Mergers and Demand-Enhancing Innovation*

Marc Bourreau† Bruno Jullien‡ Yassine Lefouili§

July 2018

Preliminary version - Comments welcome

Abstract

This paper investigates the impact of horizontal mergers on firms' incentives to invest in demand-enhancing innovation. In our baseline model, we identify three key effects of a merger on the merging firms' incentives to innovate: the margin expansion effect, the demand expansion effect, and the innovation diversion effect. The first effect is negative, while the second is positive and the third can be either positive or negative depending on the nature of the innovation. We show that the overall impact of a merger on innovation can be either positive or negative and provide sufficient conditions and specific models under which each of these two scenarios arises. Finally, we extend our model to incorporate spillovers and synergies in R&D.

Keywords: Horizontal Mergers, Innovation, Competition.

JEL Classification: D43, L13, L40.

---

*We are grateful to Giulio Federico, Massimo Motta, Volker Nocke, Jorge Padilla, Michele Polo, Patrick Rey, Mathias Reynaert, Nicolas Schultz, Tim Simcoe, Emmanuele Tarantino, Tommaso Valletti, and the participants at TSE Competition Policy Workshop and CRESSE 2018 for useful discussions. The financial support of the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No 670494) is gratefully acknowledged. Jullien and Lefouili have received funding from Baker McKenzie and Compass Lexecon for a related policy paper entitled "Horizontal Mergers and Innovation".

†Telecom ParisTech, Paris, France; Email: marc.bourreau@telecom-paristech.fr.

‡Toulouse School of Economics, CNRS, University of Toulouse Capitole, Toulouse, France; E-mail: bruno.jullien@tse-fr.eu.

§Toulouse School of Economics, University of Toulouse Capitole, Toulouse, France; E-mail: yassine.lefouili@tse-fr.eu.

1 Introduction

Motivation. The debate about the effect of horizontal mergers on innovation (Baker, 2007; Katz and Shelanski, 2007; Shapiro, 2012) has been recently invigorated by the European Commission's decision in the merger between Dow and Dupont.[1] Despite the intensity of this debate, the various effects of a merger on innovation are not fully understood and we still lack a general theoretical framework that would help decide when a merger is likely to reduce or increase investments in product innovation. This paper is an additional step towards building such a general framework.

Contribution and results. We construct a model to investigate the effect of a merger on demand-enhancing innovation. More specifically, we study the merged entity's incentives to increase its innovation efforts for given rivals' behavior; in other words, we focus on the *initial impetus* (Farrell and Shapiro, 2010; Federico et al., 2018). To ease the exposition, we consider the effect on innovation of a merger to monopoly (where the initial impetus is the only effect at work) in a symmetric context, assuming that the outcome of innovation is deterministic and that investment levels are not observed by competitors prior to price competition taking place.[2]

We first derive a necessary and sufficient condition for a merger to reduce the equilibrium level of innovation, allowing for (potential) small synergies in production.[3] We then propose a decomposition of the overall impact of a merger on the incentives to innovate. This decomposition shows that the impact of a merger is a combination of potentially opposite effects and that it can be either positive or negative, depending on the context. More specifically, in our baseline model, we identify the following effects of a merger on the merging firms' incentives to innovate. First, the merger affects the merging firms' output and, therefore, their incentives to innovate in order to increase their margin. We call this the *margin expansion effect*. This effect is negative if synergies in production are small (or absent), because the merger leads to a lower output by the merging firms for a given innovation level. Second, the merger affects the merged firm's margin and, therefore, its incentives to innovate in order to increase demand.

[1] Case M.7932 – *Dow/DuPont*. The European Commission's March 2017 decision is available at http://ec.europa.eu/competition/mergers/cases/decisions/m7932_13668_3.pdf

[2] In doing so, we follow Motta and Tarantino (2017)'s main analysis of the effects of a merger on investment in cost-reducing technologies. Note that Motta and Tarantino (2017) consider an extension where competitors observe a firm's investment prior to setting prices and show that their results become ambiguous in general under this assumption.

[3] By "small" synergies, we mean synergies that are insufficient for the merger to reduce prices for a given innovation level.

This *demand expansion effect* is positive as a merger tends to increase margins. Third, a merger induces an *innovation diversion effect*: it leads to the internalization of the impact that each merging firm's innovation investment has on the other merging firm's profit. For given prices, the sign of this externality depends on the impact of an increase in one merging firm's investment on the other merging firm's demand, and can be either positive or negative depending on the nature of the innovation. The innovation diversion effect is negative (resp., positive) when this externality is negative (resp., positive). Importantly, we show that the sign of the sum of the demand expansion effect and the innovation diversion effect is the same as the sign of the difference between the price diversion ratio, commonly used by competition authorities to perform "upward pricing analysis" (Farrell and Shapiro, 2010), and the innovation diversion ratio, which is its innovation counterpart (Farrell and Shapiro, 2010; Salinger 2016). A final element of our decomposition accounts for the effect of the merger on the return to innovation per unit of output, an effect that may mitigate or exacerbate the demand expansion effect.

We apply our decomposition of the total impact of a merger on innovation to a number of specific models commonly used in the literature. This exercise shows that the overall impact of the merger can be either positive or negative and that the sign of this effect is partially driven by the "horizontal" or "vertical" nature of innovation. Building on this, we develop a simple model in which innovation has both a horizontal dimension and a vertical one, and show that the overall effect of a merger is either positive or negative depending on which dimension matters more.

Finally, we extend our baseline model to account for technological spillovers, synergies in R&D and large synergies in production. We show that our decomposition can be adapted in a very natural way to incorporate the effects of spillovers and synergies. A key insight from our analysis is that (non-R&D) synergies in production matter not only for the effect of a merger on prices but also for its effect on the incentives to invest in R&D. In particular, the fact that synergies in production may lead to an increase in output (for a given innovation level) implies that the margin expansion effect may become *positive*.

Related literature. While there is a vast literature on the effect of competition on innovation,[4] the literature addressing the specific question of the impact of mergers on innovation is rather scarce. Motta and Tarantino (2017) focus primarily on cost-reducing investments and argue that a merger leads to less investment by the merged

[4]See Gilbert (2006) for a recent survey.

entity in the absence of spillovers and efficiency gains.[5] They also consider the subset of quality-improving investments whose formal analysis boils down to that of cost-reducing of investments. Federico et al. (2017, 2018) and Denicolò and Polo (2018a) analyze the effect of a merger on product innovation in a setting where investment in R&D affects the probability of success but *not* the value of the innovation. In particular, Denicolò and Polo (2018a) show that there are cases where a merger fosters innovation and cases where it impedes it. Denicolò and Polo (2018b) further show that a merger increase the merging firms' incentives to innovate because it allows them to share R&D knowledge and technologies. Bourreau and Jullien (2018) consider investments in coverage for a new technology and highlight the importance of the demand expansion effect. By contrast, this paper examines a broad class of investments in *demand-enhancing innovation*, which includes quality-improving investments as a special case. Loertscher and Marx (2018) analyze the competitive effects of mergers in markets with buyer power and show, in this context, that a merger increases investment incentives for rivals, and can increase investment incentives for merging parties. Davidson and Ferrett (2007) emphasize the importance of R&D synergies in shaping the profitability of a merger, while our analysis of the implications of R&D synergies focuses on the way they affect innovation. Finally, Ghandi et al. (2008) show that post-merger product repositioning can mitigate concerns about raising prices.

Organization of the paper. We lay out our baseline model and state the equilibrium conditions defining price and innovation levels in Section 2. We present our main decomposition of the overall impact of a merger on innovation in Section 3 and apply it to a number of well-known models in Section 4. In Section 6 we consider extensions of our baseline model accounting for technological spillovers, synergies in R&D and large synergies in production. Section 7 presents an alternative decomposition of the impact of a merger on innovation. Section 8 concludes.

2 Baseline model

Consider two (single-product) firms producing differentiated goods. The firms compete in prices and can invest in innovation to increase the demand for their products. Let c denote the firms' marginal cost of production and $C(\gamma_i)$ the investment cost a firm needs to incur to achieve an innovation level γ_i. We assume that $C(.)$ is convex, with

[5] See also Matsushima et al. (2013) who investigate the effects of a merger when heterogeneous oligopolists compete both in process R&D and on the product market.

$C(0) = 0$. We consider that firms set their prices and innovation levels simultaneously (or, equivalently, that a firm does not observe its rival's price before undertaking its investment in innovation).

