

Privacy Regulation and Quality Investment*

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Abstract

This paper analyzes how a privacy regulation restricting data disclosure affects quality investment by a monopoly service provider—who derives revenues solely from disclosing user data to third parties—and social welfare. In our model, a user’s gross utility from the service depends on its quality and the amount of information disclosed. We show that in a fully covered market, the regulation reduces quality investment but may still be socially desirable when quality and information are not strong complements from consumers’ perspective. In a partially covered market, the regulation may raise quality and social welfare even when quality and information are highly complementary.

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1 Introduction

Data has often been called the oil of the digital era—a highly valuable resource and a force of change and growth. Unlike oil, however, data can be “extracted, refined, valued, bought and sold in different ways” (The Economist, May 6, 2017). These exploitation activities frequently intrude on consumer privacy and occur without the knowledge or consent of the consumers who supply the resource. Regulators in many jurisdictions have, in response, sought to protect consumer privacy by requiring firms to obtain consumers’ *informed consent* for the collection and use of their data, imposing fines on firms that fail to comply.¹ But in a world increasingly dominated by big tech firms, such consent is not always voluntary. The typical *take-it-or-leave-it* offers made by these firms give consumers no meaningful choice, particularly in the absence of good alternatives to the services that these firms provide: consumers have to either consent or forgo using these services altogether. This implies that big tech firms can use their market power to impose “abusive” privacy policies, the same way traditional dominant firms are able to impose “excessive” prices. For instance, in February 2019, the German competition authority ruled that some of Facebook’s data practices constituted “exploitative abuse” and ordered the firm to curtail or end these practices within a year.² The United States Federal Trade Commission (FTC) has also renewed its calls for a national privacy law that would “regulate how big tech companies [...] collect and handle user data” (Kang, 2019).

One way to prevent abusive data practices by big tech firms is through regulatory restrictions on the ways these firms collect and use consumer data. This approach has received support from leading privacy advocates, many of whom believe that such regulatory mandates would have more a meaningful impact on the data practices of big tech firms compared to fines for privacy violations.³ Critics, however, argue that these restrictions would reduce the returns to a firm’s investment in quality (or innovation),

¹See for instance the EU General Data Protection Regulation (GDPR). The full text of the regulation is available at <https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32016R0679&from=EN>.

²For more details on the Facebook antitrust case, see <https://www.npr.org/2019/02/07/692312687/facebook-cant-gather-users-data-from-other-websites-german-antitrust-office-says> and https://www.bundeskartellamt.de/SharedDocs/Meldung/EN/Pressemitteilungen/2019/07_02_2019_Facebook.html.

³Advocates including Matt Stoller, a fellow at the Open Markets Institute, and Ashkan Soltani, a former chief technology officer at the FTC, voiced their support for these regulatory mandates in a New York Times article published on April 24, 2019. The article is available at <https://www.nytimes.com/2019/04/24/technology/facebook-ftc-fine-privacy.html?searchResultPosition=3>.

thereby stifling the firm’s investment incentives. For example, research has shown how restricting data collection for ad targeting purposes can lower advertising effectiveness and consequently, ad revenues (Goldfarb and Tucker, 2011). If the dominant firm’s operations are primarily ad-financed, this reduction in revenues may dampen its incentives to invest and innovate (Castor, 2010; Thierer, 2010; Athey, 2014). Do more stringent privacy regulations necessarily weaken a dominant firm’s incentives to invest and innovate? How would such regulations affect social welfare? Our paper aims to shed light on these questions.

We consider a setup that corresponds closely to the business model of many dominant tech firms (particularly social media platforms) in which users “pay with [their] data and [their] attention”.⁴ In our model, a monopolist offers a free service to consumers and derives revenues by disclosing user data to third parties (e.g., for targeting ads). Prior to interacting with the consumers, the monopolist sets its privacy policy (a disclosure level)⁵ and decides how much to invest in the quality of its service. The firm’s quality and disclosure levels are observed by consumers, who then decide whether to use the service (e.g., whether to create a social media account) and how much information to provide when using it (e.g., how much to reveal about themselves on their social media page; how many posts to write and photos to share). Consumers derive higher gross utility when the firm’s service quality is higher (e.g., when a social media platform provides more features and better sharing tools) and when they provide more information. Further, they perceive quality and information to be complements (e.g., the better a platform’s sharing tools, the more information consumers share). Although the monopolist’s service is free, consumers incur idiosyncratic privacy costs, which could be due to their intrinsic preference for privacy or potential adverse market outcomes (such as price discrimination and unwanted ads) that they may face as a result of the firm’s disclosure of their personal information.⁶

We analyze the social desirability of a privacy regulation which takes the form of a (binding) cap on data disclosure in this setting. The cap could correspond to a set of restrictions on the purposes for which data may be disclosed or the types of third parties

⁴The quote was taken from Facebook co-founder Chris Hughes’ opinion piece on the New York Times which appeared on May 9, 2019: <https://www.nytimes.com/2019/05/09/opinion/sunday/chris-hughes-facebook-zuckerberg.html?module=inline>

⁵For the sake of exposition, the disclosure level is defined in our model as the share of third parties the monopolist discloses personal information to. However, as discussed later, it can also be interpreted as an inverse measure of other (self-imposed) restrictions on the disclosure of data (e.g., regarding the type or share of data disclosed).

⁶See Acquisti et al. (2016) for a detailed discussion of privacy costs.

with whom data may be shared.⁷ Besides directly reducing consumers' privacy costs, a disclosure cap also generates a (strategic) change in the firm's quality investment, which raises social welfare when it is positive and lowers it otherwise. Therefore, the social desirability of the cap depends (partly) on its impact on the firm's quality investment.

A cap on disclosure affects the monopolist's incentives to invest in quality by altering its disclosure revenues, and therefore its gains from quality investment. The monopolist invests in quality to attract more consumers to use its service (extensive margin effect) and/or to induce current users to share more on the platform (intensive margin effect); both of which increase the stock of information it can monetize. A cap on disclosure reduces the firm's gain from monetizing an additional unit of information. Consequently, for a given number of users, the cap lowers the firm's incentives to induce users to share more data by investing in quality. This implies that in a fully covered market where demand is effectively fixed, a disclosure cap will lead to lower quality investment—the regulator faces a trade-off between privacy and quality. Despite this trade-off, however, the cap is socially desirable when quality and information are not strongly complementary. When the market is not fully covered (i.e., when some consumers do not use the firm's service), the cap affects the firm's incentives to invest in quality via two additional channels. First, the cap boosts demand for the firm's service, which increases its marginal benefit from investing in quality. Second, the cap alters the way the firm's demand responds to changes in quality. We find that a cap raises quality investment when it substantially increases the sensitivity of demand to quality changes. In this case, there is *no privacy-quality trade-off* and social welfare is unambiguously higher under the cap. When the demand-sensitivity does not increase sufficiently, the impact of the cap on quality further depends on the elasticity of demand with respect to disclosure. The cap reduces quality level when demand is relatively disclosure-inelastic and has an ambiguous effect otherwise. The overall social welfare impact of the cap is ambiguous in both cases.

Our findings provide insights into the factors that a regulator should consider when deciding whether to set restrictions on a dominant firm's data disclosure policy. They suggest, in particular, that the responsiveness of the firm's demand to changes in quality and disclosure levels are key determinants of the social desirability of such restrictions.

⁷U.S. regulations such as the Gramm-Leach-Bliley Act and the Fair Credit Reporting Act forbid the sharing of consumer information with non-affiliated third parties. Many countries also impose restrictions on international transfers of data, allowing for them only if the foreign-based entity has an adequate level of data protection.

When demand is essentially unresponsive,⁸ restrictions on data disclosure are likely to reduce quality investment. In this case, the regulator needs to weigh the negative effect of the cap on quality against its positive effect on consumer privacy. Our analysis further shows that the outcome of this trade-off is driven by the complementarity between quality and information from consumers’ perspective. When the dominant firm’s demand is responsive to changes in quality and/or disclosure, the regulator may not always face a privacy-quality trade-off. In particular, if the disclosure cap substantially raises the sensitivity of demand to changes in quality, the firm invests more in quality under the cap, and the cap is therefore desirable.

Finally, we present two extensions to our model. First, we consider the scenario where third parties (to which data are potentially disclosed) are heterogeneous in the privacy costs that they induce on consumers. We find that a disclosure cap is more likely to have a positive impact on quality when third parties are heterogeneous rather than homogeneous. As a second extension, we explore the case where the regulator is a consumer protection agency that cares only about consumer surplus. While the results in this case do not qualitatively differ from those derived under a welfare-maximizing regulator, we show that the consumer protection agency sets a lower disclosure cap.

Related literature. There is a growing pool of literature on the economics of privacy.⁹ The extant literature has examined the impact of competition (Casadesus-Masanell and Hervas-Drane, 2015; Dimakopoulos and Sudaric, 2018) and the effects of taxation (Bloch and Demange, 2018; Bourreau et al., 2018) on the data disclosure choices of a firm. Our paper contributes to this literature by examining the social desirability of a direct intervention—a cap on disclosure. Our modeling approach is closest to that of Casadesus-Masanell and Hervas-Drane (2015). In both models, consumers decide how much information to share with the firm, but not all information shared is disclosed by the firm. In contrast, the other studies only examine the case of full disclosure/exploitation. That said, we consider different revenue models: the firm derives revenues solely from disclosure in our paper, whereas it can also charge a positive price in Casadesus-Masanell and Hervas-Drane (2015).¹⁰ More importantly, our work is distinct from Casadesus-Masanell and Hervas-Drane (2015) (and also, the other related studies) in that we endogenize the firm’s quality choice. To the best of our knowledge,

⁸Consumers may however respond to changes in quality and/or privacy levels by changing the amount of information they provide to the firm.

⁹See Section 2.3. of Acquisti et al. (2016) for a comprehensive survey of related privacy literature.

¹⁰In Section 7.6 we discuss the scenario in which consumers are charged positive prices in our setting.

we are the first to examine from a theoretical perspective the relationship between a firm’s data practices—more precisely, its disclosure level—and its incentives to invest in quality. Understanding this relationship allows us to derive the impact of a disclosure cap on investment in quality.¹¹

Our analysis throws light on the privacy-innovation/quality debate. In particular, we find that a disclosure cap can increase quality level; i.e., there is not always a privacy-quality trade-off. Our findings echo those of Anderson (2007), who shows that an advertising cap may either decrease or increase the quality of free-to-air television. However, his findings are driven entirely by the impact of the advertising cap on the extensive margin effect of quality investment, as he considers only the (binary) participation decision of consumers, whereas our results depend also on how quality affects the amount of information provided by consumers (i.e., the intensive margin effect).¹²

The implications drawn from our analysis also complement the findings in the empirical literature that examines the interlinkage between privacy regulations and data-driven innovation. Goldfarb and Tucker (2012) analyze several empirical studies—in the healthcare (see Tucker and Miller, 2009, 2011a and 2011b) and the online advertising (see Goldfarb and Tucker, 2011) sectors—and find that privacy regulations may raise the costs and/or lower the benefits associated with data-driven innovation, hence weakening firms’ investment incentives. Our work shows, in addition, that a privacy regulation can affect the level of service innovation (quality) even when data is not a direct input for innovation.

The rest of the paper is structured as follows. Section 2 presents the model setup. Section 3 examines the consumers’ participation and information provision decisions and Section 4 compares the privately and socially optimal choices of quality and disclosure levels. Section 5 analyzes the impact of a disclosure cap on investment and social welfare. Section 6 presents two extensions to our model: heterogeneous third parties and consumer surplus maximization. Section 7 provides some general discussion and Section 8 concludes. All omitted proofs can be found in Appendix A.

¹¹Wickelgren (2015) and Campbell et al. (2015) also examine from a theoretical perspective the effects of privacy regulation but they focus on its impact on competition.

¹²Moreover, we embed our study of the effect of a disclosure cap on quality in a comprehensive welfare analysis, while Anderson (2007) does not investigate the overall welfare impact of an advertising cap.

2 Baseline Model

Consider a firm that offers a service to a unit mass of consumers at a price of zero. The firm derives revenues from disclosing its customers' personal information to (a subset of) third parties (e.g., advertisers or third-party apps) that are uniformly distributed over the interval $[0, 1]$. The firm can choose the quality level $q \geq 0$ of its service and a disclosure level $d \in [0, 1]$, which defines the extent to which personal information is shared with third parties. More precisely, information is disclosed to the third parties located in the interval $[0, d]$ and not disclosed to those in $(d, 1]$.¹³

Consumers' utility. Consumers face a trade-off when sharing their personal information with the firm (or on the firm's platform). When consumers share more information with the firm, they obtain higher utility from its service but also suffer higher privacy-related utility losses arising from the firm's disclosure of consumer data to third parties. These utility losses reflect the consumers' preference for privacy, which may arise because they value privacy intrinsically (e.g., as a right) or because they may face potential adverse market-mediated outcomes (e.g., price discrimination).¹⁴ A consumer's privacy preference is captured by an idiosyncratic privacy cost parameter, θ , which is increasing in the intensity of her preference. We assume that θ is distributed over an interval $[\underline{\theta}, \bar{\theta}] \subset \mathbb{R}_+$ according to a differentiable density function $f(\cdot)$.

