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"Secret contracting in multilateral relations"

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Secret contracting in multilateral relations*

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Abstract

We develop a flexible and tractable framework of (secret) vertical contracting between multiple upstream suppliers and downstream retailers. This framework does not put any restriction on the tariffs that can be negotiated, and yet does take account of the impact of these tariffs on downstream firms' behavior. We show that equilibrium tariffs must be cost-based; as a result, retail prices are the same as with a multi-brand oligopoly. Interestingly, this finding is in line with the empirical analysis of a recent Norwegian merger.

We then use this flexible framework to endogenize market structure as well as to analyze the effects of various vertical restraints, such as resale price maintenance and retail price parity clauses. Finally, we show that our framework also applies to the agency relationships that characterize most online platforms.

JEL classification: L13, L42, D43, K21.

Keywords: Bilateral contracting, vertical relationships, agency, resale price maintenance, price parity clauses.

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1 Introduction

Upstream markets often involve interlocking multilateral relations between suppliers and customers. For consumer goods, for instance, most supermarket chains carry the same leading brands (e.g., Coca-Cola and Pepsi can both be found in Safeway and WalMart stores). In healthcare markets, insurance companies negotiate with the same medical providers (physicians, hospitals, etc.). Likewise, in media markets, cable or satellite providers negotiate with the same TV channels or content providers. Such interlocking relationships are also frequently found in many intermediate-goods markets, where competing firms buy components or services from the same competing suppliers. For instance, PC manufacturers often use both Intel and AMD processing chips; Airbus and Boeing may offer airlines a choice of engines from General Electric, Rolls Royce or Pratt & Whitney, and deal with the same contractors (e.g., Spirit and Latécoère).

Despite the prevalence of these interlocking relationships, the literature on vertical contracting has so far focused mainly on more stylized market structures. For instance, much of the early literature focuses on the case of a monopolist either upstream or downstream,[1] or considers competing vertical structures, where each upstream firm deals with its own downstream partners (as in the case of franchise networks).[2]

Several papers have started to analyze vertical contracting in multilateral relations, but impose various restrictions. The analysis of a dominant supplier has, for instance, been extended to the case of a competitive fringe.[3] In other instances, upstream firms are assumed to offer perfect substitutes.[4] Alternatively, attention is restricted to particular types of contracts,

[1]For example, Mathewson and Winter (1984) and Rey and Tirole (1986) focus on the role of vertical restraints in improving vertical coordination between a manufacturer and its retailers, whereas Hart and Tirole (1990), O'Brien and Shaffer (1992) and McAfee and Schwartz (1994) focus on opportunistic behavior by a monopolistic supplier. Another branch of this literature has focused on the scope for exclusive dealing in the presence of a bottleneck; see, for example, Bernheim and Whinston (1985, 1986, 1998) for the case of a downstream bottleneck, and Marx and Shaffer (2007), Miklòs-Thal *et al.* (2011) and Rey and Whinston (2013) for the case of an upstream bottleneck.

[2]For example, Bonanno and Vickers (1988), Rey and Stiglitz (1988, 1995) and Gal-Or (1991) study the role of strategic delegation as a way of dampening inter-brand competition. Jullien and Rey (2007) and Piccolo and Miklòs-Thal (2012) consider instead the role of vertical contracts as facilitating tacit collusion upstream and/or downstream.

[3]This is, for instance, a common assumption in the literature on retailers' private labels; see, for example, Mills (1995) and Gabrielsen and Sørgard (2007). Other examples include Hart and Tirole (1990) and Innes and Hamilton (2009).

[4]See, for example, Salinger (1988), Ordover *et al.* (1990), de Fontenay and Gans (2005, 2014), and Nocke and White (2007, 2010).

such as linear wholesale prices[5] or two-part tariffs.[6]

Another set of literature, partly prompted by merger waves and policy debates in cable television[7] and healthcare[8] markets, has instead focused on network formation and on the division of the gains from trade within these networks.[9] This literature accounts for the externalities created by competition among the vertical channels, but it either assumes away the interplay between wholesale agreements and downstream outcomes (by restricting attention to lump-sum transfers), or accounts for it only partially (by assuming that upstream and downstream prices are set simultaneously).[10]

In this paper, we propose a tractable and flexible model of multilateral relations which does not put any restriction on the tariffs that can be negotiated, and yet takes into account the impact of these tariffs on downstream price competition.[11] For the sake of exposition, we refer to upstream firms as manufacturers and to downstream firms as retailers; however, the analysis similarly applies to other vertically related industries.

We allow for any number of manufacturers and retailers, and for any distribution of bargaining power between each manufacturer and each retailer. As wholesale negotiations are usually not publicly observable, the outcome of the negotiation between a manufacturer and a retailer (including whether or not they have reached an agreement and, if so, the terms of the contract) is considered to be private information.

Modelling secret contracting in multilateral relationships raises complex issues, even with simple bargaining games where one side of the market makes take-it-or-leave-it offers to the other side. In particular, when receiving an out-of-equilibrium offer, a firm needs to form beliefs about the contracts signed by the other vertical channels. As Bayes' rule does not restrict these off-equilibrium beliefs, there are typically many (perfect Bayesian) Nash-equilibria.

[5] See, for example, Dobson and Waterson (2007). More recently, Crawford and Yurukoglu (2012) use a similar approach in their empirical study of the welfare effects of bundling in the US cable industry.

[6] See, for example, Rey and Vergé (2010) and Allain and Chambolle (2011).

[7] For instance, Chipty and Snyder (1999) study the impact of horizontal mergers, whereas Crawford *et al.* (2015) consider the role of vertical integration.

[8] For instance, Gowrisankaran *et al.* (2015) study the impact of hospital mergers, whereas Ho and Lee (2016) analyze competition among health insurance providers.

[9] While most of the literature favors a static approach, Lee and Fong (2013) instead adopt an infinite horizon framework, in which they study Markov perfect equilibria. They also show that this approach yields qualitatively different predictions about the impact of a merger on upstream transfers and downstream prices. For an earlier analysis of buyer-seller network formation, without downstream competition, see, e.g., Kranton and Minehart (2001).

[10] For instance, among the most recent papers, Gowrisankaran *et al.* (2015) and Ho and Lee (2016) focus on lump-sum transfers, whereas Crawford *et al.* (2015) assume that linear (upstream and downstream) prices are set simultaneously.

[11] Nocke and Rey (2016) provide an analysis of multilateral relations with Cournot downstream competition.

This has led the literature to rely on "reasonable" out-of-equilibrium beliefs, such as passive or wary beliefs. Unfortunately, when downstream firms compete in prices, equilibria based on passive beliefs may fail to exist, and wary beliefs are rather intractable, even in the absence of competition in the upstream market.[12] For tractability, we rely instead on a version of the "contract equilibrium" or "Nash-in-Nash" approach, which moreover allows for balanced bargaining. This approach, first developed by Crémer and Riordan (1987) and Horn and Wolinsky (1988), focuses on outcomes such that no pair of contracting partners has an incentive to alter the terms of its own contract, taking as given the other equilibrium contracts.[13]

More specifically, we define a "bargaining equilibrium" as follows. In the downstream market, each retailer sets its prices so as to maximize its profit, given the contracts signed with manufacturers and given the other retailers' equilibrium prices. In the upstream market, each manufacturer negotiates with each retailer a contract (any non-linear tariff is admissible) that: (i) maximizes their joint profit, given the contracts negotiated with their other partners[14] as well as the other retailers' equilibrium prices, and taking into account the impact of the negotiated contract on the retailer's own downstream behavior; and (ii) shares the surplus from a successful negotiation according to some pre-determined sharing-rule.

In this framework, we first establish the existence of an equilibrium, as well as the uniqueness of the downstream equilibrium outcome (as long as tariffs induce a "smooth behavior", in a sense that will be made clear). Equilibrium tariffs are cost-based, in that marginal wholesale prices reflect marginal costs of production, and as a result, retail prices are the same as in a multi-brand oligopoly where each retailer could produce all brands. The intuition is simple: as the terms of the contracts are not observed by rivals, and thus have no impact on rivals' retail prices, pricing at marginal cost makes each retailer a residual claimant, thereby inducing it to maximize its bilateral joint profit with each manufacturer. Interestingly, this insight is in line with the empirical analysis of a recent Norwegian merger. Nilsen *et al.* (2016) find that an upstream merger between egg producers did not have any impact on consumer prices (but increased retailers' payments to producers), and conclude that the merger did not

[12] See Rey and Vergé (2004).

[13] In the vertical contracting literature, O'Brien and Shaffer (1992) apply this approach to a setting where downstream price competitors interact with a single manufacturer. Since then it has been used with various restrictions, both in the theoretical literature (e.g., Gans, 2007; Milliou and Petrakis, 2007; Allain and Chambolle, 2011) and the empirical literature (e.g., Crawford and Yurukoglu, 2012; Grennan, 2013; Gowrisankaran *et al.*, 2015). Because it combines the cooperative Nash-bargaining solution (for each vertical pair) with a non-cooperative Nash-equilibrium concept (across pairs), Collard-Wrexler *et al.* (2015) have coined the terminology "Nash-in-Nash bargaining."

[14] This approach avoids nonexistence problems that arise, even with (publicly observable) simple two-part tariffs (see Rey and Vergé, 2010).

affect marginal input prices, but only infra-marginal prices (e.g., franchise fees). Our analysis also shows that different tariffs generate different divisions of the equilibrium industry profit, more convex (resp., concave) tariffs giving a large share to manufacturers (resp., retailers).

This "bargaining equilibrium" approach (as the simultaneous Nash-bargaining approach) takes the market structure as exogenously given.[15] In order to endogenize the market structure, we introduce a preliminary stage in which manufacturers and retailers simultaneously choose which channels they are willing to activate; thus, a channel becomes active when both parties so desire. This determines the market structure and the associated bargaining equilibrium. To avoid coordination issues, we focus on the coalition-proof Nash-equilibria (CPNE)[16] of this game. In a simple symmetric setting with successive duopolies, we first show that when retailers are highly differentiated (e.g., when they operate on different geographic markets), there is a unique CPNE, which involves interlocking relationships (i.e., all channels are active). When retailers are instead very close substitutes, the unique CNPE involves either exclusive dealing (each manufacturer dealing with a different retailer) or downstream foreclosure (both manufacturers dealing with a single, common retailer). In the particular case of linear demand functions, we show that the unique CPNE involves interlocking relationships when retailers are sufficiently differentiated, and exclusive dealing otherwise. This captures the intuition that when retailers compete intensely against each other, manufacturers prefer to distribute their brand through one retailer only so as to avoid dissipating retail profits through fierce intrabrand competition. Conversely, when retailers are sufficiently differentiated, each manufacturer finds it more attractive to have its brand distributed by both retailers, so as to maximize the demand for its product.

To illustrate the flexibility of our approach, we study the impact of vertical restraints such as resale price maintenance (RPM) and price parity agreements (PPAs). As with public contracts (Rey and Vergé, 2010), there exist many equilibria when RPM is allowed. The joint profit of a manufacturer and a retailer no longer depends on their "internal" wholesale price, as wholesale prices are no longer needed to control retail prices. However, the level of this wholesale price affects their other negotiations, thus giving rise to multiple equilibria: in essence, any vector of retail prices can be sustained in equilibrium. Furthermore, price floors, but also price ceilings can **raise equilibrium** retail prices, and thus have anticompetitive effects. Restricting attention to symmetric equilibria, minimum RPM is needed to sustain supra-competitive prices **when brands are more substitutable than stores**, whereas maximal RPM can **achieve this in the opposite case**. This **finding challenges** the

[15] Indeed, in equilibrium, all channels are active. A similar feature arises in de Fontenay and Gans (2014) and Collard-Wexler *et al.* (2015).

[16] See Bernheim *et al.* (1987).

current legal status of these vertical restraints, which treats minimum RPM as highly likely to be anti-competitive, and considers maximum RPM to be often pro-competitive. It also **challenges** with the common wisdom that strong inter-brand is usually sufficient to prevent any anti-competitive effects of vertical restraints.

We then consider the role of (retail) price parity agreements (i.e., contractual provisions requiring the retailer to charge the same price for all brands), and show that contrary to the idea that PPAs are akin to RPM and should thus be banned, such clauses have no impact on retail prices in our setting. Under PPAs, pricing at marginal cost still makes the retailer the residual claimant on the joint profit generated with a given manufacturer, although this profit may now be limited due to the constraint to charge the same prices for all brands. Therefore, equilibrium contracts are again cost-based.

Finally, we show that this traditional wholesale model (where manufacturers sell their products to retailers, who resell them and set their final prices) is similar to an agency model (where manufacturers sell their goods through retail platforms, which obtain transaction-based commissions): this amounts to turning the model "upside-down". Platforms play the role of upstream firms that sell distribution services to manufacturers, who control the retail prices, and thus play the role of downstream firms. Equilibrium commissions are again cost-based (the relevant cost now being the platforms' marginal cost of distribution), and the retail outcome is similar to that of a multi-platform oligopoly in which the manufacturers directly compete against each other at various retail locations. In a symmetric setting, whether equilibrium prices are higher under the wholesale model or the agency model simply depends on whether or not interbrand competition is more fierce than intrabrand competition. Moreover, price parity agreements (requiring manufacturers to set the same prices on all platforms) have no impact on consumer prices.

The paper is organized as follows. Section 2 outlines our setting, and Section 3 characterizes the equilibrium outcomes. Several extensions are then considered. Section 4 endogenizes the market structure, while Section 5 considers vertical restraints such as Resale Price Maintenance and Price Parity Agreements, and Section 6 discusses the agency model. Section 7 provides concluding remarks.

2 The Model

We consider a multilateral vertical relations setting in which $n \geq 2$ differentiated manufacturers, $M_1, ..., M_n$, distribute their goods through $m \geq 2$ differentiated retailers, $R_1, ..., R_m$. For the sake of exposition, we assume constant returns to scale at both stages of the vertical chain and denote M_i's unit cost by c_i, for any $i \in I \equiv \{1, ..., n\}$, and R_j's unit cost by γ_j, for any $j \in J \equiv \{1, ..., m\}$. The demand for brand $i \in I$ at store $j \in J$ (i.e., for "channel" $i - j$) is given by $D_{ij}(\mathbf{p})$,[17] where $D_{ij}(.)$ is continuously differentiable in the price vector $\mathbf{p} = (p_{ij})_{i \in I, J \in J}$ whenever it is positive.[18]

We assume that wholesale contracts are purely "vertical": the contract between M_i and R_j depends on the sales of brand i at R_j's stores, but cannot depend explicitly on the sales of M_h and/or R_k, for $h \in I \setminus \{i\}$ and $k \in J \setminus \{j\}$. This, in particular, excludes exclusive dealing provisions as well as "horizontal" clauses, such as market-share discounts. However, we allow for any non-linear tariff $t_{ij}(q_{ij})$.

Building on Hart and Tirole (1990), we moreover focus on secret contracting, and assume that the terms of the contract negotiated between M_i and R_j (and whether or not they did in fact reach an agreement at all) are information that is private to the two parties. Modelling secret contracting in multilateral relationships is tricky, even in the absence of upstream competition. In particular, when receiving an out-of-equilibrium offer from M_i, R_j needs to form beliefs about the contracts that its rivals will sign. Most of the existing literature relies on "reasonable" beliefs, such as passive beliefs (i.e., R_j continues to believe that its rivals have been offered the equilibrium contract) or wary beliefs (i.e., R_j believes that M_i offered to the other retailers the contracts that maximize M_i's profit, given the out-of-equilibrium offer that M_i made to R_j). Unfortunately, as shown by Rey and Vergé (2004), even in the absence of upstream competition, passive beliefs equilibria may not exist when retailers compete in price and are close enough substitutes, because multilateral deviations are then profitable for a manufacturer.[19] And while wary beliefs can be convenient in a Cournot setting, their analysis becomes much less tractable when retailers compete in price: in particular, the most profitable contract that M_i can offer to another retailer depends on that retailer's beliefs in case of a deviant offer, which in turn calls for a fixed-point approach.[20]

To avoid these technicalities and develop a tractable setting, we instead build on the "contract equilibrium" approach pioneered by Crémer and Riordan (1987) and Horn and Wolinsky (1988), and used by O'Brien and Shaffer (1992) in a context of downstream price competition. This approach can be seen as a refinement of the perfect Bayesian equilibrium

[17] For the sake of exposition, we will systematically use subscripts i and h for manufacturers, and j and k for retailers.

[18] That is, we allow for "kinks" (as in the case of a linear demand) at prices where demand becomes zero.

[19] McAfee and Schwartz (1995) show that an equilibrium with passive beliefs may also fail to exist when retailers compete in quantities.

[20] In Rey and Vergé (2004), we considered a simpler, upstream monopolist setting and established, for instance, the existence of a unique equilibrium with polynomial wary beliefs when demand is linear; yet, even for this particular case, we could not characterize the full set of equilibria with more general wary beliefs.

concept, checking for robustness against bilateral renegotiations by any pair $M_i - R_j$. A contract equilibrium can also be interpreted as the (stationary) subgame perfect equilibrium of an infinitely repeated game in which firms are impatient (i.e., **put no weight on future profits when making their decisions**) and, in each period, one pair (re-)negotiates its contract.

Finally, we also allow for balanced bargaining between manufacturers and retailers. The timing of wholesale negotiations and retail pricing decisions is as follows:

Stage 1 Each $M_i - R_j$ pair negotiates a non-linear tariff $t_{ij}(q_{ij})$. These bilateral negotiations are simultaneous and secret.

Stage 2 Retailers simultaneously set retail prices for every brand that they carry.

We look for the "bargaining equilibria" of this two-stage game, defined as follows. In the second stage, each retailer chooses its prices assuming that its rivals set the equilibrium retail prices. In the first stage, each $M_i - R_j$ pair negotiates a tariff $t_{ij}(q_{ij})$ that: (i) maximizes its joint profit, given the other equilibrium contracts and the resulting retail pricing behavior; and (ii) gives a share $\alpha_{ij} \in [0,1]$ of the additional profit generated by a successful negotiation to the manufacturer (and thus a share $1-\alpha_{ij}$ to the retailer).

To state this formally, let us denote by $\mathbf{p}_j = (p_{ij})_{i \in I}$ the vector of R_j's prices and by $\mathbf{p}_{-j}$ the vector of prices for all retailers other than R_j. When convenient, we express the price vector as $\mathbf{p} = (\mathbf{p}_j, \mathbf{p}_{-j})$; we sometimes further decompose R_j's price vector as $\mathbf{p}_j = (p_{ij}, \mathbf{p}_{-i,j})$,[21] using the notation $\mathbf{p}_j = (\infty, \mathbf{p}_{-i,j})$ when R_j does not carry M_i's brand. An equilibrium is then defined as follows:

Definition 1 *An equilibrium is a vector of price responses $\left(\mathbf{p}_j^R(\mathbf{t}_j)\right)_{j \in J}$, together with a vector of equilibrium tariffs $\mathbf{t}^* = \left(t_{ij}^*\right)_{i \in I, j \in J}$ and a vector of equilibrium prices $\mathbf{p}^* = \left(\mathbf{p}_j^*\right)_{j \in J}$, such that:*

- *In the second stage:*
  - *For every $j \in J$ and any vector of tariffs $\mathbf{t}_j = (t_{ij})_{i \in I}$ negotiated by R_j in the first stage, R_j's pricing strategy satisfies:*

$$\mathbf{p}_j^R(\mathbf{t}_j) \in \arg\max_{\mathbf{p}_j} \left\{ \sum_{i \in I} \left[(p_{ij} - \gamma_j) D_{ij}\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right) - t_{ij}\left(D_{ij}\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right)\right) \right] \right\}.$$

  - *The equilibrium prices and tariffs satisfy $\mathbf{p}_j^* = \mathbf{p}_j^R\left(\mathbf{t}_j^*\right)$.*

[21] More generally, we will sometimes decompose a vector $\mathbf{y}_j = (y_{hj})_{h \in I}$ as $(y_{ij}, \mathbf{y}_{-i,j})$, for $i \in I$.

- *In the first stage, each tariff* t_{ij}^**:*
  - *Maximizes the joint profit of* M_i *and* R_j*, taking as given* R_j*'s other equilibrium tariffs,* $\mathbf{t}_{-i,j}^*$*, as well as rivals' equilibrium prices,* $\mathbf{p}_{-j}^*$*, and* R_j*'s pricing strategy in the second stage,* $\mathbf{p}_j^R(\mathbf{t}_j)$*; that is:*

$$t_{ij}^* \in \arg\max_{t_{ij}} \left\{ \begin{array}{l} \left(p_{ij}^R\left(t_{ij}, \mathbf{t}_{-i,j}^*\right) - c_i - \gamma_j\right) D_{ij}\left(\mathbf{p}_j^R\left(t_{ij}, \mathbf{t}_{-i,j}^*\right), \mathbf{p}_{-j}^*\right) \\ + \sum_{k \in J\setminus\{j\}} \left[t_{ik}^*\left(D_{ik}\left(\mathbf{p}_j^R\left(t_{ij}, \mathbf{t}_{-i,j}^*\right), \mathbf{p}_{-j}^*\right)\right) - c_i D_{ik}\left(\mathbf{p}_j^R\left(t_{ij}, \mathbf{t}_{-i,j}^*\right), \mathbf{p}_{-j}^*\right)\right] \\ + \sum_{h \in I\setminus\{i\}} \left[\begin{array}{c} \left(p_{hj}^R\left(t_{ij}, \mathbf{t}_{-i,j}^*\right) - \gamma_j\right) D_{hj}\left(\mathbf{p}_j^R\left(t_{ij}, \mathbf{t}_{-i,j}^*\right), \mathbf{p}_{-j}^*\right) \\ - t_{hj}^*\left(D_{hj}\left(\mathbf{p}_j^R\left(t_{ij}, \mathbf{t}_{-i,j}^*\right), \mathbf{p}_{-j}^*\right)\right) \end{array}\right] \end{array} \right\}.$$

  - *Gives* M_i *and* R_j *shares* α_{ij} *and* $1-\alpha_{ij}$ *respectively, of the additional profit generated by their relationship.*

This equilibrium concept has some features of a perfect Bayesian Nash equilibrium with passive beliefs, as in the second stage each retailer chooses its prices assuming that its rivals remain under the equilibrium contracts, even if the retailer itself has negotiated an out-of-equilibrium contract. Likewise, in the first stage, each vertical channel negotiates efficiently, assuming that the other channels stick to the equilibrium contracts. This assumption is similar in spirit to the "market-by-market bargaining" restriction of Hart and Tirole (1990) and to the "passive beliefs" or "pairwise-proofness" assumption of McAfee and Schwartz (1994). Compared with a perfect Bayesian Nash equilibrium with passive beliefs, the above bargaining equilibrium concept offers more flexibility on how to share the gains from trade, but discards the possibility of multi-sided deviations.[22]

3 Equilibrium analysis

O'Brien and Shaffer (1992) show that with secret contracting, a monopolistic supplier cannot fully exploit its market power – even if it has all the bargaining power in its bilateral negotiations with retailers – due to opportunism: as contracts are secret, when the manufacturer negotiates with one retailer, it has an incentive to free-ride on the sales of the other retailers. In this section, we show that this insight also applies when there is competition, however imperfect, on the upstream market.

