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Social long-term care insurance with two-sided altruism[1]

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Abstract

This paper studies the design of a social long-term care (LTC) insurance when altruism is two-sided. The *laissez-faire* solution is not efficient, unless there is perfect altruism. Under full information, the first-best can be decentralized by a linear subsidy on informal aid, a linear tax on bequests when the parent is dependent and state specific lump-sum transfers which provide insurance. We also study a second-best scheme comprising a LTC benefit, a payroll tax on children's earnings and an inheritance tax. This scheme redistributes resources across individuals and between the states of nature and the tax on children's labor enhances informal care to compensate for the children's possible less than full altruism.

Keywords: long-term care, social insurance, two-sided altruism
JEL: H2, H5

1 Introduction

Long-term care (LTC) is becoming a major concern for policy makers. Following the rapid aging of our societies, the needs for LTC are expected to grow and yet there is a lot of uncertainty as how to finance those needs; see Norton (2009) and Cremer, Pestieau and Ponthière (2012) for an overview. Family solidarity, which has been the main provider of LTC, is reaching a ceiling, and the market is remains rather thin.[1] Not surprisingly, one would expect that the state takes the relay.

This paper studies the design of social LTC insurance when both parents and children are altruistic towards each other. Children's altruism is however only partial. First, it is only triggered by the occurrence of dependency and second it is limited. In a pure market economy the dependent parents have to devote their resources to purchase professional care services, but also to leave some bequests to boost informal care.

One cannot be but struck by two parallel evolutions: the soaring needs for LTC and the growing share of inherited wealth in overall capital accumulation; see Piketty and Zucman (2014). LTC is surely not a problem for the very wealthy households, but for many households LTC can eat up most of their assets and make it impossible to bequeath anything. It is thus not surprising that some people have thought of using the proceeds of estate taxation to finance public long-term care. An example of this is the English green paper proposing a voluntary inheritance levy. Accordingly, people in England and Wales could pay a one-off "inheritance levy" of up to £12,000 in return for free public long-term care in their old age. The fee would either be deducted from the estates of older people when they die or be paid when they enter retirement age.[2]

[1] The economic literature has identified a number of factors that can explain the low level of LTC insurance demand. These include significant loading factors due to high administrative costs, adverse selection in the demand for insurance (Sloan and Norton, 1997; Finkelstein and McGarry, 2006; Brown and Finkelstein, 2007 and 2009) and the existence of cheaper substitutes like family care or public assistance (Pauly, 1990; Zweifel and Strüwe, 1998; Brown and Finkelstein, 2008).

[2] See http://www.theguardian.com/society/2009/jun/14/older-people-health-inheritance-levy, accessed 20th July 2015.

The aim is to avoid forcing many pensioners to sell their family homes to fund massive nursing home bills.

There is indeed a close link between bequests and long-term care. In a world without a well-developed LTC insurance market, households are forced to oversave or to self-insure. Consequently, in case of good health they end up with an excess of saving that can lead to involuntary bequests. It is then clear that if it were possible to tax those bequests, the proceeds could be used to finance long-term care. Actually if some type of insurance, private or public were available, one would end up with the same result, that is, a perfect smoothing of consumption between the two states of the world, dependency or not. Brunner and Pech (2012) made that argument by showing that a tax on bequests to finance LTC causes a smaller deadweight loss than an income or consumption tax. This holds true whether or not the parents are altruistic. The problem is that in a world with heterogeneous agents it might be impossible to distinguish bequests that are made by dependent parents from those made by autonomous parents.

The matter gets more complicated when we look at the case of formal LTC financed by saving or insurance and informal services coming from children. For the time being we assume that formal and informal care are complements. Dependent parents will have to exchange those informal services against the prospect of some bequests. If children are not altruistic, we will have a pure *quid pro quo* exchange; if they are altruistic they might help their parents even if these cannot bequeath anything. The amount of help will depend on the extent of filial altruism. In case of perfect altruism, children will provide the optimal amount of assistance to their parents.

In a world of identical individuals, the social insurance scheme will serve two purposes: it redistributes resources across the two states of nature and it induces the child to help his parent. If individuals differ in the level of their wage, social insurance must also redistribute resources across individuals.

To study these issues we consider a population consisting of one parent and one child families. Parents are pure altruists towards their child, while the child's altruism may be imperfect. Parents are retired and have accumulated some wealth. They face a probability of becoming dependent and needing LTC. The need of LTC requires expenditures of some monetary amount. In case of dependency parents would like to benefit from the aid of their children, who are ready to help their parent out of altruism but also with the expectation of some inheritance.[3]

We characterize the first-best allocation and show that the *laissez-faire* is not efficient, unless there is perfect altruism. Under full information the first-best can be decentralized by a linear subsidy on informal aid, a linear tax on bequests when the parent is dependent, and state specific lump-sum transfers which provide insurance.

