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"Mergers, investments and demand expansion"

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Mergers, investments and demand expansion*

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Abstract

In this paper, we study the impact of a merger to monopoly on prices and investments. Two single-product firms compete in prices and coverage for a new technology. In equilibrium, one firm covers a larger territory than its competitor with the new technology, leading to single-product and multi-product zones, and sets a higher uniform price. If the firms merge, the merged entity can set different prices and coverage for the two products. We find that the merger raises prices and total coverage, but reduces the coverage of the multi-product zone. We also show that the merger can increase total welfare and consumer welfare.

Keywords: horizontal mergers; investments; competition.
JEL codes: D43; L13; L40.

1 Introduction

In a number of recent merger cases in Europe among mobile network operators, the potential impact of mergers on investment has been hotly debated.[1] Operators claim that mergers in the sector can have a positive effect on investment, while the European Commission has expressed the view that mergers are detrimental to innovation absent efficiency gains.[2]

In this paper, we develop a simple model where a merger to monopoly raises total investment despite the absence of synergies. We consider a coverage-price game, where two firms decide on prices and coverage of a new technology over a territory.[3] When firms are separate, one firm covers a larger share of the territory than its rival. When a merger-to-monopoly takes place, the merged entity raises all prices, increases total coverage with a positive effect on welfare, and reduces coverage

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[1]See, for example, the European Commission decision on the Hutchison 3G/Orange merger case in Austria, COMP/M.6497 (December 2012).

[2]Genakos et al. (2017) provide evidence that concentration in the mobile market may indeed imply a trade-off between prices and investments. Based on panel data for the period 2002-2014 covering 33 countries from Europe and the OECD, they find that a 4-to-3 merger raises prices by 16% on average, but at the same time increases investments by operator by 19%. However, the evidence of a positive effect of mergers on investment is not totally conclusive, since they find that total investment is not affected significantly by the merger.

[3]In the telecommunication industry, roll-out of duplicated infrastructures occurs for mobile 4G and FTTH (for instance, in France and in Spain).

for the multi-product zone, which can either harm welfare (due to lower variety) or increase it (due to the business-stealing effect). We provide an example where total welfare and consumer surplus can increase with the merger.

Our paper is related to the broad literature on competition and innovation (see Gilbert (2006) for a recent survey). This literature has considered different measures of competition,[4] but surprisingly very few papers have considered mergers (see the discussion by Shapiro (2012)). Motta and Tarantino (2017) argue that absent spillovers or synergies, the reduction of output by the merged entity induces a reduction of cost-reducing investment. Federico et *al*. (2017a) argue in the context of a symmetric race for a new product that internalization by the merged firm of cannibalization of sales by simultaneous innovations leads to a reduction of innovation effort.[5] Denicolò and Polo (2017) show however that their conclusion holds only if the R&D technology exhibits sufficient decreasing returns to scale, and that the reverse holds otherwise. We identify a new effect that explains a positive impact of mergers on innovative investment. Other articles pointing to different channels which may lead to such a positive effect are Marshall and Para (2017), in the context of a dynamic model of leadership, and Loertscher and Marx (2017), in a model with buyer power.[6] Our paper also builds on the literature on universal service in network industries, which focuses on regulatory issues (see, among others, Valletti et *al*., 2002; Hoernig, 2006; Gautier and Wauthy, 2010).

The model is presented in Section 2 and analyzed in Section 3. All the proofs are in Appendix.

2 Model

Consider a geographic market represented by a half-line from 0 to $\bar{z}$. Two operators, 1 and 2, deploy a new technology. Initially, the market is not covered at all and there is no alternative old-generation technology. The two operators have the same development cost $c(x)$ to deploy the technology in location x, where $c(x)$ is increasing. We define as

$$C(z) = \int_0^z c(x)\, dx$$

the total cost of covering the locations from 0 to z. We assume that $c(0)$ is small enough and $\lim_{x\to\bar{z}} c(x)$ is large enough so that in a duopoly both firms invest and no firm covers the whole market (see footnote 7). We also assume that firm $i = 1, 2$ deploys the technology in all locations $x \leq z_i$ where $z_1 \geq z_2$.