The utility a consumer of type θ receives when she provides an amount of personal information $x \in [0, 1]$ is

$$U(x, \theta, q, d) \equiv V(x, q) - \theta dx - \alpha x - K, \quad (1)$$

where $V(x, q)$ is the gross utility the consumer derives from using the service, θdx is the privacy cost she incurs when her information is disclosed to a share d of third parties,¹⁵ αx is the cost of sharing personal information (e.g. the time and effort it takes to

¹³As discussed later, our model allows for other interpretations of the disclosure level d .

¹⁴We acknowledge that data disclosure can also generate positive effects for consumers (e.g., better targeted ads or a more customized newsfeed); however, we focus only on the negative effects in the baseline setting since privacy regulations are only relevant when disclosure harms consumers. Alternatively, we can interpret the utility losses as losses net of any positive effects. We discuss in greater detail the case where disclosure can have positive effects on consumer utility in Section 7.5.

¹⁵Our analysis would be qualitatively the same if the sharing of information with the monopolist also imposed a privacy cost of $\lambda\theta x$ on the consumer where $\lambda > 0$. This would simply amount to replacing d with $d + \lambda$ in the baseline model. Note also that one can interpret d as the disclosure level above a certain acceptance threshold (assumed to be the same for all consumers) as in Casadesus-Masanell and Hervas-Drane (2015). This is justified by the fact that the firm will always choose a disclosure level that is (weakly) above that threshold.

upload photos onto the platform),¹⁶ and $K > 0$ is a fixed opportunity cost of using the service.¹⁷ We further assume that $V(x, q)$ is bounded, twice continuously differentiable, increasing and concave in both its arguments. Note that the linearity of the privacy cost θdx with respect to d rests on the implicit assumption that sharing personal data with any third party induces the same privacy cost for a given consumer. In Section 6.1. we consider an extension where third parties induce heterogeneous privacy costs.

Value of personal information. We suppose that all the third parties interested in consumers' personal information have the same willingness to pay $r > 0$ for a unit of information. In addition, we assume that the firm is a monopolist in the market for (its customers') personal information. These simplifying assumptions have two straightforward implications. First, the firm always sets the unit price for access to its customers' personal information to r , independently of its other strategic choices. Second, the monopolist fully extracts the surplus of the third-party data buyers. In Section 7.2 we discuss the scenario in which third parties pay a price lower than r , thus obtaining a positive surplus.

Firm's profit and social welfare. Denote by \mathbf{x} the function mapping each $\theta \in [\underline{\theta}, \bar{\theta})$ to the amount of information $x(\theta) \in [0, 1]$ provided by a consumer of type θ .¹⁸ The firm's profit is

$$\Pi(\mathbf{x}, q, d) \equiv rd \int_{\underline{\theta}}^{\bar{\theta}} x(\theta) f(\theta) d\theta - C(q), \quad (2)$$

where $C(q)$ is the (fixed) cost of producing a service of quality level q . Assume that $C(\cdot)$ is twice differentiable, with $C(0) = 0$, $C(q) \xrightarrow{q \rightarrow +\infty} +\infty$, $C'(0) = 0$, $C'(q) > 0$ and $C''(q) > 0$ for any $q > 0$. Social welfare is defined as the sum of the firm's profit and the consumers' utility.

Before proceeding further, it is worth highlighting that the expressions of the firm's profit and the consumer's utility functions (expressions (1) and (2)) allow for an alternative interpretation of our baseline model: we could assume that the firm discloses a share $d \in [0, 1]$ of the personal information provided by each consumer to all third

¹⁶An alternative interpretation of α is provided in Section 7.1.

¹⁷If $K = 0$, all consumers would find it (weakly) optimal to use the service (i.e., there would be no extensive margin effect). This would make our model less rich and limit the insights we can draw from it.

¹⁸The amount of information $x(\theta)$ can be equal to zero either because the consumer of type θ decides not to use the service, or because she uses the service but decides to provide no personal information at all.

parties, instead of assuming that it discloses all the personal information provided by consumers to a subset $[0, d]$ of third parties.¹⁹

Interdependency between quality and information. We capture the interdependency between quality and information from consumers’ perspective through the following parameter:

$$\gamma \equiv -\frac{\frac{\partial^2 V}{\partial x \partial q}}{\frac{\partial^2 V}{\partial x^2}},$$

which we assume to be constant for the sake of simplicity.²⁰ Further, we focus on the case where quality and information are complements from consumers’ perspective; i.e., $\gamma > 0$. This is the most interesting scenario for the purpose of our analysis and is particularly relevant in the context of social media. For example, Facebook’s recent introduction of new sharing tools (including Facebook Live video and the “On this day” feature) in attempt to boost the sharing of original personal content provides evidence that consumers view the quality of the social media platform and the amount of information they share as complements.²¹ We discuss the case where quality and information are substitutes, i.e. $\gamma < 0$, in Section 7.7.

Timing. We consider the following two-stage game:

1. The firm chooses a quality level q and commits to a disclosure level d .²²
2. Consumers observe the levels of quality and disclosure. They decide then whether to patronize the firm and, if they do, how much personal information to provide.

3 Consumers’ choice

We begin our analysis with the consumers’ problem. Conditional on patronizing the firm, a consumer chooses her level of information provision so as to maximize her utility

¹⁹More generally, a lower disclosure level can be interpreted either as more (self-imposed) restrictions on the type/share of data that can be disclosed to third parties and/or more restrictions on the set of third parties that the data can be shared with.

²⁰This amounts to restricting our attention to the class of gross utility functions $V(x, q)$ for which there exists a real number γ , a twice continuously differentiable function $l(\cdot)$, and a continuously differentiable function $h(\cdot)$ such that $V(x, q) = l(q) + \int_0^x h(u - \gamma q) du$.

²¹For full story, visit <https://www.bloomberg.com/news/articles/2016-04-07/facebook-said-to-face-decline-in-people-posting-personal-content>.

²²In a number of jurisdictions (e.g. Europe), firms can be heavily fined for a privacy policy breach, which provides them with a credible commitment device. We discuss the way our results are affected when firms are unable to commit to a disclosure level in Section 7.3.

$U(x, \theta, q, d)$. Let us denote²³

$$\tilde{x}(\theta, q, d) \equiv \arg \max_{x \in [0,1]} U(x, \theta, q, d).$$

To ease the exposition, we assume throughout the paper that $\alpha > \sup_{q \geq 0} \frac{\partial V}{\partial x}(1, q)$ and $\bar{\theta} < \inf_{q \geq 0} \frac{\partial V}{\partial x}(0, q)$. These conditions ensure that, conditional on using the service, the amount of information that a consumer provides to the firm is always interior; i.e., $\tilde{x}(\theta, q, d) \in (0, 1)$. The following lemma shows the effect of quality and disclosure levels on the amount of personal information provided by a consumer.

Lemma 1 (*Comparative statics - Information amount*) *Conditional on using the service, the amount of information that a consumer provides to the firm is decreasing in the disclosure level and the idiosyncratic privacy cost parameter but increasing in the quality level. More precisely,*

$$\frac{\partial \tilde{x}}{\partial d}(\theta, q, d) = \frac{\theta}{\frac{\partial^2 V}{\partial x^2}(\tilde{x}(\theta, q, d), q)} < 0,$$

$$\frac{\partial \tilde{x}}{\partial \theta}(\theta, q, d) = \frac{d}{\frac{\partial^2 V}{\partial x^2}(\tilde{x}(\theta, q, d), q)} < 0,$$

and

$$\frac{\partial \tilde{x}}{\partial q}(\theta, q, d) = \gamma > 0.$$

The above results are intuitive. An increase in the disclosure level or the value of the consumer's idiosyncratic privacy cost parameter raises her marginal privacy cost of information provision; this leads her to provide less information. In contrast, an increase in the quality level raises her marginal gross utility from providing information; hence, she finds it optimal to provide more information.

We now consider the participation decision of a consumer. Denoting

$$\tilde{U}(\theta, q, d) \equiv U(\tilde{x}(\theta, q, d), \theta, q, d),$$

a consumer of type θ chooses to patronize the firm if and only if $\tilde{U}(\theta, q, d) > 0$.²⁴ The following lemma characterizes the demand for the service offered by the firm and shows

²³The existence and uniqueness of $\tilde{x}(\theta, q, d)$ follows from the fact that $U(x, \theta, q, d)$ is (strictly) concave in x over the compact set $[0, 1]$.

²⁴For technical reasons, we assume that a consumer who is indifferent between patronizing the firm or not decides not to patronize it.

how it is affected by the levels of quality and disclosure.

Lemma 2 (*Comparative statics - Demand*) *There exists a threshold $\tilde{\theta}(q, d) \in [\underline{\theta}, \bar{\theta}]$ such that a consumer patronizes the firm if and only if*

$$\theta < \tilde{\theta}(q, d).$$

The threshold $\tilde{\theta}(q, d)$ is weakly increasing in the quality level, but weakly decreasing in the disclosure level. Moreover, whenever $\tilde{\theta}(q, d) \in (\underline{\theta}, \bar{\theta})$, the following expressions hold:

$$\frac{\partial \tilde{\theta}}{\partial q}(q, d) = \frac{\frac{\partial V}{\partial q}(\tilde{x}(\tilde{\theta}(q, d), q, d), q)}{d\tilde{x}(\tilde{\theta}(q, d), q, d)}$$

and

$$\frac{\partial \tilde{\theta}}{\partial d}(q, d) = -\frac{\tilde{\theta}(q, d)}{d}.$$

The above lemma tells us that the demand for the service, which is given by $F(\tilde{\theta}(q, d))$, is weakly increasing in the firm's quality level but weakly decreasing in its disclosure level.

4 Private versus social incentives

We now examine the privately and socially optimal choice of quality and disclosure levels. Let us first consider the private incentives for quality investment and information disclosure. The firm's profit when consumers make their participation and information provision decisions optimally is

$$\tilde{\Pi}(q, d) = rd \int_{\underline{\theta}}^{\tilde{\theta}(q, d)} \tilde{x}(\theta, q, d) f(\theta) d\theta - C(q). \quad (3)$$

From (3), it follows that the firm's net marginal benefit from investing in quality is

$$\frac{\partial \tilde{\Pi}}{\partial q} = rd \underbrace{\int_{\underline{\theta}}^{\tilde{\theta}(q, d)} \frac{\partial \tilde{x}}{\partial q}(\theta, q, d) f(\theta) d\theta}_{\text{intensive margin effect}} + rd \underbrace{\frac{\partial \tilde{\theta}}{\partial q} \tilde{x}(\tilde{\theta}(q, d), q, d) f(\tilde{\theta}(q, d))}_{\text{extensive margin effect}} - C'(q). \quad (4)$$

The intensive margin effect captures how a change in quality level affects the firm's revenue via a change in the total amount of information provided by consumers holding the size of its user base fixed. Using Lemma 1, we can show that this effect is positive. More intuitively, this is because quality and information are complements: a higher quality level induces the firm's customers to provide more information. The extensive margin effect captures how the change in demand (i.e., in the size of the user base) resulting from a change in quality impacts the firm's profit. This effect is weakly positive as the firm's demand is weakly increasing in quality as shown in Lemma 2.

The net marginal benefit from increasing the disclosure level is

$$\frac{\partial \tilde{\Pi}}{\partial d} = r \underbrace{\int_{\theta}^{\tilde{\theta}(q,d)} \left[\tilde{x}(\theta, q, d) + d \frac{\partial \tilde{x}}{\partial d}(\theta, q, d) \right] f(\theta) d\theta}_{\text{intensive margin effect}} + \underbrace{rd \frac{\partial \tilde{\theta}}{\partial d} \tilde{x}(\tilde{\theta}(q, d), q, d) f(\tilde{\theta}(q, d))}_{\text{extensive margin effect}}.$$

The intensive and extensive margin effects of a change in disclosure level can be interpreted in a similar manner as those of a change in quality level. Lemma 1 implies that the sign of the intensive margin effect is ambiguous, while Lemma 2 shows that the extensive margin effect is weakly negative. The sign of the intensive margin effect is ambiguous because a change in the disclosure level generates two opposite effects: it raises the firm's disclosure revenues per unit of information provided but lowers each consumer's information provision level.

