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# "On Legislative Lobbying under Political Uncertainty"

Anton Tyutin and Vera Zaporozhets

Toulouse School of Economics

# On Legislative Lobbying under Political Uncertainty (preliminary)

Anton Tyutin* Vera Zaporozhets†

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**Abstract**

We study a simple influence game, in which a lobby tries to manipulate the decision of a legislature via monetary offers to one or more members. The type of a legislator is the relative weight he/she places on social welfare as compared to money. We study the equilibria of this lobbying game under political certainty and uncertainty, and examine the circumstances under which the lobby is successful, and the amount of money invested in the political process. Special attention is paid to three primitives of the environment: the budget available for lobbying, the internal organization of the legislature and the proportion of "bad" and "good" legislators in the political arena.

## 1 Introduction

The aim of this paper is to analyze how the complexity of legislative process shapes the special interest politics. To do so we consider a simple influence game, in which a single lobby tries to manipulate the decision of a legislature by making monetary offers to one or more members. We examine how the voting outcome and the monetary contributions

*RSJ, Na Florenci 2116/15, 110 00 Praha 1, Czech Republic.

†Toulouse School of Economics, INRA, University of Toulouse Capitole, 31015 Toulouse Cedex 6, France. E-mail: vera.zaporozhets@tse-fr.eu

offered to the legislators depends on the lobby's willingness to pay, the legislators' preferences and the decision-making process within the legislature. We show that the supermajorities may occur due to the uncertainty about the legislators' preferences. Moreover, the size of supermajority decreases when the lobby's willingness to pay increases, though the total spendings on capturing the legislature increases.

We depart from the voluminous literature based on the common agency setting[1] by relaxing the assumption that policies are set by a single individual or by a cohesive, well-disciplined political party. In reality, most policy decisions are made rather by a group of elected representatives acting as a legislative body. Even when the legislature is controlled by a single party (as it is necessarily the case in a two-party system if the legislature consists of a unique chamber), the delegation members do not always follow the instructions of their party leaders. In situations with multiple independent legislators, special interest groups face a subtle problem in deciding how to allocate their resources to influence policy choices. For instance, should the lobby seek to solidify support among those legislators who would be inclined to support its positions anyway, or should it seek to win over those who might otherwise be hostile to its views? The answer to this question depends on the rules of the legislative process and the optimal strategy for wielding influence would vary with the institutional setting.

In this paper we focus on the binary setting, i.e., we assume that the policy space consists of two alternatives: the status quo versus the change or reform. While simplistic, we think that many policy issues fit that formulation. In such case, there is no room for agenda setting, and the unique role of the legislature is to select one of the two options through voting. A legislature is then described by a *simple game* $(N, \mathcal{W})$ where $N$ is the set of legislators (or parties, if there is some strong party discipline) and $\mathcal{W}$ is the list of winning coalitions: the reform is adopted if and only if the coalition of legislators voting for the reform belongs to that list.

The preference of the lobby is defined by the amount of money $W_0$ that would be gained

[1] The common agency framework has been pionnered by Grossman and Helpman (1994, 2001) to study trade policy, commodity taxation and other policies.

by its members if the reform was adopted. Following Grosssman and Helpman (1994), we assume that each legislator seeks to maximize a weighted sum of social welfare and monetary contributions. Therefore, in this setting, each legislator $i$ is simply described by a single parameter $\alpha_i$ denoting the weight that he puts on social welfare[2]. This parameter is referred hereafter as being the type of the legislator. The lower the value of $\alpha_i$ is, the cheaper legislator $i$ is, and therefore there is a sense in which we can qualify politicians with low $\alpha$ as "bad" or corrupted as they are more willing to depart from social welfare when deciding upon which policy to implement[3].

In this paper the lobby does not face any competition from another group. There are evidences of a single lobby prevalence in many areas such as trade policy and regulation (e.g., Leaver and Makris, 2006 and Dal Bo, 2007). On the contrary, we focus on the conflict between the lobby and the legislature. In particular, we assume that the legislators are individually against the policy pushed forward by the lobby. In the absence of contributions any legislator would vote against the reform and support the status quo. An example may be a protectionist industrial lobby trying to introduce trade barriers.

The exogenous parameters of our strategic environment are:

· The economic stakes $W_0$ and $W_1$ that describe the respective levels of social welfare under the reform and the status quo. We assume that the policy is socially less desirable as compared to the status quo, i.e., $W_1 \geq W_0$. We call the ratio $\frac{W_0}{W_1} \leq 1$ the *efficiency threshold*, whose magnitude defines the superiority of the status quo over the reform.

· The simple game $(N, \mathcal{W})$ which describes the legislative process.

[2]The idea that $\alpha$ could be an adverse selection parameter is suggested in Grossman and Helpman (1992) and is the main motivation of Le Breton and Salanié (2003).

[3]Some empirical estimates of this parameter have been provided in the common agency setting. Interestingly, Golberg and Maggi (1999) find that the 1983 U.S. pattern of protection is consistent with the model of Grossman and Helpman and estimate the value of the parameter $\alpha$ to be between 50 and 88, a surprisingly high range of values. Gawande and Bandyopadhyay (2000) also conclude that the model of Grossman and Helpman is consistent with the data but estimate the value of $\alpha$ to be between 3 and 8. Bradford (2001) proceeds to an empirical investigation of a variant of a model of Grossman and Helpman where politicians maximize votes and finds that politicians weight a dollar of campaign contributions about 15% more than a dollar of national income. This would lead to a value of $\alpha$ very close to 1.

· The probability distribution $F$, which describes the respective frequencies of "bad" and "good" legislators.

We aim to examine the impact of each of these key parameters on the final equilibrium outcome of the political mechanism described by this influence game. The outcome has two dimensions:

· The policy which is ultimately selected by the legislators.

· The ex ante monetary offers of the lobby and their ex post implementation.

In the first part we assume that the types of the legislators are common knowledge, an environment that we call *political certainty* as all the relevant variables are known with certainty by all the players. Under that informational assumption, the legislature is described by a general simple game, and we can investigate the role of the decision-making process within the legislature. In this setting the lobbyist has an objective to obtain its preferred policy at the lowest costs. The legislators can either reject or accept the lobbyist's offer. In the absence of contributions the legislators would vote against the policy. The legislators can differ in two respects: the degree of influence reflected by voting weights as well as the minimum prices they are willing to accept for swinging their vote reflected by $\alpha$s. Clearly, making contribution to a single legislator (as soon as he/she does not have the veto power) does not guarantee the award of lobbyist's preferred policy. We show how to calculate the minimum budget the lobby needs to secure the required support as well as the distribution of this budget between the legislators. We also demonstrate the connection of the problem with the *knapsack problem* from combinatorial optimization.

In the second part we assume instead that the types of the legislators are private informations, and refer to this environment as *political uncertainty* as the lobbies when buying votes and the legislators when voting do not know with certainty the consequences of their choices. Under this informational assumption we limit our attention to a quota voting rule, which requires a certain number of votes to pass the decision (quota) . We first examine the optimal lobbying strategy and demonstrate a critical role of the efficiency threshold in explaining the feature of this strategy. We show that the supermajorities may optimally occur due to the uncertainty about the legislators' types. As the cost of a politician's support is not known for sure, the lobby may try to increase its chances of success twofold. Firstly, it may increase the amount of individual offers. Secondly, it may disperse the influence over more legislators. We show that for relatively low values of the stake $W_0$ (or efficiency threshold) the second effect dominates, and the lobby makes offers to all legislators. On the contrary, for rather large values of the stake the first effect dominates: the lobby approaches a minimal winning coalition of legislators and buys their support with certainty (treating them as the most corrupted). One surprising feature of the optimal offer is that the larger becomes the stake, the smaller is the coalition of legislators receiving an offer. To the best of our knowledge, this outcome have not been recorded before.

In the last part we analyze whether the different assumptions may change the derived conclusions. We distinguish between two possibilities concerning the behavior of the legislators. The legislators may have strong preferences about the policy outcome regardless of whether they have voted for or against this policy. Otherwise, the legislators may care about their votes per ce regardless of the outcome. We also alternate the assumption about the payments being contingent on the way the legislators vote (in favor or against the policy) versus "pivotal bribes" (payments contingent on a vote being pivotal, Dal Bo, 2007). We show that uncertainty about legislators' types makes the costless capture more difficult.

