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"Ideological Perfectionism"

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IDEOLOGICAL PERFECTIONISM

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Abstract: Studying a high-stakes field setting, we examine which individuals, on an ideological scale, conform more to the opinion of others. In the U.S. Courts of Appeals, legal precedents are set by ideologically diverse and randomly composed panels of judges. Using exogenous predictors of ideology and rich voting data we show that ideological disagreements drive dissents against the panel's decision, but ideologically extreme judges are caving in: they are the least likely to dissent and their voting records are the least correlated with their predicted ideology. Meanwhile, moderately ideological judges are dissenting the most despite evidence that they are more often determining the opinion. Our theoretical analysis shows that these findings are most consistent with a model of decision making in the presence of peer pressure with a concave cost of deviating from one's ideological convictions – perfectionism. This result presents a critique of a standard assumption in economics – that the cost of deviating from one's bliss point is convex – with fundamental implications for decision making in social and political settings and for the empirical predictions of theoretical models in these domains.

Keywords: Judicial decision making, group decision making, ideology, peer pressure.

JEL codes: D7, K0, Z1

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1 Introduction

Economics tends to gravitate toward the assumption that costs—be they economic, effort or cognitive—are convex. One rationale for this assumption is that it makes theoretical models analytically tractable. Another rationale is that it seems intuitively plausible. However, such intuition has proved fragile following a number of recent experiments suggesting that, when it comes to decision making on moral or ethical issues, individuals perceive a concave cost of deviating from what they believe is right (Kajackaite and Gneezy 2015; Abeler et al. 2016; Gino et al. 2010; Hurkens and Kartik 2009).[1] That is, individuals are perfectionist when it comes to morals or ideology as they do not distinguish much between small and large deviations from their bliss points. This has also been argued by Osborne (1995) to be realistic in ideological settings. Whether individuals perceive costs to be convex or concave has far-reaching implications for the behavior we should expect of them and, as such, it goes to the heart of the empirical predictions theoretical models provide. For instance, individuals with concave moral costs will tend to give up on their morals if they cannot follow them fully. This pattern of behavior has been popularly labeled the "what-the-hell-effect" (Ariely 2012; Baumeister and Heatherton 1996). Theoretically, the concavity of costs associated with deviations from a moral or political bliss point also affects related phenomena: Polarization in society (Kamada and Kojima 2014); optimal policy when the policy space is multi-dimensional (Eguia 2013); the sustainability of biased norms (Michaeli and Spiro in press); and whether effort to achieve high status falls when a person's relative status falls (Clark and Oswald 1998).

The question remains whether concave preferences have empirically observable implications for important real-world decision situations. Showing that they do is the purpose

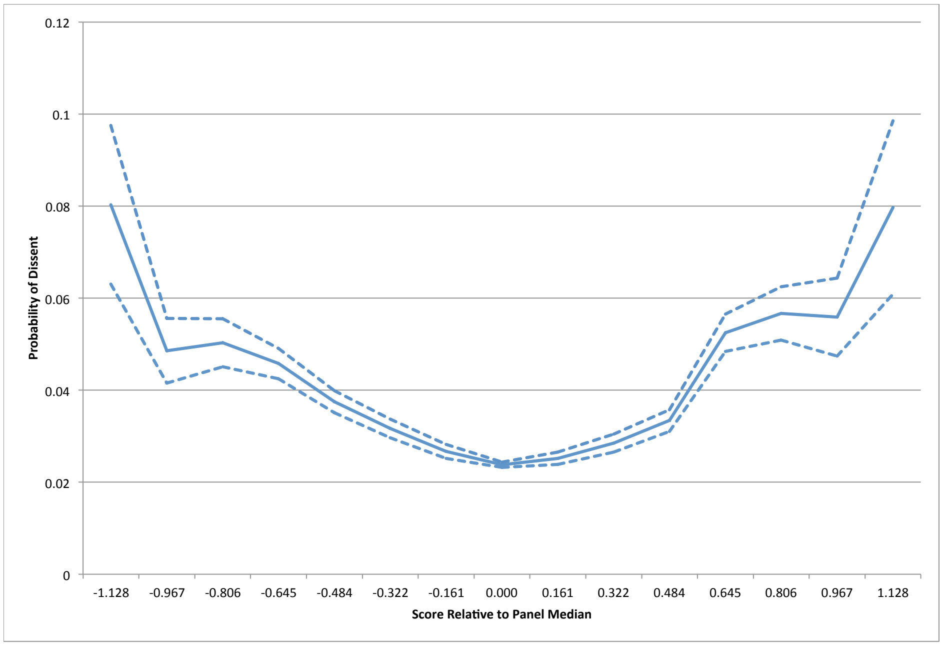
[1]Recent experimental results suggest that the intrinsic cost of lying is fixed (hence concave). Kajackaite and Gneezy (2015) show that once the incentives to lie are higher than the cost, subjects switch from telling the truth to lying to the full extent. The results of Abeler et al. (2016) are similar. Earlier research has also been suggestive of a concave cost of lying: The decision whether to lie is often insensitive to the outcome of lying once it is preferred over the outcome of being truthful (Hurkens and Kartik 2009) and so a maximal deviation from the truth will often be chosen by those deciding to lie (Gneezy et al. 2013). Likewise, using a dynamic setting, Gino et al. (2010) showed that once individuals are induced to cheat, they succumb to full-blown cheating. When it comes to deviations from one's ideological bliss point, Kendall et al. (2015) have shown by structural estimation that voters are most likely to have a concave cost of deviating from their ideological bliss point. See Merlo and De Paula (in press) for recent work on structural estimation of voter ideology.

of this paper. The setting we are interested in is one where a person interacts with peers who are ideologically distant from her when making decisions about ideologically contentious issues. How individuals behave in such a setting remains an empirically open question due to unobservability of individual ideology and due to endogeneity of the choice of whom to interact with. These two problems are resolved at the U.S. Courts of Appeals (U.S. Federal Circuit Courts): There exist commonly used and robust predictors for individual ideology, and assignment of whom an individual interacts with is random.[2] For each judicial case, a panel of three randomly assigned judges needs to decide on a verdict (affirming or overturning the lower court verdict) and to compose an "opinion" (i.e., a text) motivating the verdict. The opinion often serves as precedent for future cases and as such affects both society and policy. Furthermore, being a text rather than a binary element, the opinion can reflect the assertiveness of the panel and its ideological composition. It is well documented that ideology plays an important role in deciding the opinion (Sunstein et al. 2006; Berdejó and Chen 2014). Roughly speaking, the way by which an individual judge's action can affect outcomes is twofold: 1) the judge may have a direct effect on the verdict and opinion; and 2) if the judge does not manage to directly affect them, she may write a minority opinion (dissent or concur).[3] In this setting, using a rich set of evidence, we document a puzzling phenomenon that boils down to the following: while most judges have a clear ideological component in their actions, the judges with the most extreme ideology "cave in"—despite not affecting the ideological color of the opinion, they do not speak out (dissent or concur) against it. A more detailed account now follows (the full details appear in the next section).

[2]Several recent papers employ random assignment of judges (e.g., Aizer and Doyle 2015; Shayo and Zussman 2011; Kling 2006; Belloni, Chernozhukov, and Hansen 2011).

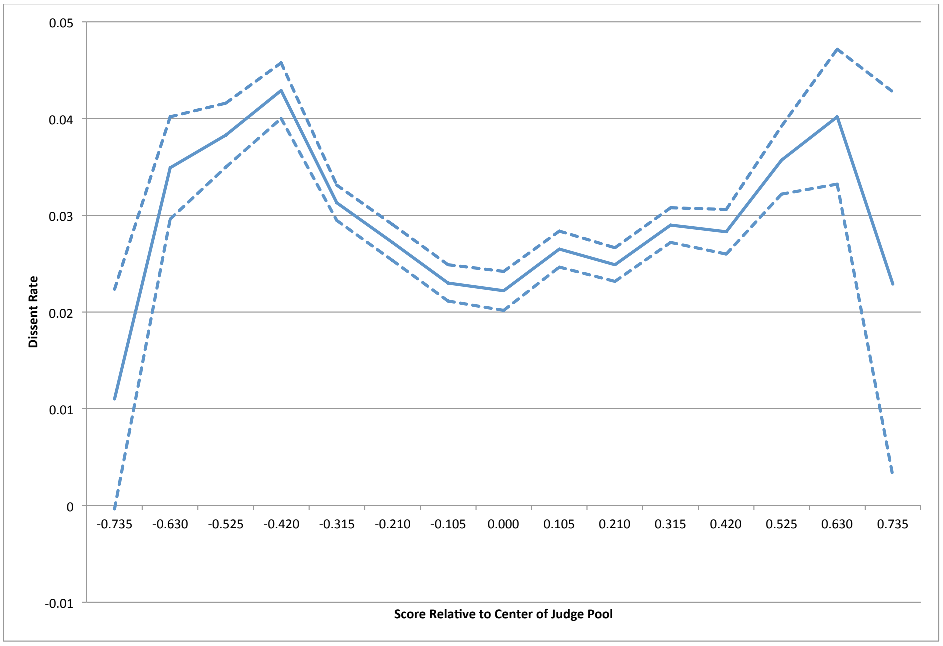
[3]Dissenting means stating a disagreement with both the verdict and the opinion. Concurring is a milder form of disagreement that is used by a judge who agrees with the verdict but disagrees with the motivation, i.e., disagrees with the opinion. See further details in Section 2.1.

FIGURE 1.— Dissent and Ideology Score Relative to Panel Median – raw data



Notes: x-axis: Ideology score of a judge demeaned by the median of the *panel* of judges assigned on the case. The x-axis is divided into 15 evenly-spaced bins (the number denotes the midpoint of the bin) from left-to-right in ideology space, where relatively more conservative scores are along the right on the x-axis. y-axis: Proportion of dissents over all votes in each bin. The dashed lines depict the 95% confidence interval. Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database.

FIGURE 2.— Dissent and Ideology Score Relative to Center of Judge Pool – raw data

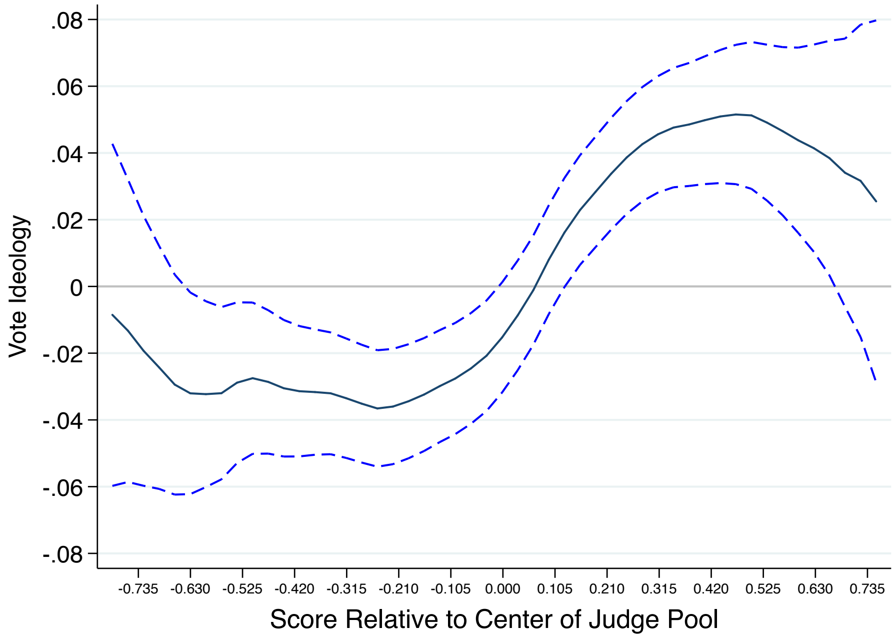


Notes: x-axis: Ideology scores of a judge demeaned by the center of the *pool* of judges available to be assigned in a Circuit-year. The x-axis is divided into 15 evenly-spaced bins (the number denotes the midpoint of the bin) from left-to-right in ideology space, where more conservative scores are along the right on the x-axis. y-axis: Average dissent rate for judges in the particular bin in that Circuit-year. The average is a weighted average to account for the number of times the judge actually appeared on cases in that Circuit-year. The dashed lines depict the 95% confidence interval. Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database.

Our starting point is the rather intuitive empirical observation that the median judge in a panel will more or less single-handedly determine the ideological component of the opinion. This has the implication (which is further corroborated empirically) that those with extreme ideology, seldom being the median in their panels, rarely affect the opinions of the cases they are part of. Given this starting point we observe the following three stylized facts that we aim to explain theoretically. The first fact—a within-judge observation—shows that ideology plays an important role in a judge's decision when to dissent: the more distanced the median of the panel is from the judge, the more likely the judge is to dissent (see Figure 1 for the raw data and Table II which includes controls for judge fixed effects). The second fact is a between-judge observation and is depicted in Figure 2 (see also Table III). It shows that judges who are centrist, relative to the *pool* of judges they encounter over the course of many cases, seldom dissent. Further, it shows that judges with moderate ideology relative to the pool (about half way to the right or left of the center) dissent the most. Finally, it shows that extremist judges dissent very little. Together this creates, on each side of the center, a hill-shaped relationship between ideology and the probability of dissent, which we aim to explain. What is particularly noteworthy with this pattern is that extremists, who by the very nature of their ideology are more often distant from their peers on a *panel*, choose to dissent very little. This is surprising given that the extremists rarely affect the opinion and given the first fact that ideological disagreements within panels drive dissent. The third fact is that the voting pattern of judges (see Figure 3 and Table IV) is such that centrists do not have a strong ideological component in their voting, moderately ideological judges have the strongest ideological voting pattern and extremist judges do not have an ideologically biased voting pattern.[4]

[4]Since there are less observations for the Ideology of Vote, the raw data cannot be used for presenting the third fact. Instead we use local polynomial estimators (Fan and Gijbels 1996) which is essentially the second best to presenting the raw data (Altman 1992).

FIGURE 3.— Ideology of Vote and Ideology Score of Judge Relative to Center of Judge Pool – local polynomial



Notes: x-axis: Ideology scores of a judge demeaned by the center of the *pool* of judges available to be assigned in a Circuit-year, where more conservative scores are along the right on the x-axis. y-axis: Predicted vote ideology, demeaned to be centered at zero. The depicted prediction is based on a local polynomial regression with an Epanechnikov kernel, where the dependent variable is ideology of a vote, which is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal. The dashed lines depict the 95% confidence interval. Data comes from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample).

To explain these three empirical observations, we construct a simple model showing that these observations are consistent with judges having a concave cost of deviating from their ideological bliss points and with the existence of peer pressure against dissent. We further test this theory against an alternative explanation—where dissent increases the chances of reversal by the U.S. Supreme Court—and find that auxiliary predictions in which the two theories differ corroborate our explanation and refute the alternative one.

The theoretical intuition behind our model is as follows. Consider a judge who is very ideologically extreme. This judge will rarely be the median judge in the panel and in fact will often be ideologically distant from the median judge. Hence, this extreme judge will rarely be in a position where she can determine the opinion and, furthermore, she will very often find herself having to decide whether or not to sign opinions very far from her ideological bliss point. Always dissenting on opinions she does not like would imply a very high collegial pressure (Epstein et al. 2011) as she will be facing such opinions virtually all the time. On the other hand, signing only some opinions while dissenting against others helps little when the cost incurred by signing few unfavorable opinions instead of many is almost the same (since the cost of deviating from one's ideology is concave). Hence, facing a sufficiently high collegial pressure, an extreme judge will tend to sign virtually all opinions—a low dissent rate. In comparison, centrist judges, often being median, are bound to often determine the panel's opinion and hence will less often need to decide whether to sign unfavorable opinions. So when they do face this problem, they might as well dissent and avoid the cost of deviating from their ideological bliss point. Hence their overall dissent rate will be relatively low too. Finally, the behavior of judges in between centrists and extremists will resemble that of the centrists—they will tend to dissent when facing an unfavorable opinion—but will face this situation more often than centrists and hence end up, in aggregate, dissenting the most of all judges.

This model of behavior predicts three empirical patterns. Firstly, to the extent that a judge dissents, she will do so when she is far from the median of the panel, hence the probability of dissent should increase with the distance between the judge's ideology and the panel median, which is consistent with the first fact (see Figure 1). Secondly, as just explained, there will be a hill-shaped relationship between ideological extremeness and dissent, which is consistent with the second fact (see Figure 2). Thirdly, the ideological component of the aggregate voting pattern should be weak for centrist judges (since they are ideologically neutral), strong for moderate leftists and rightists (since they dissent against most ideologically unfavorable opinions) and weak for extremists (since they cave in ideologically and mostly vote with the majority). This is consistent with the third fact (see Figure 3).

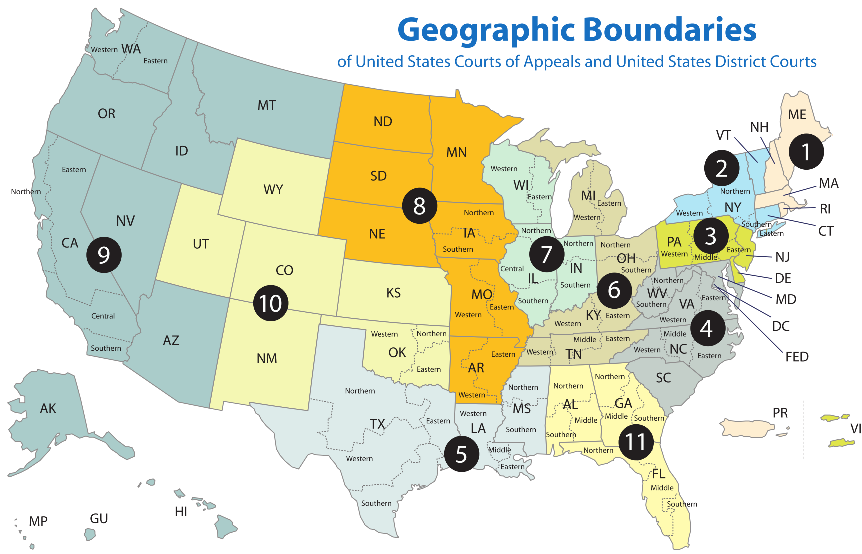
Apart from the empirical observation (and the theoretical prediction) that extreme judges surrender ideologically, two further results and implications are worth noting. First, the above logic does not hold if costs are linear or convex. We present necessary and sufficient conditions for the extent of concavity needed for the theory to align with the empirical observations. Second, our findings suggest that the ideological caving in occurs when the judge is ideologically extreme *relative* to her peers (in the pool). So the empirics and theory are essentially about the interaction between peers who disagree ideologically—it is not about extreme ideology per se.

The structure of the paper is as follows. In the next section we outline the judicial process more in detail and describe the data, estimation and main empirical facts that we aim to explain. In Section 3 we describe the model and its results, including some auxiliary predictions. In Section 4 we test these auxiliary predictions against an alternative theory. Section 5 concludes. The appendix contains all proofs, additional empirical results and a brief, yet formal, description of the alternative theory.

2 Judicial Decision-Making on Panels

2.1 Institutional background The U.S. Federal Courts are a system of local level (District Court), intermediate level (Circuit Court), and national level (Supreme Court) councils. Members of these are appointed by the U.S. President and confirmed by the U.S. Senate. They are responsible for the adjudication of disputes involving federal law. Their decisions establish precedent for adjudication in future cases in the same court and in lower courts within its geographic boundaries. Each state has 1–4 District Courts. The 94 U.S. District Courts serve as trial courts with juries. The 12 U.S. Circuit Courts (Courts of Appeals) take cases appealed from the District Courts. The Circuit Courts have no juries. Each Circuit Court presides over 3–9 states. Figure 4 displays District Court boundaries in dotted lines and Circuit Court boundaries in solid lines.

FIGURE 4.— Geographical Boundaries of U.S. Federal Courts



Notes: Boundaries of the 94 District Courts are represented in dotted lines. Numbers indicate the 12 Circuit Courts, with the Washington, D.C. Circuit being the 12th.

The Circuit Courts decide cases that provide new interpretations of prior precedents, which expand or contract the space of actions under which an actor can be found liable (Gennaioli and Shleifer 2007). State officials regularly update a set of guidelines to identify actions and regulations that may result in costly litigation after Circuit-Court decisions (Frost and Lindquist 2010; Pollak 2001). Circuit Courts, which are the empirical focus of this paper, rule on the application of federal law, such as the constitutional validity of state laws, among other things. 98% of their decisions are final.[5]

In Circuit Courts, each case has three randomly assigned judges. We refer to this as the **panel**. The judges are drawn from a **pool** of 8–40 judges (depending on the circuit) who have life tenure. The three judges decide a binary verdict (affirming or overturning the lower court verdict). A majority of two judges is needed to set the verdict.

They also compose an **opinion** (i.e., a text) motivating the verdict. The opinion serves as precedent for future cases and as such has a large impact on society and policy. Furthermore, being a text rather than a binary element, the opinion can reflect the assertiveness of the panel and its ideological composition. A judge has to write a separate (minority)

[5]In the remaining 2% that are appealed to the Supreme Court, 30% are affirmed.

opinion if she either **dissents**—votes against the binary verdict—or **concurs**—votes for the verdict but for a different reason, as manifested in her minority opinion. Both dissents and concurrences are costly in terms of time and collegiality and they cannot be cited as binding precedent. Note that, for a judge, dissenting and concurring are two mutually exclusive actions that both represent a form of dissatisfaction with the majority opinion.

2.2 Data The data we present comes from Openjurist, which contains all cases from 1950 to 2007. The data was first digitized by one of the authors (in Berdejó and Chen 2014) for whether there was a dissenting opinion and whether there was a concurring opinion. The current paper extracts the judge names and merges each judge with his/her ideology score. The ideological score we use is a standard summary measure using the voting patterns of the appointing President and home-state Senators, coming from the Judicial Common Space database (Epstein et al. 2007). This score exploits the norm of senatorial courtesy and is constructed as follows. If a judge is appointed from a state where the President and at least one home-state Senator is of the same party, the nominee is assigned a score of the home-state Senator[6] (or the average of the home-state Senators if both members of the delegation are from the President's party). If neither home-state Senator is of the President's party, the nominee receives the score of the appointing President. This score has been shown to outperform other common measures, such as the party of the appointing President or the ideology of the state from which the judge is selected (Giles et al. 2001).[7]

To examine the ideological color of the opinion and of the judge's vote on each panel, we also employ the U.S. Courts of Appeals Database Project, a random sample of roughly 5% of appeals courts decisions from 1925 to 2002.[8] This database includes additional hand-coded information on the ideological content of the opinion (liberal = -1, conservative = 1,

[6]The scores of the Senators are located in a two-dimensional space on the basis of the positions that they take in roll-call votes, but only the first of the two dimensions is salient for most purposes. The ideology scores of Presidents are then estimated along this same dimension based on the public positions that they take on bills before Congress.

[7]As a robustness check we use the party of appointing President as ideology score—see Appendix Section B. See for instance Appendix Table A.2.

[8]Documentation and data available at http://www.cas.sc.edu/poli/juri/appctdata.htm.

and mixed or unable to code = 0).[9]

Our sample contains 293,868 decisions from 1950 to 2007 (Openjurist) and 18,686 decisions for the period 1925 to 2002 (Courts of appeals database). Overall, 7.9% of opinions from 1925 to 2002 have dissents (3.6% have concurrences) while 8.5% of opinions from 1950 to 2007 have dissents (6.4% have concurrences).

We begin by calculating the average ideology score of the pool of judges in each Circuit and each year. This average score represents the center of the pool of judges available to be assigned, which we will refer to as **Center of Judge Pool**. Next, we calculate the absolute distance from each judge's ideology score to the center of her pool for each Circuit and each year.[10] We will refer to this measure as **Distance to Center of Judge Pool**. We will also consider **Score relative to Center of Judge Pool**, which refers to the ideology score of a judge that is demeaned by the center of the pool, so it can take both positive and negative values. In the Judicial Common Space, positive values are conservative and negative values are liberal.[11] A histogram of this measure is displayed in Appendix Figure A.1. We also construct a measure for the absolute distance between a judge's ideology score and the panel median and refer to it as **Distance to Panel Median**.[12] Finally, we calculate the dissent rate and concurrence rate for each judge in each Circuit-year.[13] The total number of votes per Circuit-year is calculated to account for the number of times the judge actually appeared on cases in that Circuit-year.[14]

[9]Their original coding was exactly the opposite but we flipped it to be in line with our coding of ideological scores, where positive values correspond to conservative views. The Appeals Court Database Project states that for most (but not all) issue categories, these will correspond to conventional notions of "liberal" and "conservative". For example, decisions supporting the position of the defendant in a criminal procedure case, the plaintiff who asserts a violation of her First Amendment rights, and the Secretary of Labor who sues a corporation for violation of child labor regulations would all be coded as "liberal".

[10]We use Circuit and year since this represents the ideology of the other judges a specific judge may actually be sitting with in a panel.

[11]We also merge in the Martin Quinn Supreme Court scores (Martin and Quinn 2002) and construct **Distance to Supreme Court** when we examine the alternative theory where Circuit Court judges are dissenting to signal the Supreme Court to review and revoke the panel's decision.

[12]Any analysis requiring the panel median includes only panels where there are no tied or missing scores.

[13]As robustness checks, we calculate the dissent and the concurrence rate for each judge in a 2-year bin and over a judge's lifetime.

[14]As robustness checks, we weight each judge equally but remove any visiting judge who appeared less than 10 times in a Circuit-year.

2.3 Who determines the ideological color of the opinion? We begin by examining the correlation between the conservative ideology of the opinion and the ideology score of the judges in the panel. To conduct these analyses, we turn to the U.S. Courts of Appeals Database Project (containing the ideological color of the opinion). We identify panel members according to whether they had the median ideology score in the panel. We regress the **Ideology of the Opinion** on a judge's **Score relative to Center of Judge Pool** and its interaction with whether the judge is the median of the panel:[15]

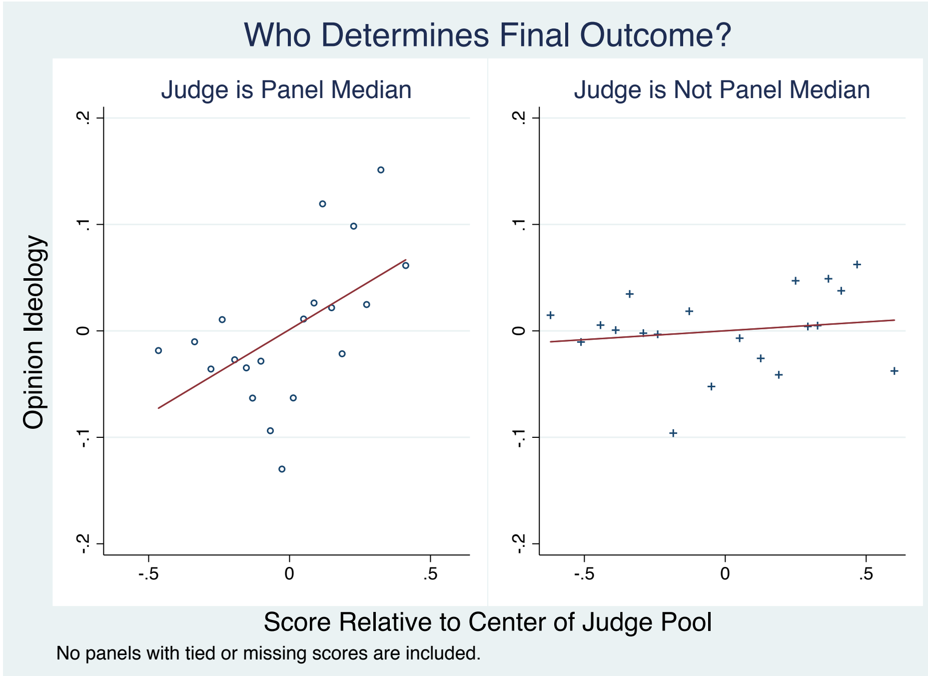
$$\begin{aligned} OpinionIdeology_{pcit} = {} & \gamma_1 \textbf{Score Relative to Center of Judge Pool}_{cit} + \\ & \gamma_2 \textbf{Score Relative to Center of Judge Pool}_{cit} * 1\left(i\, is\, median_{pcit}\right) + \\ & \gamma_3 1\left(i\, is\, median_{pcit}\right) + \nu_{pcit} \end{aligned} \tag{1}$$

for judge i on panel p in Circuit c and year t. If the ideology of a judge influences the opinion, we should expect a positive relationship between the judge's ideology score (where conservative values increase as we move to the right of the x-axis) and the likelihood of a conservative opinion. Table I confirms that only the median judge's ideology score is correlated with the opinion, and that this correlation is positive.[16] Figure 5 visualizes results from the table and shows that the median judge is driving the ideological color of the opinion.

[15]We use scores relative to center of judge pool (and not ideology scores per se) because when the entire circuit moves to the left or to the right, panelists' scores move accordingly and become slightly correlated with the ideological content of the opinion even without actually affecting it.

[16]The regression includes only three-judge panels where there are no tied or missing scores and clusters standard errors at the Circuit-year level. Appendix Table A.1 presents a robustness check of this result.

Figure 5.— Ideology of Opinion and Ideology Scores of Panel Members



Notes: x-axis: Ideology score of a judge demeaned by the center of the *pool* of judges available to be assigned in a Circuit-year, where relatively more conservative scores are along the right on the x-axis. y-axis: Opinion's ideology, which is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal, and each dot represents the average of many opinions in a bin of judges with similar ideology scores. The y-axis is demeaned to be centered around zero. Data comes from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample). Sample includes three-judge panels where there are no tied or missing scores.

TABLE I
Ideology of Opinion and Ideology Scores of Panel Members

	(1)
	Opinion Ideology
Score Relative to Center of Judge Pool	0.0166
	(0.0125)
Panel Median	0.00118
	(0.000775)
Score Relative to Center of Judge Pool	0.142***
* Panel Median	(0.0409)
N	23031
R-sq	0.001

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* $p < 0.10$; ** $p < 0.05$; *** $p < 0.01$). Data comes from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample). Sample includes three-judge panels where there are no tied or missing scores. The dependent variable is opinion ideology, which is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal.

As a consequence of the median judge being decisive in forming the opinion, extremist judges, seldom being median, have a very small impact on the opinions. We corroborate that this is indeed true in Appendix Figure A.3. We show there that the predicted ideology of the panel's opinion has an S-shaped pattern as a function of a judge's **Distance to Center of Judge Pool**. That is, when a judge becomes sufficiently extreme, there is a negative relationship between her ideology and the opinion of the panel she sits in.

