College Choice, Selection and Allocation Mechanisms: A Structural Empirical Analysis

José Raimundo Carvalho
CAEN, Universidade Federal do Ceará

Thierry Magnac
Toulouse School of Economics

Qizhou Xiong
OvGU Magdeburg and IWH

September 19, 2018

Abstract

We use rich microeconomic data on performance and choices of students at college entry to analyze interactions between the selection mechanism, eliciting college preferences through exams, and the allocation mechanism. We set up a framework in which success probabilities and student preferences are shown to be identified from data on their choices and their exam grades under exclusion restrictions and support conditions. The counterfactuals we consider balance the severity of congestion and the quality of the match between schools and students. Moving to deferred acceptance or inverting the timing of choices and exams are shown to increase welfare. Redistribution among students and among schools is also sizeable in all counterfactual experiments.

Keywords: Education, two-sided matching, school allocation mechanism, policy evaluation

JEL codes: C57, D47, I21

1 Introduction[1]

The *matching* literature provides analyses of mechanisms allocating goods or relationships between many parties in the absence of a price mechanism, and examples range from kidney exchange and marriage to school choice (see Roth and Sotomayor, 1992, Roth, 2008). The analysis of centralized mechanisms in school choice as a many-to-one match has been very popular in the recent theoretical and empirical literature (for instance Abdulkadiroğlu, Agarwal and Pathak, 2017, Agarwal, 2015, Azevedo and Leshno, 2016, Budish and Cantillon, 2012, Calsamiglia, Fu and Guell, 2018, Chen and Kesten, 2017, He, 2017, Agarwal and Somaini, 2018, among others) and has had practical value for policy implemented in primary or high schools in various countries.

College choice adds the new dimension of elicitation of college preferences over students which is of secondary importance in primary and high-school choice. This elicitation process costs time and money because of congestion if application costs are low. This market friction is large when the allocation is decentralized as in the US (Che and Koh, 2016 and Chade, Lewis and Smith, 2014) but not only. Even with centralized mechanisms, for instance, used by universities in China (Chen and Kesten, 2017) or Turkey (Balinski and Sonmez, 1999), it is costly to organize a general exam whose results determine students' ranking while keeping up with the quality of selection in decentralized systems. This is why these exams are generally composed of proofs in different fields (maths, literature etc) and at times consist of two stages. The first stage selects out students at minimal costs while the second stage allows for a costly but more precise evaluation (Hafalir, Hakimov, Kübler and Kurino, 2018).

In this paper, we analyze the interactions between allocation mechanisms and selection when college choice is centralized. College preferences are not taken as granted as in the matching literature and they are costly to elicit. We exploit an admittedly specific college choice experiment

[1]This is a much revised version of a previous paper entitled "College Choice and Entry Exams" by two of the coauthors that has been circulated since 2009. We thank the co-editor and two anonymous referees for their insighful suggestions. Useful talks and interactions with Yinghua He, Philipp Heller, Jean-Marc Robin, Bernard Salanié and comments by participants at conferences in Brown, Bristol, Atlanta, Northwestern, Shanghai, Rio de Janeiro, IAAE'15 and San Francisco as well as seminars at Oxford, CREST, CEMMAP, Cambridge, Amsterdam, Barcelona, Manchester and Bern are gratefully acknowledged. This research has received financial support from CNPq (Project 21207) and the European Research Council under the European Community's Seventh Framework Program FP7/2007-2013 grant agreement N°295298. The usual disclaimer applies.

in order to quantify some of the trade-offs that matching and selection involve. Our observational "experiment" uses observed entry exam grades and choices between schools within a Federal University in Brazil in 2004.

A mechanism called *vestibular* was in place in this University and worked as follows. During their last high school year, students chose a single specialization field or "school"[2] before taking a two-stage exam at the end of high school. The first stage is a cost-minimizing multiple question exam common to all fields and selects but the top-ranked students for a more in-depth and specialized second-stage exam. Aggregating scores of both exams yields the final rankings and admissions into each school.

This paper aims at evaluating the effects, on student allocations and their welfare, of changing the allocation mechanism and the selection device with respect to the existing *vestibular.* In the absence of experiments (Calsamiglia, Haeringer and Klijn, 2010) or quasi-experiments (Pathak and Sonmez, 2013), estimating a structural model is key to our empirical strategy. We construct such a model of college choice, exhibit conditions under which parameters are identified and derive empirical counterfactual results on outcomes and welfare.

The paper makes three original contributions.

Our first contribution is to adopt a two step empirical strategy that uses first information on performance at the two-stage exams to estimate success probabilities at each school. Second, we estimate preference parameters from observed school choices when students play strategically by taking into account their expected probabilities of success (Arcidiacono, 2005, Epple, Romano and Sieg, 2006). As far as we know, the previous empirical literature does not estimate school choice models in which students face uncertainty about their entrance exam scores. This is permitted by our rich data on exam scores as well as on an initial measure of ability obtained a year before the exams are taken.

Our second original contribution is to derive conditions under which expected success probabilities of entry and preferences are identified from observing the distribution of grades and college choices. Students play a "congestion" game in which choices of other students affect their own success probabilities. We adopt specific and admittedly strong assumptions to solve this game. The solution concept we use is a Nash equilibrium. Students have symmetric information about

[2]We use the terms "college" and "school" interchangeably for these fields in the following.

random shocks on grades i.e. they know their distribution functions only and the information set of students and econometricians is the same. We also assume that expectations are perfect in the sense that they can be obtained by infinitely repeating the game with the same players. We justify these assumptions in the specific context of our empirical application.

We show that the distribution of success probabilities can be obtained by resampling in our observed sample and by using Nash equilibrium conditions. We derive from the latter, grade thresholds for being admitted in a specific college at each exam stage and show that success probabilities are identified. We then provide a proof of non parametric identification of preference parameters by using, as in Matzkin (1993), exclusion restrictions and conditions that success probabilities fully vary over the simplex. This proof of identification specifically deals with two prevalent issues in college choice. First, data are likely to be choice-based. Second, outside options play a much more important rôle than in school choice (Agarwal and Somaini, 2018) since the number of candidates is well above the number of seats, by a factor of 15 in our data.

Our third original contribution is to analyze the aggregate and distributive effects on allocation and welfare of students and schools of three different counterfactual mechanisms that play with the trade-offs between congestion costs, the adequacy of student selection and the quality of the resulting match. These three experiments aim at analyzing salient policy issues in the current debates on school choice (Roth, 2018).

In the first experiment, we restrict the number of seats available at the second stage exam. We argue that it reduces screening costs for schools at the risk of degrading selection of adequate students. The counterfactual effect on matching quality we obtain is, however, small. In the second experiment, students are allowed to submit a list of two choices instead of a single one in order to get closer to a Gale-Shapley deferred acceptance mechanism. It indeed results in a positive aggregate effect in terms of utilitarian social welfare though it also has distributive effects. Strategic effects in the original mechanism are shown to be sizeable. Interchanging the timing of choices and the first exam is the basis of our third counterfactual experiment. We allow students to choose colleges after passing the first-stage exam instead of having them to choose before this exam. This allocation mechanism is quite popular, as in Japan for instance (Hafalir et al., 2018). As expected it has strong redistributive effects between schools and between students since it favors more opportunistic behavior.

Related Literature

This paper touches different strands of the matching and school choice literature.

Analyzing the matching of students to schools has a long history and a brief survey of the recent literature in which differences between school choice, college admission and student placement are rigorously defined is available in Sonmez and Ünver (2011). Prominently in this literature, Gale-Shapley deferred acceptance mechanisms satisfy both properties of stability and strategy-proofness (on the student side if students propose) if preferences are strict (*e.g.* Abdulkadiroğlu and Sonmez, 2003). If such a mechanism is used, the elicitation process through which schools decide on their ranking of students has very little impact on the preference lists submitted by students. The use of deferred acceptance mechanisms, however, could involve larger congestion costs than other non-stable mechanisms (He and Magnac, 2018) such as deferred acceptance with a truncated list of preferences as is the case with the mechanism we study in this paper. The truncation is severe since the list of schools is of length one.

The seminal analysis of college admissions by Balinski and Sönmez (1999) was theoretical albeit oriented towards the analysis of a specific mechanism. They studied the optimality of student placement in Turkish universities in which selection and competition among students are nationwide unlike our case. Students choose a rank-ordered list of colleges prior to writing exams in various subjects. Student rankings are constructed using exam grades, and are allowed to differ across colleges by weighting subjects differently. Grades in mathematics are given more weight by math schools.

Most of the empirical litterature on matching, however, is concerned by primary or high school choices. Abdulkadiroğlu, Pathak and Roth (2009) study the mechanisms used in the New York high school system and focus on the trade-offs between efficiency, strategy-proofness and stability. This research line on primary and secondary schools questions the relative standing of the Gale-Shapley and the Boston mechanisms (Abdulkadiroglu, Pathak, Roth and Sönmez, 2006). Others analyze the Boston mechanism as He (2017) who uses school allocation data from Beijing and evaluates the cost of strategizing for sophisticated and naive agents. The question of the importance of truncated lists of preferences used in practice in deferred acceptance mechanisms is high on the agenda in recent research about middle or high school choices (Calsamiglia et al, 2018, Fack, Grenet and He, 2017).

School and college choice, however, differ in a number of dimensions and the questions set out in this paper are more specific to college choice. College preferences over students depend on their past investments in human capital and abilities and not only on priorities given by residence and siblings. This implies in particular that colleges have strict preferences over students and the arguments underpinning the debate between the choice of allocation mechanisms such as Deferred Acceptance and Boston can be misleading for college choice. Furthermore, demand for colleges is not localized and is much larger than supply.

In the most recent literature demands for colleges are estimated in Hastings, Kane and Staiger (2009) to study how enlarging choice sets might have unintended consequences for minority students. In Agarwal (2015), medical schools and medical residents preferences are estimated using a two-sided school choice model. Fu (2014) estimates demand and supply equations when students have heterogenous abilities and preferences and when college applications are costly and uncertain. Akyol and Krishna (2017) also estimates a structural model of high school choice using Turkish data in order to understand whether the higher standing of elite schools is due to selection or to value added. As the allocation mechanism in place is deferred acceptance, preferences can be directly estimated from rank-ordered lists. It is remarkable that they find that estimates of value added are small.

To our knowledge, there is no comparative survey of college admission procedures in different countries. There exist empirical papers about the "parallel" mechanism used in China (Chen and Kesten, 2017 or Zhu, 2014) or descriptive analyses in Turkey (Dogan and Yuret, 2012) or in Egypt (Selim and Salem, 2009). Abizada and Chen (2011) analyze the eligibility restrictions to college access that gives a way of reducing costs of evaluation of students by colleges. A descriptive analysis of the mechanism centralized at the level of the country, which has been used in Brazil since 2010, is provided by Aygun and Bo (2017) and Machado and Szerman (2017).

The most obvious distinction between college admission procedures is their degree of centralization. Decentralized models of college choices, as in the United States, are studied by Chade, Lewis and Smith (2014), Che and Koh (2017) and Hafalir et al. (2018) among others. In the last paper, low and high ability students are shown to have different preferences over centralized and decentralized mechanisms and a small literature about centralization is surveyed there. Congestion is reduced by either making students pay an application cost or by making them choose

only one college. In Chade et al (2014), school preferences are noisy signals of students' abilities and college strategizing can lead to inefficient sorting of students. The use of waiting lists might lead to unstable mechanisms. In Che and Koh (2017), the uncertainty of student preferences makes schools play strategically and this leads to inefficient and unfair assignments because the management of offers and acceptance of offers is uncertain and takes time.

Centralization may avoid the costs of congestion if colleges do not have to deal with all student files. It also streamlines the competition between colleges. Yet, centralization assumes that college preferences are adequately translated by the information revealed at a general exam (Hafalir et al., 2018). In a decentralized system like in the US, many other elements than the SAT score are evaluated and the selection is multidimensional. The two-stage exam set-up tries to mitigate the reduction in selection quality. The selection mechanism used in our empirical illustration is broadly akin to the Japanese experience in which a first stage centralized exam is followed by a second-stage exam decentralized at the level of each university *on the same day* which effectively avoids congestion (see Hafalir et al., 2018). The choice of college and the sequential exams are also akin to the system now in place in South Korea (Avery, Lee and Roth, 2014). As a matter of fact, these two-step revelation procedures of school preferences are rather common (job market for PhDs, "grandes écoles" in France) although their interaction with the allocation mechanism is seldom studied in the literature (although see Lee and Schwartz, 2017).

Last, Agarwal and Somaini (2018) developed independently after us a proof of non-parametric identification of preferences in a school choice model. It either relies on exogenous variation in the environment, i.e., in expected success probabilities, as in our case, or on the existence of a special regressor, such as distance to school and quasi-linearity of preferences. Their more-in-depth analysis of the latter case is specifically suited to school choice in primary and secondary schools, while our results bear on college choice. It is indeed more credible there that success probabilities continuously vary, for instance because of grades, than in the case of school choice in which only discrete priorities matter. Conversely, a special regressor such as distance is likely to be irrelevant in college choice. Overall the intuition for both results is based on Matzkin (1992, 1993). We investigate more in depth which preference functionals are identified when there is exogenous variation in the environment and specifically because of the presence of outside options and choice-based sampling.

The paper is organized in the following way. Section 2 describes our modelling assumptions for college choices, the formation of expectations and the conditions under which preferences are identified. Section 3 presents the particulars of our empirical application, explains the estimation and computation of success probabilities and the estimation procedure of preference parameters. It also summarizes results from the estimated coefficients of grade and preference shifters. Section 4 details the results of the three counterfactual experiments. A Supplementary Appendix, available upon request, gathers the details and results of our many procedures.

2 Theoretical set-up

We start by describing a framework, encompassing our empirical application, in which we provide modelling tools and identification results. We abstract from some aspects of the empirical application, such as the two-stage nature of exams, that do not bear on general results and are clarified in the empirical Section 3.

The first subsection defines notation, formalizes the timing of events for students and describes the primitives of the decision problem and the observed variables. Students are assumed to play an imperfect information game in which information on future grades is imperfect but symmetric and its distribution known by agents. Students have no private information and we assume that the solution concept is Nash. In particular, observed characteristics and preference shocks of students are common knowledge. The construction of this set-up in terms of information sets and expectations is presented in the second subsection. We also derive the necessary conditions for a Nash equilibrium.

The final subsection provides conditions under which student preferences are identified.

2.1 Timing for the decision maker

Firstly, we adopt a simplifying framework in which students choose, according to their preferences, one and only one school among many within the University to apply to, as in our empirical application. Rank-ordered lists submitted by students are thus highly truncated and more so than in the empirical application of Agarwal and Somaini (2018) in which rank-ordered lists are of length three. The main reason for adopting such a setting is that it does not change the list

of identified objects. We retrun to this point after stating our identification results. It is worth mentioning that this is akin to the identification results of Agarwal and Somaini (2018) which are insensitive to the allocation mechanism in place provided that certain conditions are satisfied (Definitions 1-3, pp.407-8) excluding top-trading cycles. In this sense, having rank-ordered lists of length one is the minimal observational requirement for preferences to be identified. This also calls to mind that observing the ranking of alternatives in multinomial choices does not enlarge the set of identified objects but allows them to be more precisely estimated.

Second, student preferences can be monetary or non monetary and describe the consumption value of education (Alstadsæter, 2011, Jacob, McCall and Stange, 2012) as well as its investment value. The latter is derived from earnings that a degree from a specific school raises in the labor market.

We omit the individual index for readability. A random variable, say D, describes school choice and takes as realization, a specific school, j. The set of available schools is denoted by a discrete set of indices, $\mathcal{J}$, to which we add an outside option, $D = \varnothing$. We denote $J = card(\mathcal{J})$ the number of available schools. Observed student characteristics which affect preferences (respectively performance or grades) are denoted X (respectively Z) and variables X and Z can be overlapping.

We describe the assignment mechanism by a simple sequence of four steps. At each step, students obtain information or make decisions.

- **School capacities:** Every school announces the number of seats available or its capacity, n^j.

- **Choice of school:** Students apply to one and only one school among available options, $j \in \{\varnothing\} \cup \mathcal{J}$. The outside option $j = \varnothing$ means that one forfeits the opportunity to get into one of these schools and either chooses another university, searches for a job or any other alternative (waiting until next year, staying at home). After that stage, students are allocated, according to their school choices, to J sub-samples which are observed in our empirical application. We do not observe students who choose an external option and in this sense we have a choice-based sample.

- **Exam stage:** All students take a single exam or multiple exams, identical across schools, and exam grades are aggregated into a single grade denoted m and written as a function of

characteristics, Z, as:

$$m = m(Z, u; \beta)$$

in which u are random individual circumstances that affect results at these exams and β is an unknown parameter. College preferences are formed using these heterogenous grades.

- **College entry:** In each subsample, defined by $D = j$, students are ranked according to their values of grade m and the first n^j students are accepted in school j that they have chosen previously. This selection can be expressed using a threshold, T^j, that describes the set of successful students by the condition, $m \geq T^j$ (as in Azevedo and Leshno, 2016). Those who succeed receive a value, V^j, describing their preferences. Those who fail, get the value of their outside option that we normalize to 0. Individual rationality implies that j is never chosen if $V^j < 0$.

There could be additional decision nodes to consider if the value of outside options evolves over time because of the selection process. Students could leave the game after taking or passing exams because grades could give students a way to signal their ability to potential employers or other universities. This would modify the value of the outside option after the exam stage. This is why, in our empirical application, we select elite schools which are so attractive that almost no students quit after taking or passing exams.

Determining choices is now easy. Define the probability of success in school D as:

$$P^D = \Pr(m(Z, u; \beta) \geq T^D),$$

in which we delay until next section the precise definition of the probability measure for random thresholds T^D since it depends on the definition of information sets and expectations. The expected value of choosing school D at the time of the choice is given by:

$$\mathbb{E}V^D = P^D V^D,$$

and, as the outside option has value zero, choosing $j \in \mathcal{J}$ is described by the choice-based condition $\max_{k \in \mathcal{J}}(V^k) > 0$. Moreover, maximizing expected utility leads, for any $j \in \mathcal{J}$, to:

$$D = j \quad \text{iff } \max_{k \in \mathcal{J}}(V^k) > 0 \text{ and } \forall k \in \mathcal{J}/\{j\}; P^j V^j > P^k V^k. \tag{1}$$

We shall specify later on, values as functions $V^j(X, \varepsilon; \zeta)$ in which X are observed characteristics, ε is an unobservable preference random term and ζ are preference parameters. It is enough at this stage to define choices as $D(X, \varepsilon, \zeta, \{P^j\}_{j \in \mathcal{J}})$.

2.2 Expectations and Nash Equilibrium

We now state our main assumptions, formalize the timing, explain how student beliefs about success probabilities are formed and finish by the Nash equilibrium conditions.

2.2.1 Stochastic assumptions, information and solution concept

We first argue that the following assumptions are adapted to our empirical setting:

Assumptions S(etting):

(S.i) Preference shocks, ε, and grade shocks, u, are independent of (X, Z) and between each other, and both are continuously distributed.

(S.ii) The solution concept is a Nash equilibrium. Students have common knowledge of the sample-specific preferences, ε_i, characteristics, X_i and Z_i as well as common knowledge of grade equation parameters, β, and preference parameters, ζ.

(S.iii) The information of students and econometricians on the distribution of random grade shocks, u, and characteristics, X_i and Z_i is symmetric.

(S.iv) The distribution of grades is such that $\forall j \in \mathcal{J}$, $P^j > 0$ almost everywhere P_Z.

In Assumption *S.i*, independence of shocks and (X, Z) is a standard exogeneity assumption while it is key in the following that preference shocks, ε and grade shocks, u, are independent. It is akin to the usual assumption in consumer studies that income and preference shocks are independent. A relaxation of this assumption would require an instrumental strategy that is beyond the scope of this paper.[3]

Assumption *S.ii* is a complete information set-up that we adopt for two reasons. First, school choice at this University is a game which had been repeated every year over a long time span and which had high stakes for students, families, high schools and preparatory courses alike. The strategizing ability seems more acceptable in our set up than in the case of primary or high schools

[3] We test and do not reject an implication of this assumption in the empirical section, conditional on admittedly specific auxiliary conditions.

(for instance, see He, 2016). The time period over which a student ability is assessed is much longer and many other agents like parents or teachers are ready to help out students to form expectations (see Manski, 1993, for a critical appraisal of such assumptions). Second, a Bayesian-Nash solution concept would be appropriate when agents have private information about their preference shocks, ε_i. Yet, as this congestion game involves many players, it can be conjectured that strong laws of large numbers ensure that the two set-ups are close in terms of aggregate outcomes.

Assumption *S.iii* might be more controversial since it posits that students have no better knowledge of their own success probabilities than their fellow students or econometricians. First, school choices are shown below to ultimately depend on the ratio of success probabilities in the different schools. Any superior knowledge of an individual specific effect affecting success is partly wiped out by this non linear differencing. Second, we use an observable pre-exam national grade in the empirical application to control for superior knowledge. We will briefly return to the effect that the existence of superior information could have on our procedure at the end of this section.

Finally, Assumption *S.iv* makes sure that a pure strategy is optimal almost surely for all students and simplifies the analysis of the game.

2.2.2 Timing

The timing of information revelation, described in the previous section, is formalized as follows. Before schools are chosen, the number of seats in each school, $\{n^j\}_{j\in\mathcal{J}}$ are announced and the total number of participants, say $n+1$, is observed. We assume that $n+1 >> \sum_{j\in\mathcal{J}} n^j$ since the *Vestibular* exam is highly selective.

We distinguish one arbitrary applicant, indexed by 0, from all other applicants, $i = 1, ., n$, and we analyze her decision making. We can proceed this way because we are considering an independently and identically distributed (i.i.d.) setting and because the model is assumed symmetric between agents (although they differ ex-ante in their observed characteristics and ex-post in their unobserved shocks). Applicant 0 faces the n other applicants and we shall construct her best response to other players' choices, $\{D_i\}_{i=1,.,n} \equiv D_{(n)}$ since we use a Nash solution concept (Assumption *S.ii*). The information set of student 0 at the initial stage comprises at least all elements of $W_0 = (X_0, Z_0, \varepsilon_0), X_{(n)} = \{X_i\}_{i=1,.,n}, Z_{(n)} = \{Z_i\}_{i=1,.,n}$ and $D_{(n)}$.

Student 0 chooses her school ($D_0 \in \mathcal{J}$) as a function of her success probabilities, $\{P_0^j\}_{j\in\mathcal{J}}$,

and her preferences as shown in equation (1). Because of Assumption *S.iv*, student 0 plays a pure strategy almost surely. This is her best response to the aggregate behavior of other students on which success probabilities depend. In this sense this is an aggregative game (Jensen, 2010) and we will later make use of this characteristic.

After choosing one school, the exam is taken and students are selected in or out of each school, j, by retaining the best n^j students and this defines the thresholds as functions of observed grades. There are two types of risks that student 0 faces. First, the aggregate risks due to grade shocks affecting other students, $U_{(n)}$ whose elements are u_i, $i = 1, ., n$, second the individual risks due to her own grade shock, u_0. Integrating out both risks allows success probabilities to be derived as the rational expectations of success of student 0.

2.2.3 Success probabilities and best responses

Denote $Z^j_{(n)}$ the set of grade shifters of the sub-sample of students $i = 1, ., n$ applying to school $j \in \mathcal{J}$ that student 0 considers when she computes her best response to $D_{(n)}$. By construction $Z_{(n)} = (Z^j_{(n)})_{j\in\mathcal{J}}$. Similarly, we denote $U^j_{(n)}$ the corresponding components of $U_{(n)}$. We shall see in the next subsection how sub-samples are derived from primitives. Denote $T = (T^j)_{j\in\mathcal{J}}$ the random vector of exam thresholds that determine entry into each school, $j \in \mathcal{J}$ and whose realizations are observed thresholds $(t^j)_{j\in\mathcal{J}}$. These thresholds are random unknowns at the initial stage since they depend on variables, u, that are random unknowns at the initial stage.[4]

Should school j be chosen by student 0, her success would be determined, considering the sample of other students, by the binary condition

$$\mathbf{1}\{m(Z_0, u_0, \beta) \geq T^j(Z^j_{(n)}, U^j_{(n)})\}.$$

Given that the Nash solution concept, *S.ii,* fixes the sample of applicants to school j, threshold $T^j(.)$ for school $j \in \mathcal{J}$ only depends on the characteristics of applicants to this school, $Z^j_{(n)}$, and on their grade shocks, $U^j_{(n)}$. Because grades are continuously distributed (Assumption *S.i*), we can also neglect ties. The existence of thresholds resembles what Azevedo and Leshno (2016) derived in a different context of a stable equilibrium with an infinite number of applicants.

