Entry-Proofness and Market Breakdown under Adverse Selection*

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Abstract

We provide a necessary and sufficient condition for entry to be unprofitable under adverse selection; namely, that no buyer type be willing to trade at a price above the expected unit cost of serving the types who are at least as eager to trade than her. We give two applications of this result. First, we clarify the circumstances under which adverse selection causes market breakdown. Second, we consider nonexclusive active markets in which buyers can simultaneously trade on the market and with an entrant, and we fully characterize the unique entry-proof market tariff. By emphasizing the formal analogy between these two situations, our general approach offers a unified perspective on entry-proofness in adversely selected markets. We argue that estimates of upper-tail conditional expectations of unit costs are a key variable for tests of adverse selection, and we outline such a test in nonexclusive insurance markets.

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1 Introduction

It has long been recognized that markets subject to adverse selection can unravel to a no-trade equilibrium. As shown by Akerlof (1970), this can occur even if trade would always be mutually beneficial, if only the quality of the goods for sale were commonly known. This failure of the price mechanism has recently been used to interpret phenomena such as liquidity and credit freezes (Philippon and Skreta (2012), Tirole (2012)) or insurance rejections (Hendren (2013)).

The logic of market unraveling is easy to illustrate for an inactive insurance market in which no contract is initially offered. An entrant may offer a single contract consisting of a small amount of coverage at some given unit price. Under adverse selection, the consumer types who are the most eager to trade this contract are the riskiest ones. Thus the entrant's offer attracts all consumers with risk above some threshold, and the corresponding expected cost to him can be computed as the upper-tail expectation of their risks. As a result, a necessary condition for entry to be unprofitable is that, for any consumer in a risk class, her willingness to pay at the no-trade point does not exceed this cost.

Two features of this reasoning are worth emphasizing. First, it remains to clarify when entry by a *menu* of contracts is not profitable under the stated condition.[1] Second, the nonexistence of profitable trades is only defined relative to an empty set of contract offers. Yet it is natural to ask when an entrant cannot propose profitable trades that *complement* a nonempty set of contract offers. This is relevant for a wide range of nonexclusive financial and insurance markets in which a seller cannot prevent a buyer from making additional trades with his competitors.[2]

The primary goal of this paper is to propose a unified perspective on entry-proofness in adversely selected markets that applies both to inactive markets in which no contracts are offered save for those offered by the entrant, and to nonexclusive active markets in which some contracts are offered and can be traded along with those offered by the entrant. We show that these two situations are two sides of the same coin, and that the same conceptual

[1] There can be no loss of revenue for an entrant in offering a menu of contracts, rather than an arbitrary selling mechanism. Indeed, the Revelation Principle (Myerson (1979, 1982)) tells us that there is no loss of generality in restricting the entrant to offer an incentive-compatible direct-revelation mechanism, and the Taxation Principle (Hammond (1979), Guesnerie (1981), Rochet (1985)) in turn guarantees that the outcome of any such mechanism can be reproduced through a well-chosen tariff. This holds irrespective of the contracts already offered on the market.

[2] Attar, Mariotti, and Salanié (2014, 2016a) document how individuals can, and indeed do, engage in multiple trades in a large number of financial and insurance markets. In that respect, it is worth noting that the three markets empirically studied by Hendren (2013)—long-term care, disability, and life insurance—display some degree of nonexclusivity.

tools that are relevant for the case of inactive markets can be used to understand the case of nonexclusive active markets.

To this end, we consider a general adverse-selection environment in which little structure is put on the buyers' preferences beyond a single-crossing condition. Our model encompasses insurance economies—yet without assuming that consumers' preferences have an expected-utility representation, nor that there are only two possible loss levels—as well as standard trade environments, with and without wealth effects. Besides, the assumptions of the model are weak enough as to apply to buyers in a nonexclusive market, and whose preferences, therefore, incorporate the possibility of making additional trades among a set of existing contract offers.

Our main conceptual tool, Theorem 1, states a necessary and sufficient condition for the absence of profitable trade in an inactive market. This entry-proofness (EP) condition is a nonparametric version of the market-unraveling condition first formulated by Akerlof (1970) and recently extended by Hendren (2013) to the case of a Rothschild and Stiglitz (1976) insurance economy. As noticed above, the necessity of Condition EP for entry-proofness is straightforward and only requires the use of single-contract offers. By contrast, its sufficiency must account for the possibility for an entrant to offer a menu of contracts. We provide an original assumption on buyers' preferences under which entry with any menu of contracts is unprofitable as soon as entry with any single contract is unprofitable. Specifically, it requires that a positive endowment of the good for trade must not increase buyers' willingness to pay relative to the no-trade situation. This mild and intuitive assumption, which trivially holds in the insurance setting considered by Hendren (2013), holds for many preference relations, including various forms of nonexpected utility, as well as in many trade environments beyond insurance. Together with Condition EP, this assumption ensures that the entrant's revenue on each marginal quantity is less than the upper-tail expectation of the unit cost of serving the buyers who purchase this quantity; as a result, the entrant's expected profit is nonpositive, even if he can offer arbitrary menus of contracts. Finally, we show that, in spite of its mildness, this assumption is tight: we provide an example of an economy where it does not hold, and in which entry with a menu of contracts is profitable even though Condition EP is satisfied.

We provide two applications of this result.

First, we clarify the conditions under which adverse selection causes market breakdown. Market breakdown is a slightly more demanding property than unprofitable entry, as it requires that no buyer type (except perhaps the highest one) can trade with the entrant upon pain for the latter to suffer a loss. Corollary 1 states that Condition EP, together with our additional assumption on the buyer's preferences, is necessary and sufficient for market breakdown if buyers' preferences are strictly convex (except perhaps for the highest type) and satisfy the strict single-crossing condition.

Second, we consider nonexclusive markets. Nonexclusivity makes screening more difficult, as the aggregate quantity purchased by a buyer is unobservable. Sellers in nonexclusive markets then typically find it optimal to limit the maximum quantity they stand ready to trade at any given price, so as to cut their losses on the costliest buyer types. In this situation, which institutionally corresponds to the actual working of limit-order markets, buyers face a convex market tariff, resulting from the convolution of the sellers' individual tariffs. We say that such a tariff is *entry-proof* if it prevents any profitable entry by a seller whose menu offer complements the tariff, in the sense that buyers are free to combine any contract included in the entrant's menu with any trade along the tariff.

Our general approach delivers a full characterization of entry-proof convex market tariffs. We first observe that, from an entrant's viewpoint, everything happens as if he were facing buyers whose preferences, for any given trade with the entrant, are represented by indirect utility functions incorporating their optimal trades along the tariff. Convexity of the latter guarantees that these indirect utility functions inherit the regularity properties of the buyers' primitive utility functions required for the application of Theorem 1: having factored all existing trade opportunities into the buyers' preferences, we are back to the case of an inactive market from the entrant's viewpoint. It is then a direct implication of Theorem 1 that a convex tariff is entry-proof if and only if each buyer type's indirect willingness to pay is at most equal to the corresponding upper-tail expectation of unit costs. There remains to translate this abstract property into a more transparent characterization of the tariff. In Corollaries 2 and 3, we show that it singles out a unique budget-feasible allocation implemented by an entry-proof convex market tariff, and essentially a unique such tariff. This allocation and this tariff are competitive in the sense that each marginal quantity is priced at the expected cost of serving the buyer types who purchase it, and each buyer type can purchase her optimal marginal demand at that price. When preferences are linear, these competitive features lead to Akerlof (1970) pricing and an allocation corresponding to the Akerlof (1970) equilibrium that maximizes gains from trade. When preferences are strictly convex, they lead to a marginal version of Akerlof (1970) pricing and an allocation generalizing those highlighted, in specific contexts, by Jaynes (1978), Hellwig (1988), and Glosten (1994).

Our results pave the way to new tests of adverse selection, exploiting the idea that firms' costs under adverse selection depend on which consumers purchase their products. Using a simple binary-loss insurance example, we outline a nonparametric test of the consistency of coverage and premia choices with the competitive tariff. The testing procedure amounts to comparing the price of successive layers of insurance to their average cost, as measured by the empirical loss frequency of serving the consumers who purchase them. This test extends those developed by Einav, Finkelstein, and Cullen (2010) to richer environments where firms offer insurance tariffs and consumers can choose different levels of coverage, and those proposed by Hendren (2013) in the case of inactive markets. Our analysis suggests that estimates of upper-tail conditional expectations of unit costs should be a key variable for future tests of adverse selection in nonexclusive insurance markets.

Contribution to the Literature Our treatment of entry-proofness and market breakdown in inactive markets generalizes results obtained in the quasilinear case by Akerlof (1970), Glosten (1994), and Mailath and Nöldeke (2008), and in the case of a Rothschild and Stiglitz (1976) economy by Hendren (2013). Our contribution is to precisely spell out a mild and intuitive assumption on the buyers' preferences that ensures that we can dispense with quasilinearity or expected utility when checking that entry with a menu of contracts is unprofitable under Condition EP. Indeed, a strength or our general approach consists in providing a comprehensive yet elementary proof of this fact, which may be of interest for pedagogical purposes.

Our analysis of active markets does not cover exclusive markets. There are two reasons for this. First, entry-proofness in exclusive markets has been well understood since the seminal work of Rothschild and Stiglitz (1976): the unique entry-proof allocation under exclusivity is the Riley (1979) outcome, characterized by the absence of cross subsidies between types and downward-binding local incentive-compatibility constraints. Second, and more importantly, whereas the methodology that consists in evaluating a buyer's trades with an entrant through an indirect utility function is also valid for exclusive markets, in that case, the resulting indirect utility function has an upward discontinuity at the no-trade point; for, at this point, the buyer ceases to trade with the entrant and rather relies on her market outside option. This discontinuity at the origin in turn implies that the buyers' indirect utility functions do not inherit the single-crossing property from their primitive utility functions. Theorem 1 thus does not apply when competition is exclusive. This result is thus useful for the analysis of active markets only insofar as trades with an entrant are understood as complementing, rather than substituting, a set of existing contract offers.

The unique allocation that survives entry in a nonexclusive market in which supply is described by a convex tariff corresponds to the allocations characterized by Attar, Mariotti, and Salanié (2011) in the case of linear preferences, and by Jaynes (1978), Hellwig (1988), Glosten (1994), and Attar, Mariotti, and Salanié (2014, 2016a, 2016b) in the case of strictly convex preferences. While most of these papers attempt at providing a strategic foundation for this allocation, we follow Glosten (1994) in using free-entry arguments. Our contribution here consists in emphasizing the link between no-trade results in the spirit of Akerlof (1970) and the construction of an entry-proof tariff in a nonexclusive market. This duality obtains by reasoning in terms of indirect utility, an approach not followed by Glosten (1994) and which allows for a unified treatment of linear and strictly convex preferences. In the latter case, our results significantly extend those of Glosten (1994) by showing that quasilinearity of the buyers' preferences is in no way necessary. This suggests a direct way to evaluate the impact of adverse selection in nonexclusive financial and insurance markets, where wealth effects are likely to be significant.