The conclusion from this empirical analysis is that the median of the panel is determining the opinion, as in many conventional bargaining models. We will utilize this when developing the model.

2.4 Stylized fact 1: Probability of dissent and distance to panel median Figure 1 presents a non-parametric visualization of the yearly dissent rates by ideological distance of a judge to the median of a panel. For each of 15 evenly-spaced bins from the left-most to right-most score, we estimated the average dissent rate for all judge-case combinations in that bin. The dashed lines present the 95% confidence interval around the mean of the dissent rate.[17] We present similar non-parametric visualizations for concurrences in Appendix Figure A.4. These figures reveal a clear pattern. The more a judge is distant from the panel median, the more likely she is to dissent or concur. This holds on both the left and the right.

To test whether this pattern is statistically significant, we regress the dissent and concurrence rates of each judge-case combination on polynomials of the judge's **Distance to the Panel Median**—an absolute value which captures how much a judge disagrees ideologically with her panel peers. To verify that the result is not driven by the ideology scores of judges per se we add judge fixed effects. We also include Circuit (C_c) and year (T_t) fixed effects and cluster the standard errors by Circuit-year. The basic regression specification is:

$$\begin{aligned} Dissent_{pcit} &= \gamma_1 \textbf{Distance to Panel Median}_{pcit} + \\ &\gamma_2 \textbf{Distance to Panel Median}^{\mathbf{2}}{}_{pcit} + I_i + C_c + T_t + \nu_{pcit} \end{aligned} \quad (2)$$

for judge i on panel p in Circuit c and year t. Table II indicates that the pattern is robust for both dissents and concurrences even when adding judge fixed effects. This shows that

[17] We ran a regression of the dissent decision on a constant for each bin.

ideological disagreement is an important driver of dissent.

TABLE II

Dissent and Ideological Distance to Median of Panel

	(1)	(2)
	Dissent	Concur
Distance to Median of Panel	0.00425***	0.00244***
	(0.00119)	(0.000907)
Distance^2	-0.00142	-0.000868
	(0.00154)	(0.00116)
Judge Fixed Effects	Y	Y
Circuit Fixed Effects	Y	Y
Year Fixed Effects	Y	Y
N	541163	541163
R-sq	0.411	0.414

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* $p < 0.10$; ** $p < 0.05$; *** $p < 0.01$). Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database. Ideology scores are demeaned by the actual center of the panel of judges assigned on a case. The dependent variable is a dummy for whether a judge dissented (column 1) or concurred (column 2) in the panel. Fixed effects include year, circuit, and judge.

Based on these results we formulate the following stylized fact:

Fact 1 *For each judge, the probability of dissent increases the more she is distant from the ideology of the panel median.*

2.5 Stylized fact 2: Probability of dissent and distance to pool center Figure 2 presents a non-parametric visualization of the yearly dissent rates by ideological distance to the center of a judge's pool. For each of 15 evenly-spaced bins from the left-most to right-most score, we estimated the average of the yearly dissent rates of judges in that bin. The average is a weighted average to account for the number of times the judge actually appeared on cases in that Circuit-year. We also present the 95% confidence interval around the weighted mean of the dissent rate.[18]

Intuitively, one might expect that the more distant a judge is from the pool center in a Circuit-year, the more she will be inclined to write separate minority opinions (dissent or

[18]We ran a weighted regression of the dissent rate on a constant for each bin with weights being the number of votes cast by the judge in that Circuit-year.

concur).[19] But the figure reveals a surprising pattern: starting from the left, the most extreme judges rarely dissent, then there is a marked increase in dissent rates as judges become more moderate (bins 2 to 4), followed by a decrease in dissent rates towards the center of the judicial pool. A similar (yet less pronounced) pattern appears on the right. We will refer to this pattern as a *spider-pattern*, due the figure's resemblance of the body and legs of a spider. Appendix Figure A.5 shows that this pattern is robust to using the yearly number of pages of dissent a judge writes in a Circuit-year instead of the yearly rate of occurrence of dissent, so it is not the case that extremists simply put more time on one dissent.

As further evidence for the spider pattern, Appendix Figure A.6 presents the average yearly concurrence rate of judges according to the distance to the center of their respective pool of judges. We analyze concurrences separately from dissents as these are legally distinct and are mutually exclusive—a judge cannot dissent and concur at the same time. However, in our model and in the empirical tests we run, we mostly treat them together as two alternative manifestations of the same thing: A judge's decision not to sign her panel's opinion. Notably, the pattern of the spider is robust: concurrence rates are surprisingly uncommon for the most extreme judges, the rate is higher for judges at moderate distance to the center and lower again for the judges at the very center. Finding these patterns for dissents and concurrences separately decreases the likelihood that they are statistical artifacts.

We also regress the dissent and concurrence rate of each judge on polynomials of her (absolute) distance to the center of the pool of judges in her Circuit-year:

$$\begin{aligned} Dissent\ Rate_{cit} = {} & \gamma_1 \mathbf{Distance\ to\ Center\ of\ Judge\ Pool}_{cit} + \\ & \gamma_2 \mathbf{Distance\ to\ Center\ of\ Judge\ Pool^2}_{cit} + C_c + T_t + \nu_{cit} \end{aligned} \tag{3}$$

for judge i in Circuit c and year t. Circuit and year fixed effects are represented by C_c and T_t and standard errors are clustered by Circuit-year.[20] Table III indicates that the pattern is robust: according to the estimated linear and quadratic coefficients in the table the maximum dissent is obtained for distance to the center of the judge pool of 0.6 (for dissents) and 0.46

[19]In Appendix A.2 we show that judges who are more distant from the pool center have indeed a higher probability to be also more distant from the panel median.

[20]Since this is a between-judge result, we do not use judge fixed effects. See the appendix for lifetime results.

(for concurrences) which are clearly within the bounds of our distribution which goes from around -0.8 to +0.8. Additional robustness checks are reported in Appendix Tables A.2 and A.3. Appendix Table A.4 controls for personal characteristics of judges. Appendix Figure A.7 shows the pattern obtained using the estimates (including fixed effects) from the Appendix Table A.3 (column 1).[21] Based on these results we formulate the following stylized fact:

FACT 2 *There is a hill-shaped relationship between a judge's dissent rate and the absolute distance to her pool center.*

TABLE III

DISSENT AND IDEOLOGICAL DISTANCE TO CENTER OF JUDGE POOL

	(1)	(2)
	Dissent	Concur
Distance to Center of Judge Pool	0.0404***	0.0285***
	(0.00756)	(0.00570)
Distance^2	-0.0334***	-0.0313***
	(0.0118)	(0.00862)
Circuit Fixed Effects	Y	Y
Year Fixed Effects	Y	Y
N	10043	10043
R-sq	0.109	0.086

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* $p < 0.10$; ** $p < 0.05$; *** $p < 0.01$). Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database. The main independent variable is (the absolute value of) ideology score demeaned by the center of the pool of judges available to be assigned in a Circuit-year. The dependent variable is the judge's dissent rate (column 1) or concurrence rate (column 2) in a Circuit-year. Fixed effects include year and circuit. Observations are weighted by the number of votes cast by the judge in the Circuit-year.

2.6 Stylized fact 3: Ideological voting pattern and distance to pool center

Next, we investigate the relationship between a judge's ideology and the ideological color of her

[21] Appendix Table A.2 shows that the results are also robust to using an alternative measure of ideological score (i.e., the party of appointment); weighting each judge equally (rather than according to the number of votes) and dropping visiting judges; and using higher levels of aggregation (e.g., 2 year or lifetime rates, where there is one observation per judge). They are also robust to including higher-order polynomials, running logit, and controlling for Distance to Supreme Court or Distance to Median of Panel when examining the decision to dissent or concur at the panel-level. In that table, we show that when the **Distance to Center of Judge Pool** is randomly re-assigned to another observation, there is no relationship, which further mitigates the concern that the documented relationship is a statistical artifact. Appendix Table A.5 further shows that the pattern is robust to splitting the sample according to whether the case affirmed or reversed the lower court decision.

votes. The **Ideology of a Vote** was constructed as follows. The ideology of every opinion and every dissent and concur text (minority opinion) was hand-coded by an NSF-sponsored team of researchers. For example, opinions in favor of the defendant in a criminal case, or for a newspaper editor opposing an attempt at censorship, or for a union that claims that management violated labor laws when it fired a worker for union-organizing activities would all be coded as "liberal" (-1). Conservative texts are coded as +1. However, some issues are not easily categorized along a liberal/conservative dimension (e.g., attorney discipline cases).[22] Mixed outcomes, directionality that could not be determined, or outcomes that could not be classified according to any conventional outcome standards, were assigned a value of "0". Ideology of a judge's vote is set to equal the ideology of the opinion if the judge did not dissent and to equal the ideology of her own minority opinion if the judge dissented or concurred.

We test the relationship between a judge's ideology and the ideological color of her votes by using local polynomial estimators.[23] The results are presented in Figure 3. As can be seen, the strongest ideological bias is obtained for moderately ideological judges and once a judge becomes sufficiently extreme her voting becomes less ideological. To test the robustness of this result we also run the regression:

$$
\begin{gathered}
Vote_{pcit} = \gamma_1 \mathbf{Score\ Relative\ to\ Center\ of\ Judge\ Pool}_{cit} + \\
\gamma_2 \mathbf{Score\ Relative\ to\ Center\ of\ Judge\ Pool^2}_{cit} + \\
\gamma_3 \mathbf{Score\ Relative\ to\ Center\ of\ Judge\ Pool^3}_{cit} + \nu_{pcit}
\end{gathered}
$$

for judge i on panel p in Circuit c and year t.[24] The results, which confirm the local polynomial estimators, are presented in Table IV column 1 and are visualized in Figure A.8 in the

[22] The directionality codes parallel closely the directionality codes in the Spaeth Supreme Court database.

[23] Local location estimators are a set of techniques for data description, for estimating a regression curve without making strong assumptions about the shape of the true regression function, and for checking parametric models (Altman 1992). Running averages is a very simple type of smoother and the local polynomial estimator is a data-driven way of choosing the amount of smoothing (Fan and Gijbels 1996).

[24] We use polynomial of the third degree in the regression to enable testing for a U-shape on the left and a hill-shape on the right.

Appendix.[25] It is worth noting that this result disappears when considering judge scores which are not relative to the pool (Table IV column 2 and a visualization in Appendix Figure A.10). Hence, what is important is that a judge is ideologically extreme *relative* to her peers—it is not about extreme ideology per se, but about the interaction between peers who disagree ideologically. Based on this we formulate the following stylized fact:

FACT 3 *The ideological bias in judges' votes is most pronounced for judges who are moderately leftist or rightist.*

TABLE IV

IDEOLOGY OF VOTE AND IDEOLOGY SCORE OF JUDGE RELATIVE TO CENTER OF JUDGE POOL

	(1)	(2)
	Vote Ideology	
Score Relative to Center of Judge Pool	0.180***	
	(0.0308)	
Relative Score^2	0.0614	
	(0.0659)	
Relative Score^3	-0.366***	
	(0.125)	
Score		0.141***
		(0.0422)
Score^2		0.0788
		(0.0642)
Score^3		0.145
		(0.178)
N	23031	23031
R-sq	0.002	0.004

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* $p < 0.10$; ** $p < 0.05$; *** $p < 0.01$). Data on cases comes from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample). Ideology scores come from the Judicial Common Space database. Sample includes three-judge panels where there are no tied or missing scores. The dependent variable is ideology of a vote, which is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal. The main independent variables are ideology score demeaned by the center of the pool of judges available to be assigned in a Circuit-year (column 1) and ideology score (column 2).

[25] Appendix Table A.6 reports that the relationship is robust to using the lifetime average for each judge. The relationships are also robust to including higher-order polynomial terms (Appendix Table A.7). The pattern is also robust to splitting the sample according to whether the case affirmed the lower court decision (Appendix Table A.8).

3 Theoretical Model

We will now construct a theoretical model to explain Facts 1-3. We keep it as simple as possible in order to highlight the mechanism.

Three judges (t_1, t_2 and t_3) are randomly and independently drawn from a uniform distribution $F(t) \sim U(-1, 1)$ to sit together on a *panel*.[26] We call the distribution $F(t)$ *the pool of judges*. The three judges in the panel bargain and eventually produce an opinion $v \in \mathbb{R}$ through a Condorcet voting process: judges can (indefinitely) propose opinions, and any new proposal competes (by voting of the three judges) against the current leading opinion until a final decision is reached. Judges want the opinion to reflect their bliss points (t). Hence, a judge t gets a disutility $O(t)$ from sitting in panels that set opinions $v \neq t$, where $O(t)$ is increasing in the number of such opinions and in their distance from the bliss point t. Let $k(v)$ denote the overall distribution of opinions (the pdf) and let $k(v|t)$ be the distribution of opinions that judge t is facing during her judicial term, with support in $V \subset \mathbb{R}$.[27] For each opinion v, a judge needs to decide whether to sign it or whether to dissent against it.[28] When signing an opinion $v \neq t$, the judge is making a compromise and, on top of $O(t)$, has to bear the inner discomfort associated with actively approving it. This is captured by a separate cost function

$$D = D\left(\int_V |t - v|\, k(v|t)\, s(v)\, dv\right)$$

where $s(v)$ is an indicator function that equals 1 if the judge chooses to sign an opinion and equals 0 if she dissents. For tractability we assume D is a power function

$$D(x) = x^{\alpha}$$

[26] We use a uniform distribution mainly for tractability but most of our results hold under any single peaked distribution. The true empirical distribution of judges is presented in Appendix Figure A.1.

[27] The distribution of opinions faced by judge t is dependent of t since it affects who is median and hence determines the verdict through the voting process.

[28] We treat dissent and concurrence as the same thing—a refusal to sign the chosen opinion v.

where $\alpha > 0$ which means the ideological cost is increasing the larger the number of unfavorable opinions a judge signs and the more unfavorable each opinion she signs is. In total, a judge of type t has the loss function

$$L = O\left(t\right) + D\left(\int_V \left|t - v\right| k\left(v|t\right) s\left(v\right) dv\right) + W \int_V \left(1 - s\left(v\right)\right) k\left(v|t\right) dv,$$

where W is a constant marginal cost of dissent. Alternatively, W can be interpreted as capturing the effort of writing a separate opinion. The last term in the loss function thus captures the total cost of dissent arising from collegial pressure.[29]

3.1 Bargaining outcome From the model specification it is clear that for any signing strategy $s\left(v\right)$ chosen by a judge t, L is increasing in $|t - v|$. Hence, each judge will strive to minimize $|t - v|$ during the bargaining process. As a consequence, the opinion in each panel will be solely determined by the bliss point of the median judge in that panel which we denote by t_m –.

LEMMA 1 *In all panels $v = t_m$.*

This result is simple so we only provide the following heuristic proof. That the median decides is intuitive given the Condorcet voting process. To see this, note that both the other two judges cannot be better off at the same time by signing any $v \neq t_m$. Hence, these two judges cannot form a coalition and propose an opinion that would win over $v = t_m$. As discussed in the introduction (and shown in Table I and Figure 5), this result has empirical support. We next use it as our starting point for the analysis of the individual decision making.

3.2 Within-judge variation: To sign or not to sign? The deterministic nature of v in a given panel implies that the individual decision of a judge whether to sign an opinion or not can be analyzed taking v as given. Clearly, for the median judge in a panel, signing is optimal. As for the other judges in that panel, note first that $O\left(t\right)$ is independent of signing

[29] We let the dissent cost be linear in the number of dissents for tractability. The results hold qualitatively also for convex and concave collegial pressure functions and only changes the threshold values of α in the upcoming propositions.

or not. Therefore, the first argument in the loss function, $O(t)$, does not affect any signing decision made by a judge. Each of these judges therefore minimizes

$$l(s(v);t) \equiv L - O(t) = D\left(\int_V |t-v|\, k(v|t)\, s(v)\, dv\right) + W\int_V (1-s(v))\, k(v|t)\, dv.$$

We will now rewrite this expression in a form that is more convenient for analysis. This can be done by noting the following result.

PROPOSITION 1 *Every judge t has a unique cutoff $c(t)$ such that she signs opinion v if and only if $|t-v| < c(t)$.*

PROOF: See Appendix C. *Q.E.D.*

The proposition says that the optimal strategy for a judge is to dissent against opinions that are sufficiently far from her bliss point. This is of course natural since, otherwise, there would exist two opinions v_1 and v_2, such that $|t-v_1| < |t-v_2|$ yet judge t is willing to sign v_2 while refusing to sign v_1, in which case she could lower her loss by inverting this pair of choices. Put in an empirical context, the proposition says that the probability of dissent should increase the further away the judge is from the median of the panel (who, as established, sets the opinion). Hence, this result explains Fact 1.

Following this result, the choice of a judge boils down to choosing the cutoff c. Rewriting the loss function, the minimization problem for judge t is given by

$$\min_c l = \min_c \left\{ D\left(\int_{t-c}^{t+c} |t-v|\, k(v|t)\, dv\right) + W\left(1 - \int_{t-c}^{t+c} k(v|t)\, dv\right)\right\}.$$

The first argument expresses the disutility associated with signing opinions that deviate from the judge's bliss point and the second argument expresses the disutility associated with not signing the other opinions—i.e., the collegial pressure (or cost of effort) when writing a minority opinion.

We turn now to explicitly express the opinion distribution $k(v|t)$ in terms of the judges' type distribution $F(t)$ and the corresponding density function $f(t)$. For a given judge

t, the probability of having an opinion to her left is $K(t) = \Pr(t_m < t) = [F(t)]^2$, as this happens if and only if both other judges have bliss points below t. Similarly, $\Pr(v > t) = \Pr(t_m > t) = [1 - F(t)]^2$. In the remaining cases, judge t is the median, in which case $|t - v| = 0$. This means that $k(v|t) = 2F(v) f(v)$ in the range $v < t$ and $k(v|t) = 2[1 - F(t)] f(v)$ in the range $v > t$. It also follows that the probability of dissent for judge t who uses a cutoff c is given by

$$[F(t-c)]^2 + [1 - F(t+c)]^2 ,$$

and the minimization problem of judge t becomes

$$\min_c \left\{ \begin{array}{c} D\left(\int\limits_{t-c}^{t} (t-v)\, 2F(v) f(v)\, dv + \int\limits_{t}^{t+c} (v-t)\, 2(1-F(v)) f(v)\, dv \right) \\ +W\left[[F(t-c)]^2 + [1-F(t+c)]^2 \right] \end{array} \right\}.$$

3.3 Between-judge variation: Dissent pattern We move now to explain the spider pattern of Fact 2—a hill-shaped relationship between a judge's dissent rate and her absolute distance to the center of the judge pool (which is zero in our model). The probability that type t dissents is determined by two main factors. The first is the distribution of other judges' types and the location of t with respect to this distribution. For any given cutoff c, the probability of dissent is determined by the concentration of judges within and outside this cutoff. Hence, under a uniform (or a single-peaked) distribution of judges, a judge at the tail of the distribution is bound to encounter more panels where both other judges are outside a given cutoff c compared to a judge with the same cutoff but whose bliss point is at the center of the distribution. The second decisive factor is the value of the cutoff itself. For any given judge t, larger c implies less dissent. As c is endogenously chosen, the function $c(t)$ has implications for the probability of dissent of each type, denoted by $P(t)$.

LEMMA 2 *If $c(t)$ is locally decreasing in $|t|$ then $P(t)$ is locally increasing in $|t|$.*

PROOF: Follows from Lemma 4 in Appendix D. *Q.E.D.*

The lemma expresses the notion that a judge who is both more extreme and has a smaller cutoff (hence is pickier) will dissent more. The formal proof is in the appendix but the intuition follows directly from the two factors discussed above: A judge who is more extreme encounters more panels in which the median is beyond her cutoff, and more so if the cutoff is smaller. For consistency with Fact 2 it thus must hold that $c(t)$ is not locally increasing toward the edges of the distribution of judges. This turns out to imply that the function D, which aggregates the inner cost of judges, must be sufficiently concave.

PROPOSITION 2 *There exist values of W such that $P(t)$ is first increasing and the decreasing in $|t|$ if and only if $\alpha \leq \frac{2}{3}$.*

PROOF: See Appendix D. *Q.E.D.*

The proposition relates to our second stylized fact about the spider-pattern of dissent. It states that the condition for the the spider pattern of dissent to arise is that judges have to be sufficiently picky about small deviations from their ideological bliss points. We will first explain why linear and convex cost functions do not yield a spider-pattern and then why a sufficiently concave cost does.

Suppose first that D is linear. Then a judge's location on the type distribution is irrelevant for her choice whether to sign an opinion or not—only the trade-off between the fixed cost W and the distance $|t - v|$ matters.[30] This implies that c would be the same for all judges, and so judges who are more often far from the opinion (i.e., extreme judges), would tend to dissent more than others, contradicting the spider-pattern. Now suppose D is convex, so that a judge incurs an inner cost only when deviating quite a lot from her bliss point (in terms of both the number of times and the deviation size). Thus, a judge's location on the type distribution will be crucial for her choice strategy. Judges in the center of the bell-shaped distribution are rarely allocated to panels where their views are far from the opinion (i.e., far from the median judge), hence will be willing to sign the opinion in these few rare cases. Meanwhile, judges at the tails of the distribution will encounter many such

[30] To see this, note that for any given signature strategy, the linearity of D implies that signing another opinion v would impose a cost of $|t - v|$, potentially multiplied by some constant, and dissenting against v would impose a cost W.

cases, and will have to consider larger deviations on average. As a convex D implies a steep rising slope of the inner discomfort cost when compromising a lot, these judges will not be willing to compromise a lot. In particular, they will not compromise as much as needed in order to produce the spider pattern, in which these judges are supposed to dissent less than their more moderate colleagues. Hence, a convex D cannot produce a spider pattern.

As the proposition expresses, a sufficiently concave D does give rise to a spider pattern. A very concave D implies that a judge feels a very high ideological cost even if she signs only few opinions that are even a little bit distant from her bliss point. This means that a judge will either sign nothing that does not equal her bliss point, or sign almost anything. To see why the spider pattern can arise under such circumstances, note that the spider pattern requires first an increasing dissent rate when going from centrist judges (t close to 0) to moderately ideological judges (t, say, close to ± 0.5) and then a decreasing dissent rate as judges become very extreme (t close to ± 1). When $|t|$ is small, a judge will often be median, in which case she will not need to even consider signing unfavorable opinions. In the other cases, where the judge will have to make a decision whether to sign an unfavorable opinion, the marginal cost of signing will be very high since D is concave. The relatively low frequency in which this happens implies that such judges will choose to virtually always dissent when they are not median. Put differently, c will either be zero or very small. This strict adherence to ideals will hold also for judges who are more ideologically inclined yet not too extreme. The increasing probability of dissent in this range arises from the basic statistical fact that these more ideological judges will less often be the median of their panels hence will have more opportunities to dissent.

As a judge becomes sufficiently extreme, however, always dissenting becomes very costly since that would imply dissenting in virtually all panels she is part of (as she is very seldom median). Furthermore, when D is concave, once that judge signs an unfavorable opinion, she might as well sign almost anything. Hence, an extreme judge, facing a sufficiently high pressure, will give up ideologically and surrender under the collegial pressure (c will be large).[31]

The most succinct form of the spider pattern appears when α is very small. For instance, for $\alpha \lesssim 0.3$, judges with small $|t|$ have a cutoff $c = 0$, implying they dissent whenever they are not median, while judges with large $|t|$ have $c = 1 + |t|$, implying they never dissent.[32] This case is particularly tractable (see Appendix D.3.4) and, in the remainder of this theoretical section, we will use it to illustrate how the model replicates Fact 3 and provides additional empirical predictions.

3.4 Between-judge variation: Voting pattern We move now to explain our third stylized fact—an S-shaped voting pattern where the moderately ideological judges have the strongest ideological bias in their voting. To explain this fact analytically we construct a measure of the ideology $I(t)$ of a vote of judge t as follows:

$$I(t) = \begin{cases} t_m \;\; if \;\; s = 1 \\ t \;\; if \;\; s = 0 \end{cases}$$

This formulation captures that by signing the majority opinion, a judge practically votes for it. Meanwhile, the minority opinion of a dissenting judge equals her bliss point (t) because, in this case, she needs to compose a separate opinion in which she can express whatever she really thinks.[33] This way, $E[I(t)]$ captures the ideological bias of judge t's voting pattern.

Proposition 3 *There exists an* $\tilde{\alpha}$ (≈ 0.295) *such that, for each* $\alpha < \tilde{\alpha}$, *there exists a range of values of* W *for which* $|E[I(t)]|$ *is maximized for an intermediate value of* $|t|$ *(i.e.,* $argmax_t\, |E[I(t)]| \notin \{-1, 0, 1\}$*)*

Proof: See Appendix E. *Q.E.D.*

[31] Of course, if the cutoff c is infinite (or larger than $|t| + 1$) for a range of extreme judges, the dissent rate will be zero. If the cutoff is such that also the extreme judges dissent against some opinions, then, recalling the initial discussion in this subsection, the cutoff has to increase sufficiently fast in $|t|$ so that the average opinion does not become too distant for the most extreme judges. What creates a quickly increasing (or constant but very large) c is if the marginal ideological cost (D') is very small which is the case when D is very concave.

[32] This leads to a spider pattern with a discontinuous drop to zero dissent. For larger values of α the spider may be more smooth.

[33] In the data, the ideology of a vote equals the opinion if the judge signs it and otherwise equals the ideology of her own text (the minority opinion), which has been coded separately.

The proposition expresses a sufficient condition for when the ideological bias will be most pronounced for moderately ideological judges—D has to be sufficiently concave. The intuition is straightforward. As explained above, centrists and moderately ideological judges will dissent virtually whenever they are not the median, hence will have $I = t$ in almost all cases. This implies that among them we will observe an increase in ideological bias as $|t|$ increases. Conversely, extreme judges almost never dissent, hence their voting pattern mostly reflects the majority opinions they sign, which are determined by the median of their panels, hence tend to be less biased than the moderates's voting pattern.[34]

3.5 Additional predictions: The effect of collegial pressure Now that we have established that ideological perfectionism – a concave D – along with collegial pressure rationalizes the three stylized facts we set out to explain, we will move on to deriving some additional predictions which we will use for testing our model against an alternative explanation. The additional predictions pertain to how the dissent pattern is affected by a reduction in collegial pressure.

PROPOSITION 4 *Consider the equilibrium described for $\alpha < \tilde{\alpha}$. Then: (i) if $W = 0$, $P(t)$ is monotonically increasing in $|t|$; and (ii) $argmax_t P(t)$ is decreasing in W.*

PROOF: See Appendix F. *Q.E.D.*

Part (i) says that, should collegial pressure become very small, the spider pattern will disappear and the dissent rate will be a purely increasing function of a judge's extremeness. The intuition for this is that, without collegial pressure, it becomes possible also for extreme judges to be ideologically picky and hence dissent when they dislike the opinion even slightly. Then, given that they are rarely the median in their panels, they will dissent more often than the moderately ideological. Part (ii) of the proposition expresses that, as collegial pressure increases, the range of judges who dissent whenever they are not the median shrinks. This is of course natural since strictly adhering to one's morals becomes more costly.

[34]The result of Proposition 3 holds also for other values of $\alpha \leq 2/3$ but this is harder to show analytically since the pattern of dissent is more complex (see for instance appendix D.3.5).

4 Empirical Testing of Our Model against an Alternative One

This section describes an alternative model that may explain the main stylized fact (Fact 2), derives two predictions from this model that differ from the two predictions outlined in Section 3.5, and tests the different predictions against each other empirically.

The alternative theory is one where a judge dissents, at a cost of collegial pressure, in the hope that the U.S. Supreme Court (SCOTUS) will use this as a signal to review the case and overturn the (binary) verdict. Majority voting within panels implies also here that the median judge decides the verdict for the panel. In Appendix G we develop a simple model capturing this mechanism (the model is inspired by the model in Beim et al. 2014). The intuition for why this alternative SCOTUS model can produce a spider-shaped dissent pattern is as follows. A judge compares, case by case, the cost of dissent with how wrongful she thinks a certain verdict is. This means that two prerequisites need to be in place for a judge to dissent: i) she needs to think that the verdict is sufficiently bad to warrant the cost of dissent and ii) she needs to have the Supreme Court on her side as otherwise the verdict will not be overturned anyway. Here, centrists on the one hand usually have the Supreme Court on their side but on the other hand, often being the median, seldom encounter verdicts that are too far from what they think is right. Hence they rarely dissent. Conversely, extremists often dislike the verdict sufficiently to dissent but rarely have the Supreme Court on their side, hence dissent seldom too. Finally, moderately ideological judges may have a larger set of cases where they both sufficiently oppose the verdict and have the Supreme Court on their side. In the appendix we show that this may create a spider-pattern of dissent. Two additional predictions (the equivalent of Proposition 4) can be derived from the SCOTUS model.

PROPOSITION 5 *Consider the SCOTUS model. Then: (i) if $W = 0$, $P(t)$ is monotonically decreasing in $|t|$; and (ii) $argmax_{|t|} P(|t|)$ is increasing in W.*

PROOF: See Appendix G. *Q.E.D.*

Prediction (i) of the SCOTUS model says that, as the collegial cost of dissent (W)

approaches zero, the dissent rate becomes a *decreasing* function of judge's extremeness. This is intuitive since, when the collegial pressure is low, the only factor that determines whether a judge dissents is whether she has the Supreme Court on her side (because there is no collegial pressure against dissenting). This means that centrist judges will dissent very often. Furthermore, the more extreme a judge is, the less likely it is that her preferences will be aligned with the Supreme Court, which means that the dissent rate falls. This way, the prediction of the SCOTUS model is opposite to the prediction of our main (perfectionism) model where, as the collegial pressure goes to 0, the dissent rate *increases* with how extreme the judge is (Proposition 4 part i).

Prediction (ii) of Proposition 5 refers to the consequences of an increase in the cost of dissent. In the SCOTUS model, the judge at the peak of the spider-pattern is the one for whom the threshold cutoff for dissent, as determined by the cost of dissent, exactly equals her ideological distance to the Supreme Court, implying that she dissents against any verdict that both she and the Supreme Court view as biased to the "wrong" side. Judges who are more extreme dissent in these cases as well, but in total they are predicted to dissent less because, compared to the judge at the peak, they have less objection to verdicts that manifest extreme ideology on their side of the ideological spectrum, yet do not have the Supreme Court on their side for overturning verdicts they consider to be too moderate. If the cost of dissent increases, judges have to censor themselves more, hence the cutoff for dissent is larger, implying that a judge has to be more ideologically extreme to be at the peak of the spider-pattern. Therefore, the prediction of the SCOTUS model is that, as the cost of dissent increases, the spider peak would move outwards. In the main model we had the opposite prediction: an increase in the cost of dissent would have pushed the peak of the spider inwards (Proposition 4 part ii).

