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"Entry and Merger Policy"

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Abstract

This note examines the optimal merger policy when competition authorities take into account the effects of their policy on firms' entry decisions. We consider a model featuring *ex ante* uncertainty about profits and consumer surplus, and derive a simple rule governing the optimal policy in that context. More specifically, we show that the ratio between the loss in *ex post* consumer surplus and the gain in an entrant's profit induced by an *ex post* anticompetitive merger is a sufficient statistic to determine when competition authorities should be more lenient. Our findings imply in particular that competition authorities may find it optimal to commit to being more lenient towards successful, rather than unsuccessful, entrants.

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1 Introduction

Merger policy can alter firms' incentives to enter a given market. More precisely, a more lenient merger policy can lead to less product-market competition, thus making entry more attractive.[1] This dynamic effect may benefit consumers but should be weighted against the adverse static effect of a reduction in competition on consumer surplus. In this note, we study the optimal merger policy of a competition authority facing this trade-off, focusing in particular on the circumstances under which the authority should be lenient towards merging parties.

We build a simple model where a firm must decide whether to enter a market in which an incumbent operates. Entry is costly and post-entry profits depend both on the number of independent firms in the industry and on stochastic market conditions captured by the existence of two states of the world. A key player in our model is a competition authority who can commit *ex ante* to its future behavior towards merger proposals, modelled here as the probability of clearing a merger in each state of the world. The merger policy adopted by the competition authority affects the potential entrant's expected profit through its effect on market structure.

We derive the optimal merger policy focusing on mergers that are *ex post* anticompetitive, i.e. that reduce *ex post* consumer surplus.[2] We find that the competition authority should be lenient in priority in the state of the world where the ratio between the loss in *ex post* consumer surplus and the gain in the entrant's expected profit induced by the merger is the lowest. The optimal merger policy is shown to be driven by this ratio both when the competition authority knows the entry cost and when it only knows its distribution.

We then apply this general result to a scenario in which uncertainty is about the entrant's marginal cost of production. We first assume that the entrant has the same production cost as the incumbent in one state of the world, while it is less efficient in the other state of the world. When the entrant faces such a "downside" risk, we show that the optimal merger policy should be more lenient in the state of the world which is unfavorable to the entrant. This finding is somewhat reminiscent of the *failing firm defense*, where a firm on the eve of bankruptcy is allowed to merge.[3] However, this result does not always hold: the competition authority may find it optimal to be more lenient in the state of the world where the entrant is the most *successful*. More specifically, if there is an "upside" risk for the entrant, who can

[1] Note that an entrant may benefit from a higher probability that a future merger is cleared both when it is one of the merging parties and when it is an outsider. In both cases, the increase in the entrant's profit results from a decrease in the intensity of product-market competition.

[2] In the case of an *ex post* procompetitive merger, the competition policy faces no trade-off: a more lenient merger policy increases *ex post* consumer surplus and makes entry more attractive (which increases *ex ante* consumer surplus).

[3] Note that the failing firm defense is used to justify the clearance of a merger involving a failing firm on the grounds of the *ex post* effects of that merger. In contrast, our finding (in the case of an entrant facing a downside risk) justifies a more lenient treatment of a merger involving an unsuccessful entrant based on the *ex ante* effect of merger policy on entry.

be either as efficient as or more efficient than the incumbent, the competition authority shoud let the merger happen in priority when the entrant turns out to be more efficient than the incumbent. Both results follow from the fact that the *ex post* behavior of the merged entity is less harmful when the merging parties are asymmetric (in terms of efficiency). In that case, the standard negative effect of reducing the number of firms in the market is partially offset by the withdrawal of the less efficient production units.

The early literature on mergers (e.g., Salant *et al.*, 1983, Farrell and Shapiro, 1990, McAfee and Wiliams, 1992) has focused on the static effects of mergers on profits and welfare. More recently, the effects of mergers have been investigated in dynamic settings allowing for the entry of new firms and/or accounting for the reaction of competition authorities. Gowrisankaran (1999) was the first to propose a fully dynamic model of mergers, with entry, endogenous mergers and production at each period. He showed both the need for a dynamic merger analysis and the complexity of such analysis.[4] Relatedly, Nocke and Whinston (2010) derived conditions under which a simple static, i.e. myopic, merger policy could be optimal even in a dynamic framework.