Next, we study the second-best allocation when the instruments available are linear (state independent) taxes on bequests and children's labor income which finance a transfer to the dependent elderly.[4] Observe that the tax on children's labor income is in our setting effectively equivalent to a subsidy on informal care.[5]

We first consider a setting with *ex ante* identical individuals. We show that both taxes should be positive. The tax on labor which subsidizes informal care compensates for possible imperfect altruism; this is like in the first-best implementation. Both taxes also provide insurance, and that is relevant as the first-best state specific lump-sum transfers or taxes that provide full insurance are not available. Labor and bequest taxes are then used to provide (partial) insurance.

Finally, we consider a setting with *ex ante* heterogenous families, which differ in children's' productivities and parents' wealth. We show that the results obtained in the homogenous family case carry over. However, the two tax rates now also include a positive redistributive term. Throughout this paper private LTC insurance is assumed

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[3] See Pauly (1990) for the preference parents have for assistance from their children.

[4] For non-linear schemes, see Jousten *et al.* (2006) and Pestieau and Sato (2009).

[5] Labor and informal care are the sole possible usages of children's time. There is no leisure or the amount of leisure is given.

away. This is for the sake of simplicity but also because in most countries the LTC insurance market is extremely thin; see Brown and Finkelstein (2007, 2009).

2 Identical individuals

2.1 The model setup

Consider a population of a size normalized to one, consisting of one parent (subscript 'p') and one child (subscript 'c') families. Parents are pure altruists towards their child, while a child's altruism may be imperfect. Parents are retired and have accumulated wealth y. They face the probability π of becoming dependent and needing long-term care. The need of LTC requires expenditures of amount L. In case of dependency the parent would like to benefit from the aid of his child, who is ready to help his parent out of altruism but also when receiving some inheritance b. Parents choose the bequest to their children. In case of autonomy, the child inelastically supplies one unit of labor at a wage rate w. In case the parent needs long-term care, the child's time spend on the labor market is reduced by the time spend for informal care provision a and gross earnings amount to $w(1-a)$. Care provided by the child reduces the monetary loss from LTC by $h(a) \leq L$ (with $h' > 0, h'' < 0$) since then the parent requires less professional care services.

In sum, the parent's and child's expected utilities are given by

$$\begin{aligned} EU_p =& \pi \left[H(y + h(a) - L - b) + u(w(1-a) + b) \right] \\ &+ (1-\pi) \left[u(y - \widehat{b}) + u(w + \widehat{b}) \right], \end{aligned} \tag{1}$$

$$\begin{aligned} EU_c =& \pi \left[u(w(1-a) + b) + \beta H(y + h(a) - L - b) \right] \\ &+ (1-\pi) \left[u(w + \widehat{b}) + \beta u(y - \widehat{b}) \right], \end{aligned} \tag{2}$$

where $\beta \in [0, 1]$ reflects the child's degree of altruism and a $\widehat{\ }$ indicates the state of staying healthy. The utility function u is the standard one whereas, H denotes the utility for long-term care. These functions satisfy $u', H' > 0$ and $u'', H'' < 0$. Furthermore, we assume $H(x) < u(x)$ for any x, due to the loss of autonomy.

The timing of the model is as follows: the government moves first and announces its policy (stage 0). Then, the parent and the child play the following two-stage game. In stage 1, the parent chooses (for each state of nature) a level of bequest. In stage 2, after the state of nature is revealed, children decide how much informal care to provide if their parent is dependent. Otherwise, children simply consume their income plus the bequest.

To determine the subgame perfect Nash equilibrium, we solve this game by backward induction. We first study the first-best allocation which provides a benchmark against which we can compare the *laissez-faire* allocation, obtained by dropping stage 0.

2.2 First-best solution

With *ex ante* identical families, we can define the optimal allocation as the one maximizing the expected utility of a representative dynasty. Assuming that both parents and children receive equal social weights, the first-best problem can be written as

$$\max_{m,c,\widehat{m},\widehat{c},a} \quad \mathcal{W} = \pi \left[H(m) + u(c) \right] + (1-\pi) \left[u(\widehat{m}) + u(\widehat{c}) \right],$$
$$\text{s.t.} \quad (1-\pi) \left[\widehat{m} + \widehat{c} - L \right] + \pi \left[m + c \right] = y + (1-\pi) w + \pi \left[w(1-a) + h(a) \right], \qquad (3)$$

where the decision variables are informal care provision a and parent's and child's consumption in both states of nature. We denote the latter by m, $\widehat{m}$, c and $\widehat{c}$ respectively. In the first-best all variables are directly set, assuming full information and disregarding the multi-stage structure of the game. The specification of the game will of course be relevant below, when we study the decentralization of the first-best optimum. Let μ denote the Lagrange multiplier associated with the resource constraint (3). The first order conditions (FOCs) characterizing the optimal solution can be written as follows

$$H'(m) = u'(\widehat{m}) = u'(c) = u'(\widehat{c}) = \mu, \tag{4}$$

$$w = h'(a). \tag{5}$$

Equation (4) states the equality of marginal utilities of incomes across generations and states of nature (full insurance), while equation (5) describes the efficient choice of informal care. It states that the opportunity cost of informal care w equals its marginal benefit.