4.1 Testing predictions for W$\to$ 0 When testing these opposite predictions (part i of propositions 4 and 5), it is important to note that in the main model it is the **Distance to Center of Judge Pool** that measures the extremeness, while in the SCOTUS model it is the **Distance to SCOTUS** that measures a judge's extremeness. To get a measure of the **Distance to SCOTUS**, we use the Martin Quinn Supreme Court scores to put the judicial scores of the Circuit Court and the Supreme Court on the same metric (Martin and Quinn 2002).

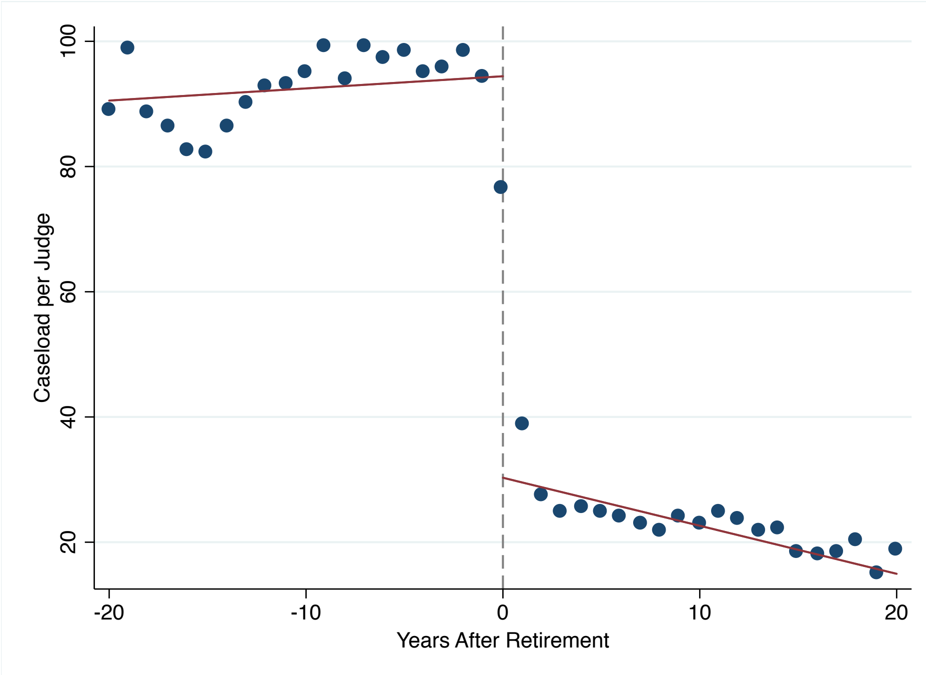
To test the predictions, we use retired status of a judge as a proxy for a very low cost of dissent. The motivation for this is as follows. Firstly, judges who have retired take a reduced caseload, hence have more time to write dissents. Secondly, they arguably have lower collegial pressure from colleagues or are less sensitive to such pressure. We first verify that judges who have retired have a discontinuous drop in caseload from about 100 per year to 30 per year (Figure 6) and that caseload continues to decline gradually thereafter. Next, we show that retired judges dissent more, discontinuously at the year of retirement (Figure 7), and verify that the increase in dissents is not due to age. In fact, older judges are less likely to dissent, which also explains the decline in dissent before retirement. Figure 7 visualizes the following regression.[35]

$$\begin{aligned} \textit{Dissent Rate}_{it} = a + b * \mathbf{1}(\textbf{Years after Retirement} \geq \mathbf{0})_{it} + \\ c * \textbf{Years after Retirement}_{it} + \\ d * \textbf{Years after Appointment}_{it} + \nu_{it} \end{aligned}$$

for judge i and year t.

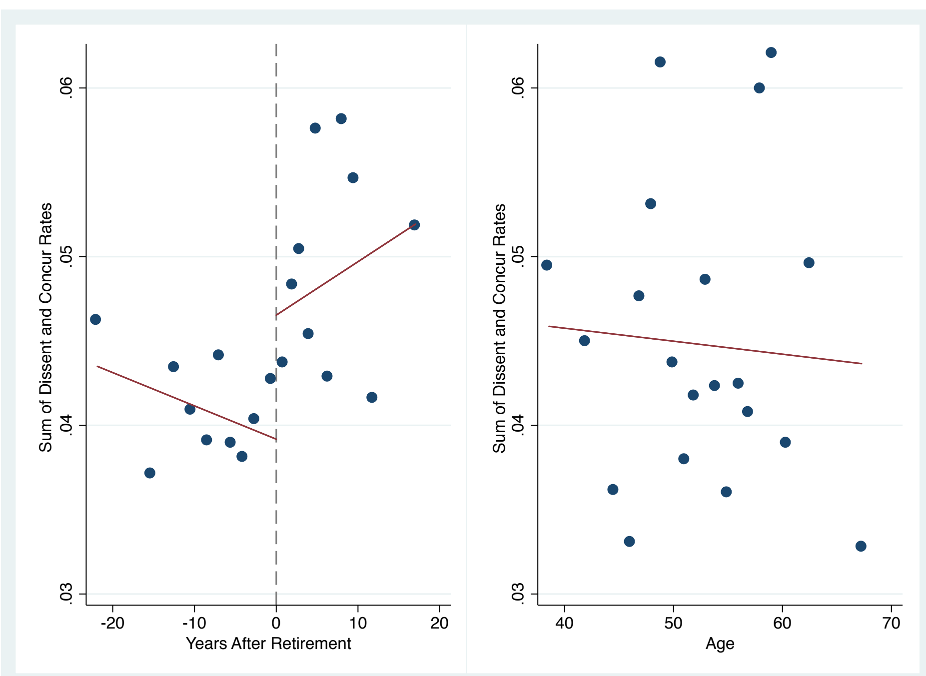
[35] In appendix B.5 we show that age and experience vary smoothly around the retirement decision and we also visualize the results of the regression presented in Table V.

FIGURE 6.— Caseload and Years from Retirement



Notes: Each dot represents the average caseload of judges with the same number of years relative to retirement. Data on cases comes from OpenJurist (1950-2007).

FIGURE 7.— Dissent or Concurrence and Years from Retirement vs. Age



Notes: Each dot represents the average sum of dissent rate and concurrence rate for judges with the same number of years relative to retirement (left panel) or the same age (right panel) in a Circuit-year. The average is a weighted average to account for the number of times the judge actually appeared on cases in that Circuit-year. Data on cases comes from OpenJurist (1950-2007).

To test the opposing prediction (i) of Propositions 4 and 5 we run the regression in equation (3), limiting the sample to retired judges only.

As can be seen in Table V, for retired judges, the rate at which judges dissent or concur is positively correlated with **Distance to Center of Judge Pool** (columns 1 and 5), which supports our main model. The rate at which judges dissent or concur is also positively correlated with **Distance to SCOTUS** (columns 3 and 7), which goes against the prediction of the SCOTUS model. Note also that the spider pattern disappears (columns 2, 4, 6 and 8), as predicted in Proposition 4. In total, these results provide support for the main model and go against the prediction of the SCOTUS model.

TABLE V

Dissent or Concurrence and Ideology Score among Retired Judges

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
				Dissent or Concur				
Distance to Center of Judge Pool	0.0323***	0.000569			0.0337***	0.0115		
	(0.00810)	(0.0244)			(0.00814)	(0.0250)		
Distance to Center of Judge Pool^2		0.0524				0.0365		
		(0.0418)				(0.0420)		
Distance to Supreme Court			0.0226***	0.0488*			0.0253***	0.0427
			(0.00849)	(0.0266)			(0.00848)	(0.0264)
Distance to Supreme Court^2				-0.0444				-0.0296
				(0.0466)				(0.0463)
Control for Age and Experience	N	N	N	N	Y	Y	Y	Y
Circuit Fixed Effects	Y	Y	Y	Y	Y	Y	Y	Y
Year Fixed Effects	Y	Y	Y	Y	Y	Y	Y	Y
Sample				Senior Judges				
N	3353	3353	3673	3673	3353	3353	3673	3673
R-sq	0.090	0.091	0.081	0.082	0.094	0.094	0.087	0.088

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* $p < 0.10$; ** $p < 0.05$; *** $p < 0.01$). Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database. Absolute values of the distance to the center of the judge pool or the Supreme Court are the main independent variables. The dependent variable is the judge's sum of dissent rate and concurrence rate in a Circuit-year. Fixed effects include year and circuit. Observations are weighted by the number of votes cast by the judge in the Circuit-year.

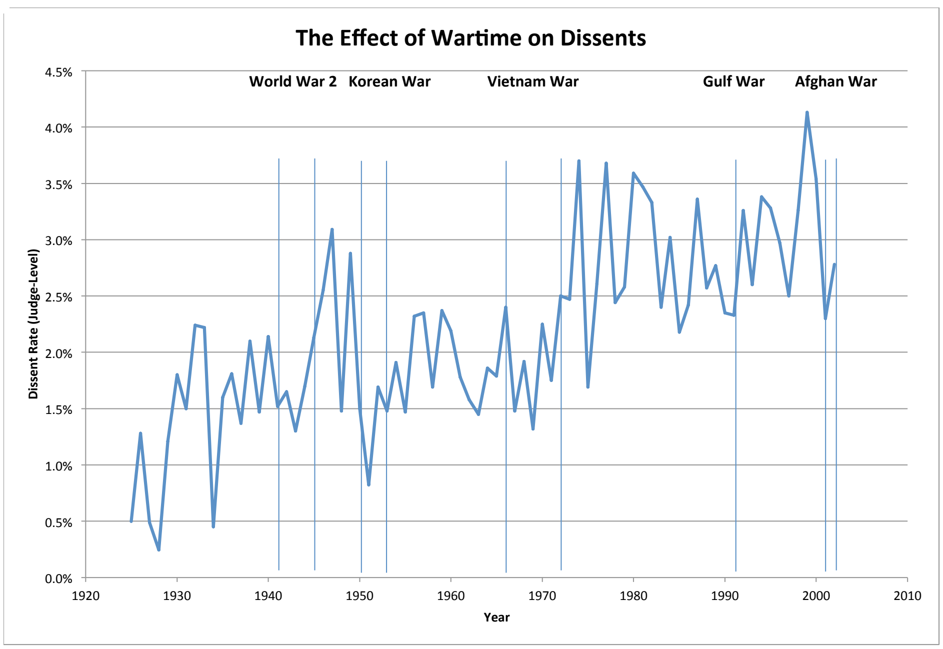
4.2 Testing predictions about the most dissenting judge Our second test examines the effect of an increase in the cost of dissent on who the most dissenting judge is (part ii of propositions 4 and 5). To test this we follow Berdejó and Chen (2014), who find that dissents decrease during wartime (Figure 8). For the purpose of our test, our assumption is that this decrease is due to wars tending to increase social cohesion (increase W). We test the main (perfectionism) model's prediction by examining how the dissent rate is affected by the interaction between a judge's **Distance to Center of Judge Pool** and wartime.

(4)

$$Dissent\ Rate_{cit} = a + b * \mathbf{Distance}_{cit} + c * \mathbf{Distance^2}_{cit} + d * \mathbf{Distance}_{cit} * wartime_t + e * \mathbf{Distance^2_{cit}} * wartime_t + f * wartime_t + \nu_{cit}$$

for judge i in Circuit c and year t.

FIGURE 8.— The Effect of Wartime on Dissents



Notes: Each dot represents the proportion of dissents over many votes on cases with the same publication year. Figure reproduced from Berdejó and Chen (2014).

We test the SCOTUS model's prediction by running the same regression but using **Distance to SCOTUS** as a measure of extremeness. The peak of the spider is determined by the first-order condition of the regression equation. Therefore, to test for a shift in the peak of the dissent rate, we test for a significant difference between $\frac{-b}{2c}$ and $\frac{-(b+d)}{2(c+e)}$. If wartime shifts the peak inwards ($\frac{-b}{2c} > \frac{-(b+d)}{2(c+e)}$), this would corroborate the main model and weaken the SCOTUS model and vice versa. The regressions, the ratios, and the test statistics for the equality of the coefficient ratios are reported in Table VI. Using **Distance to Center of Judge Pool**, Column 2 reports that during war there is a significant inward shift of the peak of the spider, which is consistent with the main model. Using **Distance to Supreme Court**, Column 4 rejects the significant outward shift that is predicted by the SCOTUS model (in fact the coefficient even has the wrong sign). Judged together, the two tests seem to refute the SCOTUS model while supporting the main model presented in this paper.

TABLE VI

DISSENT OR CONCURRENCE AND IDEOLOGY SCORE AMONG JUDGES: TESTS FOR CHANGES IN WHO DISSENTS THE MOST DUE TO INCREASE IN DISSENT COSTS DURING WARTIME

	(1)	(2)	(3)	(4)
	Dissent or Concur			
Distance^t	0.0610***		0.0758***	
	(0.0136)		(0.0121)	
Distance^2	-0.0387*		-0.0965***	
	(0.0214)		(0.0191)	
Distance * Wartime	-0.0608***		-0.0705***	
	(0.0187)		(0.0201)	
Distance^2 * Wartime	0.0790**		0.101***	
	(0.0358)		(0.0331)	
Test for Difference in		-1.584**		-1.936
Ratio of Coefficients		(0.619)		(11.74)
^{t}Distance to:	Center of Judge Pool		Supreme Court	
N	8760	8760	8760	8760
R-sq	0.025		0.017	

Notes: Results of regression 4. Robust standard errors clustered at the circuit-year level in parentheses (* p < 0.10; ** p < 0.05; *** p < 0.01). Data on cases comes from OpenJurist (1950-2007). Absolute value of the distance to the center of the judge pool (columns 1 and 2) and absolute value of the distance to supreme court (columns 3 and 4) are the main independent variables of interest. The dependent variable is the sum of a judge's rates of dissent and concurrence in a Circuit-year. Observations are weighted by the number of votes cast by the judge in the Circuit-year.

5 Conclusion

The main conclusion of this paper is that individuals are perfectionist when it comes to morals or ideology: They do not distinguish much between small and large deviations from their bliss points. The fact that individuals perceive the cost of such deviations to be concave has far-reaching implications for their behavior, and we show how this is manifested in important real-world decision making.

We study a high-stakes field setting where decisions have an ideological element, the U.S. Courts of Appeals, while exploiting the fact that judges are randomly assigned to panels of three. In this setting, we present a consistent and robust set of evidence which together suggest that judges with non-consensual world views ("extremists") cave-in ideologically: They are less likely to dissent against the panel's opinion than more moderate ones—although they are less likely to set this opinion—and their voting pattern is less ideological.

We present a simple model of judicial decision making in panels, where dissenting judges are subject to collegial pressure, and show that the observed patterns can emerge only if judges perceive a sufficiently concave cost of agreeing to opinions they find dissatisfying. A number of other model predictions are corroborated empirically. Furthermore, a competing model about dissents as a way of drawing the attention of the Supreme-Court is analyzed and rejected empirically. Our findings have direct and fundamental implications for how models related to ideology and morals should be formulated. In particular, the cost of deviating from one's principles should be concave, and this further affects the empirical predictions derived from models of ideological interaction.

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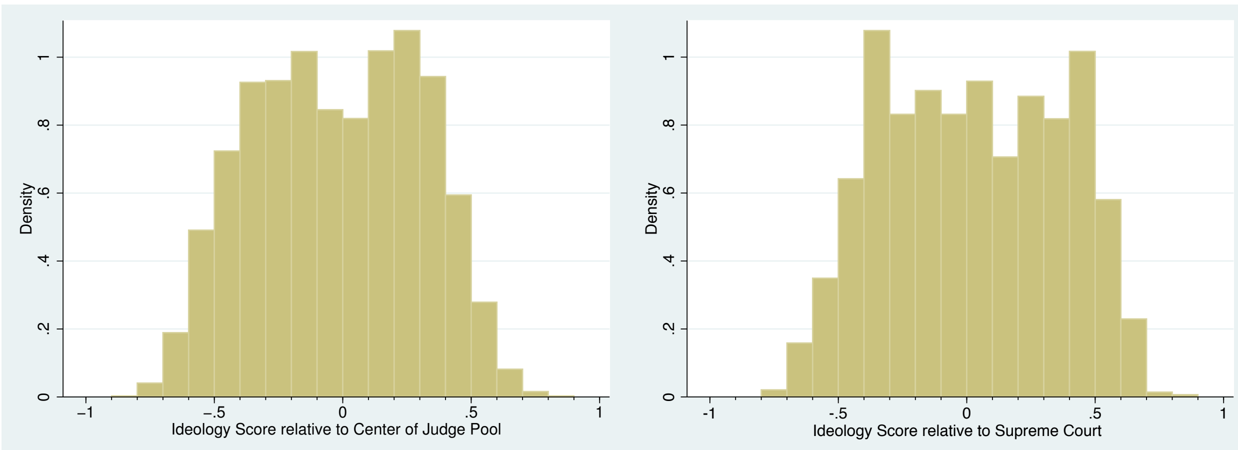
A Additional Empirical Results

This section presents some empirical results supporting the assumptions and intermediate results from the model.

A.1 Histogram of Ideology Scores An assumption in the model is that the distribution of ideology scores of judges is bell-shaped. Appendix Figure A.1 shows it roughly is for both **Distance to Center of Judge Pool** and **Distance to the Supreme Court**.

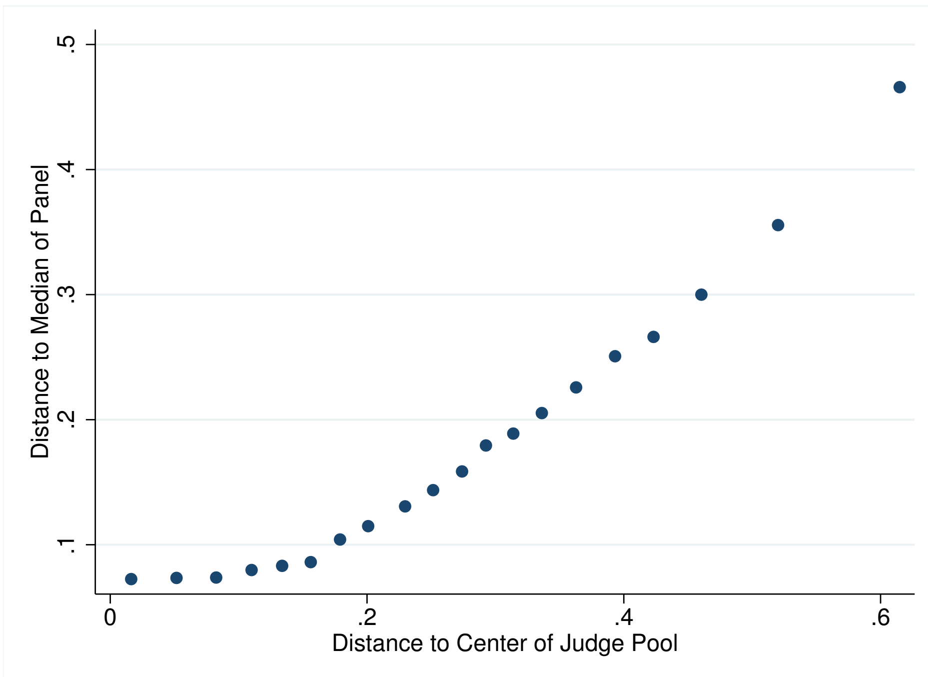
APPENDIX FIGURE A.1.— Distribution of Relative Ideology Scores



Notes: Ideology scores come from the Judicial Common Space database (Epstein et al. 2007), which provides a summary measure using the voting patterns of the appointing President and home-state Senators. Supreme Court ideology comes from Martin and Quinn (2002).

A.2 Distance to the Panel Median We check if the most extreme judges are indeed, on average, more distant from the median of their panels. Appendix Figure A.2 shows that this is the case. We plot the relationship between **Distance to Center of Judge Pool** and the average distance to the median of the judges' panels.

Appendix Figure A.2.— Distance to Panel Median and Distance to Center of Judge Pool



Notes: x-axis: Absolute value of the distance to the center of the judge pool. y-axis: Absolute value of the distance to the panel median. Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database. Sample is restricted to panels where scores are available for all three judges.

B Robustness

B.1 Median decides Appendix Table A.1 replicates the finding using a specification similar to a recent study examining group decision-making (Ambrus et al. 2015). We also control for **Center of Judge Pool** instead of subtracting it from the judge's ideology score to show the relevance of the average ideology score of the pool of judges in each Circuit and each year.

APPENDIX TABLE A.1

Ideology of Opinion and Ideology Scores of Panel Members

	(1)
	Opinion Ideology
Median of Panel Ideology Score	0.121***
	(0.0354)
Left of Panel Ideology Score	0.0499
	(0.0720)
Right of Panel Ideology Score	0.0373
	(0.0448)
Center of Judge Pool	0.251*
Ideology Score	(0.133)
N	7677
R-sq	0.008

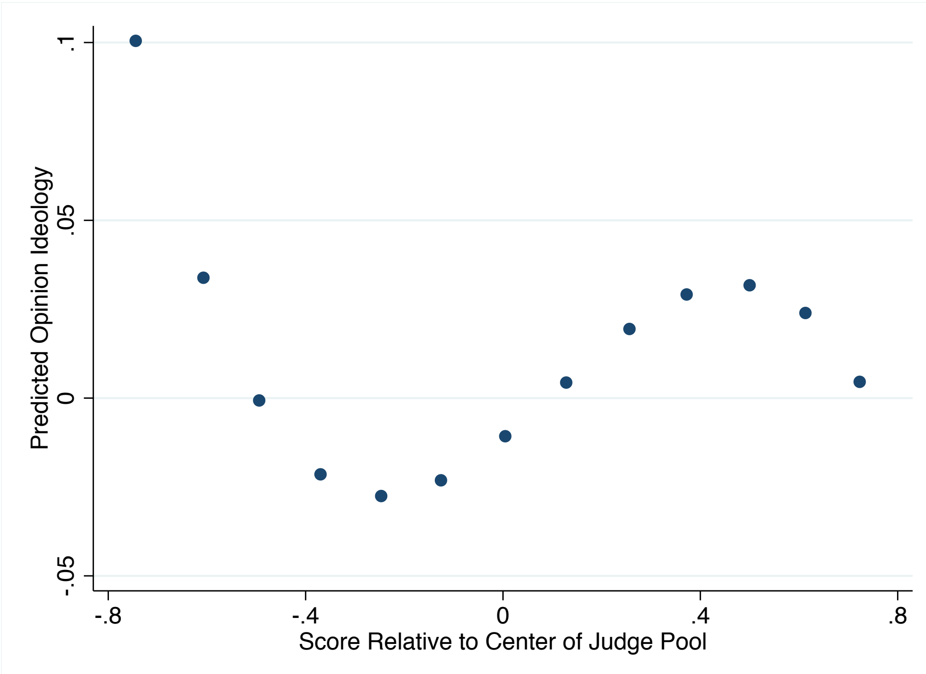
Notes: Robust standard errors clustered at the circuit-year level in parentheses (* $p < 0.10$; ** $p < 0.05$; *** $p < 0.01$). Data comes from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample). Sample includes three-judge panels where there are no tied or missing scores. The main independent variable is the (non-demeaned) ideology score of a judge. The dependent variable is ideology of opinion, which is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal.

Next, we investigate the relationship between a judge's ideology and the ideological color of the opinion. If the median of the panel is the primary driver of the opinion, then extreme judges should rarely influence the outcome of the panel. We test this by running the regression

$$\begin{aligned} \textit{Opinion Ideology}_{pcit} = \gamma_1 &\textbf{Distance to Center of Judge Pool}_{cit} + \\ \gamma_2 &\textbf{Distance to Center of Judge Pool}^{\mathbf{2}}{}_{cit} + \\ \gamma_3 &\textbf{Distance to Center of Judge Pool}^{\mathbf{3}}{}_{cit} + \nu_{pcit} \end{aligned} \tag{5}$$

for judge i on panel p in Circuit c and year t. The left schedule of Figure A.3 shows the pattern obtained using the estimates from the regression. As can be seen, the strongest ideological bias is obtained for moderate judges. Put differently extreme judges are not affecting the opinions (this result is robust to adding quartic terms and splitting the sample according to affirmance). It is worth noting that this result disappears when considering judge scores which are not relative to the panel. Hence, what is important is that a judge is ideologically extreme *relative* to her peers—it is not about extreme ideology per se, but about the interaction between peers who disagree ideologically.[36]

Appendix Figure A.3.— Ideology of Opinion and Ideology Score of Judge Relative to Center of Judge Pool – predicted pattern

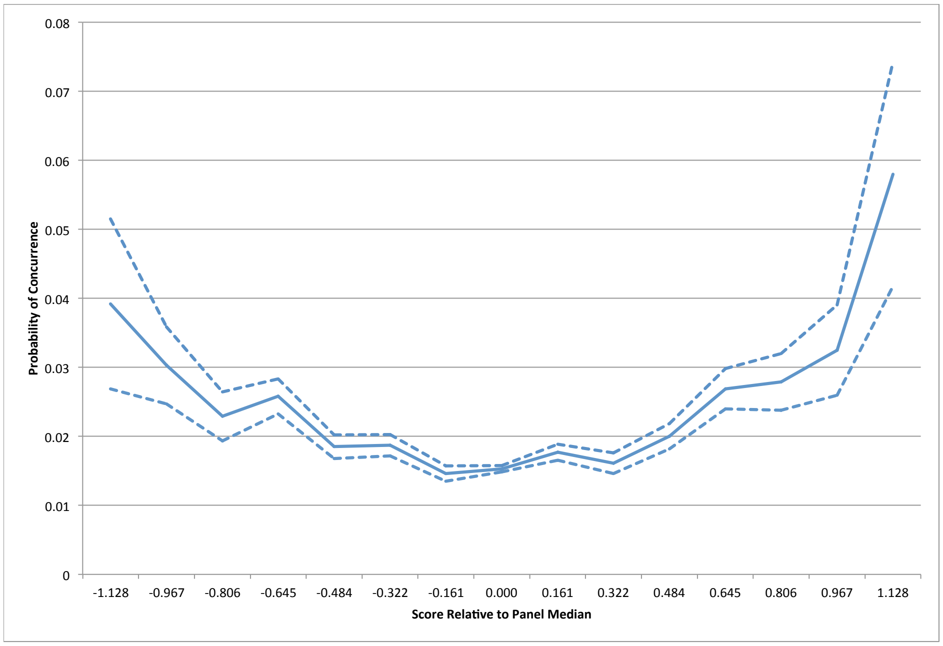


Notes: Ideology of opinion (demeaned by the center of the judge pool) from left to right as predicted by the regression in equation (5), where the dependent variable is ideology of opinion, which is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal. Aveage of predicted ideology of opinions is displayed on the y-axis for each group of judges sharing a similar score. Data comes from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample).

[36]The relationship is robust to using the lifetime average for each judge. The relationships are also robust to including higher-order polynomial terms.

B.2 Robustness for Fact 1

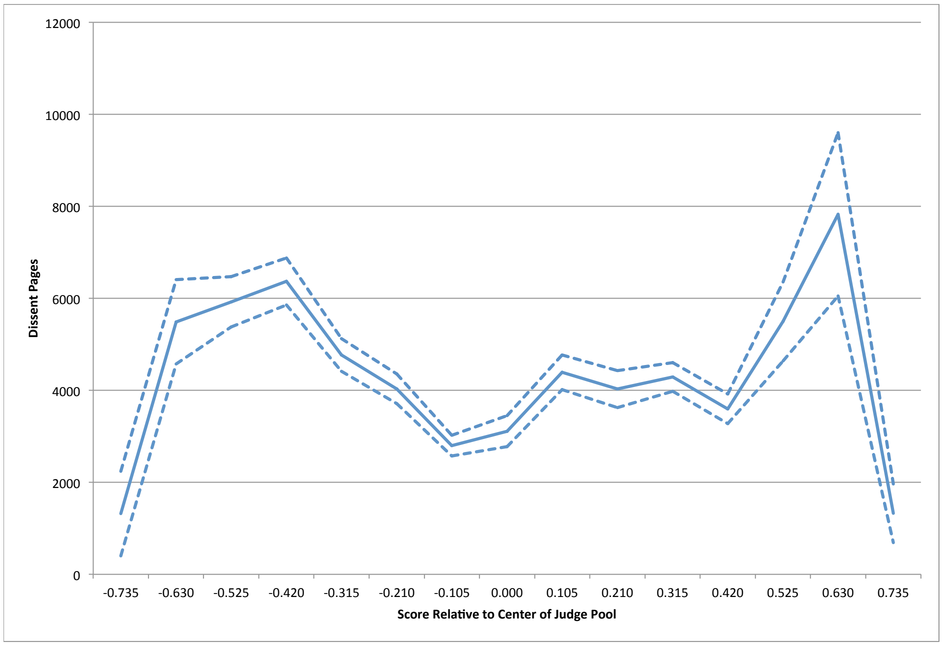
Appendix Figure A.4.— Concurrence and Ideology Score of Judge Relative to Median of Panel – raw data



Notes: x-axis: Ideology scores are demeaned by the median of the *panel* of judges assigned on a case. The x-axis is divided into 15 evenly-spaced bins (the number denotes the midpoint of the bin) from left-to-right in ideology space, where relatively more conservative scores are along the right on the x-axis. y-axis: Proportion of concurrences over all votes in each bin. The dashed lines depict the 95% confidence interval. Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database.

