

“Collusion in Two-Sided Markets”

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Abstract: This paper explores the incentives for, and the effects of, collusion in prices between two-sided platforms. We characterize the most profitable sustainable agreement when platforms collude on both sides of the market and when they collude on a single side of the market. Under two-sided collusion, prices on both sides are higher than competitive prices, implying that agents on both sides become worse off as compared to the competitive outcome. An increase in cross-group externalities makes two-sided collusion harder to sustain, and reduces the harm from collusion suffered by the agents on a given side as long as the collusive price on that side is lower than the monopoly price. When platforms collude on a single side of the market, the price on the collusive side is lower (higher) than the competitive price if the magnitude of the cross-group externalities exerted on that side is sufficiently large (small). As a result, one-sided collusion may benefit the agents on the collusive side and harm the agents on the competitive side.

Keywords: Collusion; Two-sided markets; Cross-group externalities.

JEL Codes: L41, D43.

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1 Introduction

Several cartels involving newspaper publishers have been uncovered all around the world. In 1969, a U.S. District Court convicted of monopolization the two daily newspapers of general circulation in Tucson, Arizona, for jointly setting subscription and advertising rates.¹ In 1996, several Venezuelan newspapers were convicted of forming a cartel to fix advertising rates for movie theaters.² In 2005, the Brazilian antitrust authority fined the four largest newspapers in Rio de Janeiro for forming a cartel, after a simultaneous increase in cover prices by 20%.³ In 2010, the Croatian antitrust authority established that nine publishers of daily newspapers engaged in concerted practices that translated into a uniform increase in newspapers' cover prices.⁴ In 2014, the Hungarian antitrust authority convicted the four major newspapers publishers in the country of price-fixing conspiracy.⁵ Also in 2014, the Montenegrin antitrust authority convicted the three major daily newspapers publishers in the country for price-fixing conspiracy.⁶

Newspapers are two-sided platforms that enable the interaction between two distinct types of agents: advertisers and readers. As pointed out by Evans and Schmalensee (2013, p. 2), “*a number of results for single-sided firms, which are the focus of much of the applied antitrust economics literature, do not apply directly to multi-sided platforms.*” However, the theoretical literature on collusion in two-sided markets is remarkably scarce, which is striking given the empirical evidence on collusion in these markets.⁷ In particular, our understanding of the sustainability and impact of *imperfect* collusion among two-sided platforms, i.e., collusion that does not yield the monopoly outcome, is very limited.

In this paper, we explore the incentives for, and effects of, collusion between two horizontally differentiated platforms, allowing for *any* degree of collusion. Our model is an infinitely repeated version of the canonical Armstrong (2006)'s model, with single-homing

¹Citizen Publishing Co. v. United States 394 U.S. 131 (1969).

²See page 9 of the 2005 report by the Venezuelan antitrust authority available at: <http://www.oecd.org/daf/competition/prosecutionandlawenforcement/38835563.pdf>.

³CADE - Processo Administrativo no. 08012.002097/99-81.

⁴CCA vs. daily newspaper publishers: UP/I 030-02/2008-01/072.

⁵Gazdasgi Versenyhivatal (GVH) - Case Number: Vj/23/2011.

⁶Agency for Protection of Competition - Case Number: 02-UPI-68/1-14. In this case, there was even a written agreement signed by three of the convicted publishers, where they combined to simultaneously increase the retail price of newspapers.

⁷For other examples of collusion in two-sided markets, see Ruhmer (2011) and Dewenter *et al.* (2011).

on both sides⁸ and (positive or negative) cross-group externalities.⁹

We first consider the scenario in which platforms engage in *two-sided collusion*, that is, collusion on the prices set on both sides of the market. We show that the most profitable collusive agreement involves supra-competitive prices on both sides, and a price structure that minimizes the platforms' incentives to deviate from the agreement. We also find that an increase in cross-group externalities makes two-sided collusion (at a given profit level) harder to sustain, and affects the harm from collusion suffered by the agents using the platforms in a different way depending on whether perfect collusion is sustainable or not. More specifically, an increase in cross-group externalities reduces the harm from collusion suffered by the agents on a given side as long as the collusive price on that side is lower than the monopoly price, but increases it otherwise.

Next, we consider the scenario in which platforms engage in *one-sided collusion*, i.e., they set their prices cooperatively on one side of the market and non-cooperatively on the other side. Such a collusive behavior can be explained by the existence of coordination or antitrust costs that make it optimal for platforms to collude on a single side of the market, and has been documented empirically in the case of newspapers. For instance, using data from the Italian daily newspaper market from 1976 to 2003, Argentesi and Filistrucchi (2007) found empirical evidence that the four biggest newspapers colluded on cover prices, but found no evidence for collusion on advertising rates.

One-sided collusive agreements affect the prices on the non-cooperative side of the market because of the existence of cross-group externalities. If increasing the price on the collusive side softens competition on the non-cooperative side, the most profitable one-sided collusive agreement leads to supra-competitive prices on both sides of the market. This happens when the cross-group externalities exerted on the collusive side are negative. By contrast, if increasing the price on the collusive side strengthens competition on the non-cooperative side, the price on one of the two sides will be above its static Nash level, while the price on the other side will be below its static Nash level. This scenario occurs when the cross-group externalities exerted on the collusive side are positive. Interestingly,

⁸Our model is, in particular, suitable for our leading example (the newspaper market). According to Argentesi and Filistrucchi (2007), the assumption of single-homing on both market sides is reasonable in the newspaper market. Concretely, the authors found that “*On each day of the week, more than 84% of advertisers put an advertising message in only one of the four newspapers, and only 10% in two of them. This evidence seems to give support to the assumption that multi-homing is not a widespread practice*”. Kaiser and Wright (2006) also found that in the German magazine market, from 1972 to 2003, only 8% of the readers and 17% of the advertisers multihomed. Taking this evidence into account, the authors estimated a modified version of the Armstrongs's (2006) model.

⁹There is some controversy in the literature about the readers' tastes for advertising, which we account for since we do not restrict the sign of the cross-group externalities.

if these externalities are sufficiently high (compared to the degree of differentiation between platforms), the price on the collusive side is *below* its static Nash level, while the price on the non-cooperative side is *above* its static Nash level. As a result, one-sided collusion may benefit the agents on the collusive side and harm the agents on the non-cooperative side. Note, however, that in all scenarios, one-sided collusion decreases the overall surplus of the platforms' users.

Related literature. The work by Ruhmer (2011) is the closest to our paper. She also considers a repeated version of Armstrong's model but her setting is less general than ours. First, in the context of two-sided collusion, she focuses on perfect collusion (i.e., collusion at the monopoly prices) while we allow for imperfect collusion as well. This is natural when platforms are differentiated: in this case, perfect collusion may not be sustainable while (profitable) collusion at other prices could be. The distinction between perfect and imperfect two-sided collusion turns out to be crucial, for instance, for the effects of cross-group externalities on the harm suffered by the agents using the platforms. Second, in the context of one-sided collusion, Ruhmer (2011) focuses on the profitability and sustainability of a very specific collusive agreement in which platforms set the price on the collusive side at the maximum level that allows them to fully cover that side of the market (which is above the static Nash level). In contrast, we do not restrict the type of one-sided collusive agreements that platforms can achieve and show that they may find it optimal to decrease the price on the collusive side below its static Nash level. This explains, in particular, why one-sided collusion may be unprofitable in Ruhmer's setting, while this is never the case in our setting.

Our paper is also related to the contribution of Dewenter *et al.* (2011) who build a model to investigate the welfare impacts of collusion between newspaper publishers. They consider a static setting where newspapers compete in prices in the reader market and in quantities in the advertising market, and compare the platforms' profits when there is two-sided perfect collusion, one-sided perfect collusion (on the advertising side) and two-sided competition. In contrast, we investigate, in a *dynamic* setting, the most profitable sustainable agreement, allowing for intermediate degrees of collusion and analyzing the incentives for platforms to comply with the collusive agreement. Dewenter *et al.* (2011) find that, when newspapers only collude on the advertisers' side, the price is lower on the non-cooperative side while it is higher on the collusive side (as compared to the static Nash prices). By contrast, we show that one-sided collusion may also lead to a price lower than the competitive price on the *collusive* side.¹⁰

¹⁰In both papers, the agents on the collusive side may benefit from a one-sided collusive agreement, but

Another paper our work is related to is Boffa and Filistrucchi (2014). These authors build a model of collusion between two TV channels and use it to show that prices above the two-sided monopoly price may prevail on one side of the market as a means to enhance cartel sustainability. However, they consider collusion in quantities while we study collusion in prices, which makes our papers complementary. Moreover, they focus on the case of two-sided collusion while we also deal with one-sided collusion.

Finally, our paper is also linked to the work of Choi and Gerlach (2013) on competing firms' incentives to collude when they interact in multiple markets and demands in these markets are interrelated. The main goal of Choi and Gerlach (2013) is, however, fundamentally different from ours. They focus their analysis on antitrust enforcement issues and, in particular, on whether the discovery of a cartel in one market favors the emergence or collapse of a cartel in another market. Moreover, they restrict their attention to homogeneous goods, which implies in particular that collusion at the monopoly price is sustainable whenever some collusion is sustainable. In contrast, we consider a setting with differentiated platforms and possibly imperfect collusion, and abstract away from antitrust enforcement issues.

The remainder of the paper is organized as follows. In Section 2, we lay out the model. In Section 3, we characterize the most profitable sustainable agreement under two-sided collusion. In Section 4, we make a similar analysis for the case in which platforms collude on a single side of the market. We discuss some limitations of our model in Section 5 and conclude in Section 6. Most of the proofs are relegated to the Appendix.

2 Setup

We consider an infinitely repeated version of Armstrong's (2006) model. There are two platforms in the market, A and B , that enable the interaction between two groups of agents, 1 and 2. Agents on each side are uniformly distributed along the interval $[0, 1]$ and platforms are located at the extremes: $x^A = 0$ and $x^B = 1$. Platform $i \in \{A, B\}$ sets a subscription fee p_j^i to the agents on each side of the market $j \in \{1, 2\}$. There is single-homing on both sides of the market and the utility of an agent on side j located at

the mechanisms driving this result in the two papers are different. In Dewenter *et al.* (2011), this result may hold despite the price increase on the collusive side because of an indirect feedback effect: the price decrease on the non-cooperative side leads to more participation on that side, which benefits the agents on the collusive side. In contrast, in our paper, the result that one-sided collusion may benefit the agents on the collusive side is driven by the direct impact on the price paid by these agents.

$x \in [0, 1]$ that joins platform i is:

$$u_j^i(x, p_1^i, p_2^i, p_1^{-i}, p_2^{-i}) = k_j + \alpha_j n_{-j}^i(p_1^i, p_2^i, p_1^{-i}, p_2^{-i}) - t |x^i - x| - p_j^i, \quad (1)$$

where: k_j is the intrinsic benefit that an agent on side j gets from joining a platform; α_j captures the benefit (which can be positive or negative) that an agent on side j enjoys from the existence of an agent on the other side of the market that joined the same platform; and $t > 0$ measures the degree of differentiation between platforms.

The demand addressed to platform i on side j is:¹¹

$$n_j^i(p_1^i, p_2^i, p_1^{-i}, p_2^{-i}) = \frac{1}{2} + \frac{\alpha_j(p_1^{-i} - p_2^{-i}) + t(p_1^{-i} - p_2^i)}{2(t^2 - \alpha_1\alpha_2)}. \quad (2)$$

Platforms interact for an infinite number of periods and have a common discount factor $\delta \in (0, 1)$. In each period $t \in \{0, 1, 2, \dots\}$, they simultaneously set membership fees, p_j^i . Platforms have constant marginal production costs, which, for simplicity, are set to zero. Thus, the per-period profit function of platform $i \in \{A, B\}$ is:

$$\pi^i(p_1^i, p_2^i, p_1^{-i}, p_2^{-i}) = p_1^i n_1^i(p_1^i, p_2^i, p_1^{-i}, p_2^{-i}) + p_2^i n_2^i(p_1^i, p_2^i, p_1^{-i}, p_2^{-i}). \quad (3)$$

We make the following assumptions which ensure, in particular, that the stage game has a unique (symmetric) Nash equilibrium, in which both sides of the market are fully covered.

Assumption 1 $2t > |\alpha_1 + \alpha_2|$.¹²

Assumption 2 $k_1 - \frac{\alpha_1}{2} \leq k_2 - \frac{\alpha_2}{2}$.

Assumption 3 $k_1 > \frac{3t - \alpha_1 - 2\alpha_2}{2}$

We now recall the equilibrium prices and profits when platforms compete against each other.

¹¹For details, see Armstrong (2006). Under full market coverage, the demands addressed to the two platforms on side $j \in \{1, 2\}$ are related in the following way: $n_j^i = 1 - n_j^{-i}$, for $i \in \{A, B\}$.

¹²The assumption $2t > \alpha_1 + \alpha_2$ ensures that the second-order condition corresponding to the individual profit-maximization problem is satisfied, as it implies that $t^2 > \alpha_1\alpha_2$. In addition, we assume that $2t > -\alpha_1 - \alpha_2$ for the second-order conditions of the maximization problem under two-sided collusion to be satisfied (see footnote 33).

Lemma 1 (Armstrong, 2006) *Under Assumptions 1–3, if platforms behave non-cooperatively, they set equal prices given by $p_j^N = t - \alpha_{-j}$, $j \in \{1, 2\}$, fully cover both market sides, and get equal market shares on each side. Their individual profit is given by $\pi^N = \frac{2t - \alpha_1 - \alpha_2}{2}$.*

Proof. See Armstrong (2006) for the determination of the Nash prices and profits. Market $j \in \{1, 2\}$ is fully covered if and only if:

$$u_j^i \left(\frac{1}{2}, p_1^N, p_2^N, p_1^N, p_2^N \right) \geq 0 \Leftrightarrow k_j - \frac{\alpha_j}{2} \geq \frac{3t - 2\alpha_1 - 2\alpha_2}{2}.$$

which holds by Assumptions 2 and 3. ■

3 Two-sided collusion

Suppose that, at the beginning of period $t = 0$, platforms may agree to collude using grim trigger strategies that involve a permanent reversion to the static Nash prices in case of a deviation from the collusive agreement. In this section, we consider the scenario in which platforms seek to collude on both sides of the market.

Let us first determine, for each set of parameters, the most profitable collusive agreement among those that are sustainable. We restrict our attention to symmetric agreements, i.e., such that the two platforms set equal prices on each market side ($p_j^A = p_j^B$, for $j \in \{1, 2\}$). Denote by

$$\pi(p_1, p_2) = \pi^i(p_1, p_2, p_1, p_2)$$

the profit of platform $i \in \{A, B\}$ if the two platforms set equal prices $p_j^A = p_j^B = p_j$ on each side $j \in \{1, 2\}$. The most profitable sustainable symmetric agreement involves prices that solve the following maximization program:

$$\max_{(p_1, p_2) \in \mathbb{R}^2} \pi(p_1, p_2)$$

subject to the sustainability constraint (hereafter, ICC):

$$\frac{\pi(p_1, p_2)}{1 - \delta} \geq \pi^d(p_1, p_2) + \frac{\delta}{1 - \delta} \pi^N, \quad (4)$$

where $\pi^d(p_1, p_2) = \max_{(p_1^i, p_2^i)} \pi^i(p_1^i, p_2^i, p_1, p_2)$ is the optimal deviation profit if the collusive prices are (p_1, p_2) .

3.1 Preliminaries

For any given $\delta \in (0, 1)$, denote by

$$I(\delta) = \left\{ (p_1, p_2) \in \mathbb{R}^2 \mid \frac{\pi(p_1, p_2)}{1 - \delta} \geq \pi^d(p_1, p_2) + \frac{\delta}{1 - \delta} \pi^N \right\}$$

the set of price pairs such that the ICC is satisfied, and by

$$\bar{I}(\delta) = \left\{ (p_1, p_2) \in \mathbb{R}^2 \mid \frac{\pi(p_1, p_2)}{1 - \delta} = \pi^d(p_1, p_2) + \frac{\delta}{1 - \delta} \pi^N \right\}$$

the set of price pairs such that the ICC is binding. Moreover, define

$$\pi^c(\delta) = \max_{(p_1, p_2) \in I(\delta)} \pi(p_1, p_2)$$

and

$$\delta^m = \frac{\pi^d(p_1^m, p_2^m) - \pi^m}{\pi^d(p_1^m, p_2^m) - \pi^N}$$

where (p_1^m, p_2^m) is the unique solution to the unconstrained maximization program

$$\max_{(p_1, p_2) \in \mathbb{R}^2} \pi(p_1, p_2),$$

and $\pi^m \equiv \pi(p_1^m, p_2^m)$ is the profit each firm derives from perfect collusion.

The following preliminary results are useful for the subsequent analysis. Lemma 2 shows that the collusive profit is (weakly) increasing in the platform's discount factor and that the ICC is binding for sufficiently small values of the discount factor.

Lemma 2 *The prices and profits under the most profitable sustainable agreement satisfy the following properties:*

(i) $\pi^c(\delta) \leq \pi^c(\delta') < \pi^m$, $\forall \delta, \delta' \in (0, \delta^m)$ such that $\delta < \delta'$; and $\pi^c(\delta) = \pi^m$, $\forall \delta \in [\delta^m, 1)$.

(ii) If $\delta \in (0, \delta^m)$ and $(p_1^c(\delta), p_2^c(\delta))$ is a pair of prices in $I(\delta)$ such that $\pi^c(\delta) = \pi(p_1^c(\delta), p_2^c(\delta))$, then $(p_1^c(\delta), p_2^c(\delta)) \in \bar{I}(\delta)$.

Proof. See Appendix. ■

The next lemma shows that the price structure under the most profitable sustainable agreement minimizes the platforms' incentives to deviate (among all possible price structures for a given collusive profit).

Lemma 3 *Consider $\delta \in (0, \delta^m)$ and let $(p_1^c(\delta), p_2^c(\delta))$ be a pair of prices in $I(\delta)$ such that $\pi^c(\delta) = \pi(p_1^c(\delta), p_2^c(\delta))$. Then, $(p_1^c(\delta), p_2^c(\delta))$ is necessarily a solution to the following constrained minimization program:*

$$\min_{(p_1, p_2) \in \mathbb{R}^2} \pi^d(p_1, p_2)$$

subject to

$$\pi(p_1, p_2) = \pi^c(\delta).$$

Proof. See Appendix. ■

3.2 The most profitable sustainable agreement

We now make use of the results above to characterize the most profitable sustainable agreement when platforms collude on both sides of the market.

For tractability reasons, we impose lower bounds on the agents' stand-alone values to ensure that the market is fully covered under two-sided collusion (Assumption 4). This simplifies the analysis by reducing the number of possible demand configurations under collusion.

Assumption 4

- $k_1 \geq \frac{2t - \alpha_1}{2}$ and $k_2 \geq \frac{2t - \alpha_2}{2}$;
- $2t \min\{k_1, k_2\} + (\alpha_1 + \alpha_2) \max\{k_1, k_2\} \geq \frac{(2t - \alpha_1 - \alpha_2)(2t + \alpha_1 + \alpha_2)}{2}$.