2.3 Laissez-faire allocation

2.3.1 Stage 2: Choice of children

In case of autonomy, children do not have to make any decision. They consume their income and bequest, enjoying a utility $u(w+\widehat{b})$. When their parents are in need of LTC, they solve the following problem

$$\max_{a} \ u(w(1-a)+b) + \beta H(y - L + h(a) - b).$$

The first-order condition of the above problem is given by

$$-wu'(w(1-a)+b) + \beta H'(y - L + h(a) - b)h'(a) = 0.$$

This equation defines the optimal level of a as a function of b: $a^* = a(b)$. We have

$$\frac{\partial a^*}{\partial b} = \frac{wu''(c) + \beta H''(m)h'(a)}{SOC^a} > 0, \tag{6}$$

as the second-order condition (SOC) is negative

$$SOC^a = w^2 u''(c) + \beta H'(m)h''(a) + \beta H''(m)(h'(a))^2 < 0.$$

In words, if parents increase their bequest, the willingness to provide informal care by the child also increases.

2.3.2 Stage 1: Choice of parents

The problem of the parents is to choose (in both states of nature) the level of bequests to their children. In case of dependency parents have more needs but at the same time they might leave a higher bequest to induce more assistance from their children. Their problem is the following

$$\begin{aligned} \max_{b,\widehat{b}} \; EU_p =& \pi \left[H(y - L + h(a^*) - b) + u(w(1-a^*) + b) \right] \\ &+ (1-\pi) \left[u(y - \widehat{b}) + u(w + \widehat{b}) \right]. \end{aligned} \tag{7}$$

Taking into consideration equation (6) the FOCs with respect to b and $\widehat{b}$ can be written as

$$-H'(m) \left[1 - h'(a^*) \frac{\partial a^*}{\partial b} (1-\beta) \right] + u'(c) = 0, \tag{8}$$

$$-u'(\widehat{m}) + u'(\widehat{c}) = 0. \tag{9}$$

It can be easily verified that for $\beta = 1$ the parent leaves the efficient amount of bequests so that $H'(m) = u'(c)$. This in turn goes hand in hand with an efficient amount of aid provided by the child $h'(a) = w$; see equation (5). This result is not surprising as for perfect altruism, on both the parent's and the child's side, the optimization problem of the two family members coincides with the one of the social planner. Whenever $\beta < 1$ this is no longer the case as the child puts a too low value on the parent's benefits of informal care. In this case the parent leaves a large than otherwise efficient bequest to induce the child to provide more care.

2.4 Decentralization of the first-best allocation

Assume for the time being that there is no asymmetry of information so that all relevant variables including informal aid are publicly observable. In the following we show that the FB allocation within our multi-stage setting can be decentralized by a lump-sum transfer (social LTC insurance) from the healthy to the dependent elderly $(\widehat{D}, D)$ supplemented by a tax on labor income τ_a and on bequests τ_b for those children whose parents are dependent. Public transfers must be chosen such that the government's budget constraint

$$\pi D = (1-\pi)\widehat{D} + \pi\left[\tau_a w(1-a^*) + \tau_b b^*\right] \tag{10}$$

is balanced.

2.4.1 Family's problem reconsidered

Again, in case of autonomy children do not have to make any decision; they consume their income and bequest enjoying a utility $u(w+\widehat{b})$. When their parents are in need of long-term care, they now solve the following problem

$$\max_a \; u((1-\tau_a)w(1-a) + (1-\tau_b)b) + \beta H(y + D - L + h(a) - b). \tag{11}$$

The first order condition of the above problem is given by

$$-(1-\tau_a)wu'(c) + \beta H'(m)h'(a) = 0. \tag{12}$$

The above equation yields $a^* = a(b, \tau_a, \tau_b, D)$. The comparative static effects wrt. b and the policy instruments are[6]

$$\frac{\partial a^*}{\partial b} = \frac{(1-\tau_a)wu''(c)(1-\tau_b) + \beta H'(m)h'(a)}{SOC^a} > 0, \tag{13}$$

$$\frac{\partial a^*}{\partial \tau_a} = \frac{-wu'(c) - (1-\tau_a)w^2(1-a)u''(c)}{SOC^a} \lesseqgtr 0, \tag{14}$$

$$\frac{\partial a^*}{\partial \tau_b} = \frac{-(1-\tau_b)wbu''(c)}{SOC^a} < 0, \tag{15}$$

$$\frac{\partial a^*}{\partial D} = \frac{-\beta H'(m)h'(a)}{SOC^a} < 0. \tag{16}$$

Care increases with bequests while a tax on bequest dampens it. An income tax either pushes or dampens informal care provision and public LTC decreases informal care.