Although an empirical illustration is beyond the scope of this paper, our results suggest new avenues for testing the presence of adverse selection. The empirical procedure we outline differs from the positive-correlation test of Chiappori and Salanié (2000) by making essential use of price and cost data, as in Einav, Finkelstein, and Cullen (2010) and Hendren (2013). In contrast with these papers, this procedure is specifically designed for nonexclusive markets, for which it provides a joint test of adverse selection and entry-proofness. While such markets have so far been investigated through the lens of exclusive-competition models, as in Cawley and Philipson's (1999) study of the US life-insurance market, our analysis is a first step towards a new generation of tests explicitly taking nonexclusive contracting into account.

The paper is organized as follows. Section 2 describes the model. Section 3 states our main result. Section 4 draws its implications for market breakdown. Section 5 extends our analysis to nonexclusive markets. Section 6 concludes.

2 The Economy

Consider a buyer (she) endowed with private information, and whose type i can take a finite number I of values with positive probabilities m_i.[3] Type i's preferences are represented by a utility function $u_i(q,t)$ that is continuous and quasiconcave in (q,t) and strictly decreasing

[3]The case of a seller can be handled thanks to a simple change of variables. In Appendix B, we prove that our results extend to arbitrary type distributions with bounded support over the real line. We refer to Mailath and Nöldeke (2008) for an exploration of the unbounded-support case.

in t, with the interpretation that q is the nonnegative quantity she buys and t is the payment she makes in return. Types are ordered according to the single-crossing condition (Milgrom and Shannon (1994)), which states that higher types are at least as eager to increase their purchases than lower types are:

$$\text{For all } i < j,\ q < q',\ t, \text{ and } t',\ u_i(q,t) \leq (<)\, u_i(q',t') \text{ implies } u_j(q,t) \leq (<)\, u_j(q',t').$$

To define marginal rates of substitution without assuming differentiability, let $\tau_i(q,t)$ be the supremum of the set of prices p such that

$$u_i(q,t) < \max\{u_i(q+q', t+pq') : q' \geq 0\}.$$

Thus $\tau_i(q,t)$ is the slope of type i's indifference curve at the right of (q,t). Quasiconcavity ensures that $\tau_i(q,t)$ is finite, except possibly when $q = 0$, and that it is nonincreasing along an indifference curve of type i. We additionally make the intuitive assumption that, in the absence of transfers, a positive endowment of q reduces this marginal rate of substitution.

Assumption 1 *For all i and $q > 0$, $\tau_i(q,0) \leq \tau_i(0,0)$.*

Our assumptions on the buyer's preferences hold in a Rothschild and Stiglitz (1976) insurance economy, which is the case studied by Hendren (2013); then i measures the buyer's riskiness, q is the amount of coverage she purchases, and t is the premium she pays in return. As we illustrate in Appendix C, they also hold under many alternative specifications, allowing for multiple loss levels or various forms of nonexpected utility. Finally, they encompass a broad variety of other applications, such as financial and labor markets. It should be noted that we do not require strict single crossing nor strict convexity of preferences. This choice is not motivated by an idle desire for generality. Rather, it is meant to pave the way to the analysis of nonexclusive markets provided in Section 5.

The supply side of the economy is represented by a linear technology, with unit cost $c_i > 0$ when the buyer's type is i. For each i, we denote by $\overline{c}_i$ the upper-tail conditional expectation of unit costs,

$$\overline{c}_i \equiv \mathbf{E}[c_j \,|\, j \geq i] = \frac{\sum_{j \geq i} m_j c_j}{\sum_{j \geq i} m_j}.$$

In order to highlight the role of these expectations, we only require that $\overline{c}_i$ be nondecreasing in i. This is slightly more general than the usual definition of adverse selection, which states that unit costs c_i are nondecreasing in i. In fact, the monotonicity of $\overline{c}_i$ is exactly equivalent to the following condition, which we state for further reference:

$$\text{For all } j < i,\ c_j \leq \overline{c}_i. \tag{1}$$

3 Entry-Proofness in Inactive Markets

We say that a market is *inactive* when, for whichever reason, no contracts are offered, so that each type i has reservation utility $u_i(0,0)$. The question we address in this context is under which conditions an entrant (he) can profitably trade with the buyer.

Let us first analyze the case where the entrant offers a single contract, designed so as to attract a given type i. To do so, the entrant can choose some unit price p slightly below $\tau_i(0,0)$. Then, by definition of $\tau_i(0,0)$, there exists a quantity q that strictly attracts type i at this price, that is, $u_i(q,pq) > u_i(0,0)$. As types are ordered according to the single-crossing condition, we also have $u_j(q,pq) > u_j(0,0)$ for all $j > i$. Thus any type $j > i$ is strictly attracted as well, and the entrant bears an expected unit cost $\overline{c}_i$ when trading with types $j \geq i$. Finally, some types $j < i$ may also be attracted, but (1) ensures that this can only reduce the entrant's expected unit cost. This simple reasoning shows that the following condition is necessary for entry to be unprofitable.

Condition EP *For each i, $\tau_i(0,0) \leq \overline{c}_i$.*

Now, let us turn to the case where the entrant can offer a menu of contracts so as to screen the different buyer types. We say that an inactive market is *entry-proof* if *there is no menu of contracts that yields the entrant a strictly positive expected profit no matter the buyer's best response.* We already know from the above reasoning that Condition EP is necessary for an inactive market to be entry-proof. The following result, for which a formal proof is provided in Appendix A, shows that it is also sufficient.

Theorem 1 *Under Assumption 1, an inactive market is entry-proof if and only if Condition EP is satisfied.*

The key to the proof lies in the following remark. Suppose that the entrant offers an arbitrary menu of contracts. Under single crossing, a standard monotone-comparative-statics argument implies that the buyer has a best response with nondecreasing quantities; that is, the entrant ends up trading (q_i, t_i) with each type i, with $q_i \leq q_j$ for all $i < j$. Then his expected profit is

$$\sum_i m_i(t_i - c_i q_i),$$

which, using a summation by parts in the spirit of Wilson (1993), can be rewritten as:

$$\sum_i \left(\sum_{j \geq i} m_j\right)[t_i - t_{i-1} - \overline{c}_i(q_i - q_{i-1})], \tag{2}$$

with $(q_0, t_0) \equiv (0, 0)$ by convention. Now, observe that, because type i is willing to trade $(q_i - q_{i-1}, t_i - t_{i-1})$ in addition to (q_{i-1}, t_{i-1}), each bracketed term in (2) cannot exceed $[\tau_i(q_{i-1}, t_{i-1}) - \overline{c}_i](q_i - q_{i-1})$. Thus, to prove that the entrant's expected profit is nonpositive, we only need to show that

$$\text{For each } i,\ \tau_i(q_{i-1}, t_{i-1}) \leq \overline{c}_i. \tag{3}$$

To see why this holds, assume that (q_{i-1}, t_{i-1}) lies in the nonnegative orthant, below the indifference curve of type i that goes through the origin. (This is shown in Appendix A.) As illustrated in Figure 1, the concavity of the indifference curve of type i that goes through (q_{i-1}, t_{i-1}), coupled with Assumption 1, then implies $\tau_i(q_{i-1}, t_{i-1}) \leq \tau_i(0, 0)$, from which (3) follows according to Condition EP. This shows the result.

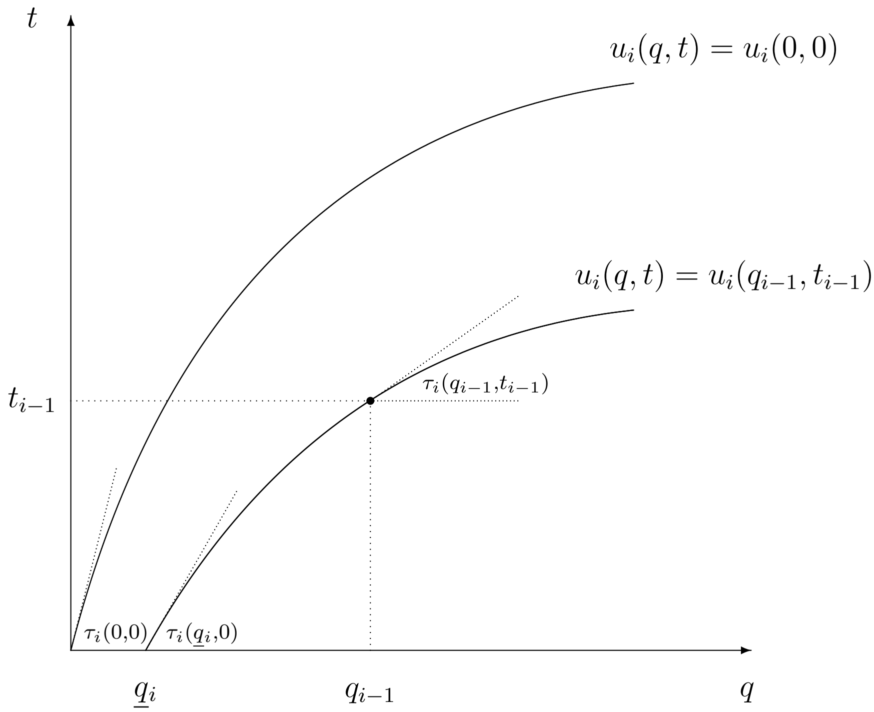

Figure 1 A graphical illustration of (3).

Although seemingly innocuous, Assumption 1 plays an important role in the proof of Theorem 1. Indeed, we show in Appendix D that, when it does not hold, we can construct examples in which entry with a menu of contracts is profitable even though Condition EP is satisfied.[4]

[4]The intuition is that the marginal rate of substitution can then take values higher than $\tau_i(0, 0)$ in the relevant area illustrated in Figure 1. Relatedly, strengthening Assumption 1 into $\tau_i(q, t) \leq \tau_i(0, 0)$ for all (q, t) such that $t \geq 0$ and $u_i(q, t) \geq u_i(0, 0)$ still allows to prove Theorem 1, even when preferences are not convex.