[4]We adopt the term random unknowns to signal that the distribution function of those unknowns are common knowledge. Measurability issues are dealt with below.

The formal construction of these thresholds is explained below after having determined choices but the intuition is clear. School j threshold that student 0 considers is equal to the grade obtained by the n^j-ranked student in $i = 1, \ldots, n$. These thresholds are not explicitly indexed by 0 although they refer to the thought experiment that student 0 performs when constructing her expectations as a function of characteristics and strategies of other students $i = 1, ., n$.

When student 0 decides upon a school to apply to, she formulates expected probabilities of success by integrating the condition of success with respect to the aggregate source of risk described by $U^j_{(n)}$ (remember that student 0 observes $Z_{(n)}$ and conditions on $D_{(n)}$) and with respect to the individual source of risk, u_0:[5]

$$\begin{aligned} P_0^j &= P^j(Z_0, Z^j_{(n)}, \beta) = E_{U^j_{(n)}, u_0}\left[\mathbf{1}\{m(Z_0, u_0, \beta) \geq T^j \mid Z_0, Z^j_{(n)}\right], \\ &= E_{U^j_{(n)}}\left[p^j(Z_0, T^j, \beta) \mid Z_0, Z^j_{(n)}\right], \end{aligned} \tag{2}$$

in which the following function results from integrating out the individual shock, u_0, only:

$$p^j(Z_0, T^j, \beta) = E_{u_0}\left[\mathbf{1}\{m(Z_0, u_0, \beta) \geq T^j \mid Z_0, T^j\right]. \tag{3}$$

Note that the only influence of $U_{(n)}$ is through thresholds which are sufficient statistics. They do not depend on the determinants of student preferences, X_0 and X, except through revealed school choices and they depend on $Z^j_{(n)}$ only through T^j that are computed below. We use the exclusion of Xs below for identification.

Denote $D_0(X_0, \varepsilon_0, \zeta, \{P_0^j\}_{j\in\mathcal{J}}) \in \mathcal{J}$ the best response of applicant 0 resulting from equation (1). Given that the sample is i.i.d and that 0 is an arbitrary representative element of the sample, we can by substitution construct the samples of applicants to school j by using:

$$Z^j_{(n)} = \{i \in \{1, ., n\}; D_i(X_i, \varepsilon_i, \zeta, \{P_i^k\}_{k\in\mathcal{J}}) = j\}.$$

It is thus clear that the application mapping $Z_{(n)}$ into $Z^j_{(n)}$ is measurable although it remains to be shown that the application mapping $Z_{(n)}$ into thresholds $(T^j)_{j\in\mathcal{J}}$ is measurable. That is what we do now.

2.2.4 The determination of the thresholds

We can now return to the determination of thresholds $(T^j)_{j\in\mathcal{J}}$, considered by agent 0. For any realization of $U_{(n)}$, the J Nash equilibrium conditions yield a realization of the thresholds, $\{t^j\}_{j\in\mathcal{J}}$,

[5] All expectations exist since integrands are measurable and bounded.

as:

$$\sum_{i=1}^{n}[\mathbf{1}\{D_i = j\}\mathbf{1}\{m(Z_i, u_i, \beta) \geq t^j] = n^j, \tag{4}$$

As usual with empirical quantiles, this system has many solutions, t^j. We retain the solution corresponding to the grades of the less well ranked applicant in each school and because ties are absent with probability one, this solution is unique and a measurable function of $Z_{(n)}$ and $U_{(n)}$. This defines the random thresholds, $\{T^j\}_{j\in\mathcal{J}}$.

Equations (1) and (4) are necessary conditions for a Nash equilibrium.[6] A sketch of proof of the existence of a Nash equilibrium is spelt out in Appendix A and builds upon tools developed for potential games with weak strategic substitutes (Dubey, Haimanko and Zapechelnyuk, 2006).

2.3 Identification of Success Probabilities and Preferences

We now study the identification of success probabilities and preferences.

2.3.1 Success probabilities

Success probabilities are expressed, using equation (2), as a function of known variables – characteristics $Z_0, Z_{(n)}$ and decisions $D_{(n)}$– and unknown variables – parameter β, the distribution of grade shocks, u, and the distribution of thresholds $\{T^j\}_{j\in\mathcal{J}}$. Firstly, the grade equation, $m = m(Z, u; \beta)$, identifies parameter β and the distribution of u. Plugging these objects into the Nash conditions (4), given $Z_{(n)}$ and $D_{(n)}$ and computing thresholds identifies the distribution of $\{T^j\}_{j\in\mathcal{J}}$. In consequence, success probabilities are identified. In the following, we denote them, $\{P^j(Z)\}_{j\in\mathcal{J}}$.

In general, the identification of $P^j(Z)$ depends on the context which may be less simple than the one we used here. Yet, it is likely in general that exogenous variation in these probabilities could be given by various measures of ability, not only of an aggregate type as here, but also by field-specific grades. In Section 3, we return to the identification of success probabilities in our empirical application.

[6]Because the number of applicants is very large with respect to the total capacity, we neglect the occurrence that seats remain unmatched. In other words, we assume that the probability that one subsample j contains less than n^j students is zero or negligible. At the University under consideration, the average rate of success is between 5% and 20% (Table S.i, Supplementary Appendix).

2.3.2 Choice-based sample and outside options

We adopt a general random utility set-up in which values are continuously distributed (Assumption S.i). By the probability integral transform, we can thus always adopt the representation in which each function V^j is monotonic in one unobservable, denoted ε^j, whose marginal distribution is uniform on $[0,1]$.

$$\forall j \in \mathcal{J}, V^j = V^j(X, \varepsilon^j) < \infty.$$

Dependence between ε_js is left unrestricted and is described by any continuous copula.

There are two issues of concern for identification that distinguishes this proof from Agarwal and Somaini (2018). The first one regards choice-based sampling since our sample comprises students interested by at least one school, so that we condition the analysis on the event that $\max_{j\in\mathcal{J}}(V^j) > 0$. Second, we have to consider that only some schools could have positive value for students and we have to condition the analysis on unobservable latent sets $\mathcal{J}_+ \subset \mathcal{J}$ of schools that provide positive utility. Namely, other schools, in the complement of $\mathcal{J}_+$ in $\mathcal{J}$, $\mathcal{J}_+^c = \mathcal{J}/\mathcal{J}_+$, are strongly dominated by the outside option with probability one.

The finite set $\mathcal{J}_+$ whose number of elements is greater or equal to one because of choice based sampling, is a random set whose distribution is induced by the distribution of random values, V^j:

$$Q(\mathcal{J}_+ \mid X) = \Pr(\forall j \in \mathcal{J}_+, V_j > 0; \forall j \in \mathcal{J}_+^c, V_j \leq 0 \mid \max_{j\in\mathcal{J}}(V^j) > 0, X).$$

Let us first derive the optimal school choice conditional on $\mathcal{J}_+$ and integrate out $\mathcal{J}_+$ in a second step. If set $\mathcal{J}_+$ is a singleton, student's choice is its single element and success probabilities do not matter. If set $\mathcal{J}_+$ has two or more elements, success probabilities affect choices through the relative values of P^jV^j (equation (1)). Students may disguise their true preferences and act strategically. These relative values are positive because set $\mathcal{J}_+$ is defined as such and because $P^j > 0$ by Assumption *S.iv.* We can rewrite the decision model when $j \in \mathcal{J}_+$ by taking the logarithm of equation (1):

$$D = j \quad \text{if } \log(P^j) + \log(V^j) > \max_{k\in\mathcal{J}_+/\{j\}}(\log(P^k) + \log(V^k)), \tag{5}$$

in which we kept the dependence of V^k on X, ε^k and of P^j on Z implicit and in which ties are of probability zero because of Assumption *S.i.* Denote $\Delta^{jk}(Z) = \log(P^j(Z)) - \log(P^k(Z))$ in the

following and express choice probabilities, by integrating out sets $\mathcal{J}_+$, as:

$$\Pr(D = j \mid Z, X) = \sum_{\mathcal{J}_+; \mathcal{J}_+ \supset \{j\}} Q(\mathcal{J}_+ \mid X) \Pr(\forall k \in \mathcal{J}_+, \Delta^{jk}(Z) > \log(V^k) - \log(V^j) \mid X, Z, \mathcal{J}_+, \mathcal{J}_+^c). \tag{6}$$

It is useful to consider the two-school example to understand the sequence of proofs below.

The two-school example When the choice set is reduced to two elements, $\mathcal{J} = \{S, F\}$ as in our empirical application,[7] Figure 1 exhibits how we solve the decision problem in each of four quadrants.

First, the south-west quadrant is composed of individuals who are excluded from the choice-based sample and its probability measure is not identified. Second, in the north-west quadrant, $V^S > 0$ and $V^F \leq 0$, $\mathcal{J}_+ = \{S\}$ and school S is necessarily chosen. The probability measure of this quadrant is

$$\delta^S(X) = Q(\{S\} \mid X) = \Pr\{V^S > 0,\, V^F \leq 0 \mid \max(V^S, V^F) > 0, X\}$$

Similarly, in the south-east quadrant, $V^F > 0$ and $V^S \leq 0$, $\mathcal{J}_+ = \{F\}$ and school F is necessarily chosen. Its probability measure is $\delta^F(X) = Q(\{F\} \mid X)$. In both regions, students reveal their true preferences and do not act strategically. Note that identification is ordinal only in these two quadrants.

This is different in the north-east quadrant since choices can change if success probabilities P^S and P^F change. Specifically, school S is chosen if and only if

$$\log(P^S) + \log(V^S) > \log(P^F) + \log(V^F).$$

Denoting $\delta^{SF}(X)$ as the probability of the north-east quadrant, the choice probability regarding the first school is derived from equation (6):

$$\Pr(D = S \mid X, Z) = \delta^S(X) + \delta^{SF}(X) \Pr\{\log(P^S) - \log(P^F) > \log(V^F) - \log(V^S) \mid X, Z\}. \tag{7}$$

Returning to the general equation (6), we now study the identification of the two following structural objects; first, the probability measure of each quadrant, $Q(\mathcal{J}_+ \mid X)$; second, the joint distribution of log-value differences, $\log(V^j) - \log(V^k)$, in each quadrant $\mathcal{J}_+$.

[7] Our empirical application deals with two schools in two cities, Fortaleza and Sobral, and we use their initials, F and S, to make easier the recollection of which school we are talking about.

2.3.3 Identification of preferences

As is well known in discrete models since Manski (1988) and Matzkin (1993), a necessary condition for identification is the full variation of some regressors, conditionally on others. Those regressors are here the success probabilities:

Assumption CV (Complete Variation): Almost everywhere (a.e.) P_X, the support of $P(Z) = (P^j(Z))_{j\in\mathcal{J}}$, conditional on X, is the set $(0,1)^J$.

Assumption CV requires first that the set of covariates Z is at least of dimension J and that their variation induces that the support of success probabilities is the full unit hypercube. Success probabilities $P(Z)$ act as prices (Azevedo and Leshno, 2016) and the effects of preference shifters cannot be identified from success probabilities absent exclusion restrictions. This is why this assumption requires that a sufficient number of grade shifters, Z, should be excluded from the list of preference shifters, X. This is akin to the exclusion of school priorities from preferences in Agarwal and Somaini (2018).

We use equation (6) and make success probabilities vary in the unit hypercube. We adopt a two-step strategy. First, we show that the probability measures of quadrants, $Q(\mathcal{J}_+ \mid X)$, are identified.

Proposition 1 *Under Assumption CV, for any non-empty $\mathcal{J}_+ \subset \mathcal{J}$, $Q(\mathcal{J}_+ \mid X)$ is identified.*

Proof. See Appendix B.1 ■

The intuition for this proof is better gained by using again the two-school example. The structural probabilities of each quadrant in Figure 1 are:

$$\{\delta^S(X), \delta^{SF}(X), \delta^F(X)\},$$

and these appear in equation (7). By Assumption CV, the support of $\Delta^{SF}(Z) = \log(P^S) - \log(P^F)$ is the full real line and we can identify δ^S by using the limit of equation (7):

$$\delta^S(X) = \lim_{\Delta^{SF}(Z)\to-\infty} \Pr(D = S \mid \Delta^{SF}(Z), X).$$

Interchanging S and F identifies δ^F and $\delta^{SF}(X) = 1 - (\delta^S(X) + \delta^F(X))$.

Returning to the general case, we now prove identification of the distribution function of log-value differences, $\log(V^j) - \log(V^k)$, in each quadrant $\mathcal{J}_+$.

Proposition 2 *Under Assumptions CV and* $\forall \mathcal{J}_+ \subset \mathcal{J}$, $Q(\mathcal{J}_+ \mid X) > 0$ *a.e.* P_X, *the joint distribution of* $\Pr((\log(V^k) - \log(V^j))_{k \in \mathcal{J}_+/\{j\}}, \mid X, \mathcal{J}_+, \mathcal{J}_+^c)$ *is identified a.e.* P_X, *for any* $\mathcal{J}_+ \subset \mathcal{J}$, *and fixing any specific* $j \in \mathcal{J}_+$ *as the "reference" alternative.*

Proof. See Appendix B.2. ■

We added the condition that $Q(\mathcal{J}_+ \mid X) > 0$ for simplicity. In the case, $Q(\mathcal{J}_+ \mid X) = 0$, preferences cannot be identified in set $\mathcal{J}_+$ but it has no importance.

The proof of the proposition is by induction over the total number of schools and we thus deal with the two-school example again to provide the intuition.

A two-school example (ct'd) From equation (7) and assuming $Q(\{S, F\} \mid X) = \delta^{SF}(X) > 0$, we can form the expression that:

$$\frac{\Pr(D = S \mid \Delta^{SF}(Z), X) - \delta^S(X)}{\delta^{SF}(X)} = \Pr(\Delta^{SF}(Z) > \log V^F - \log V^S \mid X, Z) \tag{8}$$

All terms on the left-hand side are identified and standard arguments (Matzkin, 1993) show that the distribution of $\log V^S - \log V^F$ conditional on X is identified under the condition that the support of $\Delta^{SF}(Z)$, conditional on X, is the full real line.

Returning to the main argument, it is to be emphasized that Proposition 2 states identification of a joint distribution of preferences within a quadrant. This implies that Propositions 1 and 2 have the corollary that counterfactuals, investigating alternative mechanisms, are identified and this is what we use in the empirical application. Expected utilities of a rank-ordered list of any length are derived from the success probabilities and the joint distribution of differences of values, in each set J_+ of alternatives with positive values. Generally speaking, what matters is that expected utilities are bilinear functions of the underlying values, V^k, and of the success probabilities (equation (7), Agarwal and Somaini, 2018). The corollary thus applies to all mechanisms described by Definitions 1-3 of Agarwal and Somaini (2018).

We finish by a set of remarks about extensions.

Remark 1 Most importantly, this identification proof is obtained under the restrictive condition that one school only is chosen by students. When a more informative rank-ordered list comprising several schools can be submitted by students, identified objects in Propositions 1 and

2 remain the same.[8] We proceed by providing a counter-example of a result that would state that other objects can be identified in the 2-school case.

Recall that in the 2-school case, observing rank-ordered lists of length one identifies the probability of school, say S, to be positively valued and of school, say F, of being negatively valued, as a limit result by varying success probabilities in such a way that school, F, always dominates S if both are positively valued (Proposition 1). If we can now observe rank-ordered lists of length 2, the same probability can be identified by the probability of observing a rank-ordered list which ranks S first and the empty set second.

Using length-2 rank-ordered lists, we cannot identify, however, more than this probability in the quadrant in which $V_S > 0$ and $V_F \leq 0$ and this is true as well in the other quadrant $V_S \leq 0$ and $V_F > 0$. In the quadrant $V_S > 0$ and $V_F > 0$ in which the log differences of values are identified (Proposition 2), this holds true as well. Admittedly, success probabilities change if we change the length of the rank-ordered lists but their identification still relies on using the continuous variation of grades and the cutoffs are still determined by equations similar to equation (4). In conclusion, observing longer rank-ordered lists leads to overidentification that could help increasing the precision of preference estimates although this issue is out of the scope of this paper.

Remark 2 Differences of log-values are non parametrically identified but levels are not identified. In Section 4, the evaluation of counterfactual welfare is achieved by completing identifying conditions with additional assumptions.

Remark 3 We could further adopt a linear median restriction for differences between logarithms of values such as, in the two-school example

$$\log V^S - \log V^F = X\gamma + \varepsilon$$

in which the distribution of ε, $F(. \mid X)$ is restricted as:

$$F(0 \mid X) = \frac{1}{2}. \tag{9}$$

Parameter γ and $F(\varepsilon \mid X)$ are identified.

[8] This result requires that students never rank negatively-valued schools in their rank-ordered lists because, for instance, they face an infinitesimal cost of refusing an offer (that they could receive from a negatively-valued school if they rank it).

Remark 4 It is possible to weaken Assumption CV and admit that the support of the conditional distribution of $\Delta^{jk}(Z)$ conditional on X might not be the full real line. If we keep the two-school example to make the point in a simple setting, assume for convenience that the support of Δ^{SF} for any value taken by X includes the value 0. Then as developed in Manski (1988), identification becomes partial under the median restriction (9) written above. Parameter γ is identified using the median restriction and $F(. \mid X)$ is identified in the restricted support in which $\Delta(Z) + X\gamma$ varies. Our data exhibit limited variation and this is why we adopt, in the empirical application, a parametric assumption for $F(. \mid X)$. What non parametric identification arguments above prove is that this parametric assumption is a testable assumption at least in the support in which $\Delta^{SF}(Z) + X\gamma$ varies.

Remark 5 We can now briefly return to the issue of superior information that students could have with respect to econometricians. To discuss this point, suppose that each student receives a signal, before choosing the school to apply to, about her ability, say σ_i, and which is correlated with exam grades. If we keep the complete information structure, signals are fully observed by agents. Using the same model of belief about success in each school but conditioning now on the vector of signals σ, agents use success probabilities that can be written as $\pi(Z, \sigma)$ instead of $P(Z)$. Because of the law of iterated expectations, we have that $E(\pi(Z, \sigma)|Z) = P(Z)$. Denote $W = \pi(Z, \sigma)/P(Z)$ the positive random variable standing for superior information and which is mean independent of Z by construction.[9] This is not, however, a sufficient condition to recover log-value differences and it shall be additionally assumed that W and X, Z are independent to prove that log value differences are identified up to an additive independent "measurement" error term. A common prior assumption for agents and econometricians alike is thus a strong assumption but absent any other observed decision variable that might help recover or proxy σ (see Campbell, 1987, for instance), dealing with the general case seems out of reach.

3 The empirical application

We begin with describing our empirical application and with adapting the general model described in the previous section to the particulars that Universidade Federal do Ceará (UFC from here on

[9] We take the ratio between those probabilities because the decision model in set $\mathcal{J}_+$ is written in logarithms.

out) in Northeastern Brazil used to select students in 2004. We then turn to the computation of success probabilities and give a summary of our empirical strategy. We finish by reviewing our estimation results.

We restrict, for various reasons, the empirical application to two medical schools only. They are respectively located in Sobral (denoted S), the second most populated city in the state of Ceará and Fortaleza (denoted F), the state capital. First, the content of second-stage exams differs if schools are in different fields (medicine or law for instance) and this would introduce substantial heterogeneity between schools. Second, it enables us to choose the best schools in the University for which our assumptions on information and outside options are the most likely to be satisfied. Third, the more schools are analyzed, the stronger the requirement of complete variation of success probabilities (Assumption CV) for identification is.

We chose the two best medical schools because (1) they are the schools which attract the best students among all candidates within UFC (see Table S.ii in the Supplementary Appendix) (2) on prior grounds, the best substitutes are a slightly lower quality medical school in the country side (Barbalha) or pharmacy and related fields with a lower standing in terms of cut-off grades (3) other schools of excellence are schools of law which are presumably bad substitutes. As a matter of fact, the best substitutes are outside the University: a state university and three private medical colleges in Fortaleza and Sobral; outside the state, medical schools in Recife at the closest or in Sao Paulo or Campinas further away. All substitutes are dealt within the model as an aggregate outside option.[10]

We focus on medical schools also because of their attractiveness for the best students. Almost no students desist between the two stage exams if they pass the first stage. Being accepted in those schools is extremely valuable and the care and attention of students, parents and teachers are certainly at their highest for those two schools. The school in Sobral is small and offers 40 positions only while Fortaleza is much larger since it offers 150 seats. As shown in the empirical analysis below, this asymmetry turns out to be key for evincing strategic effects.

[10] See Sections S.1 and S.2 in the Supplementary Appendix which justify these arguments and complement the empirical analysis presented here.

3.1 Timing and the two-stage exams of the *Vestibular*

The timing of the real mechanism is enriched in two ways with respect to the stylized setting that we described in Section 2.

First, students take a standardized national exam, known as $ENEM$ and measuring students' ability in different subjects (maths etc) before college choices are made and about one year before *Vestibular* exams begin. ENEM results are used by the University when computing the passing thresholds at the *Vestibular* exams. It is also a very convenient measure of ability that all students know when they choose their preferred school.

Second, exams are taken in two stages. The first stage exam is identical across schools and denoted as

$$m_1 = m_1(Z, u_1; \beta_1)$$

in which u_1 are random individual circumstances that affect results at this exam. After this first exam, students are ranked according to a weighted combination of grades $ENEM$ and m_1. Those weights are common knowledge *ex-ante* and measure the relative interest of schools in selecting students using the national and the local exam grades. The thresholds of success at the first-stage exam are given by the rule that the number of available slots is equal to 4 times the number of final seats offered by the school. Given that schools have respectively 40 and 150 seats, the number of students passing the first stage is 160 and 600 out of a total of 542 and 2,325 candidates.

We write the selection rule after the first exam as:

$$m_1 \geq t_1^j(ENEM) = \tau_1^j - a_1 ENEM,$$

in which τ_1^j is determined by the number of candidates and positions available in the school. Threshold t_1^j depends on $ENEM$ because students are ranked according to a weighted sum of m_1 and $ENEM$ whose weights are $(1, a_1)$ but we make this dependence implicit in the following.

Students who do not pass the first exam get their outside option $D = \varnothing$, with utility, $V_\varnothing$. Other students take the second-stage exam and get a second-stage grade, denoted m_2:

$$m_2 = m_2(Z, u_2; \beta_2)$$

where u_2 is an error term whose interpretation is similar to u_1 and u_2 is possibly correlated with u_1. These students are ranked according to a known weighted linear aggregator of $ENEM, m_1$ and

m_2, and this again stands for the relative importance given to each of these dimensions by schools. Students are accepted in the order of their ranks until completion of the positions available for each school. As before, we write the selection rule as:

$$m_2 \geq t_2^j(ENEM, m_1),$$

as a function of a second threshold which also depends on previous exam grades since a linear aggregator is used to rank students. Students who fail the second-stage exam get the same outside utility as students who fail the first-stage exam.

We can then extend the definition of the probability of success in school D to:

$$P^D = \Pr(m_1(Z, u_1; \beta_1) \geq T_1^D(ENEM), m_2(Z, u_2; \beta_2) \geq T_2^D(ENEM, m_1)).$$

3.2 Identification of grade equations and success probabilities

Only students who pass the first-stage exam can write the second-stage exam. Therefore in our data, the second-stage grades, m_2, are censored when first-stage grades, m_1, are not large enough i.e. $m_1 < T_1^j$ and in the absence of any restriction, the distribution of m_2 is not identified.

3.2.1 A control function approach

To proceed we shall specify that $(m_1(Z, u_1; \beta_1), m_2(Z, u_2; \beta_2))$ are linear indices of covariates with respective parameters β_1 and β_2. The estimation of β_1 proceeds under the restriction that $E(u_1|Z) = 0$. In the second-stage grade equation we use a control function approach to describe the influence of the unobservable factor derived from the first grade equation (Blundell and Powell, 2003). We assume that:

$$u_2 = g(u_1) + u_2^*$$

in which u_2^* is mean independent of u_1, $E(u_2^* \mid u_1, Z) = 0$.