Our approach is simpler in that it assumes the merger to be always beneficial for the merging firms and, therefore, always implemented if the competition authority does not block it. We rather focus on the interaction between merger policy and entry decisions when the competition authority can commit *ex ante* to its policy. In this respect, the closest paper to ours is Mason and Weeds (2013) who also study the impact of merger policy on entry and derive the optimal merger policy. In their setting, the merger policy amounts to choosing a threshold for the entrant's profit below which a merger is allowed. In that sense, their approach is built on the failing (or "ailing") firm defense story. In contrast, we consider a setting where the merger policy choice is less restricted: we do not require the competition authority to be more lenient in the state of the world which is the least favorable to the entrant. We actually show that the competition authority may find it optimal to be more lenient towards a successful, rather than an unsuccessful, entrant. Our note is therefore complementary to Mason and Weeds (2013), as it provides new insights on *how* to design merger policy when the effect of such policy on entry is taken into account.

2 Model

We consider a setting in which an incumbent, firm I, operates in a market, and a firm E considers the possibility of entering this market. To enter the market, firm E must pay a fixed cost $F \geq 0$. Profits and consumer surplus are affected by two factors: the number of firms in the industry and the realization of a random variable capturing uncertain market conditions

[4] Extending this approach, Igami and Uetake (2016) develop a dynamic game with endogenous mergers, innovation and entry-exit at each period. They find that mergers are a dominant mode of exit and sometimes generate productivity improvements.

(i.e. demand and/or cost conditions). For the sake of simplicity, we assume that there are two states of the world, $i = 1, 2$, and denote by β the (exogenous) probability that state 1 occurs.

In the state of the world $i = 1, 2$, consumer surplus is denoted CS_i^m when firm E does not enter the market, CS_i^C when firm E enters the market and competes with the incumbent, and CS_i^M when firm E enters the market and merges with the incumbent (where m stands for monopoly, C for competition, and M for merger). We focus on the scenario in which a merger between the entrant and the incumbent is anticompetitive *ex post*: consumer surplus under entry and competition is larger than consumer surplus when entry occurs and is followed by a merger, i.e. $CS_i^C > CS_i^M$. We also make the natural assumption that entry benefits consumers even when it is followed by a merger, i.e. $CS_i^M \geq CS_i^m$. These two assumptions ensure that we are in the interesting scenario in which there is a tension between the *ex ante* and *ex post* effects of merger policy on consumers.

Similarly, for each state of the world $i = 1, 2$, we denote by π_i^k and $\hat{\pi}_i^k$ the entrant's and the incumbent's profits respectively, where $k = m$ if firm E does not enter the market (with the convention that $\pi_i^m = 0$), $k = C$ if it enters the market and competes with the incumbent, and $k = M$ if it enters the market and merges with the incumbent. Without loss of generality, we assume that $\pi_1^C \leq \pi_2^C$ so that state 2 is (weakly) more favorable than state 1 for the potential entrant under competition. Finally, we suppose that a merger between the entrant and the incumbent leads to an increase in the profits of both parties in both states of the world, i.e. $\pi_i^M > \pi_i^C$ and $\hat{\pi}_i^M > \hat{\pi}_i^C$, for $i = 1, 2$. This implies that the entrant and the incumbent always want to merge in our setting.

We assume that the entrant and the incumbent need to notify their merger to a competition authority who decides whether to clear or block the merger. We define a merger policy as a commitment made *ex ante*, i.e. before the realization of the state of the world, about the decision to clear or block the merger. Formally, we assume that the competition authority commits to a probability $\rho_i \in [0, 1]$ of clearing the merger in each state $i = 1, 2$. Finally, we suppose that the authority maximizes consumer surplus.[5]

The timing of the game is as follows:

1. The competition authority decides and publicly announces its merger policy $(\rho_1, \rho_2) \in [0, 1]^2$.

2. Firm E decides whether to enter the market or not. If it does, it incurs the fixed cost F.

3. The state of the world is observed by all parties.[6]

[5] Assuming that the competition authority maximizes total welfare would not alter qualitatively our results as long as we adapt our assumptions to ensure that we are in the interesting scenario in which entry is desirable from a total welfare perspective and the merger between the entrant and the incumbent leads to a decrease in total welfare *ex post*.

[6] To justify this observability, we can include a pre-merger competition phase as in Mason ad Weeds (2013).

4. If firm E entered in stage 2, the two firms notify their merger to the competition authority who clears it with probability ρ_i in state i.