Overall, the main insight of Theorem 1 is that the upper-tail conditional expectations of unit costs play a key role when evaluating whether an inactive market is entry-proof. In that respect, everything happens as if information were complete, with each c_i turned into $\overline{c}_i$. It should be noted that Condition EP does not rule out gains from trade, in the usual first-best sense of the term. Indeed, it may well be that $\overline{c}_i \geq \tau_i(0,0) > c_i$ for any type $i < I$.[5] Rather, as the proof of Theorem 1 makes clear, what Condition EP, together with Assumption 1, rules out are gains from trade on any quantity layer $q_i - q_{i-1}$, which is all that matters for entry to be unprofitable in the second-best case where the buyer's type is unknown to the entrant.[6]

4 Market Breakdown

We have so far characterized when an inactive market is entry-proof, that is, when no seller can enter the market and earn a strictly positive expected profit no matter the buyer's best response. The existing literature often focuses instead on characterizing market breakdown, defined as a situation in which *any trade that the buyer is willing to make is loss-making, even when her best response is favorable to the entrant.* We here provide an additional result for this stronger concept. Note that, in any case, Condition EP remains necessary. However, other assumptions need to be reinforced to obtain sufficiency.

The first difficulty is that we may design menus of contracts for which the buyer has multiple best responses, some of which may be more favorable to the entrant than others. This difficulty can be overcome by requiring that types be ordered according to the strict single-crossing condition (Milgrom and Shannon (1994)), which states that higher types are strictly more eager to increase their purchases than lower types are:[7]

$$\text{For all } i < j,\ q < q',\ t, \text{ and } t',\ u_i(q,t) \leq u_i(q',t') \text{ implies } u_j(q,t) < u_j(q',t').$$

The following example illustrates that zero-expected-profit entry can take place when strict single crossing does not hold, even though Condition EP is strictly satisfied.

Example 1 Consider a two-type economy in which both types have the same preferences represented by $u(q,t) \equiv q - q^2 - t$, but different costs such that $c_1 < 1 < \overline{c}_1 < c_2$; thus

[5] Of course, this cannot be true of the highest type I, for which $c_I = \overline{c}_I$.

[6] One reason why the generality of the role of upper-tail conditional expectations of unit costs seems to have been overlooked in the literature is that, although they determine whether a monopolist's expected profit is positive or not, their values have no impact on the solution to the monopoly problem. We illustrate this point in Appendix E in the simple case of differentiable quasilinear utilities.

[7] See Footnote 16 for the corresponding modification of the proof of Theorem 1.

Condition EP is strictly satisfied. Both types are indifferent between not trading and trading the quantity $1-c_1$ at unit price c_1. An entrant offering the contract $(1-c_1, c_1(1-c_1))$ earns zero expected profit if type 1 accepts, and type 2 chooses not to trade with him.

Even under strict single crossing, it is still possible that the expected profit be exactly zero on any quantity layer $q_i - q_{i-1}$. A simple and natural way to rule out this knife-edged situation is to assume that the buyer's preferences are strictly convex. Indeed, under this additional assumption, inequalities (3), together with the downward local constraints $u_i(q_i, t_i) \geq u_i(q_{i-1}, t_{i-1})$, directly imply that the expected profit from any quantity layer $q_i - q_{i-1}$ is strictly negative whenever $q_{i-1} < q_i$. The following example illustrates that zero-expected-profit entry can take place when strict quasiconcavity does not hold, even though strict single crossing holds and Condition EP is satisfied.

Example 2 Consider, in line with Samuelson (1984), Myerson (1985), and Attar, Mariotti, and Salanié (2011), an economy in which a divisible good is traded, subject to a capacity constraint $q \in [0, \overline{q}]$. Each type i has linear preferences represented by $u_i(q,t) \equiv \overline{c}_i q - t$, where $\overline{c}_i$ is strictly increasing in i. Under this highly nongeneric assumption, strict single crossing holds and Condition EP is satisfied with equality for each type. Suppose now that the entrant offers a menu of contracts $\{(q_1, t_1), \ldots, (q_I, t_I)\}$ with strictly positive quantities q_i that are nondecreasing in i, and transfers t_i such that $t_i - t_{i-1} = \overline{c}_i(q_i - q_{i-1})$. Any such allocation yields zero expected profit for the entrant and exhibits strict gains from trade for types $i > 1$. The intuition is that Condition EP rules out gains from trade for any type i on the quantity layer $q_i - q_{i-1}$ but not necessarily, for $i > 1$, on the inframarginal quantity layers $q_j - q_{j-1}$, $j < i$. Hence, whereas strictly profitable entry is ruled out by Theorem 1, zero-expected-profit entry is possible, in many different ways, if each type i accepts to trade (q_i, t_i), even though she could as well choose to trade (q_{i-1}, t_{i-1}), with $(q_0, t_0) \equiv (0, 0)$ by convention.

Summarizing, we obtain the following result.

Corollary 1 *Suppose that, for each i, u_i is strictly quasiconcave, that Assumption 1 holds, and that types are ordered according to the strict single-crossing condition. Then there is market breakdown if and only if Condition EP is satisfied.*

Mailath and Nöldeke (2008) obtain a related result for an economy in which the buyer has quadratic quasilinear preferences, as in Glosten (1989) or Biais, Martimort, and Rochet (2000). However, they focus on competitive pricing, defined as a situation in which each quantity traded must yield zero expected profit, ruling out cross-subsidies between contracts. This is an important restriction: indeed, as Example 2 illustrates, the best pricing strategy for the entrant may not be competitive in this sense, because what matters is not the absolute profit earned on each quantity q_i, but rather the marginal profit earned on any quantity layer $q_i - q_{i-1}$.

Hendren (2013) studies a Rothschild and Stiglitz (1976) insurance economy, and his Theorem 1 is the analogue of Corollary 1 in this particular setting. As emphasized by the author, an implication of Condition EP is that the highest-risk type I must not be willing to purchase coverage at the actuarially fair rate c_I. Given that her preferences have an expected-utility representation, this is possible only if $c_I = 1$, that is, if type I incurs a loss with probability 1. In that case, type I's preferences are no longer strictly convex, and the above result becomes that all types except perhaps type I are excluded from trade.

5 Nonexclusive Markets

A striking feature of Condition EP is that we can also use it to characterize entry-proofness in active markets. For reasons explained in the Introduction, we focus on nonexclusive markets, in which a buyer can trade with several sellers. In such markets, each seller aims at limiting the risk of attracting high-cost types buying large quantities, and to do so can use limit orders—that is, offers to sell at a given unit price up to a maximum quantity. Perhaps for this reason, limit orders are one of the main instruments used on financial markets, and especially so when the market is organized as a limit-order book (Glosten (1994)). For us, the important property that we shall exploit below is that, if sellers post collections of limit orders, or, equivalently, convex tariffs, the buyer faces a convex market tariff T, obtained by convolution of those tariffs. We will use Condition EP to show that requiring that such a tariff be entry-proof singles out a unique budget-feasible allocation, the construction of which crucially hinges on upper-tail conditional expectations of unit costs.

Let us assume without further mention that the domain of the tariff T is a compact interval containing 0, with $T(0) = 0$. Each type i then selects her optimal quantity q_i so as to maximize $u_i(q, T(q))$. We say in that case that the allocation $(q_i, T(q_i))_{i=1}^{I}$ is *implemented* by the tariff T, and that it is *budget-feasible* if

$$\sum_i m_i[T(q_i) - c_i q_i] \geq 0. \tag{4}$$

As for the buyer's preferences, we will consider two polar cases, corresponding to abstract versions of the assumptions made by Akerlof (1970) and Rothschild and Stiglitz (1976). In the first, the utility functions u_i are all linear, that is, $u_i(q,t) \equiv v_i q - t$, subject to a capacity constraint $q \in [0, \overline{q}]$; in that case, given a convex tariff T, some type i may have an interval of optimal quantities. In the second, the utility functions u_i are all strictly quasiconcave; hence, given a convex tariff T, each type i now has a unique optimal quantity. In any case, we shall assume that the utility functions u_i are ordered according to the strict single-crossing condition, so that the optimal quantities q_i in any allocation implemented by the tariff T are nondecreasing in i.

Now, suppose an entrant can propose side trades to the buyer, in the form of a menu of contracts. We say that the tariff T is *entry-proof* if *there is no menu of contracts that yields the entrant a strictly positive expected profit no matter the buyer's best response, taking into account that she is free to combine these contracts with those made available by the tariff* T. Our goal is to characterize the set of budget-feasible allocations that are implementable by entry-proof convex market tariffs.

Let us first observe that, from the entrant's viewpoint, everything happens as if he were facing modified types with indirect utility functions

$$u_i^T(q', t') \equiv \max\{u_i(q + q', T(q) + t') : q\}, \tag{5}$$

reflecting that the buyer is free to combine any contract (q', t') with those made available by the tariff T.[8] In particular, $u_i^T(0,0)$ represents type i's utility when she only trades on the market and not with the entrant, and thus defines the relevant individual-rationality constraint for type i from the entrant's viewpoint.

A couple of observations are worth making at this stage. First, because the tariff T is continuous over a compact domain and the admissibility constraint implicit in (5) is continuous in (q', t'), the maximum in (5) is always attained and $u_i^T(q', t')$ is continuous in (q', t').[9] Second, because the tariff T is convex and the primitive utility functions $u_i(q,t)$ are quasiconcave in (q,t) and strictly decreasing in t, the indirect utility functions $u_i^T(q', t')$ are quasiconcave in (q', t') and strictly decreasing in t'. As a result, we can define the marginal rates of substitution $\tau_i^T(q', t')$ associated to them exactly as we did in Section 2 for the primitive utility functions. Third, because the primitive types are ordered according to the strict single-crossing condition, the modified types are ordered according to the single-

[8]In the case of strictly convex preferences, the only constraint on q is that it belong to the domain of T. In the case of linear preferences, one must add to this the requirement that $q \in [0, \overline{q} - q']$, reflecting that additional trades along the tariff T must be consistent with the buyer's capacity constraint. To simplify notation, and with no risk of confusion, we do not explicitly mention such admissibility constraints in the various maximization problems considered in this section.

[9]This follows from Berge's maximum theorem (Aliprantis and Border (2006, Theorem 17.31)).

crossing condition.[10]

These properties suggest using Theorem 1 so as to characterize the set of budget-feasible allocations that are implementable by entry-proof convex market tariffs. To do so, we need to ensure that Assumption 1 holds for the marginal rates of substitution $\tau_i^T(q',0)$. A convenient way to proceed is to impose that each type's family of primitive indifference curves satisfy a slightly stronger "fanning-out" condition than in Assumption 1.