[22] See Rey and Vergé (2004) for a complete discussion.

3.1 Two-part tariffs

We first establish the existence of an equilibrium in which each $M_i - R_j$ pair signs a *cost-based two-part tariff* of the form:

$$t_{ij}^{*}\left(q_{ij}\right) = F_{ij}^{*} + c_i q_{ij},$$

for some appropriate fixed fee F_{ij}^{*}.

By construction, any such equilibrium yields the same outcome as a "multiproduct oligopoly" in which every retailer produces all brands at cost. For the sake of exposition, we will assume that this outcome is uniquely defined and "well-behaved." Let:

$$\pi_{ij}\left(\mathbf{p}\right) \equiv \left(p_{ij} - c_i - \gamma_j\right) D_{ij}\left(\mathbf{p}\right) \text{ and } \pi_j\left(\mathbf{p}\right) \equiv \sum_{i \in I} \pi_{ij}\left(\mathbf{p}\right)$$

respectively denote R_j's profit on brand i and R_j's total profit in this multiproduct oligopoly, and let:

$$\mathbf{p}_j^r\left(\mathbf{p}_{-j}\right) \equiv \arg\max_{\mathbf{p}_j} \pi_j\left(\mathbf{p}_j, \mathbf{p}_{-j}\right)$$

denote R_j's best-response to its rivals' prices, $\mathbf{p}_{-j}$. We will maintain the following Assumption:[23]

Assumption A: Multiproduct oligopoly. There is a unique price vector $\mathbf{p}^*$ satisfying $\mathbf{p}_j^* \in \mathbf{p}_j^r\left(\mathbf{p}_{-j}^*\right)$ for every $j \in J$; this vector is moreover uniquely characterized by the first-order conditions, and such that $\mathbf{p}_j^* = \mathbf{p}_j^r\left(\mathbf{p}_{-j}^*\right)$ for every $j \in J$.[24] Furthermore, for every $i \in I$ and every $j \in J$:

(i) $D_{ij}\left(\mathbf{p}^*\right) > 0$; and,

(ii) $\sum_{h \in I \setminus \{i\}} \pi_{hj}\left(\left(\infty, \mathbf{p}_{-i,j}^*\right), \mathbf{p}_{-j}^*\right) > \sum_{h \in I \setminus \{i\}} \pi_{hj}\left(\mathbf{p}^*\right)$.

Assumption A asserts that the multiproduct oligopoly has a unique Bertrand-Nash equilibrium, with features of product differentiation. As manufacturers are imperfect substitutes: (i) in equilibrium retailers carry all brands; but (ii) if a retailer were to delist one brand then some consumers would switch to rival brands, which would increase the retailer's profit on these brands.

Under Assumption A, any equilibrium that relies on cost-based two-part tariffs yields the same equilibrium retail prices, $\mathbf{p}^*$, and thus the same industry profit. Our first Proposition

[23] This Assumption and all those following are, for instance, satisfied when demand functions are linear.

[24] That is, (i) $\mathbf{p}^*$ is the unique solution to the set of first-order conditions $\left\{\partial\pi_j / \partial p_{ij} = 0\right\}_{i \in I, j \in J}$, and (ii) best-responses to equilibrium prices are also unique.

establishes the existence of such an equilibrium, and shows that the distribution of the profit is also uniquely defined. Let:

$$\pi_{ij}^* \equiv \pi_{ij}(\mathbf{p}^*) \text{ and } \pi_j^* \equiv \pi_j(\mathbf{p}^*) = \max_{\mathbf{p}_j} \pi_j\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right)$$

denote the equilibrium profits achieved by R_j on brand i and in total, respectively. We assume that profits are also well-defined in case a negotiation breaks down, that is:[25]

Assumption B: Default options. For every $i \in I$ and every $j \in J$,

$$\pi_j^{ij} \equiv \max_{\mathbf{p}_{-i,j}} \pi_j\left(\left(\infty, \mathbf{p}_{-i,j}\right), \mathbf{p}_{-j}^*\right)$$

is well-defined.

π_j^{ij} thus denotes the profit that R_j could achieve without brand i.[26] We have:

Proposition 1 *There exists a unique equilibrium in which all contracts are cost-based two-part tariffs; in this equilibrium:*

(i) Retail prices are equal to $\mathbf{p}^*$*, as with a multiproduct oligopoly; and,*

(ii) Manufacturers' and retailers' profits are respectively equal to, for $i \in I$ *and* $j \in J$*:*

$$\Pi_{M_i}^* = \sum_{j \in J} \alpha_{ij}\left(\pi_j^* - \pi_j^{ij}\right) \quad \text{and} \quad \Pi_{R_j}^* = \pi_j^* - \sum_{i \in I} \alpha_{ij}\left(\pi_j^* - \pi_j^{ij}\right) > 0,$$

where $0 < \pi_j^* - \pi_j^{ij} < \pi_{ij}^*$.

Proof. See Appendix A. ■

The intuition is simple. First, if all other channels adopt cost-based two-part tariffs, then the joint profit of M_i and R_j (gross of fixed fees) accounts for the full margin on R_j's sales (for all brands), and for none of the margins on all other retailers' sales. To maximize this profit, it suffices to make R_j the residual claimant, which can be achieved by adopting a cost-based two-part tariff. By construction, in any such equilibrium, each retailer R_j behaves as if it were supplied at cost, and thus retail prices are the same as with a multiproduct oligopoly.

Unsurprisingly, R_j appropriates all the profit when it has all the bargaining power, that is, when $\alpha_{ij} = 0$. Interestingly, however, when $\alpha_{ij} > 0$, R_j gets a larger share of the profits than its intrinsic bargaining power would suggest. As tariffs are cost-based, should the negotiation between M_i and R_j break down (*de facto* removing M_i's brand from R_j's store), then M_i

[25] In what follows, the superscript "ij" refers to situations where all channels but $i - j$ are active.

[26] Assumption B is, for instance, satisfied if the revenue function $r_j(\mathbf{q}_j)$ introduced below is strictly quasi-concave.

would not benefit from the increase in the sales of its product through the other retailers, whereas R_j would benefit from the increase in the demand for rival brands. As a result, R_j is able to extract more than a share $1-\alpha_{ij}$ of the equilibrium channel profit π^*_{ij}. Still, as dealing with each other enables M_i and R_j to increase their joint profit (namely, by $\pi^*_j - \pi^{ij}_j$), M_i obtains a share α_{ij} of this additional profit.

3.2 Equilibrium prices

Proposition 1 establishes the existence of a unique equilibrium in cost-based two-part tariffs, which yields the same outcome as a multiproduct oligopoly. We now show that, as long as they induce a "smooth" retail behavior, equilibrium tariffs must be "cost-based", in the sense that marginal wholesale prices must reflect marginal costs of production (i.e., equilibrium tariffs t^*_{ij} and equilibrium quantities q^*_{ij} are such that $t^{*\prime}_{ij}\left(q^*_{ij}\right)=c_i$ for every $i\in I$ and $j\in J$), and thus again yield the same outcome as a multiproduct oligopoly.

We first introduce the notion of smooth retail behavior. Fix a candidate equilibrium with tariffs $\mathbf{t}^e$ and retail prices $\mathbf{p}^e$, and consider R_j's behavior given the tariffs it faces, $\mathbf{t}^e_j$, and the other retailers' equilibrium prices, $\mathbf{p}^e_{-j}$. For any $\mathbf{q}_j=(q_{ij})_{i\in I}$, let $\bar{\mathbf{p}}_j(\mathbf{q}_j)=(\bar{p}_{ij}(\mathbf{q}_j))_{i\in I}$ denote the vector of inverse residual demands; that is, $\bar{\mathbf{p}}_j=\bar{\mathbf{p}}_j(\mathbf{q}_j)$ is such that:

$$D_{ij}\left(\bar{\mathbf{p}}_j,\mathbf{p}^e_{-j}\right)=q_{ij} \text{ for every } i\in I.$$

Using these inverse demands, deriving R_j's optimal response to the tariffs $\mathbf{t}^e_j$ amounts to choosing quantities $\mathbf{q}_j$ so as to maximize:

$$r_j(\mathbf{q}_j)-\sum_{i\in I} t_{ij}(q_{ij}),$$

where

$$r_j(\mathbf{q}_j)\equiv\sum_{i\in I}\left[\bar{p}_{ij}(\mathbf{q}_j)-\gamma_j\right]q_{ij}$$

denotes the retail revenue generated by R_j, net of its retail cost. Let:

$$Q_{ij}\left(\mathbf{p}^e_{-j}\right)=\left\{q_{ij}\in\mathbb{R}^*_+ \mid \exists\mathbf{p}_j \text{ such that } D_{ij}\left(\mathbf{p}_j,\mathbf{p}^e_{-j}\right)=q_{ij}\right\}$$

denote the set of possible positive quantities for channel M_i-R_j, given the other retailers' prices, and for any $q_{ij}\in Q_{ij}\left(\mathbf{p}^e_{-j}\right)$, let:

$$\hat{\mathbf{q}}_{-i,j}(q_{ij})\equiv\arg\max_{\mathbf{q}_{-i,j}}\left\{r_j(q_{ij},\mathbf{q}_{-i,j})-\sum_{h\in I\setminus\{i\}}t_{hj}(q_{hj})\right\}$$

denote R_j's optimal quantities for the other brands. We say that the equilibrium tariffs induce a "smooth" retail behavior when these internal best-responses are well-behaved, namely:

Definition 2 *The equilibrium tariffs* $\mathbf{t}_j^e$ *induce a smooth retail behavior if they are differentiable and, for any* $i \in I$*:*

(i) *The equilibrium quantity,* q_{ij}^e*, is characterized by the first-order condition; that is:*

$$t_{ij}'\left(q_{ij}^e\right) = \frac{\partial r_j}{\partial q_{ij}}\left(\mathbf{q}_j^e\right).$$

(ii) *For any* $q_{ij} \in Q_{ij}\left(\mathbf{p}_{-j}^e\right)$*,* R_j *has a unique internal best-response* $\hat{\mathbf{q}}_{-i,j}\left(q_{ij}\right)$*, which is differentiable and characterized by first-order conditions; that is, for every* $h \in I \setminus \{i\}$*:*

$$t_{hj}'\left(\hat{q}_{hj}\left(q_{ij}\right)\right) = \frac{\partial r_j}{\partial q_{hj}}\left(q_{ij}, \hat{\mathbf{q}}_{-i,j}\left(q_{ij}\right)\right).$$

Thus, the tariffs $\mathbf{t}_j^e$ induce a smooth retail behavior if, in response to a marginal change in the sales of one brand, R_j finds it optimal to only marginally adjust the sales of the other brands. Under standard assumptions on the revenue function r_j (e.g., $r_j(\cdot)$ is twice continuously differentiable and strictly concave in $\mathbf{q}_j$), two-part tariffs induce a smooth retail behavior.[27]

Finally, for any $i \in I$ and $k \in J \setminus \{j\}$, let $\tilde{q}_{ik}\left(q_{ij}\right)$ denote the quantity sold by M_i through R_k when R_j chooses to sell a quantity q_{ij} of M_i's product, and its internal best-response $\hat{\mathbf{q}}_{-i,j}\left(q_{ij}\right)$ for the other brands; that is:

$$\tilde{q}_{ik}\left(q_{ij}\right) \equiv D_{ik}\left(\bar{\mathbf{p}}_j\left(q_{ij}, \hat{\mathbf{q}}_{-i,j}\left(q_{ij}\right)\right), \mathbf{p}_{-j}^e\right).$$

We denote by $\mathbf{\Delta}^{(i)}$ the $m \times m$ matrix such that the term in row $j \in J$ and column $k \in J$ is given by:

$$\Delta_{j,k}^{(i)} = \begin{cases} 1 & \text{if } k = j, \\ \tilde{q}_{ik}'\left(q_{ij}^e\right) & \text{otherwise.} \end{cases} \tag{1}$$

The following Proposition shows that if we restrict attention to equilibria where tariffs induce a smooth retail behavior, then tariffs are necessarily cost-based:

Proposition 2 *Whenever the equilibrium tariffs induce all retailers to adopt a smooth retail behavior:*[28]

(i) If $\left|\mathbf{\Delta}^{(i)}\right| \neq 0$*, then* M_i*'s equilibrium tariffs are "cost-based", that is,* $t_{ij}^{e\prime}\left(q_{ij}^e\right) = c_i$ *for every* $j \in J$*; and,*

(ii) If $\left|\mathbf{\Delta}^{(i)}\right| \neq 0$ *for every* $i \in I$*, then* $\mathbf{p}^e = \mathbf{p}^*$*.*

[27] More generally, any vector of tariffs faced by a retailer induces that retailer to adopt a smooth retailer behavior, when each tariff involves a non-conditional fixed fee (that is, a fee that must be paid, regardless of whether any quantity is being sold), and a variable part that is twice continuously differentiable and weakly convex.

[28] Throughout the paper, for any matrix $\mathbf{M}$, the notation $|\mathbf{M}|$ refers to the determinant of that matrix.

Proof. See Appendix B. ■

The previous intuition thus carries over to any equilibrium in which marginal considerations are relevant. As the terms of wholesale agreements are not observed by rivals, and consequently have no impact on their own behavior, pricing at marginal cost makes retailers residual claimants, thereby inducing them to maximize joint bilateral profits. It follows that the equilibrium outcome again mimics that of a multiproduct oligopoly.

Note that the condition $\left|\mathbf{\Delta}^{(i)}\right| \neq 0$ is rather innocuous. For instance, a sufficient condition is for the matrix $\mathbf{\Delta}^{(i)}$ to be diagonally dominant, which amounts to $\sum_{k\in J\setminus\{j\}} \left|\tilde{q}'_{ik}\left(q^e_{ij}\right)\right| < 1$ for any $j \in J$. This is a plausible condition, as adjusting R_j's prices so as to increase its sales of brand i is likely to decrease the sales of that brand at other stores (i.e., $\tilde{q}'_{ik}\left(q^e_{ij}\right) \leq 0$ for $k \in J \setminus \{j\}$) but nevertheless it may increase the total sales of that brand (i.e., $1 + \sum_{k\in J\setminus\{j\}} \tilde{q}'_{ik}\left(q^e_{ij}\right) > 0$). In any event, even if that sufficient condition does not hold for some retailer(s), we would expect $\left|\mathbf{\Delta}^{(i)}\right| \neq 0$ to be satisfied generically.[29]

Remark: Smooth retail behavior. In the case of a monopolistic manufacturer, O'Brien and Shaffer (1992) have shown that equilibrium tariffs must indeed induce a smooth retail behavior. While their reasoning does not readily carry over to the case of multiple upstream firms, as retailers' responses to the tariffs offered by a given manufacturer now depend also on other manufacturers' tariffs (hence, retailers' profits may no longer be "smooth" if these other profits are themselves discontinuous or non-differentiable), we suspect that equilibrium tariffs are still likely to induce a smooth retail behavior.

3.3 Division of profits

Proposition 2 shows that, as long as tariffs induce all retailers to adopt a smooth retail behavior, equilibrium prices and quantities, and thus total industry profit, are the same as in a multiproduct oligopoly. Proposition 1 shows further that the division of this profit is also uniquely defined when two-part tariffs are used. However, other tariffs can sustain different profit allocations. To see this, our next proposition considers quadratic tariffs of the form:

$$t^{\delta}_{ij}\left(q_{ij}\right) = F_{ij}\left(\delta\right) + c_i q_{ij} + \delta\left(q_{ij} - q^*_{ij}\right)^2,$$

where $q^*_{ij} = D_{ij}\left(\mathbf{p}^*\right)$ and $F_{ij}\left(\delta\right)$ remains to be determined. For the sake of exposition, we will assume that these tariffs generate a smooth retail response, even if a negotiation breaks down; that is:

[29] That is, even if $\left|\Delta^{(i)}\right| = 0$ for a particular demand specification, the condition $\left|\Delta^{(i)}\right| \neq 0$ is likely to hold for arbitrarily close demand specifications.

Assumption A'. For δ not too negative and any $j \in J$, maximizing:

$$\pi_j\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right) - \sum_{i \in I} \delta\left[D_{ij}\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right) - q_{ij}^*\right]^2$$

with respect to $\mathbf{p}_j$ yields a unique price response, characterized by first-order conditions.

Assumption B'. For δ not too negative, any $i \in I$ and any $j \in J$:

(i) Maximizing:

$$\pi_j\left(\left(\infty, \mathbf{p}_{-i,j}\right), \mathbf{p}_{-j}^*\right) - \sum_{h \in I \setminus \{i\}} \delta\left[D_{hj}\left(\left(\infty, \mathbf{p}_{-i,j}\right), \mathbf{p}_{-j}^*\right) - q_{hj}^*\right]^2$$

with respect to $\mathbf{p}_{-i,j}$ yields a unique price reaction, denoted $\mathbf{p}_{-i,j}^{ij}(\delta)$, which is a continuous function of δ;

(ii) This price reaction is such that $D_{hj}\left(\left(\infty, \mathbf{p}_{-i,j}^{ij}(0)\right), \mathbf{p}_{-j}^*\right) \neq q_{hj}^*$ for some $h \in I \setminus \{i\}$ and $D_{ik}\left(\left(\infty, \mathbf{p}_{-i,j}^{ij}(0)\right), \mathbf{p}_{-j}^*\right) \neq q_{ik}^*$ for some $k \in J \setminus \{j\}$.

Assumption A' and the first part of Assumption B' are, for instance, satisfied when the revenue function $r_j(\mathbf{q}_j)$ is strictly concave. The last part of Assumption B' simply asserts that breaking down a negotiation between a manufacturer and a retailer affects the manufacturer's sales in at least one other retailer's stores, as well as the retailer's sales of at least one other brand. We then have:

Proposition 3 *There exists $\bar{\delta} > 0$ such that:*

(i) For any δ satisfying $|\delta| < \bar{\delta}$, there exists an equilibrium in which each pair $M_i - R_j$ signs a cost-based tariff of the form $t_{ij}^{\delta}(q_{ij})$, for some $F_{ij}(\delta)$, and all retail prices are equal to $\mathbf{p}^*$; and,

(ii) Within this class of equilibria, each M_i obtains a profit $\Pi_{M_i}(\delta)$, which is such that $\Pi_{M_i}(\delta) > \Pi_{M_i}^*$ (resp., $\Pi_{M_i}(\delta) < \Pi_{M_i}^*$) for $\delta > 0$ (resp., $\delta < 0$).

Proof. See Appendix C. ■

Hence, while there is a unique retail equilibrium outcome, replicating that of a multi-product oligopoly, manufacturers and retailers can share the resulting profit in various ways. With the above quadratic tariffs, manufacturers obtain a bigger share when marginal wholesale prices increase with the quantity being traded, as this degrades the retailers' outside options in case a negotiation breaks down. To see why, start with the equilibrium two-part tariffs $t_{ij}^*(q_{ij}) = F_{ij}^* + c_i q_{ij}$ used in Proposition 1, and introduce a convex term, $\delta\left(q_{ij} - q_{ij}^*\right)^2$

with $\delta > 0$, for some $i \in I$ and $j \in J$. Modifying the tariff in this way does not affect the amount paid by R_j if it sticks to the equilibrium quantity q^*_{ij}, but increases the amount that R_j would have to pay if it were to modify its prices and/or stop carrying another brand. It follows that introducing this convex term weakens R_j's bargaining position in its negotiations with the other suppliers. Conversely, manufacturers obtain a smaller share when the tariffs are concave (i.e., when $\delta < 0$).

4 Endogenizing the market structure

In previous section, we have assumed that all channels could be active. Given our focus on bargaining equilibrium, such an assumption implies that all channels are active in equilibrium. Indeed, given that all other channels are active, the "last" negotiating pair always finds it profitable to trade. Moreover, given that equilibrium tariffs are always cost-based due to secret contracting (and the definition of a bargaining equilibrium), there is no other possible equilibrium market structure. M_i and R_j always want to deal together, whatever the set of other active channels: the manufacturer is willing to accept any non-negative fixed fee to sell its product to the retailer, and the retailer is always willing to sell the brand if the fixed payment is sufficient low.

As we have seen, there is no equilibrium where a manufacturer only deals with one retailer, although fierce intra-brand competition dissipates all profits. This thus raises the quesion of the network formation since, when retailers are close substitutes, manufacturers would like to commit to eliminate intra-brand competition by dealing exclusively with one retailer. This contrasts with Collard-Wexler et al. (2015) who show an equivalent between Nash-in-Nash bargaining (i.e., bargaining equilibrium) and an extensive forrm-game with successive rounds of offers (until the no additonal pair of upstream and downstream firms would like to deal with each other). The reason is simply that, in our framework, some links may not generate positive gains from trade (a necessary condition for their equivalence result): indeed, when intra-brand competition is fierce, dealing with a second retailer, destroys profits rather than generating additional gains.

Although we cannot use an equivalence result between our bargaining framework and a reasonable extensive form game, we could have tried to solve the extensive form of the network formation game proposed by Collard-Wexler et al. (2015) in our context. This is unfornatunetaly not as simple as it may seems, even when we assume that each bilateral contract consists of a cost-based two-part tariff. We thus propose an alternative approach, that remains tractable and still allows manufacturers to credibly commit to deal exclusvely with one retailer when intra-brand competition is too fierce. This consists in keeping unchanged our bargaining *equilibrium framework (for a given set of available channels) but introducing a preliminary stage in which the market structure is endogenously determined.*

Formally we assume that, in this preliminary stage, manufacturers and retailers choose which channels they are willing to activate, with each firm having veto power. That is, each retailer (publicly) announces which manufacturer(s) it wishes to deal with (if any), and simultaneously each manufacturer (publicly) announces which retailer(s) it wishes to deal with (if any). A channel, say $i-j$, then becomes active if and only if the manufacturer and the retailer (here M_i and R_j) both wish to deal with the other. This determines the market structure, and each market structure yields a bargaining equilibrium defined along the same lines as before.

It is well-known that in the type of game considered in this preliminary stage, coordination problems give rise to a multiplicity of equilibria. In particular, as a channel can only be active when both parties decide to participate, there always exists a trivial equilibrium in which no channel becomes active. Likewise, there also exist equilibria in which only a few unconnected channels are active. Indeed, regardless of whether or not the other channels are active, M_i and R_j always benefit from opening channel $i-j$, and thus wish to do so if all other potential partners are unwilling to deal with them. In order to avoid these coordination issues, we focus on the Coalition-Proof Nash Equilibria (hereafter, CPNE) of this market structure formation game (see Bernheim *et al.* (1987)).

