# Second-Degree Price Discrimination by a Two-Sided Monopoly Platform

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**Abstract**

We study second-degree price discrimination by a two-sided monopoly platform. The incentive constraints of the agents on the value creation side may be in conflict with internalizing externalities on the value capture side, which may render pooling optimal. Even without such conflict between the two sides, pooling may be optimal due to type-dependent Spence effects when the preferences of the marginal agents diverge from those of the average agents on the value capture side. We perform a welfare analysis of price discrimination and show that prohibiting price discrimination improves welfare when there is a strong conflict between the two sides. (*JEL codes*: D4, D82, L5, M3)

*Keywords*: platform, price discrimination, two-sided markets, nonresponsiveness, Spence effect.

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## 1 Introduction

Many two-sided platforms practice price discrimination against one or both groups of agents. For example, the world's largest on-demand streaming service, YouTube, offers the ad-free "YouTube Premium," for a subscription fee, along with the default free version containing advertisements. Many online newspapers provide a choice between paid unlimited access and free limited access. In contrast, many other platforms such as Google Search, Facebook, and Instagram do not discriminate between consumers based on price; instead, they provide services to consumers free of charge and monetize their attention by selling advertising. Despite the importance of platform markets in the modern economy, little economic analysis has yet been put forward regarding second-degree price discrimination by a two-sided platform. Given the growing global interest in regulating and establishing antitrust legislation for digital platforms,[1] it is important to understand how the logic and welfare consequences of second-degree price discrimination in a two-sided platform differ from the existing analysis, which is based on one-sided logic.[2]

Consider a media platform which provides content to consumers and generates most revenue by selling consumers' attention to advertisers. For this reason, we often call the consumer side the *value creation side* and the advertiser side the *value capture side*.[3] We show that there can be a congruence or a conflict between the two sides, in the sense that the incentive of the agents on the value creation side may be aligned or in conflict with the platform's incentive to internalize externalities on the value capture side. When conflict occurs between the two sides, this may render pooling optimal on the consumer side due to *nonresponsiveness* (Guesnerie and Laffont 1984). We also show that even without a conflict between the two sides, pooling can be optimal on the consumer side, because of a conflict between the agents on the value capture side. This conflict within the value capture side arises due to *type-dependent Spence (1975) effects* when the preferences of the marginal agents diverge from those of the average agents.

The type of an agent in a two-sided market plays a dual role by determining not only her private benefit but also the externalities she generates to the agents on the other side. This dual role of a type is essential in understanding the optimal mechanism, for it becomes the source for *nonresponsiveness* in our model. In standard principal-agent theory, nonresponsiveness refers to a clash between the incentive compatible (or implementable) allocations and the allocation the principal desires to achieve, and such a clash can arise when the agent's type and the screening instrument

[1] See for instance the reports by Crémer et al. (2019), U.K. Digital Competition Expert Panel (2019), Stigler Committee on Digital Platforms (2019), Australian Competition and Consumer Commission (2018), and Japan Fair Trade Commission (2017).

[2] For in-depth reviews on nonlinear pricing, see Wilson (1993) and Armstrong (2016).

[3] We thank Elie Ofek for suggesting these terms.

directly affect the principal's payoff.[4] A similar clash can occur in a two-sided platform that intermediates cross-group interactions. Consider the media platform described above, and suppose that there are two types of consumers, a high type and a low type, whereas all advertisers are homogeneous. The platform should design an advertising policy, that is, the amount of advertisement for each type of consumer. Assume that a high type suffers a greater nuisance from advertisement than a low type does, but the advertisers find reaching a high type more valuable than reaching a low type. In this environment, a clash could arise between the incentive constraints on the consumer side (which requires a high type to have less exposure to advertising than a low type) and the platform's incentive to maximize advertising revenue (which requires a high type to have more exposure to advertising than a low type). If such a clash arises, pooling becomes optimal (see more details in the application of Section 6.2).

To clearly identify various forces, we perform our analysis in a progressive manner. What is common throughout the whole analysis is that we consider heterogeneous agents on side $A$ (the value creation side) and focus on the screening of the types on this side with a screening instrument, which is called "quality." As is clear from the example of the advertising policy described above, the screening instrument affects not only the payoff of an agent of side $A$ to which the instrument is assigned but also that of the agent of side $B$ who interacts with an agent of side $A$.

Section 2 describes the baseline model and several examples of real-world platforms to which the model can be applied. In the baseline model, we consider two types of agents on side $A$ and homogeneous agents on side $B$. Section 3 characterizes the first-best and the second-best contracts, and compares the two. Even if the marginal benefit from quality increase is higher for a high type than for a low type, the high type can generate either more or less positive externalities to side $B$ than the low type does. We say that a conflict (a congruence) between the two sides arises when a low type generates more (less) externalities than a high type does. We also say that a quality schedule is *decreasing* when the quality chosen for a high type is lower than the quality for a low type.

We find that the first-best quality schedule is decreasing when the conflict between the two sides is larger than a threshold and that the second-best quality schedule entails pooling when the conflict between the two sides is larger than another threshold, which is larger than the first-best threshold. Pooling arises when the platform's desire to internalize externalities clashes with the

[4]For instance, suppose the principal is a benevolent regulator who cares not only about the production cost of a regulated firm but also about the amount of pollution the firm emits, which depends on the firm's type as well as the firm's output. Incentive compatibility requires that a low-cost firm should produce more output than a high-cost firm does. However, if the higher cost results from greater efforts to reduce pollution, then the principal may want to induce the high-cost firm to produce more than the low-cost firm does. Such an allocation clashes with the implementability condition.

implementability condition that calls for an increasing schedule.[5]

In Section 4, we make agents on side $B$ heterogeneous by considering binary types on each side. Therefore, when an agent of side $A$ interacts with an agent of side $B$, the benefit that each agent obtains from the interaction depends on the types of both agents. In this general model, we identify a new source for pooling even when there is congruence between the two sides. Note that because of the Spence effect in a two-sided market (Weyl 2010), the first-best quality schedule is based on the average externalities to side $B$, while the second-best quality schedule is determined by the externalities to the marginal agents of side $B$. In our model, the Spence effect is *type-dependent*, as the platform offers a menu of qualities, and both the average externalities and the marginal externalities generated by an agent of side $A$ depend on the agent's type and the quality assigned to the type. This generates a conflict within side $B$ if the average agents' preference, in terms of the preferred type on side $A$, diverges from that of the marginal agents. For instance, even when the average agents on side $B$ prefer interacting with a high type of side $A$ to a low type (hence, there is a congruence between the two sides), if the marginal agents on side $B$ strongly prefer interacting with a low type of side $A$ to a high type, this can induce the platform to desire to implement a decreasing quality schedule, which clashes with the implementability condition and thereby leads to pooling.

Section 5 provides a welfare analysis of price discrimination. We consider the two-type baseline model and compare the welfare under price discrimination with the welfare without price discrimination. We first confirm the intuitive result that low types are more likely to be excluded when price discrimination is banned than when it is allowed, regardless of whether the market is one-sided or two-sided. This result implies that price discrimination generates a higher welfare if the exclusion of low types occurs only without price discrimination.

Another interesting welfare result is obtained when the conflict between the two sides is strong enough. If the strong conflict leads to pooling, then welfare is not affected by prohibiting price discrimination. However, if the outcome under price discrimination is close enough to pooling but involves differential treatments of the types, then banning price discrimination improves welfare. Basically, under price discrimination, there is no distortion at the top (i.e., in the quality for a high type) and there is a downward distortion at the bottom (i.e., in the quality for a low type), but the presence of the strong conflict implies that the first-best quality for a low type is greater than the

[5] We also extend the analysis of the baseline model to a continuum of types on side $A$ (see details in Appendix A). When the first-best quality schedule is increasing, we provide a sufficient condition that makes the second-best quality schedule strictly increasing and hence the implementability condition never binding. When the first-best quality schedule is not increasing, we provide three different conditions, each of which renders complete pooling optimal. In addition, we analyze the optimal contract involving either a complete or a partial shutdown.

one for a high type. Hence, price discrimination entails an important distortion in the quality for a low type. Without price discrimination, the platform chooses one quality for both types, and hence banning price discrimination entails a smaller distortion in the quality for a low type, which dominates the effect from distortion in the quality for a high type, as this is of second order. We also extend the welfare analysis of the baseline model to the general model with binary types on each side.

Section 6 provides two applications to media platforms. The first applies the baseline model to an online newspaper that bundles a fixed proportion of advertising to its articles. It discusses when we expect a congruence or a conflict to arise between the two sides. The second application analyzes the advertising policy of a media platform and illustrates when pooling is optimal.

## 1.1 Related literature

This article is related to several strands of literature. First, our paper is closely related to the second-degree price discrimination in the principal-agent theory (e.g., Mussa and Rosen 1978; Maskin and Riley 1984) and to the concept of nonresponsiveness. After Guesnerie and Laffont (1984)'s pioneering work on the nonresponsiveness, Caillaud and Tirole (2004) explore it in the context of financing an essential facility; Jeon and Menicucci (2008) apply the concept to the allocation of talent between the public and private sectors. In this paper, we explore nonresponsiveness in two-sided markets. Our contribution to the literature is threefold: to microfound nonresponsiveness in a two-sided market by formalizing the notion of the congruence and conflict between two sides; to identify a new source for nonresponsiveness generated by a conflict within the value capture side; and to provide novel welfare results (for instance, banning price discrimination improves welfare when there is a strong conflict between the two sides).

This article broadly contributes to the literature studying a two-sided monopoly platform (e.g., Caillaud and Jullien 2001; Rochet and Tirole 2003; Armstrong 2006; Rochet and Tirole 2006; Hagiu 2009; Jeon and Rochet 2010). In particular, our paper complements Weyl (2010), who considers a rich type space and identifies the Spence (1975) distortion in that the platform internalizes cross-side externalities to marginal rather than average users of the other side when deciding the level of participation on one side. In our model, the platform employs a menu of screening instruments called qualities, which renders the Spence effect type-dependent. Our novelty is to show that this type-dependent Spence effect can make pooling optimal when there exists a conflict among the agents within the value capture side.

Our paper is closely related to a few recent articles studying second-degree price discrimination by a two-sided monopoly platform. Choi, Jeon, and Kim (2015) study second-degree price discrimination in the context of the debate on network neutrality regulations. They consider two types of content providers and homogeneous consumers. They assume that any surplus generated by the interaction between a content provider and a consumer is shared between the two according to an exogenous parameter, interpreted as "business model." They study how this parameter crucially affects the welfare under non-neutral networks where price discrimination is allowed and the welfare under neutral networks where price discrimination is banned. Böhme (2016) considers a model in which there are two types on each side and each agent's utility consists of two components: a type-specific intrinsic utility from access to the platform and an additional type-specific indirect network effect from interacting with agents on the opposite side of the market. We show that the model of Choi et al. (2015) can be obtained as a special case of congruence between the two sides in our baseline model (see Remark 1). In Böhme (2016), the externality generated by an agent to an agent on the other side depends only on the latter's type, such that a high type enjoys a larger externality than a low type does. For this reason, the model of Böhme (2016) belongs to a limit case in which there is neither congruence nor conflict (in a strict sense) between the two sides and there is no conflict within a side (see Remark 2). As a consequence, none of the two papers find the optimality of pooling. Hence, our result that pooling can be optimal either because of the conflict between the two sides or because of the conflict within the value capture side cannot arise in these papers.[6] Last, Lin (2020) studies advertising policy by a media platform and obtains a similar result as the application in Section 6.2.[7] Basically, Lin extends our application in an earlier version of this paper (Jeon et al., 2016), which considers heterogeneous consumers and advertisers, by allowing the platform to determine, for each type of consumer, the types of advertisers who can show advertising.[8]

Our paper is complementary to Gomes and Pavan (2016). They consider heterogeneous agents on both sides and study a centralized many-to-many matching. Like us, agents have double roles as their vertical characteristics capture "consumer value" (private benefit) and "input value" (externality) to the opposite side. The main theme of Gomes and Pavan (2016) is how matching patterns reflect optimal cross-subsidization between sides. They identify conditions on the primitives under which the optimal matching rule has a threshold structure such that each agent on one side is

[6]In a broad sense, our results on quality distortions both at the top and at the bottom are related to the literature on one-sided markets with network effects (e.g., Hahn 2003; Sundararajan 2004; Csorba 2008; Meng and Tian 2008) and the literature on countervailing incentives (e.g., Lewis and Sappington 1989; Maggi and Rodriguez-Clare 1995).

[7]Our application is broadly connected to a strand of the two-sided market literature on advertising/media platforms. For instance, it is related to Gabszewicz, Laussel, and Sonnac (2004), Anderson and Coate (2005), Peitz and Valletti (2008), Crampes, Haritchabalet, and Jullien (2009), Anderson and Gans (2011), Johnson (2013), Ambrus, Calvano, and Reisinger (2016), etc. In particular, Angelucci and Cagé (2019) study a monopoly newspaper that practices price discrimination by offering a choice between buying a subscription and buying individual issues.

[8]Jeon et al. (2016) consider a restrictive mechanism in order to focus on the question of complementarity and substitution between price discrimination on side $A$ and price discrimination on side $B$.

matched with all agents on the other side above a threshold type. By contrast, we focus on the platform's design of menu pricing when quality provision can enhance both match values under the assumption that all agents on one side interact with all agents on the other side, which is standard in the literature on two-sided markets (Armstrong 2006; Rochet and Tirole 2006; Weyl 2010).[9]

## 2 The Baseline Model

We here introduce the baseline model, which focuses on nonresponsiveness in a two-sided platform. A monopolistic two-sided platform mediates interactions between agents from two sides, $A$ and $B$. On each side there is a mass one of agents. We assume that all agents on any given side interact with all (or a random subset of) agents on the other side, following Armstrong (2006), Rochet and Tirole (2006), and Weyl (2010). We assume that the agents on side $A$ are heterogeneous, but all agents on side $B$ are homogeneous, which will be relaxed in Section 4. Throughout this paper, we consider screening on side $A$ only: the platform's mechanism design is based on a screening instrument applied to side $A$, called "quality."[10] We will first provide a general framework and then specify it to a model analyzed in Section 3.

### 2.1 A general framework

Let $\theta^A \in \mathbb{R}_{++}$ represent a generic type of the agents on side $A$ and $\Theta^A$ denote the set of types on side $A$. Let $q^A \geq 0$ represent a screening instrument called "quality." When a type $\theta^A$ agent of side $A$ interacts with an agent of side $B$, the former's gross utility from the interaction is given as

$$\text{PRIVATE BENEFIT:} \quad U^A(q^A;\theta^A),$$

where $U^A$ is strictly increasing and strictly concave in quality $q^A$ and satisfies the single-crossing condition (i.e., $\partial^2 U^A/\partial q^A \partial \theta^A > 0$). Let $c \geq 0$ denote the constant marginal cost of providing quality $q^A$.

The gross utility an agent of side $B$ obtains from the interaction with a type $\theta^A$ agent of side $A$, which is regarded as externalities generated by the latter, is given by

$$\text{CROSS-SIDE EXTERNALITIES:} \quad U^B(q^A;\theta^A),$$

[9] Gomes and Pavan (2018) introduce a new dimension that agents have horizontally differentiated preferences and focus on the issues of targeting and price customization. Since the customization cost can vary in the volume of matches and plans selected by agents, they consider both second-degree and third-degree price discrimination.

[10] When there are heterogeneous agents on both sides, we can consider screening on each side and study when the price discrimination on side $B$ substitutes for or complements the price discrimination on side $A$. But we leave this question for future research and confine the scope of this paper to screening on one side only. An earlier version of this paper (Jeon et al. 2016) addressed the above research question with some restrictive assumptions.

where we note that $U^B$ depends both on $q^A$ and $\theta^A$. $U^B$ can increase or decrease with $q^A$; similarly it can increase or decrease with $\theta^A$. Note that our framework requires $q^A$ to affect $U^B$. Otherwise, the dependency of $U^B$ on $\theta^A$ alone plays no role in determining the optimal $q^A$, implying that we are back to standard second-degree price discrimination in one-sided markets.

As we consider a two-sided platform in which agents on one side obtain utilities from interacting with agents on the other side, we can interpret $U^A$ as externalities from side $B$ to side $A$ in the same way as we interpret $U^B$ as externalities from side $A$ to side $B$. As $q^A$ affects both $U^A$ and $U^B$ in a monotonic way, $q^A$ represents the extent of cross-side externalities and the single-crossing condition implies that an agent of side $A$ experiences more benefit from externalities as $\theta^A$ increases.

Since we mainly have in mind a situation in which the platform makes the most revenue from side $B$, we often call side $A$ the value creation side and side $B$ the value capture side. In Table 1, we provide some examples of platforms to which our framework can be applied.

Table 1: Examples of model interpretation

| Platform | Agents $A$ | Agents $B$ | Quality to agents $A$ ($q^A$) | $\frac{\partial U^A}{\partial q^A}$ | $\frac{\partial U^B}{\partial q^A}$ |
|---|---|---|---|---|---|
| Online newspapers | Readers | Advertisers | Number of articles | $+$ | $+$ |
| Internet service providers | Content providers | Internet users | Quality of content delivery | $+$ | $+$ |
| Online content platforms | Users | Advertisers | Ad reduction | $+$ | $-$ |
| Social networks | Users | Advertisers | Privacy protection level | $+$ | $-$ |

In the case of online newspapers that bundle news content with advertising, side $A$ represents readers and side $B$ advertisers. $q^A$ can be the number of articles a reader reads. As the amount of advertising is proportionate to the number of articles, $q^A$ also represents the amount of advertising. In this case, both $U^A$ and $U^B$ increase with $q^A$. We analyze this example in the application of Section 6.1. For Internet service providers, side $A$ represents content providers and side $B$ users. $q^A$ can be service quality of content delivery. Still, both $U^A$ and $U^B$ increase with $q^A$. Choi et al. (2015) analyze such a situation, but our model is more general, as it can include theirs as a particular case (see Remark 1). For online content platforms financed by advertising, side $A$ represents users and side $B$ advertisers. Let $a \in [0, \overline{a}]$ represent the amount of advertising per user where $\overline{a} > 0$ is an upper bound. Then, we can define $q^A \equiv \overline{a} - a$. As users suffer from ad nuisance, $U^A$ increases with $q^A$ but $U^B$ decreases with $q^A$. We analyze this application in Section 6.2. Finally, for social media such as Facebook, side $A$ represents users and side $B$ advertisers. $q^A$ can be the level of privacy protection. Hence, $U^A$ increases with $q^A$. As long as increased privacy protection makes it more difficult to collect and use personal data for targeted advertising, $U^B$ decreases with

$q^A$.

