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“Informality and Optimal Public Policy”

David Bardey and Daniel Mejía

Toulouse School of Economics

Informality and Optimal Public Policy *

David Bardey[†] Daniel Mejía[‡]

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Abstract

This article tackles the feature of optimal public policy such as the level of enforcement and the supply of public goods in an economy characterized by a huge informal sector. We consider informality as the group of productive activities which, before hand, do not comply (totally or partially) with government regulations. The Government intervenes as a Stackelberg leader and has to decide how to allocate public expenditures, collected through the tax system, between the provision of a public good, which can only be used for formal activties, and enforcement effort, aimed at detecting informal firms that evade taxes. Taking the public policy as given, a representative family, owner of a representative firm, decides how to split a fix amount of labour supply between formal and informal activities. Our results show that the greater are the distortions in the process of tax collection, the larger is the size of the informal sector. Finally, we derive the properties of the optimal public policy. In particular, we show that the shadow cost of public fund represent the rationale of enforcement spending. We also point out that the size of the tax distortion (*e.g.* the shadow cost of public funds) is inversely related to total income, the tax rate and the provision of the public good. Our calibration results reveal that higher values of the shadow cost of public funds call for more stick (more enforcement) and less carrot (public goods).

Keywords: Informality, public good and enforcement.

JEL codes: K10, K20, K42, O17.

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†Corresponding author: Associate Profesor at the University of Los Andes (Bogota) and Visiting Fellow at Toulouse School of Economics. Email: d.bardey@uniandes.edu.co

‡Department of Economics, Universidad de los Andes. E-mail: dmejia@uniandes.edu.co. Director of Cesed.

1. Introduction

The size of the informal sector in many developing economies, as measured by Schneider (2005), is close to 40%, especially in many Latin American countries.[1] What is more worrying is that in the medium term, informality levels are on the rise in many developing economies. According to Perry *et al.* (2007) informality levels in most Latin American countries increased between the late 1980s and the begining of 2000s. Most measures of informality analyzed in Perry *et al.* (2007)[2] show a significant increase in informality levels in most Latin American countries. As pointed out in La Porta and Shleifer (2014), the informal sector has extremely low productivity compared to the formal economy since informal firms tend to be smaller and inefficient.

The model we propose in this article builds on Mejía and Posada (2011). In particular, we consider informality as the group of productive activities which, *ex-ante*, do not comply with government regulations. This non compliance with government regulations can take place in different dimensions of economic activity such as tax evasion, non compliance with social security payments, minimum wages, sanitary and environmental regulations, *etc.* Furthermore, non compliance with norms or regulations may be partial or total.[3] One of the most salient costs of being informal is the lack of access to some governement-provided services such as access to the judicial system to resolve contract-related disputes or the imposibility of participating in public trainning programs. Therefore, at the individual level, the decision to become informal can be viewed as a rational response to the system of incentives at work generated by government regulations and the provision of some public goods that can only be accessed if the firm or the individual fully comply with government regulations. In particular, firms might decide to become informal if the tax rate becomes too high for formal activities to be profitable (relative to informal activities where taxes can be partially or totally evaded). Nevertheless, although individual decisions might be optimal, this collective action problem may generate aggregate inefficiencies (Loayza, 2007).

In this article, we develop a model in which a representative individual (or household)

[1] According to La Porta and Shleifer (2014), in the poorest countries it is about 50%.

[2] Percentage of employed workers, percentage of salaried workers, percentage of the labor force that lacks pensions and percentage of self employed, among others.

[3] We focus on the informal sector and do not consider the so-called "underground economy". Namely, while the latter can be characterized as a violation to the penal code, the former is not. In particular, tax avoidance in many countries or the violation of environmental laws is not necessarily investigated and punished by the penal system.

decides the amount of time she allocates to operate in a formal firm (or technology) and in an informal firm. On the one hand, by working with the formal technology, the individual can take advantage of a public good in the production process but has to pay taxes. On the other, the individual may decide to operate with the informal firm but cannot take advantage of the public good in the production process.[4] Additionally, the firm can be detected evading taxes with a probability that is increasing in the level of government enforcement and on the size of the informal firm's activities.[5] In case of being detected, the informal firm has to pay the evaded taxes plus a fine of a given size.

We consider a non paternalistic government which maximizes the representative household's welfare.[6] We assume that the State plays as a Stackelberg leader and decides how to allocate public expenditures, collected through the tax system, between the provision of a public good (which can only be used by the firms operating in the formal sector[7]) and enforcement activities, aimed at detecting the informal firm evading taxes. When deciding the optimal provision of the public good and the enforcement level, the government takes into account how the representative family reacts to these decisions. In such a context and taking into account the shadow cost of public fund, we characterize the optimal stick and carrot policy.

The contribution of this paper is twofold. First, it contributes to the growing literature on informality by providing a new view where the size of the informal sector is endogenously determined by the interaction between the government and individual decisions. While Cerda and Saravia (2013) focus on (heterogenous) firms' decision to allocate time and factors between both sectors, we rather consider that this decision comes from workers and thus is taken by a representative consumer.[8] In this respect, we characterize the structural

[4]We assume that the public good is not a pure public good in the sense that the government can partially or totally exclude informal firms from using it. Thus, we are formally refering to a *club good*.

[5]This assumption allows us to capture the famous concept that "small is beautiful". The available evidence does show a clear negative relationship between firm size and informality levels (see Perry *et al.*, 2007).