By doing this, we are now also able to control the selection bias since u_2^* is supposed to be mean independent of u_1 and therefore $E(u_2^* \mid m_1 \geq T_1^j, Z) = 0$. This would identify parameters and the control function $g(.)$. Nonetheless, our goal is not only to estimate these parameters but also to estimate the joint distribution of (u_1, u_2). This is why in the following we assume that u_1 and u_2^* are independent of each other and of variables Z and simply use the estimated empirical distributions of u_1 and u_2 when estimating success probabilities.

3.2.2 Simulated success probabilities

To predict success probabilities, two important elements are needed: the joint distribution of random terms u_1 and u_2 and the admission thresholds for the first and second-stage grades. We already stated assumptions under which we can recover the former. The latter are derived from the definition of the final admission in each school as described by two inequalities as functions of linear combinations of initial grades and first and second-stage grades fixed by the University:

$$\begin{aligned} & m_1 + 120 * ENEM/63 \geq \tau_1^j, \\ & 0.4 * (m_1 + 120 * ENEM/63) + 0.6 * m_2 \geq \tau_2^j. \end{aligned} \tag{10}$$

Thresholds (τ_1^j, τ_2^j) are taken here as any possible realization and we construct equation (3) from the distribution of random grade shocks. Integrating out thresholds T_1^j and T_2^j comes in a second step.

Conditional success probabilities We first transcribe the inequalities (10) as functions of unobserved heterogeneity terms u_1 and u_2. For every student, passing the two exams means that the two random terms in the grade equations should be large enough as described by:

$$\begin{aligned} u_1 &\geq \tau_1^j - 120 * ENEM/63 - Z\beta_1, \\ u_2^* &\geq \frac{\tau_2^j}{0.6} - \frac{2}{3}(Z\beta_1 + u_1 + 120 * ENEM/63) - Z\beta_2 - g(u_1). \end{aligned}$$

Notice that the second inequality depends on first-stage grade shocks, u_1, because of the correlation between grades. Therefore the success probability in a school j, as defined by a function of thresholds in equation (3), can be expressed as:

$$\begin{aligned} p^j(Z, \beta, \tau_1^j, \tau_2^j) &= Pr\{u_1 \geq m_1^j - Z\beta_1, u_2^* \geq m_2^j - \frac{2}{3}Z\beta_1 - Z\beta_2 - \frac{2}{3}u_1 - g(u_1)\}, \\ &= \int_{m_1^j - Z\beta_1}^{\infty} f_{u_1}(x)(Pr\{u_2^* \geq m_2^j - \frac{2}{3}Z\beta_1 - Z\beta_2 - \frac{2}{3}x - g(x)\})dx, \\ &= \int_{m_1^j - Z\beta_1}^{\infty} f_{u_1}(x)[1 - F_{u_2^*}(m_2^j - \frac{2}{3}Z\beta_1 - Z\beta_2 - \frac{2}{3}x - g(x))]dx, \end{aligned} \tag{11}$$

in which m_1^j and m_2^j are functions of thresholds:

$$\begin{cases} m_1^j = & \tau_1^j - 120 * ENEM/63, \\ m_2^j = & \frac{\tau_2^j}{0.6} - \frac{2}{3}(120 * ENEM/63). \end{cases}$$

Unconditional success probabilities As those are derived from an expectation taken over thresholds T in equation (2), we use a simulated sample analog and compute the distribution function of T at an arbitrary level of precision using equilibrium conditions (4)[11] by simulation of $U_{(n)}$. By construction, T depends on observation 0 and thus its distribution has to be computed for every single observation. For simplicity and because this dependence matters less and less when n grows, we compute those thresholds in the empirical application using equation (4) in which the sums are taken over the full sample $i = 0, 1, ., n$ and success probabilities are estimated only once instead of $n + 1$ leave-one-out estimates.

3.3 Empirical strategy: Summary

We first estimate parameters of the grade equations and denote them $\hat{\beta}_n$. This, in turn, allows us to compute the expectation of the success probabilities conditional on thresholds $\tau_k^j, k = 1, 2, j = S, F$ as in equation (11) using the estimated distribution functions for errors in the grade equations. We then compute unconditional success probabilities by integrating out by simulation conditional success probabilities as in equation (2). Namely, for any simulation $c = 1, ., C$, draw in the distribution of $U^{(n)}$ and derive realizations of T, say t_c in the C samples of size n by fixing choices $\mathbf{1}\{D_i(Z_i, \varepsilon_i, \zeta, P_i^S, P_i^F) = S\}$, characteristics X_i and by solving the equilibrium conditions (4). Equation (2) can then be computed by integration as:

$$\hat{P}_{0,C}^j = \frac{1}{C} \sum_{c=1}^{C} p^j(Z_0, \hat{\beta}_n, t_{1,c}^j, t_{2,c}^j). \tag{12}$$

Preferences are described by the probabilities of each quadrant in Figure 1, $\{\delta^S(X), \delta^{SF}(X), \delta^F(X)\}$ and by the following parametric specification of log-value differences:

$$\log V^S - \log V^F = X\gamma + \varepsilon, \varepsilon \sim N(0, 1).$$

Preference parameters $\zeta = (\delta, \gamma)$ are estimated using a conditional maximum likelihood approach:

$$\hat{\zeta}_n = \arg\max_{\zeta} \quad l(\zeta | \hat{P}_{0,C}^S, \hat{P}_{0,C}^F).$$

This is a conditional likelihood function since $\hat{P}_{0,C}^S, \hat{P}_{0,C}^F$ depend on the first-step estimate, $\hat{\beta}_n$. Standard asymptotic arguments yield:

$$\hat{\zeta}_n \xrightarrow[n\to\infty]{P} \zeta.$$

[11] Generalizing them to the two-stage exam setting is straightforward, see equation (13) below.

We used bootstrap to obtain the covariance matrix of those estimates by replicating the complete estimation procedure as a mixture of non parametric (grade equations) and parametric bootstrap (choice equations).

3.4 A Brief Description of Estimation Results

The list of variables and descriptive statistics in the pool of applicants to the two schools we consider appear in Table 1. Looking at admission rates, one can see that Sobral admitted $40/527 = 7.6\%$ and Fortaleza $150/2340 = 6.4\%$ and this makes Fortaleza more competitive. Comparing the mean and median of initial and first-stage grades, Sobral has nonetheless better applications than Fortaleza. As to the second-stage grades, the group selected for Sobral has a slightly higher median than the one selected for Fortaleza although both groups have the same mean.

Because empirical results in this article are focussed on counterfactuals, estimates from our empirical analysis are shown and analyzed in Section S.2 in the Supplementary Appendix. We fully report and comment therein estimates of grade equations, predictions of success probabilities and estimates of preference parameters. We now discuss only briefly our most important modelling choices and our main results.

As described in Table 1, explanatory variables are those that affect exam performance or school preferences. For grade equations, all potential explanatory variables are included: a proxy for ability which is the initial grade m_0 obtained at the national exam (ENEM), age, gender, educational history, repetitions, parents' education and the undertaking of a preparatory course. Our guide for selecting variables is that a better fit of grade equations leads to a better prediction of success probabilities in the further steps of our empirical strategy.

Second, as developed in Section 2.3.3, one exclusion restriction at least is needed to identify preferences. We chose to exclude from preference shifters all variables related to past educational history. Indeed, preferences are related to the forward looking value of the schools (e.g. wages) which, conditional on the proxy for ability, is unlikely to depend on the precise educational history of the student (e.g. private/public sector history and undertaking a preparatory course). This is even more likely since we condition on ability m_0 which is assessed in the ENEM after educational history. This dynamic exclusion restriction is akin to what is assumed in panel data and posits that m_0 is a sufficient statistic for educational history. As a consequence, preferences are specified

as a function of ability, gender, age, education levels of father and mother, and the number of repetitions of the entry exam. The inclusion of gender, age and education of parents is standard in this literature. The number of repetitions reveals either the determination of a student through her strong preference for the schools or the lack of good outside options. We performed a thorough specification search and tested for overidentifying restrictions.

Third, the second-stage exam has a different format (writing essays) than the first-stage multiple choice exam and the second-stage grade equation has a much lower R^2. An interesting economic interpretation is that the first-stage exam is designed to skim out the weaker students and this multiple question exam is quite predictable (large R^2). In the second stage, the examiners can be selective in many more dimensions and try to pick out students using unobserved traits which are predictive of future behavior (success in the field of studies, drop out, etc) and that the econometrician cannot observe. This justifies the double stage nature of the exam as trying to minimize screening costs. We will return to this point below.

Fourth, Table 2 reports descriptive results on predicted probabilities of success. Means and medians of first-stage success probabilities are around 20-30% in both schools. This is close to what is observed in the sample but not exactly identical since these probabilities are partly counterfactual objects, for instance, success probabilities in Sobral for those who chose Fortaleza. The second-stage success probabilities are close to what is observed and as expected roughly four times lower than the first-stage ones.

Finally, students heavily favor Fortaleza over Sobral and this confirms that Fortaleza is the most popular medical school in the state. The ratio of those probabilities is 10 which is approximately the ratio between the populations of the two cities albeit much larger than the ratio of final seats in the two schools (150/40). Nonetheless, there is a substantial fraction of students whose utilities for both schools are positive (more than 40%).[12]

4 Evaluation of the Impact of Changes of Mechanisms

We now investigate the impact of various changes in the allocation and selection mechanisms that are discussed in academic and policy debates. To organize the presentation, some preliminary

[12] Full details and comments of our empirical analysis appear in Section S.2 of the Supplementary Appendix.

discussion of school preferences over the information obtained at the different exams is in order. We also return to the issue of substitutes.

School preferences are revealed by the type of exams and selection rules that are used for admission as described by equations (10). In the following, we will evaluate outcomes and welfare in each counterfactual by conditioning on the expected final scores used for admission.[13] The first-stage selection sums two multiple-choice exam scores – ENEM and first stage – in a roughly equivalent way.[14] Final selection however overweights the second-stage grade (.6) with respect to the compound ENEM & first-stage grade (.4). As the weight of the latter is not zero, first-stage and ENEM scores provide valuable information in addition to the selection rôle they are used for. Scores at the two stage exams presumably measure two different cognitive dimensions affecting the future career of students in the schools. Note however that there is no bottom grade requirement at the second-stage as there is at the first stage and the second stage cannot be considered as more informative even if its weight is larger.

Second, counterfactual analyses are conducted under the assumption that the choice-based sample remains the same and this implicitly means that the value of outside options does not change. We also assume that the change in pre-determined variables, for instance, the take-up of preparatory courses or the exit and entry flows of students applying to these two schools because of the change in the admission rules, is second order.

The first counterfactual experiment that we implement is to cut slots proposed at the second-stage exam by offering twice, instead of four times, the number of final seats. It is likely that a two-stage exam is used because schools want to avoid congestion and the tuning between the two stages is key. The first-stage is very easy to grade since machines can mark multiple-choice exams very quickly. The second stage is much deeper since it relies on open-ended questions and is more costly to grade. The trade off is therefore to balance these substantial screening costs with the depth of the first-stage selection that might select out good students because of the format (see also He and Magnac, 2018). There are other examples of this in other countries: in top engineering schools in France, selection is distinguished in an admissibility (written) and an admission stage (oral).

[13] As expected, success probabilities are an increasing function of this score. More interestingly, the ratio between success probabilities at Sobral relative to Fortaleza is also increasing with this score except at the very top.

[14] In equation (10), the coefficient in front of ENEM is a rescaling term that equalizes ranges of m_1 and ENEM.

Second, we experiment with enlarging the choice set of students before taking exams. They would list two ordered choices instead of a single one so as to get closer to a Gale-Shapley mechanism. This means that even if students fail the first-stage qualification in one of the two schools they may still get into the second-stage exam for the other school. This implies that the average skill level of passing students increases and that the difference between the two schools is attenuated.

Third, since having two stages in the exam allows schools to cut costs and achieve a more in-depth selection at the second stage, another experiment consists in changing the timing of choice. In the third counterfactual experiment, students would choose their final school after taking the first-exam and learning their grades. The experiment is different from the previous two since students have more information on their success probabilities when they choose. It generates however additional organization costs and delays due to the serial dictatorship mechanism that it induces after the first stage. It is also likely to generate more opportunistic behavior.

Before entering into the details of these counterfactual mechanisms, the identification of utilities from estimated preferences and success probabilities is key in these evaluations. We show that expected utilities are underidentified and we suggest how plausible bounds for counterfactual estimates can be constructed. We also explain how to compute counterfactual estimates conditional on observed choices.

4.1 Identifying Counterfactual Expected Utilities

Taking expectations with respect to grades using success probabilities P_i^S, P_i^F of ex-post utility levels, U_i, leads to:

$$\begin{aligned} E\left(U_i \mid V_i^S, V_i^F\right) &= \mathbf{1}\{V_i^S \geq 0, V_i^F < 0\} P_i^S V_i^S + \mathbf{1}\{V_i^F \geq 0, V_i^S < 0\} P_i^F V_i^F \\ &+ \quad \mathbf{1}\{V_i^F \geq 0, V_i^S \geq 0\} \left[\mathbf{1}\{D_i = S\} P_i^S V_i^S + \mathbf{1}\{D_i = F\} P_i^F V_i^F\right] \\ &= P_i^S V_i^S \left(\mathbf{1}\{V_i^S \geq 0, V_i^F < 0\} + \mathbf{1}\{V_i^F \geq 0, V_i^S \geq 0\} \mathbf{1}\{D_i = S\}\right) \\ &\quad + P_i^F V_i^F \left(\mathbf{1}\{V_i^F \geq 0, V_i^S < 0\} + \mathbf{1}\{V_i^F \geq 0, V_i^S \geq 0\} \mathbf{1}\{D_i = F\}\right). \end{aligned}$$

Even if the location parameter is fixed by the outside option, this expected utility can always be rescaled by any increasing function. This is why we choose the absolute value $\left|V_i^F\right|$ as the scale

factor to set:

$$\begin{aligned} V_i^F &= 1 \text{ if } V_i^F > 0, \\ V_i^F &= -1 \text{ if } V_i^F < 0. \end{aligned}$$

Under this normalization:

$$\begin{aligned} E\left(U_i \mid V_i^S, V_i^F\right) &= P_i^S \left(V_i^S \mathbf{1}\{V_i^S \geq 0, V_i^F < 0\} + \frac{V_i^S}{V_i^F} V_i^F \mathbf{1}\{V_i^F \geq 0, V_i^S \geq 0\} \mathbf{1}\{D_i = S\} \right) \\ &\quad + P_i^F V_i^F \left(\mathbf{1}\{V_i^F \geq 0, V_i^S < 0\} + \mathbf{1}\{V_i^F \geq 0, V_i^S \geq 0\} \mathbf{1}\{D_i = F\} \right), \\ &= P_i^S \left(V_i^S \mathbf{1}\{V_i^S \geq 0, V_i^F < 0\} + \frac{V_i^S}{V_i^F} \mathbf{1}\{V_i^F \geq 0, V_i^S \geq 0\} \mathbf{1}\{D_i = S\} \right) \\ &\quad + P_i^F \left(\mathbf{1}\{V_i^F \geq 0, V_i^S < 0\} + \mathbf{1}\{V_i^F \geq 0, V_i^S \geq 0\} \mathbf{1}\{D_i = F\} \right), \end{aligned}$$

the only unknown is V_i^S when $V_i^S \geq 0, V_i^F < 0$ since $\frac{V_i^S}{V_i^F}$ when $V_i^F \geq 0, V_i^S \geq 0$ is identified (see Section 2.3.3). This partial identification issue comes from the fact that ordinal preferences only are recovered in the case in which only one of the value function is positive and when both value functions are positive, relative cardinal utilities only can be identified.

Various assumptions are plausible. If there is some positive correlation between V_i^F and V_i^S, we would expect that

$$\begin{aligned} E\left(V_i^S \mid V_i^S \geq 0, V_i^F < 0\right) &< E\left(V_i^S \mid V_i^S \geq 0, V_i^F \geq 0\right) = E\left(\frac{V_i^S}{V_i^F} \mid V_i^S \geq 0, V_i^F \geq 0\right) \\ &< \exp(X_i\gamma) E(\exp(\varepsilon_i) \mid V_i^S \geq 0, V_i^F \geq 0) \\ &< \exp(X_i\gamma + .5), \end{aligned}$$

the last expression being obtained under normality of ε_i. This is why we assume that when $V_i^S > 0$:

$$\log V_i^S = \frac{\mu_0}{2} V_i^F + (\log \frac{V_i^S}{V_i^F} - \frac{\mu_0}{2}) \left|V_i^F\right| = \frac{\mu_0}{2} V_i^F + (X_i\gamma + \varepsilon_i - \frac{\mu_0}{2}) \left|V_i^F\right|$$

where $\mu_0 > 0$ captures the positive dependence between V_i^S and V_i^F. This is coherent with the previous equation since :

$$\begin{cases} V_i^S = \exp(X_i\gamma + \varepsilon_i) & \text{if } V_i^F = 1, \\ V_i^S = \exp(X_i\gamma + \varepsilon_i - \mu_0) & \text{if } V_i^F = -1. \end{cases}$$

We will thus evaluate $E\left(U_i \mid V_i^S, V_i^F\right)$ using bounds on $\mu = \exp(-\mu_0)$ that we make vary between 0 (the lower bound for V_i^S) and 1 (the case in which V^S and V^F are uncorrelated).

We use this measure of welfare in relative terms among students to evaluate the amount of redistribution between them of changes in the allocation mechanisms.[15]

4.2 Computing equilibria

In every counterfactual experiment, we draw unknown random terms conditional on observed choices for simulation purposes. This ensures that simulated choices are compatible with observed choices in the data. In each simulation, let $\bar{D}_i$ be the counterfactual choices of the students that depend on counterfactual expectations $\bar{P}_i^S$ and $\bar{P}_i^F$. Denote $\bar{n}_S = 2n_S$ and $\bar{n}_F = 2n_F$ the new number of seats in the cutting-seat counterfactual. In other cases $\bar{n}_S = 4n_S$ and $\bar{n}_F = 4n_F$ as in the original system.

Given that historical variables and outside option value do not change, the population of reference does not change in the counterfactual experiments since experiments affect success probabilities only. The pool of applicants remains the set of students whose utilities are such that $V^S > 0$ or $V^F > 0$ and therefore we consider the same sample $i = 0, ., n$. Consistency of choices and expectations require that the counterfactual random thresholds, $\tilde{T}_0$, as defined as the solution $(\tilde{t}_1^S, \tilde{t}_2^S, \tilde{t}_1^F, \tilde{t}_2^F)$ to the counterfactual counterpart of equation (4):

$$\begin{cases} \sum_{i=1}^{n} [\mathbf{1}\{\bar{D}_i(\bar{P}_i^S, \bar{P}_i^F) = S\}\mathbf{1}\{m_1(X_i, \beta, u_i) \geq \tilde{t}_1^S\}] = \bar{n}_S, \\ \sum_{i=1}^{n} [\mathbf{1}\{\bar{D}_i(\bar{P}_i^S, \bar{P}_i^F) = F\}\mathbf{1}\{m_1(X_i, \beta, u_i) \geq \tilde{t}_1^F\}] = \bar{n}_F, \\ \sum_{i=1}^{n} [\mathbf{1}\{\bar{D}_i(\bar{P}_i^S, \bar{P}_i^F) = S\}\mathbf{1}\{m_1(X_i, \beta, u_i) \geq \tilde{t}_1^S, m_2(X_i, \beta, u_i) \geq \tilde{t}_2^S\}] = n_S, \\ \sum_{i=1}^{n} [\mathbf{1}\{\bar{D}_i(\bar{P}_i^S, \bar{P}_i^F) = F\}\mathbf{1}\{m_1(X_i, \beta, u_i) \geq \tilde{t}_1^F, m_2(X_i, \beta, u_i) \geq \tilde{t}_2^F\}] = n_F, \end{cases} \tag{13}$$

have a distribution function that leads to the counterparts of equation (12):[16]

$$\bar{P}_0^j = \mathbb{E}(\mathbf{1}\{m_1(X_0, \beta, u_0) \geq \tilde{t}_1^j, m_2(X_0, \beta, u_0) \geq \tilde{t}_2^j\}) \tag{14}$$

We thus propose to iterate the following algorithm (we explain it for observation 0 and this extends to any index i:

[15] These welfare measures could be translated back into changes of odd ratios of expected success probabilities using the preference equation (5) but this does not add much to our evaluation.

[16] Changing the timing of choices requires to acknowledge that there are no choices to make before the first-stage. The first two equations in (13) do not depend on $\bar{D}_i$ and P_i^S, P_i^F are the conditional expectations after the second-stage. Those adaptations do not modify the main principles.

1. Initialization:

   - Draw $C = 499$ random preference shocks $\varepsilon_{(n),c}$ in their distributions conditional to observed choices, D_i, and using preference parameter estimates $\hat{\zeta}_n$. Fix those $\varepsilon_{(n),c}$ for the rest of the procedure (see Supplementary Appendix S.3.1.1 for details).
   - Draw C random vectors $U_{(n),c}$ and fix them for the rest of the procedure.
   - Set the initial $P_0^{S,0}, P_0^{F,0}$ values at their simulated values $\hat{P}_{0,C}^{j}$ derived from equation (12) in which we use $U_{(n),c}$ and the observed experiment to compute thresholds $t_{1,c}^{j}$ and $t_{2,c}^{j}$ using equations (4).

2. At step k, denote $P_i^{S,k}, P_i^{F,k}$ the expected success probabilities

   (a) Compute counterfactual choices $D_i(Z_i, \varepsilon_{i,c}, \hat{\zeta}_n, P_i^{S,k}, P_i^{F,k})$.

   (b) Compute a sequence of $\tilde{t}_c$ for $c = 1, ., C$ using $U_{(n),c}$ and equations (13).

   (c) Derive $\hat{P}_{0,C}^{j,k+1}$ from equation (14).

3. Repeat the previous step until a measure of distance $d(P^{(k+1)}, P^{(k)})$ is small enough.

If this algorithm converges then this is the fixed point we are looking for. We study in Appendix A a simplified model in which we show that a Nash equilibrium is obtained in a finite number of steps.

4.3 Cutting seats at the second-stage exam

We start with the easiest policy change that reduces admission rates to the second stage. As said, the existing *Vestibular* system allows the number of students who take the second exam to be four times the number of final seats. In the experiment, the number of available positions is kept unchanged but the number of admissions after the first-stage exam is now twice the number of seats. We explore the possible consequences of this policy and investigate two main issues – who among students benefit from this policy change and whether schools lose good students.

Some discussion about the expected effects are in order. Cutting seats in the second exam reduces schools' screening costs although this comes with the risk of losing talented students. Students may not be always consistent in their exam performance and even the most gifted may

have a strong negative shock in the first exam. Those students would be eliminated too early without being given a second chance. Nonetheless, it could also be that cutting seats protect the first-stage best achievers from competition and thus from the risk of losing ranks at the second-stage exam. A formal argument is as follows. Subpopulations defined by a specific final weighted score are now composed by more top-achievers at the first stage. The net result is however unclear theoretically because the distribution of the final weighted score changes itself and needs to be integrated out. This is why an empirical analysis is worthy of attention.

The simulation of the counterfactual and the computation of expected utility follow procedures described in Section 4.1 and Section 4.2.

4.3.1 Changes in thresholds

In Table 3 we present estimates of the new threshold distributions at both stage exams in the three counterfactual experiments. In the cutting seat experiment, the counterfactual first-stage thresholds are much higher than in the original experiment since fewer students are admitted after the first-stage exam. In contrast, the thresholds of the second-stage exam are slightly lower than in the original system because there is now less competition in the second-stage exam when half as many students are admitted. In both first- and second-stage exams, estimated thresholds in Sobral are more volatile than the ones in Fortaleza because Sobral is a much smaller school.

To evaluate how this counterfactual brings benefit to schools and students, we study in turn, changes in success probabilities and changes in students' utilities.

4.3.2 Changes in success probabilities

Schools would find that the admittance procedure has improved if abler students would get a higher chance of admission and the less gifted students would have a lower chance. This is why we evaluate changes in success probabilities in relation to an index of students' abilities. As our ability index, we use the expected final grade which is, as already said, a combination of the initial, first and second-stage grades. We also choose to focus on the top 50% of students because the lower 50% of the sample have almost no chance of getting admitted whether the original or counterfactual mechanisms are used.

