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"An Analytical Characterization of Noisy Fiscal Policy"

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An Analytical Characterization of Noisy Fiscal Policy

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Abstract

This paper provides an analytical characterization of the effects of noisy news shocks on fiscal policy. We consider a small-scale Dynamic Stochastic General Equilibrium (DSGE) model with capital accumulation and endogenous labor supply and show that noise dampens the propagation of anticipated fiscal policy over the business cycle, thus reducing the fiscal multiplier.

Keywords: Government spending shocks, Expected shocks, Noisy Information, DSGE Models.
JEL Class.: C32, E62

1 Introduction

A recent stream of literature has emphasized the role of news shocks in fiscal policy (see e.g. Leeper et al. 2013): quantitatively, news shocks are the main driver of government spending (Schmitt-Grohé and Uribe 2012, Khan and Tsoukalas 2012, Born et al. 2013). However, Fève and Pietrunti (2016) show that noisy news in government spending (in a medium–scale DSGE model with real frictions) may lead to a reduction in the size of fiscal multipliers. The aim of this note is to analytically clarify such effects. We therefore make use of a relevant small–scale DSGE model with physical capital and endogenous labor supply featuring both a backward dimension (through capital accumulation) and a forward one (expectations about the future policy). We solve it analytically and show that noise on fiscal policy may persistently affect the economy due to the reaction of private investment. Dupaigne and Fève (2016) and Fève and Pietrunti (2016)

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obtain that the response of investment is critical for government spending multiplier. We therefore concentrate our current analysis on capital accumulation.[1] The paper is organized as follows. A first section presents the DSGE model and the information structure. In a second section, we expound the analytical results and discuss the effect of noisy news on investment. A last section briefly concludes.

2 A Small-Scale DSGE Model with a Noisy Fiscal Policy

Throughout the paper we will work with the simplest and tractable possible model that is able to deliver meaningful analytical results. Hence we consider a business cycle model with capital accumulation, labor supply and exogenous government spending. The inter-temporal expected utility function of the representative household is given by

$$\hat{E}_t \sum_{i=0}^{\infty} \beta^{t+i} \left\{ \log(c_{t+i}) - \eta n_{t+i} \right\}$$

where $\eta > 0$ and $\beta \in (0,1)$ denotes the discount factor. $\hat{E}_t$ is the expectation operator conditional on the information set available as of time t. The variables c_t and n_t represent the real consumption and the household's labor supply.[2] The representative firm uses capital k_t and labor input n_t to produce a homogeneous final good y_t. Technology is represented by the following production function $y_t = A k_t^{\theta} n_t^{1-\theta}$, where $A > 0$ and $\theta \in (0,1)$. The capital stock k_t evolves according to $k_{t+1} = x_t$, where x_t denotes investment. The physical capital fully depreciates every period.[3] Finally, the final good can be either consumed, invested or devoted to government spending $y_t = c_t + x_t + g_t$. We consider that the log of g_t (in deviation from its non-stochastic steady-state) follows an autoregressive process of order one

$$\widehat{g}_t = \rho \widehat{g}_{t-1} + \varepsilon_{t-1}, \tag{1}$$

where $|\rho| \leq 1$ and $\varepsilon_t \sim N\left(0, \sigma_{\varepsilon}^2\right)$. The timing of shock is such that the government policy is declared one period in advance, for ease of illustration. The expected government policy in period $t+1$ is given by $\hat{E}_t \widehat{g}_{t+1} = \rho \widehat{g}_t + \hat{E}_t \varepsilon_t$. In a perfect information setting the change in government spending is perfectly anticipated by private agents. Hence this equation reduces to $\hat{E}_t \widehat{g}_{t+1} = \rho \widehat{g}_t + \varepsilon_t \equiv \widehat{g}_{t+1}$. Here, we make the setting more general, assuming that private agents observe a noisy signal of ε_t (i.e. a noisy news on government spending), given by

$$s_t = \varepsilon_t + \nu_t \tag{2}$$

where $\nu_t \sim N\left(0, \sigma_{\nu}^2\right)$ represents a noise shock and it is therefore uncorrelated with ε_t for any time index. If $\sigma_{\nu} \to 0$ we go back to the standard full information case. If $\sigma_{\nu} > 0$, information is noisy

[1] The positive responses of output follow the ones of investment. Check figure 2 in Appendix B for the dynamic responses of output.