The operators offer differentiated products, with product i designating firm $i = 1, 2$'s product. In each location x, the single-product monopoly demand (for product 1) is $D_s(p_1)$, while the multi-product demand is $D_1(p_1, p_2)$ for product 1 and $D_2(p_2, p_1)$ for product 2. We normalize the firms' (constant) marginal cost of production to 0.

We adopt the linear demand model of Dixit (1979) and Singh and Vives (1984). The utility of the representative consumer is given by

$$U(q_1, q_2, m) = \alpha(q_1 + q_2) - \frac{1}{2}(q_1^2 + q_2^2) - \gamma q_1 q_2 + m,$$

where m is the numeraire good and $\gamma \in [0, 1)$ represents the degree of substitutability between products 1 and 2. The products are unrelated if $\gamma = 0$ and become perfect substitutes when $\gamma \to 1$. In the paper we will assume that $\gamma \leq 0.73$ to ensure the existence of a pure-strategy equilibrium.

[4] See, for example, Vives (2008) for a theoretical analysis.

[5] Federico et *al.* (2017b) develop a more general model, allowing for pre- as well as post-innovation competition.

[6] Davidson and Ferrett (2007) and Motta and Tarantino (2017) argue that sufficient synergies may stimulate post-merger investment.

If both goods 1 and 2 are available to the consumer, utility maximization yields the following multi–product demands for firms 1 and 2 (provided that quantities are positive),

$$D_1(p_1, p_2) = \frac{\alpha - p_1 - \gamma\,(\alpha - p_2)}{1-\gamma^2} \text{ and } D_2(p_1, p_2) = \frac{\alpha - p_2 - \gamma\,(\alpha - p_1)}{1-\gamma^2}.$$

If only good 1 is available, the single-product demand for this good is

$$D_s(p_1) = \alpha - p_1.$$

For future use, we define the single-product monopoly price and the multi-product duopoly price as

$$p^m = \frac{\alpha}{2} \text{ and } p^d = \alpha\frac{1-\gamma}{2-\gamma},$$

respectively.

We follow Motta and Tarantino (2017) in considering that pricing occurs before investment decisions are publicly observed. We thus study a coverage-price game, where firms decide simultaneously on a coverage z_i for the technology and on a price p_i charged uniformly in all covered locations, with $i = 1, 2$. In the absence of merger, firm 1 and firm 2 are single-product firms. In the case of a merger, the merged entity offers the two products, 1 and 2, with potentially different coverage.

3 Analysis

We first determine the equilibrium of the coverage-price game without merger, and then with the merger. We finally compare the two equilibria to analyze the impact of a merger to monopoly on prices, coverage, and social welfare.

3.1 Without merger

Without a merger, firms 1 and 2 compete in coverage and prices.[7] Assuming that $z_1 \geq z_2$, firm 2 is competing on all its covered territory and faces the demand $z_2 D_2\,(p_2, p_1)$ over all locations, while firm 1 faces competition only on part of its territory, as it is the sole seller on all locations between z_2 and z_1, and faces the demand $z_2 D_1\,(p_1, p_2) + (z_1 - z_2)\, D_s\,(p_1)$. Firms' profits are then given by

$$\Pi_1 = z_2 p_1 D_1\,(p_1, p_2) + (z_1 - z_2)\, p_1 D_s\,(p_1) - C\,(z_1)\,, \tag{1}$$

for firm 1, and

$$\Pi_2 = z_2 p_2 D_2\,(p_1, p_2) - C\,(z_2) \tag{2}$$

for firm 2.