[6]The debate concerning the State's objective in the presence of an informal sector (where a norm is partially or totally violated) is quite complex. It might seem natural to assume that the State's objective is to maximize formal production and disincentive informal production (see for instance Mejia and Posada [2011] for a positive analysis using this assumption). However, it can also be realistic to think about a less paternalistic State that simply maximizes the representative household's utility. For instance, this assumption may reflect a democratic political system in which the government has been elected in function of the majority's preferences in terms of enforcement.

[7]In this sense this is not a pure public good as it is possible to exclude users.

[8]Another difference is that their model is intensive and extensive while we focus on an intensive approach.

parameters that determine the size of the informal sector. We particularly point out the link between the size of the informal sector and the shadow cost of public funds. This theoretical result is in line with the empirical evidence presented in Auriol and Walters (2006). Indeed, we show that the greater are these distortions, the larger is the size of the informal sector. This result is also consistent with Adaman and Mumcu's (2010) findings. In a global game framework, they show that inefficiencies and the low level of trustworthiness of the public sector induces an equilibrium with high levels of informality (see also Torgler and Schneider [2009] for empirical evidence). In spite of the fact that the State cannot observe the level of informal production, it is optimal to spend all the budget in public goods and nothing in enforcement activities as long as there is no shadow cost of public funds. Roughly speaking, our model reveals that the shadow cost of public funds constitutes the rationale for spending in enforcement activities. Using the calibration results we show that when the State cannot observe informal activities, it has to spend more in public goods and enforcement to obtain the same size of the formal sector. More precisely, our calibration results reveal that higher values of the shadow cost of public funds call for more stick (more enforcement) and less carrot (public goods).

Additionaly to the previous works already quoted, our paper is related to and borrows from a number of stands of the literature. Cremer and Gahvari (1994) determines the optimal tax design in the presence of tax evasion. They provide sufficient conditions under which tax evasion decreases the optimal tax rate, while showing that an increase in the optimal tax rate is also possible. Boadway *et al.* (2009) resumes this issue but considering explicitly the presence of an informal sector. As in our paper, the size of the informal sector is endogenous and mainly determined by State policies. Nevertheless, Cremer and Gavhari (1994) and Boadway *et al.* (2009) focus on optimal taxation issues, while our analysis is devoted to the optimal public policy analysis subject to a balanced budget constraint (*e.g.*, the choice between the carrots - *the 'public' good provision* - and the sticks - *enforcement activities*). Besfamille *et al.* (2009) analyze the relation between tax enforcement, aggregate output and government revenue when imperfectly competitive firms evade a specific output tax. They reveal that aggregate output decreases with the level of tax enforcement. Government revenue increases with enforcement when the tax rate is low but, when the tax rate is high, government revenue is either inversely U-shaped or decreasing in the level enforcement. In line with Besfamille *et al.* (2009), our paper analyzes the relationship between the tax level, enforcement and government revenue. We point out that the size of the tax distortion (*e.g.* the shadow cost of public funds) is inversely related to the size of the formal sector, total income, the optimal tax rate and the optimal provision of the 'public' good; and, is positively related to the size of the informal sector and optimal enforcement activities.

The rest of the paper is organized as follows. Section 2 presents the set-up. Section 3 characterizes the optimal public policy in various contexts. First, we solve the central planner problem when he is able to choose the time devoted by the representative family in both sectors. Next, we consider the decentralized model with tax distortions. In order to disentangle the effects of some policy variables on variables such as the size of the informal sector, optimal enforcement and the provision of the public good, we calibrate the first and second best cases using plausible parameter values. The results from the calibration excersise are presented at the end of Section 3. The last section presents some concluding remarks.

2. The model

The production of the final good in the formal sector, y_f, depends on the amount of labor allocated by the representative household to this sector, l_f, and on a public good, b, produced by the State. The production function in the formal sector is described by $y_f = f(l_f, b)$. Naturally, we assume that the formal sector is characterized by positive and decreasing returns to each input: f_{l_f}, $f_b > 0$ and $f_{l_f l_f}$, $f_{bb} \leq 0$. Additionally, we assume that formal labor and the public good are complementary in the production of the final good, that is, that $f_{l_f b} > 0$. Alternatively, the final good can be produced in the informal sector, *i.e.* $y_i = g(l_i)$, with $g_{l_i} > 0$ and $g_{l_i l_i} \leq 0$. We consider that there is no positive externality from the formal to the informal sector.[9]

We note $p(l_i, e)$ the probability that the State detects the informal firm evading taxes. This probability depends on the size of the informal firm l_i and on the State's allocation of resources to enforcement activities e. We assume $p_e > 0$, $p_{ee} \leq 0$, $p_{l_i} > 0$, and $p_{l_i l_i} \geq 0$. In words, the probability that an informal firm is detected evading taxes is increasing in the State's enforcement efforts, but with decreasing returns. The probability of detection

[9] In the informality literature, the expression public good is often used. Nevertheless, in our framework, as long as the the informal sector cannot benefit from the public good, stricly speaking, we should refer to b as a club good. Actually, it is the relative advantage generated by the public good between the two sectors and not the absolute that is relevant for the representative household´s optimal allocation decisions. As the benefit generated by the public good in the formal sector is endogenous, the assumption of no externality to the informal sector is not restrictive.

is increasing and convex in the size of the informal firm. We also assume that $p(.)$ satisfies the following Inada condition: $\partial p(0, l_i)/\partial e = +\infty$. Furthermore, we consider that $p_{el_i} \geq 0$, which means that the marginal effect of enforcement on the probability of detection is increasing in the size of the informal firm (the level of enforcement).