[2] The linearity assumption in leisure's utility greatly simplifies the computation of the solution because the real wage and the real interest rate depend only on real consumption (see Dupaigne and Fève 2016).

[3] Our results can be easily extended to incomplete depreciation (See Kass-Hanna 2016).

as private agents do not perfectly observe new policy announcements. Noise here is a shortcut for capturing both the complex political process that leads to fiscal policy changes[4] and the complex task of gathering information on newly announced policies. In this case of imperfect information, it can be shown that the conditional expectations of private agents are a linear function of the signal received: $\hat{E}_t\varepsilon_t = \alpha s_t \equiv \alpha(\varepsilon_t + \nu_t)$, where the parameter α is obtained by a linear projection of ε_t on s_t (see Hamilton 1994)

$$\alpha = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_\nu^2}.$$

When information is perfectly observed ($\sigma_\nu = 0$ and $\alpha = 1$), private agents fully incorporate the announced government policy in their consumption/saving and labor supply choices. Conversely, when information about the announced economic policy is totally noisy ($\sigma_\varepsilon/\sigma_\nu \to 0$ and $\alpha \to 0$), they will not react to the announced economic policy.

3 The Dynamic Responses to Noisy News

To get more insights about the effects of the noisy expected government spending, we derive the log–linearized version of equilibrium conditions around the non-stochastic steady state. In Appendix A, we report the solutions for output, consumption and investment. Here, we focus on the response of investment (or next period capital) to noisy fiscal shocks.[5] The solution for investment is given by:

$$\widehat{x}_t = \theta\widehat{x}_{t-1} + s_g\left(\frac{\rho-\theta}{1-\beta\theta\rho}\right)\widehat{g}_t + \alpha s_g\left(\frac{1-\beta\theta^2}{1-\beta\theta\rho}\right)s_t, \tag{3}$$

where s_g denotes the share of government spending in output at steady state. From equation (3), we can characterize the dynamic responses to both news and noise.

Proposition 1 *The dynamic responses of investment to noise (ν_t) and news (ε_t) are given by:*

$$\frac{\partial\hat{x}_{t+h}}{\partial\nu_t} = \alpha\theta^h s_g\frac{1-\beta\theta^2}{1-\beta\theta\rho} \tag{4}$$

$$\frac{\partial\hat{x}_{t+h}}{\partial\varepsilon_t} = \frac{\partial\hat{x}_{t+h}}{\partial\nu_t} + \frac{s_g}{1-\beta\theta\rho}(\rho^h - \theta^h) \tag{5}$$

for $h \geq 0$.

Proof: See Kass-Hanna (2016).

Proposition 1 shows that the responses of investment to noise and news are heavily linked. In particular, equation (5) shows that the response to news has two components: the first depends

[4]For example, it may represent the imperfect credibility and/or transparency of the government policy (See Fève and Pietrunti 2016 for a discussion).

[5]See Kass-Hanna (2016) for a complete characterization of output, consumption and investment dynamic responses. As illustration, see figure 2 for the dynamic response of output in Appendix B.

on α and corresponds to the investment response to noise; the second component is independent of α and is equivalent to the investment response to an unexpected shock that arrives one period ahead.[6] In more intuitive terms, the response to an anticipated government spending shock ε_t is twofold: a direct response to the signaling of the news that persists after the realization of the shock ($h \geq 1$), and a response to the material realization of the expected shock at $h = 1$, respectively. Notice that the second term of equation (5) is equal to 0 for $h = 0$. The presence of noise reduces the impact response of investment to expected policy by diminishing the first term of equation (5) (since $\alpha < 1$), hence weakening the impact response of investment to the signaling of the news, as noise makes news less credible.