Lemma 4 *Under Assumptions 1 and 4, both sides of the market are fully covered under the most profitable sustainable two-sided collusive agreement.*

Proof. See Appendix. ■

If platforms set equal prices, there is full coverage of side $j \in \{1, 2\}$ if and only if the utility of the indifferent consumer, located at $x = \frac{1}{2}$, is non-negative. Thus, the maximum price that platforms can charge on side j for this side to be fully covered is:¹³

$$p_j^m = p_j^N + u_j^i \left(\frac{1}{2}, p_1^N, p_2^N, p_1^N, p_2^N \right) = \frac{2k_j - t + \alpha_j}{2}. \quad (5)$$

The following lemma shows how prices under the most profitable sustainable agreement relate to the profit generated by this agreement.

Lemma 5 *Under Assumptions 1–4, for any $\delta \in (0, 1)$, there exists a unique pair of prices $(p_1^c(\delta), p_2^c(\delta)) \in I(\delta)$ satisfying $\pi^c(\delta) = \pi(p_1^c(\delta), p_2^c(\delta))$. These prices are such that:*

$$(p_1^c(\delta), p_2^c(\delta)) = \begin{cases} \left(\pi^c(\delta) + \frac{\alpha_1 - \alpha_2}{2}, \pi^c(\delta) - \frac{\alpha_1 - \alpha_2}{2} \right) & \text{if } 0 < \delta \leq \tilde{\delta} \\ (p_1^m, 2\pi^c(\delta) - p_1^m) & \text{if } \tilde{\delta} < \delta < \delta^m \\ (p_1^m, p_2^m) & \text{if } \delta^m \leq \delta < 1, \end{cases} \quad (6)$$

where $p_j^m = \frac{2k_j - t + \alpha_j}{2}$, and $\tilde{\delta}$ is the solution of $\pi^c(\tilde{\delta}) = \frac{2k_1 - t + \alpha_2}{2}$.

Proof. See Appendix. ■

This lemma implies that, for any $\delta \in (0, \tilde{\delta})$:

$$p_1^c(\delta) - p_2^c(\delta) = p_1^N - p_2^N. \quad (7)$$

Therefore, as long as the collusive price is below the monopoly level on both sides, the price structure (defined as the difference between the prices on the two sides) under two-sided collusion and under competition is the same. In other words, for sufficiently small discount factors, collusion distorts the total price, $p_1 + p_2$, but not the price structure, $p_1 - p_2$. This implies that competition authorities should not rely on changes in the price structure to detect a switch from a competitive regime to a collusive regime, and stands in sharp contrast with the case of perfect collusion (Ruhmer, 2011).

¹³Expressions for Nash prices are provided in Lemma 1.

Moreover, since the collusive profit $\pi^c(\delta)$ is (weakly) increasing in δ (by Lemma 2), Lemma 5 implies that collusive prices are (weakly) increasing in the discount factor. This leads in particular to the following result.

Proposition 1 *Under Assumptions 1–4, the collusive prices generating the most profitable two-sided sustainable agreement are (weakly) greater than the static Nash prices, i.e., for any $\delta > 0$,*

$$p_j^c(\delta) \geq p_j^N.$$

Proposition 1 implies that two-sided collusion harms the agents on both sides of the market. To show that collusive prices are *strictly* greater than the competitive prices, and derive the effects of cross-group externalities on the sustainability and profitability of two-sided collusion, we make an additional assumption. Under Assumption 5, a platform disrupting the collusive agreement would monopolize both sides of the market or none.¹⁴ Therefore, this assumption reduces the number of possible demand configurations in the deviating period.¹⁵

Assumption 5

- $k_1 > \bar{k}_1$, with $\bar{k}_1 = \frac{7t-3\alpha_1-4\alpha_2}{2}$;
- $2t^2 > \alpha_1\alpha_2 + \alpha_2^2$.

Proposition 2 *Under Assumptions 1–5, the highest sustainable profit if platforms cooperatively set the prices on both sides of the market is.¹⁶*

$$\pi^c(\delta) = \begin{cases} \frac{1+3\delta}{1-\delta} \pi^N & \text{if } 0 < \delta < \frac{1}{3} \\ \frac{2-3\delta}{1-2\delta} \pi^N & \text{if } \frac{1}{3} \leq \delta < \delta^m \\ \pi^m & \text{if } \delta^m \leq \delta < 1, \end{cases} \quad (8)$$

¹⁴Otherwise, the deviating platform could monopolize side 2 but not side 1 (see the proof of Proposition 2), in which case, we would not be able to get a closed-form solution for the most sustainable profit.

¹⁵Assumption 5 implies that the stand-alone value on side 2 is also sufficiently large: $k_2 > \frac{7t-4\alpha_1-3\alpha_2}{2}$. Notice further that Assumption 3 is implied by the condition $k_1 > \bar{k}_1$.

¹⁶For $\delta < \frac{1}{3}$, a deviating platform would not monopolize any side of the market; while, for $\delta \geq \frac{1}{3}$, a deviating platform would monopolize both market sides.

with $\pi^m = \frac{2(k_1+k_2)-(2t-\alpha_1-\alpha_2)}{4}$ and $\delta^m = \frac{2(k_1+k_2)-5(2t-\alpha_1-\alpha_2)}{4[k_1+k_2-2(2t-\alpha_1-\alpha_2)]} < \frac{1}{2}$. The collusive prices are:

$$(p_1^c(\delta), p_2^c(\delta)) = \begin{cases} \left(p_1^N + \frac{2\delta}{1-\delta}(2t - \alpha_1 - \alpha_2), p_2^N + \frac{2\delta}{1-\delta}(2t - \alpha_1 - \alpha_2) \right) & \text{if } 0 < \delta < \frac{1}{3} \\ \left(p_1^N + \frac{1-\delta}{2(1-2\delta)}(2t - \alpha_1 - \alpha_2), p_2^N + \frac{1-\delta}{2(1-2\delta)}(2t - \alpha_1 - \alpha_2) \right) & \text{if } \frac{1}{3} < \delta < \tilde{\delta} \\ \left(p_1^m, p_2^N + \frac{t(7-10\delta)-2\alpha_2(2-3\delta)-\alpha_1(3-4\delta)-2k_1(1-2\delta)}{2(1-2\delta)} \right) & \text{if } \tilde{\delta} < \delta < \delta^m \\ (p_1^m, p_2^m) & \text{if } \delta^m \leq \delta < 1. \end{cases} \quad (9)$$

with $p_j^m = \frac{2k_j-t+\alpha_j}{2}$, for $j \in \{1, 2\}$, and $\tilde{\delta} = \frac{2k_1-5t+2\alpha_1+3\alpha_2}{4k_1-8t+3\alpha_1+5\alpha_2}$.

Proof. See Appendix. ■

3.3 Impact of cross-group externalities

When the externality that side $-j$ agents exert on side j agents increases (i.e., α_j increases), agents on side j become more valuable for platforms because their willingness to pay for the service provided by the platforms increases. This is why, under competition, an increase in α_j leads platforms to set prices more aggressively on side $-j$: by decreasing p_{-j} , platforms attract more side $-j$ agents and, therefore, extract more surplus from side j agents. In sum, when α_j increases, the Nash equilibrium price on side $-j$ decreases (Armstrong, 2006).

Let us now derive the impact of cross-group externalities on the most profitable collusive prices and profit. *Ceteris paribus*, an increase in α_j would translate into a higher surplus to agents on side j . Platforms could extract this additional surplus by increasing the price charged to side j agents (and, therefore, increase collusive profits). However, if $\delta < \delta^m$, an increase in the collusive price changes the platforms' incentives to comply with the agreement. Indeed, an increase in α_j has two countervailing effects on the sustainability of collusion: (i) it increases the one-shot gain from a deviation, $\pi^d - \pi^c$, which harms collusion;¹⁷ but (ii) it also increases the severity of the punishment after a deviation, $\frac{\delta}{1-\delta}(\pi^c - \pi^N)$, which benefits collusion. Writing down the ICC for the collusive profit

¹⁷ For a given $\pi^c \in [\pi^N, \pi^m]$, the deviation profit is increasing in α_j :

$$\pi^d = \begin{cases} \frac{[2\pi^c+2t-\alpha_1-\alpha_2]^2}{8(2t-\alpha_1-\alpha_2)} & \text{if } \pi^N \leq \pi^c < \frac{3}{2}(2t - \alpha_1 - \alpha_2) \\ 2\pi^c - 2t + \alpha_1 + \alpha_2 & \text{if } \frac{3}{2}(2t - \alpha_1 - \alpha_2) < \pi^c \leq \pi^m. \end{cases}$$

$\pi^c \in [\pi^N, \pi^m)$ to be sustainable:¹⁸

$$(ICC) : \begin{cases} (3\delta + 1)(2t - \alpha_1 - \alpha_2) - 2(1 - \delta)\pi^c \geq 0 & \text{if } \pi^N \leq \pi^c < \frac{3}{2}(2t - \alpha_1 - \alpha_2) \\ 2(2\delta - 1)\pi^c + (2 - 3\delta)(2t - \alpha_1 - \alpha_2) \geq 0 & \text{if } \frac{3}{2}(2t - \alpha_1 - \alpha_2) < \pi^c \leq \pi^m, \end{cases}$$

we conclude that an increase in α_j makes the ICC more difficult to satisfy. Thus, an increase in the cross-group externalities increases more the gain from a deviation than the severity of the punishment following a deviation. As a result, if $\delta < \delta^m$, the highest sustainable profit decreases with the cross-group externalities. For $\delta > \delta^m$, the ICC is not binding and, as a result, platforms are able to sustain the highest prices ensuring full market coverage: $\pi^c(\delta) = \pi^m$.¹⁹ In this case, an increase in cross-group externalities increases the collusive profit, because platforms are able to increase the price on side j resulting from the higher willingness to pay of side j agents.

In order to study the impact of cross-group externalities on the sustainability of collusion in our model, we define $\underline{\delta}(\pi, \alpha_1, \alpha_2) \in (0, \delta^m]$ as the minimum discount factor allowing to sustain a collusive agreement generating a profit $\pi \in (\pi^N, \pi^m]$, and study how it is affected by α_1 and α_2 . Since $\pi^c(\delta)$ is increasing in α_1 and α_2 for any $\delta \in (0, \delta^m]$, the threshold $\underline{\delta}(\pi, \alpha_1, \alpha_2)$ is increasing in α_1 and α_2 for any $\pi \in (\pi^N, \pi^m]$. This means that an increase in the magnitude of network externalities makes collusion (at a given profit level) harder to sustain.

The following corollary sums up the impact of an increase in cross-group externalities on the highest sustainable profit as well as the sustainability of collusion.

¹⁸These ICCs are derived in the proof of Proposition 2.

¹⁹Recall that platforms profit more if they establish a collusive agreement that ensures full market coverage than if they only cover partially one of the sides of the market (Lemma 4).

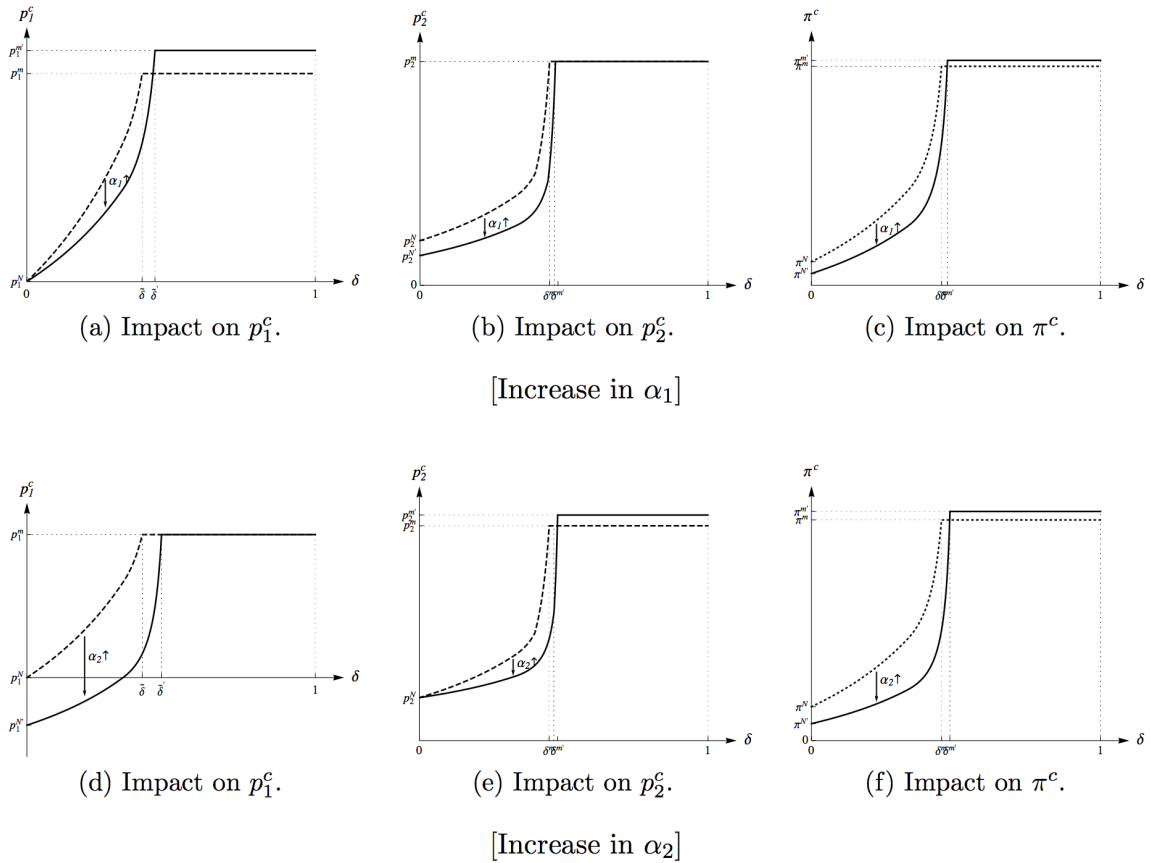


Figure 1: Impact of an increase in cross-group externalities on prices and individual profits under two-sided collusion.²⁰

Corollary 1 *Under Assumptions 1–5:*

- (i) *If perfect collusion is not sustainable ($\delta < \delta^m$), a marginal increase in the cross-group externalities on any market side decreases the highest sustainable profit.*
- (ii) *If perfect collusion is sustainable ($\delta \geq \delta^m$), a marginal increase in the cross-group externalities on any market side increases the highest sustainable profit.*
- (iii) *An increase in the magnitude of cross-group externalities makes collusion (at a given profit level) harder to sustain, i.e., $\underline{\delta}(\pi, \alpha_1, \alpha_2)$ is increasing in α_1 and α_2 .*

Proof. Follows in a straightforward way from Proposition 2 combined with Lemma 1. ■

²⁰To build these figures, we considered: $t = 1, \alpha_1 = -2, \alpha_1' = -1, \alpha_2 = 1, \alpha_2' = \frac{5}{2}, k_1 = 6, k_2 = 15$.

The last part of Corollary 1 implies that the finding by Ruhmer (2011) that an increase in cross-group externalities makes *perfect* collusion harder to sustain extends to the case of imperfect collusion. The next proposition shows, however, that the effect of cross-group externalities on the loss in users' surplus is fundamentally different under perfect and imperfect collusion. Before stating this result, note that the damage caused by collusion to the agents on side j is given by $p_j^c(\delta) - p_j^N$ because the market is fully covered both under competition and collusion.

Corollary 2 *Under Assumptions 1–5, a marginal increase in cross-group externalities may either increase or decrease the damage caused by collusion to side j agents. Precisely, a marginal increase in the cross-group externalities on any market side:*

- *decreases the harm from collusion for agents on both sides if $\delta < \tilde{\delta}$.*
- *increases the harm from collusion for side-1 agents but reduces the harm for side-2 agents if $\tilde{\delta} < \delta < \delta^m$.*
- *increases the harm from collusion for agents on both sides if $\delta > \delta^m$.*

Proof. See Appendix. ■

The harm from collusion suffered by platforms' users is affected by cross-group externalities through two channels as both the collusive price and the static Nash price depend on these externalities. When $\delta \leq \tilde{\delta}$, the collusive price on each side decreases with α_1 and α_2 , while the competitive price on side j decreases with α_{-j} but does not depend on α_j . Therefore, the way the collusive harm on side j depends on α_{-j} is *a priori* ambiguous. Corollary 2 shows that the overall effect is a decrease (with α_1 and α_2) in the harm from collusion for agents on both sides.

Moreover, Corollary 2 shows that the effect of cross-group externalities on the two sides need not be qualitatively the same. This results (partly) from the following two observations. First, the collusive price on side j is decreasing in α_1 and α_2 as long as it does not reach the monopoly price on side j ; but it decreases in α_j (and does not depend on α_{-j}) when the monopoly price is reached. Second, because of the heterogeneity in the agents' stand-alone value across sides, the collusive price on side 1 reaches the monopoly level for lower values of the discount factor than the collusive price on side 2 (this follows from Assumption 2). Therefore, there may exist a range of discount factor values ($\tilde{\delta} < \delta < \delta^m$) for which the impact of cross-group externalities on the collusive price

on one side is fundamentally different from their effect on the collusive price on the other side.²¹

4 One-sided collusion

4.1 The most profitable sustainable agreement

Let us now characterize the most profitable sustainable agreement when platforms collude on a single side of the market. Without loss of generality, we assume that platforms collude on side 1 and set prices non-cooperatively and simultaneously on side 2. We focus our analysis on symmetric collusive agreements, i.e., such that the two platforms set the same price on the collusive side ($p_1^A = p_1^B = p_1$). Thus, given $\delta \in (0, 1)$, the most profitable sustainable one-sided symmetric agreement features a price on side 1 that solves the following constrained maximization program:

$$\max_{p_1} \{ \pi^A(p_1, p_2^A, p_1, p_2^B) + \pi^B(p_1, p_2^A, p_1, p_2^B) \} \quad (10)$$

subject to the following three constraints:

$$\left\{ \begin{array}{l} p_2^A = \operatorname{argmax}_{\tilde{p}_2} \pi^A(p_1, \tilde{p}_2, p_1, p_2^B) \\ p_2^B = \operatorname{argmax}_{\tilde{p}_2} \pi^B(p_1, p_2^A, p_1, \tilde{p}_2) \\ p_1 \in I^{oc}(\delta, p_2^A, p_2^B) \equiv \left\{ p_1 \in \mathbb{R} \mid \frac{\pi^A(p_1, p_2^A, p_1, p_2^B)}{1-\delta} \geq \max_{(\tilde{p}_1, \tilde{p}_2)} \pi^A(\tilde{p}_1, \tilde{p}_2, p_1, p_2^B) + \frac{\delta}{1-\delta} \pi^N \right\} \end{array} \right. \quad (11)$$

Since the first two constraints are equivalent to

$$p_2^A = p_2^B = p_2 = \frac{t^2 - \alpha_1 \alpha_2}{t} - \frac{\alpha_1}{t} p_1 \equiv g(p_1, \alpha_1, \alpha_2),$$

we can rewrite the maximization program above as:

$$\max_{p_1} \{ \pi^A(p_1, p_2, p_1, p_2) + \pi^B(p_1, p_2, p_1, p_2) \}$$

²¹Note that this range of discount factor values is empty if the two sides of the market are perfectly symmetric (i.e., $k_1 = k_2$), since in this case $\tilde{\delta} = \delta^m$.

subject to:

$$\begin{cases} p_2 = g(p_1, \alpha_1, \alpha_2) \\ p_1 \in I^{oc}(\delta, p_2, p_2) \equiv \left\{ p_1 \in \mathbb{R} \mid \frac{\pi^A(p_1, p_2, p_1, p_2)}{1-\delta} \geq \max_{(\tilde{p}_1, \tilde{p}_2)} \pi^A(\tilde{p}_1, \tilde{p}_2, p_1, p_2) + \frac{\delta}{1-\delta} \pi^N \right\}. \end{cases} \quad (12)$$

For a given $\delta \in (0, 1)$, let $p_1^{oc}(\delta)$ denote the solution to the above maximization program, and $p_2^{oc}(\delta)$ the corresponding non-cooperative price on side 2.²² Moreover, define

$$\Delta p_j \equiv p_j^{oc}(\delta) - p_j^N$$

as the effect of one-sided collusion on the price on side $j \in \{1, 2\}$. Let us first notice that Δp_1 and Δp_2 have the same sign if $\frac{\partial g}{\partial p_1} > 0$ and have opposite signs if $\frac{\partial g}{\partial p_1} < 0$. To see why, note that:

$$\Delta p_2 = p_2^{oc}(\delta) - p_2^N = g(p_1^{oc}(\delta), \alpha_1, \alpha_2) - g(p_1^N, \alpha_1, \alpha_2) = \int_{p_1^N}^{p_1^{oc}(\delta)} \frac{\partial g}{\partial p_1} dp_1.$$

Therefore, if increasing the price on the collusive side strengthens competition on the non-cooperative side, i.e., $\frac{\partial g}{\partial p_1} < 0$, the price on one market side under one-sided collusion is higher than the static Nash price, while the other price is lower than the static Nash price. In contrast, if increasing the price on the collusive side softens competition on the non-cooperative side, i.e., $\frac{\partial g}{\partial p_1} > 0$, prices on the two sides of the market are both below or both above Nash prices. In Armstrong (2006)'s setting, platforms never find it optimal to agree on decreasing their prices below their static Nash levels on both sides of the market because the market is already fully covered under competition (by Assumptions 2 and 3).²³

The following lemma relates the magnitude of the price variations on each side of the market induced by one-sided collusion.