## 1.1 Related Literature (not complete)

Some general positive models describing the lobbying process of a legislature have been proposed by Bennedsen and Feldmann (2002), Boylan (2002), Dekel, Jackson and Wolinsky (2006, 2008), Helpman and Persson (2001), Polborn (2002) and Snyder (1991) among many others. Papers by Dal Bo (2007) and Felgenhauer and Gruner (2008) study the impact of external influence on a committee from a mechanism design angle. In particular, they compare open and closed voting and reach interesting conclusions. In contrast to this paper they model the committee choice issue as a problem with common values as in Condorcet juries.

Several papers consider a single lobbying group trying to influence a committee under certainty, as we do in the first part. The closest work addressing similar questions is due to

Young (1978c). However, in Young (1978c) the problem is considered from the legislators' point of view as they maximize the "bribe"income, while the lobbyist is the "price-taker". As a result, at equilibrium the legislators may get strictly more than their reservation prices. In contrast, in our setting, the price of a legislator is either zero (then he/she votes against the policy) or it is equal to his/her reservation price (then the legislator acts in the interest of the lobby and votes against the policy). Given the prices, the lobby's objective is to choose whom to buy in order to minimize the total costs.

Our second part, where we introduce uncertainty, generalizes a single lobby case in Zaporozhets (2006) and Le Breton and Zaporozhets (2007). Our main contribution is to explain an occurrence of supermajorities and an inverse relationship between their size and the lobby's stake. There are other models explaining formation of supermajorities, however, due to the competition of two opposed lobbying groups. Thus, Banks (2000) and Groseclose and Snyder (1996) analyze the majority game with a heterogeneous legislature and show under which conditions a supermajority is optimal. Diermeier and Myerson (1999) consider a more general case but a homogeneous legislature, and concentrate on the architecture of the legislative process that would minimize monetary offers. Contrary to this literature, we offer an alternative explanation: supermajorities may be optimal due to the uncertainty about legislators' types. The uncertainty may induce the lobby to disperse its influence over a larger group of legislators in order to increase the prospects for a successful capture. To the best of our knowledge nobody else highlighted the idea that with the increase of the stake, the lobby spends more money, however, these money are distributed between less legislators.

This work also contributes to the voluminous literature on the protection for sale initiated by Grossman and Helpman (1994).[4] We assume the same preferences for the legislators: there is a trade off between the social welfare and the contributions from the lobby. However, we relax the "unitary government" hypothesis used in this literature, the assumption that the decisions on protection policies are taken by a single politician. At the same time, the analysis under uncertainty may shed some lights to the puzzle of small contributions being

[4]See, for example, Baldwin and Magee, 2000, Mitra et al.,2002, Gawande and Bandyopadhyay, 2000, Gawande and Hoekman, 2006 among others.

able to buy the policy.[5] Many empirical studies indicate that the US government is a welfare maximizer as it puts very high weight on the social welfare (e.g., Goldberg and Maggy, 1999 and Gawande and Bandyopadhyay, 2000). This is inconsistence with large estimates of deadweight losses from the distortionary policy and low contributions.

Dal Bo (2007) shows that if there is a possibility to pay pivotal voters differently, then the lobby is able to buy the legislators essentially for free even if the legislators have strong preferences about the outcome. We show that uncertainty about legislators' types makes the costless capture more difficult.

# 2 The Model

In this section, we describe the main components of our vote-buying model. The lobbying group tries to influence the legislators to push forward the reform.[6] The lobby is willing to spend up to $W_0$ dollars to pass the bill while the amount $W_1$ would be paid to prevent the passage of the bill. Sometimes, we will refer to these two policies in competition as being policies 0 and 1. We assume that $\Delta W \equiv W_1 - W_0 > 0$, i.e., that policy 1 is the socially efficient policy. The ratio $\frac{W_0}{W_1}$ which is (by assumption) lower than 1 is called the *efficiency threshold*. It measures the intensity of the superiority of the status quo as compared to the reform.

The legislature is described by a *simple game*, a pair $(N, \mathcal{W})$ where $N = \{1, 2, ..., n\}$ is the set of legislators and $\mathcal{W}$ is the set of *winning* coalitions. The interpretation is the following. A policy is adopted if and only if the subset of legislators who voted for the bill forms a winning coalition. From that perspective, the set of winning coalitions describes the rules operating in the legislature to make decisions. A coalition $C$ is *blocking* if $N \backslash C$ is not winning: some legislators (at least one) are needed to form a winning coalition. We denote by $\mathcal{B}$ the subset of blocking coalitions[7]; from the definition, the status quo is maintained as soon as the set of legislators who voted against the policy forms a blocking coalition.

[5] Gawande and Hoekman (2006) conclude that uncertainty may explain the "paradox of high $\alpha$".

[6] The framework also covers the case of private bills as defined and analysed by Boylan (2002).

[7] In game theory, $(N, \mathcal{W})$ is called the dual game.

The simple game is called *strong* if $\mathcal{B} = \mathcal{W}$[8]. The set of minimal (with respect to inclusion) winning (blocking) coalitions will be denoted $\mathcal{W}_m(\mathcal{B}_m)$.

In this chapter, all legislators are assumed to act on behalf of social welfare, i.e., all of them vote for policy 1 against policy 0 if no other event interferes with the voting process. In contrast to Banks (2000) and Groseclose and Snyder (1996) we rule out the existence of a horizontal heterogeneity across legislators. However, legislators also value money and we introduce instead some form of vertical heterogeneity. Precisely, we assume that legislators differ according to their willingness to depart from social welfare. The type of legislator $i$, denoted by $\alpha_i$, is the minimal amount of dollars that he/she needs to receive in order to sacrifice one dollar of social welfare. Therefore, if the policy adopted generates a level of social welfare equal to $W$, the payoff of legislator $i$ if he receives a transfer $t_i$ is:

$$t_i + \alpha_i W.$$

This payoff formulation is compatible with two behavioral assumptions. Either, the component $W$ appears as soon as the legislator has voted for a policy generating a level of social welfare $W$ regardless of the fact that this policy has been ultimately selected or not: we refer to this model, as *behavioral model P*, where $P$ stands for procedural. Or, the component $W$ appears whenever the policy ultimately selected generates a level of social welfare $W$ regardless of the fact that the legislator has voted for or against this policy: we refer to this model, as *behavioral model C*, where $C$ stands for consequential. In this paper, we analyze the behavioral model $C$ and explain in the last section how to adjust the results in the case of behavioral model $P$.

To prevent passage of the bill, lobby can promise to pay money to individual legislators conditional on their support of the status quo. We denote by $t_i \geq 0$ the (conditional) offers made to legislator $i$ by the lobby. The corresponding $n$-dimensional vector is denoted by $t$.

The timing of actions and events that we consider to describe the lobbying game is the following[9].

1. Nature draws the type of each legislator.

[8] When the simple game is strong, the two competing alternatives are treated equally.

[9] Specific details and assumptions will be provided in due time.

2. Lobby makes contingent monetary offers to individual legislators.
3. Legislators vote.
4. Payments (if any) are implemented.

This game has $n + 1$ players. A strategy for a lobby is a vector in $\Re^n_+$. Each legislator can chose among two (pure) strategies: to oppose or to support the bill.

The game is not fully described as we have not precisely defined yet the information held by the players when they act. In this chapter we consider two distinct settings concerning the move of player nature, but we assume otherwise that the votes of the legislators are observable, i.e., we assume open voting[10]. The first setting to which we refer as *political certainty* corresponds to the case where the vector of legislators's types is common knowledge. This informational specification has two implications: first, the lobbies know the types of the legislators when making their offers and second, each legislator knows the type of any other legislator when voting. The second setting to which we refer as *political uncertainty* corresponds instead to the case where the type of a legislator is private information. In such case, not only the lobby ignore the types of the legislators but each potential continuation voting subgame is a Bayesian game. This means that there is an adverse selection feature in the strategic relationship between lobbies and legislators and a Bayesian feature in the strategic interaction among legislators.

To conclude the description it remains to specify the details of the decision nodes. We assume that the legislators know the offers when they are asked to vote. We examine the *subgame perfect Nash equilibria*[11] of this lobbying game. In Section 3 we investigate the case of political certainty. Then, in Section 4 we move to the case of political uncertainty.