5. The payoffs are realized.

Let us consider first firm E's decision to enter the market. Firm E enters if and only if

$$F \leq \beta \left[(1-\rho_1)\pi_1^C + \rho_1 \pi_1^M\right] + (1-\beta)\left[(1-\rho_2)\pi_2^C + \rho_2 \pi_2^M\right] \equiv \tilde{F}(\rho_1, \rho_2).$$

Using the fact that $\tilde{F}(\rho_1, \rho_2)$ is strictly increasing in (ρ_1, ρ_2) (because $\pi_i^M > \pi_i^C$), we can make the following two statements:

1. If $F > \tilde{F}(1,1)$, then firm E will never enter the market regardless of the competition authority's merger policy.

2. If $F \leq \tilde{F}(0,0)$, then firm E will enter the market regardless of the competition authority's merger policy.

In the first case, merger policy is irrelevant since there is never entry. In the second case, merger policy only affects the *ex post* expected consumer surplus:

$$\beta \left[(1-\rho_1)CS_1^C + \rho_1 CS_1^M\right] + (1-\beta)\left[(1-\rho_2)CS_2^C + \rho_1 CS_2^M\right]$$

Since the latter is decreasing in ρ_1 and ρ_2 (because $CS_i^C > CS_i^M$), the optimal merger policy is given by $(\rho_1^*, \rho_2^*) = (0,0)$ when $F \leq \tilde{F}(0,0)$. In other words, the competition authority always blocks the merger in this case.

We now focus on the intermediate (and most interesting) case where $F \in (\tilde{F}(0,0), \tilde{F}(1,1)]$. Since entry is always beneficial to consumers, the competition authority's maximization program can be written as

$$\begin{aligned} &\max_{(\rho_1,\rho_2)\in[0,1]^2} \beta \left[(1-\rho_1)CS_1^C + \rho_1 CS_1^M\right] + (1-\beta)\left[(1-\rho_2)CS_2^C + \rho_2 CS_2^M\right], \\ &\text{s.t.} \quad \tilde{F}(\rho_1, \rho_2) \geq F. \end{aligned} \tag{1}$$

Since the authority's objective function is decreasing in ρ_1 and ρ_2 while the LHS of the entry condition (1) increases in ρ_1 and ρ_2, the latter is necessarily binding at the optimum. The following proposition provides the optimal merger policy.

While this would make our model more realistic, it would not affect our results and main message because this additional phase would not be affected by the merger policy.

Proposition 1 *Assume that* $F \in (\tilde{F}(0,0), \tilde{F}(1,1)]$ *and denote* $\Delta\pi_i \equiv \pi_i^M - \pi_i^C$ *and* $\Delta CS_i \equiv CS_i^C - CS_i^M$*. Then the optimal merger policy* (ρ_1^*, ρ_2^*) *is as follows:*

1. *If* $\Delta CS_1/\Delta\pi_1 < \Delta CS_2/\Delta\pi_2$*, then*

$$(\rho_1^*, \rho_2^*) = \begin{cases} \left(\frac{F-\tilde{F}(0,0)}{\beta\Delta\pi_1}, 0\right) & \text{if} \quad \tilde{F}(0,0) < F < \tilde{F}(0,0) + \beta\Delta\pi_1 \\ \left(1, 1 - \frac{\tilde{F}(1,1)-F}{(1-\beta)\Delta\pi_2}\right) & \text{if} \quad \tilde{F}(0,0) + \beta\Delta\pi_1 \leq F < \tilde{F}(1,1) \end{cases}.$$

2. *If* $\Delta CS_1/\Delta\pi_1 > \Delta CS_2/\Delta\pi_2$*, then*

$$(\rho_1^*, \rho_2^*) = \begin{cases} \left(0, \frac{F-\tilde{F}(0,0)}{(1-\beta)\Delta\pi_2}\right) & \text{if} \quad \tilde{F}(0,0) < F < \tilde{F}(0,0) + (1-\beta)\Delta\pi_2 \\ \left(1 - \frac{\tilde{F}(1,1)-F}{\beta\Delta\pi_1}, 1\right) & \text{if} \quad \tilde{F}(0,0) + (1-\beta)\Delta\pi_2 \leq F < \tilde{F}(1,1) \end{cases}.$$

