \sigma$, which yields the result.

Proof of Proposition 4

We have

$$
g^*\left(\tilde{p}^*(\gamma),\gamma\right) = \frac{a(\gamma)\,\delta^2\gamma}{(1-\delta\gamma)(1+2\delta\gamma)} + \tau \quad \text{and} \quad g^*\left(\tilde{p}^M(\gamma),\gamma\right) = \frac{a(\gamma)\,\delta^2\gamma}{2(1-\delta\gamma)} + \tau,
$$

which implies that

$$
H_\Delta = -\frac{a(\gamma^*)^2\,\delta^2\gamma(1-2\delta\gamma^*)}{8(1-\delta\gamma^*)^2(1+2\delta\gamma^*)} < 0.
$$

Finally, straightforward computations yield

$$
H_M = -\delta\frac{a(\gamma^*)^2\,\delta\gamma^*(1-2\delta\gamma^*)}{4(1-\delta\gamma^*)^2(1+2\delta\gamma^*)^2} - \frac{a(\gamma^*)\,(1-2\delta\gamma^*)}{4(1-\delta\gamma^*)(1+2\delta\gamma^*)}\tau < 0;
$$

$$
H_D = \frac{a(\gamma^*)}{4(1-\delta\gamma^*)}\left(\frac{\delta a(\gamma^*)}{4(1-\delta\gamma^*)} + \frac{\tau}{2\delta\gamma^*}\right)(1-2\delta\gamma^*) > 0;
$$

$$
H_I = \frac{a(\gamma^*)}{4(1-\delta\gamma^*)}\left(\frac{\delta a(\gamma^*)}{4(1-\delta\gamma^*)} - \frac{\tau(1-2\delta\gamma^*)}{2\delta\gamma^*}\right) \lesseqgtr 0.
$$

Therefore, we obtain that

$$
H_M + H_D + H_I + H_\Delta = \frac{a(\gamma^*)\,(1-2\delta\gamma^*)}{4(1-\delta\gamma^*)(1+2\delta\gamma^*)}\left(\frac{a(\gamma^*)\,(1+4\delta^2\gamma^{*2})}{2(1-4\delta^2\gamma^{*2})(1-\gamma^*\delta)}\delta - \tau\right),
$$

The merger stimulates innovation if and only if

$$\frac{a\left(\gamma^*\right)\left(1+4\delta^2\gamma^{*2}\right)}{2(1-2\delta\gamma^*)(1+2\delta\gamma^*)(1-\delta\gamma^*)}\delta > \tau. \tag{12}$$

Using the fact that $a(\gamma^*) = \alpha + \tau\gamma^*$ and $\delta\gamma^* = (1-\rho^*)/2$, condition (12) is equivalent to

$$\alpha\delta\frac{(1+4\delta^2\gamma^{*2})}{2(1-2\delta\gamma^*)(1+2\delta\gamma^*)(1-\delta\gamma^*)} > \tau\left[1-\frac{\delta\gamma(1+4\delta^2\gamma^{*2})}{2(1-2\delta\gamma^*)(1+2\delta\gamma^*)(1-\delta\gamma^*)}\right],$$

that is,

$$\frac{\alpha\delta}{\tau} > -2+3\delta\gamma^* + \frac{4(1-\delta\gamma^*)}{1+4\delta^2\gamma^{*2}},$$

or

$$\frac{\alpha\delta}{\tau} > \frac{2+\rho^{*2}(5-3\rho^*)}{2\left[2-\rho^*(2-\rho^*)\right]}.$$

Plotting the term $(2+\rho^2(5-3\rho))/(2-\rho(2-\rho))$ shows that it is an increasing function of ρ.

Proof of Proposition 5

In the presence of spillovers, the counterparts to the first-order conditions (1) and (2) for the duopoly game are given by

$$(p-c)\frac{\partial D_i}{\partial p_i}\left(p,p,\hat{\gamma},\hat{\gamma}\right) + D_i\left(p,p,\hat{\gamma},\hat{\gamma}\right) = 0$$

and

$$(p-c)\left(\frac{\partial D_i}{\partial \hat{\gamma}_i} + \lambda\frac{\partial D_i}{\partial \hat{\gamma}_j}\right)\left(p,p,\hat{\gamma},\hat{\gamma}\right) = C'\left(\gamma\right),$$

respectively.