We represent changes in success probabilities in Figure 2 for Sobral. Three vertical lines are

drawn at the median of expected final grade and at the quantiles associated to the first and second-stage thresholds *in the original system* (averaged across schools). Changes in probabilities are very similar in the two schools.[17] The dispersion of these changes, conditional on expected grade, is due to the heterogeneity of observed characteristics across students.

The very top students who are above the second-stage admission quantile, have better chances in the counterfactual system since they are likely to face less competition in the second-stage exam. Our estimates of grade equations show that second-stage grades have a much larger variance than first-stage grades. The risk of failing is thus lower when fewer students participate in the second-stage exam. In contrast, for students who are between the median and first-stage threshold in terms of expected final grades, this is the converse. They are much less often admitted after the first-stage exam and even if the second-stage exam is less competitive, it is the former negative effect that dominates overall. In particular, students who are around the first-stage threshold in the current system are more likely to be selected out at the first stage.

4.3.3 Changes in students' utilities and the impact on schools

Table 4 presents summaries of changes in students' expected utility. We construct groups according to various quantiles of the distribution of the expected final grade. The closer to the top of the distribution, the smaller the groups are (two percent of the population only). As defined in Section 4.1, we set the unknown weight in utilities at $\mu = 0.8$.[18]

Consistently with changes in success probabilities, top students have significant utility improvements although this is also true for lower ranked students (above the 80% quantile). Nonetheless, focussing on means of expected utility hide very large dispersions in the 80-90% quantiles. This is best seen in the distribution of changes in utility (Supplementary Appendix S.1, Figure S.v) in which students in the 8th and 9th deciles are the ones whose changes in utility is the most dispersed. Furthermore, students just above the median tend to have lower expected utility in the counterfactual system and this is consistent with what we obtained for success probabilities. If we divide the sample by the original school choice, an indication of their preference, students who

[17] The corresponding Figure for Fortaleza appears in Section S.3 of the Supplementary Appendix (Figure S.iv).

[18] We also performed robustness checks by using weights μ varying from 0 to 1 (see Section 4.1). Results are shown in Table S.viii in the Supplementary Appendix. Differences are very small and our results are quantitatively robust to the value of μ.

chose Fortaleza tend to benefit more than the ones who opted for Sobral. The influence of the second-stage exam seems to be much larger there than in Sobral. Overall, these results about this counterfactual experiment bring out a moderate total utilitarian welfare change. Yet, there are strong distributional effects and top students are better off and less able students are worse off.

The impact of cutting seats seems favorable for schools since the most able students now have a higher chance of admission since they are protected from the competition of less able students at the second stage. This benefit comes in addition to cutting the costs of organizing and correcting the second-stage exam proofs.

4.4 Enlarging the choice set

In this experiment, students can submit an enlarged list of two schools if they wish. A choice list contains two elements d_1 and d_2 in which d_1 is the preferred school. Since our sample of interest only comprises students who positively value at least one of the schools, we have $d_1 \in \{S, F\}$. Yet, students can now apply to a second choice and $d_2 \in \{\varnothing, S, F\}/\{d_1\}$ in which $d_2 = \varnothing$ is the outside option chosen by students who do not give positive value to the second school. This mechanism belongs to the deferred-acceptance family with the additional twist that we keep the sequence of two exams as it is. The allocation of students after the first exam needs however to be adapted and this is the design that we now explain.

4.4.1 Design of the experiment

To fix ideas, consider first a student who (1) has $V_S > 0$ and $V_F > 0$ (2) chooses the list (S, F). If after the first-exam, she is above the threshold for school S, her second choice does not matter.[19] It is only if she would NOT be accepted to the second-stage exam in school S that she could compete for the second-stage exam in school F.[20] She fails altogether when her grades are lower than both thresholds.

Consider first that at equilibrium $t_1^S > t_1^F$. After the first-stage exam, there are three possible outcomes for the student:

[19] In particular, we discard the possibility of choosing a second ranked school after a success at the first stage exam.

[20] See also the third experiment in which students choose according to the information they have on their performance at the first stage for a variation around these constraints.

- $m_1 \geq t_1^S$: she takes the second exam of school S,
- $m_1 < t_1^S$ and $m_1 \geq t_1^F$: she takes the second-stage exam of school F,
- $m_1 < t_1^F$: she fails and takes the outside option.

While if $t_1^S < t_1^F$ (the probability of a tie being equal to zero),

- $m_1 \geq t_1^S$: she takes the second exam of school S;
- $m_1 < t_1^S$: she fails and takes the outside option.

This sequence is easily adapted to students choosing the list (F, S). Moreover, for students submitting a list $(d_1, \varnothing)$, the sequence of actions is the same as in the original mechanism. Students are selected into the second-stage exam for school d_1 if their grade is above its first-stage threshold.

Furthermore, given any choice among the four lists, $\{(S, F), (F, S), (S, \varnothing), (F, \varnothing)\}$ we can construct counterfactual success probabilities in each school P^S and P^F by adapting the algorithm we used before (see Supplementary Appendix S.3.2). For any value of success probabilities, we can then compute the optimal choice between $\{(S, F), (F, S), (S, \varnothing), (F, \varnothing)\}$. Details about how we get counterfactual thresholds and choices follow the lines of what was developed in Section 4.2.

4.4.2 Changes in thresholds

The new thresholds for this counterfactual experiment are also shown in Table 3. For the first stage, the threshold of Sobral is now slightly larger than the original one while the threshold of Fortaleza remains roughly unchanged. This is an indication that Sobral is admitting better students while the effect on Fortaleza is negligible. Some of the students who were failing Fortaleza before can now compete for Sobral and get admitted after the first stage. Furthermore, some of the students who were choosing Sobral for strategic reasons in the original mechanism can now at no risk choose Fortaleza first and Sobral second. Deferred acceptance mechanisms lessen strategic motives and make choices more truthful (Abdulkadiroglu and Sonmez, 2003) although the move is not necessarily Pareto-improving (Balinski and Sonmez, 1999). In the original system, students tended to choose Sobral as a "safety school" even when they truly preferred Fortaleza since success probabilities were higher at the former school. Giving students two choices attenuates the "safety school" effect although it does not eliminate it completely because of the two-stage nature of the

exam. Yet, thresholds for the school in Fortaleza remains higher than for Sobral at both stages because it attracts more top-ability (m_0) students as is shown by preference estimates (see Table S.vii).

Large standard errors for counterfactual thresholds at the second-stage exam make differences with the current ones insignificant. Even if this counterfactual experiment moves some of the relatively good students after the first-stage exam from Fortaleza to Sobral, Sobral however still attract less able students than Fortaleza in the second stage as in the first stage.

4.4.3 Changes in success probabilities

Figure 3 reports changes in success probabilities for Sobral (see Figure S.vi for Fortaleza). Unlike the previous counterfactual experiment, the changes in Sobral and Fortaleza are now somewhat different. In Fortaleza, the change in success probabilities is negligible as thresholds are constant and the reallocation of choices from Sobral to Fortaleza not strong enough. In contrast, a fraction of students below the first admission threshold and above median has a lower success probability in Sobral in the counterfactual experiment. This is because better students who fail Fortaleza switch to Sobral to compete with them and lower ranked students are evicted since first-stage thresholds are now higher in Sobral. In other words, getting Sobral if failing Fortaleza is acting as an insurance device and students just above the first-stage threshold benefit from the existence of this insurance. Last, note that the change in success probabilities is small in this counterfactual compared with cutting seats since it affects students only through the allocation mechanism.

4.4.4 Changes in expected utilities and the impact on schools

From the student perspective, this mechanism is also attractive since a majority of students – 55% – will be (strictly) better off as shown in Table 5. Moreover, top students benefit more from the change than less able students because they are more likely to pass to the second-stage exam even if they happen to fail their preferred school. Deferred acceptance restricts less the possibilities of very top students since they can keep options open. In particular, students who preferred Sobral initially, benefit much more than those who preferred Fortaleza initially, seemingly because the pressure of competition at the top in Sobral is lower since it loses its safety school status. In contrast, since Sobral has a lower threshold at the first-stage exam, students

who prefer Sobral and are ranked around the first-stage threshold suffer from more competition from evicted students from Fortaleza. However, for those who preferred Fortaleza in the original system, expected utility mainly increases because of the second chance they get to compete for Sobral when they fail Fortaleza. The effect on expected utility is thus larger than the change in success probabilities.

In summary, enlarging the choice set improves the average ability of those who pass the first-stage exam in both schools. The majority of students are better off except students ranked around the first-stage threshold in the original system and who prefer the smallest school. From the perspective of the schools, Sobral should be more favorable to this mechanism since it can now attract higher ranked students. Fortaleza's thresholds remain the same although the composition of their recruitment might have changed since Sobral lost its safety school status. This seems however to moderately affect top students.

This confirms theoretical insights that the move to a deferred acceptance mechanism is likely to make both schools and more top students better off.

4.5 Changing the timing

In the last counterfactual experiment, we try to evaluate the impact on students when they choose schools **after** learning their first-stage exam grade and no longer before this exam. Schools continue to rank students according to the same combination of $ENEM$ and m_1.

The new selection procedure is a serial dictatorship mechanism which is Pareto-optimal in the case of a single exam (for instance, Abdulkadiroglu and Sonmez, 1998). It proceeds as follows. Starting from the first-ranked student and going down the ranking afterwards, each student chooses school S or F until the number of admitted students in one of the schools, say j, reaches four times the number of final seats in this school. This defines threshold t_1^j. The sequence continues going down the ranking although choice is now restricted to the other school $D \neq j$ or to opting out until the number of admitted students in that school reaches four times the number of final seats. The allocation of students to the second-stage exam is then complete. The game continues afterwards as in the current system.

As before, utilities V^S and V^F remain the same while this new mechanism affects the probabilities of success $P^S_{m_1} = Pr\{m_2 > t_2^S | m_1\}$ and $P^F_{m_1} = Pr\{m_2 > t_2^F | m_1\}$ which are now conditional

on the first-stage grade m_1. To define choices, suppose that $t_1^S > t_1^F$ which means in practice that Sobral seats are filled in faster than Fortaleza's. A student can face three cases:

- $m_1 > t_1^S$: the choice set is complete and consists in $\{S, F\}$. Schools are chosen by comparing $P_{m_1}^S V^S$ and $P_{m_1}^F V^F$ (since either $V^S > 0$ or $V^F > 0$).
- $m_1 < t_1^S$ and $m_1 \geq t_1^F$: the choice set is restricted to F and the student either opts for the second-stage exam in F if $V^F > 0$ or the outside option if not.
- $m_1 < t_1^F$: the only choice left is the outside option.

This algorithm is easily adapted to the case in which $t_1^S < t_1^F$ prevails. Additionally, we compute the same ex-ante expected utilities by integrating out shocks in m_1.

4.5.1 Changes in thresholds

The new thresholds in this counterfactual experiment are shown in Table 3. Sobral has now a slightly lower threshold at the first stage and a slightly higher threshold at the second-stage exam while this is true but at the second stage for Fortaleza. The school in Fortaleza is overall more popular (see Table S.vii) and even more than the difference in offered seats. By making students choose in the order of first-stage grades, positions in Sobral at the second-stage exam are less likely to be filled earlier than Fortaleza's despite the one to four ratio (160/600). For instance, if more than 80% of the top 750 students prefer Fortaleza to Sobral, the 600 seats at Fortaleza would be filled in after those 750 students would reveal their choices while Sobral would still have 10 seats to fill in. Note that in simulations, such a solution can be very unstable with respect to the random draws of grade shocks and depend very much on revealed preferences and the first-stage randomness in selecting the set of students who can go to the second stage.

4.5.2 Changes in success probabilities

Changes in success probabilities in Sobral are shown in Figure 4. Success probabilities, evaluated ex-ante, now depend more on the first stage than before so that students performing well at the first stage increase their overall success probabilities while those performing worse have now lower success probabilities. There is also a large dispersion of these changes. Ex-post dispersion increases with the final expected grade because it increases with the level of the initial success probabilities

and this confirms the increasing importance of the first-stage grade. These conclusions are true for Fortaleza (see Section S.3) as well.

4.5.3 Changes in expected utilities and the impact on schools

As this mechanism introduces an element of flexibility for students since they can condition their choices on their first-stage grades, their expected utility is on average mechanically larger than in the original system. Indeed, the frequency of an increase in expected utility is the largest in the three experiments. This mechanism is mainly attractive for the top students as shown in Table 6. In a nutshell, top students in the first stage are better protected from the competition of lower ranked students.

There are clear differences in utility changes among the top students conditional on their preferences for the schools. On average, students who were choosing Fortaleza in the original system would benefit more than those who preferred Sobral. This seems to be due to the difference in the sizes of the school because of the argument presented above when we were analyzing the impact on thresholds. Sobral seats are filled less quickly than Fortaleza's.

Overall, this counterfactual seems more friendly to top students. Nonetheless, such a system seems to select students with lower future academic success as shown by the analysis of Wu and Zhong (2014) using historical data on China provinces which have changed allocation mechanisms in this direction. Our data is too limited to explore this issue.

5 Conclusion

In this paper, we use data from entry exams and an allocation mechanism to colleges to provide an evaluation of changes in those mechanisms. We first use a model of school choices as well as performance to estimate parameters governing success probabilities and preferences. Expectations of sophisticated students are obtained by sampling into the Nash equilibrium conditions. Using those estimates, we can compute in a second step the impact of three counterfactual experiments on success probabilities and expected utility of students. This shows at what benefits and costs the current mechanism could be changed, not only in terms of aggregate utilitarian welfare but also in terms of potentially strong redistributive effects between schools and between students.

These cost-benefit analyses show that the choice of an allocation mechanism has sizeable

consequences for both schools and students. The mechanism in place is neither fair nor strategic although it might be rationalized by the fact that some schools and/or groups of students would lose if it were changed. The political economy of such a choice of an allocation mechanism remains to be documented and analyzed and it would be interesting to develop the analysis of the ex-ante game between schools and/or students that leads to the adoption of such or such mechanisms of selection and allocation. As a matter of fact, Federal universities in Brazil adopted in 2010, under pressure of the Federal government, a national allocation mechanism consisting of student submissions of a list of two preferred schools and a complicated learning mechanism. Some of us are in the process of collecting data to evaluate this new system.

Nonetheless, the previous mechanism allowed schools to tailor their selection procedures to the information they had about the prerequisites for their courses and any predictors of success or drop out of the students they selected. This fine tuning is lost in the new centralized procedure which abstracts away from the question of acquiring information that determines school preferences (Coles, Kushnir and Niederle, 2013). Specifically, the new allocation mechanism used in Brazil (for instance analyzed in Machado and Szerman, 2017) is based on a single grade given by an improved version of ENEM which nevertheless remains of poorer quality than the vestibular analyzed in this paper since the additional information yielded by the two-stage exams is now lost. Universities were also reducing opportunistic behavior as shown by the last counterfactual since knowing results at the first-stage exam allows students to strategize better.

Our selection of two elite medical schools is admittedly specific and tailored to minimize departures from our simplifying assumptions. As preferences for these two schools are presumably closer than any other pair of schools, the impact of treatment on outcomes – i.e. success probabilities and school choice – might be magnified by this selection. Whether this larger impact is translated into larger welfare effects is, however, ambiguous since differences between preferences are smaller.

On the modeling side, much remains to be done. Specifically, the modelling assumptions about expectations are strong and weakening them is high on the agenda. Identification however is bound to be weak since there is nothing in our data that might indicate whether agents are sophisticated, well or badly informed or even naïve (He, 2016, Agarwal and Somaini, 2018). The analysis shall thus proceed as an analysis of robustness that could lead to partial identification of

the costs and benefits we have been describing above. It is also true that the question of why so many students are taking this exam although they have no chances to succeed remains pending. They could be overly optimistic and this relates to assumptions about expectations but they could also use the exam as a training device for the following year or for other exams of a similar type. This behaviour seems to be easier to accommodate in the current framework.

References

Abdulkadiroğlu, A., Agarwal, N., & Pathak, P. A., 2017, "The Welfare Effects of Coordinated Assignments: Evidence from the NYC High School Match", *American Economic Review,* 107(12), 3635-3689.

Abdulkadiroğlu, A., P.A. Pathak and A. Roth, 2009, "Strategy-proofness versus Efficiency in Matching with Indifferences; Redesigning the NYC High School Match", *American Economic Review*, Vol. 99, No. 5, pp. 1954-1978.

Abdulkadiroğlu, A., Pathak, P., Roth, A. E., & Sonmez, T., 2006, "Changing the Boston school choice mechanism", WP 11965, National Bureau of Economic Research.

Abdulkadiroğlu, A., & Sonmez, T., 1998, "Random serial dictatorship and the core from random endowments in house allocation problems", *Econometrica*, 66(3):689-701.

Abdulkadiroğlu, A. and T., Sonmez, 2003, "School Choice: A Mechanism Design Approach", *American Economic Review*, Vol. 93, No. 3, pp. 729-747

Abizada, A.. and S. Chen, 2011, "The College Admission Promblem with Entrance Criterion", unpublished manuscript.

Agarwal, N., 2015, "An empirical model of the medical match", *The American Economic Review*, 105(7), 1939-1978.

Agarwal, N., and P., Somaini, 2018, "Demand Analysis using Strategic Reports: An Application to a School Choice Mechanism", *Econometrica*, 86(2), 391-444.

Akyol, P., & Krishna, K., 2017, "Preferences, selection, and value added: A structural approach", *European Economic Review*, 91, 89-117.

Alstadsæter, **A.,** 2011, "Measuring the consumption value of higher education", *CESifo Economic Studies*, 57(3), 458-479.

Arcidiacono, P., 2005, "Affirmative Action in Higher Education: How Do Admission and Financial Aid Rules Affect Future Earnings?", *Econometrica*, Vol. 73, No. 5, pp. 1477-1524.

Avery, C., Lee, S., & Roth, A. E., 2014, "College Admissions as Non-Price Competition: The Case of South Korea", WP20774, National Bureau of Economic Research.

Aygün, O., & Bo, I. 2017, "College Admission with Multidimensional Privileges: The Brazilian Affirmative Action Case", SSRN Archive.

Azevedo, E.M., and J.D., Leshno, 2016, "A Supply and Demand Framework for Two-Sided Matching Markets", *Journal of Political Economy*, 124(5), 1235-1268.

Balinski M., and T., Sönmez, 1999, "A Tale of Two Mechanisms: Student Placement", *Journal of Economic Theory* 84, 73-94.

Blundell, R., and J. Powell, 2003, "Endogeneity in Non Parametric and Semi Parametric Regression Models," in *Advances in Economics and Econometrics: Theory and Applications*, Vol. II, ed. by M. Dewatripont, L. P. Hansen, and S. J. Turnovsky. Cambridge, U.K.: Cambridge University Press, 312–357.

Budish, E. and E. Cantillon, 2012, "The Multi-unit Assignment Problem: Theory and Evidence from Course Allocation at Harvard", *American Economic Review*, 102(5):2237-2271

Calsamiglia, C., C., Fu and M.Güell, 2018, "Structural Estimation of a Model of School Choices: the Boston Mechanism vs its alternatives", WP 24588, National Bureau of Economic Research.

Calsamiglia, C., Haeringer, G., & Klijn, F., 2010, "Constrained school choice: An experimental study", *The American Economic Review*, 1860-1874.

Campbell, J. Y., 1987, "Does Saving Anticipate Declining Labor Income?" *Econometrica*, 55, 1249–1273.

Chade, H., Lewis, G., & Smith, L., 2014, "Student portfolios and the college admissions problem", *The Review of Economic Studies*, 81(3), 971-1002.

Che, Y.K., and Y. Koh, 2016, "Decentralized College Admissions", *Journal of Political Economy*, 124:1295-1338.

Chen, Y., & Kesten, O., 2017, "Chinese college admissions and school choice reforms: A theoretical analysis", *Journal of Political Economy*, 125(1), 99-139.

Coles, P., A. Kushnir and M. Niederle, 2013, "Preference Signaling in Matching Markets", *American Economic Journal: Microeconomics,* 2013, 5(2): 99–134

Dogan, M. K., & Yuret, T., 2013, "Publication Performance and Student Quality of Turkish Economics Departments/Türkiye'de Iktisat Bölümlerinin Yayin Performansi ve Ögrenci Kalitesi". *Sosyoekonomi*, (1), 71.

Dubey, P., O. Haimanko and A., Zapechelnyuk, 2006, "Strategic Complements and Substitutes and Potential Games", *Games and Economic Behavior*, 54:77-94.

Epple, D., R. Romano and H. Sieg, 2006, "Admission, Tuition, and Financial Aid Policies in the Market for Higher Education", *Econometrica*, Vol. 74, No. 4, pp. 885-928

Fack, G., J., Grenet and Y., He, 2017, "Estimating Preferences in School Choice Mechanisms", unpublished manuscript.

Fu C., 2014, "Equilibrium tuition, applications, admissions, and enrollment in the college market", *Journal of Political Economy*, 122(2), 225-281.

Hafalir, I.E., R., Hakimov, D. Kübler and M. Kurino, 2018, "College Admissions with Entrance Exams: Centralized versus Decentralized", *Journal of Economic Theory*, 176, 886-934.

Hastings, J., T. J. Kane, and D. O. Staiger, 2009, "Heterogenous Preferences and the Efficacy of Public School Choice," Working paper, Yale University.

He, Y., 2017, "Gaming the School Choice Mechanism in Beijing", Toulouse School of Economics, WP 15-607, revised.

He, Y. and T., Magnac, 2018, "A Pigouvian Approach to Congestion in Matching Markets", Toulouse School of Economics, WP 17-870, revised.

Jacob, B., Mccall, B., & Stange, K., 2012, "The consumption value of education: Implications for the postsecondary market", working paper.

Jensen, M.K., 2010**,** "Aggregative games and best-reply potentials**",** *Economic Theory*, 43:45-66.

Lee, R.S. and M. Schwarz, 2017, "Interviewing in two-sided matching markets", *The RAND Journal of Economics*, 48 (3), 835-855.

Machado, C., and C., Szerman, 2017, "Centralized Admission and the Student-College Match", SSRN Archive.

Manski, C. F., 1988, "Identification of binary response models", *Journal of the American Statistical Association*, 83(403), 729-738.

Manski C., 1993, "Adolescent Econometricians: How Do Youths Infer the Returns to Schooling?"in *Studies of Supply and Demand in Higher Education*, edited by Charles T.Clotfelter and Michael Rothschild. Chicago: University of Chicago Press.

Matzkin, R. L., 1993, "Nonparametric identification and estimation of polychotomous choice models", *Journal of Econometrics*, 58(1), 137-168.

Oosterbeek, H., & Ophem, H. V., 2000, "Schooling choices: Preferences, discount rates, and rates of return", *Empirical Economics*, 25(1), 15-34.

Pathak P.A., and T., Sonmez, 2013, "Leveling the Playing Field: Sincere and Sophisticated Players in the Boston Mechanism," *American Economic Review*, 98(4), 1636–1652.

Roth, A.E, 2008, "Deferred acceptance algorithms: history, theory, practice, and open questions", *International Journal of Game Theory*, 36:537–569

Roth., A.E., 2018, "Marketplaces, Markets and Market Design", *American Economic Review,* 108(7):1609-1658.

Roth, A. E., & Sotomayor, M. A. O., 1992, *Two-sided matching: A study in game-theoretic modeling and analysis* (No. 18). Cambridge University Press.

Selim T., and S., Salem, 2009, "Student Placement in Egyptian Colleges", MPRA Paper No. 17596

Sönmez, T., & Ünver, M. U., 2011, "Matching, allocation, and exchange of discrete resources", *Handbook of Social Economics*, 1, 781-852.

Wu, B., & Zhong, X., 2014, "Matching mechanisms and matching quality: Evidence from a top university in China", *Games and Economic Behavior*, 84, 196-215.