We assume that innovation affects the demand for both products, but not their marginal cost of production. Let $D_i(p_i, p_j, \gamma_i, \gamma_j)$ denote the demand addressed to firm i when it sets its price and innovation level at p_i and γ_i, and its rival sets its price and innovation level at p_j and γ_j. Furthermore, assume that the demand functions are symmetric, i.e., $D_i(p_i, p_j, \gamma_i, \gamma_j) = D_j(p_i, p_j, \gamma_i, \gamma_j)$ for any $(p_i, p_j, \gamma_i, \gamma_j)$. As usual, we suppose that a firm's demand is decreasing in its own price and decreasing in its rival's price, i.e., $\partial D_i/\partial p_i < 0$ and $\partial D_i/\partial p_j > 0$. Moreover, we assume that an increase in a firm's innovation level leads to an increase in its own demand, i.e., $\partial D_i/\partial \gamma_i > 0$. However, we do not put any restriction on the sign of $\partial D_i/\partial \gamma_j$. Our analysis therefore applies to both demand-enhancing innovations that have a negative effect on rival's demand (which is the focus of a large part of the literature) and those that have a positive effect on rival's demand (see the example we provide in Section 4.4 and Lin and Saggi, 2002). Finally, we make the (standard) assumption that $\partial D_i/\partial p_i + \partial D_i/\partial p_j < 0$ (i.e., own effects dominate cross-effects) at symmetric price and innovation levels $p_i = p_j$ and $\gamma_i = \gamma_j$. We also make a similar (reasonable) assumption regarding the effect of a uniform increase in innovation levels: $\partial D_i/\partial \gamma_i + \partial D_i/\partial \gamma_j > 0$ at symmetric price and innovation levels $p_i = p_j$ and $\gamma_i = \gamma_j$ (note that this condition is always satisfied if $\partial D_i/\partial \gamma_j > 0$, as the cross effect has then a positive sign).[6]

Consider first the benchmark situation where the firms act independently. Assume that the pricing game derived from the price-innovation game defined above by fixing the innovation levels of the two firms to the (same) level γ has a symmetric equilibrium, which we assume to be unique for conciseness.[7] The corresponding equilibrium price $\tilde{p}^*(\gamma)$ is the solution to the following first-order condition:

$$(p - c)\frac{\partial D_i}{\partial p_i}(p, p, \gamma, \gamma) + D_i(p, p, \gamma, \gamma) = 0. \tag{1}$$

Likewise, assume that the innovation game derived from price-innovation game by fixing the prices of both firms to p has a unique symmetric equilibrium. The corresponding

[6]Note that the assumption that $\partial D_i/\partial \gamma_i + \partial D_i/\partial \gamma_{-i} > 0$ at symmetric price and innovation levels is equivalent to the assumption that an increase in one firm's innovation level (starting from a symmetric situation) has a positive effect on aggregate demand, i.e., $\partial D_i/\partial \gamma_i + \partial D_{-i}/\partial \gamma_i > 0$ at symmetric price and innovation levels.

[7]See Vives (1999) for sufficient conditions on the demand functions ensuring the existence and uniqueness of the equilibrium of the price competition game with differentiated products.

equilibrium innovation level $\tilde{\gamma}^*(p)$ is the solution to the following first-order condition:

$$(p-c)\frac{\partial D_i}{\partial \gamma_i}(p,p,\gamma,\gamma) = C'(\gamma). \tag{2}$$

Finally, we make the following assumption regarding the equilibrium of the price-innovation game.

Assumption 1: The duopoly price-innovation game has a unique symmetric equilibrium, and the corresponding equilibrium price and innovation levels p^* and γ^* are such that $\gamma^* > 0$.

Consider now a merger to monopoly. We allow the merger to induce some synergies in production; by merging, the two firms can reduce the marginal cost of their production entities from c to $c-\delta$, with $\delta \in [0,c]$. The merged entity's (monopoly) profit for levels of investments γ_1 and γ_2 is then given by

$$\Pi^M(\gamma_1,\gamma_2) \equiv \max_{p_1,p_2}(p_1-c+\delta)\left[D_1(p_1,p_2,\gamma_1,\gamma_2)+D_2(p_2,p_1,\gamma_2,\gamma_1)\right]-C(\gamma_1)-C(\gamma_2).$$

We assume that this monopoly maximization problem is well behaved in the following sense:

Assumption 2: The profit function $\Pi^M(\gamma_1,\gamma_2)$ is strictly quasi-concave in (γ_1,γ_2).

Under this assumption, the merged entity's optimal innovation strategy is symmetric, and hence, we can restrict our attention to a single innovation level for both components of the merged entity, i.e., $\gamma_1=\gamma_2=\gamma$. For any given innovation level γ that applies to both products, the merged entity's optimal symmetric price $\tilde{p}^M(\gamma)$ is defined by the following first-order condition:

$$(p-c+\delta)\left[\frac{\partial D_i}{\partial p_i}(p,p,\gamma,\gamma)+\frac{\partial D_j}{\partial p_i}(p,p,\gamma,\gamma)\right]+D_i(p,p,\gamma,\gamma)=0. \tag{3}$$

Let us make the natural assumption that, for a given innovation level, the merger leads to a higher price:

Assumption 3: $\tilde{p}^M(\gamma) > \tilde{p}^*(\gamma)$ for any $\gamma \geq 0$.

Assumption 3 means that synergies in production are sufficiently small so that their (negative) effect on prices is outweighed by the market power effect of the merger.[8]

[8]This is true in particular in the special (benchmark) case where a merger does not induce any synergies in production.

For any given symmetric prices, the merged entity's optimal innovation level is solution to the following first-order condition:

$$(p - c + \delta)\left[\frac{\partial D_i}{\partial \gamma_i}(p, p, \gamma, \gamma) + \frac{\partial D_j}{\partial \gamma_i}(p, p, \gamma, \gamma)\right] = C'(\gamma). \tag{4}$$

An optimal symmetric price-innovation pair $\left(p^M, \gamma^M\right)$ for the merged entity satisfies conditions (3) and (4).

The general idea behind the subsequent analysis is to use the first-order conditions to eliminate marginal costs and focus on equilibrium prices, innovation levels and demands. Let us first define an *independent firm's marginal gain from innovation* as

$$h^*(\gamma) \equiv -D_i(\tilde{p}^*(\gamma), \tilde{p}^*(\gamma), \gamma, \gamma)\frac{\frac{\partial D_i}{\partial \gamma_i}(\tilde{p}^*(\gamma), \tilde{p}^*(\gamma), \gamma, \gamma)}{\frac{\partial D_i}{\partial p_i}(\tilde{p}^*(\gamma), \tilde{p}^*(\gamma), \gamma, \gamma)}.$$

From (1) and (2), one can see that $h^*(\gamma)$ is firm i's marginal gain from an increase in its innovation level γ_i, when its price is set optimally, holding constant the innovation and price levels of firm j at γ and $p^*(\gamma)$, respectively.

Similarly, we define the *merged entity's marginal gain from innovation* as

$$h^M(\gamma) \equiv -D_i\left(\tilde{p}^M(\gamma), \tilde{p}^M(\gamma), \gamma, \gamma\right)\frac{\frac{\partial D_i}{\partial \gamma_i}\left(\tilde{p}^M(\gamma), \tilde{p}^M(\gamma), \gamma, \gamma\right) + \frac{\partial D_j}{\partial \gamma_i}\left(\tilde{p}^M(\gamma), \tilde{p}^M(\gamma), \gamma, \gamma\right)}{\frac{\partial D_i}{\partial p_i}\left(\tilde{p}^M(\gamma), \tilde{p}^M(\gamma), \gamma, \gamma\right) + \frac{\partial D_j}{\partial p_i}\left(\tilde{p}^M(\gamma), \tilde{p}^M(\gamma), \gamma, \gamma\right)}.$$

This expression corresponds to the slope of the merged entity's profit function with respect to γ_i (at $\gamma_i = \gamma$), when all prices are set optimally, holding constant the innovation level of the other research unit (at $\gamma_j = \gamma$).

Based on these definitions, the following proposition provides a sufficient condition for a merger to monopoly to induce an increase (resp., decrease) in innovation.

Proposition 1 *A necessary and sufficient condition for the merger to lead to an increase (resp., decrease) in innovation is that $h^M(\gamma^*)$ is greater (resp., lower) than $h^*(\gamma^*)$.*

Proof. From the positivity of γ^*, and from (1) and (2), it follows that $h^*(\gamma^*) = C'(\gamma^*)$. Moreover, from (3) and (4), it follows that $\frac{d\Pi^M}{d\gamma}(\gamma, \gamma) = 2\left[h^M(\gamma) - C'(\gamma)\right]$.

Assumption 2 implies that $\gamma^M > \gamma^*$ if and only if $\frac{d\Pi^M}{d\gamma}(\gamma^*, \gamma^*) > 0$, which yields the result. ■

3 Decomposition of the overall effect of a merger on innovation

Proposition 1 states that the effect of the merger on innovation depends on how the change in prices (i.e., the market power effect of the merger) affects the marginal gain from innovation, evaluated at the level of innovation prevailing prior to the merger. In this section, we analyze how the merger affects the marginal gain from innovation, and show that the impact of the merger on innovation depends on a combination of various effects.

To highlight the first effect of the merger on innovation, we isolate the terms that depend explicitly on the demand for the other product. Let us define the monopoly output at innovation level γ as:

$$D_i^M\left(\gamma\right) \equiv D_i\left(\tilde{p}^M\left(\gamma\right), \tilde{p}^M\left(\gamma\right), \gamma, \gamma\right).$$

Eliminating the terms related to the impact of innovation on the demand for product j in the merged entity's marginal gain from innovation $h^M\left(\gamma\right)$, we define

$$\hat{h}^M\left(\gamma\right) \equiv -D_i^M\left(\gamma\right)\frac{\frac{\partial D_i}{\partial \gamma_i}\left(\tilde{p}^M\left(\gamma\right), \tilde{p}^M\left(\gamma\right), \gamma, \gamma\right)}{\frac{\partial D_i}{\partial p_i}\left(\tilde{p}^M\left(\gamma\right), \tilde{p}^M\left(\gamma\right), \gamma, \gamma\right) + \frac{\partial D_j}{\partial p_i}\left(\tilde{p}^M\left(\gamma\right), \tilde{p}^M\left(\gamma\right), \gamma, \gamma\right)}.$$

This leads to the following decomposition:

$$h^M\left(\gamma\right) - h^*\left(\gamma\right) = \hat{h}^M\left(\gamma\right) - h^*\left(\gamma\right) + H_I,$$
$$\text{where } H_I = \hat{h}^M\left(\gamma\right)\frac{\frac{\partial D_j}{\partial \gamma_i}\left(\tilde{p}^M\left(\gamma\right), \tilde{p}^M\left(\gamma\right), \gamma, \gamma\right)}{\frac{\partial D_i}{\partial \gamma_i}\left(\tilde{p}^M\left(\gamma\right), \tilde{p}^M\left(\gamma\right), \gamma, \gamma\right)} \lesseqgtr 0.$$

The term H_I captures the internalization by the merged entity of the *diversion* of sales that (demand-enhancing) innovation in one product induces for the other product. This term, which we refer to as the *innovation diversion effect*, can be either negative or positive, depending on whether $\partial D_j/\partial\gamma_i$ is negative or positive.