As the number of potential market structures grows geometrically with the number of market participants, in this section we focus on the simplest relevant case with two symmetric manufacturers, labelled M_A and M_B for convenience, and two symmetric retailers, R_1 and R_2. The symmetry assumption implies that manufacturers' unit costs are $c_A = c_B = c$, retailers' unit costs are $\gamma_1 = \gamma_2 = \gamma$, and, for any price vector $\mathbf{p} \equiv (p_{A1}, p_{B1}, p_{A2}, p_{B2})$, any $i \neq h \in \{A, B\}$ and any $j \neq k \in \{1, 2\}$, the demand for brand i at store j is given by:

$$D_{ij}(\mathbf{p}) \equiv D(p_{ij}, p_{hj}, p_{ik}, p_{hk}),$$

where the function $D(.)$ is continuously differentiable.

In addition, we assume that the bargaining sharing rules are symmetric, that is, $\alpha_{ij} = \alpha$ for every $i \in I$ and every $j \in J$.

Finally, we assume that total industry profit, equal to:

$$\sum_{i=A,B} \sum_{j=1,2} (p_{ij} - c_i - \gamma_j) D_{ij}(\mathbf{p}),$$

is maximal for a symmetric price vector $\mathbf{p}^M$, and let $\pi^M > 0$ denote the *per* channel monopoly profit.

4.1 Bargaining equilibria

First, we briefly characterize the continuation bargaining equilibria associated with each market structure.

When every firm activates at most one channel, the equilibrium is unique. It relies on cost-based tariffs, which pins downs retail equilibrium prices, and thus the profit generated by each channel. This profit is moreover shared by the two partners according to the $(\alpha, 1-\alpha)$ sharing rule.

When a single manufacturer deals with both retailers, O'Brien and Shaffer (1992) have shown that equilibrium tariffs are necessarily cost-based. Likewise, when a single retailer deals with both manufacturers, Bernheim and Whinston (1985, 1998) have shown that equilibrium tariffs are also cost-based. Thus, in both situations the retail prices as well as the industry profit are again uniquely defined. But there exist multiple equilibrium outcomes with different divisions of the industry profit between upstream and downstream firms, as the contract signed with a partner affects the bargaining position of a firm vis-à-vis its other partner. However, there exists a unique equilibrium in which all channels rely on (cost-based) two-part tariffs.

The analysis of the case in which three channels are active is more complex and similar to that of the case analyzed in the previous section, with all channels being active. Using arguments along the lines of those in the proofs of Propositions 1 and 2, it can be shown that when restricting attention to tariffs that induce a smooth behavior, equilibrium tariffs are again cost-based and equilibrium retail prices are thus uniquely defined. Furthermore, there always exists an equilibrium relying on (cost-based) two-part tariffs, and within the class of such equilibria, all firms' equilibrium payoffs are uniquely defined.

In the light of these observations, and to ensure that firms' payoffs are properly defined for any given market structure, throughout this section we focus on continuation equilibria based on two-part tariffs.[30] For each market structure, these equilibria can be described as follows:[31]

- **Bilateral Monopoly:** A single channel is active, say $i-j$. M_i's and R_j's profits are respectively $\Pi_{M_i} = \Pi_M^m \equiv \alpha\pi^m$ and $\Pi_{R_j} = \Pi_R^m \equiv (1-\alpha)\,\pi^m$, where:

$$\pi^m \equiv \max_p \,(p-c-\gamma)\, D\,(p,\infty,\infty,\infty)$$

  denotes the bilateral monopoly profit generated by a single active channel.

[30] The analysis that follows is thus valid when only two-part tariffs are allowed or feasible. However, it remains valid when firms simply favor two-part tariffs whenever they are indifferent between two-part tariffs or more general non-linear tariffs.

[31] For a complete proof, see the Web Appendix to this paper.

- **Exclusive Dealing**: Two unconnected channels are active, say $i-j$ and $h-k$. Individual profits are then $\Pi_{M_i} = \Pi_{M_h} = \Pi_M^{ED} \equiv \alpha\pi^{ED}$ and $\Pi_{R_j} = \Pi_{R_k} = \Pi_R^{ED} \equiv (1-\alpha)\,\pi^{ED}$, where:

$$\pi^{ED} \equiv \left(p^{ED} - c - \gamma\right) D\left(p^{ED}, \infty, \infty, p^{ED}\right)$$

denotes the profit generated by each channel, and the price p^{ED} is such that:

$$p^{ED} = \arg\max_p \left(p - c - \gamma\right) D\left(p, \infty, \infty, p^{ED}\right).$$

- **Upstream Foreclosure:** A single manufacturer, say M_i, deals with both retailers. Individual profits are then $\Pi_{M_i} = \Pi_M^{UF} \equiv 2\alpha\pi^{UF}$ and $\Pi_{R_j} = \Pi_{R_k} = \Pi_R^{UF} \equiv (1-\alpha)\,\pi^{UF}$, where the profit per channel is given by:

$$\pi^{UF} \equiv \left(p^{UF} - c - \gamma\right) D\left(p^{UF}, \infty, p^{UF}, \infty\right),$$

and the price p^{UF} is such that:

$$p^{UF} = \arg\max_p \left(p - c - \gamma\right) D\left(p, \infty, p^{UF}, \infty\right).$$

- **Downstream Foreclosure:** A single retailer, say R_j, deals with both manufacturers. Individual profits are then $\Pi_{M_i} = \Pi_{M_h} = \Pi_M^{DF} \equiv \alpha\left(2\pi^{DF} - \pi^m\right)$ and $\Pi_{R_j} = \Pi_R^{DF} \equiv 2(1-\alpha)\,\pi^{DF} + 2\alpha\left(\pi^m - \pi^{DF}\right)$, where the profit per channel is equal to:

$$\pi^{DF} \equiv \max_p \left(p - c - \gamma\right) D\left(p, p, \infty, \infty\right).$$

- **Connected Structure:** Only channel, say $h-k$, remains inactive. All firms are thus directly or indirectly connected, as the two retailers have a common manufacturer (namely, M_i), and one of them (R_j) also deals with the other manufacturer (M_k). Let p_J^{CS}, p_M^{CS} and p_R^{CS} denote the retail prices, where the subscripts J, M and R refer respectively to the *joint* channel of the two multi-partner firms (here, $M_i - R_j$), the other channel of the multi-partner *manufacturer* (here, $M_i - R_k$), and the other channel of the multi-partner retailer (here, $M_h - R_j$). We will also denote by:

$$\pi_m^{CS} \equiv \left(p_J^{CS} - c - \gamma\right) D\left(p_J^{CS}, p_R^{CS}, p_M^{CS}, \infty\right) + \left(p_R^{CS} - c - \gamma\right) D\left(p_R^{CS}, p_J^{CS}, \infty, p_M^{CS}\right)$$

the profit generated by the multi-partner retailer (R_j) and by:

$$\pi_s^{CS} \equiv \left(p_M^{CS} - c - \gamma\right) D\left(p_M^{CS}, \infty, p_J^{CS}, p_R^{CS}\right)$$

the profit generated by the single-partner retailer (R_k). Finally, let:

$$\hat{\pi}_J \equiv \max_p \left(p - c - \gamma\right) D\left(p, \infty, p_M^{CS}, \infty\right) \text{ and } \hat{\pi}_R = \max_p \left(p - c - \gamma\right) D\left(p, \infty, \infty, p_M^{CS}\right)$$

denote the profit that the multi-partner retailer (R_j) could generate by focusing instead, respectively, on the joint channel ($M_i - R_j$), and on the other channel ($M_h - R_j$). Manufacturers' profits are then respectively given by:

$$\Pi_{M_i} = \Pi_{Mm}^{CS} \equiv \alpha \left(\pi_m^{CS} + \pi_s^{CS} - \hat{\pi}_R\right) \text{ and } \Pi_{M_h} = \Pi_{Ms}^{CS} \equiv \alpha \left(\pi_m^{CS} - \hat{\pi}_J\right),$$

where the subscripts Mm and Ms respectively refer to the multi-partner and single-partner manufacturers. With a similar convention, retailers' profits are respectively given by:

$$\Pi_{R_j} = \Pi_{Rm}^{CS} \equiv (1-\alpha)\,\pi_m^{CS} + \alpha \left(\hat{\pi}_J + \hat{\pi}_R - \pi_m^{CS}\right) \text{ and } \Pi_{R_k} = \Pi_{Rs}^{CS} \equiv (1-\alpha)\,\pi_s^{CS}.$$

- **Interlocking Relationships:** All channels are active, in that every manufacturer deals with every retailer. From the previous analysis, retailers charge the same price p^*, and manufacturers and retailers' profits are respectively equal to:

$$\Pi_M^* = 2\alpha \left[2\pi(p^*) - \hat{\pi}(p^*)\right] \text{ and } \Pi_R^* = 2\left\{(1-\alpha)\,\pi(p^*) + \alpha\left[\hat{\pi}(p^*) - \pi(p^*)\right]\right\},$$

where $\pi(p) \equiv (p - c - \gamma)\,D(p,p,p,p)$ and $\hat{\pi}(p) \equiv \max_{\hat{p}} (\hat{p} - c - \gamma)\,D(\hat{p}, \infty, p, p)$.

As manufacturers' brands as well as retailers' stores are imperfect substitutes, some observations can be readily made:

- *Bilateral Monopoly and Downstream Foreclosure:* In both market structures, a single retailer is active and thus monopolizes the market. In the first case, it carries a single brand, whereas in the second case it carries both brands. Comparing the maximal industry profits that can be achieved with one channel, two channels using a common retailer, and all four channels, then yields:

$$4\pi^M > 2\pi^{DF} > \pi^m > \pi^{DF} > \pi^M > 0.$$

- *Upstream Foreclosure and Exclusive Dealing:* In both market structures, both retailers are active, each carrying a single brand with a cost-based tariff. The only difference between the two market structures is that in the first case retailers carry the same brand, whereas in the second case they carry different brands. As manufacturers' brands are differentiated, it follows that:

$$\pi^{ED} > \pi^{UF} > 0.$$

4.2 Equilibrium market structure

We now study the CPNE of the market structure formation game. For expositional purposes, we restrict attention to cases where both sides have some bargaining power (i.e., $\alpha \in]0,1[$).[32]

We first note that at least two channels must be active in equilibrium. If all channels were inactive, any pair $M_i - R_j$ could generate a profit by activating their channel ($\pi^m > 0$), and such a deviation would obviously be self-enforcing as both firms would benefit from it. And if only channel $i - j$ were active, then both M_h and R_k would benefit from activating their own channel (as $\pi^{ED} > 0$), making such deviation profitable and self-enforcing.

As $\pi^{ED} > \pi^{UF}$, retailers always prefer dealing with different manufacturers when they each carry a single brand. This implies that upstream foreclosure cannot arise in a CPNE. A coalition made of the excluded supplier (say, M_h) and one of the retailers (say, R_j) would be willing to deviate and activate their channel (possibly in addition to the channel $i - j$): M_h would benefit from such a deviation as it is otherwise fully excluded (and $\Pi_M^{ED}, \Pi_{Ms}^{CS} > 0$), and R_j would also benefit as it prefers dealing with a different supplier than R_k when they each carry a single brand (and thus $\max\left\{\Pi_R^{ED}, \Pi_{Rm}^{CS}\right\} \geq \Pi_R^{ED} > \Pi_R^{UF}$).

Therefore, in any CPNE, both brands must be distributed. Intuitively, it is valuable to have them distributed by both retailers when retailers are highly differentiated, in which case interlocking relationships are likely to arise in equilibrium. Instead, when retailers are close substitutes, distributing a brand through a second channel dissipates profits, as intrabrand competition then drives prices down to marginal cost, and the second channel does not attract any significant additional demand. We would thus expect each brand to be carried by a single retailer. The following Proposition confirms this intuition by considering two polar cases where retailers are either perfect substitutes (i.e., for each brand, consumers buy from the cheapest retailer) or not competing at all against each other (e.g., they are located in different geographic territories):[33]

Proposition 4 *(i) When retailers do not compete against each other, the unique CPNE market structure involves interlocking relationships; and,*

(ii) When instead retailers are perfect substitutes:

[32] When retailers have all the bargaining power (i.e., $\alpha = 0$), manufacturers' profits are always equal to 0, and coalition-proofness has little bite (as a coalition can never convince a manufacturer to deviate); note however that our analysis selects a unique equilibrium as α tends to 0. Our findings still apply when manufacturers have all the bargaining power (i.e., $\alpha = 1$), but the formal proofs need to be adjusted (some of the deviations used below would leave retailers indifferent, but other deviations are then relevant).

[33] While we have so far ruled out these extreme cases for expositional purposes, it is straightforward to extend the previous analysis, as long as manufacturers remain imperfect substitutes.

- *If* $\pi^{ED} > 2\pi^{DF} - \pi^{m}$*, the unique CPNE market structure involves exclusive dealing; and,*
- *If* $\pi^{ED} < 2\pi^{DF} - \pi^{m}$*, the unique CPNE market structure involves downstream foreclosure.*

Proof. See Appendix D. ■

The intuition is simple. When retailers do not compete intensely against each other, it is desirable to have each brand distributed by both channels, so as to maximize their demand. By contrast, when retailers are close substitutes, each brand is distributed by a single channel, so as to avoid profit dissipation through intrabrand competition. Exclusive dealing (where the two brands are distributed by different retailers) and downstream foreclosure (where both brands are distributed by a single, common retailer) then constitute self-enforcing agreements, and among these market structures the Pareto-efficient one (from the firms' standpoint) is coalition-proof.

Maybe somewhat surprisingly, the results do not depend on the manufacturers' and retailers' relative bargaining power α. The reason is two-fold. First, in any given market structure S, manufacturers' equilibrium profits are directly proportional to their bargaining power (that is, M_i's profit is of the form $\alpha\Pi_{M_i}(S)$, where $\Pi_{M_i}(S)$ does not depend on α). Therefore, this bargaining power does not affect manufacturers' preferences over alternative market structures. Second, retailers' preferences over the relevant market structures are also not affected by their bargaining power. For instance, when retailers do not compete against each other, they always prefer to deal with both manufacturers rather than with a single one. This generates more profit, and the threat of delisting one product also provides them with a better outside option. Hence, dealing with both manufacturers enables retailers to have a bigger share of a bigger pie, regardless of their relative bargaining power. When, instead, retailers are perfect substitutes, as mentioned above, the relevant comparison is between downstream foreclosure and exclusive dealing, as manufacturers want to deal with a single retailer. However, a selected retailer always prefers downstream foreclosure, in which case competition is less intense and thus industry profit is larger, and the threat of delisting one brand again gives the retailer a higher share of that profit.

To provide further results, we restrict our attention to linear demand functions. Normalizing marginal production and distribution cost to $c = \gamma = 0$, in the remainder of this section we assume that the (symmetric) inverse demand function is given by:

$$P\left(q_{ij}, q_{hj}, q_{ik}, q_{hk}\right) = 1 - q_{ij} - \mu q_{hj} - \rho q_{ik} - \mu\rho q_{hk}, \text{ with } 0 < \mu, \rho < 1.$$

For this linear demand function, it can be checked that $\pi^{ED} > 2\pi^{DF} - \pi^{m}$. Hence, when they both deal with a single retailer, manufacturers are better off dealing with different retailers than with the same one (that is, $\Pi_M^{ED} > \Pi_M^{DF}$). It follows that downstream foreclosure cannot arise in a CPNE, as a coalition made of the excluded retailer (say, R_k) and one of the manufacturers (say, M_i) would be willing to deviate and activate their channel (possibly in addition to the channel $i-j$). Such a deviation is indeed self-enforcing, as both firms benefit from it: R_k would otherwise be fully excluded (and $\Pi_R^{ED}, \Pi_{Rs}^{CS} > 0$) and M_i benefits from dealing with a different retailer than M_h (that is, $\max\left\{\Pi_M^{ED}, \Pi_{Mm}^{CS}\right\} \geq \Pi_M^{ED} > \Pi_M^{DF}$).

The above analysis implies that there is no CPNE in which a manufacturer and/or a retailer is fully excluded. Also, for further reference, it is interesting to note that for the above linear demand, there exists a threshold $\bar{\rho}(\mu) \in]0,1[$ such that a manufacturer is indifferent between "exclusive dealing" and being the multi-partner supplier in a "connected structure" (that is, $\pi^{ED} = \pi_m^{CS} + \pi_s^{CS} - \hat{\pi}_R$) if and only if $\rho = \bar{\rho}(\mu)$. Moreover, this threshold $\bar{\rho}(\mu)$ is a decreasing function of μ. Inspecting candidate CPNE for the remaining market structures (exclusive dealing, connected structure, interlocking relationships) yields the following result:

Proposition 5 *When the demand is linear, as specified above, there exists a unique CPNE market structure, characterized as follows:*

- *If $\rho < \bar{\rho}(\mu)$, then there is a unique CPNE, which yields interlocking relationships; and,*
- *If instead $\rho \geq \bar{\rho}(\mu)$, then exclusive dealing constitutes the unique CPNE market structure.*

Proof. See Appendix E. ■

This Proposition, illustrated by Figure 1, extends (for a linear demand) the insight of Proposition 4. When retailers are sufficiently differentiated (namely, when the retail differentiation parameter satisfies $\rho < \bar{\rho}(\mu)$), the unique CPNE involves interlocking relationships. If instead retailers are close enough substitutes (namely, $\rho \geq \bar{\rho}(\mu)$), then firms avoid intrabrand competition. As $\pi^{ED} > 2\pi^{DF} - \pi^{m}$ for the linear demand specification, the unique CPNE market structure then involves exclusive dealing. Interestingly, for the same two reasons as before, the analysis does not depend on the manufacturers and retailers' relative bargaining powers (α).[34]

[34] To limit the number of parameters, we have assumed that the price sensitivity of the inverse demand

5 Vertical restraints

We now revert to our general framework with n manufacturers and m retailers, allowing for asymmetry at both stages of the vertical chain, and consider the impact of vertical restraints, namely, resale price maintenance (RPM hereafter) and price parity agreements (PPAs hereafter). These provisions are commonly observed in practice and both have triggered heated policy debates, although while RPM has been in use for a very long time, PPAs have gained importance with the development of online platforms.

5.1 Resale price maintenance

We suppose here that manufacturers and retailers can adopt RPM provisions; that is, each $M_i - R_j$ pair can contract not only on a (non-linear) tariff $t_{ij}(q_{ij})$, but also – if the two firms wish to do so – on the retail price p_{ij}. The timing of wholesale negotiations and retail pricing decisions remains as before, with the caveat that in case of an RPM agreement between M_i and R_j, the retailer simply sets the agreed retail price p_{ij} in stage 2.

We first note that allowing firms to adopt RPM provisions does not destabilize the cost-based tariff equilibria identified above. Specifically, when the other channels sign cost-based tariffs, a cost-based tariff precisely induces the retail price that maximizes the joint profit of the manufacturer and the retailer,[35] and therefore they do not need to contract on the retail price. Retailers, however, get a lower share of the industry profit when RPM is used in equilibrium. This is because they can no longer adjust the prices they charge for the rival brands if their negotiations with a manufacturer were to fail. For instance, when dealing with M_i, R_j's disagreement payoff – and therefore, the equilibrium payoff – is lower when R_j has a RPM contract with other manufacturers.

RPM can, however, be used to sustain many other outcomes. To see this, suppose that bilateral profits are well-behaved when firms rely on two-part tariffs of the form $t_{ij}(q_{ij}) = F_{ij} + w_{ij}q_{ij}$. Namely:

Assumption C. For any $i \in I$ and $j \in J$, any wholesale prices $(w_{hk})_{(h,k)\neq(i,j)\in I\times J}$ and any

across both manufacturers and retailers is the product of the price sensititivities across manufacturers (μ) and across retailers (ρ). However, the same analysis, resulting in a similar figure, obtains when normalizing, for instance, the demand (e.g., so as to ensure that $P(q,q,q,q)$ remains constant as μ and ρ evolve), or when adopting a similar specification for the demand D rather than for the inverse demand P.

[35] For instance, in the equilibrium based on two-part tariffs characterized in Proposition 1, the equilibrium contract $t^*_{ij}(q_{ij}) = F^*_{ij} + c_i q_{ij}$ induces R_j to maximize the joint profit of the pair $M_i - R_j$.

prices $(p_{hk})_{(h,k)\neq(i,j)\in I\times J}$, the profit expression:

$$\left(p_{ij}-c_i-\gamma_j\right)D_{ij}(\mathbf{p})+\sum_{k\in J\setminus\{j\}}(w_{ik}-c_i)\,D_{ik}(\mathbf{p})+\sum_{h\in I\setminus\{i\}}\left(p_{hj}-w_{hj}-\gamma_j\right)D_{hj}(\mathbf{p})$$

is strictly quasi-concave[36] in p_{ij} and maximal for a finite price level.

Let $\mathbf{\Lambda}(\mathbf{p})$ denote the $nm\times nm$ matrix such that the term in row $l(i,j)\equiv(i-1)\,m+j$ and column $l(h,k)$, for $i,j\in I$ and $j,k\in J$, is given by:

$$\Lambda_{l(i,j),l(h,k)}(\mathbf{p})=\begin{cases}\frac{\partial D_{hj}}{\partial p_{ij}}(\mathbf{p}) & \text{if } h\neq i \text{ and } k=j,\\ -\frac{\partial D_{ik}}{\partial p_{ij}}(\mathbf{p}) & \text{if } h=i \text{ and } k\neq j,\\ 0 & \text{otherwise.}\end{cases}$$

We have:

Proposition 6 *When RPM is allowed:*

(i) There exists an equilibrium based on cost-based two-part tariffs and RPM, which replicates the multiproduct oligopoly prices and quantities, but gives retailers a lower share of profit than in the absence of RPM; and,

(ii) Any price vector $\mathbf{p}$ *such that* $|\mathbf{\Lambda}(\mathbf{p})|\neq 0$ *can be sustained in equilibrium.*

Proof. See Appendix F. ■

Proposition 6 shows that with RPM, many prices can arise in equilibrium. In particular, whenever $\left|\mathbf{\Lambda}\left(\mathbf{p}^M\right)\right|\neq 0$, RPM enables the firms to sustain monopoly prices. The proof is constructive, and consists of exhibiting two-part tariffs which, together with RPM, sustain the targeted prices.