[10] The comparative analysis of closed(secret) versus open voting is the subject of several contributions among which Dal Bo (2002) and Felgenhauer and Grüner (2004).

[11] In the case of political uncertainty, the ultimate subgame is truly a Bayesian game that we solve using Bayesian-Nash equilibria. We don't use the word Bayesian subgame perfect Nash equilibrium as there is no updating operation of beliefs in our game.

# 3 Benchmark: Political Certainty

In this section, we consider the case where the vector $(\alpha_1, \alpha_2, ...., \alpha_n)$ of legislators's types is common knowledge and, without loss of generality, we assume that $\alpha_1 \leq \alpha_2 \leq ... \leq \alpha_n$.

In order to pass the proposal the lobby has to buy the support of a winning coalition. Let us denote it by $S$, then for each $i \in S$

$$t_i + \alpha_i W_0 \geq \alpha_i W_1.$$

As the lobby would like to buy the support at the lowest costs, the minimum amount the lobby should pay to legislator $i \in S$ voting in favor of the bill is[12]

$$t_i = \alpha_i \Delta W.$$

Note, that contrary to Young (1978), at equilibrium the legislators never get more than their floor prices.

The problem of the lobbyist is to find $S^* \in \mathcal{W}_m$, for which the total contribution is minimal:[13]

$$\min_{S \in \mathcal{W}_m} \sum_{i \in S} \alpha_i. \tag{1}$$

The legislators $j \notin S^*$ do not get any offers from the lobbyist, i.e., $t_j = 0$.

One may notice that if all $\alpha_i$ are identical, problem (1) is equivalent to identifying the minimal winning coalition(s) of the smallest size:

$$\min_{S \in \mathcal{W}_m} |S|,$$

where $|S|$ denotes the size of coalition $S$.

## 3.1 The Knapsack Problem

Suppose that the game $(N, \mathcal{W})$ is a *weighted majority game*, i.e., there exists an $n$-tuple $w = (w_1, ..., w_n)$ of non-negative weights with $\sum_{i \in N} w_i = 1$ and quota $q \geq 0$ such that any

[12] We assume that a legislator who is indifferent votes for the reform.

[13] The problem can be reformulated if the lobbyist is willing to block the bill instead of seeking to pass it. Then we substitue $S^* \in \mathcal{W}_m$ for $T^* \in \mathcal{B}_m$.

$S \in \mathcal{W}$ if and only if $\sum_{i \in S} w_i \geq q$.[14] Then, the problem of finding $S^*$ can be formulated as the combinatorial problem called a *knapsack problem* (e.g., Pisinger, 1995 and Kellerer et al., 2004):

$$\begin{gathered}\min_{z_i} \sum_{i=1}^{n} \alpha_i z_i \\ \text{subject to the constraints} \\ \sum_{i=1}^{n} w_i z_i \geq q \\ z_i \in \{0,1\}\,. \end{gathered} \tag{2}$$

In the formulation (2) we refer to packing of $n$ items into a knapsack. Each object $i = 1, .., n$ is characterized by a pair $(w_i, \alpha_i)$, where $\alpha_i$ is the value and $w_i$ is the weight of object $i$. Integer $z_i$ indicates whether the object $i$ is included in the knapsack ($z_i = 1$) or not ($z_i = 0$). The objective is to minimize the total weight of the knapsack $\sum\limits_{i=1}^{n} \alpha_i z_i$ while maintaining the total value $\sum\limits_{i=1}^{n} w_i z_i$ above the threshold $q$.

There is strong theoretical evidence that for the knapsack problem no polynomial time algorithm exists for computing its optimal solution (e.g., Kellerer et al., 2004). In fact, the knapsack problem belongs to a class of so-called $\mathcal{NP}$-hard optimization problems, for which there does not exist any polynomial time algorithm to find an optimal solution. However, if we consider the linear relaxation

$$z_i \in [0,1] \;\text{ for all } i = 1, ..., n,$$

things become simpler. Indeed, let us consider the impact of a small change $(dz_i, dz_j)$ leaving the constraint unchanged, i.e., such that $w_i dz_i + w_l dz_l = 0$. The change in the objective is equal to

$$\alpha_i dz_i + \alpha_j dz_j = dz_i w_i \left( \frac{\alpha_i}{w_i} - \frac{\alpha_j}{w_j} \right).$$

For $\frac{\alpha_i}{w_i} > \frac{\alpha_i}{w_l}$ the change is positive if $dz_i$ is positive and negative otherwise. This suggests the following optimal solution. Order the numbers $\left(\frac{\alpha_i}{w_i}\right)_{1 \leq i \leq n}$ in increasing order. Let $\sigma$ be that order. Then, define

[14] Consequently, any $T \in \mathcal{B}$ if and only if $\sum_{i \in T} w_i \geq 1 - q$.

$$z_{\sigma(i)} = 1 \text{ for all } i = 1, .., i^* - 1$$

and

$$z_{\sigma(i^*)} = q - \sum_{i=1}^{i^*-1} w_{\sigma(i)} z_{\sigma(i)},$$

where

$$i^* = \inf_{1 \leq i \leq n} \left\{ i : \sum_{i=1}^{i^*-1} w_{\sigma(i)} z_{\sigma(i)} \geq q \right\}.$$

This algorithm, called *greedy algorithm*, is simple but its performance under the integer constraints is not clear. Thus, Kellerer et al. (2004) show that greedy solutions can be arbitrary bad as compared to the optimal solution. Clearly, for small $n$ one can find the solution by elementary checking as we illustrate in the examples below.

The problem has a straightforward solution in the symmetric case, when $w_i = 1$ for all $i = 1, ..., n$. Suppose for simplicity that $\alpha_1 \leq \alpha_2 \leq ...\alpha_n$. In such a case:

$$z_i = \begin{cases} 1 \text{ if } i = 1, 2, ..., q, \\ 0 \text{ otherwise.} \end{cases}$$

In general, the determination of a closed-form solution may be complicated because of the trade-off between the voting weight $w_i$ of player $i$ and his reservation price $\alpha_i$.

# 4 Political Uncertainty (behavioral model C)

In this section, we analyze the lobbying game under political uncertainty in a special case where the simple game is the qualified majority game. Precisely, there is an odd number $n = 2k + 1$ of legislators and the quota $q \in [k + 1, 2k + 1]$, i. e., at least $q$ votes is required to pass the decision. If $q = k + 1$ we have a simple majority game, and if $q = 2k + 1$ we have a unanimity game. We assume that the types $\alpha_i$ of the legislators are independently and identically distributed from a continuous cumulative distribution function $F$ with the bounded support $[\underline{\alpha}, \bar{\alpha}]$ where $0 \leq \underline{\alpha} < \bar{\alpha}$.[15] We denote by $f$ the probability density function, which is assumed to be strictly positive on the whole interval $[\underline{\alpha}, \bar{\alpha}]$. Finally, we assume that the hazard rate $\frac{F}{f}$ is increasing and that the hazard rate $\frac{1-F}{f}$ is decreasing.

### 4.1 The Optimal Strategy of the Lobby

The contractual problem faced by the lobby amounts to the selection of a vector $t \in \Re_{+}^{n}$ conditional on verifiable information. Given our observability assumptions, this information consists of the $n$-dimensional vector of individual votes. In principle, the lobby could make the payment to legislator $i$ contingent upon the votes of other legislators as well or a general statistic depending upon the whole profile of votes. We assume here that the reward to legislator $i$ is simply based on his own vote: legislator $i$ receives $t_i$ if and only if he voted against the bill. This excludes, for instance, the ingenious contractual solution of Dal Bo (2002) where a given legislator is paid only in the event where his vote has been decisive.

The rest of this section is devoted to a complete analysis of this principal-agent(s) problem, i.e., to a characterization of the main features of the optimal strategy $t$. Let $N_0 \equiv \{i \in N : t_i^* > 0\}$ be the set of legislators who have been promised to receive bribes by the lobby in the optimal strategy, and we denote by $n_0^*$ the number of those legislators.