We now turn to three specific cases to further illustrate the effect and propagation of noisy fiscal policy over time.

First, we consider the peculiar case where investment does not react to contemporaneous fiscal policy (ie. $\rho = \theta$).[7] Under such assumption, its dynamics simplify to

$$\widehat{x}_t = \theta \widehat{x}_{t-1} + \alpha s_g s_t.$$

Investment responds only to the signaling of the fiscal policy and can thus be equally explained by news and noise shocks when $\sigma_\varepsilon = \sigma_\nu$ (ie. $\alpha = 1/2$). From the above equation we obtain that the information friction (initially) created by noisy news ($\alpha < 1$) will persist along the transition path toward the non–stochastic steady-state. Indeed, the dynamic responses of investment are such that

$$\frac{\partial \widehat{x}_{t+h}}{\partial \varepsilon_t} \equiv \frac{\partial \widehat{x}_{t+h}}{\partial \nu_t} = \alpha \theta^h s_g$$

for $h \geq 0$. It follows immediately that the relative difference between the full and partial information settings is given by the size of the initial information friction $1 - \alpha$, and is invariant to the horizon. This result ensues because information friction changes the impact responses without modifying the model's dynamic properties.

Second, consider the case where $\rho \neq \theta$, and public policy is very noisy ($\alpha \to 0$). In this case, the dynamics of investment are governed by

$$\widehat{x}_t = \theta \widehat{x}_{t-1} + s_g \left(\frac{\rho - \theta}{1 - \beta \theta \rho} \right) \widehat{g}_t,$$

which is equivalent to the case of an unexpected shock (see Dupaigne and Fève 2016) but one step ahead. Indeed, because public policy is (almost) perfectly noisy, private agents do not react to

[6]Notice that the response of investment to an unexpected shock that arrives at $t+1$ would be equivalent to $\frac{\partial \hat{x}_{t+h}}{\partial \hat{g}_{t+1}} = \frac{s_g}{1-\beta\theta\rho}(\rho^h - \theta^h)$.

[7]Given standard calibrations for θ around 0.4 and ρ larger than 0.9, this assumption seems hard to swallow. Note however that in general, the parameter of interest is not θ per se, but rather $\mu = \frac{\theta}{\theta + (1-\theta)(1-\beta(1-\delta))}$ (the stable root of the model), which happens to be equal to θ in our complete depreciation case ($\delta = 1$). Under an incomplete depreciation scenario whereby $\delta = 0.015$, $\beta = 0.98$, and $\theta = 0.4$, the parameter of interest μ equals 0.95, a value close to the standard calibration for ρ, rendering our first considered case more relevant, especially under incomplete depreciation. Indeed, the estimates of the first-order autoregressive coefficient of US government spending are found to be large (0.97 in Smets and Wouters (2007) and Leeper et al. (2010)).

signals. It follows that this extreme case of noisy fiscal policy deeply impacts the short-run effect of news shocks on government spending, because it introduces a systematic delay in the diffusion of the policy to the private sector. With time, the effect of this delay progressively disappears.

Third, we consider a more realistic case of a highly persistent process for government spending ($\rho \to 1$) such that $\rho \neq \theta$. Accordingly, the dynamics of investment are governed by

$$\Delta\widehat{x}_t = \theta\Delta\widehat{x}_{t-1} + s_g\left(\frac{1-\theta}{1-\beta\theta}\right)\varepsilon_{t-1} + \alpha s_g\left(\frac{1-\beta\theta^2}{1-\beta\theta}\right)\Delta s_t,$$

where Δ is the first difference operator. Notice that during the implementation phase of government spending (i.e. $q = 1$) news and noise shocks equally explain the variance of investment. However, when the news shock realizes, expected government policy begins to matter while the contribution of noise gradually fades out. Nevertheless, the impact of noise is not extinguished at the realization of the shock, it rather propagates over time. As in the cases discussed above, the response on impact is directly linked to the value of α:

$$\frac{\partial\widehat{x}_t}{\partial\varepsilon_t} = \alpha s_g\frac{1-\beta\theta^2}{1-\beta\theta}.$$

