The value of incumbency when platforms face heterogeneous customers*

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Abstract

We study competition for the market in a dynamic model with network externalities, focusing on the efficiency of market outcomes. We propose a representation of the strategic advantages of incumbency and embed it in a dynamic framework with heterogeneous consumers. Then, we completely identify the conditions under which inefficient equilibria with several platforms emerge at equilibrium; explore the reasons why these inefficient equilibria arise; compute the value of incumbency and analyze why static models generally exaggerate it.

1 Introduction

On 31 March 2019, the eight most valuable publicly traded firms in the world were, in order of market capitalization, Microsoft, Apple, Amazon.com, Alphabet Inc., Berkeshire Hathaway, Facebook, Alibaba Group and Tencent, seven of them "platform" firms.[1] Yet, despite the economic and social importance of these large platforms, we do not fully understand the way they compete and either the reasons or consequences of their dominance.

These firms create a tension in the analysis of competition, both from a theoretical and a policy viewpoint. With network externalities, monopolies (or, with product differentiation, a limited number of platforms) are generally efficient. However, as we discuss in section 5 where we review the literature, economists who have studied competition between platforms, have generally studied competition *in* the market by making assumptions on platform heterogeneity and horizontal consumer differentiation which ensure the co-existence of several platforms. From a policy viewpoint, competition authorities do not know how much competition is desirable or even possible in these industries. Our contribution is to provide a unified framework to study both competition *for* the market when only one platform is present and competition *within* the market, when two platforms choose strategies which induce different types of consumers to join different platforms.

We construct a model in which at the outset a single platform controls the market. There are positive network externalities so that consumers prefer, other things being equal, to be on the same platform as other consumers. We study dynamic competition, assuming that there are (at least two) potential entrants in each of an infinite sequence of periods,[2] both when consumers are all similar to each other and when there are two types of consumers.

We use the model to tackle three issues. A) Does competition among platforms enhance or reduce social welfare? B) What strategies will platforms use to compete? C) What is the value of incumbency?

The first question arises because, in the presence of network externalities, competition is a double edged sword. It lowers prices, which benefits consumers. However, it can also induce different groups of consumers to choose different platforms, which is inefficient (see Weyl and White (2014) for an analysis along these lines). In a dynamic setup, we fully characterize the circumstances under which several platforms can co-exist (see Proposition 1) and derive some important lessons. First, the presence of entrants generally

[1] *List of public corporations by market capitalization*, accessed 5 June 2019, `https://en.wikipedia.org/wiki/List_of_public_corporations_by_market_capitalization`.

[2] We make this assumption for expositional reasons. All our results hold with two infinitely lived entrants at the start of the game.

increases social welfare. Not only does it decrease prices and ensures that more consumers join a platform, but it also makes it more likely that they join the same platform. Accordingly, entry deterring strategies by an incumbent cannot be justified on the grounds that they help consumers coordinate on the same platform. Second, static models generally exaggerate the inefficiencies: they predict that consumers join different platforms more often than do dynamic models. Thus, it is important to think through the long run consequences of competition.

Turning to the second question, our analysis yields rich insights about the strategies platforms use to defend their incumbency advantage. In particular, we show that the incumbent faces a tension in his treatment of consumers who are not very sensitive to network externalities. On the one hand, their presence on its own platform increases what the other consumers are willing to pay. On the other hand, letting them be captured by an entrant might decrease the intensity of competition.

Finally, our analysis of the value of incumbency focuses on a comparison between static and dynamic settings. As in the switching cost models of Biglaiser, Crémer and Dobos (2013, 2016), when consumers are either identical or not too different from each other, the profits which would be computed for the incumbent in a one period model are *exactly* the same as those obtained from a fully dynamic model. In the first period, entrants price low enough in order to try to attract clients that competition "eats up" all the incumbent's future profits. On the other hand, when consumers are heterogenous, the one period model underestimates the profits computed through a fully dynamic model. However, this difference is relatively limited and, in any case, the dynamic profits are always strictly less than the value of a flow of one period profits. That is, the value of incumbency is more limited than what a naive analysis would predict. This should give policy makers pause before they react too aggressively in markets with network externalities. (Of course, we abstract from a host of other factors which would enter in a full analysis of the benefits of being an incumbent, such as, for instance, the possession of consumer data.)

To conduct this analysis, we develop a new and, we believe, more convenient way to represent the reluctance of consumers to migrate from one platform to the other. In policy discussions economists often argue that network effects make consumers reluctant to migrate and hence provide a strong advantage to incumbents: Levin's (2013) statement that "It may be difficult for an innovative new platform to gain market share, even if its underlying attributes and technology are better" provides a typical example. However, formal models of competition between platforms do not naturally lead to this conclusion. Absent switching costs, there is no reason for all the members of an incumbent platform *not* to purchase from a new entrant who would offer better conditions.

It is impossible to study the constraints that potential entry puts on the strategies of incumbents without formalizing the above intuition. As we discuss in the literature review of section 5, much of the small amount of work which has been conducted on this issue tackles the problem by modelling the beliefs of consumers. This approach has been used in a dynamic model by Hałaburda, Jullien and Yehezkel (2018) with homogenous consumers, but cannot readily be extended to the case of heterogenous consumers.

In section 2, we present a simple model and propose a new solution concept to represent the coordination of consumers through what we call *Attached Consumers* (AC) equilibria. We essentially assume that consumers only change platforms when it is individually rational for them to do so: they are very bad at coordinating their moves even when it would be Pareto efficient. This enables us to select an equilibrium of the game played by the consumers when they choose which platform to join. This equilibrium depends on the prices charged by the platforms and also on the initial allocation of consumers among the various platforms; our equilibrium is tractable even with several types of consumers. We show that AC equilibria always exist and that they are generally unique. Because this equilibrium concept gives a great deal of power to the incumbent(s) and can be viewed as choosing the best equilibrium from their point of view, it makes our results that the profits of incumbent(s) are quite limited in the dynamic model all the more striking.

In section 3, we provide a complete characterization of the circumstances where there are, inefficiently, several platforms in equilibrium, while in section 4 we characterize the circumstances where there is, efficiently, only one platform in equilibrium. In order to facilitate the comparison of our paper to previous contributions, we discuss the literature in section 5, after presenting our analysis. Section 6 concludes by presenting some open research questions. Proofs are in the Appendix.

2 The model and equilibrium

2.1 Preferences and competition

We consider an economy with two types of consumers who must decide which platform to join. There is a mass α_h of *High Network Effects* (HNE) consumers and a mass α_ℓ of *Low Network Effects* (LNE) consumers.[3] We refer to h and ℓ

[3]For the purpose of this section, the fact that some consumers derive more utility than the others from the presence of other consumers play no role.

as the "types" of the consumers.

Using, as we will throughout the paper, θ' to denote "the other type," different from θ, a consumer of type θ derives utility $u_\theta(\gamma_{i\theta}, \gamma_{i\theta'})$ from belonging to platform i to which $\gamma_{i\theta}$ consumers of type θ and $\gamma_{i\theta'}$ consumers of type θ' also belong. The functions u_θ are differentiable and satisfy $u_\theta(0,0) = 0$.[4] Note that the utility of consumers depends on the platform which they join only through the identities and numbers of other consumers on that same platform.

Consumers prefer to have more consumers of both types on the platform to which they belong, and also prefer a marginal increase in the number of consumers of their own type to a marginal increase in the number of consumers of the other type:

$$\partial u_\theta(\gamma_{i\theta}, \gamma_{i\theta})/\partial\gamma_{i\theta} > \partial u_\theta(\gamma_{i\theta}, \gamma_{i\theta'})/\partial\gamma_{i\theta'} \geq 0. \tag{1}$$

Thus, we are not studying a two-sided market where agents care primarily about the number of the other type of agents on the platform which they join — we discuss briefly this hypothesis in section 6, the conclusion. We make no concavity or convexity assumptions on the utility functions.

It will be useful to have notation for consumer utilities when all consumers of a given type are on the same platform:

$$u_\theta^\theta = u_\theta(\alpha_\theta, 0),\ u_\theta^{\theta'} = u_\theta(0, \alpha_{\theta'}),\ u_\theta^{\theta\theta'} = u_\theta(\alpha_\theta, \alpha_{\theta'}).$$

Hence, u_θ^θ is the utility of the consumers of type θ when they all belong to a platform to which no consumer of type $\theta' \neq \theta$, belong. Similarly, $u_\theta^{\theta'}$ is the utility of one of these consumers who belong to the same platform as all the consumers of the other type, and $u_\theta^{\theta\theta'}$ their utility if all consumers belong to the same platform.

Consumers of any type prefer to be on a platform with *all* consumers of the same type rather than on another platform with *all* the consumers of the other type:

$$u_\theta^\theta > u_\theta^{\theta'} \text{ for } \theta \in \{h, \ell\}. \tag{2}$$

Condition (1) implies (2) if $\alpha_h = \alpha_\ell$. However, if α_ℓ were much larger than α_h, HNE consumers might rather belong to the same platform as LNE consumers than belong to the same platform as other HNE consumers. Condition (2) assumes this away.

[4]All our results still hold true if $u_h(0,0) = u_\ell(0,0) > 0$. On the other hand, we would have to change our analysis, but in non-essential ways, if HNE and LNE consumers had different stand-alone utilities for the platforms, *i.e.,* if $u_h(0,0) \neq u_\ell(0,0)$.

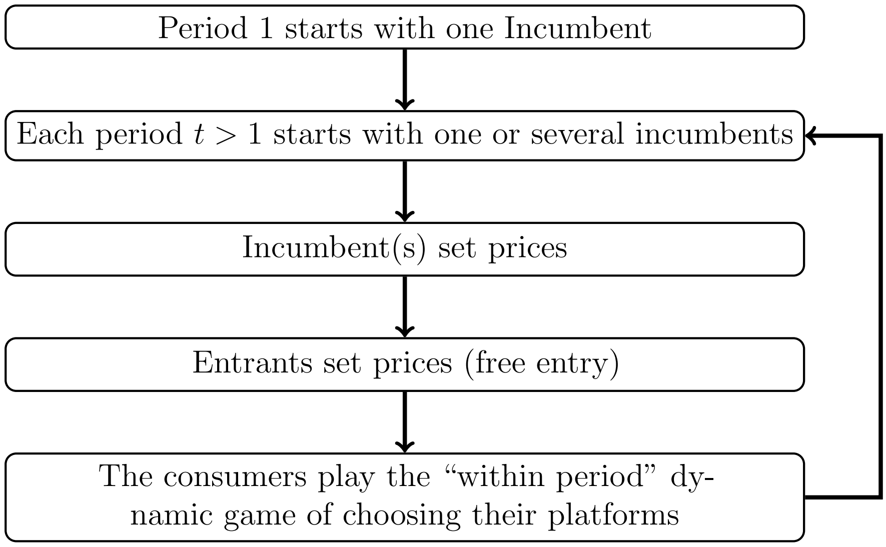

Figure 1: The dynamic model.

Condition (1) also implies $u_\theta^{\theta\theta'} \geq u_\theta^\theta$ and $u_\theta^{\theta\theta'} > u_\theta^{\theta'}$ and therefore

$$u_\theta^{\theta\theta'} \geq u_\theta^\theta > u_\theta^{\theta'} \geq 0 \text{ for } \theta \in \{h, \ell\}.$$

We are representing the fact that the HNE consumers value network effects more than LNE consumers by the following conditions:

$$u_h^{h\ell} > u_\ell^{\ell h} \text{ and } u_h^h > u_\ell^\ell.$$

Finally, it is convenient to divide the parameter space according to the following criterion:

$$u_\ell^{\ell h} \leq u_h^h - u_h^\ell. \tag{3}$$

The right hand side is the amount that an HNE consumer would be willing to pay to move from a platform to which all the LNE consumers belong to a platform to which all the (other) HNE consumers belong — (2) implies that it is strictly positive. The left hand side is the willingness of an LNE consumer to be on the same platform as all other LNE and HNE consumers. Written $u_h^\ell < u_h^h - u_\ell^{\ell h}$, the inequality puts an upper bound on u_h^ℓ. When (3) does not hold there is only one platform in equilibrium whether in a one period or in an infinite horizon model (see section 3).

We now turn to a description of the game played by the platforms and the consumers, which is illustrated by Figure 1. At the beginning of period 1 there is one incumbent, which we will call the "Incumbent'; we assume in the un-modeled period 0 it sold to all the consumers, maybe because it developed the market or maybe because of intellectual property protection.[5,6] In each subsequent period, there can be one or more incumbents: the firms that sold to a strictly positive measure of consumers in the previous period. There will also be $n_E \geq 2$ (new) entrants in each period. For simplicity, we assume Stackelberg timing where all the incumbents first set prices simultaneously and then the entrants, having seen these prices, choose their own prices.[7] Having observed the prices, the consumers choose their platforms; we discuss how in section 2.2. We assume that platforms with no consumers at the end of a period "drop out"of the game.[8] The game then moves to period $t + 1$. All the agents, platforms and consumers have a discount rate of δ.

2.2 Modeling incumbency

We now describe how, within each period, the consumers react to the prices posted by the platforms and choose the platform they join.

An *allocation* γ of consumers among m platforms is a $2 \times m$ vector of nonnegative real numbers $\{\gamma_{ih}, \gamma_{i\ell}\}_{i=1,\ldots,m}$ with $\sum_i \gamma_{i\theta} = \alpha_\theta$ for $\theta \in \{h, \ell\}$, where $\gamma_{i\theta}$ is the measure of consumers of type θ on platform i. A minimal requirement is that the allocation of users among platforms is a Nash equilibrium of the game in which they would simultaneously choose a platform, *i.e.*, that (suppressing the period indices) we have

$$\gamma_{i\theta} > 0 \Longrightarrow u_\theta(\gamma_{i\theta}, \gamma_{i\theta'}) - p_i = \max_j \left[u_\theta(\gamma_{j\theta}, \gamma_{j\theta'}) - p_j\right] \text{ for all } i \text{ and all } \theta,$$

where p_i is the price charged by platform i.