[6] The second order condition is $SOC^a = ((1-\tau_a)w)^2 u''(c) + \beta H'(m)h''(a) + \beta H''(m)(h'^2 < 0$.

Comparison of equation (12) with the first-best allocation (equations (4) and (5)) shows that to implement the first-best level of care we first need a tax on child's income, i.e. $\tau_a = 1 - \beta$. A tax on income implicitly subsidizes the child's informal care provision so that the trade-off between the child's marginal costs and marginal benefit of care is the efficient one.

The problem for the parents is now the following

$$\begin{aligned} \max_{b,\widehat{b}} \; U_p = & \pi \left[H(y + D - L + h(a^*) - b) + u((1-\tau_a)w(1-a^*) + (1-\tau_b)b) \right] \\ & + (1-\pi)\left[u(y - \widehat{D} - \widehat{b}) + u(w + \widehat{b}) \right]. \end{aligned} \tag{17}$$

Using equation (12), the FOCs with respect to b, and $\widehat{b}$ are

$$-H'(m)\left[1 - h'(a^*)\frac{\partial a^*}{\partial b}(1-\beta)\right] + (1-\tau_b)u'(c) = 0, \tag{18}$$

$$-u'(\widehat{m}) + u'(\widehat{c}) = 0. \tag{19}$$

Equations (18) and (19) yield $b^* = b(D, \widehat{D}, \tau_a, \tau_b)$ and $\widehat{b}^* = \widehat{b}(D, \widehat{D}, \tau_a, \tau_b)$. When the transfers $(D, \widehat{D})$ are chosen such that the parent (and thus also the child) is fully insured implying $H'(m) = u'(\widehat{m})$ we have $H'(m) = u'(\widehat{c})$. The tax on bequests must then be chosen such that its effect on informal care is offset, *i.e.*

$$\tau_b = h'(a^*)\frac{\partial a^*}{\partial b}(1-\beta) > 0.$$

That is, we have to tax bequests when the parent ends up being dependent. This tax offsets the parent's incentives to induce informal care above its first-best level.[7]

To decentralize the first-best optimum we need three types of tools: a distortionary tax τ_a that fosters child's assistance in case of impure altruism ($\beta < 1$), a tax τ_b on bequests when the parent is dependent and lump-sum transfers acting as redistributive device between the two states of nature $(D, \widehat{D})$.

[7] The expression for $\partial a^*/\partial b$ is not independent of τ_b.

Note that we have assumed away private insurance. If private insurance were available, its role would depend on the existence and the size of loading costs. With zero loading costs, a first best could be achieved simply by taxing child's labor at the appropriate rate, namely $1-\beta$, and by taxing bequests when the parent is dependent. With positive loading cost, we also need the lump-sum transfers, which then completely crowd out private insurance.

3 Second-best solution

3.1 Identical children

We now turn to the second-best. We assume that the government can use only linear (proportional) taxes on bequests τ and on labor earnings t, to finance a LTC benefit g. Tax rates cannot be conditioned on the parents' health status. We use the notation t, τ and g rather then τ_a, τ_b and D to avoid confusion with the first-best implementation.

With our instruments we write the revenue constraint as

$$\pi g = t(1-\pi a^*)w + \tau\left[\pi b^* + (1-\pi)\widehat{b}^*\right].$$

3.1.1 The child's problem

The child solves

$$\max_a \; u\left((1-t)w(1-a) + (1-\tau)b\right) + \beta H\left(y - L - b + g - h(a)\right), \tag{20}$$

which apart from the change in notation is equivalent, to (11) so that the FOC continues to be given by (12) and can be written as

$$(1-t)wu'(c) = \beta H'(m)h'(a^*). \tag{21}$$

The solution is given by $a^* = a(b,t,\tau,g)$. The comparative statics properties of this function follow those described by equations (13)–(16). Using subscripts to denote partial derivatives, we thus have $a_b^* > 0, a_\tau^* < 0, a_t^* \gtrless 0, a_g^* < 0$.