Let us assume that $\tilde{\Pi}(\cdot, \cdot)$ is strictly quasi-concave in each of its arguments. The privately optimal level of quality for a given level of disclosure and the privately optimal level of disclosure for a given level of quality are defined as follows:²⁵

$$q^M(d) \equiv \arg \max_{q \in [0, +\infty)} \tilde{\Pi}(q, d);$$

$$d^M(q) \equiv \arg \max_{d \in [0, 1]} \tilde{\Pi}(q, d).$$

We further assume that the privately optimal pair of quality and disclosure levels

$$\left(\tilde{q}^M, \tilde{d}^M \right) \equiv \arg \max_{(q, d) \in [0, +\infty) \times [0, 1]} \tilde{\Pi}(q, d)$$

²⁵The existence and uniqueness of $q^M(d)$ follows from the fact that $\tilde{\Pi}(q, d)$ is continuous and strictly quasi-concave in q and $\tilde{\Pi}(q, d) \xrightarrow{q \rightarrow +\infty} -\infty$, while the existence and uniqueness of $d^M(q)$ follows from the fact that $\tilde{\Pi}(q, d)$ is continuous and strictly quasi-concave in d over the compact set $[0, 1]$.

is unique.²⁶

Let us now consider the social incentives for quality provision and information disclosure. The social planner aims to maximize the sum of the firm's profit and consumer surplus:²⁷

$$\tilde{W}(q, d) \equiv \tilde{\Pi}(q, d) + \underbrace{\int_{\underline{\theta}}^{\tilde{\theta}(q, d)} \tilde{U}(\theta, q, d) f(\theta) d\theta}_{\equiv \tilde{CS}(q, d)}.$$

Assuming that $\tilde{W}(\cdot, \cdot)$ is strictly quasi-concave in each of its arguments, the socially optimal quality level for a given level of disclosure and the socially optimal disclosure level for a given level of quality are defined as follows:²⁸

$$q^W(d) \equiv \arg \max_{q \in [0, +\infty)} \tilde{W}(q, d);$$

$$d^W(q) \equiv \arg \max_{d \in [0, 1]} \tilde{W}(q, d).$$

In the following lemma, we compare the socially and privately optimal levels of quality for a given level of disclosure and those of disclosure for a given level of quality.²⁹

Lemma 3 (*Private vs social incentives*)

- For a given disclosure level, the monopolist under-provides quality from a social welfare perspective: $q^M(d) \leq q^W(d)$ for any $d \in [0, 1]$.
- For a given quality level, the monopolist over-discloses information from a social welfare perspective: $d^M(q) \geq d^W(q)$ for any $q \in [0, +\infty)$.

The under-provision (rep. over-disclosure) result stems from the fact that the firm does not fully internalize the increase (resp. decrease) in consumers' utility that results from higher quality (resp. disclosure). Under the regularity conditions that we have

²⁶The existence of the optimal pair of quality and disclosure levels follows from the fact that $\tilde{\Pi}(q, d)$ is continuous in (q, d) and $\tilde{\Pi}(q, d)$ goes to $-\infty$ uniformly with respect to d when $q \rightarrow -\infty$ (which allows to reduce the maximization over $[0, +\infty) \times [0, 1]$ to a maximization over a compact set).

²⁷Recall that third parties' surplus is zero in the baseline model.

²⁸The existence and uniqueness of $q^W(d)$ follows from the fact that $\tilde{W}(q, d)$ is continuous and strictly quasi-concave in q and $\tilde{W}(q, d) \xrightarrow{q \rightarrow +\infty} -\infty$, while the existence and uniqueness of $d^W(q)$ follows from the fact that $\tilde{W}(q, d)$ is continuous and strictly quasi-concave in d over the compact set $[0, 1]$.

²⁹Note that the comparison of the privately optimal pair $(\tilde{q}^M, \tilde{d}^M)$ with its socially optimal counterpart is not relevant for our policy analysis because the regulator has a *single* instrument (i.e., a cap on the disclosure level).

imposed, Lemma 3 can help us determine the direction of the impact of a change in quality (resp. disclosure) level, holding disclosure (resp. quality) level fixed, on the firm's profit or on social welfare. These results will come in handy for the analysis of the privacy regulation examined in the next section.

5 Privacy regulation

We now turn to the core question of our paper: is it socially desirable to regulate the information disclosure level? The first step to answering this question, as will become apparent, is understanding the impact of such a regulation on the firm's quality choice.

We consider a scenario where the only available regulatory instrument is a cap on the disclosure level and study the decision of a social-welfare-maximizing regulator to implement such a cap.

The timing of the game is as follows:

- First, the regulator decides whether to impose a cap on the disclosure level and sets the value of that cap \bar{d} if it does so.
- Second, the firm decides on its disclosure and quality levels.
- Third, consumers decide whether to patronize the firm and how much information to provide if they do so.

Let us first analyze the firm's behavior in the second stage when the regulator imposes a cap \bar{d} in the first stage. The firm's optimal disclosure level maximizes $\tilde{\Pi}(q^M(d), d)$ subject to the constraint $d \leq \bar{d}$. If $\bar{d} \geq \tilde{d}^M$, the constraint is not binding. In this case, the firm's decision will be the same as in the unregulated scenario; i.e., the firm will set its disclosure level at $d = \tilde{d}^M$. If $\bar{d} < \tilde{d}^M$, the constraint binds. Under the additional assumption that $\tilde{\Pi}(q^M(d), d)$ is strictly quasi-concave in d , which we make in the remainder of the paper, the firm will choose $d = \bar{d}$ whenever $\bar{d} < \tilde{d}^M$.

Let us now consider the regulator's decision in the first stage. Notice first that setting no cap or setting a cap $\bar{d} > \tilde{d}^M$ is the same as setting a cap $\bar{d} = \tilde{d}^M$. In all of these scenarios, the cap does not affect the firm's behavior, and therefore has no impact on social welfare. We focus on the (interesting) scenario in which (i) the firm invests in quality in the absence of a disclosure cap (i.e. $\tilde{q}^M > 0$) and (ii) the regulator does not find it optimal to impose a disclosure cap that induces no investment in quality at all

(i.e., a cap such that $q^M(\bar{d}) = 0$). Then, denoting³⁰

$$\underline{d} \equiv \inf \left\{ d \in [0, \tilde{d}^M] \mid q^M(d) > 0 \right\},$$

we can restrict the analysis to disclosure caps \bar{d} between \underline{d} and \tilde{d}^M . More precisely, the regulator's maximization program can be written as

$$\max_{\bar{d} \in [\underline{d}, \tilde{d}^M]} \hat{W}(\bar{d}) \equiv \tilde{W}(q^M(\bar{d}), \bar{d}) = \tilde{\Pi}(q^M(\bar{d}), \bar{d}) + \int_{\underline{\theta}}^{\bar{\theta}(q^M(\bar{d}), \bar{d})} \tilde{U}(\theta, q^M(\bar{d}), \bar{d}) f(\theta) d\theta.$$

Further, assume that $\hat{W}(\cdot)$ is strictly quasi-concave over $[\underline{d}, \tilde{d}^M]$. Under this regularity assumption, the regulator finds it strictly optimal to set a binding cap; i.e., there exists $\bar{d} < \tilde{d}^M$ such that $\hat{W}(\bar{d}) > \hat{W}(\tilde{d}^M)$, if and only if

$$-\left. \frac{\partial \hat{W}}{\partial \bar{d}} \right|_{\bar{d}=\tilde{d}^M} > 0.$$

In other words, setting a binding disclosure cap is strictly socially desirable if and only if a marginal decrease in disclosure level starting from the unregulated level leads to an increase in social welfare. The following lemma provides a useful decomposition of the welfare effect of a marginal decrease in disclosure level.

Lemma 4 *The marginal impact of a decrease in disclosure level on social welfare at the unregulated level is given by*

$$\begin{aligned} -\left. \frac{\partial \hat{W}}{\partial \bar{d}} \right|_{\bar{d}=\tilde{d}^M} &= \underbrace{\int_{\underline{\theta}}^{\bar{\theta}(\tilde{q}^M, \tilde{d}^M)} \theta \tilde{x}(\theta, \tilde{q}^M, \tilde{d}^M) f(\theta) d\theta}_{\text{direct effect}} \\ &\quad - \underbrace{\int_{\underline{\theta}}^{\bar{\theta}(\tilde{q}^M, \tilde{d}^M)} \frac{\partial V}{\partial q}(\tilde{x}(\theta, \tilde{q}^M, \tilde{d}^M), \tilde{q}^M) \left. \frac{\partial q^M}{\partial d} \right|_{d=\tilde{d}^M} f(\theta) d\theta}_{\text{strategic effect}}. \end{aligned}$$

³⁰Our focus on the case in which $q^M(\tilde{d}^M) = \tilde{q}^M$ is positive, combined with the fact that $q^M(d)$ is continuous at $d = \tilde{d}^M$ (which follows from Berge's Theorem of the Maximum) ensures that the considered set is not empty and that its lower bound \underline{d} is well-defined in $[0, \tilde{d}^M]$.

This lemma shows that a (marginal) decrease in disclosure level starting from the unregulated level generates two effects: a *direct* effect on the privacy costs incurred by consumers (holding the quality level constant), and a *strategic* effect, which captures how a decrease in the disclosure level (due to the disclosure cap) alters the firm's quality choice. The direct effect is always positive because there is over-disclosure by the firm from a social perspective, while the sign of the strategic effect depends on how the firm's quality choice responds to the reduction in disclosure level. Since the firm under-provides quality from a social perspective, a marginal increase in quality level at the unregulated equilibrium raises social welfare. Therefore, the strategic effect is (weakly) positive if the firm (weakly) increases its quality level when disclosure level is decreased; i.e., if $-\frac{\partial q^M}{\partial d}\Big|_{d=\tilde{d}^M} \geq 0$. When the strategic effect is positive, the overall social welfare effect of a marginal decrease in disclosure level from its unregulated level is unambiguously positive, which implies that it is strictly socially desirable to set a binding disclosure cap. However, if $-\frac{\partial q^M}{\partial d}\Big|_{d=\tilde{d}^M} < 0$, the strategic effect is negative and the overall social welfare effect of a disclosure cap on becomes *a priori* ambiguous.³¹

Let us now analyze the effect of a disclosure cap on quality, which determines the sign of the strategic effect described above. The following lemma relates the effect of a change in disclosure level on the firm's optimal quality level to the cross-effect of quality and disclosure on the firm's profit.

Lemma 5 (*Effect of a decrease in disclosure level on quality*) *If $q^M(d) > 0$, then*

$$-\frac{\partial q^M}{\partial d} = \frac{\frac{\partial^2 \tilde{\Pi}}{\partial q \partial d}(q^M(d), d)}{\frac{\partial^2 \tilde{\Pi}}{\partial q^2}(q^M(d), d)}.$$

From Lemma 5, we see that the effect of a decrease in disclosure level on the firm's choice of quality has the opposite sign of the cross-effect of quality and disclosure on the firm's profit, i.e. $\frac{\partial^2 \tilde{\Pi}}{\partial q \partial d}(q^M(d), d)$.³² More intuitively, the Lemma tells us that if the marginal benefit of investing in quality is increasing (resp. decreasing) in the level of disclosure, the firm will invest less (resp. more) at lower levels of disclosure.

For our analysis, we distinguish between the scenarios where the market is fully covered and where it is partially covered. A change in quality level only generates an intensive margin effect when the market is fully covered, but it also creates an extensive

³¹This shows more fundamentally that although the disclosure cap unambiguously mitigates the market failure of over-disclosure of personal information, it can either mitigate or amplify the market failure of under-provision of quality.

³²This follows from the second-order condition $\frac{\partial^2 \tilde{\Pi}}{\partial q^2}(q^M(d), d) < 0$.

margin effect (arising from a change in demand) when the market is partially covered. Consequently, the cross-effect of quality and disclosure on the firm's profit will depend on whether the market is fully covered.

5.1 Full market coverage

Suppose that $\tilde{\theta}(q^M(d), d) = \bar{\theta}$ for any $d \in [0, 1]$.³³ Under this assumption, the market is fully covered regardless of the regulator's decision in the first stage of the game.

Consider the firm's optimal choice of quality for a given level of disclosure. The firm's marginal benefit from investing in quality is provided by (4), with the term capturing the effect of disclosure on the extensive margin effect equal to zero under full market coverage. The impact of a disclosure cap on quality is given by the following proposition.

Proposition 1 (*Effect of a disclosure cap on quality under full market coverage*) *When the market is fully covered, a disclosure cap $\bar{d} \in (\underline{d}, \tilde{d}^M)$ has a negative impact on quality.*

The intuition behind the above proposition is as follows. In a fully covered market, the firm invests in quality because its existing customers provide more information when its service is of better quality; this allows the firm to generate higher disclosure revenues. A cap on disclosure level weakens the firm's ability to monetize the additional data that consumers provide when it offers a higher quality level, thereby lowering its marginal benefit of quality investment. The firm's investment in quality is therefore lower if a binding disclosure cap is implemented. A direct implication of Proposition 1 is that there always exists a trade-off between privacy and quality when the market is fully covered.