Zhu, M., 2014, "College admissions in China: A mechanism design perspective", *China Economic Review* 30 (2014) 618–631

A Existence of a Nash equilibrium and convergence to an equilibrium

When using the current mechanism or counterfactual experiments, the question of the existence of a Nash equilibrium is pending. This equilibrium is defined as the solution to the best response equations (1) and success probabilities that are mutually compatible and compatible with the equilibrium conditions (4). We rely on the theory of pseudo potential games as developed in Dubey et al (2006)

In this discussion, we sketch the proof in a simpler game restricted to two schools $j \in \{S, F\}$ and a single stage exam and imposing some weak conditions. The extension to mores schools or two exams complicates notation but does not affect the intuition. Conditions (4) become:

$$\begin{aligned}
&\sum_{i=1}^{n}[\mathbf{1}\{D_i = S\}\mathbf{1}\{m_1(Z_i, u_i, \beta) \geq t_1^S] = 4n^S, \\
&\sum_{i=1}^{n}[\mathbf{1}\{D_i = F\}\mathbf{1}\{m_1(Z_i, u_i, \beta) \geq t_1^F] = 4n^F.
\end{aligned}$$

We will also assume that both schools are overdemanded by students who do not value positively both schools i.e.

$$\begin{aligned}
&\sum_{i=1}^{n}\mathbf{1}\{V_i^S > 0 \geq V_i^F\} > 4n^S, \\
&\sum_{i=1}^{n}\mathbf{1}\{V_i^F > 0 \geq V_i^S] > 4n^F,
\end{aligned} \tag{15}$$

so that thresholds in (4) are always defined by equalities.

Setting $\lambda_j(D_{(n)}) = \frac{4n^j}{\sum_{i=1}^{n}\mathbf{1}\{D_i=j\}}$, we can write an explicit definition of the thresholds as the empirical $(1-\lambda_j)-$quantile of the distribution of grades in the sample of applicants to j:

$$T_1^j(Z_{(n)}^j, U_{(n)}^j) = F^{-1}_{\{m_1(Z_i,u_i,\beta),D_i=j\}}(1-\lambda_j).$$

Note that the strategies of other students affect λ_j as well as the quantile so that expected success probabilities can be written as:

$$P_0^j(D_{(n)}) = \mathbb{E}(\mathbf{1}\{m_1(Z_0, u_i, \beta) \geq T_1^j(Z_{(n)}^j, U_{(n)}^j)\}).$$

It is easy to formulate deep assumptions about the distribution function of grades that imply that the success probabilities strictly decrease when adding an additional competitor to the set of applicants to d. Indeed, let order the strategy set $\{S, F\}$ as $S > F$. Extend the order to a partial order in strategies $D_{(n)}$ in the sample by positing that:

$$D_{(n)} > D'_{(n)} \text{ iff } D_i \geq D'_i \text{ and for at least one } i \ D_i > D'_i.$$

If the distribution of grade shocks is unbounded, adding competitors creates congestion and we have that:

$$D_{(n)} > D'_{(n)} \Longrightarrow P_0^S(D_{(n)}) < P_0^S(D'_{(n)}) \text{ and } P_0^F(D_{(n)}) > P_0^F(D'_{(n)}).$$

It is now straighforward to prove that the game satisfies the *dual strong single crossing property.* Suppose indeed that $V_0^S > 0$ and that:

$$P_0^S(D'_{(n)})V_0^S \leq P_0^F(D'_{(n)})V_0^F.$$

This implies

$$P_0^S(D_{(n)})V_0^S < P_0^S(D'_{(n)})V_0^S \leq P_0^F(D'_{(n)})V_0^F < P_0^F(D_{(n)})V_0^F.$$

This is also trivially satisfied when $V_0^S \leq 0$ and $V_0^F > 0$.

As this property of *dual strong single crossing* implies that this is a game of weak strategic substitutes with aggregation (Dubey et al, 2006) , it is a pseudo potential game (Theorem 1, p.81) and it has a Nash equilibrium (Proposition 1, p.84). Furthermore, since the strategy set is finite, there are no best response cycles in the game. "If players start with an arbitrary strategic profile and each player (one at a time) unilaterally deviates to his unique best reply then the process terminates in a Nash equilibrium after finitely many steps" (Remark 1, p.85)

B Proofs in Section 2

B.1 Proof of Proposition 1

Fix $\mathcal{J}_0 \subset \mathcal{J}$, a set of non-empty indices. The probability that the observed choice belongs to $\mathcal{J}_0$, $\Pr(D \in \mathcal{J}_0 \mid Z, \max_{j\in\mathcal{J}}(V^j) > 0, X)$ is identified a.e. $P_{X,Z}$ and by equation (6) is equal to:

$$\sum_{(j,\mathcal{J}_+);\mathcal{J}_+\supset\{j\}\subset\mathcal{J}_0} Q(\mathcal{J}_+ \mid X) \Pr(\forall l \in \mathcal{J}_+, \Delta^{jl}(Z) > \log(V^l) - \log(V^j) \mid X, Z, \mathcal{J}_+, \mathcal{J}_+^c).$$

Because of Assumption CV, the support of any vector $\{\Delta^{jk}(Z)\}_{k\in\mathcal{J}/\{j\}}$ is $(-\infty, +\infty)^{card(\mathcal{J})-1}$. Consider the limit when, for all $j \in \mathcal{J}_0$ and all $k \in \mathcal{J}_0^c$, $\Delta^{jk}(Z)$ tends to $-\infty$

$$\lim_{\forall(j,k)\in\mathcal{J}_0^c\times\mathcal{J}_0^c, \Delta^{jk}(Z)\to-\infty} \Pr(D \in \mathcal{J}_0 \mid Z, \max_{j\in\mathcal{J}}(V^j) > 0, X)$$

For all $k \in \mathcal{J}_0^c$, conditions $\Delta^{jk}(Z) > \log(V^k) - \log(V^j)$ are never satisfied and the limit above is thus equal to:

$$\sum_{(j,\mathcal{J}_+);\mathcal{J}_+\supset\{j\}\subset\mathcal{J}_0} Q(\mathcal{J}_+ \mid X) \Pr(\forall l \in \mathcal{J}_+ \cap \mathcal{J}_0, \Delta^{jl}(Z) > \log(V^l) - \log(V^j) \mid X, Z, \mathcal{J}_+, \mathcal{J}_+^c)$$

$$= \sum_{\mathcal{J}_+;\mathcal{J}_+\subset\mathcal{J}_0} Q(\mathcal{J}_+ \mid X) \equiv Q^*(\mathcal{J}_0 \mid X)$$

because the terms in the first line, $\Pr(\forall l \in \mathcal{J}_+ \cap \mathcal{J}_0, \ldots)$ sum to one over $j \in \mathcal{J}_0$ for all $\mathcal{J}_+ \subset \mathcal{J}_0$. In consequence, $\forall \mathcal{J}_0 \subset \mathcal{J}$ and $\mathcal{J}_0$ non empty, $Q^*(\mathcal{J}_0 \mid X)$ is identified a.e. P_X.

Consider now that $\mathcal{J}_0 = \{j\}$ is a singleton. Then $Q(\{j\} \mid X) = Q^*(\{j\} \mid X)$ is identified. By induction suppose that for $K \geq 2$, $Q(\mathcal{J}_K \mid X)$ is identified for all $\mathcal{J}_K$ such that $card(\mathcal{J}_K) = K$. Consider $\mathcal{J}_{K+1}$ with $card(\mathcal{J}_{K+1}) = K + 1$ and:

$$Q^*(\mathcal{J}_{K+1} \mid X) = \left[\sum_{\mathcal{J}_K; \mathcal{J}_K \subset \mathcal{J}_0, card(\mathcal{J}_K)=K} Q(\mathcal{J}_K \mid X) \right] + Q(\mathcal{J}_{K+1} \mid X)$$

which proves that $Q(\mathcal{J}_{K+1} \mid X)$ is identified. As this is true for $K = 1$, and if true for K, true for $K + 1$, $Q(\mathcal{J}_0 \mid X)$ is identified for all $\mathcal{J}_0 \subset \mathcal{J}$.■

B.2 Proof of Proposition 2

We proceed by induction over the number of schools, J. The two-school case is proved in the text. To design the proof at the simplest level, we first derive the proof for $J = 3$ and $\mathcal{J} = \{1, 2, 3\}$. The general proof will follow the same lines but at a more abstract level and will show that if it is true for J, this is also true for $J + 1$.

Stage 1: from two schools to $J = 3$ Write the observed choice probabilities in equation (6) when $\Delta^{13}(Z) \to -\infty$, which is permitted by assumption CV:

$$\begin{aligned} \Pr(D &= 1 \mid Z, X) = Q(\{1\} \mid X) \\ +Q(\{1,2\} &\mid X) \Pr(\Delta^{12}(Z) > \log(V^2) - \log(V^1) \mid X, Z, \mathcal{J}_+ = \{1,2\}, \mathcal{J}_+^c = \{3\}), \end{aligned}$$

since alternative 1 is always dominated by alternative 3 when both V_1 and V_3 are positive. Given that $Q(.)$ is identified and different from zero, this identifies

$$\Pr(\Delta^{12}(Z) > \log(V^2) - \log(V^1) \mid X, Z, \mathcal{J}_+ = \{1,2\}, \mathcal{J}_+^c = \{3\}).$$

By generalizing this line of argument to any pair $\{j, k\}$ in $\{1, 2, 3\}$ this proves the identification of distributions in all quadrants of reduced dimension, $J = 2$.

We can return to equation (6)

$$\Pr(D = j \mid Z, X) = \sum_{\mathcal{J}_+ \supset \{j\}} Q(\mathcal{J}_+ \mid X) \Pr(\forall k \in \mathcal{J}_+, \Delta^{jk}(Z) > \log(V^k) - \log(V^j) \mid X, Z, \mathcal{J}_+, \mathcal{J}_+^c)$$

in which all terms are identified except when set $\mathcal{J}_+ = \mathcal{J}$. As $Q(\mathcal{J} \mid X)$ is positive by assumption, we derive from this equation an expression for $\Pr(\forall k \in \mathcal{J}, \Delta^{jk}(Z) > \log(V^k) - \log(V^j) \mid X, Z, \mathcal{J}_+ = \{1, 2, 3\}, \mathcal{J}_+^c = \varnothing)$ for all $j \in \mathcal{J}$ as a function of identified terms. By the complete variation assumption CV of $\Delta^{jk}(Z)$, this ensures the identification of the joint distribution $\Pr((\log(V^k) - \log(V^1))_{\forall k \in \mathcal{J}/\{1\}} \mid X, Z, \mathcal{J})$ if 1 is taken as the reference alternative. The property under induction is thus true for $J = 3$.

Stage 2: from J to $J+1$ Assume now that the property is true for J. We now show that the property is true for $J+1$. It follows the same steps as above:

(i) Assume that for all $j \in \mathcal{J}/\{l\}$, $\Delta^{jl}(Z) \to -\infty$ which identifies, through equation (6), for any $j \in \mathcal{J}/\{l\}$:

$$\Pr(\forall k \in \mathcal{J}/\{j,l\}, \Delta^{jk}(Z) > \log(V^j) - \log(V^k) \mid X, Z, \mathcal{J}_+ = \mathcal{J}/\{l\}, \mathcal{J}_+^c = \{l\}).$$

(ii) Return to equation (6)

$$\Pr(D = j \mid Z, X) = \sum_{\mathcal{J}_+ \supset \{j\}} Q(\mathcal{J}_+ \mid X) \Pr(\forall k \in \mathcal{J}_+, \Delta^{jk}(Z) > \log(V^k) - \log(V^j) \mid X, Z, \mathcal{J}_+, \mathcal{J}_+^c)$$

in which all terms are identified except the one corresponding to $\mathcal{J}_+ = \mathcal{J}$. As $Q(\mathcal{J} \mid X)$ is positive, we can derive an expression for $\Pr(\forall k \in \mathcal{J}/\{j\}, \Delta^{jk}(Z) > \log(V^k) - \log(V^j) \mid X, Z, \mathcal{J})$ for all $j \in \mathcal{J}$. By the complete variation assumption CV of $\Delta^{jk}(Z)$, this ensures the identification of the joint distribution $\Pr((\log(V^k) - \log(V^1))_{k \in \mathcal{J}/\{1\}} \mid X, Z, \mathcal{J})$ if 1 is taken as the reference alternative. Identification of differences between log-values is thus true for $J+1$.■

Table 1: Descriptive statistics in the two medical majors

Sobral: 40 positions

Variable	Mean	Median	Std. Dev.	Min.	Max.	N
Grade: National Exam (m_0)	50.43	52.00	7.29	18.00	61.00	527
Grade: First stage	71.67	73.00	15.74	20.00	103.00	527
Grade: Second stage	240.0	246.5	33.98	94.3	296.6	160
Female	0.47	0	0.50	0	1	527
Age	19.58	21.50	2.48	16.00	25.00	527
Private High School	0.87	1	0.33	0	1	527
Repetitions	0.99	1	0.88	0	2	527
Preparatory Course	0.71	1	0.45	0	1	527
Father's education	2.09	2	1.03	0	3	527
Mother's education	2.21	3	0.98	0	3	527

Fortaleza: 150 positions

Variable	Mean	Median	Std. Dev.	Min.	Max.	N
Grade: National Exam (m_0)	49.16	52.00	10.03	12.00	63.00	2340
Grade: First stage	70.06	72.00	20.01	20.01	110.00	2340
Grade: Second stage	240.0	245.1	34.37	48.3	311.1	600
Female	0.54	1	0.50	0	1	2340
Age	19.13	17.50	2.43	16.00	25.00	2340
Private High School	0.77	1	0.41	0	1	2340
Repetitions	0.69	1	0.83	0	2	2340
Preparatory Course	0.59	1	0.49	0	1	2340
Father's education	2.13	2	1.00	0	3	2340
Mother's education	2.15	2	0.98	0	3	2340

Source: Vestibular cross section data in 2004.

Table 2: Simulated success probabilities

	Sobral		Fortaleza	
	Stage 1	Final Success	Stage 1	Final Success
Min.	0.000	0.000	0.000	0.000
25%	0.001	0.001	0.000	0.000
Median	0.088	0.011	0.012	0.004
Mean	0.314	0.076	0.203	0.062
75%	0.676	0.103	0.360	0.071
Max.	1.000	0.934	1.000	0.920

[1] Success probabilities are constructed using 1000 Monte Carlo simulations.

Table 3: Thresholds of the Counterfactuals

	School		Sobral	Fortaleza
Stage 1	Original system	Mean Thresholds	184.48	189.88
		Standard Errors	(1.257)	(0.401)
	Cutting seats	Mean Thresholds	195.79	201.04
		Standard Errors	(0.996)	(0.506)
	Two-Choices	Mean Thresholds	186.98	190.13
		Standard error	(0.564)	(0.458)
	Timing-Change	Mean Thresholds	183.05	190.11
		Standard error	(0.859)	(0.447)
	School		Sobral	Fortaleza
Stage 2	Original system	Mean Thresholds	235.41	241.44
		Standard Errors	(1.669)	(0.898)
	Cutting seats	Mean Thresholds	233.34	237.77
		Standard Errors	(3.094)	(1.603)
	Two-Choices	Mean Thresholds	235.38	241.19
		Standard error	(2.589)	(1.302)
	Timing-Change	Mean Thresholds	239.07	244.30
		Standard error	(2.722)	(1.408)

[1] The coefficients and their standard errors are computed by using the 499 bootstrapped estimates of preference and grade parameters and applying the procedure in the text.

[2] The cutting seats counterfactual has a few cases in which the computation developed in Section 5.2 does not converge after many repetitions, and we have excluded those bootstrap values that do not converge after 500 iterations.

Table 4: Cutting seats: Expected utility changes

Expected	ALL		D=Sobral		D=Fortaleza	
Final Grade	mean	s.d.	mean	s.d.	mean	s.d.
0% -50%	-0.00029	0.00116	-0.00109	0.00185	-0.00011	0.00084
50%-60%	0.00001	0.00744	-0.00733	0.00563	0.00252	0.00622
60%-70%	0.00674	0.01655	-0.01125	0.00931	0.01206	0.01433
70%-80%	0.03122	0.02770	-0.00919	0.01272	0.03843	0.02306
80%-82%	0.04070	0.02813	-0.00096	0.00413	0.05493	0.01574
82%-84%	0.05491	0.02896	0.00540	0.00539	0.06567	0.01887
84%-86%	0.07304	0.03128	0.00107	0.00866	0.08482	0.01123
86%-88%	0.06124	0.03374	0.00257	0.00837	0.07762	0.01367
88%-90%	0.07932	0.03072	0.00813	0.00484	0.09027	0.01308
90%-92%	0.09239	0.03272	0.00617	0.00969	0.10224	0.01463
92%-94%	0.08806	0.04041	0.00991	0.00905	0.10696	0.01227
94%-96%	0.11009	0.03125	0.00834	0.00213	0.11839	0.01114
96%-98%	0.11178	0.03456	0.01104	0.00531	0.12249	0.01008
98%-100%	0.08939	0.04669	0.00760	0.00533	0.11185	0.01989
$\mathbf{E}(\Delta U_i)$	0.01966		-0.00267		0.02487	
$\mathbf{s.d.}(\Delta U_i)$	0.03785		0.00817		0.04009	
Frequency$(\ \Delta U_i > 0)$	0.4363		0.2084		0.4894	

[1] ALL contains all the students no matter what the original choices are.

[2] D=Sobral means the sub-population of those who choose Sobral in the original system; and D=Fortaleza means the sub-population of those who choose Fortaleza in the original system.

[3] $\mathbf{E}(\Delta U_i)$ (resp. $\mathbf{s.d.}(\Delta U_i)$) is the sample average (resp. standard deviation) of the total utilitarian welfare change.

[3] $\mathbf{Pr}(\Delta U_i > 0)$ is the frequency of students whose expected utility changes are positive

Table 5: Two choices: Expected utility changes

Expected	ALL		D=Sobral		D=Fortaleza	
Final Grade	mean	s.d.	mean	s.d.	mean	s.d.
0% -50%	0.00002	0.00076	0.00048	0.00144	-0.00009	0.00043
50%-60%	0.00176	0.00372	0.00537	0.00531	0.00052	0.00174
60%-70%	0.00727	0.01006	0.02106	0.01084	0.00320	0.00487
70%-80%	0.01706	0.01907	0.05619	0.01400	0.01008	0.00843
80%-82%	0.03719	0.03064	0.08629	0.00523	0.02042	0.01126
82%-84%	0.03140	0.03081	0.09163	0.00784	0.01831	0.01291
84%-86%	0.02817	0.03303	0.10573	0.00620	0.01548	0.01003
86%-88%	0.04673	0.03837	0.11457	0.00713	0.02780	0.01406
88%-90%	0.04323	0.03573	0.12548	0.00781	0.03058	0.01563
90%-92%	0.03728	0.03984	0.14298	0.01055	0.02520	0.01731
92%-94%	0.05830	0.04879	0.14871	0.00588	0.03643	0.02148
94%-96%	0.04137	0.04184	0.17341	0.00454	0.03059	0.01799
96%-98%	0.05055	0.04687	0.18008	0.00424	0.03677	0.02042
98%-100%	0.05964	0.06562	0.17849	0.01288	0.02702	0.02069
$\mathbf{E}(\Delta U_i)$						
	0.01143		0.03074		0.00693	
$\mathbf{s.d.}(\Delta U_i)$						
	0.02705		0.05073		0.01400	
$\mathbf{Frequency}(\Delta U_i > 0)$						
	0.5431		0.7029		0.5058	

[1] ALL contains all students no matter what the original choices are.

[2] D=Sobral means the sub-population of those who choose Sobral in the original system; and D=Fortaleza means the sub-population of those who choose Fortaleza in the original system.

[3] Notes: See notes of Table 4.

Table 6: Timing change: Expected utility changes

Expected	ALL		D=Sobral		D=Fortaleza	
Final Grade	mean	s.d.	mean	s.d.	mean	s.d.
0% -50%	0.00053	0.00158	-0.00019	0.00065	0.00070	0.00168
50%-60%	0.00731	0.00584	-0.00064	0.00182	0.01003	0.00394
60%-70%	0.01798	0.01034	0.00196	0.00291	0.02271	0.00612
70%-80%	0.03551	0.01394	0.00691	0.00491	0.04062	0.00722
80%-82%	0.04164	0.01984	0.00954	0.00386	0.05260	0.00654
82%-84%	0.04821	0.01877	0.01087	0.00588	0.05633	0.00683
84%-86%	0.05375	0.01971	0.00818	0.00574	0.06121	0.00674
86%-88%	0.05348	0.02207	0.01425	0.00571	0.06443	0.00745
88%-90%	0.06125	0.02169	0.01046	0.00434	0.06907	0.00865
90%-92%	0.06933	0.02109	0.01364	0.00633	0.07569	0.00932
92%-94%	0.06514	0.02591	0.01646	0.00808	0.07691	0.00989
94%-96%	0.07891	0.02258	0.01366	0.00598	0.08424	0.01289
96%-98%	0.07912	0.02262	0.01773	0.00570	0.08566	0.01054
98%-100%	0.06413	0.02878	0.01581	0.00784	0.07739	0.01454
$\mathbf{E}(\Delta U_i)$						
	0.01862		0.00288		0.02229	
$\mathbf{s.d.}(\Delta U_i)$						
	0.02703		0.00609		0.02865	
$\mathbf{Frequency}(\ \Delta U_i > 0)$						
	0.6557		0.5166		0.6881	

[1] ALL contains all the students no matter what the original choices are.

[2] D=Sobral means the sub-population of those who choose Sobral in the original system; and D=Fortaleza means the sub-population of those who choose Fortaleza in the original system.