Clearly, the pricing decision of firm 2 is the same as in the standard multi-product duopoly game, leading to the best-reply

$$p_2 = BR\,(p_1) = \frac{\alpha + \gamma\,(p_1 - \alpha)}{2}. \tag{3}$$

[7]For an interior solution, we assume that $c\,(0) < \alpha^2\,(1-\gamma)\,/\,(2-\gamma)^2\,(1+\gamma)$ and $c\,(\bar{z}) > \alpha^2/4$.

Firm 1 sets a uniform price for all its covered territory, balancing the revenue from the single-product part of it, where it faces no competition, and the revenue from the multi-product part, where it competes with firm 2. We find that:

$$p_1 = BR_1\left(p_2, z_1, z_2\right) = \frac{\alpha + \theta\gamma\left(p_2 - \alpha\right)}{2}, \tag{4}$$

where

$$\theta \equiv \frac{\frac{\frac{z_2}{z_1 - z_2}}{1-\gamma^2}}{1 + \frac{\frac{z_2}{z_1 - z_2}}{1-\gamma^2}} \in [0, 1).$$

Thus, the larger the coverage by firm 2, the closer is p_1 to the duopoly best-reply price.

Considering the choice of coverage by the two firms, we first notice that (see Appendix), because expanding coverage yields higher returns for locations above z_2 (single-product) than below z_2 (multi-product), there is no equilibrium with symmetric coverage.

With a smaller coverage, firm 2 trades off the investment cost with the revenue of additional coverage in the multi-product zone. The equilibrium coverage for firm 2 is then the solution of

$$c(z_2) = p_2 D_2(p_1, p_2). \tag{5}$$

Firm 1, however, takes into account the difference in revenue between the single-product and multi-product zones. As the multi-product zone is infra-marginal, the marginal return on coverage is given by its profit in single-product locations, which yields:

$$c(z_1) = p_1 D_s(p_1). \tag{6}$$

We then obtain:

Proposition 1 *Under separation and sufficient product differentiation, a unique pure strategy equilibrium exists with* $z_1^S > z_2^S$ *and* $p^d < p_2^S < p_1^S < p^m$.

The firm with the largest coverage trades off between charging a high price to exploit its market power in single-product locations, and charging a low price to compete with its rival in multi-product locations. It thus sets a price in between its multi-product best-reply and the single-product monopoly price p^m, which is higher than the price set by the firm with a lower coverage. By strategic complementarity, both firms then set a price that is higher than the multi-product duopoly price p^d. Formally, prices and coverage levels are:

$$c\left(z_1^S\right) = p_1^S\left(\alpha - p_1^S\right) = \frac{\alpha^2\left(2 + \theta\gamma\right)\left[2 - \theta\gamma\left(1 + \gamma\right)\right]}{\left(4 - \theta\gamma^2\right)^2} \tag{7}$$

and

$$c\left(z_2^S\right) = p_2^S D_2(p_1^S, p_2^S) = \left(\alpha\frac{2 - \gamma - \theta\gamma^2}{4 - \theta\gamma^2}\right)^2 \frac{1}{1-\gamma^2}, \tag{8}$$

with $\theta = \theta\left(z_1^S, z_2^S\right)$.

3.2 Merger-to-monopoly

If firms 1 and 2 merge, the merged entity has two products, 1 and 2, for which it can decide on different coverage levels. Assume that product 1 has a larger coverage, that is, $z_1 \geq z_2$. Given coverage z_1 and z_2 and prices p_1 and p_2 for products 1 and 2, respectively, the firm's profits are:

$$\Pi = (z_1 - z_2)p_1 D_s(p_1) + z_2 \left[p_1 D_1(p_1, p_2) + p_2 D_2(p_1, p_2)\right] - C(z_1) - C(z_2).$$

We find that the profit-maximizing prices are $p_1^M = p_2^M = p^m = \alpha/2$. The equilibrium coverage levels for products 1 and 2 are then given by

$$c\left(z_1^M\right) = \pi^m = \frac{\alpha^2}{4},\ c\left(z_2^M\right) = \max\left\{\left(\frac{1-\gamma}{1+\gamma}\right)\pi^m, c\left(0\right)\right\}.$$

The following proposition summarizes this analysis.

