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and Energy Efficiency Rates

During the Transition Toward the Green Economy

Abstract

The global economy produces energy from two sources: a polluting non-renewable resource and a renewable resource. Transforming crude energy into ready-to-use energy services requires costly processes and more efficient energy transformation rates are more costly to achieve. Renewable energy is in competition with food production for land acreage but the food productivity rate of land can also be improved at some cost. The exploitation of non-renewable energy releases polluting emissions in the atmosphere. To avoid catastrophic climate damages, the pollution stock is mandated to stay below a given cap. In the interesting case where the economy would be constrained by the carbon cap at least temporarily, we show the following. When the economy is not constrained by the cap, the efficiency rates of energy transformation increase steadily until the transition toward the ultimate green economy; when renewable energy is exploited, its land acreage rises at the expense of food production; food productivity increases together with the land rent but food production drops; the prices of useful energy and food increase and renewables substitute for non-renewable energy. During the constrained phase, the economy follows a constant path of prices, quantities, efficiency rates, food productivity and land rent, a phenomenon we call the generalized ceiling paradox.

Keywords: energy efficiency; carbon pollution; non-renewable resources; renewable resources; land use.

JEL classifications: Q00, Q32, Q43, Q54.

1 Introduction

Meeting the ambitious objectives of the COP 21 recent agreement will require huge changes of the present energy systems throughout the next decades. The transition toward a 'green' economy depends on both significant energy efficiency improvements and a progressive substitution from polluting fossil fuels to carbon free renewable energy sources. Furthermore, energy transition is both a scale problem: in which proportion different 'greening' options should be combined, and a time problem: what should be their ordering throughout time.

Many constraints affect the transition toward a green economy. First are time to build issues: replacing fossil fuels by renewables like wind or solar energy implies the accumulation of dedicated capital goods. Second the transition may require significant technological improvements and innovations in energy production, storage and delivery systems.[1] Third renewable energy sources may be competing for inputs with other production systems. This is especially the case for land with respect to food production.

The objectives of this paper are two-fold. First we want to stress the importance of energy efficiency dynamics during the transition from a fossil-fuel based economy toward a carbon-free renewable economy. The improvement of the conversion rates of fossil fuel energy into energy services is a way to reduce the carbon content of final output. Fossil fuels are relatively cheap and abundant today. Satisfying the energy needs while polluting less thanks to more efficient transformation processes of fossil energy into ready-to-use energy stands as an appealing option in that respect. Renewables can also benefit from parallel enhancements in energy efficiency. While technical progress is usually offered as the main road to reduce the cost gap between renewable energy and fossil energy, more efficient devices are generally more costly to operate, raising the issue of the relative competitiveness dynamics of renewables with respect to fossils.

If the substitution option has received considerable attention in the eco-

[1] Recent studies of capital building constraints and time to build issues are Amigues, Ayong and Moreaux (2015), Kollenbach (2015), Gronwald, Long and Röpke (2013). The literature on the role of technical progress in the energy transition is immense. For a recent contribution, see Acemoglu et al. (2015).

nomic literature on climate change management, this is much less true for energy efficiency dynamics. Most models assume given and fixed efficiency rates for different energy sources or introduce exogenous trends of efficiency improvements.[2]

Second we want to examine in some detail the problem of competition for land between renewable energy and food production. This issue has raised a significant attention in the preceding decade when the bio-fuel US regulation has been charged of inducing price hikes on the corn market, the so-called 'tortilla problem'. It is today recognized that the main cause of the food price increase during this period was more the dramatic changes experienced by the world agricultural production system facing high food demand from emerging countries. However the current population prospects coupled with the need to expand considerably the use of land for renewable energy production raises the issue of possible shortcomings in food provision because of the world arable land scarcity, a problem itself exacerbated by the potential harmful consequences of climate change on crop yields.

A vast empirical literature has explored the land competition issue between food and energy provision recently, mostly through large scale simulation models (Hertel *et al.*, 2010, Rosegrant *et al.*, 2008, Chakravorty *et al.*, 2014). The theoretical contributions are more rare. The present work extends the original study of Chakravorty, Magné and Moreaux (2008) which assumed fixed and exogenous efficiency rates instead of endogenous rates as in the present paper.

We consider an economy producing ready-to-use energy from two crude energy sources. The first one is a non-renewable and polluting fossil fuel, the second one is renewable and carbon free but requires some land acreage, one can think to bio-fuels production, solar farms or wind turbines as typical examples. The transformation of crude energy into useful energy from any source is a costly process and higher conversion rates are more costly to achieve. Burning fossil fuels to produce useful energy releases polluting

[2]On the theoretical side, main original contributions are Tahvonen (1991), Tahvonen and Kuuluvainen (1991), Farzin and Tahvonen (1996), Withagen (1994), Tahvonen and Withagen (1996), Toman and Withagen (2000). For more recent contributions see Golosov *et al.* (2014), Van der Ploeg and Withagen (2014). The applied literature has intensively used IAM's to assess climate policy options. Prominent contributions are Gerlagh, Van der Zwaan (2006), Stern (2007), Nordhaus (2008).

emissions in the atmosphere. To prevent excessive climate damages, the atmospheric carbon concentration must be maintained below some cap, or ceiling, as in Chakravorty, Magné, Moreaux (2006).

Land may be used either to produce renewable energy or food. The efficiency rate of energy transformation from the renewable source may be improved over time, meaning an increased land productivity for energy production. The same applies to food production, farmers being able to rise food land productivity but higher agricultural performances are also more costly.

In the interesting situation where fossil fuels would be so abundant that the economy is eventually constrained by the atmospheric carbon cap, we ask the following questions. What should be the optimal substitution path from fossil fuels to renewable energy? What are the dynamics of the energy efficiency rates? How should evolve the land allocation to food and energy production respectively? What should be the trends of food productivity and the land rent? How the prices of useful energy and food should evolve, together with the shadow cost of carbon pollution, equivalently the 'social cost' of carbon?

To answer these questions we develop a highly stylized model able to catch the main ingredients of the inter-temporal arbitrage problems faced by the society, between producing energy from fossil resources or renewables, between producing food or renewable energy. Our main findings are the following. The energy transition is a sequence of three time phases. During a first phase, the economy accumulates carbon until its faces the cap constraint. Then the cap binds, implying a constant rate of fossil fuel extraction. The fossil resource being exhaustible, there must exist some time when even without a carbon constraint, the economy would choose to extract fossil resources at a lower rate than what is mandated by the cap. Thus the economy escapes from the cap and enters an unconstrained phase until the end of fossil fuel exploitation. Last begins the ultimate green economic regime when the society produces only renewable energy and food.

During the unconstrained phases, the useful energy price rises together with the efficiency rate of fossil energy production. When renewable energy is produced jointly with non-renewable energy, its efficiency rate rises also and it takes an increasing share in the energy mix. Furthermore the land acreage devoted to renewable energy expands at the expense of the land acreage for food production and the land rent increases. The food sector reacts to this trend by increasing its productivity, although insufficiently to prevent the decline of the food production rate and thus the rise of the food price. Before the constrained phase, the shadow cost of carbon rises while it is nil after the constrained phase.

During the constrained phase, the exploitation rate of fossil fuels must stay constant. This induces a constant price of useful energy and constant efficiency rates. Implied by this constancy, the land sharing between renewable energy production and food production remains constant, together with the land rent level. The food production sector then maintains constant its productivity resulting in a constant level of the food production rate and the food price. We call this phenomenon the 'generalized ceiling paradox'. This is during the period when the economy faces actually the climate constraint that it should stop improving its energy performance, stabilizing in turn the food delivery conditions.

The paper is organized as follows. The next section 2 presents the model. In section 3 we lay down the optimality problem faced by the society and characterize the main features of the variables dynamics. The optimal paths are described in Section 4. The last Section 5 concludes.

2 The model

The model extends Amigues and Moreaux (2015-a) to incorporate land competition dynamics between food and energy production. We consider a stationary economy producing both food and energy services.[3] Food production requires both land and other inputs. Energy services can be produced from either the exploitation of a polluting non-renewable resource (oil) or the ex-

[3]From a pure energy perspective food is an *energy service*, a vital one. However according to the *Guide Michelin* other characteristics than the pure energy content of food should be taken into account. We neglect the qualitative characteristics of food given the problem at stake.

ploitation of land (solar) both with other inputs.[4]

2.1 Food and energy needs and gross surplus

Let us denote respectively by q_e and q_f the instantaneous consumption rates of energy and food. To simplify we first assume that the gross surplus generated by any pair of instantaneous consumption rates, $u(q_e, q_f)$, is additively separable and may be written as $u(q_e, q_f) = u_e(q_e) + u_f(q_f)$, each function u_i, $i = e, f$, satisfying the standard following assumption A.1.

Assumption A. 1 *For any $i = e, f$, $u_i : \mathbb{R}_+ \to \mathbb{R}$ is twice continuously differentiable, strictly increasing, $u_i' \equiv du_i/dq_i > 0$, strictly concave, $u_i''(q_i) \equiv d^2u_i/dq_i^2 < 0$, and satisfies the basic Inada condition: $\lim_{q_i \downarrow 0} u_i'(q_i) = +\infty$.*

Assumption A.1 is not innocuous, asserting that a smaller food diet may be compensated by a larger energy consumption. The other extreme assumption would be to assume that energy services and food are strictly complementary goods.

We denote alternatively by $p_i(q_i)$ the marginal gross surplus function or inverse demand function, $p_i(q_i) \equiv u_i'(q_i)$, and by $q_i^d(p_i)$ its inverse, the direct demand function.

2.2 The oil energy sector

The oil energy sector includes two industries: The extraction or mining industry produces extracted oil from the underground resource and the transformation industry produces energy services from extracted oil.

The extractive industry

[4]We subsume as 'solar energy', different renewable energy sources requiring space and/or sun energy like solar PVs, wind energy or biofuels.

Let $X(t)$ denote the underground stock of oil at time t measured in energy units, X^0 the initial endowment, $X(0) = X^0$, and $x(t)$ the instantaneous extraction rate or extracted oil production: $\dot{X}(t) = -x(t)$. The unitary extraction cost depends on the grade under exploitation, as in Heal (1976) and Hanson (1980). Let $a(X)$ denote this unitary cost. The function $a(.)$ satisfies the following standard assumption.[5]

Assumption A. 2 *$a : (0, X^0] \to \mathbb{R}_+$ is twice continuously differentiable on $(0, X^0)$, strictly decreasing, $a'(X) \equiv da(X)/dX < 0$, strictly convex, $a''(X) \equiv d^2a(X)/dX^2 > 0$, with $a(0^+) = +\infty$ and $a'(0^+) = +\infty$.*

Under $a'(0^+) = +\infty$ and the below assumptions on the solar energy costs, some part of the oil endowment is left underground.