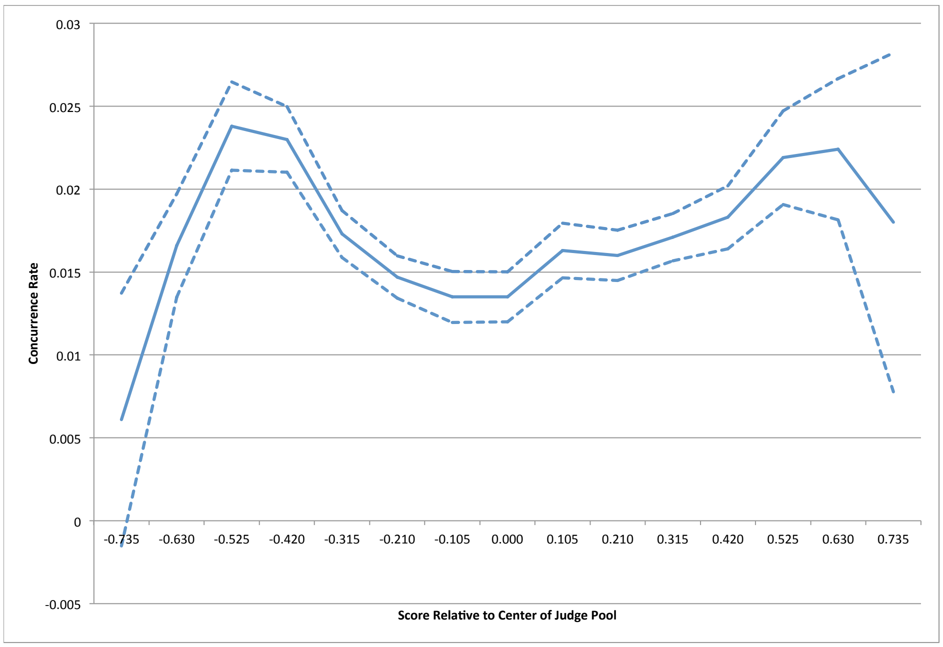
B.3 Robustness for Fact 2

Appendix Figure A.5.— Dissent Pages and Ideology Score of Judge Relative to Center of Judge Pool – raw data



Notes: x-axis: Ideology scores are demeaned by the center of the *pool* of judges available to be assigned in a Circuit-year. The x-axis is divided into 15 evenly-spaced bins (the number denotes the midpoint of the bin) from left-to-right in ideology space, where relatively more conservative scores are along the right on the x-axis. y-axis: Average of number of pages of dissent written for judges in the particular bin in a Circuit-year. The average is a weighted average to account for the number of times the judge actually appeared on cases in that Circuit-year. The dashed lines depict the 95% confidence interval. Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database.

Appendix Figure A.6.— Concurrence and Ideology Score of Judge Relative to Center of Judge Pool – raw data



Notes: Ideology scores of a judge demeaned by the center of the *pool* of judges available to be assigned in a Circuit-year. The x-axis is divided into 15 evenly-spaced bins (the number denotes the midpoint of the bin) from left-to-right in ideology space, where relatively more conservative scores are along the right on the x-axis. y-axis: Average concurrence rate for judges in the particular bin in that Circuit-year. The average is a weighted average to account for the number of times the judge actually appeared on cases in that Circuit-year. The dashed lines depict the 95% confidence interval. Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database.

We check if the spider is present under different weighting and scores. Appendix Table A.2 Column 1 repeats the main specification using **Distance to Center of Judge Pool**. Column 2 includes a cubic term. Column 3 weights each judge equally but excludes judges who vote less than 10 times. Column 4 uses a 2-year binned dissent or concur rate. Column 5 uses the lifetime average rate. Column 6 randomly assigns **Distance to Center of Judge Pool** to a different judge to mitigate the concern of spurious significance. Column 7 uses ideology scores that are simply a dummy indicator for the party of appointment. Column 8 uses the **Distance to the Supreme Court**. Column 9 uses both **Distance to Center of Judge Pool** and **Distance to the Supreme Court**. Column 10 uses distance to median of panel. Column 11 includes both distance to panel median and **Distance to Center of Judge Pool**. Notably, distance to panel median does not yield a negative quadratic term in the quadratic regression, indicating that our results are about the interaction with the peers in the Circuit rather than something about ideology per se or interaction with a particular panel.

Appendix Table A.2.— Robustness to Alternative Scores

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
						Dissent or Concur					
Distance to Center of Judge Pool	0.0664***	-0.0140	0.0460***						0.0636***		0.0538***
Scores based on Ideology	(0.0103)	(0.0196)	(0.0121)						(0.0105)		(0.0109)
Distance^2	-0.0649***	0.243***	-0.0433**						-0.0349**		-0.0891***
	(0.0156)	(0.0696)	(0.0188)						(0.0157)		(0.0165)
Distance^3		-0.314***									
		(0.0697)									
Distance to Center of Judge Pool				0.0678***							
2-year bin				(0.0132)							
Distance^2				-0.0670***							
				(0.0197)							
Distance to Center of Judge Pool					0.103***						
Lifetime average					(0.0318)						
Distance^2					-0.114**						
					(0.0516)						
Distance to Center of Judge Pool						0.00325					
Resampled						(0.00897)					
Distance^2						-0.00482					
						(0.0141)					
Distance to Center of Judge Pool							0.0727***				
Scores based on Party of Appointment							(0.0213)				
Distance^2							-0.0552**				
							(0.0222)				
Distance to Supreme Court								0.0622***	0.0416***		
								(0.00956)	(0.00960)		
Distance^2								-0.0897***	-0.0975***		
								(0.0143)	(0.0150)		
Distance to Median of Panel										0.0299***	0.0259***
										(0.00475)	(0.00506)
Distance^2										0.0144**	0.0215***
										(0.00646)	(0.00670)
Circuit Fixed Effects	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Year Fixed Effects	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
N	10043	10043	7744	5836	424	10387	10033	10592	10043	541182	509022
R-sq	0.124	0.126	0.111	0.147	0.267	0.111	0.120	0.114	0.131	0.008	0.008

Notes: Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database. Absolute value of the distance to the center of the judge pool is the main independent variable. The dependent variable is the judge's sum of dissent rate and concurrence rate in this Circuit-year (with the exception of column 4, which is the rate calculated over two years; column 5, which is the lifetime rate; and columns 10 and 11, which is the decision to dissent or concur on this panel). Fixed effects include circuit and year (year of appointment for column 5). Observations are weighted by the number of votes cast by the judge in the time-unit of observation (with the exception of column 3, which does not weight but excludes judges with less than 10 votes in a Circuit-year). Distance to Panel Median is not an alternative score, but is presented as a rejection of judge-specific mechanisms. Resampled Distance to Center of Judge Pool is presented as a rejection of spurious significance, where judicial scores have been randomly reassigned. All columns use robust standard errors clustered at the Circuit-year level, except column 4, which clusters at the Circuit-2-year-bin level, and column 5, which clusters at the Circuit level (* $p < 0.10$; ** $p < 0.05$; *** $p < 0.01$).

APPENDIX TABLE A.3

Dissent and Quartic Polynomial in Ideology Score of Judge Relative to Center of Judge Pool

	(1)	(2)
	Dissent	Concur
Distance to Center of Judge Pool	0.0434	0.0225
	(0.0303)	(0.0216)
Distance^2	-0.186	-0.102
	(0.174)	(0.124)
Distance^3	0.577	0.353
	(0.366)	(0.257)
Distance^4	-0.554**	-0.369**
	(0.250)	(0.174)
Circuit Fixed Effects	Y	Y
Year Fixed Effects	Y	Y
N	10043	10043
R-sq	0.111	0.088

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* $p < 0.10$; ** $p < 0.05$; *** $p < 0.01$). Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database. The main independent variable is (the absolute value of) ideology score demeaned by the center of the pool of judges available to be assigned in a Circuit-year. The dependent variable is the judge's dissent rate (column 1) or concurrence rate (column 2) in a Circuit-year. Fixed effects include year and circuit. Observations are weighted by the number of votes cast by the judge in the Circuit-year.

Appendix Table A.4 controls for biographical characteristics of the judge. These are controlled as dummy indicators for party of appointment, whether the judge and appointing president were of the same or different political parties, whether government (Congress and president) was unified or divided at the time of appointment, whether the judge was Protestant, Evangelical Protestant, Mainline Protestant, Catholic, Jewish, or non-religious, whether the judge was Black, non-white, or female, whether the judge received a law degree from a public institution, a bachelor's degree from a public institution, a bachelor's degree from within the state of appointment, or obtained further graduate studies in law (LLM or SJD), was born in the 1910s, 1920s, 1930s, 1940s, or 1950s, had previous experience as federal district judge, law professor, U.S. attorney, assistant U.S. attorney, Solicitor-General, mayor, state governor, Attorney-General, Deputy or assistant district/county/city attorney, Bankruptcy judge, U.S. Magistrate, Congressional counsel, District/County/City Attorney, Local/municipal court judge, Sub-cabinet secretary, Cabinet secretary, Special prosecutor, State lower court judge, State high court judge, or Local/municipal court judge, or had experience in City council, Department of Justice, Solicitor-General's office, or served as a member of the State house, State senate, U.S. House of Representatives, or had previous experience in private practice, in government, or in other federal capacity, and if the judge was elevated from the district courts, the party of the president who made the district bench appointment, and receiving an exceptional rating from the American Bar Association.

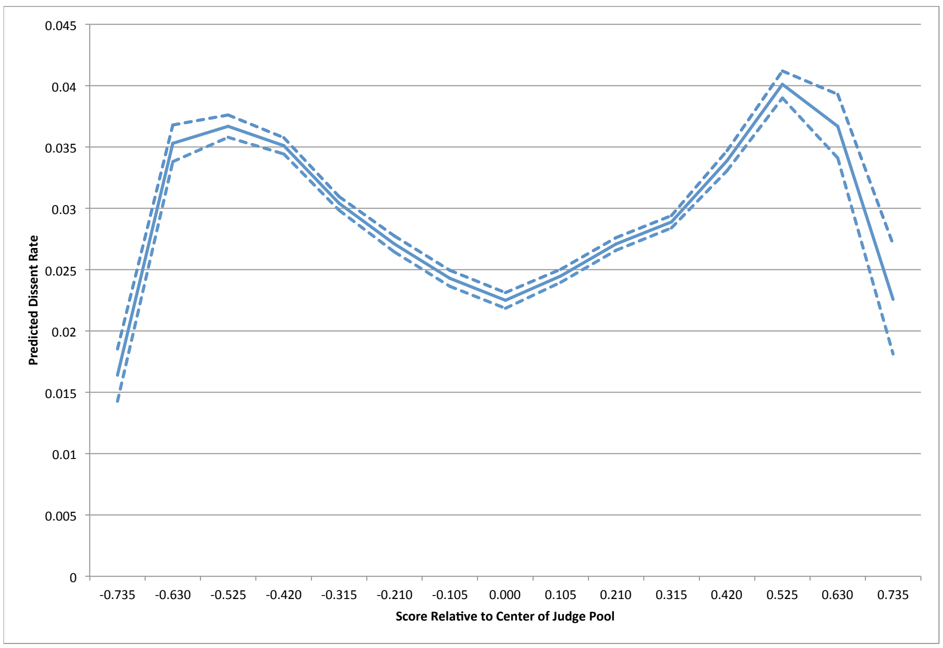
APPENDIX TABLE A.4

Dissent and Ideology Score of Judge Relative to Center of Judge Pool

	(1)	(2)
	Dissent	Concur
Distance to Center of Judge Pool	0.0459***	0.0288***
	(0.00784)	(0.00586)
Distance^2	-0.0403***	-0.0324***
	(0.0125)	(0.00886)
Judge Characteristics	Y	Y
Circuit Fixed Effects	Y	Y
Year Fixed Effects	Y	Y
N	8692	8692
R-sq	0.183	0.173

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* $p < 0.10$; ** $p < 0.05$; *** $p < 0.01$). Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database. Judicial characteristics come from Federal Judiciary Center/Attributes of U.S. Federal Judges Database and controlled for as binary indicators. The main independent variable is (the absolute value of) ideology score demeaned by the center of the pool of judges available to be assigned in a Circuit-year. The dependent variable is the judge's dissent rate (column 1) or concurrence rate (column 2) in a Circuit-year. Fixed effects include year and circuit. Observations are weighted by the number of votes cast by the judge in the Circuit-year.

Appendix Figure A.7.— Dissent and Ideology Score of Judge Relative to Center of Judge Pool – predicted pattern



Notes: x-axis: Ideology scores are demeaned by the center of the *pool* of judges available to be assigned in a Circuit-year. The x-axis is divided into 15 evenly-spaced bins (the number denotes the midpoint of the bin) from left-to-right in ideology space, where relatively more conservative scores are along the right on the x-axis. y-axis: Predicted dissent rate (from Appendix Table A.3 column 1) in a Circuit-year. Average of predicted dissent rate is displayed on the y-axis for each bin. The average is a weighted average to account for the number of times the judge actually appeared on cases in that Circuit-year. Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database.

Appendix Table A.5 presents the main specification but splits the sample according to whether the decision affirmed the lower court opinion. The sample size differs slightly when there are no affirmances or all affirmances for a judges in a Circuit-year.

APPENDIX TABLE A.5

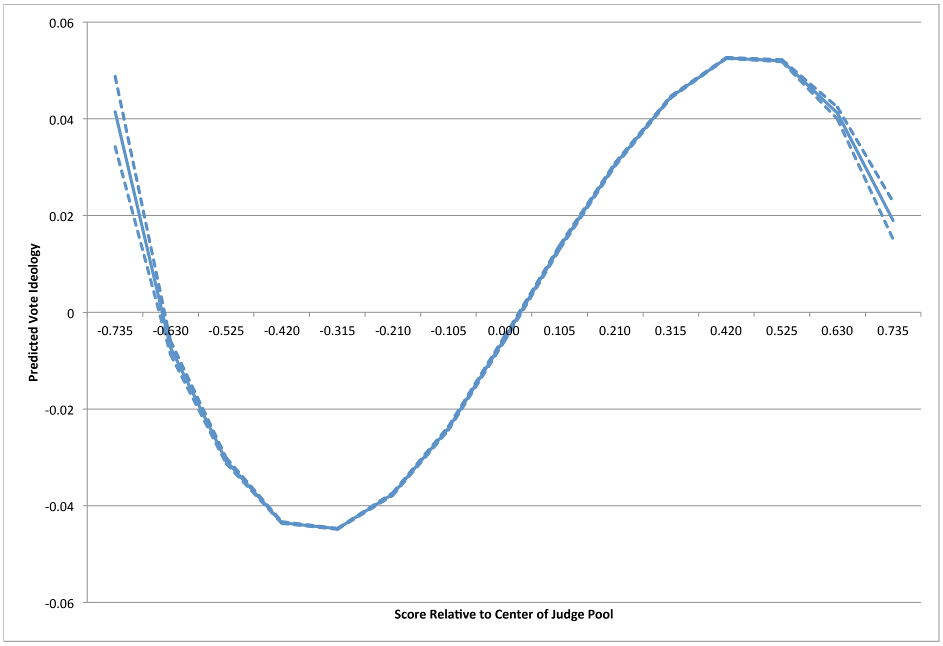
Robustness to Splitting the Sample by Affirmance

	(1)	(2)
	Dissent or Concur	
Distance to Center of Judge Pool	0.0760***	0.0522***
	(0.0109)	(0.0136)
Distance^2	-0.0750***	-0.0495**
	(0.0167)	(0.0215)
Circuit Fixed Effects	Y	Y
Year Fixed Effects	Y	Y
Sample	Affirmed	Not Affirmed
N	9577	9622
R-sq	0.091	0.072

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* $p < 0.10$; ** $p < 0.05$; *** $p < 0.01$). Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database. The main independent variable is Ideology score demeaned by the center of the pool of judges available to be assigned in a Circuit-year. Column 1 uses only the sample of decisions that affirmed the lower court opinion and Column 2 uses only the sample of decisions that did not. The dependent variable is the judge's sum of dissent rate and concurrence rate in this Circuit-year. Fixed effects include year and circuit. Observations are weighted by the number of votes cast by the judge in the sample of cases in the Circuit-year.

B.4 Robustness for Fact 3

Appendix Figure A.8.— Ideology of Vote and Ideology Score of Judge Relative to Center of Judge Pool – predicted pattern



Notes: x-axis: Ideology scores of a judge demeaned by the center of the *pool* of judges available to be assigned in a Circuit-year. The x-axis is divided into 15 evenly-spaced bins (the number denotes the midpoint of the bin) from left-to-right in ideology space, where more conservative scores are along the right on the x-axis. y-axis: Predicted vote ideology, demeaned to be centered at zero. The predicted value is obtained using Table IV column 1, where the dependent variable is ideology of a vote, which is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal. Average of predicted ideology of votes is displayed on the y-axis for each bin. The dashed lines depict the 95% confidence interval. Data comes from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample).

We check if the S-shape is present using lifetime average values for judges. Appendix Table A.6 Column 1 shows that the cubic term is still negative and significant when using the **Score Relative to Center of Judge Pool**. Column 2 shows that the cubic term is neither negative nor significant when using the judge's own ideology score implying that the results are driven by ideological disagreement rather than ideology per se.

APPENDIX TABLE A.6

Ideology of Vote and Ideology Score Relative to Center of Judge Pool

	(1)	(2)
	Vote Ideology	
Score Relative to Center of Judge Pool	0.217***	
Lifetime average	(0.0443)	
Relative Score^2	0.0557	
	(0.0860)	
Relative Score^3	-0.489**	
	(0.166)	
Score		0.141**
		(0.0528)
Score^2		0.0788
		(0.114)
Score^3		0.145
		(0.262)
N	421	421
R-sq	0.056	0.112

Notes: Robust standard errors clustered at the circuit level in parentheses (* $p < 0.10$; ** $p < 0.05$; *** $p < 0.01$). Data on cases comes from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample). Ideology scores come from the Judicial Common Space database. Main independent variables are Ideology scores demeaned by the center of the pool of judges available to be assigned in a Circuit-year and averaged over a judge's career in column 1 or, as placebo, not demeaned in column 2. Sample includes three-judge panels where there are no tied or missing scores. The dependent variable is lifetime-average ideology of vote (each vote is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal). Regression weights are the number of votes cast over a judge's career. Vote is the vote of the judge.

We next check if the S-shape is present when using quartic terms. Appendix Table A.7 reports the regression coefficients. Appendix Figure A.9 shows the predicted values when using **Score Relative to Center of Judge Pool**.

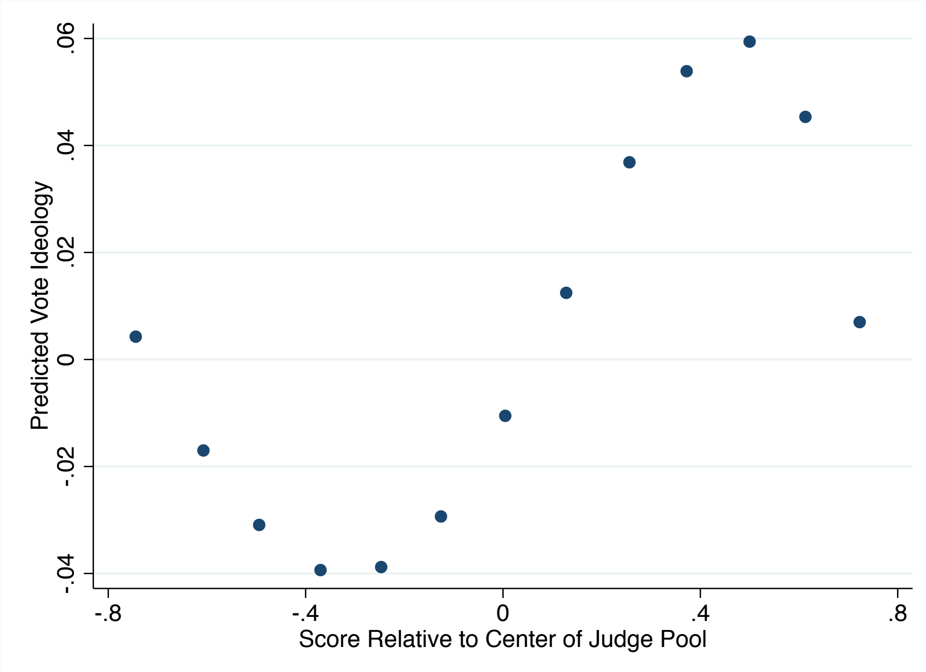
APPENDIX TABLE A.7

Ideology of Vote and Ideology Score of Judge Relative to Center of Judge Pool

	(1)	(2)
	Vote Ideology	
Score Relative to Center of Judge Pool	0.170***	
	(0.0331)	
Relative Score^2	0.175	
	(0.138)	
Relative Score^3	-0.317**	
	(0.142)	
Relative Score^4	-0.278	
	(0.262)	
Score		0.179***
		(0.0444)
Score^2		0.481***
		(0.177)
Score^3		-0.0646
		(0.188)
Score^4		-1.231**
		(0.512)
N	23031	23031
R-sq	0.002	0.005

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* $p < 0.10$; ** $p < 0.05$; *** $p < 0.01$). Data on cases comes from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample). Ideology scores come from the Judicial Common Space database. Main independent variables are Ideology scores demeaned by the center of the pool of judges available to be assigned in a Circuit-year and averaged over a judge's career in column 1 or, as placebo, not demeaned in column 2. Sample includes three-judge panels where there are no tied or missing scores. The dependent variable is ideology of a vote, which is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal. Vote is the vote of the judge.

Appendix Figure A.9.— Ideology of Vote and Ideology Score of Judge Relative to Center of Judge Pool – predicted pattern from quartic regression



Notes: Data comes from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample). Predicted ideology of vote (from Table A.7 column 1) is plotted for evenly spaced bins of ideology score (demeaned by the center of the judge pool) from left to right. The dependent variable is ideology of a vote, which is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal. Average of predicted ideology of votes is displayed for each bin.

We next check if the S-shape in the voting pattern is present when splitting the sample according to whether the decision affirmed the lower court opinion. Appendix Table A.8 reports the regression coefficients.

APPENDIX TABLE A.8

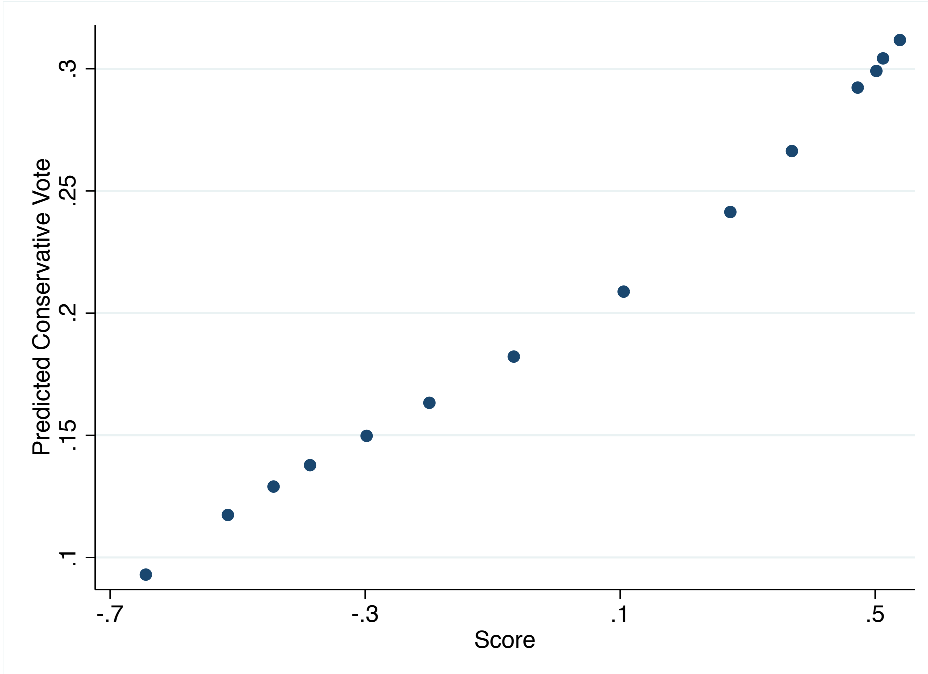
Ideology of Vote and Ideology Score of Judge Relative to Center of Judge Pool, split sample

	(1)	(2)
	Vote Ideology	
Score Relative to Center of Judge Pool	0.148***	0.190***
	(0.0384)	(0.0483)
$Score^2$	0.128*	0.0298
	(0.0750)	(0.0967)
$Score^3$	-0.387**	-0.346*
	(0.163)	(0.199)
Sample	Affirmed	Not Affirmed
N	12918	10113
R-sq	0.001	0.002

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* $p < 0.10$; ** $p < 0.05$; *** $p < 0.01$). Data on cases comes from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample). Ideology scores come from the Judicial Common Space database. The main independent variable is ideology score demeaned by the center of the pool of judges available to be assigned in a Circuit-year. Sample includes three-judge panels where there are no tied or missing scores. The dependent variable is ideology of vote, which is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal. Column 1 restricts to sample of cases that affirmed the lower court opinion and column 2 restricts to cases that did not.

Finally, we show that the result disappears when considering judge scores which are not relative to the pool (Table IV column 2 and a visualization in Appendix Figure A.10). Hence, what is important is that a judge is ideologically extreme *relative* to her peers—it is not about extreme ideology per se, but about the interaction between peers who disagree ideologically. This result is robust also to quartic terms and life voting (see the second columns in Tables A.7 and A.6).

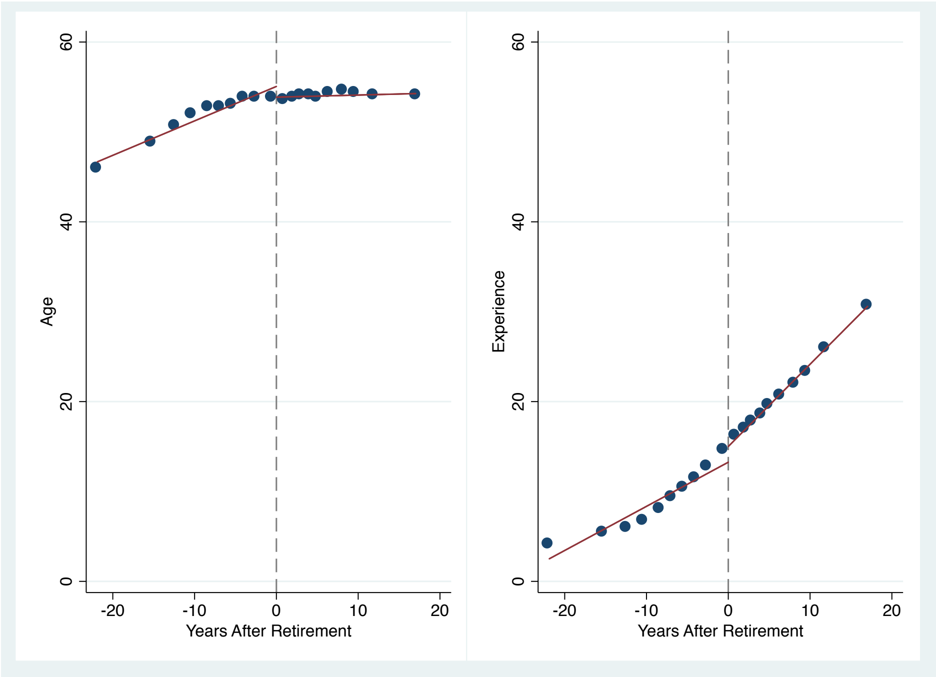
Appendix Figure A.10.— Ideology of Vote and (non-relative) Ideology Score – predicted pattern



Notes: Predicted ideology of vote is plotted for evenly spaced bins of (non-demeaned) ideology score from left to right. The predicted value is obtained using Table IV column 2, where the dependent variable is ideology of a vote, which is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal. Average of predicted ideology of votes is displayed on the y-axis for each bin. Data comes from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample).

B.5 Robustness regarding retired judges Appendix Figure A.11 shows that age and experience vary smoothly at the time of retirement.

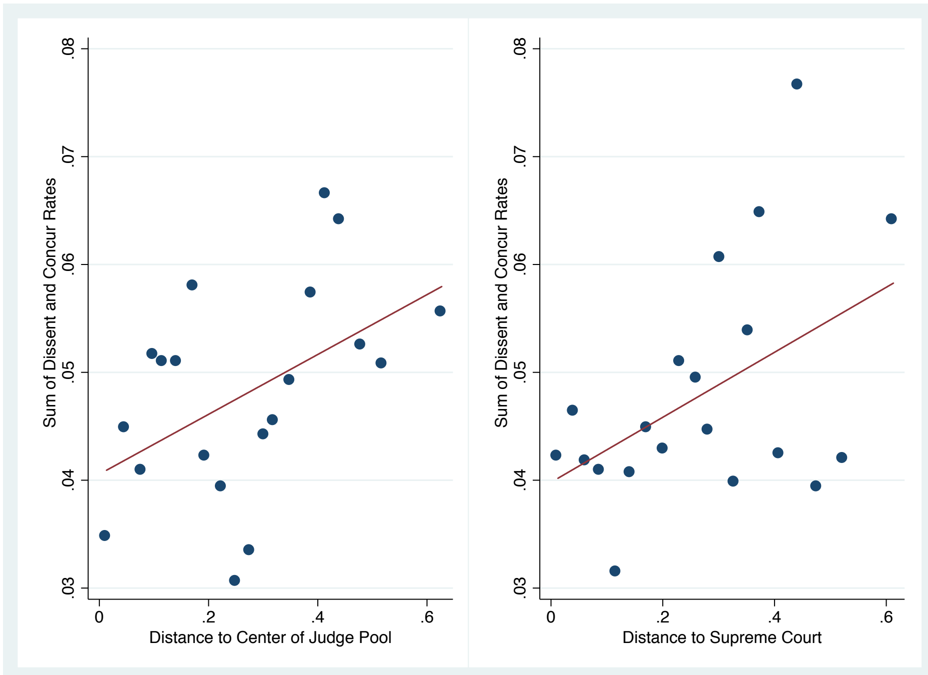
APPENDIX FIGURE A.11.— Age and Experience at Retirement



Notes: Each dot represents the average age (left panel) or experience (right panel) of judges with the same number of years relative to retirement. Data comes from biographical data on judges (1950-2007).

Appendix Figure A.12 shows that the relationship between rate of dissent or concurrence and ideology score among retired judges does not appear to be driven by outliers.

Appendix Figure A.12.— Retired Judges: Dissent or Concurrence and Ideological Distance



Notes: Rates of dissent or concurrence are calculated at the Circuit-year level. Each dot represents the average yearly sum of dissent rate and concurrence rate for retired judges sharing a similar score. The average is a weighted average to account for the number of times the judge actually appeared on cases in that Circuit-year. On the left panel, the score is ideological distance to the center of the judge pool and on the right panel, the score is ideological distance to the Supreme Court. Data on cases comes from OpenJurist (1950-2007).