Proposition 2 *The merged firm sets* $p_1^M = p_2^M = p^m$*, and* $0 \leq z_2^M < z_1^M = c^{-1}\left(\pi^m\right)$*.*

As product substitutability decreases, the firm expands the range where it offers both products, while still offering only one product in the more costly locations.

3.3 Impact of merger on prices, coverage, and social welfare

We can now compare the equilibrium with and without the merger, in terms of prices and coverage.

Proposition 3 *A merger to monopoly raises prices and total coverage, and reduces the coverage of the multi-product zone.*

Thus, a merger to monopoly has a priori an ambiguous effect on social welfare. On the one hand, it leads to higher prices as expected. On the other, it raises the incentives to invest in total coverage. The multi-product coverage is however lower with the merger, and therefore there are locations where consumers lose the benefit from a larger variety of products.

Let us define welfare in a location x, denoted by w, as the sum of firms' profits (π_i) and consumer surplus (cs) in the location.

Without the merger, total equilibrium welfare is

$$W^S = z_1^S w^s(p_1^S) - C(z_1^S) + z_2^S \underbrace{\left(w^d(p_1^S, p_2^S) - w^s(p_1^S)\right)}_{>0} - C(z_2^S),$$

whereas if the firms merge, total welfare is

$$W^M = z_1^M w^s(p^m) - C(z_1^M) + z_2^M \underbrace{\left(w^d(p^m, p^m) - w^s(p^m)\right)}_{>0} - C(z_2^M).$$

The impact of the merger on total welfare is thus given by the difference:

$$\begin{aligned} W^M - W^S \;\; = \;\; & (z_1^M - z_1^S)w^s(p^m) - \left(C(z_1^M) - C(z_1^S)\right) \\ & + (z_2^M - z_2^S)\left(w^d(p^m, p^m) - w^s(p^m)\right) - \left(C(z_2^M) - C(z_2^S)\right) \\ & - \left(z_1^S - z_2^S\right)\left(w^s(p_1^S) - w^s(p^m)\right) - z_2^S\left(w^d(p_1^S, p_2^S) - w^d(p^m, p^m)\right). \end{aligned}$$

The first term corresponds to the positive[8] welfare gain from the total coverage expansion allowed by the merger, evaluated at constant prices. The second term corresponds to the effect of the reduction of the multi-product zone (again evaluated at constant prices). This term may be positive or negative depending on whether the incremental social value of the second product is smaller or larger than the incremental profit from it. Indeed, due to a standard business stealing effect (see Tirole, 1988), there may be excessive coverage by firm 2 in the duopoly game.[9]

Finally, the last line subsumes the traditional negative effect of the merger on prices in both regions, holding coverage constant.

To summarize, the merger expands total coverage, but has a negative impact on prices and variety. It has a positive effect on welfare if the expansion of total coverage is large enough and/or variety is excessive. We provide below a specific example where the merger leads to higher welfare.[10]

Proposition 4 *Suppose that* $C(z) = c \log\left[(\beta + e^z)/(1+\beta)\right]$ *and* $\gamma < 0.73$*. Then, a merger raises total welfare if* c *is not too high and* β *is not too small.*

Figure 1 shows the area where a merger raises welfare (i.e., $W^M > W^S$) in the $\{\gamma, c\}$ space, for $\alpha = \beta = 1$ and our specific cost function.[11] This area corresponds to cases where, due to the low degree of differentiation: (i) competition between firms decreases total coverage substantially; (ii) the business stealing effect is strong. For consumers, we find that the set of parameter values such that consumer surplus increases with the merger is smaller but also non empty.