3. *If* $\Delta CS_1/\Delta\pi_1 = \Delta CS_2/\Delta\pi_2$*, then the optimal merger policies* (ρ_1^*, ρ_2^*) *are the solutions to the equation* $\tilde{F}(\rho_1, \rho_2) = F$*.*

Proof. See Appendix. ∎

The optimal merger policy is the outcome of a cost/benefit analysis of rent shifting in both states of the world. An increase in the probability that the merger is cleared in either state leads to both an increase in the potential entrant's *ex ante* expected profit and a decrease in *ex post* consumer surplus. In the special case where the ratio of the loss in consumer surplus over the gain in the entrant's profit is the same in both states of the world (scenario 3), the instruments ρ_1 and ρ_2 are perfect substitutes from the competition authority's perspective: a given marginal increase in the entrant's profit requires the same marginal loss in *ex post* consumer surplus in both states of the world. Therefore, in this scenario, any combination (ρ_1, ρ_2) that makes the entrant just break even is optimal. However, when the relevant ratio is not the same in both states of the world (scenarios 1 and 2), the instruments ρ_1 and ρ_2 become *imperfect* substitutes. In that case, the competition authority should favor the instrument associated with the lowest ratio, and use the other instrument only if it is necessary to induce entry. For instance, if the ratio is lower in state 1 (scenario 1), then the competition authority should be lenient in state 1 (i.e. should set $\rho_1 > 0$) *in priority*, and should be lenient in state 2 (i.e. should set $\rho_2 > 0$) only if committing to clear the merger for sure in state 1 (i.e. setting $\rho_1 = 1$) is not sufficient to provide firm E with enough incentives to enter the market.

3 Extension : Asymmetric information

In our baseline model we assume that the competition authority has the same information as the potential entrant. In particular, we suppose that the competition authority knows the entry cost F, which allows it to make its merger policy dependent on that cost. In this extension, we assume instead that it only knows the distribution of F, and show that our main finding is robust to this change: the rule governing the choice of the state of the world in which the competition authority should be more lenient still applies.

Denoting $G(.)$ the c.d.f. of the entry cost F, merger policy affects the expected consumer surplus through the probability $G(\tilde{F}(\rho_1, \rho_2))$ that entry occurs and through its effect on *ex post* consumer surplus when entry occurs. More precisely, the expected consumer surplus is given by

$$\begin{aligned} ECS(\rho_1,\rho_2) &= G(\tilde{F}(\rho_1,\rho_2))\left[\beta\left[(1-\rho_1)CS_1^C+\rho_1 CS_1^M\right]+(1-\beta)\left[(1-\rho_2)CS_2^C+\rho_1 CS_2^M\right]\right] \\ &\quad +\left(1-G(\tilde{F}(\rho_1,\rho_2))\right)\left[\beta CS_1^m+(1-\beta)CS_2^m\right] \\ &= G(\tilde{F}(\rho_1,\rho_2)\left[\beta\left(CS_1^C-\rho_1\Delta CS_1-CS_1^m\right)+(1-\beta)\left(CS_2^C-\rho_2\Delta CS_2-CS_2^m\right)\right] \\ &\quad +\beta CS_1^m+(1-\beta)CS_2^m. \end{aligned}$$

The competition authority seeks to maximize $ECS(\rho_1, \rho_2)$ which we assume to be globally concave in (ρ_1, ρ_2). The next proposition shows that the ratio of the effect of the merger on *ex post* consumer surplus over its effect on the entrant's profit is still a sufficient statistic to determine the state of the world in which the competition authority should be more lenient.[7]

Proposition 2 *Assume that the competition authority only knows the distribution of the entry cost. Then the optimal merger policy is such that:*

1. *If* $\Delta CS_1/\Delta\pi_1 < \Delta CS_2/\Delta\pi_2$ *then* $\rho_1^* \geq \rho_2^*$ *and* $\rho_1^* = 1$ *whenever* $\rho_2^* > 0$.
2. *If* $\Delta CS_1/\Delta\pi_1 > \Delta CS_2/\Delta\pi_2$ *then* $\rho_1^* \leq \rho_2^*$ *and* $\rho_2^* = 1$ *whenever* $\rho_1^* > 0$.
3. *If* $\Delta CS_1/\Delta\pi_1 = \Delta CS_2/\Delta\pi_2$ *then there are multiple optimal pairs* (ρ_1^*, ρ_2^*) *and one of them is such that* $\rho_1^* = \rho_2^*$.