3.1.2 Parent's problem

Turning to the parent's problem, it is given by

$$\max_{b,\widehat{b}} \; EU_p = \pi \left[H\left(y - L - b + g + h(a^*)\right) + u\left((1-t)w(1-a^*) + (1-\tau)b\right)\right] \\ + (1-\pi)\left[u(y-\widehat{b}) + u\left((1-t)w + (1-\tau)\widehat{b}\right)\right]. \tag{22}$$

The two FOCs wrt. $\widehat{b}$ and b are

$$u'(\widehat{m}) = u'\left(\widehat{c}\right)(1-\tau), \tag{23}$$

$$H'(m)\left[1 - h'(a^*)a_b^*\right] = u'(c)[(1-\tau) - (1-t)wa_b^*]. \tag{24}$$

Using (21), equation (24) can be rewritten as

$$H'(m)\left[1 - h'(a^*)(1-\beta)a_b^*\right] = u'(c)(1-\tau).$$

These conditions define the bequests supply functions, *i.e.* $b^* = b(t,\tau,g)$ and $\widehat{b}^* = b(t,\tau)$. We implicitly assume positive bequests in either state of nature. Substituting for b into a^* yields the level of aid as a function of the government's instruments $a^* = a\left(b(t,\tau,g),t,\tau,g\right) \equiv \tilde{a}(t,\tau,g)$.

3.1.3 The government's problem

The second-best problem can now be expressed by the following Lagrangean

$$\mathcal{L}(g,t,\tau) = \pi\left[H\left(y - L - b^* + g + h(a^*)\right) + u\left((1-t)w(1-a^*) + (1-\tau)b^*\right)\right] \\ + (1-\pi)\left[u\left(y - \widehat{b}^*\right) + u\left((1-t)w + (1-\tau)\widehat{b}^*\right)\right] \\ - \mu\left[\pi g - t(1-\pi a^*)w - \tau(\pi b^* + (1-\pi)\widehat{b}^*)\right] \tag{25}$$

Using the envelope theorem, we can write the FOCs wrt. g, t and τ as

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial g} =&\pi\left[H'(m)h'(a^*) - u'(c)(1-t)w\right]\tilde{a}_g + \pi H'(m) - \mu\left[\pi + \pi t w \tilde{a}_g - \tau\pi b_g\right] = 0,\\
\frac{\partial \mathcal{L}}{\partial t} =&\pi\left[H'(m)h'(a^*) - u'(c)(1-t)w\right]\tilde{a}_t - \left[\pi u'(c)\left(1-a\right) + (1-\pi)u'(\widehat{c})\right]w\\
&+\mu\left[w(1-\pi a^*) - \pi t w\tilde{a}_t + \tau\left[\pi b_t + (1-\pi)\widehat{b}_t\right]\right] = 0,\\
\frac{\partial \mathcal{L}}{\partial \tau} =&\pi\left[H'(m)h'(a^*) - u'(c)(1-t)w\right]\tilde{a}_\tau - \left[\pi u'(c)b^* + (1-\pi)u'(\widehat{c})\widehat{b}^*\right]\\
&+\mu\left[\pi b^* + (1-\pi)\widehat{b}^* - \pi t w\tilde{a}_\tau + \tau\left[\pi b_\tau + (1-\pi)\widehat{b}_\tau\right]\right] = 0.
\end{aligned}$$

These FOCs can be rearranged using the compensated form with

$$\frac{\mathrm{d}g}{\mathrm{d}\tau} = \frac{\pi b^* + (1-\pi)\widehat{b}^*}{\pi}, \quad \text{and} \quad \frac{\mathrm{d}g}{\mathrm{d}t} = \frac{(1-\pi a^*)w}{\pi}.$$

In other words, we keep the tax rates t and τ as sole decision variables, while accounting of course for their impact on g, via the government's budget constraint. This yields[8]

$$\begin{aligned}
\frac{\partial \mathcal{L}^c}{\partial \tau} =& \frac{\partial \mathcal{L}}{\partial \tau} + \frac{\partial \mathcal{L}}{\partial g}\frac{\mathrm{d}g}{\mathrm{d}\tau} = \pi(1-\beta)H'(m)h'(a^*)\tilde{a}^c_\tau - \left[\pi u'(c)b^* + (1-\pi)u'(\widehat{c})\widehat{b}^*\right]\\
&+ H'(m)(\pi b^* + (1-\pi)\widehat{b}^*) - \mu\left[t w\tilde{a}^c_\tau - \tau\left(\pi b^c_\tau + (1-\pi)\widehat{b}^c_\tau\right)\right] = 0, \qquad (26)\\
\frac{\partial \mathcal{L}^c}{\partial t} =& \frac{\partial \mathcal{L}}{\partial t} + \frac{\partial \mathcal{L}}{\partial g}\frac{\mathrm{d}g}{\mathrm{d}t} = \pi(1-\beta)H'(m)h'(a^*)\tilde{a}^c_t - \left[\pi u'(c)\left(1-a^*\right) + (1-\pi)u'(\widehat{c})\right]w\\
&+ H'(m)(1-\pi a^*)w - \mu\left[t w\tilde{a}^c_t - \tau\left(\pi b^c_t + (1-\pi)\widehat{b}^c_t\right)\right] = 0. \qquad (27)
\end{aligned}$$