Let us now turn to the social desirability of a disclosure cap. Recall from the preceding discussion that it is strictly optimal for the regulator to set a disclosure cap if and only if a marginal decrease in disclosure level from its unregulated level $\bar{d} = \tilde{d}^M$ increases social welfare. From Lemmas 4 and 5, it follows that

$$-\frac{\partial \hat{W}}{\partial \bar{d}} \Big|_{\bar{d}=\tilde{d}^M} = \int_{\underline{\theta}}^{\bar{\theta}} \theta \tilde{x}(\theta, \tilde{q}^M, \tilde{d}^M) f(\theta) d\theta - \frac{r\gamma}{C''(\tilde{q}^M)} \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial V}{\partial q}(\tilde{x}(\theta, \tilde{q}^M, \tilde{d}^M), \tilde{q}^M) f(\theta) d\theta.$$

³³A sufficient—but not necessary—condition for this to hold is that $K < V(0, 0)$.

Since \tilde{q}^M , \tilde{d}^M and $\tilde{x}(\theta, \cdot, \cdot)$ also depend on γ , it is *a priori* unclear how the expression above depends on γ . However, it can be easily shown that it is positive for γ below a certain threshold $\tilde{\gamma}$.³⁴ Thus, a cap on the disclosure level is strictly socially desirable whenever $\gamma < \tilde{\gamma}$; i.e., when the complementarity between quality and information is not too strong. In other words, under this condition, there exists a cap \bar{d} between \underline{d} and \tilde{d}^M that leads to a strict increase in social welfare relative to the unregulated scenario. However, when quality and information are sufficiently strong complements (i.e., $\gamma > \tilde{\gamma}$), the sign of $-\frac{\partial \hat{W}}{\partial d} \Big|_{\bar{d}=\tilde{d}^M}$ is ambiguous and therefore, so is the impact of a disclosure cap on social welfare.³⁵ In this case, one cannot exclude the possibility that the decrease in social welfare due to the reduction in quality (the strategic effect) may outweigh the increase in welfare resulting from the reduction in privacy costs (the direct effect).

More intuitively, the full market coverage scenario we consider here is essentially one in which the demand for the firm's service is unresponsive to changes in quality and disclosure levels—no consumers join or leave the firm. The firm's existing customers react to these changes only by adjusting the amount of information they share. A disclosure cap reduces the quality of the firm's service, and consumers share less information as a result. Moreover, the stronger the complementarity between quality and information, the larger the reduction in information sharing. This reduction in the amount of information provided decreases both the gross utility obtained by the consumers and the disclosure revenues of the firm, thereby lowering social welfare. When quality and information are relatively weak complements ($\gamma < \tilde{\gamma}$), the negative welfare impact from the fall in quality level is dominated by the positive impact from the reduction consumers' privacy costs, and a disclosure cap is socially desirable. When quality and information are relatively strong complements, the impact from the decrease in quality level may dominate, and the effect of the disclosure cap on social welfare is ambiguous.

The following proposition summarizes the above analysis.

³⁴To see why, define

$$\tilde{\gamma} \equiv \left\{ \gamma' > 0 \mid \frac{\partial \hat{W}}{\partial d} \Big|_{\bar{d}=\tilde{d}^M} < 0 \text{ for all } \gamma \in (0, \gamma') \right\}.$$

From the fact that $\frac{\partial \hat{W}}{\partial d} \Big|_{\bar{d}=\tilde{d}^M}$ is continuous in γ and (strictly) negative when $\gamma \rightarrow 0$, it follows that the above set is not empty, which ensures that its upper bound $\tilde{\gamma}$ is well-defined in $\mathbb{R}_{++} \cup \{+\infty\}$. Note that one cannot exclude *a priori* that $\tilde{\gamma}$ takes an infinite value.

³⁵If $\tilde{\gamma}$ takes an infinite value, privacy regulation would be socially desirable whatever the level of complementarity between quality and information.

Proposition 2 (*Social desirability of a disclosure cap under full market coverage*)
When the market is fully covered, a privacy regulation taking the form of a binding disclosure cap is strictly socially desirable if the complementarity between quality and information is not too strong (i.e., $\gamma < \tilde{\gamma}$). Otherwise, the social desirability of such a regulation is ambiguous.

5.2 Partial market coverage

We now consider the scenario in which $\tilde{\theta}(q^M(d), d) < \bar{\theta}$ for any $d \in [0, 1]$. In this case, the market is only partially covered whatever the regulator's decision in the first stage.

Let $F(\cdot)$ denote the cumulative distribution function of θ . It follows from expression (4) and Lemmas 1 and 2 that the firm's net marginal benefit from investing in quality is

$$\frac{\partial \tilde{\Pi}}{\partial q} = \underbrace{rd\gamma F(\tilde{\theta}(q, d))}_{\text{intensive margin effect}} + r \underbrace{\frac{\partial V}{\partial q}(\tilde{x}(\tilde{\theta}(q, d), q, d), q) f(\tilde{\theta}(q, d))}_{\text{extensive margin effect}} - C'(q) \quad (5)$$

whenever the market is partially covered; i.e., $\tilde{\theta}(q, d) \in (\underline{\theta}, \bar{\theta})$.

The effect of the disclosure level on the firm's optimal choice of quality is driven by the cross-effect of quality and disclosure on the firm's profit. Differentiating expression (5) with respect to d and using the results from Lemmas 1 and 2,³⁶ we obtain the effect of a marginal decrease in disclosure level on the marginal benefit from investing in quality:

$$-\frac{\partial^2 \tilde{\Pi}}{\partial q \partial d} = \underbrace{-r\gamma F(\tilde{\theta}(q, d))}_{\equiv A} - r d \gamma \underbrace{\frac{\partial \tilde{\theta}}{\partial d} f(\tilde{\theta}(q, d))}_{\equiv B} - r \underbrace{\frac{\partial V}{\partial q} \frac{\partial \tilde{\theta}}{\partial d} f'(\tilde{\theta}(q, d))}_{\equiv C}. \quad (6)$$

Term $A + B$ shows how the intensive margin effect of investment in quality is affected by a decrease in disclosure level. More specifically, term A corresponds to the effect of a decrease in the disclosure level on the marginal benefit from investing in quality for a *given* demand for the service (i.e., for a fixed size of user base); it captures how the users of the firm's service adjust the amount of information they share in response to a decrease in disclosure level. This is the only term that appears in our analysis of the full market coverage scenario (where $\tilde{\theta}(q, d) = \bar{\theta}$) and its sign is negative. Under

³⁶We use in particular the fact that $\frac{\partial \tilde{x}}{\partial \theta} \frac{\partial \tilde{\theta}}{\partial d} + \frac{\partial \tilde{x}}{\partial d} = 0$. For notational convenience, we drop the arguments of $\frac{\partial V}{\partial q}(q, \tilde{x}(\tilde{\theta}(q, d), q, d))$ and $\frac{\partial^2 V}{\partial q \partial x}(q, \tilde{x}(\tilde{\theta}(q, d), q, d))$.

partial market coverage, a change in disclosure level also affects the intensive margin effect by changing the level of demand (at every given quality level). This effect, which is captured by term B , is positive because a decrease in disclosure level leads to an expansion of the user base. Finally, term C shows how the extensive margin effect of quality investment changes if the disclosure level decreases. Since the extensive margin effect is proportional to the density of consumers at the margin and the impact of disclosure on demand is negative, term C is positive (resp. negative) if the density function is locally increasing (resp. decreasing). The following result shows that the sign of the net effect $A + B + C$ depends on the elasticity and the curvature of the cumulative distribution function $F(\cdot)$.

Lemma 6 (*Impact of a decrease in disclosure level on the intensive and extensive margin effects*)

- If $F(\cdot)$ is relatively inelastic (resp. elastic), i.e., $\frac{\theta f(\theta)}{F(\theta)} < 1$ (resp. > 1) for all $\theta \in [\underline{\theta}, \bar{\theta}]$, the impact of a decrease in disclosure level on the intensive margin effect of investment in quality is negative (resp. positive).

- If $F(\cdot)$ is convex (resp. concave) over $[\underline{\theta}, \bar{\theta}]$, the impact of a decrease in disclosure level on the extensive margin effect of investment in quality is positive (resp. negative).

The elasticity of $F(\cdot)$ is related to the shape of the demand for the service. We show in Appendix B that this corresponds to the elasticity of demand with respect to disclosure level (holding the amount of information shared constant). We also establish that the convexity/concavity of $F(\cdot)$ is related to the elasticity of the marginal effect of quality on demand with respect to disclosure level, holding the amount of information constant; it is greater (resp. less) than 1 if $F(\cdot)$ is convex (resp. concave). Therefore, the curvature of $F(\cdot)$ tells us how the responsiveness of demand to a change in quality level is affected by changes in the disclosure level.

We can now state our result on the impact of a disclosure cap on quality.

Proposition 3 (*Effect of a disclosure cap on quality under partial market coverage*)
Assume that the market is partially covered.

- If $F(\cdot)$ is weakly convex, the effect of a disclosure cap $\bar{d} \in (\underline{d}, \tilde{d}^M)$ on quality is positive.

- If $F(\cdot)$ is concave and relatively inelastic, the effect of a disclosure cap $\bar{d} \in (\underline{d}, \tilde{d}^M)$ on quality is negative.

- If $F(\cdot)$ is concave and relatively elastic, the effect of a disclosure cap $\bar{d} \in (\underline{d}, \tilde{d}^M)$ on quality is negative if the complementarity between quality and information is not too strong, and is ambiguous otherwise.

Unlike in the full market coverage scenario—where a cap always reduces quality level—the cap may either decrease or increase quality level under the partial coverage scenario. Proposition 3 characterizes how a disclosure cap affects quality level depending on the shape of the distribution of privacy costs and the degree of complementarity between quality and information. Under partial market coverage, a disclosure cap generates two additional effects relative to the full market coverage scenario: an expansion of the firm’s user base (or demand) and a change in the sensitivity of demand to quality; both of these impact the marginal benefit of investing in quality. A bigger user base implies a larger increase in the amount of user-provided data when the firm (marginally) increases its quality level; this raises the marginal benefit of quality investment. The change in the sensitivity of demand to quality may either increase or decrease the marginal benefit of investment depending on whether this sensitivity increases substantially under a disclosure cap. The overall effect of a disclosure cap on the firm’s incentives to invest in quality is thus the combination of a negative effect (due to the decline in the firm’s ability to monetize user data via disclosure), a positive effect (due to demand expansion) and an ambiguous effect (due to a change in the sensitivity of demand to quality). A disclosure cap raises quality investment if the second effect is large enough and/or the third effect is positive or large enough. Proposition 3 shows that this holds when $F(\cdot)$ is weakly convex. In contrast, when $F(\cdot)$ is concave, the impact of the cap on quality level is either negative or ambiguous, depending on the elasticity of the demand with respect to disclosure level and the degree of complementarity between quality and information.

The results above may be better understood with the aid of a simple illustration. Consider the special case of the uniform distribution (i.e., $\theta \sim \mathcal{U}[\underline{\theta}, \bar{\theta}]$). The uniform distribution is weakly convex, and therefore case (i) of Proposition 3 applies: a disclosure cap leads to more quality investment. To see why, notice first that term C in Equation (6) is equal to zero in the uniform distribution case; this implies that a disclosure cap does not affect the extensive margin effect of quality investment, and we do not have the ambiguous effect described above. The cap only generates the negative effect—a reduction in the firm’s disclosure revenue for any given user base—and the positive effect—an expansion in the firm’s user base. In the uniform distribution

case, the positive effect of the cap dominates: the increase in disclosure revenues from the expansion in the firm’s user base more than compensates for the fall in disclosure revenues for a given user base. Overall, the firm has stronger incentives to invest in quality.

Combining Proposition 3 with the fact that the direct effect of a disclosure cap on social welfare is always (strictly) positive leads us to the following result about the social desirability of a cap on the disclosure level.

Proposition 4 (*Social desirability of a disclosure cap under partial market coverage*)
Assume that the market is partially covered.