The oil transformation industry

Let us denote by η_x the fraction of extracted oil energy which is converted into useful energy by the oil transformation industry, what we call also the efficiency rate of the industry. Converting a higher fraction of extracted oil requires more sophisticated processes, that is more costly ones. Let $b(\eta_x)$ be the unitary processing cost of the extracted oil input necessary to achieve the efficiency rate η_x, equal to the marginal cost. Processing x units of oil with the efficiency rate η_x allows to obtain $q_x = \eta_x x$ units of useful energy at the processing cost $b(\eta_x)x$. Hence the average production cost of useful oil energy amounts to $b(\eta_x)/\eta_x$, also equal to the marginal cost. We assume that $b(\eta_x)/\eta_x$ is increasing, hence also $b'(\eta_x) \equiv db(\eta_x)/d\eta_x$.[6]

Assumption A. 3 *$b : [0, 1) \to \mathbb{R}_+$ is twice continuously differentiable on $(0, 1)$, strictly increasing, $b'(\eta_x) > 0$, strictly convex, $b''(\eta_x) \equiv d^2b(\eta_x)/d\eta_x^2 >$*

[5]For any function $f(x)$ defined on $X \subseteq \mathbb{R}$, and for any value $\bar{x} \in X$, we denote by $f(\bar{x}^-)$ and $f(\bar{x}^+)$ respectively, the limits $\lim_{x\uparrow\bar{x}} f(x)$ and $\lim_{x\downarrow\bar{x}} f(x)$ when such limits exist.

[6]Differentiating the average production cost $b(\eta_x)/\eta_x$ yields:

$$\frac{d}{d\eta_x}\frac{b(\eta_x)}{\eta_x} = \frac{1}{\eta_x}\left[b'(\eta_x) - \frac{b(\eta_x)}{\eta_x}\right].$$

Hence $b'(\eta_x) > 0$ is necessary for $d\left(b(\eta_x)/\eta_x\right)/d\eta_x > 0$.

0, with $b(0^+) = 0$, $b'(0^+) > 0$, $b(1^-) = +\infty$ and $b'(1^-) = +\infty$. The average production cost of useful oil energy, and so the marginal cost, is a strictly increasing function of η_x: $b'(\eta_x) > b(\eta_x)/\eta_x$ and $\lim_{\eta_x \downarrow 0} b(\eta_x)/\eta_x > 0$.[7]

Producing useful energy from oil requires other costly inputs, hence a strictly positive marginal cost at 0^+: $\lim_{\eta_x \downarrow 0} b(\eta_x)/\eta_x > 0$. The assumptions $b(1^-) = +\infty$ and $b'(1^-) = +\infty$ mean that a complete conversion of the energy content of the extracted oil is not physically possible.

Carbon pollution

Burning oil to produce useful energy generates in the atmosphere a pollution flow proportional to the flow of the extracted oil input used in the transformation industry. Let ζ be the unitary pollution content of oil, hence a pollution flow $\zeta x(t)$ at time t, feeding the atmospheric pollution stock. Denote by $Z(t)$ the size of this pollution stock at time t and by Z^0 the stock inherited from the past, $Z(0) = Z^0$. The pollution stock self-regenerates at a proportional rate α, assumed constant to simplify. Hence the dynamics of $Z(t)$ is given by $\dot{Z}(t) = \zeta x(t) - \alpha Z(t)$.

The atmospheric pollution concentration is constrained to be kept at most equal to some cap, or ceiling, $\bar{Z}$, as in Chakravorty, Magné and Moreaux (2006), to prevent excessive climate damages. As far as the average earth temperature level is an increasing function of the atmospheric carbon concentration, such a cap may be seen as another formulation of the $+2^0$ target assuming that $+2^0$ is an effective constraint and not a mere wish. In order that the model makes sense we must assume that $Z^0 < \bar{Z}$. When the ceiling constraint binds, then there is a cap on the oil input in the transformation industry that we denote by $\bar{x}$: $\bar{x} = \alpha \bar{Z}/\zeta$.

2.3 The solar energy sector

The solar energy sector uses some part L_e of the available land $\bar{L}$ to produce useful energy. Are included in this sector all the activities required to

[7] Note that $b'(\eta_x) > b(\eta_x)/\eta_x$, $\eta_x \in (0, 1)$, implies that $b'(0^+) \geq \lim_{\eta_x \downarrow 0} b(\eta_x)/\eta_x$, hence $b'(0^+) > 0$.

bring ready-to-use energy to the final users. Thus it may include agricultural activities when, for example, ethanol is produced from sugar cane, together with all the industrial processes necessary to transform the sugar cane into ethanol.

The available land is assumed to be homogeneous and to receive y^m units of solar energy by acreage unit. The problem of the solar energy sector is to convert y^m into useful energy. Let η_y be the conversion or efficiency rate, so that the useful energy produced by the solar sector, denoted by q_y, amounts to $\eta_y y^m L_e$.

Choosing higher conversion rates implies to bring into operation more elaborate techniques, that is more costly ones, like in the extracted oil transformation industry. Let us denote by $h(\eta_y)$ the average conversion cost per unit of natural energy required to work with an efficiency rate η_y. Since a cost $h(\eta_y)y^m$ allows to produce $\eta_y y^m$ units of useful energy then the unitary cost of useful energy is equal to $h(\eta_y)/\eta_y$. We assume that this unitary cost is increasing, implying that $h(\eta_y)$ itself is also increasing.

Assumption A. 4 $h : [0,1) \to \mathbb{R}_+$ *is twice continuously differentiable on* $(0,1)$*, strictly increasing,* $h'(\eta_y) \equiv dh/d\eta_y > 0$*, strictly convex,* $h''(\eta_y) \equiv d^2h/d\eta_y^2 > 0$*, with* $h(0) = 0$ *and* $h'(0^+) > 0$*, and* $h(1^-) = +\infty$ *and* $h'(1^-) = +\infty$*. The average cost of useful energy and so the marginal cost is a strictly increasing function of* η_y*:* $h'(\eta_y) > h(\eta_y)/\eta_y$*, and* $\lim_{\eta_y \downarrow 0} h(\eta_y)/\eta_y > 0$*.*

The rationale for A.4 is the same than the rationale for A.3.

2.4 The food sector

The food sector includes not only the agricultural activities but also all the industrial activities necessary to bring ready-to-eat food to the consumers. Let L_f be the acreage of land devoted to the production of food and denote by φ the average production of food per unit of land so that the total food production amounts to φL_f. Let $f(\varphi)$ be the average production cost per unit of land hence an average cost $f(\varphi)/\varphi$ per unit of food. Clearly the land productivity is bounded from above. Let φ_u be this upper bound. The following assumption A.5 is for the production of food, similar to the assumptions A.3 and A.4 for the production of useful energy.

Assumption A. 5 *$f : [0, \varphi_u) \to \mathbb{R}_+$ is twice continuously differentiable on $(0, \varphi_u)$, strictly increasing, $f'(\varphi) \equiv df/d\varphi > 0$, strictly convex, $f''(\varphi) \equiv d^2f/d\varphi^2 > 0$, with $f(0) = 0$ and $f'(0^+) > 0$, and $f(\varphi_u^-) = +\infty$ and $f'(\varphi^-) = +\infty$. The average cost of food and so the marginal cost is a strictly increasing function of φ: $f'(\varphi) > f(\varphi)/\varphi$, and $\lim_{\varphi \downarrow 0} f(\varphi)/\varphi > 0$.*

We assume that the allocation of land to the production of energy and to the production of food may be adjusted instantaneously and costlessly.

2.5 The optimal land use and the land rent

A strong implication of the assumption A.5 is the following proposition.

Proposition P. 1 *Under the assumptions A.1 and A.5, the land rent is always positive, the land is scarce: $L_e + L_f = \bar{L}$.*

Proof : Let L_e^*, $L_e^* \in [0, \bar{L})$, be the optimal acreage of land devoted to energy production, then the optimal management of the food sector is this pair (φ, L_f) solving the following food sector problem $(F.S.P)$:[8]

$$
\begin{array}{lll}
(F.S.P) & \max_{\varphi, L_f} & u_f(\varphi L_f) - f(\varphi) L_f \\
& \text{s.t.} & \bar{L} - L_e^* - L_f \geq 0 \ .
\end{array}
$$

Denote by λ_L the multiplier associated to the land availability constraint and by $\mathcal{L}_f$ the Lagrangian:

$$
\mathcal{L}_f = u_f(\varphi L_f) - f(\varphi) L_f + \lambda_L \left[\bar{L} - L_e^* - L_f \right] \ .
$$

[8]We neglect the non-negativity constraints on φ, L_f and the upper bound constraint of φ, all of which are satisfied.

The f.o.c's are:

$$\frac{\partial \mathcal{L}_f}{\partial \varphi} = 0 \implies u'_f(\varphi L_f) = f'(\varphi) \ , \tag{2.1}$$

$$\frac{\partial \mathcal{L}_f}{\partial L_f} = 0 \implies u'_f(\varphi L_f)\,\varphi = f(\varphi) + \lambda_L \ , \tag{2.2}$$

together with the complementary slackness condition:

$$\lambda_L \geq 0 \ , \ \bar{L} - L_e^* - L_f \geq 0 \quad \text{and} \quad \lambda_L \left[\bar{L} - L_e^* - L_f\right] = 0 \ . \tag{2.3}$$

Let $(\varphi^*, L_f^*, \lambda_L^*)$ be a solution of the system (2.1)-(2.3) such that $\lambda_L^* = 0$ so that (2.2) may be simplified to get $u'_f\left(\varphi^* L_f^*\right) = f(\varphi^*)/\varphi^*$, hence by (2.1): $f'(\varphi^*) = f(\varphi^*)/\varphi^*$. However according to A.5: $f'\varphi) > f(\varphi)/\varphi$ for all $\varphi \in (0, \varphi_u)$, hence for φ^*, a contradiction. ■

The intuition behind this result is quite clear. Assumption A.5 means that more extensive agricultures are less costly than more intensive ones. Assume that a quantity of food q_f is produced thank to an acreage L_f and a productivity φ: $q_f = \varphi L_f$. Assume also that the land constraint is slack: $L_e^* + L_f < \bar{L}$. Then the same quantity of food can be produced on a larger acreage $L'_f > L_f$, $\bar{L} - L_e^* - L'_f \geq 0$, with a lower productivity $\varphi' < \varphi$: $q_f = \varphi' L'_f$. Since a more extensive land exploitation is less costly, the total cost of q_f is smaller with (φ', L'_f) than with (φ, L_f) and (φ, L_f) cannot be optimal.