The intuition is simple. By construction, the joint profit of M_i and R_j does not depend on the "internal" wholesale price w_{ij}. As it is no longer needed to "drive" the retail price p_{ij} (which can now be directly agreed upon through RPM), this internal wholesale price w_{ij} can thus be fixed in any arbitrary way, adjusting the fixed fee F_{ij} so as to share the profit as desired. However, this internal price affects M_i's negotiation with every other retailer R_k, as well as R_j's negotiation with every other manufacturer M_h, and can thus be set so as to sustain the targeted retail prices. As there are nm "instruments"(the wholesale prices) for nm "targets"(the retail prices), it follows that, generically, an equilibrium can be constructed around any profile of retail prices.

[36] If the demand for the channel $i-j$ drops to zero when the price p_{ij} is high enough, then the strict quasi-concavity should hold in the price range where $D_{ij}(\cdot)>0$. A similar comment applies to Assumptions D and E.

More precisely, in the absence of RPM and with cost-based tariffs, R_j takes into consideration the full margin on its sales; it thus chooses p_{ij} so as to maximize:

$$\pi_j(\mathbf{p}) = \sum_{h \in I} \left(p_{hj} - c_h - \gamma_j\right) D_{hj}(\mathbf{p}).$$

Let:

$$\mu_{ij}(\mathbf{p}) \equiv \frac{\partial \pi_j(\mathbf{p})}{\partial p_{ij}} = D_{ij}(\mathbf{p}) + \sum_{h \in I} \left(p_{hj} - c_h - \gamma_j\right) \frac{\partial D_{hj}}{\partial p_{ij}}(\mathbf{p}) \tag{2}$$

denote the impact of a marginal increase in p_{ij} on the above profit. In the absence of RPM, Assumption A implies that equilibrium retail prices $\mathbf{p}^*$ are such that $\mu_{ij}(\mathbf{p}^*) = 0$ for every $i \in I$ and every $j \in J$.

With RPM, p_{ij} is instead chosen by M_i and R_j, who, for given vectors of wholesale prices $\mathbf{w}_{i,-j} = (w_{ik})_{k \in J \setminus \{j\}}$ and $\mathbf{w}_{-i,j} = (w_{hj})_{h \in I \setminus \{i\}}$, now ignore the upstream margin on R_j's sales of any rival brand h, $w_{hj} - c_h$, but do account for the upstream margin on M_i's sales through any rival store k, $w_{ik} - c_i$. Hence, under Assumption C, and for any given retail price profile $\mathbf{p}$, to ensure that M_i and R_j stick to p_{ij} it suffices to pick $\mathbf{w}_{i,-j}$ and $\mathbf{w}_{-i,j}$ so as to satisfy their first-order condition. This amounts to satisfy:

$$\sum_{h \in I \setminus \{i\}} (w_{hj} - c_h) \frac{\partial D_{hj}}{\partial p_{ij}}(\mathbf{p}) - \sum_{k \in J \setminus \{j\}} (w_{ik} - c_i) \frac{\partial D_{ik}}{\partial p_{ij}}(\mathbf{p}) = \mu_{ij}(\mathbf{p}). \tag{3}$$

That is, the differential impact of a marginal increase of p_{ij} on the upstream margins of the channels $M_i - R_k$, for $k \in J \setminus \{j\}$, and $M_h - R_j$, for $h \in I \setminus \{i\}$, should off-set $\mu_{ij}(\mathbf{p})$. The condition $|\mathbf{\Lambda}(\mathbf{p})| \neq 0$ ensures the existence of a wholesale price vector $\mathbf{w}$ satisfying the above equations for every $i \in I$ and every $j \in J$, in which case these wholesale prices are moreover uniquely defined. Fixed fees are then uniquely determined through the bargaining sharing rule.

So far we have considered "full RPM," where a retailer is required to charge the exact price negotiated with the manufacturer. The analysis can also shed some light on the role of minimum RPM (i.e., price floors) and maximum RPM (i.e., price caps). For the sake of exposition, we will focus here on symmetric manufacturers and retailers,[37] and on symmetric equilibria, where $w_{ij} = w$ and $p_{ij} = p$ for every $i \in I$ and every $j \in J$. The condition $|\mathbf{\Lambda}(\mathbf{p})| \neq 0$ then amounts to $\lambda_M(p) \neq \lambda_R(p)$, where, letting $\mathfrak{p}$ denote the vector of prices such

[37] Symmetry among manufacturers means $c_i = c$ and $D_{ij}(\mathbf{p}) = D_{hj}\left(\sigma_{ih}^M(\mathbf{p})\right)$ for any $j \in J$ and any $i, h \in I$, where $\sigma_{ih}^M(\mathbf{p})$ is derived from $\mathbf{p}$ by swapping the prices of brands i and h in each retailer's stores. Likewise, symmetry among retailers means $\gamma_j = \gamma$ and $D_{ij}(\mathbf{p}) = D_{ik}\left(\sigma_{jk}^R(\mathbf{p})\right)$ for any $i \in I$ and any $j, k \in J$, where $\sigma_{jk}^R(\mathbf{p})$ is derived from $\mathbf{p}$ by swapping R_j's and R_k's prices for each brand.

that $p_{ij} = p$ for every $i \in I$ and every $j \in J$:

$$\lambda_M(p) \equiv \sum_{h \in I \setminus \{i\}} \frac{\partial D_{hj}}{\partial p_{ij}}(\mathfrak{p}) \text{ and } \lambda_R(p) \equiv \sum_{k \in J \setminus \{j\}} \frac{\partial D_{ik}}{\partial p_{ij}}(\mathfrak{p})$$

denote the impact of a change in the price of brand i in store j on, respectively, the sales of the others brands at store j (interbrand price sensitivity of demand) and on the sales of brand i in the other stores (intrabrand price sensitivity of demand).[38] Thus, whenever $\lambda_M(p) \neq \lambda_R(p)$, there exists an equilibrium based on two-part tariffs and RPM, in which all retail prices are equal to p and all wholesale prices are equal to $w = \bar{w}(p)$, where (using (3)):

$$\bar{w}(p) \equiv c + \frac{\mu(p)}{\lambda_M(p) - \lambda_R(p)}, \tag{4}$$

where $\mu(p) = \mu_{ij}(\mathfrak{p})$ denotes, as before, the marginal impact given by (2), of an increase in one retailer's price on the profit generated by that retailer when it faces cost-based tariffs.

To ensure that price caps or price floors induce the expected outcomes, we introduce the following regularity conditions:

Assumption D. For any $p > p^*$ such that $\lambda_M(p) \neq \lambda_R(p)$:

(i) For any $j \in J$, R_j's gross profit $\sum_{i \in I} (p_{ij} - \bar{w}(p) - \gamma) D_{ij}(\mathbf{p}_j, \mathfrak{p}_{-j})$ is strictly quasi-concave in $\mathbf{p}_j = (p_{ij})_{i \in I}$; and,

(ii) the function $\mu(p)$ satisfies $\mu'(p) < 0$.

Finally, to rule out large deviations in the bilateral negotiations, we introduce another technical assumption. For any $p > p^*$ and $w \neq c$, for any $i \in I$ and any $j \in J$, let denote by $\hat{\mathbf{p}}^{ij}(p_{ij}; w, p) = \left(\hat{p}_h^{ij}(p_{ij}; w, p)\right)_{h \in I \setminus \{i\}}$ the prices that R_j would like to charge on the other brands, conditional on charging p_{ij} for brand i and on facing price caps (if $w > c$) or price floors (if $w < c$) set to p on the other brands; that is:

- If $w > c$,

$$\hat{\mathbf{p}}^{ij}(p_{ij}; w, p) \equiv \begin{array}{c} \arg\max\limits_{\mathbf{p}_{-i,j}} \sum\limits_{h \in I} (p_{hj} - w - \gamma) D_{hj}((p_{ij}, \mathbf{p}_{-i,j}), \mathfrak{p}_{-j}) \\ \text{s.t. } p_{hj} \leq p \text{ for any } h \in I \setminus \{i\}. \end{array}$$

- If $w < c$,

$$\hat{\mathbf{p}}^{ij}(p_{ij}; w, p) \equiv \begin{array}{c} \arg\max\limits_{\mathbf{p}_{-i,j}} \sum\limits_{h \in I} (p_{hj} - w - \gamma) D_{hj}((p_{ij}, \mathbf{p}_{-i,j}), \mathfrak{p}_{-j}) \\ \text{s.t. } p_{hj} \geq p \text{ for any } h \in I \setminus \{i\}. \end{array} .$$

[38] The symmetry assumptions ensure that these parameters are also symmetric.

We now assume that, when all other retailers are charging prices equal to p, and R_j faces price caps (if $w > c$) or price floors (if $w < c$) set to p all brands except brand i, the joint-profit of M_i and R_j remains well-behaved, even if the price cap/floor faced by R_j on at least one of the other brands is no longer binding. Namely:

Assumption E. For any $i \in I$ and $j \in J$, any wholesale price w and any retail price p, the profit expression:

$$\left(p_{ij} - c - \gamma\right) D_{ij}\left(\left(p_{ij}, \hat{\mathbf{p}}^{ij}\left(p\right)\right), \mathfrak{p}_{-j}\right) + \sum_{k \in J \setminus \{j\}} \left(w - c\right) D_{ik}\left(\left(p_{ij}, \hat{\mathbf{p}}^{ij}\left(p\right)\right), \mathfrak{p}_{-j}\right)$$
$$+ \sum_{h \in I \setminus \{i\}} \left(\hat{p}_h^{ij}\left(p_{ij}; w, p\right) - w - \gamma\right) D_{hj}\left(\left(p_{ij}, \hat{\mathbf{p}}^{ij}\left(p\right)\right), \mathfrak{p}_{-j}\right),$$

is strictly quasi-concave in p_{ij} and maximal for a finite price level.

We then have:

Proposition 7 *Restricting attention to symmetric equilibria, minimum RPM (resp., maximum RPM) can be used to sustain higher equilibrium prices than in the absence of any price restraint when there is more (resp., less) substitution among manufacturers' brands than among retailers' stores, that is, when $\lambda_M(p) > \lambda_R(p)$ (resp., $\lambda_M(p) < \lambda_R(p)$).*

Proof. See Appendix G. ■

To understand the underlying intution, consider first the retail pricing decisions. If retailers were free to set their prices, they would do so taking into consideration their downstream margins but ignoring their partners' upstream margins. Hence, if upstream margins are positive, classic double marginalization problems arise: the price of any brand at any store would be higher than what would maximize the joint profit of the manufacturer and the retailer, and price caps are therefore needed. Conversely, if upstream margins are negative, retailers would be tempted to adopt too low prices, and price floors are thus needed.

The next step is to determine whether positive or negative upstream margins are needed to sustain supra-competitive retail prices. If tariffs were cost-based, each negotiating pair would aim at maximizing the profit generated by the retailer's sales (on all brands); but then, each pair would have an incentive to undercut the others' prices.[39] When relying instead on a wholesale price $w \neq c$, each pair moreover takes into account the impact of their joint decision on the manufacturer's margins earned on the sales of its brand at the other stores, which, in

[39] To see this formally, consider a situation where all retail prices are equal to $p > p^*$. By construction, $\mu(p^*) = 0$, and thus, from Assumption D(ii), $\mu(p) < 0$ for $p > p^*$.

a symmetric situation, is given by

$$(w-c) \sum_{k \in J \setminus \{j\}} \frac{\partial D_{ik}}{\partial p_{ij}} (\mathfrak{p}) = (w-c) \lambda_R (p),$$

but however ignores the impact of their decision on the upstream margins earned on the retailer's sales of the other brands, which is given by

$$(w-c) \sum_{h \in I \setminus \{i\}} \frac{\partial D_{hj}}{\partial p_{ij}} (\mathfrak{p}) = (w-c) \lambda_M (p).$$

Therefore, in order to sustain the equilibrium price (i.e., discourage undercutting it), the net balance of these two effects should be positive, which amounts to

$$(w-c) [\lambda_R (p) - \lambda_M (p)].$$

It follows that in order to raise prices above p^*, negative upstream margins are required when $\lambda_M (p) > \lambda_R (p)$, in which case price floors are needed to counter retailers' excessive incentives to lower prices; when instead $\lambda_M (p) < \lambda_R (p)$, positive upstream margins are required, and price caps are then needed to counter retailers' excessive incentives to raise prices.[40]

Remark: Price caps and price floors. Moving from full RPM to price floors or price caps may also affect the division of profit, as R_j's disagreement payoffs may be affected. If the negotiation between M_i and R_j were to fail, R_j would be tempted to react by optimally revising the retail prices $\mathbf{p}_{-i,j}$ it charges the other brands. Such adjustment is impossible under full RPM, but may become feasible under a price floor or price ceiling. When such a change is indeed feasible, R_j's disagreement payoffs – and thus the equilibrium division of profit – are affected.

5.2 Price Parity Agreements

We now turn to the role of price parity agreements (PPAs). A PPA is a contractual provision requiring the retailer to price the manufacturer's brand at the same level as competing brands. Variants of such PPAs may be slightly less restrictive and simply prevent the retailer from charging less for competing brands, or more for competing brands.

These provisions have recently triggered debates about their potential anti-competitive effects. In April 2010, the UK Office of Fair Trading (OFT) imposed £225 million fines against tobacco manufacturers and retailers over retail pricing strategies. The OFT considered that

[40] Price floors thus have no effect in this case; by contrast, Allain and Chambolle (2011) find that industry-wide price floors are always anticompetitive.

manufacturers and retailers had entered into bilateral agreements linking the retail price of a tobacco brand to the prices of competing brands (at the same stores). Those retail price parity agreements were deemed to be anti-competitive by the OFT, who judged that they had the same adverse effects as RPM.[41]

We now show that in our framework, a PPA is not a substitute for RPM. To see this, we adapt the previous two-stage game of wholesale negotiations and retail pricing decisions as follows:

- In the first stage, each $M_i - R_j$ pair can also adopt a PPA (in addition to agreeing on a tariff $t_{ij}(q_{ij})$); and,
- In the second stage, a retailer that has accepted a PPA must set the same retail price for all the brands it carries.

Obviously, imposing uniform prices across brands can affect retailers' pricing behavior when they would otherwise wish to charge asymmetric prices. In particular, the "internal best responses" introduced in Section 3.2 are now given by:

$$\bar{q}_{hj}(q_{ij}) = D_{hj}\left(\bar{\mathbf{p}}_j(q_{ij}), \mathbf{p}^e_{-j}\right),$$

where $\mathbf{p}^e = \left(p^e_{ij}\right)_{i\in I, j\in J}$ is the vector of equilibrium prices and the price vector $\bar{\mathbf{p}}_j(q_{ij}) = (\bar{p}_j(q_{ij}), ..., \bar{p}_j(q_{ij}))$ is such that:

$$D_{ij}\left(\bar{\mathbf{p}}_j(q_{ij}), \mathbf{p}^e_{-j}\right) = q_{ij}.$$

Assumption F. For every $i \in I$ and every $j \in J$, whenever it is positive, the demand function $D_{ij}(\mathbf{p})$ satisfies:

(i) $\sum_{h\in I} \partial D_{ij}(\mathbf{p}) / \partial p_{hj} < 0$;

(ii) $\sum_{h\in I} \partial D_{ij}(\mathbf{p}) / \partial p_{hk} > 0$ for any $k \in J \setminus \{j\}$; and,

(iii) In addition, $\sum_{h\in I} \sum_{k\in J} \partial D_{ik}(\mathbf{p}) / \partial p_{hj} < 0$.

Assumption F is rather innocuous and simply relies on products being differentiated. Part (i) requires that R_j's sales of M_i's brand decrease when R_j uniformly increases the price of all brands, whereas part (ii) assumes that the same sales increase when a rival retailer uniformly increases its prices. Finally, part (iii) ensures that when R_j uniformly increases all of its prices, the total sales of M_i's brand through all retailers decreases (i.e., the direct effects on the sales through R_j dominates).

[41] See Decision CA98/01/2010 of the Office of Fair Trading, Case CE/2596-03: Tobacco, 15 April 2010. This decision was later quashed by the Competition Appeals Tribunal (see the CAT Judgement [2011] CAT 41, 12 December 2011), who however did not discuss the possible anticompetitive effects of PPAs.

The following proposition shows that firms cannot strategically use PPAs to depart from cost-based tariffs, and thus cannot affect the equilibrium outcome beyond imposing symmetry:

Proposition 8 *In the class of equilibria based on differentiable tariffs and price parity agreements where all equilibrium quantities are positive:*

(i) Equilibrium tariffs are all cost-based, that is, marginal wholesale prices reflect marginal costs of production; and,

(ii) If firms are symmetric at both stages of the vertical chain,[42] *then all prices are the same as if in the absence of any price parity agreement.*

Proof. See Appendix H. ■

The adoption of PPAs thus does not affect the previous analysis. Pricing at marginal cost again makes a retailer the residual claimant for the profit it can generate together with a given manufacturer – even if this profit is limited due to the imposition of uniform prices – and thus induces the retailer to maximize this joint profit (possibly subject to the uniform price restriction). It follows that in equilibrium, all contracts are cost-based.

Remark: Smooth tariffs. Proposition 8 is more general than Proposition 2 as it applies to all equilibria based on differentiable tariffs, regardless of whether or not they would induce a "smooth retail behavior" in the absence of PPAs. The reason is that by imposing uniform prices across brands, PPAs *de facto* ensure that retail behavior will be "smooth." By the same token, the assumption $\left|\mathbf{\Delta}^{(i)}\right| \neq 0$ (or, more precisely, its equivalent, replacing $\tilde{q}_{ik}(q_{ij})$ with $D_{ik}\left(\bar{\mathbf{p}}_j(q_{ij}), \mathbf{p}^e_{-j}\right)$) always holds when retailers are subject to PPAs.[43]

Remark: Price caps and price floors. The above analysis focuses on "pure" PPAs, which require retailers to charge the same price for all brands; any manufacturer can thus unilaterally impose this price uniformity. As mentioned above, in practice a variant consists of preventing retailers from charging prices that exceed those of rival brands. Obviously, the outcome is the same as with pure PPAs when all manufacturers adopt this variant, as retailers are then *de facto* constrained to charge the same price for all brands. While this paper does not formally study the case where a limited number of manufacturers adopt this variant, it should be clear that the proof of Proposition 8 readily extends to this case. A similar comment applies when retailers are instead required to charge no less than for rival brands, or when a limited number of retailers are subject to a PPA or one of its variants.

[42] See Footnote 37 for a precise expression of this symmetry assumption.

[43] That is, while Proposition 2 relies on the analysis of the "internal best response" $\hat{\mathbf{q}}_{-i,j}(q_{ij})$, Proposition 8 relies instead on the mechanical impact that a change in the quantity q_{ij} will have on the quantities $\bar{\mathbf{q}}_{-i,j}$ of the other brands sold by R_j, given that R_j has to charge the same price $\bar{p}_j(q_{ij})$ for all brands.

6 Agency model

We have been focussing so far on the "resale" business model, where the distributor buys the goods and/or services from the suppliers, and then resells them to consumers (hence, absent RPM, it is the distributor who sets consumer prices). If such a model is standard for "brick-and-mortar" retailers, online retail platforms often adopt instead an "agency" business model in which the supplier remains the owner of its goods and/or services, and chooses the prices at which it offers them on the platforms; each distributor then obtains commissions on the sales made through its platform.

To study this agency business model within our framework, in this section we adapt the timing of negotiations and pricing decisions as follows:

1. Each $M_i - R_j$ pair negotiates a (possibly non-linear) commission schedule $\tilde{t}_{ij}(q_{ij})$, based on the volume of sales q_{ij} achieved by M_i through R_j's platform. As before, these bilateral negotiations are simultaneous and secret; and,

2. Each M_i sets the retail prices for its product on each platform that carries it; in this section we will refer to M_i's prices as $\tilde{\mathbf{p}}_i = (\tilde{p}_{ij})_{j \in J}$.

The bargaining equilibria of this game are defined accordingly. In the second stage (retail pricing decisions), each manufacturer chooses its prices assuming that its rivals set the equilibrium retail prices, $\tilde{\mathbf{p}}^*_{-i} = \left(\tilde{p}^*_{hj}\right)_{h \in I \setminus \{i\}, j \in J}$. In the first stage, each $M_i - R_j$ pair negotiates a schedule $\tilde{t}_{ij}(q_{ij})$ that: (i) maximizes its joint profit, given the other equilibrium contracts and the resulting retail pricing behavior; and (ii) gives a share $\alpha_{ij} \in [0,1]$ of the additional profit generated by a successful negotiation to the manufacturer (and thus a share $1 - \alpha_{ij}$ to the retailer).

Formally, a bargaining equilibrium is a vector of price responses $\left(\tilde{\mathbf{p}}^R_i\left(\tilde{\mathbf{t}}_i\right)\right)_{i \in I}$, together with a vector of equilibrium commission schedules $\tilde{\mathbf{t}}^* = \left(\tilde{t}^*_{ij}\right)_{i \in I, j \in J}$ and a vector of equilibrium prices $\tilde{\mathbf{p}}^* = (\tilde{\mathbf{p}}^*_i)_{i \in I}$ such that:

- In the second stage:
  - For every $i \in I$ and any vector of schedules $\tilde{\mathbf{t}}_i = \left(\tilde{t}_{ij}\right)_{j \in J}$ negotiated by M_i in the first stage, M_i's pricing strategy is given by:

$$\tilde{\mathbf{p}}^R_i\left(\tilde{\mathbf{t}}_i\right) \in \arg\max_{\tilde{\mathbf{p}}_i} \left\{ \sum_{j \in J} \left[(\tilde{p}_{ij} - c_i) D_{ij}\left(\tilde{\mathbf{p}}_i, \tilde{\mathbf{p}}^*_{-i}\right) - \tilde{t}_{ij}\left(D_{ij}\left(\tilde{\mathbf{p}}_i, \tilde{\mathbf{p}}^*_{-i}\right)\right) \right] \right\}.$$

  - The equilibrium prices and commission schedules satisfy $\tilde{\mathbf{p}}^*_i = \tilde{\mathbf{p}}^R_i\left(\tilde{\mathbf{t}}^*_i\right)$; and,

- In the first stage, each schedule $\tilde{t}^*_{ij}$:
  - Maximizes the joint profit of M_i and R_j, taking as given M_i's other equilibrium schedules, $\tilde{\mathbf{t}}^*_{i,-j}$, rivals' equilibrium prices, $\tilde{\mathbf{p}}^*_{-i}$, and M_i's pricing strategy in the second stage, $\tilde{\mathbf{p}}^R_i(\tilde{\mathbf{t}}_i)$:

$$\begin{aligned}\tilde{t}^*_{ij} \in \arg\max_{\tilde{t}_{ij}} \; & \Big\{ \left(\tilde{p}^R_{ij}\left(\tilde{t}_{ij}, \tilde{\mathbf{t}}^*_{i,-j}\right) - c_i - \gamma_j\right) D_{ij}\left(\tilde{\mathbf{p}}^R_i\left(\tilde{t}_{ij}, \tilde{\mathbf{t}}^*_{i,-j}\right), \tilde{\mathbf{p}}^*_{-i}\right) \\ & + \sum_{h\in I\setminus\{i\}} \left[\tilde{t}^*_{hj}\left(D_{hj}\left(\tilde{\mathbf{p}}^R_i\left(\tilde{t}_{ij}, \tilde{\mathbf{t}}^*_{i,-j}\right), \tilde{\mathbf{p}}^*_{-i}\right)\right) - \gamma_j D_{hj}\left(\tilde{\mathbf{p}}^R_i\left(\tilde{t}_{ij}, \tilde{\mathbf{t}}^*_{i,-j}\right), \tilde{\mathbf{p}}^*_{-i}\right)\right] \\ & + \sum_{k\in J\setminus\{j\}} \left[\begin{array}{c} \left(\tilde{p}^R_{ik}\left(\tilde{t}_{ij}, \tilde{\mathbf{t}}^*_{i,-j}\right) - c_i\right) D_{ik}\left(\tilde{\mathbf{p}}^R_i\left(\tilde{t}_{ij}, \tilde{\mathbf{t}}^*_{i,-j}\right), \tilde{\mathbf{p}}^*_{-i}\right) \\ -\tilde{t}^*_{ik}\left(D_{ik}\left(\tilde{\mathbf{p}}^R_i\left(\tilde{t}_{ij}, \tilde{\mathbf{t}}^*_{i,-j}\right), \tilde{\mathbf{p}}^*_{-i}\right)\right) \end{array}\right]\Big\}.\end{aligned}$$

  - Gives M_i and R_j shares α_{ij} and $1-\alpha_{ij}$, respectively, of the additional profit generated by their relationship.