[3] See notes of Table 4

Figure 1: Choice space

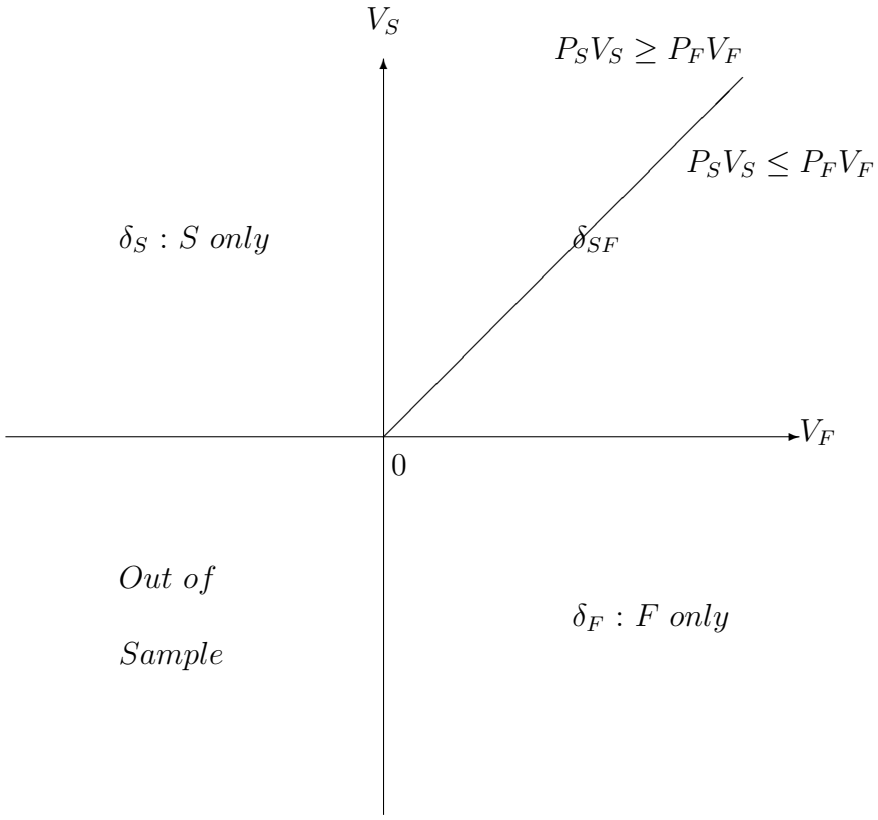
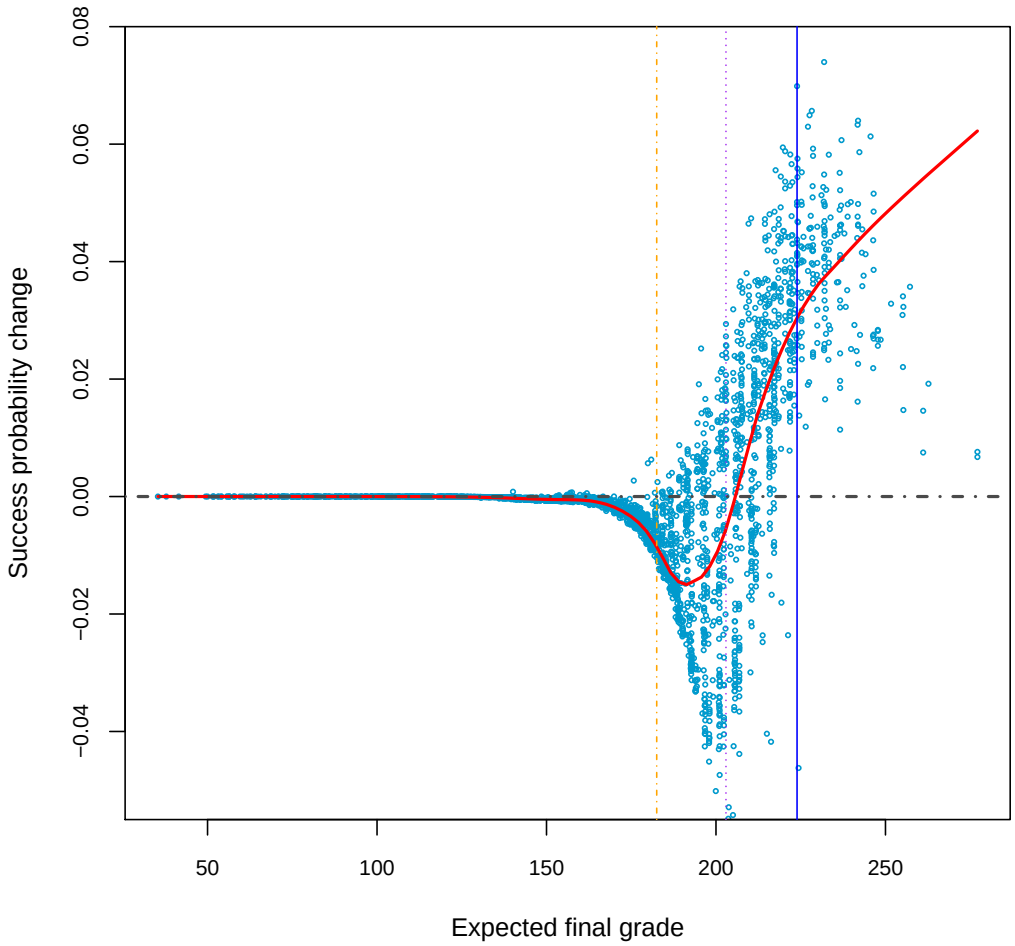
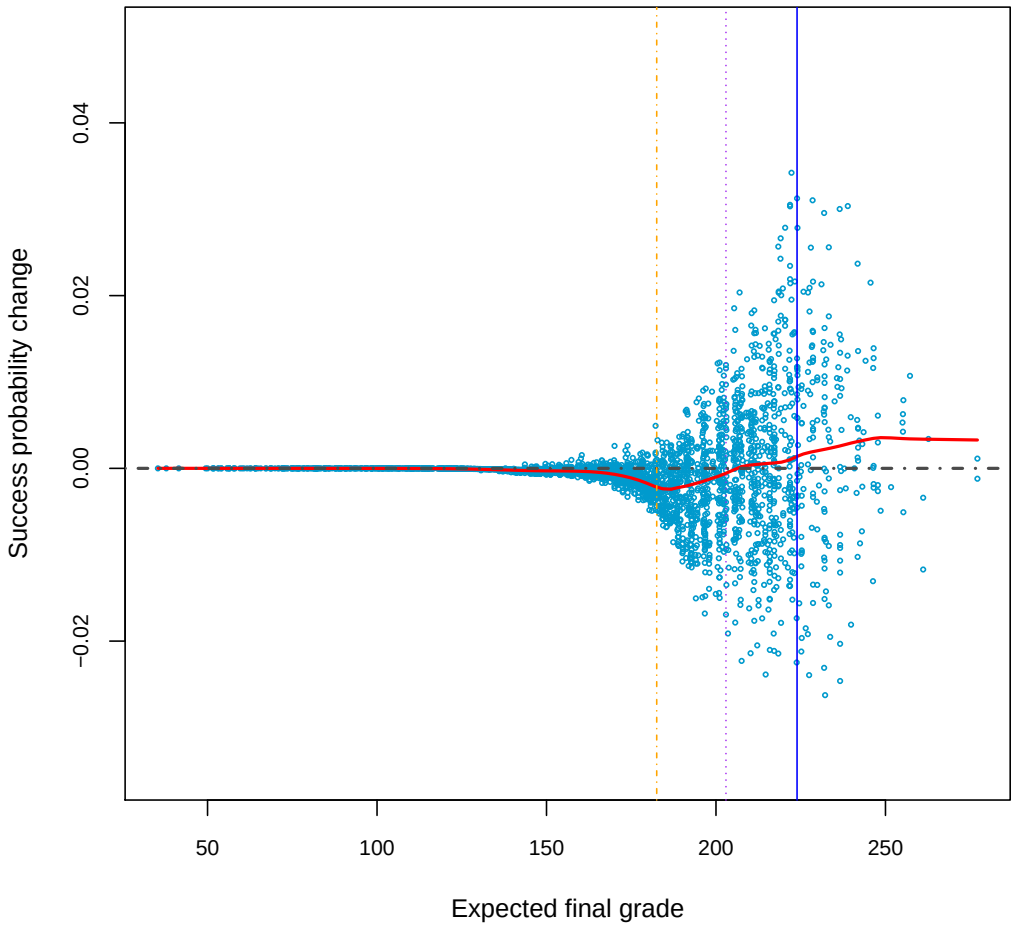


Figure 2: Cutting seats: Changes of success probabilities in Sobral



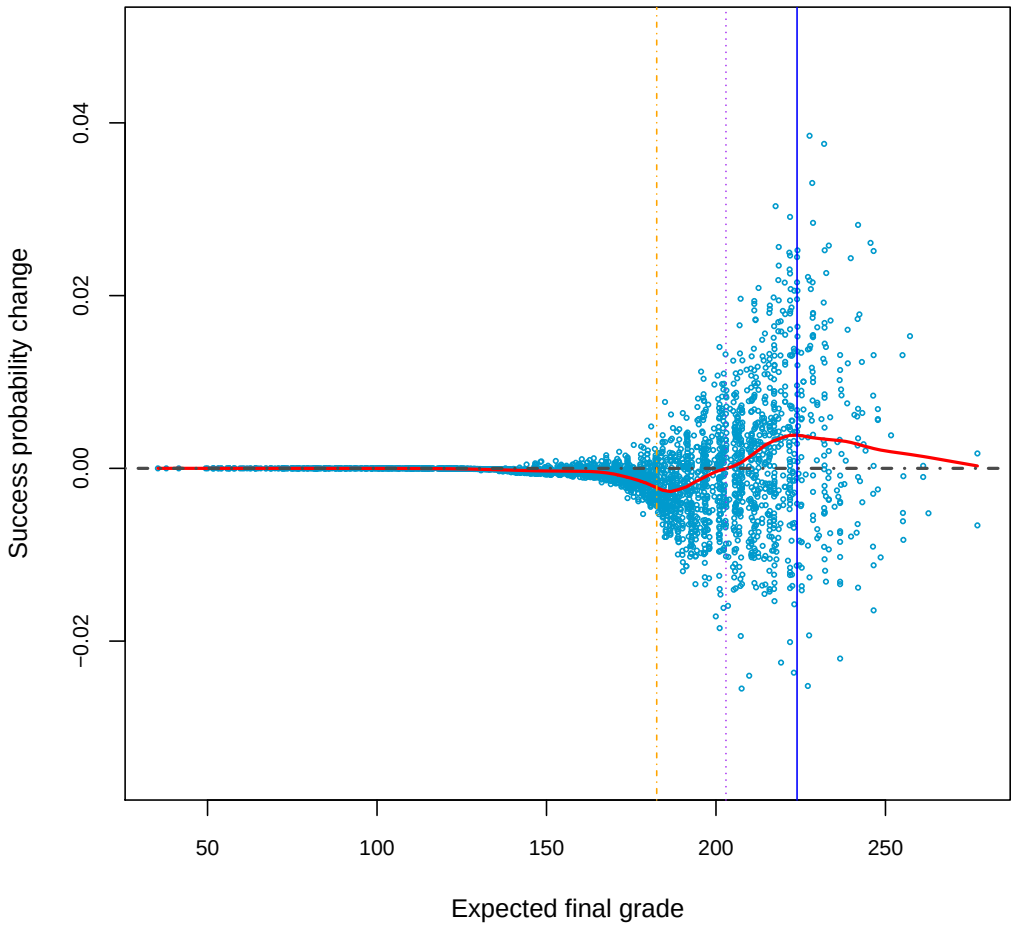
[1] The circles plot individual success probability changes vs expected final grades; [2] From left to right, 1) the first vertical line is the median, 2) the second vertical line is the average of quantiles for 1st stage admission – $(1-\frac{4(nos+nof)}{nobs})\times 100\%$, and 3) the third line is the average of quantiles for 2nd stage admission – $(1-\frac{(nos+nof)}{nobs})\times 100\%$ in which nos is the number of final seats in Sobral, nof is the number of final seats in Fortaleza and $nobs$ is the number of total applicants; [3] The solid fitted curve is obtained by lowess smoothing.

Figure 3: Two choices: Success probability change in Sobral



Notes: See notes of Figure 2

Figure 4: Timing change: Success probability changes in Sobral



Notes: See notes of Figure 2

Supplementary Appendix available upon request

S.1 Data appendix

Matching students with university schools in Brazil is a very competitive process and in particular in public federal universities which are mostly the best institutions. More than two millions of students competed to access one of the 331,105 seats in 2006. In some schools, medicine or law for instance, the ratio of applications to available seats can be as high as 20 or more (INEP, 2008). Fierce competition is by no means the exclusivity of Brazilian universities. What made Brazil specific in the years 2000s was the formality of the selection process at the level of each university. In contrast to countries such as the United States where the predominant selection system uses multiple criteria (for instance, Arcidiacono, 2005), selection using only objective performance under the form of grades at exams is pervasive in Brazil. More than 88% of available seats are allocated through a *vestibular* as is called the sequence of exams taken by applicants to university degrees (INEP, 2008). Moreover, in contrast to countries such as Turkey (Balinski and Sonmez, 1999), the organization of selection was decentralized at the level of universities until 2010.

ENEM is a non-mandatory Brazilian national exam, which evaluates high school education in Brazil. Until 2008, the exam consisted in two tests: a 63 multiple-choice test on different subjects (Portuguese, History, Geography, Math, Physics, Chemistry and Biology) and writing an essay.

S.1.1 Description

The Vestibular, an entrance exam whereby different universities develop their own format of testing students restricted by some federal constraints, has its roots in the creation of the first undergraduate course in Brazil 200 hundred years ago. Only in 1970, with the creation of the National Commission of the Vestibular, the system started to develop a regulatory background in order to rationalize the increasing demand for undergraduate education in the country. The final step that shaped the format of the Vestibular in place in 2004 was taken in 1996 with the approval of the Law of Directives and Basis of the National Education (LDB). The LDB, among other things, set the minimum requirements of the exam and explicit constraints regarding the form and

content that universities must obey if they choose to select their students through a Vestibular. Olive (2002) asserts that LDB introduced a regular and systematic process of evaluation and credentialing that initiated a new era of meritocracy in Brazilian universities. Even though LDB reinforced regulation and as a consequence brought about many new restrictions, law abiding universities still have in practice a lot of degrees of freedom to adapt their entrance exams to their needs.

S.1.2 The Vestibular at UFC

The Vestibular at UFC shares the same features described above regarding its protocol. However, we give a detailed description of some of its feature in order to gain insight when developing and estimating econometrics models. First, all entrance exams in public universities must be preceded, by law, by the release of a document called Edital which contains the whole set of regulations regarding the exam: among others, a specific timeline for exams, a detailed list of syllabus for all disciplines required in the exams, the schools offered as well as the available spots in each one, how scores are calculated, how students are ranked, forbidden actions that may cause elimination from the exams, minimum requirements in terms of grades and so on. Accordingly to Brazilian law the Edital is a document that possesses the status of legislation, i.e., any dispute of rights with respect to details of the Vestibular must use the contents of the Edital as a first guiding line in order to settle the dispute.

The first stage, called General Knowledge (GK), is composed of a unique 63 objective questions (multiple choice, with five alternatives A, B, C, D and E) exam whose content is exactly the core high school curricula, i.e., Portuguese (Grammar and Writing), Geography, History, Biology, Chemistry, Mathematics, Physics and Foreign Language.

Adding up all "standardized" scores gives the total standardized score X_s^{GK}. In order to pass to the following second stage and take the so called Specific Knowledge (SK) exam, the student must obey the following rules:

1. Get a grade in each subject appearing in the GK exam;

2. After being ranked accordingly to his/her overall standardized score X_s^{GK}, the student must be placed in a position equal or above the threshold specific to his/her chosen school. This threshold is calculated based on the following rule: Let N be the number of available places in a specific school previously shown in the Edital. Let r be defined as the ratio of the number of students choosing the school and the number of available seats in the school. If $r < 10$ then the threshold is $3N$, otherwise it is $4N$.

The second-stage exam is comprised of two separated sub-exams (realized in two consecutive days apart only two weeks after the release of first-stage exam results) and they are set according to the requirements of each school. The sum of all standardized scores taken in the second stage gives the second-stage grade. The sum of all first-stage standardized scores and all second-stage standardized scores gives the final grade. All students are ranked again and available seats are allocated to the best ranked students.

S.1.3 Descriptive analysis

The complete original database comprises 41377 students who took the Vestibular exam in 2004. There are several groups of variables in the database that are useful for this study:

- Grades at the various exams – the initial national high school evaluation exam (ENEM), the first and second stage of the Vestibular system as well as the number of repetitions of the entry exams.
- Basic demographic variables – gender, age by discrete values (16, 17.5, 21 and 25) and the education levels of father and mother.
- Education history – public or private primary or high school as described by discrete values indicating the fraction of time spent in private schools and undertaking of a preparatory course
- Choices of schools

In total there are 58 schools that students may consider at Universidade Federal do Ceará. We grouped these schools into broad groups according to the type of second-stage exams that students take to access these schools. Table S.i reports the number of student applications, available positions and the rate of success at stages 1 and 2 in each of those school fields. These fields are quite different not only in terms of organization and in terms of contents but also regarding the ratio of the number of applicants to the number of positions. At one extreme lie Physics and Chemistry in which the number of applications is low and the final pass rates reasonably high (20%). At a lesser degree this is also true for Accountancy, Agrosciences and Engineering. At the other extreme, lie Law, Medicine, Other humanities and Pharmacy, Dentist and Other in which the final pass rate is as low as 5 or 6% that is one out of 16 students passes the exam.

Medicine is one of the most difficult school to enter as can be seen in Table S.ii which reports summary statistics in each school field and the grades obtained at the first stage of the college

exam.[S.1] We report statistics on the distribution of the first-stage grades in three samples:[S.2] the complete sample, the sample of students who passed the first stage and the sample of students who passed the second stage and thus are accepted in the schools. school fields are ranked according to the median grade among those who passed the final exam in that school field. These statistics are very informative. Distributions remain similar across groups. Minima (column1) tend to be ordered as the median of students who pass (column 6). The first columns also reveal that some groupings might be artificial. The whole distribution is for example scattered out in mathematics from a minimum of 70 to a maximum of 222 while in medicine the range is 189 to 224. Other details are worth mentioning. Medicine and Law are ranked the highest and the difference with other school fields is large. The minimum grade in medicine to pass to the second stage is close to the maximum that was obtained by a successful student in Other fields and somewhat less than in Agrosciences. The first-stage grade among those who passed in Medicine (resp. Law) has a median of 206 (resp. 189) while the next two are Pharmacy, Dentist and Other (175) and Engineering (171) and the minimum is for Agrosciences at 142.

This is why eventually, we chose to analyze only two medical schools in Sobral and Fortaleza.

S.2 Empirical Analysis: Estimates of Grade and Preference Equations

We present here the results of the estimation of grade equations, success probabilities and preferences.

S.2.1 Descriptive statistics

Table S.iii summarises the distribution of grades in the two medical schools Sobral and Fortaleza in three samples: the complete sample, the sample of students who passed the first stage and the sample of students who pass the final stage. Fortaleza is the most competitive one since the median of the first-stage grade of those who passed is equal to 209 while it remains around 200 for Sobral. In conclusion, Fortaleza is more popular among students who apply to a medical school

[S.1] We do not report the second stage grades as they consist in grades in specific fields that are not necessarily comparable across majors.

[S.2] We report for the complete sample the 10th percentile instead of the minimum in order to have a less noisy view of whom are the applicants. There are also a few zeros in the distribution of the initial grades.

although it is not clear whether this popularity comes from preferences or is the result of strategic behavior of students. Our model is an attempt to disentangle those effects.

There are also other interesting differences among applicants to the two schools regarding gender, age, private high school and preparatory course as appears in Table 1. There are more female applicants to Fortaleza than to Sobral. Sobral candidates are older on average and repeat more exams than Fortaleza candidates do and these two variables are highly correlated. The average time spent in private high school is higher in Sobral and it is more likely for a Sobral candidate to have taken a preparatory course.

Among explanatory variables, the initial grade obtained at the national exam $ENEM$ receives a special treatment. When missing (in 5% of cases), we imputed for ability the predicted value of the initial grade $ENEM$ obtained by using all exogenous variables and we denote the result as m_0 to distinguish it from $ENEM$ which is used when computing the passing grades. The administrative rule is to impute 0 when $ENEM$ is missing.

S.2.2 Estimates of grade equations

S.2.2.1 First-stage exam

We report in Table S.iv the results of linear regressions of the first grade equation using three different specifications. We pay special attention to the flexibility of this equation as a function of the ability proxy m_0, which is the observed ranking of each student with respect to his or her fellow students and the best proxy for the success probability at the exams. We use splines in this variable although other non-parametric methods such as Robinson (1988) could be used. A thorough specification search made us adopt a 2-term spline specification, which is reported in the first column of Table S.iv. This specification is used later to predict success probabilities in both schools.

Estimates show that more talented students tend to have better grades in exams, since m_0 has significant positive effects on the first-stage grades although this dependence is slightly non linear as represented in Figure S.ii. Among other explanatory variables, age has a significant negative coefficient in all specifications and this indicates that older students who might have taken one gap year or more are relatively less successful in the first-stage exam. Taking a preparatory course and repeating the entry exam have positive and significant effects on grades by presumably increasing abilities and experience of applicants. In the second specification, we tested for the joint exclusion of parents' education and it is not rejected by a F-test. In the third specification, we restrict the

term in m_0 to be linear. It shows that results related to other coefficients are stable and robust. The set of explanatory variables we choose yields a large R^2 at around 0.72, and this does not vary much across different specifications.

S.2.2.2 Second-stage exam

In the second-stage grade equation, we again sought for flexibility with respect to two variables – the initial stage grade m_0 and the residual from the first-stage grade equation $\hat{u}_1$ as it controls for dependence between stages. Using both non-parametric and spline methods, we found that a two term spline in the initial stage grade m_0 and a linear term in $\hat{u}_1$ were enough in terms of predictive power. Results are reported in Table S.v. First of all, there exists a strong positive correlation between u_1 and u_2, which indicates that unobservable factors on top of the ability proxy affect both equations. All other things being equal, students are more likely to perform well in the second exam if they perform well in the first exam. This may due to some unobservable effort difference or emotional resilience difference between students. The clear significance of the first-stage residual signals that effort for studying might have been exerted by students during the year separating the initial stage exam revealing m_0 and the proper entry exam that we analyze. Yet, our attempts in previous work to construct a more sophisticated model including endogenous effort failed in the sense that the influence of effort never came out significantly. This is why we decided to use the current simpler model. As for other demographic variables, they affect similarly the second-stage grade as the first-stage grade except for gender. Results suggest that females perform significantly better than males in the second-stage exam, while in the first-stage grade gender differences are not significant.

Regarding robustness checks, another concern is heteroskedasticity. We perform Breusch-Pagan tests to see whether there is substantial heteroskedasticity in the grade equations. For the first grade equation, gender is negatively correlated with squared residuals although the global F-test does not reject homoskedasticity at a 1% level (p-value of 3.4%). For the second grade equation, the test rejects homoskedasticity at the 1% level and shows that age, private high school and repetition are significant in explaining squared residuals. This is consistent with the common sense that better high school education and more experience makes your performance steadier. However, in the rest of the paper, we adopt the homoskedasticity assumption since we checked that heteroskedasticity does not generate large differences in the prediction of success probabilities.

S.2.2.3 Success probabilities

Success probabilities are simulated using the empirical distributions of $\hat{u}_1$ and $\hat{u}_2$ and of the thresholds. We run $n_S = 2000$ sets of n simulations by drawing into the estimated empirical distribution of errors, $\hat{u}_1$ and $\hat{u}_2$. We then compute thresholds by solving equation (4) for each of the previous n_S set of simulators. We then replace the integration with respect to the thresholds as in equation (12) and the integration in equation (11) by summing over the set of n_S simulators. We experimented with different numbers of simulations to make sure that simulation error is negligible. This allows to compute simulated success probabilities for each student at both stages of the exam and in both schools.

In addition to summaries of predicted probabilities reported in the text in Table 2, we break down the simulated probability to see the difference between students choosing Fortaleza and choosing Sobral in the original data. In order to see how student choices depend on their actual success probabilities, we compute the odds ratio of success probabilities at both stages. We rank the population with respect to their first-stage grades and construct the grid of odd ratios at all percentiles for both stages. The result is shown in Table S.vi. Some critical quantiles at the top are provided for more detail. The two most important range of percentiles are indeed the 70/75th and 93/95th percentiles since the admission rate at the first exam is slightly less than 30% and the admission rate at the second exam is around 5/7%. Odds ratios are generally larger than 1 and odds ratios are the largest at the middle percentiles for both stages of the exam. It suggests that students who are not at the top of the rankings are making decisions that are affected more by success probabilities than by preferences and might play more strategically. For top students, odd ratios are closer to 1 because preferences matter more for those whose success probabilities are large and strategic effects are less important.

S.2.3 Estimates of school preferences

We build our estimation procedure on the identification results developed in Section 2.3.3 although we adopt two parametric assumptions. First, the distribution of random preferences is assumed to be a normal distribution when both schools yield positive utility to students. Second, the probabilities that only one school has positive utility are described by logistic functions which depend on a smaller set of covariates. Following the notation of Section 2.3.3, we write the probability measure of the regions in Figure 1, for instance the north-east quadrant (that is

$V^S > 0, V^F > 0$) as:

$$\delta^{SF}(X) = \frac{1}{1 + \exp(X\delta^{SF})}.$$

The choice probability is thus derived from equation (7):

$$\Pr(D = S \mid \Delta(Z), X) = \delta^S(X) + \delta^{SF}(X)\Phi(\log(P^S) - \log(P^F) + X\gamma)$$

in which $\Phi(.)$ is the zero mean unit normal distribution[S.3] and the success probabilities P^d are to be replaced by their simulated predictions using grade equations (column 1 of Table S.iv and column 2 of Table S.v) as developed in the previous Section S.2.2.3. In the first part of Table S.vii, we report the estimated preference coefficients and in the second part we present more readable summary statistics of the estimated probabilities of each region, $\delta^{SF}(X)$. There are three different specifications included in this table. The key difference is how explanatory variables enter the specification of δ^S and δ^{SF}. We chose to use two main variables, ability m_0 and Living in Fortaleza as the main drivers of these probabilities and the three columns of Table S.vii include one or both of these variables.

The results are very stable across specifications. As far as δ parameters are concerned, ability significantly affects the probability of the region of jointly positive values, (S, F) (and as a consequence of adding up, also the preference for F alone). Living in Fortaleza decreases preferences for Sobral alone (δ^S) or jointly with Fortaleza (δ^{SF}). The second part of Table S.vii shows that the average probability of preferring Sobral alone (resp. Fortaleza alone) to the outside option is around 0.06 (respetively 0.55). These frequencies stay almost invariant across specifications. These results lead to what is commented in the text.