The other assumption having strong implications on the agricultural sector is the additive separability of the gross surplus function.

Proposition P. 2 *Under the assumption of additive separability of the gross surplus function and the assumption A.5, the productivity in the food sector and the food production rate are both constant when the land acreage devoted to the energy production is constant.*

Proof : If $L_e(t)$ is constant during some time interval then the constraint and the objective function of the problem $(F.S.P.)$ are the same at any time of the interval hence also its solution. ■

Without the separability assumption, it must be pointed out that the objective function of the problem $(F.S.P)$ would have to be written as : $u\left(q_e(t), q_f(t)\right) - f\left(\varphi(t)\right) L_f(t)$, to be maximized under the constraints $\bar{L} - L_e - L_f(t) \geq 0$ and $\varphi(t)L_f(t) - q_f(t) \geq 0$, where L_e is the constant acreage allocated to energy production. The f.o.c's would be now:

$$\frac{\partial u\left(q_e(t), q_f(t)\right)}{\partial q_f(t)} = f'(\varphi(t)) \tag{2.4}$$

$$\frac{\partial u\left(q_e(t), q_f(t)\right)}{\partial q_f(t)}\varphi(t) = f(\varphi(t)) + \lambda_L(t) \ . \tag{2.5}$$

Without the separability assumption, $\partial^2 u/\partial q_e \partial q_f \neq 0$, hence (2.4) and (2.5) cannot be reduced to (2.1) and (2.2). With L_e held constant, but not $q_e(t)$, L_f would be constant because Proposition 1 still holds, but the productivity in the food sector would be no more constant.

Another implication of Proposition 2 is that, when the energy needs are fed only by oil, the food consumption rate, the food price level and the land rent should be constant for two reasons. On the one hand, $L_e = 0$ implies that $L_f(t) = \bar{L}$, a constant, and on the other hand, the food productivity rate, φ, should also be constant under the separability assumption, hence $q_f = \varphi\bar{L}$ is constant together with $p_f = u'_f(q_f)$ and $\lambda_L = p_f\varphi - f(\varphi)$, independently of the possible time evolutions of the oil extraction rate, $x(t)$, the useful energy production rate, $q_e(t) = q_x(t)$, and the energy price, $p_e(t)$.

2.6 The ultimate green economy

Let $\bar{t}_x$ be the time at which ends the oil exploitation, that is the time at which ends the transition toward the green renewable economy. Whatever the pollution stock level at this time, either $\bar{Z}$ or a lower level, from $\bar{t}_x$ onwards the pollution stock constraint can be put aside since $Z(t)$ is now decreasing: $Z(t) = Z(\bar{t}_x)e^{-\alpha(t-\bar{t}_x)}$, $t \geq \bar{t}_x$. Since the land allocation is freely adjustable, once at $\bar{t}_x$ the best at each future time is to choose the transformation rate $\tilde{\eta}_y$, the productivity level $\tilde{\varphi}$ and the land allocation $(\tilde{L}_e, \tilde{L}_f)$ solving the following static green economy problem $(G.E.P)$:[9]

$$(G.E.P) \quad \max_{\eta_y,\varphi,L_e,L_f} \quad u_e(\eta_y y^m L_e) + u_f(\varphi L_f) - h(\eta_y) y^m L_e - f(\varphi) L_f$$
$$\text{s.t.} \quad \bar{L} - L_e - L_f \geq 0 \ .$$

Keeping the notation λ_L for the multiplier associated to the land availability constraint, the Lagrangian of the problem, denoted by $\mathcal{L}_g$, reads:

$$\mathcal{L}_g = u_e(\eta_y y^m L_e) + u_f(\varphi L_f) - h(\eta_y) y^m L_e - f(\varphi) L_f + \lambda_L(\bar{L} - L_e - L_f) \ ,$$

and the f.o.c's are, after elementary simplifications:

$$\frac{\partial \mathcal{L}_g}{\partial \eta_y} = 0 \implies u_e'(\eta_y y^m L_e) = h'(\eta_y) \tag{2.6}$$

$$\frac{\partial \mathcal{L}_g}{\partial L_e} = 0 \implies u_e'(\eta_y y^m L_e)\eta_y y^m = h(\eta_y) y^m + \lambda_L \tag{2.7}$$

$$\frac{\partial \mathcal{L}_g}{\partial \varphi} = 0 \implies u_f'(\varphi L_f) = f'(\varphi) \tag{2.8}$$

$$\frac{\partial \mathcal{L}_g}{\partial L_f} = 0 \implies u_f'(\varphi L_f)\varphi = f(\varphi) + \lambda_L \ , \tag{2.9}$$

together with the complementary slackness condition:

$$\lambda_L \geq 0 \ , \ \bar{L} - L_e - L_f \geq 0 \quad \text{and} \quad \lambda_L \left[\bar{L} - L_e - L_f\right] = 0 \ . \tag{2.10}$$

Taking $\tilde{\eta}_e$ and $\tilde{L}_e$ as given, the problem $(G.E.P)$ is nothing but than the problem $(F.S.P)$ and the conditions (2.8)-(2.10) are the conditions (2.1)-(2.3). Now λ_L appears in both (2.7) for the energy sector and (2.9) for the food sector, mirroring their competition for the land attribution. By Proposition 1 the optimal value of the land rent $\tilde{\lambda}_L$ is positive and the land availability constraint is tight. Denote by $\lambda_L^f(L_e)$ the implicit relationship between the optimized value of the land rent in the problem $(F.S.P.)$ and the land acreage devoted to solar energy production, L_e. Substituting $\lambda_L^f(L_e)$ for λ_L in (2.7) expresses (2.6)-(2.7) as a system in two unknowns, η_y and L_e. Under our assumptions, this system admits a unique solution, the optimal value of $\tilde{L}_e$ allowing in turn to determine the unique triplet $(\tilde{\varphi}, \tilde{L}_f, \tilde{\lambda}_L)$ solution of (2.8)-(2.10). Thus the problem $(G.E.P)$ admits a unique solution.

[9] We omit the constraints $0 \leq \eta_y \leq 1$, $0 \leq \varphi \leq \varphi_u$, $0 \leq L_e$ and $0 \leq L_f$ which are satisfied as strict inequalities under the assumptions A.1, A.4 and A5.

In what follows, we denote by $\tilde{W}$ the capitalized value of the green economy, measured in current value at the time $\bar{t}_x$ at which it begins:

$$\begin{aligned}\tilde{W} &= \int_{\bar{t}_x}^{\infty} \left\{ u_e(\tilde{\eta}_y y^m \tilde{L}_e) + u_f(\tilde{\varphi}\tilde{L}_f) - h(\tilde{\eta}_y) y^m \tilde{L}_e - f(\tilde{\varphi})\tilde{L}_f \right\} e^{-\rho(t-\bar{t}_x)} dt \\ &= \frac{1}{\rho} \left\{ u_e(\tilde{\eta}_y y^m \tilde{L}_e) + u_f(\tilde{\varphi}\tilde{L}_f) - h(\tilde{\eta}_y) y^m \tilde{L}_e - f(\tilde{\varphi})\tilde{L}_f \right\} . \qquad (2.11)\end{aligned}$$

3 The social planner problem

3.1 The problem

The social planner determines the duration of the transition $\bar{t}_x$ and the paths of oil extraction, $x(t)$, of the transformation rates $\eta_x(t)$ and $\eta_y(t)$, of the productivity of land for food production, $\varphi(t)$, and of the land allocation, $L_e(t)$ and $L_f(t)$, during the transition toward the green economy, $t \in [0, \bar{t}_x)$, which maximize the social welfare. She/he solves the following problem $(S.P.1)$:[10]

[10]We omit the constraints $0 \leq X(t)$, $\eta_x(t) < 1$, $\eta_y(t) < 1$, $0 \leq \varphi(t) < \varphi_u$ and $0 \leq L_f(t)$ which are all satisfied as strict inequalities under the assumptions A.1 to A.5. Note also that under instantaneously and costlessly adjustable land allocation, the land allocation at time $t = 0$ is endogenously determined and not inherited from the past, contrary to the oil stock and the pollution stock.

$(S.P.1)$

$$\max_{\substack{\{x(t),\eta_x(t),\eta_y(t)\\ \varphi(t),L_e(t),L_f(t)\}_0^{\bar{t}_x}\\ \text{and } \bar{t}_x}} \int_0^{\bar{t}_x} \{u_e\left(\eta_x(t)x(t)+\eta_y(t)y^m L_e(t)\right)+u_f\left(\varphi(t)L_f(t)\right)$$

$$-a(X(t))x(t)-b(\eta_x(t))x(t)$$

$$-h(\eta_y(t))y^m L_e(t)-f(\varphi(t))L_f(t)\}\, e^{-\rho t}dt$$

$$+\tilde{W}e^{-\rho\bar{t}_x}$$

s.t.

$$\dot{X}(t)=-x(t)\ ,\ X(0)=X^0 \text{ given}$$

$$\bar{L}-L_e(t)-L_f(t)\geq 0\ , L_f(t)\geq 0\ , L_e(t)\geq 0$$

$$\dot{Z}(t)=\zeta x(t)-\alpha Z(t)\ ,\ Z(0)=Z^0<\bar{Z} \text{ given}$$
$$\text{and } \bar{Z}-Z(t)\geq 0$$

$$x(t)\geq 0\ ,\ \eta_x(t)\geq 0 \text{ and } \eta_y(t)\geq 0\ \ .$$

The social planner problem may be alternatively written as a problem in which the upper bound of the integral of the objective function is extended up to infinity and the second term of the objective function is deleted. Then $\bar{t}_x$ formally disappears from the set of arguments with respect to which the optimization is performed. In this alternative formulation, that we call problem $(S.P.2)$, the constraints are the same.