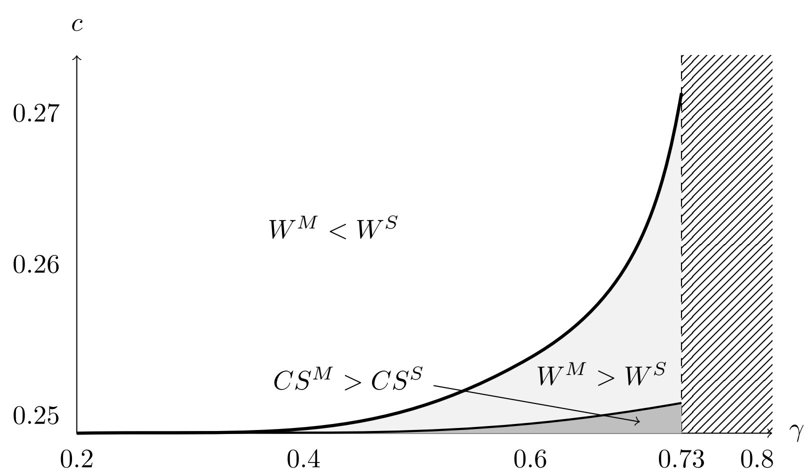

Figure 1: Impact of merger on total welfare and consumer surplus ($\alpha = \beta = 1$).

[8]It is positive because $w^s(p^m) > c\left(z_1^M\right) = \pi^m \geq c(z)$ for $z \leq z_1^M$.

[9]While $w^d(p^m, p^m) - w^s(p^m) > c(z_2^M)$, it may be the case that $w^d(p^m, p^m) - w^s(p^m) < c(z_2^S)$, so this term cannot be signed.

[10]We ran simulations with alternative cost functions. Welfare was systematically lower with the merger for the following cost functions: (i) $c(z) = cz^\beta$; and (ii) $c(z) = c\ \log(1+\beta z)$. With the cost functions (iii) $c(z) = cz^\beta/(1+z^\beta)$, and (iv) $c(z) = \beta c(e^z - 1)/((1+\beta)(e^z + \beta)$, we found similar results than the one given in the proposition.

[11]Since $\pi^m = \alpha^2/4 = 1/4$, our assumption $c(0) < \pi^m < c(\infty)$ implies that $c \in (1/4, 1/2)$.

4 Conclusion

In the recent Dow/Dupont decision,[12] the European Commission discussing the impact of mergers on innovation stated that "the effect of a less intense product market competition on innovation is potentially ambiguous." While our model aims at explaining the scale of adoption of a new technology, the effect identified should apply to a more general setup with demand-enhancing investment. Our paper thus illustrates one channel through which a merger may have a positive effect on innovation: by raising equilibrium margins, a merger raises incentives to invest to expand demand. In a merger case involving investment and/or innovation, this positive effect on the incentives to invest has to be balanced with other countervailing effects, either related to the restriction of output induced by a merger for any given demand and costs (Motta and Tarantino, 2017) or to the internalization of the business stealing effect associated with product innovation (Federico et *al.*, 2017).

A lot remains to be done to understand the interplay of these effects. To quote the current European Commission's Chief Competition Economist, "There's a lot of policy work, but there's very little explicit theoretical work about this. There's a huge body of research on the relationship between concentration and innovation, and on the relationship between market integration and innovation, but when it comes specifically to mergers and innovation, there's much less."[13] Hence, a global analysis that would help authorities identifying circumstances under which a merger fosters or impedes investment remains to be done.

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[12] Case M7932, Annex 4 to the Commission Decision.

[13] Quoted in "Mergers Are Bad for Innovation," ProMarket, September 29, 2017, by Asher Schechter, `https://promarket.org/mergers-bad-innovation`.