We now turn to parameters γ that affect preferences of students who prefer both schools to the outside option in the north-east quadrant of Figure 1. The variables, "Living in Fortaleza", Age, Gender (female) and ability, m_0, have a negative impact on the preference for Sobral, the smaller school. In contrast, the number of repetitions have a positive impact on choosing the medical school in Sobral. A well educated father affects positively preferences for the bigger school in Fortaleza while mother's education does not have any significant influence on preferences. This is probably because of the colinearity between parents' educations.

Finally, we tested the maintained hypothesis that performance shocks and preference shocks are independent by introducing the residual $\hat{u}_1$ in this preference equation. The hypothesis cannot be rejected at the 10% level (the p-value is equal to 0.184).

[S.3] As the range of the log probability difference is not the whole real line as in Section 2.3.3, the scale of the error is not identified and its variance is thus normalized to one.

S.3 Complements to the Counterfactual Analysis

S.3.1 Simulated preferences conditional on observed choices

Recall that we describe three groups of students according to their preferences: those only interested in Sobral, those only interested in Fortaleza and those interested in both. The probability of each of these three groups are denoted as $\delta_i^S, \delta_i^F, \delta_i^{SF}$ and these probabilities are heterogeneous across students since they depend on X_i. Let $\varepsilon_i = (\varepsilon_i^{(1)}, \varepsilon_i^{(2)})$ be such that $\varepsilon_i^{(1)} \sim U[0,1]$ and $\varepsilon_i^{(2)} \sim N(0,1)$. The first random term allocates student 0 to one of the three groups i.e. $\varepsilon_i^{(1)} \leq \delta^S(X_i)$ means that she prefers Sobral only to the outside option and $\varepsilon_i^{(1)} \geq \delta^S(X_i) + \delta^{SF}(X_i)$ means that she prefers Fortaleza only to the outside option. If $\varepsilon_i^{(1)} \in (\delta^S, \delta^S + \delta^{SF})$, both schools bring positive utility to her. It is only in the latter case that expected success probabilities matter. Let the function of X_i and the second random term:

$$\ln(V^F(X_i, \varepsilon_i, \zeta)/V^S(X_i, \varepsilon_i, \zeta)) = X_i\gamma + \varepsilon_i^{(2)}$$

be the relative utility in logarithms of Sobral and Fortaleza. Using success probabilities $P_i^S(Z_i, \beta)$ and $P_i^F(Z_i, \beta)$, the decision is determined by:

$$\begin{aligned} D_0(X_i, \varepsilon_i, \zeta, P_i^S, P_i^F) &= S \iff \ln(V^S(X_i, \varepsilon_i, \zeta)/V^F(X_i, \varepsilon_i, \zeta)) + \ln(P_i^S/P_i^F) \geq 0, \\ D_0(X_i, \varepsilon_i, \zeta, P_i^S, P_i^F) &= F \iff \ln(V^S(X_i, \varepsilon_i, \zeta)/V^F(X_i, \varepsilon_i, \zeta)) + \ln(P_i^S/P_i^F) < 0. \end{aligned}$$

S.3.1.1 Simulations of $\varepsilon_{(i)}$ conditional on choices

We shall simulate $\varepsilon_{i,c}$ in its distribution conditional on the observed choice $D_i = S$ (say). This necessarily means that $\varepsilon_i^{(1)} \sim U[0,1]$ conditional on $\varepsilon_i^{(1)} < \delta^S(X_i) + \delta^{SF}(X_i)$ so that we can write:

$$\varepsilon_{i,c}^{(1)} = (\delta^S(X_i) + \delta^{SF}(X_i))\tilde{\varepsilon}_{i,c}^{(1)}$$

in which $\tilde{\varepsilon}_{i,c}^{(1)} \sim U[0,1]$. Then, if $\varepsilon_{i,c}^{(1)} < \delta^S(X_i)$ the observed choice is necessarily $D_i = S$. In the other case, if $\varepsilon_{i,c}^{(1)} > \delta^S(X_i)$, we should condition the drawing of $\varepsilon_0^{(2)}$ on the restriction that:

$$X_i\gamma + \varepsilon_i^{(2)} + \ln(P_i^S/P_i^F) > 0$$

as derived from equation (5). This is easily done by drawing in a truncated normal distribution. Draw $\tilde{\varepsilon}_{i,c}^{(2)}$ into a $U[0,1]$ and write:

$$\varepsilon_{i,c}^{(2)} = \Phi^{-1}(\Phi(-\ln(P_i^S/P_i^F) - X_i\gamma) + (1 - \Phi(-\ln(P_i^S/P_i^F) - X_i\gamma))\tilde{\varepsilon}_{i,c}^{(2)}),$$

or equivalently:

$$\varepsilon_{i,c}^{(2)} = -\Phi^{-1}(\Phi(\ln(P_i^S/P_i^F) + X_i\gamma)(1 - \tilde{\varepsilon}_{i,c}^{(2)})).$$

Adaptations should be made to this construction when the choice is $D_i = F$. In this case,

$$\varepsilon_{i,c}^{(1)} = \delta^S(X_i) + (1 - \delta^S(X_i)\tilde{\varepsilon}_{i,c}^{(1)}, \tilde{\varepsilon}_{i,c}^{(1)} \sim U[0,1],$$

$$\varepsilon_{i,c}^{(2)} = \Phi^{-1}(\Phi(-\ln(P_i^S/P_i^F) - X_i\gamma)(1 - \tilde{\varepsilon}_{i,c}^{(2)})), \tilde{\varepsilon}_{i,c}^{(2)} \sim U[0,1].$$

S.3.2 The counterfactual experiment with lists of two choices

Here we describe how to compute the model of choice between two schools, S and F. This allows four possible choices: (S,F), (F,S), $(S,\varnothing)$, $(F,\varnothing)$ and their respective expected values: U^{SF}, U^{FS}, U^S, U^F. Those values depend on probabilities of success and on thresholds in the following way.

Starting with the singleton lists $(d,\varnothing)$, we have that:

$$U^d = V^d \Pr\{m_1 > t_1^d, m_2 > t_2^d\}$$

as before. For the lists $(d_1,d_2) \in \{(S,F),(F,S)\}$, we use the description of the text to state that:

$$U^{d_1d_2} = V^{d_1} \Pr\{m_1 > t_1^{d_1}, m_2 > t_2^{d_1}\} + V^{d_2} \Pr\{m_1 \in [t_1^{d_1}, t_1^{d_2}), m_2 > t_2^{d_2}\}$$

in which $\Pr\{m_1 \in [t_1^{d_1}, t_1^{d_2})\} = 0$ if $t_1^{d_2} < t_1^{d_1}$. The choice model can now be described by four success probabilities:

$$\begin{cases} P^d = \Pr\{m_1 > t_1^d, m_2 > t_2^d\}, d = S,F \\ P^{d_1d_2} = \Pr\{m_1 \in [t_1^{d_1}, t_1^{d_2}), m_2 > t_2^{d_2}\}, (d_1,d_2) \in \{(S,F),(F,S)\}, \end{cases}$$

which are functions of thresholds t_1^d, t_2^d. Those thresholds remain sufficient statistics in order to derive success probabilities.

S.3.3 Additional Tables and Figures

Figure S.i reports the estimated density of grades distinguishing Sobral and Fortaleza applicants. The first-stage grade density function in Sobral has a regular unimodal shape while Fortaleza has a somewhat irregular modal shape and a fat tail on the left. The second-stage grade density functions, both in Fortaleza and Sobral, are unimodal and the Sobral density function has a fatter

tail on the left-hand side. The truncation at the first-stage plays an important role in removing the fat tails of both densities on the left-hand side.

Figure S.iii shows a picture of those odds ratios at all percentiles. We can visualize individual changes in expected utility in the cutting seat counterfactual in Figure S.v . Figure S.vi (respectively Figure S.viii) report changes in success probabilities for Fortaleza in the two choice experiment (resp. timing change). Changes in expected utility for the two choice experiment (resp. timing change) are graphed in Figure S.vii (resp. Figure S.ix).

Other references:

Instituto Nacional de Estudos e Pesquisas (INEP), 2008, "Sinopses estatísticas da educação superior", available at http://www.inep.gov.br/superior/censosuperior/sinopse/.

Olive, A. C., 2002, "Histórico da educação superior no Brasil", in: Soares, M. S. A. (coord.). *Educação superior no Brasil.* Brasília, p. 31-42.

Robinson, P. M., 1988, "Root-N-consistent semiparametric regression", *Econometrica,* 56:931-954.

Table S.i: Number of applications, number of positions and success probabilities

Groups of majors	Applications	% Pass 1st stage	% Pass 2nd stage	Positions
Accountancy	1,374	40%	13%	185
Administration	2,474	29%	8%	200
Agrosciences	2,996	41%	13%	390
Economics	1,516	37%	11%	160
Engineering	2,648	40%	14%	360
Humanities	4,897	17%	9%	430
Law	3,625	20%	5%	180
Mathematics	2,425	37%	11%	269
Medicine	4,024	23%	6%	230
Other	2,778	21%	6%	165
Pharmacy, Dentist & Other	5,312	24%	6%	320
Physics & Chemistry	1,734	58%	20%	349
Social Sciences	5,574	26%	7%	385

Source: Vestibular cross section data in 2004.

Table S.ii: Summary statistics of first stage grades in the samples of (1) all, (2) pass after first stage (3) definite pass after second stage (The order of subgroups is given by the median of the first stage grades in the pass sample, column 6)

Subgroup	10th percentile	Min	Min	Median			Maximum		
	All	Firststage	Pass	All	First stage	Pass	All	First stage	Pass
Agrosciences	71.1	91.2	100.1	106.9	128.1	141.6	192.6	192.6	192.6
Other	66.1	102.1	104.8	102.0	136.7	143.3	187.5	187.5	187.5
Physics & Chemistry	76.8	33.0	50.0	115.2	128.9	144.6	210.2	210.2	210.2
Humanities	67.9	96.3	99.2	104.2	133.6	147.1	203.3	203.3	203.3
Social Sciences	68.9	101.0	102.0	109.4	138.6	147.9	214.3	214.3	214.3
Accountancy	80.5	120.5	122.9	120.3	139.9	151.5	200.7	200.7	198.6
Economics	71.8	113.3	121.1	110.9	133.8	152.3	209.2	209.2	209.2
Administration	68.6	108.5	121.0	108.7	140.9	154.2	212.3	212.3	212.3
Mathematics	75.8	70.3	73.0	122.1	151.7	158.9	222.1	222.1	222.1
Engineering	84.3	130.2	137.6	133.7	156.3	170.8	210.5	210.5	210.5
Pharmacy, Dentist & Other	73.8	142.0	143.8	123.0	160.2	175.1	208.1	208.1	208.1
Law	77.4	165.5	168.0	139.5	179.4	189.5	215.2	215.2	215.2
Medicine	89.6	182.0	186.9	169.0	200.2	206.4	224.3	224.3	224.3

Source: Vestibular cross section data in 2004.

Table S.iii: Summary statistics of initial grades in the samples of (1) all, (2) pass after first stage (3) definite pass after second stage (Medicine sample composed by two majors: Sobral and Fortaleza)

Major	10th percentile	Min	Min	Median			Maximum			Observations
	All	Firststage	Pass	All	First stage	Pass	All	First stage	Pass	
Sobral	121.57	185.05	186.86	171.76	196.52	200.76	214.38	214.38	214.19	542
Fortaleza	93.05	193.67	193.86	172.95	202.57	208.57	224.29	224.29	224.29	2325

Source: Vestibular cross section data in 2004.

Table S.iv: First stage exam grade equation

	Specification 1	Specification 2	Specification 3
(Intercept)	27.28	26.59	78.00
	(3.59)***	(3.66)***	(2.23)***
Female	0.54	0.47	0.44
	(0.40)	(0.40)	(0.40)
Age	-0.86	-0.86	-0.87
	(0.11)***	(0.11)***	(0.11)***
Special high school	-6.54	-6.46	-6.65
	(1.73)***	(1.74)***	(1.75)***
Private high school	2.67	1.99	2.14
	(0.56)***	(0.67)***	(0.65)***
Preparatory course	1.67	1.51	1.51
	(0.48)***	(0.50)***	(0.50)***
Repetitions	2.83	2.86	2.87
	(0.35)***	(0.37)***	(0.37)***
Ability(m_0)			12.96
			(0.65)***
Spline(1)(m_0 Residual)	48.18	48.72	
	(4.03)***	(4.00)***	
Spline(2)(m_0 Residual)	89.17	89.20	
	(4.54)***	(4.49)***	
Living in Fortaleza	3.72	3.69	3.60
	(0.66)***	(0.67)***	(0.67)***
Living in Fortaleza*Ability	2.02	1.98	1.93
	(0.68)***	(0.66)***	(0.66)***
Mother's education		0.11	0.10
		(0.31)	(0.31)
Father's education		0.33	0.33
		(0.29)	(0.29)
R^2	0.7196	0.7199	0.7198

[1] Living in Fortaleza is a dummy which indicates whether the student is currently living in Fortaleza.

[2] Standard errors are between brackets and * (resp. ** and ***) denotes significance at a 10 (resp 5 and 1) percent level.

[3] The coefficients and their standard errors are computed by bootstrapping the procedure 499 times using the empirical distribution of residuals.

Table S.v: Second stage exam grade equation

	Specification 1	Specification 2
(Intercept)	232.65	171.69
	(13.72)***	(20.08)***
Female	7.36	7.16
	(2.27)***	(2.28)***
Age	-3.90	-3.96
	(0.75)***	(0.74)***
Special high school	-11.48	-12.68
	(21.76)	(20.25)
Private high school	8.82	9.11
	(4.15)***	(4.27)***
Preparatory course	9.15	8.95
	(3.38)***	(3.44)***
Repetitions	13.91	14.14
	(2.21)***	(2.25)***
u_1 (m_1 residual)	2.51	
	(0.18)***	
Spline(1)(m_1 residual)		68.09
		(28.38)***
Spline(2)(m_1 residual)		153.07
		(11.47)***
Ability (m_0)	35.23	35.05
	(3.52)***	(2.63)***
R^2	0.2284	0.2286

[1] Standard errors are computed by bootstrapping 499 times using both grade equations and the empirical distributions of residuals.
[2] Standard errors are between brackets and starred signs are defined as in Table S.iv.

Table S.vi: Odds ratio of success probabilities

Percentile	First stage	Second stage
10	1.00	2.66
20	1.00	1.60
30	1.47	1.08
40	0.86	1.61
50	1.07	2.26
60	1.33	3.43
70	1.29	5.34
75	1.18	5.62
80	1.15	5.22
85	1.14	4.41
90	1.10	3.73
95	1.03	3.37
100	1.00	1.74

[1] The first column reports the odds ratio of success probabilities at the first stage between subsamples of those who choose Sobral and choose Fortaleza $\frac{p1sob|d_i=s}{p1fort|d_i=s} / \frac{p1sob|d_i=f}{p1fort|d_i=f}$.

[2] The second column reports the odds ratio of final success probability at the second stage between subsamples of those who choose Sobral and choose Fortaleza $\frac{psob|d_i=s}{pfort|d_i=s} / \frac{psob|d_i=f}{pfort|d_i=f}$.

[3] Percentiles in rows are computed using first stage exam grades.

Table S.vii: Estimated preferences for Sobral's medical school

Parameters

	Specification 1	Specification 2	Specification 3
δ_0^S	-2.782	-1.132	-1.167
	(0.303)***	(0.309)***	(0.277)***
$\delta_{m_0}^S$	0.261	0.166	
	(0.189)*	(0.146)*	
$\delta_{LivinginFortaleza}^S$		-1.815	-1.586
		(0.522)***	(0.283)***
δ_0^{SF}	-0.453	0.521	0.484
	(0.271)*	(0.312)**	(0.296)**
$\delta_{m_0}^{SF}$	0.979	1.062	
	(0.198)***	(0.179)***	
$\delta_{LivinginFortaleza}^{SF}$		-1.314	-1.225
		(0.326)***	(0.393)***
Intercept	0.075	0.334	0.0482
	(0.707)	(0.387)	(0.393)
Ability (m_0)	-1.079	-0.977	-0.020
	(0.261)***	(0.247)***	(0.095)
Living in Fortaleza		-0.248	-0.558
		(0.301).	(0.314)**
Female	-0.325	-0.240	-0.373
	(0.139)***	(0.152)***	(0.186)***
Age	-0.038	-0.045	-0.048
	(0.039)	(0.027)**	(0.026)**
Repetitions	0.688	0.851	0.911
	(0.144)***	(0.141)***	(0.210)***
Father's education	-0.278	-0.257	-0.341
	(0.111)***	(0.119)***	(0.154)***
Mother's education	0.084	0.046	0.216
	(0.106)	(0.114)	(0.145)

Proportions

		Specification 1	Specification 2	Specification 3
δ^S	Min	0.022	0.021	0.050
	Mean	0.060	0.057	0.066
	Max	0.122	0.248	0.196
δ^{SF}	Min	0.015	0.016	0.365
	Mean	0.385	0.412	0.386
	Max	0.816	0.852	0.559
δ^F	Min	0.062	0.027	0.245
	Mean	0.555	0.531	0.548
	Max	0.963	0.962	0.585

[1] The second part of the table reports summaries of the probabilities of being in one of the three regions of Figure 1.
[2] The coefficients and their standard errors are computed by bootstrapping 499 times the whole procedure (including grade equations).
[3] Standard errors are between brackets and starred signs are defined as in Table S.iv.

Table S.viii: Cutting seats: Robustness

Expected	$\mu = 0.8$		$\mu = 0$		$\mu = 1$	
Final Grade	mean	s.d.	mean	s.d.	mean	s.d.
0% -50%	-0.00029	0.00116	-0.00026	0.00105	-0.00030	0.00119
50%-60%	0.00001	0.00744	0.00032	0.00697	-0.00007	0.00756
60%-70%	0.00674	0.01655	0.00715	0.01594	0.00664	0.01671
70%-80%	0.03122	0.02770	0.03143	0.02727	0.03117	0.02781
80%-82%	0.04070	0.02813	0.04074	0.02806	0.04069	0.02815
82%-84%	0.05491	0.02896	0.05478	0.02917	0.05494	0.02891
84%-86%	0.07304	0.03128	0.07302	0.03129	0.07305	0.03128
86%-88%	0.06124	0.03374	0.06117	0.03381	0.06126	0.03372
88%-90%	0.07932	0.03072	0.07917	0.03106	0.07936	0.03063
90%-92%	0.09239	0.03272	0.09230	0.03293	0.09241	0.03267
92%-94%	0.08806	0.04041	0.08779	0.04087	0.08812	0.04029
94%-96%	0.11009	0.03125	0.11000	0.03153	0.11011	0.03118
96%-98%	0.11178	0.03456	0.11163	0.03498	0.11181	0.03446
98%-100%	0.08939	0.04669	0.08917	0.04707	0.08945	0.04660
$\mathbf{E}(\Delta U_i)$						
	0.01966		0.01975		0.01964	
$\mathbf{s.d.}(\Delta U_i)$						
	0.03785		0.03775		0.03787	
$\mathbf{Pr}(\Delta U_i > 0)$						
	0.4363		0.4363		0.4363	

[1] Results as in Table 4 using different values of μ.
[2] See notes of Table 4

Figure S.i: Density plots of the grades

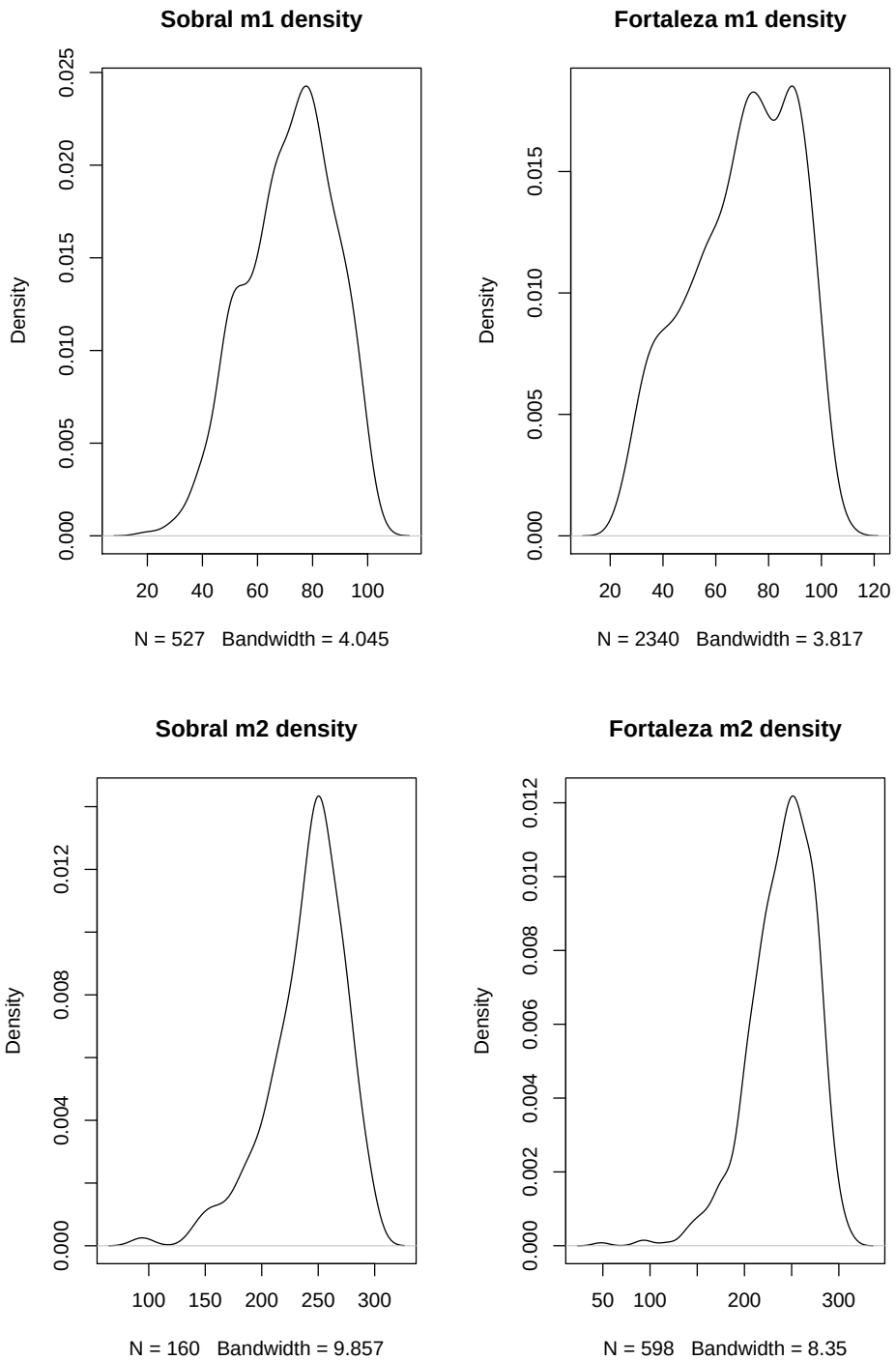
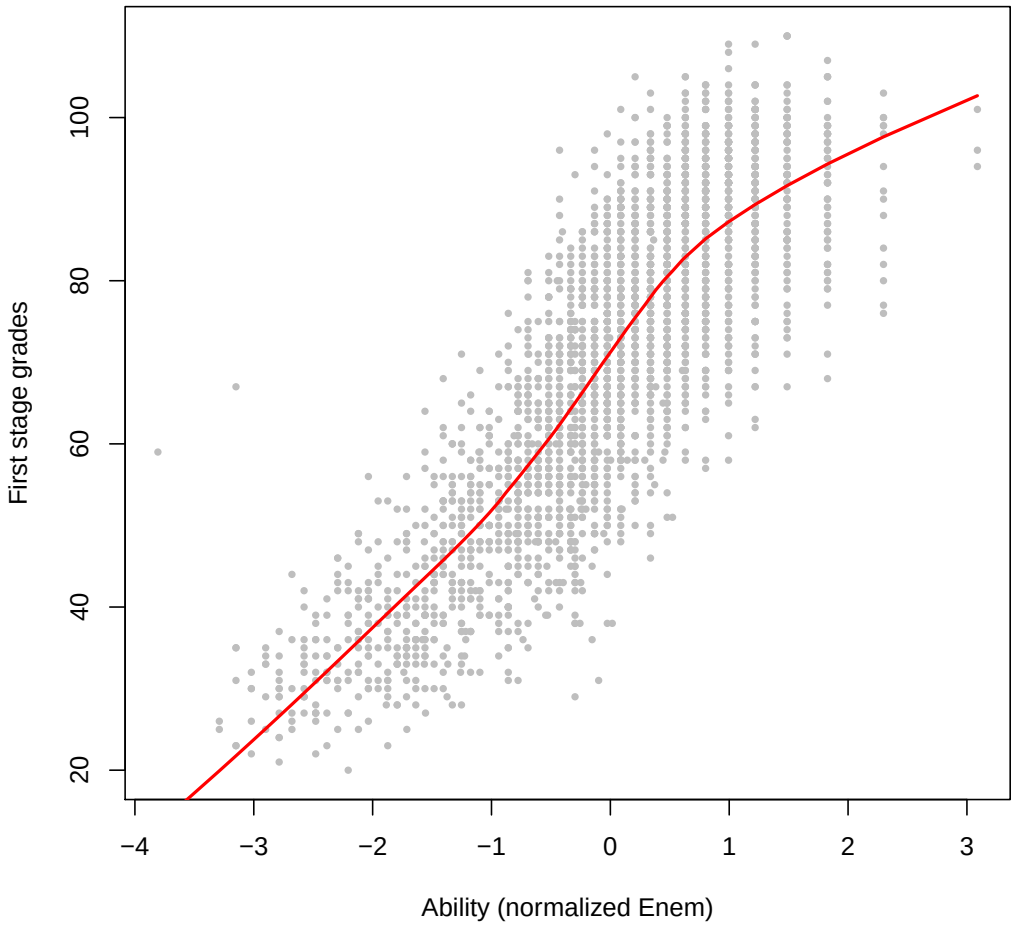
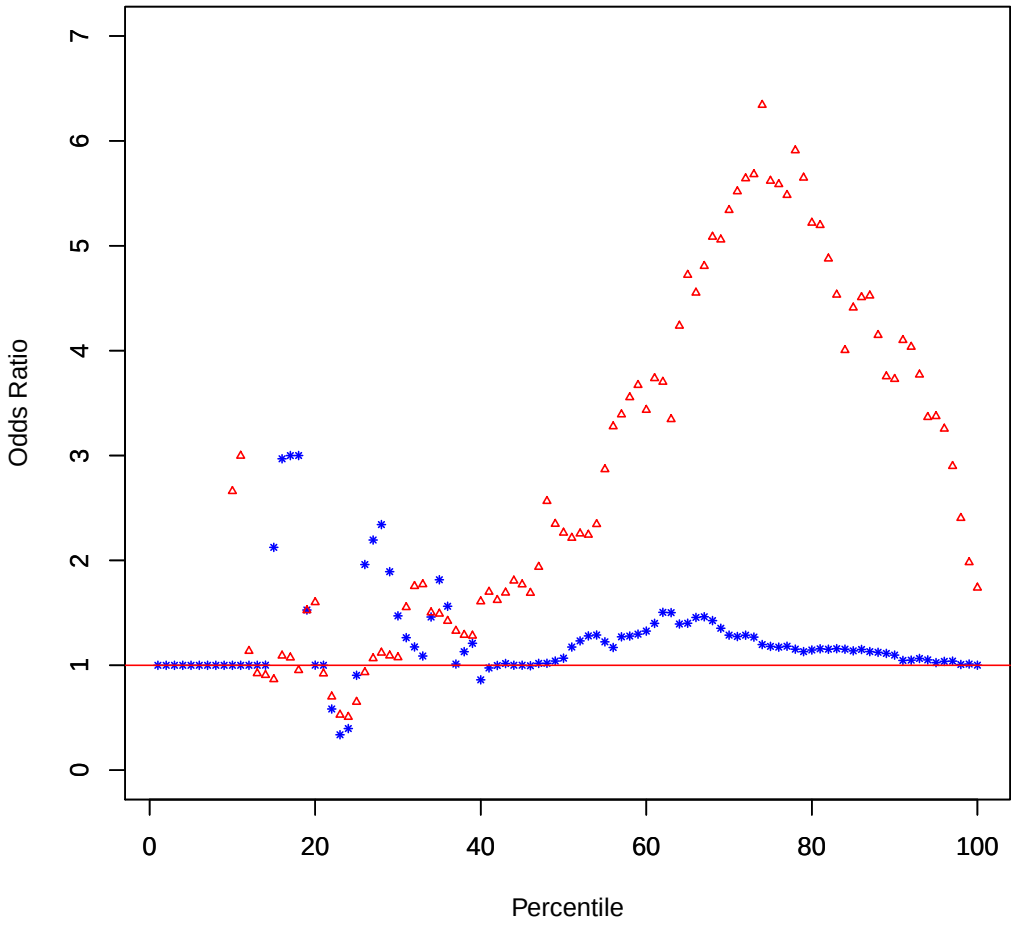