We assume that the cost to an informal firm of being detected evading taxes consists of a fine of size $\phi g(l_i)$. This fine may be interpreted as the opportunity cost of not being able to produced if the firm is closed down for a certain amount of time or as a pure fine that the firm has to pay if it is detected.[10] The representative household takes the strategic variables of the State (tax rate, enforcement, and public good provision) as given. We consider that the household's total labor supply, $\bar{L}_s$, is exogenously given. Then, the household's decision consists only in allocating the total labor supply between the formal and the informal sector in order to maximize his expected income. The problem faced by the representative household can be written as:

$$\max_{\{l_f, l_i\}} \ (1-\tau) f(l_f, b) + g(l_i) - p(e, l_i) g(l_i) \phi, \tag{1}$$

$$\text{s.t} \quad \bar{L}_s = l_f + l_i, \tag{2}$$

where $(1-\tau)f(l_f, b)$ represents the net income (after tax) earned in the formal sector, while $g(l_i)$ denotes the income earned in the informal sector. These income levels must be reduced by $p(e, l_i)g(l_i)\phi$, the expected cost of the fine if $l_i > 0$.

The optimal allocation of labor between the formal and the informal sector is given by[11]:

$$g_{l_i}(l_i)\left(1 - p(e, l_i)\phi\right) - p_{l_i}(e, l_i) g(l_i)\phi = (1-\tau) f_{l_f}(l_f, b). \tag{3}$$

The household's optimal allocation of labor between the formal and the informal sector is such that the expected net marginal benefit from allocating an extra unit of time to the informal sector is equal to its net marginal benefit in the formal sector. On the one hand, the marginal benefit from allocating an extra unit of time to the informal sector (the left hand side of equation 3) is given by the marginal productivity in the informal sector $g_{l_i}(l_i)$ times $(1 - p(e, l_i)\phi)$, minus the marginal increase in the probability of being detected, p_{l_i}, times the size of the fine that has to be paid if the firm is detected, $g(l_i)\phi$. On the other

[10] In this paper, since we consider a risk-neutral representative household, we do not focus on the trade-off between enforcement effort and the size of the fine. Finally, we consider that the fine takes a finite value due to a limited liability argument. Otherwise, the solution would be straightforward and would have followed an argument a la Becker: the fine would be equal to infinity.

[11] We adopt an intensive approach and thus focus our attention on interior solution.

hand, the net marginal benefit from allocating an extra unit of time to the formal sector (right hand side of equation 3) is simply composed of the marginal productivity of labor in the formal sector net of taxes, $(1-\tau)f_{l_f}$.

Remark 1. *All other things equal, the level of production in the formal sector increases with the level of provision of public goods and enforcement activities.*

Proof. Applying the implicit function theorem directly gives the result. ■

A higher provision of the public good makes it relatively more attractive, *ceteris paribus,* to allocate time to the formal sector because labor and the public good are complementary in the production process. As the amount of enforcement efforts increases, the incentive to allocate time to the informal sector decreases because the probability of being caught evading taxes and having to pay the fine increases. Therefore, both instruments work into the same direction and allow the State to reduce the size of the informal sector.

3. The optimal public policy

In this section we distinguish between several cases. We start with a first-best analysis where the State acts as a Central Planner and can directly choose the representative household's labor supply between the formal and informal sectors, l_f and l_i respectively. In the second-best allocation, we consider the situation in which the State cannot choose or impose the household labor supply between the two sectors. Nevertheless, it still behaves as a Stackelberg leader in the sense that it chooses the vector (b, e, τ) taking into account the optimal reaction of the representative household to its choices.

3.1. First-best analysis

In a first-best analysis, the State can directly choose the household's allocation of time between the formal and the informal sector. In such a context, enforcement activities, as captured by e, should be interpreted as the cost generated by the burden of the proof. The State's objective is the maximization of the household's utility, that is:

$$\max_{\{l_f,b,e\}} W^{FB} = (1-\tau)f(l_f,b) + g(\bar{L}_s - l_f)\left(1 - p(e, \bar{L}_s - l_f)\phi\right), \tag{4}$$

subject to the following budget constraint:

$$\tau(1-\lambda)f(l_f,b) + p(e,\bar{L}_s - l_f)g(\bar{L}_s - l_f)\phi \geq b + e. \tag{5}$$

The budget constraint says that the tax collected plus the fine earned through the detection process is equal to the sum of the expenditures in the public good and detection efforts. Following Laffont and Tirole (1993), an easy way to capture these distortions is to consider that for each unit of tax collected, a proportion λ is lost.[12] Because a fraction λ of the taxes collected is lost, the first term in the left hand side of the State's budget constraint is scaled down by a fraction $1-\lambda$. As the tax and the detection efforts intervene negatively in the objective function, it is straightforward that the budget constraint is binding. Therefore, we write the tax rate τ that ensures that the budget constraint holds with equality as:

$$\tau = \frac{b+e-p(e,\bar{L}_s-l_f)g(\bar{L}_s-l_f)\phi}{(1-\lambda)\,f(l_f,b)}. \tag{6}$$

Replacing (6) in the State's objective function, the State's program becomes:

$$\max_{\{l_f,b,e\}} W^{FB} = f(l_f,b)+g(\bar{L}_s-l_f)-\frac{b+e}{1-\lambda}+\frac{\lambda}{1-\lambda}p(e,\bar{L}_s-l_f)g(\bar{L}_s-l_f)\phi. \tag{7}$$

The objective function is then composed of the sum of the production in both sectors, minus the State's expenditures in enforcement and the public good $(b+e)$, scaled down by a fraction $1-\lambda$. Additionally, the fine collected $p(e,\bar{L}_s-l_f)g(\bar{L}_s-l_f)\phi$ must be added as it relaxes the budget constraint. It is worth noticing that the expected fine collected, $p(e,\bar{L}_s-l_f)g(\bar{L}_s-l_f)\phi$, is multiplied by λ because for each 1\$ coming from the fine (*e.g.*, not coming from tax distortions), λ\$ is saved. In the first-best scenario, in addition to maximize the welfare function with respect to its policy instruments b and e, the State is able to chose the amount of labor supply in the formal sector (l_f). Moreover, since the tax rate is defined by the budget constraint, it is equivalent to maximize with respect to τ or one of these instruments, *i.e.* b and e.