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Appendix

Lemma 1 *There is no equilibrium with symmetric coverage.*

Proof. With symmetric coverage $z_1 = z_2 = z$, we have $p_1 = p_2 = p^d = \alpha(1-\gamma)/(2-\gamma)$ and

$$p^d D_1\left(p^d, p^d\right) < p^d D_s\left(p^d\right),$$

implying that the right derivative of Π_1 in z_1 is strictly larger than its left derivative. As a consequence, if an equilibrium exists, it must be such that $z_1 > z_2$. ■

Proof of Proposition 1. Solving the first-order conditions (3) and (4) for the equilibrium prices $P_1(\theta)$ and $P_2(\theta)$, we find that for given coverage z_1 and z_2:

$$P_1(\theta) = \alpha\left(\frac{2-\theta\gamma-\theta\gamma^2}{4-\theta\gamma^2}\right) \quad \text{and} \quad P_2(\theta) = \alpha\left(\frac{2-\gamma-\theta\gamma^2}{4-\theta\gamma^2}\right). \tag{9}$$

Plugging the equilibrium prices into the demand functions, we obtain:

$$D_1(P_1, P_2) = \frac{\alpha\left(2-2\gamma-\gamma^2+\theta\gamma\right)}{(1-\gamma^2)(4-\theta\gamma^2)} \quad \text{and} \quad D_2(P_1, P_2) = \frac{\alpha\left(2-\gamma-\theta\gamma^2\right)}{(1-\gamma^2)(4-\theta\gamma^2)}. \tag{10}$$

We have $D_2(P_1, P_2) \geq 0$ for all $\theta \in [0,1]$ and $\gamma \in [0,1)$, while $D_1(P_1, P_2) \geq 0$ for all $\theta \in [0,1]$ requires that $2 - 2\gamma - \gamma^2 \geq 0$, which is true since $\gamma \leq 0.73$ under our assumptions. For higher values of γ, that is, if the products are sufficiently homogenous, firm 1 has an incentive to retreat on its single-product area by setting a high (monopoly) price. In this case, an equilibrium in pure strategy may fail to exist (see Valletti et *al*. (2002) and Gautier and Wauthy (2010) for details on this case).

The equilibrium prices $P_1(\theta)$ and $P_2(\theta)$ satisfy the following properties: (i) $P_i(\theta)$ is decreasing; (ii) $P_1(0) = p^m$; (iii) $P_2(0) = BR(p^m) < p^m$; and (iv) $P_1(1) = P_2(1) = p^d$. Moreover, the relative price of product 1 decreases with the relative coverage of product 1, i.e., with θ.

The first-order conditions for the coverage levels, (5) and (6), can then be rewritten as (we discuss the second-order conditions below):

$$\begin{aligned} c(z_1) &= P_1(\theta) D_s(P_1(\theta)), \\ c(z_2) &= P_2(\theta) D_2(P_1(\theta), P_2(\theta)). \end{aligned}$$

These two equations define positive coverage levels $Z_1(\theta)$ and $Z_2(\theta)$, for $\theta \in [0,1]$, both decreasing in θ. Since $Z_1(\theta) > Z_2(\theta)$, $Z_2(0) > 0$ and $Z_2(1) > 0$, we have

$$\frac{\frac{Z_2(0)}{Z_1(0)-Z_2(0)}\frac{1}{1-\gamma^2}}{1+\frac{Z_2(0)}{Z_1(0)-Z_2(0)}\frac{1}{1-\gamma^2}} > 0 \quad \text{and} \quad \frac{\frac{Z_2(1)}{Z_1(1)-Z_2(1)}\frac{1}{1-\gamma^2}}{1+\frac{Z_2(1)}{Z_1(1)-Z_2(1)}\frac{1}{1-\gamma^2}} < 1.$$

Hence, there exists $\theta \in (0,1)$ such that

$$\frac{\frac{Z_2(\theta)}{Z_1(\theta)-Z_2(\theta)}\frac{1}{1-\gamma^2}}{1+\frac{Z_2(\theta)}{Z_1(\theta)-Z_2(\theta)}\frac{1}{1-\gamma^2}} = \theta.$$

We thus have shown the existence of a solution to the system of first-order conditions, (3)-(6).