The relative difference on impact between the full and partial information cases depends only on α. Nonetheless, the difference persists even after the news shock realizes. Indeed, the response to news at $t+1$ is given by

$$\frac{\partial\widehat{x}_{t+1}}{\partial\varepsilon_t} = s_g(1-\theta)\frac{1-\alpha(1-\beta\theta^2)}{1-\beta\theta},$$

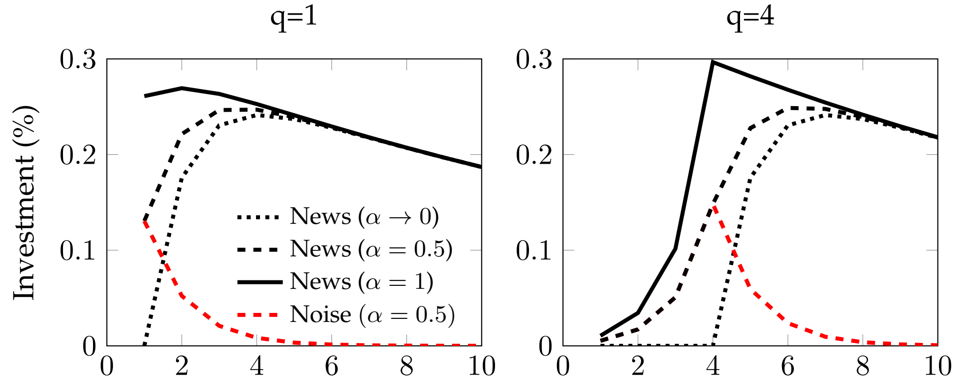
whereby the relative difference between full and partial information equals $(1-\alpha(1-\beta\theta^2)/(\beta\theta^2)$. Notice that the difference after the realization of the shock does not only depend on α, but also on the interaction of imperfect information with deep parameters representing preferences and technology.

The argument made so far can be extended to the case in which fiscal shocks are generically declared $q > 1$ periods in advance. In this case, the government spending process rewrites

$$\widehat{g}_t = \rho\widehat{g}_{t-1} + \varepsilon_{t-q}. \tag{6}$$

In Figure 1 we plot the Impulse Response Functions (IRFs) of investment to news shocks under alternative scenarios. In the left panel we plot the IRF of investment when $q = 1$ for three different values of α, ranging from the perfect information case ($\alpha = 1$) to the extremely noisy one ($\alpha \to 0$). Note that for $\alpha > 0$ agents start accumulating new capital stock since the announcement of the policy, at a pace that reflects the quality of the information received. However, when information is completely noisy, the capital stock stands still until the news realizes. Such a shock comes indeed unexpected for the agents populating this economy. In the right panel, we plot the response when news is announced four periods in advance ($q = 4$). Notice that a gradual accumulation of capital – increasing with α – occurs directly after the announcement of the policy and before the realization of the shock. We also plot (in red dashed line) the response to noise under the assumption of an equal share of news and noise in the economy ($\sigma_\varepsilon = \sigma_\nu$, hence $\alpha = 0.5$). We observe that up to the realization of the shock in period q, the reaction to both shocks is exactly the same. Also notice that the effect of noisy fiscal policy persists due to the backward–looking behavior of capital accumulation (See also the Figure 2 in Appendix B).

Figure 1: IRFs of investment to news and noise shocks



Note: Parameters are set at the following values: $\beta = 0.98$, $\theta = 0.4$ and $\rho = 0.95$.

4 Conclusion

In this paper, we have shown in a small–scale DSGE model that news shocks on fiscal policy mainly propagate through capital accumulation. Such dynamics are hampered when imperfect information is introduced to the setting and the effect of noise in the reaction of investment is long lasting, as it persists even after a news shock materializes.