Lemma 6 *Under Assumptions 1–3, the most profitable one-sided sustainable agreement leads to price variations on both sides that are related as follows:*

$$\Delta p_2 = -\frac{\alpha_1}{t} \Delta p_1. \quad (13)$$

²²The existence and uniqueness of the solution are established later.

²³In Section 5, we discuss the implications of relaxing the assumption of inelastic demand.

Proof. We have: $\Delta p_2 = \int_{p_1^N}^{p_1^{oc}(\delta)} \frac{\partial g}{\partial p_1} dp_1 = -\frac{\alpha_1}{t} [p_1^{oc}(\delta) - p_1^N] = -\frac{\alpha_1}{t} \Delta p_1$. ■

Even though platforms collude only on one side of the market, the price on the non-cooperative side is also affected by collusion due to the existence of cross-group externalities. Suppose that $p_1^{oc} > p_1^N$. Collusion makes side-1 agents more valuable to platforms. As a result, platforms would like to increase their market share on side 1 (as compared to the competitive scenario). As p_1 is fixed by the collusive agreement, the only way for a platform to conquer more side-1 agents without triggering a punishment from the rival platform is to increase the attractiveness of its platform to these agents by changing the number of agents on side 2. If $\alpha_1 > 0$, side-1 agents enjoy the presence of more side-2 agents, and platforms have, therefore, incentives to decrease p_2 . By contrast, if $\alpha_1 < 0$, platforms have incentives to increase p_2 to attract less side-2 agents, and increase their attractiveness to side-1 agents. If $p_1^{oc} < p_1^N$, the reasoning is exactly the opposite: as collusion makes side-1 agents less valuable to platforms (as compared to the competitive scenario), they use p_2 to decrease the value that side-1 agents get from joining a platform.

To gain further insights we need to distinguish between the scenario in which the ICC

$$\frac{\pi^A(p_1, g(p_1, \alpha_1, \alpha_2), p_1, g(p_1, \alpha_1, \alpha_2))}{1 - \delta} \geq \max_{(\tilde{p}_1, \tilde{p}_2)} \pi^A(\tilde{p}_1, \tilde{p}_2, p_1, g(p_1, \alpha_1, \alpha_2)) + \frac{\delta}{1 - \delta} \pi^N$$

is binding (*imperfect one-sided collusion*) and the scenario in which it is not (*perfect one-sided collusion*). Given $\delta \in (0, 1)$, defining $\pi^{oc}(\delta)$ as the highest sustainable profit under one-sided collusion and π^{om} as a firm's profit in the absence of the ICC, it is straightforward to show that $\pi^{oc}(\delta)$ is (weakly) increasing in δ ,²⁴ and that there exists a unique threshold $\delta^{om} \in (0, 1)$ such that $\pi^{oc}(\delta) < \pi^{om}$ if and only if $\delta < \delta^{om}$.

Let us first consider that perfect one-sided collusion is sustainable, i.e., $\delta \geq \delta^{om}$. In this scenario, firms can increase or decrease p_1 (with respect to its Nash level) without any constraint. Let p_1^{om} denote the firms' optimal price on side 1 in this case, i.e., $p_1^{om} = p_1^{oc}(\delta), \forall \delta \geq \delta^{om}$. If $\alpha_1 < 0$, a decrease in p_1 leads to a decrease in p_2 and, therefore, decreasing p_1 below its Nash level is surely unprofitable. This implies that $p_1^{om} > p_1^N$. In contrast, if $\alpha_1 > 0$, an increase in p_1 is followed by a decrease in p_2 . In this case, increasing p_1 is profitable if and only if the gain on side 1, $p_1^{om} - p_1^N$, outweighs the loss on side 2, i.e., $\frac{\alpha_1}{t} (p_1^{om} - p_1^N)$, which is the case if and only if $t > \alpha_1$. If, instead, $t < \alpha_1$, side-1 agents are so valuable that platforms decrease p_2 so much (to increase their attractiveness on side 1)

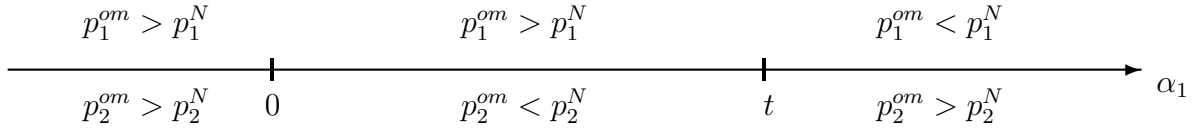
²⁴This follows from the fact that an increase in δ does not affect the firms' objective function but relaxes the constraints (or, equivalently, widens the subspace of prices over which firms maximize their joint profits).

that the profit loss on side 2 offsets the profit gain on side 1. This leads to the following proposition.

Proposition 3 *Suppose that Assumptions 1–3 hold and consider $\delta \geq \delta^{om}$.*

- (i) *If $\alpha_1 < 0$, the prices under the most profitable one-sided agreement are above their static Nash levels on both sides of the market.*
- (ii) *In contrast, if $\alpha_1 > 0$, the prices under the most profitable one-sided agreement are such that the price on one side is above its static Nash level while the price on the other side is below its static Nash level.*

More precisely, the following holds:



When the cross-group externalities exerted on the collusive side are positive, the relative price variation on the two sides due to collusion depends on the ratio between the strength of these externalities and the degree of differentiation on the collusive side, $\frac{\alpha_1}{t}$. If $|\alpha_1| > t$, the price variation due to collusion is higher in the non-cooperative side, $|\Delta p_2| > |\Delta p_1|$. In contrast, if $|\alpha_1| < t$, the price variation is higher in the collusive side: $|\Delta p_1| > |\Delta p_2|$. This is due to the fact that an additional side-2 agent attracts $\frac{\alpha_1}{t}$ additional side-1 agents to a platform (Armstrong, 2006). If $t = \alpha_1$, any price change in the collusive side is followed by a change of the same magnitude but on the opposite direction on side 2. Therefore, if $t = \alpha_1$, the (one-period) collusive profit coincides with the static Nash profit, corresponding to the conjecture of Evans and Schmalensee (2008) that if platforms “agree to fix prices on one side only, the cartel members will tend to compete the supracompetitive profits away on the other side.” (p. 689) We prove, however, that this only happens in a very particular case.²⁵

To show that the comparison between the prices under one-sided collusion and competition provided by Proposition 3 extends to the case of imperfect one sided-collusion (i.e., for $\delta < \delta^{om}$), we make Assumption 6, which ensures full market coverage under one-sided collusion.

²⁵Dewenter *et al.* (2011) also find that the claim by Evans and Schmalensee (2008) is true only in a very special case in their model.

Assumption 6

- If $t \geq \alpha_1$, let $k_1 > t - \alpha_1$.
- If $t \leq \alpha_1$, let $k_2 > \frac{2\alpha_1 t - \alpha_1 \alpha_2 - t^2}{2\alpha_1}$.

Lemma 7 Under Assumptions 1–3 and 6, the most profitable sustainable one-sided agreement is such that both market sides are fully covered.

Proof. See Appendix. ■

Lemma 8 Let $\tilde{u}_j^N \equiv u_j^i(\frac{1}{2}, p_1^N, p_2^N, p_1^N, p_2^N)$ denote the utility of the side- j agent located at $x = \frac{1}{2}$ if platforms set the static Nash prices. Under Assumptions 1–3 and 6, the prices under the most profitable sustainable one-sided agreement are such that:

1. If $-\frac{\tilde{u}_2^N}{\tilde{u}_1^N}t \leq \alpha_1 < t$:

$$(p_1^{oc}(\delta), p_2^{oc}(\delta)) = \begin{cases} \left(\frac{2t\pi^{oc}(\delta) - t^2 + \alpha_1\alpha_2}{t - \alpha_1}, \frac{t^2 - \alpha_1\alpha_2 - 2\alpha_1\pi^{oc}(\delta)}{t - \alpha_1} \right) & \text{if } 0 < \delta \leq \tilde{\delta}^{om} \\ \left(p_1^m, t - \frac{\alpha_1(2k_1 - t + \alpha_1 + 2\alpha_2)}{2t} \right) & \text{if } \tilde{\delta}^{om} \leq \delta < 1, \end{cases} \quad (14)$$

where p_1^m is given by (5), and $\tilde{\delta}^{om}$ is the value of δ^{om} in this scenario and is given by $\pi^{oc}(\tilde{\delta}^{om}) = \frac{2k_1(t - \alpha_1) + t(t + 2\alpha_1) - \alpha_1(\alpha_1 + 2\alpha_2)}{4t}$.

2. If $0 < t < \alpha_1$ or $\alpha_1 < -\frac{\tilde{u}_2^N}{\tilde{u}_1^N}t < t$:

$$(p_1^{oc}(\delta), p_2^{oc}(\delta)) = \begin{cases} \left(\frac{2t\pi^{oc}(\delta) - t^2 + \alpha_1\alpha_2}{t - \alpha_1}, \frac{t^2 - \alpha_1\alpha_2 - 2\alpha_1\pi^{oc}(\delta)}{t - \alpha_1} \right) & \text{if } 0 < \delta \leq \hat{\delta}^{om} \\ \left(\frac{3t^2 - t\alpha_2 - 2\alpha_1\alpha_2 - 2tk_2}{2\alpha_1}, p_2^m \right) & \text{if } \hat{\delta}^{om} \leq \delta < 1, \end{cases} \quad (15)$$

where p_2^m is given by (5), and $\hat{\delta}^{om}$ is the value of δ^{om} in this scenario and is given by $\pi^{oc}(\hat{\delta}^{om}) = \frac{2k_2(\alpha_1 - t) + t(3t - \alpha_1 - \alpha_2) - \alpha_1\alpha_2}{4\alpha_1}$.

Proof. See Appendix. ■

Using the fact that the highest sustainable profit under one-sided collusion, $\pi^{oc}(\delta)$, is increasing in δ over $(0, \delta^{om})$ (where δ^{om} is equal to either $\tilde{\delta}^{om}$ or $\hat{\delta}^{om}$ depending on the

considered scenario), it is straightforward to derive from the lemma above the monotonicity of the prices under one-sided collusion with respect to δ . This allows, in particular, to compare these prices to the static Nash prices (recall that $p_1^N = p_1^{oc}(0)$ and $p_2^N = p_2^{oc}(0)$).

Proposition 4 *Under Assumptions 1–3 and 6 and for $\delta < \delta^{om}$, the prices under the most profitable one-sided collusive agreement are as follows:*

- (i) *If $\alpha_1 < 0$, platforms charge prices that are increasing in δ on both sides. Therefore, the prices are above their static Nash level on both sides.*
- (ii) *If $0 < \alpha_1 < t$, the price on side 1 is increasing in δ while the price on side 2 is decreasing in δ . Therefore, the price on side 1 is above its static Nash level while the price on side 2 is below its static Nash level.*
- (iii) *If $\alpha_1 > t$, the price on side 1 is decreasing in δ while the price on side 2 is increasing in δ . Therefore, the price on side 1 is below its static Nash level while the price on side 2 is above its static Nash level.*

We can now state the effect of one-sided collusion on the surplus of the agents on each side of the market.

Corollary 3 *Under Assumptions 1–3 and 6, the impact of one-sided collusion on the surplus of side $j \in \{1, 2\}$ agents is equal to the price change on this side, Δp_j . Thus:*

- (i) *If $\alpha_1 < 0$, agents on both market sides are damaged. Side-1 agents are the most damaged iff $|\alpha_1| > t$.*
- (ii) *If $0 < \alpha_1 < t$, collusion damages side-1 agents and benefits side-2 agents.*
- (iii) *If $0 < t < \alpha_1$, collusion benefits side-1 agents and damages side-2 agents.*
- (iv) *Collusion always decreases the joint surplus of side-1 and side-2 agents.*

Proof. The first three statements follow in a straightforward way from Proposition 4. The fourth statement follows from the fact that $\Delta p_1 + \Delta p_2$ is positive (since this also represents the increase in the platforms' joint profits due to collusion). ■

To derive the effect of cross-group externalities on the sustainability of one-sided collusion, we provide a closed-form expression for collusive profits under an additional assumption on the parameters of the model. More precisely, we impose lower bounds on the stand-alone values, k_1 and k_2 , to get the richest possible setting.²⁶

Assumption 7

- If $-\frac{\tilde{u}_2^N}{\tilde{u}_1^N}t \leq \alpha_1 < t$, let $k_1 > \frac{14t^3 - (2t^2 - \alpha_1^2 - \alpha_1\alpha_2)(\alpha_1 + 2\alpha_2) - t(\alpha_1 + \alpha_2)(5\alpha_1 + 2\alpha_2)}{2(2t^2 - \alpha_1^2 - \alpha_1\alpha_2)}$;
- If $0 < t < \alpha_1$ or $\alpha_1 < -\frac{\tilde{u}_2^N}{\tilde{u}_1^N}t < t$, let $k_2 > \frac{t^2(11\alpha_1 - 3\alpha_2) - (\alpha_1 - \alpha_2)(2\alpha_1 + \alpha_2)t - 2\alpha_1(\alpha_1 + \alpha_2)^2}{2t(\alpha_1 - \alpha_2)}$.

We present in the next proposition the expressions for the highest sustainable profit under one-sided collusion.

Proposition 5 *Under Assumptions 1–3 and 7, given $\delta \in (0, 1)$, the most sustainable profit if platforms collude on side 1 and set prices non-cooperatively on side 2 is:*

1. If $-\frac{\tilde{u}_2^N}{\tilde{u}_1^N}t \leq \alpha_1 < t$:

$$\pi^{oc}(\delta) = \begin{cases} \frac{(2t - \alpha_1 - \alpha_2) \left[t(t^2 - \alpha_1\alpha_2) + [(t^3 - t^2(3\alpha_1 - \alpha_2) - t\alpha_1\alpha_2 + \alpha_1^2(\alpha_1 + \alpha_2))\delta] \right]}{2t(t^2 - \alpha_1\alpha_2)(1 - \delta)} & \text{if } 0 < \delta < \tilde{\delta}^{oc} \\ \tilde{\pi}^{oc}(\delta) & \text{if } \tilde{\delta}^{oc} \leq \delta < \tilde{\delta}^{om} \\ \tilde{\pi}^{om} & \text{if } \tilde{\delta}^{om} \leq \delta < 1, \end{cases} \quad (16)$$

$$\text{with } \tilde{\delta}^{oc} = \frac{t(t^2 - \alpha_1\alpha_2)}{t(t^2 - \alpha_1\alpha_2) + (t - \alpha_1)(2t^2 - \alpha_1^2 - \alpha_1\alpha_2)} \text{ and } \tilde{\pi}^{om} = \frac{2k_1(t - \alpha_1) + t(t + 2\alpha_1) - \alpha_1(\alpha_1 + 2\alpha_2)}{4t}.$$

²⁶As shown in the proof of Proposition 5, if k_1 is large enough, $\exists \bar{\pi}^{oc} \in [\pi^N, \tilde{\pi}^{om})$ such that: a deviating platform would not monopolize any side of the market if $\pi^{oc}(\delta) < \bar{\pi}^{oc}$; it would monopolize the collusive side 1 (but not the non-cooperative side 2) if $\pi^{oc}(\delta) \geq \bar{\pi}^{oc}$. For lower values of k_1 , the deviating platform would never monopolize any side of the market. In that case, the second branch of (16) would disappear. Similarly, if k_2 is not sufficiently large, a deviating platform would never monopolize the non-cooperative side 2, and the expression for the one-sided collusive profit would be given by (17) without the second branch. Note that assumption 7 is more demanding than Assumption 6.

²⁷The expressions for $\tilde{\pi}^{oc}(\delta)$ and $\tilde{\delta}^{om}$ are, respectively, given by (48) and (49) in the Appendix.

2. If $0 < t < \alpha_1$ or $\alpha_1 < -\frac{\tilde{u}_2^N}{\tilde{u}_1^N}t < t$:

$$\pi^{oc}(\delta) = \begin{cases} \frac{(2t-\alpha_1-\alpha_2)[t(t^2-\alpha_1\alpha_2)] + [(t^3-t^2(3\alpha_1-\alpha_2)-t\alpha_1\alpha_2+\alpha_1^2(\alpha_1+\alpha_2))\delta]}{2t(t^2-\alpha_1\alpha_2)(1-\delta)} & \text{if } 0 < \delta < \hat{\delta}^{oc} \\ \hat{\pi}^{oc}(\delta) & \text{if } \hat{\delta}^{oc} \leq \delta < \hat{\delta}^{om} \\ \hat{\pi}^{om} & \text{if } \hat{\delta}^{om} \leq \delta < 1, \end{cases} \quad (17)$$

$$\text{with } \hat{\delta}^{oc} = \frac{t^2 - \alpha_1\alpha_2}{t^2 - \alpha_1\alpha_2 + (\alpha_1 - \alpha_2)(\alpha_1 - t)} \text{ and } \hat{\pi}^{om} = \frac{2k_2(\alpha_1 - t) + t(3t - \alpha_1 - \alpha_2) - \alpha_1\alpha_2}{4\alpha_1}. \text{ }^{28}$$

Proof. See Appendix. ■

4.2 Effects of cross-group externalities

The mathematical expression for the highest sustainable one-sided collusive profit makes it difficult to derive the effect of a unilateral increase in the externalities exerted by one side of the market on the other side on profits. Nevertheless, we are able to sign the effect of a joint increase in cross-group externalities when they are symmetric.