The concept of Nash equilibrium is not fully satisfactory for our purposes, for two reasons. First, if the prices charged by the platforms are not too

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[5]We will assume if some consumers did not buy before entry occurred, they think of the Incumbent as a natural source when making their purchasing decisions once entry occurs.

[6]As we discuss on page 3.1, our results are essentially not changed if we assume that at the outset consumers of different types are on different platforms.

[7]The same basic results regarding profits hold with Nash timing, where firms simultaneously set prices. There would only exist mixed strategy equilibria, but the equilibrium profits of the platforms would be the same (see Biglaiser et al. (2013, 2016) for discussion of similar issues in a model of switching costs). In equilibrium the Incumbent trades off higher revenues due to higher prices with the probability of losing all the consumers. For the entrants a lower price increases the probability of attracting the HNE consumers (which is a plus) but also implies a higher loss when they they attract only the LNE consumers. Along the equilibrium path, consumers switch more between platforms with Nash than with Stackelberg timing.

[8]Formally, this would be done by assuming that in any period $\tau > t$, their strategy set is a singleton, and that purchasing from these firms is not in the consumers strategy set.

different from each other, there will be multiple Nash equilibria, as is standard in models with network externalities. Second, the concept of Nash equilibrium has no role for incumbency. In the rest of this section, we propose an equilibrium selection strategy which solves these two problems. As discussed in the introduction, this equilibrium selection models consumers who would find it very difficult to coordinate their migration to a superior platform. The selected equilibrium, which we call *Attached Consumers* or AC equilibrium, depends on the incumbency pattern.

We do not call the concept of AC equilibria a refinement because we do not attempt to find the most "reasonable" equilibrium given the rules of the game but rather select one using extraneous information. We do so by describing how consumers migrate from their origin platforms to destination platforms, and show that the final allocation is a Nash equilibrium.

In keeping with the focus of this paper we define the migration process and AC equilibria with two types of consumers, but the definition of equilibrium and the results of this section trivially extend to any finite number of types. We also assume that the only choice that the consumers face is which platform to join, but there is no difficulty extending the definitions to situations where one of the choices is to join no platform (and indeed we do so in section 3).

Loosely speaking, we think of the within period migration process as follows: Consumers evaluate their utility of staying on the platform from the previous period and migrating to a new platform under the assumption that no other consumer migrates. We rank the consumers from the most to gain to the least. If some consumers obtain a positive gain, a small measure of those consumers who have the most to gain from their current platform migrate to the best alternative platform. This process is repeated until no consumer has a strictly positive gain from moving. The migration process and definition of AC equilibria are illustrated on Figure 2. Note that, for expository purposes, it is easier to proceed through a rather mechanical definition but the reader should keep in mind that, as we show at the end of the section, our consumers are in no way irrational.

Formally, a migration path from an allocation β to another allocation γ is a sequence $\{\eta^t\}_{t=0,1\ldots,T}$ of allocations

$$\sum_i \eta^t_{i\theta} = \alpha_\theta \text{ for all } t = 0, 1, \ldots T \text{ and all } \theta,$$

which *leads from* β to γ in T steps, so that we have

$$\eta^0_{i\theta} = \beta_{i\theta} \text{ and } \eta^T_{i\theta} = \gamma_{i\theta}, \text{ for all } i \text{ and all } \theta. \tag{4}$$

At each step $t = 1, 2, \ldots, T$ the consumers who migrate all have the same type, the *type transferred* $\theta(t)$, and all belong to the same platform, the

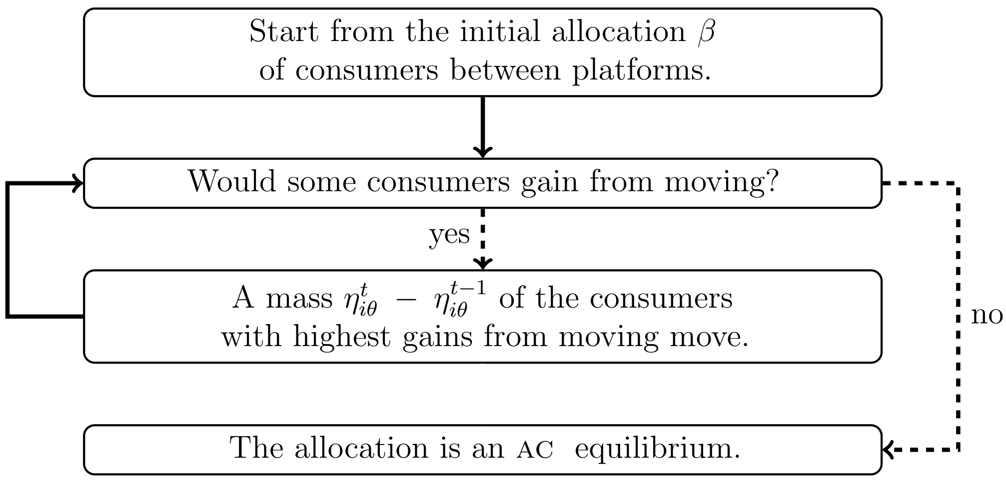

Figure 2: This figure represents in algorithmic form the definition of AC equilibria.

source platform $s(t)$. All these consumers also move to the same *destination platform* $d(t)$. Formally:

$$\eta_{d(t)\theta(t)}^{t} - \eta_{d(t)\theta(t)}^{t-1} = \eta_{s(t)\theta(t)}^{t-1} - \eta_{s(t)\theta(t)}^{t} > 0 \tag{5a}$$

and

$$\eta_{i\theta}^{t} = \eta_{i\theta}^{t-1} \text{ unless } \{i, \theta\} \text{ is equal to either } \{d(t), \theta(t)\} \text{ or to } \{s(t), \theta(t)\}. \tag{5b}$$

Note that the definition puts no constraint on the mass of consumers who migrate at each step; Lemmas 2 and 4 show that it has no influence.

Along a migration path, consumers only migrate if they would have *strictly* benefit from the migration had they migrated alone:[9]

$$u_{\theta(t)}\left(\eta_{d(t)\theta(t)}^{t-1}, \eta_{d(t)\theta'(t)}^{t-1}\right) - p_{d(t)} > u_{\theta(t)}\left(\eta_{s(t)\theta(t)}^{t-1}, \eta_{s(t)\theta'(t)}^{t-1}\right) - p_{s(t)}. \tag{6}$$

[9]Through the strict inequality in (6) we assume that consumers stay on their current platform when they are indifferent between doing so and joining another one. This considerably simplifies the proofs without changing the qualitative results: if we replaced the strict inequality by a weak inequality we would have to use the type of limit pricing arguments standard in the study of Bertrand competition with different marginal costs when analyzing the price setting strategy of platforms. We do relax this condition in the study of single platform equilibria in section 4, for off the equilibrium path moves, in order to guarantee the existence of a certain class of equilibria.

Because of positive network externalities, whenever (6) holds, we have

$$\begin{aligned} u_{\theta(t)}\left(\eta^{t-1}_{d(t)\theta(t)}+\mu(t),\eta^{t-1}_{d(t)\theta'(t)}\right)-p_{d(t)} &> u_{\theta(t)}\left(\eta^{t-1}_{s(t)\theta(t)},\eta^{t-1}_{s(t)\theta'(t)}\right)-p_{s(t)} \\ &> u_{\theta(t)}\left(\eta^{t-1}_{s(t)\theta(t)}-\mu(t),\eta^{t-1}_{s(t)\theta'(t)}\right)-p_{s(t)}. \end{aligned} \tag{7}$$

The utility of all the consumers who migrate strictly increases (first line) and is larger than if they had not migrated (second line).

Finally, we assume that there is no other migration of one consumer, either by consumers of type $\theta(t)$ or by any other consumer, which would yield greater gains in utility: [10]

$$\begin{aligned} &\{\theta(t), d(t), s(t)\} \\ &\qquad \in \arg\max_{\theta,i,j}\left\{\left[u_\theta\left(\eta^{t-1}_{i\theta},\eta^{t-1}_{i\theta'}\right)-p_i\right]-\left[u_\theta\left(\eta^{t-1}_{j\theta},\eta^{t-1}_{j\theta'}\right)-p_j\right]\right\}. \end{aligned} \tag{8}$$

Equation (8) states that the consumers who migrate are among those who would have gained the most by migrating alone. However, because several consumers move at the same time, this does not necessarily imply that the migration which takes place is the one which generates the largest increase in the utility of the migrating consumers. We come back to this point after lemma 4 and show that (8) can be given very natural interpretations.

We gather the conditions we impose on migration paths in the following definition.

Definition 1 (migration path). *A migration path from an allocation β to another allocation γ is a sequence $\{\eta^t\}_{t=0,1\ldots,T}$ of allocations which satisfy equations* (4)*,* (5a)*,* (5b)*,* (6) *and* (8)*.*

The allocation γ is a *final allocation* if there is no migration path leading from γ to another allocation.

Definition 2 (AC equilibrium). *An allocation γ is an* AC *equilibrium if it is on a migration path from the original allocation and is a final allocation.*

[10] An alternative set of assumptions would dispense with (8). The only constraint on migration would be (6): at each step, the utility increase due to migration is strictly positive. This is not sufficient to prove our results. Indeed, we have built an example using this relaxed assumption where in the initial allocation the HNE consumers are on one platform and the LNE consumers on another. The migration leads to a Nash equilibrium where all the HNE consumers and some of the LNE consumers migrate to one platform and the rest of the LNE consumers to another one. Lemma 3 does not hold.

We discuss how our equilibrium concept differs from others in the literature in section 5. At this time, we do note that it is related to the pessimistic beliefs approach of Caillaud and Jullien (2003), but our concept can handle situations where there are multiple incumbents while there is no natural way to do so with their solution concept.

If an initial allocation is a final allocation, then there can be no other allocation that can be reached by a migration path of length 1. It is straightforward to see that this implies that the allocation is an AC equilibrium and therefore proves the following lemma.

Lemma 1. *All* AC *equilibria are Nash equilibria. An initial allocation is an* AC *equilibrium if and only if it is a Nash equilibrium. Furthermore, if an initial allocation is a Nash equilibrium, it is the only* AC *equilibrium.*

Lemma 1 has two consequences for the interpretation of AC equilibria. First, trivially from a technical viewpoint (this is a consequence of the fact that there is a continuum of consumers) but importantly for interpretation, it would be straightforward to write down a proper dynamic game played by the consumers for which the sequence of moves from an initial allocation to an AC equilibrium would be a (perfect Nash) equilibrium.[11] Second, Lemma 1 also enables us to think of the description of moves in Figure 2 as a representation of the way in which consumers think about the choices of the other consumers, in the spirit of fictitious play. In "real" time, the actual migrations would take place simultaneously.

We will call a migration path a *migration path through large steps* if at every step all the consumers of type $\theta(t)$ on platform $s(t)$ migrate to platform $d(t)$: $\eta^t_{s(t)\theta(t)} = 0$ for all t. The following lemma makes easier both the identification and proof of existence of AC equilibria.

Lemma 2. *The set of* AC *equilibria is not changed if we impose the restriction that the migration path is a migration path through large steps.*

Lemma 2, whose proof can be found in the appendix, holds because migration gives rise to a 'snowballing' effect: if in any migration process not all the consumers of type $\theta(t)$ migrate from platform $s(t)$ to platform $d(t)$, then we must have $\theta(t+1) = \theta(t)$, $s(t+1) = s(t)$ and $d(t+1) = d(t)$. Indeed, for consumers of type $\theta(t)$, the utility of being on platform $d(t)$ has strictly increased while the utility of being on platform $s(t)$ has decreased; there is no other move between two platforms which would yield a greater increase

[11]Note that this depends on the fact that the migration takes place fast enough that there is no utility generated during the migration.

the utility of a consumer of type $\theta(t)$. From Condition 1, this also holds for consumers of type $\theta'(t)$, but for a smaller gain than for those of type $\theta(t)$.

The following lemma, an immediate consequence of Lemma 2, is used extensively in the rest of the paper.

Lemma 3. *If all the consumers of type θ belong to the same platform in the initial allocation (i.e., if $\beta_{i\theta} = \alpha_\theta$ for some i), then they also belong to the same platform in any* AC *equilibrium.*

Since condition (7) holds for any non-negative $\mu(t)$, we have the following lemma for which we can interpret our migration paths as a sequence of "individual moves".

Lemma 4. *The final allocation of consumers of the migration path is not changed if we add the restriction that, for all t, $\eta^t_{d(t)\theta(t)} - \eta^{t-1}_{d(t)\theta(t)} = \eta^{t-1}_{s(t)\theta(t)} - \eta^t_{s(t)\theta(t)}$ must be smaller than some $\epsilon > 0$.*

Lemma 4 is proved by "cutting" each step of a large step migration into smaller steps with the same source and destination platforms and the same migrating type. It shows that we can think of migration paths as approximating a process in which the consumers move "one by one" from one platform to the other; in each stage the consumer who moves is one of the consumers who gains the most from moving.

By Lemma 4, our results would not change under the assumption that at any stage only a small mass of consumers migrate. Then, our assumption that it is the consumers who would have the most to gain if they migrated alone who migrate is basically equivalent to the maybe more natural assumption that it is the consumers who have the most to gain from the actual migration who migrate.

It is easy to show that large step migrations must eventually stop at an AC equilibrium, which proves the following lemma.

Lemma 5. *Whatever the initial allocation $\{\beta_{ih}, \beta_{i\ell}\}_{i=1,\dots,m}$ and prices p_i charged by the platforms, there exists an* AC *equilibrium.*

The interested reader will find an extensive comparison between our approach and other approaches proposed in the literature in section 5.

2.3 Entry and efficiency in a one period model: a digression

Before turning to the analysis in the equilibria in the infinite horizon model, it is useful to briefly discuss the AC equilibria in one period models. First, as a benchmark, suppose that there is no entry: the Incumbent announces a price and the consumers decide whether or not to stay on its platform. They all stay if the Incumbent charges $u_\ell^{\ell h}$ or less, and its profit is then $(\alpha_h+\alpha_\ell)u_\ell^{\ell h}$. If the Incumbent charges u_h^h, which is greater than $u_\ell^{\ell h}$ by (3), its only clients are the HNE consumers and its profit is $\alpha_h u_h^h$.[12] This proves the following lemma.