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Appendix

A The Equilibrium with Noisy Fiscal Policy

The dynamic equilibrium of this economy is summarized by the following equations

$$\begin{aligned} k_{t+1} &= y_t - c_t - g_t && \text{(A.1)} \\ y_t &= k_t^{\theta} n_t^{1-\theta} && \text{(A.2)} \\ \eta &= \frac{1}{c_t}(1-\theta)\frac{y_t}{n_t} && \text{(A.3)} \\ \frac{1}{c_t} &= \beta\theta \hat{E}_t\left[\left(\frac{y_{t+1}}{k_{t+1}}\right)\frac{1}{c_{t+1}}\right] && \text{(A.4)} \end{aligned}$$

Equation (A.1) defines the law of motion of physical capital with complete depreciation, while equations (A.2) and (A.3) are the production function and the marginal rate of substitution between consumption and leisure at equilibrium. The last equation (A.4) represents the Euler equation on consumption. The log–linearization of (A.1)-(A.4) around the non-stochastic steady state yields

$$\begin{aligned} \widehat{k}_{t+1} &= \frac{y}{k}\widehat{y}_t - s_c\frac{y}{k}\widehat{c}_t - s_g\frac{y}{k}\widehat{g}_t && \text{(A.5)} \\ \widehat{y}_t &= \theta\widehat{k}_t + (1-\theta)\widehat{n}_t && \text{(A.6)} \\ \widehat{n}_t &= \widehat{y}_t - \widehat{c}_t && \text{(A.7)} \\ \hat{E}_t\widehat{c}_{t+1} &= \widehat{c}_t + \hat{E}_t(\widehat{y}_{t+1} - \widehat{k}_{t+1}) && \text{(A.8)} \end{aligned}$$

where $y/k = 1/(\beta\theta)$, $s_c = c/y \equiv 1 - \beta\theta - s_g$ and $s_g = g/y$. After substitution of (A.7) into (A.6), one gets

$$\widehat{y}_t - \widehat{k}_t = -\frac{1-\theta}{\theta}\widehat{c}_t$$

Now, using the above expression, (A.5) and (A.8) rewrite

$$\hat{E}_t\widehat{c}_{t+1} \quad = \quad \mu\widehat{c}_t \quad \text{with} \qquad \mu = \theta \in (0,1) \tag{A.9}$$

$$\widehat{k}_{t+1} \quad = \quad \nu_1\widehat{k}_t - \nu_2\widehat{c}_t - \nu_3\widehat{g}_t \ , \tag{A.10}$$

with

$$\nu_1 = \frac{1}{\beta\mu} > 1 \ \ , \ \ \nu_2 = (y/k)(((1-\theta)/\theta) + s_c) > 0 \ \text{ and } \ \nu_3 = s_g(y/k) > 0,$$

As $\mu < 1$ and $\mu\nu_1 = 1/\beta > 1$, we immediately deduce $\nu_1 > 1$. Then, equation (A.10) must be solved forward

$$\widehat{k}_t = \left(\frac{\nu_2}{\nu_1}\right)\lim_{T\to\infty}\hat{E}_t\sum_{i=0}^{T}\left(\frac{1}{\nu_1}\right)^i\widehat{c}_{t+i} + \left(\frac{\nu_3}{\nu_1}\right)\lim_{T\to\infty}\hat{E}_t\sum_{i=0}^{T}\left(\frac{1}{\nu_1}\right)^i\widehat{g}_{t+i} + \lim_{T\to\infty}\hat{E}_t\left(\frac{1}{\nu_1}\right)^T\widehat{k}_{t+T}.$$

Excluding explosive pathes, i.e. $\lim_{T\to\infty}\hat{E}_t\,(1/\nu_1)^T\,\widehat{k}_{t+T} = 0$ and taking the limit, we have:

$$\widehat{k}_t = \left(\frac{\nu_2}{\nu_1}\right)\hat{E}_t\sum_{i=0}^{\infty}\left(\frac{1}{\nu_1}\right)^i\widehat{c}_{t+i} + \left(\frac{\nu_3}{\nu_1}\right)\hat{E}_t\sum_{i=0}^{\infty}\left(\frac{1}{\nu_1}\right)^i\widehat{g}_{t+i}.$$