This is an important feature of the strategy as it provides an answer to the question: how large is the supermajority bought by the lobby? A second feature is the total amount of money paid by the lobby. From its perspective, this is a risky prospect, as it does not know for sure what will be the behavioral response of the legislators. Therefore, the amount $M_0^* \equiv \sum_{i \in N} t_i^*$ just represents the upper bound of the range of this random variable. Other parameters of interest are the first $E_0^*$ and second $V_0^*$ moments of this random variable. The expected rate of return of this "investment" is then given by:

$$\frac{W_0 - E_0^*}{E_0^*}.$$

The third and last feature of the strategy that deserves to be investigated is the distribution of $M_0^*$ across legislators. We have seen in section 3 that, when the simple game is

[15] Therefore, the probability that any legislator has a type less than or equal to some $\alpha$ is $F(\alpha)$.

not symmetric, i.e., when some legislators are more powerful than others or when they are not perfect substitutes, we should expect some differentials in the way they will be treated by the lobby. However, when the game is symmetric, they are all offered the same amount. Our assumption that the legislators are all identical ex ante together with the fact that the majority game is symmetric suggest that it will happen here too. This is not straightforward and calls for a proof, as the behavioral responses of the legislature following any possible history of offers is now more complicated. In cases where uniformity across the bribed legislators is shown to be optimal, we can, without loss of generality, limit ourselves to strategies defined by two dimensions: an integer $n_0^*$ and a real number $t$.

#### 4.1.1 The Voting Subgame(s)

Given any profile of offers $t$, a Bayesian strategy for legislator $i$ in the continuation voting subgame is a mapping $\sigma_i$ from the set of types $[\underline{\alpha}, \bar{\alpha}]$ into $\{0,1\} : \sigma_i(T_0, \alpha_i) = 0$ means that legislator $i$ votes for the status quo when $T_0$ is the vector of standing offers and his type is $\alpha_i$.

A key determinant of legislator $i$ strategic evaluation is the probability $p_i$ of being pivotal. Legislator $i$ of type $\alpha_i$ with an offer equal to $t_i$ votes for policy 0 if and only if

$$t_i + p_i \alpha_i W_0 \geq p_i \alpha_i W_1. \tag{3}$$

The Bayesian decision rule is therefore described by a cut point $\widehat{\alpha_i}$: legislator $i$ votes for the reform if his type $\alpha_i$ is below the cut point and votes for the status quo. The cut point $\widehat{\alpha_i}$ is defined as

$$\widehat{\alpha_i} = \max\left\{\underline{\alpha}, \min\left\{\frac{t_i}{p_i \Delta W}, \bar{\alpha}\right\}\right\}. \tag{4}$$

Under the restriction that offers are uniform i.e. $t_i \equiv t$ for all $i \in N_0$, all legislators in $N_0$ face the same decision problem. Hereafter, we will restrict our attention here to symmetric equilibria i.e. we assume that these legislators use the same decision rule. We will denote by $\widehat{\alpha}$ the cut point describing this strategy and by $p$ the probability of being pivotal for any of them. For the legislators outside $N_0$, voting for the reform is a dominant strategy.

For any legislator $i$ in $N_0$ the probability $p$ of being pivotal is simply the probability that exactly $q-1$ other legislators vote for the status quo. Since the legislators in $N \setminus N_0$ always vote for the reform, this is the probability of the event that exactly $q-1$ legislators from $N_0 \setminus \{i\}$ vote for the status quo. Given the cut point $\widehat{\alpha}$, it is possible to write down explicitly the formula for $p$:

$$p = p(t, n_0, \widehat{\alpha}) = B_{q-1}\left[n_0 - 1, F(\widehat{\alpha})\right], \tag{5}$$

where $B_k[n,p] = C_n^k p^k (1-p)^{n-k}$ denotes the probability of the event $k$ for a binomial random variable with parameters $n$ and $p$. The pivotal probability depends upon the voting strategies played by the other legislators. The equilibrium pivotal probability will be a solution of (5) when $\widehat{\alpha}$ is the equilibrium cut point. Since the equilibrium cut point is itself dependent upon the equilibrium pivotal probability, we are left with an existence issue which is covered by the following proposition[16].

**Proposition 1** *For any given $t \geq 0$ and $n_0$, the continuation voting subgame has two interior symmetric equilibria $\underline{\alpha} < \widehat{\alpha}_L < \widehat{\alpha}_R < \overline{\alpha}$ and $\overline{\alpha}$ as a corner equilibrium. The low cut point equilibrium $\widehat{\alpha}_L$ is increasing in $t$, and it Pareto dominates[17] the two other equilibria.*

**Proof.** The proof of the first assertion is divided into two cases.

(i) $n_0 = q$, i.e., the lobby offers positive transfers to a qualified majority of voters.

In this case the unique cut-off level exists. Applying (5) one gets that $p = F^{q-1}(\widehat{\alpha})$. Substituting it into (4) it follows that for $t \in (\bar{\alpha}\Delta W, \infty)$ the cut point $\widehat{\alpha} = \bar{\alpha}$, and for $t \in [0, \bar{\alpha}\Delta W]$ it is defined by

$$\widehat{\alpha} F^{q-1}(\widehat{\alpha}) = t/\Delta W. \tag{6}$$

From the assumptions on the distribution function it follows that the LHS of this equality is

[16] A game with similar features has been examined by Palfrey and Rosenthal (1985) as describing the decision to vote in an election given that voters incur a private cost to do so. In their model voters compare this cost to the expected differential benefit. They also face the issue of multiplicity of equilibria.

[17] Some warning is needed about what we mean by Pareto dominance. Precisely, we refer to unanimity in restriction to the coalition $N_0$ of legislators. It represents a way to solve the coordination issue faced by this subset of players.

a strictly increasing function of $\widehat{\alpha}$, therefore $\widehat{\alpha}$ is uniquely defined by (6). One can see that $\widehat{\alpha}$ is an increasing function of $t$.

(ii) $n_0 > q$, i.e., the number of voters receiving positive offers from the lobby is higher than a qualified majority.

In this case there can be 3, 2 or 1 equilibrium cut-off levels. From (5) the probability of being pivotal is

$$C_{n_0-1}^{q-1} F^{q-1}(\widehat{\alpha})(1-F(\widehat{\alpha}))^{n_0-q}.$$

First, let us consider the function

$$\alpha F^{q-1}(\alpha)(1-F(\alpha))^{n_0-q}.$$

One can see that on the interval $[\underline{\alpha}, \bar{\alpha}]$ it is non-negative: it is equal to zero at $\underline{\alpha}$ and $\bar{\alpha}$, and it is strictly positive elsewhere on the interval. It has exactly one maximum at $\overline{\alpha}_{n_0} \in (\underline{\alpha}, \bar{\alpha})$, where $\overline{\alpha}_{n_0}$ is defined by

$$\frac{\partial}{\partial \alpha}\left[\alpha F^{q-1}(\alpha)(1-F(\alpha))^{n_0-q}\right] = 0,$$

or

$$F^{q-2}(\alpha)(1-F(\alpha))^{n_0-q-1}\left[F(\alpha)(1-F(\alpha)) + \alpha f(\alpha)\left[(q-1)-(n_0-1)F(\alpha)\right]\right] = 0, \tag{7}$$

To see that $\overline{\alpha}_{n_0}$ is uniquely defined on the interval $(\underline{\alpha}, \bar{\alpha})$ let us rewrite the expression in the brackets in (7) as

$$\alpha F(\alpha)(1-F(\alpha))\left[\frac{1}{\alpha} + (q-1)\frac{f}{F}(\alpha) - (n_0-q)\frac{f}{1-F}(\alpha)\right] = 0.$$

From the assumptions on the hazard rates it follows that the function in the brackets is monotonically decreasing, and for $\alpha \to \underline{\alpha}$ it approaches to $+\infty$ and for $\alpha \to \bar{\alpha}$ it approaches to $-\infty$. Therefore, it can be equal to zero exactly at one point $\overline{\alpha}_{n_0} \in (\underline{\alpha}, \bar{\alpha})$.

For convenience let

$$t_{\max} = C_{n_0-1}^{q-1}\overline{\alpha}_{n_0}\Delta W F^{q-1}(\overline{\alpha}_{n_0})(1-F(\overline{\alpha}_{n_0}))^{n_0-q}.$$

From (4), (5) for $t \in [0, t_{\max})$ there are two solutions for $\widehat{\alpha}$ defined by

$$\alpha F^{q-1}(\alpha)(1-F(\alpha))^{n_0-q} = \frac{t}{C_{n_0-1}^{q-1}\Delta W}. \tag{8}$$

We denote them by $\widehat{\alpha}_L$ and $\widehat{\alpha}_R$ and assume that $\widehat{\alpha}_L \leq \widehat{\alpha}_R$. For all $t \in (0, \infty)$ there is also the solution $\widehat{\alpha} = \bar{\alpha}$.