Proposition 6 *Under Assumptions 1–3 and 7, if cross-group externalities are symmetric across sides ($\alpha_1 = \alpha_2 = \alpha$), an increase in α leads to a decrease of the highest sustainable one-sided collusive profit:*

$$\frac{\partial \pi^{oc}}{\partial \alpha} < 0.$$

As a result, an increase in cross-group externalities makes one-sided collusion at a given profit level harder to sustain.

Proof. See Appendix. ■

Proposition 6 shows that the effect of an increase in cross-group externalities on one-sided collusive profits (with symmetric externalities) is qualitatively the same as under imperfect two-sided collusion.

²⁸The expressions for $\hat{\pi}^{oc}(\delta)$ and $\hat{\delta}^{om}$ are, respectively, given by (58) and (61) in the Appendix.

²⁹To build these figures, we considered: $t = 3, \alpha = 1, \alpha' = 2, k_1 = 10, k_2 = 4$.

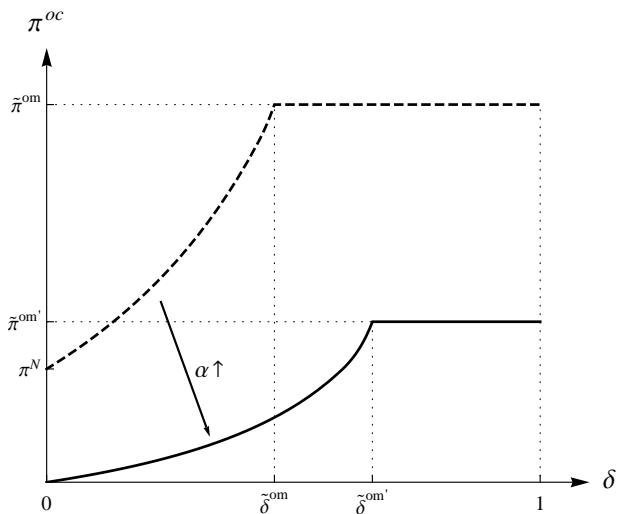


Figure 2: Impact of a joint increase in cross-group externalities when they are symmetric.²⁹

5 Discussion

Demand expansion

In our model, both sides of the market are fully covered and split equally between the two platforms under competition and collusion. Thus, collusion does not affect total demand (and individual demand in a symmetric equilibrium). While this feature allows to have a tractable model and derive neat results, it also imposes limitations. In particular, in a setting where collusion would affect the size of demand, the result that two-sided collusion always leads to higher prices on both sides of the market may not hold. The reason is that a decrease in price on one side would expand demand on that side and would, therefore, increase the willingness to pay of the agents on the other side if cross-group externalities are positive. If the magnitude of this effect (which does not exist in our setting) is relatively large, it might be the case that collusion leads to a lower price on one side of the market than competition. Note, however, that the striking result that, under one-sided collusion, the price on the collusive side may be lower than the static Nash price on that side would be strengthened if we allowed for demand expansion. To see why, note that, in our setting, firms' incentives to set a price below the Nash level are solely driven by the incentive to soften competition on the other side. In an environment where, following a decrease in the price on side j , there is a demand expansion on that side and potentially also on side $-j$ (if $\alpha_{-j} > 0$), our result is even more likely to hold.

Endogenous choice of the collusive side(s)

In our setting, platforms can always sustain some degree of collusion in equilibrium both when they collude on the two market sides, and when they just collude on one side of the market. Our model also suggests that platforms should always prefer to collude on both sides (since this is the most profitable scenario). However, as mentioned before, there is evidence of platforms being convicted of just coordinating the price on one market side. Coordination costs and the possibility of them being (prohibitively) higher when platforms coordinate two prices instead of one may underlie actual platforms' choice.³⁰ Relatedly, platforms may engage in one-sided collusion to attempt to reduce the risk of being caught and punished by antitrust authorities.

In the context of one-sided collusion, a natural question that arises concerns the choice of the collusive side. While a general treatment of this issue is outside the scope of this paper, we provide two special cases where we are able to determine the platforms' choice. Collusion on side 1 yields the same outcome as competition if $\alpha_1 = t$, and the same outcome as two-sided collusion if $\alpha_1 = -t$ and $\delta < \delta^m$.³¹ Therefore, platforms (weakly) prefer to collude on side 2 in the former case while they (weakly) prefer to collude on side 1 in the latter case. There are other reasons outside our model that may also affect the choice of the side to collude on. For instance, it may be harder for platforms to coordinate prices on one side of the market than on the other one. For example, in the case of newspapers, coordinating cover prices may probably be easier than coordinating ad prices (as the latter are likely to be more heterogeneous). Moreover, monitoring might be easier on one side of the market than on the other one. Considering again the newspapers example, cover prices are typically more transparent and, therefore, easier to monitor, than ad prices.

Multi-homing

In the standard competitive bottleneck model (Armstrong, 2006), there is no strategic interaction on the multi-homing side. A first implication is that there is no incentive to engage in collusion on the multi-homing side.³² Thus, in that context, platforms may

³⁰One (perhaps simplistic) way of incorporating these ingredients in our model would be to introduce a fixed coordination cost. It follows straightforwardly that: if this cost is not much higher when platforms coordinate prices on both sides of the market than when they just coordinate one price, platforms will settle a two-sided collusive agreement; while, if the coordination cost is larger enough under two-sided collusion, platforms will settle a one-sided collusive agreement. For intermediate values of this coordination cost, platforms' choice may depend on the discount factor.

³¹Both results follow from Lemma 6.

³²In particular, two-sided collusion and one-sided collusion on the single-homing side of the market yield the same outcome.

collude on the single-homing side. An agreement on supra-competitive (infra-competitive) prices on the single-homing side generates three potential effects: *(i)* higher (lower) margins on the single-homing side; *(ii)* potentially lower (higher) demand on the single-homing side; and *(iii)* potentially lower (higher) prices on the multi-homing side. Therefore, whether platforms increase or decrease the price on the (collusive) single-homing side, relative to its static Nash level, depends on whether the price effect on the single-homing side outweighs or is outweighed by the combination of the demand effect on the single-homing side and the price effect on the multi-homing side. In a setting where the single-homing side takes the form of a Hotelling line, fully covered under both competition and collusion, the latter two effects do not exist (as collusion on the single-homing side does not affect the demand on that side and, therefore, does not affect the price on the multi-homing side). Thus, in that special case, collusion on the single-homing side would lead to an increase in price on that side and leave the price on the multi-homing side unaffected.

6 Conclusion

We investigate collusion between two-sided platforms in a repeated version of Armstrong (2006)'s canonical model. Considering first the scenario in which collusion occurs on both sides of the market, we show that platforms use the structure of prices as an instrument to minimize the incentives to deviate. Moreover, we establish that collusion (at a given profit level) is harder to sustain if cross-group externalities increase. Turning to the case where firms collude only on one side of the market, we show that the price on the collusive (non-cooperative) side may be lower (higher) than the competitive price, thus leading to an increase (decrease) in the surplus of this side's agents.

Our results also provide interesting insights into the effects of collusion on prices in a multi-product setting with demand linkages. When the parameters capturing cross-group externalities in our model are positive, the latter can be reinterpreted as a model in which two firms selling two complementary products compete against each other. Our results show in particular that single-product collusion in such an environment can lead to a decrease in the price of the product for which there is collusion and an increase in the price of the product for which there is competition.

Appendix

Proof of Lemma 2.

Note first that ICC (4) can be rewritten as:

$$\frac{1}{\delta} [\pi^d(p_1, p_2) - \pi(p_1, p_2)] \leq \pi^d(p_1, p_2) - \pi^N.$$

(i) From the fact that $\pi^d(p_1, p_2) - \pi(p_1, p_2) \geq 0$, it follows that $\delta < \delta' \Rightarrow I(\delta) \subseteq I(\delta') \Rightarrow \pi^c(\delta) \leq \pi^c(\delta')$. For $\delta \in (0, \delta^m)$, $(p_1^m, p_2^m) \notin I(\delta)$. This, combined with the fact that (p_1^m, p_2^m) is the unique solution to the unconstrained maximization program $\max_{(p_1, p_2) \in \mathbb{R}^2} \pi(p_1, p_2)$ implies then that $\pi^c(\delta) < \pi^m$. For $\delta \in [\delta^m, 1)$, $(p_1^m, p_2^m) \in I(\delta)$ and, therefore, $\pi^c(\delta) = \pi^m$.

(ii) Let $\delta \in (0, \delta^m)$ and assume by way of contradiction that

$$\frac{1}{\delta} [\pi^d(p_1^c(\delta), p_2^c(\delta)) - \pi(p_1^c(\delta), p_2^c(\delta))] < \pi^d(p_1^c(\delta), p_2^c(\delta)) - \pi^N,$$

i.e., the constraint is not binding at the optimum. Then, by a (standard) continuity argument, there exists $\epsilon > 0$ such that

$$\frac{1}{\delta} [\pi^d(p_1, p_2) - \pi(p_1, p_2)] < \pi^d(p_1, p_2) - \pi^N$$

for any $(p_1, p_2) \in [p_1^c(\delta) - \epsilon, p_1^c(\delta) + \epsilon] \times [p_2^c(\delta) - \epsilon, p_2^c(\delta) + \epsilon]$. This implies that the pair of prices $(p_1^c(\delta), p_2^c(\delta))$ is a local maximum of $\pi(p_1, p_2)$. However, straightforward computations show that $\pi(p_1, p_2)$ does not have a local maximum but its global maximum, which is uniquely reached at (p_1^m, p_2^m) . This implies that $(p_1^c(\delta), p_2^c(\delta)) = (p_1^m, p_2^m)$, and contradicts (i). \square

Proof of Lemma 3.

Assume, by way of contradiction, that $(p_1^c(\delta), p_2^c(\delta))$ is not a solution to the constrained minimization program. Denoting $(\hat{p}_1(\delta), \hat{p}_2(\delta))$ a solution to that program, we then have

$$\pi^d(\hat{p}_1(\delta), \hat{p}_2(\delta)) < \pi^d(p_1^c(\delta), p_2^c(\delta)).$$

Therefore

$$\begin{aligned}\frac{\pi(\hat{p}_1(\delta), \hat{p}_2(\delta))}{1-\delta} &= \frac{\pi(p_1^c(\delta), p_2^c(\delta))}{1-\delta} = \\ &= \pi^d(p_1^c(\delta), p_2^c(\delta)) + \frac{\delta}{1-\delta}\pi^N > \pi^d(\hat{p}_1(\delta), \hat{p}_2(\delta)) + \frac{\delta}{1-\delta}\pi^N,\end{aligned}$$

which implies that

$$\frac{1}{\delta} [\pi^d(\hat{p}_1(\delta), \hat{p}_2(\delta)) - \pi(\hat{p}_1(\delta), \hat{p}_2(\delta))] < \pi^d(\hat{p}_1(\delta), \hat{p}_2(\delta)) - \pi^N.$$

Again, by a continuity argument, there exists $\mu > 0$ such that

$$\frac{1}{\delta} [\pi^d(p_1, p_2) - \pi(p_1, p_2)] < \pi^d(p_1, p_2) - \pi^N$$

for any $(p_1, p_2) \in [\hat{p}_1(\delta) - \mu, \hat{p}_1(\delta) + \mu] \times [\hat{p}_2(\delta) - \mu, \hat{p}_2(\delta) + \mu]$. There are only two possible scenarios, which both lead to a contradiction:

- If $\pi(p_1, p_2)$ reaches a local maximum at $(\hat{p}_1(\delta), \hat{p}_2(\delta))$ then it is necessarily the case that $(\hat{p}_1(\delta), \hat{p}_2(\delta)) = (p_1^m, p_2^m)$, and, therefore, $\pi(\hat{p}_1(\delta), \hat{p}_2(\delta)) = \pi^m > \pi^c(\delta)$ because $\delta \in (0, \delta^m)$, a contradiction.

- If $\pi(p_1, p_2)$ does not reach a local maximum at $(\hat{p}_1(\delta), \hat{p}_2(\delta))$ then there exists $(\check{p}_1, \check{p}_2) \in [\hat{p}_1(\delta) - \mu, \hat{p}_1(\delta) + \mu] \times [\hat{p}_2(\delta) - \mu, \hat{p}_2(\delta) + \mu]$ such that

$$\pi(\check{p}_1, \check{p}_2) > \pi(\hat{p}_1(\delta), \hat{p}_2(\delta)) = \pi^c(\delta)$$

Since $(\check{p}_1, \check{p}_2) \in I(\delta)$, this contradicts the fact that $\pi^c(\delta) = \max_{(p_1, p_2) \in I(\delta)} \pi(p_1, p_2)$. \square

Proof of Lemma 4.

We are focusing on symmetric collusive agreements, i.e., such that platforms set the same cooperative price on each side of the market, i.e., $p_j^A = p_j^B = p_j$, $j \in \{1, 2\}$. Let \tilde{x}_j denote the consumer on side j that is indifferent between joining platform A and not joining any platform.

1. We start by deriving the conditions that ensure that, if platforms fully serve side 2, they will also prefer to fully serve side 1.

Given $\tilde{x}_2 = \frac{1}{2}$:

$$u_1^A(\tilde{x}_1, p_1, p_2, p_1, p_2) = 0 \Leftrightarrow \tilde{x}_1 = \frac{2k_1 - 2p_1 + \alpha_1}{2t}.$$

If $\tilde{x}_1 \leq \frac{1}{2}$, the individual collusive profit is:

$$\pi^c(p_1, p_2) = p_1 \tilde{x}_1 + \frac{p_2}{2} = \frac{p_1(2k_1 - 2p_1 + \alpha_1)}{2t} + \frac{p_2}{2}. \quad (18)$$

As π^c is strictly increasing in p_2 , platforms will choose the highest price that leaves the consumer \tilde{x}_2 with zero utility:

$$u_2^A(\frac{1}{2}, p_1, p_2, p_1, p_2) = 0 \Leftrightarrow p_2 = k_2 + \alpha_2 \tilde{x}_1 - \frac{t}{2} \Leftrightarrow p_2 = k_2 + \frac{\alpha_2(2k_1 - 2p_1 + \alpha_1)}{2t} - \frac{t}{2}.$$

Combining this expression with the FOC corresponding to the maximization of π^c with respect to p_1 , $\frac{\partial \pi^c}{\partial p_1} = 0$, we obtain:

$$p_1 = \frac{2k_1 + \alpha_1}{4} \quad \text{and} \quad p_2 = \frac{4k_2 t - 2t^2 + 2k_1 \alpha_2 + \alpha_1 \alpha_2}{4t}.$$

Given these prices:

$$\tilde{x}_1 = \frac{2k_1 + \alpha_1}{4t}.$$

As a result, there is a local maximum of π^c with partial coverage of side 1 if:

$$\tilde{x}_1 < \frac{1}{2} \Leftrightarrow k_1 < t - \frac{\alpha_1}{2}.$$

Similarly, there is a local maximum of π^c with partial coverage of side 2 if:

$$k_2 < t - \frac{\alpha_2}{2}.$$

2. Let us now see under which conditions platforms prefer to partially serve both sides instead of fully serving them.

For $j \in \{1, 2\}$, we have:

$$k_j + \alpha_j \tilde{x}_{-j} - t \tilde{x}_j - p_j = 0.$$

Solving this system of two equations, we obtain:

$$\tilde{x}_1 = \frac{\alpha_1(k_2 - p_2) + t(k_1 - p_1)}{t^2 - \alpha_1 \alpha_2} \quad \text{and} \quad \tilde{x}_2 = \frac{\alpha_2(k_1 - p_1) + t(k_2 - p_2)}{t^2 - \alpha_1 \alpha_2}.$$

If $\tilde{x}_j \leq \frac{1}{2}$, the individual collusive profit is given by:

$$\pi^c = p_1 \tilde{x}_1 + p_2 \tilde{x}_2.$$

Solving the FOCs corresponding to the maximization of π^c , we obtain:

$$p_1 = \frac{k_2 t (\alpha_1 - \alpha_2) + k_1 (2t^2 - \alpha_2 \alpha_1 - \alpha_2^2)}{(2t - \alpha_1 - \alpha_2)(2t + \alpha_1 + \alpha_2)} \quad \wedge \quad p_2 = \frac{k_1 t (\alpha_2 - \alpha_1) + k_2 (2t^2 - \alpha_2 \alpha_1 - \alpha_1^2)}{(2t - \alpha_1 - \alpha_2)(2t + \alpha_1 + \alpha_2)}.$$

Given these prices:

$$\tilde{x}_j = \frac{2k_j t + k_{-j}(\alpha_j + \alpha_{-j})}{(2t - \alpha_1 - \alpha_2)(2t + \alpha_1 + \alpha_2)}.$$

There is an interior local maximum (with partial coverage of both market sides) iff $\tilde{x}_1 < \frac{1}{2} \wedge \tilde{x}_2 < \frac{1}{2}$, i.e.:

$$\tilde{x}_j < \frac{1}{2} \Leftrightarrow 2tk_j + (\alpha_1 + \alpha_2)k_{-j} < \frac{(2t - \alpha_1 - \alpha_2)(2t + \alpha_1 + \alpha_2)}{2}.$$

Thus, the two platforms prefer to fully cover the two market sides iff:

$$\begin{aligned} k_1 \geq \frac{2t - \alpha_1}{2} \quad \wedge \quad k_2 \geq \frac{2t - \alpha_2}{2} \quad \wedge \\ \wedge \quad 2t \min\{k_1, k_2\} + (\alpha_1 + \alpha_2) \max\{k_1, k_2\} \geq \frac{(2t - \alpha_1 - \alpha_2)(2t + \alpha_1 + \alpha_2)}{2}. \end{aligned}$$

Under Assumption 1, a sufficient condition for the last inequality to hold is:

$$k_1 + k_2 \geq \frac{(2t - \alpha_1 - \alpha_2)(2t + \alpha_1 + \alpha_2)}{2(\alpha_1 + \alpha_2)}.$$

□

Proof of Lemma 5.