Now, using (1), (2) and (A.9), one gets

$$\begin{aligned}\left(\frac{\nu_2}{\nu_1}\right)\hat{E}_t\sum_{i=0}^{\infty}\left(\frac{1}{\nu_1}\right)^i\widehat{c}_{t+i} \quad &= \quad \frac{\nu_2}{\nu_1-\mu}\widehat{c}_t \\ \left(\frac{\nu_3}{\nu_1}\right)\hat{E}_t\sum_{i=0}^{\infty}\left(\frac{1}{\nu_1}\right)^i\widehat{g}_{t+i} \quad &= \quad \frac{\nu_3}{\nu_1-\rho}\widehat{g}_t + \alpha\frac{\nu_3}{\nu_1(\nu_1-\rho)}s_t,\end{aligned}$$

from which we deduce the decision rule on consumption:

$$\widehat{c}_t = \left(\frac{\nu_1-\mu}{\nu_2}\right)\widehat{k}_t - \left(\frac{\nu_3(\nu_1-\mu)}{\nu_2(\nu_1-\rho)}\right)\widehat{g}_t - \alpha\left(\frac{\nu_3(\nu_1-\mu)}{\nu_1\nu_2(\nu_1-\rho)}\right)s_t. \tag{A.11}$$

After substituting (A.11) into (A.10), the dynamics of capital is given by:

$$\widehat{k}_{t+1} = \mu\widehat{k}_t + \left(\frac{\nu_3(\rho-\mu)}{\nu_1-\rho}\right)\widehat{g}_t + \alpha\left(\frac{\nu_3(\nu_1-\mu)}{\nu_1(\nu_1-\rho)}\right)s_t. \tag{A.12}$$

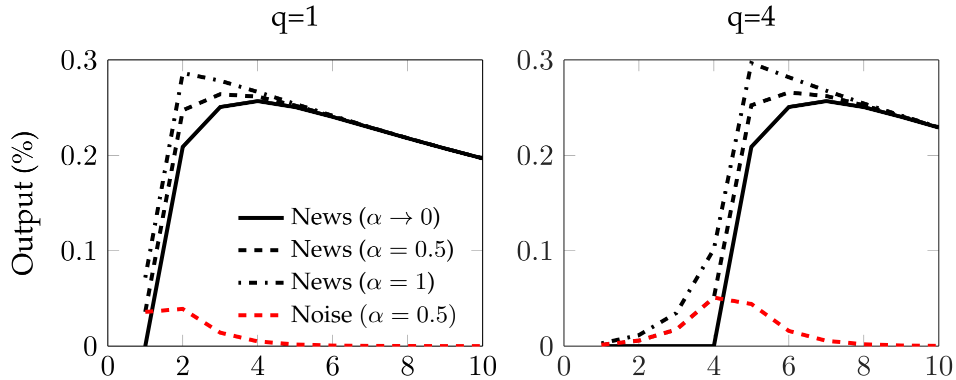
The persistence properties of the model are thus governed by the parameter $\mu \in (0,1)$ and the response of the physical capital to the announced government spending policy critically depends on α and ρ. Now, using the expressions for μ, ν_1, ν_2 and ν_3, and the dynamics of capital (or equivalently investment, as with full depreciation we have $\widehat{x}_t = \widehat{k}_{t+1}$) (A.12), the consumption function (A.11) and the production function

$$\widehat{y}_t = \widehat{k}_t - \frac{1-\theta}{\theta}\widehat{c}_t,$$

we can characterize the dynamics of this economy in response to noisy news on government spending (See Kass-Hanna (2016) for a complete characterization of the dynamic responses of each variable).

B The Dynamic Responses of Output

Figure 2: IRFs of output to news and noise shocks



Note: Parameters are set at the following values: $\beta = 0.98$, $\theta = 0.4$ and $\rho = 0.95$.