Let λ_X and $-\lambda_Z$ be the co-state variables associated to X and Z respectively.[11] The current value Hamiltonian of the problems $(S.P.1)$ and $(S.P.2)$, denoted by $\mathcal{H}$, reads:[12]

$$\begin{aligned}\mathcal{H} &= u_e\left(\eta_x x+\eta_y y^m L_e\right)+u_f\left(\varphi L_f\right)-a(X)x\\ &\quad -b(\eta_x)x-h(\eta_y)y^m L_e-f(\varphi)L_f-\lambda_X x-\lambda_Z[\zeta x-\alpha Z]\ .\end{aligned}$$

Denote by λ_L the Lagrange multiplier associated to the land availability constraint, by ν_e the multiplier associated to the non-negativity constraint on

[11] By choosing $-\lambda_Z$ as the co-state variable of Z, we may interpret λ_Z as the shadow marginal cost of the pollution stock.

[12] We omit the time index when this causes no confusion.

the land acreage devoted to energy production, by ν_Z the multiplier associated to the cap constraint on the pollution stock and by γ_x, $\gamma_{\eta x}$, and $\gamma_{\eta y}$ the multipliers associated to the non-negativity constraints on x, η_x and η_y respectively. Let $\mathcal{L}$ be the current value Lagrangian of the problem:

$$\begin{aligned}\mathcal{L} &= \mathcal{H} + \lambda_L[\bar{L} - L_e - L_f] + \nu_e L_e + \nu_Z[\bar{Z} - Z] \\ &\quad +\gamma_x x + \gamma_{\eta x}\eta_x + \gamma_{\eta y}\eta_y \ .\end{aligned}$$

For both $(S.P.1)$ and $(S.P.2)$ the f.o.c's are:

$$\frac{\partial \mathcal{L}}{\partial x} = 0 \implies u_e'(\eta_x x + \eta_y y^m L_e)\eta_x = a(X) + \lambda_X + b(\eta_x) + \zeta\lambda_Z - \gamma_x \quad (3.1)$$

$$\frac{\partial \mathcal{L}}{\partial \eta_x} = 0 \implies u_e'(\eta_x x + \eta_y y^m L_e)x = b'(\eta_x)x - \gamma_{\eta x} \quad (3.2)$$

$$\frac{\partial \mathcal{L}}{\partial L_e} = 0 \implies u_e'(\eta_x x + \eta_y y^m L_e)\eta_y y^m = h(\eta_y)y^m + \lambda_L - \nu_e \quad (3.3)$$

$$\frac{\partial \mathcal{L}}{\partial \eta_y} = 0 \implies u_e'(\eta_x x + \eta_y y^m L_e)y^m L_e = h'(\eta_y)y^m L_e - \gamma_{\eta y} \quad (3.4)$$

$$\frac{\partial \mathcal{L}}{\partial L_f} = 0 \implies u_f'(\varphi L_f)\varphi = f(\varphi) + \lambda_L \quad (3.5)$$

$$\frac{\partial \mathcal{L}}{\partial \varphi} = 0 \implies u_f'(\varphi L_f) = f'(\varphi) \ , \quad (3.6)$$

together with the usual complementary slackness conditions.

The dynamics of the co-state variables must satisfy if time differentiable:

$$\dot{\lambda}_X = \rho\lambda_X - \frac{\partial \mathcal{L}}{\partial X} \implies \dot{\lambda}_X(t) = \rho\lambda_X(t) + a'(X(t))\,x(t) \quad (3.7)$$

$$\dot{\lambda}_Z = \rho\lambda_Z + \frac{\partial \mathcal{L}}{\partial Z} \implies \dot{\lambda}_Z(t) = (\rho + \alpha)\lambda_Z(t) - \nu_Z(t) \quad (3.8)$$

$$\nu_Z(t) \geq 0, \bar{Z} - Z(t) \geq 0 \quad \text{and} \quad \nu_Z(t)\left[\bar{Z} - Z(t)\right] = 0 \ . \quad (3.9)$$

Last, for the problem $(S.P.1)$ we must have at the end of the transition:

$$\begin{aligned}\mathcal{H}(\bar{t}_x^-) &\equiv \mathcal{H}\left(x(\bar{t}_x^-), \eta_x(\bar{t}_x^-), \eta_y(\bar{t}_x^-), \varphi(\bar{t}_x^-), L_e(\bar{t}_x^-), L_f(\bar{t}_x^-), \lambda_X(\bar{t}_x^-), \lambda_Z(\bar{t}_x^-)\right) \\ &= \rho\tilde{W} \\ &= u_e(\tilde{\eta}_y y^m \tilde{L}_e) + u_f(\tilde{\varphi}\tilde{L}_f) - h(\tilde{\eta}_y)y^m\tilde{L}_e - f(\tilde{\varphi})\tilde{L}_f \ . \end{aligned} \quad (3.10)$$

For the problem $(S.P.2)$, the transversality condition at infinity is:

$$\lim_{t\uparrow\infty}\left[\lambda_X(t)X(t) + \lambda_Z(t)Z(t)\right]e^{-\rho t} = 0 \ . \quad (3.11)$$

Mining rent and full marginal cost of oil exploitation

According to (3.7) the path of the mining rent is not necessarily monotonous since $a'(X) < 0$. However, the full marginal cost of the extracted oil, that is the marginal cost augmented by the mining rent, is increasing. Let us denote by $\mu(t)$ this full marginal cost: $\mu(t) = a(X(t)) + \lambda_X(t)$. Time differentiating and substituting (3.7) for $\dot{\lambda}_X$ yields:

$$\dot{\mu}(t) \quad = \quad -a'(X(t))x(t) + \rho\lambda_X(t) + a'(X(t))x(t) \quad = \quad \rho\lambda_X(t) > 0 \ . \tag{3.12}$$

Along the optimal path, the behavior of the marginal cost of extracted coal is dominated by the increase of its marginal extraction cost component: $sign\, \dot{\mu}(t) = sign\, \dot{a}(X(t)) = -sign\, a'(X(t))x(t) > 0$.

Land use and land rent

The land allocation is determined by the sub-system (3.3)-(3.6). The link between the two alternatives uses of the land is given by the land rent, λ_L, which appears in (3.3) for the energy production and in (3.5) for the food production. The link between the land allocation problem and the competitiveness of the oil sector is given by the price of the useful energy, $p_e = u'_e$, which appears in (3.3) and (3.4) for the solar energy sector and in (3.1) and (3.2) for the oil sector.

In order that the land be divided between energy and food production the marginal net surplus must be the same in both production sectors, that is (c.f. (3.3) and (3.5)):

$$(p_e\eta_y - h(\eta_y))\, y^m \quad = \quad \lambda_L = p_f\varphi - f(\varphi) \ , \tag{3.13}$$

where η_y and φ are optimally determined, that is solve respectively (3.4): $p_e = h'(\eta_y)$, and (3.6): $p_f = f'(\varphi)$. The *l.h.s.* of the equality (3.13) is the marginal net surplus generated by the allocation of an additional unit of land to the energy production and the *r.h.s.* is the marginal net surplus generated by its allocation to the food production.

There exist two benchmark prices of the useful energy. The first one, we denote by $\underline{p}_e^h$, is the price under which the transformation cost of natural solar energy into useful energy cannot be recovered. Since the average transformation cost $h(\eta_y)/\eta_y$ is an increasing function of η_y, then $\underline{p}_e^h = \lim_{\eta_y \downarrow 0} h(\eta_y)/\eta_y$. This benchmark depends upon the inner cost structure of the solar energy production.

That p_e be higher than $\underline{p}_e^h$ does not justify by itself solar energy production. For $p_e > \underline{p}_e^h$ the *l.h.s.* of (3.13) is positive but not necessarily matching λ_L, and another problem is: which λ_L to match? When the solar energy production is nil, then $L_f = \bar{L}$ as shown in Section 2. Then the productivity in the food sector is this level $\underline{\varphi}$ of φ which solves (3.6) (equivalently (2.1)) with $L_f = \bar{L}$: $u_f'(\underline{\varphi}\bar{L}) = f'(\underline{\varphi})$. Then the food price is $\underline{p}_f = u_f'(\underline{\varphi}\bar{L})$ and the land rent amounts to $\underline{\lambda}_L = \underline{p}_f \underline{\varphi} - f(\underline{\varphi})$, which is the level of the land rent to be matched. We define the benchmark $\underline{p}_e^f$ as the level of the useful energy price for which:

$$\left(p_e \underline{\varphi} - h(\eta_y)\right) y^m = \underline{\lambda}_L \quad \text{and } \eta_y \text{ solves (3.3)} : p_e = h'(\eta_y) \ .$$

For energy prices $p_e > \underline{p}_e^f$ some acreage must be allocated to energy production and (3.13) holds.

Shadow marginal cost of pollution

Let us denote by $\underline{t}_Z$ and $\bar{t}_Z$ the times at which respectively begins and ends the period during which the constraint is tight, assuming that the constraint actually binds along the optimal path. Since initially the pollution stock is smaller than the cap, $Z^0 < \bar{Z}$, there must exist some pre-ceiling period $[0, \underline{t}_Z)$, $0 < \underline{t}_Z$, during which the constraint does not bind, hence $\nu_Z(t) = 0$, $t < \underline{t}_Z$, and $\dot{\lambda}_Z = (\rho + \alpha)\lambda_Z$, so that:

$$\lambda_Z(t) \;=\; \lambda_Z^0 e^{(\rho+\alpha)t} \ , \ t \in [0, \underline{t}_Z) \ , \ \lambda_Z^0 \equiv \lambda_Z(0) \ . \tag{3.14}$$

After $\bar{t}_Z$, since the constraint will be never anymore active, then:

$$\lambda_Z(t) = 0 \quad , \quad t \;\in\; [\bar{t}_Z, \infty) \ . \tag{3.15}$$

It remains to determine the path of $\lambda_Z(t)$ when the ceiling constraint binds. We show in the Subsection 3.3 that $\lambda_Z(t)$ decreases during the period at the ceiling so that $\lambda_Z(t)$ is single-peaked and the peak occurs at the precise time at which the constraint becomes binding.

Necessity of a post-ceiling period preceding the ultimate green period

Given that the problem $(S.P.1)$ and $(S.P.2)$ are convex problems, then the shadow prices $p_e = u'_e$, $p_f = u'_f$ and the land rent, λ_L, all three for any $t \geq 0$, and the mining rent, λ_X, for $t \in [0, \bar{t}_x)$, are continuous functions of time, although maybe not differentiable. These continuity properties imply that the economy cannot switch directly from the period at the ceiling to the ultimate green state characterized in the Sub-section 2.6 *supra*.