Consider now the second assertion. The expected utility of agent $i$ is

$$U_i\left(\alpha_i, \widehat{\alpha}\right) = \begin{cases} P^1(\widehat{\alpha})\alpha_i W_1 + (1 - P^1(\widehat{\alpha}))\alpha_i W_0, \text{ for } \alpha_i \geq \widehat{\alpha} \\ P^0(\widehat{\alpha})\alpha_i W_0 + (1 - P^0(\widehat{\alpha}))\alpha_i W_1 + t, \text{ for } \alpha_i \leq \widehat{\alpha} \end{cases},$$

where $P^1$ (respectively $P^0$) is the probability that at least $k$ other agents from $N_0$ choose 1 (respectively 0).

First, let us consider the case $\alpha_i \leq \widehat{\alpha}$. The expected utility can be written as

$$U_i(\alpha_i, \widehat{\alpha}) = \alpha_i W_1 - \Delta W P^0(\widehat{\alpha})\alpha_i + t.$$

The probability $P^0$ can be written as

$$P^0(\widehat{\alpha}) = \sum_{i=q-1}^{n_0-1} C_{n_0-1}^{i} F^i(\widehat{\alpha})\left(1 - F(\widehat{\alpha})\right)^{n_{0.}-1-i}.$$

From lemma 1 it follows that

$$\frac{\partial P^0}{\partial \widehat{\alpha}} = f(\widehat{\alpha})(n_0 - k)C_{n_0-1}^{q-2} F^k(\widehat{\alpha})(1 - F(\widehat{\alpha}))^{n_0-q} \geq 0.$$

Thus, $P^0(\widehat{\alpha})$ is increasing and therefore, expected utility is decreasing in $\widehat{\alpha}$. Then, $U_i(\alpha_i, \widehat{\alpha}_L) \geq U_i(\alpha_i, \widehat{\alpha}_R)$, i.e., in the equilibrium $\widehat{\alpha}_L$ utility of each agent $i$ is at least as high as in equilibrium $\widehat{\alpha}_R$. The case $\alpha_i \geq \widehat{\alpha}$ is similar. ■

In solving backward the whole game, we solve each terminal voting subgames following a pair $(t, n_0)$ by considering the equilibrium $\widehat{\alpha}_L = \widehat{\alpha}_L(t, n_0)$ which will be denoted simply by $\widehat{\alpha} = \widehat{\alpha}(t, n_0)$ without risk of confusion. In what follows we will also use the fact that $\widehat{\alpha}_L \in [0, \overline{\alpha}_{n_0}]$.

### 4.1.2 The Optimal Offer of the Lobby

We are now in position to investigate the two dimensions of the optimal strategy of the lobby. Given $t$ and $N_0$, the probability of accepting the contribution by any legislator in $N_0$ is simply $F(\widehat{\alpha})$ and the probability of success for the lobby is

$$G(\widehat{\alpha}, n_0) = \sum_{j=q}^{n_0} B_j\left[n_0, F(\widehat{\alpha})\right]. \tag{9}$$

Therefore, the expected payoff of the lobby is

$$G(\widehat{\alpha}, n_0)W_0 - n_0 F(\widehat{\alpha})t.$$

First, we as $t$ is a non-decreasing function of $\alpha$ on the interval $[0, \overline{\alpha}_{n_0}]$, we may optimize the expected payoff with respect to $\alpha$ instead of optimizing with respect to $t$. Second, for convenience we divide the expected payoff by the positive constant $\Delta W$ and consider the following function:

$$\Pi(\widehat{\alpha}, n_0) = G(\widehat{\alpha}, n_0)r - T(\widehat{\alpha}, n_0), \tag{10}$$

where we denote by

$$r = \frac{W_0}{\Delta W}$$

and

$$T(\alpha, n_0) = \frac{\alpha F^q(\alpha)(1 - F(\alpha))^{n_0 - q}}{B(q, n_0 + 1 - q)}$$

is the expected total transfer from the lobby.

In what follows we often use the following function:

$$\phi(\alpha, n_0) = \frac{\alpha F^{q-1}(\alpha)(1 - F(\alpha))^{n_0 - q}}{B(q, n_0 + 1 - q)}, \tag{11}$$

and we can re-express expected total transfer as:

$$T(\alpha, n_0) = F(\alpha)\phi(\alpha, n_0).$$

By our assumptions $\frac{\partial \phi(\alpha, n_0)}{\partial \alpha} \geq 0$. Then, the maximization problem is considered for the following range of the variables: $\Omega = \left\{ (\alpha, n_0) : \ n_0 \in \{q, ..., n\}, \ \alpha \in [\underline{\alpha}, \overline{\alpha}] \text{ and } \frac{\partial \phi(\alpha, n_0)}{\partial \alpha} \geq 0 \right\}$[18] and the parameter $r \in (0, +\infty]$.

**Remark 1.** If we arrange random variables $\alpha_1, \alpha_2, ...\alpha_n$ in order of magnitude and write them down as $\alpha_{(1)} \leq \alpha_{(2)} \leq ... \leq \alpha_{(n)}$, where $\alpha_{(i)}$ is called the $i$th order statistic. Then we may notice that the expression (9) is the cdf of the $q$th order statistic for $\alpha_{(1)} \leq \alpha_{(2)} \leq$

[18] By our assumptions $\phi(\alpha, n_0)$ is non-decreasing in $\alpha$ on the interval, and the equilibrium value of our interest $\widehat{\alpha}_L$.

... $\leq \alpha_{(n_0)}$, and it can be rewritten in terms of incomplete beta function (e.g., David and Nagaraja, 2003):

$$F_{(q,n_0)}(\widehat{\alpha}) = \frac{\int\limits_0^{F(\widehat{\alpha})} t^{q-1}(1-t)^{n_0-q}dt}{B(q, n_0-q-1)}, \tag{12}$$

where $B(a,b) \equiv \int\limits_0^1 t^{a-1}(1-t)^{b-1}dt$ is beta-function. One may notice that if $F(\widehat{\alpha})$ is the cdf of the uniform distribution on $[0,1]$ then $F_{(q,n_0)}(\widehat{\alpha})$ is the cdf of beta distribution.

The following proposition describes the optimal offer $t$ when the lobby buys a minimal winning coalition.

**Proposition 2** *When $n_0 = q$, the equilibrium offer $t$ is uniquely defined:*

*(i) for $r \in [0, q\underline{\alpha}]$ the equilibrium offer $t = 0$;*

*(ii) for $r \in \left(q\underline{\alpha}, q\bar{\alpha} + \frac{1}{1+f(\bar{\alpha})}\right)$ the equilibrium offer $t = aF^{q-1}(a)\Delta W < \Delta W\bar{\alpha}$, where $a \in (\underline{\alpha}, \bar{\alpha})$ is the unique solution to the equation:*

$$r - qa = \frac{F(a)}{f(a)}.$$

*(iii) for $r \in \left[q\bar{\alpha} + \frac{1}{1+f(\bar{\alpha})}, +\infty\right)$ the equilibrium offer $t = \Delta W\bar{\alpha}$.*

**Proof.** After substituting for $t$ from (6) the expected payoff of the lobby 0 becomes

$$\Pi(q, \widehat{\alpha}) = F^q(\widehat{\alpha})\left(r - q\widehat{\alpha}\right).$$

The first-order condition with respect to $\widehat{\alpha}$ is:

$$\frac{\partial\Pi(q,\widehat{\alpha})}{\partial\widehat{\alpha}} = qF^{q-1}(\widehat{\alpha})f(\widehat{\alpha})\left[r - q\widehat{\alpha} - \frac{F(\widehat{\alpha})}{f(\widehat{\alpha})}\right] = 0.$$

First, consider the equation

$$r - q\widehat{\alpha} = \frac{F(\widehat{\alpha})}{f(\widehat{\alpha})} \tag{13}$$

on the interval $\widehat{\alpha} \in (\underline{\alpha}, \bar{\alpha})$. By the assumption on the hazard rate the RHS is an increasing function of $\widehat{\alpha}$, and the LHS is a decreasing one. Therefore, these two functions can intersect at most once on the interval $(\underline{\alpha}, \bar{\alpha})$. It is easy to see that interior solution $a_{\max}(q) \in (\underline{\alpha}, \bar{\alpha})$ exists if and only if $r - q\underline{\alpha} > 0$ and $r - q\bar{\alpha} < \frac{1}{f(\bar{\alpha})}$. ■

Of course, it is not necessarily optimal for the lobby to buy a minimal winning coalition. It may prefer to buy a supermajority. Given the fact that the function $\Pi$ is continuous with respect to $t$ (equivalently, $\alpha$), and that $n_0$ takes a finite number of values, an optimal strategy is always well defined. In what follows, we investigate the following general questions concerning the equilibrium strategy of the lobby.