Lemma 6. *In a one period model without entry, if* (3) *holds and we have*

$$u_h^h \geq \frac{\alpha_h+\alpha_\ell}{\alpha_h} u_\ell^{\ell h}. \tag{9}$$

the incumbent charges u_h^h and sells only to HNE *consumers for a profit of $\alpha_h u_h^h$.*

Otherwise, i.e., if either (3) *or* (9) *does not hold, the incumbent charges $u_\ell^{\ell h}$, sells to both types of consumers, and its profit is $(\alpha_h+\alpha_\ell)u_\ell^{\ell h}$.*

Let us now assume that there is at least two entrants. Entrants will never charge less than 0, and competition among them implies that any entrant who attracts consumers will do so at a price of 0. The price at which the incumbent keeps all the consumers is $u_\ell^{\ell h}$, as without entry. On the other hand, assuming that (3) holds, entry will force the incumbent to offer a lower price (at most $u_h^h - u_h^\ell$) to retain only the HNE consumers as they are attracted by the presence of the LNE consumers on one of the entrants. This implies the following lemma.

Lemma 7. *In a one period model with entry, if*

$$u_h^h - u_h^\ell \geq \frac{\alpha_h+\alpha_\ell}{\alpha_h} u_\ell^{\ell h} \tag{10}$$

(which implies that (3) *holds) the incumbent sells only to the* HNE *customers at the price $u_h^h - u_h^\ell$ and its profit is $\alpha_h(u_h^h - u_h^\ell)$.*

If (10) *does not hold, and in particular if* (3) *does not hold, the incumbent sells to all consumers at price $u_\ell^{\ell h}$ and its profit is $(\alpha_h+\alpha_\ell)u_\ell^{\ell h}$.*

Lemmas 6 and 7 together imply the following corollary.

Corollary 1. *In a one period model, entry makes the separation of* LNE *and* HNE *consumers less likely and therefore improves efficiency.*[13] *When the incumbent sells to both types of consumers the price it charges and its profit are the same with or without entry.*

[12] All other prices are dominated byone of these two prices.

[13] Formally, there are some parameter values for which separation occurs with entry but not without entry, and none for which the opposite is true.

Note the reason why entry improves efficiency: the incumbent finds it more costly to let the LNE consumers "go" to an entrant, since it will be more costly to keep the HNE consumers. This result has ramifications for policy, since it says, somewhat counter-intuitively, that entry keeps more consumers on an incumbent's platform despite network effects. This is a gain in efficiency in two senses. First, all consumers belong to the same platform. Second, without entry, instead of joining another platform LNE consumers opt out of the market. In section 3.2, we revisit the role of entry in a dynamic framework and generally confirm the results of Corollary 1.

2.4 Equilibrium in the infinite horizon game

We now turn to the definition of equilibrium in an our infinite horizon model. We focus on Markov equilibria where on and off the equilibrium path, in any period t, the price charged by the incumbent(s) i depends only on the mass of consumers which they have inherited from past periods, the $\beta^t_{i\theta}$s; the prices charged by the entrants depend only on the $\beta^t_{i\theta}$s and on the the prices charged by the incumbent(s); and the equilibrium of the game played between the consumers depends on the $\beta^t_{i\theta}$s and on the prices charged by the platforms, incumbent(s) and entrants.[14,15]

In Appendix B we prove the following "myopia principle", which also holds in the belief based analysis of Hałaburda et al. (2018), and plays an important role in the sequel.

Lemma 8 (Myopia principle)**.** *Given the prices chosen by the firms, the set of equilibria of the game played by the consumers in any period t of a dynamic game is the same as if the game were a one period game.*

The myopia principle *does not imply* that the prices charged by the platforms will be the same in a multi-period game as in a one period game — it is only the consumers who act *as if* they are "myopic", not the firms. This is a consequence of two aspects of our model. First, there is a continuum of consumers and, as a consequence, no individual consumer can affect the other players in the game. Second, consumers have no switching costs, their future utility is unaffected by the platform which they joined in period t.

[14]More precisely, we are considering anonymous and measurable equilibria.

[15]Our assumptions are sufficient to exclude "collusive" equilibria which would arise if we assumed a finite number of infinitely lived entrants. See Biglaiser and Crémer (2011) for discussion of such outcomes in a switching cost framework.

3 Two platform equilibria

In this section, we study the conditions under which two platforms coexist at equilibrium; they are formally stated in Proposition 1.

3.1 Main results

In the first period, the Incumbent charges p_2, when it has both types of consumers. After the first period, there will be two platforms on the equilibrium path. If, off equilibrium, in some period t all the consumers belong to one platform, by the Markov hypothesis in period $t+1$ they would again split among two platforms as described in the previous paragraph. In any period, on and off equilibrium, in which there is only one incumbent, it will keep only the HNE consumers, because $u_h^h > u_\ell^{\ell h}$.

Thus, we need only distinguish the following equilibrium prices and profits for the incumbents:

p_h: the price charged by a firm, which we will call an H incumbent, whose clients in the previous period were (only) the HNE consumers. Its equilibrium total discounted profit will be $\Pi_h = \alpha_h p_h/(1-\delta)$.

p_ℓ: the price charged by a firm, which we will call an L incumbent, whose clients in the previous period were (only) the LNE consumers. Its equilibrium total discounted profit will be $\Pi_\ell = \alpha_\ell p_\ell/(1-\delta)$;.

p_2: the price charged by a firm, which we will call a 2 incumbent, who sold to both types of clients in the previous period with total discounted profit of $\Pi_2 = \alpha_h p_2 + \delta \Pi_h$. These are also the price and profit of the Incumbent.

There can be a two platform equilibrium only if in the first period at least one of the entrants charges a price p_E which enables it to attract one type of consumer, which must be the LNE consumers, since

$$-p_E - (u_\ell^{\ell h} - p_2) > -p_E - (u_h^{h\ell} - p_2).$$

In the appendix, we prove the following proposition.

Proposition 1. *There exists a two platform equilibrium if and only if the following condition, which implies* (3)*, holds:*

$$u_h^h - u_h^\ell \geq \frac{(1-\delta)\alpha_\ell + \alpha_h}{(1-\delta)\alpha_h}(u_\ell^{\ell h} - \delta u_\ell^\ell). \qquad \text{(2NetCond)}$$

In the equilibrium of Proposition 1 starting in period 2, we have

$$p_\ell = u_\ell^\ell (1-\delta) \tag{11}$$

and

$$\Pi_\ell = \alpha_\ell u_\ell^\ell . \tag{12}$$

It is easy to show that this would be the price and the profits if charged by the Incumbent if there were only LNE consumers.

The first period Incumbent and in subsequent periods H incumbents and 2 incumbents charge the same price,

$$p_2 = p_h = \frac{(1-\delta)(\alpha_h + \alpha_\ell)(u_h^h - u_h^\ell)}{(1-\delta)\alpha_\ell + \alpha_h}, \tag{13}$$

and we have,

$$\Pi_2 = \Pi_h = \frac{\alpha_h \times p_h}{1-\delta} = \frac{\alpha_h(\alpha_h + \alpha_\ell)(u_h^h - u_h^\ell)}{(1-\delta)\alpha_\ell + \alpha_h}. \tag{14}$$

The difficult part of the proof is proving that (2NetCond) is a necessary and sufficient condition and that equations (12) to (14) hold — this is done in Appendix C. The binding deviation for the existence of a two platform equilibrium is the attempt by the Incumbent to keep all the consumers. By (12) the lowest price that entrants are willing to charge is $-\delta u_\ell^\ell$. Because the LNE consumers are the most eager to change platforms, the Incumbent has to charge at most $u_\ell^{\ell h} - \delta u_\ell^\ell$ if it wants to keep all the consumers. The profits resulting from repeating this strategy forever are

$$\Pi_D = \frac{(\alpha_\ell + \alpha_h)(u_\ell^{\ell h} - \delta u_\ell^\ell)}{1-\delta}. \tag{15}$$

Condition (2NetCond) is equivalent to

$$\Pi_D \leq \Pi_2 .$$

The following results are an immediate consequence of Proposition 1 and equations (11) to (14).

Corollary 2. *In a two platform equilibrium, the profit of the Incumbent*

a) is increasing in u_h^h, decreasing in u_h^ℓ and independent of $u_h^{h\ell}$, $u_\ell^{\ell h}$, u_ℓ^ℓ and u_ℓ^h;

b) is increasing in α_h and in α_ℓ;

c) is less than $(\alpha_\ell + \alpha_h)(u_h^h - u_h^\ell)$;

d) and (for parameters where there exist two platforms equilibria in both the static and dynamic models) is greater than the profit of the Incumbent of the two platform equilibrium in the one period model, $\alpha_h(u_h^h - u_h^\ell)$, and smaller than the value of a flow of this one period profit, $\alpha_h(u_h^h - u_h^\ell)/(1-\delta)$.

Point a of the Corollary shows that the profit of the Incumbent is independent of the preferences of the LNE consumers; indeed these preferences do not affect p_2: when the Incumbent does not sell to the LNE consumers the price it charges is not affected by then how much they value the presence of others. But these preferences do play a role in the existence of a two platform equilibrium, as they influence Π_D, through the maximal price the Incumbent can charge while keeping all the consumers.

Entrants cannot attract the HNE consumers without also attracting the LNE consumers, who take advantage of the below cost price and leave in the next period. Therefore, the LNE consumers get in the way of entrants who want to attract only the HNE consumers and, as a consequence, they have value for the Incumbent: the more numerous they are, the less aggressive the entrants and the higher the price and profits of the Incumbent — this is true even if the LNE consumers derive no utility from joining platforms ($u_\ell^{\ell h} = u_\ell^\ell = u_\ell^h = 0$).

Points c and d of Corollary 2 put bounds on the Incumbent's profit. In particular, its profit is increasing in δ, and, as stated in point c, always smaller than $(\alpha_h + \alpha_\ell)(u_h^h - u_h^\ell)$, and *a fortiori* smaller than $(\alpha_h + \alpha_\ell)u_h^h$, its profit in the one period model if all the consumers were HNE consumers. Thus, an incumbent's long run profits are smaller than the greatest profit it could make in the static model if it was constraint to keep only the HNE consumers. Section 4 will show that this result also holds true when there is only one platform in equilibrium (see page 21).

We have assumed that all the consumers are on the Incumbent at the start of the first period. If condition (2NetCond) is satisfied, the equilibrium that we have described above in this section is also, *mutatis mutandis*, a two platform stationary Markov equilibria if at the outset the HNE consumers are on one platform and the LNE consumers on the other. It is also clear that in that case (2NetCond) is also necessary for the existence of such an equilibrium, as the analysis of existence can be conducted starting from an out of equilibrium state of nature with all the consumers on one platform.

In 3.2, we examine in greater details the positive and normative consequences of Proposition 1.

3.2 Existence and welfare implications

We now turn to a detailed discussion of the existence of two platform equilibria; this is of policy importance since, in our setup with positive network externalities, it is always more efficient to have one rather than two platforms. We will show that, depending on the parameter values, a two platform equilibrium exist either for δ not too large or δ neither too small nor too large. We use this analysis to show that entry increases welfare, even though a one network equilibrium is efficient. Finally, we offer some comments on the difference in predictions between static and dynamic models.

For the rest of this subsection, we assume $u_\ell^{\ell h} > u_\ell^\ell$ and treat the special case when $u_\ell^{\ell h} = u_\ell^\ell$ in a note at the end. The following corollary is a direct consequence of (2NetCond).

Corollary 3. *If $u_\ell^{\ell h} > u_\ell^\ell$, there exists a $\widetilde{\delta}$ such that a two platform equilibrium does not exist when $\delta \in \left(\widetilde{\delta}, 1\right)$.*

There are two complementary ways to think about this result. First, in industrial organization economics, the discount factor is thought of as being influenced both by the interest rate and the probability of the "end of the world", which for our model would be interpreted as the appearance of a new disruptive technology. Our results indicate that efficiency, under the form of the existence of a single platform, is more likely in a more stable world. Second, one can view δ as a proxy for the frequency with which platforms can change their prices; the more often this happens, the more intense the competition between platforms.

To study the entire range of discount factors, it is useful to rewrite (2NetCond) as $u_h^h - u_h^\ell \geq g(\delta)$, where

$$g(\delta) \stackrel{\text{def}}{=} \frac{(1-\delta)\alpha_\ell + \alpha_h}{(1-\delta)\alpha_h}(u_\ell^{\ell h} - \delta u_\ell^\ell),$$

is strictly convex[16] in δ and satisfies $\lim_{\delta \to 1} g(\delta) = +\infty$.

The condition $u_h^h - u_h^\ell > g(0)$ holds if and only if the rightmost inequality in (10) does not hold — this is a necessary and sufficient for the existence of a two network equilibrium in the static model with entry whose equilibrium is described in lemma 7.

The sign of

$$g'(0) = u_\ell^{\ell h} - \frac{\alpha_\ell + \alpha_h}{\alpha_h} u_\ell^\ell$$

is key to the existence of a two network equilibrium as a function of δ.