The counterparts to (3) and (4) for the multi-product monopoly are[24]

$$(p-c+\sigma)\left(\frac{\partial D_i}{\partial p_i} + \frac{\partial D_j}{\partial p_i}\right)\left(p,p,\hat{\gamma},\hat{\gamma}\right) + D_i\left(p,p,\hat{\gamma},\hat{\gamma}\right) = 0,$$

and

$$(p-c+\sigma)\left(1+\lambda\right)\left(\frac{\partial D_i}{\partial \hat{\gamma}_i} + \frac{\partial D_j}{\partial \hat{\gamma}_i}\right)\left(p,p,\hat{\gamma},\hat{\gamma}\right) = C'\left(\gamma\right).$$

respectively.

These first-order conditions show that the analysis leading to Proposition 1 in the base-

[24]To simplify the exposition, we assume away any synergies in production, i.e., $\sigma = 0$.

line model extends to an environment with spillovers as long as $\frac{\partial D_i}{\partial \gamma_i}$ and $\frac{\partial D_j}{\partial \gamma_i}$ are replaced with $\frac{\partial D_i}{\partial \hat{\gamma}_i} + \lambda \frac{\partial D_i}{\partial \hat{\gamma}_j}$ and $\frac{\partial D_j}{\partial \hat{\gamma}_i} + \lambda \frac{\partial D_j}{\partial \hat{\gamma}_j}$, respectively, and the arguments $(\tilde{p}^*(\gamma), \tilde{p}^*(\gamma), \gamma, \gamma)$ and $\left(\tilde{p}^M(\gamma), \tilde{p}^M(\gamma), \gamma, \gamma\right)$ are replaced with $(\tilde{p}^*(\hat{\gamma}), \tilde{p}^*(\hat{\gamma}), \hat{\gamma}, \hat{\gamma})$ and $\left(\tilde{p}^M(\hat{\gamma}), \tilde{p}^M(\hat{\gamma}), \hat{\gamma}, \hat{\gamma}\right)$, respectively.

Sign of the strategic effect on rival's price $\partial \tilde{p}_j^* / \partial \gamma_i$

Differentiating the first-order condition

$$(p_i^* - c) \frac{\partial D_i}{\partial p_i} \left(p_i^*, p_j^*, \gamma_i, \gamma_j\right) + D_i \left(p_i^*, p_j^*, \gamma_i, \gamma_j\right) = 0$$

with respect to γ_i and rearranging terms leads to

$$\left[2\frac{\partial D_i}{\partial p_i} + (\tilde{p}_i^* - c) \frac{\partial^2 D_i}{\partial p_i^2}\right] \frac{\partial \tilde{p}_i^*}{\partial \gamma_i} + \left[\frac{\partial D_i}{\partial p_j} + (\tilde{p}_i^* - c) \frac{\partial^2 D_i}{\partial p_i \partial p_j}\right] \frac{\partial \tilde{p}_j^*}{\partial \gamma_i} = -(\tilde{p}_i^* - c) \frac{\partial^2 D_i}{\partial p_i \partial \gamma_i} - \frac{\partial D_i}{\partial \gamma_i}.$$

Similarly, differentiating the same first-order condition with respect to γ_j and rearranging terms, we obtain

$$\left[\frac{\partial D_i}{\partial p_j} + (\tilde{p}_i^* - c) \frac{\partial^2 D_i}{\partial p_i \partial p_j}\right] \frac{\partial \tilde{p}_i^*}{\partial \gamma_j} + \left[2\frac{\partial D_i}{\partial p_i} + (\tilde{p}_i^* - c) \frac{\partial^2 D_i}{\partial p_i^2}\right] \frac{\partial \tilde{p}_j^*}{\partial \gamma_j} = -(\tilde{p}_i^* - c) \frac{\partial^2 D_i}{\partial p_i \partial \gamma_j} - \frac{\partial D_i}{\partial \gamma_j}.$$