The remaining term in the decomposition, $\hat{h}^M - h^*$, compares the pre-merger and post-merger incentives to raise the demand of product i through innovation. Therefore, it captures how an increase in *market power* due to the merger affects the incentives to innovate. When a firm increases its investment in innovation from γ_i to $\gamma_i + d\gamma_i$, its return depends on how prices are adjusted. If prices are only slightly adjusted, the main motivation for innovation is to expand sales volume. By contrast, if prices are adjusted such that the volume is only slightly affected by innovation, then the motivation is mostly to expand one's margin. In the case where innovation is mostly motivated by margin expansion, a lower supply following the merger reduces the merging firms' incentives to innovate in order to increase their prices and, therefore, their margins. We refer to this effect as the *margin expansion effect*. In the case where prices are maintained when innovation increases, a higher margin following the merger raises the merging firms' incentives to innovate in order to raise demand. We refer to this effect as the *demand expansion effect*.[9]

More formally, we can write the gain from changing the price and innovation levels from (p_i, γ_i) to (p_i', γ_i'), holding the price and innovation levels of the other firm constant at $(p_j, \gamma_j) = (p, \gamma)$, as:

$$\begin{aligned} & (p_i' - c) D_i (p_i', p, \gamma_i', \gamma) - (p_i - c) D_i (p_i, p, \gamma_i, \gamma) \\ = \; & (p_i' - c) (D_i (p_i', p, \gamma_i', \gamma) - D_i (p_i, p, \gamma_i, \gamma)) + (p_i' - p_i) D_i (p_i, p, \gamma_i, \gamma) . \end{aligned}$$

The margin expansion effect dominates the demand expansion effect if $D_i (p_i', p, \gamma_i', \gamma) - D_i (p_i, p, \gamma_i, \gamma)$ is (relatively) small, while the opposite is true if $p_i' - p_i$ is (relatively) small. Which effect dominates ultimately depends on the price-elasticity of demand at γ_i and γ_i'. In particular, when this elasticity is not affected by innovation, demand expansion is the main driver of innovation efforts.

Let us now define the (pre-merger) *per unit return to innovation* as

$$g^* (p, \gamma) \equiv -\frac{\frac{\partial D_i}{\partial \gamma_i} (p, p, \gamma, \gamma)}{\frac{\partial D_i}{\partial p_i} (p, p, \gamma, \gamma)}.$$

This ratio measures the price increase that the firm can achieve when it increases innovation at the margin and raises prices so as to maintain the volume of sales constant. Therefore, it can be interpreted as the return to innovation per unit of output. The *independent firm's marginal gain from innovation* can then be written as the product of the volume of output and the per unit return to innovation:

$$h^* (\gamma) = D_i (\tilde{p}^* (\gamma), \tilde{p}^* (\gamma), \gamma, \gamma) \, g^* (\tilde{p}^* (\gamma), \gamma) .$$

Defining $D_i^* (\gamma) \equiv D_i (\tilde{p}^* (\gamma), \tilde{p}^* (\gamma), \gamma, \gamma)$, the effect of the merger on the firms'

[9]An original form of the demand expansion effect can be found in Loertscher and Marx (2018).

investments in demand-enhancing innovation, which is given by $\hat{h}^M(\gamma) - h^*(\gamma)$ (net of H_I), can be decomposed as follows:

$$\hat{h}^M(\gamma) - h^*(\gamma) = \underbrace{\left[D_i^M(\gamma) - D_i^*(\gamma)\right] g^*(\tilde{p}^*(\gamma), \gamma)}_{\equiv H_M} + \left[\left(-\frac{\frac{\partial D_i}{\partial \gamma_i}}{\frac{\partial D_i}{\partial p_i} + \frac{\partial D_j}{\partial p_i}}\right) - g^*(\tilde{p}^*(\gamma), \gamma)\right] D_i^M(\gamma),$$

where the derivatives are evaluated at $\left(\tilde{p}^M(\gamma), \tilde{p}^M(\gamma), \gamma, \gamma\right)$.

The first term H_M measures the effect of the change in output on innovation incentives. To obtain further insights, we decompose the remaining term into two terms, H_p and H_Δ, to derive our final decomposition:

Proposition 2 *The change in innovation incentives induced by the merger can be decomposed as follows:*

$$h^M(\gamma^*) - h^*(\gamma^*) = H_M + H_D + H_I + H_\Delta,$$

where

$$H_M \equiv \left[D_i^M(\gamma^*) - D_i^*(\gamma^*)\right] g^*(p^*, \gamma^*) < 0,$$

$$H_D \equiv \hat{h}^M(\gamma^*) \times \left(\frac{\frac{\partial D_j}{\partial p_i}}{-\frac{\partial D_i}{\partial p_i}}\right) > 0, \; H_I \equiv \hat{h}^M(\gamma^*) \times \left(\frac{\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial \gamma_i}}\right),$$

$$H_\Delta \equiv D_i^M(\gamma^*) \left[g^*\left(p^M(\gamma^*), \gamma^*\right) - g^*(p^*, \gamma^*)\right] \lesseqgtr 0,$$

and all the derivatives are evaluated at $\left(\tilde{p}^M(\gamma^*), \tilde{p}^M(\gamma^*), \gamma^*, \gamma^*\right)$.

The term H_M captures the *margin expansion effect* discussed above. It is always negative under our assumption that the merger raises prices, i.e., $\tilde{p}^M(\gamma) > \tilde{p}^*(\gamma)$. The term H_D is always positive and is larger the greater the magnitude of the derivative $\partial D_j / \partial p_i$, which drives the merged entity's incentives to increase prices (for a given innovation level) with respect to the situation in which firms set their prices independently. Since a higher price (and, therefore, a higher margin) provides the merged entity with higher incentives to increase demand, we interpret term H_D as capturing the *demand expansion effect*. Finally, the term H_Δ is a correction for the demand expansion effect due to the fact that the effect is not evaluated at the same level of prices pre-merger and post-merger. It measures the change in the per unit return to innovation and can be positive or negative depending on whether $g^*(p, \gamma^*)$ is increasing or decreasing in p.

Importantly, the term $H_D + H_I$, i.e., the combination of the demand expansion effect and the innovation diversion effect, has the same sign as the difference between the price diversion ratio[10] and the innovation diversion ratio, i.e.,

$$\underbrace{\frac{\frac{\partial D_j}{\partial p_i}}{-\frac{\partial D_i}{\partial p_i}}}_{\text{price diversion ratio}} - \underbrace{\frac{-\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial \gamma_i}}}_{\text{innovation diversion ratio}},$$

evaluated at $\left(\tilde{p}^M(\gamma^*), \tilde{p}^M(\gamma^*), \gamma^*, \gamma^*\right)$. This difference is always positive when the innovation diversion ratio is negative (i.e., when an increase in the innovation effort of a firm leads to an increase in its rival's demand). However, when the innovation diversion ratio is positive, the difference between the two ratios can be either positive or negative depending on their relative magnitudes.

Based on the analysis in this section, we can now provide a sufficient condition for the merger to reduce innovation.

Proposition 3 *The merger reduces innovation if the price diversion ratio is smaller than the innovation diversion ratio and the pre-merger per unit return to innovation $g^*(p, \gamma)$ is non-increasing in prices.*

4 Applications

In this section we apply our decomposition to various standard classes of models. First, this allows us to show that our approach is operational for these settings. Second, it makes it possible to identify models in which we obtain unambiguous predictions on the impact of a merger on innovation. In particular, we provide examples for both the case where the overall impact is negative and the case where it is positive. Finally, for those models where a merger impedes innovation absent synergies, our approach shows that this conclusion extends to the case with synergies as long as the merger raises prices at fixed innovation levels.

[10] This ratio is used in particular in upward pricing pressure (UPP) analysis (see, e.g., Farrell and Shapiro, 2010).