Proposition P. 3 *Assume that along the optimal path, there exists a time period at the ceiling, $[\underline{t}_Z, \bar{t}_Z]$, $\underline{t}_Z < \bar{t}_Z$, then there must also exist a post-ceiling period $(\bar{t}_Z, \bar{t}_x)$, $\bar{t}_Z < \bar{t}_x$, during which oil is exploited before the ultimate green period $[\bar{t}_x, \infty)$.*

Proof: Assume that such a period does not exist: $\bar{t}_Z = \bar{t}_x \equiv \bar{t}$. Since the optimal path of the shadow price p_e must be time continuous at $\bar{t}$, then $u'_e(\eta_x(\bar{t}^-))\bar{x} + \eta_y(\bar{t}^-)y^m L_e(\bar{t}^-)) = u'_e(\tilde{\eta}_y y^m \tilde{L}_e)$, hence $\tilde{\eta}_y \tilde{L}_e - \eta_y(\bar{t}^-)L_e(\bar{t}^-) = \eta_x(\bar{t}^-)\bar{x}/y^m > 0$, so that either $\tilde{L}_e - L_e(\bar{t}^-) > 0$, or $\tilde{\eta}_y - \eta_y(\bar{t}^-) > 0$, or both, the case $\tilde{L}_e - L_e(\bar{t}^-) < 0$ together with $\tilde{\eta}_y - \eta_y(\bar{t}^-) < 0$ being excluded.

Assume first that $\tilde{L}_e - L_e(\bar{t}^-) > 0$, equivalently that $L_f(\bar{t}^-) > \tilde{L}_f$. Then from (3.6): $\varphi(\bar{t}^-) < \tilde{\varphi}$ and $f'(\tilde{\varphi}) > f'(\varphi(\bar{t}^-))$ as illustrated in Figure 1, so that u'_f would jump upward at $t = \bar{t}$, contradicting the time continuity of the optimal path of p_f.

Assume now that $\tilde{\eta}_y - \eta_y(\bar{t}^-) > 0$ and $\tilde{L}_e - L_e(\bar{t}^-) > 0$. Thus $L_f(\bar{t}^-) > \tilde{L}_f$. Hence from (3.6), $\varphi(\bar{t}^-) > \tilde{\varphi}$ and $f'(\tilde{\varphi}) < f'(\varphi(\bar{t}^-))$ so that u'_f would jump downwards, contradicting once again the time continuity of p_f. ■

Figure 1 about here

To characterize the dynamics of the different variables during the transition, it is useful to distinguish the periods of unconstrained oil exploitation from the period of constrained oil exploitation. The reason is that during the unconstrained periods, the path of the shadow cost of pollution is well characterized (c.f. (3.14) and (3.15)) while during the constrained period, it has to be determined, however in this later case, the oil extraction rate is known, $x(t) = \bar{x}$.

3.2 Periods of unconstrained oil exploitation

Let us first determine what happens whether oil is the only exploited resource or both oil and solar are simultaneously exploited, oil exploitation being not constrained by the cap on the pollution stock.

Let us start from the f.o.c's (3.1) and (3.2) written as follows:

$$u_e'(q_e)\eta_x = \mu + b(\eta_x) + \zeta\lambda_Z \tag{3.16}$$
$$u_e'(q_e) = b'(\eta_x) \ . \tag{3.17}$$

Consider first the unconstrained period preceding the ceiling period, when $\lambda_Z > 0$.

Time differentiating (3.16), using (3.12), (3.14) and (3.17), we obtain:

$$\dot{q}_e(t) = \frac{\rho\lambda_X(t) + \zeta(\rho+\alpha)\lambda_Z(t)}{u_e''(q_e(t))\eta_x(t)} < 0 \ . \tag{3.18}$$

After the ceiling period, $\lambda_Z(t) = 0$ through (3.15) and still $\dot{q}_e(t) = \rho\lambda_X/(u_e''\eta_x) < 0$. Time differentiating (3.17) and making use of (3.18) yields:

$$\dot{\eta}_x(t) = \frac{u_e''(q_e(t))}{b''(\eta_x(t))}\dot{q}_e(t) > 0 \ . \tag{3.19}$$

Only oil is exploited

In this case: $q_e = \eta_x x$, hence:

$$\dot{x}(t) = \frac{1}{\eta_x(t)}\left(\dot{q}_e(t) - \dot{\eta}_x(t)x(t)\right) < 0 \ . \tag{3.20}$$

Both energy resources are exploited

Since now $L_e > 0$ and $\eta_y > 0$, then we may write (3.4) as follows: $u'_e(q_e) = h'(\eta_y)$. Time differentiating and making use of (3.18), we obtain:

$$\dot{\eta}_y(t) \;=\; \frac{u''_e(q_e(t))}{h''(\eta_y(t))}\dot{q}_e(t) \;>\; 0 \;. \tag{3.21}$$

Since $L_e > 0$, then (3.3) may be written as: $(u'_e(q_e)\eta_y - h(\eta_y))\, y^m = \lambda_L$. Again time differentiating, then by (3.18) and (3.4), $u'_e - h' = 0$, we get:

$$\dot{\lambda}_L(t) \;=\; u''_e(q_e(t))\eta_y(t)y^m\dot{q}_e(t) > 0 \;. \tag{3.22}$$

Last, consider the food sub-system (3.5)-(3.6):

$$\begin{aligned} u'_f(\varphi L_f)\varphi &= f(\varphi) + \lambda_L \\ u'_f(\varphi L_f) &= f'(\varphi) \;. \end{aligned}$$

Time differentiating and using (3.6), $u'_f - f' = 0$, we can express $\dot{\varphi}$ and $\dot{L}_f$ as the following functions of $\dot{\lambda}_L$:

$$\begin{bmatrix} u''_f\varphi L_f & u''_f\varphi^2 \\ u''_f L_f - f'' & u''_f\varphi \end{bmatrix} \begin{bmatrix} \dot{\varphi} \\ \dot{L}_f \end{bmatrix} = \begin{bmatrix} \dot{\lambda}_L \\ 0 \end{bmatrix} .$$

Hence:

$$\dot{\varphi}(t) = \frac{1}{f''(\varphi(t))\varphi(t)}\dot{\lambda}_L(t) > 0 \;, \tag{3.23}$$

$$\dot{L}_f(t) = -\frac{u''_f(q_f(t))L_f(t) - f''(\varphi(t))}{u''_f(q_f(t))f''(\varphi(t))\varphi^2(t)}\dot{\lambda}_L(t) < 0 \;, \tag{3.24}$$

$$\dot{q}_f(t) = \frac{1}{u''_f(q_f(t))\varphi(t)}\dot{\lambda}_L(t) < 0 \;, \tag{3.25}$$

$$\dot{L}_e(t) > 0 \text{ and } \dot{q}_y(t) = \eta_y(t)\dot{L}_e(t) + \dot{\eta}_y(t)L_e(t) > 0 \;, \tag{3.26}$$

$$\dot{q}_x(t) = \dot{q}_e(t) - \dot{q}_y(t) < 0 \;\text{ and }\; \dot{x}(t) = \frac{1}{\eta_x(t)}\left[\dot{q}_x(t) - \dot{\eta}_x(t)x(t)\right] < 0 \;. \tag{3.27}$$

Taking stock:

Proposition P. 4 *Along the optimal path, during the periods of unconstrained exploitation:*

a. Whether oil is the only exploited resource or both oil and solar energy are exploited:

a.i. The production of useful energy decreases, hence its price increases;

a.ii. The production of useful oil energy decreases and the transformation rate of extracted oil into useful energy increases, hence the production of extracted oil decreases.

b. When both oil and solar energies are exploited:

b.i. The useful solar energy production increases due to both the increase of the land acreage allocated to the production of this energy and the increase of the transformation rate of solar energy into useful energy;

b.ii. The land acreage allocated to food production decreases and the food productivity of land increases but not sufficiently to compensate for the acreage decrease so that the food production decreases and its price increases;

b.iii. The land rent increases.

3.3 Periods of constrained oil exploitation

During the period, the dynamics of λ_Z is not yet known because $\dot{\lambda}_Z = (\rho + \alpha)\lambda_Z - \nu_Z$ and $\nu_Z > 0$ since the constraint $\bar{Z} - Z(t) \geq 0$ is tight. At this stage, the only qualitative information that we have is that $\dot{\mu}(t) > 0$ (c.f. (3.12)) and $\dot{x}(t) = 0$ since $x(t) = \bar{x}$. To characterize the constrained period, we express the dynamics of all the other variables as function of $\dot{\mu}$, taking care that $x = \bar{x}$. We may have two types of constrained oil exploitation periods according to solar energy is simultaneously exploited with oil or not.

Exclusive exploitation of oil: $q_e = q_x = \eta_x \bar{x}$.

Let us start from the f.o.c's (3.1) and (3.2) written now as follows:

$$u_e'(\eta_x\bar{x})\eta_x = \mu + b(\eta_x) + \zeta\lambda_Z \quad (3.28)$$
$$u_e'(\eta_x\bar{x}) = b'(\eta_x) \ . \quad (3.29)$$

Time differentiating and using $u_e' - b' = 0$, we obtain the following system:

$$\begin{bmatrix} u_e''\bar{x}\eta_x & -\zeta \\ & \\ u_e''\bar{x} - b'' & 0 \end{bmatrix} \begin{bmatrix} \dot{\eta}_x \\ \\ \dot{\lambda}_Z \end{bmatrix} = \begin{bmatrix} \dot{\mu} \\ \\ 0 \end{bmatrix} .$$

Hence:

$$\dot{\eta}_x(t) = 0 \quad \text{and} \quad \dot{\lambda}_Z(t) = -\frac{1}{\zeta}\dot{\mu}(t) < 0 \ . \tag{3.30}$$

Let us denote by $\bar{\eta}_x$ the constant level of the efficiency rate η_x. Then q_e is constant, $q_e(t) = q_x(t) = \bar{\eta}_x\bar{x}$, and also the price of useful energy: $p_e(t) = \bar{p}_e = u_e'(\bar{\eta}_x\bar{x})$.

The increase of the full marginal cost of the extracted oil is exactly balanced by the decrease of the shadow marginal cost of the pollution stock: $\dot{\mu}(t) + \zeta\dot{\lambda}_Z(t) = 0$. The full marginal cost of useful energy, the input cost of the oil transformation industry, is constant and equal to $b'(\bar{\eta}_x)$, itself equal to $\bar{p}_e$.