Figure S.ii: The relation between ability and first stage grades



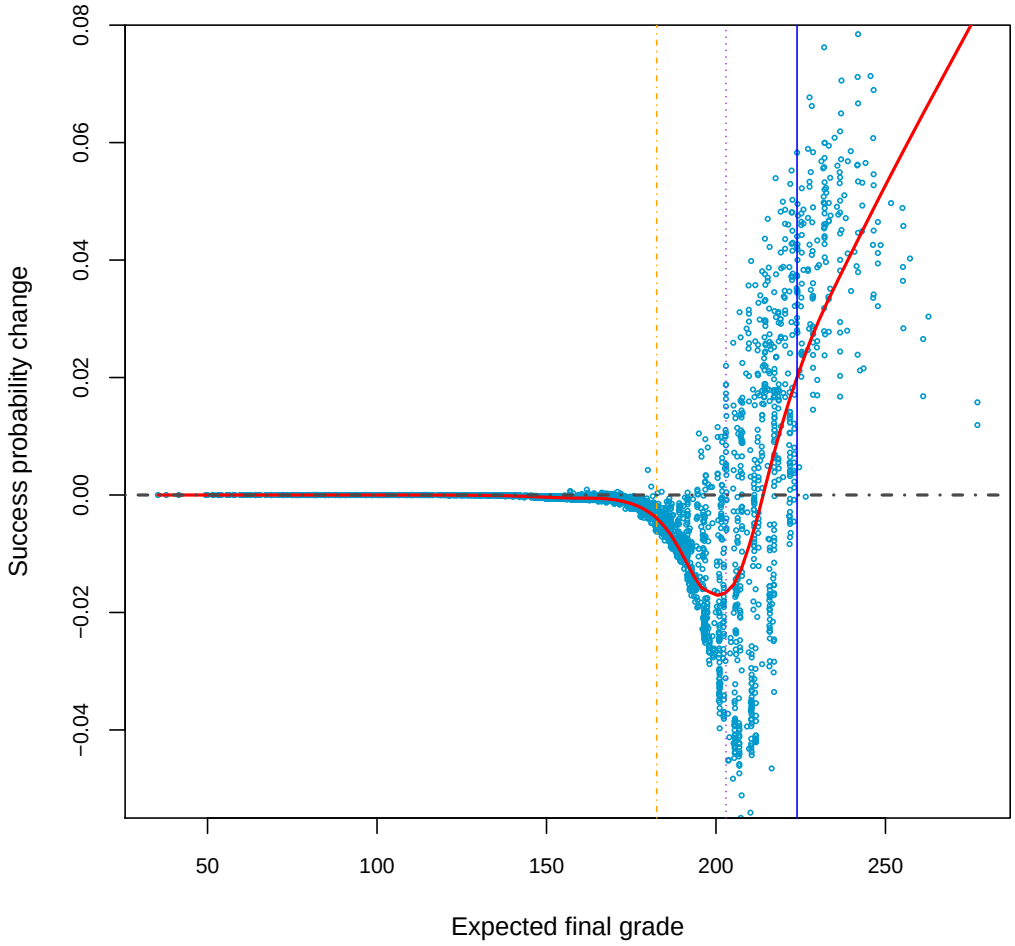
[1] The round grey points are the scatter plots of first stage grade on ability (normalized Enem); [2] The curve is the LOWESS curve of first stage grade on ability (normalized Enem).

Figure S.iii: The Odds ratio plot of simulated success probabilities



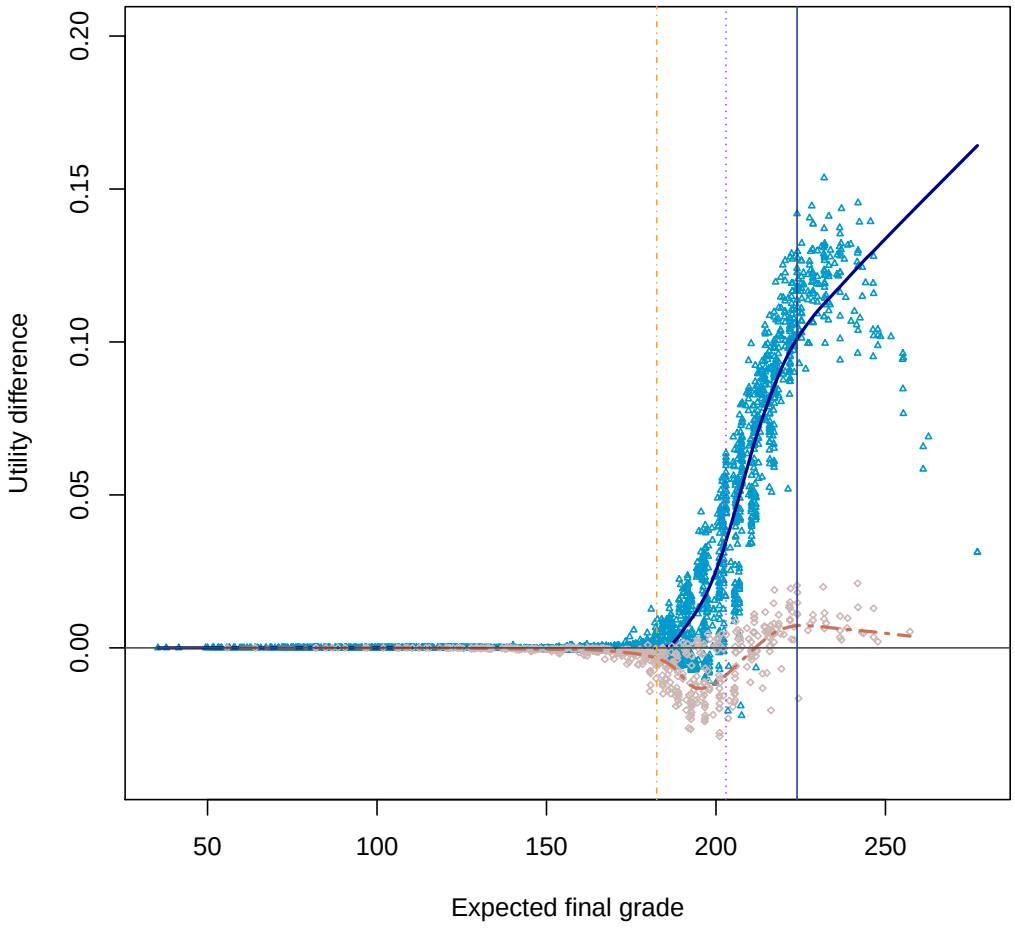
[1] The star points are odds ratio at the first stage ; [2] the triangular points are the odds ratio at the second stage; [3] percentiles are computed using first stage grades.

Figure S.iv: Cutting seats: Changes of success probabilities in Fortaleza



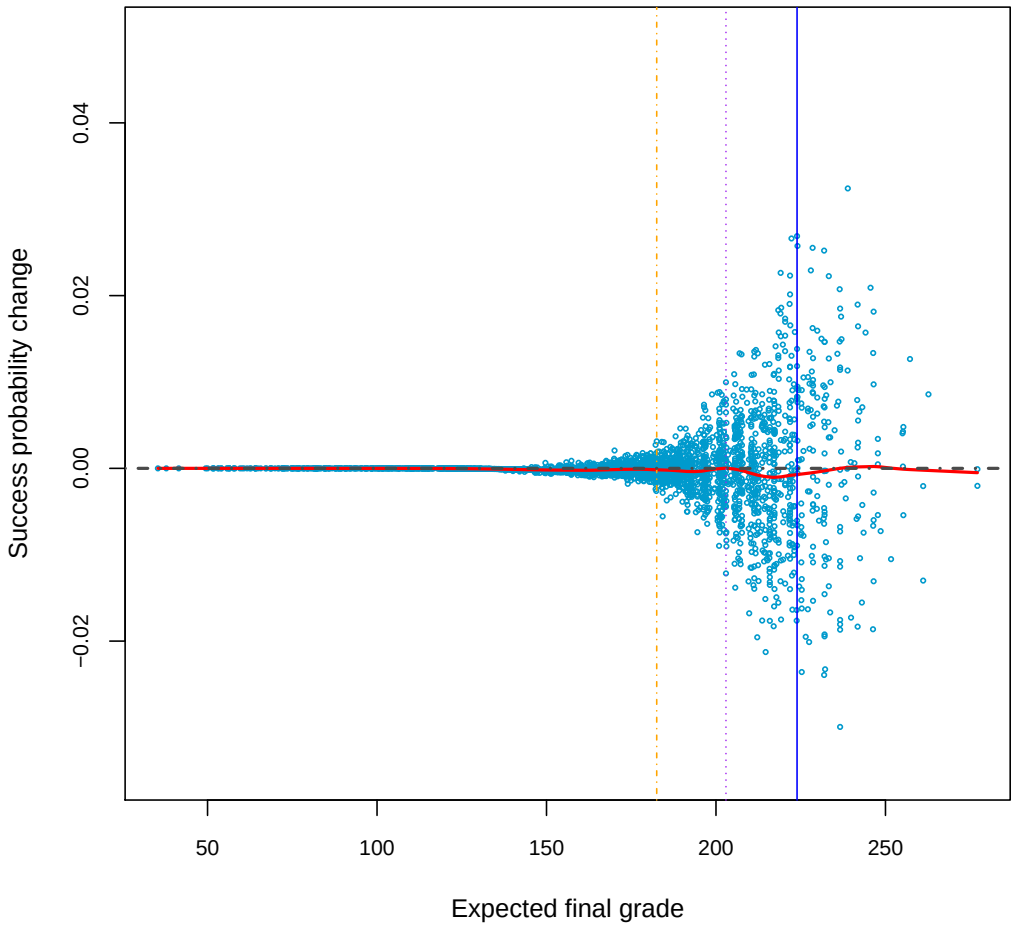
See notes of Figure 2

Figure S.v: Cutting seats: Expected utility changes



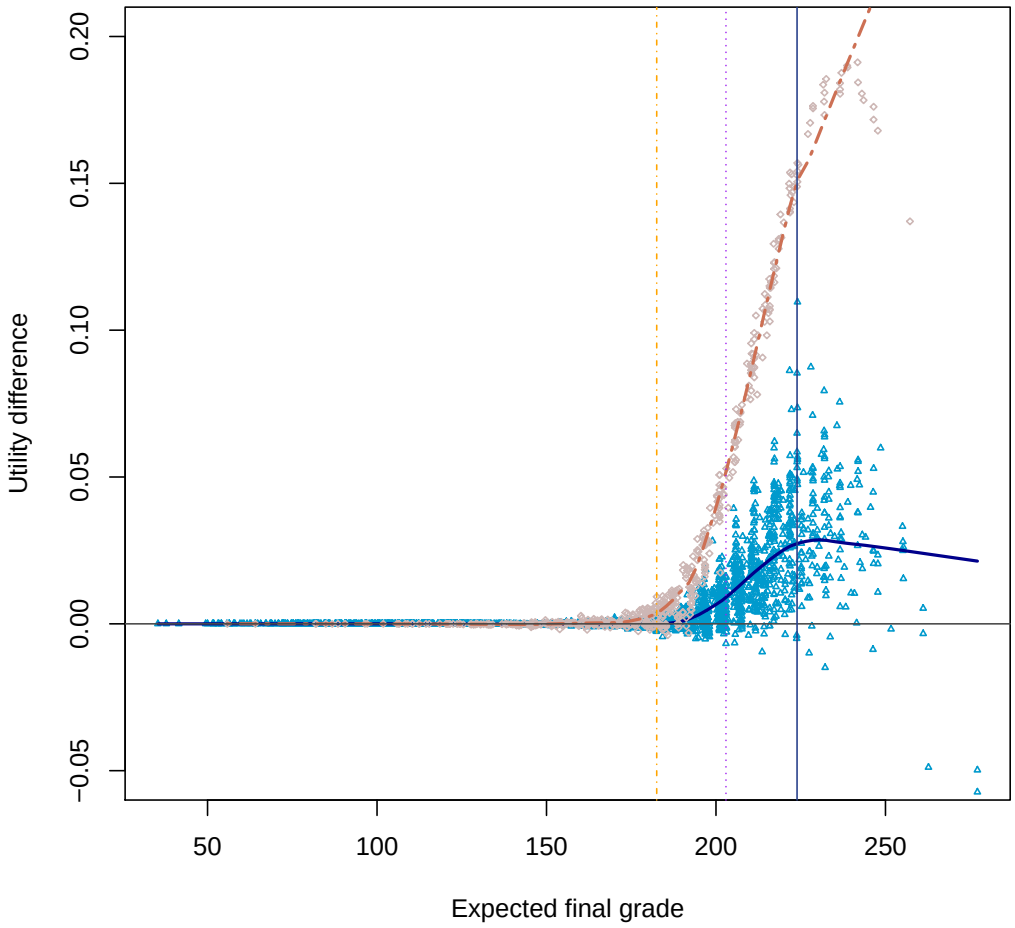
[1] the grey squares (resp. blue triangles) report changes in expected utilities and expected final grades for those who choose Sobral (resp. Fortaleza) in the original system. [2] the red line is the 0 level; [3] the vertical lines are as in Figure S.iv.

Figure S.vi: Two choices: Success probability change in Fortaleza



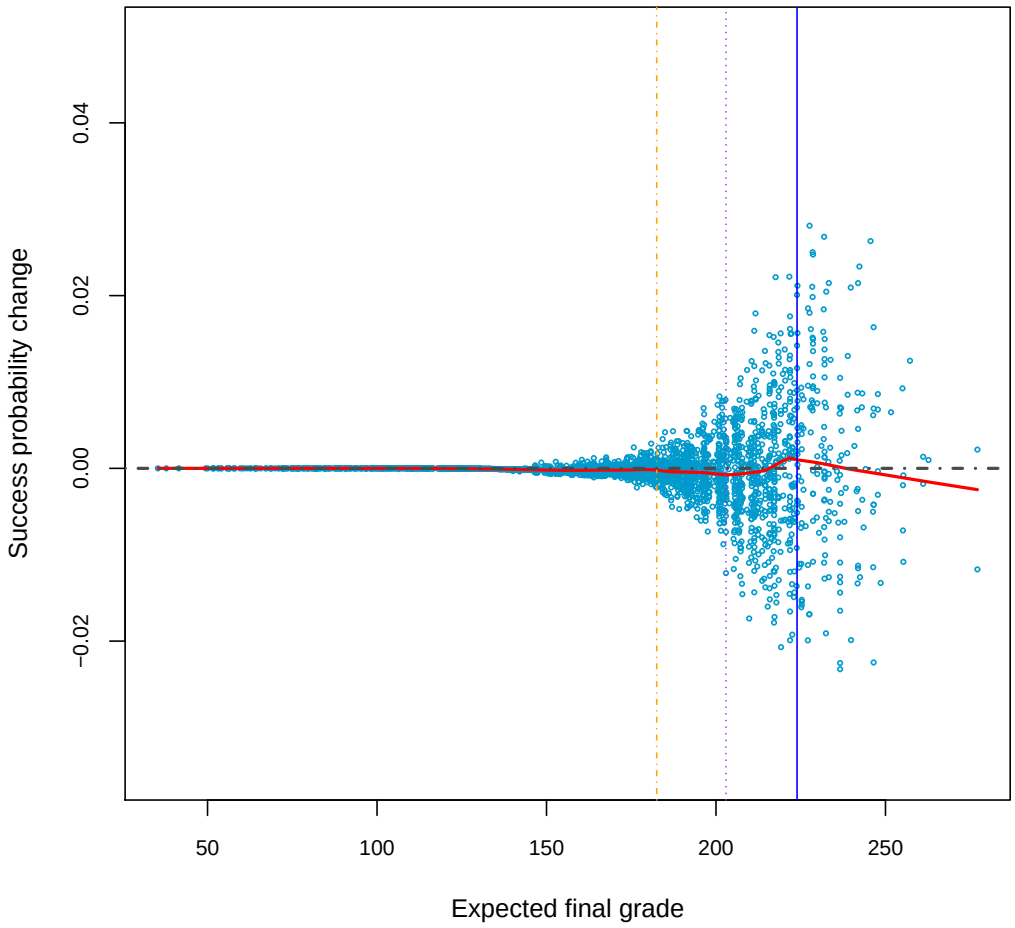
Notes: See notes of Figure 2

Figure S.vii: Two choices: Expected utility changes



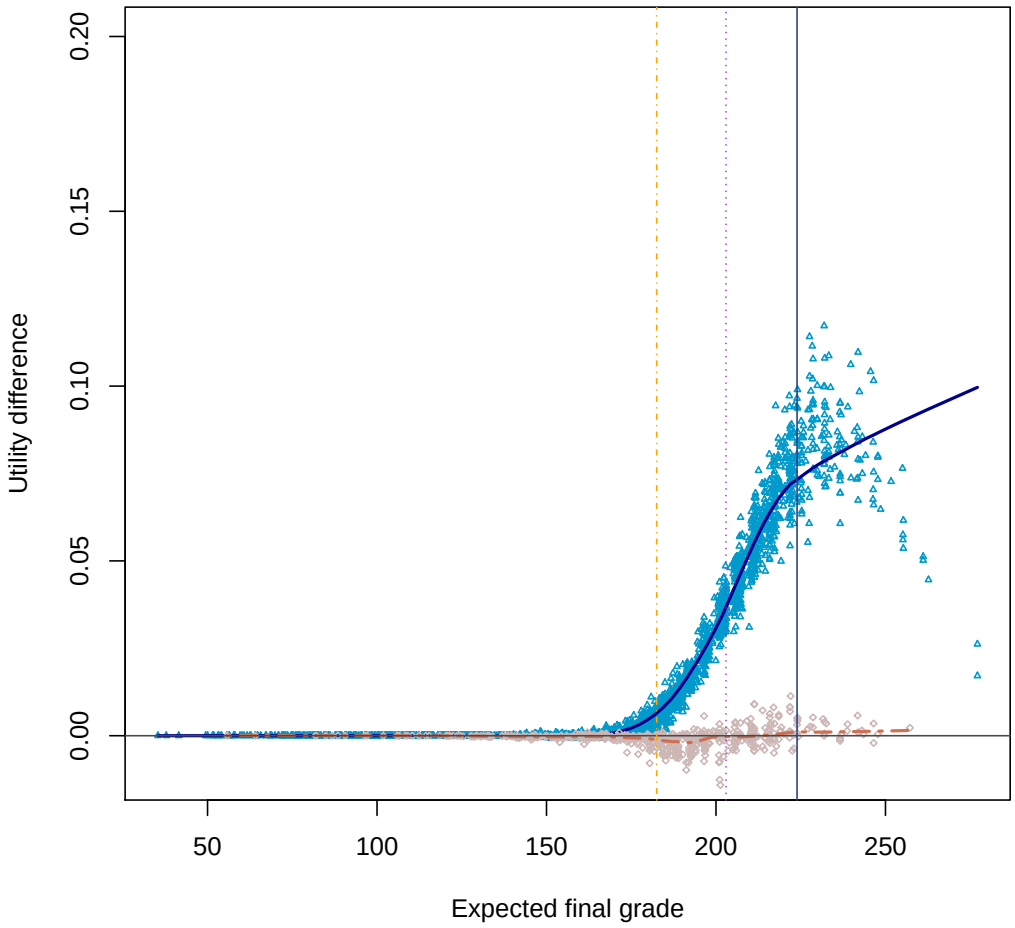
Notes: See notes of Figure S.v

Figure S.viii: Timing change: Success probability changes in Fortaleza



Notes: See notes of Figure 2

Figure S.ix: Timing change: Expected utility changes



Notes: See notes of Figure S.v