Proposition 1. *The first-best allocation is characterized by:*

i) $\frac{\partial f(\bar{L}_s-l_f^*,b^*)}{\partial l_f}-\frac{\partial g(\bar{L}_s-l_f^*)}{\partial l_f}=\frac{\lambda\phi}{1-\lambda}\left(\frac{\partial p(e^*,\bar{L}_s-l_f^*)}{\partial l_f}g(\bar{L}_s-l_f^*)+\frac{\partial g(\bar{L}_s-l_f^*)}{\partial l_f}p(e^*,\bar{L}_s-l_f^*)\right)$,

ii) $\frac{\partial f(\bar{L}_s-l_f^*,b^*)}{\partial b}=\frac{1}{1-\lambda}$,

iii) $\frac{\partial p(e^*,\bar{L}_s-l_f^*)}{\partial e}g(\bar{L}_s-l_f^*)=\frac{1-\lambda}{\lambda\phi}$.

Condition (i) is an *efficiency condition*: if the State is able to choose the household's allocation of time between the formal and the informal sectors on the one hand, and there is no tax distortions on the other, *i.e.* $\lambda=0$, it would choose an allocation such that

[12] In some extent, $1-\lambda$ captures the tax system quality: when 1\$ is collected through tax, λ is lost.

the marginal productivity of labor is the same in both sectors. Moreover, this condition reveals that, for a given level of provision of the public good, the State's optimal allocation of labor to the formal sector decreases with the presence of tax distortions. Because taxes create distortions and the fines collected constitute an alternative source of revenue, the State is more lenient on informality compared to the case of no distortions. Condition (ii) says that the State chooses the optimal provision of the public good in such a way that the marginal productivity of the public good is equal to the marginal cost of providing it, weighted by the size of the tax distortion. All other things equal, for higher values of λ, the spending on the public good becomes lower. The third condition (iii) points out that even in the case where the State can impose the household's allocation of time between the formal and the informal sector, it allocates positive levels of resources to enforcement in order to increase the probability of detecting informal firms and, thus, increase revenue without tax distortions. Moreover, as the probability of detection is increasing and concave in e, the optimal level of enforcement is increasing in the tax distortion λ. Obviously, it also increases with the fine rate, ϕ.

Let us now define the following marginal rate of substitution between enforcement and public good provision:

$$MRS_{e/l_f} = \frac{\frac{\partial p\left(e,\bar{L}_s - l_f\right)}{\partial l_f} g(\bar{L}_s - l_f) + \frac{\partial g(\bar{L}_s - l_f)}{\partial l_f} p(e, \bar{L}_s - l_f)}{\frac{\partial p\left(e,\bar{L}_s - l_f\right)}{\partial e} g(\bar{L}_s - l_f)}.$$

This marginal rate of substitution tells us about the trade-off the State faces when choosing l_f and e. For a given level of fines collected, the State can choose to increase the level of enforcement $g(\bar{L}_s - l_f)\left(\partial p(e, \bar{L}_s - l_f)/\partial e\right)$ or it can increase the level of informality l_i. In this case, there are two effects at work. First, for a given level of informal production, $g(\bar{L}_s - l_f)$, it marginally increases the probability of detection $\partial p(e, \bar{L}_s - l_f)/\partial l_f$. Second, for a given level of detection of informal firms, it marginally increases the size of the fine collected $\partial g(\bar{L}_s - l_f)/\partial l_f$.

Finally, combining the three first order conditions, we obtain:

Remark 2. $$\frac{\frac{\partial f\,(l_f, b)}{\partial l_f} - \frac{\partial g(l_i)}{\partial l_i}}{\frac{\partial f\,(l_f, b)}{\partial b}} = (1-\lambda)\, MRS_{e/l_f}(e, l_f).$$

To understand the previous equality, consider the following function:

$$\Gamma(l_f, b) = \frac{\partial f\,(l_f, b)}{\partial l_f} - \frac{\partial g(l_i)}{\partial l_i}.$$

It is worth noticing that with λ equal to 0, we have $\Gamma(l_f^*, b^*) = 0$, which can be considered as a "pure" efficiency condition. The term "pure" refers to the fact that in such a case, the time devoted to each sector is chosen to equate their marginal productivities. Technically, it comes from the fact that $e^* = 0$, implying that $\partial p(e^* = 0, l_i^*)/\partial e \rightarrow +\infty$ and therefore $MRS_{e/l_f}(e, l_f) = 0$. In words, it means that the presence of tax distortions obliges the State to alter the "pure" efficiency condition. These distortions depend on the marginal rate of substitution between the size of the informal sector and the level of enforcement efforts. This marginal rate of substitution is informative about how much the State needs to increase its spending in enforcement to outweigh a reduction of the size of the informal sector in order to keep the expected fine constant.