- *If $F(\cdot)$ is weakly convex, a privacy regulation taking the form of a binding disclosure cap is strictly socially desirable.*

- *If $F(\cdot)$ is concave, a privacy regulation taking the form of a binding disclosure cap has an ambiguous effect on social welfare.*

Thus, when the market is partially covered, a disclosure cap may be socially desirable even when quality and information are strong complements from consumers’ perspective. In particular, this is the case if the reduction in disclosure level substantially increases the responsiveness of demand to an increase in quality level (i.e., if $F(\cdot)$ is weakly convex). In this scenario, the regulator can improve *both* consumer privacy and service quality by setting a disclosure cap.³⁷

6 Extensions

6.1 Heterogeneous third parties

In this extension, we analyze the scenario where third parties are heterogeneous in the privacy cost that they induce for consumers.³⁸ This heterogeneity could reflect, for instance, differences in data use practices of these third parties. Consider the case of

³⁷Note that we do not consider in this paper the scenario in which the market can be fully or partially covered *depending* on the regulator’s decision in the first stage. In such a scenario, the regulator’s problem is less smooth than in the two cases we considered. Because of that, solving this problem would require us to determine the optimal disclosure cap under each of the two regimes (i.e., full market coverage and partial market coverage) and compare the corresponding social welfare values. While this substantially complicates the analysis, it is unclear whether it would provide additional insights into the desirability of privacy regulation.

³⁸See Bergemann and Bonatti (2015) and Bergemann et al. (2018) for models in which a monopolist sells personal information to third parties that are heterogeneous along other dimensions.

third-party advertisers. Some advertisers may choose to target their advertisements using data at a more aggregated level than others (e.g., based on demographic groups instead of individual characteristics), thereby resulting in lower privacy costs.

Let us assume that disclosing a unit of personal information to a third party of type $s \in [0, 1]$ induces a privacy cost $2\theta s$ for the consumer of type θ (instead of a privacy cost θ in the baseline model). The total privacy cost incurred by a consumer of type θ when an amount x of her personal information is disclosed to third parties located in $[0, d]$ is then given by $\int_0^d 2\theta s x ds = \theta d^2 x$. Correspondingly, the consumer's utility function is

$$U(x, \theta, q, d) = V(x, q) - (\alpha + \theta d^2) x - K.$$

Under the full market coverage scenario, the impact of a disclosure cap on quality and social welfare are qualitatively the same as in our baseline model: a disclosure cap lowers the investment in quality but is socially desirable if the complementarity between quality and information is not too strong.

However, under the partial market coverage scenario, the results are qualitatively affected by the assumption that third parties are heterogeneous. More specifically, straightforward computations show that the effect of a decrease in disclosure level on the marginal benefit from investing in quality has the following form:

$$-\frac{\partial^2 \tilde{\Pi}}{\partial q \partial d} = \frac{1}{d} (A + B + C) + \underbrace{\left(1 - \frac{1}{d}\right) A}_{\geq 0} + \underbrace{D}_{\geq 0},$$

whereas it is equal to $A + B + C$ in our baseline model. This implies that the effect of a disclosure cap on quality is positive in this extension whenever it was positive in the baseline model. Therefore, we get the following result:

Proposition 5 (*Effect of a disclosure cap with heterogeneous third parties*)

- Under full market coverage, a disclosure cap $\bar{d} \in (\underline{d}, \tilde{d}^M)$ has a negative effect on quality. However, setting a binding disclosure cap is socially desirable if quality and information are not strong complements.

- Under partial market coverage, the effect of a disclosure cap $\bar{d} \in (\underline{d}, \tilde{d}^M)$ on quality is weakly positive if $F(\cdot)$ is weakly convex. Consequently, setting a binding disclosure cap is socially desirable if $F(\cdot)$ is weakly convex.

6.2 Consumer-surplus-maximizing regulator

In the analysis of privacy regulation presented previously, we considered the decision problem of a regulator who seeks to maximize social welfare, which is given by the sum of the firm's profit and consumer surplus.³⁹ We now examine the case where the regulator is a consumer protection agency, whose objective is to maximize consumer surplus. Let us focus again on the scenario in which the regulator does not find it optimal to choose a disclosure cap that induces no investment in quality.⁴⁰ The regulator seeks to maximize

$$\widehat{CS}(\bar{d}) \equiv \widetilde{CS}(q^M(\bar{d}), \bar{d}) = \int_{\underline{\theta}}^{\bar{\theta}(q, \bar{d})} \tilde{U}(\theta, q^M(\bar{d}), \bar{d}) f(\theta) d\theta$$

over $[\underline{d}, \tilde{d}^M]$. Assuming that $\widehat{CS}(\cdot)$ is strictly quasi-concave over this interval, setting a binding cap on disclosure is strictly desirable from the perspective of the consumer agency if the marginal benefit of lower disclosure to the consumers is positive when evaluated at $\bar{d} = \tilde{d}^M$; i.e., $-\left. \frac{\partial \widehat{CS}}{\partial \bar{d}} \right|_{\bar{d}=\tilde{d}^M} > 0$.

We now compare the desirability of a disclosure cap for a consumer protection agency and for a social-welfare-maximizing planner.

Proposition 6 (*Consumer-surplus-maximizing vs socially optimal disclosure cap*)

- A privacy regulation taking the form of a disclosure cap is strictly desirable for a consumer protection agency if and only if it is strictly socially desirable.
- When such a regulation is strictly desirable for the consumer protection agency, its optimal disclosure cap is lower than the socially optimal disclosure cap.

The circumstances under which a disclosure cap is desirable from consumers' perspective are the same as those under which it is socially desirable. The reason behind this finding is that a marginal decrease of the disclosure level starting from the privately optimal disclosure level \tilde{d}^M has a second-order effect on the firm's profit but a first-order effect on consumer surplus.

³⁹Recall that third parties do not get any surplus in our baseline setting.

⁴⁰If the consumer protection agency finds it optimal to choose a disclosure cap that leads to no investment in quality, it will choose $\bar{d} = 0$ (as this will minimize the privacy costs incurred by consumers).

7 Discussion

7.1 Alternative interpretation of the model

In our analysis, we interpreted the consumer’s input, x , as the amount of information that a consumer provides to the firm. This interpretation is appropriate when considering a matching website, where each user decides on the preferences she reveals, or in the context of services involving user-generated content, where each user decides on the amount of content to share (e.g., on a social media platform). This interpretation may, however, be less suited to other forms of Internet services, such as email and search. In these contexts, consumers do not directly supply information to the firm; instead, information is generated as a result of their use of the firm’s service. The relevant consumer choice variable (and hence the appropriate interpretation of x) is therefore *usage intensity*, rather than information provision level. Correspondingly, α , which formerly captured the marginal cost of providing information, can be interpreted more generally as the marginal opportunity cost of using the firm’s service.

7.2 Positive surplus for third parties

We assumed in our baseline model that the firm was able to appropriate all the surplus generated by the access of a third party to its customers’ personal information. Let us relax this assumption by allowing third parties to capture a positive share of that surplus. More precisely, we assume that the unit price of personal information is βr , where $\beta \in (0, 1]$.

The firm’s profit function in this variant of our model can be derived from the one in our baseline setting by replacing r with βr . This implies in particular that the effect of a disclosure cap on firm’s choice of quality is (qualitatively) the same as in the baseline model. More precisely, Propositions 1 and 3 still hold.

That said, when it comes to the effect of a disclosure cap on social welfare—defined as the sum of the firm’s profit, the third parties’ surplus and consumer surplus—it becomes more complicated to derive unambiguous results in the current setting. To see why, consider first the *direct effect* of a marginal decrease in the disclosure level (starting from the firm’s optimal disclosure level) on social welfare. In our baseline model (i.e., $\beta = 1$) this effect is always positive. However, when the surplus of third parties with whom personal information is shared is positive (i.e., $\beta < 1$), this need not be true. A decrease in the disclosure level still leads to an increase in consumer

surplus (through a decrease in privacy costs), but it also results in a decrease in the surplus of third parties. Moreover, when the surplus of third parties is positive, there is an additional indirect effect, besides the strategic effect we identified in our baseline model. This additional effect corresponds to the positive impact of a disclosure cap on the amount of information provided by consumers and, therefore, on the surplus of the third parties with whom personal data is shared. When this indirect effect and the strategic effect we identified in our baseline model do not have the same sign (which is the case when a disclosure cap has a negative effect on quality), the sign of the overall indirect effect of a disclosure cap depends on their relative magnitudes.

Importantly, notice that if the regulator maximizes consumer surplus instead of social welfare, allowing third parties to make a positive surplus out of their access to customers' personal data does not affect the desirability of a disclosure cap (as long as r is replaced with βr in the analysis).

7.3 Inability to commit to a disclosure level

We assumed in our baseline model that the firm is able to commit to its disclosure level. We now suppose that the firm is unable to commit, which implies that consumers need to anticipate its disclosure level when making their participation and information provision decisions. Since the firm finds it optimal to choose the highest possible disclosure level for any *given* consumer beliefs, an equilibrium with rational expectations necessarily features $d = 1$ absent any regulation.⁴¹ In such an environment, regulation serves two purposes. First, it allows the firm to commit to a maximum disclosure level.⁴² Second, it constrains the firm's behavior by limiting its choice set.⁴³

Our analysis and results on the impact of a disclosure cap on quality investment in the baseline setting carries over to the no-commitment scenario.⁴⁴ The only difference in the no-commitment setting is that Propositions 1 and 3 hold for any disclosure cap $\bar{d} \in (\underline{d}, 1)$ instead of $\bar{d} \in (\underline{d}, \tilde{d}^M)$.

⁴¹This is due partly to the static nature of the game. See Jullien et al. (2018) for a model where a firm cannot commit to its privacy policy in a setting where it interacts repeatedly with its customers.

⁴²This can be made possible through heavy sanctions against firms that violate the regulation. For instance, the fine for violating the European General Data Protection Regulation is up to €20 million or 4% of the company's global annual turnover of the previous financial year, whichever is higher.

⁴³While the second effect has a negative impact on the firm, the first effect positively affects the firm. In other words, if the regulation were to serve as a commitment device only, the firm would always welcome it.

⁴⁴The reason is that the sign of the cross-effect of quality and disclosure levels on profit is what matters, regardless of whether the firm is able to commit to a disclosure level or not.

The analysis of the social desirability of privacy regulation is, however, more complex than in the baseline model. That said, under the regularity assumption that $\hat{W}(\cdot)$ is strictly quasi-concave in \bar{d} , we can show that regulation is socially desirable under the no-commitment scenario whenever it is socially desirable under the commitment scenario.⁴⁵ This implies that the scope for a welfare-enhancing privacy regulation is larger under the former scenario than the latter.

7.4 Network externalities

Assume now that the utility of each customer increases with the amount of information provided by the other customers. This form of network externalities exists for instance in social media platforms. To incorporate these externalities, we augment the consumer's utility in the baseline model by σX , where $X \equiv \int_{\underline{\theta}}^{\bar{\theta}} x(\theta) d\theta$ is the amount of personal information provided by all (other) consumers⁴⁶ and $\sigma \geq 0$ is a parameter that captures the intensity of the externalities.

Observe first that, conditional on using the service, consumers' optimal level of information provision does not depend on the network externalities σX and is, therefore, the same as in the baseline model; i.e., $\tilde{x}(\theta, q, d)$. This implies in particular that the presence of network externalities does not affect our results in the full market coverage scenario: a disclosure cap leads to a decrease in quality investment but is still socially desirable if quality and information are strongly complementary for consumers.

Although network externalities do not affect consumers' information provision decision, they alter their participation decision. This changes the firm's marginal benefit from investing in quality under the partial market coverage scenario, and therefore quantitatively affects our analysis. Specifically, it can be shown that in a fulfilled expectations equilibrium (i.e., a situation where each consumer anticipates correctly the decisions of all other consumers), there exists as in the baseline model a threshold $\tilde{\theta}(q, d, \sigma)$ such that only consumers with type $\theta < \tilde{\theta}(q, d, \sigma)$ use the service.⁴⁷ The effect of a marginal decrease in disclosure level on the marginal benefit of investing in

⁴⁵To see why, notice that the quasi-concavity of $\hat{W}(\cdot)$ with respect to \bar{d} implies that $-\frac{\partial \hat{W}}{\partial \bar{d}} \Big|_{\bar{d}=1} > 0$ whenever $-\frac{\partial \hat{W}}{\partial \bar{d}} \Big|_{\bar{d}=\bar{d}^M} > 0$.

⁴⁶Since a consumer is of zero measure in our setting, the total amount of personal information provided by all consumers and the total amount of information provided by all consumers but one coincide.

⁴⁷This threshold and, therefore, the demand for the service are increasing in the intensity of network externalities σ (and are therefore larger than in the baseline model, i.e. $\sigma = 0$). This intuitive result follows from the fact that consumers' utility function is increasing in σ and X .

quality—which determines the impact of a disclosure cap on quality—can again be split in three terms: the impact of an decrease in disclosure level on i) the intensive margin effect for a fixed demand, ii) the intensive margin effect via the change in demand it induces, and iii) the extensive margin effect. As in the baseline model, the first term is negative, the second term is positive, and the sign of the third term is ambiguous. This implies that a disclosure cap has again three effects on the firm’s incentives to invest in quality: a negative effect through the decrease in the returns of investment for a fixed demand, a positive effect stemming from the boost in demand induced by the cap, and an ambiguous effect capturing the impact of the cap on the responsiveness of demand to quality. Although the magnitude of these effects is not the same as in the baseline model, the result still holds qualitatively: that the cap raises quality if it leads to a large enough increase in the responsiveness of demand to quality.