· Is it the case that lobbying activities are normal goods, i.e., exhibiting positive income effects?

· Is it the case that the size of the coalition of legislators approached by the lobby decreases as the stake becomes larger?

Not surprisingly, the larger is $r$ (or, equivalently, the stake $W_0$), the more money the lobby spends to buy votes. What is more intriguing, however, is that this money is spent on less legislators, i.e., the size of the coalition to which offers are made becomes smaller. When the lobby finds it optimal to bribe the simple majority it offers the maximum possible bribe to be sure the offers are accepted, i.e., the legislators are treated as being of the highest type $\overline{\alpha}$.

There exist thresholds $\underline{r}$ and $\overline{r}$, such that

·for $r \in [0, \underline{r}]$ the lobby does not bribe anybody;

·for $r \in (\underline{r}, \overline{r})$ the optimal $n_0$ is non-increasing and $t$ is non-decreasing;

·for $r \in [\overline{r}, \infty)$ the optimal $n_0$ is $q$ (a qualified majority) and $t = \Delta W \bar{\alpha}$.

The proposition below establishes a lower bound for $\underline{r}$.

**Proposition 3** *For $r \leq q\underline{\alpha}$ the lobby does not intervene, i.e., the optimal transfers are $t = 0$.*

**Proof.** One may check that

$$\begin{aligned} \frac{\partial \Pi(\alpha, n_0)}{\partial \alpha} &= C_{n_0}^{q}(1 - F(\alpha))^{n_0 - q - 1} \times \\ &\left[(1 - F(\alpha))\frac{\partial \Pi(\alpha, q)}{\partial \alpha} + q\alpha(n_0 - q)F^q(\alpha)f(\alpha)\right]. \end{aligned} \tag{14}$$

Therefore,

$$\lim_{\alpha \to \underline{\alpha}} \frac{\partial \Pi(\alpha, n_0)}{\partial \alpha} = C^q_{n_0} \frac{\partial \Pi(\alpha, q)}{\partial \alpha}.$$

From Proposition 2 the derivative $\frac{\partial \Pi(\alpha,q)}{\partial \alpha} \leq 0$ for the indicated values of $r$. ■

In what follows we apply monotone comparative statics theorems (e.g., Topkis, 1998 and Milgrom and Shanon, 1999) in order to state our main result.

**Proposition 4** *If*

$$\frac{\partial \Pi(\alpha, n_0)}{\partial \alpha} = 0$$

*then*

$$\frac{\partial^2 \Pi(\alpha, n_0)}{\partial \alpha \partial n_0} \geq 0.$$

**Proof.** Remark 1 implies

$$\frac{\partial G(\alpha, n_0)}{\partial \alpha} = \frac{F^{q-1}(\alpha)\,(1 - F(\alpha))^{n_0 - q}\, f(\alpha)}{B(q, n_0 - q - 1)}. \tag{15}$$

Then, we deduce that

$$\frac{\partial^2 G(\alpha, n_0)}{\partial \alpha \partial n_0} = \frac{\partial G(\alpha, n_0)}{\partial \alpha} \left[\ln(1 - F(\alpha)) + \psi(n_0 + 1) - \psi(n_0 + 1 - q)\right], \tag{16}$$

where $\psi(x)$ is the logarithmic derivative of the gamma function.[19]

Similarly,

$$\begin{aligned} \frac{\partial^2 T(\alpha, n_0)}{\partial \alpha \partial n_0} &= \frac{\partial T(\alpha, n_0)}{\partial \alpha} \left[\ln(1 - F(\alpha)) + \psi(n_0 + 1) - \psi(n_0 + 1 - q)\right] \\ &\quad - T(\alpha, n_0) \frac{f(\alpha)}{1 - F(\alpha)}. \end{aligned} \tag{17}$$

Combining (16) and (17) we get that

$$\begin{aligned} \frac{\partial^2 \Pi(\alpha, n_0)}{\partial \alpha \partial n_0} &= \frac{\partial \Pi(\alpha, n_0)}{\partial \alpha} \left[\ln(1 - F(\alpha)) + \psi(n_0 + 1) - \psi(n_0 + 1 - q)\right] \\ &\quad + T(\alpha, n_0) \frac{f(\alpha)}{1 - F(\alpha)}. \end{aligned}$$

If $\frac{\partial \Pi(\alpha, n_0)}{\partial \alpha} = 0$ the above expression is equal to $T(\alpha, n_0) \frac{f(\alpha)}{1 - F(\alpha)}$, which is non-negative on the interval $[\underline{\alpha}, \overline{\alpha}]$. ■

[19] It is called *digamma function* (e.g., Andrews et al., 1999).

**Proposition 5** *The following inequalities holds true:*

$$\frac{\partial^2 \Pi(\alpha, n_0)}{\partial \alpha \partial r} \geq 0$$

*and*

$$\frac{\partial^2 \Pi(\alpha, n_0)}{\partial r \partial n_0} \geq 0.$$

**Proof.** 1. From the expression (10) for the expected payoff

$$\frac{\partial^2 \Pi(\alpha, n_0)}{\partial \alpha \partial r} = \frac{\partial G(\alpha, n_0)}{\partial \alpha},$$

which is calculated in (15). Obviously, it is non-negative on the interval $[\underline{\alpha}, \overline{\alpha}]$.

2. One may notice that

$$\frac{\partial^2 \Pi(\alpha, n_0)}{\partial r \partial n_0} = \frac{\partial G(\alpha, n_0)}{\partial n_0}.$$

The idea is to show that the function $\frac{\partial G(\alpha, n_0)}{\partial n_0}$ is increasing in $\alpha$ for $\alpha \in (\underline{\alpha}, \widetilde{\alpha})$ and it is decreasing for $\alpha \in (\widetilde{\alpha}, \overline{\alpha})$ for some $\widetilde{\alpha}$. On top of that, this function is equal to zero for $\alpha = \underline{\alpha}$ and $\overline{\alpha}$. This would imply that $\frac{\partial G(\alpha, n_0)}{\partial n_0} \geq 0$ on the interval $[\underline{\alpha}, \overline{\alpha}]$.

From the expression (16) we deduce that $\widetilde{\alpha} \in (\underline{\alpha}, \overline{\alpha})$ is defined from:

$$\ln(1 - F(\alpha)) + \psi(n_0 + 1) - \psi(n_0 + 1 - q) = 0. \tag{18}$$

Lemma 1 implies that

$$\psi(n_0 + 1) - \psi(n_0 + 1 - q) = \ln\left(\frac{\gamma + n_0}{\gamma + n_0 - q}\right).$$

Therefore, $\widetilde{\alpha} = F^{-1}\left(\frac{q}{\gamma + n_0}\right)$. ■

**Corollary 6** *If the function $\Pi(\alpha, n_0)$ has an inferior maximum then the optimal $\alpha(r)$ and $n_0(r)$ are increasing functions.*

**Proof.** From Propositions 4 and 5 we may conclude that the second-order partial derivatives of the function $\Pi(\alpha, n_0, r)$ are non-negative in the whole region $\Omega$. Then, the result follows from Theorem 4 (Milgrom and Shanon, 1994). ■

**Proposition 7** *The derivative*

$$\frac{\partial \Pi(\alpha, n_0)}{\partial n_0} \geq \Pi(\alpha, n_0)\left[\ln(1 - F(\alpha)) + \psi(n_0 + 1) - \psi(n_0 + 1 - q)\right].$$