4.1 Model with hedonic prices

A first commonly used class of models assumes that there exist functions $Q(.,.)$ and $v(.)$ such that

$$D_i(p_i, p_j, \gamma_i, \gamma_j) = Q(p_i - v(\gamma_i), p_j - v(\gamma_j)).$$

With this formulation, the innovation level γ_i can be interpreted as the quality of product i, and the demand for each product depends on quality-adjusted (hedonic) prices. We find that the terms $H_D + H_I$ and H_Δ are equal to *zero* under this specification. To see why, note that

$$\frac{\frac{\partial D_i}{\partial \gamma_i}(p,p,\gamma,\gamma)}{-\frac{\partial D_i}{\partial p_i}(p,p,\gamma,\gamma)} = \frac{-\frac{\partial D_j}{\partial \gamma_i}(p,p,\gamma,\gamma)}{\frac{\partial D_j}{\partial p_i}(p,p,\gamma,\gamma)} = v'(\gamma)$$

does not depend on p, which implies that $H_\Delta = 0$, and that

$$\frac{-\frac{\partial D_j}{\partial \gamma_i}(p,p,\gamma,\gamma)}{\frac{\partial D_i}{\partial \gamma_i}(p,p,\gamma,\gamma)} = \frac{\frac{\partial D_j}{\partial p_i}(p,p,\gamma,\gamma)}{-\frac{\partial D_i}{\partial p_i}(p,p,\gamma,\gamma)},$$

which means that the innovation diversion ratio is equal to the price diversion ratio, and hence, $H_D + H_I = 0$.

Therefore, in this class of models, our decomposition boils down to $h^M(\gamma^*) - h^*(\gamma^*) = H_M < 0$, which implies that the merger leads to a reduction of investments in demand-enhancing innovation. The analysis above shows that this decrease in investment is solely driven by the margin expansion effect, as would be the case for cost-reducing investments.[11]

4.2 Quality-adjusted model

Consider now a system of demand functions for which there exists $Q(.,.)$ such that

$$D_i(p_i, p_j, \gamma_i, \gamma_j) = \frac{1}{\gamma_i} Q\left(\frac{p_i}{\gamma_i}, \frac{p_j}{\gamma_j}\right).$$

Such a system of demand functions is considered for instance by Motta and Tarantino (2017). Denote by $s_i = p_i/\gamma_i$ the price-quality ratio, where γ_i is interpreted again as

[11] The only difference between the margin expansion effect in a cost-reducing innovation setting (Motta and Tarantino, 2017) and our demand-enhancing innovation setting is that firms increase their margins by decreasing their marginal costs instead of increasing their prices.

the quality of firm i's product. We have then

$$D_i\left(p_i, p_j, \gamma_i, \gamma_j\right) = \frac{1}{\gamma_i} Q(s_i, s_j).$$

Let us denote by Q_l the derivative of Q with respect to the lth variable. We assume that $Q_1 < 0 < Q_2$ and $Q + sQ_1 < 0$, so that innovation raises demand. The price diversion ratio and the innovation diversion ratio at symmetric price and innovation levels are given by

$$\frac{\frac{\partial D_j}{\partial p_i}\left(p, p, \gamma, \gamma\right)}{-\frac{\partial D_i}{\partial p_i}\left(p, p, \gamma, \gamma\right)} = \frac{\frac{1}{\gamma^2} Q_2\left(s, s\right)}{-\frac{1}{\gamma^2} Q_1\left(s, s\right)} = \frac{Q_2\left(s, s\right)}{-Q_1\left(s, s\right)} > 0,$$

and

$$\frac{-\frac{\partial D_j}{\partial \gamma_i}\left(p, p, \gamma, \gamma\right)}{\frac{\partial D_i}{\partial \gamma_i}\left(p, p, \gamma, \gamma\right)} = \frac{\frac{p}{\gamma^3} Q_2\left(s, s\right)}{-\frac{p}{\gamma^3} Q_1\left(s, s\right) - \frac{1}{\gamma^2} Q\left(s, s\right)} = \frac{sQ_2\left(s, s\right)}{-sQ_1\left(s, s\right) - Q\left(s, s\right)},$$

which is positive under our assumptions on Q_1 and Q_2. Since the innovation diversion ratio is greater than the price diversion ratio, we have $H_D + H_I < 0$.

The term H_Δ depends on the slope of $g^*\left(p, \gamma\right)$ where

$$g^*\left(p, \gamma\right) = -\frac{\frac{\partial D_i}{\partial \gamma_i}\left(p, p, \gamma, \gamma\right)}{\frac{\partial D_i}{\partial p_i}\left(p, p, \gamma, \gamma\right)} = \frac{-\frac{p}{\gamma^3} Q_1\left(s, s\right) - \frac{1}{\gamma^2} Q\left(s, s\right)}{-\frac{1}{\gamma^2} Q_1\left(s, s\right)} = s + \frac{Q\left(s, s\right)}{Q_1\left(s, s\right)}.$$

This term is increasing in p for many usual demand functions. In this case, the overall effect is *a priori* ambiguous because of two opposite effects, i.e., $H_M + H_D + H_I < 0$, while $H_\Delta > 0$.

To evaluate the overall impact of the merger on innovation, let us notice that the equilibrium competitive and monopoly prices for $\gamma = \gamma^*$ satisfy

$$\begin{aligned} \gamma^*\left(s^* + \frac{Q\left(s^*, s^*\right)}{Q_1\left(s^*, s^*\right)}\right) &= c \\ \gamma^*\left(s^{M*} + \frac{Q\left(s^{M*}, s^{M*}\right)}{Q_1\left(s^{M*}, s^{M*}\right) + Q_2\left(s^{M*}, s^{M*}\right)}\right) &= c - \delta, \end{aligned}$$

where $s^* = \tilde{p}^*\left(\gamma^*\right)/\gamma^*$ and $s^{M*} = \tilde{p}^M\left(\gamma^*\right)/\gamma^*$. The marginal gains of innovation, pre- and post-merger, can then be written as

$$h^*\left(\gamma^*\right) = \frac{Q\left(s^*, s^*\right)}{\gamma^*}\left(s^* + \frac{Q\left(s^*, s^*\right)}{Q_1\left(s^*, s^*\right)}\right) = \frac{cQ\left(s^*, s^*\right)}{\left(\gamma^*\right)^2}$$

and

$$h^M(\gamma^*) = \left(s^{M*} + \frac{Q\left(s^{M*}, s^{M*}\right)}{Q_1\left(s^{M*}, s^{M*}\right) + Q_2\left(s^{M*}, s^{M*}\right)} \right) \frac{1}{\gamma^*} Q\left(s^M, s^M\right) = \frac{(c-\delta)\, Q\left(s^{M*}, s^{M*}\right)}{(\gamma^*)^2}.$$

Thus, the innovation incentives (at $\gamma = \gamma^*$) are related to the volume of sales, and more precisely to (total) variable production costs. Therefore, under this specification, any merger that raises equilibrium prices (or reduces total variable costs) also reduces innovation.[12]

4.3 Model with CES demand

Federico et al. (2018) consider a system of demand functions (among others) such that

$$D_i(p_i, p_j, \gamma_i, \gamma_j) = \frac{\left(\frac{p_i}{\gamma_i}\right)^{-\alpha-1}}{p_i\left(\frac{p_i}{\gamma_i}\right)^{-\alpha} + p_j\left(\frac{p_j}{\gamma_j}\right)^{-\alpha}},$$

where $\alpha > 0$. As above, let us denote $s_i = p_i/\gamma_i$ the price-quality ratio, where γ_i represents the quality of firm i's product. We have then

$$D_i(p_i, p_j, \gamma_i, \gamma_j) = \frac{(s_i)^{-\alpha-1}}{p_i(s_i)^{-\alpha} + p_j(s_j)^{-\alpha}} \equiv Q(p_i, p_j, s_i, s_j).$$

Denoting by Q_l the derivative with respect to the l^{th} variable, the following holds at a symmetric equilibrium:

$$Q(p, p, s, s) = \frac{\gamma}{2p^2},$$

$$Q_1(p, p, s, s) = Q_2(p, p, s, s) = -sQ(p, p, s, s)^2,$$

$$Q_3(p, p, s, s) = -\left(1 + \frac{\alpha}{2}\right)\frac{Q(p, p, s, s)}{s} \text{ and } Q_4(p, p, s, s) = \frac{\alpha}{2}\frac{Q(p, p, s, s)}{s}.$$

The price diversion ratio and the innovation diversion ratio at symmetric price and innovation levels are given by

$$\frac{\frac{\partial D_j}{\partial p_i}}{-\frac{\partial D_i}{\partial p_i}} = \frac{Q_4 + \gamma Q_2}{-Q_3 - \gamma Q_1} \text{ and } \frac{-\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial \gamma_i}} = \frac{Q_4}{-Q_3},$$

[12] Motta and Tarantino's analysis shows through a direct argument that, absent any production synergies ($\delta = 0$), the overall effect of the merger on innovation is negative under this specification.

which implies that

$$H_D + H_I = \hat{h}^M(\gamma^*) \frac{\gamma^*(-Q_3 + Q_4) Q_1}{(Q_3 + \gamma^* Q_1) Q_3} < 0.$$

Furthermore, we have

$$g^*(p, \gamma) = -\frac{Q_3 \frac{\partial s}{\partial \gamma}}{Q_1 + Q_3 \frac{\partial s}{\partial p}} = \frac{\left(1 + \frac{\alpha}{2}\right) \frac{Q}{s} \frac{p}{\gamma^2}}{sQ^2 + \left(1 + \frac{\alpha}{2}\right) \frac{Q}{s} \frac{1}{\gamma}} = \frac{2 + \alpha}{3 + \alpha} \frac{p}{\gamma},$$

which is increasing in p. Therefore, we have $H_\Delta > 0$.