7.5 Positive effect of disclosure on consumers

Our baseline model abstracts away from any positive effect of disclosure to third parties on consumers. Assume now that disclosure brings about benefits to consumers which we capture through an additional term βdx in the consumer utility function where $\beta \geq 0$.⁴⁸ Thus, the *net* cost of disclosure for consumer of type θ is $(\theta - \beta) dx$.

Note first that if $\beta \leq \underline{\theta}$, i.e. all consumers are harmed by the disclosure of their personal information in net terms, then it is straightforward that it is sufficient to replace θ by $\theta - \beta$ for our analysis to carry over. Suppose now that $\beta > \underline{\theta}$ so that at least some consumers benefit from the disclosure of their data to third parties. In this case, our analysis still applies if aggregate consumer surplus is locally decreasing in the disclosure level at $(\tilde{q}^M, \tilde{d}^M)$, which means that there is overdisclosure of personal information when the firm is not regulated.⁴⁹ If this condition is not satisfied then there is no case for a regulation taking the form of a disclosure cap.

7.6 Positive prices for consumers

Our baseline model has focused on the case in which the firm offers the service for free to consumers. While this is a widespread scenario in practice, it may also be the case that the firm finds it optimal to charge consumers a positive price (in addition to disclosing

⁴⁸We assume for the sake of exposition that β is the same for all consumers.

⁴⁹This condition is met if the share of consumers that are harmed (in net terms) by the disclosure of their personal information is sufficiently large.

their personal data).⁵⁰ Incorporating this possibility in our model would complicate the analysis substantially. To see why, notice that in a simpler setting without investment in quality and without an intensive margin (i.e., a setting where consumers would not be able to choose the amount of data they provide), we would expect such a cap to lead to an increase in the price charged to consumers. This is due to the so-called *seesaw effect* in two-sided markets (Rochet and Tirole, 2006): a cap on data disclosure would reduce the marginal benefit from attracting an additional consumer, which would make it optimal for the website to increase the price on the consumer side. However, in our setting, a cap on data disclosure would lead to a joint adjustment of both price and quality. A quick inspection of the relevant cross-derivatives of the website’s profit function suggests that the effects of a disclosure cap on both price and quality are quite complex, and are likely to be ambiguous.

7.7 Substitutability between quality and information

For many Internet services, it is natural to think of the firm’s quality level and the consumer information provision level as complements (i.e., $\gamma > 0$): the better the quality of a firm’s service, the higher the level of usage or information provision by consumers. There may also be cases, however, where the firm’s quality level and the consumer’s information exhibit substitutability (i.e., $\gamma < 0$). User authentication is one such scenario. Interpreting the firm’s quality level as its ability to verify its user identity without the use of personal information provided by the consumer,⁵¹ the higher the firm’s quality level, the lower the consumer’s utility from providing additional pieces of personal information (phone number, secondary email address, etc.) for authentication purposes.

Note that when quality and information are substitutes, a cap on the level of disclosure is always socially desirable under the full market coverage scenario. There is no trade-off between privacy protection and quality provision in this case because the firm never invests in quality. When the market is partially covered, however, a trade-off between privacy and quality may also exist in the substitutes case.

⁵⁰Note that the parameter K in our baseline model could be interpreted as an *exogenous* price.

⁵¹For example, the firm could make use of the IP address or geographical location to assess if a login attempt is potentially fraudulent.

7.8 Regulating quality instead of disclosure

Regulating the disclosure level is one way of addressing (partially) the market failure identified in Section 4. An alternative way of doing so is to regulate the quality level. In Appendix C, we study the effect of setting an *ex ante* minimum quality requirement and analyze the social desirability of such a regulation. We show that the sign of the effect of a minimum quality requirement on the disclosure level is the opposite of the sign of the effect of a disclosure cap on quality: whenever a disclosure cap leads to a lower (resp. higher) quality, a minimum quality requirement leads to a higher (resp. lower) disclosure level. The reason is that both effects are driven by the (same) cross-effect of quality and disclosure on profit. This “duality” between quality and disclosure extends to the welfare effects of a regulation targeting one of them. More precisely, because the direct effect of a marginal increase in quality above the unregulated level on social welfare is positive (and, therefore, has the same sign as the direct effect of a disclosure cap on social welfare), the conditions under which setting a minimum quality requirement is socially desirable turn out to be qualitatively similar to those under which setting a disclosure cap is desirable.

8 Conclusion

In this paper, we study how a privacy regulation—specifically, a cap on data disclosure—affects a monopolist’s incentives to invest in the quality of its service and social welfare. We find that the impact of a reduction in disclosure level on the monopolist’s optimal choice of quality is negative when the market is fully covered, and depends on the effect of disclosure on the sensitivity of demand to quality when the market is partially covered. Under full market coverage, a cap on the disclosure level is socially desirable when the degree of complementarity between quality and information is not too strong. Under partial market coverage, a cap is desirable when the marginal effect of quality on demand is (sufficiently) elastic with respect to disclosure. As extensions, we also analyzed the case where third parties are heterogeneous and the scenario in which the regulator’s objective is consumer surplus maximization.

Our analysis has implications for the regulation of dominant firms’ disclosure policy. It shows in particular that regulators should distinguish between firms whose demand is (essentially) unresponsive to changes in quality and/or disclosure and those whose demand is (significantly) responsive to such changes. For firms of the former type,

regulators are likely to face a privacy-quality trade-off and will have to weigh the direct privacy gains for consumers against lower investment in quality. The level of complementarity between quality and information from consumers' perspective turns out to be the key determinant of the desirability of privacy regulation in this scenario. For firms of the latter type, our results provide conditions under which a disclosure cap raises investment in quality, and therefore unambiguously increases social welfare.

In addition to a disclosure cap, another privacy regulation that can be explored using our framework is the taxation of disclosure revenues. The taxation of digital monopoly platforms have been studied, for instance, by Bloch and Demange (2018) and Bourreau et al. (2018). However, to the best of our knowledge, no paper has considered the impact of taxation on the firm's incentives to invest in quality. A (unit) tax on the monopolist's disclosure revenues would translate to a reduction in the value of information in our model. Since this reduction affects both the optimal quality and disclosure levels of the firm, the impact of a tax is *a priori* unclear.

Finally, our model may also be interpreted more generally than one of privacy and quality. For example, the value of information can be thought of (more broadly) as the value that the firm derives from the exploitation of consumer data.⁵² Correspondingly, the level of disclosure could instead be interpreted as the degree of data exploitation and the privacy cost parameter as a more general parameter reflecting the cost of sharing information with the firm. One can even take a step further and consider other types of inputs (besides personal information) that consumers may provide. For instance, consumers could provide time or attention rather than personal information. The interpretation of the consumers' cost parameter (which captured the intensity of privacy preferences in the case of information provision) would then change depending on the input that we are considering.

⁵²Bloch and Demange (2018) provide several interpretations for the degree of data exploitation.

A Appendix: Proofs

Proof of Lemma 1. Differentiating

$$\frac{\partial U}{\partial x}(\tilde{x}(\theta, q, d), \theta, q, d) = \frac{\partial V}{\partial x}(\tilde{x}(\theta, q, d), q) - (\alpha + \theta d) = 0 \quad (7)$$

with respect to d yields

$$\frac{\partial^2 V}{\partial x^2}(\tilde{x}(\theta, q, d), q) \frac{\partial \tilde{x}}{\partial d}(\theta, q, d) - \theta = 0$$

and therefore

$$\frac{\partial \tilde{x}}{\partial d}(\theta, q, d) = \frac{\theta}{\frac{\partial^2 V}{\partial x^2}(\tilde{x}(\theta, q, d), q)} < 0.$$

Differentiating (7) with respect to θ and q leads to

$$\frac{\partial \tilde{x}}{\partial \theta}(\theta, q, d) = \frac{d}{\frac{\partial^2 V}{\partial x^2}(\tilde{x}(\theta, q, d), q)} < 0,$$

$$\frac{\partial \tilde{x}}{\partial q}(\theta, q, d) = -\frac{\frac{\partial^2 V}{\partial x \partial q}(\tilde{x}(\theta, q, d), q)}{\frac{\partial^2 V}{\partial x^2}(\tilde{x}(\theta, q, d), q)} = \gamma.$$

Proof of Lemma 2. Since $U(x, \theta, q, d)$ is decreasing in θ then $\tilde{U}(\theta, q, d) = \max_{x \in [0,1]} U(x, \theta, q, d)$ is decreasing in θ (by the Envelope Theorem). Therefore, there exists $\tilde{\theta}(q, d) \in [\underline{\theta}, \bar{\theta}]$ such that, for any $\theta \in [\underline{\theta}, \bar{\theta})$, the following equivalence holds:

$$\tilde{U}(\theta, q, d) > 0 \iff \theta < \tilde{\theta}(q, d).$$

Moreover, whenever $\tilde{\theta}(q, d)$ is in the open interval $(\underline{\theta}, \bar{\theta})$, it is defined by

$$\tilde{U}(\tilde{\theta}(q, d), q, d) = 0.$$

Differentiating the latter with respect to q and d , and using the Envelope Theorem, we get that

$$\frac{\partial \tilde{\theta}}{\partial q} = -\frac{\frac{\partial \tilde{U}}{\partial q}}{\frac{\partial \tilde{U}}{\partial \theta}} = -\frac{\frac{\partial U}{\partial q}}{\frac{\partial U}{\partial \theta}} = \frac{\frac{\partial V}{\partial q}}{d\tilde{x}(\tilde{\theta}(q, d), q, d)} > 0,$$

$$\frac{\partial \tilde{\theta}}{\partial d} = -\frac{\frac{\partial \tilde{U}}{\partial d}}{\frac{\partial \tilde{U}}{\partial \theta}} = -\frac{\frac{\partial U}{\partial d}}{\frac{\partial U}{\partial \theta}} = -\frac{\tilde{\theta}(q, d)}{d} < 0.$$

Proof of Lemma 3. The net marginal social benefit from investing in quality is

$$\begin{aligned} \frac{\partial \tilde{W}}{\partial q} &= \frac{\partial \tilde{\Pi}}{\partial q} + \int_{\underline{\theta}}^{\tilde{\theta}(q, d)} \frac{\partial \tilde{U}}{\partial q}(\theta, q, d) f(\theta) d\theta + \frac{\partial \tilde{\theta}}{\partial q} \tilde{U}(\tilde{\theta}(q, d), q, d) f(\tilde{\theta}(q, d)) \quad (8) \\ &= \frac{\partial \tilde{\Pi}}{\partial q} + \underbrace{\int_{\underline{\theta}}^{\tilde{\theta}(q, d)} \frac{\partial V}{\partial q}(\tilde{x}(\theta, q, d), q) f(\theta) d\theta}_{>0} + \underbrace{\frac{\partial \tilde{\theta}}{\partial q} \tilde{U}(\tilde{\theta}(q, d), q, d) f(\tilde{\theta}(q, d))}_{=0 \text{ by definition of } \tilde{\theta}(q, d)}. \end{aligned}$$

The second term is obtained by applying the Envelope Theorem and the last term is equal to zero both when $\tilde{\theta}(q, d) < \bar{\theta}$ and when $\tilde{\theta}(q, d) = \bar{\theta}$ (When $\tilde{\theta}(q, d) < \bar{\theta}$, this follows from the fact that $\tilde{U}(\tilde{\theta}(q, d), q, d) = 0$, and when $\tilde{\theta}(q, d) = \bar{\theta}$ it follows from the fact that $\frac{\partial \tilde{\theta}}{\partial q} = 0$).

Likewise, the net marginal social benefit from information disclosure is

$$\begin{aligned} \frac{\partial \tilde{W}}{\partial d} &= \frac{\partial \tilde{\Pi}}{\partial d} + \int_{\underline{\theta}}^{\tilde{\theta}(q, d)} \frac{\partial \tilde{U}}{\partial d}(\theta, q, d) f(\theta) d\theta + \frac{\partial \tilde{\theta}}{\partial d} \tilde{U}(\tilde{\theta}(q, d), q, d) f(\tilde{\theta}(q, d)) \\ &= \frac{\partial \tilde{\Pi}}{\partial d} - \underbrace{\int_{\underline{\theta}}^{\tilde{\theta}(q, d)} \theta \tilde{x}(\theta, q, d) f(\theta) d\theta}_{<0} + \underbrace{\frac{\partial \tilde{\theta}}{\partial d} \tilde{U}(\tilde{\theta}(q, d), q, d) f(\tilde{\theta}(q, d))}_{=0}. \end{aligned}$$

The first part of the lemma follows directly from the fact that

$$\frac{\partial \tilde{\Pi}}{\partial q} < \frac{\partial \tilde{W}}{\partial q}$$

and the strict quasi-concavity of $\tilde{\Pi}$ and \tilde{W} with respect to q . Likewise, the second part of the lemma follows directly from the fact that

$$\frac{\partial \tilde{\Pi}}{\partial d} > \frac{\partial \tilde{W}}{\partial d}$$

and the strict quasi-concavity of $\tilde{\Pi}$ and \tilde{W} with respect to d .