It is straightforward to see that this definition of a bargaining equilibrium amounts to turning the previous framework "upside-down": manufacturers are now downstream (they control retail prices), whereas retailers/platforms are upstream. As before, however, commissions are non-linear payment schedules paid by downstream firms (here, the manufacturers) to their upstream partners (the retailers).

We thus simply need to adapt our initial assumptions to conclude that as long as commissions induce a "smooth" retail pricing behavior by manufacturers, equilibrium commissions are cost-based and the outcome is similar to that of a multi-store oligopoly in which n firms directly compete against each other at m retail locations. Formally, the modified assumption is:

Assumption $\tilde{\mathbf{A}}$: Multi-store oligopoly. There is a unique price vector $\tilde{\mathbf{p}}^*$ satisfying $\tilde{\mathbf{p}}_i \in \tilde{\mathbf{p}}^r_i(\tilde{\mathbf{p}}_{-i}) \equiv \arg\max_{\tilde{\mathbf{p}}_i}\left\{\sum_{j\in J}\left(\tilde{p}_{ij} - c_i - \gamma_j\right) D_{ij}(\tilde{\mathbf{p}})\right\}$ for every $i \in I$; it is characterized by first-order conditions and such that $\tilde{\mathbf{p}}^*_i = \tilde{\mathbf{p}}^r_i\left(\tilde{\mathbf{p}}^*_{-i}\right)$ for every $j \in J$, and $D_{ij}(\tilde{\mathbf{p}}^*) > 0$ for every $i \in I$ and every $j \in J$.

Under Assumption $\tilde{\text{A}}$, and in the class of contracts inducing the manufacturers to adopt a smooth pricing behavior, all commission schedules must be cost-based, in the sense that marginal commission rates must reflect marginal costs of distribution; hence, the equilibrium outcome replicates that of direct competition between multi-store firms (that is, $\tilde{\mathbf{p}} = \tilde{\mathbf{p}}^*$). Moreover in this framework, price parity agreements (i.e., agreements between manufacturers and retailers requiring that manufacturers set the same prices on all platforms) have no impact on the equilibrium outcome beyond imposing symmetry. More precisely, equilibrium tariffs are once again cost-based in the sense that marginal commissions reflect marginal costs of distribution (i.e., the intermediaries' costs). In addition, when firms are symmetric at both stages of the vertical chains (and the equilibrium prices are symmetric in the absence of PPAs), then price parity agreements do not affect the equilibrium retail prices either.

The result that Price Parity Agreements (PPAs) have no impact on prices in the agency model contrasts with the recent literature on these agreements. However, so far this literature has focused on either linear commissions[44] or constant revenue-sharing rules,[45] which generate contractual inefficiencies; instead, we allow for general non-linear commissions and thus for efficient bilateral contracting. Foros *et al.* (2016) also consider constant revenue-sharing rules but study the platforms' choice between setting final prices (traditional wholesale model) or delegating these pricing decisions to suppliers (agency model). They show that a coordination failure may arise, whereby the agency model may fail to be adopted (even though it would increase all firms' profits); PPAs can then be used to facilitate the adoption of the agency model, thus leading to higher prices for consumers.

7 Conclusion

In this paper, we develop a framework for the analysis of multilateral vertical relations. The key features are secret bilateral negotiations of wholesale tariffs, followed by downstream price competition. The setting is sufficiently flexible to allow for any number of firms at each stage of the vertical chain, differentiated demand for each channel, and any given bargaining power within each channel. In addition, no restriction is imposed on the tariffs that can be signed. Finally, while for the sake of exposition we refer to upstream firms as manufacturers and to downstream firms as retailers, the framework can be applied to other vertical chains, for example, aircraft or car manufacturers and their subcontractors, media content and TV channels, hospitals and health insurance providers, and so forth.

An appealing feature of this framework lies in its tractability. We show that equilibrium tariffs are cost-based whenever they are differentiable and induce downstream firms to adopt a "smooth" behavior (i.e., a small change in the quantity sold for one brand by a retailer triggers only small changes in the quantities sold for the other brands by that same retailer). The equilibrium retail prices and quantities are thus uniquely defined and correspond to the outcome of price competition between differentiated multiproduct firms. However, the division of profit depends on the shape of the equilibrium tariffs: downstream firms get a

[44] See Boik and Corts (2016) and Johansen and Vergé (2016).
[45] See Johnson (2015).

higher (resp., lower) share of the industry profit when tariffs are convex (resp., concave).

To illustrate the versatility of this framework, we then consider several extensions. We first endogenize the market structure by introducing a preliminary stage in which upstream and downstream firms choose which channels they are willing to activate. To obtain a more complete characterization, we restrict attention to successive symmetric duopolies. In the polar case where downstream firms are local monopolies, the unique (coalition-proof) equilibrium market structure involves interlocking relationships. When downstream firms are instead perfect substitutes, the unique structure involves either exclusive dealing (each upstream firm dealing with a different downstream firm) or downstream foreclosure (both upstream firms deal with a common downstream firm). Likewise, when demand is linear, there is again a unique (coalition-proof) equilibrium market structure, where all channels are active if retailers are sufficiently differentiated, otherwise exclusive dealing arises.

We also use our framework to analyze the competitive effects of vertical restraints. We first consider resale price maintenance (RPM) provisions, where the retail price of a product is contractually set by its manufacturer. We show that even purely vertical, bilateral RPM agreements drastically affect competition; in particular, they can sustain industry-wide monopoly prices, thus eliminating inter-brand as well as intrabrand competitive pressures. We also find that, maximum RPM can be used to sustain supra-competitive prices when there is more substitution among strores than among brands, whereas minimum RPM can instead be used to achieve this when there is more substitution among brands than among strores. This suggests that maximum RPM can have anti-competitive effects, and that minimum RPM is not necessarily harmful. These insights are at odds with the current legal treatment of RPM provisions. In Europe, for example, minimum RPM is seen as a hardcore restriction (and is thus close to being *per se* illegal), whereas maximum RPM tends to be seen as pro-competitive.

We then turn to price parity agreements (PPAs) that restrict a retailer's pricing policy across competing brands. For instance, in a recent case, the UK Office of Fair Trading (OFT) considered a provision whereby, according the OFT, a cigarette manufacturer required tobacconists to charge no more for its brands than for competing brands. While the OFT viewed these price parity agreements as a restriction of competition, similar to minimum RPM, in our setting these contractual clauses are instead rather ineffective – they do not substantially affect the equilibrium outcome, beyond imposing symmetry.

Finally, we use our framework to study the agency business model (where the suppliers keep ownership of the products and set the price at which their products are sold on the platforms) which is widely used by online retailers and intermediation platforms. This amounts to turning the initial resale setting upside-down: manufacturers are now downstream and retailers (or intermediation platforms) are upstream. The analysis also applies to this modified setting. In particular, as long as firms can negotiate non-linear commissions, these must be cost-based. Thus, retail equilibrium prices are again uniquely defined and correspond to the outcome of direct competition between multi-platform firms. Likewise, price parity agreements (linking the prices of a product across distribution platforms) do not substantially affect the equilibrium outcome, beyond imposing symmetry.

The assumption that the terms of wholesale agreements are private information and thus not observable by rival suppliers or customers appears plausible in many upstream markets. This assumption also allows for a simple characterization of the equilibrium tariffs, even without any restriction on the class of tariffs that can be negotiated. Interestingly, our insight that tariffs are then "cost-based" (which pins down consumer prices), is in line with the empirical analysis of Nilsen *et al.* (2016) who find that an upstream merger between Norwegian egg producers did not have any impact on consumer prices, but only on the division of profits between producers and retailers.

Yet, other markets may be more transparent. It may therefore be useful to consider the case of public contracting. This appears particularly difficult in the absence of any restriction on admissible tariffs. However, it would be relatively easy to extend, for instance, the above analysis to the case of public two-part tariffs. Likewise, the case of secret or public linear contracts could be considered as well.

Finally, while – following most of the literature – we consider a simple static game to endogenize the network formation (together with coalition-proofness as an equilibrium selection device), it would be interesting to compare the predictions with those of alternative approaches, such as the dynamic approach developed by Lee and Fong (2013) (using Markov-perfection as an equilibrium selection device).

References

[1] Allain, Marie-Laure, and Claire Chambolle (2011), "Anti-competitive effects of resale-below-cost laws," *International Journal of Industrial Organization*, 29(4):373-385.

[2] Bernheim, B. Douglas, Bezalel Peleg and Michael D. Whinston (1987), "Coalition-Proof Nash Equilibria I. Concepts," *Journal of Economic Theory*, 42(1):1-12.

[3] Bernheim, B. Douglas, and Michael D. Whinston (1985), "Common Marketing Agency as a Device for Facilitating Collusion," *Rand Journal of Economics*, 16(2):269-281.

[4] Bernheim, B. Douglas, and Michael D. Whinston (1986), "Common Agency," *Econometrica*, 54(4):923-942.

[5] Bernheim, B. Douglas, and Michael D. Whinston (1998), "Exclusive Dealing," *Journal of Political Economy*, 106:64-103.

[6] Boik, Andre and Kenneth Corts (2015), "The Effects of Platform Most-Favored-Nation Clauses on Competition and Entry," *Journal of Law and Economics*, 59(1):105-134.

[7] Bonanno, Giacomo, and John Vickers (1988), "Vertical Separation," *Journal of Industrial Economics*, 36:257-265.

[8] Chipty, Tasneem, and Christopher M. Snyder (1999), "The role of firm size in bilateral bargaining: A study of the cable television industry," *Review of Economics and Statistics*, 81(2):326-340.

[9] Collard-Wexler, Allan, Robin S. Lee and Gautam Gowrisankaran (2015), "'Nash-in-Nash' Bargaining: A Microfoundation for Applied Work," NBER Working Paper No. 20641.

[10] Crawford, Gregory S., and Ali Yurukoglu (2012), "The Welfare Effects of Bundling in Multichannel Television Markets," *The American Economic Review*, 102(2):643-685.

[11] Crawford, Gregory S., Robin S. Lee, Michael D. Whinston, and Ali Yurukoglu (2015), "The Welfare Effects of Vertical Integration in Multichannel Television Markets," NBER Working Paper No. 21832.

[12] Crémer, Jacques, and Michael Riordan (1987), "On Governing Multilateral Transactions with Bilateral Contracts," *Rand Journal of Economics*, 18(3):436-451.

[13] de Fontenay, Catherine, and Joshua Gans (2005), "Vertical Integration in the Presence of Upstream Competition," *Rand Journal of Economics*, 35(3):544-572.

[14] de Fontenay, Catherine, and Joshua Gans (2014), "Bilateral Bargaining with Externalities," *Journal of Industrial Economics*, 62(4):756-788.

[15] Dobson, Paul, and Michael Waterson (2007), "The Competition Effects of Industry-wide Vertical Price Fixing in Bilateral Oligopoly," *International Journal of Industrial Organization*, 25(5):935-962.

[16] Foros, Øystein, Hans Jarle Kind and Greg Shaffer (2016), "Apple's Agency Model and the Role of Most-Favored Nation Clauses, " mimeo.

[17] Gabrielsen, Tommy, and Lars Sørgard (2007), "Private labels, price rivalry, and public policy," *European Economic Review*, 51(2):403-424.

[18] Gal-Or, Esther (1991), "Duopolistic Vertical Restraints," *European Economic Review*, 35(6):1237-1253.

[19] Gans, Joshua (2007), "Vertical Contracting When Competition For Orders Precedes Procurement," *Journal of Industrial Economics*, 55:325-346.

[20] Gowrisankaran, Gautam, Aviv Nevo and Robert Town (2015), "Mergers When Prices Are Negotiated: Evidence from the Hospital Industry," *The American Economic Review*, 175(1):172-203.

[21] Grennan, Matthew (2013), "Price Discrimination and Bargaining: Empirical Evidence from Medical Devices," *The American Economic Review*, 103(1):145-177.

[22] Hart, Oliver, and Jean Tirole (1990), "Vertical Integration and Market Foreclosure," *Brookings Papers on Economic Activity: Microeconomics*, 205-286.

[23] Ho, Kate, and Robin S. Lee (2016), "Insurer Competition in Health Care Markets," NBER Working Paper No. 19401.

[24] Horn, Henrick, and Asher Wolinsky (1988), "Bilateral monopolies and incentives for merger," *Rand Journal of Economics*, 19(3):408-419.

[25] Innes, Robert, and Stephen Hamilton (2009), "Vertical Restraints and Horizontal Control," *Rand Journal of Economics*, 40(1):120-143.

[26] Johansen, Bjørn Olav, and Thibaud Vergé (2016), "Platform Price Parity Clauses and Direct Sales," mimeo.

[27] Johnson, Justin (2015), "The Agency Model and MFN Clauses," mimeo, available at http://dx.doi.org/10.2139/ssrn.2217849 .

[28] Jullien, Bruno, and Patrick Rey (2007), "Resale Price Maintenance and Collusion," *Rand Journal of Economics*, 38(4):983-1001.

[29] Kranton, Rachel E. and Deborah F. Minehart (2001), "A Theory of Buyer-Seller Networks", *The American Economic Review*, 91(3):485-508.

[30] Lee, Robin S., and Kyna Fong (2013), "Markov-Perfect Network Formation: An Applied Framework for Bilateral Oligopoly and Bargaining in Buyer-Seller Networks," mimeo, available at http://www.people.fas.harvard.edu/~robinlee/papers/MPNENetworkFormation.pdf.

[31] Marx, Leslie M., and Greg Shaffer (2007), "Upfront Payments and Exclusion in Downstream Markets," *Rand Journal of Economics*, 38(3):823-843.

[32] Mathewson, Frank, and Ralph Winter (1984), "An Economic Theory of Vertical Restraints," *Rand Journal of Economics*, 15(1):27-38.

[33] McAfee, Preston, and Marius Schwartz (1994), "Opportunism in Multilateral Vertical Contracting: Nondiscrimination, Exclusivity and Uniformity," *The American Economic Review*, 84(1):210-230.

[34] McAfee, Preston, and Marius Schwartz (1995), "The Non-Existence of Pairwise-Proof Equilibrium," *Economics Letters*, 49(3):251-259.

[35] Miklòs-Thal, Jeanine, Patrick Rey and Thibaud Vergé (2011), "Buyer Power and Intrabrand Coordination," *Journal of the European Economic Association*, 9(4):721-741.

[36] Milliou, Chrysovalantou and Emmanuel Petrakis (2007), "Upstream Horizontal Mergers, Vertical Contracts, and Bargaining," *International Journal of Industrial Organization*, 25:963-987.

[37] Mills, David (1995), "Why retailers sell private labels," *Journal of Economics and Management Strategy*, 4(3):509-528.

[38] Nilsen, Øivind Anti , Lars Sørgard, Simen A. Ulsaker (2016), "Upstream merger in a successive oligopoly: Who pays the price,"*International Journal of Industrial Organization*, 48:143–172.

[39] Nocke, Volker, and Patrick Rey (2016), "Exclusive Dealing and Vertical Integration in Interlocking Relationships," TSE Working Paper N° 14-515, available at http://www.tse-fr.eu/publications/exclusive-dealing-and-vertical-integration-interlocking-relationships.

[40] Nocke, Volker, and Lucy White (2007), "Do Vertical Mergers Facilitate Upstream Collusion?," *The American Economic Review*, 97(4):1321-1339.

[41] Nocke, Volker, and Lucy White (2010), "Vertical merger, collusion, and disruptive buyers," *International Journal of Industrial Organization*, 28(4):350-354.

[42] O'Brien, Daniel, and Greg Shaffer (1992), "Vertical Control with Bilateral Contracts," *Rand Journal of Economics*, 23(3):299-308.

[43] Ordover, Janusz, Garth Saloner and Steven C. Salop (1990), "Equilibrium Vertical Foreclosure," *The American Economic Review*, 80(1):127-142.

[44] Piccolo, Salvatore, and Jeanine Miklòs-Thal (2012), "Colluding through Suppliers," *Rand Journal of Economics*, 43(3):492-513.

[45] Rey, Patrick, and Joseph E. Stiglitz (1988), "Vertical Restraints and Producers Competition," *European Economic Review*, 32(2-3):561-568.

[46] Rey, Patrick, and Joseph E. Stiglitz (1995), "The Role of Exclusive Territories in Producer's Competition," *Rand Journal of Economics*, 26(3):431-451.

[47] Rey, Patrick, and Jean Tirole (1986), "The Logic of Vertical Restraints," *The American Economic Review,* 76(5):921-939.

[48] Rey, Patrick, and Thibaud Vergé (2004), "Bilateral Control with Vertical Contracts," *Rand Journal of Economics*, 35(4):728-746.

[49] Rey, Patrick, and Thibaud Vergé (2010), "Resale Price Maintenance And Interlocking Relationships," *Journal of Industrial Economics*, 58(4):928-961.

[50] Rey, Patrick, and Michael Whinston (2013), "Does retailer power lead to exclusion?," *Rand Journal of Economics*, 44(1):75-81.

[51] Salinger, Michael A. (1988), "Vertical Mergers and Market Foreclosure," *The Quarterly Journal of Economics*, 77:345–356.

[52] Rubinstein, Ariel (1982), "Perfect Equilibrium in a Bargaining Model," *Econometrica*, 50(1):97-110.

Appendix

A Proof of Proposition 1

To establish existence, fix a candidate equilibrium in which each $M_i - R_j$ pair, for $i \in I$ and $j \in J$, signs the cost-based two-part tariff $t_{ij}^*(q_{ij}) = F_{ij}^* + c_i q_{ij}$, where:

$$F_{ij}^* = \alpha_{ij}\left(\pi_j^* - \pi_j^{ij}\right),$$

and retail prices are equal to $\mathbf{p}^*$. Consider the negotiation between M_i and R_j. Given their other equilibrium tariffs, $(t_{ik}^*)_{k\in J\setminus\{j\}}$ and $\left(t_{hj}^*\right)_{h\in I\setminus\{i\}}$, and the other retailers' equilibrium prices, $\mathbf{p}_{-j}^*$, they seek to maximize their joint profit, equal to:

$$\begin{aligned}&\left(p_j - c_i - \gamma_j\right) D_{ij}\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right) + \sum_{k\in J\setminus\{j\}} F_{ik}^* \\ &+ \sum_{h\in I\setminus\{i\}} \left[\left(p_{hj} - c_h - \gamma_j\right) D_{hj}\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right) - F_{hj}^*\right].\end{aligned}$$

As the variable part of this joint profit coincides with $\pi_j\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right)$, Assumption A ensures that it is maximal for $\mathbf{p}_j^* = \mathbf{p}_j^r\left(\mathbf{p}_{-j}^*\right)$. However, given R_j's other equilibrium tariffs, $\mathbf{t}_{-i,j}^*$, adopting a tariff t_{ij} leads R_j to maximize its own profit, equal to:

$$\begin{aligned}&\left(p_{ij} - \gamma_j\right) D_{ij}\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right) - t_{ij}\left(D_{ij}\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right)\right) \\ &+ \sum_{h\in I\setminus\{i\}} \left[\left(p_{hj} - c_i - \gamma_j\right) D_{hj}\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right) - F_{hj}^*\right].\end{aligned}$$

A cost-based two-part tariff in the form $t_{ij}(q_{ij}) = F_{ij} + c_i q_{ij}$ is then obviously optimal, as it makes R_j's profit equal – up to a constant – to the joint profit of M_i and R_j, and thus induces R_j to charge $\mathbf{p}_j^*$.

Hence, given their other equilibrium tariffs and the other retailers' equilibrium prices, each $M_i - R_j$ pair is willing to sign a cost-based two-part tariff and to stick to the equilibrium retail prices. To complete the proof of existence, it suffices to show that the fixed fees satisfy the Nash bargaining rule.

In the candidate equilibrium, manufacturers derive their profits from fixed fees, whereas retailers are residual claimants; hence, M_i and R_j respectively obtain:

$$\Pi_{M_i}^* = \sum_{k\in J} F_{ik}^* \text{ and } \Pi_{R_j}^* = \pi_j^* - \sum_{h\in I} F_{hj}^*.$$

If the negotiation between M_i and R_j were to break down, M_i would still obtain the other retailers' fixed fees, whereas R_j would keep selling the other brands, and would moreover however adjust prices so as to maximize its retail profit. That is, they would respectively obtain:

$$\Pi_{M_i}^{ij} = \sum_{k \in J \setminus \{j\}} F_{ik}^* \text{ and } \Pi_{R_j}^{ij} = \pi_j^{ij} - \sum_{h \in I \setminus \{i\}} F_{hj}^*.$$

The change in profit generated by a successful negotiation is therefore equal to:

$$\Pi_{M_i}^* + \Pi_{R_j}^* - \left(\Pi_{M_i}^{ij} + \Pi_{R_j}^{ij}\right) = \pi_j^* - \pi_j^{ij},$$

which is positive:

$$\pi_j^* - \pi_j^{ij} = \max_{\mathbf{p}_j} \pi_j\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right) - \max_{\mathbf{p}_{-i,j}} \pi_j\left(\left(\infty, \mathbf{p}_{-i,j}\right), \mathbf{p}_{-j}^*\right) > 0, \tag{5}$$

where the strict inequality stems from the fact that: (i) In the determination of π_j^{ij}, R_j is constrained to set p_{ij} to a prohibitively high level (consistent with $q_{ij} = 0$); and (ii) from Assumption A, maximizing $\pi_j\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right)$ with respect to $\mathbf{p}_j$ leads to a unique best response $\mathbf{p}_j^* = \mathbf{p}_j^r\left(\mathbf{p}_{-j}^*\right)$, which is such that $D_{ij}(\mathbf{p}^*) > 0$.