To interpret these formulas, we make the standard assumption that the cross-derivatives are negligible. Formally, $b^c_t = \widehat{b}^c_t = \tilde{a}^c_\tau = 0$. Then, we have

$$\tau = \frac{-\pi\Delta b^* - (1-\pi)\widehat{\Delta}\widehat{b}^*}{-\mu\left[\pi b^c_\tau + (1-\pi)\widehat{b}^c_\tau\right]}, \qquad (28)$$

$$t = \frac{\pi(1-\beta)H'(m)h'(a^*)\tilde{a}^c_t - \pi\Delta y - (1-\pi)\widehat{\Delta}w}{\mu w\tilde{a}^c_t}, \qquad (29)$$

[8] We define $\tilde{a}^c_x = \frac{\partial \tilde{a}}{\partial x} + \frac{\partial \tilde{a}}{\partial g}\frac{\mathrm{d}g}{\mathrm{d}x}$, $b^c_x = \frac{\partial b^*}{\partial x} + \frac{\partial b^*}{\partial g}\frac{\mathrm{d}g}{\mathrm{d}x}$ and $\widehat{b}^c_x = \frac{\partial \widehat{b}^*}{\partial x} + \frac{\partial \widehat{b}^*}{\partial g}\frac{\mathrm{d}g}{\mathrm{d}x}$, where $x = \tau, t$.

where $\Delta \equiv u'(c) - H'(m) \gtrless 0$ and $\widehat{\Delta} \equiv u'(\widehat{c}) - H'(m) < 0$.[9] The denominators in these two formulas are standard. They reflect the efficiency effect of taxes. Normally one expects $\tilde{a}_t^c > 0$ and

$$\pi b_\tau^c + (1-\pi)\widehat{b}_\tau^c < 0.$$

To interpret the numerator of equation (28), we have to return to our assumption of positive bequests and on the inequalities $\Delta \equiv u'(c) - H'(m) \gtrless 0$ and $\widehat{\Delta} \equiv u'(\widehat{c}) - H'(m) < 0$. As shown above in the first-best, $\Delta = \widehat{\Delta} = 0$. The numerator will thus be positive if π is not too high and if $\left|\widehat{\Delta}\right| > \Delta > 0$, which is likely if L is large enough. Turning to the numerator of equation (29), note that the first term in the nominator vanishes if $\beta = 1$. Otherwise, it represents the need to foster child's assistance. This term mirrors the first-best implementation but has a different structure in the second-best. The second and third terms are the same as in the previous condition. They reflect the insurance benefits provided by the tax. They give the welfare gap between the young and the dependent elderly in the two states of nature. If the marginal utility of the dependent is high relative to that of the young, then there is a need for a higher tax to finance the social benefit g.

Private insurance would not change our qualitative results. Even with zero loading cost, the first-best cannot be achieved with the instruments considered here; it would require a state-contingent inheritance tax. With a positive loading cost individuals buy less than full insurance and the two taxes continue to make up for part of the incompleteness of private insurance protection.

3.2 Heterogenous families

We now assume that families differ *ex ante* in the children's productivity levels and in the parents' resources. We assume that both (w_i and y_i) are positively correlated so that our assumption of positive bequests is valid across families. Again, the government

[9] This means that $\widehat{m} > \widehat{c} > c$.

levies a flat tax t on earnings and τ on bequests to finance a uniform LTC benefit g. Within such a framework the social insurance scheme pursues an additional objective, namely redistribution.

The government's revenue constraint has to be modified such that

$$\pi g = \sum_i n_i \left[t(1-\pi a_i^*)w_i + \tau(\pi b_i^* + (1-\pi)\widehat{b}_i^*) \right],$$

where n_i is the relative number of families with productivity w_i and y_i. The optimizing behavior of children and parents remains the same as described above; each child and parent of type-i solves the problem given in (20) and (22). From this we obtain optimal long-term care provision $a_i^* = \tilde{a}(t, \tau, g)$ with $\tilde{a}_{i\tau} < 0, \tilde{a}_{it} \gtreqless 0$ and $\tilde{a}_{ig} < 0$ and optimal bequests $b_i^* = b(t, \tau, g)$ and $\widehat{b}_i^* = b(t, \tau)$.