A deviating platform (say A) sets prices (p_1^d, p_2^d) that maximize its individual profit given that platform B charges the collusive prices (p_1^c, p_2^c) :

$$\pi^d(p_1^d, p_2^d) = \frac{-t(p_1^d)^2 - t(p_2^d)^2 - (\alpha_1 + \alpha_2)p_1^d p_2^d + p_1^d (tp_1^c + \alpha_1 p_2^c + t^2 - \alpha_1 \alpha_2) + p_2^d (tp_2^c + \alpha_2 p_1^c + t^2 - \alpha_1 \alpha_2)}{2(t^2 - \alpha_1 \alpha_2)}. \quad (19)$$

The FOCs corresponding to the maximization of π^d are ($j \in \{1, 2\}$):

$$\frac{\partial \pi^d}{\partial p_j^d} = 0 \Leftrightarrow \frac{1}{2} - \frac{2tp_j^d + (\alpha_1 + \alpha_2)p_{-j}^d - tp_j^c - \alpha_j p_{-j}^c}{2(t^2 - \alpha_1 \alpha_2)} = 0.$$

Combining the two FOCs, we obtain:³³

$$p_j^d(p_1^c, p_2^c) = \frac{(2t^2 - \alpha_1\alpha_2 - \alpha_{-j}^2)p_j^c + t(\alpha_j - \alpha_{-j})p_{-j}^c + (2t - \alpha_1 - \alpha_2)(t^2 - \alpha_1\alpha_2)}{(2t - \alpha_1 - \alpha_2)(2t + \alpha_1 + \alpha_2)}. \quad (20)$$

Replacing these prices in (19), we obtain the deviation profit (for given p_1^c and p_2^c):

$$\pi^d(p_1^c, p_2^c) = \frac{1}{2(2t + \alpha_1 + \alpha_2)} \left[\frac{t}{2t - \alpha_1 - \alpha_2} (p_1^c + p_2^c)^2 + t(p_1^c + p_2^c) + \alpha_1 p_2^c + \alpha_2 p_1^c - p_1^c p_2^c + t^2 - \alpha_1 \alpha_2 \right]. \quad (21)$$

From Lemma 4, if platforms charge prices (p_1, p_2) , their individual per-period collusive profit is:

$$\pi^c(p_1, p_2) = \frac{p_1 + p_2}{2}. \quad (22)$$

From Lemma 3, collusive prices (p_1^c, p_2^c) solve the following constrained minimization program:

$$\min_{(p_1, p_2) \in R^2} \pi^d(p_1, p_2) \quad \text{s.t.} \quad \pi^c = \frac{p_1 + p_2}{2}.$$

Solving this program, we obtain:³⁴

$$(p_1^c(\delta), p_2^c(\delta)) = \left(\frac{2\pi^c(\delta) + \alpha_1 - \alpha_2}{2}, \frac{2\pi^c(\delta) - \alpha_1 + \alpha_2}{2} \right). \quad (23)$$

Given these prices, the utility of the consumer on side $j \in \{1, 2\}$ located at $x = \frac{1}{2}$ that is indifferent between joining platform A or platform B is:

$$u_j^i\left(\frac{1}{2}, p_1^c, p_2^c, p_1^c, p_2^c\right) = \frac{2k_j - 2\pi^c(\delta) - t + \alpha_{-j}}{2},$$

for $i \in \{A, B\}$ and $j \in \{1, 2\}$. Under Assumption 2, the utility of the marginal consumer on side 1 is lower than that of the marginal consumer on side 2. Thus, prices (23) are valid iff:

$$u_1^i\left(\frac{1}{2}, p_1^c, p_2^c, p_1^c, p_2^c\right) \geq 0 \Leftrightarrow \pi^c(\delta) \leq \frac{2k_1 - t + \alpha_2}{2} \equiv \tilde{\pi}.$$

Combining Lemma 2 with $\pi^c(0) = \pi^N$ and the continuity of $\pi^c(\delta)$, we conclude that, as long as $\tilde{\pi} \in (\pi^N, \pi^m)$, $\exists \tilde{\delta} \in (0, 1)$ such that $\pi^c(\delta) \leq \tilde{\pi}, \forall \delta \leq \tilde{\delta}$. For $\pi^c(\delta) > \tilde{\pi}$, the price on side 1 is no longer given by (23). Replacing $p_1^m = \frac{2k_1 - t + \alpha_1}{2}$ in (22), we obtain:

³³Under Assumption 1, the second-order conditions are satisfied since: $\frac{\partial^2 \pi^d}{\partial (p_j^d)^2} = -\frac{t}{t^2 - \alpha_1 \alpha_2} < 0$ and $\frac{\partial^2 \pi^d}{\partial (p_1^d)^2} \frac{\partial^2 \pi^d}{\partial (p_2^d)^2} - \left(\frac{\partial^2 \pi^d}{\partial p_1^d \partial p_2^d} \right)^2 = \frac{(2t - \alpha_1 - \alpha_2)(2t + \alpha_1 + \alpha_2)}{4(t^2 - \alpha_1 \alpha_2)^2} > 0$.

³⁴The second-order is satisfied, $\frac{d^2 \pi^d}{dp_1^c} (p_1^c, 2\pi^c - p_1^c) > 0$, meaning that our candidate is, indeed, a minimum.

$p_2^c(\delta) = 2\pi^c(\delta) - p_1^m = \frac{4\pi^c(\delta) - 2k_1 + t - \alpha_1}{2}$. For values of $\pi^c(\delta)$ above a given threshold, the indifferent consumer on side 2 would get negative utility. Thus, to ensure full coverage of side 2, the maximum price that platforms can charge on this side is $p_2^m = \frac{2k_2 - t + \alpha_2}{2}$. Thus, the maximum profit with full coverage of both market sides is:

$$\pi^m = \frac{2(k_1 + k_2) - (2t - \alpha_1 - \alpha_2)}{4}. \quad (24)$$

Combining Assumptions 1 and 4, we obtain $k_1 + k_2 \geq \frac{3}{2}(2t - \alpha_1 - \alpha_2)$, which implies that $(\pi^N, \pi^m] \neq \emptyset$. \square

Proof of Proposition 2.

In this proof we assume, w.l.o.g., that the deviating platform is platform A .

1. We start by analyzing the case of $\pi^c(\delta) \in (\pi^N, \tilde{\pi})$ with $\tilde{\pi} = \frac{2k_1 - t + \alpha_2}{2}$.

Replacing (6) in (20), we obtain the prices charged by the deviating platform:

$$p_j^d(\delta) = \frac{2\pi^c(\delta) + 2t - \alpha_1 - \alpha_2}{2(2t - \alpha_1 - \alpha_2)}(t - \alpha_{-j}).$$

Let us check whether, for these prices, the deviating platform monopolizes any side of the market. Replacing $p_j^i = p_j^d(\delta)$ and $p_j^{-i} = p_j^c(\delta)$, $j \in \{1, 2\}$, in the demand functions (2), we obtain:

$$n_j^d(\delta) = \frac{2\pi^c(\delta) + 2t - \alpha_1 - \alpha_2}{4(2t - \alpha_1 - \alpha_2)}, \quad j \in \{1, 2\}. \quad (25)$$

Notice that despite the deviating platform charging different prices on the two market sides, it gets the same market share on both. Thus, the deviating platform does not monopolize any side of the market iff:

$$n_j^d(\delta) < 1 \Leftrightarrow \pi^c(\delta) < \frac{3}{2}(2t - \alpha_1 - \alpha_2) \equiv \hat{\pi}. \quad (26)$$

As $k_1 \geq \bar{k}_1$ (Assumption 5), we have $\hat{\pi} \leq \tilde{\pi}$. Thus, for $\pi^c(\delta) \in (\pi^N, \tilde{\pi})$, the deviating platform may or may not monopolize both sides of the market.

1.1. If $\pi^c(\delta) < \hat{\pi}$, the deviating platform does not monopolize any side of the market. Replacing (6) in (21), we obtain the deviation profit:

$$\pi^d(\delta) = \frac{[2\pi^c(\delta) + 2t - \alpha_1 - \alpha_2]^2}{8(2t - \alpha_1 - \alpha_2)}.$$

Replacing the expressions for profits in the ICC (4), and after simple algebraic manipulation, we obtain that the collusive profit π^c is sustainable iff:

$$\frac{2\pi^c - 2t + \alpha_1 + \alpha_2}{8(2t - \alpha_1 - \alpha_2)} \left\{ (3\delta + 1)(2t - \alpha_1 - \alpha_2) - 2(1 - \delta)\pi^c \right\} \geq 0.$$

For collusion to be profitable, we must have $\pi^c \geq \pi^N$. Thus: $2\pi^c - 2t + \alpha_1 + \alpha_2 \geq 0$, and the ICC is satisfied iff:

$$\pi^c \leq \frac{1 + 3\delta}{2(1 - \delta)}(2t - \alpha_1 - \alpha_2).$$

Recall that this analysis is only valid for $\pi^c(\delta) < \hat{\pi}$ and:

$$\frac{1 + 3\delta}{2(1 - \delta)}(2t - \alpha_1 - \alpha_2) < \hat{\pi} \Leftrightarrow \delta < \frac{1}{3}.$$

Thus, for $\delta \in (0, \frac{1}{3})$, the highest sustainable collusive profit is:

$$\pi^c(\delta) = \frac{1 + 3\delta}{2(1 - \delta)}(2t - \alpha_1 - \alpha_2). \quad (27)$$

1.2. If $\hat{\pi} \leq \pi^c(\delta) \leq \tilde{\pi}$, the deviating platform monopolizes both sides of the market. In this case, the deviating platform charges the highest prices that guarantee that the furthest consumer on each market side does not prefer to join the rival platform. Let p_j^d denote the deviating price on side $j \in \{1, 2\}$. The utility of the consumer on side j located at $x = 1$ that joins platform A is:

$$u_j^{Ad}(1) = k_j + \alpha_j - t - p_j^d.$$

If, instead, this consumer joined platform B (charging the collusive prices), her utility would be:

$$u_j^{Bc}(1) = k_j - p_j^c.$$

Thus, the deviating platform A monopolizes both sides of the market iff:

$$u_j^{Ad}(1) \geq u_j^{Bc}(1) \Leftrightarrow p_j^d \leq p_j^c - t + \alpha_j.$$

As the deviating profit, $\pi^d(p_1, p_2) = p_1 + p_2$, is strictly increasing in p_1 and p_2 , the platform

will charge the highest possible prices $p_j^d(\delta) = p_j^c(\delta) - t + \alpha_j$, and the deviation profit is:³⁵

$$\pi^d(\delta) = 2\pi^c(\delta) - 2t + \alpha_1 + \alpha_2.$$

In this case, the ICC for collusion sustainability is satisfied iff:

$$2(2\delta - 1)\pi^c + (2 - 3\delta)(2t - \alpha_1 - \alpha_2) \geq 0. \quad (28)$$

1.2.1. If $\delta < \frac{1}{2}$, the above ICC can be written as:

$$\pi^c \leq \frac{2 - 3\delta}{2(1 - 2\delta)}(2t - \alpha_1 - \alpha_2). \quad (29)$$

Recall that this analysis is only valid if $\pi^c \in (\hat{\pi}, \tilde{\pi})$ and:

$$\frac{2 - 3\delta}{2(1 - 2\delta)}(2t - \alpha_1 - \alpha_2) < \tilde{\pi} \Leftrightarrow \delta < \frac{2k_1 - 5t + 2\alpha_1 + 3\alpha_2}{4k_1 - 8t + 3\alpha_1 + 5\alpha_2} \equiv \tilde{\delta},$$

with $\tilde{\delta} < \frac{1}{2}$ (under Assumption 2). Thus, for $\delta \in \left(\frac{1}{3}, \tilde{\delta}\right)$, the highest sustainable collusive profit is:

$$\pi^c(\delta) = \frac{2 - 3\delta}{2(1 - 2\delta)}(2t - \alpha_1 - \alpha_2).$$

1.2.2. If $\frac{1}{2} \leq \delta < \frac{2}{3}$, ICC (28) is trivially satisfied $\forall \pi^c > 0$. As this analysis is only valid for $\pi^c \in (\hat{\pi}, \tilde{\pi})$, we conclude that, $\forall \delta \in \left(\frac{1}{2}, \frac{2}{3}\right)$, the highest sustainable collusive profit is $\pi^c(\delta) = \tilde{\pi}$.

1.2.3. If $\delta \geq \frac{2}{3}$, the ICC (28) can be written as: $\pi^c \geq \frac{3\delta - 2}{2(2\delta - 1)}(2t - \alpha_1 - \alpha_2)$. Notice that, for $\delta \in \left(\frac{2}{3}, 1\right)$, we have $\frac{3\delta - 2}{2(2\delta - 1)} < \frac{1}{2}$. As a result, given any $\delta \in \left(\frac{2}{3}, 1\right)$, a collusive profit $\pi^c > \frac{1}{2}(2t - \alpha_1 - \alpha_2) = \pi^N$ is sustainable. Again, as this analysis is only valid for $\pi^c \in (\hat{\pi}, \tilde{\pi})$, we conclude that $\pi^c(\delta) = \tilde{\pi}$ is the highest sustainable collusive profit $\forall \delta \in \left(\frac{2}{3}, 1\right)$.

2. Consider now that $\pi^c(\delta) \in (\tilde{\pi}, \pi^m)$.

Replacing $p_1^c = p_1^m$ and $p_2^c = \frac{4\pi^c - 2k_1 + t - \alpha_1}{2}$ in (20), we obtain the expressions for the deviating

³⁵The expression for π^d is strictly positive since we are assuming that the monopolization condition, $\pi^c \geq \frac{3}{2}(2t - \alpha_1 - \alpha_2)$, holds.

prices:

$$p_1^d = \frac{(2t - \alpha_1 - \alpha_2) [(t + \alpha_1)(t - \alpha_2) + 2k_1(t + \alpha_2)] + 4t(\alpha_1 - \alpha_2)\pi^c}{2(2t - \alpha_1 - \alpha_2)(2t + \alpha_1 + \alpha_2)}$$

$$p_2^d = \frac{(2t - \alpha_1 - \alpha_2) [3t^2 - 2k_1(t + \alpha_1) - \alpha_1(\alpha_1 + 2\alpha_2)] + 4 [2t^2 - \alpha_1(\alpha_1 + \alpha_2)] \pi^c}{2(2t - \alpha_1 - \alpha_2)(2t + \alpha_1 + \alpha_2)}.$$

Given these prices, the demand of the deviating platform is:

$$n_1^d = \frac{4(\alpha_1 + \alpha_2)\pi^c + (2t - \alpha_1 - \alpha_2)(2k_1 + t + \alpha_1 + 2\alpha_2)}{4(2t - \alpha_1 - \alpha_2)(2t + \alpha_1 + \alpha_2)}$$

$$n_2^d = \frac{8t\pi^c - (2k_1 - 3t - \alpha_1)(2t - \alpha_1 - \alpha_2)}{4(2t - \alpha_1 - \alpha_2)(2t + \alpha_1 + \alpha_2)}.$$

The deviating platform monopolizes side 2 iff:

$$n_2^d \geq 1 \Leftrightarrow \pi^c \geq \frac{(2t - \alpha_1 - \alpha_2)(2k_1 + 5t + 3\alpha_1 + 4\alpha_2)}{8t} \equiv \tilde{\pi}^n.$$

Under Assumption 5:

$$\tilde{\pi}^n < \tilde{\pi} \Leftrightarrow -\frac{(2k_1 - 7t + 3\alpha_1 + 4\alpha_2)(2t + \alpha_1 + \alpha_2)}{8t} < 0, \quad (30)$$

which implies that the deviating platform always monopolizes side 2. As we are assuming $\tilde{\pi} \leq \pi^c$, we conclude that $n_1^d - n_2^d = \frac{\tilde{\pi} - \pi^c}{2t + \alpha_1 + \alpha_2} \leq 0$. As a result, the deviating platform will choose prices that solve:

$$\begin{cases} \frac{\partial \pi^d(p_1^d, p_2^d, p_1^m, \frac{4\pi^c - 2k_1 + t - \alpha_1}{2})}{\partial p_1^d} = 0 \\ n_2^A(p_1^d, p_2^d, p_1^m, \frac{4\pi^c - 2k_1 + t - \alpha_1}{2}) = 1 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} p_1^d(\pi^c) = \frac{t^3 + t^2(2k_1 + 3\alpha_1 + \alpha_2) + t\alpha_2(2k_1 + \alpha_2) - (2k_1 + 3\alpha_1)\alpha_2(\alpha_1 + \alpha_2) - 4t\alpha_2\pi^c}{2(2t^2 - \alpha_1\alpha_2 - \alpha_2^2)} \\ p_2^d(\pi^c) = \frac{4(2t^2 - \alpha_1\alpha_2)\pi^c - 2t^3 - t^2(4k_1 + 2\alpha_1 + 3\alpha_2) + 2\alpha_2t(k_1 + 2\alpha_1) + \alpha_1\alpha_2(2k_1 + \alpha_1 + 2\alpha_2)}{2(2t^2 - \alpha_1\alpha_2 - \alpha_2^2)}. \end{cases}$$

Given these prices, the deviating platform also monopolizes side 1 iff:

$$n_1^d(p_1^d, p_2^d, p_1^m, \frac{4\pi^c - 2k_1 + t - \alpha_1}{2}) \geq 1 \Leftrightarrow \pi^c \geq \frac{7t^2 - 2k_1(t - \alpha_2) - t(3\alpha_1 - \alpha_2) - 3\alpha_1\alpha_2 - 2\alpha_2^2}{4\alpha_2} \equiv \tilde{\pi}^t.$$

Under Assumption 5:

$$\tilde{\pi}^t - \tilde{\pi}^n = -\frac{(2t^2 - \alpha_1\alpha_2 - \alpha_2^2)(2k_1 - 7t + 3\alpha_1 + 4\alpha_2)}{8t\alpha_2} < 0.$$

From (30), we have $\tilde{\pi}^n < \tilde{\pi}$ and, therefore: $\tilde{\pi}^t < \tilde{\pi}$. As a result, $\forall \pi^c \in (\tilde{\pi}, \pi^m)$, the deviating platform monopolizes both sides of the market. Consequently, the deviating platform prefers to choose prices (p_1^d, p_2^d) such that:

$$\begin{cases} n_1^d(p_1^d, p_2^d, p_1^m, \frac{4\pi^c - 2k_1 + t - \alpha_1}{2}) = 1 \\ n_2^d(p_1^d, p_2^d, p_1^m, \frac{4\pi^c - 2k_1 + t - \alpha_1}{2}) = 1 \end{cases} \Leftrightarrow \begin{cases} p_1^d = \frac{2k_1 - 3t + 3\alpha_1}{2} \\ p_2^d = \frac{4\pi^c - 2k_1 - t - \alpha_1 + 2\alpha_2}{2} \end{cases}$$

and the deviating platform is:

$$\pi^d(\delta) = 2\pi^c(\delta) - 2t + \alpha_1 + \alpha_2.$$

Replacing the expressions for profits in the ICC (4), we conclude that, given $\delta \in (\tilde{\delta}, \delta^m)$, the collusive profit $\pi^c \in (\tilde{\pi}, \pi^m)$ is sustainable iff:

$$2(2\delta - 1)\pi^c \geq (3\delta - 2)(2t - \alpha_1 - \alpha_2). \quad (31)$$

2.1. For $\delta < \frac{1}{2}$, the ICC can be rewritten as follows:

$$\pi^c \leq \frac{2-3\delta}{2(1-2\delta)}(2t - \alpha_1 - \alpha_2). \quad (32)$$

From Lemma 4, platforms fully cover both sides of the market under collusion. As the maximum (collusive) profit that ensures full market coverage is π^m , it is the upper bound for π^c . Notice further that:³⁶

$$\frac{2-3\delta}{2(1-2\delta)}(2t - \alpha_1 - \alpha_2) \leq \pi^m \Leftrightarrow \delta \leq \frac{2(k_1 + k_2) - 5(2t - \alpha_1 - \alpha_2)}{4[k_1 + k_2 - 2(2t - \alpha_1 - \alpha_2)]} \equiv \delta^m.$$

It is easy to check that, under Assumption 2, we have $\delta^m < \frac{1}{2}$. Thus: if $\delta \in (\tilde{\delta}, \delta^m)$, the highest sustainable collusive profit is given by:

$$\pi^c(\delta) = \frac{2-3\delta}{1-2\delta}(2t - \alpha_1 - \alpha_2)$$

while, for $\delta \in (\delta^m, \frac{1}{2})$, it is $\pi^c(\delta) = \pi^m$.