Therefore, $U^A$ increases with $q^A$ in all the examples described above, whereas $U^B$ can increase or decrease with $q^A$. In what follows, we specify the model such that $U^A$ and $U^B$ increase with $q^A$. But the result on nonresponsiveness obtained from this specification is valid even when $U^B$ decreases with $q^A$ (see Section 6.2).

## 2.2 Specification of the Model

As we prefer a baseline model similar to that of Mussa and Rosen (1978), for the analysis of Section 3, we specify $U^A$ and $U^B$ as follows:

$$U^A(q^A;\theta^A) = \theta^A u^A(q^A), \qquad U^B(q^A;\theta^A) = e(\theta^A)u^B(q^A), \tag{1}$$

where $e(\theta^A) > 0$ represents the intensity of the positive externalities. We make no assumptions on the monotonicity of the function $e$, but assume that both $u^A$ and $u^B$ are strictly increasing, strictly concave, and $u^A(0) = u^B(0) = 0$. A one-sided market benchmark is captured by the special case of $e(\theta^A) = 0$ for any $\theta^A \in \Theta^A$, which essentially makes our model collapse into the classic model of Mussa and Rosen (1978).

Importantly, we can see the dual role played by the type, $\theta^A$. Beyond its usual role as a taste intensity, $\theta^A$ is a determinant of externalities to the agents on side $B$.

The platform offers on side $A$ a menu of quality-price pairs $\left\{(q^A(\theta^A), p^A(\theta^A))\right\}_{\theta^A \in \Theta^A}$, while it offers only a subscription price $p^B$ on side $B$. We say a quality schedule $q^A : \Theta^A \to \mathbb{R}_+$ is increasing (decreasing) if $q^A$ is a weakly increasing (decreasing) function of $\theta^A$.

In Section 3, we analyze a two-type model, whereas we analyze a model with a continuum of types in Appendix A. Consider a two-type case: $\Theta^A = \left\{\theta_L^A, \theta_H^A\right\}$ with $\theta_H^A > \theta_L^A > 0$ and $\Delta\theta^A \equiv \theta_H^A - \theta_L^A$ where a $\theta_L^A$ type is called a low type and a $\theta_H^A$ type is called a high type. Let $\nu^A \in (0,1)$ represent the probability that an agent on side $A$ has type $\theta_L$.

# 3 Analysis of the baseline model: nonresponsiveness

In this section, we analyze the baseline model introduced in Section 2. Since there are types only on side $A$, in this section we drop the superscript $A$ for qualities, types, and payments. For simple notations, we define $q_L \equiv q(\theta_L)$, $p_L \equiv p(\theta_L)$, and $e_L \equiv e(\theta_L)$; $q_H$, $p_H$, and $e_H$ are similarly defined. Hence, the menu of contracts offered by the platform on side $A$ is $\{(q_L, p_L), (q_H, p_H)\}$.

### 3.1 First-best

Given the menu $\{(q_L, p_L), (q_H, p_H)\}$ and $p^B$, the individual rationality constraints are written as follows: for a low-type agent on side $A$,

$$(2) \qquad (IR_L) \quad \theta_L u^A(q_L) - p_L \geq 0;$$

for a high-type agent on side $A$,

$$(3) \qquad (IR_H) \quad \theta_H u^A(q_H) - p_H \geq 0;$$

for an agent on side $B$,

$$(IR^B) \quad \nu^A e_L u^B(q_L) + (1-\nu^A) e_H u^B(q_H) - p^B \geq 0.$$

In the first-best case, the platform can extract the full surplus of all agents of both sides through the prices. Hence the platform's profit, which is equal to the welfare $W(q_L, q_H)$, is given by

$$W(q_L, q_H) \equiv \nu^A \left( \theta_L u^A(q_L) + e_L u^B(q_L) - cq_L \right) + (1-\nu^A) \left( \theta_H u^A(q_H) + e_H u^B(q_H) - cq_H \right).$$

Let $\left(q_L^{FB}, q_H^{FB}\right)$ denote the first-best quality schedule. We assume interior solutions. Then, $\left(q_L^{FB}, q_H^{FB}\right)$ is characterized by

$$(4) \qquad \theta_L u^{A\prime}(q_L^{FB}) + e_L u^{B\prime}(q_L^{FB}) = c, \qquad \theta_H u^{A\prime}(q_H^{FB}) + e_H u^{B\prime}(q_H^{FB}) = c.$$

Given a type, the first-best quality equalizes the sum of the marginal surplus generated on each side to the marginal cost. In a one-sided market with $e_L = e_H = 0$, the first-best quality schedule is strictly increasing, that is, $q_L^{FB} < q_H^{FB}$, since $\theta_L < \theta_H$. By contrast, in a two-sided market with $e_L > 0$ and $e_H > 0$, the relationship $q_L^{FB} < q_H^{FB}$ does not necessarily hold. Below, we report when an increasing quality schedule arises in the first-best and when it does not.

**Proposition 1** *Consider the baseline model. The first-best quality schedule $\left(q_L^{FB}, q_H^{FB}\right)$ is determined by (4). Given $(\theta_H, \theta_L, e_H)$, there is a threshold $\Delta^{FB} > 0$ such that $q_L^{FB} \leq q_H^{FB}$ if and only if $e_L - e_H \leq \Delta^{FB}$.*

**Example 1** *Suppose $u^B(q) = \beta u^A(q)$ for some $\beta > 0$, for each $q \geq 0$. Then, $q_L^{FB} \leq q_H^{FB}$ if and only if $e_L - e_H \leq (\theta_H - \theta_L)/\beta$.*

### 3.2 Second-best

In the presence of asymmetric information about $\theta$, the platform faces incentive constraints on the value creation side, which creates what is called the implementability condition such that the platform can implement only increasing quality schedules, i.e., those satisfying $q_L \leq q_H$. This condition is not binding if $e_H \geq e_L$ holds, because then both extracting the surplus on side $A$ and internalizing externalities on side $B$ require the platform to offer an increasing quality schedule. In contrast, if $e_H < e_L$ holds, internalizing externalities requires the platform to offer a decreasing quality schedule and therefore can create a conflict with the implementability condition. Therefore, we introduce the following definition to distinguish between two situations.

**Definition (Congruence vs. Conflict between the two sides)** There is a *congruence* (*conflict*) between the value creation side and the value capture side if $e_H \geq e_L$ ($e_H < e_L$).

In what follows, we show that the optimal mechanism can differ greatly, depending on whether there exists a congruence or a conflict between the two sides.

In the second-best, the menu of contracts $\{(q_L, p_L), (q_H, p_H)\}$ offered by the platform needs to satisfy the following incentive constraints, in addition to the individual rationality constraints introduced previously: for a low-type agent on side $A$,

$$(5) \qquad (IC_L) \quad \theta_L u^A(q_L) - p_L \geq \theta_L u^A(q_H) - p_H;$$

for a high-type agent on side $A$,

$$(6) \qquad (IC_H) \quad \theta_H u^A(q_H) - p_H \geq \theta_H u^A(q_L) - p_L.$$

The platform maximizes its profit

$$(7) \qquad \pi = \nu^A (p_L - cq_L) + (1 - \nu^A)(p_H - cq_H) + p^B$$

subject to the incentive constraints $(IC_L)$ and $(IC_H)$, and the individual rationality constraints $(IR_L)$, $(IR_H)$, and $(IR^B)$.

The platform fully extracts the surplus on side $B$. Standard arguments show that on side $A$: (i) $(IR_L)$ and $(IC_H)$ bind in the optimum and hence $p_L = \theta_L u^A(q_L)$ and $p_H = \theta_H u^A(q_H) - \Delta\theta u^A(q_L)$; (ii) $(IC_L)$ reduces to

$$(8) \qquad (M) \quad q_L \leq q_H,$$

where $(M)$ refers to the *monotonicity constraint*, also called the *implementability condition* (Laffont and Martimort 2002). The monotonicity constraint means that the platform can implement only increasing quality schedules. Then the platform's profit is given by

$$\text{(9)} \quad \Pi(q_L, q_H) = \nu^A \left( \theta_L^v u^A(q_L) + e_L u^B(q_L) - cq_L \right) + (1 - \nu^A) \left( \theta_H u^A(q_H) + e_H u^B(q_H) - cq_H \right),$$

where $\theta_L^v = \theta_L - \frac{1-\nu^A}{\nu^A}\Delta\theta$ is the virtual valuation of the low type, and we assume $\theta_L^v > 0$. The platform maximizes $\Pi$ with respect to $(q_L, q_H)$ subject to (8). Let $\left(q_L^{SB}, q_H^{SB}\right)$ denote the second-best quality schedule and $(q_L^*, q_H^*)$ denote the solution that maximizes $\Pi$ when the monotonicity constraint is neglected: $(q_L^*, q_H^*)$ satisfies the first-order conditions for the maximization of (9):

$$\text{(10)} \qquad \theta_L^v u^{A\prime}(q_L^*) + e_L u^{B\prime}(q_L^*) = c, \qquad \theta_H u^{A\prime}(q_H^*) + e_H u^{B\prime}(q_H^*) = c.$$

Comparing (4) and (10) reveals that the only difference between the two is that $\theta_L$ is replaced by $\theta_L^v$ in (10), which implies $q_H^* = q_H^{FB}$ and $q_L^* < q_L^{FB}$. If $q_L^* \leq q_H^{FB}$, the optimal second-best quality schedule is given as $q_L^{SB} = q_L^*$ and $q_H^{SB} = q_H^{FB}$. Otherwise (i.e., if $q_L^* > q_H^{FB}$), the monotonicity constraint is binding and we have $q_L^{SB} = q_H^{SB} = q^P$, where $q^P$ is characterized by

$$\text{(11)} \qquad \theta_L u^{A\prime}(q^P) + (\nu^A e_L + (1 - \nu^A) e_H) u^{B\prime}(q^P) = c.$$

**Proposition 2** *Consider the baseline model. The second-best quality schedule* $\left(q_L^{SB}, q_H^{SB}\right)$ *is characterized as follows. Given* $(\theta_H, \theta_L, e_H)$*, there is a threshold* $\Delta^{SB}$ $(> \Delta^{FB})$ *such that:*

*(i)* $q_L^{SB} < q_H^{SB} (= q_H^{FB})$ *if and only if* $e_L - e_H < \Delta^{SB}$*, where* $q_L^{SB} = q_L^*$ *is given by (10);*

*(ii) Otherwise, pooling is optimal:* $q_L^{SB} = q_H^{SB} = q^P$ *where* $q^P$ *satisfies (11).*

**Example 2** *Suppose* $u^A(q) = u^B(q) = \ln(1+q)$ *and* $c = 1$*. Then we have* $q_L^{FB} = \theta_L + e_L - 1$, $q_H^{FB} = \theta_H + e_H - 1$ *and hence* $q_L^{FB} \leq q_H^{FB}$ *if and only if* $e_L - e_H \leq \Delta^{FB} = \Delta\theta$*. Moreover,* $q_L^* = \theta_L^v + e_L - 1 \leq q_H^{FB}$ *if and only if* $e_L - e_H \leq \Delta^{SB} = \frac{\Delta\theta}{\nu^A}$.

In the case of a conflict between the two sides, $e_L - e_H$ measures the degree of conflict. The proposition shows that pooling is optimal in the case of strong conflict (i.e., $e_L - e_H \geq \Delta^{SB}$). $\Delta^{SB} > \Delta^{FB}$ follows from the fact that $\theta_L^v < \theta_L$ makes $q_L^* < q_L^{FB}$ in (10). The pooling arises because of *nonresponsiveness* (Guesnerie and Laffont 1984; Laffont and Martimort 2002), which refers to a clash between the allocation the principal desires to achieve and incentive compatible (or implementable) allocations. In a standard principal-agent model such a conflict may arise when the agent's type and quality directly affect the principal's utility. Our model shows that nonresponsiveness can arise naturally in a two-sided platform because of the double role played by an agent's type: an agent's type determines not only the agent's private benefit (as usual in the principal-agent literature), but also the externalities the agent generates to the other side.

Comparing the first-best quality schedule with the second-best one by distinguishing different levels of conflict between two sides yields:

**Proposition 3** *Consider the baseline model. When we compare the first-best quality schedule with the second-best one, we find:*

*(i) In the case of a congruence or a low level of conflict between the two sides (i.e., $e_L - e_H \leq \Delta^{FB}$), $q_L^{SB} < q_L^{FB} \leq q_H^{SB} = q_H^{FB}$;*

*(ii) In the case of an intermediate level of conflict between the two sides (i.e., $\Delta^{FB} < e_L - e_H < \Delta^{SB}$), $q_L^{SB} < q_H^{SB} = q_H^{FB} < q_L^{FB}$;*[11]

*(iii) In the case of a high level of conflict between the two sides (i.e., $e_L - e_H > \Delta^{SB}$), $q_H^{FB} < q_L^{SB} = q_H^{SB} < q_L^{FB}$.*

In the case of a congruence or a low level of conflict between the two sides, the first-best and the second-best schedules have the same order (both are increasing). In contrast, in the case of an intermediate level of conflict, the order is reversed: the first-best schedule is decreasing while the second-best one is increasing. There is a downward distortion at the bottom (i.e., $q_L^{SB} < q_L^{FB}$) in all cases.[12] Interestingly, there is an upward distortion at the top when pooling arises: $q_H^{SB} > q_H^{FB}$ for $e_L - e_H > \Delta^{SB}$.

**Remark 1** *Choi et al. (2015) consider two types of content providers on side A and homogeneous consumers on side B. Our model captures their model as a special case as follows:*

$$e_H = \frac{1-\alpha}{\alpha}\theta_H, e_L = \frac{1-\alpha}{\alpha}\theta_L,$$

*where $\alpha \in (0,1]$ is a parameter capturing the business model. They assume $\theta_H > \theta_L$, which always leads to $e_H > e_L$. Hence, the first-best quality schedule is always strictly increasing; neither the case of conflict nor nonresponsiveness is considered in their model.*

[11] When $e_L - e_H = \Delta^{SB}$, we have $q_L^{SB} = q_H^{FB} = q_H^{SB} < q_L^{FB}$.

[12] Hence, we can have $q_L^{SB} = 0 < q_L^{FB}$: the exclusion of low types can occur in the second-best while it does not in the first-best. We provide analysis of shutdown in Section 5 where we consider binary types, and in Appendix A where we consider a continuum of types.

## 4 A General Model: Type-dependent Spence Effects

We now consider a general model that extends the baseline model of Section 2 by introducing heterogeneity on side $B$, so that multiple types exist on both sides. The main goal of this extension is to show that type-dependent Spence effects can lead to pooling even if there is a congruence between the two sides (more generally, even if the first-best quality schedule is strictly increasing), which is not possible without the heterogeneity on side $B$.

On side $A$, we maintain the same model as the two-type case in the previous section. On side $B$, we introduce two types: each agent on side $B$ has a type $\theta^B$ that belongs to the set $\Theta^B = \left\{\theta_l^B, \theta_h^B\right\}$, where we use $l$ ($h$) for a low (high) type on side $B$. $\nu^B \in (0,1)$ denotes the probability that $\theta^B$ is equal to $\theta_l^B$.

When a type $\theta^A$ agent of side $A$ interacts with a type $\theta^B$ agent of side $B$ and obtains quality $q$ from the platform, his gross surplus depends on both types $\theta^A$ and $\theta^B$ as follows:

$$b(\theta^A, \theta^B)u^A(q).$$

Under our assumption that each agent of a given side interacts with all (or a random subset of) agents on the other side, the total gross surplus that a type $\theta^A$ agent of side $A$ obtains from interacting with all agents of side $B$ is given by

$$\text{PRIVATE BENEFIT:} \quad \left[\nu^B b(\theta^A, \theta_l^B) + (1-\nu^B) b(\theta^A, \theta_h^B)\right] u^A(q).$$

The term $\nu^B b(\theta^A, \theta_l^B) + (1-\nu^B) b(\theta^A, \theta_h^B)$ is what matters for an agent on side $A$; hence *with some abuse of notation*, we use $\theta^A$ to denote it. This enables us to maintain the same notation as in Section 3, where $\theta_l^B = \theta_h^B$. We assume $0 < \theta_L^A < \theta_H^A$: a high type obtains a greater benefit of interaction than a low type does.

On side $B$, the externality term depends on both types $\left(\theta^A, \theta^B\right)$ and $q^A$ such that the gross utility of an agent of side $B$ is given by

$$\text{CROSS-SIDE EXTERNALITIES:} \quad e(\theta^B, \theta^A) u^B(q^A),$$

where $e(\theta^B, \theta^A) > 0$.

We consider that the platform uses the following mechanism $\left\{ \left(q^A(\theta^A), p^A(\theta^A)\right)_{\theta^A \in \left\{\theta_L^A, \theta_H^A\right\}}, p^B \right\}$ where the platform chooses a single price for side $B$. This implies that the quality assigned to a type $\theta^A$ agent depends only on his own type but does not depend on the types of the agents on side $B$. We can consider an alternative mechanism in which the quality depends on both types

$\{q^A(\theta^A,\theta^B), p^A(\theta^A), p^B(\theta^B)\}$ for $\theta^A \in \{\theta^A_L, \theta^A_H\}$ and $\theta^B \in \{\theta^B_l, \theta^B_h\}$. Depending on the applications, one mechanism makes more sense than the other. If $q^A$ represents the number of newspaper articles or the quality of content delivery or the level of privacy protection in Table 1 of Section 2, a mechanism specifying $q^A(\theta^A)$ is appropriate. In contrast, in the case of an advertising platform in which $q^A$ decreases with amount of advertising, a mechanism specifying $q^A(\theta^A,\theta^B)$ is appropriate if the platform can determine the amount of advertising that a $\theta^B$ type of advertiser can show to a $\theta^A$ type of consumer (see Lin (2020) for a related analysis).

Therefore, the total gross surplus that a type $\theta^B$ agent of side $B$ obtains from interacting with all agents of side $A$ is given by

$$\nu^A e(\theta^B,\theta^A_L)u^B(q^A(\theta^A_L)) + (1-\nu^A)e(\theta^B,\theta^A_H)u^B(q^A(\theta^A_H)).$$

As there are two types on each side, we simplify notation by defining

$$e_{jk} \equiv e(\theta^B,\theta^A) \quad \text{if } \theta^B = \theta^B_j \text{ and } \theta^A = \theta^A_k, \text{ for } j = l,h \text{ and } k = L,H.$$

For instance, $e_{lH}$ is the externality to a low-type agent of side $B$ from a high-type agent of side $A$. Generically, either $\nu^A e_{hL} + (1-\nu^A)e_{hH} > \nu^A e_{lL} + (1-\nu^A)e_{lH}$ or $\nu^A e_{hL} + (1-\nu^A)e_{hH} < \nu^A e_{lL} + (1-\nu^A)e_{lH}$ holds. In the latter case, we can rename types and call a low type (of side $B$) a high type and vice versa. Therefore, without loss of generality, we can restrict attention to the case in which the following inequality holds:

$$\nu^A e_{hL} + (1-\nu^A)e_{hH} > \nu^A e_{lL} + (1-\nu^A)e_{lH}. \tag{12}$$

The above inequality means that the total marginal surplus from a unit increase in $u^B$ is greater for a high type than for a low type on side $B$.