Consequently, the overall effect of the merger on innovation in this case is *a priori* ambiguous because of two opposite effects (i.e., $H_M + H_D + H_I < 0$, while $H_\Delta > 0$). But direct computations show that

$$\begin{aligned} H_M + H_\Delta &= \frac{2 + \alpha}{3 + \alpha} \frac{p^*}{\gamma^*} \left(\frac{\gamma^*}{2\left(\tilde{p}^M(\gamma^*)\right)^2} - \frac{\gamma^*}{2(p^*)^2} \right) + \frac{\gamma^*}{2\left(\tilde{p}^M(\gamma^*)\right)^2} \left(\frac{\tilde{p}^M(\gamma^*)}{\gamma^*} - \frac{p^*}{\gamma^*} \right) \frac{2 + \alpha}{3 + \alpha} \\ &= \frac{2 + \alpha}{2(3 + \alpha)} \left(\frac{1}{\tilde{p}^M(\gamma^*)} - \frac{1}{p^*} \right) < 0. \end{aligned}$$

Hence, in this model, any merger that raises prices also reduces innovation incentives.

4.4 Extended Hotelling model

We now provide an example where the total effect of the merger on innovation is positive. Consider an extended (sufficiently long) Hotelling line with uniformly distributed consumers. Absent any investment in innovation, the firms cannot differentiate. They then both locate at the center of the line (at $x = 0$) and obtain zero profits. In order to differentiate from its rival and locate at $-\gamma_1 < 0$, firm 1 needs to invest $C(\gamma_1)$. Similarly, to locate at $\gamma_2 > 0$, firm 2 needs to invest $C(\gamma_2)$. A consumer located at x has a unit demand and derives utility

$$U_1 = v - p_1 - t\,|x + \gamma_1|$$

from buying one unit of product 1 and utility

$$U_2 = v - p_2 - t\,|x - \gamma_2|$$

from buying one unit of product 2. The demand addressed to firm i is then

$$D_i\left(p_i, p_j, \gamma_i, \gamma_j\right) = \frac{p_j - p_i}{2t} + \frac{\gamma_i + \gamma_j}{2} + \frac{v - p_i}{t}.$$

Note that in this example, the innovation diversion effect is positive, as $\partial D_j/\partial \gamma_i > 0$. Straightforward computations yield

$$D_i^*\left(\gamma\right) = \frac{3}{5}\left(\gamma + \frac{v}{t}\right), \quad D_i^M\left(\gamma\right) = \frac{1}{2}\left(\gamma + \frac{v}{t}\right), \quad g^*\left(p, \gamma\right) = \frac{t}{3}.$$

This leads to

$$H_M = -\frac{t}{30}\left(\gamma^* + \frac{v}{t}\right), \; H_D = \frac{t}{12}\left(\gamma^* + \frac{v}{t}\right), \; H_I = \frac{t}{4}\left(\gamma^* + \frac{v}{t}\right), \; H_\Delta = 0 \; .$$

Therefore, we obtain that $H_M + H_D + H_I + H_\Delta > 0$, which implies that the merger leads to an increase in the merging firms' investments in innovation.

5 Combining horizontal and vertical differentiation

The predictions of the models that we have analyzed in the previous section are very different depending on whether innovation is purely vertical, such as in the model with hedonic prices, or purely horizontal, such as in the Hotelling model. In practice, innovation may involve both dimensions. We therefore investigate the effect of a merger in a context where innovation has both a horizontal and a vertical nature, using the linear demand model of Singh and Vives (1984).

The utility of the representative consumer is given by

$$U(q_1, q_2, m) = a_1 q_1 + a_2 q_2 - (q_1^2 + q_2^2)/2 - \rho q_1 q_2 + m,$$

where (q_1, q_2) is the vector of quantities, m is the numeraire good, and $\rho \in [0, 1]$ represents the degree of substitutability between the product of firm 1 and the product of firm 2. Products are independent if $\rho = 0$ and perfect substitutes if $\rho = 1$.

We assume that R&D raises product quality as in the model with hedonic prices, which we capture by assuming that $a_i = a\left(\gamma_i\right) = \alpha + \tau\gamma_i$. We also suppose that R&D allows firms to increase the differentiation between their products as in Lin and Saggi (2002). Formally, the degree of substitutability is given by $\rho(\gamma_1, \gamma_2) = 1 - \delta(\gamma_1 + \gamma_2)$, and we assume that R&D costs are sufficiently high so that $\delta\left(\gamma_1 + \gamma_2\right) < 1$ in equilibrium (i.e., $2\delta\gamma < 1$ in the symmetric equilibrium).

The demand for firm i is then given by

$$D_i\left(p_i, p_j, \gamma_i, \gamma_j\right) = \frac{(\alpha + \tau\gamma_i) - (\alpha + \tau\gamma_j)\,\rho - p_i + \rho p_j}{1 - \rho^2}.$$

With this demand specification, we find that

$$D_i^*\left(\gamma\right) = \frac{a\left(\gamma\right)}{2(1-\delta\gamma)(1+2\delta\gamma)}, \quad D_i^M\left(\gamma\right) = \frac{a\left(\gamma\right)}{4(1-\delta\gamma)}, \quad g^*\left(p, \gamma\right) = \frac{(a(\gamma) - p)\delta^2\gamma}{1-\delta\gamma} + \tau.$$

As $g^*\left(p, \gamma\right)$ is decreasing in p, we have $H_\Delta < 0$. More precisely, we find that

$$g^*\left(\tilde{p}^*\left(\gamma\right), \gamma\right) = \frac{a\left(\gamma\right)\delta^2\gamma}{(1-\delta\gamma)(1+2\delta\gamma)} + \tau \quad \text{and} \quad g^*\left(\tilde{p}^M\left(\gamma\right), \gamma\right) = \frac{a\left(\gamma\right)\delta^2\gamma}{2(1-\delta\gamma)} + \tau,$$

which implies that

$$H_\Delta = -\frac{a\left(\gamma^*\right)^2 \delta^2\gamma(1-2\delta\gamma^*)}{8(1-\delta\gamma^*)^2(1+2\delta\gamma^*)} < 0.$$

Finally, straightforward computations yield

$$H_M = -\delta\frac{a\left(\gamma^*\right)^2 \delta\gamma^*(1-2\delta\gamma^*)}{4(1-\delta\gamma^*)^2(1+2\delta\gamma^*)^2} - \frac{a\left(\gamma^*\right)(1-2\delta\gamma^*)}{4(1-\delta\gamma^*)(1+2\delta\gamma^*)}\tau < 0;$$

$$H_D = \frac{a\left(\gamma^*\right)}{4(1-\delta\gamma^*)}\left(\frac{\delta a\left(\gamma^*\right)}{4(1-\delta\gamma^*)} + \frac{\tau}{2\delta\gamma^*}\right)(1-2\delta\gamma^*) > 0;$$

$$H_I = \frac{a\left(\gamma^*\right)}{4(1-\delta\gamma^*)}\left(\frac{\delta a\left(\gamma^*\right)}{4(1-\delta\gamma^*)} - \frac{\tau(1-2\delta\gamma^*)}{2\delta\gamma^*}\right) \lesseqgtr 0.$$

Therefore, we obtain that

$$H_M + H_D + H_I + H_\Delta = \frac{a\left(\gamma^*\right)(1-2\delta\gamma^*)}{4(1-\delta\gamma^*)(1+2\delta\gamma^*)}\left(\frac{a\left(\gamma^*\right)(1+4\delta^2\gamma^{*2})}{2(1-2\delta\gamma^*)(1+2\gamma^*\delta)(1-\gamma^*\delta)}\delta - \tau\right),$$

which implies that the merger leads to an increase in product innovation if δ/τ is not too small.

Proposition 4 *In the augmented Singh and Vives model, the merger spurs innovation if the horizontal effect is relatively strong (i.e., δ is not too small, given ρ^*, α and τ), if there is little differentiation (ρ^* is small, given τ, δ and α), or if demand is large (α is large, given ρ^*, τ and δ).*

Proof. The merger stimulates innovation if and only if

$$\frac{a(\gamma^*)(1+4\delta^2\gamma^{*2})}{2(1-2\delta\gamma^*)(1+2\delta\gamma^*)(1-\delta\gamma^*)}\delta > \tau. \tag{5}$$

Using the fact that $a(\gamma^*) = \alpha + \tau\gamma^*$ and $\delta\gamma^* = (1-\rho^*)/2$, condition (5) is equivalent to

$$\alpha\delta\frac{(1+4\delta^2\gamma^{*2})}{2(1-2\delta\gamma^*)(1+2\delta\gamma^*)(1-\delta\gamma^*)} > \tau\left[1-\frac{\delta\gamma(1+4\delta^2\gamma^{*2})}{2(1-2\delta\gamma^*)(1+2\delta\gamma^*)(1-\delta\gamma^*)}\right],$$

that is,

$$\frac{\alpha\delta}{\tau} > -2 + 3\delta\gamma^* + \frac{4(1-\delta\gamma^*)}{1+4\delta^2\gamma^{*2}},$$

or

$$\frac{\alpha\delta}{\tau} > \frac{2+\rho^{*2}(5-3\rho^*)}{2\left[2-\rho^*(2-\rho^*)\right]}.$$

Plotting the term $(2+\rho^2(5-3\rho))/(2-\rho(2-\rho))$ shows that it is an increasing function of ρ.