Assumption 2 *For all i and t, $\tau_i(q,t)$ is nonincreasing in q.*

Assumption 2 is quite weak, and notably holds in the illustrative examples provided in Appendix C. Linear preferences correspond to the limiting case where the marginal rate of substitution is constant over the consumption space, so that Assumption 2 is trivially satisfied. An important implication of Assumption 2 is that, if the buyer faces a convex market tariff, then she does not become strictly more willing to trade a higher quantity along the tariff when her endowment of q increases. In Appendix A, we use this property to establish the following result.

Lemma 1 *Assumption 1 holds for $\tau_i^T(q',0)$ if Assumption 2 holds for $\tau_i(q,t)$.*

Consequently, we can now use Theorem 1, and we obtain that a tariff T is entry-proof if and only if the following holds:

$$\text{For each } i, \ \tau_i^T(0,0) \leq \bar{c}_i. \tag{6}$$

To see what this abstract condition entails for T and the allocation it implements, observe that, according to (5), $\tau_i^T(0,0)$ is the supremum of the set of prices p such that

$$u_i(q_i,T(q_i)) = u_i^T(0,0) < \max\{u_i^T(q',pq') : q'\} = \max\{u_i(q+q',T(q)+pq') : q,q'\}.$$

Thus, according to (6), we have

$$\text{For each } i, \ u_i(q_i,T(q_i)) \geq \max\{u_i(q+q',T(q)+\bar{c}_i q') : q,q'\}. \tag{7}$$

In particular, letting $q = q_{i-1}$ and $q' = q_i - q_{i-1}$ in (7) yields

$$\text{For each } i, \ T(q_i) \leq T(q_{i-1}) + \bar{c}_i(q_i - q_{i-1}), \tag{8}$$

with $q_0 \equiv 0$ by convention. Now, rewriting the expected profit (4) as in (2), and imposing

[10] See Attar, Mariotti, and Salanié (2016b, Appendix) for a proof of the second and third observations.

that the allocation $(q_i, T(q_i))_{i=1}^I$ be budget-feasible, we have

$$\sum_i \left(\sum_{j \geq i} m_j \right) [T(q_i) - T(q_{i-1}) - \overline{c}_i(q_i - q_{i-1})] \geq 0.$$

Thus (8) must hold as an equality,

$$\text{For each } i, \ T(q_i) = T(q_{i-1}) + \overline{c}_i(q_i - q_{i-1}),$$

which, in turn, implies, according to (7),

$$\text{For each } i, \ u_i(q_i, T(q_i)) = \max \{u_i(q_{i-1} + q', T(q_{i-1}) + \overline{c}_i q') : q'\}.$$

The following lemma summarizes this discussion.

Lemma 2 *Any budget-feasible allocation $(q_i, T(q_i))_{i=1}^I$ that is implemented by an entry-proof convex market tariff T satisfies the recursive system defined by $(q_0, T(q_0)) \equiv (0, 0)$ and*

$$q_i - q_{i-1} \in \arg\max \{u_i(q_{i-1} + q', T(q_{i-1}) + \overline{c}_i q') : q'\} \tag{9}$$

$$T(q_i) - T(q_{i-1}) \equiv \overline{c}_i(q_i - q_{i-1}) \tag{10}$$

for all i. In particular, $(q_i, T(q_i))_{i=1}^I$ is budget-balanced.

Lemma 2 gives a characterization of the allocations that are implementable by entry-proof convex market tariffs. What can be said in turn about such tariffs? The following lemma offers a partial answer to this question.

Lemma 3 *Let T be an entry-proof convex market tariff T that implements a budget-feasible allocation $(q_i, T(q_i))_{i=1}^I$. Then T is piecewise linear, and for each i such that $q_i > q_{i-1}$,*

$$\tau_i^T(0, 0) = \overline{c}_i = \partial^- T(q_i) = \frac{T(q_i) - T(q_{i-1})}{q_i - q_{i-1}}. \tag{11}$$

In particular, an entry-proof tariff is competitive in the sense that any marginal quantity $\mathrm{d}q$ is priced at the expected cost of serving the buyer types who purchase it, which can be interpreted as a marginal version of Akerlof (1970) pricing.

To further characterize entry-proof tariffs and the allocations they implement, we now distinguish the two cases considered for the buyer's preferences.

Linear Preferences Suppose first that the utility functions u_i are all linear, subject to a capacity constraint. To avoid discussing nongeneric cases, let us henceforth assume that

$v_i \neq \overline{c}_i$ for all i.[11] If $v_i < \overline{c}_i$ for all i, there is market breakdown. Thus assume that $v_i > \overline{c}_i$ for some i, and let i^* be the smallest such i. Given that the buyer's preferences are linear, it follows from (9)–(10) that any type $i \geq i^*$ trades up to capacity at unit price $\overline{c}_{i^*}$, whereas any type $i < i^*$ does not trade at all at this price. Hence the unique candidate for a budget-feasible allocation that is implementable by an entry-proof convex market tariff is

$$q_i^* \equiv \overline{q}1_{\{i \geq i^*\}}, \tag{12}$$

$$T^*(q_i^*) \equiv \overline{c}_{i^*}\overline{q}1_{\{i \geq i^*\}}, \tag{13}$$

and, according to (11), the corresponding tariff is linear,

$$\text{For each } q \in [0, \overline{q}],\ T^*(q) \equiv \overline{c}_{i^*} q. \tag{14}$$

The allocation (12)–(13) and the tariff (14) correspond to the Akerlof (1970) equilibrium that maximizes gains from trade, and will be henceforth referred to as the *A allocation* and the *A tariff.* This is in line with Attar, Mariotti, and Salanié (2011), who show that the A allocation is the only one consistent with equilibrium in a nonexclusive framework with linear preferences up to a capacity constraint. By construction, we have $\tau_i^{T^*}(0,0) = v_i < \overline{c}_i$ for all $i < i^*$ and $\tau_i^{T^*}(0,0) = \partial^- T^*(\overline{q}) = \overline{c}_{i^*} \leq \overline{c}_i$ for all $i \geq i^*$, so that we can conclude from Theorem 1 that the A tariff is entry-proof. Summarizing, the following result holds.

Corollary 2 *Suppose that, for each i, u_i is linear subject to a capacity constraint, that $v_i \neq \overline{c}_i$ for all i, and that types are ordered according to the strict single-crossing condition. Then the A allocation (12)–(13) is the unique budget-feasible allocation that is implementable by an entry-proof convex market tariff, and this tariff is the A tariff (14).*

Strictly Convex Preferences Suppose next that the utility functions u_i are all strictly quasiconcave. Then (9)–(10) pin down a unique candidate for a budget-feasible allocation that is implementable by an entry-proof convex market tariff, and which is recursively defined by $(q_0^*, T^*(q_0^*)) \equiv (0,0)$ and, for each i,

$$q_i^* - q_{i-1}^* \equiv \arg\max\{u_i(q_{i-1}^* + q', T^*(q_{i-1}^*) + \overline{c}_i q') : q'\}, \tag{15}$$

$$T^*(q_i^*) - T^*(q_{i-1}^*) \equiv \overline{c}_i(q_i^* - q_{i-1}^*). \tag{16}$$

Following Attar, Mariotti, and Salanié (2014, 2016a), we label this allocation, which was originally introduced, in different contexts, by Jaynes (1978), Hellwig (1988), and Glosten (1994), the *JHG allocation.* It is well defined, for instance, when the Inada condition

$$\text{For all } i,\ (q,t), \text{ and } p > 0,\ \arg\max\{u_i(q + q', t + pq') : q'\} < \infty \tag{17}$$

[11] See Example 2 above for an illustration of a nongeneric case.

is satisfied. The JHG allocation is easily shown to be incentive-compatible: the downward local constraints are by construction satisfied, which implies global incentive compatibility as the quantities q_i^* are nondecreasing in i. Finally, according to (11), there exists an essentially unique convex tariff implementing the JHG allocation,[12] namely, the *JHG tariff* defined by

$$T^*(q) \equiv T^*(q_{i-1}^*) + \overline{c}_i(q - q_{i-1}^*) \text{ if } q_{i-1}^* \leq q \leq q_i^*. \tag{18}$$

Thus the JHG tariff consists of a sequence of segments with slopes $\overline{c}_i$, and an upward kink at each quantity $q_i^* > 0$ whenever $\overline{c}_i$ is strictly increasing in i.[13] By construction, we have $\tau_i^{T^*}(0,0) = \partial^- T^*(q_i^*) = \overline{c}_i$ for all i such that $q_i^* > q_{i-1}^*$ and $\tau_i^{T^*}(0,0) = \tau_i(q_i^*, T^*(q_i^*)) \leq \overline{c}_i$ for all i such that $q_i^* = q_{i-1}^*$, so that we can conclude from Theorem 1 that the JHG tariff is entry-proof. Summarizing, the following result holds.

Corollary 3 *Suppose that, for each i, u_i is strictly quasiconcave, that the Inada condition (17) is satisfied and Assumption 2 holds, and that types are ordered according to the strict single-crossing condition. Then the JHG allocation (15)–(16) is the unique budget-feasible allocation that is implementable by an entry-proof convex market tariff, and this tariff is the JHG tariff (18).*

The generality of this result deserves special emphasis. First, the strict single-crossing condition on the primitive utility functions can be relaxed into the standard single-crossing condition, provided we focus on allocations with nondecreasing quantities. Next, the buyer's preferences need not be quasilinear, unlike in Glosten's (1994) original analysis. Instead, we only require that the weaker Assumption 2 hold, according to which a higher quantity traded reduces the buyer's willingness to pay. This intuitive assumption, which is satisfied by a large variety of preference relations, is particularly relevant in the context of nonexclusive markets, where trades with other sellers shift the aggregate quantity purchased by the buyer. This paves the way for applications to health- or life-insurance markets, which are often nonexclusive in practice, and where wealth effects are likely to be significant. By contrast, the assumption that the market tariff is convex plays an important role in our analysis by ensuring that the modified types are ordered according to the single-crossing condition, and seems difficult to relax. A positive result in that direction is provided by Attar, Mariotti, and Salanié (2016a), who prove a result similar to Corollary 3 for general market tariffs; however, their analysis is limited to the two-type case.

Our analysis suggests a fundamental analogy between, on the one hand, the A allocation

[12] We may variously extend the tariff (18) beyond q_I^*, but such changes are inessential.