³⁶Assumption 5 implies that $k_1 + k_2 > \frac{7}{2}(2t - \alpha_1 - \alpha_2)$.

2.2. Consider now $\delta \geq \frac{1}{2}$. If $\delta \leq \frac{2}{3}$, the ICC (31) is trivially satisfied $\forall \pi^c \geq 0$. As a result, the highest sustainable collusive profit (that ensures full market coverage) is $\pi^c(\delta) = \pi^m$. Finally, if $\delta > \frac{2}{3}$, the ICC (31) can be rewritten as: $\pi^c \geq \frac{3\delta-2}{2(2\delta-1)}(2t - \alpha_1 - \alpha_2)$. Notice, however, that:

$$\frac{3\delta - 2}{2(2\delta - 1)}(2t - \alpha_1 - \alpha_2) \leq \frac{2t - \alpha_1 - \alpha_2}{2} = \pi^N \leq \pi^m,$$

which implies that $\pi^c(\delta) = \pi^m$ is sustainable $\forall \delta \geq \frac{2}{3}$.

Summing up:

$$\pi^c(\delta) = \begin{cases} \frac{1+3\delta}{2(1-\delta)}(2t - \alpha_1 - \alpha_2) & \text{if } 0 < \delta < \frac{1}{3} \\ \frac{2-3\delta}{2(1-2\delta)}(2t - \alpha_1 - \alpha_2) & \text{if } \frac{1}{3} \leq \delta < \delta^m \\ \pi^m & \text{if } \delta^m \leq \delta < 1. \end{cases} \quad (33)$$

To obtain the expressions for prices, simply replace these expressions in (6). □

Proof of Corollary 2.

Using (9), we obtain:

$$(\Delta p_1(\delta), \Delta p_2(\delta)) = \begin{cases} \left(\frac{2\delta}{1-\delta}(2t - \alpha_1 - \alpha_2), \frac{2\delta}{1-\delta}(2t - \alpha_1 - \alpha_2) \right) & \text{if } 0 < \delta < \frac{1}{3} \\ \left(\frac{1-\delta}{2(1-2\delta)}(2t - \alpha_1 - \alpha_2), \frac{1-\delta}{2(1-2\delta)}(2t - \alpha_1 - \alpha_2) \right) & \text{if } \frac{1}{3} < \delta < \tilde{\delta} \\ \left(\frac{2k_1-3t+\alpha_1+2\alpha_2}{2}, \frac{2(2k_1-5t+2\alpha_1+3\alpha_2)\delta-2k_1+7t-3\alpha_1-4\alpha_2}{2(1-2\delta)} \right) & \text{if } \tilde{\delta} < \delta < \delta^m \\ \left(\frac{2k_1-3t+\alpha_1+2\alpha_2}{2}, \frac{2k_2-3t+2\alpha_1+\alpha_2}{2} \right) & \text{if } \delta^m \leq \delta < 1. \end{cases}$$

The sign of $\frac{\partial \Delta p_j}{\partial \alpha_k}$ follows straightforwardly when $0 < \delta < \tilde{\delta}$ and when $\delta \geq \delta^m$. When $\tilde{\delta} < \delta < \delta^m$, the sign of $\frac{\partial(\Delta p_1)}{\partial \alpha_k}$ is also straightforward and:

$$\frac{\partial \Delta p_2}{\partial \alpha_1} = -\frac{3-4\delta}{2(1-2\delta)} < 0 \quad \text{and} \quad \frac{\partial \Delta p_2}{\partial \alpha_2} = -\frac{2-3\delta}{1-2\delta} < 0.$$

□

Proof of Lemma 7.

Let us analyze under which conditions platforms prefer to fully cover the market when they collude only on side 1. We focus on symmetric collusive agreements such that platforms set the same price on the collusive side, i.e., $p_1^A = p_1^B = p_1$, $j \in \{1, 2\}$. Let \tilde{x}_j denote the consumer on side j that is indifferent between joining platform A and not joining any platform.

1. If $t \geq \alpha_1$, π^{oc} is increasing in p_1 . If $n_2^i = \frac{1}{2}$, the maximum price that platforms can set on side 1 for this side be fully covered is p_1^m , given in (5). As $p_1^{oc} > p_1^N$, it follows that $p_2^{oc} < p_2^N$ (Lemma 6). Thus, if $p_1^{oc} \leq p_1^m$, there will be full coverage of both market sides (under the condition for full coverage under Nash competition).

Let us analyze the incentives for platforms to set $p_1^{oc} > p_1^m$ and not fully cover side 1. We start by studying the incentives for small deviations, i.e., to set $p_1^{oc} = p_1^m + \epsilon$, for sufficiently small $\epsilon > 0$. If platforms set $p_1^{oc} = p_1^m$, the price on side 2 is such that $p_2(p_1^m) < p_2^N$, and the utility of side-2 agents is higher than their utility when platforms engage in Nash competition. Therefore, the utility of side-2 agents under collusion is strictly positive. Thus, a slightly increase in p_1^{oc} will decrease the network size on side 1, but not up to a point that side-2 agents get negative utility if joining a platform (notice further that $p_2^{oc}(p_1) < p_2^{oc}(p_1^m)$ if $p_1 > p_1^m$). As a result, when platforms slightly increase p_1 above p_1^m , side 2 remains fully covered, i.e., $n_2^i = \frac{1}{2}$. Thus, the demand of platform A on side 1 is equal to \tilde{x}_1 that solves:

$$u_1^A(\tilde{x}_1, p_1^{oc}, p_2^{oc}, p_1^{oc}, p_2^{oc}) = 0 \Leftrightarrow k_1 + \frac{\alpha_1}{2} - t\tilde{x}_1 - p_1^{oc} = 0 \Leftrightarrow \tilde{x}_1 = \frac{2k_1 + \alpha_1 - 2p_1^{oc}}{2t}.$$

Combining the first-order conditions corresponding to the maximization problems in the constraints of the maximization program (10), we obtain that, for a given p_1 , platforms charge the same price on side 2, given by:³⁷

$$p_2(p_1, \alpha_1, \alpha_2) = \frac{t^2 - \alpha_1\alpha_2}{t} - \frac{\alpha_1}{t}p_1. \quad (34)$$

Using (34), the profit of platform A can be written as follows:

$$\pi^A(p_1^{oc}) = p_1^{oc}\tilde{x}_1 + \frac{p_2^{oc}(p_1^{oc})}{2} = \left(\frac{2k_1 + \alpha_1 - 2p_1^{oc}}{2t} \right) p_1^{oc} + \frac{t^2 - \alpha_1\alpha_2 - \alpha_1 p_1^{oc}}{2t}.$$

³⁷The second-order condition is satisfied since: $\frac{\partial^2 \pi^i}{\partial (p_2^i)^2} = -\frac{t}{t^2 - \alpha_1\alpha_2} < 0$.

Using (5):

$$\left. \frac{\partial \pi^A}{\partial p_1^{oc}} \right|_{p_1^{oc}=p_1^m} \leq 0 \Leftrightarrow \frac{k_1 - 2p_1^m}{t} \leq 0 \Leftrightarrow k_1 \geq t - \alpha_1$$

As π^A is concave, if $k_1 \geq t - \alpha_1$, we have $\pi^A(p_1^{oc}) < \pi^A(p_1^m)$ for any price $p_1^{oc} > p_1^m$ (that ensures full coverage of side 2). As a result, a greater increase in p_1 that compromises full coverage of side 2 is even less profitable, since demand on side 2 decreases, and, consequently, revenues on this side also decreases.³⁸

2. If $t \leq \alpha_1$, π^{oc} is decreasing in p_1 and, therefore, platforms want to set the lowest possible price on side 1. In particular, platforms will set $p_1^{oc} < p_1^N$. Thus, if side 2 is fully covered, full coverage of side 1 is implied by the condition for full market coverage under Nash competition (3). However, as $p_2(p_1^{oc}) > p_2^N$ (Lemma 6), the condition for full coverage with Nash competition is not sufficient to ensure full coverage of side 2 under one-sided collusion.

Following the same steps as above, we have that, if $p_2^{oc} > p_2^m$, the demand of each platform on side 2 is:

$$\tilde{x}_2 = \frac{2k_2 + \alpha_1 - 2p_2^{oc}}{2t}.$$

Replacing $\Delta p_j = p_j^{oc} - (t - \alpha_{-j})$ in (13) and solving the equation with respect to p_1^{oc} , we obtain: $p_1^{oc}(p_2^{oc}) = \frac{t^2 - \alpha_1 \alpha_2}{\alpha_1} - \frac{t}{\alpha_1} p_2^{oc}$. Thus, the profit of platform A can be written:

$$\pi^A(p_2^{oc}) = \frac{p_1^{oc}(p_2^{oc})}{2} + p_2^{oc} \tilde{x}_2 = \frac{t^2 - \alpha_1 \alpha_2 - t p_2^{oc}}{2\alpha_1} + p_2^{oc} \left(\frac{2k_2 + \alpha_1 - 2p_2^{oc}}{2t} \right).$$

As we are assuming $t \leq \alpha_1$, we necessarily have $\alpha_1 > 0$ and, therefore, using (5):

$$\left. \frac{\partial \pi^A}{\partial p_2^{oc}} \right|_{p_2^{oc}=p_2^m} < 0 \Leftrightarrow \frac{\alpha_1(2k_2 - 4p_2^m + \alpha_2) - t^2}{2t\alpha_1} < 0 \Leftrightarrow k_2 > \frac{2\alpha_1 t - \alpha_1 \alpha_2 - t^2}{2\alpha_1}.$$

Again, as π^A is concave, if $k_2 > \frac{2\alpha_1 t - \alpha_1 \alpha_2 - t^2}{2\alpha_1}$, we have $\pi^A(p_2^{oc}) < \pi^A(p_2^m)$ for any $p_2^{oc} > p_2^m$ (that ensures full coverage of side 1). Additionally, any greater increase in p_2 that compromises full coverage of side 1 is even less profitable. \square

Proof of Lemma 8.

Let (p_1^{oc}, p_2^{oc}) denote the (unique) solution of the maximization program (10).

³⁸From Lemma 5: $p_2(p_1^{oc}) < p_2(p_1^m)$, for $p_1^{oc} > p_1^m$.

1. If $t > \alpha_1 \geq 0$, platforms are willing to collude iff $p_1^{oc} \geq p_1^N$. As $\alpha_1 > 0$, we conclude that $p_2^{oc} < p_2^N$. As platforms charge equal prices on each side of the market, they equally share both sides of the market. Thus, both sides of the market are fully covered iff the (indifferent) consumers located at $x = \frac{1}{2}$ have non-negative utilities:

$$u_1^i \left(\frac{1}{2}, p_1^{oc}, t - \frac{\alpha_1(p_1^{oc} + \alpha_2)}{t}, p_1^{oc}, t - \frac{\alpha_1(p_1^{oc} + \alpha_2)}{t} \right) \geq 0 \Leftrightarrow k_1 \geq p_1^{oc} + \frac{t - \alpha_1}{2}, \quad (35)$$

$$u_2^i \left(\frac{1}{2}, p_1^{oc}, t - \frac{\alpha_1(p_1^{oc} + \alpha_2)}{t}, p_1^{oc}, t - \frac{\alpha_1(p_1^{oc} + \alpha_2)}{t} \right) \geq 0 \Leftrightarrow k_2 \geq \frac{3t - \alpha_2}{2} - \frac{\alpha_1(p_1^{oc} + \alpha_2)}{t}. \quad (36)$$

Recall that the condition for the market side 2 to be fully covered when platforms charge the Nash price on each side is $k_2 - \frac{\alpha_2}{2} > \frac{3t - 2\alpha_1 - 2\alpha_2}{2}$. Thus, if $\alpha_1 > 0$, the condition for full coverage of side 2 is implied by the condition for full coverage of side 2 when platforms set the Nash prices on both sides of the market iff $p_1^{oc} > t - \alpha_2 = p_1^N$.

Using (35), we conclude that there is full coverage of the (two sides of the) market iff $p_1^{oc} \leq p_1^m$ with $p_1^m = \frac{2k_1 - t + \alpha_1}{2}$. If platforms set price p_1^{oc} on side 1 and price p_2^{oc} given in (34), their individual profit is:

$$\pi^{oc}(p_1^{oc}) = \frac{t^2 - \alpha_1\alpha_2}{2t} + \frac{t - \alpha_1}{2t} p_1^{oc}.$$

Equivalently:

$$p_1^{oc}(\pi^{oc}) = \frac{2t\pi^{oc} - t^2 + \alpha_1\alpha_2}{t - \alpha_1}. \quad (37)$$

Replacing this expression in (34), we conclude that, for a given δ , the price on side 2 is:

$$p_2^{oc}(\delta) = \frac{t^2 - \alpha_1\alpha_2 - 2\alpha_1\pi^{oc}(\delta)}{t - \alpha_1}. \quad (38)$$

However, for side 1 to be fully covered, the condition $p_1^{oc}(\pi^{oc}) \leq p_1^m$ must be satisfied:

$$p_1^{oc}(\pi^{oc}) \leq p_1^m \Leftrightarrow \frac{-2k_1(t - \alpha_1) - t^2 - 2t\alpha_1 + \alpha_1^2 + 2\alpha_1\alpha_2 + 4t\pi^{oc}}{2(t - \alpha_1)} \leq 0.$$

Thus, for $t > \alpha_1$, the highest collusive profit with full market coverage is:

$$\pi^{om} = \frac{2k_1(t - \alpha_1) + t(t + 2\alpha_1) - \alpha_1(\alpha_1 + 2\alpha_2)}{4t}.$$

To obtain the expressions for p_2^{oc} , simply replace $p_1^{oc} = p_1^m$ in (34).

2. Consider now $0 < t < \alpha_1$. As in the previous case, both market sides are fully covered iff conditions (35) and (36) are satisfied. Recall that side 1 is fully covered when platforms charge Nash prices iff $k_1 > p_1^N + \frac{t-\alpha_1}{2}$. As $t < \alpha_1$, collusion on side 1 is profitable iff $p_1^{oc} < p_1^N$. As a result: $k_1 > p_1^N + \frac{t-\alpha_1}{2} > p_1^{oc} + \frac{t-\alpha_1}{2}$, which implies that the condition for full coverage of side 1 under Nash competition is sufficient for full coverage with collusion on side 1. Thus, there is full coverage of both market sides iff:

$$u_2^i \left(\frac{1}{2}, p_1^{oc}, t - \frac{\alpha_1(p_1^{oc} + \alpha_2)}{t}, p_1^{oc}, t - \frac{\alpha_1(p_1^{oc} + \alpha_2)}{t} \right) \geq 0 \Leftrightarrow p_1^{oc} \geq \frac{3t^2 - t\alpha_2 - 2\alpha_1\alpha_2 - 2tk_2}{2\alpha_1}.$$

For any $\delta < \delta^{om}$, the collusive prices on side 1 and 2 are given by (37) and (38), respectively. For $\delta \geq \delta^{om}$, platforms set the highest price on side 2 that allows full coverage of this side, i.e., $p_2^{oc} = p_2^m$. The price on side 1 is obtained by replacing $p_2^{oc} = p_2^m$ in (34).

3. Finally, consider the case of $\alpha_1 < 0 < t$. Expressions (37) and (38) for prices are valid as long as consumers located at $x = \frac{1}{2}$ on each market side get positive utility. If $t > \alpha_1$, the collusive profit is strictly increasing in p_1 . As a result, platforms would like to increase p_1^{oc} as much as possible. However, for $p_j^{oc} > p_j^m$, with p_j^m given in (5), side $j \in \{1, 2\}$ would not be fully covered. If $\alpha_1 < 0$, an increase in p_1^{oc} is followed by an increase in p_2^{oc} (Lemma 6). Thus, using (34), market 2 is not fully covered iff:

$$p_2^{oc}(p_1^{oc}) > p_2^m \Leftrightarrow p_1^{oc} > \frac{t^2 - \alpha_1\alpha_2}{\alpha_1} - \frac{t}{\alpha_1}p_2^m \equiv \bar{p}_1$$

Using (5), we can write: $\bar{p}_1 = p_1^N - \frac{t}{\alpha_1}\tilde{u}_2^N$. In sum, expression (37) for p_1^{oc} is valid iff $p_1^{oc} < \min\{p_1^m, \bar{p}_1\}$, with:

$$p_1^m < \bar{p}_1 \Leftrightarrow \alpha_1 > -\frac{\tilde{u}_2^N}{\tilde{u}_1^N}t.$$

Hence:

- If $\alpha_1 \geq -\frac{\tilde{u}_2^N}{\tilde{u}_1^N}t$, platforms cannot set $p_1^{oc} > p_1^m$ (otherwise, side 1 would not be fully covered). As a result, given δ , the most collusive prices are given by (14).
- If $\alpha_1 < -\frac{\tilde{u}_2^N}{\tilde{u}_1^N}t$, platforms cannot set $p_1^{oc} > \bar{p}_1$ (otherwise, side 2 would not be fully covered). As a result, given δ , the most collusive prices are given by (15).

□

Proof of Proposition 5.

Let us assume, without loss of generality, that the deviating platform is platform A .