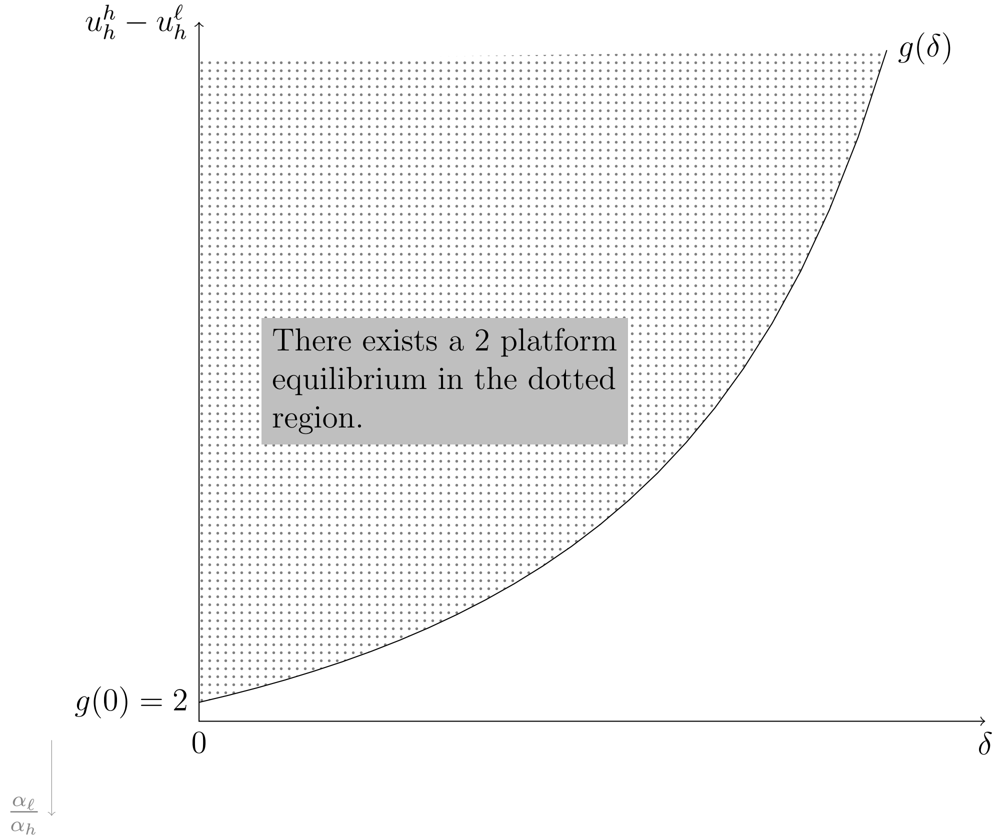

Figure 3: This figure illustrates the existence of a two platform equilibrium when $g'(0) > 0$ with $\alpha_\ell = \alpha_h$, $u_\ell^{\ell h} = 1$ and $u_\ell^\ell = .2$. The right hand side of (2NetCond) is then equal to $(2-\delta)(1-.2\delta)/(1-\delta)$, which is equal to 2 when $\delta = 0$. There exists a two platform equilibrium whenever δ is small enough.

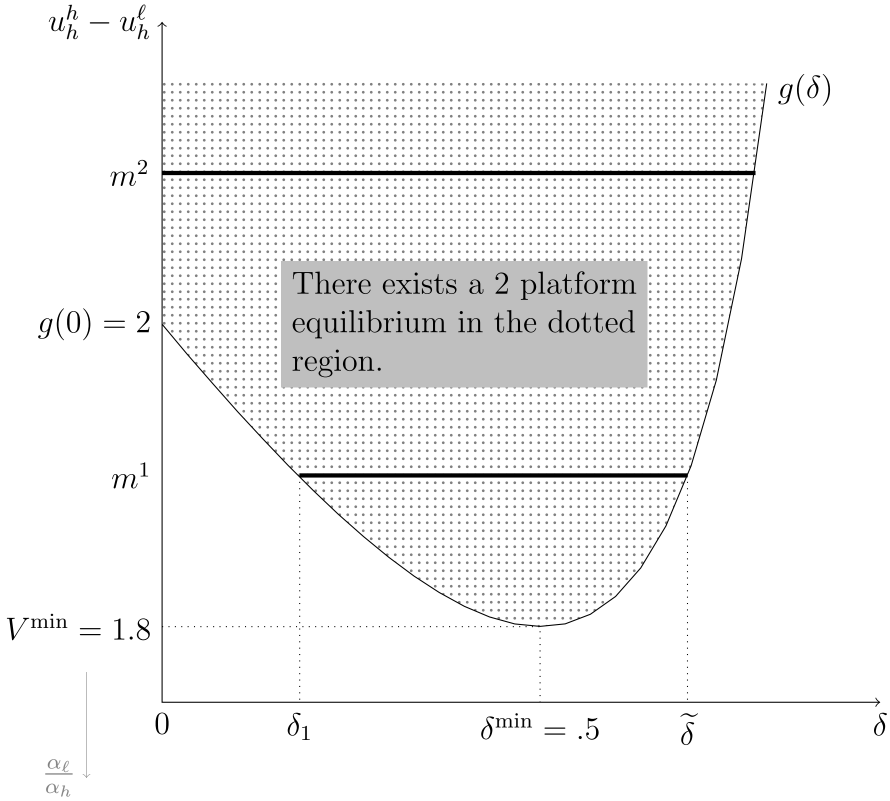

Figure 4: This figure illustrates the existence of a two platform equilibrium when $g'(0) < 0$ with $\alpha_\ell = \alpha_h$, $u_\ell^{\ell h} = 1$ and $u_\ell^\ell = .8$. The right hand side of (2NetCond) is then equal to $(2-\delta)(1-.8\delta)/(1-\delta)$, which is equal to 2 when $\delta = 0$. Its minimum, $V^{\min}$, which is obtained for $\delta^{\min} = .5$, is equal to 1.8. For any $m^1 = u_h^h - u_h^\ell \in [1.8, 2]$, there exists a two platform equilibrium when $\delta \in [\delta_1, \widetilde{\delta}] \subset (0, 1)$. For any $m^2 = u_h^h - u_h^\ell > 2$, there exists a two platform equilibrium if and only if δ is small enough.

The case $g'(0) > 0$ is illustrated by Figure 3. If a two network equilibrium exists in the static model it only exists in the dynamic model if $\delta \leq \widetilde{\delta}$. Therefore per period welfare is higher in the dynamic than in the static setting, as consumers are less likely to be allocated to different platforms.

The case $g'(0) < 0$ is illustrated by Figure 4. The function g reaches a minimum $V^{\min}$ for some $\delta^{\min} < 1$. As when $g'(0) > 0$, two network equilibria do not exist for large discount factors and for small discount factors they do not exist in the dynamic model whenever they do not exist in the static model. However, for some intermediate values of the discount factor there can exist a two network equilibrium in the dynamic model when there does not exist one in the static model. This requires that $u_\ell^{\ell h} - u_\ell^\ell$, the additional value that LNE consumers obtain from being on the same platform as HNE consumers, be small. Intuitively, for small discount factors, LNE consumers derive little additional benefit from belonging to the same platform as the HNE consumers, and the entrants price aggressively to attract the LNE consumers. In contrast, the incentive for the incumbent to deviate and keep all the consumers increases in the discount rate and eventually swamps this effect.

Finally, it is interesting to see whether entry enhances welfare in the dynamic model as is the case in the static model. Observing that in the no entry model, the Incumbent's *per period* payoffs are exactly the same in the static and the dynamic models, we have the following corollary:

Corollary 4. *For δ close to 0 or 1, entry makes the separation of* LNE *and* HNE *consumers less likely in the dynamic model and therefore improves efficiency.*

As explained in section 2.3, entry improves welfare in the static model. The situation is more complicated in the dynamic model, confirming the remark of Fudenberg and Tirole (2000, p. 375) that "in equilibrium, the welfare effects of inducing additional entry are ambiguous". Without entry, the leftmost inequality of (10) must be satisfied in both in the static and dynamic models for the Incumbent to keep only the HNE consumers while with entry Condition (2NetCond) is necessary and sufficient for a two platform equilibrium to exist in the dynamic model. When δ is close to either 0 or 1, entry makes it more likely that all consumers remain on the Incumbent network, just as it does in the static model. For intermediate values of δ, however, for some values of the parameters there exists a two platform equilibrium in the dynamic model without entry but not in the dynamic model with entry. On the one hand, the Incumbent finds it more costly to

[16] Indeed, $g(\delta) = \frac{\alpha_\ell}{\alpha_h}(u_\ell^{\ell h} - \delta u_\ell^\ell) + \frac{u_\ell^{\ell h} - \delta u_\ell^\ell}{1-\delta} = \frac{\alpha_\ell}{\alpha_h}(u_\ell^{\ell h} - \delta u_\ell^\ell) + u_\ell^\ell + \frac{u_\ell^{\ell h} - u_\ell^\ell}{1-\delta}$, which is the sum of an affine and a strictly convex function.

keep the LNE consumers in the dynamic model where entrants are willing to price down to $-\delta u_\ell^\ell$ while in the static model they are not willing to offer a negative price. On the other hand, without entry the profit in the two platform dynamic model is equal to $1/(1-\delta)$ times the profit in the static model, but with entry when a two platform equilibrium exists in the dynamic model, the profit is smaller than $1/(1-\delta)$ times the static profit. This leads to Corollary 4.

Note on the case $u_\ell^{\ell h} = u_\ell^\ell$. When the LNE consumers obtain no additional utility from belonging to the same platform as the HNE consumers, if there exists a two platform equilibrium in the static model, then there is a two platform equilibrium in the dynamic model, but the converse is not always the case. Why is there such a sharp difference with the $u_\ell^{\ell h} > u_\ell^\ell$ case? From (15), Π_D is increasing in δ when $u_\ell^{\ell h} > u_\ell^\ell$ and independent of δ when $u_\ell^{\ell h} = u_\ell^\ell$, so Π_D and the efficiency gains of having all consumers on the same platform do not become infinite when δ is close to 1. Because (2NetCond) is equivalent to $\Pi_D \leq \Pi_2$, this explains the sharp contrast between the two cases.

4 Analysis of equilibria with one platform

In this section, we present our main results on the existence and properties of single platform equilibria paying particular attention to δ close to 1, leaving a full analysis to Appendix E. First, we have

Corollary 5. *There exists a $\overline{\delta}$ such that for any $\delta \geq \overline{\delta}$:*

a) there exists a single platform equilibrium;

b) in all single platform equilibria the Incumbent's profit is $(\alpha_h + \alpha_\ell)(u_h^h - u_h^\ell)$;

c) the profit of the Incumbent is larger than in the static model, but smaller than the value of a flow of one period profits.

Corollary 5 implies that for large δ there exists only a single platform equilibrium, since, as Corollary 3 and the comment that follow it demonstrate, there does not exist a two platform equilibrium for large δ.

From point b, the Incumbent's profit is equal to the product of the total number of consumers, $\alpha_h + \alpha_\ell$, and the price that it would charge to maximize its profit while selling only to HNE consumers in the static model, $u_h^h - u_h^\ell$. This is equal to the upper bound on profits in the two platform equilibrium (see point c of Corollary 2).

For small δ the profit of the incumbent can be smaller, but never larger, than the profit as when the discount factor is large, $(\alpha_h + \alpha_\ell)(u_h^h - u_h^\ell)$.

Corollary 6. *The Incumbent's equilibrium profit in both single and two platform equilibria of the dynamic model never exceeds*

$$(\alpha_h + \alpha_\ell)(u_h^h - u_h^\ell).$$

In the Appendix, we demonstrate that the bound can actually be lower for small δ. In particular, the profit cannot be larger than $u_\ell^{\ell h}/(1-\delta)$. Therefore, taking into consideration Proposition 1 and Corollaries 5 and 6, profits in the dynamic model are never much larger than the profits in the one period model.

For a small set of parameters there exists both a single platform and a two platform equilibrium.[17] As the following corollary states, the Incumbent always prefers the single platform equilibrium, which is also welfare maximizing.[18]

Corollary 7. *For parameter values such that both a single platform equilibrium and a two platform equilibrium exist, the profit of the incumbent is larger in the single platform equilibrium.*

5 Literature

We have built our model to focus on the role of incumbency advantage when platforms compete for the market and subgroups of consumers differ in their tastes for network externalities. This has enabled us to present a) clean conditions under which competition will lead, inefficiently, to the presence of two platforms and b) crisp comparisons of the difference of the incumbent's profits in dynamic and static models.

The relevant literature is vast — the interested reader will find a very complete survey up to about 2005 in Farrell and Klemperer (2007) and newer references in Cabral (2011); we will selectively discuss some papers to highlight what we believe is new in our approach.

5.1 Dynamic competition between platforms

First, our paper is related to the recent literature on dynamic competition between platforms. This literature has focused on oligopoly models where

[17]More precisely, for a small set of parameters there exist both a two platform equilibrium and a "T type" equilibrium (see definition in the Appendix). However, whatever δ, there nearly never exist both a two platform equilibrium and an "S type" one platform equilibrium.

[18]Corollary 7 is a direct consequence of Proposition 1 and Lemma E-6 (and Lemma E-4 for the set of measure zero in which there exist both a two platform equilibrium and a S2 one platform equilibrium).

platforms are in somewhat stable competition with each other. Cabral (2011) provides an interesting and representative example. In every period, a new consumer chooses to join one of two differentiated platforms; once a consumer has joined a platform he stays with this platform until "death" — this is equivalent to assuming infinite switching costs. The dynamics of the model are driven by the interplay of two forces. First, platforms would like to price low to attract consumers, which increases both their current profits and their future attractiveness. On the other hand, as they become larger, they have incentives to increase their prices in order to reap high profits. The analysis focuses on the dynamics of dominance and stresses that convergence to monopoly is unlikely despite the fact that platforms will be of unequal size. Because in each period only one consumer chooses a platform, the issue of coordination between consumers does not arise. Some authors have used similar models to study compatibility between platforms (for instance, Chen, Doraszelski and Harrington, 2009) or introduce some simple version of two sided competition (Zhu and Iansiti, 2012; Laussel and Resende, 2014). In this tradition, several platforms compete over an infinite horizon. To ensure that they both survive, they must attract consumers even when they are smaller than their rivals. To obtain this result it is typically assumed that horizontal differentiation is "stronger" than platform effects.

Contrary to this strand of literature, we are interested in the dynamics of competition *for* the market rather than competition *in* the market. We study the persistence of market power and endogenize the number of firms at equilibrium. Consumers can switch platforms and the platforms can attract each other's customers.