Simultaneous exploitation of both oil and solar energies

Making use of $L_f = \bar{L} - L_e$, we have to determine six variables as a function of $\dot{\mu}$: $\dot{\eta}_x$, $\dot{\eta}_y$, $\dot{L}_e$, $\dot{\lambda}_Z$, $\dot{\varphi}$ and $\dot{\lambda}_L$.

Let us consider the system of the six f.o.c's obtained after some simplifications and substitution of $\bar{L} - L_e$ for L_f:

$$\begin{aligned} u_e'\left(\eta_x\bar{x} + \eta_y y^m L_e\right)\eta_x &= \mu + b(\eta_x) + \zeta\lambda_Z && (3.31)\\ u_e'\left(\eta_x\bar{x} + \eta_y y^m L_e\right) &= b'(\eta_x) && (3.32)\\ u_e'\left(\eta_x\bar{x} + \eta_y y^m L_e\right) &= h'(\eta_y) && (3.33)\\ u_e'\left(\eta_x\bar{x} + \eta_y y^m L_e\right)\eta_y y^m &= h(\eta_y)y^m + \lambda_L && (3.34)\\ u_f'\left(\varphi\left[\bar{L} - L_e\right]\right) &= f'(\varphi) && (3.35)\\ u_f'\left(\varphi\left[\bar{L} - L_e\right]\right)\varphi &= f(\varphi) + \lambda_L \ . && (3.36) \end{aligned}$$

Time differentiating results in the following system:

$$\begin{bmatrix} & ; & -\zeta \\ & ; & 0 \\ & ; & 0 \\ M & ; & 0 \\ & ; & 0 \\ & ; & 0 \end{bmatrix} \begin{bmatrix} \dot{\eta}_x \\ \dot{\eta}_y \\ \dot{L}_e \\ \dot{\varphi} \\ \dot{\lambda}_L \\ \dot{\lambda}_Z \end{bmatrix} = \begin{bmatrix} \dot{\mu} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tag{3.37}$$

where M is a 5×6 sub-matrix the details of which are given in Appendix A.1.

Let us denote by Δ the determinant on the system and by $\Delta_{(k,l)}$ the sub-determinant obtained by deleting the k^{th} line and the l^{th} column. Thus $\Delta = \zeta\Delta_{(1,6)}$. Note that $\Delta_{(1,l)} = 0$, for $l = 1, \cdots, 5$, so that:

$$\dot{\eta}_x(t) = \dot{\eta}_y(t) = \dot{L}_e(t) = \dot{\varphi}(t) = \dot{\lambda}_L(t) = 0 \quad \text{and} \quad \dot{\lambda}_Z(t) = -\frac{1}{\zeta}\dot{\mu}(t) \ . \tag{3.38}$$

Like in the case of exclusive oil exploitation all the variables of the model are constant, excepted the shadow marginal cost of pollution and the full marginal cost of extracted oil, the decrease of the first balancing the increase of the second. Thus the full marginal cost of the extracted oil input of the oil transformation industry is constant. Facing a constant output price and a constant input price the oil transformation industry must stay with a constant efficiency rate $\bar{\eta}_x$.

Thus although constrained by the pollution stock upper bound, absent any abatement option, it is optimal to undertake no additional effort to improve the efficiency rates η_x, η_y, φ, a situation that Amigues and Moreaux (2015) have called the *ceiling paradox* in another context.

Proposition P. 5 Generalized ceiling paradox.

Along the optimal path, whether oil is the only exploited energy or both oil and solar energies are simultaneously exploited, the economy moves along a stationary trajectory during the period at the ceiling: Efficiency rates, land productivity, production levels of extracted oil, useful energies and food, and *the land allocation are all constant. The only changes come from the increase of the full marginal cost of extracted oil, exactly balanced by the decrease of the shadow marginal cost of the pollution stock, so that the full marginal cost of the useful oil energy is constant. The other costs, the marginal cost of the solar energy when exploited and the marginal cost of food, the prices of the useful energy and food, and the land rent are all constant.*

An immediate implication of the Proposition 5 is the following corollary:

Corollary 1 *Along the optimal path the solar energy becomes to be exploited either before the arrival at the ceiling or after the end of the period at the ceiling, but never within the period.*

4 Optimal paths

All the optimal paths share a common strong qualitative structure determined by the increasing scarcity of the non-renewable resource and the ceiling constraint when the constraint is effective during some time period, and the constraint is tight provided that the oil endowment be sufficiently abundant.

Assuming that it is the case, there exists only one main type of optimal paths along which the characteristics of the different periods identified in the Propositions 1 to 5 hold together with the time continuity of the price paths, a necessary condition for optimality in the present context. The only possible differences within this main type come from the time, denoted by $\underline{t}_y$, at which begins the exploitation of the solar energy either before or after the period at the ceiling, and when before the ceiling period, possibly at the beginning time of the planning period, $t = 0$.

We describe the type of optimal path along which the solar energy is not immediately exploited but begins to be exploited before the time $\underline{t}_Z$ at which the pollution stock $Z(t)$ begins to be constrained by the carbon cap, $\bar{Z}$. In this scenario, the optimal path is a five periods path. The implicit price paths of useful energy and food are illustrated in Figure 2, the efficiency rates paths in Figure 3 and the land allocation and land use paths in Figure 4.

Figure 2 about here

Figure 3 about here

Figure 4 about here

First period $[0, t_y)$ *: Pre-ceiling phase of exclusive exploitation of the oil energy source.*

During this period, the available land $\bar{L}$ is allocated to the sole production of food. The food sector productivity is constant, $\varphi(t) = \underline{\varphi}$, together with the food output $q_f(t) = \underline{q}_f = \underline{\varphi}\bar{L}$, the food implicit price $p_f(t) = \underline{p}_f = u_f(\underline{q}_f)$, and the land rent, $\lambda_L(t) = \underline{\lambda}_L = \underline{p}_f\underline{\varphi} - f(\underline{\varphi})$.

The production of extracted oil, $x(t)$, decreases, the transformation rate $\eta_x(t)$ increases and the production of useful energy, $q_e(t) = q_x(t)$ decreases, hence the implicit price of useful energy, $p_e(t)$ increases. The flow of polluting emissions, $\zeta x(t)$, decreases but remains larger than the self-regeneration flow, $\alpha Z(t)$, so that the pollution stock increases and the shadow cost of the pollution stock, $\lambda_Z(t)$, rises.

In order that such an initial time period exists, it must be the case that the initial price of useful energy, $p_e(0)$, be lower than the benchmark price $\underline{p}_e^f$. The period ends at time $\underline{t}_y$ when the useful energy price attains the benchmark level.

Second period $(\underline{t}_y, \underline{t}_Z)$ *: Pre-ceiling period of simultaneous exploitation of both oil and solar energy.*

The acreage of land, $L_e(t)$, allocated to solar energy production expands continuously and the transformation rate of solar energy, $\eta_y(t)$, improves.

Hence the production of solar energy, $q_y(t)$, increases. The productivity in the food sector, $\varphi(t)$, increases, but not sufficiently to balance the decrease of the acreage, $L_f(t)$, allocated to food production, hence the food production, $q_f(t)$, drops and its price, $p_f(t)$, increases.

The useful energy needs are now satisfied by both oil and solar energies. The solar energy production, $q_y(t)$, increases, due to the simultaneous increase of the land acreage, $L_e(t)$, allocated to its production and of its transformation rate, $\eta_y(t)$. The oil extraction rate, $x(t)$, decreases and the oil efficiency rate, $\eta_x(t)$, increases, but as in the preceding period, not sufficiently to match the drop of the extraction rate so that the production of oil useful energy, $q_x(t)$, decreases. Moreover, the rise of the solar energy production rate does not balance the fall of oil energy production, hence the aggregate useful energy production, $q_e(t)$, decreases as during the first time period and its price increases.

The useful energy needs are less and less fed by oil useful energy, hence a stronger pressure on land allocation and a progressive rise of the land rent, $\lambda_L(t)$.

Although time decreasing, the emission flow, $\zeta x(t)$, is still larger than the regeneration flow, $\alpha Z(t)$, so that the pollution stock, $Z(t)$, increases and the shadow marginal cost of the pollution stock, $\lambda_Z(t)$, rises. The second period ends when the pollution stock attains the cap level, $\bar{Z}$, and its shadow marginal cost attains its maximum.

Third period $[\underline{t}_Z, \bar{t}_Z]$ *: Ceiling period.*

When constrained by the cap, the economy extracts oil at a constant rate, $x(t) = \bar{x}$, the emission flow thus generated, $\zeta\bar{x}$, balancing the constant self-regeneration flow of the maintained constant pollution stock, $\alpha Z(t) = \alpha\bar{Z}$. The other variables of the model are also constant, excepted the marginal/average extraction cost, $a(X(t))$, the mining rent, $\lambda_X(t)$, and the shadow marginal cost of the pollution stock, $\lambda_Z(t)$. The sum $a(X(t)) + \lambda_X(t)$, that is the full marginal cost of oil extraction, $\mu(t)$, increases, $\dot{\mu}(t) = \rho\lambda_X(t)$ according to (3.12), and the shadow cost of pollution decreases by an amount $\dot{\lambda}_Z(t) = -\dot{\mu}(t)/\zeta$, so that the full marginal cost of the extracted oil input in the oil transformation industry is also constant.

The constant values of η_x, η_y, L_e, L_f, φ and λ_L solve the system (3.32)-(3.36).[13] The period ends at $t = \bar{t}_Z$ when $\lambda_Z(t)$ has decreased down to 0.

Fourth period $(\bar{t}_Z, \bar{t}_x)$ *: Last period of oil exploitation.*

This is another period of unconstrained oil exploitation during which solar energy is also exploited, like during the second period, excepted that now, the shadow marginal cost of the pollution stock is nil. Due to the declining rate of oil exploitation, the polluting emission flow, $\zeta x(t)$, falls below $\zeta \bar{x}$ and the pollution stock $Z(t)$ remains below the cap level $\bar{Z}$. Since the atmospheric carbon concentration constraint is never more active, the shadow marginal cost of carbon pollution is nil forever.

The qualitative properties of the dynamics of the production rates, prices, efficiency rates, land allocation and land rent are the same as their qualitative properties during the second period. Thus after an halt during the ceiling period, the land acreage devoted to solar energy production rises again at the expense of food production, while the land rent grows.