C Proof of Proposition 1

Suppose by negation that type t has a strategy that violates the conditions of the proposition, i.e., there is not a cutoff strategy. Suppose first that at least on one side of t there are both opinions that t signs and opinions that she dissents against, and without loss of generality suppose that the violation of the conditions of the proposition happens on her left side (where t is not necessarily larger than 0). Thus, to the left of t, there must exist two non-overlapping ranges Δv_s and Δv_d such that (1) t signs all opinions at Δv_s and dissents against all opinions at Δv_d; (2) $\max\{|t-v| \text{ s.t. } v \in \Delta v_d\} < \min\{|t-v| \text{ s.t. } v \in \Delta v_s\}$; and (3) $\int_{\Delta v_s} 2F(v) f(v)\, dv = \int_{\Delta v_d} 2F(v) f(v)\, dv$. Condition (1) is just a definition of the two ranges; condition (2) states that there are opinions that t dissents against and yet are closer to her compared to other opinions she signs, and this is satisfied because any strategy that is not a cutoff strategy as described in the proposition must contain two such ranges; condition (3) equates the probability of the events of having an opinion at Δv_s and at Δv_d, and this can always be satisfied by cutting down the region that initially corresponds to a more likely event until both events have equal probabilities. Conditions (2) and (3) together imply that by switching her strategy to signing all opinions at Δv_d and dissenting against all opinions at Δv_s, judge t can strictly decrease her D cost without changing W. Thus this supposed strategy cannot be her optimal one. Finally, the only non-degenerate strategy that violates the conditions of the proposition and such that, on any side of her, t either always signs the opinion or always dissents against it is a strategy in which t signs v iff $t \geq v$ or iff $t \leq v$. In this case we can still require that ranges Δv_s and Δv_d satisfy conditions (1) and (2), and replace condition (3) with an equivalent one. Without loss of generality, suppose that t signs v iff $t \geq v$. Then replace condition (3) with condition (3') as follows: $\int_{\Delta v_s} 2F(v) f(v)\, dv = \int_{\Delta v_d} 2F(1-v) f(v)\, dv$. The rest of the proof stays the same – by switching her strategy to signing all opinions at Δv_d and dissenting against all opinions at Δv_s, judge t can strictly decrease her D cost without changing W and so this strategy cannot be her optimal one.

D Proof of Proposition 2

The proposition is proven:

- For $\alpha > 1/2$ in Section D.2.
- For $\alpha < 1/2$ in Section D.3.
- For $\alpha = 1/2$ in Section D.4.

Before we get to these parts we will show some useful first properties of the model. Let the distribution of judges be single-peaked and symmetric around 0 and w.l.o.g consider judge $t \geq 0$.

For a cutoff strategy c of judge t, the following probabilities are instrumental for calculating the judge's loss. The probability that t is the median is

(6) $$P_m \equiv 2F(t)\,[1-F(t)]\,.$$

The probability that the opinion is more than c away from t is

$$P(t) = [F(t-c)]^2 + [1 - F(t+c)]^2 . \tag{7}$$

The pattern of a spider is defined as follows.

Definition 1 A spider pattern is when $P(t)$ is first increasing and then decreasing.

Here is a first useful lemma.

Lemma 3 *Suppose the cutoff c is constant at a certain range of types and for each type t at this range $c < 1 - t$. Then the probability of dissent is increasing in t in that range.*

Proof: Differentiating $P(t)$ and using the properties of a uniform distribution yields

$$P'(t) = 2\{F(t-c)f(t-c) - [1 - F(t+c)]f(t+c)\} \geq 0,$$

because symmetry implies that $F(t-c) \geq [1 - F(t+c)]$ and single-peaked distribution implies that $f(t-c) \geq f(t+c)$. *Q.E.D.*

Lemma 4 *Let $\alpha > 0$. At the range $1-t < c < 1+t$, the probability of dissent decreases if and only if $\frac{dc}{dt} > 1$ (when $\frac{dc}{dt}$ is well defined).*

Proof: Noting that $1 - t \leq c$, hence the dissent occurs only for opinions to the left of t, we get that

$$\begin{aligned} P'(t) &= 2F(t-c)f(t-c)\left(1 - \frac{dc}{dt}\right) - 2[1 - F(t+c)]f(t+c)\left(1 + \frac{dc}{dt}\right) \\ &= \frac{1}{2}(1+t-c)\left(1 - \frac{dc}{dt}\right), \text{ when } t \sim U(-1,1) \end{aligned}$$

i.e., $P'(t) < 0$ iff $\frac{dc}{dt} > 1$. *Q.E.D.*

D.1 Analyzing $L(c)$ Denoting

$$z \equiv \int_{t-c}^{t} (t-v)2F(v)f(v)dv + \int_{t}^{t+c} (v-t)2[1 - F(v)]f(v)dv, \tag{8}$$

we can express the loss of judge t with cutoff strategy c as follows

$$L(c) = \left[\int_{t-c}^{t} (t-v)\,2F(v)\,f(v)\,dv + \int_{t}^{t+c} (v-t)\,2(1-F(v))\,f(v)\,dv\right]^{\alpha} + WP(t) = z^{\alpha} + WP(t) \tag{9}$$

Differentiating by c yields

$$\frac{dL}{dc} = \alpha z^{\alpha-1}\frac{dz}{dc} + W\frac{dP(t)}{dc} \rightarrow$$

$$\frac{dL}{dc} = 2cM\alpha z^{\alpha-1} - 2WM = 2M\left[c\alpha z^{\alpha-1} - W\right], \tag{10}$$

where

$$M \equiv F(t-c)f(t-c) + [1 - F(t+c)]\, f(t+c). \tag{11}$$

M is positive since $F \in [0,1]$ and $f \geq 0$.

Let the distribution of judges be $U(-1,1)$, so that $f(t) = 0.5, F(t) = \frac{1+t}{2}$. Then, for $t \geq 0$, we have three possible regions for c:

D.1.1 Case 1: $c \geq 1+t$

$$z = \frac{1}{2}\left[\int_{-1}^{t}(t-v)(1+v)\,dv + \int_{t}^{1}(v-t)(1-v)\,dv\right] = ...$$

$$z = \frac{1}{2}\left[t^2 + \frac{1}{3}\right] \tag{12}$$

Note that $M = 0$ (in 11) since $F(t-c) = f(t+c) = 0$ under a uniform distribution $c \geq 1+t$. Hence (from (10))

$$\frac{dL}{dc} = 0.$$

D.1.2 Case 2: $1-t \leq c < 1+t$

$$\begin{aligned} z &= \int_{t-c}^{t}(t-v)\,2F(v)\,f(v)\,dv + \int_{t}^{1}(v-t)\,2\,(1-F(v))\,f(v)\,dv \\ &= \int_{t-c}^{t}(t-v)\,\frac{1+v}{2}dv + \int_{t}^{1}(v-t)\,\frac{1-v}{2}dv = ... \\ &= \frac{1}{2}\left[\frac{(t-c)^3}{3} - (t-1)\frac{(t-c)^2}{2} + \left(c - \frac{1}{2}\right)t + \frac{1}{6}\right] \end{aligned}$$

$$\Rightarrow \frac{dL}{dc} = 2M\left[\alpha c\left(\frac{1}{2}\left[\frac{(t-c)^3}{3} - (t-1)\frac{(t-c)^2}{2} + \left(c - \frac{1}{2}\right)t + \frac{1}{6}\right]\right)^{\alpha-1} - W\right].$$

Substituting

$$M = \frac{1}{2}\left[\frac{1+t-c}{2}\right]$$

into $\frac{dL}{dc}$ yields

$$\frac{dL}{dc} = \left(\frac{1+t-c}{2}\right)\left(2^{1-\alpha}\alpha c\left[\frac{(t-c)^3}{3} - (t-1)\frac{(t-c)^2}{2} + \left(c-\frac{1}{2}\right)t + \frac{1}{6}\right]^{\alpha-1} - W\right). \tag{13}$$

We will interchangeably use different formulations for z:

$$2z = \frac{(t-c)^3}{3} - (t-1)\frac{(t-c)^2}{2} + \left(c-\frac{1}{2}\right)t + \frac{1}{6} = ... \tag{14}$$

$$2z = -\frac{1}{3}c^3 - \frac{1}{6}t^3 + \frac{1}{2}tc^2 + \frac{1}{2}t^2 + \frac{1}{2}c^2 - \frac{1}{2}t + \frac{1}{6} = ... \tag{15}$$

$$2z = \frac{1}{6}(1-t)^3 - \frac{1}{3}c^3 + \frac{1}{2}tc^2 + \frac{1}{2}c^2. \tag{16}$$

D.1.3 Case 3: $c \leq 1-t$

$$\begin{aligned}
z &= \int_{t-c}^{t} (t-v)\,2F(v)\,f(v)\,dv + \int_{t}^{t+c} (v-t)\,2(1-F(v))\,f(v)\,dv \\
&= \int_{t-c}^{t} (t-v)\,\frac{1+v}{2}dv + \int_{t}^{t+c} (v-t)\,\frac{1-v}{2}dv = ... \\
&= c^2\left[\frac{1}{2} - \frac{1}{3}c\right]
\end{aligned}$$

$$\Rightarrow \frac{dL}{dc} = 2M\left[\alpha c\left(c^2\left[\frac{1}{2} - \frac{1}{3}c\right]\right)^{\alpha-1} - W\right] \tag{17}$$

$$= 2M\left[\alpha c^{2\alpha-1}\left[\frac{1}{2} - \frac{1}{3}c\right]^{\alpha-1} - W\right].$$

Using the uniform distribution in equation (11) for M

$$\begin{aligned}
M &= \frac{1}{2}\left[\frac{1+t-c}{2} + \frac{1-t-c}{2}\right] \\
&= \frac{1}{2}(1-c)
\end{aligned}$$

and substituting into (17) yields

$$\frac{dL}{dc} = (1-c)\left[\alpha c^{2\alpha-1}\left[\frac{1}{2} - \frac{1}{3}c\right]^{\alpha-1} - W\right], \tag{18}$$

$$\lim_{c\to 0}\frac{dL}{dc} = \begin{cases} +\infty \text{ if } \alpha < 1/2 \\ 2^{-1/2} - W \text{ if } \alpha = 1/2 \\ -W \text{ if } \alpha > 1/2 \end{cases}. \tag{19}$$

Lemma 5 *$\frac{dL}{dc}$ is continuous everywhere.*

Proof: It is immediate that L is continuous within each range so we only need to check the transitions.

At $c = 1 - t$:

$$\begin{aligned}
\frac{dL}{dc} &= \left(\frac{1+t-c}{2}\right)\left(2^{1-\alpha}\alpha c\left[\frac{(t-c)^3}{3} - (t-1)\frac{(t-c)^2}{2} + \left(c-\frac{1}{2}\right)t + \frac{1}{6}\right]^{\alpha-1} - W\right) \\
&= (1-c)\left(2^{1-\alpha}\alpha c\left[-\frac{2}{3}c^3 + c^2\right]^{\alpha-1} - W\right) \\
&= (1-c)\left(2^{1-\alpha}\alpha c^{2\alpha-1}\left[1-\frac{2}{3}c^3\right]^{\alpha-1} - W\right) \\
&= (1-c)\left[\alpha c^{2\alpha-1}\left[\frac{1}{2}-\frac{1}{3}c\right]^{\alpha-1} - W\right]
\end{aligned}$$

which equals dL/dc at case 3 (see equation 18). At $c = 1 + t$:

$$\frac{dL}{dc} = \left(\frac{1+t-c}{2}\right)\left(2^{1-\alpha}\alpha c\left[\frac{(t-c)^3}{3} - (t-1)\frac{(t-c)^2}{2} + \left(c-\frac{1}{2}\right)t + \frac{1}{6}\right]^{\alpha-1} - W\right) = 0$$

which equals dL/dc at case 1 (see equation 18). *Q.E.D.*

D.1.4 Analyzing dL/dc In order to investigate $\frac{dL}{dc}$, define the function $g(c,t)$ as follows:

$$(20) \qquad g(c,t) \equiv \begin{cases} c^{2\alpha-1}\left[1-\frac{2}{3}c\right]^{\alpha-1} & \text{if } 0 < c < 1-t \\ c\left[-\frac{1}{3}c^3 - \frac{1}{6}t^3 + \frac{1}{2}tc^2 + \frac{1}{2}t^2 + \frac{1}{2}c^2 - \frac{1}{2}t + \frac{1}{6}\right]^{\alpha-1} & \text{if } 1-t \le c \le 1+t \end{cases}.$$

Lemma 6 *The sign of* $\frac{dL}{dc}$ *equals the sign of* $2^{1-\alpha}\alpha g(c,t) - W$.

Proof: Follows immediately from substituting the values of $g(c,t)$ for the ranges $0 < c < 1-t$ and $1-t \le c < 1+t$ in the corresponding expressions of $\frac{dL}{dc}$. *Q.E.D.*

Lemma 7 *The sign of* $\frac{\partial g(c,t)}{\partial c}$ *is determined by the sign of*

$$(21) \qquad E \equiv \begin{cases} \left[2\alpha(1-c) - 1 + \frac{4}{3}c\right] & \textit{if } 0 < c < 1-t \\ \left[\frac{2}{3}c^3 - \frac{1}{6}t^3 - \frac{1}{2}tc^2 + \frac{1}{2}t^2 - \frac{1}{2}c^2 - \frac{1}{2}t + \frac{1}{6} + \alpha\left(tc^2 + c^2 - c^3\right)\right] & \textit{if } 1-t \le c < 1+t \end{cases}.$$

Proof: Note that $g(c,t)$ is continuous (since g is part of dL/dc which is continuous by Lemma 5) and

$$\frac{\partial g(c,t)}{\partial c} = \frac{\partial}{\partial c}\begin{cases} c^{2\alpha-1}\left[1-\frac{2}{3}c\right]^{\alpha-1} & \text{if } 0 < c < 1-t \\ c\left[-\frac{1}{3}c^3 - \frac{1}{6}t^3 + \frac{1}{2}tc^2 + \frac{1}{2}t^2 + \frac{1}{2}c^2 - \frac{1}{2}t + \frac{1}{6}\right]^{\alpha-1} & \text{if } 1-t \le c < 1+t \end{cases} = ...$$

$$(22) \qquad \frac{\partial g(c,t)}{\partial c} = \begin{cases} c^{2\alpha-2}\left[2\alpha(1-c) - 1 + \frac{4}{3}c\right]\left[1-\frac{2}{3}c\right]^{\alpha-2} & \text{if } 0 < c < 1-t \\ \left[\frac{2}{3}c^3 - \frac{1}{6}t^3 - \frac{1}{2}tc^2 + \frac{1}{2}t^2 - \frac{1}{2}c^2 - \frac{1}{2}t + \frac{1}{6} + \alpha\left(tc^2 + c^2 - c^3\right)\right] * & \\ \left[\frac{1}{6}(1-t)^3 - \frac{1}{3}c^3 + \frac{1}{2}tc^2 + \frac{1}{2}c^2\right]^{\alpha-2} & \text{if } 1-t \le c < 1+t \end{cases}$$

Note first that $\left[1-\frac{2}{3}c\right]\geq 0$ when $0<c<1-t$. Then note from equation (16) that $\left[\frac{1}{6}\left(1-t\right)^{3}-\frac{1}{3}c^{3}+\frac{1}{2}tc^{2}+\frac{1}{2}c^{2}\right]=2z\geq 0$ (since z by definition (8) is a sum of two positive integrals). Thus, the sign of $\frac{\partial g(c,t)}{\partial c}$ is solely determined by

$$E=\begin{cases}\left[2\alpha(1-c)-1+\frac{4}{3}c\right] \text{ if } 0<c<1-t\\ \left[\frac{2}{3}c^{3}-\frac{1}{6}t^{3}-\frac{1}{2}tc^{2}+\frac{1}{2}t^{2}-\frac{1}{2}c^{2}-\frac{1}{2}t+\frac{1}{6}+\alpha\left(tc^{2}+c^{2}-c^{3}\right)\right] \text{ if } 1-t\leq c<1+t\end{cases}.$$

Q.E.D.

D.2 Convexity, linearity and weak concavity $\alpha>\frac{1}{2}$ This case is divided into three subcases which are proven:

- For $\alpha\geq 1$ in Section D.2.1.
- For $2/3\leq\alpha<1$ in Section D.2.2.
- For $1/2<\alpha<2/3$ in Section D.2.3.

But we start with presenting some useful results.

When $\alpha>\frac{1}{2}$ we have $\lim_{c\to 0}\frac{dL}{dc}<0$, implying that $c=0$ is not a potential solution. Every type has either an inner solution or chooses the corner solution of no dissent ($c=1+t$).

For $\alpha=\frac{1}{2}$ equation (21) yields

$$E=\begin{cases}\frac{1}{3}c \text{ if } 0<c<1-t\\ \frac{1}{6}\left[c^{3}+\left(1-t\right)^{3}\right] \text{ if } 1-t\leq c<1+t\end{cases}$$

which is strictly positive. It then follows, given that E increases in α at the range $1-t\leq c<1+t$,[37] that for any $\alpha\geq\frac{1}{2}$, $\frac{\partial g(c,t)}{\partial c}$ is positive and hence $g(c,t)$ increases in c at the whole range for which it is defined, hence equals $\frac{W}{2^{1-\alpha}\alpha}$ at most once. Hence $L(c)$ has at most one inner local min point (where $\frac{dL}{dc}=0$).

Corollary 1 *Let $\alpha>\frac{1}{2}$. Type t chooses $c=1+t$ if $W\geq\alpha 2^{1-\alpha}g\left(1+t,t\right)$, and otherwise she has an inner solution $c\in\left]0,1+t\right[$.*

Proof: We know from Lemma 5. If $W\geq\alpha 2^{1-\alpha}g\left(c,t\right)$ at $c=1+t$ then L' never equals zero, hence no inner solution. If $W<\alpha 2^{1-\alpha}g\left(1+t,t\right)$ then she has an inner solution since $L'\left(1+t\right)>0$ and since $\lim_{c\to 0}L'<0$. *Q.E.D.*

Lemma 8 *Let $\alpha\geq\frac{1}{2}$. Suppose that some type t' has an inner solution $c\left(t'\right)\leq 1-t'$. Then any $t<t'$ has an inner solution $c\left(t\right)=c\left(t'\right)\leq 1-t'$ too.*

[37] Since $\left(tc^{2}+c^{2}-c^{3}\right)=c^{2}\left(1+t-c\right)>0$ when $c<1+t$.

Proof: Lemma 6 implies that, in inner solutions, $W = \alpha 2^{1-\alpha} g\left(c\left(t'\right), t'\right)$. For any $t < t'$ we have $g\left(c, t\right) = g\left(c, t'\right)$ at the range $c \in \left[0, 1-t'\right]$, hence $W = \alpha 2^{1-\alpha} g\left(c\left(t'\right), t\right)$ where $c\left(t'\right) \leq 1-t'$ implies that $c\left(t'\right) < 1-t$. In other words, t has an inner solution at $c\left(t'\right)$ as well. *Q.E.D.*

Lemma 9 *Let*

$$h\left(t\right) \equiv g(1+t, t) = \left(1+t\right)\left(\frac{1}{3}+t^2\right)^{\alpha-1}.$$

Then $h\left(t\right)$ is monotonically increasing if $\alpha \geq \frac{2}{3}$ but has a unique max point at

$$t_{\max} \equiv \frac{1-\alpha-\sqrt{(\alpha-2)(\alpha-\frac{2}{3})}}{2\alpha-1} \tag{23}$$

if $\alpha \in \left]\frac{1}{2}, \frac{2}{3}\right[$.

Proof:

$$\begin{aligned}
\frac{dh}{dt} &= \left(\frac{1}{3}+t^2\right)^{\alpha-1} + 2t\left(\alpha-1\right)\left(1+t\right)\left(\frac{1}{3}+t^2\right)^{\alpha-2} = \ldots \\
&= \left[\left(2\alpha-1\right)t^2 + 2\left(\alpha-1\right)t + \frac{1}{3}\right]\left(\frac{1}{3}+t^2\right)^{\alpha-2} \qquad (24)
\end{aligned}$$

which is positive if $\alpha \geq 1$. For $1/2 < \alpha < 1$ note that the bracket determines the sign, it is U-shaped in t and switches sign for t such that

$$\begin{aligned}
\left(2\alpha-1\right)t^2 + 2\left(\alpha-1\right)t + \frac{1}{3} &= 0 \\
t_{1,2} &= \frac{-2\left(\alpha-1\right) \pm \sqrt{4\left(\alpha-1\right)^2 - 4\frac{1}{3}\left(2\alpha-1\right)}}{2\left(2\alpha-1\right)} \\
&= \frac{1-\alpha \pm \sqrt{(\alpha-2)(\alpha-\frac{2}{3})}}{2\alpha-1}.
\end{aligned}$$

which has no real solution, and hence the bracket is strictly positive, if $\alpha \in \left[\frac{2}{3}, 2\right]$. Hence $\frac{dh}{dt} \geq 0$. Otherwise, if $\alpha \in \left]\frac{1}{2}, \frac{2}{3}\right[$, we first note that

$$\alpha < \frac{2}{3} \Rightarrow 1-\alpha > 2\alpha-1 \Rightarrow t_1 > 1,$$

meaning that the bracket crosses the zero-line at most once for $t \in [0,1]$. It crosses it exactly once if there exists a $t \in [0,1]$ for which $\frac{dh}{dt} < 0$:

$$\begin{aligned}
\frac{1-\alpha-\sqrt{(\alpha-2)(\alpha-\frac{2}{3})}}{2\alpha-1} &< 1 \Leftrightarrow \\
\left(2\alpha-1\right)\left(\alpha-\frac{2}{3}\right) &< 0
\end{aligned}$$

which indeed holds when $\alpha \in \left]\frac{1}{2},\frac{2}{3}\right[$. Finally, noting that

$$\begin{aligned}\frac{1-\alpha-\sqrt{(\alpha-2)(\alpha-\frac{2}{3})}}{2\alpha-1} &> 0 \Leftrightarrow \\ \alpha &> \frac{1}{2},\end{aligned}$$

we get that, for any $\alpha \in \left]\frac{1}{2},\frac{2}{3}\right[$, $h(t)$ has a max point at $t_2 = \frac{1-\alpha-\sqrt{(\alpha-2)(\alpha-\frac{2}{3})}}{2\alpha-1} \equiv t_{\max}$, while $\frac{dh}{dt} > 0$ $\forall t \in [0,1]$ if $\alpha \geq \frac{2}{3}$. *Q.E.D.*

LEMMA 10 *Let $\alpha \geq \frac{2}{3}$. A necessary condition for getting the pattern of a spider is to have a range of types $[\underline{t},1]$ with cutoffs $c \in [1-t,1+t]$ and such that among them the probability of dissent is decreasing in t.*

PROOF: Corollary (19) implies a type has an inner solution iff $W < \alpha 2^{1-\alpha} g(1+t,t)$. Lemma 9 says that, when $\alpha \geq \frac{2}{3}$, $h(t) \equiv g(1+t,t)$ is monotonically increasing. Hence if some type t' has an inner solution c, then any type $t > t'$ has an inner solution too. If W is sufficiently large so that no type has an inner solution, it follows that no type ever dissents, and this is a degenerate case with no spider. Otherwise, the pattern of a spider requires that among types with sufficiently large t the probability of dissent will be decreasing. To complete the proof, we will show that a decrease in dissent cannot happen if the inner solutions are such that $c(t) < 1-t$. To see that, note that if a given type t'' has an inner solution $c(t'') = \tilde{c} < 1-t''$, then it must be that all types $t < t''$ have $c(t) = \tilde{c}$ as their solution too, because $\forall t \leq t''$ we have $\tilde{c} < 1-t$ and because the part of $g(c,t)$ at the range $c \in [0,1-t'']$ is identical for all these types (see equation 20 in the first range). This implies by Lemma 3 that at the range $t \in [0,t'']$ the probability of dissent is increasing in t. Hence, a necessary condition for getting the pattern of a spider is a decrease in the probability of dissent among a range of types whose cutoffs are such that $1-t < c < 1+t$. *Q.E.D.*

LEMMA 11 *Let $\alpha > \frac{1}{2}$ and suppose there exists a range of types with inner solutions $c \in [1-t,1+t]$. Then, in this range of inner solutions, $\frac{dc}{dt} \leq 0$ if and only if $\alpha \geq 1$*

PROOF: For type t with an inner solution at $1-t < c < 1+t$ the first order condition (13) must hold:

$$2^{1-\alpha}\alpha c\left[\frac{(t-c)^3}{3} - (t-1)\frac{(t-c)^2}{2} + \left(c-\frac{1}{2}\right)t + \frac{1}{6}\right]^{\alpha-1} - W = 0.$$

Using the implicit function theorem we get

$$\begin{aligned}\frac{dc}{dt} &= -\frac{\frac{d}{dt}c\left[-\frac{1}{3}c^3-\frac{1}{6}t^3+\frac{1}{2}tc^2+\frac{1}{2}t^2+\frac{1}{2}c^2-\frac{1}{2}t+\frac{1}{6}\right]^{\alpha-1}}{\frac{d}{dc}c\left[-\frac{1}{3}c^3-\frac{1}{6}t^3+\frac{1}{2}tc^2+\frac{1}{2}t^2+\frac{1}{2}c^2-\frac{1}{2}t+\frac{1}{6}\right]^{\alpha-1}} \to \ldots \\ \text{(25)}\qquad \frac{dc}{dt} &= c\frac{(1-\alpha)\left[-\frac{1}{2}t^2+\frac{1}{2}c^2+t-\frac{1}{2}\right]}{\frac{2}{3}c^3-\frac{1}{6}t^3-\frac{1}{2}tc^2+\frac{1}{2}t^2-\frac{1}{2}c^2-\frac{1}{2}t+\frac{1}{6}+\alpha\left(tc^2+c^2-c^3\right)}.\end{aligned}$$

Since we look at the case of $\alpha > \frac{1}{2}$, and noting that

$$tc^2 + c^2 - c^3 = c^2 (t + 1 - c) > 0,$$

it is enough to show that $\left[\frac{2}{3}c^3 - \frac{1}{6}t^3 - \frac{1}{2}tc^2 + \frac{1}{2}t^2 - \frac{1}{2}c^2 - \frac{1}{2}t + \frac{1}{6} + \alpha\left(tc^2 + c^2 - c^3\right)\right]$ is positive at $\alpha = \frac{1}{2}$ (since the expression increases in α)[38] in order to conclude that the denominator is positive. Indeed, this expression with $\alpha = 1/2$ can be shown to equal $\frac{1}{6}\left[c^3 + (1-t)^3\right] > 0$. It thus follows that for $\alpha \in \left]\frac{1}{2}, 1\right[$ the sign of $\frac{dc}{dt}$ equals the sign of $-\frac{1}{2}t^2 + \frac{1}{2}c^2 + t - \frac{1}{2}$ while for $\alpha > 1$ the sign of $\frac{dc}{dt}$ is the opposite of the sign of $-\frac{1}{2}t^2 + \frac{1}{2}c^2 + t - \frac{1}{2}$:

$$\begin{aligned} -\frac{1}{2}t^2 + \frac{1}{2}c^2 + t - \frac{1}{2} &\geq -\frac{1}{2}t^2 + \frac{1}{2}(1-t)^2 + t - \frac{1}{2} = ... \\ &= -\frac{1}{2}t^2 + \frac{1}{2} - t + \frac{1}{2}t^2 + t - \frac{1}{2} = 0. \end{aligned} \tag{26}$$

It thus follows that $\frac{dc}{dt} \leq 0$ if $\alpha \geq 1$ and $\frac{dc}{dt} >$ if $\alpha \in]1/2, 1[$. *Q.E.D.*

LEMMA 12 *Suppose there exists a range of types with inner solutions $c \in [1-t, 1+t]$. Then $\frac{dc}{dt} > 1$ iff*

$$G \equiv c^2 \left[(3\alpha - 1)c - (2\alpha - 1)3(1+t)\right] - (1-t)^2 \left[(1-t) + (1-\alpha)3c\right] > 0. \tag{27}$$

PROOF: Using $\frac{dc}{dt}$ from (25), $\frac{dc}{dt} > 1$ holds iff

$$\begin{aligned} c\frac{(1-\alpha)\left[-\frac{1}{2}t^2 + \frac{1}{2}c^2 + t - \frac{1}{2}\right]}{\frac{2}{3}c^3 - \frac{1}{6}t^3 - \frac{1}{2}tc^2 + \frac{1}{2}t^2 - \frac{1}{2}c^2 - \frac{1}{2}t + \frac{1}{6} + \alpha\left(tc^2 + c^2 - c^3\right)} &> 1 \Leftrightarrow ... \\ c^2 \left[(3\alpha - 1)c - (2\alpha - 1)3(1+t)\right] - (1-t)^2 \left[(1-t) + (1-\alpha)3c\right] &> 0. \end{aligned}$$

Q.E.D.

D.2.1 Convexity and linearity $\alpha \geq 1$

PROPOSITION 6 *There cannot be a spider pattern when $\alpha \geq 1$*

PROOF: Lemma 10 implies that a necessary condition for getting the pattern of a spider is a decrease in the probability of dissent at the range $1 - t < c < 1 + t$. It thus follows from Lemmas 11 and 4 that there cannot be a spider when $\alpha \geq 1$. *Q.E.D.*

D.2.2 Very weak concavity $\alpha \in \left[\frac{2}{3}, 1\right[$

PROPOSITION 7 *There cannot be a spider pattern when $\alpha \in \left[\frac{2}{3}, 1\right[$.*

PROOF: Lemma 10 implies that a necessary condition for getting the pattern of a spider is a decrease in the probability of dissent at the range $1 - t < c < 1 + t$, which together with Lemma 4 implies that G

[38]Since $\left(tc^2 + c^2 - c^3\right) = c^2(1 + t - c) > 0$ when $c < 1 + t$.

(defined in equation (27)) must be strictly positive somewhere in the range to maintain the possibility of a spider. Investigating G, note first that if $[(3\alpha-1)\,c-(2\alpha-1)\,3\,(1+t)]\leq 0$ then $G\leq 0$. Otherwise, if $[(3\alpha-1)\,c-(2\alpha-1)\,3\,(1+t)]>0$, then it gets its max value for $c=1+t$, while the part that is deducted from it, $(1-t)^2\,[(1-t)+(1-\alpha)\,3c]$, is positive and increases in c hence is minimal when $c=1-t$. By plugging these values correspondingly we get that

$$G<(1+t)^3\,[2-3\alpha]-(1-t)^3\,[4-3\alpha]\leq 0,$$

when $\alpha\in\left[\frac{2}{3},1\right[$. Hence there cannot be a spider pattern. *Q.E.D.*

D.2.3 Mildly weak concavity $\alpha\in\left]\frac{1}{2},\frac{2}{3}\right[$ We will show that for any $\alpha\in\left]\frac{1}{2},\frac{2}{3}\right[$ the following pattern of spider exists (though other kinds of spider can be generated too: most types, including those close to 0 and 1, never dissent, while there exists a non-empty range of types in-between that do dissent sometimes.