Proof. See Appendix. ■

The intuition behind this result is similar to the one when the entry cost is observable to the competition authority: the latter has two instruments to provide (additional) incentives to enter the market and should use the instrument that induces the lowest loss in *ex post* consumer surplus for a given increase in post-entry profits.

[7] Note that this result applies not only to the case where the competition authority does not know the entry cost but also to the scenario in which the competition authority knows the entry cost but cannot make the merger policy depend on it.

4 Application: Uncertainty about the entrant's production cost

We now apply our main result to a scenario in which the demand and the incumbent's cost of production are deterministic but the entrant's cost of production is uncertain. We conduct our analysis with a general reduced-form model of competition and rely on the special case of Cournot competition with linear demand to provide clear-cut findings when the general model offers ambiguous results.

Denote $\pi^C(x,y)$ the (equilibrium) duopoly profit of a firm producing at marginal cost x and facing a rival producing at marginal cost y, and $\pi^m(x)$ the monopoly profit of a firm producing at marginal cost x. Further, denote $CS^C(x,y) = CS^C(y,x)$ the consumer surplus (in equilibrium) when the industry is duopolistic with one firm producing at marginal cost x and the other producing at marginal cost y, and $CS^m(x)$ consumer surplus when the industry is monopolistic and the marginal cost of production is x.

Let us make the following natural assumptions on the equilibrium profits and consumer surplus:

A1 $\pi^C(x,y)$ is (weakly) decreasing in x and and (weakly) increasing in y.
A2 $\pi^C(x,y) + \pi^C(y,x) < \pi^m(\min(x,y))$ whenever $\min(\pi^C(x,y), \pi^C(y,x)) > 0$
A3 $CS^C(x,y)$ is (weakly) decreasing in both x and y.
A4 $CS^C(x,y) \geq CS^m(\min(x,y))$.

Denote c_i (resp. $\hat{c}_i$) the entrant's (resp. incumbent's) marginal cost in state $i = 1, 2$. Assume that, in both states of the world, both firms are active if they compete. Moreover, suppose that the merged entity's marginal cost of production is given by $\min(c_i, \hat{c}_i)$. Finally, denote δ the share of the joint profit gain from the merger $\pi^m(\min(c_i,\hat{c}_i)) - \left(\pi^C(c_i,\hat{c}_i) + \pi^C(\hat{c}_i,c_i)\right)$ captured by the entrant (which we assume to be the same in the two states of the world). With these notations and assumptions we have:

$$\frac{\Delta CS_i}{\Delta \pi_i} = \frac{CS^C(c_i,\hat{c}_i) - CS^m(\min(c_i,\hat{c}_i))}{\delta\left[\pi^m(\min(c_i,\hat{c}_i)) - (\pi^C(c_i,\hat{c}_i) + \pi^C(\hat{c}_i,c_i))\right]}$$

The existence of potential productive efficiency gains (because of the potential asymmetry between the incumbent and the entrant) makes it possible that a merger in state i generates both less consumer surplus loss *and* more profits for the industry than a merger in state $j \neq i$, i.e. $\Delta CS_i < \Delta CS_j$ and $\Delta \pi_i > \Delta \pi_j$. In that case we have unambiguously

$$\frac{\Delta CS_i}{\Delta \pi_i} < \frac{\Delta CS_j}{\Delta \pi_j}$$

which implies that the competition authority should be more lenient in state i. This preliminary remark is useful for the analysis of the two scenarios that we consider next. In the first scenario the entrant faces the risk of being less efficient than the incumbent, whereas in the second it can be more efficient than the incumbent (with some probability). One can interpret the first scenario as corresponding to a mature industry, in which a latecomer (the entrant) can at best catch up with an incumbent. In contrast, the second scenario would be a best fit for an innovative industry in which newcomers can be more efficient than incumbents.