In the earlier literature, Katz and Shapiro (1986) built a two period, two platforms model which tackled similar issues. The platforms are identical except for their costs. Consumers, who all have the same utility function, are unattached at the start; some choose a platform in the first period, some in the second. Katz and Shapiro study the circumstances under which first and second period consumers join the same platform. They show that firms compete aggressively in the first period, to benefit of the incumbency advantage in the second period — in the switching cost context Klemperer (1995) named this strategy, also discussed in Beggs and Klemperer (1992), "invest and harvest".

Fudenberg and Tirole (2000) present an infinite horizon model which builds on Katz and Shapiro's. Consumers live for two periods and the new consumers, who are all identical, coordinate on the best platform from their viewpoint. In some periods all the consumers purchase from the incumbent. In others, a low cost entrant will price low enough to attract the new consumers. However, despite the fact that the incumbent has an installed based advantage, the entrant technology is assumed to be better than the incumbent in such a way that, at equal benefits from network externalities, it provides more utility to the consumers. This technological gap caused by entry is larger than the network externalities. As a consequence, the focus of the paper is more on the way in which the incumbent uses network externalities to defend against entry of a technically superior entrant than on the way in which incumbency advantage can be exploited.

In particular they show that despite the fact that in a static model the incumbent would price high enough to exclude consumers with a low standalone valuation for platform services, in a dynamic model, it charges a lower price to make its services more attractive to the new consumers.

Some authors have studied the incentives for a monopolist to allocate its customers among several platforms that it owns (Board, 2009; Veiga, 2018): controlling several platforms opens the door to price discrimination but lessens the benefits of platform externalities.

5.2 Modeling incumbency advantage

Policy makers have been quite concerned about the long lasting market power of platforms as consumers find it difficult to coordinate on joining other platform; yet, the literature proposes very few theoretical explorations of the sources of incumbency advantage. In their early survey of platform effects, Katz and Shapiro (1994) say "Asymmetries involving reputation, product differentiation, and installed base[19] are especially likely when one of the firms is an entrant and the other an incumbent." Notice that according to this view, it is not incumbency by itself that induces the competitive advantage; however, we believe that most economists would feel that incumbency advantage would persist even if the entrants were subsidiaries of large firms with established reputation, even if products were similar and even if there were no switching costs so that there would be no "hard wired" installed base. It is this view which we have taken in this paper, despite the fact that we do agree that the points raised by Katz and Shapiro are often relevant.

Most papers which try to model incumbency advantage have done so by modelling the beliefs of the consumers. In a static framework, the pioneering work in this strand of literature is Caillaud and Jullien (2003) who assume, in a two-sided framework, that, out of equilibrium, the agents on both sides of the market coordinate on the equilibrium which is the less favorable to the entrant. Hagiu (2006) and Hałaburda and Yehezkel (2013) follow a similar strategy.

[19] In Crémer, Rey and Tirole (2000) incumbency is defined by the presence of "trapped" consumers who cannot leave the Incumbent.

Jullien (2011) extended the idea of favorable expectations for the incumbent to multi-sided platforms.[20] The only restriction that these authors put on migration processes is that consumers, of which they have only one type, migrate as long as it is a dominant strategy for them to do so; as explained in section 2.2, we impose the stronger condition that, essentially, the consumers with the most to gain from migration migrate first. This enables us to select one equilibrium when there several types of consumers and, as a consequence, to tackle multi-period models and also multiple possible incumbents.

Recently, Hałaburda et al. (2018) have built a dynamic infinite horizon duopoly model with heterogenous firms where incumbency, modeled through a "belief approach", plays an essential role. They assume that last period's incumbent is "focal" in the beliefs of the consumers, who all have the same preferences. They show that a firm can stay dominant even with lower quality. Because of their belief based approach, there can be several equilibria when the horizon is infinite.

Other authors use strategies closer to those of the present paper, modelling either the way in which consumers would migrate from an incumbent to an entrant or the way in which they would choose to join a platform where none existed previously. For instance, Ochs and Park (2010) assume that consumers are uncertain about the tastes of other consumers and go through several rounds of choosing whether or not to join a firm. Farrell and Saloner (1985, 1986, 1988) study games where consumers choose one after the other whether or not to join a platform; there can be either too much or too little migration. Biglaiser, Crémer and Veiga (2018) study the attempt of consumers to spend as little time as possible on a "small" platform, the incumbent if they migrate too early or the entrant if they migrate too late. They find that there can be an incumbency advantage due to the "free rider" problem of waiting for others to move (see also Ostrovsky and Schwarz (2005) for a similar approach).

[20] A number of authors have constructed models of coordination without a role for incumbency. For instance Ambrus and Argenziano (2009) use the concept of coalitional rationalizability to essentially assume that consumers can coordinate on "good" equilibria from their point of view. Argenziano (2008) adopts the global game approached introduced in the economic literature by Carlsson and Van Damme (1993) to study competition between symmetric platforms but in her framework there is only one "threshold equilibrium"; Gunay (2013) follow her lead but assumes that consumers have better information about the incumbent platform than about the entrant. In a recent working paper, Akerlof, Holden, and Rayo (2018) examine an equilibrium refinement based on k-level thinking in settings with network externalities.

5.3 Switching costs and network externalities

As we discussed in the introduction, network externalities have often been called "social switching costs." Biglaiser et al. (2013, 2016) examine models with free entry, an incumbent, and consumer switching costs. As in the current paper, the Incumbent's profit does not grow very much when expanding the time horizon from one to an infinite number of periods. Furthermore, the result of equality of static and dynamic profit with homogenous consumers holds in both the case of switching costs and network effects. This can be seen by examining Corollary 6 with $\alpha_\ell = u_h^\ell = 0$. Then, the profit is $\alpha_h u_h^h$ which is the same as the profit in the static model with only HNE consumers. When consumers are all similar, competition with the entrants dissipates all the rents of the Incumbent. It is the diversity of consumers that is the source of profits.

While some of the results in the current paper are similar to the results obtained with switching costs, there are subtle but important differences in their consequences for the strategies of firms and consumers. For example, the myopia principle does not hold in switching cost models unless future switching costs are uncorrelated with current switching costs: otherwise, high switching cost consumers try to "hide among" low switching cost consumers who induce firms to charge low prices — the high switching cost consumers are willing to incur higher costs in the current period in order to do so (Biglaiser et al., 2016). Furthermore, with network externalities, a consumer's current utility is directly affected by the choice of platform of the other consumers, while with switching cost it is only by the consequences on future prices that consumers choices affect each other. These differences are analyzed in Crémer and Biglaiser (2012), where we also conduct a very preliminary analysis of a model with both network externalities and switching cost.

6 Conclusion

We have studied the value of incumbency in a market with network externalities and free entry. Competition *for* the market greatly limits the additional profits in a dynamic model relative to the static market outcome. Consumer heterogeneity can have great strategic value and even consumers who never join the incumbent's platform enhance the incumbent's profits. In order to study the value of incumbency we have proposed a criterion for equilibrium selection based on a model of migration between platforms.

We assumed that entry is costless. We focussed on platforms with identical qualities in order to isolate the effects of competition for the behavior of platforms. We believe that our methodology can be extended to take into account the benefits derived by the incumbent from learning by doing and from the acquisition of data. From a policy point of view, it should be stressed that the importance of these benefits is not clear and vigorously debated. For instance, even in the search engine industries where fixed costs would be thought to be enormous, Google and Bing have a number of smaller competitors: Qwant, DuckDuckGo, Exelead, and Gigablast (it should be noted that some buy their search results from the two bigger engines).

Many markets with platform externalities are two-sided markets. Using our selection approach in two sided settings introduces some interesting possibilities. The 'snowballing' effect discussed in section 2.2 when examining migration through large steps would not necessarily arise. Also, many two sided platforms offer multiple functionalities on at least one side. They may therefore compete on some dimensions and not others (for instance, eBay and Amazon compete for the sales of some goods, but not on the e-book market). Combined with the fact that consumers often multi-home, this opens up a very rich area for investigation which has not been sufficiently explored. We plan to use our approach to explore two-sided markets in the future.

We can allow for horizontal or vertical differentiation between platforms. If this differentiation is small relative to the network effects, nothing much changes.[21] On the other hand, large horizontal or vertical quality differences could substantially affect the results. For example, we have focussed our attention on one type of inefficiency with network effects, that consumers could inefficiently choose different platforms. Differentiation could lead to other types of inefficiencies; for instance that consumers join the wrong type of platform. However, it is also conceivable that competition could enhance efficiency along these dimensions.

Finally, we have focussed on one aspect of the way in which platforms could exploit their market power, through excessive pricing, in a framework where monopolization induces efficiency. There are other issues on which much more work needs to be done. In particular, platforms may or may not offer the efficient quality of the product.

[21] In a working paper version, we showed that many of the insights can apply when entrants' platforms have an added fixed term quality advantage over the incumbent.

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Appendix

A Proofs of results in sections 2.2 and 2.3

Proof of Lemma 1. As stated in the text, straightforward from the definitions. □

Proof of Lemma 2. Assume that a migration path which leads to an AC equilibrium is not a migration path through large steps. Then, there exists a t such that $\eta^{t+1}_{\theta(t)s(t)} > 0$ (at the beginning of step $t+1$ there remains some consumers of type $\theta(t)$ on platform $s(t)$). It is easy to see that

$$\{\theta(t+1), d(t+1), s(t+1)\} = \{\theta(t), d(t), s(t)\}$$

as at step $t+1$, the migration will involve the same type of consumers moving from the same platform to the same platform as at step t. Indeed, in all platforms the utility of agents of type $\theta(t)$ is the same at the end and at the beginning of step t, except for the fact that it is strictly higher in $d(t)$ and strictly smaller in $s(t)$. For agents of type $\theta'(t)$, the "other type", the same property holds true; however by (1), the increase in the utility they derive from $d(t)$ and the the decrease in the utility they derive from $s(t)$ are smaller than for agents of type $\theta(t)$. Hence, Condition (6) holds true when the superscript $t-1$ is replaced by t. We can therefore construct a new migration path, which will lead to the same final allocation by replacing steps t and $t+1$ by one "larger" step with the same θ, d and s. Iterating on this procedure will lead to a migration path through large steps which leads to the same allocation as the original path. □

Proof of Lemma 3. As stated in the text, this is an immediate consequence of Lemma 2. □

Proof of Lemma 4. It is sufficient to show that a migration path through large steps can be replaced by a migration path with $\eta^{t}_{d(t)\theta(t)} - \eta^{t-1}_{d(t)\theta(t)} < \varepsilon$ for all t.

Let $\left\{\bar{\theta}(t), \bar{d}(t), \bar{s}(t)\right\}_{t=1,\overline{T}}$ define a large step migration path. We construct a new migration path in the following way. Let $\underline{\theta}(1) = \bar{\theta}(1)$, $\underline{d}(1) = \bar{d}(1)$, $\underline{s}(1) = \bar{s}(1)$, and $\underline{\eta}$ such that

$$0 < \underline{\eta}^{1}_{\underline{\theta}(1)\underline{d}(1)} - \underline{\eta}^{0}_{\underline{\theta}(1)\underline{d}(1)} = \underline{\eta}^{0}_{\underline{\theta}(1)\underline{s}(1)} - \underline{\eta}^{1}_{\underline{\theta}(1)\underline{s}(1)} < \varepsilon.$$

At the end of step 1 on the new migration path, by the same reasoning as in the proof of Lemma 2,

$$u_{\underline{\theta}(1)}\left(\underline{\eta}^1_{\underline{d}(1)\underline{\theta}(1)}, \underline{\eta}^1_{\underline{d}(1)\underline{\theta}(1)}\right) - p_{\underline{d}(1)} - \left[u_{\underline{\theta}(1)}\left(\underline{\eta}^1_{\underline{s}(1)\underline{\theta}(1)}, \underline{\eta}^1_{\underline{s}(1)\underline{\theta}(1)}\right) - p_{\underline{s}(1)}\right] \\ > u_{\widetilde{\theta}}\left(\underline{\eta}^1_{i\widetilde{\theta}}, \underline{\eta}^1_{i,\widetilde{\theta}'}\right) - p_i - \left[u_{\widetilde{\theta}}\left(\underline{\eta}^1_{j\widetilde{\theta}}, \underline{\eta}^1_{j,\theta'}\right) - p_j\right]$$

for all $(\widetilde{\theta}, i, j) \neq (\underline{\theta}(1), \underline{d}(1), \underline{s}(1))$. Therefore

$$\{\underline{\theta}(2), \underline{d}(2), \underline{s}(2)\} = \{\underline{\theta}(1), \underline{d}(1), \underline{s}(1)\},$$

and by an easy recursion it is possible to build a new migration path which after a finite number t^*_1 of steps will rejoin the original migration path: $\underline{\eta}^{t^*_1}_{\theta j} = \overline{\eta}^1_{\theta j}$ for all θ and j. We can then take $\underline{\theta}(t^*_1+1) = \overline{\theta}(2)$, $\underline{d}(t^*_1+1) = \overline{d}(2)$ and $\underline{s}(t^*_1+1) = \overline{s}(2)$. By the same reasoning as in the previous paragraph there will exist t^*_2 such that after t^*_2 steps the new migration path will have the same allocation as the original migration path at $t = 2$. The result is proved by noticing that we can repeat the process until convergence to the final allocation along the original path. □

Proof of Lemma 5. We have defined migration paths by the fact that they lead from one initial allocation to a final allocation. To show that there exists a final allocation whatever the initial allocation define the following large steps migration path. At every step, check whether there exist a $\{\theta(t), d(t), s(t)\}$ satisfying (6). If there is, move all the consumers of type $\theta(t)$ from $s(t)$ to $d(t)$. If there is not, we have identified a final allocation and therefore an AC equilibrium.