The surplus sharing rule then yields:

$$\Pi_{M_i}^* = \Pi_{M_i}^{ij} + \alpha_{ij}\left(\pi_j^* - \pi_j^{ij}\right),$$

leading to:

$$F_{ij}^* = \Pi_{M_i}^* - \Pi_{M_i}^{ij} = \alpha_{ij}\left(\pi_j^* - \pi_j^{ij}\right).$$

The candidate equilibrium thus indeed constitutes an equilibrium, in which all equilibrium contracts are cost-based tariffs. Conversely, in any such equilibrium:

- Given its rivals' equilibrium prices, $\mathbf{p}_{-j}^e$, R_j's profit (gross of the fixed fees) coincides with $\pi_j\left(\mathbf{p}_j, \mathbf{p}_{-j}^e\right)$, and thus its equilibrium prices must satisfy $\mathbf{p}_j^e \in \mathbf{p}_j^r\left(\mathbf{p}_{-j}^e\right)$; Assumption A therefore ensures that retail prices are equal to $\mathbf{p}^e = \mathbf{p}^*$;

- The Nash bargaining rule then uniquely pins down the equilibrium fixed fees.

We now turn to the last part of the Proposition. Manufacturers obtain:

$$\Pi_{M_i}^* = \sum_{j \in J} \alpha_{ij}\left(\pi_j^* - \pi_j^{ij}\right),$$

which, from (5), is positive as long as $\alpha_{ij} > 0$. However, they obtain less than a share α_{ij} of the equilibrium channel profit, that is:

$$\pi_j^* - \pi_j^{ij} < \pi_{ij}^*. \tag{6}$$

To see this, it suffices to note that:

$$\begin{aligned}\pi_j^{ij} &= \max_{\mathbf{p}_{-i,j}} \sum_{h\in I\setminus\{i\}} \pi_{hj}\left(\left(\infty,\mathbf{p}_{-i,j}\right),\mathbf{p}_{-j}^*\right) \\ &\geq \sum_{h\in I\setminus\{i\}} \pi_{hj}\left(\left(\infty,\mathbf{p}_{-i,j}^*\right),\mathbf{p}_{-j}^*\right) \\ &> \sum_{h\in I\setminus\{i\}} \pi_{hj}\left(\mathbf{p}^*\right) \\ &= \pi_j^* - \pi_{ij}^*,\end{aligned}$$

where the strict inequality stems from Assumption A. It follows that retailers get more than a share $1-\alpha_{ij}$ of the profits they generate. In particular, they obtain a positive profit:

$$\Pi_{R_j}^* = \pi_j^* - \sum_{i\in I} F_{ij}^* = \pi_j^* - \sum_{i\in I} \alpha_{ij}\left(\pi_j^* - \pi_j^{ij}\right) \geq \pi_j^* - \sum_{i\in I}\left(\pi_j^* - \pi_j^{ij}\right) > \pi_j^* - \sum_{i\in I}\pi_{ij}^* = 0,$$

where the strict inequality derives from (6).

B Proof of Proposition 2

Part (i). Consider the negotiation between M_i and R_j, given the equilibrium tariffs negotiated for the other brands, $\mathbf{t}_{-i,j}^e$, and the other retailers' equilibrium prices, $\mathbf{p}_{-j}^e$. Choosing the tariff t_{ij} that maximizes the joint profit of the pair $M_i - R_j$ is equivalent to choosing the quantity q_{ij} sold by R_j at the retail competition stage, anticipating the associated volume of sales by R_j for the other brands, $\hat{\mathbf{q}}_{-i,j}(q_{ij})$, as well as the sales of M_i's brand by other retailers, $\{\tilde{q}_{ik}(q_{ij})\}_{k\in J\setminus\{j\}}$. The equilibrium quantity q_{ij}^e thus maximizes:

$$r_j\left(q_{ij},\hat{\mathbf{q}}_{-i,j}(q_{ij})\right) - c_i q_{ij} - \sum_{h\in I\setminus\{i\}} t_{hj}^e\left(\hat{q}_{hj}(q_{ij})\right) + \sum_{k\in J\setminus\{j\}}\left[t_{ik}^e\left(\tilde{q}_{ik}(q_{ij})\right) - c_i\tilde{q}_{ik}(q_{ij})\right],$$

where:

$$\hat{\mathbf{q}}_{-i,j}(q_{ij}) \equiv \arg\max_{\mathbf{q}_{-i,j}}\left\{r_j\left(q_{ij},\mathbf{q}_{-i,j}\right) - \sum_{h\in I\setminus\{i\}} t_{hj}^e\left(q_{hj}\right)\right\},$$

and:

$$\tilde{q}_{ik}(q_{ij}) \equiv D_{ik}\left(\bar{\mathbf{p}}_j\left(q_{ij},\hat{\mathbf{q}}_{-i,j}(q_{ij})\right),\mathbf{p}_{-j}^e\right).$$

It therefore satisfies:

$$\frac{\partial r_j}{\partial q_{ij}}\left(\mathbf{q}_j^e\right) - c_i + \sum_{h\in I\setminus\{i\}}\left[\frac{\partial r_j}{\partial q_{hj}}\left(\mathbf{q}_j^e\right) - t_{hj}^{e\prime}\left(q_{hj}^e\right)\right]\hat{q}_{hj}'\left(q_{ij}^e\right) + \sum_{k\in J\setminus\{j\}}\left[t_{ik}^{e\prime}\left(q_{ik}^e\right) - c_i\right]\tilde{q}_{ik}'\left(q_{ij}^e\right) = 0. \quad (7)$$

As the equilibrium tariffs $\mathbf{t}_j^e$ are smooth, R_j's equilibrium behavior is characterized by the first-order conditions: For every $h\in I$,

$$\frac{\partial r_j}{\partial q_{hj}}\left(\mathbf{q}_j^e\right) = t_{hj}^{e\prime}\left(q_{hj}^e\right).$$

Using this, condition (7) simplifies to:

$$t_{ij}^{e\prime}\left(q_{ij}^{e}\right)-c_{i}+\sum_{k\in J\backslash\{j\}}\left[t_{ik}^{e\prime}\left(q_{ik}^{e}\right)-c_{i}\right]\tilde{q}_{ik}^{\prime}\left(q_{ij}^{e}\right)=0.$$

That is, for every $i\in I$, M_i's equilibrium margins:

$$u_{ij}^{e}\equiv t_{ij}^{e\prime}\left(q_{ij}^{e}\right)-c_{i},$$

must satisfy:

$$\boldsymbol{\Delta}^{(i)}\cdot\left[\begin{array}{c}u_{i1}^{e}\\ \vdots\\ u_{im}^{e}\end{array}\right]=0,$$

where the matrix $\boldsymbol{\Delta}^{(i)}$ is given by (1). Hence, if this matrix is invertible, M_i's equilibrium tariffs must be cost-based: $t_{ij}^{e\prime}\left(q_{ij}^{e}\right)=c_i$, for every $j\in J$.

Part (ii). When all tariffs are cost-based and induce smooth retail behaviors, the equilibrium prices satisfy the first-order conditions of each retailer's profit maximization program, that is, for $i\in I$ and $j\in J$:

$$\begin{aligned}0&=D_{ij}\left(\mathbf{p}^{e}\right)+\sum_{h\in I}\left[p_{hj}-t_{hj}^{e\prime}\left(q_{hj}^{e}\right)-\gamma_{j}\right]\frac{\partial D_{hj}}{\partial p_{ij}}\left(\mathbf{p}^{e}\right)\\&=D_{ij}\left(\mathbf{p}^{e}\right)+\sum_{h\in I}\left(p_{hj}-c_{h}-\gamma_{j}\right)\frac{\partial D_{hj}}{\partial p_{ij}}\left(\mathbf{p}^{e}\right)\\&=\frac{\partial\pi_{j}}{\partial p_{ij}}\left(\mathbf{p}^{e}\right).\end{aligned}$$

These conditions thus coincide with those characterizing $\mathbf{p}^*$ and Assumption A then ensures that retail prices are $\mathbf{p}^e=\mathbf{p}^*$.

C Proof of Proposition 3

Consider a candidate equilibrium where retail prices are equal to $\mathbf{p}^*$ and each M_i-R_j pair signs a contract:

$$\mathbf{t}_{ij}^{\delta}\left(q_{ij}\right)=F_{ij}\left(\delta\right)+c_{i}q_{ij}+\delta\left(q_{ij}-q_{ij}^{*}\right)^{2},$$

for an appropriately chosen $F_{ij}(\delta)$.

We first check that $\mathbf{p}^*$ constitutes a retail price equilibrium when these contracts are in place. In response to $\mathbf{p}_{-j}^*$, R_j chooses its prices $\mathbf{p}_j$ so as to maximize:

$$\pi_{j}\left(\mathbf{p}_{j},\mathbf{p}_{-j}^{*}\right)-\sum_{i\in I}\delta\left[D_{ij}\left(\mathbf{p}_{j},\mathbf{p}_{-j}^{*}\right)-q_{ij}^{*}\right]^{2}.$$

It follows that $\mathbf{p}_j^* = \mathbf{p}_j^r\left(\mathbf{p}_{-j}^*\right)$, which maximizes $\pi_j\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right)$ and leads to $D_{ij}\left(\mathbf{p}^*\right) = q_{ij}^*$, satisfies the first-order conditions: For every $h \in I$:

$$\begin{aligned} \frac{\partial}{\partial p_{hj}}\left\{\pi_j\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right) - \sum_{i\in I}\delta\left[D_{ij}\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right) - q_{ij}^*\right]^2\right\}\Bigg|_{\mathbf{p}_j=\mathbf{p}_j^*} &= \frac{\partial \pi_j}{\partial p_{hj}}\left(\mathbf{p}^*\right) - 2\delta\sum_{i\in I}\left(q_{ij}^* - q_{ij}^*\right)\frac{\partial D_{ij}}{\partial p_{hj}}\left(\mathbf{p}^*\right) \\ &= 0. \end{aligned}$$

Assumption A' then ensures that $\mathbf{p}_j = \mathbf{p}_j^*$ constitutes R_j's unique best-response when it faces the tariffs $\mathbf{t}_j^\delta$.

In the negotiation between M_i and R_j, given their other equilibrium tariffs, $\left(t_{ik}^\delta\right)_{k\in J\setminus\{j\}}$ and $\left(t_{hj}^\delta\right)_{h\in I\setminus\{i\}}$, and the other retailers' equilibrium prices, $\mathbf{p}_{-j}^*$, the two firms seek to maximize their joint profit, which is now equal to:

$$\begin{aligned} &\left(p_j - c_i - \gamma_j\right)D_{ij}\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right) + \sum_{k\in J\setminus\{j\}}\left\{F_{ik}\left(\delta\right) + \delta\left[D_{ik}\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right) - q_{ik}^*\right]^2\right\} \\ &+ \sum_{h\in I\setminus\{i\}}\left[\left(p_{hj} - c_h - \gamma_j\right)D_{hj}\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right) - \left\{F_{hj}\left(\delta\right) + \delta\left[D_{hj}\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right) - q_{hj}^*\right]^2\right\}\right]. \end{aligned}$$

By construction, $\mathbf{p}_j^* = \mathbf{p}_j^r\left(\mathbf{p}_{-j}^*\right)$ satisfies the associated first-order conditions for $\delta = 0$. As charging $\mathbf{p}_j = \mathbf{p}_j^*$ leads to $D_{hk}\left(\mathbf{p}^*\right) = q_{hk}^*$ for every $h \in I$ and every $k \in J$, it follows that $\mathbf{p}_j = \mathbf{p}_j^*$ still satisfies these first-order conditions for $\delta \neq 0$. Furthermore, for $\delta = 0$, the joint profit is uniquely maximal for $\mathbf{p}_j = \mathbf{p}_j^*$. It follows that it remains maximal at $\mathbf{p}_j^*$ for $|\delta|$ low enough.

Likewise, from (5), M_i and R_j have an incentive to deal with each other when $|\delta|$ is low enough. The tariffs $\mathbf{t}_j^\delta$ then sustain an equilibrium in which retail prices are set to $\mathbf{p}^*$ and each channel $i - j$ generates a profit π_{ij}^*, to be shared according to the Nash bargaining rule.

Let us now evaluate the impact of δ on the division of profit. In equilibrium, each M_i derives all of its profit through the fixed fees:

$$\Pi_{M_i}\left(\delta\right) = \sum_{j\in J} F_{ij}\left(\delta\right),$$

whereas each R_j obtains $\Pi_{R_j}\left(\delta\right) = \pi_j^* - \sum_{i\in I} F_{ij}\left(\delta\right)$. If the negotiation with M_i were to break down, R_j would adjust its prices $\mathbf{p}_{-i,j}$ so as to maximize:

$$\pi_j\left(\left(\infty, \mathbf{p}_{-i,j}\right), \mathbf{p}_{-j}^*\right) - \sum_{h\in I\setminus\{i\}}\delta\left[D_{hj}\left(\left(\infty, \mathbf{p}_{-i,j}\right), \mathbf{p}_{-j}^*\right) - q_{hj}^*\right]^2.$$

From Assumption B', this yields a unique price response, $\mathbf{p}_{-i,j}^{ij}\left(\delta\right)$, which is a continuous function of δ. Letting:

$$\pi_j^{ij}\left(\delta\right) \equiv \pi_j\left(\left(\infty, \mathbf{p}_{-i,j}^{ij}\left(\delta\right)\right), \mathbf{p}_{-j}^*\right) - \sum_{h\in I\setminus\{i\}}\delta\left[D_{hj}\left(\left(\infty, \mathbf{p}_{-i,j}^{ij}\left(\delta\right)\right), \mathbf{p}_{-j}^*\right) - q_{hj}^*\right]^2$$

denote the associated value, and $q_{ik}^{ij}(\delta) \equiv D_{ik}\left(\left(\infty, \mathbf{p}_{-i,j}^{ij}(\delta)\right), \mathbf{p}_{-j}^{*}\right)$ denote M_i's sales through every other retailer R_k, M_i's and R_j's disagreement payoffs are respectively equal to:

$$\Pi_{M_i}^{ij}(\delta) = \sum_{k \in J \setminus \{j\}} \left\{F_{ik}(\delta) + \delta \left[q_{ik}^{ij}(\delta) - q_{ik}^{*}\right]^2\right\} \text{ and } \Pi_{R_j}^{ij}(\delta) = \pi_j^{ij}(\delta) - \sum_{h \in I \setminus \{i\}} F_{hj}(\delta).$$

Comparing the expressions of $\Pi_{M_i}(\delta)$ and $\Pi_{M_i}^{ij}(\delta)$ yields:

$$F_{ij}(\delta) = \Pi_{M_i}(\delta) - \Pi_{M_i}^{ij}(\delta) + \delta \sum_{k \in J \setminus \{j\}} \left[q_{ik}^{ij}(\delta) - q_{ik}^{*}\right]^2,$$

where, from the surplus sharing rule:

$$\begin{aligned} \Pi_{M_i}(\delta) - \Pi_{M_i}^{ij}(\delta) &= \alpha_{ij}\left[\Pi_{M_i}(\delta) + \Pi_{R_j}(\delta) - \Pi_{M_i}^{ij}(\delta) - \Pi_{R_j}^{ij}(\delta)\right] \\ &= \alpha_{ij}\left\{\pi_j^{*} - \pi_j^{ij}(\delta) - \delta \sum_{k \in J \setminus \{j\}} \left[q_{ik}^{ij}(\delta) - q_{ik}^{*}\right]^2\right\}. \end{aligned}$$

Therefore:

$$F_{ij}(\delta) = \alpha_{ij}\left\{\pi_j^{*} - \pi_j^{ij}(\delta) - \delta \sum_{k \in J \setminus \{j\}} \left[q_{ik}^{ij}(\delta) - q_{ik}^{*}\right]^2\right\} + \delta \sum_{k \in J \setminus \{j\}} \left[q_{ik}^{ij}(\delta) - q_{ik}^{*}\right]^2,$$

and thus:

$$\Pi_{M_i}(\delta) = \sum_{j \in J}\left\{\alpha_{ij}\left[\pi_j^{*} - \pi_j^{ij}(\delta)\right] + (1 - \alpha_{ij})\,\delta \sum_{k \in J \setminus \{j\}} \left[q_{ik}^{ij}(\delta) - q_{ik}^{*}\right]^2\right\}.$$

Assumption B' ensures that this expression is a continuously differentiable function of δ. Furthermore, using the envelope theorem yields:

$$\frac{d\pi_j^{ij}}{d\delta}(0) = -\sum_{h \in I \setminus \{i\}} \left[q_{hj}^{ij}(0) - q_{hj}^{*}\right]^2.$$

We thus have:

$$\Pi_{M_i}'(0) = \sum_{j \in J}\left\{\alpha_{ij} \sum_{h \in I \setminus \{i\}} \left[q_{hj}^{ij}(0) - q_{hj}^{*}\right]^2 + (1 - \alpha_{ij}) \sum_{k \in J \setminus \{j\}} \left[q_{ik}^{ij}(0) - q_{ik}^{*}\right]^2\right\} > 0,$$

where the strict inequality follows from Assumption B': for $\alpha_{ij} > 0$, $q_{hj}^{ij}(0) \neq q_{hj}^{*}$ for some $h \neq j$, and for $\alpha_{ij} = 0$, $q_{ik}^{ij}(0) \neq q_{ik}^{*}$ for some $k \neq j$.

It follows that $\Pi_{M_i}'(\delta) > 0$ for δ close to 0; hence, in that range, $\Pi_{M_i}(\delta) > \Pi_{M_i}^{*} = \Pi_{M_i}(0)$ (resp., $\Pi_{M_i}(\delta) < \Pi_{M_i}^{*}$) for $\delta > 0$ (resp., $\delta < 0$).

D Proof of Proposition 4

We consider the two polar cases in turn.

No retail competition

Consider first the case where retailers are active in independent geographic markets. Each geographic market can then be analyzed separately and, building on the analysis already presented in the text, in any CPNE both brands must be carried in each market. Finally, it is straightforward to check that this indeed constitutes a CPNE.

Consider the geographic market of R_j, say. In the candidate CPNE, R_j carries both brands, each channel generates π^M, and firms' profits are respectively given by $\Pi_A = \Pi_B = \alpha\left(2\pi^M - \pi^m\right) (> 0)$ and $\Pi_{R_j} = 2(1-\alpha)\pi^M + 2\alpha\left(\pi^m - \pi^M\right) (> 0)$. Obviously, in the preliminary stage manufacturers have no incentive to deviate (either unilaterally, or as a coalition), as they can only change the market structure by exiting the market. Likewise, the retailer has no incentive to exit the market, and a deviation involving the "grand coalition" (i.e., R_j together with both manufacturers) would either have no effect (if all firms remain active) or require the exit of one firm, which the firm would reject. Finally, suppose that R_j deviates with one manufacturer. To make the deviation profitable for the manufacturer, it must exclude the other brand. In the continuation bargaining game, the remaining active channel generates π^m and R_j obtains $(1-\alpha)\pi^m < \Pi_{R_j}$, making the deviation unprofitable for R_j. It follows that "interlocking relationships" (i.e. here, both brands being carried in each retailer's territory) indeed constitutes a CPNE.

Perfect retail substitutes

Consider now the case where retailers are perfect substitutes.

We first note that each brand will be carried by a single retailer. To see this, consider a candidate equilibrium in which M_i, say, deals with both retailers. As tariffs are cost-based, retailers face the same marginal cost, and intrabrand competition leads them to simply pass on this cost to consumers. As a result, retailers derive zero profit from the sales of M_i's product, and thus M_i obtains zero profit as well. But then, M_i would profitably deviate by refusing to deal with one retailer: the other retailer would then generate a profit from selling M_i's product, and M_i would obtain a share of that profit.

As both brands must be sold (from the reasoning at the beginning of Section 4.2), it follows that the only candidate CPNE market structures are "exclusive dealing" and "downstream foreclosure".

In the case of exclusive dealing, each firm has a single trading partner, and thus its outside option in case of disagreement yields zero profit. The channel profit π^{ED} is thus simply shared in proportion $(\alpha, 1-\alpha)$. Each manufacturer obtains $\Pi_M^{ED} \equiv \alpha\pi^{ED}$ and each retailer obtains $\Pi_R^{ED} \equiv (1-\alpha)\,\pi^{ED}$. In case of downstream foreclosure, each manufacturer again has a single trading partner, but now one retailer carries both brands.[46] As a result, in case of disagreement with one manufacturer, the retailer would still obtain a share of the bilateral monopoly profit π^m. As a result, manufacturers now obtain $\Pi_M^{DF} \equiv \alpha\left(2\pi^{DF} - \pi^m\right)$, whereas the selected retailer obtains $\Pi_R^{DF} \equiv 2\,(1-\alpha)\,\pi^{DF} + 2\alpha\left(\pi^m - \pi^{DF}\right)$.

Note that when starting from a candidate CPNE involving either exclusive dealing or downstream foreclosure:

- Deviations by a coalition activating more than two channels are irrelevant: At least one manufacturer (who has to be part of the deviating coalition) would be dealing with both retailers, and this manufacturer would have an incentive to (unilaterally) deviate from the coalition so as to deal instead with a single retailer;

- All active firms obtain a positive profit, and thus none of them has an incentive to deviate by simply refusing to deal. In the same vein, in case of downstream foreclosure, the active retailer has no incentive to close down any channel. With only one active channel (that is, under bilateral monopoly) the retailer would only obtain $\Pi_R^m = (1-\alpha)\,\pi^m$, whereas with both active channels (downstream foreclosure) the retailer obtains:

$$\Pi_R^{DF} = 2\,(1-\alpha)\,\pi^{DF} + 2\alpha\left(\pi^m - \pi^{DF}\right) > (1-\alpha)\,2\pi^{DF} > (1-\alpha)\,\pi^m = \Pi_R^m,$$

  where the last inequality stems from the fact that the retailer generates more profit when it carries both brands.

We now consider the other potential deviations for each of the two candidate equilibrium market structures.