Therefore, under the assumption of symmetric demand functions, the following system of linear equations in $\left(\partial \tilde{p}_i^* / \partial \gamma_i, \partial \tilde{p}_j^* / \partial \gamma_i\right)$ is satisfied in a symmetric equilibrium:

$$\begin{cases} A \frac{\partial \tilde{p}_i^*}{\partial \gamma_i} + B \frac{\partial \tilde{p}_j^*}{\partial \gamma_i} = E \\ B \frac{\partial \tilde{p}_i^*}{\partial \gamma_i} + A \frac{\partial \tilde{p}_j^*}{\partial \gamma_i} = F \end{cases} \tag{13}$$

where $A \equiv 2\frac{\partial D_i}{\partial p_i} + (\tilde{p}^* - c) \frac{\partial^2 D_i}{\partial p_i^2}$, $B \equiv \frac{\partial D_i}{\partial p_j} + (\tilde{p}^* - c) \frac{\partial^2 D_i}{\partial p_i \partial p_j}$, $E \equiv -(\tilde{p}^* - c) \frac{\partial^2 D_i}{\partial p_i \partial \gamma_i} - \frac{\partial D_i}{\partial \gamma_i}$ and $F \equiv -(\tilde{p}^* - c) \frac{\partial^2 D_i}{\partial p_i \partial \gamma_j} - \frac{\partial D_i}{\partial \gamma_j}$.

The solution to (13) is given by

$$\begin{cases} \frac{\partial \tilde{p}_i^*}{\partial \gamma_i} = \frac{AE - BF}{A^2 - B^2} \\ \frac{\partial \tilde{p}_j^*}{\partial \gamma_i} = \frac{AF - BE}{A^2 - B^2} \end{cases}$$

Assume that $\frac{\partial^2 D_i}{\partial p_i^2} \leq 0$, $\frac{\partial^2 D_i}{\partial p_i^2} + \frac{\partial^2 D_i}{\partial p_i \partial p_j} \geq 0$, and $\frac{\partial^2 D_i}{\partial p_i \partial \gamma_i} + \frac{\partial^2 D_i}{\partial p_i \partial \gamma_j} \geq 0$. This implies in particular that $A < 0 < B$, $E < 0 < F$, $A + B < 0$ and $E + F < 0$. It is straightforward to show that this set of inequalities implies that $\frac{\partial \tilde{p}_j^*}{\partial \gamma_i} < 0$.

Proof of Proposition 8

First, note that (γ_1^*, γ_2^*) is the unique solution of the following system of equations

$$\begin{cases} h_1^*(\gamma_1,\gamma_2) = C_1'(\gamma_1) \\ h_2^*(\gamma_1,\gamma_2) = C_2'(\gamma_2) \end{cases}, \tag{14}$$

and that $\left(\gamma_1^M,\gamma_2^M\right)$ is the unique solution of:

$$\begin{cases} h_1^M(\gamma_1,\gamma_2) = C_1'(\gamma_1) \\ h_2^M(\gamma_1,\gamma_2) = C_2'(\gamma_2) \end{cases}. \tag{15}$$

Denote by $R_1^*(\gamma_2)$ the unique solution of $h_1^*(\gamma_1,\gamma_2) = C_1'(\gamma_1)$ in γ_1 (the uniqueness is guaranteed by the assumption that $h_1^*(\gamma_1,\gamma_2) - C_1'(\gamma_1)$ is decreasing in γ_1) and by $R_2^*(\gamma_1)$ the unique solution of $h_2^*(\gamma_1,\gamma_2) = C_2'(\gamma_2)$ in γ_2. The functions $R_1^*(.)$ and $R_2^*(.)$ can be interpreted as the pre-merger reaction functions of a game in which firms choose their innovation levels and prices are set at their equilibrium levels.

Likewise, denote by $R_1^M(\gamma_2)$ the unique solution of $h_1^M(\gamma_1,\gamma_2) = C_1'(\gamma_1)$ in γ_1 (the uniqueness is guaranteed by Assumption 2') and by $R_2^*(\gamma_1)$ the unique solution of $h_2^*(\gamma_1,\gamma_2) = C_2'(\gamma_2)$ in γ_2.