To prove that this is an equilibrium, we now exhibit conditions for the profit function of each firm to be quasi-concave.

Consider first firm 2. Its profit function, which is given by (2), is concave in price. Thus, its profit can be written as $\Pi_2(z_2) = z_2 \max_{p_2} p_2 D_2(p_1, p_2) - C(z_2)$, with a second derivative $\Pi_2''(z_2) = -c'(z_2) < 0$. Hence, firm 2's profit function is quasi-concave.

Consider now firm 1's profit, which can be written as

$$\Pi_1(z_1) = \max_{p_1} \{z_2 p_1 D_1(p_1, p_2) + (z_1 - z_2) p_1 D_s(p_1)\} - C(z_1).$$

We have

$$\Pi_1'(z_1) = p_1 D_s(p_1) - c(z_1),$$

with $p_1 = BR_1(p_2, z_1, z_2)$. The second derivative can then be written as:

$$\begin{aligned} \Pi_1''(z_1) &= \left(p_1 D_s'(p_1) + D_s(p_1)\right) \frac{\partial BR_1(p_2, z_1, z_2)}{\partial z_1} - c'(z_1) \\ &= (\alpha - 2p_1) \frac{\gamma}{2} (\alpha - p_2) \frac{\left(1-\gamma^2\right) z_2}{\left[z_1(1-\gamma^2) + z_2\gamma^2\right]^2} - c'(z_1). \end{aligned}$$

We look for a sufficient condition for the second derivative to be negative, therefore we look for an upper bound for $\Pi_1''(z_1)$. Since $p_i \geq p^d$, we have $(\alpha - 2p_1)(\alpha - p_2) \leq \alpha^2\gamma/(2-\gamma)^2$. Furthermore, since $z_1 \geq z_2$, we have

$$\frac{z_2}{\left[z_1(1-\gamma^2) + z_2\gamma^2\right]^2} \leq \frac{1}{z_2}.$$

Therefore, a sufficient condition for $\Pi_1''(z_1) \leq 0$ is that

$$\frac{1}{2}\frac{\alpha^2\gamma^2\left(1-\gamma^2\right)}{(2-\gamma)^2 z_2} \leq c'(z_1).$$

Finally, we have $z_1 > z_2 \geq c^{-1}\left(p^d D_2\left(p^d, p^d\right)\right) = c^{-1}\left(\pi^d\right)$, where $\pi^d \equiv \alpha^2(1-\gamma)/((2-\gamma)^2(1+\gamma))$ represents the multi-product duopoly profit. Therefore, we have $\Pi_1''(z_1) \leq 0$ if

$$\frac{1}{2}\frac{\alpha^2\gamma^2\left(1-\gamma^2\right)}{(2-\gamma)^2 c^{-1}\left(\pi^d\right)} \leq c'\left(c^{-1}\left(\pi^d\right)\right).$$

This condition can be written as

$$\frac{1}{2}\gamma^2(1+\gamma)^2 \leq \frac{c'\left(c^{-1}\left(\pi^d\right)\right)c^{-1}\left(\pi^d\right)}{\pi^d},$$

which holds for $\gamma \leq 0.73$ if at $x = c^{-1}\left(\pi^d\right)$ we have:

$$0.79746 \leq \frac{c'(x)\,x}{c(x)}.$$

Finally, since in equilibrium $\theta \in (0,1)$, then $p^d < P_2(\theta) < P_1(\theta) < p^m$. Furthermore, $\theta > 0$ implies that $z_1^S > z_2^S$. ■