1. Let us first assume that $-\frac{\tilde{u}_2^N}{\tilde{u}_1^N}t \leq \alpha_1 < t$.

If platform A deviated from the collusive agreement, it would set prices that solved $\max_{(p_1, p_2)} \pi^A(p_1, p_2, p_1^{oc}, p_2^{oc})$. Replacing (14) in (3):

$$\pi^A(p_1, p_2, p_1^{oc}, p_2^{oc}) = \frac{1}{2(t - \alpha_1)(t^2 - \alpha_1\alpha_2)} \left\{ -p_1^2 t(t - \alpha_1) + p_1(t - \alpha_1)[2(t + \alpha_1)\pi^{oc} - p_2(\alpha_1 + \alpha_2)] - p_2 \left[t(t - \alpha_1)p_2 + 2t[\alpha_1\alpha_2 + (\alpha_1 - \alpha_2)\pi^{oc}] - 2t^3 + (\alpha_1 + \alpha_2)(t^2 - \alpha_1\alpha_2) \right] \right\}. \quad (39)$$

Solving the corresponding FOCs, we obtain:³⁹

$$p_1^{od}(\pi^{oc}) = \frac{2t(2t^2 - \alpha_1^2 - \alpha_2^2)\pi^{oc} - (2t - \alpha_1 - \alpha_2)(\alpha_1 + \alpha_2)(t^2 - \alpha_1\alpha_2)}{(t - \alpha_1)(2t - \alpha_1 - \alpha_2)(2t + \alpha_1 + \alpha_2)}, \quad (40)$$

$$p_2^{od}(\pi^{oc}) = \frac{2[t^2(\alpha_2 - 3\alpha_1) + \alpha_1^2(\alpha_1 + \alpha_2)]\pi^{oc} + 2t(2t - \alpha_1 - \alpha_2)(t^2 - \alpha_1\alpha_2)}{(t - \alpha_1)(2t - \alpha_1 - \alpha_2)(2t + \alpha_1 + \alpha_2)}.$$

However, these are the actual deviation prices iff $n_j^A(p_1^{od}, p_2^{od}, p_1^{oc}, p_2^{oc}) \leq 1$, for $j \in \{1, 2\}$. Notice that:⁴⁰

$$n_1^A(p_1^{od}, p_2^{od}, p_1^{oc}, p_2^{oc}) - n_2^A(p_1^{od}, p_2^{od}, p_1^{oc}, p_2^{oc}) = \frac{(t + \alpha_1)(2\pi^{oc} - 2t + \alpha_1 + \alpha_2)}{2(t - \alpha_1)(2t + \alpha_1 + \alpha_2)} > 0. \quad (41)$$

Thus: if $t > \alpha_1$ and the deviating platform does not monopolize side 1, it also does not monopolize side 2. Replacing expressions (40) in (2), we obtain:

$$n_1^A(p_1^{od}, p_2^{od}, p_1^{oc}, p_2^{oc}) \leq 1 \Leftrightarrow \pi^{oc} \leq \frac{(2t - \alpha_1 - \alpha_2)[4t^2 - t(\alpha_1 - \alpha_2) - 2\alpha_1(\alpha_1 + \alpha_2)]}{2(2t^2 - \alpha_1^2 - \alpha_1\alpha_2)} \equiv \bar{\pi}^{oc}.$$

Under Assumption 1, we have $2t^2 - \alpha_1^2 - \alpha_1\alpha_2 = (t - \alpha_1)(t + \alpha_1) + t^2 - \alpha_1\alpha_2 > 0$. Thus:

$$\bar{\pi}^{oc} > \pi^N \Leftrightarrow \frac{(t - \alpha_1)(2t - \alpha_1 - \alpha_2)(2t + \alpha_1 + \alpha_2)}{2(2t^2 - \alpha_1^2 - \alpha_1\alpha_2)} > 0,$$

³⁹ The second-order conditions are satisfied, since: $\frac{\partial^2 \pi^A}{\partial p_j^2} = -\frac{t}{t^2 - \alpha_1\alpha_2} < 0$ and $\frac{\partial^2 \pi^A}{\partial p_1^2} \frac{\partial^2 \pi^A}{\partial p_2^2} - \left(\frac{\partial^2 \pi^A}{\partial p_1 \partial p_2} \right)^2 = \frac{(2t - \alpha_1 - \alpha_2)(2t + \alpha_1 + \alpha_2)}{4(t^2 - \alpha_1\alpha_2)^2} > 0$.

⁴⁰For collusion to be profitable, we must have $\pi^{oc} > \pi^N \Leftrightarrow 2\pi^{oc} - 2t + \alpha_1 + \alpha_2 > 0$.

is always satisfied and:⁴¹

$$\bar{\pi}^{oc} > \tilde{\pi}^{om} \Leftrightarrow k_1 < \frac{14t^3 - (2t^2 - \alpha_1^2 - \alpha_1\alpha_2)(\alpha_1 + 2\alpha_2) - t(\alpha_1 + \alpha_2)(5\alpha_1 + 2\alpha_2)}{2(2t^2 - \alpha_1^2 - \alpha_1\alpha_2)} \equiv \hat{k}_1.$$

In order to get a richer setting, we assume that $k_1 > \hat{k}_1$.⁴² As a result, the deviating platform does not monopolize any side of the market if $\pi^{oc} \in [\pi^N, \hat{\pi}^{oc})$, and monopolizes side 1 if $\pi^{oc} \in [\hat{\pi}^{oc}, \tilde{\pi}^{om}]$. If $\pi^{oc} \geq \hat{\pi}^{oc}$, the deviating platform chooses (p_1, p_2) such that:

$$\begin{aligned} & \begin{cases} n_1^A(p_1, p_2, p_1^{oc}, p_2^{oc}) = 1 \\ \frac{\partial \pi^{od}(p_1, p_2, p_1^{oc}, p_2^{oc})}{\partial p_2} = 0 \end{cases} \Leftrightarrow \\ & \Leftrightarrow \begin{cases} p_1^{od}(\pi^{oc}) = \frac{2t(2t^2 - \alpha_1^2 - \alpha_1\alpha_2)\pi^{oc} - (t^2 - \alpha_1\alpha_2)[2t(2t - \alpha_1) - \alpha_1(\alpha_1 + \alpha_2)]}{(t - \alpha_1)(2t^2 - \alpha_1^2 - \alpha_1\alpha_2)} \\ p_2^{od}(\pi^{oc}) = \frac{(2t^2 + t\alpha_1 - 2\alpha_1^2 + t\alpha_2 - 2\alpha_1\alpha_2)(t^2 - \alpha_1\alpha_2) - 2\alpha_1(2t^2 - \alpha_1^2 - \alpha_1\alpha_2)\pi^{oc}}{(t - \alpha_1)(2t^2 - \alpha_1^2 - \alpha_1\alpha_2)}. \end{cases} \end{aligned} \quad (42)$$

Replacing these expressions in the demand function (2), we obtain:

$$n_2^A(p_1^{od}, p_2^{od}, p_1^{oc}, p_2^{oc}) = \frac{(t - \alpha_1)(2t + \alpha_1 + \alpha_2)}{2(2t^2 - \alpha_1^2 - \alpha_1\alpha_2)} < 1,$$

which means that there is never monopolization of side 2 in the deviating period.

1.1. Start by assuming that $\pi^{oc} < \hat{\pi}^{oc}$. Replacing expressions (40) for prices in (39), we obtain the deviation profit:

$$\pi^{od}(\pi^{oc}) = \frac{t(2t - \alpha_1 - \alpha_2)^2(t^2 - \alpha_1\alpha_2) - 2(2t - \alpha_1 - \alpha_2)(3t^2\alpha_1 - \alpha_1^3 - t^2\alpha_2 - \alpha_1^2\alpha_2)\pi^{oc} + 4t(t^2 - \alpha_1\alpha_2)(\pi^{oc})^2}{2(t - \alpha_1)^2(2t - \alpha_1 - \alpha_2)(2t + \alpha_1 + \alpha_2)}. \quad (43)$$

Replacing this expression and $\pi^N = \frac{2t - \alpha_1 - \alpha_2}{2}$ in the ICC (11), we find that one-sided collusion is sustainable iff:⁴³

$$\frac{2\pi^{oc} - (2t - \alpha_1 - \alpha_2)}{2(t - \alpha_1)^2(2t - \alpha_1 - \alpha_2)(2t + \alpha_1 + \alpha_2)} \left\{ (2t - \alpha_1 - \alpha_2) \left[t(t^2 - \alpha_1\alpha_2) + [(t^3 - t^2(3\alpha_1 - \alpha_2) - t\alpha_1\alpha_2 + \alpha_1^2(\alpha_1 + \alpha_2))\delta] \right] - 2t(t^2 - \alpha_1\alpha_2)(1 - \delta)\pi^{oc} \right\} \geq 0.$$

⁴¹See Lemma 8 for the expression for $\tilde{\pi}^{om}$.

⁴²If $k_1 < \hat{k}_1$, the deviating platform does not monopolize any side of the market $\forall \pi^{oc} \in [\pi^N, \tilde{\pi}^{om}]$.

⁴³Recall that, for platforms to be willing to collude, we must have $\pi^{oc} \geq \pi^N = \frac{2t - \alpha_1 - \alpha_2}{2}$.

Under Assumption 1, the above ICC is equivalent to:

$$\pi^{oc} \leq \frac{(2t - \alpha_1 - \alpha_2) \left[t(t^2 - \alpha_1\alpha_2) + [(t^3 - t^2(3\alpha_1 - \alpha_2) - t\alpha_1\alpha_2 + \alpha_1^2(\alpha_1 + \alpha_2))\delta] \right]}{2t(t^2 - \alpha_1\alpha_2)(1 - \delta)} \equiv \check{\pi}^{oc}. \quad (44)$$

Recall, however, that this analysis is only valid if $\pi^{oc} < \hat{\pi}^{oc}$ and:

$$\begin{aligned} \check{\pi}^{oc} < \bar{\pi}^{oc} &\Leftrightarrow \frac{(t - \alpha_1)(2t - \alpha_1 - \alpha_2)(2t + \alpha_1 + \alpha_2)}{2t(t^2 - \alpha_1\alpha_2)(2t^2 - \alpha_1^2 - \alpha_1\alpha_2)(1 - \delta)} \left\{ -t(t^2 - \alpha_1\alpha_2) + \right. \\ &\quad \left. + [(t - \alpha_1)(2t^2 - \alpha_1^2 - \alpha_1\alpha_2) + t(t^2 - \alpha_1\alpha_2)]\delta \right\} < 0 \\ &\Leftrightarrow \delta < \frac{t(t^2 - \alpha_1\alpha_2)}{t(t^2 - \alpha_1\alpha_2) + (t - \alpha_1)(2t^2 - \alpha_1^2 - \alpha_1\alpha_2)} \equiv \tilde{\delta}^{oc}. \end{aligned}$$

1.2. Suppose now that $\pi^{oc} \geq \bar{\pi}^{oc}$, or, equivalently, that $\delta \geq \tilde{\delta}^{oc}$. Replacing expressions (42) in (39), we obtain:

$$\begin{aligned} \pi^{od}(\pi^{oc}) &= \frac{1}{2(t - \alpha_1)(2t^2 - \alpha_1^2 - \alpha_1\alpha_2)^2} \left[2(2t^2 - \alpha_1^2 - \alpha_1\alpha_2)(4t^3 - 2t^2\alpha_1 - t\alpha_1^2 + \alpha_1^3 - 3t\alpha_1\alpha_2 + \alpha_1^2\alpha_2)\pi^{oc} - \right. \\ &\quad \left. - t(2t - \alpha_1 - \alpha_2)(t^2 - \alpha_1\alpha_2)(6t^2 - t\alpha_1 - 3\alpha_1^2 + t\alpha_2 - 3\alpha_1\alpha_2) \right]. \end{aligned} \quad (45)$$

Replacing in (11), we find that platforms are willing to collude iff:

$$\begin{aligned} &\frac{1}{2(t - \alpha_1)(2t^2 - \alpha_1^2 - \alpha_1\alpha_2)^2} \times \\ &\times \left\{ (2t - \alpha_1 - \alpha_2) \left[t(t^2 - \alpha_1\alpha_2) \left[3(2t^2 - \alpha_1^2 - \alpha_1\alpha_2) + t(\alpha_2 - \alpha_1) \right] - \right. \right. \\ &\quad \left. \left. - \left[10t^5 - (5\alpha_1 - \alpha_2)t^4 - \alpha_1(7\alpha_1 + 13\alpha_2)t^3 + \alpha_1(4\alpha_1^2 + 5\alpha_1\alpha_2 - \alpha_2^2)t^2 + \alpha_1^2(\alpha_1 + \alpha_2)(\alpha_1 + 4\alpha_2)t - \alpha_1^3(\alpha_1 + \alpha_2)^2 \right] \delta \right] - \right. \\ &\quad \left. - 2(2t^2 - \alpha_1^2 - \alpha_1\alpha_2) \left[2t(t^2 - \alpha_1\alpha_2) - [(t^2 - \alpha_1\alpha_2)(3t - \alpha_1) + (t - \alpha_1)^2(t + \alpha_1)]\delta \right] \pi^{oc} \right\} \geq 0 \end{aligned} \quad (46)$$

The coefficient of π^{oc} is positive iff:

$$\delta < \frac{2t(t^2 - \alpha_1\alpha_2)}{(t^2 - \alpha_1\alpha_2)(3t - \alpha_1) + (t - \alpha_1)^2(t + \alpha_1)} \equiv \bar{\delta}^{oc}. \quad (47)$$

Recall, however, that we are assuming $\delta \geq \tilde{\delta}^{oc}$ and:

$$\bar{\delta}^{oc} > \tilde{\delta}^{oc} \Leftrightarrow \frac{t(t - \alpha_1)(t^2 - \alpha_1\alpha_2)(2t^2 - \alpha_1(\alpha_1 + \alpha_2))}{[(t - \alpha_1)(2t^2 - \alpha_1^2 - \alpha_1\alpha_2) + t(t^2 - \alpha_1\alpha_2)][(t^2 - \alpha_1\alpha_2)(3t - \alpha_1) + (t - \alpha_1)^2(t + \alpha_1)]} > 0$$

is always satisfied.

1.2.1. If $\delta \in (\tilde{\delta}^{oc}, \bar{\delta}^{oc})$, the signal of the coefficient of π^{oc} in (46) is positive and the ICC is equivalent to $\pi^{oc} \leq \tilde{\pi}^{oc}(\delta)$ with:

$$\tilde{\pi}^{oc}(\delta) = \frac{N_{\tilde{\pi}^{oc}}(\delta)}{D_{\tilde{\pi}^{oc}}(\delta)} \quad (48)$$

with:

$$\begin{aligned} N_{\tilde{\pi}^{oc}}(\delta) = & (2t - \alpha_1 - \alpha_2) \left\{ t(t^2 - \alpha_1\alpha_2) \left[3(2t^2 - \alpha_1^2 - \alpha_1\alpha_2) + t(\alpha_2 - \alpha_1) \right] - \right. \\ & - \left[10t^5 - (5\alpha_1 - \alpha_2)t^4 - \alpha_1(7\alpha_1 + 13\alpha_2)t^3 + \alpha_1(4\alpha_1^2 + 5\alpha_1\alpha_2 - \alpha_2^2)t^2 + \alpha_1^2(\alpha_1 + \alpha_2)(\alpha_1 + 4\alpha_2)t - \right. \\ & \left. \left. - \alpha_1^3(\alpha_1 + \alpha_2)^2 \right] \delta \right\}, \end{aligned}$$

$$D_{\tilde{\pi}^{oc}}(\delta) = 2(2t^2 - \alpha_1^2 - \alpha_1\alpha_2) \left[2t(t^2 - \alpha_1\alpha_2) - [(t^2 - \alpha_1\alpha_2)(3t - \alpha_1) + (t - \alpha_1)^2(t + \alpha_1)]\delta \right].$$

As $\lim_{\delta \rightarrow (\bar{\delta}^{oc})^-} \tilde{\pi}^{oc}(\delta) = +\infty$ and:

$$\frac{\partial \tilde{\pi}^{oc}}{\partial \delta} = \frac{t(t - \alpha_1)^2(2t - \alpha_1 - \alpha_2)(2t + \alpha_1 + \alpha_2)(t^2 - \alpha_1\alpha_2)}{2[\alpha_1^2(\alpha_1 + \alpha_2)\delta - 2t^2\alpha_1\delta - 2t^3(1 - 2\delta) - t\alpha_1^2\delta + t\alpha_1\alpha_2(2 - 3\delta)]^2} > 0,$$

we conclude that $\exists! \tilde{\delta}^{om} \in (\tilde{\delta}^{oc}, \bar{\delta}^{oc})$ such that $\tilde{\pi}^{oc}(\delta) > \tilde{\pi}^{om}, \forall \delta > \tilde{\delta}^{om}$. More precisely:

$$\tilde{\pi}^{oc}(\tilde{\delta}^{om}) = \tilde{\pi}^{om} \Leftrightarrow \tilde{\delta}^{om} = \frac{N_{\tilde{\delta}^{om}}}{D_{\tilde{\delta}^{om}}}, \quad (49)$$

with:

$$N_{\tilde{\delta}^{om}} = 2t(t^2 - \alpha_1\alpha_2) \left[10t^3 - t(\alpha_1 + \alpha_2)(4\alpha_1 + \alpha_2) - (2t^2 - \alpha_1^2 - \alpha_1\alpha_2)(\alpha_1 + 2\alpha_2 + 2k_1) \right],$$

$$\begin{aligned} D_{\tilde{\delta}^{om}} = & 32t^6 + \alpha_1^3(\alpha_1 + \alpha_2)^2(\alpha_1 + 2\alpha_2) - 4t^5(5\alpha_1 + 4\alpha_2) - t^2\alpha_1(\alpha_1 + \alpha_2)(\alpha_1^2 - 3\alpha_1\alpha_2 - 2\alpha_2^2) - \\ & - 2t\alpha_1^2(\alpha_1 + \alpha_2)(2\alpha_1^2 + 4\alpha_1\alpha_2 + 3\alpha_2^2) - 2t^4(8\alpha_1^2 + 17\alpha_1\alpha_2 + \alpha_2^2) + 2t^3\alpha_1(9\alpha_1^2 + 17\alpha_1\alpha_2 + 10\alpha_2^2) - \\ & - 2k_1(2t^2 - \alpha_1^2 - \alpha_1\alpha_2) \left[4t^3 - 2t^2\alpha_1 + \alpha_1^2(\alpha_1 + \alpha_2) - t\alpha_1(\alpha_1 + 3\alpha_2) \right]. \end{aligned}$$

Recalling that $\tilde{\pi}^{om}$ is the maximum collusive profit for side 1 to be fully covered, we conclude that $\pi^{oc}(\delta) = \tilde{\pi}^{om}, \forall \delta \in (\bar{\delta}^{oc}, \bar{\delta}^{oc})$.

1.2.2. If $\delta \in (\bar{\delta}^{oc}, 1)$, the coefficient of π^{oc} in (46) is negative. Thus, the ICC is equivalent to $\pi^{oc} \geq \check{\pi}^{oc}(\delta)$ with $\check{\pi}^{oc}(\delta) = \frac{-N\tilde{\pi}^{oc}(\delta)}{-D_{\tilde{\pi}^{oc}}(\delta)}$. Notice further that:

$$\frac{\partial \check{\pi}^{oc}}{\partial \delta} = \frac{\partial \tilde{\pi}^{oc}}{\partial \delta} > 0 \quad \text{and} \quad \check{\pi}^{oc}(1) = \pi^N.$$

Thus, any profit above the Nash profit is surely sustainable for $\delta > \bar{\delta}^{oc}$. However, if $\pi^{oc} > \tilde{\pi}^{om}$, side 1 would not be fully covered (which is not optimal from Lemma 7). Thus: $\pi^{oc}(\delta) = \tilde{\pi}^{om}, \forall \delta \in (\bar{\delta}^{oc}, 1)$.

2. Assume now that $0 < t < \alpha_1$. Start by noticing that Assumption 1 and condition $t < \alpha_1$ are only compatible if $\alpha_2 < \alpha_1$.