PROPOSITION 8 *For any* $\alpha\in\left]\frac{1}{2},\frac{2}{3}\right[$, *there exist values of* W *for which dissent has the pattern of a spider*

PROOF: From Lemma 9 we know that $h(t)=g\,(1+t,t)$ has a hill-shape for any $\alpha\in\left]\frac{1}{2},\frac{2}{3}\right[$ with a peak at $t_{\max}$ (defined in equation (23)). We will prove the proposition for $W=\alpha 2^{1-\alpha}\,[g\,(1+t_{\max},t_{\max})-\varepsilon]$, where ε is very small. In this case, since $W<\alpha 2^{1-\alpha}g\,(1+t_{\max},t_{\max})$, Corollary 1 says inner solutions exist for a small range of types $t\in(t_{\max}-\delta_1,t_{\max}+\delta_2)$, with c very close to $1+t_{\max}$. For a given α, they also depend on ε as a parameter. From corollary 1, we know that all other types, including those close to 0 or close to 1, choose a corner solution $c=1+t$ (always dissent). To get the pattern of a spider we need to verify that at the range of inner solutions the probability of dissent is either monotonic or first increases and then decreases. Given W has been set such that type $t=0$ has a corner solution, Lemma 8 implies that any type t in the range of types with inner solutions has a solution at the range $c\in[1-t,1+t]$. In this range, we know from Lemma 4 that the probability of dissent decreases if and only if $\frac{dc}{dt}>1$.

Let us denote the inner min point of $L\,(c)$ for type $t\in(t_{\max}-\delta_1,t_{\max}+\delta_2)$ by $c_0\,(t;\varepsilon)$. Then we can expand $c_0\,(t;\varepsilon)$ into Taylor series in ε and t around $\varepsilon=0$ and $t=t_{\max}$ as follows:

$$\Delta c\equiv c_0\,(t;\varepsilon)-(1+t_{\max})=\left(\frac{\partial c_0}{\partial t}\right)_{\substack{\varepsilon=0\\ t=t_{\max}}}\Delta t+\frac{1}{2}\left(\frac{\partial^2 c_0}{\partial t^2}\right)_{\substack{\varepsilon=0\\ t=t_{\max}}}(\Delta t)^2+\left(\frac{\partial c_0}{\partial \varepsilon}\right)_{\substack{\varepsilon=0\\ t=t_{\max}}}\varepsilon+...,\tag{28}$$

where $\Delta t\equiv t-t_{\max}$. We know, since $t_{\max}$ is the peak of $h\,(t)$, that at $t_{\max}$ and $c=1+t_{\max}$ we have

$$\frac{dh}{dt}=\frac{\partial g(c,t)}{\partial t}+\frac{\partial g(c,t)}{\partial c}\frac{d}{dt}\,(1+t)=\frac{\partial g(c,t)}{\partial t}+\frac{\partial g(c,t)}{\partial c}=0,$$

hence at $t_{\max}$ we have (from the implicit function theorem)

$$(29)\qquad \left(\frac{\partial c_0}{\partial t}\right)_{\substack{\varepsilon=0\\ t=t_{\max}}} = -\left[\frac{\partial g(c,t)}{\partial t}\right]_{\substack{t=t_{\max}\\ c=1+t_{\max}}} / \left[\frac{\partial g(c,t)}{\partial c}\right]_{\substack{t=t_{\max}\\ c=1+t_{\max}}} = 1.$$

To find the other derivatives in equation (28) we will expand into series the defining equation

$$g(c_0(t), t) - g(1+t_{\max}, t_{\max}) = -\varepsilon.$$

The expansion will be

$$\begin{aligned}
& \left(\frac{\partial g}{\partial t}\right)_{\substack{t=t_{\max}\\ c=1+t_{\max}}} \Delta t + \left(\frac{\partial g}{\partial c}\right)_{\substack{t=t_{\max}\\ c=1+t_{\max}}} \Delta c + \frac{1}{2}\left(\frac{\partial^2 g}{\partial t^2}(\Delta t)^2 + 2\frac{\partial^2 g}{\partial t\partial c}(\Delta t)(\Delta c) + \frac{\partial^2 g}{\partial c^2}(\Delta c)^2\right)_{\substack{t=t_{\max}\\ c=1+t_{\max}}} + ... \\
= & \left(\frac{\partial g}{\partial t}\right)_{\substack{t=t_{\max}\\ c=1+t_{\max}}} \Delta t + \left(\frac{\partial g}{\partial c}\right)_{\substack{t=t_{\max}\\ c=1+t_{\max}}} \left[\Delta t + \frac{1}{2}\left(\frac{\partial^2 c_0}{\partial t^2}\right)_{\substack{\varepsilon=0\\ t=t_{\max}}} (\Delta t)^2 + \left(\frac{\partial c_0}{\partial \varepsilon}\right)_{\substack{\varepsilon=0\\ t=t_{\max}}} \varepsilon\right] \\
& + \frac{1}{2}\left(\frac{\partial^2 g}{\partial t^2}(\Delta t)^2 + 2\frac{\partial^2 g}{\partial t\partial c}(\Delta t)(\Delta c) + \frac{\partial^2 g}{\partial c^2}(\Delta c)^2\right)_{\substack{t=t_{\max}\\ c=1+t_{\max}}} + ... \\
= & -\varepsilon
\end{aligned}$$

Equation (29) implies that $\Delta c \simeq \Delta t$ hence we can write[39]

$$\begin{aligned}
& \left(\frac{\partial g}{\partial t}\right)_{\substack{t=t_{\max}\\ c=1+t_{\max}}} \Delta t + \left(\frac{\partial g}{\partial c}\right)_{\substack{t=t_{\max}\\ c=1+t_{\max}}} \left[\Delta t + \frac{1}{2}\left(\frac{\partial^2 c_0}{\partial t^2}\right)_{\substack{\varepsilon=0\\ t=t_{\max}}} (\Delta t)^2 + \left(\frac{\partial c_0}{\partial \varepsilon}\right)_{\substack{\varepsilon=0\\ t=t_{\max}}} \varepsilon\right] \\
& + \frac{1}{2}\left(\frac{\partial^2 g}{\partial t^2} + 2\frac{\partial^2 g}{\partial t\partial c} + \frac{\partial^2 g}{\partial c^2}\right)_{\substack{t=t_{\max}\\ c=1+t_{\max}}} (\Delta t)^2 + ... = -\varepsilon
\end{aligned}$$

Given that the RHS is independent of Δt or $(\Delta t)^2$, and that the coefficient of Δt in the LHS,

$$\left(\frac{\partial g}{\partial t}\right)_{\substack{t=t_{\max}\\ c=1+t_{\max}}} + \left(\frac{\partial g}{\partial c}\right)_{\substack{t=t_{\max}\\ c=1+t_{\max}}} = \left(\frac{dh}{dt}\right)_{\substack{t=t_{\max}\\ c=1+t_{\max}}}$$

is zero, we can conclude from the coefficient of $(\Delta t)^2$ that

$$\left(\frac{\partial g}{\partial c}\right)_{\substack{t=t_{\max}\\ c=1+t_{\max}}} \left(\frac{\partial^2 c_0}{\partial t^2}\right)_{\substack{\varepsilon=0\\ t=t_{\max}}} = -\left(\frac{\partial^2 g}{\partial t^2} + 2\frac{\partial^2 g}{\partial t\partial c} + \frac{\partial^2 g}{\partial c^2}\right)_{\substack{t=t_{\max}\\ c=1+t_{\max}}}$$

hence

$$\left(\frac{\partial^2 c_0}{\partial t^2}\right)_{\substack{\varepsilon=0\\ t=t_{\max}}} = -\frac{\left(\frac{\partial^2 g}{\partial t^2} + 2\frac{\partial^2 g}{\partial t\partial c} + \frac{\partial^2 g}{\partial c^2}\right)_{\substack{t=t_{\max}\\ c=1+t_{\max}}}}{\left(\frac{\partial g}{\partial c}\right)_{\substack{t=t_{\max}\\ c=1+t_{\max}}}}$$

[39] Note that we plug in $\Delta c = \Delta t$ only in the expressions for $(\Delta t)(\Delta c)$ and $(\Delta c)^2$, where elements of size $\varepsilon\Delta t$ or ε^2 in the expansion can be ignored.

The denominator is positive because $g(c,t)$ increases in c (even for $c = 1+t$ – see Lemma 14 below, which applies to any α). The numerator is the explicit expression of $\left(\frac{d^2h}{dt^2}\right)_{\substack{t=t_{\max}\\c=1+t_{\max}}}$, which is negative given that $t_{\max}$ is a max point of $h(t)$. It thus follows that

$$\left(\frac{\partial^2 c_0}{\partial t^2}\right)_{\substack{\varepsilon=0\\t=t_{\max}}} > 0$$

hence it follows from a Taylor expansion of $\frac{\partial c_0}{\partial t}$ around $t_{\max}$, using equation (29), that

$$\left(\frac{\partial c_0}{\partial t}\right)_{\substack{\varepsilon=0\\t=t_{\max}}} = 1 + \left(\frac{\partial^2 c_0}{\partial t^2}\right)_{\substack{\varepsilon=0\\t=t_{\max}}} \Delta t + ... \gtrless 1 \text{ for } \Delta t \gtrless 0.$$

Hence, by Lemma 4, the dissent is first increasing and then decreasing as t passes $t_{\max}$. We get a spider of the following kind: when t goes from 0 to 1 the probability of dissent is first 0, then it jumps to some strictly positive probability, then it first increases and then decreases, and finally the probability of dissent decreases abruptly back to 0 and stays there. *Q.E.D.*

D.3 Strong concavity $\alpha < \frac{1}{2}$ This case is divided into two subcases depending on whether α is larger or smaller than an α^* which is defined in Section D.3.3. The proof is provided

- For $\alpha \leq \alpha^*$ in Section D.3.4.
- For $\alpha^* < \alpha < 1/2$ in Section D.3.4.

We start with some first useful results. When $\alpha < \frac{1}{2}$ we have by equation (19) $\lim_{c\to 0} \frac{dL}{dc} = \infty$, implying that $\forall t$, $c = 0$ is a potential solution. Since $\frac{dL}{dc}$ is continuous everywhere, there can be an inner solution only if $\frac{dL}{dc} = 0$ more than once. In what follows we study some properties of $g(c,t)$ before splitting the strict concavity case into two subcases.

D.3.1 The shape of $g(c,t)$ In this subsection we study the properties of $g(c,t)$ as a function of c.

Lemma 13 *The sign of* $\frac{\partial g(c,t)}{\partial c}|_{c\overset{-}{\to}1-t}$ *equals the sign of* $\frac{\partial g(c,t)}{\partial c}|_{c\overset{+}{\to}1-t}$.

Proof: We start by revisiting Lemma 7 which showed that the sign of $\frac{\partial g(c,t)}{\partial c}$ is solely determined by E (defined in equation 21) which we rewrite as follows

$$(30) \qquad E = \begin{cases} \left[2\alpha(1-c) - 1 + \frac{4}{3}c\right] & \text{if } 0 < c < 1-t \\ c^2\left[\left(\frac{2}{3}-\alpha\right)c + \left(\alpha - \frac{1}{2}\right)(1+t)\right] + \frac{1}{6}(1-t)^3 & \text{if } 1-t \leq c < 1+t \end{cases}.$$

Thus, the sign of $\frac{\partial g(c,t)}{\partial c}|_{c\overset{+}{\to}1-t}$ equals the sign of

$$\begin{aligned} & (1-t)^2\left[\left(\frac{2}{3}-\alpha\right)(1-t)+\left(\alpha-\frac{1}{2}\right)(1+t)\right]+\frac{1}{6}(1-t)^3 = ... \\ = \ & (1-t)^2\left[\frac{1}{3}-\frac{4}{3}t+2\alpha t\right], \end{aligned}$$

where the bracket equals $E|_{c\overset{-}{\to}1-t}$. Hence both limits have the same sign. *Q.E.D.*

Lemma 14 $\frac{\partial g(c,t)}{\partial c}|_{c\overset{-}{\to}1-t} > 0$.

Proof: Using Lemma 7 and equation (30) the sign of $\frac{\partial g(c,t)}{\partial c}|_{c\overset{-}{\to}1-t}$ equals the sign of

$$\begin{aligned} & (1+t)^2\left[\left(\frac{2}{3}-\alpha\right)(1+t)+\left(\alpha-\frac{1}{2}\right)(1+t)\right]+\frac{1}{6}(1-t)^3 \\ = \ & \frac{1}{6}(1+t)^3+\frac{1}{6}(1-t)^3 > 0 \end{aligned}$$

Q.E.D.

Lemma 15 *$\frac{\partial g(c,t)}{\partial c}$ has at most one local min point (with respect to c) at the range $[1-t,1+t]$.*

Proof: By Lemma 7 follows that $\frac{\partial g(c,t)}{\partial c}$ has a local min point only if $dE/dc=0$. Differentiating equation (30) by c yields

$$E' = \begin{cases} \frac{4}{3}-2\alpha \text{ if } 0<c<1-t \\ c\left[(2-3\alpha)c+(2\alpha-1)(1+t)\right] \text{ if } 1-t\le c<1+t \end{cases}. \tag{31}$$

To learn the sign of $\frac{\partial g(c,t)}{\partial c}$ throughout the range $1-t\le c<1+t$, we differentiate the expression in (31) at this range, yielding

$$E'' = 2(2-3\alpha)c+(2\alpha-1)(1+t) > 0 \tag{32}$$

Thus, equation (31) implies that for $c>0$, since $\alpha<1/2$, E which by Lemma 7 determines the sign of $\frac{\partial g(c,t)}{\partial c}$ at the range $1-t\le c<1+t$ has at most one local extremum, at

$$c=\frac{(1-2\alpha)(1+t)}{2-3\alpha}, \tag{33}$$

and equation (32) implies that this is a min point. *Q.E.D.*

Lemma 16 *$g(c,t)$ does not have a local max point at $0<c<1-t$*

Proof: We know from equation (31) Lemma 7 that $\frac{\partial g(c,t)}{\partial c}$ cannot turn from positive to negative at the range $0<c<1-t$ (since $\alpha<1/2$), hence $g(c,t)$ cannot have a local max point there. *Q.E.D.*

LEMMA 17 *If* $c_b \equiv \frac{1-2\alpha}{\frac{4}{3}-2\alpha} > 1-t$, *then* $g(c,t)$ *has a U-shape at the range* $[0, 1+t]$

PROOF: By Lemma 7 we know $\frac{\partial g(c,t)}{\partial c} = 0$ when $E = 0$. Setting $E = 0$ in equation (30) yields

$$\begin{cases} c = \frac{1-2\alpha}{\frac{4}{3}-2\alpha} \text{ if } 0 < c < 1-t \\ c^2 \left[\left(\frac{2}{3} - \alpha\right) c + \left(\alpha - \frac{1}{2}\right)(1+t) \right] = -\frac{1}{6}(1-t)^3 \text{ if } 1-t \leq c < 1+t \end{cases}.$$

Starting with the first region ($c < 1-t$), we get by (21) that $\lim_{c \to 0} E = 2\alpha - 1 < 0$ (when $\alpha < 1/2$). Hence (by Lemma 7) the function $g(c,t)$ is decreasing initially. Then it keeps on decreasing until $c = c_b$ at which $\frac{\partial g(c,t)}{\partial c} = 0$. Thus, if $c_b \equiv \frac{1-2\alpha}{\frac{4}{3}-2\alpha} > 1-t$, we get that $\frac{\partial g(c,t)}{\partial c}$ stays negative throughout the first region, and from Lemma 13 we know that it does not change signs at $c = 1-t$. Then, Lemmas 15 and 14 imply that $\frac{\partial g(c,t)}{\partial c}$ changes sign exactly once, from negative to positive, and so, overall, $g(c,t)$ has a U-shape at the range $[0, 1+t]$. *Q.E.D.*

LEMMA 18 $g(c,t)$ *may have a local max point at the range* $1-t \leq c < 1+t$ *only if* $\alpha \in \left[\frac{1}{3}, \frac{1}{2}\right]$ *and* $t \in \left[\frac{1-\alpha}{3-5\alpha}, \frac{1}{4-6\alpha}\right]$

PROOF: If $c_b \leq 1-t$ (as defined in Lemma 17), then $g(c,t)$ has a local min point at c_b and $\frac{\partial g(c,t)}{\partial c}|_{c \to^{-} 1-t} \geq 0$. In that case, Lemmas 15 and 14 imply that, at the range $1-t \leq c < 1+t$, it might be that $g(c,t)$ first increases, then decreases, and then increases again. In this case $g(c,t)$ would have a local max point at this range. A necessary condition for $g(c,t)$ to have a local max point is that

$$c_b \leq 1-t \Rightarrow t \leq 1 - \frac{1-2\alpha}{\frac{4}{3}-2\alpha} = \frac{1}{4-6\alpha}$$

Moreover, the min point ($c = \frac{(1-2\alpha)(1+t)}{2-3\alpha}$, as given by equation (33), must exist within the range $[1-t, 1+t]$. Otherwise the fact that both $\frac{\partial g(c,t)}{\partial c}|_{c \to^{-} 1-t}$ and $\frac{\partial g(c,t)}{\partial c}|_{c \to^{-} 1+t}$ are positive would imply that $g(c,t)$ increases throughout the range $1-t \leq c < 1+t$ and so cannot have a local max point. The condition $\frac{(1-2\alpha)(1+t)}{2-3\alpha} < 1+t$ is indeed fulfilled, given that

$$\frac{1-2\alpha}{2-3\alpha} = 1 - \frac{1-\alpha}{2-3\alpha} < 1$$

when $\alpha < 1/2$. The condition $\frac{(1-2\alpha)(1+t)}{2-3\alpha} > 1-t$ can be rewritten as

$$\begin{aligned} (1-2\alpha)(1+t) &> (1-t)(2-3\alpha) \Leftrightarrow ... \\ t &> \frac{1-\alpha}{3-5\alpha}. \end{aligned}$$

For $c \in [1-t, 1+t]$ we thus get the necessary condition

$$\begin{aligned}
\frac{1-\alpha}{3-5\alpha} &\leq \frac{1}{4-6\alpha} \Leftrightarrow ... \\
&\Leftrightarrow \alpha \in \left[\frac{1}{3}, \frac{1}{2}\right]
\end{aligned}$$

Q.E.D.

D.3.2 $g(1+t,t)$ **as a function of** t We now turn to study further the properties of $g(c,t)$ at $c = 1+t$, as implied by the function $h(t)$ defined in Lemma 9.

Lemma 19 *Let* $\alpha \in \left]0, \frac{1}{2}\right[$. *Then, as defined in Lemma 9,* $h(t) = g(1+t,t) = (1+t)\left(\frac{1}{3}+t^2\right)^{\alpha-1}$ *has a unique inner global max point at* $t_{\max} \equiv \frac{1-\alpha-\sqrt{(\alpha-2)(\alpha-\frac{2}{3})}}{2\alpha-1}$.

Proof: The analysis of $h(t)$ follows the same steps as in Lemma 9 (which was performed for the case of $\alpha > 1/2$), up to the analysis of the two roots of the square brackets in (24), which determine the sign of $h(t)$,

$$(2\alpha-1)\,t^2 + 2\,(\alpha-1)\,t + \frac{1}{3}$$

Then, when $\alpha \in \left]0, \frac{1}{2}\right[$, we get that the first root $t_1 = \frac{1-\alpha+\sqrt{(\alpha-2)(\alpha-\frac{2}{3})}}{2\alpha-1} < 0$, hence $h(t)$ has a max point at $t_2 = t_{\max} = \frac{1-\alpha-\sqrt{(\alpha-2)(\alpha-\frac{2}{3})}}{2\alpha-1}$ if this value falls within the range $[0,1]$. We thus have

$$\begin{aligned}
\frac{1-\alpha-\sqrt{(\alpha-2)(\alpha-\frac{2}{3})}}{2\alpha-1} &< 1 \Leftrightarrow ... \\
(2\alpha-1)\left(\alpha-\frac{2}{3}\right) &> 0
\end{aligned}$$

which holds for any $\alpha \in \left]0, \frac{1}{2}\right[$. Finally $t_{\max} > 0$ iff

$$\frac{1-\alpha-\sqrt{(\alpha-2)(\alpha-\frac{2}{3})}}{2\alpha-1} > 0$$

which can be verified to hold for $\alpha < 1/2$. *Q.E.D.*

Lemma 20 $t_{\max} \equiv \frac{1-\alpha-\sqrt{(\alpha-2)(\alpha-\frac{2}{3})}}{2\alpha-1}$ *increases in* α *at the range* $\alpha \in \left[0, \frac{1}{2}\right[$.

PROOF: The statement holds iff

$$\begin{aligned}
\frac{dt_{\max}}{d\alpha} &= \frac{d}{d\alpha}\left\{\frac{1-\alpha-\sqrt{(\alpha-2)(\alpha-\frac{2}{3})}}{2\alpha-1}\right\} > 0 \\
&\Leftrightarrow\ 2\left(1-\alpha-\sqrt{\left(\alpha-2)(\alpha-\frac{2}{3}\right)}\right) < (1-2\alpha)\left(1+\frac{\alpha-4/3}{\sqrt{(\alpha-2)(\alpha-\frac{2}{3})}}\right) \\
&\Leftrightarrow\ 2\left((1-\alpha)\sqrt{\left(\alpha-2)(\alpha-\frac{2}{3}\right)}-(\alpha-2)\left(\alpha-\frac{2}{3}\right)\right) < (1-2\alpha)\left(\sqrt{\left(\alpha-2)(\alpha-\frac{2}{3}\right)}+(\alpha-4/3)\right) \\
&\Leftrightarrow\ \sqrt{(\alpha-2)\left(\alpha-\frac{2}{3}\right)} < (1-2\alpha)(\alpha-4/3)+2(\alpha-2)\left(\alpha-\frac{2}{3}\right) = \frac{4-5\alpha}{3}.
\end{aligned}$$

Since both sides are positive, this amounts to proving

$$\begin{aligned}
\alpha^2-\frac{8}{3}\alpha+\frac{4}{3} &< \frac{25\alpha^2-40\alpha+16}{9} \\
&\Leftrightarrow\ 0 < 16\alpha^2-16\alpha+4 = 4(2\alpha-1)^2
\end{aligned}$$

which is evident since $\alpha < 1/2$. *Q.E.D.*

LEMMA 21 *Let $\alpha \in \left[0, \frac{1}{2}\right[$. Then $t_{\max} \leq \frac{1}{3}$.*

PROOF: Lemma 20 implies that $t_{\max} = \frac{1-\alpha-\sqrt{(\alpha-2)(\alpha-\frac{2}{3})}}{2\alpha-1}$ reaches its max value for $\alpha \in \left[0, \frac{1}{2}\right[$ when $\alpha = \frac{1}{2}$. Using L'Hopital we get

$$\begin{aligned}
&\lim_{\alpha\to\frac{1}{2}} \frac{1-\alpha-\sqrt{(\alpha-2)(\alpha-\frac{2}{3})}}{2\alpha-1} \\
=\ &\lim_{\alpha\to\frac{1}{2}}\left\{-\frac{1}{2}-\frac{1}{4}\frac{2\alpha-8/3}{\sqrt{(\alpha-2)(\alpha-\frac{2}{3})}}\right\} = \frac{1}{3}
\end{aligned}$$

Q.E.D.

D.3.3 Splitting the strong concavity case into two subcases

LEMMA 22 *Let $\alpha \in \left[0, \frac{1}{2}\right[$ and define $\Omega\left(t_{\max}(\alpha), \alpha\right) \equiv \frac{\alpha(1+t_{\max})}{1+3t_{\max}^2}$. Then $\frac{d\Omega(t_{\max}(\alpha),\alpha)}{d\alpha} > 0$.*

PROOF: By construction (see Lemma 19) $t_{\max}$ is the solution to $\frac{dh}{dt} = 0$. Setting equation (24) to zero and solving for α yields

$$\alpha\left(t_{\max}\right) = \frac{\left(t_{\max}+1+2/\sqrt{3}\right)\left(t_{\max}+1-2/\sqrt{3}\right)}{2t_{\max}\left(1+t_{\max}\right)}.$$

Using this in $\Omega\left(t_{\max}(\alpha), \alpha\right)$ yields

$$\Omega\left(t_{\max}\right) = \frac{\left(t_{\max}+1+2/\sqrt{3}\right)\left(t_{\max}+1-2/\sqrt{3}\right)}{2t_{\max}\left(1+3t_{\max}^2\right)}$$

where $t_{\max} \in \left[2/\sqrt{3}-1, 1/3\right]$ (the lower limit follows from substituting $\alpha = 0$ in $t_{\max} = \frac{1-\alpha-\sqrt{(\alpha-2)(\alpha-\frac{2}{3})}}{2\alpha-1}$ while noting that $t_{\max}$ is increasing in α, as shown in Lemma 20, and the upper limit follows from Lemma 21, and from). We will now show that $\Omega\left(t_{\max}\right)$ increases in $t_{\max}$, which will imply (by Lemma 20) that $\Omega\left(t_{\max}\left(\alpha\right), \alpha\right)$ increases in α.

$$B\left(t_{\max}\right) \equiv \frac{d \ln \Omega\left(t_{\max}\right)}{d t_{\max}} = B_1\left(t_{\max}\right) + B_2\left(t_{\max}\right) + B_3\left(t_{\max}\right) + B_4\left(t_{\max}\right)$$

for

$$\begin{aligned}
B_1\left(t_{\max}\right) &\equiv \frac{1}{\left(t_{\max}+1-2/\sqrt{3}\right)}, \\
B_2\left(t_{\max}\right) &\equiv \frac{1}{\left(t_{\max}+1+2/\sqrt{3}\right)},\ B_2'\left(t_{\max}\right) = \frac{-1}{\left(t_{\max}+1+2/\sqrt{3}\right)^2},\ B_2''\left(t_{\max}\right) = \frac{2}{\left(t_{\max}+1+2/\sqrt{3}\right)^3} > 0, \\
B_3\left(t_{\max}\right) &\equiv -\frac{1}{t_{\max}},\ B_3'\left(t_{\max}\right) = \frac{1}{t_{\max}^2},\ B_3''\left(t_{\max}\right) = -\frac{2}{t_{\max}^3} < 0, \\
B_4\left(t_{\max}\right) &\equiv \frac{-6t_{\max}}{\left(1+3t_{\max}^2\right)},\ B_4'\left(t_{\max}\right) = 6\frac{3t_{\max}^2-1}{\left(1+3t_{\max}^2\right)^2},\ B_4''\left(t_{\max}\right) = \frac{108 t_{\max}\left(1-t_{\max}^2\right)}{\left(1+3t_{\max}^2\right)^2} > 0.
\end{aligned}$$

According to the signs of the second order derivatives, and designating $t_{low} \equiv 2/\sqrt{3}-1$, $t_{high} \equiv 1/3$, we have for $t_{\max} \in \left[t_{low}, t_{high}\right]$

$$\begin{aligned}
B_2\left(t_{\max}\right) &\geq B_2\left(t_{low}\right) + B_2'\left(t_{low}\right)\left(t_{\max}-t_{low}\right) = \frac{\sqrt{3}}{4} - \frac{3}{16}\left(t_{\max}-2/\sqrt{3}+1\right) \\
B_3\left(t_{\max}\right) &\geq B_3\left(t_{low}\right) + \frac{B_3\left(t_{high}\right)-B_3\left(t_{low}\right)}{t_{high}-t_{low}}\left(t_{\max}-t_{low}\right) = -3-2\sqrt{3}+3\left(3+2\sqrt{3}\right)\left(t_{\max}-2/\sqrt{3}+1\right) \\
B_4\left(t_{\max}\right) &\geq B_4\left(t_{low}\right) + B_4'\left(t_{low}\right)\left(t_{\max}-t_{low}\right) = -\sqrt{3}/2 - \frac{3}{4}\left(3+2\sqrt{3}\right)\left(t_{\max}-2/\sqrt{3}+1\right).
\end{aligned}$$

Let us designate $z \equiv t_{\max}-2/\sqrt{3}+1$. Then $B\left(t_{\max}\right) \geq N\left(z\right) = 1/z - \left(3+2.25\sqrt{3}\right) + \left(6.5625+4.5\sqrt{3}\right)z$ for $z \in \left[0, 4/3-2/\sqrt{3}\right]$. Since $N'\left(z\right) < 0$ for $z < 4/3-2/\sqrt{3}$ and $N\left(4/3-2/\sqrt{3}\right) > 0$ it follows that $N\left(z\right) > 0$ for all z involved, and so $B\left(t_{\max}\right) > 0$ implying $\Omega\left(t_{\max}\right)$ increases in $t_{\max}$ and so finally $\Omega\left(t_{\max}\left(\alpha\right), \alpha\right)$ increases in α. *Q.E.D.*

LEMMA 23 *The functions $\alpha 2^{1-\alpha} h\left(t_{\max}\right)$ and $2\left(1/6\right)^{\alpha}$ have one intersection point, denoted by α^*, at the range $\alpha \in \left]0, \frac{1}{2}\right[$. Furthermore we have*

$$\begin{cases} \alpha 2^{1-\alpha} h\left(t_{\max}\right) < 2\left(1/6\right)^{\alpha} & \text{if } \alpha \in \left(0, \alpha^*\right) \\ \alpha 2^{1-\alpha} h\left(t_{\max}\right) > 2\left(1/6\right)^{\alpha} & \text{if } \alpha \in \left(\alpha^*, \frac{1}{2}\right) \end{cases}$$

PROOF: Using $h(t)$ as defined in Lemma 9 we get

$$\begin{aligned}
\alpha 2^{1-\alpha} h(t_{\max}) &= 2(1/6)^{\alpha} \\
&\Leftrightarrow \alpha(1+t_{\max})\left(\frac{1}{6}+\frac{1}{2}t_{\max}^2\right)^{\alpha-1} = \frac{1}{3}(1/6)^{\alpha-1} \\
&\Leftrightarrow 3\left(1+3t_{\max}^2\right)^{\alpha-1} = \frac{1}{\alpha(1+t_{\max})} \\
&\Leftrightarrow 3\left(1+3t_{\max}^2\right)^{\alpha} = \frac{1+3t_{\max}^2}{\alpha(1+t_{\max})}
\end{aligned}$$

The RHS is the inverse of $\Omega(t_{\max}(\alpha),\alpha)$ as defined in Lemma 22, and we know from that lemma that $\Omega(t_{\max}(\alpha),\alpha)$ increases in α hence the RHS decreases in α.