3.3 Government's problem

The second-best problem of a utilitarian government with heterogenous families can be expressed by the following Lagrangean expression

$$\begin{aligned}\mathcal{L}(g,t,\tau) = & \sum_i n_i \Big\{ \pi H\left(y_i - L - b_i^* + g + h(a_i^*)\right) + u\left((1-t)w_i(1-a_i^*) + (1-\tau)b_i^*\right) \\ & + (1-\pi)\left[u\left(y_i - \widehat{b}_i^*\right) + u\left((1-t)w_i + (1-\tau)\widehat{b}_i^*\right)\right] \\ & - \mu\left[\pi g - t(1-\pi a_i^*)w_i - \tau\left(\pi b_i^* + (1-\pi)\widehat{b}_i^*\right)\right]\Big\}. \end{aligned} \quad (30)$$

Maximizing this expression with respect to g, t and τ yields the following FOCs

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial g} =& E\Big\{[\pi\left[H'(m)h'(a^*) - u'(c)(1-t)w\right]\tilde{a}_g + \pi H'(m) \\ & - \mu\left[\pi + \pi t w \tilde{a}_g - \tau\pi b_g\right]\Big\} = 0, \\ \frac{\partial \mathcal{L}}{\partial t} =& E\Big\{\pi\left[H'(m)h'(a^*) - u'(c)(1-t)w\right]\tilde{a}_t - \left[\pi u'(c)\left(1-a^*\right) + (1-\pi)u'(\widehat{c})\right]w \\ & + \mu\left[w(1-\pi a^*) - \pi t w \tilde{a}_{\tau_a} + \tau\left(\pi b_t + (1-\pi)\widehat{b}_t\right)\right]\Big\} = 0, \\ \frac{\partial \mathcal{L}}{\partial \tau} =& E\Big\{\pi\left[H'(m)h'(a^*) - u'(c)(1-t)w\right]\tilde{a}_\tau - \left[\pi u'(c)b^* + (1-\pi)u'(\widehat{c})\widehat{b}^*\right] \\ & + \mu\left[\pi b^* + (1-\pi)\widehat{b}^* - \pi t w \tilde{a}_\tau + \tau\left(\pi b_\tau + (1-\pi)\widehat{b}_\tau\right)\right]\Big\} = 0,\end{aligned}$$

where we use the operator E for $\sum_i n_i$. As for identical families we can rearrange the above FOCs in terms of compensations with $\mathrm{d}g/\mathrm{d}\tau$ and $\mathrm{d}g/\mathrm{d}t$; see Appendix (A.2). By assuming that the cross derivatives are nil, *i.e.* $b^c_{it} = \widehat{b}^c_{it} = \tilde{a}^c_{i\tau} = 0$, we have

$$\begin{aligned}\tau =& \frac{-\pi\,\mathrm{cov}(\Delta, b^*) - (1-\pi)\,\mathrm{cov}(\widehat{\Delta}, \widehat{b}^*)}{-\mu E\left[\pi b^c_\tau + (1-\pi)\widehat{b}^c_\tau\right]} \\ & + \frac{-\pi E\Delta E b^* - (1-\pi)E\widehat{\Delta}E\widehat{b}^*}{-\mu E\left[\pi b^c_\tau + (1-\pi)\widehat{b}^c_\tau\right]},\end{aligned} \tag{31}$$

and

$$\begin{aligned}t =& \frac{\pi(1-\beta)E\left[H'(m)h'(a^*)\tilde{a}^c_t\right] - \pi\,\mathrm{cov}(\Delta, w(1-a^*)) - (1-\pi)\,\mathrm{cov}(\widehat{\Delta}, w)}{\mu E w \tilde{a}^c_t} \\ & + \frac{-\pi E\Delta E w(1-a^*) - (1-\pi)E\widehat{\Delta}Ew}{\mu E w \tilde{a}^c_t}\end{aligned} \tag{32}$$

The interpretation of these two formulas is the same as in the case of identical individuals with one exception, namely the covariance terms that are expected to be negative. This is because with higher income/wealth the difference in marginal utilities between states of nature decreases. In other words, earnings inequality pushes for more taxation of either earnings or bequests.

4 Conclusion

The starting point of this paper is the concomitance of two trends: the increasing needs for LTC, and unusual growth of inheritance. In a *laissez faire* economy with neither public nor private insurance, dependency would bite a big chunk of accumulated wealth with the consequence that bequests would greatly differ for kids with dependent parents and kids with healthy parents. Children's care can only partially mitigate those differences. Given that the market for LTC insurance is typically thin or even absent, one has to rely on public action to restore some balance between the two states of nature.

In a first-best world, a subsidy on children's care, state-dependent bequest taxes and a state-contingent lump sum tax could achieve the optimum and provide full insurance. If for obvious reasons individualized lump sum taxes are not available, one has to rely on second-best schemes. In this paper a wage tax and a linear inheritance tax contribute to finance a uniform LTC benefit.

The main conclusion is that under plausible conditions linear taxes on both inheritances and wages to finance a public LTC flat rate benefit are desirable. The tax on children's labor income effectively subsidizes informal care, like in the first-best implementation. It is required when altruism is less than full. Both taxes also provide insurance and, in the case of heterogenous families (differing in children's productivity and parent's wealth) redistributive benefits. We have seen that we can end up with a subsidy of bequests or even earnings in the special case where the welfare of the dependent would be quite high relative to that of his children and the probability of dependency would be particularly high.