Exclusive dealing. Consider a candidate CPNE in which, say, M_i deals with R_j whereas M_h deals with R_k. In the light of the above remarks, deviations leading to fewer, or to more active channels are irrelevant. Likewise, a coalition deviating to upstream foreclosure is irrelevant (as intrabrand competition would then dissipate all profits). Therefore, the only relevant deviation is for a coalition to move to downstream foreclosure. Suppose, for instance, that M_i and R_k agree to open their channel (in addition to the $h-k$ channel) and foreclose R_j (that is, M_i and R_k now deal with each other, whereas M_i stops dealing with R_j but R_k keeps dealing with M_h):

[46] As retailers are perfect substitutes here, the active retailer generates the industry-wide monopoly profit (that is, $2\pi^{DF} = \Pi^M$).

- This deviation is always profitable for R_k, whose profit increases from $\Pi_R^{ED} = (1-\alpha)\,\pi^{ED}$ to:

$$\Pi_R^{DF} = 2(1-\alpha)\,\pi^{DF} + 2\alpha\left(\pi^m - \pi^{DF}\right) > (1-\alpha)\,2\pi^{DF} > (1-\alpha)\,\pi^{ED} = \Pi_R^{ED},$$

where the first inequality stems from the fact that a channel profit is maximal when all other channels are inactive (and thus $\pi^m > \pi^{DF}$), whereas the second inequality stems from the fact that industry-wide profit is larger when the two brands are carried by the same retailer (so that $2\pi^{DF} > \pi^{ED}$);

- By contrast, this deviation is profitable for M_i if and only if:

$$\Pi_M^{DF} = \alpha\left(2\pi^{DF} - \pi^m\right) > \Pi_M^{ED} = \alpha\pi^{ED}.$$

It follows that exclusive dealing is a CPNE market structure if and only if $\pi^{ED} \geq 2\pi^{DF} - \pi^m$.

Downstream foreclosure. Consider now a candidate CPNE in which the two manufacturers deal with a single common retailer, say, R_j. Using the same reasoning as above, the only relevant deviation is now for a coalition to move to exclusive dealing. Suppose, for instance, that M_h stops dealing with R_i and forms a coalition with R_k to open their channel (that is, M_h and R_k now deal with each other, whereas R_j keeps dealing with M_i but no longer deals with M_h):

- This deviation is always profitable for R_k, whose profit is now positive whereas it would otherwise be excluded;

- By contrast, this deviation is profitable for M_h if and only if:

$$\Pi_M^{ED} = \alpha\pi^{ED} > \Pi_M^{DF} = \alpha\left(2\pi^{DF} - \pi^m\right).$$

It follows that downstream foreclosure is a CPNE market structure if and only if $\pi^{ED} \leq 2\pi^{DF} - \pi^m$.

In summary, exclusive dealing constitutes the unique CPNE market structure if $\pi^{ED} > 2\pi^{DF} - \pi^m$, whereas downstream foreclosure constitutes the unique CPNE market structure if instead $\pi^{ED} < 2\pi^{DF} - \pi^m$ (in the limit case where $\pi^{ED} = 2\pi^{DF} - \pi^m$, both market structures can arise in a CPNE).

E Proof of Proposition 5

We already know that no firm can be fully excluded in equilibrium, which leaves us with only three candidate market structures for a CPNE: exclusive dealing; connected structures; and interlocking relationships. We consider them in turn.

Exclusive dealing

Consider a candidate CPNE yielding exclusive dealing. Without loss of generality, we can restrict attention to candidate strategies where firms are willing to deal with a single partner, as this minimizes the number of alternative market structures that a coalition could achieve. Thus, consider a candidate equilibrium in which M_i and R_j, on the one hand, and M_h and R_k, on the other hand, only want to deal with each other.

We first note that these strategies constitute indeed a Nash-equilibrium of the market structure formation game as, by unilaterally deviating, a firm can affect the market structure only by excluding itself from the market. Furthermore, given these equilibrium strategies, the coalition of manufacturers, the coalition of retailers and the coalition consisting of M_i and R_j (resp., M_h and R_k) cannot profitably deviate. Indeed, any deviation affecting the market structure would involve the exclusion of at least one coalition member.

Finally, given these strategies, any market structure that can be achieved by a deviating coalition of three firms can also be achieved by a two-firm coalition.

Let us now consider deviations by the coalition consisting of M_i and R_k (by symmetry, the same analysis applies to the coalition consisting of M_h and R_j). Looking for self-enforcing deviations by that coalition amounts to looking for Pareto-undominated Nash-equilibria of the two-player game between M_i and R_k, keeping fixed the strategies of M_h and R_j – i.e., taking as given that M_h only wants to deal with R_k, and R_j only wants to deal with M_i.

As noted above, M_i and M_h dealing exclusively with R_j and R_k respectively, constitutes a Nash equilibrium of this two-player game. And as M_i and R_k obtain a positive profit in this exclusive dealing market structure, we can restrict attention to alternative Nash equilibria in which they both have at least one trading partner. Furthermore, we have:

(i) If M_i is willing to deal only with R_k, then R_k's best-response is to deal with both manufacturers (as downstream foreclosure gives R_k a greater profit than bilateral monopoly);

(ii) If M_i is willing to deal with both retailers, then R_k prefers dealing exclusively with M_h to dealing exclusively with M_i (as competition is softer when the retailers carry different brands);

(iii) If R_k is willing to deal with both suppliers, then M_i prefers dealing exclusively with R_j to dealing exclusively with R_k, as the condition $\pi^{ED} > 2\pi^{DF} - \pi^m$ implies $\Pi_M^{ED} > \Pi_M^{DF}$.

The first two observations imply that there is no Nash equilibrium in which R_k deals exclusively with M_i. The third one implies that there is no Nash equilibrium in which R_k deals with both suppliers and R_j is excluded from the market. Therefore, besides exclusive dealing (with channels $i-j$ and $h-k$ being active), the only other market structure that may arise in a Nash-equilibrium of the two-player game is a connected structure, where only channel $h-j$ remains inactive.

In addition, the above observations imply that, starting from a candidate Nash equilibrium yielding the connected structure, for each partner the only relevant deviation consists of switching to exclusive dealing, by refusing to deal with its other trading partner. Therefore, the connected structure constitutes a Nash equilibrium if and only if M_i and R_k both (weakly) prefer it to exclusive dealing, that is, if and only if:

$$\pi_m^{CS} + \pi_s^{CS} - \hat{\pi}_R \geq \pi^{ED} \text{ and } (1-\alpha)\,\pi_m^{CS} + \alpha\left(\hat{\pi}_J + \hat{\pi}_R - \pi_m^{CS}\right) \geq (1-\alpha)\,\pi^{ED}. \tag{8}$$

For the linear demand specified above: (i) The first condition in (8) amounts to $\rho \leq \bar{\rho}(\mu)$, where the threshold $\bar{\rho}(\mu)$ is the unique solution to $\pi^{ED} = \pi_m^{CS} + \pi_s^{CS} - \hat{\pi}_R$, and is such that $\bar{\rho}(\mu) \in]0,1[$; and (ii) when this first condition holds, then $\pi_m^{CS} > \pi^{ED}$, and thus the second condition in (8) holds strictly for any $\alpha \in [0,1]$.

Therefore:

- When $\rho < \bar{\rho}(\mu)$, both exclusive dealing and the connected structure can be supported as a Nash-equilibrium of the two-player game, and the connected structure in which M_i is the multi-partner supplier is strictly preferred by both M_i and R_k;

- When $\rho = \bar{\rho}(\mu)$, both structures can be supported as a Nash-equilibrium of the two-player game, but M_i is indifferent between the exclusive dealing structure, and being the multi-partner supplier in a connected structure;

- Finally, when $\rho > \bar{\rho}(\mu)$, exclusive dealing is the unique market structure that can be supported as a Nash-equilibrium of the two-player game.

It follows from these observations that, when $\rho < \bar{\rho}(\mu)$, starting from the candidate Nash-equilibrium with exclusive dealing, there exists a self-enforcing profitable deviation for the coalition made of M_i and R_k. When instead $\rho \geq \bar{\rho}(\mu)$, there is no self-enforcing profitable deviation for this coalition (as at least one firm – namely, M_i – would not strictly benefit from such a deviation); there thus exists a CPNE leading to exclusive dealing in this case.

Interlocking relationships

Consider now a candidate CPNE leading to interlocking relationships (i.e., where all channels are active). By construction, in such an equilibrium all firms must be willing to deal with both of their trading partners. It follows that any deviating market structure that could be achieved by a coalition made of the manufacturers and at least one retailer (resp., the retailers and at least one manufacturer) could also be achieved by the coalition of manufacturers (resp., retailers). Hence, there is no need to consider deviations by coalitions of three or more firms, and we can instead restrict attention to unilateral deviations and deviations by two-firm coalitions.

As exiting the market is not profitable (as all firms are profitable in the equilibrium generated by interlocking relationships), to rule out unilateral deviations, it suffices to check that no firm prefers dealing with a single partner, which amounts to:

$$2\left[2\pi\left(p^*\right)-\hat{\pi}\left(p^*\right)\right] \geq \pi_m^{CS}-\hat{\pi}_J \text{ and } 2\left(1-\alpha\right)\pi\left(p^*\right)+2\alpha\left[\hat{\pi}\left(p^*\right)-\pi\left(p^*\right)\right] \geq \left(1-\alpha\right)\pi_s^{CS}. \quad (9)$$

For the linear demand specification:

- $2\pi\left(p^*\right) > \pi_s^{CS}$, and thus the second condition in (9) holds strictly for any $\alpha \in [0,1]$;
- the first condition in (9) holds instead if and only if $\rho \leq \bar{\rho}(0)$.

Therefore, there exists a Nash-equilibrium leading to interlocking relationships if and only if $\rho \leq \bar{\rho}(0)$. Next, we consider (self-enforcing) deviations by two-firm coalitions.

Consider first deviations by the coalition of manufacturers. Such deviations are self-enforcing if they constitute Pareto-undominated Nash-equilibria of the two-player game between M_A and M_B, taking R_1 and R_2' strategies as given. As retailers are willing to deal with both suppliers, in this two-player game each manufacturer freely determines which of its two distribution channels will be active.

Exiting the market is again never a best-response. Furthermore, from the above observation, in response to M_h dealing with both retailers, M_i is also willing to deal with both retailers when $\rho \leq \bar{\rho}(0)$, and strictly prefers doing so (rather than dealing exclusively with one retailer) if $\rho < \bar{\rho}\left(0\right)$. If instead M_h chooses to deal with one retailer only (say, R_k):

- M_i prefers to deal exclusively with R_j (so as to induce the "exclusive dealing" market structure) to dealing exclusively with R_k (as this would lead to the foreclosure of R_j, which is less profitable for M_i, as $\pi^{ED} > 2\pi^{DF}-\pi^m$ for the linear demand specification);
- and M_i strictly prefers dealing with both retailers rather than dealing exclusively with R_j whenever $\pi_m^{CS}+\pi_s^{CS}-\hat{\pi}_R > \pi^{ED}$, that is, whenever $\rho < \bar{\rho}(\mu)$.

As $\bar{\rho}(\mu)$ is a decreasing function of μ, it follows from the above observations that, when $\rho < \bar{\rho}(\mu)$, there exists a unique Nash-equilibrium of the above two-player manufacturer game, and this equilibrium induces interlocking relationships.

When instead $\rho \geq \bar{\rho}(\mu)$, there also exists a Nash-equilibrium of the two-player game leading to exclusive dealing. It can furthermore be checked that, for the linear demand specification, manufacturers then prefer the outcome generated by exclusive dealing to the outcome generated by interlocking relationships; that is, $\rho \geq \bar{\rho}(\mu)$ implies $\pi^{ED} > 2\left[2\pi(p^*) - \hat{\pi}(p^*)\right]$. Hence, even when interlocking relationships can be supported as a Nash-equilibrium (which is the case when $\rho \leq \bar{\rho}(0)$), there exists a self-enforcing deviation (to exclusive dealing) for the coalition of manufacturers. In what follows, we thus focus on the case $\rho < \bar{\rho}(\mu)$.

Next, we consider deviations by the coalition of retailers. Such deviations are self-enforcing if they constitute Pareto-undominated equilibria of the two-player game between R_1 and R_2, taking M_A and M_B's strategies as given. As manufacturers are willing to deal with both distributors, in this two-player game each retailer freely determines which of the two brands it will carry. Building on the previous observations, exiting the market is never a best-response and, if a retailer chooses to carry both brands, then the other retailer strictly prefers carrying both brands as well. In addition, $\rho < \bar{\rho}(\mu)$ implies $\Pi_{Rm}^{CS} > \Pi_R^{ED}$ (that is, the second part of in (8) holds); hence, if a retailer chooses to carry a single brand, the other retailer strictly prefers carrying both brands. Carrying both brands thus constitutes a strictly dominant strategy for each retailer, implying that, starting from the Nash-equilibrium with interlocking relationships, there is no self-enforcing profitable deviation by the coalition of retailers.

Finally, consider a coalition made of a supplier (say, M_i) and a retailer (say, R_j). When $\rho < \bar{\rho}(\mu)$:

- When M_i (resp., R_j) deals with both retailers (resp., manufacturers), R_j's (resp., M_i's) best-response is to deal with both manufacturers (resp., retailers);
- When R_j is willing to deal exclusively with M_i, M_i's (unique) best-response is to deal with both retailers.

Moreover, when M_i deals exclusively with R_k, R_j has two best-responses (dealing with M_h exclusively, or accepting to deal with both manufacturers) that yield the same market structure (connected structure, with channel $i-j$ remaining inactive). Likewise, when R_j deals exclusively with M_h, M_i has two best-responses (dealing with R_k exclusively, or accepting to deal with both retailers) leading to the same market structure.

This implies that this two-player game has two Nash-equilibria, one leading to interlocking relationships and one leading to a connected structure (with channel $i-j$ remaining inactive).

But in this last case, M_i and R_j would strictly prefer to activate channel $i-j$. Hence, the equilibrium with a connected structure is strictly Pareto-dominated, implying that there is no self-enforcing deviation for the coalition $M_i - R_j$.

In summary, there exists a CPNE with interlocking relationships (i.e., all links are active in equilibrium) if and only if $\rho < \bar{\rho}(\mu)$.

Connected structure

We finally show that there never exists a CPNE with three active channels (i.e., with a connected structure). To see this, consider a candidate CPNE with a connected structure in which channel $h-k$, say, is inactive.

When $\rho < \bar{\rho}(0)$, we have seen that both conditions in (9) strictly hold. It follows that there exists a self-enforcing deviation for the coalition $M_h - R_k$, which consists of activating the fourth channel (in addition to the other ones).

Furthermore, when $\rho > \bar{\rho}(\mu)$, we have seen that condition in (8) is violated. Therefore, M_i would find it profitable to unilaterally deviate and deal exclusively with R_k.

As $\bar{\rho}(\mu)$ is a decreasing function of μ, the above analysis implies that there always exists either a profitable unilateral deviation (when $\rho > \bar{\rho}(\mu)$), or a self-enforcing deviation by a two-firm coalition (when $\rho < \bar{\rho}(0)$). Hence, there never exists a CPNE with three active channels.

F Proof of Proposition 6

Part (i). Assuming that all other channels, $h-k$ (i.e., for every $h \neq i$ and every $k \neq j$) sign cost-based two-part tariffs $\bar{t}^*_{hk}(q) = \bar{F}^*_{hk} + c_h q_{hk}$ and agree, through RPM, to set retail prices $p_{hk} = p^*_{hk}$, the joint profit of M_i and R_j is given by:

$$\begin{aligned}\Pi_{M_i - R_j} &= (p_{ij} - c_i - \gamma_j) D_{ij}\left(\left(p_{ij}, \mathbf{p}^*_{-i,j}\right), \mathbf{p}^*_{-j}\right) + \sum_{k \in J \setminus \{j\}} \bar{F}^*_{ik} \\ &\quad + \sum_{h \in I \setminus \{i\}} \left[\left(p^*_{hj} - c_h - \gamma_j\right) D_{hj}\left(\left(p_{ij}, \mathbf{p}^*_{-i,j}\right), \mathbf{p}^*_{-j}\right) - \bar{F}^*_{hj}\right].\end{aligned}$$

As the variable part of this profit coincides with $\pi_j\left(\mathbf{p}_j, \mathbf{p}^*_{-j}\right)$, Assumption A ensures that it is maximized for $p_{ij} = p^*_{ij}$. Therefore, M_i and R_j can maximize their joint profit by agreeing to charge p^*_{ij}. Furthermore, as this joint profit does not depend on their own tariff (in particular, the tariff t_{ij} no longer affects R_j's prices, which are here set through RPM), they can also sign a cost-based two-part tariff.

If such an equilibrium exists, for every $i \in I$ and every $j \in J$, the fixed fee $\bar{F}_{ij}^*$ needs to be such that given the fees $\bar{F}_{hk}^*$ signed by all other pairs $M_h - R_k$, M_i and R_j respectively get shares α_{ij} and $1 - \alpha_{ij}$ of the additional profit generated by a successful negotiation. If their negotiation succeeds, the profits of M_i and R_j are given by:

$$\bar{\Pi}_{M_i}^* = \sum_{k \in J} \bar{F}_{ik}^* \text{ and } \bar{\Pi}_{R_j}^* = \pi_j^* - \sum_{h \in I} \bar{F}_{hj}^*.$$

If instead the negotiation between M_i and R_j were to break down, M_i would still obtain the fixed fees from the other retailers, whereas R_j would keep selling the other brands, but could no longer adjust its prices here. M_i's and R_j's disagreement profits are thus given by:

$$\bar{\Pi}_{M_i}^{ij} = \sum_{k \in J \setminus \{j\}} \bar{F}_{ik}^* \text{ and } \bar{\Pi}_{R_j}^{ij} = \bar{\pi}_j^{ij} - \sum_{h \in I \setminus \{i\}} F_{hj}^*, \text{ where } \bar{\pi}_j^{ij} = \pi_j \left(\left(\infty, \mathbf{p}_{-i,j}^* \right), \mathbf{p}_{-j}^* \right).$$

The additional profit generated by a successful negotiation is thus now given by $\pi_j^* - \bar{\pi}_j^{ij}$. As retailers cannot adjust their prices in case of disagreement, it is (weakly) larger than in the absence of RPM:

$$\begin{aligned} \pi_j^* - \bar{\pi}_j^{ij} &= \pi_j^* - \pi_j \left(\left(\infty, \mathbf{p}_{-i,j}^* \right), \mathbf{p}_{-j}^* \right) \\ &\geq \pi_j^* - \max_{\mathbf{p}_{-i,j}} \pi_j \left(\left(\infty, \mathbf{p}_{-i,j} \right), \mathbf{p}_{-j}^* \right) = \pi_j^* - \pi_j^{ij} > 0, \end{aligned}$$

where the strict inequality stems from (5).

The bargaining rule implies that:

$$\bar{F}_{ij}^* = \bar{\Pi}_{M_i}^* - \bar{\Pi}_{M_i}^{ij} = \alpha_{ij} \left(\pi_j^* - \bar{\pi}_j^{ij} \right),$$

which ensures that fixed fees are uniquely defined and there thus exists an equilibrium where firms negotiate cost-based two-part tariffs and RPM is used (and retail prices are equal to $\mathbf{p}^*$). M_i's and R_j's equilibrium profits are then given by:

$$\bar{\Pi}_{M_i}^* = \sum_{j \in J} \alpha_{ij} \left(\pi_j^* - \bar{\pi}_j^{ij} \right) \text{ and } \bar{\Pi}_{R_j}^* = \pi_j^* - \sum_{i \in I} \alpha_{ij} \left(\pi_j^* - \bar{\pi}_j^{ij} \right).$$

It follows that, as long as $\alpha_{ij} > 0$, manufacturers obtain a positive profit, which is moreover (weakly) greater than what they would obtain in the absence of RPM (namely, $\Pi_{M_i}^* = \sum_{j \in J} \alpha_{ij} \left(\pi_j^* - \pi_j^{ij} \right)$). However, they still obtain less than a share α_{ij} of the equilibrium channel profit:

$$\pi_j^* - \bar{\pi}_j^{ij} < \pi_{ij}^*.$$

To see this, it suffices to note that, from Assumption A(ii):

$$\bar{\pi}_j^{ij} = \sum_{h \in I \setminus \{i\}} \pi_{hj} \left(\left(\infty, \mathbf{p}_{-i,j}^* \right), \mathbf{p}_{-j}^* \right) > \sum_{h \in I \setminus \{i\}} \pi_{hj} \left(\mathbf{p}^* \right) = \pi_j^* - \pi_{ij}^*.$$

It follows that retailers still get more than a share $1-\alpha_{ij}$ of the profits they generate, and thus obtain a positive profit.

Part (ii). Fix a price vector $\mathbf{p}$ satisfying $|\mathbf{\Lambda}(\mathbf{p})| \neq 0$ and consider a candidate equilibrium in which each pair $M_i - R_j$ agrees on setting the retail price to p_{ij}, and on a two-part tariff based on some wholesale price w_{ij}. Note that the condition $|\mathbf{\Lambda}(\mathbf{p})| \neq 0$ implies that all quantities are positive. Indeed, if we had $D_{ij}(\mathbf{p}) = 0$ for some $(i,j) \in I \times J$, then an increase in p_{ij} could not affect the demand for any other channel (that is, we would have $\partial D_{hj}/\partial p_{ij}(\mathbf{p}) = \partial D_{ik}/\partial p_{ij}(\mathbf{p}) = 0$ for any $h \neq i$ and any $k \neq j$); hence, the row $l(i,j) \equiv (i-1)m + j$ would only have zeros, implying $|\mathbf{\Lambda}(\mathbf{p})| = 0$.