The period ends when the useful energy price level attains its second benchmark level, $\tilde{p}_e$, and $x(t)$ has decreased down to 0.[14] The grade at which ends oil exploitation, $\tilde{X} \equiv X(\bar{t}_x)$, is given together with the oil transformation rate, $\eta_x(\bar{t}_x^-)$, as the solution of the system (3.1)-(3.2) in which $\lambda_X(\bar{t}_x^-) = 0$ because the mining rent on the last exploited grade must be nil, and $\lambda_Z(\bar{t}_x^-) = 0$, that is:

$$\tilde{p}_e \eta_x(\bar{t}_x^-) = a(\tilde{X}) + b(\eta_x(\bar{t}_x^-)) \quad \text{and} \quad \tilde{p}_e = b'(\eta_x(\bar{t}_x^-)) \ .$$

Fifth period, $[\bar{t}_x, \infty)$ *: Ultimate green economy.*

The last period is the ultimate green economy characterized in the subsection 2.6, in which all the variables are constant excepted the pollution stock which progressively disappears: $Z(t) = Z(\bar{t}_x) e^{-\alpha(t-\bar{t}_x)}$.

[13]Note that (3.31) is deleted because $\dot{\lambda}_Z = -\dot{\mu}(t)/\zeta$ implies that its *r.h.s.* reduces to $b(\eta_x)$ and λ_Z disappears from the system (3.31)-(3.36). Only five variables have to be determined and one equation, either (3.31) or (3.32), has to be deleted.

[14]That $x(t)$ converges down to 0 is an implication of the necessary time continuity of the price paths $p_e(t)$ and $p_f(t)$.

5 Concluding remarks

Facing the increasing scarcity of fossil fuels and the pollution problems raised by their exploitation, the economy should improve the efficiency of useful energy production from any energy source, fossils or renewables. However, efficiency gains being costly, the production cost of renewable energy should increase throughout time, absent any technical progress able to reverse the trend. This does not prevent renewables to take progressively a larger share in the energy mix while expanding their land acreage despite the induced rise of the land rent. Under the standard assumptions on costs and demands retained in the present paper, the development of renewables is both an intensive process, through continuous efficiency gains, and an extensive one, through a larger occupation of space. Being in competition for land access with renewables, the food production sector intensifies its activity on less land but not sufficiently to counterbalance the acreage decrease. Hence the food output decreases, inducing an increase of food prices.

The above dynamics of the energy and food sectors stops when the economy is actually constrained by the cap on the atmospheric carbon concentration. The cap implies a constant rate of exploitation of fossil fuels, absent any abatement option. This is a general property of ceiling models not resulting from our simplifying assumption of a constant self-regeneration rate of carbon in the atmosphere, but the non-availability of an outside option, like a pollution abatement device, permitting to temporarily relax the ceiling constraint, as in Amigues and Moreaux, (2015-b). Any law of motion of carbon concentration of the form $\dot{Z} = G(\zeta x, Z)$ should admit under mild assumptions a unique root of the equation $G(\zeta x, \bar{Z}) = 0$ when the economy must satisfy the carbon cap. This constancy of the fossil fuel exploitation rate has strong consequences over the variables dynamics. Facing a constant supply of extracted fossils, the transformation industry makes no more any effort to improve its conversion efficiency performance and delivers useful energy at a constant rate. The energy price being now constant, the renewables sector also stops making efficiency gains and no more increases its land occupation. This stabilizes in turn the economic conditions in the food sector and the land rent level.

From a policy viewpoint, a carbon pricing scheme implementing the first best can be to charge the shadow cost of carbon in the useful energy price.

The analysis shows that the carbon price should rise before the attainment of the ceiling. However, during the ceiling period, the carbon price must decline in order to keep constant the net surplus from fossil fuels exploitation, just balancing the rise of the full marginal cost of extracted fossil fuels, the sum of their extraction cost and mining rent. Such a carbon pricing device is sufficient to restore optimality in a decentralized context and no subsidies to renewable energy production or land acquisition for this activity are required.

In contrast with Chakravorty, Magné and Moreaux, (2008), (CMM thereafter), the land rent is always positive and either increases or stabilizes in two situations, when oil is the only exploited energy source and/or the carbon cap constraint binds. The CMM model combines a Ricardian description of land use for renewables and food production with Hotelling dynamics for fossil fuels exploitation. The land productivities in both activities are assumed to be constant resulting in the possibility that some fraction of the land remains fallow. With differentiated land productivities, a Ricardian order of the spatial dynamics of land exploitation emerges, highest quality lands being used in priority. With adjustable productivities and land free disposal, the spatial Ricardian ordering is replaced by a pure time ordering. As in CMM, the land sharing between food and renewables production should stay constant absent the Hotelling logic of increasing fossil resources prices, e.g. in the ultimate green economy. But contrarily to CMM, the whole available land should be cultivated for food production if renewables are not yet competitive. This is a consequence of the possibility for farmers to extensify their activity, reduce their productivity and thus their production costs per acre.

When renewables are competitive, their land acreage expands in the CMM framework because of the growing profitability of producing clean and renewable energy when the economy faces an increasing scarcity of fossil fuels and carbon pollution problems. In the present model, the Hotelling logic plays more indirectly on the renewables production dynamics. This is because the rising price of useful energy makes attractive to use more costly, but also more efficient, techniques that the renewables sector improves its energy conversion performance per unit of land, this improvement allowing in turn to undertake renewables production on larger areas despite increasing land prices.

Our work may be extended in several directions. Instead of assuming that the land allocation may be adjusted instantaneously without cost, the land conversion from agriculture to renewable energy production could be subjected to specific conversion costs. Such costs are expected to rise with the speed of land conversion. The result should be a smoothing of the energy transition, the land conversion to renewables having to begin earlier. It may even be the case that land conversion should begin before renewable energy becomes competitive with respect to fossil fuels energy.

The literature has explored the optimal management of carbon sinks, like forests, to alleviate climate change. In the present model context, this is equivalent to assume that first, some fraction of the carbon emissions may be indirectly abated and stored into forest lands, and second, that food and renewable energy production will have to compete with forests for land access. Alternatively, it may be assumed that the self-regeneration capacity of the environment depends on the size of the forest areas. We leave this problem for future research.

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Appendix

A.1 Details of the matrix M of equation (3.37)

$$
\mathrm{M} \equiv \begin{bmatrix}
u_e''\bar{x}\eta_x & u_e''\eta_x y^m L_e & u_e''\eta_x\eta_y y^m & 0 & 0 \\
u_e''\bar{x} - b'' & u_e'' y^m L_e & u_e''\eta_y y^m & 0 & 0 \\
u_e''\bar{x} - h'' & u_e'' y^m L_e & u_e''\eta_y y^m & 0 & 0 \\
u_e''\bar{x}\eta_y y^m & u_e''\eta_y (y^m)^2 & u_e''(\eta_y y^m)^2 & 0 & -1 \\
0 & 0 & -u_f''\varphi & u_f''\left[\bar{L} - L_e\right] - f'' & 0 \\
0 & 0 & -u_f''\varphi^2 & u_f' + u_f'' q_f - f'' & -1
\end{bmatrix}
$$

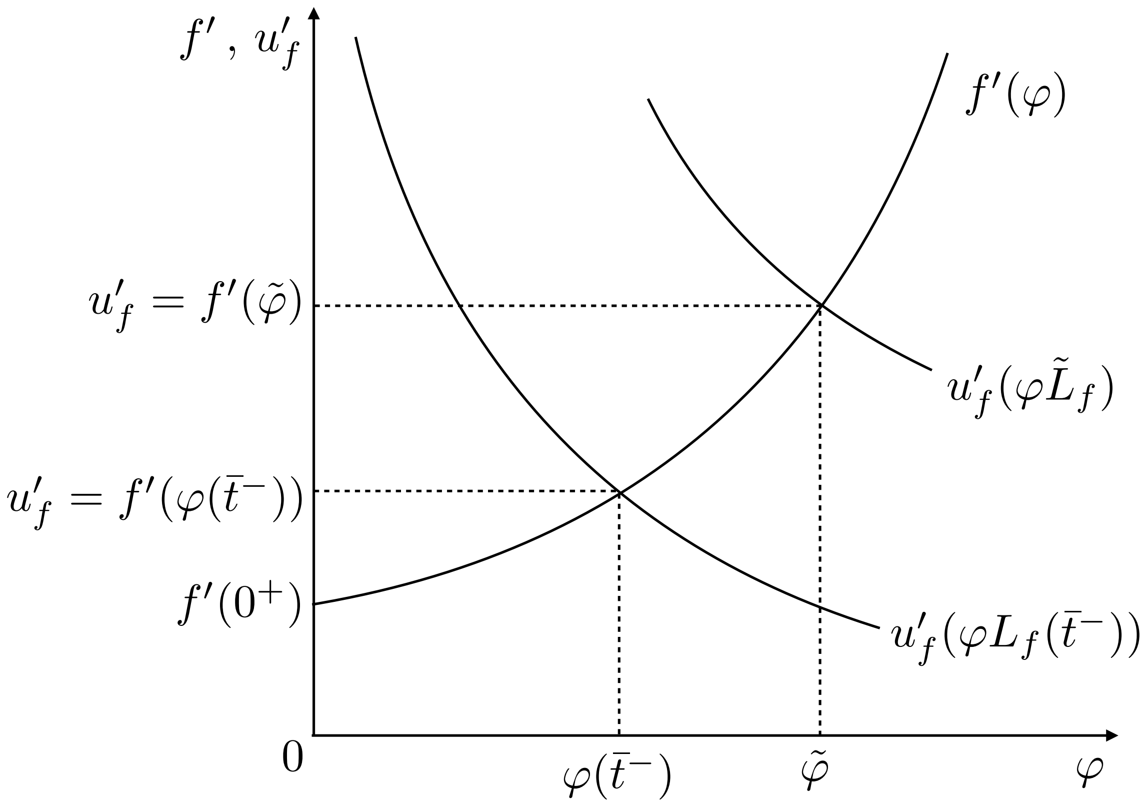
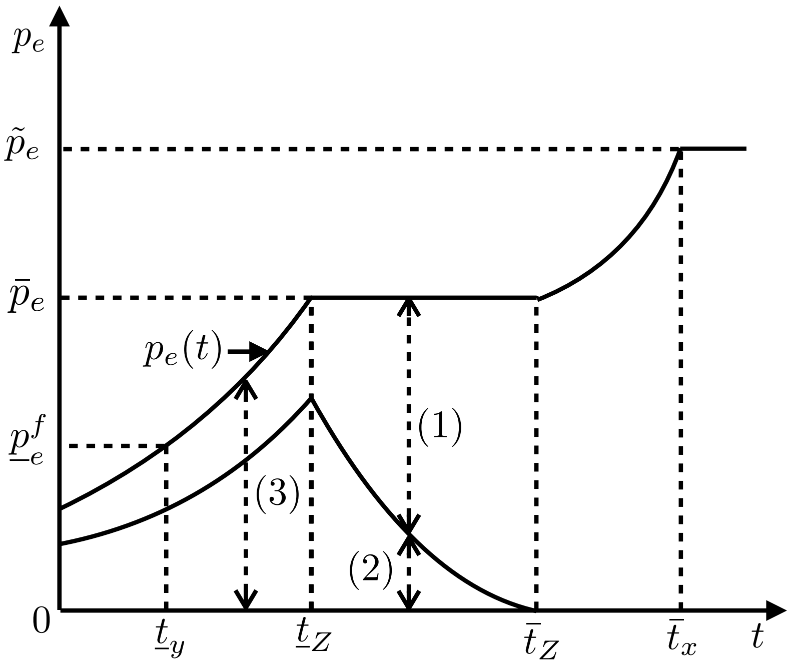