Thus, we have

$$\gamma_1^* = R_1^*(\gamma_2^*) \quad ; \quad \gamma_2^* = R_2^*(\gamma_1^*)$$

and

$$\gamma_1^M = R_1^M\left(\gamma_2^M\right) \quad ; \quad \gamma_2^M = R_2^M\left(\gamma_1^M\right).$$

Differentiating $h_1^*(R_1^*(\gamma_2),\gamma_2) = C_1'(R_1^*(\gamma_2))$ with respect to γ_2 yields

$$\frac{dR_1^*}{d\gamma_2} = \frac{-\frac{\partial h_1^*}{\partial \gamma_2}(R_1^*(\gamma_2),\gamma_2)}{\frac{\partial h_1^*}{\partial \gamma_2}(R_1^*(\gamma_2),\gamma_2) - C_1''(R_1^*(\gamma_2))},$$

which has the same sign as $\frac{\partial h_1^*}{\partial \gamma_2}(R_1^*(\gamma_2),\gamma_2)$ (since the denominator is negative by assumption). Likewise $\frac{dR_2^*}{d\gamma_1}$, $\frac{dR_1^M}{d\gamma_2}$, $\frac{dR_2^M}{d\gamma_1}$ have the same signs as $\frac{\partial h_2^*}{\partial \gamma_1}(\gamma_1, R_2^*(\gamma_1))$, $\frac{\partial h_1^M}{\partial \gamma_2}\left(R_1^M(\gamma_2),\gamma_2\right)$, and $\frac{\partial h_2^M}{\partial \gamma_1}\left(\gamma_1, R_2^M(\gamma_1)\right)$, respectively.

(i) Assume that $h_1^*(\gamma_1,\gamma_2)$ is increasing in γ_2 and $h_2^*(\gamma_1,\gamma_2)$ is increasing in γ_1, and that $h_i^M(\gamma_1,\gamma_2) < h_i^*(\gamma_1,\gamma_2)$ for $i = 1,2$ and any (γ_1,γ_2). This implies that $R_1^*(.)$ and $R_2^*(.)$ are increasing and that $R_1^*(\gamma_2) > R_1^M(\gamma_2)$ and $R_2^*(\gamma_1) > R_2^M(\gamma_1)$ for any γ_1,γ_2. To see why the latter inequalities hold, notice that

$$h_1^M(R_1^*(\gamma_2),\gamma_2) - C_1'(R_1^*(\gamma_2)) < h_1^*(R_1^*(\gamma_2),\gamma_2) - C_1'(R_1^*(\gamma_2)) = 0 = h_1^M\left(R_1^M(\gamma_2),\gamma_2\right) - C_1'\left(R_1^M(\gamma_2)\right)$$

which, combined with the assumption that $h_1^M(\gamma_1,\gamma_2) - C_1'(\gamma_1)$ is decreasing in γ_1, leads to $R_1^*(\gamma_2) > R_1^M(\gamma_2)$.

Let us now show by way of contradiction that $\gamma_1^* > \gamma_1^M$. Note first that

$$R_1^* \circ R_2^*(\gamma_1^*) = \gamma_1^*.$$

Moreover, it holds that $R_1^* \circ R_2^*(\gamma_1) > \gamma_1$ for any $\gamma_1 < \gamma_1^*$ and $R_1^* \circ R_2^*(\gamma_1) < \gamma_1$ for any $\gamma_1 > \gamma_1^*$. To see why, notice that if the latter inequalities did not hold, then the equation $R_1^* \circ R_2^*(\gamma_1) = \gamma_1$ would have more than one solution because $R_1^* \circ R_2^*(0) \geq 0$, which would mean that there is more than one equilibrium; that would violate Assumption 1'.