Proof of Proposition 3. Taking the firm's profits gross of investment costs and dividing by $z_1 - z_2$, the firm sets its prices p_1 and p_2 to maximize

$$p_1(\alpha - p_1) + \frac{z_2}{z_1 - z_2}\left\{p_1\left(\frac{\alpha - p_1 - \gamma(\alpha - p_2)}{1-\gamma^2}\right) + p_2\left(\frac{\alpha - p_2 - \gamma(\alpha - p_1)}{1-\gamma^2}\right)\right\}.$$

From Propositions 1 and 2, we have $p_i^S < p^m = p_i^M$. Furthermore, since the right-hand side of (7) is decreasing in θ and $\theta\left(z_1^S, z_2^S\right) > 0$, we have $c\left(z_1^S\right) < c\left(z_1^M\right) = \pi^m$. Therefore, total coverage increases with the merger. Finally, note that the right-hand side of (8) is decreasing in θ, hence, it is minimum at $\theta = 1$. We find that

$$\left(\alpha\frac{2-\gamma-\gamma^2}{4-\gamma^2}\right)^2\frac{1}{1-\gamma^2} \geq \left(\frac{1-\gamma}{1+\gamma}\right)\pi^m,$$

and hence $z_2^S \geq z_2^M$. ■

Proof of Proposition 4. In the case of a merger, we have prices $p_1^M = p_2^M = \alpha/2$ and quantities

$$q_1^{sM} = \frac{\alpha}{2} \text{ and } q_1^{dM} = q_2^{dM} = \frac{\alpha}{2}\frac{1}{1+\gamma},$$

leading to a point-wise welfare $w^s(p^m) = 3\alpha^2/8$ for the single-product zone and $w^d(p^m, p^m) = 3\alpha^2/\left[4(1+\gamma)\right]$ for the multi-product zone. The single-product and multi-product coverage are then given by

$$\begin{aligned} z_1^M &= c^{-1}\left(\frac{\alpha^2}{4}\right), \\ z_2^M &= c^{-1}\left(\frac{\alpha^2}{4}\frac{1-\gamma}{1+\gamma}\right) \text{ if } \frac{c}{1+\beta} < \frac{\alpha^2}{4}\frac{1-\gamma}{1+\gamma} \text{ and } z_2^M = 0 \text{ otherwise,} \end{aligned}$$

where $c^{-1}(y) = \log(\beta y/(c-y))$.

Consider now the case of separation. Equilibrium prices and quantities in the multi-product zone are given by equations (9) and (10), where θ is endogenous. The quantity in the single-product zone is

$$q_1^s = \alpha - p_1^S = \alpha \left(\frac{2+\theta\gamma}{4-\theta\gamma^2} \right).$$

Finally, the single-product and multi-product coverage are given by

$$c\left(z_1^S\right) = \frac{\alpha^2 (2+\theta\gamma)[2-\theta\gamma(1+\gamma)]}{(4-\theta\gamma^2)^2}$$

and

$$c\left(z_2^S\right) = \alpha^2 \frac{\left(2-\gamma-\theta\gamma^2\right)^2}{(4-\theta\gamma^2)^2} \frac{1}{1-\gamma^2},$$

respectively, with $c(z) = c/(1+\beta e^{-z})$.

From the definition of θ, we have

$$\frac{z_2^S}{z_1^S} = \frac{\theta\left(1-\gamma^2\right)}{1-\theta\gamma^2}.$$

Combining these three equations leads to the following condition for θ:

$$\frac{c^{-1}\left(\alpha^2 \frac{\left(2-\gamma-\theta\gamma^2\right)^2}{(4-\theta\gamma^2)^2} \frac{1}{1-\gamma^2}\right)}{c^{-1}\left(\frac{\alpha^2(2+\theta\gamma)[2-\theta\gamma(1+\gamma)]}{(4-\theta\gamma^2)^2}\right)} = \frac{\theta\left(1-\gamma^2\right)}{1-\theta\gamma^2}.$$

This allows to compute numerically all equilibrium quantities. ∎