■

Figure 1 below provides a numerical example of Proposition 4. We set $\alpha = 1$ and $C(\gamma) = (3/2)\gamma^2$, and then compute the R&D levels at the equilibrium of the price-innovation game, pre-merger (γ^*) and post-merger (γ^M). The horizontal axis corresponds to the horizontal dimension of the innovation, δ, and the vertical axis to the vertical dimension, τ. The gray area shows the set of parameters for which the merger increases innovation (i.e., $\gamma^M > \gamma^*$), which corresponds to high values of δ and/or low values of τ.

6 Extensions

In this section, we discuss how our decomposition in the baseline model should be adjusted to account for technological spillovers, synergies in R&D, and synergies in production.

6.1 Technological spillovers

Assume that a firm's R&D benefits its rivals through technological spillovers (d'Aspremont and Jacquemin, 1988; Bloom et al., 2013; Lopez and Vives, 2016). More precisely,

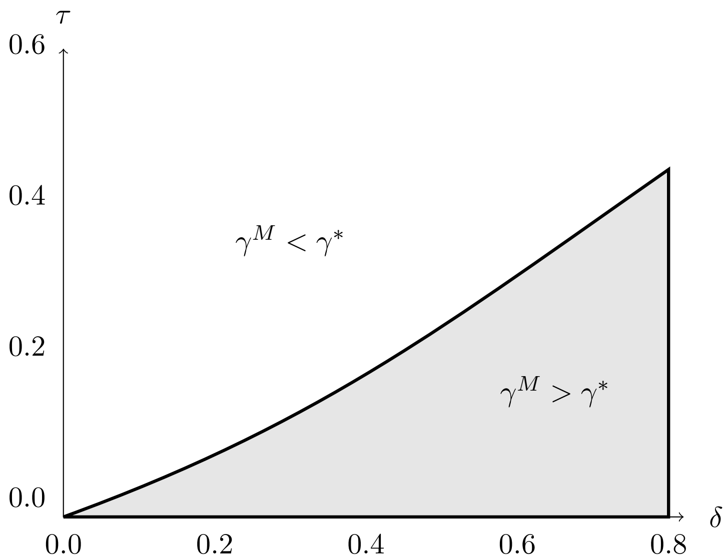

Figure 1: Impact of the merger to monopoly on innovation.

let us assume that there exists a degree of spillovers $\lambda \in [0,1]$ such that the demand addressed to firm i is given by $D_i(p_i, p_j, \gamma_i + \lambda\gamma_j, \gamma_j + \lambda\gamma_i)$. Let $\hat{\gamma}_i \equiv \gamma_i + \lambda\gamma_j$ and $\hat{\gamma} \equiv (1+\lambda)\gamma$. The counterparts to first-order conditions (1) and (2) for the duopoly game in this setting are given by

$$(p-c)\frac{\partial D_i}{\partial p_i}(p, p, \hat{\gamma}, \hat{\gamma}) + D_i(p, p, \hat{\gamma}, \hat{\gamma}) = 0$$

and

$$(p-c)\left(\frac{\partial D_i}{\partial \hat{\gamma}_i} + \lambda \frac{\partial D_i}{\partial \hat{\gamma}_j}\right)(p, p, \hat{\gamma}, \hat{\gamma}) = C'(\gamma),$$

respectively. Note that at a symmetric equilibrium, the latter condition is equivalent to

$$(p-c)\left(\frac{\partial D_i}{\partial \hat{\gamma}_i} + \lambda \frac{\partial D_j}{\partial \hat{\gamma}_i}\right)(p, p, \hat{\gamma}, \hat{\gamma}) = C'(\hat{\gamma}).$$

The counterparts to (3) and (4) for the multi-product monopoly are (to simplify the exposition, we assume away any synergies in production, i.e., $\delta = 0$)

$$(p-c)\left(\frac{\partial D_i}{\partial p_i} + \frac{\partial D_j}{\partial p_i}\right)(p, p, \hat{\gamma}, \hat{\gamma}) + D_i(p, p, \hat{\gamma}, \hat{\gamma}) = 0,$$

and

$$(p-c)(1+\lambda)\left(\frac{\partial D_i}{\partial \hat{\gamma}_i} + \frac{\partial D_j}{\partial \hat{\gamma}_i}\right)(p, p, \hat{\gamma}, \hat{\gamma}) = C'(\gamma).$$

respectively. Notice that the duopoly and monopoly prices are given by $\tilde{p}^*_\lambda(\gamma) = \tilde{p}^*(\hat{\gamma})$ and $\tilde{p}^M_\lambda(\gamma) = \tilde{p}^M(\hat{\gamma})$ in this setting and, therefore, the competitive price is lower than the monopoly price, i.e., $\tilde{p}^*_\lambda(\gamma) < \tilde{p}^M_\lambda(\gamma)$.

Proposition 1 still applies as long as the following replacements are made:

1. The arguments of $D_i(p, p, \gamma, \gamma)$, $\frac{\partial D_i}{\partial p_i}(p, p, \gamma, \gamma)$ and $\frac{\partial D_j}{\partial p_i}(p, p, \gamma, \gamma)$ should be replaced with $(p, p, \hat{\gamma}, \hat{\gamma})$.

2. The derivatives $\frac{\partial D_i}{\partial \gamma_i}$ and $\frac{\partial D_j}{\partial \gamma_i}$, evaluated at (p, p, γ, γ), should be replaced with $\frac{\partial D_i}{\partial \hat{\gamma}_i} + \lambda \frac{\partial D_i}{\partial \hat{\gamma}_j}$ and $\frac{\partial D_j}{\partial \hat{\gamma}_i} + \lambda \frac{\partial D_j}{\partial \hat{\gamma}_j}$, respectively, evaluated at $(p, p, \hat{\gamma}, \hat{\gamma})$.

Denoting by $g^*_\lambda(p, \hat{\gamma})$, $h^*_\lambda(\hat{\gamma})$, $h^M_\lambda(\hat{\gamma})$ and $\hat{\gamma}^*$ the counterparts in this setting of $g^*(p, \gamma)$, $h^*(\gamma)$, $h^M(\gamma)$ and γ^* respectively, we have

$$h^M_\lambda(\hat{\gamma}^*) - h^*_\lambda(\hat{\gamma}^*) = H_{M\lambda} + H_{D\lambda} + H_{I\lambda} + H_{\Delta\lambda} + E_\lambda$$

with

$$H_{M\lambda} \equiv \left[D^M_i(\hat{\gamma}^*, \lambda) - D^*_i(\hat{\gamma}^*, \lambda)\right] g^*_\lambda(\tilde{p}^*(\hat{\gamma}^*), \hat{\gamma}^*) < 0,$$

$$H_{D\lambda} \equiv \hat{h}^M(\hat{\gamma}^*) \times \left(\frac{\frac{\partial D_j}{\partial p_i}}{-\frac{\partial D_i}{\partial p_i}}\right) > 0 \text{ and } H_{I\lambda} \equiv \hat{h}^M(\hat{\gamma}^*) \left(\frac{\frac{\partial D_j}{\partial \hat{\gamma}_i}}{\frac{\partial D_i}{\partial \hat{\gamma}_i}}\right) \lesseqgtr 0,$$

$$H_{\Delta\lambda} \equiv D^M_i(\hat{\gamma}^*, \lambda) \left[g^*_\lambda\left(\tilde{p}^M(\hat{\gamma}^*), \hat{\gamma}^*\right) - g^*_\lambda(\tilde{p}^*(\hat{\gamma}^*), \hat{\gamma}^*)\right] \lesseqgtr 0,$$

$$E_\lambda \equiv \lambda \hat{h}^M(\hat{\gamma}^*) \left[\frac{\frac{\partial D_j}{\partial \hat{\gamma}_i}}{\frac{\partial D_i}{\partial \hat{\gamma}_i}} \times \frac{\frac{\partial D_j}{\partial p_i}}{-\frac{\partial D_i}{\partial p_i}} + 1\right] > 0,$$

where all the partial derivatives are evaluated at $\left(\tilde{p}^M_\lambda(\hat{\gamma}^*), \tilde{p}^M_\lambda(\hat{\gamma}^*), \hat{\gamma}^*, \hat{\gamma}^*\right)$ and

$$D^M_i(\hat{\gamma}, \lambda) \equiv D_i\left(\tilde{p}^M(\hat{\gamma}), \tilde{p}^M(\hat{\gamma}), \hat{\gamma}, \hat{\gamma}\right),$$

$$D^*_i(\hat{\gamma}, \lambda) \equiv D_i(\tilde{p}^*(\hat{\gamma}), \tilde{p}^*(\hat{\gamma}), \hat{\gamma}, \hat{\gamma}).$$

Notice that the function $\hat{h}^M$ is unchanged but is now evaluated at $\hat{\gamma}$.

The terms $H_{M\lambda}$, $H_{D\lambda}$, $H_{I\lambda}$ capture the margin expansion effect, the demand expansion effect and the innovation diversion effect, respectively, as in the baseline model. The term $H_{\Delta\lambda}$ measures the effect of price increase on the unit return to innovation. Finally, the term E_λ captures a new *spillover effect*, and is positive. To see why, note that, regardless of the sign of $\partial D_j/\partial\hat{\gamma}_i$, the following holds:

$$\frac{\partial D_j}{\partial \hat{\gamma}_i} \times \frac{\frac{\partial D_j}{\partial p_i}}{-\frac{\partial D_i}{\partial p_i}} + \frac{\partial D_i}{\partial \hat{\gamma}_i} > \min\left[\frac{\partial D_i}{\partial \hat{\gamma}_i}, \frac{\partial D_j}{\partial \hat{\gamma}_i} + \frac{\partial D_i}{\partial \hat{\gamma}_i}\right] > 0,$$

because $\partial D_i/\partial p_i + \partial D_i/\partial p_j$ is negative and $\partial D_i/\partial\hat{\gamma}_i + \partial D_i/\partial\hat{\gamma}_j$ is positive.