To finish the proof, we only need to show that any such migration path will eventually find itself at a stage where no $\{\theta, d, s\}$ satisfies (6). At every step, either the destination platform already has clients or it charges a strictly lower price than the source platform, or both. To each platform which has a strictly positive mass of consumers, associate an index equal to the number of platforms which charge strictly lower prices multiplied by either 1 if it has a positive mass of only one type of consumers and 2 if it has a positive mass of both types of consumers. The sum of these platform indexes decreases by at least one at each stage of the migration. Given that this sum cannot be smaller than 0, the result is proved. □

Proofs of Lemmas 6 and 7. See text above statements of the lemmas. □

Proof of Corollary 1. Straightforward. □

B The myopia principle: Proof of Lemma 8

In order to prove Lemma 8, we must first define formally the extension of the notion of AC equilibrium appropriate for dynamic games. In each period t, there is a set of incumbents $\{1, 2, \ldots, n_I^t\}$ (in equilibrium, n_I^t will actually be equal to either 1 or 2), and a set of entrants $\{1, 2, \ldots, n_E\}$. In period 1, all consumers are allocated to the Incumbent. In future periods, the initial allocation is the allocation of consumers at the end of the previous period. For incumbent i in period t, we call β_{ih}^t and $\beta_{i\ell}^t$ the mass of HNE and LNE consumers in his initial clientele; because it is an incumbent, we must have $\beta_{ih}^t + \beta_{i\ell}^t > 0$.

The purchasing decisions of the consumers depend on the $\beta_{i\theta}^t$s, on the prices charged by the firms, and on their expectations of the decisions of other consumers. We call $W_{i\theta,t+1}\left(\beta^{t+1}\right)$ the expected discounted utility measured at the beginning of period $t+1$, before incumbents have chosen their prices, of a consumer of type θ who has purchased from platform i in period t.

Proof of Lemma 8. Because consumers are "small" and do not affect the market through their individual choices and there are no switching costs, $W_{i\theta,t+1}\left(\beta^{t+1}\right)$ does not depend on i, and can therefore be written $W_{\theta,t+1}\left(\beta^{t+1}\right)$. If the equilibrium allocation of consumers in period t has γ_{ih}^t HNE consumers and $\gamma_{i\ell}^t$ LNE consumers in platform j, the utility of a consumer of type θ who purchases from platform i which charges p_i^t will be

$$u_\theta(\gamma_{i\theta}^t, \gamma_{i\theta'}^t) - p_i^t + \delta W_{\theta,t+1}\left(\{\gamma_{jh}, \gamma_{j\ell}\}_{j\in\mathcal{I}(t+1)}\right),$$

where $\mathcal{I}(t+1)$ is the set of incumbents at stage t.

We can apply the same reasoning as in section 2.2 to define migration paths within period t. At each step τ, the consumers who change platforms are those consumers of type $\theta(\tau)$ such that there exists source and destination platforms, $s(\tau)$ and $d(\tau)$, which are solution of

$$\begin{aligned}\max_{\theta',i,i'}\Bigg\{&\left[u_{\theta'}(\eta_{i\theta'}^{\tau-1}, \eta_{i,-\theta'}^{\tau-1}) + W_{\theta',t}\left(\left\{\eta_{j\theta'}^{\tau-1}, \eta_{j,-\theta'}^{\tau-1}\right\}_{j\in\mathcal{I}(t)}\right) - p_i^t\right]\\ &- \left[u_{\theta'}(\eta_{i'\theta'}^{\tau-1}, \eta_{i',-\theta'}^{\tau-1}) + W_{\theta',t}\left(\left\{\eta_{j\theta'}^{\tau-1}, \eta_{j,-\theta'}^{\tau-1}\right\}_{j\in\mathcal{I}(t)}\right) - p_{i'}^t\right]\Bigg\}, \quad \text{(B-1)}\end{aligned}$$

as long as the value of this solution is strictly positive. The same W term appears in both terms of this expression and therefore solving (B-1) is equivalent to solving (8). □

C Proof that condition (2NetCond) is necessary for a two platform equilibrium

This section of the appendix is devoted to the proof of Proposition 1.

One would expect the net surplus of HNE consumers to be larger than the net surplus of LNE consumers in any equilibrium:

$$u_\ell^\ell - p_\ell \le u_h^h - p_h. \tag{C-2}$$

Under this condition, along the equilibrium path once the two types of consumers are on different networks, an entrant cannot attract the HNE consumers without first attracting the LNE consumers.[22] It turns out that the proof of (C-2) is not trivial and we prove it below, starting on page App-8.

We first show that we can strengthen (C-2).

Lemma C-1. *If* (C-2) *holds, then in any two platform equilibrium*

$$u_h^\ell + u_\ell^\ell - p_\ell < u_h^h - p_h. \tag{C-3}$$

Proof of Lemma C-1. Assume that the H incumbent charges a price p_h which satisfies (C-2) and $u_h^\ell + u_\ell^\ell - p_\ell \ge u_h^h - p_h$.

An entrant which charges $p_E \ge -(u_\ell^\ell - p_\ell)$ attracts no consumer as (C-2) implies $-p_E \le u_\ell^\ell - p_\ell \le u_h^h - p_h$.

An entrant which charges $p_E < -(u_\ell^\ell - p_\ell)$ attracts all the consumers: the LNE consumers as $-p_E > u_\ell^\ell - p_\ell$, and, once it has attracted the LNE consumers, the HNE consumers as $u_h^\ell - p_E > u_h^\ell + u_\ell^\ell - p_\ell \ge u_h^h - p_h$. In equilibrium, this must not be profitable; a sufficient and necessary condition for this is $-(\alpha_h + \alpha_\ell)(u_\ell^\ell - p_\ell) + \delta\Pi_2 \le 0$. This condition does not depend on p_h. Therefore, the H incumbent could increase p_h without affecting the demand for its services and therefore increase its profits. □

Condition (C-3) obviously implies $u_h^\ell - p_\ell < u_h^h - p_h$: along the equilibrium path, HNE consumers *strictly* prefer to purchase from the H incumbent than from the L incumbent. Indeed the L incumbent finds it less attractive to attract the HNE consumers than do the entrants, as its opportunity cost to do so is greater because it obtains a positive profit from the LNE consumers that it has attracted in a previous period.

Condition (C-3) implies that the continuation equilibria in the consumers' game as a function of p_E are as represented on Figure C-1.

[22] If (C-2) holds as an equality, then an entrant charging $-p_E = u_h^\ell - p_\ell$, will obtain no consumers. If $-p_E > u_h^\ell - p_\ell$, the entrant will attract all consumers. It will not matter which type of consumers moves first in the migration to the entrant.

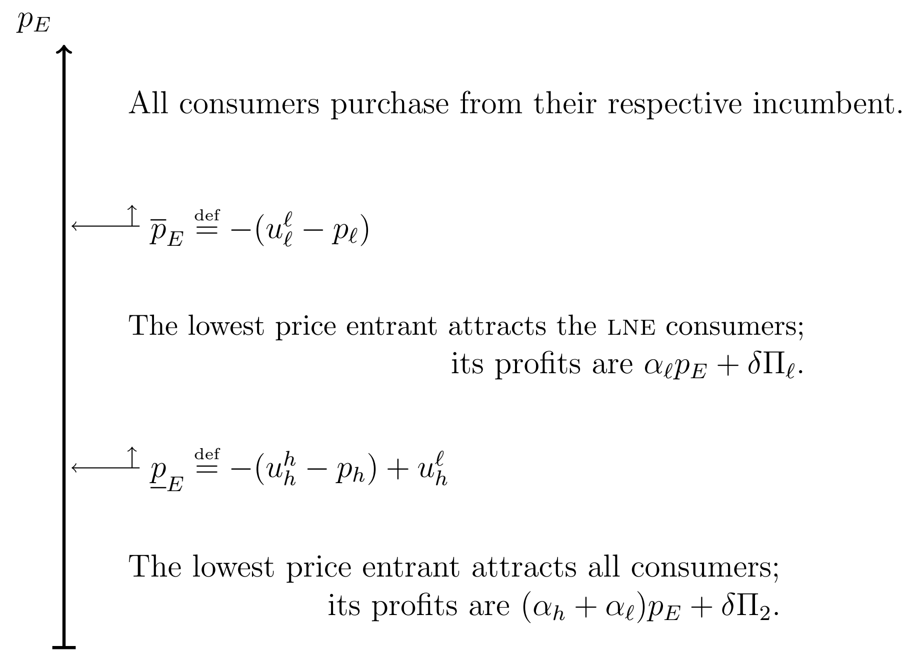

Figure C-1: The response of consumers to entry when (C-3) holds.

From the definitions of the cutoff prices $\underline{p}_E$ and $\overline{p}_E$ in Figure C-1, the following conditions are necessary to ensure that there is no profitable entry:

— an entrant cannot profitably attract only the LNE consumers:

$$\alpha_\ell \overline{p}_E + \delta\Pi_\ell \leq 0; \tag{C-4}$$

— an entrant cannot profitably attract all consumers:

$$(\alpha_h + \alpha_\ell)\underline{p}_E + \delta\Pi_2 \leq 0. \tag{C-5}$$

It is relatively intuitive, and proved in Claim C-1, that (C-4) is binding at equilibrium: otherwise, the L incumbent could raise its price and keep its consumers. Because $\Pi_\ell = \alpha_\ell p_\ell/(1-\delta)$, equations (11) and (12) hold. Thus, once the two groups are separated, the L incumbent behaves in the same way and obtains the same profit as if it where the Incumbent with only the LNE consumers present (see the discussion following equation (12)). Similarly, (C-5) is binding, as shown in Claim C-2: if not, the H incumbent could raise its price and increase its profit.

Claim C-1. *If* (C-2) *holds, then* (C-4) *is binding:*

$$-\alpha_\ell(u_\ell^\ell - p_\ell) + \delta\Pi_\ell = 0.$$

Proof. From (C-4), it is sufficient to prove that we cannot have $-\alpha_\ell(u_\ell^\ell - p_\ell) + \delta\Pi_\ell < 0$. Assume this were the case. In any period after the first, the L incumbent could increase its profit by charging $p'_\ell \in (p_\ell, u_\ell^\ell - \delta\Pi_\ell/\alpha_\ell)$. Indeed, in order to attract the LNE consumers an entrant would have to charge at most $\overline{p}'_E = -(u_\ell^\ell - p'_\ell) < -\delta\Pi_\ell/\alpha_\ell$ and would therefore make negative profits, $\alpha_\ell \overline{p}'_E + \delta\Pi_\ell$. □

Claim C-2. *If* (C-2) *holds, then* (C-5) *is binding:*

$$-(\alpha_h + \alpha_\ell)[u_h^h - p_h - u_h^\ell] + \delta\Pi_2 = 0. \tag{C-6}$$

Proof. By (C-3) and (C-5), if the claim does not hold, there exists $p'_h > p_h$ which satisfies both

$$-(\alpha_h + \alpha_\ell)(u_h^h - p'_h - u_h^\ell) + \delta\Pi_2 < 0 \tag{C-7}$$

and

$$u_h^h - p'_h > u_h^\ell + u_\ell^\ell - p_\ell \Longrightarrow u_h^h - p'_h > u_h^\ell - p_\ell. \tag{C-8}$$

We will show that a deviation by the H incumbent to such a p'_h would be profitable.

The H and L incumbents announce their prices simultaneously; therefore the deviation by the H incumbent would not affect p_ℓ. By (C-8), after such a deviation the LNE consumers would respond by purchasing either from the lowest price entrant or from the L incumbent, as in Figure C-1 (replacing, of course, p_h by p'_h). Therefore, the deviation would be unprofitable for the H incumbent only if an entrant could profitably attract *all* the consumers. It could do this only by charging a price p'_E which satisfies $u_h^\ell - p'_E > u_h^h - p_h$, which by (C-7) implies $p'_E \leq -(u_h^h - p'_h - u_h^\ell) < -\delta\Pi_2/(\alpha_h + \alpha_\ell)$. The profits of the entrant, $(\alpha_h + \alpha_\ell)p'_E + \delta\Pi_2$, would be strictly negative, which proves the result. □

We can now demonstrate the following lemma.

Lemma C-2. *Equations* (11) *and* (13) *are necessary and sufficient for the fact that once* LNE *and* HNE *consumers have purchased from different platforms they will continue to do so in the continuation equilibrium.*

Proof. Only the sufficiency part is left to prove. From Figure C-1 an entrant could try either

a) to attract only the LNE consumers by charging a price strictly smaller than $-(u_\ell^\ell - p_\ell)$, but, by Claim C-1, this is not profitable as $\alpha_\ell(-(u_\ell^\ell - p_\ell)) + \delta\Pi_\ell = \alpha_\ell u_\ell^\ell(-1 + (1-\delta) + \delta) = 0$,
or

b) to attract all consumers by charging a price strictly smaller that $-(u_h^h - p_h) + u_h^\ell$, but this is not profitable by (C-6). $\square$

We now turn to the study of the first period and on the incentives of the agents to create two platforms out of one. First, we must have

$$-(\alpha_h + \alpha_\ell)\left[p_2 - (u_h^h - u_h^\ell)\right] + \delta\Pi_2 \leq 0; \tag{C-9}$$

otherwise, in the first period an entrant could attract all the consumers by charging a price “slightly below” $p_2 - (u_h^h - u_h^\ell)$ and make a strictly positive profit.

Proof of equation (C-9). Let p_E be price charged by the first period entrant which attracts the LNE consumers (it is the lowest price offered by any entrant). Because the entrant attracts no HNE consumers, we must have $u_h^h - p_2 \geq u_h^\ell - p_E$ and therefore $p_E \geq p_2 - (u_h^h - u_h^\ell)$. If (C-9) did not hold, then an entrant could choose a p' “close to” but smaller than $p_2 - (u_h^h - u_h^\ell)$, attract all the consumers and make a strictly positive profit. $\square$

Claim C-3 shows that in equilibrium (C-9) must be binding: if not, the Incumbent could profitably increase its price in period 1. Along with (C-5) this implies $p_2 = p_h$ and therefore $\Pi_2 = \Pi_h$. With $\Pi_h = \alpha_h p_h/(1-\delta)$ this implies (13) and (14).