If the deviating platform A did not monopolize any market side, it would charge prices (40) and its profit would be equal to (43). In addition, from (41): $n_1^A(p_1^{od}, p_2^{od}, p_1^{oc}, p_2^{oc}) < n_2^A(p_1^{od}, p_2^{od}, p_1^{oc}, p_2^{oc})$. Thus, if platform A did not monopolize side 2, it did not monopolize side 1 as well. Replacing expressions for prices (40) in the demand function (2), we obtain:

$$n_2^A(p_1^{od}, p_2^{od}, p_1^{oc}, p_2^{oc}) \leq 1 \Leftrightarrow \pi^{oc} \leq \frac{(2t - \alpha_1 - \alpha_2)[\alpha_1(\alpha_1 + \alpha_2) + 2t(\alpha_1 - t - \alpha_2)]}{2t(\alpha_1 - \alpha_2)} \equiv \check{\pi}^{oc}. \quad (50)$$

If Assumption 1 and condition $t < \alpha_1$ hold, we have:

$$\check{\pi}^{oc} - \pi^N = \frac{(\alpha_1 - t)(2t - \alpha_1 - \alpha_2)(2t + \alpha_1 + \alpha_2)}{2t(\alpha_1 - \alpha_2)} > 0. \quad (51)$$

In addition:⁴⁴

$$\check{\pi}^{oc} < \hat{\pi}^{om} \Leftrightarrow k_2 > \frac{t^2(11\alpha_1 - 3\alpha_2) - (\alpha_1 - \alpha_2)(2\alpha_1 + \alpha_2)t - 2\alpha_1(\alpha_1 + \alpha_2)^2}{2t(\alpha_1 - \alpha_2)} \equiv \hat{k}_2. \quad (52)$$

To get a richer setting, we assume that $k_2 > \hat{k}_2$. Thus: if $\pi^{oc} \in [\pi^N, \check{\pi}^{oc})$, the deviating platform does not monopolize any market side; while, if $\pi^{oc} \in [\check{\pi}^{oc}, \hat{\pi}^{om}]$, it monopolizes

⁴⁴The expression for $\hat{\pi}^{om}$ is given in Lemma 8.

side 2. If $\pi^{oc} \geq \hat{\pi}^{oc}$, the deviating platform chooses prices that solve:

$$\begin{cases} n_2^A(p_1, p_2, p_1^{oc}, p_2^{oc}) = 1 \\ \frac{\partial \pi^A}{\partial p_2} = 0 \end{cases} \Leftrightarrow \begin{cases} p_1^{od}(\pi^{oc}) = \frac{(3\alpha_1 - 2t - \alpha_2)(t^2 - \alpha_1\alpha_2) - 2t(\alpha_1 - \alpha_2)\pi^{oc}}{(\alpha_1 - t)(\alpha_1 - \alpha_2)} \\ p_2^{od}(\pi^{oc}) = \frac{(t^2 - \alpha_1\alpha_2)[2t\alpha_2 - \alpha_1(\alpha_1 + \alpha_2)] + 2t\alpha_1(\alpha_1 - \alpha_2)\pi^{oc}}{t(\alpha_1 - t)(\alpha_1 - \alpha_2)}. \end{cases} \quad (53)$$

Replacing these expressions in (2), we conclude that there is never monopolisation of side 1 in the deviating period, since:

$$n_1^A(p_1^{od}, p_2^{od}, p_1^{oc}, p_2^{oc}) - 1 = -\frac{(t + \alpha_1)(2t - \alpha_1 - \alpha_2)}{2t(\alpha_1 - \alpha_2)} < 0.$$

2.1. If $\pi^{oc} < \check{\pi}^{oc}$, the analysis is quite similar to the case **1.1**. More precisely, any collusive profit $\pi^{oc}(\delta) \leq \check{\pi}^{oc}$, with $\check{\pi}^{oc}$ given in (44), is sustainable. However, this analysis is only valid if $\pi^{oc} < \check{\pi}^{oc}$, given in (50) and:

$$\begin{aligned} \check{\pi}^{oc} < \hat{\pi}^{oc} &\Leftrightarrow \\ \Leftrightarrow \frac{(\alpha_1 - t)(2t - \alpha_1 - \alpha_2)(2t + \alpha_1 + \alpha_2)}{2t(\alpha_1 - \alpha_2)(t^2 - \alpha_1\alpha_2)(1 - \delta)} \left\{ - (t^2 - \alpha_1\alpha_2) + [t^2 - \alpha_1\alpha_2 + (\alpha_1 - \alpha_2)(\alpha_1 - t)] \delta \right\} &< 0 \\ \Leftrightarrow \delta < \frac{t^2 - \alpha_1\alpha_2}{t^2 - \alpha_1\alpha_2 + (\alpha_1 - \alpha_2)(\alpha_1 - t)} \equiv \hat{\delta}^{oc}. & \quad (54) \end{aligned}$$

2.2. If $\pi^{oc} \geq \check{\pi}^{oc}$ (or, equivalently, $\delta \geq \hat{\delta}^{oc}$), there is monopolisation of side 2 in the deviating period. Using (53), the deviation profit is:

$$\begin{aligned} \pi^{od}(\pi^{oc}) &= \frac{(2t - \alpha_1 - \alpha_2)(t^2 - \alpha_1\alpha_2)}{2t(\alpha_1 - t)(\alpha_1 - \alpha_2)^2} \times \\ &\times \left\{ [2t^2 - 3t(\alpha_1 - \alpha_2) - \alpha_1(\alpha_1 + \alpha_2)] + 2t(\alpha_1 - \alpha_2) [2t^2 - t(\alpha_1 - \alpha_2) + \alpha_1(\alpha_1 - 3\alpha_2)] \pi^{oc} \right\}. \end{aligned} \quad (55)$$

Replacing this expression and $\pi^N = \frac{2t - \alpha_1 - \alpha_2}{2}$ in the ICC (11), we obtain:

$$\begin{aligned} \frac{1}{2t(\alpha_1 - t)(\alpha_1 - \alpha_2)^2} \left\{ (2t - \alpha_1 - \alpha_2) \left[- (t^2 - \alpha_1\alpha_2) [2t^2 - 3t(\alpha_1 - \alpha_2) - \alpha_1(\alpha_1 + \alpha_2)] + \right. \right. \\ \left. \left. + [2t^4 - 3t^3(\alpha_1 - \alpha_2) - \alpha_2t^2(5\alpha_1 - \alpha_2) - t\alpha_1(\alpha_1 - 4\alpha_2)(\alpha_1 - \alpha_2) + \alpha_1^2\alpha_2(\alpha_1 + \alpha_2)] \delta \right] - \right. \\ \left. - 2t(\alpha_1 - \alpha_2) [2(t^2 - \alpha_1\alpha_2) - [2(t^2 - \alpha_1\alpha_2) + (\alpha_1 - \alpha_2)(\alpha_1 - t)] \delta] \pi^{oc} \right\} \geq 0 \end{aligned} \quad (56)$$

The signal of the coefficient of π^{oc} is positive iff:

$$\delta > \frac{2(t^2 - \alpha_1\alpha_2)}{2(t^2 - \alpha_1\alpha_2) + (\alpha_1 - \alpha_2)(\alpha_1 - t)} = \bar{\delta}. \quad (57)$$

Under Assumption 1 and condition $t < \alpha_1$, we have:

$$\bar{\delta} - \hat{\delta}^{oc} = \frac{(\alpha_1 - t)(\alpha_1 - \alpha_2)(t^2 - \alpha_1\alpha_2)}{[2(t^2 - \alpha_1\alpha_2) + (\alpha_1 - \alpha_2)(\alpha_1 - t)][t^2 - \alpha_1\alpha_2 + (\alpha_1 - \alpha_2)(\alpha_1 - t)]} > 0.$$

2.2.1. If $\delta \in (\hat{\delta}^{oc}, \bar{\delta})$, the coefficient of π^{oc} in (46) is negative and the ICC can be written as $\pi^{oc} \leq \hat{\pi}^{oc}(\delta)$:

$$\hat{\pi}^{oc}(\delta) = \frac{N_{\hat{\pi}^{oc}}(\delta)}{D_{\hat{\pi}^{oc}}(\delta)} \quad (58)$$

with:

$$\begin{aligned} N_{\hat{\pi}^{oc}}(\delta) = & (2t - \alpha_1 - \alpha_2) \left\{ - (t^2 - \alpha_1\alpha_2) [2t^2 - 3t(\alpha_1 - \alpha_2) - \alpha_1(\alpha_1 + \alpha_2)] + \right. \\ & \left. + [2t^4 - 3t^3(\alpha_1 - \alpha_2) - \alpha_2t^2(5\alpha_1 - \alpha_2) - t\alpha_1(\alpha_1 - 4\alpha_2)(\alpha_1 - \alpha_2) + \alpha_1^2\alpha_2(\alpha_1 + \alpha_2)] \delta \right\}, \end{aligned} \quad (59)$$

$$D_{\hat{\pi}^{oc}}(\delta) = 2t(\alpha_1 - \alpha_2) \left[2(t^2 - \alpha_1\alpha_2) - [2(t^2 - \alpha_1\alpha_2) + (\alpha_1 - \alpha_2)(\alpha_1 - t)] \delta \right] \quad (60)$$

As $\lim_{\delta \rightarrow \bar{\delta}^-} \hat{\pi}^{oc}(\delta) = +\infty$ and:

$$\frac{\partial \hat{\pi}^{oc}}{\partial \delta} = \frac{(\alpha_1 - t)^2 (t^2 - \alpha_1\alpha_2) (2t - \alpha_1 - \alpha_2) (2t + \alpha_1 + \alpha_2)}{2t \left[2t^2(1 - \delta) + t(\alpha_1 - \alpha_2)\delta - \alpha_1 [\alpha_2(2 - 3\delta) + \alpha_1\delta] \right]^2} > 0,$$

we conclude that $\exists! \hat{\delta}^{om} \in (\hat{\delta}^{oc}, \bar{\delta})$ such that $\hat{\pi}^{oc}(\delta) > \hat{\pi}^{om}, \forall \delta > \hat{\delta}^{om}$. More precisely:

$$\hat{\pi}^{oc}(\hat{\delta}^{om}) = \hat{\pi}^{om} \Leftrightarrow \hat{\delta}^{om} = \frac{N_{\hat{\delta}^{om}}}{D_{\hat{\delta}^{om}}}, \quad (61)$$

with:

$$N_{\hat{\delta}^{om}} = 2(t^2 - \alpha_1\alpha_2) \left[2k_2t(\alpha_1 - \alpha_2) + \alpha_1(\alpha_1 + \alpha_2)^2 + t(\alpha_1 - \alpha_2)(2\alpha_1 + \alpha_2) - t^2(7\alpha_1 - 3\alpha_2) \right],$$

$$\begin{aligned} D_{\hat{\delta}^{om}} = & 2k_2t(\alpha_1 - \alpha_2) [2t^2 + \alpha_1(\alpha_1 - 3\alpha_2) - t(\alpha_1 - \alpha_2)] - 2t^4(7\alpha_1 - 3\alpha_2) - 2\alpha_1^2\alpha_2(\alpha_1 + \alpha_2)^2 + \\ & + t\alpha_1(\alpha_1 - 3\alpha_2)(\alpha_1 - \alpha_2)(2\alpha_1 + \alpha_2) + t^3(7\alpha_1^2 - 8\alpha_1\alpha_2 + \alpha_2^2) - t^2(3\alpha_1^3 - 27\alpha_1^2\alpha_2 + 7\alpha_1\alpha_2^2 + \alpha_2^3) \alpha_2. \end{aligned}$$

Recall that side 2 is fully covered iff $\pi^{oc}(\delta) \leq \hat{\pi}^{om}$. Thus, from Lemma 7, we conclude that: $\pi^{oc}(\delta) = \hat{\pi}^{om}$, $\forall \delta \in (\tilde{\delta}^{oc}, \hat{\delta}^{oc})$.

2.2.2. Following the same steps as in **1.2.2.**, it follows that $\pi^{oc}(\delta) = \hat{\pi}^{om}$, $\forall \delta \in (\hat{\delta}^{oc}, 1)$.

3. Finally, assume that $\alpha_1 < -\frac{\tilde{u}_2^N}{\tilde{u}_1^N}t < t$.

As in case **2.**, if the deviating platform does not monopolize any market side, it charges prices (40) and its profit is given by (43). From (41): $n_1^A(p_1^{od}, p_2^{od}, p_1^{oc}, p_2^{oc}) < n_2^A(p_1^{od}, p_2^{od}, p_1^{oc}, p_2^{oc})$. From (50): $n_2^A(p_1^{od}, p_2^{od}, p_1^{oc}, p_2^{oc}) \leq 1 \Leftrightarrow \pi^{oc} \leq \check{\pi}^{oc}$, with $\check{\pi}^{oc}$ given in (50). If $k_2 > \hat{k}_2$, we have $\check{\pi}^{oc} < \hat{\pi}^{om}$. In addition, as $\alpha_2 > \alpha_1$, it follows from (51) that $\check{\pi}^{oc} \geq \pi^N$.

3.1 If $\pi^{oc} < \check{\pi}^{oc}$, there is no monopolisation of any market side in the deviating period and the analysis is identical to case **1.1**. More precisely, any collusive profit such that $\pi^{oc}(\delta) \leq \check{\pi}^{oc}$, with $\check{\pi}^{oc}$ given in (44), is sustainable. However, this analysis is only valid if $\pi^{oc} < \check{\pi}^{oc}$, given in (50). From (54), $\pi^{oc}(\delta) = \check{\pi}^{oc}(\delta)$ iff $\delta < \hat{\delta}^{oc} = \frac{t^2 - \alpha_1 \alpha_2}{t^2 - \alpha_1 \alpha_2 + (\alpha_2 - \alpha_1)(t - \alpha_1)}$.

3.2. If $\pi^{oc} \geq \check{\pi}^{oc}$ (or, equivalently, $\delta \geq \hat{\delta}^{oc}$), there is monopolization of side 2 in the deviating period. The ICC for collusion to be sustainable is given by (56). As $t > \alpha_1$ and $\alpha_2 > \alpha_1$, the signal of the coefficient of π^{oc} is positive iff $\delta > \bar{\delta}$, with $\bar{\delta} \in (\hat{\delta}^{oc}, 1)$ given in (57).

3.2.1. If $\delta \in (\hat{\delta}^{oc}, \bar{\delta})$, the coefficient of π^{oc} in (46) is negative and the ICC can be written as $\pi^{oc} \leq \hat{\pi}^{oc}(\delta)$ with: $\hat{\pi}^{oc}(\delta) = \frac{-N_{\hat{\pi}^{oc}(\delta)}}{-D_{\hat{\pi}^{oc}(\delta)}}$, where $N_{\hat{\pi}^{oc}(\delta)}$ and $D_{\hat{\pi}^{oc}(\delta)}$ are given in (59) and (60), respectively. Following the same steps as in **2.2.1**, we conclude that: $\pi^{oc}(\delta) = \hat{\pi}^{om}$, $\forall \delta \in (\tilde{\delta}^{oc}, \hat{\delta}^{oc})$.

3.2.2. As in **2.2.2**, it follows that $\pi^{oc}(\delta) = \pi^{om}$, $\forall \delta \in (\hat{\delta}^{oc}, 1)$. □

Proof of Proposition 6.

Assume that $\alpha_1 = \alpha_2 \equiv \alpha$. For Assumption 1 to be satisfied, we must have $-t < \alpha < t$. Let us determine the sign of the derivative of π^{oc} , given in (16), with respect to α .

1. If $0 < \delta < \tilde{\delta}^{oc}$, with $\tilde{\delta}^{oc} = \frac{t}{3t-2\alpha}$:

$$\frac{\partial \pi^{oc}}{\partial \alpha} < 0 \Leftrightarrow -\frac{t + (3t - 4\alpha)\delta}{t(1 - \delta)} < 0 \Leftrightarrow (3t - 4\alpha)\delta > -t.$$

If $t \geq \frac{4}{3}\alpha$, the last inequality is trivially satisfied. If $t < \frac{4}{3}\alpha$, we have that $\frac{\partial \pi^{oc}}{\partial \alpha} < 0 \Leftrightarrow \delta < \frac{t}{4\alpha-3t}$. Notice, however, that we are assuming $\delta < \tilde{\delta}^{oc}$ and $\tilde{\delta}^{oc} < \frac{t}{4\alpha-3t}$.

1.2. If $\tilde{\delta}^{oc} < \delta \leq \tilde{\delta}^{om}$ with $\tilde{\delta}^{om} = \frac{t(2k_1-5t+3\alpha)}{2k_1(2t-\alpha)-8t^2+9t\alpha-3\alpha^2}$:

$$\frac{\partial \pi^{oc}}{\partial \alpha} < 0 \Leftrightarrow -\frac{3t^2 - 2t(5t - 2\alpha)\delta + (9t^2 - 8t\alpha + 2\alpha^2)\delta^2}{2(t + \alpha\delta - 2t\delta)^2} < 0.$$

Notice that $9t^2 - 8t\alpha + 2\alpha^2 > 8t(t - \alpha) + 2\alpha^2 > 0$. As $3t^2 - 2t(5t - 2\alpha)\delta + (9t^2 - 8t\alpha + 2\alpha^2)\delta^2$ has no real roots, we conclude that $\frac{\partial \pi^{oc}}{\partial \alpha} < 0$.

1.3. If $\tilde{\delta}^{om} < \delta < 1$:

$$\frac{\partial \pi^{oc}}{\partial \alpha} < 0 \Leftrightarrow -\frac{k_1 - t + 3\alpha}{2t} < 0 \Leftrightarrow k_1 > t - 3\alpha.$$

Notice, however, that for the collusive profit to be given by (16), we must have $k_1 > \frac{7t-3\alpha}{2}$. As $t - 3\alpha < \frac{7t-3\alpha}{2}$, we conclude that $\frac{\partial \pi^{oc}}{\partial \alpha} < 0$. \square

References

- [1] Argentesi, E., & Filistrucchi, L. (2007). Estimating market power in a two-sided market: The case of newspapers. *Journal of Applied Econometrics*, 22(7): 1247-1266.
- [2] Armstrong, M. (2006). Competition in twosided markets. *RAND Journal of Economics*, 37(3): 668-691.
- [3] Boffa, F., & Filistrucchi, L. (2014). Optimal Cartel Prices in Two-Sided Markets. NET Institute Working Paper 14-19.
- [4] Choi, J. P., & Gerlach, H. (2013). Multimarket collusion with demand linkages and antitrust enforcement. *The Journal of Industrial Economics*, 61(4): 987-1022.
- [5] Dewenter, R., Haucap, J., & Wenzel, T. (2011). Semi-collusion in media markets. *International Review of Law and Economics*, 31(2), 92-98.

- [6] Evans, D. S., & Schmalensee, R. (2008). Markets with two-sided platforms. *Issues in Competition Law and Policy* (ABA Section of Antitrust Law), 1.
- [7] Evans, D. S., & Schmalensee, R. (2013): The antitrust analysis of multi-sided platform businesses. NBER Working Paper No. 18783, February.
- [8] Thomadsen, R., & Rhee, K. E. (2007). Costly collusion in differentiated industries. *Marketing Science*, 26(5): 660-665.
- [9] Kaiser, U., and J. Wright. (2006). Price structure in two-sided markets: Evidence from the magazine industry. *International Journal of Industrial Organization*, 24(1): 1-28.
- [10] Ruhmer, I. (2011). Platform collusion in two-sided markets. University of Mannheim.