To sum-up, in the presence of tax distortions, the State may optimally tolerate a larger level of informality in order to reduce the negative impact of distortions on the State's revenues. On the one hand, more informality is associated to less expenditures in the public good, reducing the effect of tax distortions. On the other hand, because the probability of detection is increasing in the size of the informal firm, more informality implies more fines collected by the State and still less tax distortions. These results are resumed in the following corollary:

Corollary 2. *The presence of tax distortions in developing countries may provide a "rationale" for a relatively large informal sector.*

This remark is consistent with the estimations provided in Auriol and Walters (2006) for African developing countries. Indeed, applying the standard Devarajan *et al*'s "1-2-3" model[13] to a data base of 38 countries, these authors reveal that there is a strong positive relationship between the marginal cost of public funds and the informality levels for the countries in their sample. Our first-best allocation results allow us to provide an explanation to this evidence. In spite of the fact that we consider that the State can choose the household's allocation of time between the formal and the informal sectors, it may be optimal for the State to tolerate a larger informal sector in the presence of higher tax distortions. The shadow cost of public funds may constitute a piece of the puzzle that explains the so-called "broken contract" between the State and the citizens in developing countries that are characterized by high levels of informality (see Perry *et al.*, 2007).

[13] Devarajan *et al.* (1994).

3.2. Second-best analysis

In the second-best analysis, we consider a decentralized economy in which the State cannot choose the household's allocation of labor between the formal and the informal sectors. However, we assume that the State is a Stackelberg leader and chooses the optimal allocation of tax revenues between the provision of the public good and enforcement efforts, taking into account the household's reaction to these choices.

The second-best analysis is organized in three parts. First, we analyze the reaction function of the representative household. The second part is devoted to the normative issue in which the State's program consists in choosing the allocation of public funds between enforcement activities and the provision of the public good. The third part provides a calibration exercise which sheds light on the interplay between policy variables and other endogenous variables of the model.

3.2.1. Optimal public policy with tax distortions and non-observable informality

In this case, the State objective is:

$$\max_{\{b,e\}} W^{SB} = f(\hat{l}_f, b) + g(\bar{L}_s - \hat{l}_f) + \frac{\lambda}{1-\lambda} p(e, \bar{L}_s - \hat{l}_f) g(\bar{L}_s - \hat{l}_f)\phi - \frac{1}{1-\lambda}(b+e) \tag{8}$$

where, $\hat{l}_f\ (e, \tau, b)$ is implicitly determined by the following household's reaction function:

$$g_{l_i}(l_i)\left(1 - p(e, l_i)\phi\right) - p_{l_i}(e, l_i) g(l_i)\phi = (1-\tau) f_{l_f}(\hat{l}_f, b).$$

The first order conditions are:

$$\frac{\partial f(.)}{\partial b} - \frac{1}{1-\lambda} = -\left[\frac{\partial f(.)}{\partial \hat{l}_f\,(.)} - \frac{\partial g(.)}{\partial \hat{l}_f\,(.)} - \frac{\lambda\phi}{1-\lambda}\left(\frac{\partial p(.)}{\partial \hat{l}_f\ (.)} g(l_i) + \frac{\partial g(.)}{\partial \hat{l}_f\,(.)} p(e, l_i)\right)\right] \frac{\partial \hat{l}_f\,(.)}{\partial b}$$

and,

$$\frac{\lambda \frac{\partial p(.)}{\partial e} g(l_i)\phi - 1}{1-\lambda} = -\left[\frac{\partial f(.)}{\partial \hat{l}_f\,(.)} - \frac{\partial g(.)}{\partial \hat{l}_f\,(.)} - \frac{\lambda\phi}{1-\lambda}\left(\frac{\partial p(.)}{\partial \hat{l}_f\ (.)} g(l_i) + \frac{\partial g(.)}{\partial \hat{l}_f\,(.)} p(e, l_i)\right)\right] \frac{\partial \hat{l}_f\,(.)}{\partial e}.$$

In order to understand some features of the optimal policy, consider first that the State wants to implement the same level of formality (resp. informality) in the first-best as that obtained in the second-best (*e.g.* $l_f^* = l_f^{**}$). In this case, we have:

Lemma 3. *If $l_f^* = l_f^{**}$, then the optimal public policy consists of $e^{**} > e^*$ and $b^{**} > b^*$.*

Proof. See the appendix. ■

Lemma 3 says that, in order to implement the same level of formality as in the first-best, the State has to spend more resources in both, the public good provision and enforcement, *i.e.* in b and e respectively. It is due to the fact that as pointed out in Remark 3, the levels of formal activities decided by the household increases in b and e.

Let us consider the case where $\lambda = 0$.

Remark 3. *For $\lambda = 0$, we have $e^{**} = 0$.*

This remark points out that if taxes do not generate distortions, the State still prefers to spend all its budget in the public good and nothing in enforcement activities. This result comes from the same mechanism at work that was revealed in Proposition 1 and Remark 1. It says that spending in the public good is a sufficient instrument in the absence of tax distortions to maximize the representative household's welfare. Conversely, the shadow cost of public funds constitutes the rationale behind the State's investment in enforcement activities. From the State's perspective, all other things being equal, it becomes more profitable to spend in enforcement activities when λ increases. Indeed, in the second-best, the first-order condition in e implies that the term $1 - (\partial p/\partial e)\,\lambda\phi$ is negative, which means that the marginal cost of enforcement activities is lower than its marginal benefit, inducing the State to spend a positive amount of resources on enforcement activities.