Scenario 1: Insuring an unsuccessful entrant

Assume that $\hat{c}_1 = \hat{c}_2 = c$ and that $c_1 = \bar{c} > c_2 = c$. In this scenario, the entrant faces a downside risk in the sense that it will be at best as efficient as the incumbent. The cost-benefit ratio in the state of the world i is given by

$$\frac{\Delta CS_i}{\Delta \pi_i} = \frac{CS^C(c, c_i) - CS^m(c)}{\delta\left[\pi^m(c) - \left(\pi^C(c, c_i) + \pi^C(c_i, c)\right)\right]}.$$

As long as the competitive industry profits are higher when the entrant is more efficient (that is, in state 2), i.e.

$$\pi^C(c, \bar{c}) + \pi^C(\bar{c}, c) < 2\pi^C(c, c) \tag{2}$$

the following inequality holds

$$\frac{\Delta CS_1}{\Delta \pi_1} < \frac{\Delta CS_2}{\Delta \pi_2}$$

which implies that the competition authority should be more lenient in state 1. The gain in productive efficiency in that state makes the merger less detrimental to consumers and more profitable to the entrant. Note that condition (2) requires that product-market rivalry when the industry is symmetric is somewhat limited.[8]

On the contrary, if competitive industry profits are lower when the entrant is more efficient (that is, when condition (2) does not hold), then the merger is less harmful to consumers in state 1 (because it generates productive efficiency gains in that state) but it is also less profitable to the firms in that state (because the asymmetry between the firms relaxes competition). Therefore, the comparison of the two relevant ratios is *a priori* ambiguous.

However, in the special case, examined in the Appendix, in which firms compete *à la Cournot* and face a linear demand, we are able to compare the two cost-benefit ratios both when condition (2) holds or not. More precisely, we show that the inequality

$$\frac{\Delta CS_1}{\Delta \pi_1} < \frac{\Delta CS_2}{\Delta \pi_2}$$

always holds and, therefore, the competition authority should be more lenient in state 1.

This result could be interpreted as an *ex ante* variant of the *failing firm defense*. We show indeed that, when the entrant faces a downside risk, competition authorities should be more lenient with that firm when it turns out to be inefficient. This implies that, in this scenario,

[8] In particular, this condition would not hold under Bertrand competition with homogeneous products.

the optimal merger policy should provide the potential entrant with some insurance against the realization of the bad state of the world.

Scenario 2: Rewarding a successful entrant

Assume again that there is no uncertainty regarding the incumbent marginal's cost, i.e. $\hat{c}_1 = \hat{c}_2 = c$, but that $c_1 = c > c_2 = \underline{c}$. In this scenario, the entrant faces an upside risk in the sense that it will be at worst as efficient as the incumbent. The cost/benefit ratio in each state of the world can be written as

$$\frac{\Delta CS_1}{\Delta \pi_1} = \frac{CS^C(c,c) - CS^m(c)}{\delta\left[\pi^m(c) - 2\pi^C(c,c)\right]}$$

and

$$\frac{\Delta CS_2}{\Delta \pi_2} = \frac{CS^C(c,\underline{c}) - CS^m(\underline{c})}{\delta\left[\pi^m(\underline{c}) - \left(\pi^C(\underline{c},c) + \pi^C(c,\underline{c})\right)\right]}.$$

Comparing these two ratios in our general setting is difficult. However, intuition suggests that $\Delta CS_2/\Delta \pi_2$ should be smaller than $\Delta CS_1/\Delta \pi_1$ because the productive efficiency gains generated by the merger in state 2 tend to reduce the loss in consumer surplus and to amplify the gain in profits resulting from the merger. This intuition is confirmed in the special case, examined in the Appendix, in which firms compete à la Cournot and face a linear demand function. In that case, straightforward computations show indeed that

$$\frac{\Delta CS_1}{\Delta \pi_1} > \frac{\Delta CS_2}{\Delta \pi_2}$$

implying that the competition authority should be more lenient in state 2.

This finding is at odds with the failing firm defense story (adapted to the *ex ante* view taken in this note). Indeed, it implies that the competition authority should be more lenient with an entrant that turns out to be particularly *efficient* (more precisely, an entrant that is more efficient than the incumbent). In other words, in this scenario, the competition authority should prefer to increase the reward of an entrant in case it is successful (i.e. efficient) rather than reduce its loss in case it is unsuccessful (i.e. inefficient).

5 Conclusion

We build a simple model to examine the optimal design of merger policy when competition authorities maximize consumer surplus taking into account the effect of their policy on entry. Focusing on the treatment of mergers that are anticompetitive *ex post* (i.e. after entry occurs), we show that the ratio between the loss in consumer surplus and the gain in an entrant's profit induced by the merger is a key determinant of the optimal policy. More specifically, this ratio is a sufficient statistic that determines when competition authorities should be the most lenient.

Our analysis extends to a more general problem than the one considered in this note: it applies to any situation in which a public authority seeks to induce an investment beneficial to consumers by using an instrument that results in losses in consumer surplus *ex post*. In particular, they are relevant for the optimal use of patent policy to encourage R&D investments when there is uncertainty about the state of the world that prevails once the innovation is achieved.