[13] In the case of the limit-order book, this corresponds to a family of limit orders with quantities $q_i^* - q_{i-1}^*$ and unit prices $\overline{c}_i$.

and the A tariff, and, on the other hand, the JHG allocation and the JHG tariff. Intuitively, in the latter, one can think of each $(q^*_{i-1}, q^*_i]$ as a differentiated market segment. On this segment, each type $j \geq i$ has indirect preferences, given the JHG tariff, that are locally linear: in particular, type i is indifferent between all the trades she could make with the entrant at unit price $\overline{c}_i$, up to capacity $\overline{q}_i \equiv q^*_i - q^*_{i-1}$. Hence types $j < i$ are excluded from trade while types $j \geq i$ trade up to capacity at this price, and the resulting allocation over the market segment $(q^*_{i-1}, q^*_i]$ is the A allocation, with the marginal type i playing the role of type i^* in (12)–(13). The JHG tariff can then be interpreted as a family of A tariffs, indexed by the relevant marginal types. The impossibility of screening more finely over each market segment reflects nonexclusivity, together with the locally linear structure of indirect preferences induced by the market tariff. Conversely, in the limiting case where the buyer's primitive preferences are linear, subject to a capacity constraint, the A allocation can be seen as a degenerate version of the JHG allocation, featuring a single market segment.

A Remark on Exclusivity At this point, the reader may wonder whether our approach can be fruitfully used in the exclusive-competition case considered by Rothschild and Stiglitz (1976). A first difficulty is that, because of exclusivity, indirect utility functions exhibit a discontinuity. Indeed, if the buyer trades $(q, t) \neq (0, 0)$ with the entrant, then her indirect utility is $u^T_i(q, t) = u_i(q, t)$; on the other hand, if the buyer prefers to trade along the tariff T, her utility is $u^T_i(0, 0) = \max\{u_i(q, T(q)) : q\}$, which is typically different from $u_i(0, 0)$. This discontinuity at the origin in particular implies that the indirect utility functions do not inherit the single-crossing property from the primitive utility functions, precisely because the fact that type i prefers to trade with the entrant, that is, $u_i(q, t) > u^T_i(0, 0)$, does not necessarily imply that a higher type would also do so: for $j > i$, is may well be that $u_j(q, t) < u^T_j(0, 0)$. Therefore, an entrant may attract type i without attracting all types $j > i$, which allows for a better targeting of each type. Theorem 1 thus does not apply when competition is exclusive. A consequence is that entry-proof allocations only exist under some conditions under exclusivity, whereas they always exist under nonexclusivity.

Empirical Tests Our analysis suggests that a natural test of the presence of adverse selection in a nonexclusive market consists in comparing the price of successive quantity layers to the average cost of serving the consumers who purchase them. As pointed out by Einav, Finkelstein, and Cullen (2010), insurance markets provide a natural empirical framework to perform such a test, because cost data can be recovered from loss occurrences and insurance claims. To fix ideas, let us thus consider the case of a binary-loss insurance economy, and suppose that we are given a survey of consumers $n = 1, \ldots, N$, which provides information about each consumer's aggregate amount of coverage Q_n, the aggregate premium T_n she pays in return, and her eventual loss $L_n \in \{0, 1\}$. Given this data, a natural two-step empirical procedure runs as follows.

1. Using the observations on coverage choices $(Q_n)_{n=1}^N$ and premia $(T_n)_{n=1}^N$, we can first nonparametrically estimate the marginal price schedule $p \equiv \partial^- T$. A relevant statistical model is a nonparametric regression $T_n = T(Q_n) + \varepsilon_n$ with one-sided error terms ε_n, reflecting that consumers may make errors in combining the firms' offers into the (unobservable) market tariff T.

2. Equipped with an estimator $\hat{p}^N$ of p and with observations on losses $(L_n)_{n=1}^N$, we can then proceed to test whether these observations are consistent with the JHG tariff. To do so, we need only compare, for each coverage level q, the estimated marginal price $\hat{p}^N(q)$ with the empirical loss frequency

$$\hat{\bar{c}}^N(q) \equiv \frac{\sum_n 1_{\{Q_n \geq q, L_n = 1\}}}{\sum_n 1_{\{Q_n \geq q\}}}$$

of the consumers whose aggregate coverage is at least q.

Note that prices and costs play a crucial role in this empirical procedure. This contrasts with standard tests of the positive-correlation property, which only rely on the observation that, under adverse selection, there should be a positive correlation between the coverage purchased by a consumer and her risk (Chiappori and Salanié (2000)). The above test is closer to the one proposed by Einav, Finkelstein, and Cullen (2010) in a setting where consumers have a zero-one demand: evidence of adverse selection is obtained if the average cost of serving the consumers choosing to buy an additional layer of insurance is affected by the price of that layer. Our analysis suggests that the upper-tail conditional expectation function is the generalization of the firms' cost function in Einav, Finkelstein, and Cullen (2010) to richer environments where firms are offering insurance tariffs and consumers can choose different levels of coverage, giving rise to more conditioning events. Observe finally that a strength of the above test is that it is fully nonparametric: there is no need to make assumptions about the consumers' underlying utility functions nor about the distribution of their private information.

6 Concluding Remarks

In this paper, we provide a unified perspective on entry-proofness in adversely selected markets, which is relevant both for inactive markets and for active markets in which buyers cannot be prevented from conducting additional trades with an entrant. These two scenarios appear to be intimately linked: indeed, in our approach, the second scenario reduces to the first one as soon as buyers' utilities are modified in order to incorporate optimal trades along the market tariff. An alternative but equivalent approach would have consisted to start from the second scenario, and to observe that market breakdown occurs if the highest type does not trade in the JHG allocation. In general, this allocation and the nonlinear tariff that implements it emerged as the natural extension of Akerlof (1970) pricing to a large class of convex preferences. This result paves the way to new tests of adverse selection, based on price and cost data, that explicitly take nonexclusive contracting into account.

To conclude, it is appropriate to signal an important limitation of our results. Following a tradition initiated by Akerlof (1970), Pauly (1974), and Rothschild and Stiglitz (1976), we have assumed that the buyers' private information is one-dimensional and that buyer types are ordered according to a single-crossing condition. These restrictions stand in contrast with the important role of multi-dimensional private information emphasized in the recent empirical literature.[14] There are in comparison few theoretical analyses of this question, and they have focused on exclusive-contracting environments.[15] Extending our general approach to entry-proofness so as to incorporate multiple dimensions of buyers' private information is an important challenge for future research.

[14] In the case of insurance, heterogeneity in both risk and in risk aversion has been documented in long-term care (Finkelstein and McGarry (2006)), health (Fang, Keane, and Silverman (2008)), and annuity markets (Einav, Finkelstein, and Schrimpf (2010)).

[15] See, for instance, Azevedo and Gottlieb (2017) and Guerrieri and Shimer (2017).

Appendix A: Proofs of the Main Results

Proof of Theorem 1. The proof consists of three steps.

Step 1 We first formulate the entrant's problem. From the Revelation and Taxation Principles, there is no loss of generality in letting the entrant offer a menu of contracts $\{(q_1, t_1), \ldots, (q_I, t_I)\}$ that is incentive-compatible,

$$\text{For all } i \text{ and } j, \; u_i(q_i, t_i) \geq u_i(q_j, t_j),$$

and individually rational,

$$\text{For each } i, \; u_i(q_i, t_i) \geq u_i(0, 0).$$

We claim that, for any such menu, the buyer has a best response with quantities that are nondecreasing in her type. Indeed, if i optimally trades (q, t) and $j > i$ optimally trades (q', t'), then it must be that $u_i(q, t) \geq u_i(q', t')$ and $u_j(q', t') \geq u_j(q, t)$. Now, suppose that $q > q'$. Because $i < j$ and $q > q'$, applying single crossing to the first inequality yields $u_j(q, t) \geq u_j(q', t')$, which, along with the second inequality, implies $u_j(q, t) = u_j(q', t')$. So type j could optimally trade (q, t) as well.[16] The same reasoning applies to any such pair (i, j) for which quantities are decreasing, which proves the claim.

Because we want entry to be profitable no matter the buyer's best response, we are thus allowed to add the monotonicity constraint that quantities q_i be nondecreasing in i to the entrant's profit-maximization problem, as announced in the text. We can further relax this problem by focusing on the downward local constraints, that is, the downward local incentive-compatibility constraints of types $i > 1$ and the individual-rationality constraint of type $i = 1$. The entrant's expected profit is thus bounded above by

$$\max \left\{ \sum_i m_i(t_i - c_i q_i) : q_i \text{ is nondecreasing in } i \text{ and } u_i(q_i, t_i) \geq u_i(q_{i-1}, t_{i-1}) \text{ for all } i \right\},$$

with $(q_0, t_0) \equiv (0, 0)$ by convention. We call $\mathcal{P}$ this relaxed problem.

Step 2 We now prove that we can focus in $\mathcal{P}$ on menus with nonnegative transfers. Indeed, suppose that a menu $\{(q_1, t_1), \ldots, (q_I, t_I)\}$ satisfies all the constraints in $\mathcal{P}$, and is such that at least one type makes a strictly negative payment. Let i be the smallest such type. Then we can build a new menu by assigning (q_{i-1}, t_{i-1}) to both types $i - 1$ and i. Let us check that this new menu satisfies all the constraints in $\mathcal{P}$. First, because the original

[16] Assuming that the strict single-crossing condition holds would enable us to reach a contradiction at this point, so that each best response exhibits nondecreasing quantities.

menu displays nondecreasing quantities, so does the new menu. Second, the downward local constraint for type i is now an identity. Third, the downward local constraint for type $i+1$, if such type exists, now writes as $u_{i+1}(q_{i+1}, t_{i+1}) \geq u_{i+1}(q_{i-1}, t_{i-1})$, which holds under single crossing because the initial menu satisfies $q_i \geq q_{i-1}$, $u_{i+1}(q_{i+1}, t_{i+1}) \geq u_{i+1}(q_i, t_i)$, and $u_i(q_i, t_i) \geq u_i(q_{i-1}, t_{i-1})$. So all the constraints in $\mathcal{P}$ are satisfied by the new menu, as claimed. The resulting variation in expected profit is, up to multiplication by m_i,

$$(t_{i-1} - c_i q_{i-1}) - (t_i - c_i q_i) = t_{i-1} - t_i + c_i(q_i - q_{i-1}),$$

which is strictly positive as $t_{i-1} \geq 0 > t_i$ by construction and $q_i \geq q_{i-1}$. It follows that the initial menu cannot be solution to $\mathcal{P}$. The entrant's expected profit is thus bounded above by the value of the problem $\mathcal{P}_+$ obtained by adding to $\mathcal{P}$ the constraints $t_i \geq 0$ for all i.