Claim C-3. *If* (C-2) *holds, then* (C-9) *is binding:*

$$-(\alpha_h + \alpha_\ell)(u_h^h - p_2 - u_h^\ell) + \delta\Pi_2 = 0. \tag{C-10}$$

Proof. Because (C-9) holds, it is sufficient to show that if $-(\alpha_h + \alpha_\ell)(u_h^h - p_2 - u_h^\ell) + \delta\Pi_2 < 0$, then a deviation by the period 1 incumbent to a price $p_2' > p_2$ satisfying

$$-(\alpha_h + \alpha_\ell)(u_h^h - p_2' - u_h^\ell) + \delta\Pi_2 < 0 \tag{C-11}$$

would be profitable. At the original p_2, there was profitable entry by attracting only the LNE consumers; *a fortiori*, it will also be profitable to attract the LNE consumers when the price is p_2'. Therefore, the deviation by the period 1 incumbent is unprofitable only if an entrant could profitably attract *all* the consumers when the price is p_2'. In order to attract the HNE consumers as well as the LNE consumers, an entrant must charge a price p_E' which satisfies $p_E' < -(u_h^h - p_2') + u_h^\ell$, which, by (C-11), yields strictly negative profits $((\alpha_h + \alpha_\ell)p_E' + \delta\Pi_2 < 0)$ and proves the result. $\square$

Summarizing the discussion so far, we have proved that if there is an equilibrium satisfying (C-3), then the prices must satisfy equations (11) and (13). Furthermore, if the prices satisfy these equations, then there is no profitable entry by (C-4) and (C-5).

In the first period, the entrant who attracts the LNE consumers must charge less than $p_2 - u_\ell^{\ell h}$ and its profit will be less than $-\alpha_\ell(u_\ell^{\ell h} - p_2) + \delta\Pi_\ell = \alpha_\ell(p_2 - (u_\ell^{\ell h} - \delta u_\ell^\ell))$. Thus, no entrant will be willing to attract the LNE consumers unless $p_2 > u_\ell^{\ell h} - \delta u_\ell^\ell$, which is therefore a necessary condition for the existence of a two platform equilibrium. If $(\alpha_h + \alpha_\ell)(u_\ell^{\ell h} - \delta u_\ell^\ell) > \alpha_h p_2$, the Incumbent finds it profitable to charge $u_\ell^{\ell h} - \delta u_\ell^\ell + \varepsilon$ and keep all the consumers. Hence, $(\alpha_h + \alpha_\ell)(u_\ell^{\ell h} - \delta u_\ell^\ell) \leq \alpha_h p_2$ must hold, which is equivalent to (2NetCond) and also implies $p_2 > u_\ell^{\ell h} - \delta u_\ell^\ell$.

We have used (C-2) and (3) to prove that (2NetCond) is necessary for the existence of a two platform equilibrium. It is easy to show (see Claim C-4) that both of these conditions hold whenever (2NetCond) holds; we have therefore proved that (2NetCond) is necessary for the existence of a two platform equilibrium.

Claim C-4. *If* (2NetCond) *holds, then a)* (3) *holds and b) the prices defined by* (11) *and* (13) *satisfy Condition* (C-2).

Proof. a) Because $u_\ell^{\ell h} - \delta u_\ell^\ell \geq (1-\delta)u_\ell^{\ell h}$, (2NetCond) implies (3).

b)
$$\begin{aligned} u_h^h - p_h &= u_h^h - \frac{(1-\delta)(\alpha_h + \alpha_\ell)(u_h^h - u_h^\ell)}{(1-\delta)\alpha_\ell + \alpha_h} \geq \delta \frac{\alpha_h}{(1-\delta)\alpha_\ell + \alpha_h}(u_h^h - u_h^\ell) \\ &\geq \delta \frac{\alpha_h}{(1-\delta)\alpha_\ell + \alpha_h} \times \frac{(1-\delta)\alpha_\ell + \alpha_h}{(1-\delta)\alpha_h}(u_\ell^{\ell h} - \delta u_\ell^\ell) \text{ (by (2NetCond))} \\ &= \frac{\delta}{1-\delta}(u_\ell^{\ell h} - \delta u_\ell^\ell) \geq \delta u_\ell^\ell = u_\ell^\ell - p_\ell. \end{aligned}$$
□

Claim C-5. *Condition* (C-2) *must hold.*

We have assumed that Condition (C-2) holds. In this part of the appendix, we show that this must indeed be the case whenever a two platform equilibrium exists.

We proceed by contradiction. If (C-2) did not hold, we would have $u_h^h - p_h < u_\ell^\ell - p_\ell$. We first show that the results which we obtained based on the fact that consumers do not change platforms from period 2 onwards hold with h and ℓ inverted. Then, we show that these results are incompatible with the separation of the consumers into two different platforms in the first period.

The proof of Lemma C-1 can be reproduced with h and ℓ inverted[23] and therefore

$$u_\ell^h + u_h^h - p_h < u_\ell^\ell - p_\ell. \tag{C-12}$$

Similarly, adapting the reasoning which leads to Claim C-2 we obtain:

$$-(\alpha_h + \alpha_\ell)[u_\ell^\ell - p_\ell - u_\ell^h] + \delta\Pi_2 = 0; \tag{C-13}$$

$$-\alpha_h(u_h^h - p_h) + \delta\Pi_h = 0. \tag{C-14}$$

Equation (C-14) implies $p_h = u_h^h(1-\delta)$. Along with (C-12) and (C-13), this implies

$$u_h^h < \frac{\Pi_2}{\alpha_h + \alpha_\ell}. \tag{C-15}$$

To compute Π_2, we eliminate p_2 from the system composed of the two equations a) $\Pi_2 = \alpha_h p_2 + \delta\alpha_h u_h^h$ and b) (C-10), which still holds as Claim C-3, whose proof is based on period 1 deviations which attract all consumers, is still valid as it stands. Substituting into (C-15), we obtain

$$u_h^h((1+\delta)\alpha_h + \alpha_\ell) < \alpha_h(1+\delta)u_h^h - \alpha_h u_h^\ell \Longleftrightarrow \alpha_\ell u_h^h + \alpha_h u_h^\ell < 0,$$

which establishes the contradiction.

D Condition (2NetCond) is a sufficient condition for a two platform equilibrium

We now show that (2NetCond) is sufficient for the existence of a two platform equilibrium. To do so, we show that when it holds, there is no profitable deviation from the equilibrium described in Proposition 1.

First period and any subsequent period where, off equilibrium, there is only one incumbent

Incumbent: By the reasoning leading to (C-9), if the Incumbent increased its price an entrant would find it profitable to attract all the consumers. Decreasing the price to $p_2' \geq u_\ell^{\ell h} - \delta u_\ell^\ell$ would not change the demand facing the Incumbent and hence would lower its profit. Decreasing the price below $u_\ell^{\ell h} - \delta u_\ell^\ell$ would enable the Incumbent to keep all the consumers, but, by (2NetCond), at the cost of lower profits.

Entrants: Competition between the entrants will lead them to charge a price equal to $-\delta\Pi_\ell/\alpha_\ell = -\delta u_\ell^\ell$. At that price, LNE consumers find it

[23] It is sufficient to note that the proof of Lemma C-1 depends on the relative sizes of the platform effect only through (C-2).

profitable to purchase from the entrants. The proof of Proposition 1 shows that no entrant will find it profitable to attract all the consumers.

Subsequent periods with two incumbents

Incumbents The same reasoning as for period 1 shows that the H incumbent has no incentive to deviate. The incentives of the L incumbent are the same as entrants is the first period. In order to attract the HNE consumers it would have to choose a price smaller to $p_2 - (u_h^h - u_h^\ell)$, which is unprofitable for the same reason that it would be unprofitable for an entrant to attract all the consumers in the first period.

Entrants For the same reasons that they cannot profitably attract the LNE consumers, they cannot attract profitably all the consumers.

E One platform equilibria

In this section of the appendix, we provide a full analysis of one platform equilibria. For simplicity, we take it as granted that Condition (C-2), $u_\ell^\ell - p_\ell \leq u_h^h - p_h$, holds whenever, out of equilibrium, LNE and HNE consumers have joined different platforms. This would be the case whenever $\delta > u_\ell^\ell / u_h^h$.[24]

We first discuss the structure of one platform equilibria, which is complicated by the fact that they differ along two dimensions. The first dimension describes what happens off the equilibrium path if the consumers ever get "separated" into two different platforms: consumers can either stay separated in subsequent periods — the S (for **S**eparated) equilibria, or they can all purchase from the same platform in the period after they have split so that two platforms coexist for only one period — the T (for **T**ogether) equilibria.

The second dimension is the entry constraint which binds on the Incumbent along the equilibrium path when it sells to both types of consumers: either preventing profitable entry which would attract only the LNE consumers or preventing profitable entry which would attract all consumers.

As a consequence, there are four types of single platform equilibria which are represented in Figure E-2. For large δ the binding constraint is preventing entry which attracts both LNE and HNE consumers: equilibria are either of type S2 or T2. To attract both types of consumers, an entrant must charge

[24] If $u_h^h - p_h < u_\ell^\ell - p_\ell$, then by the same argument as in the proof of Claim C-1, $p_h = u_h^h(1-\delta)$ and therefore $u_h^h - p_h = \delta u_h^h$. Since $p_\ell \geq 0$, $u_h^h - p_h < u_\ell^\ell - p_\ell$ is possible only if $u_\ell^\ell > \delta u_h^h$. Assuming that Condition (C-2) holds is only relevant in the analysis of "S equilibria" (definition below). From Claim C-1, when (C-2) holds the profits of an entrant who attracts the LNE consumers are $\alpha_\ell u_\ell^\ell$. From Appendix C, when (C-2) fails, they would be greater. We conjecture that this will make entrants more aggressive along the equilibrium path and lead to lower profits for the Incumbent.

		Entry constraints	
		LNE consumers	both types
After separation	keep separated	Sℓ	S2
	back together	Tℓ	T2

Figure E-2: The type of equilibria in the one platform case.

a price p_E which satisfies $u_h^\ell - p_E > u_h^h - p_2$.[25] This is unprofitable only if $(\alpha_h + \alpha_\ell)p_E + \delta\Pi_2 < 0$, which is equivalent to $p_E + \delta p_2/(1-\delta) < 0$ because $\Pi_2 = (\alpha_h + \alpha_\ell)p_2/(1-\delta)$. Therefore, to prevent this type of entry p_2 must satisfy

$$p_2 \leq (1-\delta)(u_h^h - u_h^\ell). \tag{E-1}$$

Along the equilibrium path, constraint (E-1) is binding and this yields the profits of Corollary 5.

The study of the existence of T equilibria raises some difficulties. In section 2.2, we assumed that consumers left their current platform only if this *strictly* increased their utility (this is the strict inequality in (6)). If we maintain this assumption, no T equilibrium exist: after, out of equilibrium, some consumers have purchased from an entrant, the Incumbent would have to choose the highest possible price that makes them strictly prefer to come back to its platform, but no such highest price exists, as the set of prices that make consumers prefer to purchase from an entrant than from the incumbent has a supremum, but no maximum. To finesse this issue, *only in the analysis of T equilibria*, we assume that, when the consumers are separated and indifferent between changing platform and not, they change whenever the platform they are purchased from last period would generate negative profits if it lowered its price, whereas the destination platform would still make positive profits if it decreased its price by a small enough amount.[26]

[25] By (3), this condition is sufficient for the entrant to attract first the LNE consumers and then the HNE consumers.

[26] A more fundamentalist approach would conduct a full Bertrand game analysis, including the continuation game played by the consumers.

E.1 S equilibria: consumers stay separated after they split

In S equilibria, if, off the equilibrium path, HNE and LNE consumers join different platforms in some period, then they stay on these platforms in subsequent periods.

As proven in Claim C-1, when Condition (C-2) holds $\Pi_\ell = \alpha_\ell u_\ell^\ell$ and $p_\ell = u_\ell^\ell(1-\delta)$. Along the equilibrium path, in order to attract only the LNE consumers, an entrant must charge a price p_E which satisfies $-p_E > u_\ell^{\ell h} - p_2$ as well as $u_h^\ell - p_E \le u_h^h - p_2$; such a p_E exists by (3). This is profitable if $\alpha_\ell p_E + \delta\Pi_\ell > 0$, which is equivalent to $-p_E < \delta\Pi_\ell/\alpha_\ell = \delta u_\ell^\ell$.

To make this type of entry impossible the incumbent must ensure that if $p_E = -\delta u_\ell^\ell$ the LNE consumers choose not to purchase from the entrant. Therefore, it must choose p_2 such that

$$p_2 \le u_\ell^{\ell h} - \delta u_\ell^\ell. \tag{E-2}$$

To attract both types of consumers, an entrant must charge a price p_E which satisfies $u_h^\ell - p_E > u_h^h - p_2$.[27] This is unprofitable only if $(\alpha_h + \alpha_\ell)p_E + \delta\Pi_2 < 0$, which is equivalent to $p_E + \delta p_2/(1-\delta) < 0$ because $\Pi_2 = (\alpha_h + \alpha_\ell)p_2/(1-\delta)$. Therefore, to prevent this type of entry p_2 must satisfy

$$p_2 \le (1-\delta)(u_h^h - u_h^\ell). \tag{E-3}$$

Along the equilibrium path, both constraints (E-2) and (E-3) must be met, and at least one of them must be binding. When (E-2) is binding, we have an Sℓ equilibrium; when (E-3) is binding we have an S2 equilibrium. This implies the following lemma.

Lemma E-3. *In S type single platform equilibria the profit of the Incumbent is*

$$(\alpha_h + \alpha_\ell)\min\left[\frac{u_\ell^{\ell h} - \delta u_\ell^\ell}{1-\delta}, u_h^h - u_h^\ell\right].$$

The full characterization of the conditions under which these equilibria exist, which we present in Lemmas E-4 and E-5, is rather complicated. It is a direct consequence of the following two facts: a) S equilibria exist only for "small" $u_h^h - u_h^\ell$ and b) when (2NetCond) holds, *i.e.*, when a two platform equilibrium exists, no S equilibrium exists except on a set of parameters of measure zero.[28] Furthermore, the set of parameters for which there exist

[27] By (3), this condition is sufficient for the entrant to attract first the LNE consumers and then the HNE consumers.