Proof of Lemma 4. The marginal effect of reducing the disclosure cap on social welfare is

$$\begin{aligned}
-\frac{\partial \hat{W}}{\partial \bar{d}} &= -\frac{\partial \tilde{\Pi}}{\partial d}(q^M(\bar{d}), \bar{d}) - \int_{\underline{\theta}}^{\tilde{\theta}(q^M(\bar{d}), \bar{d})} \left[\frac{\partial \tilde{U}}{\partial q}(\theta, q^M(\bar{d}), \bar{d}) \frac{\partial q^M}{\partial d} + \frac{\partial \tilde{U}}{\partial d}(\theta, q^M(\bar{d}), \bar{d}) \right] f(\theta) d\theta \\
&\quad - \underbrace{\left[\frac{\partial \tilde{\theta}}{\partial q} \frac{\partial q^M}{\partial d} + \frac{\partial \tilde{\theta}}{\partial d} \right] \tilde{U}(\tilde{\theta}(q^M(\bar{d}), \bar{d}), q^M(\bar{d}), \bar{d}) f(\tilde{\theta}(q^M(\bar{d}), \bar{d}))}_{=0} \\
&= -\frac{\partial \tilde{\Pi}}{\partial d}(q^M(\bar{d}), \bar{d}) - \int_{\underline{\theta}}^{\tilde{\theta}(q^M(\bar{d}), \bar{d})} \left[\frac{\partial V}{\partial q}(\tilde{x}(\theta, q^M(\bar{d}), \bar{d}), q^M(\bar{d})) \frac{\partial q^M}{\partial d} - \theta \tilde{x}(\theta, q^M(\bar{d}), \bar{d}) \right] f(\theta) d\theta,
\end{aligned}$$

where the second equality is obtained by applying the Envelope Theorem. Evaluating this at $\bar{d} = \tilde{d}^M$ and using the fact that

$$\frac{\partial \tilde{\Pi}}{\partial d}(q^M(\tilde{d}^M), \tilde{d}^M) = \frac{\partial \tilde{\Pi}}{\partial d}(\tilde{q}^M, \tilde{d}^M) = 0,$$

we obtain the result.

Proof of Lemma 5. Assume that $q^M(d) > 0$. Then, by continuity, $q^M(d') > 0$ for d' sufficiently close to d . Therefore, for d' sufficiently close to d , $q^M(d')$ is an interior solution given by the first-order condition

$$\frac{\partial \tilde{\Pi}}{\partial q}(q^M(d'), d') = 0.$$

Differentiating this with respect to d' and evaluating it at $d' = d$ yields

$$\frac{\partial^2 \tilde{\Pi}}{\partial q^2}(q^M(d), d) \frac{\partial q^M}{\partial d} + \frac{\partial^2 \tilde{\Pi}}{\partial q \partial d}(q^M(d), d) = 0,$$

which leads to the result.

Proof of Proposition 1. Substituting $\frac{\partial \tilde{x}}{\partial q}$ by its expression in Lemma 1 and using the fact that $C'(0) = 0$ we obtain that the marginal benefit of investing in quality is

positive when evaluated at $q = 0$ for any $d > 0$:

$$\left. \frac{\partial \tilde{\Pi}}{\partial q} \right|_{q=0} = rd\gamma.$$

This implies that the firm's optimal choice of quality level for a given level of disclosure, $q^M(d)$, is positive for all levels of disclosure $d > 0$ (and is zero for $d = 0$).⁵³ From Lemma 5 it follows that

$$\frac{\partial q^M}{\partial d} = \frac{r\gamma}{C''(q^M(d))} > 0.$$

This leads us to the result.

Proof of Lemma 6. Using Lemmas 1 and 2 again, we can rewrite (6) as

$$-\frac{\partial^2 \tilde{\Pi}}{\partial q \partial d} = \underbrace{-r\gamma F(\tilde{\theta}(q, d)) \left[1 - \frac{\tilde{\theta}(q, d) f(\tilde{\theta}(q, d))}{F(\tilde{\theta}(q, d))} \right]}_{A+B} + \underbrace{r \frac{\partial V}{\partial q} \frac{\tilde{\theta}(q, d)}{d} f'(\tilde{\theta}(q, d))}_{C}.$$

The result follows immediately from this decomposition.

Proof of Proposition 3. Lemma 6 suggests that we should distinguish between four scenarios. However, there are only three possible scenarios because the elasticity of $F(\cdot)$ is always greater than 1 if $F(\cdot)$ is (globally) convex. To see why, notice that

$$\frac{\theta f(\theta)}{F(\theta)} = 1 + \frac{\frac{\theta}{f(\theta)} - \int_{\theta}^{\theta} [f(u) - f(\theta)] du}{F(\theta)},$$

which is greater than 1 if $f(\cdot)$ is increasing. Before stating the main result of this section, let us consider the limiting case where quality and information are independent; i.e., $\gamma = 0$. In this scenario, the intensive margin effect is zero regardless of the disclosure level, while the extensive margin effect is increasing (resp. decreasing) in the disclosure level if $F(\cdot)$ is concave (resp. convex). Consequently, a decrease in disclosure level leads to a decrease (resp. increase) in the firm's marginal benefit from investing in quality if $F(\cdot)$ is concave (resp. convex).

Using Lemmas 5 and 6 and the observations above for the case $\gamma = 0$ (which extend

⁵³This implies that $\underline{d} = 0$ under full market coverage.

by continuity to sufficiently small values of γ), we obtain the result.

Proof of Proposition 6. Since

$$\frac{\partial \widehat{CS}}{\partial \bar{d}} \Big|_{\bar{d}=\bar{d}^M} = \frac{\partial \widehat{W}}{\partial \bar{d}} \Big|_{\bar{d}=\bar{d}^M} - \underbrace{\frac{\partial \tilde{\Pi}}{\partial d} (q^M(\bar{d}^M), \bar{d}^M)}_{=0} = \frac{\partial \widehat{W}}{\partial \bar{d}} \Big|_{\bar{d}=\bar{d}^M},$$

the condition under which a disclosure cap is desirable from the perspective of the consumer protection agency is the same as that under which it is desirable from the perspective of a social-welfare-maximizing regulator.

Let us now assume that setting a disclosure cap is strictly socially desirable and compare the optimal cap \bar{d}^W for a social-welfare-maximizing regulator and the optimal cap \bar{d}^C for a consumer-surplus-maximizing regulator. Since the disclosure cap is binding and $\tilde{\Pi}(q^M(d), d)$ is strictly quasi-concave with respect to d ,

$$\frac{\partial \tilde{\Pi}}{\partial d} (q^M(\bar{d}^W), \bar{d}^W) > 0.$$

Therefore,

$$\frac{\partial \widehat{CS}}{\partial \bar{d}} \Big|_{\bar{d}=\bar{d}^W} < \frac{\partial \widehat{W}}{\partial \bar{d}} \Big|_{\bar{d}=\bar{d}^W} \leq 0,$$

where the second inequality follows from the fact that $\bar{d}^W > 0$.⁵⁴ From the strict quasi-concavity of $\widehat{CS}(\cdot)$, it then follows that

$$\bar{d}^C < \bar{d}^W.$$

B Appendix: Elasticities

Denote

$$\check{V}(x, q) \equiv V(x, q) - \alpha x.$$

⁵⁴Under the assumption that a social-welfare-maximizing regulator does not find it optimal to set a disclosure cap that leads to no investment in quality, it must hold that $\bar{d}^W > 0$.

The demand addressed to the firm when the amount of information is chosen optimally by consumers is

$$\tilde{D}(q, d) = F\left(\tilde{\theta}(q, d)\right) = F\left(\min\left(\frac{\check{V}\left(\tilde{x}\left(\tilde{\theta}(q, d), q, d\right), q\right) - K}{d\tilde{x}\left(\tilde{\theta}(q, d), q, d\right)}, \bar{\theta}\right)\right).$$

Consider the following function:

$$D(x, q, d) = F\left(\min\left(\frac{\check{V}(x, q) - K}{dx}, \bar{\theta}\right)\right),$$

which can be interpreted as the demand addressed to the firm if all consumers using the service (are required to) provide the same amount of information x . Notice that

$$\tilde{D}(q, d) = D\left(\tilde{x}\left(\tilde{\theta}(q, d), q, d\right), q, d\right).$$

The elasticity of $D(x, q, d)$ with respect to d is (in absolute value)

$$-d \frac{\partial D}{\partial d} = \frac{d \frac{1}{d^2} \frac{\check{V}(x, q) - K}{x} f\left(\frac{\check{V}(x, q) - K}{dx}\right)}{F\left(\frac{\check{V}(x, q) - K}{dx}\right)} = \frac{\frac{\check{V}(x, q) - K}{dx} f\left(\frac{\check{V}(x, q) - K}{dx}\right)}{F\left(\frac{\check{V}(x, q) - K}{dx}\right)}$$

whenever $D(x, q, d) \in (0, 1)$. In particular, under partial (positive) market coverage,

$$-d \frac{\partial D}{\partial d} \Bigg|_{x=\tilde{x}(\tilde{\theta}(q, d), q, d)} = \frac{\tilde{\theta}(q, d) f\left(\tilde{\theta}(q, d)\right)}{F\left(\tilde{\theta}(q, d)\right)},$$

which shows that the elasticity of demand holding the amount of information constant (at the level of the marginal consumer) is the same as the elasticity of the cumulative distribution function (computed for the marginal type).

Similarly, assuming that $\frac{\partial^2 D}{\partial q \partial d} \leq 0$, straightforward algebraic manipulations show that the elasticity of $\frac{\partial D}{\partial q}$ with respect to d is given by:

$$-d \frac{\frac{\partial^2 D}{\partial q \partial d}}{\frac{\partial D}{\partial q}} = 1 + \frac{\frac{\check{V}(x, q) - K}{dx} f'\left(\frac{\check{V}(x, q) - K}{dx}\right)}{f\left(\frac{\check{V}(x, q) - K}{dx}\right)}$$

whenever $D(x, q, d) \in (0, 1)$. In particular, under partial (positive) market coverage,

$$-d \frac{\frac{\partial^2 D}{\partial q \partial d}}{\frac{\partial D}{\partial q}} \Bigg|_{x=\tilde{x}(\tilde{\theta}(q,d),q,d)} = 1 + \frac{\tilde{\theta}(q,d) f'(\tilde{\theta}(q,d))}{f(\tilde{\theta}(q,d))} = 1 + \underbrace{\frac{\tilde{\theta}(q,d) F''(\tilde{\theta}(q,d))}{F'(\tilde{\theta}(q,d))}}_{\text{curvature of } F(\cdot)}.$$

This implies that the elasticity of $\frac{\partial D}{\partial q}$ with respect to d holding the amount of information constant is related to the curvature of $F(\cdot)$. It is greater (resp. less) than 1 if $f'(\cdot)$ is positive (resp. negative), that is, if $F(\cdot)$ is convex (resp. concave).

C Appendix: Minimum quality requirement

In this section, we investigate the social desirability of a policy whereby the authority does not regulate the disclosure level but, instead, regulates the quality level *ex ante* (i.e., before the disclosure level is set by the firm). Specifically, we study the decision of a social-welfare-maximizing regulator whose only instrument is a minimum quality standard. This requires, in particular, an understanding of the effect of a minimum quality requirement on the disclosure level chosen by the firm.

More precisely, consider the following game:

- First, the regulator decides whether to impose a minimum quality requirement, and sets the value of that requirement \underline{q} if it does so.
- Second, the firm decides on its disclosure and quality levels.
- Third, consumers decide whether to patronize the firm and how much information to provide if they do.

Let us first analyze the firm's behavior for a given regulator's choice. The firm's optimal quality level maximizes $\tilde{\Pi}(q, d^M(q))$, which we assume to be quasi-concave in q , subject to the constraint $q \geq \underline{q}$. If $\underline{q} \leq \tilde{q}^M$, the constraint is not binding; this means that the firm's decision will be the same as in the unregulated scenario. If $\underline{q} > \tilde{q}^M$, however, the constraint is binding. From the quasi-concavity of $\tilde{\Pi}(q, d^M(q))$ with respect to q , it then follows that the firm will choose $q = \underline{q}$.