Step 3 Fix a menu $\{(q_1, t_1), \ldots, (q_I, t_I)\}$ that satisfies all the constraints in $\mathcal{P}_+$ and, for any type i, consider the trade (q_{i-1}, t_{i-1}). For $i = 1$, we clearly have $u_i(q_{i-1}, t_{i-1}) \geq u_i(0, 0)$ as $(q_0, t_0) = (0, 0)$. For $i > 1$, we know that type $i-1$ weakly prefers (q_{i-1}, t_{i-1}) to $(0, 0)$. By single crossing, so does type i. Thus, in any case, we have $u_i(q_{i-1}, t_{i-1}) \geq u_i(0, 0)$. Because $t_{i-1} \geq 0$, this shows that the indifference curve of type i going through (q_{i-1}, t_{i-1}) must cross the q-axis at some point $(\underline{q}_i, 0)$, with $\underline{q}_i \in [0, q_{i-1}]$. Under quasiconcavity, type i's marginal rate of substitution is nonincreasing with respect to the quantity purchased along this indifference curve. Therefore, $\tau_i(q_{i-1}, t_{i-1}) \leq \tau_i(\underline{q}_i, 0)$. Assumption 1 further implies $\tau_i(\underline{q}_i, 0) \leq \tau_i(0, 0)$, and we can finally use Condition EP to get (3). The argument in the text then shows that $\sum_i m_i(t_i - c_i q_i) \leq 0$, which implies that the value of $\mathcal{P}_+$ is nonpositive. Hence the result. ■

Proof of Corollary 1. Suppose, by way of contradiction, that there is some trade, so that $q_i > q_{i-1}$ for some type i, and that the entrant earns zero expected profit. Then, as type i's preferences are strictly convex, and as $u_i(q_i, t_i) \geq u_i(q_{i-1}, t_{i-1})$, inequality (3) implies $t_i - t_{i-1} - \bar{c}_i(q_i - q_{i-1}) < 0$, so that the entrant's expected profit (2) is strictly negative, a contradiction. Hence the result. ■

Proof of Lemma 1. For the sake of clarity, the index i is hereafter omitted. The proof consists of three steps.

Step 1 We begin by studying the statement $p < \tau^T(q', t')$. By definition, $\tau^T(q', t')$ is the supremum of the set of prices p such that

$$\max\{u(q + q', T(q) + t') : q\} = u^T(q', t')$$

$$
\begin{aligned}
&< \max\{u^T(q'+q'', t'+pq'') : q''\} \\
&= \max\{u(q+q'+q'', T(q)+t'+pq'') : q, q''\} \\
&= \max\{u(q+q', T \,\square\, T_p(q)+t') : q\},
\end{aligned}
$$

where T_p is the linear tariff with slope p and $T \,\square\, T_p(q) \equiv \min\{T(q')+T_p(q-q') : q' \in [0,q]\}$ is the infimal convolution of T and T_p (Rockafellar (1970, Theorem 5.4)). Hence the statement $p < \tau^T(q', t')$ is equivalent to the following:

$$
\max\{u(q+q', T(q)+t') : q\} < \max\{u(q+q', T \,\square\, T_p(q)+t') : q\}. \tag{19}
$$

Notice that $T \,\square\, T_p \leq T$, and that both tariffs coincide up to some quantity q_p, above which the inequality is strict. Two cases can arise. Either the maximization problem on the right-hand side of (19) admits one solution at most equal to q_p, and then (19) is an equality. Or all the solutions to this problem are strictly above q_p: then (19) cannot be an equality because, if it were, then there would exist a solution to the maximization problem on the left-hand side of (19) at most equal to q_p, and thus this solution would also be a solution to the maximization problem on the right-hand side of (19), a contradiction; therefore, (19) must hold because in any case $T \,\square\, T_p \leq T$. Overall, we have shown that the statement $p < \tau^T(q', t')$ is equivalent to the statement that all the solutions to the maximization problem on the right-hand side of (19) are strictly above q_p.

Step 2 Next, fix t' and choose two quantities q'_0 and q'_1 such that $q'_0 < q'_1$. Then define the following quasiconcave functions:

$$
v_0(q,t) \equiv u(q+q'_0, t+t') \text{ and } v_1(q,t) \equiv u(q+q'_1, t+t').
$$

Assumption 2 expresses that the indifference curves for v_0 are everywhere steeper than the indifference curves for v_1. Therefore, if two buyers with utilities v_0 and v_1 face the same tariff $t = T(q)$, then the lowest optimal quantity choice for the buyer with utility v_0 is at least as large as the lowest optimal quantity choice for the buyer with utility v_1.

Step 3 Now, suppose that $p < \tau^T(q'_1, t')$. From Step 1, we first obtain that all the solutions to the maximization problem on the right-hand side of (19) (in which q' is replaced by q'_1) are strictly above q_p. From Step 2, we next obtain that all the solutions to the maximization problem on the right-hand side of (19) (in which q' is replaced by q'_0) are strictly above q_p. From Step 1 again, we finally obtain $p < \tau^T(q'_0, t')$. This shows that $\tau^T(q'_1, t') \leq \tau^T(q'_0, t')$ for all t' and $q'_0 < q'_1$, and, therefore, that the property expressed by Assumption 2 is inherited from $\tau(q,t)$ by $\tau^T(q', t')$; a fortiori, Assumption 1 holds for $\tau^T(q', 0)$. The result follows. ■

Proof of Lemma 3. For each i such that $q_i > q_{i-1}$, we have

$$\partial^- T(q_i) \geq \frac{T(q_i) - T(q_{i-1})}{q_i - q_{i-1}} = \overline{c}_i \geq \tau_i^T(0,0),$$

where the first inequality follows from the convexity of T, the equality follows from (10), and the second inequality follows from (6). Hence we only need to check that $\tau_i^T(0,0) \geq \partial^- T(q_i)$. Suppose that $p \geq \tau_i^T(0,0)$. According to the proof of Lemma 1, this implies

$$\max\{u_i(q, T(q)) : q\} = \max\{u_i(q, T \,\square\, T_p(q)) : q\}, \tag{20}$$

and in this case there exists a solution $\tilde{q}_i$ to these two maximization problems that is at most equal to q_p; in particular, $p \geq \partial^- T(\tilde{q}_i)$. If u_i is strictly quasiconcave, then $\tilde{q}_i$ coincides with q_i, the unique solution to the maximization problem on the left-hand side of (20), and we have $p \geq \partial^- T(q_i)$. If u_i is linear, both maximization problems may have an interval of solutions, and so it may be that $\tilde{q}_i < q_i$. But then it must be that T is linear and coincides with $T \,\square\, T_p$ over $[\tilde{q}_i, q_i]$, so that $p = \partial^- T(q_i)$. Thus, in any case, $p \geq \partial^- T(q_i)$, which shows that $\tau_i^T(0,0) \geq \partial^- T(q_i)$. The result follows. ■

Proof of Corollary 2. We need to check that (6) holds for the tariff (14). For each $i < i^*$, we have $v_i < \overline{c}_{i^*}$ and hence $q_i^* = 0$ according to (12). Therefore, $\tau_i^{T^*}(0,0)$ is the supremum of the prices p such that

$$u_i(0,0) < \max\{u_i(q + q', \overline{c}_{i^*} q + pq') : q, q'\},$$

and is thus equal to v_i, which is strictly less than $\overline{c}_i$ by definition of i^*. For each $i \geq i^*$, we have $v_i > v_{i^*} > \overline{c}_{i^*}$ and hence $q_i^* = \overline{q}$ according to (12). Therefore, $\tau_i^{T^*}(0,0)$ is the supremum of the prices p such that

$$u_i(\overline{q}, \overline{c}_{i^*}\overline{q}) < \max\{u_i(q + q', \overline{c}_{i^*} q + pq') : q, q'\},$$

and is thus equal to $\overline{c}_{i^*}$, which in turn is at most equal to $\overline{c}_i$. Hence the result. ■

Proof of Corollary 3. We need to check that (6) holds for the tariff (18). For each i, $\tau_i^{T^*}(0,0)$ is the supremum of the prices p such that

$$u_i(q_i^*, T^*(q_i^*)) < \max\{u_i(q + q', T^*(q) + pq') : q, q'\}.$$

If $q_i^* > q_{i-1}^*$, this is equal to $\partial^- T^*(q_i^*)$, which is equal to $\overline{c}_i$ by construction. If $q_i^* = q_{i-1}^*$, this is equal to $\tau_i(q_i^*, T^*(q_i^*))$, which, according to (15), must be at most equal to $\overline{c}_i$ for q_i^* not to be strictly above q_{i-1}^*. Hence the result. ■

Appendix B: Arbitrary Distributions

In this appendix, we extend Theorem 1 to arbitrary distributions of types with bounded support $\mathcal{I}$ over the real line. Denote by i the buyer's type, and by $\mathbf{m}$ the corresponding distribution; $\mathbf{m}$ may be continuous, discrete, or mixed. It will sometimes be convenient to think of any point in $\overline{\mathcal{I}} \equiv [\min \mathcal{I}, \max \mathcal{I}]$ as a type, even if it does not belong to $\mathcal{I}$. We impose the same conditions on the utility functions u_i and on the upper-tail conditional expectations of unit costs $\overline{c}_i^{\mathbf{m}} \equiv \mathbf{E}^{\mathbf{m}}[c_j \,|\, j \geq i]$ as in Section 2, and we moreover assume that $u_i(q, t)$ is jointly continuous in (i, q, t) and that c_i is continuous in i.

The proof that Condition EP is necessary for entry-proofness is exactly the same as in Section 3, and need not be repeated. There only remains to show that Condition EP is sufficient for entry-proofness. The Taxation Principle tells us there is no loss of generality in letting the entrant offer a tariff specifying a transfer $T(q)$ to be paid as a function of the quantity q demanded by the buyer, with $T(0) = 0$. We assume that the domain of T is a compact set containing 0, and that T is bounded from below and lower semicontinuous. These minimal regularity conditions ensure that any type i's maximization problem

$$\max\{u_i(q, T(q)) : q \geq 0\} \tag{21}$$

has a solution. The following result then holds.

Lemma 4 *There exists for each i a solution q_i to (21) such that*

(i) The mapping $i \mapsto q_i$ is nondecreasing.

(ii) The mapping $i \mapsto T(q_i) - c_i q_i$ is bounded from below and lower semicontinuous.

Proof. As in Step 1 of the proof of Theorem 1, the single-crossing condition ensures that we can select the buyer's best response in such a way that the mapping $i \mapsto q_i$ is nondecreasing. Hence (i). As for (ii), observe first that, because T has a compact domain and is bounded from below, the mapping $i \mapsto T(q_i) - c_i q_i$ is bounded from below no matter the buyer's best response. To show that the buyer's best response can be chosen in such a way that this mapping is lower semicontinuous, it is useful to fix a best response $i \mapsto q_i$ and some type $i_0 \in \overline{\mathcal{I}}$, and then to distinguish two cases.