**Definition (type reversals)** Given (12), we can identify three sub-cases depending on the signs of $e_{hH} - e_{lH}$ and of $e_{hL} - e_{lL}$. We say that on side $B$ there is

$$\begin{cases} \text{no type reversal} & \text{if } e_{hH} - e_{lH} > 0 \text{ and } e_{hL} - e_{lL} > 0; \\ \text{type reversal with positive sorting} & \text{if } e_{hH} - e_{lH} > 0 > e_{hL} - e_{lL}; \\ \text{type reversal with negative sorting} & \text{if } e_{hL} - e_{lL} > 0 > e_{hH} - e_{lH}. \end{cases}$$

For instance, the type reversal with positive sorting occurs if, conditional on interacting with a high type on side $A$, the agent on side $B$ prefers to have a high type, but conditional on interacting with a low type on side $A$, the agent on side $B$ prefers to have a low type.

The total gross externalities a type $\theta^A$ agent generates to all agents of side $B$ are given by

$$\left[\nu^B e(\theta_l^B, \theta^A) + (1-\nu^B) e(\theta_h^B, \theta^A)\right] u^B(q^A(\theta^A)).$$

In what follows, we use simpler notations. First, we define

$$e_L^a \equiv \nu^B e_{lL} + (1-\nu^B) e_{hL} \qquad \text{and} \qquad e_H^a \equiv \nu^B e_{lH} + (1-\nu^B) e_{hH}$$

where $e_k^a$ represents the *average externality* exerted by a type $k$ agent of side $A$ ($k = L, H$). We also succinctly notate

$$q_L \equiv q^A(\theta_L^A) \qquad \text{and} \qquad q_H \equiv q^A(\theta_H^A).$$

## 4.1 First-best

The total welfare is given by

$$\nu^A \left(\theta_L^A u^A(q_L) + e_L^a u^B(q_L) - c q_L\right) + (1-\nu^A)\left(\theta_H^A u^A(q_H) + e_H^a u^B(q_H) - c q_H\right).$$

The first-best quality schedule is characterized by the two first-order conditions:

$$\theta_L^A u^{A\prime}(q_L^{FB}) + e_L^a u^{B\prime}(q_L^{FB}) = c, \qquad \theta_H^A u^{A\prime}(q_H^{FB}) + e_H^a u^{B\prime}(q_H^{FB}) = c. \tag{13}$$

which is analogous to (4), after replacing $e_L$ with $e_L^a$ and $e_H$ with $e_H^a$. That is, in the first-best, when one computes the marginal surplus from interaction between a given type agent of side $A$ with all agents of side $B$, one should consider the benefit of the agent on side $A$ and the *average externality* he exerts to side $B$. Applying Proposition 1 to (13) yields:

**Proposition 4** *Consider the general model in which the agents on each side are heterogeneous. The first-best quality schedule* $\left(q_L^{FB}, q_H^{FB}\right)$ *is determined by (13). Given* $(\theta_H^A, \theta_L^A, e_H^a)$*, there is a threshold* $\Delta^{FB} > 0$ *such that* $q_L^{FB} \le q_H^{FB}$ *if and only if* $e_L^a - e_H^a \le \Delta^{FB}$*. The threshold* $\Delta^{FB}$ *is identical to the one in Proposition 1 as long as* $(\theta_H, \theta_L, e_H) = (\theta_H^A, \theta_L^A, e_H^a)$

## 4.2 Second-best

The aim of the second-best analysis is to show that pooling can arise due to conflict within the value capture side even when the first-best quality schedule is increasing. Since pooling never arises if the first-best quality schedule is increasing in the case of type reversal with negative sorting (see

Lemma 2 in Appendix B), we here consider only no type reversal and type reversal with a positive sorting and relegate the analysis of type reversal with negative sorting to Appendix B.[13]

Based on Proposition 4, we adapt the previous definition of congruence (and conflict) between the two sides as follows:

**Definition (Congruence vs. Conflict between the two sides)** There is a *congruence* (*conflict*) between the value creation side and the value capture side if $e^a_H \geq e^a_L$ ($e^a_H < e^a_L$).

We also define a *congruence (and conflict) within* side $B$ between the average agent and the marginal agent. We consider that in the definition, the marginal agent has the low type and later on show that this is true.

**Definition (Congruence vs. Conflict within a side)**

- There is a *congruence within side B* between the average agent and the marginal agent if we have either $e^a_H > e^a_L$ and $e_{lH} > e_{lL}$, or $e^a_H < e^a_L$ and $e_{lH} < e_{lL}$;
- There is a *conflict within side B* between the average agent and the marginal agent if we have either $e^a_H > e^a_L$ and $e_{lH} < e_{lL}$, or $e^a_H < e^a_L$ and $e_{lH} > e_{lL}$.

The platform proposes a menu $\{(q_L, p^A_L), (q_H, p^A_H)\}$ to side $A$ and a single subscription price $p^B$ to side $B$ under asymmetric information about types on both sides. We assume that the platform serves both types of agents on each side. Then the profit of the platform is given by $\pi$ in (7). The platform maximizes $\pi$ subject to the incentive constraints on side $A$ (i.e., (5) and (6)), the individual rationality constraints on side $A$ (i.e., (2) and (3)), and the individual rationality constraints on side $B$ (i.e., $(IR^B_h)$, and $(IR^B_l)$), where

$$(IR^B_j) \quad \nu^A e_{jL} u^B(q_L) + (1-\nu^A) e_{jH} u^B(q_H) - p^B \geq 0 \text{ for } j = l, h.$$

As in Section 3, $(q_L, q_H)$ need to satisfy the monotonicity constraint (8).

In the case of no type reversal or type reversal with positive sorting, we find that $(IR^B_l)$ is necessary and sufficient to induce full participation on side $B$, since $q_L \leq q_H$ must hold because of (8). Therefore, the price on side $B$ is

$$p^B = \nu^A e_{lL} u^B(q_L) + (1-\nu^A) e_{lH} u^B(q_H).$$

[13] Interestingly, we show that in the case of type reversal with negative sorting, the individual rationality constraint on side $B$ can bind for low type only, for high type only, or for both types.

Then the platform maximizes the following profit, subject to (8):

(14)

$$\Pi(q_L,q_H) = \nu^A \left(\theta_L^{Av} u^A(q_L) + e_{lL} u^B(q_L) - cq_L\right) + (1-\nu^A)\left(\theta_H^A u^A(q_H) + e_{lH} u^B(q_H) - cq_H\right)$$

where $\theta_L^{Av}$ is the same as the virtual valuation $\theta_L^v$ we saw in Section 3. Let $(q_L^*, q_H^*)$ denote the solution that maximizes $\Pi$ when (8) is neglected: $(q_L^*, q_H^*)$ satisfies the first-order conditions:

$$\theta_L^{Av} u^{A\prime}(q_L^*) + e_{lL} u^{B\prime}(q_L^*) = c, \qquad \theta_H^A u^{A\prime}(q_H^*) + e_{lH} u^{B\prime}(q_H^*) = c. \tag{15}$$

Comparing (15) with (10), the first-order conditions characterizing $(q_L^*, q_H^*)$ in Section 3, reveals that the only difference lies in that $e_L$ is replaced by $e_{lL}$ and $e_H$ by $e_{lH}$. This is because the externality is now evaluated with the valuation of the marginal type on side $B$, that is, the low type. In contrast, under complete information on side $B$, the externality is evaluated with the average valuation. This is clearer when we compare (13) with (15): then $e_L^a$ is replaced by $e_{lL}$ and $e_H^a$ by $e_{lH}$. This distortion is known as the Spence effect (1975). Weyl (2010) identifies the Spence (1975) distortion in a two-sided platform in terms of the level of participation on one side. In our model, the Spence effect is type-dependent because the platform offers a menu of qualities on side $A$ and hence the distortion in quality generated by the effect differs depending on whether $q = q_L$ or $q = q_H$. Our novelty is to show that this type-dependent Spence effect can render pooling optimal even when the first-best quality schedule is increasing.[14]

Note that the optimal pooling quality $q^P$ is determined as follows:

$$\theta_L^A u^{A\prime}(q^P) + (\nu^A e_{lL} + (1-\nu^A) e_{lH}) u^{B\prime}(q^P) = c. \tag{16}$$

Applying Proposition 2 to (15) yields:

**Proposition 5** *Consider the case of no type reversal or type reversal with positive sorting in the general model. The second-best quality schedule $\left(q_L^{SB}, q_H^{SB}\right)$ is characterized as follows. Given $(\theta_H^A, \theta_L^A, e_{lH})$, there is a threshold $\Delta^{SB}$, which is the same as the one in Proposition 2 as long as $(\theta_H, \theta_L, e_H) = (\theta_H^A, \theta_L^A, e_{lH})$, such that*

*(i) $q_L^{SB} < q_H^{SB}$ if and only if $e_{lL} - e_{lH} < \Delta^{SB}$, where $q_L^{SB} = q_L^*$ and $q_H^{SB} = q_H^*$ from (15);*

[14] One might wonder whether Spence distortions arise because of the assumption that the platform cannot price discriminate side $B$. Suppose that we allow the platform to price discriminate side $B$ by using a screening instrument $q^B$. Then, we should discuss whether $q^A$ is substitute to or independent of or complementary to $q^B$ when they enter into the payoff function of each agent on side $i$ ($i = A, B$). This adds complexity to the analysis. However, in an earlier version (Jeon et al. 2016), we studied the case in which $q^A$ and $q^B$ enter into the payoff function in a separable way and obtained the same first-order conditions as (15) regarding $\left(q_L^{A*}, q_H^{A*}\right)$.

*(ii) Otherwise, pooling is optimal:* $q_L^{SB} = q_H^{SB} = q^P$*, where* $q^P$ *satisfies (16).*

From Proposition 5 (and Proposition 4), we obtain the following corollary.

**Corollary 1** *Consider the case of no type reversal or type reversal with positive sorting in the general model.*

*(i) If* $e_{lL} - e_{lH} \geq \Delta^{SB}$ *and* $e_L^a \leq e_H^a$*, pooling is optimal even if there is a congruence between the two sides. Under the condition, there is a strong conflict within side B.*

*(ii) If* $e_{lL} - e_{lH} \geq \Delta^{SB}$ *and* $e_L^a - e_H^a < \Delta^{FB}$*, pooling is optimal even if the first-best quality schedule is strictly increasing.*

As the congruence between the two sides implies that the first-best quality schedule is strictly increasing, the condition in Corollary 1(i) is more restrictive than the condition in Corollary 1(ii). The condition in Corollary 1(i) clearly shows that pooling occurs because of the conflict within side $B$: even if the average agents of side $B$ prefer interacting with a high type to interacting with a low type, the marginal type of side $B$ strongly prefers interacting with a low type to interacting with a high type. This implies that $q_L^*$ is larger than $q_H^*$ and hence the platform chooses pooling. Below, we provide an example in which pooling arises under congruence between the two sides.

**Example 3** *Consider* $u^A(q) = u^B(q) = \ln(1+q)$ *and* $c = 1$ *(as in Example 2). Suppose* $\nu^A = 1/2$*,* $\theta_L = 17$*,* $\theta_H = 18$*, which implies* $\theta_L^\nu = 16$*. Suppose also* $\nu^B = 0.55$*,* $e_{lL} = 6$*,* $e_{hL} = 7$*,* $e_{lH} = 3$ *and* $e_{hH} = 11$*, implying that there is no type reversal. As* $e_L^a = 6.45 < e_H^a = 6.6$*, there is a congruence between the two sides and thus the first-best quality schedule is strictly increasing. Precisely, we have* $q_L^{FB} = 22.45 < q_H^{FB} = 23.6$*.* $e_{lL} - e_{lH} = 3 >> e_H^a - e_L^a = 0.15$ *implies that there is a strong conflict within side B.*

*Under asymmetric information about types, from (15),* $q_L^* = 21 > q_H^* = 20$*. Therefore, pooling arises in the second-best and we have* $q^P = 20.5$*. In this analysis of pooling,* $p^B$ *is chosen to induce full participation of both types (i.e., the IR constraint binds for low types) on side B and the profit is 45.463. When we consider an alternative in which* $p^B$ *is chosen to induce participation of high types only on side B,*[15] *it is optimal to choose* $q_L = 18.15$ *and* $q_H = 21.95$*, which generates a profit of 44.173. Therefore, pooling with full participation on side B is optimal.*

[15] Recall that in this section, we assume that the platform induces full participation by agents on side $B$. However, in this example, we verify that full participation is indeed optimal.

**Remark 2** *Böhme (2016) considers two types on both sides and screening on both sides as we did in an earlier version of our paper (see footnote 14). He assumes that type j on side k has an intrinsic utility $\theta_j^k u(q_j^k)$ from joining the platform, and an indirect network effect from interacting with the agents on the other side, side h, which (in terms of our notation) is equal to $\nu^h e_{jL}^k q_L^h + (1-\nu^h) e_{jH}^k q_H^h$ with $e_{lL}^k = e_{lH}^k < e_{hL}^k = e_{hH}^k$; hence, the network effect depends on the quality received by the agent on side h he interacts with, but does not depend on that agent's type, since $e_{lL}^k = e_{lH}^k$ and $e_{hL}^k = e_{hH}^k$. From $e_{lL}^k = e_{lH}^k < e_{hL}^k = e_{hH}^k$, we see that Böhme (2016) considers only a special case of no type reversal in which no agent directly cares about the type of the agent he interacts with. This in turn implies that there is no conflict within a side in terms of the preferred type to interact with. Moreover, as both types generate the same level of externalities to the other side, he considers the limit case in which neither congruence nor conflict (in a strict sense) exists between the two sides. As a consequence, pooling never arises in his model.*

## 5 Welfare Analysis

In this section, we study the effects of price discrimination on welfare. We first consider the baseline model of Section 2, in which agents are heterogeneous on side $A$ but homogeneous on side $B$. Then we consider the model of Section 4, in which agents are heterogeneous on both sides.

### 5.1 Heterogeneous agents on side $A$ only

We here consider the baseline model of Section 2. We first extend to a two-sided market the conventional wisdom that prohibiting price discrimination can decrease welfare by inducing the exclusion of some consumers. And then, we provide two other interesting welfare results.

Given $(q_L, q_H)$, the welfare is given by

$$(17)\quad W(q_L,q_H) \equiv \nu\left(\theta_L u^A(q_L) + e_L u^B(q_L) - cq_L\right) + (1-\nu)\left(\theta_H u^A(q_H) + e_H u^B(q_H) - cq_H\right),$$

where $\nu = \nu^A$. Under price discrimination, the quality schedule is determined by Proposition 2 and welfare is $W^{PD} \equiv W(q_L^{SB}, q_H^{SB})$. We compare $W^{PD}$ with the welfare that arises when price discrimination is prohibited, denoted by $W^N$. Since no difference arises if pooling is optimal in the second-best, we focus on the case of $q_H^*(= q_H^{SB} = q_H^{FB}) > q_L^*(= q_L^{SB})$ (see (10)) and therefore $W^{PD} = W(q_L^*, q_H^*)$. Note that we consider $\nu > \hat{\nu} = \frac{\theta_H - \theta_L}{\theta_H} > 0$, so as to make the virtual valuation $\theta_L^\nu$ strictly positive.

We first investigate the question of how price discrimination affects the exclusion of consumers. Consider the case of no price discrimination. Then, the platform offers a single contract $(q, p)$ and a type $\theta$ consumer accepts it if and only if $\theta u^A(q) - p \geq 0$. The platform can serve either both types or high types only.

- In the first case, $p = \theta_L u^A(q)$. Then, the optimal $q$ coincides with $q^P$ in (11). Let $\pi_L$ denote the resulting profit:

$$\pi_L \equiv \theta_L u^A(q^P) + (\nu e_L + (1-\nu)e_H)u^B(q^P) - cq^P.$$

  The corresponding welfare is

$$W(q^P, q^P) = \nu \left(\theta_L u^A(q^P) + e_L u^B(q^P) - cq^P\right) + (1-\nu)\left(\theta_H u^A(q^P) + e_H u^B(q^P) - cq^P\right) \quad (18)$$

- In the second case, $p = \theta_H u^A(q)$. Then, the optimal $q$ is $q_H^{FB}$. The resulting profit is $(1-\nu)\pi_H$, where

$$\pi_H \equiv \theta_H u^A(q_H^{FB}) + e_H u^B(q_H^{FB}) - cq_H^{FB}.$$

  The corresponding welfare is equal to $(1-\nu)\pi_H$ because of full surplus extraction on both sides.

Therefore, the choice of the platform between the two cases is determined by the sign of $D(\nu) = (1-\nu)\pi_H - \pi_L$, a function defined in the interval $[\hat{\nu}, 1)$.[16] Immediately, we see that $D(\nu) < 0$ if $\nu$ is close to 1 and we show, in the proof of Proposition 6, that $D$ is strictly decreasing in $\nu$. Hence, if $D(\hat{\nu}) > 0$, there exists a unique threshold $\nu^N \in (\hat{\nu}, 1)$ satisfying $D(\nu) = 0$ such that the platform serves both types when $\nu \geq \nu^N$ but excludes low types when $\hat{\nu} < \nu < \nu^N$. In the opposite case of $D(\hat{\nu}) \leq 0$, the platform serves both types for any $\nu > \hat{\nu}$ ; then we set $\nu^N$ equal to $\hat{\nu}$.

In the case of price discrimination, let $\nu^{PD}(\geq \hat{\nu})$ represent the threshold value of $\nu$, above which the platform serves both types. $q_L^* > 0$ if and only if $\theta_L^\nu u^{A\prime}(0) + e_L u^{B\prime}(0) > c$. Since $\theta_L^\nu > 0$, $e_L \geq \frac{c}{u^{B\prime}(0)}$ is sufficient to make $q_L^* > 0$. In other words, if $e_L \geq \frac{c}{u^{B\prime}(0)}$, the platform serves both types for any $\nu > \hat{\nu}$; in this case, we set $\nu^{PD} = \hat{\nu}$. If, instead, $e_L < \frac{c}{u^{B\prime}(0)}$, then the inequality $\theta_L^\nu u^{A\prime}(0) + e_L u^{B\prime}(0) > c$ is equivalent to

$$\nu > \frac{\theta_H - \theta_L}{\theta_H - \frac{c}{u^{A\prime}(0)} + \frac{u^{B\prime}(0)}{u^{A\prime}(0)} e_L} \equiv \nu^{PD}(> \hat{\nu}).$$

[16] In order to simplify the presentation, from now on we consider the case of $e_L \geq e_H$. But the proof of Proposition 6 also covers the case of $e_L < e_H$.