Furthermore, we find that the sum of the innovation diversion effect, the demand expansion effect and the spillover effect, $H_{D\lambda} + H_{I\lambda} + E_\lambda$, has the same sign as:

$$\underbrace{\frac{\frac{\partial D_j}{\partial p_i}}{-\frac{\partial D_i}{\partial p_i}}}_{\text{price diversion ratio}} - \underbrace{\frac{-\frac{\frac{\partial D_j}{\partial \hat{\gamma}_i}}{\frac{\partial D_i}{\partial \hat{\gamma}_i}} - \lambda}{1 + \lambda\frac{\frac{\partial D_j}{\partial \hat{\gamma}_i}}{\frac{\partial D_i}{\partial \hat{\gamma}_i}}}}_{\text{spillover-adjusted innovation diversion ratio}}$$

The denominator of the spillover-adjusted innovation diversion ratio is always positive. This follows from the assumptions that $\lambda \in [0, 1]$ and $\partial D_i/\partial\hat{\gamma}_i + \partial D_i/\partial\hat{\gamma}_j > 0$. Thus, the sign of the spillover-adjusted innovation diversion ratio is given by the sign of the difference between the innovation diversion ratio and the spillover rate:

$$\left(-\frac{\frac{\partial D_j}{\partial \hat{\gamma}_i}}{\frac{\partial D_i}{\partial \hat{\gamma}_i}}\right) - \lambda.$$

The sign of the spillover-adjusted innovation diversion ratio can be related to the magnitude of the "net innovation pressure" (NIP) defined in Salinger (2016):

$$NIP = \frac{\left(1 + \frac{\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial \gamma_i}}\right)(1+\lambda)}{1 + \lambda\frac{\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial \gamma_i}}}$$

Indeed, we have $NIP > 1$ if and only if $\left(1 + \frac{\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial \gamma_i}}\right)(1+\lambda) > 1 + \lambda\frac{\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial \gamma_i}}$, which is equivalent to $\lambda + \frac{\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial \gamma_i}} > 0$.

Remark 1 *The spillover-adjusted innovation diversion ratio is positive (negative) whenever the magnitude of the NIP is less (greater) than* 1*.*

6.2 Synergies

6.2.1 Synergies in R&D

In this section, we assume that the merger induces synergies that reduce the cost of R&D investments. We formalize this by assuming that the post-merger R&D cost function is given by $\frac{C(\gamma)}{1+\mu}$, where μ is a measure of the magnitude of R&D synergies. The only first-order condition that is affected by R&D synergies is the one associated to the merged entity's innovation level, i.e., equation (4), which becomes

$$(1+\mu)(p-c)\left[\frac{\partial D_i}{\partial \gamma_i}(p,p,\gamma,\gamma)+\frac{\partial D_j}{\partial \gamma_i}(p,p,\gamma,\gamma)\right]=C'(\gamma).$$

Therefore, the equilibrium price and the monopoly price for a given (symmetric) innovation level are still given by $\tilde{p}^*(\gamma)$ and $\tilde{p}^M(\gamma)$, respectively. Proposition 1 holds as long as $h^M\left(\tilde{p}^M(\gamma),\gamma\right)$ is replaced with $(1+\mu)\,h^M\left(\tilde{p}^M(\gamma),\gamma\right)$

Proposition 5 *In the presence of R&D synergies, the merger leads to an increase in innovation if and only if* $(1+\mu)\,h^M(\gamma^*)\geq h^*(\gamma^*)$.

As in the baseline model, we can decompose the impact of the merger on innovation as a combination of different effects. We have

$$(1+\mu)\,h^M(\gamma^*)-h^*(\gamma^*)=H_M+H_D+H_I+H_\Delta+\mu h^M(\gamma^*),$$

where H_M, H_D, H_I and H_Δ are defined in the baseline model. Developing the term $\mu h^M(\gamma^*)$ allows us to rewrite the decomposition as follows:

$$(1+\mu)\,h^M(\gamma^*)-h^*(\gamma^*)=H_M+H_{D\mu}+H_{I\mu}+H_{\Delta\mu},$$

where H_M is the same as in the baseline model, while the other terms are adjusted as follows:

$$H_{D\mu}\equiv\hat{h}^M(\gamma^*)\left(\mu+\frac{\frac{\partial D_j}{\partial p_i}}{-\left(\frac{\partial D_i}{\partial p_i}\right)}\right)>0,$$

$$H_{I\mu}\equiv(1+\mu)\,H_I=\hat{h}^M(\gamma^*)\,(1+\mu)\,\frac{\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial \gamma_i}}\lesseqgtr 0,$$

$$H_{\Delta\mu}\equiv(1+\mu)\,H_\Delta=(1+\mu)\,D_i^M(\gamma^*)\left[g^*\left(\tilde{p}^M(\gamma^*),\gamma^*\right)-g^*(p^*,\gamma^*)\right]\lesseqgtr 0.$$

The sum of the adjusted demand expansion effect and the adjusted innovation diversion effect $H_{D\mu} + H_{I\mu}$ has the same sign as the difference between a synergy-adjusted price diversion ratio and the innovation diversion ratio:

$$\underbrace{\frac{\frac{\frac{\partial D_j}{\partial p_i}}{-\frac{\partial D_i}{\partial p_i}} + \mu}{1+\mu}}_{\text{synergy-adjusted price diversion ratio}} \quad - \quad \underbrace{\frac{-\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial \gamma_i}}}_{\text{innovation diversion ratio}} .$$

Note that, since $\partial D_i/\partial p_i + \partial D_i/\partial p_j < 0$ from our assumptions, the synergy-adjusted price diversion ratio is increasing in μ.

6.2.2 Synergies in production

Synergies at the production level matter not only for the effect of a merger on prices, but also for its effect on innovation, as we now show.

Recall that we assume that the merger brings synergies that allow the merging firms to decrease their marginal cost by $\delta \in [0, c]$, and that Equation (3) leads to an optimal price $\tilde{p}^M(\gamma, \delta)$. While we have focused so far on the case where $\tilde{p}^M(\gamma, \delta) > \tilde{p}^*(\gamma)$ (Assumption 3), the opposite may be true if synergies are strong enough.

Let us make the natural assumption that $\tilde{p}^M(\gamma, \delta)$ is decreasing in δ. Suppose furthermore that there exists $\tilde{\delta}(\gamma) \in (0, c)$ such that $\tilde{p}^M(\gamma, \delta) = p^*(\gamma)$. Thus, $\tilde{p}^M(\gamma, \delta)$ is smaller (resp., greater) than $\tilde{p}^*(\gamma)$ if δ is greater (resp., smaller) than $\tilde{\delta}(\gamma)$. Given (3) and (4), and the fact that the pre-merger behavior is not affected by the synergies from the merger, Proposition 1 still holds for $\tilde{p}^M(\gamma, \delta) < \tilde{p}^*(\gamma)$ and our decomposition remains valid in this case as well. A key difference with the baseline model, however, is that the margin expansion effect H_M is positive if production synergies are strong enough so that the monopoly price is smaller than the competitive prices at the pre-merger innovation level, i.e., if $\delta > \tilde{\delta}(\gamma^*)$. In this case, the coefficient H_Δ would also be of the opposite sign compared to the baseline model, provided that g^* is monotonic in p.

Proposition 6 *Suppose that $\delta = \tilde{\delta}(\gamma^*)$. Then, the merger fosters innovation if and only if the price diversion ratio is larger than the innovation diversion ratio.*

Proof. In this special case, we have $H_M = H_\Delta = 0$. Therefore, the merger fosters innovation if and only if $H_D + H_I > 0$. ■

7 Alternative decomposition

To illustrate further the effect of a merger on the incentives to expand output we present an alternative decomposition of the overall impact of the merger on innovation. This decomposition focuses more on margins than output. Define

$$\mu^*(p,\gamma) \equiv -\frac{D_i(p,p,\gamma,\gamma)}{\frac{\partial D_i}{\partial p_i}(p,p,\gamma,\gamma)}$$

and

$$\mu^M(p,\gamma) \equiv -\frac{D_i(p,p,\gamma,\gamma)}{\frac{\partial D_i}{\partial p_i}(p,p,\gamma,\gamma)+\frac{\partial D_j}{\partial p_i}(p,p,\gamma,\gamma)}.$$

These functions relate the duopoly and monopoly margins to innovation and equilibrium prices as follows:

$$\tilde{p}^*(\gamma)-c=\mu^*(\tilde{p}^*(\gamma),\gamma) \text{ and } \tilde{p}^M(\gamma)-c+\delta=\mu^M\left(\tilde{p}^M(\gamma),\gamma\right).$$

An independent firm's marginal gain from innovation is then given by

$$h^*(\gamma) \equiv \frac{\partial D_i}{\partial \gamma_i}(\tilde{p}^*(\gamma),\tilde{p}^*(\gamma),\gamma,\gamma)\,\mu^*(\tilde{p}^*(\gamma),,\gamma),$$

and the merged entity's marginal gain from innovation net of the innovation diversion effect H_I is

$$\hat{h}^M(\gamma) \equiv \frac{\partial D_i}{\partial \gamma_i}\left(\tilde{p}^M(\gamma),\tilde{p}^M(\gamma),\gamma,\gamma\right)\mu^M\left(\tilde{p}^M(\gamma),\gamma\right).$$