Case 1 Suppose first that $i \mapsto q_i$ is continuous at i_0. Then, as T is lower semicontinuous and c_i is continuous in i, we have $\liminf_{i \to i_0} \{T(q_i) - c_i q_i\} \geq T(q_{i_0}) - c_{i_0} q_{i_0}$.

Case 2 Suppose next that $i \mapsto q_i$ is discontinuous and left continuous at i_0. (The other types of jump discontinuities can be treated in a similar way.) Observe that, because the domain of T is a compact set, it must include $q_{i_0}^+$; moreover, T must be right continuous at $q_{i_0}^+$, for, otherwise, some type $i > i_0$ would be strictly better off purchasing $q_{i_0}^+$ than q_i. Now, observe that type i_0 must be indifferent between the trades $(q_{i_0}, T(q_{i_0}))$ and $(q_{i_0}^+, T(q_{i_0}^+))$. Indeed, we clearly have $u_{i_0}(q_{i_0}, T(q_{i_0})) \geq u_{i_0}(q_{i_0}^+, T(q_{i_0}^+))$ and, if we had $u_{i_0}(q_{i_0}, T(q_{i_0})) > u_{i_0}(q_{i_0}^+, T(q_{i_0}^+))$, then, by continuity of u_i in i, some type $i > i_0$ would be strictly better off purchasing q_{i_0} than q_i, a contradiction. We can thus select the trade of type i_0 in such a way that $\liminf_{i \to i_0} \{T(q_i) - c_i q_i\} \geq T(q_{i_0}) - c_{i_0} q_{i_0}$. The result follows. ■

The next step of the analysis consists in checking that any distribution that satisfies Condition EP can be weakly approximated by a sequence of discrete distributions that satisfy Condition EP. Specifically, the following result holds.

Lemma 5 *If* $\mathbf{m}$ *satisfies Condition EP, there exists a sequence of discrete distributions* $(\mathbf{m}_n)_{n=1}^{\infty}$ *that weakly converges to* $\mathbf{m}$ *and such that*

$$\text{For all } n \text{ and } i,\ \overline{c}_i^{\mathbf{m}_n} \geq \overline{c}_i^{\mathbf{m}}.$$

Proof. The proof is a simple adaptation of Hendren (2013, Supplementary Material, Lemma A.7), using the fact that c_i is continuous in i and that, as $\overline{c}_i^{\mathbf{m}}$ is nondecreasing in i, $c_{\max \mathcal{I}} \geq c_i$ for all i. Hendren's (2013) proof establishes that the sequence of cumulative distribution functions associated to the sequence $(\mathbf{m}_n)_{n=1}^{\infty}$ can be chosen so as to uniformly converge to the cumulative distribution function associated to $\mathbf{m}$. The result follows. ■

We are now ready to complete the proof of Theorem 1 for arbitrary distributions. Let $\mathbf{m}$ be a distribution that satisfies Condition EP. Fix a tariff T as above and, for each i, a solution q_i to (21) such that conditions (i)–(ii) in Lemma 4 are satisfied. Lemma 5 implies that there exists a sequence of discrete distributions $(\mathbf{m}_n)_{n=1}^{\infty}$ that weakly converges to $\mathbf{m}$ and such that each $\mathbf{m}_n$ satisfies Condition EP. Taking advantage of the fact that the mapping $i \mapsto q_i$ is nondecreasing, we can apply the version of Theorem 1 for discrete distributions provided in the main text to get

$$\text{For each } n,\ \int [T(q_i) - c_i q_i]\, \mathbf{m}_n(\mathrm{d}i) \leq 0.$$

Because the mapping $i \mapsto T(q_i) - c_i q_i$ is bounded from below and lowersemicontinuous, the weak convergence of the sequence $(\mathbf{m}_n)_{n=1}^{\infty}$ to $\mathbf{m}$ then yields

$$\int [T(q_i) - c_i q_i]\, \mathbf{m}(\mathrm{d}i) \leq \liminf_{n \to \infty} \left\{ \int [T(q_i) - c_i q_i]\, \mathbf{m}_n(\mathrm{d}i) \right\} \leq 0$$

according to a corollary of the Portmanteau Theorem (Aliprantis and Border (2006, Theorem 15.5)). Hence, if the distribution $\mathbf{m}$ satisfies Condition EP, no tariff can guarantee the entrant a strictly positive expected profit, which is the desired result.

Appendix C: Illustrative Examples

The following examples for the buyer's preferences illustrate the range of possible applications of our model.

Quasilinear Utility We may first suppose, as in the models of trade on financial markets studied by Glosten (1994) and Mailath and Nöldeke (2008), that each type i' preferences are quasilinear,

$$u_i(q,t) \equiv U_i(q) - t,$$

for some concave utility function U_i. The single-crossing condition is satisfied if, for each q, $\partial^+ U_i(q)$ is nondecreasing in i, and the concavity of U_i ensures that Assumption 1 holds.

Expected Utility Consider next a Rothschild and Stiglitz (1976) insurance economy. The buyer has initial wealth w_0 and faces the risk of a loss l with a probability c_i that defines her type. Each type i's preferences over coverage-premium pairs (q,t) have an expected-utility representation,

$$u_i(q,t) \equiv c_i u(w_0 - l + q - t) + (1 - c_i) u(w_0 - t),$$

for some strictly increasing and strictly concave von Neumann–Morgenstern utility function u. The single-crossing condition is satisfied if c_i is nondecreasing in i, and the concavity of u ensures that Assumption 1 holds.

We can modify this and the following examples to allow for multiple loss levels if we focus on coinsurance contracts requiring that a fraction q of the loss be covered for a premium t. The buyer has initial wealth w_0 and faces the risk of a loss l distributed according to a density f_i that defines her type. Each type i's preferences then have the following representation:

$$u_i(q,t) \equiv \int u(w_0 - (1-q)l - t) f_i(l)\,\mathrm{d}l.$$

The single-crossing condition is satisfied whenever the densities f_i are increasing in the monotone-likelihood ratio order, that is, whenever higher types are relatively more likely to incur large losses than lower types are (Attar, Mariotti, and Salanié (2016a, Appendix B)). This, in turn, implies that the former are more costly to serve than the latter, as required.

Rank-Dependent Expected Utility Consider again a Rothschild and Stiglitz (1976) insurance economy but, following Quiggin (1982), suppose that each type i's preferences over coverage-premium pairs (q,t) have a rank-dependent expected-utility representation,

$$u_i(q,t) \equiv [w(1) - w(1-c_i)]u(w_0 - l + q - t) + w(1-c_i)u(w_0 - t),$$

for some strictly increasing and strictly concave von Neumann–Morgenstern utility function u and some strictly increasing weighting function w such that $w(0) = 0$ and $w(1) = 1$. Because w is strictly increasing, the single-crossing condition is satisfied if c_i is nondecreasing in i, and the concavity of u ensures that Assumption 1 holds.

Robust Control Our framework also encompasses ambiguity. Consider again a Rothschild and Stiglitz (1976) insurance economy but, following Hansen and Sargent (2007), suppose that each type i recognizes that the true probability distribution over outcomes $\tilde{\mathbf{c}}_i \equiv (\tilde{c}_i, 1-\tilde{c}_i)$ is uncertain and may differ from $\mathbf{c}_i \equiv (c_i, 1-c_i)$. Then type i's preferences over coverage-premium pairs (q,t) have a robust-control representation,

$$u_i(q,t) \equiv \min\left\{\tilde{c}_i u(w_0 - l + q - t) + (1-\tilde{c}_i)u(w_0 - t) + \alpha e(\tilde{\mathbf{c}}_i, \mathbf{c}_i) : \tilde{\mathbf{c}}_i\right\},$$

for some strictly increasing and strictly concave von Neumann–Morgenstern utility function u, where $e(\tilde{\mathbf{c}}_i, \mathbf{c}_i)$ is the relative entropy function that penalizes distortions from $\mathbf{c}_i$,

$$e(\tilde{\mathbf{c}}_i, \mathbf{c}_i) \equiv \tilde{c}_i \log_2\left(\frac{\tilde{c}_i}{c_i}\right) + (1-\tilde{c}_i)\log_2\left(\frac{1-\tilde{c}_i}{1-c_i}\right).$$

As u_i is a minimum of concave functions, it is itself concave. For each (q,t), let us denote by $\tilde{\mathbf{c}}_i(q,t) = (\tilde{c}_i(q,t), 1-\tilde{c}_i(q,t))$ the unique solution to the minimization problem that defines $u_i(q,t)$. Taking first-order conditions yields

$$\frac{1-\tilde{c}_i(q,t)}{\tilde{c}_i(q,t)} = \frac{1-c_i}{c_i} 2^{u(w_0-l+q-t)-u(w_0-t)}.$$

Thus, for each (q,t), $\tilde{c}_i(q,t)$ and c_i are comonotonic in i, and $\tilde{c}_i(q,t)$ is strictly decreasing in q for all t. Assuming that u is differentiable, we can apply the envelope theorem to derive the marginal rate of substitution of u_i,

$$\tau_i(q,t) = \left[1 + \frac{1-\tilde{c}_i(q,t)}{\tilde{c}_i(q,t)} \frac{u'(w_0-t)}{u'(w_0-l+q-t)}\right]^{-1}.$$

Hence, because $\tilde{c}_i(q,t)$ and c_i are comonotonic in i, the single-crossing condition is satisfied if c_i is nondecreasing in i. Finally, the fact that $\tilde{c}_i(q,0)$ is strictly decreasing in q, together with the concavity of u, ensures that Assumption 1 holds.