Analyzing the LHS:

Let

$$\Phi(t_{\max}(\alpha),\alpha) \equiv \left(1+3t_{\max}^2\right)^{\alpha}$$

In Lemma 20 we showed that $t_{\max}$ increases in α. To show that $\Phi(t_{\max}(\alpha),\alpha)$ is increasing in α it is therefore enough to show that the two partial derivatives of $\Phi(t_{\max}(\alpha),\alpha)$ with respect to its two arguments, $t_{\max}(\alpha)$ and α, are both positive.

$$\begin{aligned}
&\frac{\partial \Phi(t_{\max},\alpha)}{\partial \alpha} \\
=\ &\left(1+3t_{\max}^2\right)^{\alpha}\ln\left(1+3t_{\max}^2\right) > 0
\end{aligned}$$

$$\begin{aligned}
&\frac{\partial \Phi(t_{\max},\alpha)}{\partial t_{\max}} \\
=\ &6t_{\max}\alpha\left(1+3t_{\max}^2\right)^{\alpha-1} > 0
\end{aligned}$$

Thus, the LHS increases in α while the RHS decreases in α, implying that there is a unique intersection point α^*. To find which of the functions $\alpha 2^{1-\alpha}h(t_{\max})$ and $2(1/6)^{\alpha}$ is larger below and above α^* we can plug in specific values of α. When $\alpha = 0$ the former function goes to 0 (recall that h is bounded) while the latter equals 2 hence is larger. When $\alpha \xrightarrow{-} \frac{1}{2}$ we know from Lemma 21 that $t_{\max}$ approaches $\frac{1}{3}$ hence the former function approaches

$$\frac{1}{2}2^{1/2}\frac{4}{3}\left(\frac{4}{9}\right)^{-1/2} = \frac{4}{3}\left(\frac{9}{8}\right)^{1/2} = \sqrt{2}$$

while the latter equals $2(1/6)^{1/2} = \sqrt{2/3}$ hence is smaller. This also implies $\alpha^* \in]0,1/2[$. *Q.E.D.*

We will show the pattern of a spider separately for $\alpha \in (0,\alpha^*)$ and $\alpha \in \left(\alpha^*,\frac{1}{2}\right)$. The value of α^* can be numerically calculated to be ≈ 0.3.

D.3.4 Very strong concavity $\alpha \in]0, \alpha^*]$

Lemma 24 *If $\alpha \in]0, 1/3[$ then $g(c,t)$ has exactly one local min point with respect to c.*

Proof: We have two cases to consider. Case i) is where $c_b > 1-t$ (c_b is defined in Lemma 17). Lemma 17 then states that $g(c,\cdot)$ has a U-shape, that is, exactly one local min point. Case ii) is where $c_b \leq 1-t$. Then (by the proof of Lemma 17) $g(c,\cdot)$ has one local min point at $c_b \leq 1-t$, and $\frac{\partial g(c,t)}{\partial c}|_{c \overset{-}{\to} 1-t} \geq 0$. Continuity of $\frac{\partial g(c,\cdot)}{\partial c}$at $c = 1-t$ (see Lemma 13) then implies $\frac{\partial g(c,\cdot)}{\partial c}|_{c \overset{+}{\to} 1-t} \geq 0$. Lemma 14 states that $\frac{\partial g(c,\cdot)}{\partial c}|_{c \overset{-}{\to} 1+t} \geq 0$. Finally, from Lemma 18 we know that, when $\alpha < 1/3$, $g(c,\cdot)$ does not have a local max point in the range $c \in [1-t, 1+t]$, hence it must be that $\frac{\partial g(c,t)}{\partial c} \geq 0$ in this range. Thus, in case ii) there is one local min point. *Q.E.D.*

Lemma 25 *Type t has an inner solution only if $h(t) > \frac{W}{2^{1-\alpha}\alpha}$.*

Proof: We (from Lemma 24) know that $g(c,t)$ has a U-shape for all types (because $\alpha^* < \frac{1}{3}$). Remembering (from Lemma 6) that

$$\frac{dL}{dc} > 0 \Leftrightarrow g(c,t) > \frac{W}{2^{1-\alpha}\alpha},$$

and that (by equation 19)

$$\lim_{c \to 0} \frac{dL}{dc} = +\infty,$$

we get that in order for type t to have an inner solution it must be that $h(t) = g(1+t,t) > \frac{W}{2^{1-\alpha}\alpha}$. *Q.E.D.*

We will now show the spider for the range of W that satisfy the following conditions:

(i) $t = 0$ prefers $c = 0$ over $c = 1+t$:

$$\begin{aligned} L(0,0) &= \frac{1}{2}W < \left(\frac{1}{2}\left[t^2 + \frac{1}{3}\right]\right)^\alpha = L(t+1,0) \\ W &< 2(1/6)^\alpha \end{aligned}$$

(ii) $t = 1$ prefers $c = 1+t$ over $c = 0$:

$$L(c) = z^\alpha = \left(\frac{1}{2}\left[t^2 + \frac{1}{3}\right]\right)^\alpha = (2/3)^\alpha < W$$

(iii) $\forall t \in [0,1]$, $\frac{dL}{dc}|_{c=1+t} < 0$. From Lemma 6 this is equivalent to:

$$\begin{aligned}
\forall t \quad \in \quad & [0,1]\,, \quad h\left(t\right) < \frac{W}{2^{1-\alpha}\alpha} \\
\Rightarrow \quad & W > \alpha 2^{1-\alpha} * \max_{t}\left\{h\left(t\right)\right\} \\
\Rightarrow \quad & \{\text{by Lemma 19}\} W > \alpha 2^{1-\alpha} h(t_{\max}) \\
\Rightarrow \quad & W > \alpha 2^{1-\alpha}(1+t_{\max})\left(\frac{1}{3}+t_{\max}^{2}\right)^{\alpha-1}
\end{aligned}$$

Lemma 26 *For any $\alpha \in \left]0,\alpha^*\right]$ there exists a range of W that satisfy conditions (i)-(iii)*

Proof: The range of W that satisfy conditions (i)-(iii) is the intersection of the range of W that satisfy conditions (i) and (ii) and the range of W that satisfy conditions (i) and (iii) so it is enough to show that, for any $\alpha \in \left]0,\alpha^*\right]$, none of these ranges is empty. Starting with conditions (i) and (ii), we note that

$$\frac{(2/3)^{\alpha}}{2\left(1/6\right)^{\alpha}} = \frac{4^{\alpha}}{2} < 1 \;\; \forall \alpha < \frac{1}{2},$$

hence the range of W that satisfy conditions (i) and (ii) is not empty. Next, the fact that the range of W that satisfy conditions (i) and (iii) is not empty when $\alpha < \alpha^*$ follows directly from Lemma 23. *Q.E.D.*

Lemma 27 *If $\frac{dL}{dc}|_{c=1+t} < 0$ for type $t=0$ then $\Delta L \equiv L(0) - L(1+t)$ increases in t.*

Proof: From (12) and (9) we get

$$\begin{aligned}
\Delta L &= L(0) - L(1+t) = W\left[1-P_m\right] - \left(\frac{1}{2}\left[t^2+\frac{1}{3}\right]\right)^{\alpha} \\
\{\text{by } 6\} &= W\left(1 - 2F(t)\left[1-F(t)\right]\right) - \left(\frac{1}{2}\left[t^2+\frac{1}{3}\right]\right)^{\alpha} \\
\{\text{by } t \sim U\left(-1,1\right)\} &= W\left[1-\frac{1}{2}\left(1+t\right)\left(1-t\right)\right] - \left(\frac{1}{2}\left[t^2+\frac{1}{3}\right]\right)^{\alpha} \\
&= \frac{1}{2}W\left(1+t^2\right) - \left(\frac{1}{2}\left[t^2+\frac{1}{3}\right]\right)^{\alpha}
\end{aligned}$$

$$\frac{d}{dt}\Delta L = \left[W - \alpha\left(\frac{1}{2}\left[t^2+\frac{1}{3}\right]\right)^{\alpha-1}\right] t$$

The term $\left[W - \alpha\left(\frac{1}{2}\left[t^2+\frac{1}{3}\right]\right)^{\alpha-1}\right]$ increases in t (since $\alpha < 1$) and at $t=0$ it equals $W - 6\alpha\left(\frac{1}{6}\right)^{\alpha}$. If $\frac{dL}{dc}|_{c=1+t} < 0$ for type $t=0$ then

$$\begin{aligned}
\frac{dL}{dc}|_{c=1+t=1} \quad &< \quad 0 \Rightarrow \{\text{Lemma 6}\} \Rightarrow \\
2^{1-\alpha}g\left(1,t\right) \quad &< \quad \frac{W}{\alpha} \Rightarrow W > \alpha\left(\frac{1}{6}\right)^{\alpha-1} = 6\alpha\left(\frac{1}{6}\right)^{\alpha}
\end{aligned}$$

Thus, $\left[W - \alpha\left(\frac{1}{2}\left[t^2+\frac{1}{3}\right]\right)^{\alpha-1}\right] > 0 \;\; \forall t \in [0,1]$ implying that ΔL increases in t. *Q.E.D.*

Lemma 28 *Condition (iii) implies that ΔL increases in t.*

Proof: Follows directly from applying condition (iii) to $t = 0$ and using Lemma 27. *Q.E.D.*

Proposition 9 *For any $\alpha \in \left]0, \alpha^*\right]$ there is a spider for any W at the range of values that satisfy conditions (i)-(iii).*

Proof: Lemma 25 and Condition (iii) imply that every type has a corner solution, either at $c = 0$ or at $c = 1 + t$. Then, conditions (i) and (ii) and Lemma 28 imply that there exists a unique switching point at the range $[0, 1]$ such that types below it choose $c = 0$ while types above it choose $c = 1 + t$. Finally, Lemma 3 implies that dissent rate is increasing in t when t is below the switching point and then it abruptly falls to 0 at the switching point and stays there. *Q.E.D.*

D.3.5 Mildly strong concavity $\alpha \in \left]\alpha^*, \frac{1}{2}\right[$

Lemma 29 *Suppose that $g(c,t)$ has a local max point at some $c \in [1-t, 1+t]$. Then c is strictly smaller than* 1.

Proof: A necessary condition for $g(c,t)$ to have a local max point at c is that $\frac{\partial g(c,t)}{\partial c}$ has a min point at some $c' > c$ (because between $1-t$ and the max point $g(c,t)$ is always increasing while after the max point it starts decreasing). The value of c at the min point of $\frac{\partial g(c,t)}{\partial c}$ at the range $[1-t, 1+t]$ is given by equation (33). We will show that this value is smaller than 1. To show that, we need to show

$$\begin{aligned} \frac{1-2\alpha}{2-3\alpha}(1+t) &< 1 \Leftrightarrow ... \\ t &< \frac{1-\alpha}{1-2\alpha}. \end{aligned}$$

Lemma 18 implies that $g(c,t)$ has a local max point only if $t \leq \frac{1}{4-6\alpha}$. Noting that

$$\begin{aligned} \frac{1}{4-6\alpha} &< \frac{1-\alpha}{1-2\alpha} \Leftrightarrow ... \\ &\Leftrightarrow 6\alpha^2 - 8\alpha + 3 > 0, \end{aligned}$$

which can be verified to hold for all α, we indeed get that $t \leq \frac{1}{4-6\alpha}$ implies that $t < \frac{1-\alpha}{1-2\alpha}$, hence the local max point is strictly smaller than 1. *Q.E.D.*

Lemma 30 *Suppose that $g(c,t)$ has a local max point at some $c_m \in [1-t, 1+t]$. Then the value of $g(c,t)$ at the local max point is strictly smaller than $g(1+t, t)$.*

Proof: Lemma 18 implies that if $g(c,t)$ has a local max point for some $c_m \in [1-t, 1+t]$ then $\alpha \in \left[\frac{1}{3}, \frac{1}{2}\right]$ and $t \in \left[\frac{1-\alpha}{3-5\alpha}, \frac{1}{4-6\alpha}\right]$. So we focus on these values in the remainder of the proof. Lemma 29 further implies that $c_m < 1$. We will prove that the value of $g(c_m, t) < g(1, t)$. This is sufficient because, if this holds, then together with the fact that $c_m < 1$ it implies that at the range $c > 1$ the function $g(c,t)$ must be increasing[40] and so $g(1,t) < g(1+t,t)$. Focusing on the range $1 - t \le c < 1 + t$, where (by equation (20))

$$g(c,t) = c\left[\left(-\frac{1}{3}c + \frac{1}{2}t + \frac{1}{2}\right)c^2 + \frac{1}{6}(1-t)^3\right]^{\alpha-1},$$

let

$$X(c;\alpha,t) \equiv [g(c)]^{\frac{1}{\alpha-1}} = c^{\frac{1}{\alpha-1}}\left[-\frac{1}{3}c^3 + \frac{1}{2}(t+1)c^2 + \frac{1}{6}(1-t)^3\right] \tag{34}$$

be defined in the domain (α,t) which corresponds to $\alpha \in \left[\frac{1}{3},\frac{1}{2}\right]$ and $t \in \left[\frac{1-\alpha}{3-5\alpha},\frac{1}{4-6\alpha}\right]$. Noting that

$$t = \frac{1}{4-6\alpha} \Rightarrow \alpha = \frac{2}{3} - \frac{1}{6t}$$

$$t = \frac{1-\alpha}{3-5\alpha} \Rightarrow t = \frac{2/5}{3-5\alpha} + \frac{1}{5} \Rightarrow \alpha = \frac{3}{5} - \frac{2}{25t-5},$$

the domain can alternatively be described as $t \in \left[\frac{1}{2},1\right]$ and $\alpha \in [\alpha_1(t), \alpha_2(t)]$, where $\alpha_1(t) = \frac{2}{3} - \frac{1}{6t}$ and $\alpha_2(t) = \frac{3}{5} - \frac{2}{25t-5}$. We will show that $X(c;\alpha,t)$ has an inner min point exactly where $g(c,t)$ has an inner max point, and that $X(c;\alpha,t)$ at this min point is larger than $X(1;\alpha,t)$, which is equivalent to showing that the value of $g(c,t)$ at the inner max point of $g(c,t)$ is smaller than $g(1,t)$.[41] Holding t constant, denoting the value of c at the inner min point of $X(c;\alpha,t)$ by c_0 (if it exists) and exploiting the fact that $\frac{\partial X}{\partial c} = 0$ at this min point, we get

$$\frac{dX(c;\alpha,t)}{d\alpha}\Big|_{\substack{c=c_0\\ t=const}} = \frac{\partial X}{\partial c}\frac{\partial c}{\partial \alpha} + \frac{\partial X}{\partial \alpha} = \frac{\partial X}{\partial \alpha} > 0,$$

implying that $X(c_0;\alpha,t)$ for given t reaches its minimum at $\alpha = \alpha_1(t)$. Hence, given that $X(1;\alpha,t)$ is independent of α, it is necessary and sufficient to show that $X(c_0;\alpha,t) > X(1;\alpha,t)$ for $\alpha = \alpha_1(t)$.

The partial derivative $\frac{\partial X(c;\alpha,t)}{\partial c}$ is

$$\frac{\partial X(c;\alpha,t)}{\partial c} = -\frac{c^{-\frac{2-\alpha}{1-\alpha}}}{1-\alpha}\left\{c^2\left[\left(\frac{2}{3}-\alpha\right)c + \left(\alpha - \frac{1}{2}\right)(1+t)\right] + \frac{1}{6}(1-t)^3\right\},$$

where the expression in the curly brackets is the one determining the sign of $\frac{\partial g(c,t)}{\partial c}$ (see equation (30)), hence the sign of $\frac{\partial X(c;\alpha,t)}{\partial c}$ is opposite to that of $\frac{\partial g(c,t)}{\partial c}$, and so indeed $X(c;\alpha,t)$ has an inner min point exactly where $g(c,t)$ has an inner max point. Substituting $\alpha = \alpha_1(t)$ into the expression for $\frac{\partial X(c;\alpha,t)}{\partial c}$ and equating

[40] To see why it must be increasing, note that g has at most one inner local max point (this follows from Lemma 15 since when there is one local min point of $\partial g/\partial c$ there can be at most two inner extrema where only one is a max point). Thus, as c_m is the only inner max point of g and $g(c_m,t) < g(1,t)$ then g must be increasing at $c = 1$ and beyond.

[41] Since in the expression for X the power $\frac{1}{\alpha-1} < 0$.

to zero to find a local min point we get

$$\begin{aligned}
\left(\frac{2}{3}-\frac{2}{3}+\frac{1}{6t}\right)c^3+\left(\frac{2}{3}-\frac{1}{6t}-\frac{1}{2}\right)(1+t)\,c^2+\frac{1}{6}(1-t)^3 &= 0 \Leftrightarrow ... \\
[c-(1-t)]\left[\frac{1}{t}c^2-(1-t)\,c-(1-t)^2\right] &= 0,
\end{aligned}$$

whose roots in increasing order are

$$c_{1,2,3,}=\frac{1}{2}t\,(1-t)\left[1-\sqrt{1+\frac{4}{t}}\right],1-t,\frac{1}{2}t\,(1-t)\left[1+\sqrt{1+\frac{4}{t}}\right].$$

It can be verified that at $c_2=1-t$ the sign of $\frac{\partial g(c,t)}{\partial c}$ turns from positive to negative,[42] hence this is a local max point of $g(c,t)$ and a local min point of $X(c;\alpha,t)$. Let us now substitute $c=c_2$ back into $X(c;\alpha,t)$:

$$X(c=1-t;\alpha_1(t),t)=\frac{1}{3}(1-t)^{\frac{2-2t}{2t+1}}(2t+1)\,.$$

We need to show that this is larger than

$$X(c=1;\alpha_1(t),t)=-\frac{1}{3}+\frac{1}{2}(t+1)+\frac{1}{6}(1-t)^3=\frac{2+3t^2-t^3}{6}.$$

Let

$$\begin{aligned}
Y(t) &\equiv 3\left[X(1-t;\alpha_1(t),t)-X(1;\alpha_1(t),t)\right] \\
&= (1-t)^{\frac{2-2t}{2t+1}}(2t+1)-\frac{2+3t^2-t^3}{2}.
\end{aligned}$$

Then the proof boils down to showing that $Y(t)>0\ \ \forall t\in\left[\frac{1}{2},1\right]$. Define

$$y\equiv-\frac{2-2t}{2t+1}\ln(1-t)$$

Then $y\geq 0$ for $t\in\left[\frac{1}{2},1\right]$ and the Taylor-Lagrange formula for e^{-y} implies that $e^{-y}\geq 1-y$. Hence

$$\begin{aligned}
Y(t) &\geq Y_1(t)\equiv(2t+1)\left[1+\frac{2-2t}{2t+1}\ln(1-t)\right]-\frac{2+3t^2-t^3}{2} \\
&= 2t+1+2(1-t)\ln(1-t)-\frac{2+3t^2-t^3}{2}.
\end{aligned}$$

[42] Plugging $c=1-t$ and the value of $\alpha_1(t)$ into equation (31), which is the derivative of the expression determining the sign of $\frac{\partial g(c,t)}{\partial c}$, yields $\frac{1}{6t}(1-t)^2[1-2t]$, which is negative for $t\in\left[\frac{1}{2},1\right]$, implying that the sign of $\frac{\partial g(c,t)}{\partial c}$ turns from positive to negative at $c=1-t$.

Let us now investigate $Y_1(t)$. We have

$$\begin{aligned} Y_1'(t) &= -2\left[1+\ln(1-t)\right] \\ Y_1''(t) &= \frac{2}{1-t} > 0, \end{aligned}$$

and it can be verified that $Y_1(1/2) < 0$ while $Y_1(5/9) > 0$, which implies that $Y_1(t) > 0 \;\; \forall t \geq 5/9$, implying also that $Y(t) > 0 \;\; \forall t \geq 5/9$. So the only thing left to show is that $Y(t) > 0 \;\; \forall t \in \left[\frac{1}{2}, \frac{5}{9}\right]$. In the interval $t \in \left[\frac{1}{2}, \frac{5}{9}\right]$ we have

$$Y(t) \geq Y_2(t) \equiv (2t+1)(1-t)^{1/2} - \frac{2+3t^2-t^3}{2}$$

Changing variables as follows: $q \equiv \sqrt{1-t}$, we get

$$\begin{aligned} Y_2(q) &\equiv q\left[1+2\left(1-q^2\right)\right] - \frac{2+3\left(1-q^2\right)^2-\left(1-q^2\right)^3}{2} \\ &= -2+3q+\frac{3}{2}q^2-2q^3+q^6 \end{aligned}$$

for $q \in \left[2/3, \sqrt{1/2}\right]$. Now, we have $Y_2(2/3) > 0$ and $Y_2'(2/3) > 0$, so it is sufficient to show that $Y_2''(q) > 0$. Indeed,

$$Y_2''(q) = 3 - 12q + 30q^4 > 0 \text{ for } q \in \left[2/3, \sqrt{1/2}\right],$$

because at the range $q \in \left[2/3, \sqrt{1/2}\right]$ we have

$$Y_2'''(q) = 12\left(10q^3-1\right) \geq 12\left(10\left(\frac{2}{3}\right)^3 - 1\right) > 0$$

and

$$Y_2''\left(\frac{2}{3}\right) = 3 - 12\left(\frac{2}{3}\right) + 30\left(\frac{2}{3}\right)^4 > 0.$$

We have thus showed that $Y_2(q) > 0$ for $q \in \left[2/3, \sqrt{1/2}\right]$ and hence $Y(q) > 0$ for $q \in \left[2/3, \sqrt{1/2}\right]$. *Q.E.D.*

LEMMA 31 *For any $\alpha \in \left]\alpha^*, \frac{1}{2}\right[$ and any $t \in [0,1]$, the loss function $L(c)$ does not have a local min point if $W \geq \alpha 2^{1-\alpha} h(t)$*

PROOF: Lemma 6 implies that the sign of $\frac{dL}{dc}$ is determined by the sign of $2^{1-\alpha}\alpha g(c,t) - W$. Recalling that $\lim_{c\to 0}\frac{dL}{dc} = \infty$, this implies that $L(c)$ may have a local min point only if there are at least two different values of $c \in \left]0, 1+t\right[$ for which $2^{1-\alpha}\alpha g(c,t) = W$. Turning now the focus to $g(c,t)$, the fact that $\lim_{c\to 0}\frac{dL}{dc} = \infty$ implies further that $2^{1-\alpha}\alpha g(0,t) > W$, while it is given that $2^{1-\alpha}\alpha h(t) \leq W$. We thus get that a necessary (though insufficient) condition for $L(c)$ to have a local min point is for $g(c,t)$ to have a local max point in which its value exceeds that of $h(t)$. This condition does not hold at the range $c \in \left]0, 1-t\right[$ because $g(c,t)$ does not have a local max point there (see Lemma 16), and it also does not hold at the range $c \in [1-t, 1+t]$, because Lemma 30 states that the value of $g(c,t)$ at the local max point, if it exists, is strictly smaller than $h(t)$. *Q.E.D.*

We will show the spider for the range of W that satisfy the following conditions (where the inequalities in I-III follow from Lemma 31):

(I) There exist types with a potential inner solution (i.e. local min point):

$$W < \alpha 2^{1-\alpha} h(t_{\max})$$

(II) The type $t = 0.34$ does not have an inner solution:

$$\alpha 2^{1-\alpha} h(0.34) \leq W$$

(III) The type $t = 0$ does not have an inner solution:

$$\alpha 2^{1-\alpha} h(0) < W$$

(IV) The type $t = 0$ strictly prefers $c = 1 + t$ over $c = 0$ which implies (by plugging the corner options into equation (9)):

$$\begin{aligned} \frac{1}{2}W &> \left(\frac{1}{2}\left[t^2 + \frac{1}{3}\right]\right)^{\alpha} \\ W &> 2(1/6)^{\alpha} \end{aligned}$$

(V) There is no type t for whom $g(c,t)$ has a local max point in which $\alpha 2^{1-\alpha} g(c,t) > W$: Let

$$W_c \equiv \alpha 2^{1-\alpha} \max_t \{g(c,t) | c \text{ is local max of } g(c,t)\}.$$

Then we require $W > W_c$.

Lemma 32 *For any $\alpha \in \left]\alpha^*, \frac{1}{2}\right[$ there exist a range of W that satisfy conditions (I)-(V)*

Proof: Condition (I) sets an upper bound on W while the other four conditions set lower bounds, hence we need to show that the intersections of condition (I) and each of the other conditions are not empty. Conditions (I) and (II): Lemma 21 shows that $t_{\max}$ is weakly smaller than $1/3$, hence $h(t_{\max}) > h(0.34)$. Conditions (I) and (III): Lemma 19 establishes that $t_{\max} > 0$, hence $h(t_{\max}) > h(0)$. Conditions (I) and (IV): Lemma 23 implies that $2(1/6)^{\alpha} < \alpha 2^{1-\alpha} h(t_{\max})$ at the range $\alpha \in \left]\alpha^*, \frac{1}{2}\right[$. Conditions (I) and (V):

Let t_c be a type for whom $W_c = \alpha 2^{1-\alpha} g(c,t)$ at the local max point of $g(c,t)$. Then Lemma 16 implies that $g(c,t_c)$ reaches that local max point at some $c \in [1-t, 1+t]$, and from Lemma 30 we get that the value of $g(c,t_c)$ at this local max point is strictly smaller than $h(t_c) \leq h(t_{\max})$ hence $W_c < \alpha 2^{1-\alpha} h(t_{\max})$. *Q.E.D.*

LEMMA 33 *Let $\alpha \in \left]\alpha^*, \frac{1}{2}\right[$ and let W satisfy conditions (I-V). Then there exists a (non-singleton) neighborhood of $t_{\max}$ s.t. types at this neighborhood are choosing an inner solution while any other type is choosing $c = 1+t$.*

PROOF: Lemma 31 implies that any t for whom $W \geq \alpha 2^{1-\alpha} g(1+t,t)$ has a corner solution to the minimization problem, while condition (V) implies that any t for whom $W < \alpha 2^{1-\alpha} h(t)$ has at most one local inner min point.[43] Condition (IV) states that type $t = 0$ strictly prefers $c = 1+t$ over $c = 0$, and condition (III) states that $\alpha 2^{1-\alpha} h(0) < W$, hence $\frac{dL}{dc}|_{c=1+t} < 0$ for $t = 0$ (Lemma 6), which by Lemma 27 implies that ΔL increases in t, hence all types prefer $c = 1+t$ over $c = 0$. Thus any type with a corner solution chooses $c = 1+t$, while all the types for whom $W < \alpha 2^{1-\alpha} h(t)$ (which by condition (I) contain more than a singleton) choose their unique local min point as a solution because Lemma 6 implies that $\frac{dL}{dc}|_{c=1+t} > 0$ for these types hence $c = 1+t$ cannot be their global min point. Finally, Lemma 19 implies that $h(t)$ has a hill shape with a peak at $t_{\max}$, hence the types for whom $W < \alpha 2^{1-\alpha} h(t)$ form a neighborhood around $t_{\max}$. *Q.E.D.*

LEMMA 34 *Let $\alpha \in \left]\alpha^*, \frac{1}{2}\right[$ and suppose condition (V) holds and some type t' has an inner solution $c(t') \leq 1-t'$. Then any $t < t'$ has an inner solution $c(t') \leq 1-t'$ too.*

PROOF: Lemma 6 implies that, in inner solutions, $W = \alpha 2^{1-\alpha} g(c(t'), t')$. For any $t < t'$ we have (by equation (20)) $g(c,t) = g(c,t')$ at the range $c \in [0, 1-t']$, hence $W = \alpha 2^{1-\alpha} g(c(t'), t)$ where $c(t') \leq 1-t'$ implies that $c(t') < 1-t$. In other words, t has a local min point of L at $c(t')$ as well. Condition (V) ensures that this local min point is unique.[44] Furthermore, at an inner solution, $\frac{dL}{dc}$ switches sign from negative to positive, hence $g(c,t)$ is increasing (by Lemma 6). Condition (V) ensures that this increasing part of $g(c,t)$ continues until $c = 1+t$, implying that $W < \alpha 2^{1-\alpha} h(t)$ hence $\frac{dL}{dc}|_{c=1+t} > 0$ and so the local min point is also the global min point. *Q.E.D.*

PROPOSITION 10 *For any $\alpha \in \left]\alpha^*, \frac{1}{2}\right[$ there is a spider for any W at the range of values that satisfy conditions (I)-(IV)*

PROOF: Lemma 32 established that the conditions imply a non-empty set of W. Lemma 33 established that there exists a non-singleton neighborhood of $t_{\max}$ where types have inner solutions, while types outside

[43]In inner solutions $\alpha 2^{1-\alpha} g(c,t) = W$ and g is increasing (so that $\alpha 2^{1-\alpha} g(c,t)$ crosses the W-line from below). $g(c,t)$ can be either U-shaped or double-U-shaped. In the latter case $\alpha 2^{1-\alpha} g(c,t)$ may cross the W-line from below twice. Condition (V) ensures that such a crossing from below happens at most once.