Given the agreements signed by the other channels, M_i and R_j are willing to reach an agreement, as they can replicate the no-agreement outcome by agreeing on a prohibitively high price for their channel (together with a tariff satisfying $t_{ij}(0) = 0$). Furthermore, if M_i and R_j were to deviate to some $\check{t}_{ij}$ and to a different retail price $\check{p}_{ij} \neq p_{ij}$, their joint profit (gross of fixed fees) would be given by:

$$
\begin{aligned}
\Pi_{M_i-R_j}(\check{p}_{ij}) &= \left(\check{p}_{ij} - c_i - \gamma_j\right) D_{ij}\left(\left(\check{p}_{ij}, \mathbf{p}_{-i,j}\right), \mathbf{p}_{-j}\right) + \sum_{k \in J \setminus \{j\}} \left(w_{ik} - c_i\right) D_{ik}\left(\left(\check{p}_{ij}, \mathbf{p}_{-i,j}\right), \mathbf{p}_{-j}\right) \\
&\quad + \sum_{h \in I \setminus \{i\}} \left(p_{hj} - w_{hj} - \gamma_j\right) D_{hj}\left(\left(\check{p}_{ij}, \mathbf{p}_{-i,j}\right), \mathbf{p}_{-j}\right),
\end{aligned}
$$

which depends only on the deviating retail price $\check{p}_{ij}$, and not on the deviating wholesale tariff $\check{t}_{ij}(q_{ij})$. Hence, M_i and R_j have no incentive to deviate from the specified wholesale tariff. Furthermore, under Assumption C, this joint profit has a unique maximum, characterized by the first-order condition. Hence, M_i and R_j have no incentive to deviate from the specified retail price, p_{ij}, whenever, $\Pi'_{M_i-R_j}(p_{ij}) = 0$, that is:

$$
\begin{aligned}
& D_{ij}(\mathbf{p}) + \left(p_{ij} - c_i - \gamma_j\right) \frac{\partial D_{ij}}{\partial p_{ij}}(\mathbf{p}) \\
& + \sum_{k \in J \setminus \{j\}} \left(w_{ik} - c_i\right) \frac{\partial D_{ik}}{\partial p_{ij}}(\mathbf{p}) + \sum_{h \in I \setminus \{i\}} \left(p_{hj} - w_{hj} - \gamma_j\right) \frac{\partial D_{hj}}{\partial p_{ij}}(\mathbf{p}) = 0,
\end{aligned}
$$

which can be rewritten as:

$$
\sum_{h \in I \setminus \{i\}} \left(w_{hj} - c_h\right) \frac{\partial D_{hj}}{\partial p_{ij}}(\mathbf{p}) - \sum_{k \in J \setminus \{j\}} \left(w_{ik} - c_i\right) \frac{\partial D_{ik}}{\partial p_{ij}}(\mathbf{p}) = \mu_{ij}(\mathbf{p}),
$$

where:

$$
\mu_{ij}(\mathbf{p}) \equiv D_{ij}(\mathbf{p}) + \sum_{h \in I} \left(p_{hj} - c_h - \gamma_j\right) \frac{\partial D_{hj}}{\partial p_{ij}}(\mathbf{p}).
$$

It follows that, if $|\mathbf{\Lambda}(\mathbf{p})| \neq 0$, there exists a unique vector of wholesale prices, $\mathbf{w}(\mathbf{p})$ satisfying the above equations for every $i \in I$ and every $j \in J$.

Finally, the equilibrium fixed fees $F_{ij}(\mathbf{p})$ (for every $i \in I$ and every $j \in J$) can be determined using the $(\alpha_{ij}, 1-\alpha_{ij})$ surplus-sharing rule. If the negotiation between M_i and R_j succeeds, their respective (equilibrium) profits are:

$$\begin{cases} \Pi_{M_i}(\mathbf{p}) &= \sum_{k \in J} \{[w_{ik}(\mathbf{p}) - c_i] D_{ik}(\mathbf{p}) + F_{ik}(\mathbf{p})\}, \\ \Pi_{R_j}(\mathbf{p}) &= \sum_{h \in I} \left\{ \left[p_{hj} - w_{hj}(\mathbf{p}) - \gamma_j\right] D_{hj}(\mathbf{p}) - F_{hj}(\mathbf{p}) \right\}. \end{cases}$$

If instead their negotiation were to break down, M_i's and R_j's disagreement payoffs would be given by:

$$\begin{cases} \bar{\Pi}^{ij}_{M_i}(\mathbf{p}) &= \sum_{k \in J \setminus \{j\}} \{[w_{ik}(\mathbf{p}) - c_i] D_{ik}((\infty, \mathbf{p}_{-i,j}), \mathbf{p}_{-j}) + F_{ik}(\mathbf{p})\}, \\ \bar{\Pi}^{ij}_{R_j}(\mathbf{p}) &= \sum_{h \in I \setminus \{i\}} \left\{ \left[p_{hj} - w_{hj}(\mathbf{p}) - \gamma_j\right] D_{hj}((\infty, \mathbf{p}_{-i,j}), \mathbf{p}_{-j}) - F_{hj}(\mathbf{p}) \right\}. \end{cases}$$

The surplus-sharing rule then uniquely identifies the equilibrium fixed fee $F_{ij}(\mathbf{p})$. This rule indeed implies:

$$\Pi_{M_i}(\mathbf{p}) - \bar{\Pi}^{ij}_{M_i}(\mathbf{p}) = \alpha_{ij} \left[\Pi_{M_i}(\mathbf{p}) + \Pi_{R_j}(\mathbf{p}) - \bar{\Pi}^{ij}_{M_i}(\mathbf{p}) - \bar{\Pi}^{ij}_{R_j}(\mathbf{p}) \right],$$

where the right-hand side term is independent of fixed fees and the left-hand side term depends only on $F_{ij}(\mathbf{p})$.

Conversely, starting from an equilibrium in which each channel $i - j$ agrees on a wholesale unit price equal to $w_{ij}(\mathbf{p})$ (associated with the corresponding fixed fee $F_{ij}(\mathbf{p})$) and a retail price equal to p_{ij}, no manufacturer-retailer pair has an incentive to deviate to another wholesale and/or retail price.

G Proof of Proposition 7

Among symmetric equilibria where channels sign the same two-part tariff $t(q) = F + wq$ and agree to set the same retail price p, Assumption C ensures that for any retail price p, the equilibrium wholesale price $w = \bar{w}(p)$ is uniquely defined and characterized by equation (4).[47] Furthermore, starting from such an equilibrium, adjusting the price p_{ij} would give R_j a profit equal to (where $\mathfrak{p}$ is the vector such that $p_{ij} = p$ for every $i \in I$ and every $j \in J$):

$$[p_{ij} - \bar{w}(p) - \gamma] D_{ij}((p_{ij}, \mathfrak{p}_{-i,j}), \mathfrak{p}_{-j}) + \sum_{h \in I \setminus \{i\}} [p - \bar{w}(p) - \gamma] D_{hj}((p_{ij}, \mathfrak{p}_{-i,j}), \mathfrak{p}_{-j}) - nF(p).$$

[47] The equilibrium fixed fee $F(p)$ is also uniquely defined and determined by the surplus-sharing rule.

Thus, the impact of a marginal increase in one retailer's price on that retailer's profit is given by:

$$\begin{aligned} D(\mathfrak{p}) - [p - \bar{w}(p) - \gamma][\lambda(p) - \lambda_M(p)] &= \mu(p) + [\bar{w}(p) - c][\lambda(p) - \lambda_M(p)] \\ &= [\bar{w}(p) - c][\lambda(p) - \lambda_R(p)], \end{aligned}$$

where:

$$\lambda(p) \equiv -\frac{\partial D_{ij}}{\partial p_{ij}}(\mathfrak{p}) > 0$$

denotes the own-price sensitivity of demand, and the last equality in the above equation comes from (4). As retailers are differentiated, and thus imperfect substitutes, $\lambda_R(p) < \lambda(p)$ (that is, when the price of a particular brand increases in one store, and thus consumers buy less of that brand in that store, consumers only partially report the lost demand for the brand to different stores). Given that under Assumption D(i), R_j's profit is strictly quasi-concave in its prices ($\mathbf{p}_j$), it follows that retailers have an incentive to lower their prices if $\bar{w}(p) < c$, and to raise them if $\bar{w}(p) > c$. In other words, price floors are needed to sustain p if $\bar{w}(p) < c$, and price caps are instead needed to sustain p if $\bar{w}(p) > c$. Morever, given that $[\bar{w}(p) - c][\lambda(p) - \lambda_R(p)] \neq 0$ whenever $\bar{w}(p) \neq c$, it follows that the constraints (price caps or price floors) imposed by manufacturers on retailers are binding.

By continuity, this remains true even if some manufacturer, say M_i, imposes to some retailer, say R_j, a price p_{ij} that slightly departs from the symmetric price p. Even when such a (marginal) deviation occurs, the constraints imposed by the other manufacturer continue to bind, and R_j thus continues to charges prices equal to p for the other brands.

Consider now the negotiation between M_i and R_j. When comtemplating a small deviation in the retail price p_{ij}, they anticipates that R_j's other prices will not be affected and will remain equal to p (as are the prices set by the other retailers). Exactly as for the case of fixed RPM, such a marginal deviation is not profitable for the pair if and only if condition (4) holds, i.e.,

$$\bar{w}(p) = c + \frac{\mu(p)}{\lambda_M(p) - \lambda_R(p)}.$$

By construction, $\bar{w}(p) = c$ and $\mu(p) = 0$ for $p = p^*$, the symmetric equilibrium price absent RPM. Hence, from Assumption D(ii), $\mu(p) < 0$ for $p > p^*$, and it follows that, in order to sustain supra-competitive prices, $\bar{w}(p) < c$ if $\lambda_M(p) > \lambda_R(p)$, whereas $\bar{w}(p) > c$ if $\lambda_M(p) < \lambda_R(p)$.

To conclude the proof, and thus ensure that price floors (resp., price caps) are needed when $\lambda_M(p) > \lambda_R(p)$ (resp., $\lambda_M(p) < \lambda_R(p)$), it remains to be checked that "large deviations" in

p_{ij} cannot be profitable. If the deviation in p_{ij} is no longer marginal, it may now be the case that the price caps (if $w > c$) or price floors (if $w < c$) set to p on the other brands are no longer binding (at least for one brand). In this case, R_j will charge prices $\hat{\mathbf{p}}^{ij}(p_{ij}; w, p)$ on other brands, where

- If $w > c$,

$$\hat{\mathbf{p}}^{ij}(p_{ij}; w, p) \equiv \begin{array}{c} \arg\max_{\mathbf{p}_{-i,j}} \sum_{h \in I} (p_{hj} - w - \gamma) D_{hj}((p_{ij}, \mathbf{p}_{-i,j}), \mathfrak{p}_{-j}) \\ \text{s.t. } p_{hj} \leq p \text{ for any } h \in I \setminus \{i\}. \end{array}$$

- If $w < c$,

$$\hat{\mathbf{p}}^{ij}(p_{ij}; w, p) \equiv \begin{array}{c} \arg\max_{\mathbf{p}_{-i,j}} \sum_{h \in I} (p_{hj} - w - \gamma) D_{hj}((p_{ij}, \mathbf{p}_{-i,j}), \mathfrak{p}_{-j}) \\ \text{s.t. } p_{hj} \geq p \text{ for any } h \in I \setminus \{i\}. \end{array}.$$

When negotiating over the price p_{ij}, M_i and R_j thus maximize their joint profit which is now given by the expression:

$$(p_{ij} - c - \gamma) D_{ij}\left(\left(p_{ij}, \hat{\mathbf{p}}^{ij}(p)\right), \mathfrak{p}_{-j}\right) + \sum_{k \in J \setminus \{j\}} (w - c) D_{ik}\left(\left(p_{ij}, \hat{\mathbf{p}}^{ij}(p)\right), \mathfrak{p}_{-j}\right)$$
$$+ \sum_{h \in I \setminus \{i\}} \left(\hat{p}_h^{ij}(p_{ij}; w, p) - w - \gamma\right) D_{hj}\left(\left(p_{ij}, \hat{\mathbf{p}}^{ij}(p)\right), \mathfrak{p}_{-j}\right),$$

which, under Assumption E, is strictly quasi-concave in p_{ij} and maximal for a finite price level. Given that the definition of $\hat{\mathbf{p}}^{ij}(p_{ij}; w, p)$ also applies to prices p_{ij} close to p, the strict quasi-concavity assumption ensures that if marginal deviations are unprofitable, so are larger deviations.

H Proof of Proposition 8

Part (i). Consider a candidate equilibrium where the equilibrium tariffs are t_{ij}^e for every $i \in I$ and every $j \in J$, and all equilibrium quantities are positive and the equilibrium retail prices are given by the price vector $\mathbf{p}^e$ such that, for every $j \in J$, $p_{ij}^e = p_j^e$ for all $i \in I$.

If such an equilibrium exists, it must be such that when it faces the tariffs $\mathbf{t}_j^e$ and anticipates that each rival retailer R_k, for any $k \neq j \in J$ sets retail prices equal to $p_{hk}^e = p_k^e$ for every $h \in I$, R_j chooses the price p_j^e so as to maximize its profit, that is:

$$p_j^e \in \arg\max_{p_j} \left\{ \sum_{h \in I} \left[(p_j - \gamma_j) D_{hj}\left(\mathbf{p}_j, \mathbf{p}_{-j}^e\right) - t_{hj}^e\left(D_{hj}\left(\mathbf{p}_j, \mathbf{p}_{-j}^e\right)\right) \right] \right\}.$$

Alternatively, one can write R_j's maximizing program as choosing a quantity q_{ij} for M_i's brand. Under the price parity requirement choosing a quantity q_{ij} amounts to choosing the price $\bar{\mathbf{p}}_j(q_{ij}) = (\bar{p}_j(q_{ij}), \ldots, \bar{p}_j(q_{ij}))$, such that:

$$D_{ij}\left(\bar{\mathbf{p}}_j(q_{ij}), \mathbf{p}^e_{-j}\right) = q_{ij}. \tag{10}$$

Assumption E ensures that such a price $\bar{p}_{ij}(q_{ij})$ exists and is continuously differentiable as long as $q_{ij} \leq q^{\max}\left(\mathbf{p}^e_{-j}\right) \equiv D_{ij}\left((0,\ldots,0), \mathbf{p}^e_{-j}\right)$.

Therefore, when it faces the tariffs $\mathbf{t}_j = \left(t_{ij}, \mathbf{t}^e_{-i,j}\right)$ and anticipates that its rivals set their equilibrium prices, $\mathbf{p}^e_{-j}$, R_j chooses the quantity q_{ij} that maximizes its profit:

$$\bar{\pi}_j(q_{ij}) \equiv \left[\bar{p}_j(q_{ij}) - \gamma_j\right] q_{ij} - t_{ij}(q_{ij}) + \sum_{h \in I\setminus\{i\}} \left\{\left[\bar{p}_j(q_{ij}) - \gamma_j\right] \bar{q}_{hj}(q_{ij}) - t^e_{hj}(\bar{q}_{hj}(q_{ij}))\right\},$$

where R_j's sales of the M_h's brand, for any $h \neq i \in I$, $\bar{q}_{hj}(q_{ij})$ is given by:

$$\bar{q}_{hj}(q_{ij}) \equiv D_{hj}\left(\bar{\mathbf{p}}_j(q_{ij}), \mathbf{p}^e_{-j}\right).$$

To maximize their joint profit, subject to the PPA, M_i and R_j should adopt a tariff t_{ij} inducing the quantity q_{ij} that maximizes:

$$\bar{\pi}_j(q_{ij}) + t_{ij}(q_{ij}) - c_i q_{ij} + \sum_{k \in J\setminus\{j\}} \left[t^e_{ik}(\bar{q}_{ik}(q_{ij})) - c_i \bar{q}_{ik}(q_{ij})\right],$$

where:

$$\bar{q}_{ik}(q_{ij}) \equiv D_{ik}\left(\bar{\mathbf{p}}_j(q_{ij}), \mathbf{p}^e_{-j}\right). \tag{11}$$

Therefore, to induce the quantity $q^e_{ij} > 0$ that maximizes their joint profit, M_i and R_j need to agree on an equilibrium tariff t^e_{ij} that satisfies (using $\bar{q}_{ik}\left(q^e_{ij}\right) = q^e_{ik}$):

$$t^{e\prime}_{ij}\left(q^e_{ij}\right) - c_i + \sum_{k \in J\setminus\{j\}} \left[t^{e\prime}_{ik}(q^e_{ik}) - c_i\right] \bar{q}'_{ik}\left(q^e_{ij}\right) = 0.$$

For any $i \in I$, the equilibrium upstream margins $u^e_{ij} = t^{e\prime}_{ij}\left(q^e_{ij}\right) - c_i$, for $j \in J$, thus satisfy:

$$\bar{\mathbf{\Delta}}^{(i)} \cdot \begin{bmatrix} u^e_{i1} \\ \vdots \\ u^e_{im} \end{bmatrix} = 0, \tag{12}$$

where $\bar{\mathbf{\Delta}}^{(i)}$ denotes the $m \times m$ matrix such that the term in row $j \in J$ and column $k \in J$ is given by:

$$\bar{\Delta}^{(i)}_{j,k} = \begin{cases} 1 & \text{if } k = j, \\ \bar{q}'_{ik}\left(q^e_{ij}\right) & \text{otherwise.} \end{cases}$$

Conversely, to induce R_j to sell a given quantity q_{ij}, it suffices to adopt a continuously differentiable tariff $t_{ij}(\cdot)$ that is sufficiently convex and satisfies $\bar{\pi}_j'(q_{ij}) = 0$.

We now conclude the proof by showing that the matrix $\bar{\mathbf{\Delta}}^{(i)}$ is invertible. Differentiating (11), yields:

$$\bar{q}_{ik}'\left(q_{ij}^e\right) = \sum_{h\in I} \frac{\partial D_{ik}}{\partial p_{hj}}\left(\mathbf{p}^e\right) \bar{p}_j'\left(q_{ij}^e\right). \tag{13}$$

Differentiating (10), we get:

$$\bar{p}_j'\left(q_{ij}\right) = \frac{1}{\sum_{h\in I} \frac{\partial D_{ij}}{\partial p_{hj}}\left(\bar{\mathbf{p}}_j\left(q_{ij}\right), \mathbf{p}_{-j}^e\right)} < 0. \tag{14}$$

Using (14), equation (13) rewrites as:

$$\bar{q}_{ik}'\left(q_{ij}^e\right) = \frac{\sum_{h\in I} \frac{\partial D_{ik}}{\partial p_{hj}}\left(\mathbf{p}^e\right)}{\sum_{h\in I} \frac{\partial D_{ij}}{\partial p_{hj}}\left(\mathbf{p}^e\right)} < 0,$$

where the inequality stems from Assumption F. Indeed, parts (i) and (ii) of that assumption respectively imply that $\sum_{h\in I} \frac{\partial D_{ij}}{\partial p_{hj}}\left(\mathbf{p}^e\right) < 0$ and $\sum_{h\in I} \frac{\partial D_{ik}}{\partial p_{hj}}\left(\mathbf{p}^e\right) > 0$.

The matrix $\bar{\mathbf{\Delta}}^{(i)}$ is diagonally dominant, since for every $j \in J$ we have:

$$\begin{aligned}
\left|\bar{\Delta}_{j,j}^{(i)}\right| - \sum_{k\in J\setminus\{j\}} \left|\bar{\Delta}_{j,k}^{(i)}\right| &= 1 - \sum_{k\in J\setminus\{j\}} \frac{\sum_{h\in I} \frac{\partial D_{ik}}{\partial p_{hj}}\left(\mathbf{p}^e\right)}{-\sum_{h\in I} \frac{\partial D_{ij}}{\partial p_{hj}}\left(\mathbf{p}^e\right)} \\
&= \frac{-\sum_{h\in I} \frac{\partial D_{ij}}{\partial p_{hj}}\left(\mathbf{p}^e\right) - \sum_{k\in J\setminus\{j\}} \sum_{h\in I} \frac{\partial D_{ik}}{\partial p_{hj}}\left(\mathbf{p}^e\right)}{-\sum_{h\in I} \frac{\partial D_{ij}}{\partial p_{hj}}\left(\mathbf{p}^e\right)} \\
&= \frac{-\sum_{h\in I} \left[\frac{\partial D_{ij}}{\partial p_{hj}}\left(\mathbf{p}^e\right) + \sum_{k\in J\setminus\{j\}} \frac{\partial D_{ik}}{\partial p_{hj}}\left(\mathbf{p}^e\right)\right]}{-\sum_{h\in I} \frac{\partial D_{ij}}{\partial p_{hj}}\left(\mathbf{p}^e\right)} \\
&= -\frac{\sum_{h\in I} \left[\sum_{k\in J} \frac{\partial D_{ik}}{\partial p_{hj}}\left(\mathbf{p}^e\right)\right]}{-\sum_{h\in I} \frac{\partial D_{ij}}{\partial p_{hj}}\left(\mathbf{p}^e\right)} \\
&> 0,
\end{aligned}$$

where the inequality stems from Assumption F (parts (i) and (iii)). It follows that the matrix $\bar{\mathbf{\Delta}}^{(i)}$ is invertible, and thus (12) yields $t_{ij}^{e\prime}\left(q_j^e\right) = c_i$ for every $i \in I$ and every $j \in J$.

Part (ii). Given the equilibrium tariffs $\mathbf{t}^e$, the equilibrium prices must be such that for any $j \in J$, p_j^e maximizes R_j profit, that is:

$$p_j^e \in \arg\max_{p_j} \left\{\sum_{h\in I} \left[\left(p_j - \gamma_j\right) D_{hj}\left(\mathbf{p}_j, \mathbf{p}_{-j}^e\right) - t_{hj}^e\left(D_{hj}\left(\mathbf{p}_j, \mathbf{p}_{-j}^e\right)\right)\right]\right\}.$$

This maximization program also writes as:

$$p_j^e \in \arg\max_{p_j} \left\{ \pi_j \left(\mathbf{p}_j, \mathbf{p}_{-j}^e \right) + \sum_{i \in I} \left[t_{ij}^e \left(D_{ij} \left(\mathbf{p}_j, \mathbf{p}_{-j}^e \right) \right) - c_i D_{ij} \left(\mathbf{p}_j, \mathbf{p}_{-j}^e \right) \right] \right\}.$$

Given that we focus here on interior symmetric equilibria, the equilibrium retail price p_j^e must satisfy the first-order condition:

$$\sum_{i \in I} \left\{ \frac{\partial \pi_j}{\partial p_{ij}} \left(\mathbf{p}^e \right) + \left[t_{ij}^{e\prime} \left(q_{ij}^e \right) - c_i \right] \frac{\partial D_{ij}}{\partial p_{ij}} \left(\mathbf{p}^e \right) \right\} = 0 \quad \Longleftrightarrow \quad \sum_{i \in I} \frac{\partial \pi_j}{\partial p_{ij}} \left(\mathbf{p}^e \right) = 0. \tag{15}$$

By definition the prices $\mathbf{p}^*$ satisfy this last condition, since $\partial \pi_j \left(\mathbf{p}^* \right) / \partial p_{ij} = 0$ for every $i \in I$. Moreover, when firms are symmetric at both stages of the vertical chain, the equilibrium price vector $\mathbf{p}^*$ is symmetric, in the sense that for every $j \in J$, $p_{ij}^* = p_j^*$. Therefore, $\mathbf{p}^*$ is a solution to the set of first-order conditions given by equation (15) for every $j \in J$.

Finally, using symmetry, equation (15) simplifies to $\partial \pi_j \left(\mathbf{p}^e \right) / \partial p_{ij} = 0$. Under Assumption A, this system of first-order conditions has a unique solution, which ensures that we must have $p^e = p^*$.