Assume that $\gamma_1^* \leq \gamma_1^M$. This implies that

$$R_1^* \circ R_2^*\left(\gamma_1^M\right) \leq \gamma_1^M.$$

However, since $R_1^*(\gamma_2) > R_1^M(\gamma_2)$ and $R_2^*(\gamma_1) > R_2^M(\gamma_1)$ for any γ_1, γ_2, we have

$$R_1^* \circ R_2^*(\gamma_1) > R_1^M \circ R_2^M(\gamma_1)$$

for any γ_1; in particular

$$R_1^* \circ R_2^*\left(\gamma_1^M\right) > R_1^M \circ R_2^M\left(\gamma_1^M\right) = \gamma_1^M,$$

which yields a contradiction. Hence, $\gamma_1^* > \gamma_1^M$. Likewise, $\gamma_2^* > \gamma_2^M$.

Part (ii) can be proven in the same way.

Condition under which the post-merger innovation efforts are strategic complements in the Singh-Vives model

Consider the demand function

$$D_i\left(p_i, p_j, \gamma_i, \gamma_j\right) = \frac{\alpha + \tau\gamma_i - \left(\alpha + \tau\gamma_j\right)\rho - p_i + \rho p_j}{1-\rho^2}.$$

Denoting $a_i = \alpha + \tau\gamma_i$ and $a_j = \alpha + \tau\gamma_j$, the post-merger first-order condition with respect to p_i is

$$a_i - a_j\rho - p_i + \rho p_j - p_i + c_i + \rho\left(p_j - c_j\right) = 0,$$

which leads to the following post-merger prices

$$p_i = \frac{1}{2}\left(a_i + c_i\right);\ p_j = \frac{1}{2}\left(a_j + c_j\right)$$

Denoting $A_i = \alpha + \tau\gamma_i - c_i$, the profit with optimal prices is

$$\begin{aligned}\Pi\left(a_i, a_j, \rho\right) &= \frac{\frac{1}{4}A_i^2 + \frac{1}{4}A_j^2 - \frac{1}{2}A_iA_j\rho}{1-\rho^2} \\ &= \frac{1}{4}\frac{\left(A_i - A_j\right)^2}{1-\rho^2} + \frac{1}{2}\frac{A_iA_j}{1+\rho}.\end{aligned}$$

Hence

$$\frac{\partial\Pi}{\partial\rho} = \frac{2\rho}{\left(1-\rho^2\right)^2}\frac{1}{4}\left(A_i - A_j\right)^2 - \frac{1}{2}\frac{A_iA_j}{\left(1+\rho\right)^2}$$

and

$$\frac{\partial^2\Pi}{\partial\rho^2} = \frac{1+3\rho^2}{\left(1-\rho^2\right)^3}\frac{1}{2}\left(A_i - A_j\right)^2 + \frac{A_iA_j}{\left(1+\rho\right)^3}.$$

Considering now the derivative with respect to γ_i, we have

$$\frac{\partial\Pi}{\partial\gamma_i} = -\delta\frac{\partial\Pi}{\partial\rho} + \frac{\tau}{2}\left(\frac{A_i - A_j}{1-\rho^2} + \frac{A_j}{1+\rho}\right).$$

Differentiating this with respect to ρ yields

$$\frac{\partial^2\Pi}{\partial\rho\partial\gamma_j} = -\frac{2\rho}{\left(1-\rho^2\right)^2}\frac{\tau}{2}\left(A_i - A_j\right) - \frac{\tau}{2}\frac{A_i}{\left(1+\rho\right)^2}$$

Thus, a sufficient condition for post-merger strategic complementarity, i.e., $\frac{\partial^2\Pi}{\partial\gamma_i\partial\gamma_j} > 0$ is

$$\delta\frac{A_i + A_j}{1+\rho} > \tau\frac{\rho}{1-\rho},$$

where $\rho = 1 - \delta\left(\gamma_1 + \gamma_2\right)$. Using the expressions of A_i and A_j, this condition can be written as

$$2\alpha - c_1 - c_2 > \frac{\tau}{\delta}\left(\frac{3\rho - 1}{1-\rho}\right). \tag{16}$$

It is easy to see that Condition (16) is more likely to hold the smaller ρ and the larger α. Note that it is compatible with the condition under which $H_I < 0$, i.e.,

$$2\alpha < \frac{\tau}{\delta}2\left(\frac{-1 + 4\rho + \rho^2}{1-\rho}\right).$$

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