We establish in the proof of Proposition 6(i) that

$$v^{PD} \leq v^N, \text{ with equality if and only if } v^{PD} = v^N = \hat{v}. \tag{19}$$

Therefore, if the exclusion of low types occurs under price discrimination, it also occurs without price discrimination; but the reverse does not hold. Conditional on the exclusion of low types, $W^{PD} = W^N$ holds, since high types consume $q_H^{FB}$ in both regimes. But if the exclusion occurs only under no price discrimination, $W^{PD} > W^N$ holds, because high types still consume $q_H^{FB}$ in both regimes but the welfare from low types' consumption of $q_L^* > 0$ is strictly positive under price discrimination. Therefore, our finding extends to a two-sided market the conventional wisdom that prohibiting price discrimination can reduce welfare by inducing the exclusion of some consumers.

We also identify another case in which $W^{PD} > W^N$ (Proposition 6(ii)) and the opposite case in which $W^{PD} < W^N$, showing that prohibiting price discrimination increases welfare (Proposition 6(iii)).

**Proposition 6** *Consider the baseline model with binary types on side A.*

*(i) If exclusion of low types occurs under price discrimination, then it occurs also without price discrimination, and therefore banning price discrimination does not affect welfare: $W^{PD} = W^N$. However, it is possible that the exclusion occurs only when price discrimination is banned. In this case, welfare is higher under price discrimination than without price discrimination: $W^{PD} > W^N$.*

*(ii) The welfare under price discrimination is higher than the welfare without price discrimination if most agents are of a low type: $W^{PD} > W^N$ if $v$ is close to 1.*

*(iii) Consider the case of conflict between the two sides. Then, prohibiting price discrimination yields a higher welfare, i.e., $W^N > W^{PD}$ if $q_L^*$ and $q_H^*$ from (10) are so close that pooling is almost optimal under price discrimination.*

***Proof.*** *See Appendix C.* ■

Let us first explain Proposition 6(iii), which we find the most interesting. It shows that prohibiting price discrimination improves welfare in the case of strong conflict between the two sides. If the strong conflict leads to pooling under price discrimination, then prohibiting price discrimination has no consequence. Hence, Proposition 6(iii) focuses on the case in which the outcome under price discrimination is close to pooling; in this case, prohibiting price discrimination strictly increases welfare. To fix the ideas, consider parameters such that $q_L^* = q_H^*$, which means that pooling "just" occurs with price discrimination and $q^P = q_L^{SB} = q_H^{SB} = q_L^* = q_H^* = q_H^{FB} < q_L^{FB}$. Then consider a small decrease in $e_L$,[17] and notice that this implies

(a) (under no price discrimination) a decrease in $q^P$ because $\nu e_L + (1-\nu)e_H$ decreases;

(b) (under price discrimination) (1) an even stronger decrease in $q_L^{SB}$ because the externality term associated with $q_L^{SB}$ is $e_L u^B(q)$, whereas the externality term associated with $q^P$ is $(\nu e_L + (1-\nu)e_H)u^B(q)$; (2) no change in $q_H^{FB}$.

Hence, under no price discrimination, (a) implies that the small decrease in $e_L$ reduces $W^N$ mainly through welfare reduction from low types, because welfare reduction from high types is negligible as $q^P$ is initially equal to $q_H^{FB}$. Similarly, under price discrimination, (b) implies that the small decrease in $e_L$ reduces $W^{PD}$ only through welfare reduction from low types. However, $q_L^{SB}$ decreases more than $q^P$ does, implying that welfare reduction in $W^{PD}$ is larger than welfare reduction in $W^N$.

Proposition 6(ii) is rather straightforward. Recall that we consider $q_L^*(= q_L^{SB}) < q_H^*(= q_H^{FB})$. Given that $\nu$ is close to 1, we have that $\theta_L^\nu$ is close to $\theta_L$ and $\nu e_L + (1-\nu)e_H$ is close to $e_L$, implying that both $q_L^*(= q_L^{SB})$ and $q^P$ are close to $q_L^{FB}$. This together with $\theta_L u^{A\prime}(q_L^{FB}) + e_L u^{B\prime}(q_L^{FB}) - c = 0$ implies that welfare from low types under price discrimination is equal to welfare from low types without price discrimination, up to a second order term in $1-\nu$. In contrast, welfare from high types is maximized under price discrimination since $q_H^* = q_H^{FB}$, whereas without price discrimination, high types consume $q^p$, which is strictly smaller than $q_H^{FB}$. Therefore, the welfare gain from high types generated by price discrimination is of the first order in $1-\nu$ and dominates the second order effect mentioned above, implying $W^{PD} > W^N$.

## 5.2 Heterogeneous agents on both sides

Consider now the model of Section 4, in which there are two types on each side. The welfare is given as follows:

$$
\begin{aligned}
W(q_L, q_H) \equiv\ & \nu^A \left[\theta_L u^A(q_L) + (\nu^B e_{lL} + (1-\nu^B)e_{hL})u^B(q_L) - cq_L\right] \\
& + (1-\nu^A)\left[\theta_H u^A(q_H) + (\nu^B e_{lH} + (1-\nu^B)e_{hH})u^B(q_H) - cq_H\right].
\end{aligned}
$$

[17] This requires that initially $e_L$ is positive, but in fact it is impossible to have $q_L^* = q_H^*$ if $e_L = 0$.

It is useful to decompose the welfare as follows:

$$W(q_L,q_H) = \overline{W}(q_L,q_H) + (1-\nu^B)d(q_L,q_H), \tag{20}$$

in which

$$\begin{aligned}
\overline{W}(q_L,q_H) &= \nu^A\left(\theta_L u^A(q_L) + e_{lL}u^B(q_L) - cq_L\right) + (1-\nu^A)\left(\theta_H u^A(q_H) + e_{lH}u^B(q_H) - cq_H\right) \\
d(q_L,q_H) &= \nu^A(e_{hL}-e_{lL})u^B(q_L) + (1-\nu^A)(e_{hH}-e_{lH})u^B(q_H).
\end{aligned}$$

$\overline{W}(q_L,q_H)$ can be interpreted as the welfare defined in (17) when agents on side $B$ are homogeneous, where the externality terms $e_L$ and $e_H$ are replaced by $e_{lL}$ and $e_{lH}$. The term $d(q_L,q_H)$ represents the difference between the gross surplus of a high type and the gross surplus of a low type on side $B$. Moreover, we define

$$\begin{aligned}
W^{PD} &= \overline{W}(q_L^*,q_H^*) + (1-\nu^B)d(q_L^*,q_H^*); \\
W^N &= \overline{W}(q^P,q^P) + (1-\nu^B)d(q^P,q^P)
\end{aligned}$$

in which $q_L^*$ and $q_H^*$ are determined by (15), and $q^P$ satisfies (16).

By using the definition of type reversal with positive (or negative) sorting, we immediately have the following lemma:

**Lemma 1** *Consider the general model with binary types on each side. Suppose that $q_L^* < q^P < q_H^*$.*

*(i) Under type reversal with positive sorting, if price discrimination increases $\overline{W}$ (i.e., $\overline{W}(q_L^*,q_H^*) \geq \overline{W}(q^P,q^P)$), then price discrimination increases welfare as well ($W^{PD} > W^N$).*

*(ii) Under type reversal with negative sorting, if no price discrimination increases $\overline{W}$ (i.e., $\overline{W}(q^P,q^P) \geq \overline{W}(q_L^*,q_H^*)$), then no price discrimination increases welfare as well ($W^N > W^{PD}$).*

For instance, consider the case that $\overline{W}(q^P,q^P) \geq \overline{W}(q_L^*,q_H^*)$ and type reversal occurs with negative sorting. Because this means $e_{hL} - e_{lL} > 0 > e_{hH} - e_{lH}$ and $q_L^* < q^P < q_H^*$, it follows that $d(q^P,q^P) > d(q_L,q_H)$, which in turn implies $W^N > W^{PD}$.

Combining Lemma 1 with Proposition 6(ii) and (iii), we can extend the welfare results obtained with homogeneous agents on side $B$ to the case of heterogeneous agents on side $B$ as Proposition

7(ii) and (iii), below.[18] Importantly, Proposition 7(i) extends Proposition 6(i) to the case of no type reversal.

**Proposition 7** *Consider the general model with binary types on each side.*

*(i) Suppose there is no type reversal. If exclusion of low types occurs under price discrimination, then it occurs also without price discrimination, and in this case $W^{PD} = W^N$. However, it is possible that the exclusion occurs only when price discrimination is banned. In this case, $W^{PD} > W^N$.*

*(ii) $W^{PD} > W^N$ if $\nu$ is close to 1 and there is type reversal with positive sorting.*

*(iii) $W^N > W^{PD}$ if there is type reversal with negative sorting and pooling is almost optimal under price discrimination.*

***Proof.*** *See Appendix C.* ■

## 6 Applications to media platforms

In this section, we provide two applications of our baseline model to media platforms. The first one applies the model specified in (1) to online newspapers. We assume that newspaper articles are bundled with a fixed proportion of advertisements per article and examine when we should see a conflict or a congruence between the two sides. The second one is different from the model specified in (1), although it belongs to the general framework of the baseline model presented in Section 2.1. It analyzes the advertising policy of a media platform and illustrates when pooling is optimal. Therefore, the two applications are complementary.

### 6.1 Application I: online newspapers

Consider the model specified in (1). Suppose that $q$ represents the number of articles in an online newspaper a consumer can consume. Since newspaper articles are bundled with a fixed proportion of advertisements per article, we here assume that the amount of advertising linearly increases with $q$. Hence, for expositional simplicity, we let $q$ represent both the number of articles and the amount of advertising. Agents on side $A$ are consumers and agents on side $B$ are advertisers. Consumers enjoy reading articles but suffer from advertising nuisance.

[18] By Lemma 2 in Appendix B, in order for almost pooling to be almost optimal, as required by Proposition 7(iii), it is necessary that there is conflict between the two sides. Moreover, the fact that almost pooling is optimal implies that $(IR_l)$ binds on side $B$, thus the quality schedule is determined by (15).

We consider two types of consumers, rich and poor. Given $q$ amount of advertising, an advertiser's ad revenue from a rich consumer is $e^{rich}u^B(q)$ and his ad revenue from a poor consumer is $e^{poor}u^B(q)$. We assume

$$e^{rich} > e^{poor} > 0.$$

The assumption means that a rich consumer is more valuable than a poor consumer to advertisers is. Given $q$ amount of articles, a consumer's utility from reading net of ad nuisance is $\eta^{rich}u^A(q)$ or $\eta^{poor}u^A(q)$, depending on her type. Our main assumption is that $\eta^{rich}$ and $\eta^{poor}$ can be decomposed as follows:

$$\eta^{rich} = b^{rich} - d^{rich} > 0 \qquad \text{and} \qquad \eta^{poor} = b^{poor} - d^{poor} > 0,$$

where for instance $b^{rich}$ represents a rich consumer's benefit from reading articles and $d^{rich}$ her disutility from exposure to advertisement; $b^{poor}$ and $d^{poor}$ are similarly defined. We assume

$$b^{rich} > b^{poor} \qquad \text{and} \qquad d^{rich} > d^{poor}$$

The assumption means that a rich consumer obtains a larger benefit from reading but suffers a larger ad nuisance than a poor consumer does. However, the assumption does not allow us to determine the sign of $\eta^{rich} - \eta^{poor}$ .

When $\eta^{rich} > \eta^{poor}$ holds, we define type $H$ as a rich consumer and type $L$ as a poor consumer, and hence we have:

$$\theta_H = \eta^{rich} > \theta_L = \eta^{poor} \qquad \text{and} \qquad e_H = e^{rich} > e_L = e^{poor}.$$

In this case, we have a congruence between the two sides. In contrast, when $\eta^{rich} < \eta^{poor}$ holds, we define type $H$ as a poor consumer and type $L$ as a rich consumer, and thus we have:

$$\theta_H = \eta^{poor} > \theta_L = \eta^{rich} \qquad \text{and} \qquad e_H = e^{poor} < e_L = e^{rich}.$$

In this case we have a conflict between the two sides.

When there is a congruence or a minor conflict between the two sides, the second-best quantity schedule exhibits $q_H^{SB} > q_L^{SB}$, implying that high types consume more news articles and thereby have more exposure to advertising than low types do. This is consistent with the real world practice of newspapers. For instance, paid subscribers to the *Wall street Journal* and the *New York Times* have unlimited access to the content and thus have more ad exposure than do unsubscribed consumers having limited access to content.

## 6.2 Application II: advertising policy of a media platform

### 6.2.1 The model

A media platform provides an online content service to consumers and monetizes consumers' attention by selling advertising. We study the design of its advertising policy. The model is similar to the baseline two-type model in Section 2. There are two types of consumers, $\theta(=\theta^A) \in \{H, L\}$ of a mass one. A consumer's type captures her sensitivity to ads nuisance. Let $\nu \in (0,1)$ denote the proportion of low type consumers with $\theta = L$. Let $a \in [0,1]$ index the advertising amount per consumer, ranging from the ad-free environment $a = 0$ to the highest possible number of ads $a = 1$.

Given $a$, a consumer of type $\theta \in \{L, H\}$ earns the following utility from joining the platform:

$$u_0 - C_\theta(a),$$

where $u_0 > 0$ is a constant surplus from consuming the content and $C_\theta(a)$ is the disutility from ad nuisance. We assume that $C_\theta(\cdot)$ is strictly increasing and strictly convex with $C_\theta(0) = 0$ for $\theta \in \{L, H\}$.

Given $a$, the platform's advertising revenue from a consumer of type $\theta \in \{L, H\}$ is given by

$$R_\theta(a),$$

where we assume that $R_\theta(\cdot)$ is strictly increasing and strictly concave in $a$ with $R_\theta(0) = 0$.

Regarding how a consumer type affects the disutility and the revenue, we make the following assumptions:

$$C'_H(a) > C'_L(a), \quad \text{for } a \in [0,1]; \tag{21}$$

$$R'_H(a) > R'_L(a), \quad \text{for } a \in [0,1]. \tag{22}$$

On the one hand, a high type experiences a larger marginal nuisance than a low type; this implies $C_H(a) > C_L(a)$, for $a \in [0,1]$. On the other hand, advertisement to a high type generates a larger marginal revenue than advertisement to a low type; this implies $R_H(a) > R_L(a)$ for $a \in [0,1]$.

To make clear the connection with the general framework presented in Section 2, we define

$q^A \equiv 1-a$. Then, in the application, we have

$$U^A(q^A;\theta^A) = u_0 - C_{\theta^A}(1-q^A) \quad U^B(q^A;\theta^A) = R_{\theta^A}(1-q^A).$$

On side $A$, as in the baseline model, $U^A$ increases with $q^A$ and a high type experiences a larger benefit from quality increase than a low type does. On side $B$, there are three differences with respect to the baseline model with $U^B(q^A;\theta^A) = e(\theta^A)u^B(q^A)$. First, $U^B$ decreases with $q^A$ in the application, while $U^B$ increases with $q^A$ in the baseline model. Second, we make the level of participation on side $B$ endogenous.[19] Last, by assumption, high types are more valuable than low types to advertisers are. In other words, there is a conflict between the two sides because maximizing externality on side $B$ requires $q^A_H < q^A_L$, which is in conflict with the implementability condition on side $A$ (i.e., $q^A_H \geq q^A_L$).

The platform proposes a menu of contracts $\{(t_H, a_H), (t_L, a_L)\}$ to consumers, where $t_\theta$ is the payment from a consumer of type $\theta$ to the platform, and $a_\theta$ is the amount of advertising for a type $\theta$ consumer. Then, the platform's profit is

$$\pi = \nu \{t_L + R_L(a_L)\} + (1-\nu)\{t_H + R_H(a_H)\}.$$

The timing of events is given as follows.

Stage 1: The platform proposes $\{(t_H, a_H), (t_L, a_L)\}$.

Stage 2: Each consumer simultaneously decides to accept or reject the offer. If a consumer accepts the offer, she chooses a contract $(t_\theta, a_\theta)$ in the menu and accordingly pays $t_\theta$, and the platform chooses the advertising level $a_\theta$.

### 6.2.2 First-best

Suppose that the platform knows the consumer types. Then the profit-maximizing advertising level for type $\theta$ is a solution to the following problem:

$$\max_{a_\theta} [R_\theta(a_\theta) - C_\theta(a_\theta)]$$

[19] This is the case if there is a mass one of advertisers and each can show at most one unit of advertising per consumer. Then, choosing $a_L$ ($a_H$) is equivalent to choosing the number of advertisers who can show ads to low (high) type consumers. To be precise, suppose for instance that advertisers are heterogeneous only in terms of reservation utility. Then, there is a cut-off level of reservation utility corresponding to $a_L$ (respectively, another cut-off corresponding to $a_H$) such that all advertisers whose reservation utility is smaller than the cut-off will show ads to low (high) type consumers, and in this sense the level of participation is endogenous.

Let $\left(a_L^{FB}, a_H^{FB}\right)$ denote the first-best allocation, which is characterized by

$$R_L'(a_L^{FB}) = C_L'(a_L^{FB}), \quad R_H'(a_H^{FB}) = C_H'(a_H^{FB}). \tag{23}$$

As a high type generates a higher revenue but also suffers a higher nuisance from ads than a low type, both $a_L^{FB} \leq a_H^{FB}$ and $a_L^{FB} > a_H^{FB}$ are possible.

### 6.2.3 Second-best

In the presence of asymmetric information, the platform offers a menu of contracts $\{(t_H, a_H), (t_L, a_L)\}$ to maximize its profit

$$\pi = \nu \{t_L + R_L(a_L)\} + (1-\nu)\{t_H + R_H(a_H)\} \tag{24}$$

subject to

$$\begin{aligned} \mathrm{IR}_L: &\ u_0 - C_L(a_L) - t_L \geq 0 \\ \mathrm{IR}_H: &\ u_0 - C_H(a_H) - t_H \geq 0 \\ \mathrm{IC}_L: &\ u_0 - C_L(a_L) - t_L \geq u_0 - C_L(a_H) - t_H \\ \mathrm{IC}_H: &\ u_0 - C_H(a_H) - t_H \geq u_0 - C_H(a_L) - t_L \end{aligned}$$

Adding $\mathrm{IC}_L$ and $\mathrm{IC}_H$ and using (21) yields the implementability condition

$$a_H \leq a_L. \tag{25}$$

Using $q^A \equiv 1 - a$, the implementability condition (25) can be equivalently written as $q_H^A \geq q_L^A$, which is exactly the condition (8) we had in Section 3.