Throughout this paper we have assumed that parents in both states of nature were leaving some bequests. This implies that parents are relatively wealthier than their children, and that dependency does not represent a large financial loss. In an extension of this work we would like to explore cases where some parents do not leave any bequest or they only bequeath in the healthy state. It is expected that if bequests in the healthy state were much higher than those in the state of dependency and if the wealthy families were leaving high bequests whereas the poorer families were bequest-constrained, estate taxation would play an even more important role. Among other extensions, we would like to introduce private insurance with loading costs and study the possibility of non-linear LTC policies.

A Appendix

A.1 Identical families: second-best

Using a shorter notation, we can write

$$-\left[\pi u'(c)b^* + (1-\pi)u'(\widehat{c})\widehat{b}^*\right] + H'(m)(\pi b^* + (1-\pi)\widehat{b}^*) = -\pi\Delta b^* - (1-\pi)\,\widehat{\Delta}\widehat{b}^*$$

$$-\left[\pi u'(c)\,(1-a^*) + (1-\pi)u'(\widehat{c})\right]w + H'(m)(1-\pi a^*)w = -\pi\Delta(1-a^*)w - (1-\pi)\,\widehat{\Delta}w$$

We now rewrite equations (26) and (27) as

$$\pi(1-\beta)H'(m)h'(a^*)\tilde{a}^c_\tau - \pi\Delta b^* - (1-\pi)\,\widehat{\Delta}\widehat{b}^* = \mu\left[tw\tilde{a}^c_\tau - \tau\left(\pi b^c_\tau + (1-\pi)\widehat{b}^c_\tau\right)\right],$$

$$\pi(1-\beta)H'(m)h'(a^*)\tilde{a}^c_t - \pi\Delta(1-a^*)w - (1-\pi)\,\widehat{\Delta}w = \mu\left[tw\tilde{a}^c_t - \tau\left(\pi b^c_t + (1-\pi)\widehat{b}^c_t\right)\right].$$

A.2 Heterogenous families: second-best

$$\begin{aligned}\frac{\partial\mathcal{L}^c}{\partial\tau} =& E\left\{\pi(1-\beta)H'(m)h'(a^*)\tilde{a}^c_\tau - \left[\pi u'(c)b^* + (1-\pi)u'(\widehat{c})\widehat{b}^*\right]\right.\\ &\left. + H'(m)(\pi b^* + (1-\pi)\widehat{b}^*) - \mu\left[tw\tilde{a}^c_\tau - \tau\left[\pi b^c_\tau + (1-\pi)\widehat{b}^c_\tau\right]\right]\right\} = 0, \qquad (33)\\ \frac{\partial\mathcal{L}^c}{\partial t} =& E\left\{\pi(1-\beta)H'(m)h'(a^*)\tilde{a}^c_t - \left[\pi u'(c)\,(1-a^*) + (1-\pi)u'(\widehat{c})\right]w\right.\\ &\left. + H'(m)(1-\pi a^*)w - \mu\left[tw\tilde{a}^c_t - \tau\left[\pi b^c_t + (1-\pi)\widehat{b}^c_t\right]\right]\right\} = 0. \qquad (34)\end{aligned}$$

We use $\Delta \equiv u'(c) - H'(m) \gtrless 0$ and $\widehat{\Delta} \equiv u'(\widehat{c}) - H'(m) < 0$, to represent the gap between the child's marginal utility of consumption and that of the dependent parent.

We now rewrite the FOC's as

$$\pi(1-\beta)E\left[H'(m)h'(a^*)\tilde{a}_t^c\right] - \pi\,\mathrm{cov}(\Delta, w(1-a^*)) - (1-\pi)\,\mathrm{cov}(\widehat{\Delta}, w)$$

$$-\,\pi E\Delta E w(1-a^*) - (1-\pi)E\widehat{\Delta}Ew = \mu E\left[tw\tilde{a}_t^c - \tau\left[\pi b_t^c + (1-\pi)\widehat{b}_t^c\right]\right], \qquad (35)$$

$$\pi(1-\beta)E\left[H'(m)h'(a^*)\tilde{a}_\tau^c\right] - \pi\,\mathrm{cov}(\Delta, b^*) - (1-\pi)\,\mathrm{cov}(\widehat{\Delta}, \widehat{b}^*)$$

$$-\,\pi E\Delta E b^* - (1-\pi)E\widehat{\Delta}E\widehat{b}^* = \mu E\left[tw\tilde{a}_\tau^c - \tau\left[\pi b_\tau^c + (1-\pi)\widehat{b}_\tau^c\right]\right]. \qquad (36)$$

Solving for t and τ yields the expressions in the text.

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