**Proof.** It can be shown that

$$\frac{\partial T(\alpha, n_0)}{\partial n_0} = T(\alpha, n_0)\left[\ln(1 - F(\alpha)) + \psi(n_0 + 1) - \psi(n_0 + 1 - q)\right].$$

Similarly,

$$\frac{\partial G(\alpha, n_0)}{\partial n_0} = \frac{\int\limits_0^{F(\alpha)} t^{q-1}(1-t)^{n_0-q}\ln(1-t)dt}{B(q, n_0 - q - 1)} + R(\alpha, n_0)\left[\psi(n_0 + 1) - \psi(n_0 + 1 - q)\right].$$

The mean-value theorem implies that there is $\xi \in (0, F(\alpha))$ such that

$$\frac{\int\limits_0^{F(\alpha)} t^{q-1}(1-t)^{n_0-q}\ln(1-t)dt}{B(q, n_0 - q - 1)} = \ln(1-\xi)G(\alpha, n_0) > \ln(1 - F(\alpha))G(\alpha, n_0).$$

Therefore,

$$\frac{\partial G(\alpha, n_0)}{\partial n_0} > G(\alpha, n_0)\left[\ln(1 - F(\alpha)) + \psi(n_0 + 1) - \psi(n_0 + 1 - q)\right]$$

and the result follows. ■

Proposition 7 implies that for $\alpha$ such that $\Pi(\alpha, n_0) \geq 0$ and $\alpha \in (\underline{\alpha}, \widetilde{\alpha})$ (where $\widetilde{\alpha}$ is defined by (18)) the derivative $\frac{\partial \Pi(\alpha, n_0)}{\partial n_0} \geq 0$, which means that the optimal $n_0$ is maximal possible in $\Omega$. This conclusion together with Corollary 6 leads us to the main result:

**Proposition 8** *The optimal pairs $(\alpha, n_0)$ lie on the border of the set $\Omega$.*

Our results imply that when we move along the part of the border, on which $n_0 = n$, the optimal $\alpha$ should increase with respect to $r$. As the individual offer $t$ is also an increasing function of $r$, then, the function $T$ should be also increasing in $r$ on this part of the border.

In what follows we establish what happens when we get on another part of the border, $l = \left\{(\alpha, n_0) : \frac{\partial \phi(\alpha, n_0)}{\partial \alpha} = 0\right\}$. Below we show that the optimal $n_0$ is a decreasing function of $r$. The monotone comparative statics theorems suggest that we need to check if the second-order derivative $\frac{\partial^2 \Pi(\alpha, n_0)}{\partial r \partial \alpha} > 0$ for $(\alpha, n_0) \in l$. This is equivalent to the following statement.

**Proposition 9** *The derivative*

$$\frac{d}{d\alpha}T(\alpha, n_0^*(\alpha)) > 0,$$

*where*[20]

$$n_0^*(\alpha) = \frac{1 - F(\alpha)}{\alpha f(\alpha)} + \frac{q-1}{F(\alpha)} + 1. \tag{19}$$

The proof can be found in the Appendix.

**Remark 2.** From the proof it follows that the assumption on the hazard rate can be weakened. In fact, we need just that $\frac{1-F(\alpha)}{\alpha f(\alpha)}$ be decreasing in $\alpha$ and $\frac{\alpha F(\alpha)}{f(\alpha)}$ be increasing in $\alpha$.

Proposition 9 implies that along the border $l$ the total expected contributions $T$ from the lobby is an increasing function of $r$. Combining this with the earlier observation, we may conclude that $T$ is increasing in $r$ on the whole border of $\Omega$.

**Proposition 10** *For $r > q\bar{\alpha} + \frac{1}{1+f(\bar{\alpha})}$ the optimal $n_0 = q$ and the optimal $\alpha = \overline{\alpha}$.*

**Proof.** The result follows from Proposition 2 and (19). ■

## 4.2 Pivotal Bribes

Following Dal Bo (2007), in this subsection we would like to investigate a situation in which the lobby has the possibility to offer different rewards to the pivotal legislators.

One may notice that under certainty the costless capture is possible. Suppose for simplicity $\alpha$s are arranged in a non-decreasing order. Then, the lobby offers $\Delta W \alpha_i$ to $(q+1)$ "cheapest" legislators. We assume that the offers should be credible, i.e., the budget $W_0$ should be at least

$$\sum_{i=1}^{q+1} \Delta W \alpha_i. \tag{20}$$

Under uncertainty, the costless capture is possible if the lobby offers $t = \Delta W \bar{\alpha}$ to $q+1$ voters. Therefore, for being credible, the lobby should possess at least $(q+1)\Delta W \bar{\alpha}$, which is higher than (20).

[20] This expression is rewritten (7).

One may notice that when considering the voting subgame(s), legislator $i$ of type $\alpha_i$ with an offer $t_i$ votes for the status quo if and only if

$$p_i t_i + p_i \alpha_i W_0 \geq p_i \alpha_i W_1.$$

Therefore, the expression for the cut point $\widehat{\alpha}_i$ becomes:

$$\widehat{\alpha}_i = \max\left\{\underline{\alpha}, \min\left\{\frac{t_i}{\Delta W}, \bar{\alpha}\right\}\right\}.$$

Following the above steps we derive the following expected payoff function for the lobby:

$$\Pi(\widehat{\alpha}, n_0) = G(\widehat{\alpha}, n_0)r - T(\widehat{\alpha}, n_0),$$

where

$$r = \frac{W_0}{\Delta W}.$$

The expected transfers are

$$T(\alpha, n_0) = qC_{n_0}^q \alpha F^q(\alpha)(1 - F(\alpha))^{n_0 - q},$$

reflect that there are only pivotal members are paid. One may notice that the optimization problem is equivalent the problem already analyzed with the less restricted set of variables: $n_0 \in \{q, ..., n\}$ and $\widehat{\alpha} \in [\underline{\alpha}, \overline{\alpha}]$. Along the lines of the previous section, we establish the following result.

**Proposition 11** *(?)Suppose that $r \leq (q+1)\bar{\alpha}$ (i.e., the costless capture is not possible).Then, there exist thresholds $\underline{r}$ and $\overline{r}$, such that*

*for $r \in [0, \underline{r}]$ the lobby does not bribe anybody;*

*for $r \in (\underline{r}, \overline{r})$ the optimal $n_0 = n$; and $t$ is non-decreasing;*

*for $r \in [\overline{r}, (q+1)\bar{\alpha})$ the optimal $n_0 = q$ and $t = \Delta W \bar{\alpha}$.*

## 4.3 Procedurial versus Consequential

Before we assumed that the legislators had strong preferences about the policy outcome regardless of whether they have voted for or against this policy. In this subsection, to the contrary, we assume that the legislators care about their votes per ce regardless of the outcome.

Following the lines of Section 4, let us consider first the voting subgames. In contrast to the model $C$, it does not matter whether the legislator is pivotal. A legislator $i$ of type $\alpha_i$ with an offer equal to $t_i$ votes for the status quo if and only if:

$$t_i + \alpha_i W_0 \geq \alpha_i W_1.$$

The decision rule is therefore described by a cut point $\widehat{\alpha}_i$ : legislator $i$ votes for the status quo if his type $\alpha_i$ is below the cut point and votes for the reform otherwise. The cut point $\widehat{\alpha}_i$ is defined as

$$\widehat{\alpha}_i = \max\left\{\underline{\alpha}, \min\left\{\frac{t_i}{\Delta W}, \bar{\alpha}\right\}\right\} \tag{21}$$

Recall $N_0 \equiv \{i \in N : t_i > 0\}$. Under the restriction that offers are uniform, i. e., $t_i \equiv t$ for all $i \in N_0$, all legislators in $N_0$ face the same decision problem. Again, we restrict our attention here to symmetric equilibria and denote the cut point describing this strategy by $\widehat{\alpha}$. For the legislators outside $N_0$, voting for the reform is a dominant strategy.