Since $C_H(a) > C_L(a)$, standard arguments show that we can neglect $\mathrm{IR}_L$, but $\mathrm{IR}_H$ and $\mathrm{IC}_L$ bind in the optimum. From the binding constraints, we obtain an expression for $t_L$ and an expression for $t_H$. Inserting them into $\pi$ in (24) gives the following objective function that the platform maximizes

$$\begin{aligned} u_0 \quad + &\quad (1-\nu)\{R_H(a_H) - C_H(a_H)\} \\ + &\quad \nu\{R_L(a_L) - C_L(a_L) - [C_H(a_H) - C_L(a_H)]\} \end{aligned}$$

subject to (25).

Let $(a_L^{SB}, a_H^{SB})$ denote the second-best allocation, which is the solution to the maximization problem. Let $(a_L^*, a_H^*)$ denote the solution to the maximization problem when (25) is neglected.

Then $(a_L^*, a_H^*)$ satisfies

$$a_L^* = a_L^{FB} \qquad R'_H(a_H^*) = C'_H(a_H^*) + \frac{\nu}{1-\nu}[C'_H(a_H^*) - C'_L(a_H^*)] \tag{26}$$

Thus, we have $a_L^* = a_L^{FB}$ and $a_H^* < a_H^{FB}$.

If $a_H^{FB} \leq a_L^{FB}$ holds, we have $a_H^* < a_H^{FB} \leq a_L^* = a_L^{FB}$, implying that $(a_L^*, a_H^*)$ satisfies (25), and hence $(a_L^{SB}, a_H^{SB}) = (a_L^*, a_H^*)$. However, even if $a_H^{FB} > a_L^{FB}$, it is possible that the downward distortion in $a_H^*$ makes $(a_L^*, a_H^*)$ satisfy (25) and still $(a_L^{SB}, a_H^{SB}) = (a_L^*, a_H^*)$. If, instead, $a_H^* > a_L^*$ (which requires $a_H^{FB} > a_L^{FB}$), then $(a_L^{SB}, a_H^{SB})$ exhibits pooling; that is, $a_L^{SB} = a_H^{SB} = a^P$, where $a^P$ is characterized by

$$(1-\nu)R'_H(a^P) + \nu R'_L(a^P) = C'_H(a^P) \tag{27}$$

The R.H.S. in (27) is written only with respect to the high type's nuisance cost because $\text{IR}_H$ binds.

We have:

**Proposition 8** *Consider the application to the advertising policy of a media platform. The second-best advertising schedule $(a_L^{SB}, a_H^{SB})$ is characterized as follows.*

*(i) If $(a_L^*, a_H^*)$ in (26) satisfies the implementability condition (25), then*

$$a_L^{SB} = a_L^{FB} \geq a_H^{SB} = a_H^* \textit{ (and } a_H^{SB} < a_H^{FB}\textit{)}$$

*(ii) If $(a_L^*, a_H^*)$ in (26) violates the implementability condition (25), then pooling is optimal:*

$$a_H^{FB} > a_H^{SB} = a_L^{SB} = a^P > a_L^{FB},$$

*where $a^P$ is characterized in (27).*

Consider the following quadratic setting: for $\theta = L, H$

$$\begin{aligned} C_\theta(a) &= \alpha_\theta^C a + \frac{1}{2}a^2, \quad \text{with } \alpha_\theta^C > 0 \text{ and } \Delta\alpha^C = \alpha_H^C - \alpha_L^C > 0; \\ R_\theta(a) &= \alpha_\theta^R a - \frac{1}{2}a^2, \quad \text{with } \alpha_\theta^R > 1 \text{ and } \Delta\alpha^R = \alpha_H^R - \alpha_L^R > 0. \end{aligned}$$

To guarantee that $a_\theta^{FB} \in (0,1)$ for $\theta = L, H$, we assume

$$0 < \alpha_H^R - \alpha_H^C < 2, \qquad 0 < \alpha_L^R - \alpha_L^C < 2.$$

Then, we have

$$a_\theta^{FB} = \frac{\alpha_\theta^R - \alpha_\theta^C}{2} \text{ for } \theta = L, H,$$

and

$$a_H^{FB} \gtreqless a_L^{FB} \text{ if and only if } \Delta\alpha^R \gtreqless \Delta\alpha^C.$$

$a_H^*$ is given by

$$a_H^* = a_H^{FB} - \frac{\nu}{1-\nu}\frac{\Delta\alpha^C}{2} < a_H^{FB}.$$

Hence, at $(a_L^*, a_H^*)$, (25) reduces to

$$\frac{\Delta\alpha^C}{(1-\nu)} \geq \Delta\alpha^R.$$

Given $\Delta\alpha^C$, $\Delta\alpha^R$ measures the degree of conflict between the two sides. Hence, in the case of strong conflict (i.e., $\Delta\alpha^R \geq \frac{\Delta\alpha^C}{(1-\nu)}$), pooling is optimal, with $a^P = \nu a_L^{FB} + (1-\nu)a_H^*$.

## 7 Concluding Remarks

In this article, we have studied optimal nonlinear pricing by a two-sided platform that mediates interactions between multiple groups. In this situation, each agent in a group plays a dual role in obtaining the private benefit from interactions and generating externalities to the agents of the other group(s). We show that this dual role is a key to the platform's mechanism design, particularly regarding when the platform finds it better to offer a single contract instead of multiple contracts to different types of agents on one side.

There are two directions in which our work can be further developed. First, it would be interesting to extend our model to competing platforms with nonlinear tariffs. Second, we think it important and interesting to study when price discrimination on one side substitutes for price discrimination on the other side, and when they form complements. In particular, one may be interested in the conflict situation in which the monotonicity constraint binds on side $A$ under no price discrimination on side $B$, but introducing a well-designed price discrimination on side $B$ may relax the conflict such that the implementability condition on side $A$ is relaxed. We hope that the current work serves as a foundation for more studies to understand price discrimination in multisided markets.

# A Appendix: Analysis for a continuum of types

Here we consider a continuum of types: $\theta$ is a realization of a random variable, which is distributed according to a smooth CDF $F$ and associated density $f = F'$ over the support $\Theta^A = [\underline{\theta}, \overline{\theta}]$. In this case, the virtual valuation for type $\theta$ is $\theta^v = \theta - \frac{1-F(\theta)}{f(\theta)}$. We assume that $\theta^{vu} > 0$ for each $\theta$ and that the hazard rate of $F$, $\frac{f(\theta)}{1-F(\theta)}$, is increasing in $\theta$, and hence $\theta^v$ is increasing.

## A.1 First-best

In the first-best, the platform extracts the whole surplus from the homogeneous agents on side $B$ and also from the agents on side $A$, by setting $p(\theta) = \theta u^A(q(\theta))$ for each $\theta$. Hence the first-best schedule $q^{FB}$ maximizes the welfare

$$\int_{\underline{\theta}}^{\overline{\theta}} \left(\theta u^A(q(\theta)) + e(\theta) u^B(q(\theta)) - cq(\theta)\right) f(\theta) d\theta$$

and is characterized by

$$\text{(A.1)} \qquad q^{FB}(\theta) = \arg\max_{q \geq 0} [\theta u^A(q) + e(\theta) u^B(q) - cq] \qquad \text{for each } \theta \in [\underline{\theta}, \overline{\theta}]$$

As in the case of two types, $q^{FB}$ is increasing with respect to $\theta$ if $e$ is increasing, otherwise $q^{FB}$ may not be increasing. For instance, suppose that $u^B(q) = \beta u^A(q)$ for some $\beta > 0$. Then $q^{FB}$ is increasing if and only if $\theta + \beta e(\theta)$ is increasing, which fails to hold if $\beta e'(\theta) < -1$ for some $\theta$. This occurs when the externality generated on side $B$ by an agent on side $A$ is decreasing with respect to $\theta$ and $\beta$ is large, or when the externality is quickly decreasing. In the next example we illustrate this point.

**Example 4** *Suppose that $\theta$ is uniformly distributed in the interval $[3,5]$; in the following we represent this by writing $\theta \sim \mathscr{U}[3,5]$. Moreover, $u^A(q) = u^B(q) = \ln(1+q)$, $c = 1$, and $e(\theta) = 10 - \delta\left(\theta - 3 + \frac{1}{2}(\theta-3)^2\right)$ with $\delta \in (0, \frac{5}{2})$. It is immediate that $e$ decreases with $\theta$ as $e'(\theta) = -\delta(\theta-2) < 0$, and that it decreases more quickly, the greater $\delta$ becomes. From (A.1) we obtain*

$$\text{(A.2)} \qquad q^{FB}(\theta) = \theta + e(\theta) - 1$$

*and $q^{FB\prime}(\theta) = 1 - \delta(\theta - 2)$. Therefore, $q^{FB}$ is increasing if $\delta \leq \frac{1}{3}$, is decreasing if $\delta \geq 1$, and is first increasing and then decreasing if $\delta \in (\frac{1}{3}, 1)$.*

In what follows, we say that a quality schedule is *single-peaked* if in the interval $[\underline{\theta},\overline{\theta}]$ it is first increasing and then decreasing, and we use $\theta_m$ to denote the peak, that is, $\theta_m$ is the maximum point and $q(\theta_m)$ is the maximum value for the schedule.

## A.2 Second-best

In the second-best, the menu of contracts $\{(q(\theta),p(\theta))\}_{\theta\in[\underline{\theta},\overline{\theta}]}$ offered by the platform needs to satisfy the incentive constraints:

$$\theta q(\theta)-p(\theta)\geq\theta q(\theta')-p(\theta') \qquad \text{for each } \theta,\theta' \text{ in } [\underline{\theta},\overline{\theta}]. \tag{A.3}$$

It is well known from Mussa and Rosen (1978) that $\{(q(\theta),p(\theta))\}_{\theta\in[\underline{\theta},\overline{\theta}]}$ satisfies (A.3) if and only if $q$ is increasing and

$$p(\theta)=\theta u^A(q(\theta))-\int_{\underline{\theta}}^{\theta}u^A(q(s))ds-U(\underline{\theta}) \qquad \text{for each } \theta \text{ in } [\underline{\theta},\overline{\theta}]$$

in which $U(\underline{\theta})=\underline{\theta}q(\underline{\theta})-p(\underline{\theta})$ is the expected rent of type $\underline{\theta}$. In order to reduce the rent of the agents on side $A$ without violating participation constraints, it is optimal to set $p(\underline{\theta})$ such that $U(\underline{\theta})=0$. Hence, $p(\theta)=\theta u^A(q(\theta))-\int_{\underline{\theta}}^{\theta}u^A(q(s))ds$ and the platform's expected profit is

$$\underbrace{\int_{\underline{\theta}}^{\overline{\theta}}\left(\theta u^A(q(\theta))-\int_{\underline{\theta}}^{\theta}u^A(q(s))ds-cq(\theta)\right)f(\theta)d\theta}_{\text{profit from side } A}+\underbrace{\int_{\underline{\theta}}^{\overline{\theta}}e(\theta)u^B(q(\theta))f(\theta)d\theta}_{\text{profit from side } B} \tag{A.4}$$

$$=\int_{\underline{\theta}}^{\overline{\theta}}\left[\theta^v u^A(q(\theta))+e(\theta)u^B(q(\theta))-cq(\theta)\right]f(\theta)d\theta.$$

The second-best schedule $q^{SB}$ maximizes (A.4) subject to $q$ increasing. Let $q^*$ be the schedule that maximizes (A.4) pointwise, that is

$$q^*(\theta)=\arg\max_{q\geq 0}[\theta^v u^A(q)+e(\theta)u^B(q)-cq] \qquad \text{for each } \theta\in[\underline{\theta},\overline{\theta}]. \tag{A.5}$$

If $q^*$ is increasing, then $q^{SB}(\theta)=q^*(\theta)$ for each $\theta\in[\underline{\theta},\overline{\theta}]$ and $q^{SB}(\theta)<q^{FB}(\theta)$ for each $[\underline{\theta},\overline{\theta})$, $q^{SB}(\overline{\theta})=q^{FB}(\overline{\theta})$. If $q^*$ is not increasing, then determining $q^{SB}$ is less immediate and requires some "ironing" of $q^*$: see, for instance, Guesnerie and Laffont (1984).

Consider first the case in which $q^{FB}$ is increasing. In the setting with two types, we have seen that if $q^{FB}$ is increasing then $q^*$ is also increasing so that $q^{SB}$ coincides with $q^*$. In the current setting with a continuum of types, this implication does not hold. However, in the next proposition we show that $q^{FB}$ increasing implies that $q^*$ is increasing as long as $\frac{u^{A\prime}}{u^{B\prime}}$ is decreasing; this loosely means that the marginal utility on side $A$ decreases more quickly than the marginal utility on side $B$.[20]

**Proposition 9** *Consider the baseline model with a continuum of types. Suppose that $q^{FB}$ in (A.1) is increasing and that the hazard rate of F is increasing, that is, $\frac{u^{A\prime}}{u^{B\prime}}$ is decreasing. Then, $q^*$ in (A.5) is increasing, and subsequently $q^{SB}(\theta) = q^*(\theta)$ for each $\theta \in [\underline{\theta}, \overline{\theta}]$.*
***Proof.*** *See Appendix C.* ■

When there are only two types, the inequality $q_L^{FB} \leq q_H^{FB}$ implies $q_L^* < q_H^*$, because $q_H^* = q_H^{FB}$ and $q_L^*$ is determined by using $\theta_L^v < \theta_L$ instead of $\theta_L$. With a continuum of types, the above argument applies if we compare $q^*(\underline{\theta})$ with $q^*(\overline{\theta})$, that is, the values of $q^*$ at the extremes of the interval $[\underline{\theta}, \overline{\theta}]$. But when we check the monotonicity of $q^*$ in the interval $(\underline{\theta}, \overline{\theta})$, we need to examine how increasing $\theta$ affects $\theta^v u^{A\prime}(q) + e(\theta)u^{B\prime}(q)$ (see (A.5)), knowing that it also affects $\theta u^{A\prime}(q) + e(\theta)u^{B\prime}(q)$ (see (A.1)) in such a way as to make $q^{FB}$ increasing.

The implicit function theorem reveals that

(a) $q^{FB}$ is increasing if and only if $\frac{u^{A\prime}(q^{FB}(\theta))}{u^{B\prime}(q^{FB}(\theta))} + e'(\theta) \geq 0$;

(b) $q^*$ is increasing if and only if $\frac{d\theta^v}{d\theta}\frac{u^{A\prime}(q^*(\theta))}{u^{B\prime}(q^*(\theta))} + e'(\theta) \geq 0$.

Given that $\frac{d\theta^v}{d\theta} \geq 1$ (as the hazard rate of $F$ is increasing) and $q^*(\theta) < q^{FB}(\theta)$, if $\frac{u^{A\prime}}{u^{B\prime}}$ is decreasing, then the inequality in (a) implies the inequality in (b). But if $\frac{u^{A\prime}}{u^{B\prime}}$ is increasing (and $\frac{d\theta^v}{d\theta}$ is not much larger than one), then the inequality in (a) may hold even though the inequality in (b) is violated.

**Example 5** *Continue to work with Example 4. Then (A.5) yields*

(A.6) $$q^*(\theta) = \theta^v + e(\theta) - 1$$

*and since $\theta^v = 2\theta - 5$, we have that $q^{*\prime}(\theta) = 2 - \delta(\theta - 2)$. Therefore $q^*$ is increasing if $\delta \leq \frac{2}{3}$, is decreasing if $\delta \geq 2$, and is non-monotone (but single-peaked) if $\delta \in (\frac{2}{3}, 2)$. Recall from Example 3 that $q^{FB}$ is increasing if $\delta \leq \frac{1}{3}$. Consistently with Proposition 9, $\delta \leq \frac{1}{3}$ is a sufficient condition for $q^*$ to be increasing.*

[20]This property is equivalent to the property that $u^B$ is less concave than $u^A$, according to the Arrow-Pratt risk aversion measure, that is, $-\frac{u^{B\prime\prime}(q)}{u^{B\prime}(q)} \leq -\frac{u^{A\prime\prime}(q)}{u^{A\prime}(q)}$ for each $q \geq 0$.

Consider now the situation in which $q^{FB}$ is decreasing in at least one interval in $[\underline{\theta}, \bar{\theta}]$. Then Proposition 9 does not apply, $q^*$ may not be increasing and some pooling may be optimal; indeed, complete pooling is optimal in some cases. Below we identify some sufficient conditions for $q^{SB}$ to be constant.

Define $G(q,\theta)$ as follows, from (A.5):

$$G(q,\theta) \equiv \theta^v u^A(q) + e(\theta)u^B(q) - cq$$

and let $g(q,\theta) \equiv \frac{\partial G}{\partial q}(q,\theta)$. Since $G$ is strictly concave in $q$, $g$ is strictly decreasing with $q$. Then $q^*(\theta) = \arg\max_{q\geq 0} G(q,\theta)$ implies

$$\begin{cases} g(q,\theta) > 0 & \text{if } q < q^*(\theta) \\ g(q,\theta) < 0 & \text{if } q > q^*(\theta) \end{cases} \tag{A.7}$$

We use $q_k$ to denote the constant schedule at the quality level $k$, i.e., $q_k$ satisfies $q_k(\theta) = k$ for each $\theta \in [\underline{\theta}, \overline{\theta}]$. The platform's profit with $q_k$ is $\int_{\underline{\theta}}^{\overline{\theta}} G(k,\theta)f(\theta)d\theta$, and we let

$$h(k) \equiv \frac{d}{dk}\left(\int_{\underline{\theta}}^{\overline{\theta}} G(k,\theta)f(\theta)d\theta\right) = \int_{\underline{\theta}}^{\overline{\theta}} g(k,\theta)f(\theta)d\theta$$

denote the derivative of this profit with respect to the constant level of quality. In order for the schedule $q_k$ to be optimal, it is necessary (but not sufficient) that $k$ satisfies $h(k) = 0$. Using these tools, we provide some results on the optimality of a constant schedule under the assumption that $q^*$ has at most one change in monotonicity, e.g., when $q^*$ is single-peaked with peak at $\theta_m \in (\underline{\theta}, \overline{\theta})$. In such a case, from Guesnerie and Laffont (1984) it follows that if $q^{SB}$ is not constant, then

$$q^{SB}(\theta) = \begin{cases} q^*(\theta) & \text{if } \theta \in [\underline{\theta}, \theta') \\ z & \text{if } \theta \in [\theta', \overline{\theta}] \end{cases} \tag{A.8}$$

with $\theta'$ and $z$ such that $\theta' \in (\underline{\theta}, \theta_m)$, $z = q^*(\theta')$, and

$$\int_{\theta'}^{\overline{\theta}} g(z,\theta)f(\theta)d\theta = 0. \tag{A.9}$$

**Proposition 10** *Consider the baseline model with a continuum of types. Under the second-best, complete pooling is optimal under any of the following circumstances:*

*(i)* $q^*$ *is decreasing in* $[\underline{\theta}, \bar{\theta}]$*;*

*(ii)* $q^*$ *is single-peaked and* $h(q^*(\underline{\theta})) \leq 0$*;*

*(iii)* $q^*$ *is first decreasing, then increasing, and* $h(q^*(\overline{\theta})) \geq 0$.