Rearranging the first order conditions gives:

$$\frac{\partial \hat{l}_f\,(.)}{\partial b} = \frac{\left[\dfrac{1}{1-\lambda} - \dfrac{\partial f(.)}{\partial b}\right]}{\left[\dfrac{\partial f(.)}{\partial \hat{l}_f\,(.)} + \dfrac{\partial g(.)}{\partial \hat{l}_f\,(.)} + \dfrac{\lambda}{1-\lambda}\dfrac{\partial p(.)}{\partial \hat{l}_f\,(.)}\phi\right]}, \tag{9}$$

$$\frac{\partial \hat{l}_f\,(.)}{\partial e} = \frac{\left[\dfrac{1}{1-\lambda} - \dfrac{\lambda}{1-\lambda}\dfrac{\partial p(.)}{\partial e}\phi\right]}{\left[\dfrac{\partial f(.)}{\partial \hat{l}_f\,(.)} + \dfrac{\partial g(.)}{\partial \hat{l}_f\,(.)} + \dfrac{\lambda}{1-\lambda}\dfrac{\partial p(.)}{\partial \hat{l}_f\,(.)}\phi\right]}. \tag{10}$$

Combining the last two conditions leads to the following proposition:

Proposition 4. *In the presence of distorsive taxes and incentives problem, the optimal public policy follows:*

$$\frac{\dfrac{\partial \hat{l}_f\,(.)}{\partial b}}{\dfrac{\partial \hat{l}_f\,(.)}{\partial e}} = \frac{\dfrac{1}{1-\lambda} - \dfrac{\partial f(.)}{\partial b}}{\dfrac{1}{1-\lambda} - \dfrac{\lambda}{1-\lambda}\dfrac{\partial p(.)}{\partial e}\phi}. \tag{11}$$

In order to understand Proposition 4, let us first focus on the case when $\lambda = 0$. The previous condition becomes:

$$\frac{\partial f(.)}{\partial b} = 1 - MRS_{e/b}.$$

We can easily recognize the equality obtained in the first best case between the marginal productivity of the public good in the formal sector and its marginal cost. Nevertheless, this equality is distorted by the marginal rate of substitution between e and b. Indeed, as long as the State cannot choose directly the level of informality, it has to set the amounts e^{**} and b^{**} taking into account their relative impact on the size of the formal (resp. informal) sectors.

It is more tedious to disentangle the effects of λ and the no observability issues arising from the condition given in Proposition 4. In the following section we calibrate the model in order to highlight these two combining effects on $\hat{l}_f$, b^{**} and e^{**}.

3.3. Calibration of the model and results

In order to asses how changes in some of the structural parameters of the model affect equilibrium choices, we proceed by calibrating the model using functional forms that satisfy the assumptions made througout the paper. In the appendix, we present the equations used in the calibration of the model. The functional forms that we use in this calibration exercise are:[14]

$$f(l_f, b) = l_f^{\alpha} b^{\psi}, \qquad g(l_i) = \gamma l_i, \qquad p(e, l_i) = e^{\beta} l_i, \tag{12}$$

$$\text{with} : \alpha, \psi, \beta \in (0,1), \text{ and } \gamma > 0. \tag{13}$$

According to these functional forms, α and ψ represent the relative importance of labor and the public good in production in the formal sector.[15] We assume, without loss of generality, that informal production is a linear function of the amount of informal labor, with a productivity captured by the parameter γ.[16] Finally, the probability that an informal firm is detected evading taxes is an increasing and concave function of the government's

[14] It is straightforward to show that these functional forms satisfy all assumptions made throughout the paper.

[15] We assume that $\alpha + \psi \leq 1$.

[16] Behind the assumption that labor has decreasing returns to scale in the formal sector and linear in the informal sector lies the (not modeled here) potential use of a fixed amount of physical capital in the formal sector that induces decreasing returns to the allocation of labor to this sector.

enforcement efforts, e, and a linear function of the size of the informal sector, as captured by l_i.

In order to check the robustness and stability of the calibration results, we conduct 10,000 Montecarlo calibrations where we obtain the parameters used in each round of calibrations from a pre-defined distribution function for each parameter.[17] While some of the parameters were chosen relying on literature estimates, others were chosen in order to match some outcomes of Latin American countries such as informality rates and tax collection as a percentage of GDP. Table 1 presents the mean value (and support) of the parameters used in the calibration excersise.

[INSERT Table 1 here]

Table 2 presents the results of the calibration of the model under the first-best (second column) and the second-best (third column). In particular, this table presents the mean calibrated value for the endogenous variables of the model as well as its 95% confidence interval (presented in parenthesis).

[INSERT Table 2 here]

According to the results presented in Table 2, formal labor accounts for about 63% in the first-best and 59% in the second-best. The provision of the public good accounts for about 4.5% of this economy total production ($y_f + y_i$) in the first-best and 4.3% in the second-best, while enforcement accounts for 0.04% in the first-best and 0.06% in the second best. According to the calibration results, the probability that an informal firm is detected is about 7.2% in the first-best and 8.7% in the second-best. These calibration results are aimed at showing that the calibration of the model in a baseline scenario reproduces plausible results.

A perhaps more interesting dimension to exploit with the calibration results is to undertake simulation exercises, where we exogenously change a parameter of interest and recover the change in the endogenous variables of the model. More precisely, we are interested in doing simulations with respect to λ, the shadow cost of public funds. The results of these simulations are presented in Figure 1. The panels in Figure 1 show how the endogenous variables of the model change as the shadow cost of public funds, λ increases in the range from 0.2 to 0.6. In particular, the panels in Figure 1 show a negative relation between the shadow cost of public funds and expected income (panel A); the optimal tax rate (B); the size of the formal sector, as captured both by l_f (C); the provision of the public good

[17] We use the F-Solve routine in Matlab in order to calibrate the model. We use uniform distributions as the underlying distribution behind each of the parameters of the model.