[28] This arises when (2NetCond) and the right hand side of (E-4) are both binding.

either a two platform equilibrium or an S equilibrium is quite large. S2 equilibria are described in Lemma E-4, while Sℓ equilibria are described in Lemma E-5.

Lemma E-4. *If $\delta\alpha_\ell - (1-\delta)\alpha_h > 0$, then an S2 equilibrium exists if and only if*

$$u_h^h - u_h^\ell \le \min\left[\frac{u_\ell^{\ell h} - \delta u_\ell^\ell}{1-\delta}, \frac{(\alpha_h+\alpha_\ell)(\delta u_\ell^\ell - u_\ell^h)}{\delta\alpha_\ell - (1-\delta)\alpha_h}\right].$$

If $\delta\alpha_\ell - (1-\delta)\alpha_h < 0$, then an S2 equilibrium exists if and only if

$$\frac{(\alpha_h+\alpha_\ell)(\delta u_\ell^\ell - u_\ell^h)}{\delta\alpha_\ell - (1-\delta)\alpha_h} \le u_h^h - u_h^\ell \le \frac{u_\ell^{\ell h} - \delta u_\ell^\ell}{1-\delta}.$$

In both cases, the profit of the Incumbent is $(\alpha_h+\alpha_\ell)(u_h^h - u_h^\ell)$.

Proof. By definition of S2 equilibria, (E-3) is binding, and therefore, trivially, $\Pi_2 = (\alpha_h+\alpha_\ell)(u_h^h - u_h^\ell)$ which proves the last sentence of the lemma. Along with (E-2) this implies $u_h^h - u_h^\ell \le (u_\ell^{\ell h} - \delta u_\ell^\ell)/(1-\delta)$. By the discussion on page App-8, after LNE and HNE consumers are separated Claim C-2 holds. This implies $p_h = p_2$ and $\Pi_h = \alpha_h(u_h^h - u_h^\ell)$. In order to attract the LNE consumers the H incumbent would have to charge a price p_h' which satisfies $v_\ell - p_h' > u_\ell - p_\ell = \delta u_\ell^\ell$. This is unprofitable only if

$$\Pi_h \ge (\alpha_h+\alpha_\ell)(u_\ell^h - \delta u_\ell^\ell) + \delta\Pi_2,$$

which is equivalent to

$$(\alpha_h+\alpha_\ell)(\delta u_\ell^\ell - u_\ell^h) \ge (\delta\alpha_\ell - (1-\delta)\alpha_h)(u_h^h - u_h^\ell),$$

and proves the lemma. □

Lemma E-5. *An Sℓ equilibrium exists if and only if $u_h^h - u_h^\ell \ge (u_\ell^{\ell h} - \delta u_\ell^\ell)/(1-\delta)$ and if*

$$\frac{(1-\delta)(\alpha_\ell+\alpha_h)}{\alpha_h}(u_\ell^h - \delta u_\ell^\ell) + \frac{\delta[\alpha_h(2-\delta)+\alpha_\ell(1-\delta)]}{(1-\delta)\alpha_h}(u_\ell^{\ell h} - \delta u_\ell^\ell) \le u_h^h - u_h^\ell \le \frac{(1-\delta)\alpha_\ell + \alpha_h}{(1-\delta)\alpha_h}(u_\ell^{\ell h} - \delta u_\ell^\ell). \quad \text{(E-4)}$$

The profit of the Incumbent is then $(\alpha_h+\alpha_\ell)(u_\ell^{\ell h} - \delta u_\ell^\ell)/(1-\delta)$.

Proof. By definition of Sℓ equilibria, (E-2) is binding which immediately yields $p_2 = (u_\ell^{\ell h} - \delta u_\ell^\ell)$ and the profit of the Incumbent.

The lowest price that an entrant is willing to charge in order to attract all the consumers is $-\delta\Pi_2/(\alpha_h + \alpha_\ell)$, and therefore the Incumbent can price up to $(u_h^h - u_h^\ell) - \delta\Pi_2/(\alpha_h + \alpha_\ell)$ and sell only to the HNE consumers. In subsequent periods, it would set the same price by Claim C-2. Therefore, this deviation is unprofitable, only if

$$\Pi_2 \geq \frac{1}{1-\delta} \times \alpha_h \times \left(u_h^h - u_h^\ell - \delta \frac{\Pi_2}{\alpha_\ell + \alpha_h} \right), \tag{E-5}$$

which is equivalent to the right most inequality of (E-4).

Off the equilibrium path, consumers stay separated. In order to attract the LNE consumers away from the L incumbent, the H incumbent would have to announce a price not larger than $u_\ell^h - u_\ell^\ell + p_\ell = u_\ell^h - \delta u_\ell^\ell$. This is unprofitable only if $\Pi_h \geq (\alpha_h + \alpha_\ell)(u_\ell^h - \delta u_\ell^\ell) + \delta\Pi_2$, where Π_h, the profit of the H incumbent, is equal to the right hand side of (E-5). This inequality is equivalent to the left most inequality in (E-4).[29]

Out of equilibrium, once the consumers are separated, for either the L incumbent or an entrant to attract all consumers by charging p' would yield a profit of $(\alpha_h + \alpha_\ell)p' + \delta\Pi_2$. For such a strategy to be profitable, we must have $p' \geq -\delta\Pi_2/(\alpha_h + \alpha_\ell)$. It is only feasible if $p' + (u_h^h - u_h^\ell) < 0$ — otherwise the H incumbent can profitably ensure the fidelity of the HNE consumers. By (E-3), these two bounds on p' cannot hold simultaneously and therefore no such deviation is possible. □

E.2 T equilibria: consumers come back together after they split

In a T equilibrium, we must have

$$p_2 \leq u_\ell^{\ell h}. \tag{E-6}$$

Otherwise, by charging, for instance, $(p_2 - u_\ell^{\ell h})/2 > 0$, an entrant would attract at least the LNE consumers and make positive profits even if it "lost" all these consumers in the following period. An entrant must also find it unprofitable to attract all the consumers. This occurs if and only if $u_h^h - p_2 \geq u_h^\ell + \delta\Pi_2/(\alpha_\ell + \alpha_h)$, which is equivalent to

$$p_2 \leq (1-\delta)(u_h^h - u_h^\ell). \tag{E-7}$$

[29] There are other possible deviations. It is possible to show that they are not profitable.

because $\Pi_2 = (\alpha_h + \alpha_\ell)p_2/(1-\delta)$.

When (E-6) is binding, we have a Tℓ equilibrium; when (E-7) is binding we have a T2 equilibrium. Because at least one of these two constraints is always binding, we have the following lemma.

Lemma E-6. *In a T type single platform equilibria the profit of the incumbent is*

$$(\alpha_h + \alpha_\ell) \min \left[\frac{u_\ell^{\ell h}}{1-\delta}, u_h^h - u_h^\ell \right].$$

Contrary to what happens for S equilibria, there can exist both a two platform equilibrium and a T equilibrium: this is true, for instance, when $\delta = 1/2$, $\alpha_\ell = 3\alpha_h/2$ and $u_\ell^\ell = u_\ell^{\ell h} = 3u_\ell^h/2 = 8(u_h^h - u_h^\ell)/15$. Then, (3), (2NetCond) and the conditions of Lemma E-7 below are satisfied.

The following fact is both economically interesting and technically important for the characterization of T equilibria: if, off the equilibrium path, the consumers are separated in two platforms, in the next period they will all purchase from the H incumbent, not from the L incumbent. The L incumbent profitably attracts all the consumers only if $p_\ell(\alpha_h + \alpha_\ell) \geq -\delta\Pi_2$. It will attract the HNE consumers if for all non negative prices the consumers prefer to "leave" the H incumbent (otherwise, the H incumbent could deviate and profitably keep its consumers). Thus, we must have $p_\ell < -(u_h^h - u_h^\ell)$. But, from Lemma E-6, $-(\alpha_h + \alpha_\ell)(u_h^h - u_h^\ell) + \delta\Pi_2 < 0$, and these two bounds on p_ℓ cannot be satisfied simultaneously. A similar argument shows that no entrant can attract all consumers to its platform.

The following two lemmas provide the conditions for existence and the profits for each of the types of T equilibria.

Lemma E-7. *A T2 equilibrium exists if and only if*

$$\frac{(\alpha_\ell + \alpha_h)(u_\ell^\ell - u_\ell^h)}{\delta\alpha_\ell - (1-\delta)\alpha_h} \leq u_h^h - u_h^\ell \leq \frac{u_\ell^{\ell h}}{1-\delta}. \tag{E-8}$$

The equilibrium profit of the Incumbent is $(\alpha_h + \alpha_\ell)(u_h^h - u_h^\ell)$.

Proof. In a T2 equilibrium, the Incumbent cannot profitably raise its price and it sells only to the HNE consumers at price $p_2 = (1-\delta)(u_h^h - u_h^\ell) \leq u_\ell^{\ell h}$ (by (E-6)). $\Pi_2 = (\alpha_h + \alpha_\ell)(u_h^h - u_h^\ell)$ and the right hand side of (E-8) follow immediately.

Off the equilibrium path, we need to find conditions for the Incumbent to be willing and able to attract the LNE consumers if the consumers are ever separated. The Incumbent attracts the LNE consumers, by charging a p_h smaller than or equal to $u_\ell^h - u_\ell^\ell$, since the L incumbent is willing to charge any positive price to keep the LNE consumers.[30,31] Furthermore, the Incumbent must choose a price that will induce the HNE consumers to stay on its platform instead of joining an entrant platform. Entrants are willing to price down to $-\delta(u_h^h - u_h^\ell)$ in order to attract all the consumers and the incumbent must therefore charge a price p_h smaller than or equal to $(u_h^h - u_h^\ell)(1-\delta)$ to keep the HNE consumers. Because $u_\ell^h - u_\ell^\ell \leq 0 \leq (1-\delta)(u_h^h - u_h^\ell)$, the binding constraint is $p_h \leq u_\ell^h - u_\ell^\ell$.

The most profitable deviation which allows the H to keep only the HNE consumers is to charge $(1-\delta)(u_h^h - u_h^\ell)$. Thus, for the H incumbent to prefer to attract the LNE consumers to deviating and keeping only the HNE consumers we must have $(\alpha_\ell + \alpha_h)(u_\ell^h - u_\ell^\ell + \delta(u_h^h - u_h^\ell)) \geq \alpha_h(u_h^h - u_h^\ell)$ which is equivalent to the left hand side of the necessary and sufficient condition. □

Lemma E-8. *A $T\ell$ equilibrium exists if and only if:*

$$\frac{u_\ell^{\ell h}}{1-\delta} \leq u_h^h - u_h^\ell \leq \min\left[\frac{(\alpha_\ell + \alpha_h)\left\{(1-\delta)(u_\ell^h - u_\ell^\ell) + \delta u_\ell^{\ell h}\right\}}{\alpha_h} + \frac{\delta u_\ell^{\ell h}}{1-\delta}, \frac{(\alpha_\ell + \alpha_h)\{u_\ell^{\ell h}(1+\delta) - \delta(u_\ell^h - u_\ell^\ell)\}}{\alpha_h}\right]. \tag{E-9}$$

The Incumbent's equilibrium profit is $(\alpha_\ell + \alpha_h)u_\ell^{\ell h}/(1-\delta)$.

Proof. The binding pricing constraint when the incumbent has all the consumers is (E-6) and the left hand side of Condition (E-9) reflects this. It follows immediately that $\Pi_2 = (\alpha_\ell + \alpha_h)u_\ell^{\ell h}/(1-\delta)$.

Off the equilibrium path, in order to attract back the LNE consumers, the Incumbent must offer a price less than or equal to $u_\ell^h - u_\ell^\ell$, since the L incumbent will price at 0.[32] The lowest price an entrant is willing to offer to attract all the consumers is $-\delta u_\ell^{\ell h}/(1-\delta)$. Thus, the Incumbent's price must not exceed $u_h^h - u_h^\ell - \delta u_\ell^{\ell h}/(1-\delta)$. Since $u_\ell^h - u_\ell^\ell < 0 < u_h^h - u_h^\ell - \delta u_\ell^{\ell h}/(1-\delta)$, it is the first constraint which is binding. The most profitable deviation which would allow the Incumbent to keep only the HNE consumers is to

[30] We assume that firms do not use weakly dominated strategies and it is clear that if the L firm charged a positive price and lost consumers that it could profitably deviate and lower its price.

[31] Recall that we are using the weak inequality definition of AC equilibria for this class of equilibria.

[32] See proof of Lemma E-7.

charge $u_h^h - u_h^\ell - \delta u_\ell^{\ell h}/(1-\delta)$. Thus, for the Incumbent to prefer to bring all consumers onto its platform we must have

$$(\alpha_\ell + \alpha_h)\left[u_\ell^h - u_\ell^\ell + \delta u_\ell^{\ell h}/(1-\delta)\right] \geq \frac{\alpha_h}{1-\delta}\left[u_h^h - u_h^\ell - \frac{\delta u_\ell^{\ell h}}{1-\delta}\right], \qquad \text{(E-10)}$$

which is the first term of the right hand side of (E-9).

On the equilibrium path, the Incumbent must prefer to keep all consumers to just keeping the HNE consumers and then bringing them back the LNE onto its platform the following period at a price of $u_\ell^h - u_\ell^\ell$. Hence, we must have

$$(\alpha_\ell + \alpha_h)u_\ell^{\ell h}/(1-\delta) \geq \alpha_h(u_h^h - u_h^\ell) + \delta(\alpha_\ell + \alpha_h)\left[u_\ell^h - u_\ell^\ell + \delta u_\ell^{\ell h}/(1-\delta)\right],$$

where the left hand side is the equilibrium profit, and the right hand side is the sum of profits in the defection period plus the discounted left hand side of expression (E-10). This can be rewritten

$$\frac{(\alpha_\ell + \alpha_h)u_\ell^{\ell h}(1+\delta) - \delta(\alpha_\ell + \alpha_h)\left[u_\ell^h - u_\ell^\ell\right]}{\alpha_h} \geq u_h^h - u_h^\ell,$$

which is the second term of the right hand side of (E-9). $\square$