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7 Appendix

Proof of Proposition 1

Denote $\mu_1 = \beta\rho_1$ and $\mu_2 = (1-\beta)\rho_2$. Since it is optimal for the competition authority to induce entry, its maximization program can be rewritten as the following (constrained) minimization program

$$\begin{gathered}\min_{(\mu_1,\mu_2)\in[0,\beta]\times[0,1-\beta]} \mu_1 \Delta CS_1 + \mu_2 \Delta CS_2 \\ \text{s.t. } \mu_1\Delta\pi_1 + \mu_2\Delta\pi_2 = F - \tilde{F}(0,0)\end{gathered}$$

where we use the (previously made) observation that the constraint is necessarily binding at the optimum. Plugging the value of μ_2 given by the constraint into the objective function yields the following unidimensional minimization program

$$\begin{array}{c}\min\limits_{\mu_1\in[0,\beta]}\mu_1\Delta CS_1+\frac{F-\tilde{F}(0,0)-\mu_1\Delta\pi_1}{\Delta\pi_2}\Delta CS_2\\ \text{s.t. } \frac{F-\tilde{F}(0,0)-\mu_1\Delta\pi_1}{\Delta\pi_2}\in[0,1-\beta]\end{array}$$

which can be rewritten as

$$\begin{array}{c}\min\limits_{\mu_1}\mu_1\left(\Delta CS_1-\frac{\Delta\pi_1}{\Delta\pi_2}\Delta CS_2\right)\\ \text{s.t. } \mu_1\in\left[\max\left(0,\frac{F-\tilde{F}(0,0)-(1-\beta)\Delta\pi_2}{\Delta\pi_1}\right),\min\left(\beta,\frac{F-\tilde{F}(0,0)}{\Delta\pi_1}\right)\right]\end{array}$$

If $\Delta CS_1/\Delta\pi_1>\Delta CS_2/\Delta\pi_2$ then $\Delta CS_1-\frac{\Delta\pi_1}{\Delta\pi_2}\Delta CS_2>0$ and, therefore, the solution to this program is $\mu_1^*=\max\left(0,\frac{F-\tilde{F}(0,0)-(1-\beta)\Delta\pi_2}{\Delta\pi_1}\right)$, which implies that the optimal value of μ_2 is $\mu_2^*=\min\left(\frac{F-\tilde{F}(0,0)}{\Delta\pi_2},(1-\beta)\right)$. This, combined with the observation that $\frac{F-\tilde{F}(0,0)-(1-\beta)\Delta\pi_2}{\beta\Delta\pi_1}=1-\frac{\tilde{F}(1,1)-F}{\beta\Delta\pi_1}$ leads to the result in this scenario. The result in the case $\Delta CS_1/\Delta\pi_1<\Delta CS_2/\Delta\pi_2$ can then be derived by symmetry. Finally, in the limiting case $\Delta CS_1/\Delta\pi_1=\Delta CS_2/\Delta\pi_2$, all the pairs (ρ_1,ρ_2) that satisfy the (binding) constraint are solutions to the program.

Proof of Proposition 2

Denoting $g=G'$ the density function of the entry cost, and omitting the arguments of $\tilde{F}$, we have

$$\begin{aligned}\frac{\partial ECS}{\partial\rho_1} &= \frac{\partial\tilde{F}}{\partial\rho_1}g(\tilde{F})\left[\beta\left(CS_1^C-\rho_1\Delta CS_1-CS_1^m\right)+(1-\beta)\left(CS_2^C-\rho_2\Delta CS_2-CS_2^m\right)\right]\\ &\quad -G(\tilde{F})\beta\Delta CS_1\\ &= \beta\Delta\pi_1 G(\tilde{F})\left(\frac{g}{G}(\tilde{F})[\beta\left(CS_1^C-\rho_1\Delta CS_1-CS_1^m\right)\right.\\ &\qquad\qquad\left.+(1-\beta)\left(CS_2^C-\rho_2\Delta CS_2-CS_2^m\right)]-\frac{\Delta CS_1}{\Delta\pi_1}\right).\end{aligned}$$