(F); total government expenditures, $b + e$ (G); and total production (D). Also, panels D and E depict a positive relationship between the size of tax distortions and the size of the informal sector, and the optimal level of enforcement, respectively. Finally, the message to take away is that higher values of the shadow cost of public funds require more stick (more enforcement) and less carrot (public goods).

4. Concluding remarks

This paper develops a model where the size of the informal sector is endogenously determined by the interaction of a representative individual and the government. On the one hand, the representative individual has to decide the allocation of time between the formal and the informal sectors. While in the formal sector the individual can make use of a public good provided by the government but has to pay taxes with probability one, in the informal sector the individual only pays taxes with an endogenously determined probability (which is lower than one) but cannot benefit from the use of the government-provided good in production. The government, on the other hand, has to decide the allocation of resources (collected through the tax system and fines imposedon those informal firms that are detercted) between enforcement activities to detect and penalize informal activities (the 'sticks') and the provision of a public (club) good that can only be used in the formal sector. We make special emphasis on the role of tax distortions (the shadow cost of public funds) in the determination of the size of the informal sector, the optimal tax rate and total production, among other endogenous variables.

We make use of calibrations and simulations of the model using plausible parameter values in order to disentangle the relationship between the size of tax distortions and other endogenous variables of the model. In particular, our results point out a negative relationship between the shadow cost of public funds and the optimal tax rate. Consequently, the higher is the shadow of public funds, the lower are the size of the formal sector, the provision of the public good and the total government expenditure. Our results also exhibit a positive relationship between the size of tax distortions and the size of the informal sector. When tax distortions increase, it is optimal to reduce the amount of public good in order to spend in enforcement.

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5. Appendix

5.1. Proof of Lemma 3

Consider the following function:

$$\Psi\left(e,b,l_f,\lambda\right)=\frac{\partial f(.)}{\partial\hat{l}_f\,(.)}-\frac{\partial g(.)}{\partial\hat{l}_f\,(.)}-\frac{\lambda\phi}{1-\lambda}\left(\frac{\partial p(.)}{\partial\hat{l}_f\,(.)}g(l_i)+\frac{\partial g(.)}{\partial\hat{l}_f\,(.)}p(e,l_i)\right).$$

At the first-best, we have $\Psi\left(e^{**},b^{**},l_f^{**},\lambda\right)=0$. Moreover, we have:

$$\left\{\begin{array}{r}\frac{\partial f(L-l_f^*,b^*)}{\partial b}=\frac{1}{1-\lambda},\\ g(L-l_f^*)=\frac{\lambda\phi}{1-\lambda}\frac{1}{\frac{\partial p\left(e^*,L-l_f^*\right)}{\partial e}}.\end{array}\right.$$

The second-best is characterized by:

$$\left\{\begin{array}{r}\frac{\partial f(L-\hat{l}_f,b^{**})}{\partial b}=\frac{1}{1-\lambda}-\Psi\left(e^{**},b^{**},\hat{l}_f,\lambda\right)\frac{\partial\hat{l}_f(.)}{\partial b},\\ g(L-\hat{l}_f)=\left(\frac{\lambda\phi}{1-\lambda}-\Psi\left(e^{**},b^{**},\hat{l}_f,\lambda\right)\frac{\partial\hat{l}_f(.)}{\partial e}\right)\frac{1}{\frac{\partial p\left(e^{**},L-\hat{l}_f\right)}{\partial e}}.\end{array}\right.$$

Therefore, we obtain that

$$\begin{aligned}g(L-\hat{l}_f)-g(L-l_f^*)&=\left(\frac{\lambda\phi}{1-\lambda}-\Psi\left(e^{**},b^{**},\hat{l}_f,\lambda\right)\frac{\partial\hat{l}_f\,(.)}{\partial e}\right)\frac{1}{\frac{\partial p\left(e^*,L-\hat{l}_f\right)}{\partial e}}\\&\quad-\left(\frac{\lambda\phi}{1-\lambda}\frac{1}{\frac{\partial p\left(e^*,L-l_f^*\right)}{\partial e}}\right)\\&=\frac{\lambda\phi}{1-\lambda}\left(\frac{1}{\frac{\partial p\left(e^{**},L-\hat{l}_f\right)}{\partial e}}-\frac{1}{\frac{\partial p\left(e^*,L-l_f^*\right)}{\partial e}}\right)\\&\quad-\Psi\left(e^{**},b^{**},\hat{l}_f,\lambda\right)\frac{\partial\hat{l}_f\,(.)}{\partial e}\frac{1}{\frac{\partial p\left(e^*,L-\hat{l}_f\right)}{\partial e}}.\end{aligned}$$

We have

$$g(L-\hat{l}_f)-g(L-l_f^*)$$

$$\Leftrightarrow$$

$$\frac{1}{\frac{\partial p\left(e^{**},L-\hat{l}_f\right)}{\partial e}}-\frac{1}{\frac{\partial p\left(e^{*},L-l_f^*\right)}{\partial e}} \leq \Psi\left(e^{**},b^{**},\hat{l}_f,\lambda\right)\frac{\partial \hat{l}_f\left(.\right)}{\partial e}\frac{1}{\frac{\partial p\left(e^{*},L-\hat{l}_f\right)}{\partial e}}$$

It implies that

$$\frac{1}{\frac{\partial p\left(e^{**},L-\hat{l}_f\right)}{\partial e}} \leq \frac{1}{\frac{\partial p\left(e^{*},L-l_f^*\right)}{\partial e}}.$$

According to that $\partial^2 p/\partial e \partial l_i > 0$, we have $e^{**} \geq e^*$.