Consider now the regulator's decision in the first stage. Note first that the regulator must take account of the firm's participation constraint, i.e.,

$$\tilde{\Pi}(\underline{q}, d^M(\underline{q})) \geq 0.$$

It can be easily shown that there exists $\bar{q} > \tilde{q}^M$ such that the participation constraint above holds if and only if $q \leq \bar{q}$.⁵⁵ Therefore, the regulator seeks to maximize

$$\check{W}(\underline{q}) \equiv \check{W}(\underline{q}, d^M(\underline{q})) = \tilde{\Pi}(\underline{q}, d^M(\underline{q})) + \int_{\underline{\theta}}^{\tilde{\theta}(\underline{q}, d^M(\underline{q}))} \tilde{U}(\theta, \underline{q}, d^M(\underline{q})) f(\theta) d\theta$$

with respect to $\underline{q} \in [\tilde{q}^M, \bar{q}]$. Assuming that $\check{W}(\cdot)$ is quasi-concave over this interval, the regulator finds it strictly optimal to set a binding minimum quality requirement if and only if

$$\left. \frac{\partial \check{W}}{\partial \underline{q}} \right|_{\underline{q}=\tilde{q}^M} > 0.$$

Moreover, we have

$$\begin{aligned} \frac{\partial \check{W}}{\partial \underline{q}} &= \frac{\partial \tilde{\Pi}}{\partial q}(\underline{q}, d^M(\underline{q})) + \int_{\underline{\theta}}^{\tilde{\theta}(\underline{q}, d^M(\underline{q}))} \left[\frac{\partial \tilde{U}}{\partial d}(\theta, \underline{q}, d^M(\underline{q})) \frac{\partial d^M}{\partial q} + \frac{\partial \tilde{U}}{\partial q}(\theta, \underline{q}, d^M(\underline{q})) \right] f(\theta) d\theta \\ &\quad + \left[\frac{\partial \tilde{\theta}}{\partial q} + \frac{\partial \tilde{\theta}}{\partial d} \frac{\partial d^M}{\partial q} \right] \underbrace{\tilde{U}(\tilde{\theta}(\underline{q}, d^M(\underline{q})), \underline{q}, d^M(\underline{q})) f(\tilde{\theta}(\underline{q}, d^M(\underline{q})))}_{=0} \\ &= \frac{\partial \tilde{\Pi}}{\partial q}(\underline{q}, d^M(\underline{q})) + \int_{\underline{\theta}}^{\tilde{\theta}(\underline{q}, d^M(\underline{q}))} \left[-\theta \tilde{x}(\theta, \underline{q}, d^M(\underline{q})) + \frac{\partial V}{\partial q}(\tilde{x}(\theta, \underline{q}, d^M(\underline{q})), \underline{q}) \right] f(\theta) d\theta, \end{aligned}$$

where the second equality follows from the application of the Envelope Theorem. Evaluating this at $\underline{q} = \tilde{q}^M$ and using the fact that $\tilde{d}^M = d^M(\tilde{q}^M)$ yields

$$\begin{aligned} \left. \frac{\partial \check{W}}{\partial \underline{q}} \right|_{\underline{q}=\tilde{q}^M} &= \underbrace{\frac{\partial \tilde{\Pi}}{\partial d}(\tilde{q}^M, \tilde{d}^M)}_{=0} + \\ &\quad \int_{\underline{\theta}}^{\tilde{\theta}(\tilde{q}^M, \tilde{d}^M)} \left[-\theta \tilde{x}(\theta, \tilde{q}^M, \tilde{d}^M) \left. \frac{\partial d^M}{\partial q} \right|_{q=\tilde{q}^M} + \frac{\partial V}{\partial q}(\tilde{x}(\theta, \tilde{q}^M, \tilde{d}^M), \tilde{q}^M) \right] f(\theta) d\theta \end{aligned}$$

⁵⁵This follows from the fact that $\tilde{\Pi}(q, d^M(q))$ is continuous and decreasing in q over $[\tilde{q}^M, +\infty)$ (because it is quasi-concave in q and reaches its maximum at $q = \tilde{q}^M$).

or, equivalently,

$$\begin{aligned} \frac{\partial \check{W}}{\partial q} \Big|_{q=\tilde{q}^M} &= \underbrace{\int_{\underline{\theta}}^{\tilde{\theta}(\tilde{q}^M, \tilde{d}^M)} \frac{\partial V}{\partial q} \left(\tilde{x} \left(\theta, \tilde{q}^M, \tilde{d}^M \right), \tilde{q}^M \right) f(\theta) d\theta}_{\text{direct effect}} \\ &\quad - \underbrace{\int_{\underline{\theta}}^{\tilde{\theta}(\tilde{q}^M, \tilde{d}^M)} \theta \tilde{x} \left(\theta, \tilde{q}^M, \tilde{d}^M \right) \frac{\partial d^M}{\partial q} \Big|_{q=\tilde{q}^M} f(\theta) d\theta}_{\text{indirect effect}}. \end{aligned}$$

This shows that a (marginal) increase in the quality level starting from the unregulated level has two effects: a *direct* effect on the value of the service for consumers (keeping the disclosure level constant), and an *indirect* effect capturing how an increase in the quality level alters the firm's choice of disclosure level. The direct effect is always positive, while the sign of the indirect effect depends on whether the firm's optimal disclosure level increases or decreases in response to an increase in quality. This indirect effect is weakly positive if the firm weakly decreases its disclosure level when quality level is increased (starting from $q = \tilde{q}^M$), i.e., if

$$\frac{\partial d^M}{\partial q} \Big|_{q=\tilde{q}^M} \leq 0.$$

In this case, the overall effect of a marginal increase in quality (starting from the unregulated level) on social welfare is unambiguously positive, which implies that it is strictly socially desirable to set a minimum quality requirement (under our regularity conditions). However, if

$$\frac{\partial d^M}{\partial q} \Big|_{q=\tilde{q}^M} > 0,$$

the indirect effect is negative and, therefore, the overall effect of a minimum quality requirement is *a priori* ambiguous. The following lemma relates the effect of a change in quality level on the firm's optimal disclosure level to the cross-effect of quality and disclosure on the firm's profit.

Lemma 7 (*Effect of the quality level on disclosure*) *If $d^M(q) \in (0, 1)$, then*

$$\frac{\partial d^M}{\partial q} = - \frac{\frac{\partial^2 \tilde{\Pi}}{\partial q \partial d} (q, d^M(q))}{\frac{\partial^2 \tilde{\Pi}}{\partial d^2} (q, d^M(q))}.$$

Proof. Similar to the proof of Lemma 5. ■

From Lemma 7, we see that the effect of a change in disclosure level on the firm's choice of quality has the same sign as the cross-effect of quality and disclosure on the firm's profit.⁵⁶

We now study the sign of the effect of a change in quality level on disclosure, which in turn determines the sign of the indirect effect of a minimum quality level.

We first focus on the scenario in which the market is fully covered, and then turn to the scenario in which the market is partially covered.

C.1 Full market coverage

Suppose that $\tilde{\theta}(q, d^M(q)) = \bar{\theta}$ for any $q \in [0, \bar{q}]$. Under this assumption, the market is fully covered whatever the regulator's decision in the first stage of the game.

Consider the firm's optimal choice of disclosure for a given level of quality. The firm's marginal benefit from increasing disclosure is given by (4), with the term capturing the effect of disclosure on the extensive margin effect equal to zero (due to full market coverage). Under the assumption that the firm's optimal choice of disclosure level, $d^M(q)$, is interior for any quality level q (in the relevant range), which we make in this extension, Lemma 7 implies that

$$\frac{\partial d^M}{\partial q} = -\frac{\frac{\partial^2 \tilde{\Pi}}{\partial q \partial d}(q, d^M(q))}{\frac{\partial^2 \tilde{\Pi}}{\partial d^2}(q, d^M(q))} = -\frac{r\gamma}{\frac{\partial^2 \tilde{\Pi}}{\partial d^2}(q, d^M(q))} > 0.$$

Therefore, the firm's optimal level of quality is increasing in the level of disclosure. This leads to the following result.

Proposition 7 (*Effect of an ex ante minimum quality requirement on disclosure under full market coverage*) *When the market is fully covered, a binding minimum quality requirement $\underline{q} \in (\tilde{q}^M, \bar{q}]$ leads to an increase in the disclosure level chosen by the firm.*

Let us now study the social desirability of a minimum quality requirement. Recall that it is strictly optimal for the regulator to set such a requirement if and only if $\left. \frac{\partial \tilde{W}}{\partial q} \right|_{q=\underline{q}^M} > 0$. The social marginal benefit of increasing the quality level, evaluated at

⁵⁶We use the fact that $\frac{\partial^2 \tilde{\Pi}}{\partial d^2}(q, d^M(q)) < 0$, which is given by the second-order condition of the maximization of $\tilde{\Pi}(q, d)$ with respect to d .

$\underline{q} = \tilde{q}^M$, is

$$\frac{\partial \check{W}}{\partial \underline{q}} \Big|_{\underline{q}=\tilde{q}^M} = \int_{\underline{\theta}}^{\tilde{\theta}(\tilde{q}^M, \tilde{d}^M)} \frac{\partial V}{\partial q} \left(\tilde{x} \left(\theta, \tilde{q}^M, \tilde{d}^M \right), \tilde{q}^M \right) f(\theta) d\theta - \int_{\underline{\theta}}^{\tilde{\theta}(\tilde{q}^M, \tilde{d}^M)} \theta \tilde{x} \left(\theta, \tilde{q}^M, \tilde{d}^M \right) \frac{\partial d^M}{\partial q} \Big|_{q=\tilde{q}^M} f(\theta) d\theta.$$

Let

$$\check{\gamma} = \sup \left\{ \gamma' > 0 \mid \frac{\partial \check{W}}{\partial q} \Big|_{q=\tilde{q}^M} > 0 \text{ for all } \gamma < \gamma' \right\}.$$

Using the fact that $\frac{\partial \check{W}}{\partial \underline{q}} \Big|_{\underline{q}=\tilde{q}^M}$ is continuous in γ and (strictly) positive when $\gamma \rightarrow 0$, it follows that $\check{\gamma}$ is well defined in $\mathbb{R}_{++} \cup \{+\infty\}$. By definition of $\check{\gamma}$, we obtain that setting a minimum quality requirement is socially desirable when $\gamma < \check{\gamma}$. Thus, a minimum quality requirement is socially desirable whenever the complementarity between quality and information is not too strong. When quality and information are sufficiently strong complements (i.e., $\gamma > \check{\gamma}$), the sign of $\frac{\partial \check{W}}{\partial q} \Big|_{q=\tilde{q}^M}$ is ambiguous and, therefore, so is the impact of a minimum quality requirement on social welfare. The following proposition summarizes the above analysis.

Proposition 8 (*Social desirability of an ex ante minimum quality requirement under full market coverage*) *When the market is fully covered, an ex ante regulation taking the form of a minimum quality requirement is socially desirable if the complementarity between quality and information is not too strong (i.e., $\gamma < \check{\gamma}$). Otherwise, such a regulation may be not be socially desirable.*

C.2 Partial market coverage

We now assume that $\tilde{\theta}(q, d^M(q)) < \bar{\theta}$ for any $q \in [0, \bar{q}]$. Under this assumption, the market is only partially covered whatever the regulator's decision in the first stage of the game. The sign of the effect of a higher quality on the firm's optimal disclosure level is given by the sign of the cross-effect $\frac{\partial^2 \tilde{\Pi}}{\partial q \partial d}$, which has already been studied in the analysis of the disclosure regulation. Therefore, we get the following result which is the counterpart of Proposition 3 when the authority regulates quality instead of privacy.

Proposition 9 (*Effect of an ex ante minimum quality requirement on disclosure under partial market coverage*) *Assume that the market is partially covered.*

- *If $F(\cdot)$ is weakly convex, a minimum quality requirement $\underline{q} \in (\tilde{q}^M, \bar{q}]$ leads to a lower disclosure level.*

- If $F(\cdot)$ is concave and relatively inelastic, a minimum quality requirement $\underline{q} \in (\tilde{q}^M, \bar{q}]$ leads to a higher disclosure level.
- If $F(\cdot)$ is concave and relatively elastic, the effect of a minimum quality requirement $\underline{q} \in (\tilde{q}^M, \bar{q}]$ is negative if quality and information are weak complements, and is ambiguous otherwise.

Combining Proposition 9 with the fact that the direct effect of a minimum quality requirement on social welfare is always positive, we arrive at the following result.

Proposition 10 (*Social desirability of an ex ante minimum quality requirement under partial market coverage*) Assume that the market is partially covered.

- If the distribution of the idiosyncratic privacy cost exhibits a weakly increasing density function (i.e., $F(\cdot)$ is weakly convex), a regulation taking the form of a binding minimum quality requirement is strictly socially desirable.
- If the distribution of the idiosyncratic privacy cost exhibits a decreasing density function (i.e., $F(\cdot)$ is concave), a regulation taking the form of a binding minimum quality requirement may not be socially desirable.

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