Following the steps of the previous section, the expected payoff of the lobby for given $t$ and $n_0$ is

$$\Pi(\widehat{\alpha}, n_0) = G(\widehat{\alpha}, n_0)r - T(\widehat{\alpha}, n_0),$$

where $G(t, n_0)$ and $r$ are defined as before and

$$T(\widehat{\alpha}, n_0) = n_0 \widehat{\alpha} F(\widehat{\alpha}).$$

One may notice that the expressions for

$$\frac{\partial^2 \Pi(\alpha, n_0)}{\partial \alpha \partial r} \text{ and } \frac{\partial^2 \Pi(\alpha, n_0)}{\partial r \partial n_0}$$

the same as in the model $C$.

# 5 Conclusion

# 6 Appendix

**Lemma 1** *There exists* $\gamma \equiv \gamma(n_0, q)$ *such that* $\gamma \in (1/2, 1)$ *and*

$$\psi(n_0+1) - \psi(n_0+1-q) = \ln(\gamma + n_0) - \ln(\gamma + n_0 - q). \tag{22}$$

**Proof.** First, we notice that the right-hand side of (22) can be rewritten as

$$\ln\left(1 + \frac{q}{\gamma + n_0 - q}\right),$$

which is a decreasing function of $\gamma$.

Next, the left-hand side can be rewritten as

$$\sum_{i=0}^{q-1} \frac{1}{n_0 - i} \text{ (e.g., Andrews et al., 1999)}$$

or, it is equivalent to

$$\sum_{i=n_0-q+1}^{n_0} \frac{1}{i}.$$

Lastly, we are going to show that

$$\ln\left(\frac{\gamma + n_0}{\gamma + n_0 - q}\right) - \sum_{i=n_0-q+1}^{n_0} \frac{1}{i} \begin{cases} \leq 0, \text{ if } \gamma = 1 \\ \geq 0, \text{ if } \gamma = \frac{1}{2} \end{cases}. \tag{23}$$

It is enough to prove the result since the function in (23) is continuous and decreasing in $\gamma$.

Notice that

$$\ln\left(\frac{\gamma + n_0}{\gamma + n_0 - q}\right) = \sum_{i=n_0-q+1}^{n_0} \ln\left(1 + \frac{1}{\gamma + i - 1}\right). \tag{24}$$

For $\gamma = 1$ the sum in (24) boils down to

$$\sum_{i=n_0-q+1}^{n_0} \ln\left(1 + \frac{1}{i}\right) \leq \sum_{i=1}^{q} \frac{1}{i}.$$

This proves the first part of (23).

In order to prove the necessary inequality for $\gamma = 1/2$, we use the following lower bound:

$$\ln x \geq 2\frac{x-1}{x+1}.$$

Then, the expression (24) can be evaluated as

$$\sum_{i=n_0-q+1}^{n_0} \ln\left(1 + \frac{1}{i-1/2}\right) \geq \sum_{i=1}^{q} \frac{1}{i},$$

which proves the second part of (23). ■

**Proof of Proposition 9.**

One may check that

$$\frac{d}{d\alpha}T(\alpha, n_0^*(\alpha)) = \phi(\alpha, n_0^*(\alpha))\left[f(\alpha) + \frac{dn_0^*}{d\alpha}F(\alpha)\frac{\partial \ln T(\alpha, n_0^*(\alpha))}{\partial n_0}\right].$$

For convenience we denote the expression in the brackets by $H$. Then, the proof will follow the following steps.

1. Show that $\frac{dn_0^*}{d\alpha} < 0$;
2. It is possible to find some constant $A$ such that
$$-\frac{dn_0^*}{d\alpha}F(\alpha) \leq A;$$
3. It is possible to find some constant $B$ such that
$$\frac{\partial \ln T(\alpha, n_0^*(\alpha))}{\partial n_0} \leq B;$$
4. If $B \geq 0$ then $AB < 1$.

One can notice that if
$$\frac{\partial \ln T(\alpha, n_0^*(\alpha))}{\partial n_0} < 0$$
then obviously $H > 0$. If the opposite inequality is true, i.e., then
$$\frac{dn_0^*(\alpha)}{d\alpha}F(\alpha)\frac{\partial \ln T(\alpha, n_0^*(\alpha))}{\partial n_0} \leq AB < 1,$$
which implies $H > 0$. Let us proceed step by step.

1. Follows from the expression (19) for $n_0^*$ and the assumption on the hazard rate.

2. For further convenience, let us introduce the following variable

$$\delta(\alpha) = \frac{F(\alpha)}{\alpha f(\alpha)}.$$

Then, (19) implies

$$F(\alpha) = \frac{\delta(\alpha) + q - 1}{\delta(\alpha) + n_0^*(\alpha) - 1}. \tag{25}$$

We can calculate the derivative

$$\frac{dn_0^*(\alpha)}{d\alpha} = f(\alpha)\left[-\frac{1}{\alpha f(\alpha)} - \frac{q-1}{F^2(\alpha)}\right] - (1 - F(\alpha))\frac{1}{(\alpha f(\alpha))^2}\left[f(\alpha) + \alpha f'(\alpha)\right]. \tag{26}$$

The assumption on the hazard rate can be reformulated as

$$f'(\alpha) \leq \frac{f^2(\alpha)}{F(\alpha)},$$

and then (26) implies:

$$-\frac{dn_0^*(\alpha)}{d\alpha} \leq f(\alpha)\left[\frac{q-1}{F^2(\alpha)} + \frac{1-F(\alpha)}{(\alpha f(\alpha))^2} + \frac{1}{\alpha f(\alpha)F(\alpha)}\right]. \tag{27}$$

Substituting for

$$\delta = \frac{F(\alpha)}{\alpha f(\alpha)},$$

one gets

$$-\frac{dn_0^*(\alpha)}{d\alpha}F(\alpha) \leq f(\alpha)\left[\frac{q-1}{F^2(\alpha)} + \delta^2\left(\frac{1}{F(\alpha)} - 1\right) + \frac{\delta}{F(\alpha)}\right].$$

Then, we define $A$ as:

$$A = \left(\frac{\delta^2}{\delta + q - 1} + 1\right)(\delta + n_0^*(\alpha) - 1).$$

3. One may check that

$$\frac{\partial \ln T(\alpha, n_0^*(\alpha))}{\partial n_0} = \psi(n_0 + 1) - \psi(n_0 + 1 - q) + \ln(1 - F(\alpha)).$$

Applying Lemma and (25) we get

$$\frac{\partial \ln T(\alpha, n_0^*(\alpha))}{\partial n_0} = \ln\left(\frac{\gamma + n_0^*(\alpha)}{\gamma + n_0^*(\alpha) - q}\frac{n_0^*(\alpha) - q}{\delta + n_0^*(\alpha) - 1}\right).$$

We denote by $C$ the argument of the ln above. Then,

$$C = 1 + \frac{1}{\delta + n_0^*(\alpha) - 1}\left(\gamma\frac{1 - \frac{\delta}{n_0^*(\alpha) - q} - \frac{n_0^*(\alpha) - 1}{n_0^*(\alpha) - q}}{\frac{\gamma}{n_0^*(\alpha) - q} + 1} + (1 - \delta)\frac{1}{\frac{\gamma}{n_0^*(\alpha) - q} + 1}\right).$$

Using the fact that

$$\frac{n_0^*(\alpha) - 1}{n_0^*(\alpha) - q} \geq 2$$

and that

$$\frac{1}{\frac{\gamma}{n_0^*(\alpha) - q} + 1} \leq 1,$$

we obtain that, $C \leq 1 - \delta - \gamma$. Therefore,

$$\frac{\partial \ln T(\alpha, n_0^*(\alpha))}{\partial n_0} \leq \ln \left(1 + \frac{1 - \delta - \gamma}{\delta + n_0^*(\alpha) - 1}\right).$$

Further,

$$\frac{\partial \ln T(\alpha, n_0^*(\alpha))}{\partial n_0} \leq \frac{1 - \delta - \gamma}{\delta + n_0^*(\alpha) - 1}.$$

Finally, we denote by

$$B = \frac{1 - \delta - \gamma}{\delta + n_0^*(\alpha) - 1}.$$

4. The expressions for $A$ and $B$ imply

$$AB = \left(\frac{\delta^2}{\delta + q - 1} + 1\right)(1 - \delta - \gamma).$$

One may check that $B \geq 0$ if and only if $\delta \leq 1 - \gamma < 1/2$. Then,

$$AB < \left(\frac{1/4}{q - 1} + 1\right)\frac{1}{2} < 1.$$

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