*In each case (i)-(iii),* $q^*(\overline{\theta}) < q^*(\underline{\theta})$ *and the optimal schedule is* $q_k$ *for a unique* $k$ *between* $q^*(\overline{\theta})$ *and* $q^*(\underline{\theta})$ *such that* $h(k) = 0$.
***Proof.*** *See Appendix C.* ■

Note first that $q^*$ decreasing in an interval implies that $q^{FB}$ is decreasing in the same interval, by Proposition 9, as long as the hazard rate is increasing and $u^{A\prime}/u^{B\prime}$ is decreasing. Result (i) is straightforward. As result (iii) is symmetric to result (ii), we briefly discuss result (ii). Suppose that $q^*$ is single-peaked with the peak at $\theta_m$. In this case, $h(q^*(\underline{\theta})) \leq 0$ implies $q^*(\overline{\theta}) < q^*(\underline{\theta})$, because if $q^*(\overline{\theta}) \geq q^*(\underline{\theta})$ then $q^*(\underline{\theta}) < q^*(\theta)$ for each $\theta \in (\underline{\theta}, \overline{\theta})$ and (A.7) imply $h(q^*(\underline{\theta})) > 0$. Therefore, although we are not assuming that $q^*$ is decreasing in the whole interval $[\underline{\theta}, \overline{\theta}]$, the inequality $h(q^*(\underline{\theta})) \leq 0$ implies that $q^*$ is decreasing enough in $(\theta_m, \overline{\theta})$ to make $q^*(\overline{\theta})$ smaller than $q^*(\underline{\theta})$.[21] Then $h(q^*(\underline{\theta})) \leq 0$ and (A.7) imply $\int_{\theta'}^{\overline{\theta}} g(q^*(\theta'), \theta) f(\theta) d\theta < 0$ for each $\theta' \in (\underline{\theta}, \theta_m)$, hence (A.9) cannot hold. In what follows, we apply the results of Proposition 10 to the same example we have previously worked on.

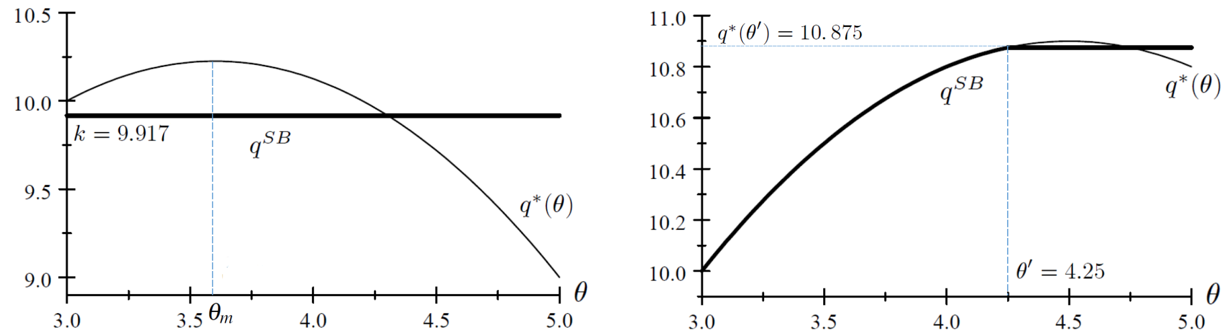
**Example 6** *Consider the setting of Example 4:* $\theta \sim \mathscr{U}[3,5]$, $u^A(q) = u^B(q) = \ln(1+q)$, $c = 1$, *and* $e(\theta) = 10 - \delta\left(\theta - 3 + \frac{1}{2}(\theta-3)^2\right)$ *with* $\delta \in (0, \frac{5}{2})$.

- *From Example 4,* $q^*$ *defined in (A.6) is increasing if* $\delta \leq \frac{2}{3}$. *Hence,* $q^{SB}$ *coincides with* $q^*$.

- *When* $\delta > \frac{2}{3}$*, we can apply Proposition 10-(i) if* $q^*$ *is decreasing, which occurs when* $\delta \geq 2$. *Therefore,* $q^{SB}$ *is constant if* $\delta \geq 2$. *Since* $h(k) = \frac{13 - \frac{5}{3}\delta}{k+1} - 1$, *it follows that* $q^{SB} = q_k$ *with* $k = 12 - \frac{5}{3}\delta$.

- *For the intermediate case with* $\delta \in (\frac{2}{3}, 2)$, $q^*$ *is single-peaked with the peak at* $\theta_m = 2 + \frac{2}{\delta}$ *and Proposition 10-(ii) applies if* $h(q^*(\underline{\theta} = 3))$ *is negative or zero. Since* $h(q^*(3)) = \frac{1}{33}(6 - 5\delta) \leq 0$ *is equivalent to* $\delta \geq \frac{6}{5}$*, it follows that* $q^{SB} = q_k$*, with* $k = 12 - \frac{5}{3}\delta$*, if* $\delta \in [\frac{6}{5}, 2)$. *Figure 1-(a) illustrates* $q^{SB}$ *and* $q^*$ *when* $\delta = \frac{5}{4}$ *and* $c = 1$.

[21] Intuitively, we could think of the case in which $\theta_m$ is close to $\underline{\theta}$; therefore, $q^*$ is increasing in a small interval and then decreasing in a much wider interval.




(a) $\delta = \frac{5}{4}$ and $c = 1$ (b) $\delta = \frac{4}{5}$ and $c = 1$

Figure A.1: Complete and partial pooling under the second-best

- *Conversely, if* $\delta \in (\frac{2}{3}, \frac{6}{5})$ *then* $h(q^*(3)) > 0$ *and (A.8) applies:*

$$q^{SB}(\theta) = \begin{cases} q^*(\theta) & \text{if } \theta \in [3, \theta') \\ z & \text{if } \theta \in [\theta', 5] \end{cases}$$

*with* $\theta' = \frac{1}{2} + \frac{3}{\delta}$, $z = q^*(\theta')$. *Figure 1-(b) illustrates* $q^{SB}$ *and* $q^*$ *when* $\delta = \frac{4}{5}$ *and* $c = 1$.

## A.3 Shutdown

Up to now, we have implicitly assumed $q^*(\theta) > 0$, which implies $q^{FB}(\theta) > 0$ for each $\theta$. Here we allow for the shutdown possibility, $q^*(\theta) = 0$ for some $\theta$. We show that if $q^{FB}$ is decreasing in some interval in $[\underline{\theta}, \overline{\theta}]$, then a complete shutdown may occur even if shutdown occurs in the first-best only for $\theta$ very close to $\overline{\theta}$. Moreover, it is also possible that the pattern of shutdowns changes completely such that in the first-best it occurs for $\theta$ close to $\overline{\theta}$ but in the second-best it occurs for $\theta$ close to $\underline{\theta}$.

**Proposition 11** *Consider the baseline model with a continuum of types.*

*(i) Suppose there exists* $\theta_0 \in (\underline{\theta}, \overline{\theta})$ *such that (a)* $q^*(\theta) > 0$ *for each* $\theta \in [\underline{\theta}, \theta_0)$ *and* $q^*$ *is either decreasing or single-peaked in* $[\underline{\theta}, \theta_0)$*; (b)* $q^*(\theta) = 0$ *for each* $\theta \in [\theta_0, \overline{\theta}]$. *If* $h(0) \leq 0$*, then a complete shutdown is optimal:* $q^{SB}(\theta) = 0$ *for any* $\theta \in [\underline{\theta}, \overline{\theta}]$.

*(ii) Suppose there exist* $\theta_0$ *and* $\theta_1$ *such that (a) for* $\theta \in (\theta_0, \theta_1)$, $q^*(\theta) > 0$ *and* $q^*$ *is single-peaked with the peak at* $\theta_m$*; (b)* $q^*(\theta) = 0$ *for each* $\theta \in [\underline{\theta}, \theta_0] \cup [\theta_1, \overline{\theta}]$. *If* $\int_{\theta_0}^{\overline{\theta}} g(0, \theta) f(\theta) d\theta >$

0*, then*

(A.10)
$$q^{SB}(\theta) = \begin{cases} 0 & \text{if } \theta \in [\underline{\theta}, \theta_0) \\ q^*(\theta) & \text{if } \theta \in [\theta_0, \theta'] \\ z & \text{if } \theta \in (\theta', \overline{\theta}] \end{cases}$$

*with* $(\theta', z)$ *such that* $\theta' \in (\theta_0, \theta_m)$*,* $z = q^*(\theta')$ *and (A.9) holds. If* $\int_{\theta_0}^{\overline{\theta}} g(0,\theta) f(\theta) d\theta \leq 0$*, then a complete shutdown is optimal:* $q^{SB}(\theta) = 0$ *for any* $\theta \in [\underline{\theta}, \overline{\theta}]$*.*

***Proof.*** *See Appendix C.* ■

Proposition 11(i) considers the case in which it is optimal to serve the low types (i.e., $q^*(\theta) > 0$ for low $\theta$), but not the high types ($q^*(\theta) = 0$ for high $\theta$) if we had neglected the monotonicity constraint. In this situation, the monotonicity constraint can lead to a complete shutdown, i.e., $q^{SB}(\theta) = 0$ for any $\theta \in [\underline{\theta}, \overline{\theta}]$, because serving the high types without serving the low types is not possible. This result, which requires $e$ to decrease quickly at least for high $\theta$, is in a stark contrast to the usual outcome in a one-sided market where $e(\theta)$ is a constant equal to zero; then if $q^*(\theta) > 0$ for at least some $\theta$, we have $q^*(\overline{\theta}) > 0$ and $q^{SB}(\overline{\theta}) > 0$ and hence there is never a complete shutdown. Also notice that $q^*(\theta) > 0$ implies $q^{FB}(\theta) > 0$, and $q^*(\bar{\theta}) = 0$ requires $\overline{\theta} u^{A\prime}(0) + e(\overline{\theta}) u^{B\prime}(0) < c$,[22] which implies $q^{FB}(\theta) = 0$ for $\theta$ close to $\bar{\theta}$. Thus it is possible that $q^{FB}(\theta) > 0$ except for a set of types close to $\bar{\theta}$, but nevertheless, a complete shutdown is optimal.

Proposition 11(ii) reports another interesting shutdown pattern. Suppose that the platform serves some medium types but neither high types nor low types, when the monotonicity constraint was neglected. Precisely, $q^*(\theta) = 0$ for $\theta \in [\underline{\theta}, \theta_0] \cup [\theta_1, \overline{\theta}]$, hence $\overline{\theta} u^{A\prime}(0) + e(\overline{\theta}) u^{B\prime}(0) < c$ even though $\theta^v u^{A\prime}(0) + e(\theta) u^{B\prime}(0) > c$ for medium values of $\theta$, thus $q^{FB}(\theta) = 0$ for $\theta$ close to $\overline{\theta}$. Proposition 11(ii) establishes circumstances under which $q^{SB}(\theta)$ is positive and constant in an interval that includes $[\theta_1, \overline{\theta}]$. Therefore, a set of types close to $\overline{\theta}$ is excluded under the first-best, whereas only a set of types close to $\underline{\theta}$ is excluded under the second-best. Hence, the exclusion pattern is reversed as we move from the first-best to the second-best.[23]

[22] In fact, $q^*(\bar{\theta}) = 0$ implies $\overline{\theta} u^{A\prime}(0) + e(\overline{\theta}) u^{B\prime}(0) \leq c$, but for the sake of the argument here we consider the strict inequality.

[23] A setting in which Proposition 11(i) (Proposition 11 (ii)) applies is the one of Example 3 with $\delta = \frac{19}{10}$, $c = 10$ ($\delta = 1$, $c = 11.25$). For space's sake, the detailed analysis of these examples is omitted but is available from the authors upon request.

## B Appendix: Type reversal with negative sorting

Type reversal with negative sorting is different from no type reversal or type reversal with positive sorting in one important aspect: the most restrictive participation constraint on side $B$ is not necessarily $(IR_l^B)$. This is because $e_{lH} > e_{hH}$ makes it possible that $(IR_h^B)$ becomes more restrictive than $(IR_l^B)$ if $q_H$ is sufficiently larger than $q_L$.

Consider first the case in which the first-best quality schedule is increasing. Then, we find that pooling is never optimal in the case of type reversal with negative sorting:

**Lemma 2** *If the first-best quality schedule is increasing, pooling does not arise in the case of type reversal with negative sorting.*

***Proof.*** *Suppose that $(IR_h^B)$ does not bind. Then $(IR_l^B)$ binds, and since negative type reversal implies $e_L^a > e_{lL}$ and $e_{lH} > e_H^a$, from (13) and (15) we see that $q_H^* > q_L^*$; hence, no pooling occurs. Therefore, pooling requires that $(IR_h^B)$ binds. But still, $(IR_l^B)$ needs to be satisfied, and this plus (12) imply that $(IR_h^B)$ is slack when $q_H = q_L$.* ■

We have three cases to examine in terms of the binding participation constraint(s) on side $B$.

Case I. As with Section 3.2, the quality schedule determined by (15) is the second-best schedule as long as

$$\nu^A e_{lL} u^B(q_L^*) + (1-\nu^A) e_{lH} u^B(q_H^*) \leq \nu^A e_{hL} u^B(q_L^*) + (1-\nu^A) e_{hH} u^B(q_H^*), \tag{B. 1}$$

which means that $(IR_h^B)$ is satisfied given that $(q_L, q_H) = (q_L^*, q_H^*)$ and $(IR_l^B)$ is the binding participation constraint (hence $p^B = \nu^A e_{lL} u^B(q_L^*) + (1-\nu^A) e_{lH} u^B(q_H^*)$). Under negative sorting, $e_{lL} < e_L^a$ and $e_{lH} > e_H^a$ make (15) yield a downward distortion for the low type and an upward distortion for the high type. Hence, (B. 1) might be violated due to these distortions.

Case II. When (B. 1) is violated, we consider the case that $(IR_h^B)$ binds with $p^B = \nu^A e_{hL} u^B(q_L) + (1-\nu^A) e_{hH} u^B(q_H)$. The profit to maximize changes to

$$\hat{\Pi}(q_L, q_H) = \nu^A \left(\theta_L^{A\nu} u^A(q_L) + e_{hL} u^B(q_L) - c q_L\right) + (1-\nu^A)\left(\theta_H^A u^A(q_H) + e_{hH} u^B(q_H) - c q_H\right)$$

Let $(\hat{q}_L, \hat{q}_H)$ be the qualities that maximize $\hat{\Pi}$. The first-order conditions are

$$\theta_L^{A\nu} u^{A\prime}(\hat{q}_L) + e_{hL} u^{B\prime}(\hat{q}_L) = c, \qquad \theta_H^A u^{A\prime}(\hat{q}_H) + e_{hH} u^{B\prime}(\hat{q}_H) = c. \tag{B. 2}$$

$(\hat{q}_L, \hat{q}_H)$ is established as the second-best schedule as long as

(B. 3) $$\nu^A e_{hL} u^B(\hat{q}_L) + (1-\nu^A) e_{hH} u^B(\hat{q}_H) \le \nu^A e_{lL} u^B(\hat{q}_L) + (1-\nu^A) e_{lH} u^B(\hat{q}_H)$$

which means that $(IR_l^B)$ is satisfied given $(q_L, q_H) = (\hat{q}_L, \hat{q}_H)$ and $p^B = \nu^A e_{hL} u^B(\hat{q}_L) + (1-\nu^A) e_{hH} u^B(\hat{q}_H)$. Unlike (15), now we find that ( B. 2) implies an upward distortion for the low type and a downward distortion for the high type, which may violate (B. 3).

Case III. When both (B. 1) and (B. 3) are violated, both $(IR_l^B)$ and $(IR_h^B)$ bind; no agent on side $B$ earns any information rent. In this case the second-best schedule is determined by maximizing (14) subject to

(B. 4) $$\nu^A (e_{hL} - e_{lL}) u^B(q_L) - (1-\nu^A)(e_{lH} - e_{hH}) u^B(q_H) = 0$$

Then the first-order conditions for $q_L, q_H$ are given by

(B. 5)
$$\theta_L^{A\nu} u^{A\prime}(q_L) + ((1-\lambda) e_{lL} + \lambda e_{hL}) u^{B\prime}(q_L) = c, \quad \theta_H^A u^{A\prime}(q_H) + ((1-\lambda) e_{lH} + \lambda e_{hH}) u^{B\prime}(q_H) = c$$

in which $\lambda$ is the Lagrange multiplier for the constrained problem. Notice that from (B. 5) we obtain $q_L^*, q_H^*$ if $\lambda = 0$, $\hat{q}_L, \hat{q}_H$ if $\lambda = 1$. In the optimum it is necessary that $\lambda \in (0,1)$ because

- if $\lambda \le 0$, then (B. 5) yields $q_L \le q_L^*$, $q_H \ge q_H^*$ and the left-hand side of (B. 4) is negative because (B. 1) is violated;
- if $\lambda \ge 1$, then (B. 5) yields $q_L \ge \hat{q}_L$, $q_H \le \hat{q}_H$ and the left-hand side of (B. 4) is positive because (B. 3) is violated.

Thus, $\lambda \in (0,1)$ is such that (B. 5) yields $(q_L, q_H)$, which satisfy (B. 4). As a result, we have $q_L^{SB} \in (q_L^*, \hat{q}_L)$, $q_H^{SB} \in (\hat{q}_H, q_H^*)$.

Consider now the case in which the first-best quality schedule is decreasing. First, pooling can be optimal when agents are homogeneous on side $B$ by Proposition 2, and in such a case, by continuity, pooling would be still optimal even after introducing a very small degree of heterogeneity to the agents on side $B$. However, pooling implies that only $(IR_l^B)$ binds on side $B$ under (12). Hence, the optimal pooling contract is the one characterized by Proposition 5(ii). Second, if pooling is not optimal, then the above analysis applies and we can have three different situations in terms of the binding individual rationality constraint(s) on side $B$.