Smooth Ambiguity Our framework also encompasses smooth ambiguity. Consider again a Rothschild and Stiglitz (1976) insurance economy but, following Klibanoff, Marinacci, and Mukerji (2005), suppose that each type i's preferences over coverage-premium pairs (q, t) have a smooth-ambiguity-aversion representation,

$$u_i(q,t) \equiv \int \phi(cu(w_0 - l + q - t) + (1-c)u(w_0 - t)) f_i(c)\,\mathrm{d}c,$$

for some strictly increasing and strictly concave von Neumann–Morgenstern utility function u and some strictly increasing and strictly concave function ϕ capturing ambiguity aversion regarding the true distribution over outcomes $\mathbf{c} \equiv (c, 1-c)$, the distribution of which is itself represented by a continuous density f_i over $[0,1]$. As ϕ and u are concave, so is u_i. Assuming that ϕ and u are differentiable, with bounded derivatives over the relevant domain, we can express the marginal rate of substitution of u_i as

$$\tau_i(q,t) = \left[1 + \frac{u'(w_0 - t)}{u'(w_0 - l + q - t)} \times \frac{\int \phi'(cu(w_0 - l + q - t) + (1-c)u(w_0 - t))(1-c) f_i(c)\,\mathrm{d}c}{\int \phi'(cu(w_0 - l + q - t) + (1-c)u(w_0 - t)) c f_i(c)\,\mathrm{d}c}\right]^{-1}.$$

We claim that the single-crossing condition is satisfied if the densities f_i are increasing in the monotone-likelihood-ratio order. To see this, observe that

$$\frac{\int \phi'(cu(w_0 - l + q - t) + (1-c)u(w_0 - t))(1-c) f_i(c)\,\mathrm{d}c}{\int \phi'(cu(w_0 - l + q - t) + (1-c)u(w_0 - t)) c f_i(c)\,\mathrm{d}c} = \frac{1}{\int c\,\mathrm{d}G_i(c)} - 1,$$

where G_i is a distribution with density

$$g_i(c) = \frac{\phi'(cu(w_0 - l + q - t) + (1-c)u(w_0 - t)) f_i(c)}{\int \phi'(cu(w_0 - l + q - t) + (1-c)u(w_0 - t)) f_i(c)\,\mathrm{d}c}$$

with respect to Lebesgue measure. As the densities f_i are increasing in the monotone-likelihood ratio order, so are the densities g_i. This implies that the ratio $1/\int c\,\mathrm{d}G_i(c)$ is decreasing in i, which proves the claim given the expression for $\tau_i(q,t)$. Finally, there remains to determine circumstances under which Assumption 1 holds. Letting $t \equiv 0$ and proceeding as above yields

$$\frac{\int \phi'(cu(w_0 - l + q) + (1-c)u(w_0))(1-c) f_i(c)\,\mathrm{d}c}{\int \phi'(cu(w_0 - l + q) + (1-c)u(w_0)) c f_i(c)\,\mathrm{d}c} = \frac{1}{\int c\,\mathrm{d}G_i(c\,|\,q)} - 1,$$

where, for each q, $G_i(\cdot\,|\,q)$ is a distribution with density

$$g_i(c\,|\,q) = \frac{\phi'(cu(w_0 - l + q) + (1-c)u(w_0)) f_i(c)}{\int \phi'(cu(w_0 - l + q) + (1-c)u(w_0)) f_i(c)\,\mathrm{d}c}$$

with respect to Lebesgue measure. It follows that

$$\frac{g_i(c\,|\,0)}{g_i(c\,|\,q)} \propto \frac{\phi'(cu(w_0 - l) + (1-c)u(w_0))}{\phi'(cu(w_0 - l + q) + (1-c)u(w_0))},$$

up to multiplicative constants. In particular

$$\frac{\partial}{\partial c}\left[\frac{g_i(c\,|\,0)}{g_i(c\,|\,q)}\right] \propto -\frac{\phi''(cu(w_0 - l) + (1-c)u(w_0))}{\phi'(cu(w_0 - l) + (1-c)u(w_0))}\,[u(w_0) - u(w_0 - l)] + \frac{\phi''(cu(w_0 - l + q) + (1-c)u(w_0))}{\phi'(cu(w_0 - l + q) + (1-c)u(w_0))}\,[u(w_0) - u(w_0 - l + q)],$$

which is strictly positive for $q > 0$ if the function ϕ exhibits nonincreasing concavity in the sense that $-\phi''/\phi'$ is nonincreasing. Under these circumstances, $g_i(c\,|\,0)$ dominates $g_i(c\,|\,q)$ in the monotone-likelihood-ratio order and, as a result, $\int c\,\mathrm{d}G_i(c\,|\,q) < \int c\,\mathrm{d}G_i(c\,|\,0)$. Combining this with the straightforward observation that $u'(w_0 - l + q) < u'(w_0 - l)$ by strict concavity of u, we obtain, using the above expression for $\tau_i(q,t)$, that $\tau_i(q,0) < \tau_i(0,0)$ for all $q > 0$, so that Assumption 1 holds.

It is easy to check that the slightly stronger Assumption 2 used in Section 5 holds in the above examples. Moreover, many other families of preferences, involving, for instance, first-order risk aversion (Segal and Spivak (1990)), also fit within our general framework.

Appendix D: On the Role of Assumption 1

The following example justifies the claim that, when Assumption 1 does not hold, entry with a menu of contracts can be profitable even though Condition EP is satisfied.

Example 3 Consider a two-type economy in which each type has preferences represented by $u_i(q,t) \equiv (q+1)(\theta_i q - t)$, for $\theta_2 > \theta_1 > 0$. These preferences are convex, with

$$\tau_i(q,t) = \theta_i\left(1 + \frac{q}{q+1}\right) - \frac{t}{q+1}, \tag{22}$$

so that the strict single-crossing condition is satisfied. However, from (22), $\tau_i(q,0)$ is strictly increasing in q, so that Assumption 1 does not hold. Now, fix quantities $q_2 > q_1 > 0$. For some small $\eta > 0$, consider an entrant offering a menu $\{(q_1,t_1),(q_2,t_2)\}$ such that

$$t_1 \equiv \theta_1 q_1 - \eta,$$

so that type 1 earns a small rent above $u_1(0,0) = 0$, and

$$t_2 \equiv \theta_2 q_2 - \frac{q_1+1}{q_2+1}\,(\theta_2 q_1 - t_1) - \eta,$$

so that type 2 has a slight preference for (q_2, t_2) over (q_1, t_1). Each type has a unique best response, and the entrant's expected profit is $m_1(t_1 - c_1 q_1) + m_2(t_2 - c_2 q_2)$. To compute this expected profit, choose $\varepsilon > 0$ and set up costs so that $\overline{c}_1 = \theta_1 + \varepsilon$ and $\overline{c}_2 = \theta_2 + \varepsilon$. Note that, from (22) again, Condition EP is satisfied. According to (2), the entrant's expected profit can be rewritten as $t_1 - \overline{c}_1 q_1 + m_2[t_2 - t_1 - \overline{c}_2(q_2 - q_1)]$; this in turn simplifies into

$$m_1 m_2 (c_2 - c_1)(q_2 - q_1)\,\frac{q_1}{q_2 + 1} - \varepsilon(m_1 q_1 + m_2 q_2) - \eta\left(1 + m_2\,\frac{q_1 + 1}{q_2 + 1}\right),$$

which is strictly positive for arbitrary quantities $q_2 > q_1 > 0$ whenever ε and η are small enough. This proves the claim. Notice that the entrant makes a profit when trading with type 1 and a loss when trading with type 2; but he also incurs an expected loss on the quantity layer q_1, which he more than recoups on the quantity layer $q_2 - q_1$.

Appendix E: Wilson's Demand-Profile Approach

In this appendix, we draw the link between our approach and Wilson's (1993) demand-profile approach to second-order price discrimination. Suppose that the buyer's preferences are quasilinear,

$$u_i(q, t) \equiv U_i(q) - t.$$

We throughout assume that the utility functions U_i are strictly concave and differentiable and that the tariff T is absolutely continuous. Thus the marginal price schedule $p \equiv \partial T^-$ is defined almost everywhere and, denoting by q_i the quantity optimally purchased by type i given the tariff T, the monopolist's expected profit is, supposing that the first-order approach is valid, equal to

$$\begin{aligned}\sum_i m_i[T(q_i) - c_i q_i] &= \sum_i m_i \int_0^{q_i} [p(q) - c_i]\,\mathrm{d}q \\ &= \int_0^{\infty} \sum_{\{i\,:\,U_i'(q) \geq p(q)\}} m_i[p(q) - c_i]\,\mathrm{d}q \\ &= \int_0^{\infty} \left(\sum_{\{i\,:\,U_i'(q) \geq p(q)\}} m_i\right) \{p(q) - \mathbf{E}[c_i \,|\, U_i'(q) \geq p(q)]\}\,\mathrm{d}q, \qquad (23)\end{aligned}$$

where the second inequality follows from switching the sum and the integral signs, using the fact that type i, having already purchased the quantity $q - \mathrm{d}q$, is willing to purchase the marginal quantity $\mathrm{d}q$ if and only if $U_i'(q) \geq \partial^- T(q) = p(q)$. By single crossing, the set $\{i : U_i'(q) \geq p(q)\}$ of types who purchase at least the quantity q is of the form $\{i(q), \ldots, I\}$, or is empty if q is too large, in which case $i(q) \equiv \infty$ by convention. We can always assume that $U'_{i(q)}(q) = p(q)$,[17] so that we can rewrite the monopolist's expected profit (23) as

$$\int_0^\infty \left(\sum_{j \geq i(q)} m_j \right) [U'_{i(q)}(q) - \overline{c}_{i(q)}] \,\mathrm{d}q. \tag{24}$$

It is apparent from (24) that the upper-tail conditional expectations of unit costs determine the *sign* of the expected profit earned on each marginal quantity dq, as in (2). On the other hand, the *solution* to the monopoly problem is characterized by

$$U'_i(q_i) = c_i + \frac{\sum_{j>i} m_j}{m_i} [U'_{i+1}(q_i) - U'_i(q_i)], \tag{25}$$

from which the upper-tail conditional expectations of unit costs have disappeared.[18] Notice that (25) is the discrete-type counterpart of

$$U'_i(q_i) = c_i + \frac{1 - F(i)}{f(i)} \frac{\partial U'_i}{\partial i}(q_i),$$

which differs from the standard characterization of the optimal monopoly schedule (Wilson (1993)) only in that costs are type-dependent.

[17]This is obvious if T is differentiable at q. If T has an upward kink at q, with types $i(q), \ldots, j(q)$ bunched at q, we can increase the expected profit by slightly increasing the tariff T in such a way that the new tariff $\hat{T}$ satisfies $\partial^- \hat{T}(q) = U'_{i(q)}(q)$ and $\partial^+ \hat{T}(q) = U'_{j(q)}(q)$, so that all these types remain bunched at q but now pay a higher price.

[18]To derive (25), observe that, if there is no bunching, so that $i(q_i) = i$ for all i, pointwise optimization of the integrand in (24) yields $\Delta_i(q) \geq 0$ for all $q \leq q_i$ and $\Delta_i(q) < 0$ for all $q > q_i$, where

$$\Delta_i(q) \equiv \sum_{j \geq i+1} m_j \, U'_{i+1}(q) - \sum_{j \geq i+1} m_j c_j - \sum_{j \geq i} m_j \, U'_i(q) + \sum_{j \geq i} m_j c_j.$$

This is continuous in q, so that $\Delta_i(q_i) = 0$, which is (25).

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