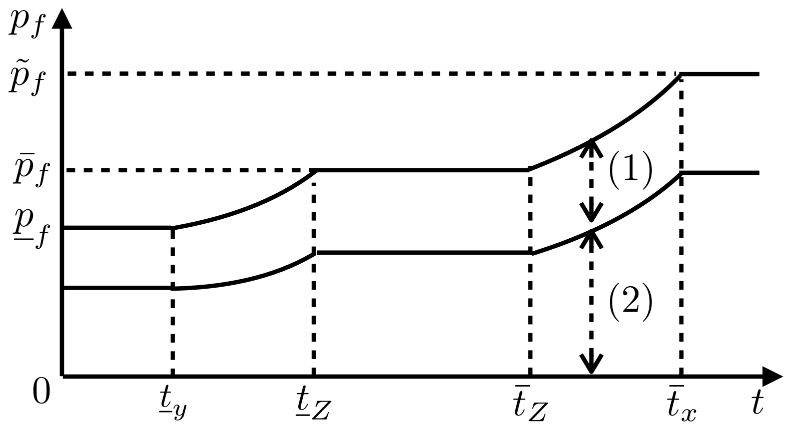
Figure 1: **Jump of u'_f at time $\bar{t}$. The case $L_f(\bar{t}^-) > \tilde{L}_f$.**



(1) Average cost of useful oil energy including the mining rent: $(a(X) + \lambda_X + b(\eta_x))/\eta_x$

(2) Shadow average pollution cost of useful oil energy: $\zeta\lambda_Z/\eta_x$.

(3) Average cost of useful solar energy including the land rent: $(h(\eta_y)y^m + \lambda_L)/\eta_y$



(1) Average production cost of food: $f(\varphi)/\varphi$.

(2) Land rent per unit of food: λ_L/φ.

Figure 2: **Optimal price paths:**
Top: Useful energy price
Bottom: Food price.

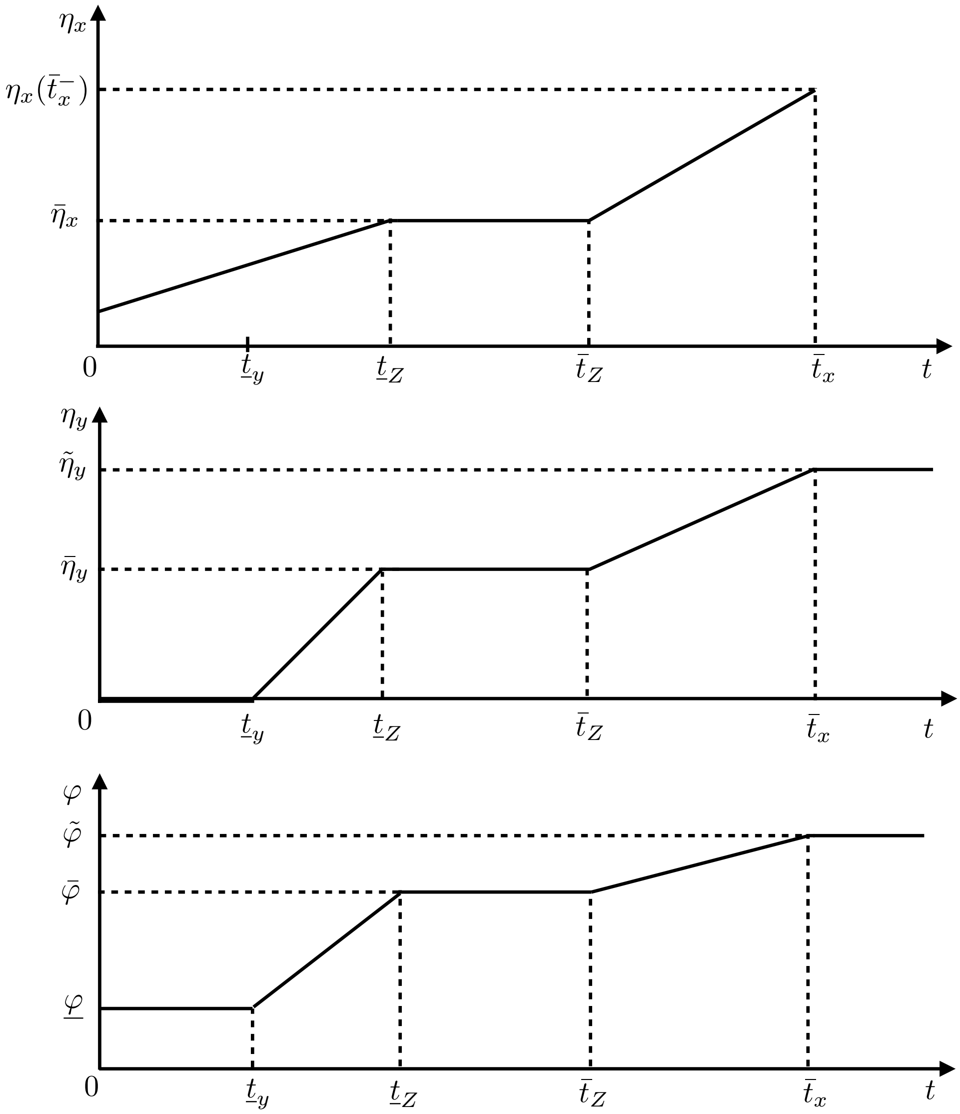


Figure 3: **Paths of efficiency and productivity rates:**
Top: Efficiency rate in the oil transformation industry
Middle: Efficiency rate in the solar energy sector
Bottom: Productivity rate in the food sector.

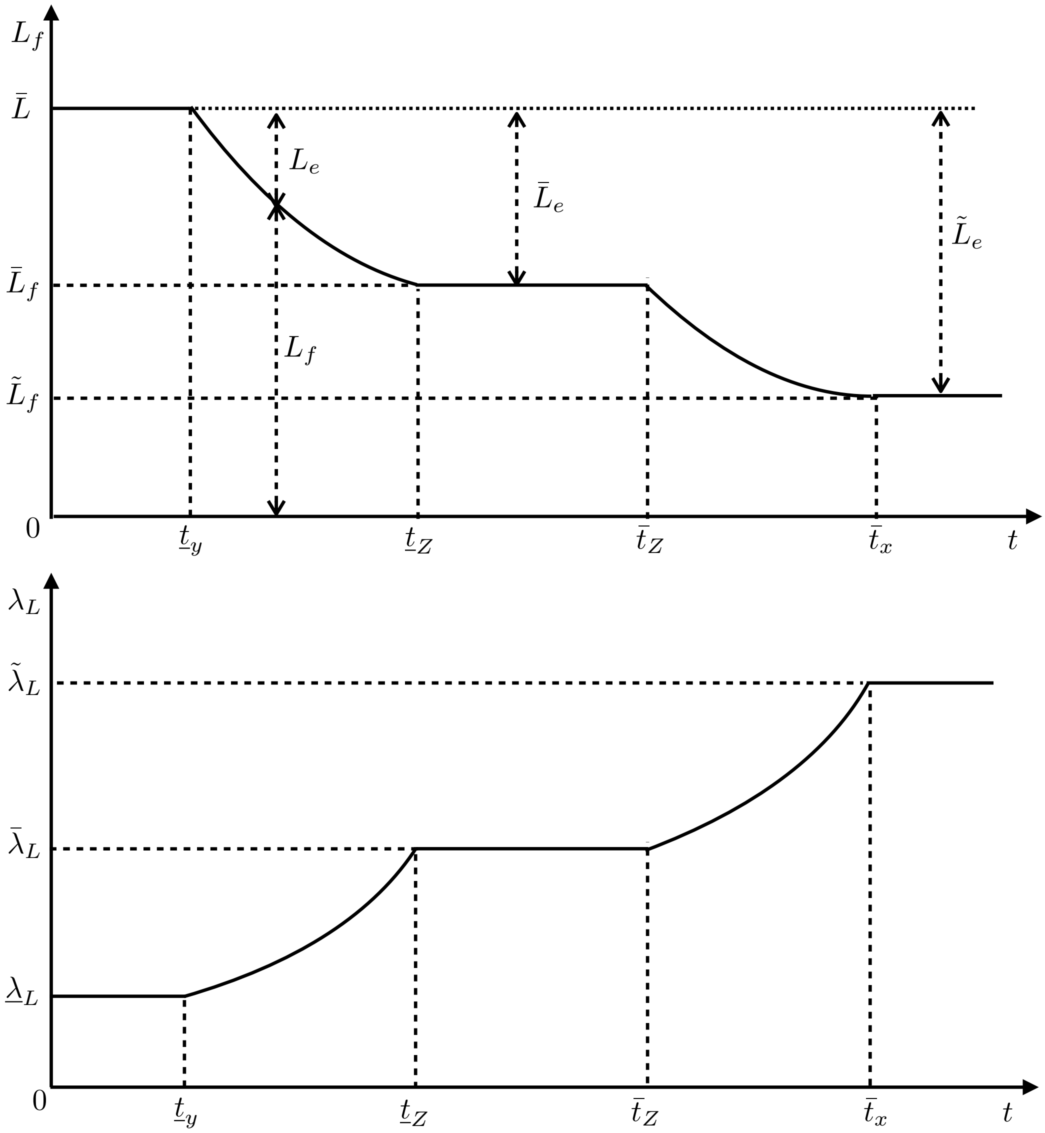


Figure 4: **Paths of land allocation and land rent:**
Top: Land allocation
Bottom: Land rent.