The same reasoning applies for the public spending. *Q.E.D.*

5.2. Calibration of the model

5.2.1. Functional Forms

In this appendix we describe the functional forms as well as the equations that we use to calibrate the model.

The functional forms that we use to calibrate the model are:[18]

$$f\left(l_f,b\right)=l_f^{\alpha}b^{\psi}, \qquad g(l_i)=\gamma l_i, \qquad p(e,l_i)=e^{\beta}l_i \tag{A1}$$

5.2.2. First-Best

The solution in this case is described in Proposition 2. When we combine the optimality conditions from Proposition 2 with the functional forms in **??**, we obtain the following four equations that are used to calibrate the model under the first-best allocation:

$$\alpha l_f^{\alpha-1}b^{\psi}-\gamma=\left(\frac{\lambda\phi}{1-\lambda}\right)2\gamma e^{\beta}l_i \tag{A9}$$

$$\psi l_f^{a}b^{\psi-1}=\left(\frac{1}{1-\lambda}\right) \tag{A10}$$

$$\beta\gamma e^{\beta-1}l_i^2=\frac{1}{\lambda\phi} \tag{A11}$$

[18] It is straigtforward to show that these functional forms satisfy all assumptions made throughout the paper.

$$1 = l_f + l_i \tag{A12}$$

5.2.3. Second-Best

As it is explained in section 3.3.1, the household's reaction functions arise from the implicit function $\Omega(l_f, b, e)$, which, using the functional forms in **??** becomes:

$$\Omega(l_f, b, e) = (1-\tau)\alpha l_f^{\alpha-1} b^{\psi} - \gamma + 2\gamma\phi e^{\beta}(1-l_f) \tag{A13}$$

The reaction functions from equations (11) and (12) become:

$$\frac{\partial \hat{l}_f(.)}{\partial b} = \frac{-(1-\tau)\alpha\psi l_f^{\alpha-1} b^{\psi-1}}{(1-\tau)\alpha(\alpha-1) l_f^{\alpha-2} b^{\psi} - 2\gamma\phi e^{\beta}} \tag{A14}$$

$$\frac{\partial \hat{l}_f(.)}{\partial e} = \frac{-2\gamma\phi\beta e^{\beta-1}(1-l_f)}{(1-\tau)\alpha(\alpha-1) l_f^{\alpha-2} b^{\psi} - 2\gamma\phi e^{\beta}} \tag{A15}$$

The equations that we use to calibrate the solution under the second-best allocation are:

$$\gamma - 2\gamma\phi e^{\beta} l_i = (1-\tau)\alpha l_f^{\alpha-1} b^{\psi} \tag{A16}$$

$$1 = l_f + l_i$$

$$\frac{1}{1-\lambda} - \psi l_f^{\alpha} b^{\psi-1} = \frac{\partial \hat{l}_f(.)}{\partial b}\left[\alpha l_f^{\alpha-1} b^{\psi} - \gamma - \frac{2\lambda\phi\gamma e^{\beta} l_i}{1-\lambda}\right] \tag{A17}$$

$$\frac{\lambda\phi\gamma\beta e^{\beta-1} l_i^2 - 1}{1-\lambda} = -\frac{\partial \hat{l}_f(.)}{\partial e}\left[\alpha l_f^{\alpha-1} b^{\psi} - \gamma - \frac{2\lambda\phi\gamma e^{\beta} l_i}{1-\lambda}\right], \tag{A18}$$

where the terms $\partial\hat{l}_f(.)/\partial b$ and$\partial\hat{l}_f(.)/\partial e$ are obtained from equations **??** and **??** respectively.

Table 1: Parameter Values and Underlying Support

Parameter	Mean	Support
α	0.6	[0.575, 0.625]
β	0.2	[0.175, 0.225]
γ	0.5	[0.49, 0.51]
ψ	0.1	[0.09, 0.11]
ϕ	0.25	[0.225, 0.275]
λ	0.4	[0.35, 0.45]

Table 2: Calibration Results and Conf. Intervals

Variable	Second Best	First Best
b	0.0317	0.0323
	[0.0268, 0.0366]	[0.0268, 0.0379]
e	0.0004307	0.000276
	[0.0003104, 0.0005509]	[0.00002131, 0.0005307]
l_f	0.59	0.63
	[0.51, 0.67]	[0.51, 0.76]
l_i	0.41	0.37
	[0.33, 0.49]	[0.24, 0.49]
$p(e, l_i)$	0.0873	0.0717
	[0.0636, 0.111]	[0.0288, 0.115]
y_f	0.52	0.54
	[0.46, 0.57]	[0.46, 0.62]
y_i	0.21	0.18
	[0.17, 0.25]	[0.12, 0.25]
$b + e$	0.0321	0.0326
	[0.0273, 0.037]	[0.0272, 0.038]
$l_i/(l_f + l_i)$	0.41	0.367
	[0.334, 0.486]	[0.242, 0.491]
$y_i/(y_f + y_i)$	0.285	0.254
	[0.224, 0.345]	[0.161, 0.348]

Figure 1: Simulation results with respect to the shadow cost of public funds