Thus, $\frac{\partial ECS}{\partial\rho_1}=0$ if and only if

$$\frac{g}{G}(\tilde{F})\left[\beta\left(CS_1^C-\rho_1\Delta CS_1-CS_1^m\right)+(1-\beta)\left(CS_2^C-\rho_2\Delta CS_2-CS_2^m\right)\right]=\frac{\Delta CS_1}{\Delta\pi_1},\tag{3}$$

and similarly $\frac{\partial ECS}{\partial\rho_2}=0$ if and only if

$$\frac{g}{G}(\tilde{F})\left[\beta\left(CS_1^C-\rho_1\Delta CS_1-CS_1^m\right)+(1-\beta)\left(CS_2^C-\rho_2\Delta CS_2-CS_2^m\right)\right]=\frac{\Delta CS_2}{\Delta\pi_2}.\tag{4}$$

Since the left-hand sides of (3) and (4) are the same, it follows that the solution (ρ_1^*,ρ_2^*) to the competition authority's maximization program cannot be such that $0<\rho_1^*<1$ and $0<\rho_2^*<1$ unless $\Delta CS_1/\Delta\pi_1=\Delta CS_2/\Delta\pi_2$. More specifically, if $\Delta CS_1/\Delta\pi_1<\Delta CS_2/\Delta\pi_2$ then the only three possible scenarios are the following: $\rho_1^* = \rho_2^* = 0$; $\rho_1^* > 0$ and $\rho_2^* = 0$; $\rho_1^* = 1$ and $\rho_2^* > 0$. The possible outcomes in the case $\Delta CS_1/\Delta\pi_1 > \Delta CS_2/\Delta\pi_2$ can be obtained by symmetry.

Scenario 1 vs Scenario 2: Cournot competition with linear demand

Assume that the incumbent and the entrant compete *à la* Cournot (if they do not merge) and that the demand function is given by $D(p) = 1 - p$.

Scenario 1:

Assume that $\hat{c}_1 = \hat{c}_2 = c = 0$ and $c_1 = \bar{c} > c_2 = 0$, where $\bar{c} < 1/2$. Note first that, in this setting, condition (2) is satisfied if $0 < \bar{c} < 2/5$ and is not satisfied if $2/5 \leq \bar{c} < 1/2$. Straightforward computations yield

$$\pi^m(0) = \frac{1}{4};\ CS^m(0) = \frac{1}{8};\ \pi^C(0,\bar{c}) = \frac{(1+\bar{c})^2}{9};\ \pi^C(\bar{c},0) = \frac{(1-2\bar{c})^2}{9};\ CS^C(0,\bar{c}) = \frac{(2-\bar{c})^2}{18}$$

which implies that

$$\frac{\Delta CS_1}{\Delta\pi_1} = \frac{\frac{1}{72}\left(7 - 16\bar{c} + 4\bar{c}^2\right)}{\frac{1}{72}\delta\left(1 + 8\bar{c} - 20\bar{c}^2\right)} = \frac{\left(\bar{c} - \frac{1}{2}\right)(4\bar{c} - 14)}{\delta\left(\bar{c} - \frac{1}{2}\right)(-2 - 20\bar{c})} = \frac{7 - 2\bar{c}}{\delta(1 + 10\bar{c})}$$

The latter is decreasing in $\bar{c}$, which implies that its is less than its value for $\bar{c} = 0$, which is equal to $\frac{\Delta CS_2}{\Delta\pi_2}$. Thus,

$$\frac{\Delta CS_1}{\Delta\pi_1} < \frac{\Delta CS_2}{\Delta\pi_2}.$$

Scenario 2:

Assume that $\hat{c}_1 = \hat{c}_2 = c > 0$ and $c_1 = c > c_2 = \underline{c} = 0$, where $c < 1/2$. In that case,

$$\pi^m(c) = \frac{(1-c)^2}{4};\ \pi^C(c,c) = \frac{(1-c)^2}{9}; CS^C(c,c) = \frac{2}{9}(1-c)^2;\ CS^m(c) = \frac{(1-c)^2}{8}$$

which yields $\frac{\Delta CS_1}{\Delta\pi_1} = \frac{7}{\delta}$. Moreover, we can derive from our computations in scenario 1 that $\frac{\Delta CS_2}{\Delta\pi_2} = \frac{7-2c}{\delta(1+10c)}$, which is decreasing in c and therefore less than 7. Thus,

$$\frac{\Delta CS_1}{\Delta\pi_1} > \frac{\Delta CS_2}{\Delta\pi_2}.$$