**Proposition 12** *Consider type reversal with negative sorting in the general model.*

*(i) Consider first the case in which the first-best quality schedule is increasing. Then, pooling is never optimal and the second-best quality schedule* $\left(q_L^{SB}, q_H^{SB}\right)$ *is characterized as follows.*

- $\left(q_L^{SB}, q_H^{SB}\right)$ *is equal to* $(q_L^*, q_H^*)$ *if (B. 1) holds; in this case* $(IR_l^B)$ *binds.*
- $\left(q_L^{SB}, q_H^{SB}\right)$ *is equal to* $(\hat{q}_L, \hat{q}_H)$ *if (B. 3) holds; in this case* $(IR_h^B)$ *binds.*
- *If both (B. 1) and (B. 3) are violated,* $\left(q_L^{SB}, q_H^{SB}\right)$ *is characterized by (B. 5) for a suitable* $\lambda \in (0,1)$ *such that (B. 4) holds; in this case* $(IR_l^B)$ *and* $(IR_h^B)$ *both bind.*

*(ii) Consider now the case in which the first-best quality schedule is decreasing. If the optimal quality schedule obtained by neglecting the monotonicity constraint is strictly increasing, then it is the second-best schedule. Otherwise, pooling is optimal and the optimal pooling contract is the one described by Proposition 12(i).*

**Example 7** *Suppose that* $u^A(q) = u^B(q) = \ln(1+q)$, $c = 1$, $\nu^A = \frac{1}{2}$, $\nu^B = \frac{1}{2}$ *and* $\theta_L = 16$, $\theta_H = 20$, $e_{lL} = 4$, $e_{lH} = 10$, $e_{hH} = 5$, $e_{hL} \in [9,19]$ *under which (12) is satisfied, the first-best quality schedule is increasing as* $q_H^{FB} = 26.5 \geq q_L^{FB} = 17 + \frac{1}{2}e_{hL}$, *and type reversal with negative sorting occurs. Then we can derive:*

$$\begin{aligned} q_L^* &= 15, & q_H^* = 29, \\ \hat{q}_L &= 11 + e_{hL}, & \hat{q}_H = 24. \end{aligned}$$

- *(B. 1) holds if and only if* $e_{hL} \geq 10.217$; $(q_L^{SB}, q_H^{SB}) = (15, 29)$.
- *(B. 3) holds if and only if* $e_{hL} \leq 9.28$; $(q_L^{SB}, q_H^{SB}) = (11 + e_{hL}, 24)$.
- *If* $e_{hL}$ *belongs to the interval* $(9.28, 10.217)$ *then* $(q_L^{SB}, q_H^{SB}) = (15 - 4\lambda + \lambda e_{hL}, 29 - 5\lambda)$ *for a suitable* $\lambda \in (0,1)$ *such that (B. 4) holds. For instance, if* $e_{hL} = 10$ *then* $\lambda = 0.18428$ *and* $(q_L^{SB}, q_H^{SB}) = (16.106, 28.079)$.[24]

[24] A case of type reversal with negative sorting in which pooling is optimal is such that $u^A(q) = u^B(q) = \ln(1+q)$, $c = 1$, $\nu^A = \frac{1}{2}$, $\nu^B = \frac{1}{2}$ and $\theta_L = 19$, $\theta_H = 20$, $e_{lL} = 7$, $e_{lH} = 4$, $e_{hH} = 3$, $e_{hL} = 9$. Then $q_L^* = 24 > q_H^* = 23$, $\hat{q}_L = 26 > \hat{q}_H = 22$, and $q_L^{SB} = q_H^{SB} = 23.5$.

## C Appendix: Mathematical Proofs

Here we collect all mathematical proofs that are not offered in the main text and in Appendices A and B.

**Proof of Proposition 6(i)** We first provide the proof for the case in which $e_L \geq e_H$.

**Step 1** *The function $D(\nu) = (1-\nu)\pi_H - \pi_L$ is strictly decreasing.*

Since $D'(\nu) = -\pi_H - (e_L - e_H)u^B(q^P)$, it is immediate that $D'(\nu) < 0$.

In the rest of this proof, we use $\alpha = u^{A\prime}(0)$, $\beta = u^{B\prime}(0)$.

**Step 2** *If $e_L \geq \frac{c}{\beta}$, then $\nu^{PD} = \hat{\nu} \leq \nu^N$.*

If $e_L \geq \frac{c}{\beta}$, then $\theta_L^{\nu}\alpha + e_L\beta \geq c$ for each $\nu \geq \hat{\nu}$, hence $\nu^{PD} = \hat{\nu}$. However, whether $\nu^N$ is equal to $\hat{\nu}$ or $\nu^N > \hat{\nu}$ depends on the parameters.[25] In particular, if $e_L = \frac{c}{\beta}$ then $D(\hat{\nu}) = (1-\hat{\nu})\pi_H - \pi_L > 0$, hence $\nu^N > \hat{\nu}$. Precisely,

$$\begin{aligned}
\pi_L &= \max_q \left[\theta_L u^A(q) + \left(\left(1 - \frac{\theta_L}{\theta_H}\right)\frac{c}{\beta} + \frac{\theta_L}{\theta_H}e_H\right)u^B(q) - cq\right] \\
&< \max_q \left[\theta_L u^A(q) + \left(1 - \frac{\theta_L}{\theta_H}\right)\frac{c}{\beta}\beta q + \frac{\theta_L}{\theta_H}e_H u^B(q) - cq\right] \\
&= \max_q \left[\theta_L u^A(q) + \frac{\theta_L}{\theta_H}e_H u^B(q) - \frac{\theta_L}{\theta_H}cq\right] \\
&= \frac{\theta_L}{\theta_H}\max_q \left[\theta_H u^A(q) + e_H u^B(q) - cq\right] = (1-\hat{\nu})\pi_H
\end{aligned}$$

**Step 3** *If $e_L < \frac{c}{\beta}$, then $\nu^{PD} = \frac{\theta_H - \theta_L}{\theta_H - \frac{c}{\alpha} + \frac{\beta}{\alpha}e_L}$ is greater than $\hat{\nu}$ but is smaller than $\nu^N$.*

We use $\delta$ to denote the difference $(1-\nu^{PD})\pi_H - \pi_L$, that is $\delta = D(\nu^{PD})$, and we show $\delta > 0$. Therefore, $(1-\nu)\pi_H - \pi_L > 0$ at $\nu = \nu^{PD}$ and $\nu^N$ must be larger than $\nu^{PD}$ by Step 1.

**Step 3.1** *$\delta$ is increasing with respect to $e_H$.*

We have that $\frac{\partial\delta}{\partial e_H} = (1-\nu^{PD})(u^B(q_H^{FB}) - u^B(q^P))$, which is positive since $q_H^{FB} > q^P$. Thus $\frac{\partial\delta}{\partial e_H} > 0$, and in the following we prove that $\delta > 0$ when $e_H = 0$.

**Step 3.2** *$\delta > 0$ given $e_H = 0$ and $e_L \in (0, \frac{c}{\beta})$.*

[25] In some cases with $e_L > \frac{c}{\beta}$, we have that $(1-\hat{\nu})\pi_H - \pi_L < 0$, hence $\nu^N > \hat{\nu}$. Precisely, if $u^A(q) = u^B(q) = \ln(1+q)$, $c = 1$, $\theta_L = 7$, $\theta_H = 10$, $e_L = 4$, $e_H = 0$, then $\hat{\nu} = 0.3$ and $(1-\hat{\nu})\pi_H - \pi_L = 0.7\max_q[10\ln(1+q) - q] - \max_q[(7+0.3\cdot 4)\ln(1+q) - q] = 9.8181 - 10.054 < 0$.

In order to prove that $\delta > 0$, we pick $u^B(q) = \beta q$, which increases $\pi_L$ but does not affect $\pi_H$ since $e_H = 0$. Hence

$$\pi_L = \max_q \left[\theta_L u^A(q) + \nu^{PD} e_L \beta q - cq\right] = \frac{c - \nu^{PD} e_L \beta}{c} P\left(\frac{c}{c - \nu^{PD} e_L \beta}\theta_L\right),$$

with $P(\theta) \equiv \max_q[\theta u^A(q) - cq]$. Thus,

$$\delta = (1 - \nu^{PD})\left(\pi_H - \frac{c - \nu^{PD} e_L \beta}{c(1 - \nu^{PD})} P(b\theta_L)\right)$$

with $b = \frac{c}{c - e_L \nu^{PD} \beta}$. When $e_L = \frac{c}{\beta}$, we have $\nu^{PD} = \hat{\nu}$, $\frac{c - \nu^{PD} e_L \beta}{c(1 - \nu^{PD})} = 1$ , $b\theta_L = \theta_H$, hence $\delta = (1 - \hat{\nu})[P(\theta_H) - P(\theta_H)] = 0$. Now we prove that $-\frac{c - \nu^{PD} e_L \beta}{c(1 - \nu^{PD})} P(b\theta_L)$ *is decreasing with respect to* $e_L$ *in the interval* $(0, \frac{c}{\beta})$ *;*
*therefore* $\delta > 0$ *for each* $e_L \in (0, \frac{c}{\beta})$ .
First we notice that with some algebra, we have

$$-\frac{c - \nu^{PD} e_L \beta}{c(1 - \nu^{PD})} = -\frac{\kappa}{c\,(\alpha\theta_L + e_L\beta - c)}, \qquad b = \frac{c\,(\alpha\theta_H + e_L\beta - c)}{\kappa}$$

where $\kappa \equiv c\alpha\theta_H + ce_L\beta + e_L\alpha\beta\theta_L - e_L\alpha\beta\theta_H - c^2$. Then we have

$$-\frac{c - \nu^{PD} e_L \beta}{c(1 - \nu^{PD})} P(b\theta_L) = -\frac{\kappa}{c\,(\alpha\theta_L + e_L\beta - c)} P\left(\frac{c\,(\alpha\theta_H + e_L\beta - c)}{\kappa}\theta_L\right)$$

which has derivative with respect to $e_L$ equal to

$$\begin{aligned}
&= \frac{\alpha^2\beta\theta_L(\theta_H - \theta_L)}{c\,(\alpha\theta_L + e_L\beta - c)^2} P(b\theta_L) - \frac{\alpha\beta\theta_L\,(\theta_H - \theta_L)\,(\alpha\theta_H - c)}{(\alpha\theta_L + e_L\beta - c)\,\kappa} P'(b\theta_L) \\
&= \frac{\alpha\beta\theta_L\,(\theta_H - \theta_L)}{\alpha\theta_L + e_L\beta - c}\left(\frac{\alpha}{c\,(\alpha\theta_L + e_L\beta - c)} P(b\theta_L) - \frac{\alpha\theta_H - c}{\kappa} P'(b\theta_L)\right) \\
&= \frac{\alpha\beta\theta_L\,(\theta_H - \theta_L)}{\alpha\theta_L + e_L\beta - c}\left[\frac{\alpha}{c\,(\alpha\theta_L + e_L\beta - c)}\left(b\theta_L u^A(\tilde{q}(b\theta_L)) - c\tilde{q}(b\theta_L)\right) - \frac{\alpha\theta_H - c}{\kappa} u^A(\tilde{q}(b\theta_L))\right]
\end{aligned}$$

in which $\tilde{q}(\theta) \equiv \arg\max_q [\theta u^A(q) - cq]$. Hence

$$\begin{aligned} & \frac{\alpha\beta\theta_L(\theta_H-\theta_L)}{\alpha\theta_L+e_L\beta-c}\left[\frac{\alpha}{c(\alpha\theta_L+e_L\beta-c)}\Big(b\theta_L u^A(\tilde{q}(b\theta_L))-c\tilde{q}(b\theta_L)\Big)-\frac{\alpha\theta_H-c}{\kappa}u^A(\tilde{q}(b\theta_L))\right] \\ = & \frac{\alpha\beta\theta_L(\theta_H-\theta_L)}{(\alpha\theta_L+e_L\beta-c)^2}(u^A(\tilde{q}(b\theta_L))-\alpha\tilde{q}(b\theta_L)) \end{aligned}$$

which is negative as $u^{A\prime}(0) = \alpha$ and $u$ is concave. QED

Now we provide the proof for the case of $e_L < e_H$. First we suppose that $e_L \geq \frac{c}{\beta}$. Then Step 2 above establishes that no exclusion occurs under PD, for each $\nu \geq \hat{\nu}$. Hence, exclusion is weakly more frequent under no PD.

Then we suppose that $e_L < \frac{c}{\beta}$. In this case $\nu^{PD} > \hat{\nu}$, hence, exclusion occurs under PD for each $\nu$ in $[\hat{\nu}, \nu^{PD}]$, and now we present a series of steps that show that exclusion occurs also under no PD for each $\nu$ between $\hat{\nu}$ and $\nu^{PD}$: (i) from the proof of Step 2 above we know that if $e_L = \frac{c}{\beta}$, then $D(\hat{\nu}) > 0$; (ii) $D(\hat{\nu})$ is decreasing with respect to $e_L$, hence, $D(\hat{\nu}) > 0$ for each $e_L < \frac{c}{\beta}$; (iii) from the proof of Step 3 we know that $D(\nu^{PD}) > 0$; (iv) $q^P$ from (11) is such that $\frac{dq^P}{d\nu} = \frac{(e_L-e_H)u^{B\prime}(q^P)}{-\theta_L u^{A\prime\prime}(q^P)-(\nu e_L+(1-\nu)e_H)u^{B\prime\prime}(q^P)} < 0$ as $e_L < e_H$; (v) from $D'(\nu) = -\pi_H + (e_H - e_L)u^B(q^P)$ (see Step 1 above) it follows that $D$ is a concave function of $\nu$; (vi) $D(\hat{\nu}) > 0$, $D(\nu^{PD}) > 0$, $D$ concave imply $D(\nu) > 0$ for each $\nu \in [\hat{\nu}, \nu^{PD}]$ (in fact, we can prove that there exists $\nu^N > \nu^{PD}$ such that exclusion under no PD occurs if and only if $\nu < \nu^N$). QED

**Proof of Proposition 6(ii)** At $\nu^A = 1$, we have $q^P = q_L^*$ (also equal to $q_L^{FB}$) and $W^{PD} = W^N$. We prove below that $\frac{\partial(W^{PD}-W^N)}{\partial\nu^A}$ is negative at $\nu^A = 1$, which implies that $W^{PD} > W^N$ for $\nu^A$ close to 1. From (18) we obtain

$$\begin{aligned} \frac{dW^N}{d\nu^A} &= \theta_L u^A(q^P) + e_L u^B(q^P) - cq^P - [\theta_H u^A(q^P) + e_H u^B(q^P) - cq^P] \\ &+ \left(\Big(\nu^A\theta_L + (1-\nu^A)\theta_H\Big)u^{A\prime}(q^P) + (\nu^A e_L + (1-\nu^A)e_H)u^{B\prime}(q^P) - c\right)\frac{dq^P}{d\nu^A}, \end{aligned}$$

but since $\theta_L u^{A\prime}(q^P) + (\nu^A e_L + (1-\nu^A)e_H)u^{B\prime}(q^P) - c = 0$, it follows that

$$(\nu^A\theta_L + (1-\nu^A)\theta_H)u^{A\prime}(q^P) + (\nu^A e_L + (1-\nu^A)e_H)u^{B\prime}(q^P) - c = (1-\nu^A)(\Delta\theta)u^{A\prime}(q^P),$$

which is zero at $\nu^A = 1$. Hence, at $\nu^A = 1$,

$$\frac{dW^N}{d\nu^A} = (\theta_L - \theta_H)u^A(q_L^*) + (e_L - e_H)u^B(q_L^*).$$

From $W^{PD} \equiv W(q_L^*, q_H^*)$ we obtain,

$$\begin{aligned} \frac{dW^{PD}}{d\nu^A} &= \theta_L u^A(q_L^*) + e_L u^B(q_L^*) - cq_L^* - [\theta_H u^A(q_H^{FB}) + e_H u^B(q_H^{FB}) - cq_H^{FB}] \\ &+ \nu^A(\theta_L u^{A\prime}(q_L^*) + e_L u^{B\prime}(q_L^*) - c)\frac{dq_L^*}{d\nu^A}, \end{aligned}$$

but since $\theta_L^\nu u^{A\prime}(q_L^*) + e_L u^{B\prime}(q_L^*) - c = 0$, it follows that

$$\nu^A(\theta_L u^{A\prime}(q_L^*) + e_L u^{B\prime}(q_L^*) - c) = (1-\nu^A)(\Delta\theta)u^{A\prime}(q_L^*),$$

which is zero at $\nu^A = 1$. Hence, at $\nu^A = 1$,

$$\frac{dW^{PD}}{d\nu^A} = \theta_L u^A(q_L^*) + e_L u^B(q_L^*) - cq_L^* - [\theta_H u^A(q_H^{FB}) + e_H u^B(q_H^{FB}) - cq_H^{FB}]$$

$$\frac{d(W^{PD} - W^N)}{d\nu^A} = [\theta_H u^A(q_L^*) + e_H u^B(q_L^*) - cq_L^*] - [\theta_H u^A(q_H^{FB}) + e_H u^B(q_H^{FB}) - cq_H^{FB}].$$

Since we are considering the case in which pooling does not arise under price discrimination, it must be the case that $q_L^* < q_H^*$, that is, $q_L^* < q_H^{FB}$. Hence, by definition of $q_H^{FB}$, it follows that $\frac{d(W^{PD}-W^N)}{d\nu^A} < 0$ when $\nu^A$ is close to 1. QED

**Proof of Proposition 6(iii)** Suppose that $q_L^* = q_H^*$; then $q^P = q_L^*$ and $W^N = W^{PD}$. Now we prove that $\frac{dW^{PD}}{de_L} > \frac{dW^N}{de_L}$ and $\frac{dW^N}{de_H} > \frac{dW^{PD}}{de_H}$. Precisely, we use

$$\begin{cases} \frac{dq^P}{de_H} = \frac{(1-\nu^A)u^{B\prime}(q^P)}{-\theta_L u^{A\prime\prime}(q^P) - (\nu^A e_L + (1-\nu^A)e_H)u^{B\prime\prime}(q^P)} \\ \frac{dq^P}{de_L} = \frac{\nu^A u^{B\prime}(q^P)}{-\theta_L u^{A\prime\prime}(q^P) - (\nu^A e_L + (1-\nu^A)e_H)u^{B\prime\prime}(q^P)} \\ \frac{dq_L^*}{de_L} = \frac{u^{B\prime}(q_L^*)}{-\theta_L^\nu u^{A\prime\prime}(q_L^*) - e_L u^{B\prime\prime}(q_L^*)} \end{cases}$$

$$\frac{dW^{PD}}{de_L} = \nu^A u^B(q_L^*) + \nu^A(\theta_L u^{A\prime}(q_L^*) + e_L u^{B\prime}(q_L^*) - c)\frac{dq_L^*}{de_L}$$

(C. 1)
$$= \nu^A u^B(q_L^*) + \frac{(1-\nu^A)(\Delta\theta)u^{A\prime}(q_L^*)u^{B\prime}(q_L^*)}{-\theta_L^\nu u^{A\prime\prime}(q_L^*) - e_L u^{B\prime\prime}(q_L^*)}$$

(C. 2)
$$\frac{dW^{PD}}{de_H} = (1-\nu^A)u^B(q_H^{FB})$$

and

$$\frac{dW^N}{de_L} = \nu^A u^B(q^P) + \frac{dW^N}{dq^P}\frac{dq^P}{de_L}$$

(C. 3)
$$= \nu^A u^B(q^P) + \frac{\nu^A(1-\nu^A)(\Delta\theta)u^{A\prime}(q^P)u^{B\prime}(q^P)}{-\theta_L u^{A\prime\prime}(q^P) - (\nu^A e_L + (1-\nu^A)e_H)u^{B\prime\prime}(q^P)}$$

$$\frac{dW^N}{de_H} = (1-\nu^A)u^B(q^P) + \frac{dW^N}{dq^P}\frac{dq^P}{de_H}$$

(C. 4)
$$= (1-\nu^A)u^B(q^P) + \frac{(1-\nu^A)^2(\Delta\theta)u^{A\prime}(q^P)u^{B\prime}(q^P)}{-\theta_L u^{A\prime\prime}(q^P) - (\nu^A e_L + (1-\nu^A)e_H)u^{B\prime\prime}(q^P)}.$$

Given $q_L^* = q_H^* = q^P$, we prove that $\frac{d(W^N - W^{PD})}{de_H} > 0 > \frac{d(W^N - W^{PD})}{de_L}$. Therefore, a small increase in $e_H$ implies $W^N > W^{PD}$; by the same reasoning, a small decrease in $e_L$ implies $W^N > W^{PD}$. The inequality $\frac{d(W^N - W^{PD})}{de_H} > 0$ is immediate from (C. 2) and (C. 4), given $q^P = q_L^* = q_H^* = q_H^{FB}$. Then we use (C. 1) and (C. 3) and again $q^P = q_L^*$ to see that $0 > \frac{d(W^N - W^{PD})}{de_L}$ is equivalent to $0 < -\theta_H u^{A\prime\prime}(q_L^*) - e_H u^{B\prime\prime}(q_L^*)$, which is satisfied. QED

**Proof of Proposition 7**

(i) In case the single contract is designed to be accepted by both types on side $A$, $p = \theta_L u(q)$ and the profit is $\theta_L u^A(q) + (\nu^A e_{lL} + (1-\nu^A)e_{lH})u^B(q) - cq$.[26] The optimal $q$ is $q^P$ that satisfies (16). We denote with $\pi_L$ the resulting profit. In this case, welfare is

$$\overline{W}(q^P, q^P) + (1-\nu^B)d(q^P, q^P).$$

In case the single contract is designed to be accepted only by type $H$ on side $A$, then $p = \theta_H u(q)$, the profit is $(1-\nu^A)(\theta_H u^A(q) + e_{lH}u^B(q) - cq)$,[27] and the optimal $q$ is $q_H^*$ as de-

[26] This follows from the assumption of no type reversal, which implies that the participation constraint of type $L$ on side $B$ binds.