The effect of a merger on the merging firms' incentives to invest in demand-enhancing innovation, $\hat{h}^M(\gamma)-h^*(\gamma)$, net of the innovation diversion effect, can be decomposed as follows:

$$\hat{h}^M(\gamma^*)-h^*(\gamma^*)=H_\mu+H_\sigma,$$

where

$$\begin{aligned}
H_\mu &\equiv \left[\mu^M-\mu^*\right]\frac{\partial D_i}{\partial \gamma_i}(\tilde{p}^*(\gamma^*),\tilde{p}^*(\gamma^*),\gamma^*,\gamma^*)>0,\\
H_\sigma &\equiv \mu^M\frac{\partial}{\partial \gamma_i}\left(D_i\left(\tilde{p}^M(\gamma^*),\tilde{p}^M(\gamma^*),\gamma^*,\gamma^*\right)-D_i(\tilde{p}^*(\gamma^*),\tilde{p}^*(\gamma^*),\gamma^*,\gamma^*)\right)\lesseqgtr 0,
\end{aligned}$$

where μ^M and μ^* are the post-merger and pre-merger margins, respectively, for an innovation level γ^*

The first term measures the effect of a change in margins on the incentives to innovate. The second term measures how the diversion of sales by innovation is affected by the merger. We thus obtain the following final decomposition:

$$h^M(\gamma^*) - h^*(\gamma^*) = H_\mu + H_\sigma + H_I.$$

Note that the innovation diversion effect H_I can be written as

$$H_I = \hat{h}^M(\gamma^*) \frac{\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial \gamma_i}} = \mu^M\left(\tilde{p}^M(\gamma^*), \gamma^*\right) \frac{\partial D_j}{\partial \gamma_i}.$$

From this decomposition, we obtain the following result.

Proposition 7 *The merger fosters innovation if it raises margins (i.e., $\tilde{p}^M(\gamma^*) + \delta > \tilde{p}^*(\gamma^*)$) and*

$$\frac{d}{d\gamma}\left(D^M(\gamma^*) - D^*(\gamma^*)\right) \geq -\frac{\partial D_j}{\partial \gamma_i}\left(\tilde{p}^*(\gamma^*), \tilde{p}^*(\gamma^*), \gamma^*, \gamma^*\right).$$

Proof. We have $H_\mu > 0$ if and only if $\mu^M\left(\tilde{p}^M(\gamma^*), \gamma^*\right) > \mu^*\left(\tilde{p}^*(\gamma^*), \gamma^*\right)$, which is equivalent to $\tilde{p}^M(\gamma^*) + \delta > \tilde{p}^*(\gamma^*)$. Then, we have $H_\sigma + H_I \geq 0$ if

$$\frac{\partial(D_i + D_j)}{\partial \gamma_i}\left(\tilde{p}^M(\gamma^*), \tilde{p}^M(\gamma^*), \gamma^*, \gamma^*\right) \geq \frac{\partial D_i}{\partial \gamma_i}\left(\tilde{p}^*(\gamma), \tilde{p}^*(\gamma), \gamma^*, \gamma\right),$$

or, equivalently,

$$\frac{\partial(D_i + D_j)}{\partial \gamma_i}\left(\tilde{p}^M(\gamma^*), \tilde{p}^M(\gamma^*), \gamma^*, \gamma^*\right) \geq \left(\frac{\partial(D_i + D_j)}{\partial \gamma_i} - \frac{\partial D_j}{\partial \gamma_i}\right)\left(\tilde{p}^*(\gamma), \tilde{p}^*(\gamma), \gamma^*, \gamma\right).$$

Using $\frac{\partial}{\partial \gamma_i}(D_i + D_j)(p, p, \gamma, \gamma) = \frac{\partial}{\partial \gamma_j}(D_i + D_j)(p, p, \gamma, \gamma)$, we have that

$$\frac{\partial}{\partial \gamma_i}(D_i + D_j)\left(\tilde{p}^M(\gamma^*), \tilde{p}^M(\gamma^*), \gamma^*, \gamma^*\right) = \frac{dD^M}{d\gamma}(\gamma^*).$$

Similarly,

$$\frac{\partial}{\partial \gamma_i}(D_i + D_j)\left(\tilde{p}^*(\gamma^*), \tilde{p}^*(\gamma^*), \gamma^*, \gamma^*\right) = \frac{dD^*}{d\gamma}(\gamma^*),$$

which yields the result. ■

8 Conclusion

Our analysis sheds light on the impact of a merger on investment in demand-enhancing innovation. We offer a decomposition of the various effects at work and show that the overall impact on the merging firms' incentives to innovate can be either positive or negative. The most intuitive effects we identified are the *margin expansion effect* that is particularly relevant when innovation raises margins but has little effect on the volume of sales, the *demand expansion effect* that is particularly relevant when innovation raises demand but has little effect on margins, and the *innovation diversion effect* that arises when the innovation affects sales of other products.

This work focuses on the "initial impetus" when investment in innovation leads to a deterministic outcome and is not observed by competitors at the moment price competition takes place. We plan to extend our analysis in three directions to get a more complete understanding of the way mergers affect demand-enhancing innovation. The first direction is to investigate the effect of a merger between a subset of competitors on merging and non-merging firms' incentives to innovate. The second one is to analyze the case where price competition occurs after firms observe their rivals' innovations. The third one is to introduce uncertainty in the R&D process.

Finally, while the effect of mergers on innovation is interesting in itself, the ultimate question that U.S. and EU competition authorities are seeking to answer when assessing a merger is its effect on consumer surplus. Therefore, it would be interesting to embed our analysis in a full-fledged assessment of the effect of a merger on consumers.

References

d'Aspremont, C. and A. Jacquemin (1988), "Cooperative and Noncooperative R&D in Duopoly with Spillovers," *American Economic Review*, 78, 1133–1137.

Baker, J. (2007), "Beyond Schumpeter vs. Arrow: How Antitrust Fosters Innovation," *Antitrust Law Journal*, 74, 575–602.

Bloom, N., Schankerman, M. and J. Van Reenen (2013), "Identifying Technology Spillovers and Product Market Rivalry," *Econometrica*, 81, 1347–1393.

Bourreau, M. and B. Jullien (2018), "Mergers, Investment and Demand Expansion,"*Economics Letters*, 167, 136-141.

Davidson, C. and B. Ferrett (2007), “Mergers in Multidimensional Competition”, *Economica*, 74, 695–712.

Denicolò, V. and M. Polo (2018a), “Duplicative Research, Mergers and Innovation,” *Economics Letters*, 166, 56-59.

Denicolò, V. and M. Polo (2018b), “Mergers and Innovation Sharing,” mimeo.

Farrell, J. and C. Shapiro (2010), “Antitrust Evaluation of Horizontal Mergers: An Economic Alternative to Market Definition,” *B.E. Journal of Theoretical Economics*, 10, Article 9.

Federico, G., Langus, G. and T. Valletti, (2017), “A Simple Model of Mergers and Innovation,” *Economics Letters*, 157, 136–140.

Federico, G., Langus, G. and T. Valletti, (2018), “Horizontal Mergers and Product Innovation,” *International Journal of Industrial Organization*, 59, 1-23.

Ghandi, A., Froeb, L., Tschantz, S. and G.J. Werden (2008),“Post-Merger Product Repositioning,” *The Journal of Industrial Economics*, 56, 49–67.

Gilbert, R. (2006). “Looking for Mr. Schumpeter: Where Are We in the Competition-Innovation Debate?” In *Innovation Policy and the Economy* (A. B. Jae, J. Lerner and S. Stern (eds)), The MIT Press, Cambridge, MA.

Katz, M. and H. Shelanski (2007), “Mergers and Innovation,” *Antitrust Law Journal*, 74, 1–86.

Lin, P. and K. Saggi (2002), “Product Differentiation, Process R&D, and the Nature of Market Competition,” *European Economic Review*, 46, 201–211.

Loertscher, S. and L. M. Marx (2018), “Merger Review for Markets with Buyer Power,” working paper.

Lopez A. L. and X. Vives (2016), “Cross-Ownership, R&D Spillovers and Antitrust Policy,” CEPR Discussion Paper 11315.

Matsushima, N., Sato, Y., and K. Yamamoto (2013), “Horizontal Mergers, Firm Heterogeneity, and R&D Investments,” *B.E. Journal of Economic Analysis & Policy,* 13, 959–990.

Motta, M. and E. Tarantino (2017), "The Effect of Horizontal Mergers, When Firms Compete in Prices and Investments," mimeo.

Salinger, M. A. (2016), "Net Innovation Pressure in Merger Analysis," mimeo.

Shapiro, C. (2012), "Competition and Innovation: Did Arrow hit the bull's eye?" Chapter 7 of Lerner, J. and S. Stern (eds.), *The Rate and Direction of Inventive Activity Revisited*, 361–404.

Singh, N. and X. Vives (1984), "Price and Quantity Competition in a Differentiated Duopoly," *RAND Journal of Economics*, 15, 546–554.

Vives, X. (1999), *Oligopoly pricing: old ideas and new tools*, The MIT Press, Cambridge, MA.