[44]See proof of Lemma 33 for explanation.

that neighborhood choose $c = 1 + t$, thus do not dissent. Condition (III) implies that type $t = 0$ is among these latter types, and condition (II) implies the same for all types with $t \geq 0.34$. Thus we know that the neighborhood of $t_{\max} \subset [0, 1]$. Since $P = 0$ for all types who do not dissent, to get the pattern of a spider we need to verify that at the range of inner solutions the probability of dissent is either monotonic or first increases and then decreases. First note that condition (III) implies that type $t = 0$ has no inner solution, and so condition (V) and Lemma 34 imply that any type t in the range of types with inner solutions has a solution at the range $c \in [1 - t, 1 + t]$. In this range, we know from Lemma 4 that the probability of dissent decreases if and only if $\frac{dc}{dt} > 1$. Equation (25) gives us the expression of $\frac{dc}{dt}$,

$$\frac{dc}{dt} = c\frac{(1-\alpha)\left[-\frac{1}{2}t^2 + \frac{1}{2}c^2 + t - \frac{1}{2}\right]}{\frac{2}{3}c^3 - \frac{1}{6}t^3 - \frac{1}{2}tc^2 + \frac{1}{2}t^2 - \frac{1}{2}c^2 - \frac{1}{2}t + \frac{1}{6} + \alpha\left(tc^2 + c^2 - c^3\right)},$$

where equation (26) implies that the numerator is positive. The denominator is the expression that determines the sign of $\frac{\partial g(c,t)}{\partial c}$ (by Lemma 7) at the range $c \in [1 - t, 1 + t]$, hence is positive too given that at an inner solution $\frac{dL}{dc}$ switches signs from negative to positive, which by Lemma 6 implies that $g(c, t)$ must be increasing. It thus follows that $\frac{dc}{dt} > 0$. Furthermore

$$\begin{aligned} \frac{dc}{dt} &> 1 \Leftrightarrow \{\text{as established in Lemma 12}\} \\ G &= c^2\left[(3\alpha - 1)c - (2\alpha - 1)3(1+t)\right] - (1-t)^2\left[(1-t) + (1-\alpha)3c\right] > 0. \end{aligned}$$

To show the pattern of the spider it is thus sufficient to show that

$$\frac{dG}{dt} = \frac{\partial G}{\partial t} + \frac{\partial G}{\partial c}\frac{dc}{dt} \geq 0,$$

which, given that $\frac{dc}{dt} > 0$, holds if both partial derivatives, $\frac{\partial G}{\partial t}$ and $\frac{\partial G}{\partial c}$, are positive. Indeed,

$$\frac{\partial G}{\partial t} = 3\left[(1-t)^2 + 2(1-\alpha)c(1-t) - (2\alpha - 1)c^2\right] > 0$$

for $\alpha \leq 1/2$.

$$\frac{\partial G}{\partial c} = 3\left[(3\alpha - 1)c^2 - 2(2\alpha - 1)(1+t)c - (1-t)^2(1-\alpha)\right]$$

Fixing t and analyzing the behavior of $\frac{\partial G}{\partial c}$ as a function of c, we get

$$\begin{aligned} \frac{\partial^2 G}{\partial c^2} &= 6\left[(3\alpha - 1)c - (2\alpha - 1)(1+t)\right] = 0 \\ &\Rightarrow c = \frac{(2\alpha - 1)(1+t)}{(3\alpha - 1)} \end{aligned}$$

If $\alpha \in [1/3, 1/2]$ then $\frac{\partial G}{\partial c}$ is U-shaped with min point at $c = \frac{(2\alpha-1)(1+t)}{(3\alpha-1)} < 0$ (i.e., outside the permissible range), implying that at the range $c \in [1-t, 1+t]$ it reaches its min at $c = 1-t$ where it equals

$$\begin{aligned}\frac{\partial G}{\partial c} &= 3\left[(3\alpha-1)(1-t)^2 - 2(2\alpha-1)(1+t)(1-t) - (1-t)^2(1-\alpha)\right] \\ &= -12(2\alpha-1)(1-t)t > 0\end{aligned}$$

Alternatively, if $\alpha < 1/3$, then $\frac{\partial G}{\partial c}$ is hill-shaped and

$$c = \frac{1-2\alpha}{1-3\alpha}(1+t) > 1+t$$

is a max point, implying again that at the range $c \in [1-t, 1+t]$ the function $\frac{\partial G}{\partial c}$ reaches its min at $c = 1-t$ where it was just shown to be positive for any $\alpha < 1/2$. Thus, for any $\alpha \in \left]\alpha^*, \frac{1}{2}\right[$, G is increasing, implying that at the range of inner solutions the probability of dissent is either monotonic or first increases and then decreases, as required for getting the pattern of a spider. Overall, we get a spider of the following kind: when t goes from 0 to 1 the probability of dissent is first 0, then jumps to some strictly positive probability, then it either increases or decreases, or first increases and then decreases, and finally the probability of dissent decreases abruptly to 0 and stays there. *Q.E.D.*

D.4 The special case of $\alpha = \frac{1}{2}$

PROPOSITION 11 *Suppose $\alpha = \frac{1}{2}$. Then there exists a non-empty set of W such that there is a spider pattern.*

PROOF: We will prove a spider pattern of the following kind exists for some W: when t goes from 0 to 1 the probability of dissent is first 0, then jumps to some strictly positive probability, then it either increases or decreases, or first increases and then decreases, and finally the probability of dissent decreases abruptly to 0 and stays there. By equation (10) we have

$$\frac{dL}{dc} = 2M\left[c\frac{1}{2}z^{-1/2} - W\right],$$

and by 19

$$\lim_{c \to 0}\frac{dL}{dc} = \frac{\sqrt{2}}{2} - W.$$

Next, note that by (20)

$$g(c,t) = \begin{cases} \left[1 - \frac{2}{3}c\right]^{-1/2} > 0 \text{ if } 0 < c < 1-t \\ c\left[\left(-\frac{1}{3}c + \frac{1}{2}t + \frac{1}{2}\right)c^2 + \frac{1}{6}(1-t)^3\right]^{-1/2} \text{ if } 1-t \le c < 1+t \end{cases}$$

$$\frac{\partial g(c,t)}{\partial c}=\begin{cases}\frac{1}{3}\left[1-\frac{2}{3}c\right]^{-3/2} \text{ if } 0<c<1-t \\ \frac{1}{6}\left[c^3+(1-t)^3\right]\left[\frac{1}{6}(1-t)^3-\frac{1}{3}c^3+\frac{1}{2}tc^2+\frac{1}{2}c^2\right]^{-3/2} \text{ if } 1-t\leq c<1+t\end{cases}>0$$

We know the expression is positive since $\left[\frac{1}{6}(1-t)^3-\frac{1}{3}c^3+\frac{1}{2}tc^2+\frac{1}{2}c^2\right]=2z\geq 0$ (since z by definition (8) is a sum of two positive integrals). So $g(c,t)$ increases everywhere. If $W<\frac{\sqrt{2}}{2}$ so that $\lim_{c\to 0}\frac{dL}{dc}>0$, then $L(c)$ is always increasing in c (by Lemma 6 and since $\frac{\partial g(c,t)}{\partial c}>0$) and so all types choose $c=0$ hence dissent rate is monotonically increasing – no spider. If $W>\frac{\sqrt{2}}{2}$ so that $\lim_{c\to 0}\frac{dL}{dc}<0$, then $L(c)$ is at least initially decreasing and so all types have either a unique (by Lemma 6 and since $\frac{\partial g(c,t)}{\partial c}>0$) inner solution or a corner solution at $c=1+t$. Moreover, since $g(c,t)$ increases everywhere, we know from Lemma 6 that type t has an inner solution if and only if

$$h(t)=g(1+t,t)>\frac{W}{2^{1-\alpha}\alpha}=\sqrt{2}W$$

Using $h(t)=g(1+t,t)=(1+t)\left(\frac{1}{3}+t^2\right)^{-1/2}$ (see Lemma 9), we get

$$\begin{aligned}\frac{dh}{dt} &= \left(\frac{1}{3}+t^2\right)^{-1/2}-t(1+t)\left(\frac{1}{3}+t^2\right)^{-3/2}=...\\ &= \left(\frac{1}{3}-t\right)\left(\frac{1}{3}+t^2\right)^{-3/2}.\end{aligned}$$

So $h(t)$ is hill-shaped with a peak at $t_{\max}=1/3$, where it equals

$$\frac{4}{3}\left(\frac{4}{9}\right)^{-1/2}=2.$$

Thus, by setting $W<\sqrt{2}$ we can guarantee that at least types close to $t_{\max}$ have inner solutions. The hill-shape of $h(t)$ further implies that we can set W to be strictly greater than

$$\max\left\{\frac{\sqrt{2}}{2}h(0),\frac{\sqrt{2}}{2}h(1)\right\}=\sqrt{3/2}$$

yet strictly smaller than $\sqrt{2}$, so that, by corollary 1, types close to 0 or to 1 choose $c=1+t$ while there exists a non-empty range of types in-between with inner solutions. To get the pattern of a spider we need to verify that at the range of inner solutions the probability of dissent is either monotonic or first increases and then decreases. As W has been set such that type $t=0$ has no inner solution, Lemma 8 implies that any type t in the range of types with inner solutions has a solution at the range $c\in[1-t,1+t]$. In this range, we know from Lemma 4 that the probability of dissent decreases if and only if $\frac{dc}{dt}>1$. At the range

$c \in [1-t, 1+t]$ (setting $g(c,t) = 0$ for inner solutions and applying the implicit function theorem) we have

$$\begin{aligned}
\frac{dc}{dt} &= -\frac{\frac{d}{dt}c\left[-\frac{1}{3}c^3 - \frac{1}{6}t^3 + \frac{1}{2}tc^2 + \frac{1}{2}t^2 + \frac{1}{2}c^2 - \frac{1}{2}t + \frac{1}{6}\right]^{-1/2}}{\frac{d}{dc}c\left[-\frac{1}{3}c^3 - \frac{1}{6}t^3 + \frac{1}{2}tc^2 + \frac{1}{2}t^2 + \frac{1}{2}c^2 - \frac{1}{2}t + \frac{1}{6}\right]^{-1/2}} \\
&= -c\frac{-\frac{1}{2}\left[-\frac{1}{2}t^2 + \frac{1}{2}c^2 + t - \frac{1}{2}\right]}{\frac{2}{3}c^3 - \frac{1}{6}t^3 - \frac{1}{2}tc^2 + \frac{1}{2}t^2 - \frac{1}{2}c^2 - \frac{1}{2}t + \frac{1}{6} + \frac{1}{2}\left(tc^2 + c^2 - c^3\right)}.
\end{aligned}$$

This expression was shown before to be positive (from equation (26) we know the numerator is positive, and the denominator is positive (by Lemma 7) because it has the sign of $\frac{\partial g(c,t)}{\partial c}$ which is positive), hence $\frac{dc}{dt} \geq 0$, implying the possibility of a spider.

The probability of dissent $(P(t))$ decreases in the range of types with inner solutions iff $\frac{dc}{dt} \geq 1$ (see Lemma 4).

$$\begin{aligned}
\frac{dc}{dt} &= c\frac{\frac{1}{2}\left[-\frac{1}{2}t^2 + \frac{1}{2}c^2 + t - \frac{1}{2}\right]}{\frac{2}{3}c^3 - \frac{1}{6}t^3 - \frac{1}{2}tc^2 + \frac{1}{2}t^2 - \frac{1}{2}c^2 - \frac{1}{2}t + \frac{1}{6} + \frac{1}{2}\left(tc^2 + c^2 - c^3\right)} \geq 1 \\
&\Leftrightarrow \quad c^3 \geq (1-t)^2\left[2(1-t) + 3c\right]
\end{aligned}$$

Let

$$H(t,c) \equiv c^3 - (1-t)^2\left[2(1-t) + 3c\right].$$

Then

$$\frac{\partial H(t,c)}{\partial t} = 6(1-t)\left[(1-t) + c\right] \geq 0$$

and

$$\frac{\partial H(t,c)}{\partial c} = 3\left[c^2 - (1-t)^2\right] \geq 0,$$

(since we established earlier in the proof that $c > 1-t$ in inner solutions) and so

$$\frac{dH}{dt} = \frac{\partial H}{\partial t} + \frac{\partial H}{\partial c}\frac{dc}{dt} \geq 0,$$

implying that at the range of inner solutions the probability of dissent is either monotonic or first increases and then decreases, as required for getting the pattern of a spider. *Q.E.D.*

E Proof of Proposition 3

LEMMA 35 *The functions $\alpha 2^{1-\alpha} h\left(t_{\max}\right)$ and $1.8\,(2/9)^{\alpha}$ have one intersection point, denoted by $\tilde{\alpha}$, at the range $\alpha \in \left]0, \frac{1}{2}\right[$. Furthermore we have*

$$\begin{cases} \alpha 2^{1-\alpha} h\left(t_{\max}\right) < 1.8\,(2/9)^{\alpha} & \text{if } \alpha \in (0, \tilde{\alpha},) \\ \alpha 2^{1-\alpha} h\left(t_{\max}\right) > 1.8\,(2/9)^{\alpha} & \text{if } \alpha \in \left(\tilde{\alpha}, \frac{1}{2}\right) \end{cases}$$

PROOF: Using $h(t)$ as defined in Lemma 9 we get

$$\begin{aligned} \alpha 2^{1-\alpha} h\left(t_{\max}\right) &= 1.8\,(2/9)^{\alpha} \\ \Leftrightarrow \quad & \alpha 2^{1-\alpha}\left(1+t_{\max}\right)\left(\frac{1}{3}+t_{\max}^{2}\right)^{\alpha-1} = \frac{9}{5}\left(\frac{2}{9}\right)^{\alpha} \\ \Leftrightarrow \quad & \alpha(1+t_{\max})\left(\frac{1}{6}+\frac{1}{2}t_{\max}^{2}\right)^{\alpha-1} = \frac{2}{5}\left(\frac{2}{9}\right)^{\alpha-1} \\ \Leftrightarrow \quad & \frac{5}{2}\left(\frac{3}{4}+\frac{9}{4}t_{\max}^{2}\right)^{\alpha-1} = \frac{1}{\alpha(1+t_{\max})} \\ \Leftrightarrow \quad & \frac{5}{2}\left(\frac{3}{4}+\frac{9}{4}t_{\max}^{2}\right)^{\alpha} = \frac{3}{4}\frac{1+3t_{\max}^{2}}{\alpha(1+t_{\max})} \\ \Leftrightarrow \quad & \frac{10}{3}\left(\frac{3}{4}+\frac{9}{4}t_{\max}^{2}\right)^{\alpha} = \frac{1+3t_{\max}^{2}}{\alpha(1+t_{\max})} \end{aligned}$$

The RHS is the inverse of $\Omega\left(t_{\max}(\alpha), \alpha\right)$ which is defined in Lemma 22, where it is also shown to be increasing in α, hence the RHS decreases in α.

Analyzing the LHS. Let

$$\xi\left(t_{\max}(\alpha), \alpha\right) \equiv \left(\frac{3}{4}+\frac{9}{4}t_{\max}^{2}\right)^{\alpha}.$$

In Lemma 20 we showed that $t_{\max}$ increases in α. To show that $\xi\left(t_{\max}(\alpha), \alpha\right)$ is increasing in α it is therefore sufficient to show that the two partial derivatives of $\xi\left(t_{\max}(\alpha), \alpha\right)$ with respect to its two arguments, $t_{\max}(\alpha)$ and α, are both positive.

$$\frac{\partial \xi\left(t_{\max}, \alpha\right)}{\partial \alpha} = \left(\frac{3}{4}+\frac{9}{4}t_{\max}^{2}\right)^{\alpha} \ln\left(\frac{3}{4}+\frac{9}{4}t_{\max}^{2}\right) > 0$$

where the inequality follows since the smallest possible $t_{\max}$ is $\frac{2}{3}\sqrt{2}\sqrt{3}-1$ (to see this recall that $t_{\max}$ increases α and hence plug in $\alpha=0$ in equation 23) which implies $\frac{3}{4}+\frac{9}{4}t_{\max}^{2} > 1$.

$$\frac{\partial \xi\left(t_{\max}, \alpha\right)}{\partial t_{\max}} = 4.5 t_{\max}\alpha\left(\frac{3}{4}+\frac{9}{4}t_{\max}^{2}\right)^{\alpha-1} > 0$$

Thus, the LHS increases in α while the RHS decreases in α, implying that there is a unique intersection point $\tilde{\alpha}$. To find which of the functions $\alpha 2^{1-\alpha} h(t_{\max})$ and $1.8\,(2/9)^{\alpha}$ is larger below and above $\tilde{\alpha}$, we can plug in specific values of α. When $\alpha = 0$ the former function goes to 0 (recall that h is bounded) while the latter equals 1.8 hence is larger. When $\alpha \xrightarrow{-} \frac{1}{2}$ we know from Lemma 21 that $t_{\max}$ approaches $\frac{1}{3}$ hence the former function approaches

$$\frac{1}{2}2^{1/2}\frac{4}{3}\left(\frac{4}{9}\right)^{-1/2} = \frac{4}{3}\left(\frac{9}{8}\right)^{1/2} = \sqrt{2}$$

while the latter equals $1.8\,(2/9)^{1/2} = \frac{3}{5}\sqrt{2}$ hence is smaller. This also implies $\tilde{\alpha} \in]0, 1/2[$. The value of $\tilde{\alpha}$ can be numerically calculated to be ≈ 0.295. *Q.E.D.*

E.1 Proof of the proposition We prove the proposition for $t \geq 0$. Equivalent statements can be made for $t \leq 0$.

The average vote of type t, denoted by $J(t) \equiv E[I(t)]$, is a weighted sum of her own type, with probability $P_m + P(t)$, and the median of the judges' panel otherwise.

$$\begin{aligned} J(t) &= [P_m + P(t)]\,t + \left[\int_{t-c}^{t} 2vF(v)\,f(v)\,dv + \int_{t}^{t+c} 2v\,(1-F(v))\,f(v)\,dv\right] \\ &= \left\{2F(t)\,[1-F(t)] + [F(t-c)]^2 + [1-F(t+c)]^2\right\}t + \int_{t-c}^{t} 2vF(v)\,f(v)\,dv + \int_{t}^{t+c} 2v\,(1-F(v))\,f(v)\,dv \end{aligned}$$

Let

$$Z \equiv \int_{t-c}^{t} 2vF(v)\,f(v)\,dv + \int_{t}^{t+c} 2v\,(1-F(v))\,f(v)\,dv$$

We analyze separately the three possible regions of c (for $t \geq 0$):

$\mathbf{c \geq 1 + t}$

$$\begin{aligned} Z &= \frac{1}{2}\left[\int_{-1}^{t} v\,(1+v)\,dv + \int_{t}^{1} v\,(1-v)\,dv\right] \\ &= \frac{1}{2}\left[\int_{-1}^{t} \left(v+v^2\right)dv + \int_{t}^{1} \left(v-v^2\right)dv\right] = ... \\ &= \frac{t^3}{3} \end{aligned}$$

(35) $$\Rightarrow J(t) = \frac{1}{2}\left(1-t^2\right)t + \frac{t^3}{3} = \frac{1}{2}t - \frac{1}{6}t^3$$

$\mathbf{1 - t \leq c < 1 + t}$

$$
\begin{aligned}
Z &= \int_{t-c}^{t} v\frac{1+v}{2}dv + \int_{t}^{1} v\frac{1-v}{2}dv \\
&= \frac{1}{2}\left[\int_{t-c}^{t}\left(v+v^2\right)dv + \int_{t}^{1}\left(v-v^2\right)dv\right] = ... \\
&= \frac{t^3}{6} + \frac{1}{12} - \frac{1}{4}t^2 + \frac{1}{2}ct - \frac{1}{4}c^2 + \frac{1}{2}ct^2 - \frac{1}{2}c^2t + \frac{1}{6}c^3 \\
\Rightarrow \quad J(t) &= \left\{\frac{1}{2}\left(1-t^2\right) + \left[\frac{1}{2}(1+t-c)\right]^2\right\}t + \frac{t^3}{6} + \frac{1}{12} - \frac{1}{4}t^2 + \frac{1}{2}ct - \frac{1}{4}c^2 + \frac{1}{2}ct^2 - \frac{1}{2}c^2t + \frac{1}{6}c^3 \\
&= ... = t + \frac{1}{12}(1-t)^3 + c^2\left(\frac{1}{6}c - \frac{1}{4}t - \frac{1}{4}\right)
\end{aligned}
$$

$\mathbf{c < 1 - t}$

$$
\begin{aligned}
Z &= \int_{t-c}^{t} 2vF(v)f(v)\,dv + \int_{t}^{t+c} 2v(1-F(v))f(v)\,dv \\
&= \frac{1}{2}\left[\int_{t-c}^{t}\left(v+v^2\right)dv + \int_{t}^{t+c}\left(v-v^2\right)dv\right] = ... \\
&= tc(1-c)
\end{aligned}
$$

(36)
$$
\begin{aligned}
\Rightarrow \quad J(t) &= \left\{\frac{1}{2}\left(1-t^2\right) + \left[\frac{1}{2}(1+t-c)\right]^2 + \left[\frac{1}{2}(1-t-c)\right]^2\right\}t + tc(1-c) \\
&= ... = \left(1 - \frac{1}{2}c^2\right)t
\end{aligned}
$$

Note: the corner solution of $c = 0$ implies that the judge never signs $v \neq t$ hence $J(t) = t$. The corner solution of never dissenting ($c = 1 + t$) implies that the judge always votes according to the median of the panel, which equals $\frac{1}{2}t - \frac{1}{6}t^3$.

An equivalent statement of the proposition is that $argmax(J(t)) \notin \{0, 1\}$. In Proposition 9 we showed that, for $\alpha < \alpha^*$ (where $0.3 \approx \alpha^* > \tilde{\alpha} \approx 0.295$), there exists a unique switching point at the range $[0, 1]$ such that types below it dissent whenever they are not the median of their panel ($c = 0$) while types above it never dissent ($c = 1 + t$) under the conditions

(37) $$W < 2(1/6)^{\alpha}$$

(38) $$(2/3)^{\alpha} < W$$

(39) $$\alpha 2^{1-\alpha} h(t_{\max}) < W$$

(see (23) for a definition of $t_{\max}$) and that these conditions hold for a non-empty set of W. This dissent pattern implies (by (36) and (35)) that there exists some $\bar{t} < 1$ such that

$$
J(t) = \begin{cases} t \text{ for } t \leq \bar{t} \\ \frac{1}{2}t - \frac{1}{6}t^3 \text{ for } t > \bar{t} \end{cases}
$$

and hence (since $t \leq 1$) that $J(t)$ is first increasing for $t \leq \bar{t}$, then drops sharply at $\bar{t}$, and finally increases again for $t > \bar{t}$. To show that $argmax(J(t)) \notin \{0,1\}$ it is thus necessary and sufficient that

$$J(\bar{t}) \quad > \quad J(1) \Leftrightarrow \bar{t} < \frac{1}{3},$$

hence that the type $t = 1/3$ prefers $c = 0$ over $c = 1 + t$:

$$\begin{aligned} L(c = 0; t = 1/3) \quad &< \quad L(c = 1 + 1/3; t = 1/3) \leftrightarrow \\ & \quad \text{using (7) with a uniform distribution, (9) and (12)} \\ W\left(\left[\frac{1+t}{2}\right]^2 + \left[1 - \frac{1+t}{2}\right]^2\right) \quad &< \quad \left(\frac{1}{2}\left[t^2 + \frac{1}{3}\right]\right)^{\alpha} \leftrightarrow \\ W \quad &< \quad \frac{9}{5}\left(\frac{2}{9}\right)^{\alpha}. \end{aligned} \tag{40}$$

First we note that $\frac{9}{5}\left(\frac{2}{9}\right)^{\alpha} < 2(1/6)^{\alpha}$ for any $\alpha < \tilde{\alpha}$, hence we must show that condition (40) holds along with conditions (38) and (39), i.e., that the set of W fulfilling the conditions is non-empty. Conditions (40) and (38) yield a non-empty set if

$$\begin{aligned} (2/3)^{\alpha} \quad &< \quad \frac{9}{5}\left(\frac{2}{9}\right)^{\alpha} \leftrightarrow \\ \alpha \quad &< \quad \frac{\ln\left(\frac{5}{9}\right)}{\ln\left(\frac{6}{18}\right)} \approx 0.54, \end{aligned}$$

which is fulfilled since $\tilde{\alpha} \approx 0.295$. Conditions (40) and (39) hold together for any $\alpha < \tilde{\alpha}$ by Lemma 35.

F Proof of Proposition 4

We prove the proposition for $t \geq 0$. Equivalent statements can be made for $t \leq 0$. In Proposition 9 we showed that, for $\alpha < \alpha^*$ (where $0.3 \approx \alpha^* > \tilde{\alpha} \approx 0.295$), there exists a unique switching point at the range $[0,1]$ such that types below it dissent whenever they are not the median of their panel ($c = 0$) while types above it never dissent ($c = 1 + t$) under a non-empty set of W.

Proof of part (i): When $W = 0$ the objective function for all types becomes $min\, D$ which clearly is achieved by dissenting whenever not being median ($c = 0$). Using (7) with a uniform distribution and $c = 0$ we get $P(t) = \left[\frac{1+t}{2}\right]^2 + \left[1 - \frac{1+t}{2}\right]^2 = t^2 + 1$ which clearly increases in t.

Proof of part (ii): The dissent pattern just described implies that $\widetilde{t} = argmax P(t)$ equals the largest t that dissents when not being median. Since under these conditions the alternative is to never dissent, this type must be indifferent between these two options: $L\left(c = 0; \widetilde{t}\right) = L\left(c = 1 + \widetilde{t}; \widetilde{t}\right)$ .Using (7) with a uniform distribution together with (9) and (12) yields:

$$W\left(\left[\frac{1+\widetilde{t}}{2}\right]^2 + \left[1 - \frac{1+\widetilde{t}}{2}\right]^2\right) = \left(\frac{1}{2}\left[\widetilde{t}^2 + \frac{1}{3}\right]\right)^{\alpha}.$$

Solving for W it is easy to show that $\widetilde{t}$ decreases in W if $\alpha < \tilde{\alpha} \approx 0.295$.

G The SCOTUS model

In this section we present a hierarchical model that is able to produce the spider-shaped pattern of dissent rate as a function of ideological distance to SCOTUS and we derive the predictions that are presented in Proposition 5 and are tested empirically in Section 4. The hierarchical model presented here is developed along the lines of the model of Beim et al. (2014).

Every period, three judges are randomly and independently drawn from a uniform distribution of types $t \sim U\ (-1, 1)$ to sit together on a panel. The panel produces a *binary* verdict ("conservative" or "liberal") for a case with characteristics x, where $x \sim (-1, 1)$. A judge of type t prefers a conservative verdict over a liberal one iff $x < t$. The panel determines the verdict by a majority voting, implying that the verdict is conservative if and only if the median judge, denoted t_m, is such that $x < t_m$.

A panel member may also dissent. Upon noticing a dissent, the Supreme Court may decide to review the case. The bliss point of the Supreme Court is normalized to 0. Thus, the Supreme Court rules conservatively on a reviewed case iff $x < 0$. The cost of dissenting is denoted W (and represents writing costs of the minority opinion or collegial pressure). When a judge t is able to reverse the binary verdict in case x her utility gain is $|t - x|$. Hence, a judge will never dissent if $|t - x| \leq W$. We can calculate the type-dependent probability of dissent $P(t)$ while considering only a judge with $t > 0$ (by symmetry the same applies to judges $t < 0$).

Under this framework, judge $t > 0$ may dissent in two scenarios:

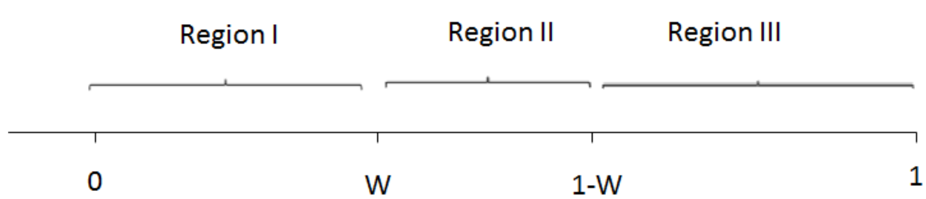
1. $t_m > 0$ and $x \in [t, t_m]$, so that both t and the Supreme Court prefer a liberal verdict while the panel produces a conservative verdict.

2. $t_m < 0$ and $x \in [t_m, 0]$, so that both t and the Supreme Court prefer a conservative verdict while the panel produces a liberal verdict.

Under scenario 1, the judge indeed dissents if $x - t > W$, i.e., if $x \in [t + W, t_m]$. Under scenario 2, the judge indeed dissents if $t - x > W$, i.e., if $x \in [t_m, \min\{0, t - W\}]$. We will now show that the model produces a spider-shaped pattern of dissent rate for $W < 1/2$. In this case, we have $W < 1 - W$.

Figure A.13 is helpful in distinguishing between three regions of t.

Appendix Figure A.13.— Regions in SCOTUS model



A judge t in region I dissents if either $x \in [t + W, t_m]$ or $x \in [t_m, t - W]$.

A judge t in region II dissents if either $x \in [t + W, t_m]$ or $x \in [t_m, 0]$.

A judge t in region III dissents if $x \in [t_m, 0]$.

Calculating the type-dependent probability of dissent $P(t)$, we get (each line represents one region in the graph)

$$P(t) = \begin{cases} \frac{1}{3}\left[\frac{1}{2}\left[1-(t+W)\right]\right]^3 + \frac{1}{3}\left[\frac{1}{2}\left[(t-W)-(-1)\right]\right]^3 \text{ if } t \in [0, W] \\ \frac{1}{3}\left[\frac{1}{2}\left[1-(t+W)\right]\right]^3 + \frac{1}{3}\left(\frac{1}{2}\right)^3 \text{ if } t \in [W, 1-W] \\ \frac{1}{3}\left(\frac{1}{2}\right)^3 \text{ if } t \in [1-W, 1] \end{cases}$$

To understand the calculations of the expression of $P(t)$, note first that the event $x \in [t_m, 0]$ is independent of t and it occurs iff $\min t < t_m < x < 0$. As $\min t, t_m$ and x are all drawn from a uniform distribution over $[-1, 1]$, the probability that all three of them are negative is $\left(\frac{1}{2}\right)^3$, and the probability that x is the largest among the three is 1/3, yielding the expression $\frac{1}{3}\left(\frac{1}{2}\right)^3$. Next, the event $x \in [t_m, t-W]$ is an adjustment of this calculation for the event $x \in [t_m, t-W]$. In particular, we now need $\min t < t_m < x < t-W$, so x needs to be the largest of the three uniformly-distributed variables, which all need to be in the region $[-1, t-W]$, and this event corresponds to probability $\frac{1}{3}\left[\frac{1}{2}\left[(t-W)-(-1)\right]\right]^3$ (i.e., the probability of being negative, 1/2, is replaced with the probability of being smaller than $t-W$, which is $\frac{1}{2}\left[(t-W)-(-1)\right]$). Finally, the event $x \in [t+W, t_m]$ occurs iff $t+W < x < t_m < \max t$. So x needs to be the smallest of three uniformly-distributed variables, which all need to be in the region $[t+W, 1]$, and this event has probability $\frac{1}{3}\left[\frac{1}{2}\left[1-(t+W)\right]\right]^3$.

Differentiating with respect to t yields

$$\frac{dP(t)}{dt} = \begin{cases} -\frac{1}{8}\left[1-(t+W)\right]^2 + \frac{1}{8}\left[(t-W)+1\right]^2 \text{ if } t \in [0, W] \\ -\frac{1}{8}\left[1-(t+W)\right]^2 \text{ if } t \in [W, 1-W] \\ 0 \text{ if } t \in [1-W, 1] \end{cases}$$

It is immediate to see that $dP(t)/dt$ is negative in region II. To get the sign of $dP(t)/dt$ in region I, note that $t > 0$ implies that $t+W$ is closer to the right edge of the type distribution (1) than $t-W$ is to the left edge of the type distribution (-1), implying that

$$\left[1-(t+W)\right]^2 < \left[(t-W)+1\right]^2,$$

implying that $dP(t)/dt > 0$ in region I. Overall, we get that $P(t)$ increases in region I and then decreases in region II and stays flat in region III, implying a spider-shaped pattern of dissent rate.

G.1 Proof of Proposition 5 (i) When $W = 0$, regions I and III disappear and we are left only with region II where $dP(t)/dt$ is negative. Symmetry implies that for any type t, $P(t)$ is decreasing in $|t|$.

(ii) The value of t for which $P(t)$ is maximal is the border between regions I and II, i.e. $t = W$. It is thus immediate that $\arg\max_t P(t)$ increases in W.