[27] The assumption of no type reversal implies that on side $B$ the participation constraint that binds is the one of type $L$, hence the platform's revenue from side $B$ is $(1-\nu^A)e_{lH}u^B(q)$.

termined by (15). The resulting profit is $(1-\nu^A)\pi_H$ with $\pi_H = \theta_H u^A(q_H^*) + e_{lH} u^B(q_H^*) - c q_H^*$ and also welfare is equal to $(1-\nu^A)\pi_H$.

After replacing $e_L, e_H$ with $e_{lL}, e_{lH}$, we can argue as in the proof of Proposition 6(i) to show that if exclusion occurs under PD, then it occurs also under no PD, but the reverse is untrue. If exclusion occurs under no PD but does not occur under PD, then

$$\begin{aligned} W^{PD} &= \nu^A[\theta_L u^A(q_L^*) + e_L^a u^B(q_L^*) - c q_L^*] \\ &+ (1-\nu^A)[\theta_H u^A(q_H^*) + e_H^a u^B(q_H^*) - c q_H^*] \end{aligned}$$

is larger than

$$W^N = (1-\nu^A)[\theta_H u^A(q_H^*) + e_H^a u^B(q_H^*) - c q_H^*]$$

because $q_L^* = \arg\max_q[\theta_L^\nu u^A(q) + e_{lL} u^B(q) - cq]$ and $\theta_L^\nu < \theta_L$, $e_{lL} < e_L^a$ (given no type reversal), hence

$$\theta_L u^A(q_L^*) + e_L^a u^B(q_L^*) - c q_L^* > \theta_L^\nu u^A(q_L^*) + e_{lL} u^B(q_L^*) - c q_L^* > 0.$$

(ii) The proof of Proposition 6(ii) applies, after replacing $e_L, e_H$ with $e_{lL}, e_{lH}$ to show that $\overline{W}(q_L^*, q_H^*) > \overline{W}(q^P, q^P)$, and type reversal with positive sorting means that $e_{hL} - e_{lL} < 0 < e_{hH} - e_{lH}$. Hence $q_L^* < q^P < q_H^*$ implies $\nu^A(1-\nu^B)(e_{hL} - e_{lL})u^B(q_L^*) + (1-\nu^A)(1-\nu^B)(e_{hH} - e_{lH})u^B(q_H^*) > \nu^A(1-\nu^B)(e_{hL} - e_{lL})u^B(q^P) + (1-\nu^A)(1-\nu^B)(e_{hH} - e_{lH})u^B(q^P)$, and from (20) we conclude that $W^{PD} > W^N$.

(iii) The proof of Proposition 6(iii) applies, after replacing $e_L, e_H$ with $e_{lL}, e_{lH}$ to show that $\overline{W}(q^P, q^P) > \overline{W}(q_L^*, q_H^*)$, and type reversal with negative sorting means that $e_{hL} - e_{lL} > 0 > e_{hH} - e_{lH}$. Hence $q_L^* < q^P < q_H^*$ implies $\nu^A(1-\nu^B)(e_{hL} - e_{lL})u^B(q^P) + (1-\nu^A)(1-\nu^B)(e_{hH} - e_{lH})u^B(q^P) > \nu^A(1-\nu^B)(e_{hL} - e_{lL})u^B(q_L^*) + (1-\nu^A)(1-\nu^B)(e_{hH} - e_{lH})u^B(q_H^*)$, and from (20) we conclude that $W^N > W^{PD}$. QED

**Proof of Proposition 9** We know that $q^{FB}(\theta)$ satisfies $\theta u^{A\prime}(q^{FB}(\theta)) + e(\theta)u^{B\prime}(q^{FB}(\theta)) - c = 0$, hence, the implicit function theorem reveals that

$$\frac{dq^{FB}(\theta)}{d\theta} = \frac{u^{A\prime}(q^{FB}(\theta)) + e'(\theta)u^{B\prime}(q^{FB}(\theta))}{-\theta u^{A\prime\prime}(q^{FB}(\theta)) - e(\theta)u^{B\prime\prime}(q^{FB}(\theta))}.$$

Since $-\theta u^{A\prime\prime}(q^{FB}(\theta)) - e(\theta)u^{B\prime\prime}(q^{FB}(\theta)) > 0$ and $\frac{dq^{FB}(\theta)}{d\theta} \geq 0$ by assumption, we deduce that

$$e'(\theta) \geq -\frac{u^{A\prime}(q^{FB}(\theta))}{u^{B\prime}(q^{FB}(\theta))} \tag{C. 5}$$

Now we consider $q^*(\theta)$, which satisfies $\theta^v u^{A\prime}(q^*(\theta)) + e(\theta)u^{B\prime}(q^*(\theta)) - c = 0$. The implicit function theorem implies that

$$\frac{dq^*(\theta)}{d\theta} = \frac{\frac{d\theta^v}{d\theta}u^{A\prime}(q^*(\theta)) + e'(\theta)u^{B\prime}(q^*(\theta))}{-\theta^v u^{A\prime\prime}(q^*(\theta)) - e(\theta)u^{B\prime\prime}(q^*(\theta))},$$

and from $-\theta^v u^{A\prime\prime}(q^*(\theta)) - e(\theta)u^{B\prime\prime}(q^*(\theta)) > 0$, $u^{B\prime}(q^*(\theta)) > 0$, and (C. 5) we obtain

$$\frac{dq^*(\theta)}{d\theta} \geq \frac{\frac{d\theta^v}{d\theta}u^{A\prime}(q^*(\theta)) - \frac{u^{A\prime}(q^{FB}(\theta))}{u^{B\prime}(q^{FB}(\theta))}u^{B\prime}(q^*(\theta))}{-\theta u^{A\prime\prime}(q^*(\theta)) - e(\theta)u^{B\prime\prime}(q^*(\theta))}$$

Hence, a sufficient condition for $\frac{dq^*(\theta)}{d\theta} \geq 0$ is to have

$$\frac{d\theta^v}{d\theta}u^{A\prime}(q^*(\theta)) - \frac{u^{A\prime}(q^{FB}(\theta))}{u^{B\prime}(q^{FB}(\theta))}u^{B\prime}(q^*(\theta)) \geq 0,$$

which can be rewritten as

$$\frac{d\theta^v}{d\theta} \geq \left(\frac{u^{A\prime}(q^{FB}(\theta))}{u^{B\prime}(q^{FB}(\theta))}\right) / \left(\frac{u^{A\prime}(q^*(\theta))}{u^{B\prime}(q^*(\theta))}\right).$$

This inequality holds, since (i) $\frac{f(\theta)}{1-F(\theta)}$ increasing implies $\frac{d\theta^v}{d\theta} \geq 1$; (ii) $q^{FB}(\theta) \geq q^*(\theta)$ and $\frac{u^{A\prime}}{u^{B\prime}}$ decreasing imply

$$\left(\frac{u^{A\prime}(q^{FB}(\theta))}{u^{B\prime}(q^{FB}(\theta))}\right) / \left(\frac{u^{A\prime}(q^*(\theta))}{u^{B\prime}(q^*(\theta))}\right) \leq 1.$$

QED

**Proof of Proposition 10(ii)** From Guesnerie and Laffont (1984), $q^{SB}$ is either given by (A.8) or is a constant schedule. In this proof, we show that if $h(q^*(\underline{\theta})) > 0$, then no constant schedule is optimal, whereas if $h(q^*(\underline{\theta})) \leq 0$ then $q_k$ is optimal, with $k$ such that $h(k) = 0$. Recall that $\int_{\underline{\theta}}^{\overline{\theta}} G(k,\theta)f(\theta)d\theta$ is the platform's profit given the constant schedule $q_k$, and $h(k) = \int_{\underline{\theta}}^{\overline{\theta}} g(k,\theta)f(\theta)d\theta$ is the derivative of $\int_{\underline{\theta}}^{\overline{\theta}} G(k,\theta)f(\theta)d\theta$ with respect to $k$.

First, suppose $h(q^*((\underline{\theta})) > 0$, and notice that $h(q^*(\theta_m)) < 0$, since $q^*(\theta) \leq q^*(\theta_m)$ for each

$\theta \in [\underline{\theta}, \overline{\theta}]$ (by definition of $\theta_m$), hence, $0 = g(q^*(\theta_m), \theta_m) \geq g(q^*(\theta_m), \theta)$ for each $\theta \in [\underline{\theta}, \overline{\theta}]$ by (A.7) (the strict inequality holds for each $\theta \neq \theta_m$). Hence, the $k$ that satisfies $h(k) = 0$ is between $q^*(\underline{\theta})$ and $q^*(\theta_m)$. But $q_k$ is inferior to the following schedule $\tilde{q}$, which is constant only for high values of $\theta$:

$$\tilde{q}(\theta) = \begin{cases} q^*(\theta) & \text{if } \theta \in [\underline{\theta}, \theta') \\ k & \text{if } \theta \in [\theta', \overline{\theta}] \end{cases}$$

In $\tilde{q}$, the value $\theta' \in (\underline{\theta}, \theta_m)$ is such that $q^*(\theta') = k$. Precisely, with $\tilde{q}$ the profit is equal to

$$\int_{\underline{\theta}}^{\theta'} G(q^*(\theta), \theta) f(\theta) d\theta + \int_{\theta'}^{\overline{\theta}} G(k, \theta) f(\theta) d\theta;$$

with $q_k$ the profit is

$$\int_{\underline{\theta}}^{\theta'} G(k, \theta) f(\theta) d\theta + \int_{\theta'}^{\overline{\theta}} G(k, \theta) f(\theta) d\theta.$$

The former is greater than the latter as

$$\int_{\underline{\theta}}^{\theta'} G(q^*(\theta), \theta) f(\theta) d\theta > \int_{\underline{\theta}}^{\theta'} G(k, \theta) f(\theta) d\theta$$

by definition of $q^*$. Basically, $\tilde{q}$ maximizes the profit for $\theta$ in the interval $[\underline{\theta}, \theta')$, hence it is superior to $q_k$ for $\theta \in [\underline{\theta}, \theta')$. Moreover, $\tilde{q}$ and $q_k$ coincide for $\theta \in [\theta', \overline{\theta}]$.

Now suppose that $h(q^*(\underline{\theta})) \leq 0$. We prove that each schedule in (A.8) violates (A.9). Hence, no schedule in (A.8) can be optimal, therefore, the optimal schedule is constant. In detail, we show that $\int_{\theta'}^{\overline{\theta}} g(z, \theta) f(\theta) d\theta < 0$ for each $\theta' \in [\underline{\theta}, \theta_m)$ and $z = q^*(\theta')$:

(i)

$$\int_{\theta'}^{\overline{\theta}} g(z, \theta) f(\theta) d\theta < \int_{\theta'}^{\overline{\theta}} g(q^*(\underline{\theta}), \theta) f(\theta) d\theta$$

since for each $\theta' \in (\underline{\theta}, \theta_m)$, $z = q^*(\theta')$ is larger than $q^*(\underline{\theta})$ and $g$ is decreasing in $q$;

(ii)

$$0 \geq h(q^*(\underline{\theta})) = \int_{\underline{\theta}}^{\overline{\theta}} g(q^*(\underline{\theta}), \theta) f(\theta) d\theta = \int_{\underline{\theta}}^{\theta'} g(q^*(\underline{\theta}), \theta) f(\theta) d\theta + \int_{\theta'}^{\overline{\theta}} g(q^*(\underline{\theta}), \theta) f(\theta) d\theta;$$

(iii) $\int_{\underline{\theta}}^{\theta'} g(q^*(\underline{\theta}), \theta) f(\theta) d\theta > 0$ by (A.7) because $q^*(\underline{\theta}) < q^*(\theta)$ for each $\theta \in (\underline{\theta}, \theta')$ and $g(q^*(\theta), \theta) = 0$ . Hence, from (ii) we see that $\int_{\theta'}^{\overline{\theta}} g(q^*(\underline{\theta}), \theta) f(\theta) d\theta < 0$. This, together with (i), implies $\int_{\theta'}^{\overline{\theta}} g(z, \theta) f(\theta) d\theta < 0$.

Also notice that $\min q^* = \min_{\theta \in [\underline{\theta}, \overline{\theta}]} q^*(\theta) = \min\{q^*(\underline{\theta}), q^*(\overline{\theta})\}$ and (A.7) implies that $h(\min q^*) > 0$. Since $h(q^*(\underline{\theta})) \leq 0$ by assumption, it follows that $q^*(\underline{\theta}) > q^*(\bar{\theta})$ and $q^{SB}$ coincides with $q_k$ for a suitable $k$ such that $q^*(\overline{\theta}) < k < q^*(\underline{\theta})$. QED

**Proof of Proposition 10(iii)** The proof is omitted for space's sake because the proof proceeds as a mirror to the proof for Proposition 10(ii).

**Proof of Proposition 11(i)** From Guesnerie and Laffont (1984), it follows that $q^{SB}$ is given by (A.8) or is a constant schedule. Since $h(0) \leq 0$, then there exists no $k > 0$ such that $h(k) = 0$, and it is impossible that $q^{SB} = q_k$ for some $k > 0$. If $q^{SB}$ satisfies (A.8), then $q^*$ need to be single-peaked in $(\underline{\theta}, \theta_0)$ and $\theta_m \in (\underline{\theta}, \theta_0)$ denotes the peak, $\theta', z$ are such that $\theta' \in (\underline{\theta}, \theta_m)$, $z = q^*(\theta')$, and (A.9) holds. We now show that this is impossible:

(i) $h(0) \leq 0$ implies $h(q^*(\underline{\theta})) < 0$;

(ii) for each $\theta' \in (\underline{\theta}, \theta_m)$ , $h(q^*(\underline{\theta})) = \int_{\underline{\theta}}^{\theta'} g(q^*(\underline{\theta}), \theta) f(\theta) d\theta + \int_{\theta'}^{\overline{\theta}} g(q^*(\underline{\theta}), \theta) f(\theta) d\theta$;

(iii) from (i)-(ii) and $\int_{\underline{\theta}}^{\theta'} g(q^*(\underline{\theta}), \theta) f(\theta) d\theta > \int_{\underline{\theta}}^{\theta'} g(q^*(\theta), \theta) f(\theta) d\theta = 0$, we obtain $\int_{\theta'}^{\overline{\theta}} g(q^*(\underline{\theta}), \theta) f(\theta) d\theta < 0$.

Finally, $z = q^*(\theta') > q^*(\underline{\theta})$ implies $\int_{\theta'}^{\overline{\theta}} g(z, \theta) f(\theta) d\theta < \int_{\theta'}^{\overline{\theta}} g(q^*(\underline{\theta}), \theta) f(\theta) d\theta < 0$, which violates (A.9). QED

**Proof of Proposition 11(ii)** Since $q^*(\theta) = 0$ in $[\underline{\theta}, \theta_0]$, it follows that also $q^{SB}(\theta)$ is zero in $[\underline{\theta}, \theta_0]$. For $\theta > \theta_0$, it is suboptimal to set $q^{SB}(\theta) = 0$ if $\int_{\theta_0}^{\overline{\theta}} g(0, \theta) f(\theta) d\theta > 0$, as a superior alternative is $q^{SB}(\theta) = k$ for each $\theta \in (\theta_0, \overline{\theta}]$, for a small $k > 0$. Since $q^*$ is increasing in $(\theta_0, \theta_m)$, $q^{SB}(\theta)$ coincides with $q^*(\theta)$ for $\theta \in (\theta_0, \theta']$ for some $\theta' \in (\theta_0, \theta_m)$ and is flat in $(\theta', \overline{\theta}]$, with $\theta'$ such that $\int_{\theta'}^{\overline{\theta}} g(q^*(\theta'), \theta) f(\theta) d\theta = 0$. If, instead, $\int_{\theta_0}^{\overline{\theta}} g(0, \theta) f(\theta) d\theta \leq 0$, then $q^{SB} = q_0$ as no $\theta' \in (\theta_0, \theta_m)$ satisfies $\int_{\theta'}^{\overline{\theta}} g(q^*(\theta'), \theta) f(\theta) d\theta = 0$. QED

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