“Investment Price Rigidities and Business Cycles”

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INVESTMENT PRICE RIGIDITIES
AND BUSINESS CYCLES

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Abstract. I incorporate investment price rigidity in a two-sector monetary model of business cycles. Fit to quarterly U.S. time series, the model suggests that price sluggishness in the investment sector is the single most empirically relevant friction to match the data. Sticky investment prices constitutes an important propagation mechanism to understand the sources of aggregate fluctuations, the dynamic effects of technology shocks, and the properties of the relative price of investment goods.

JEL Codes: E3, E5.

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Price stickiness matters for macroeconomic outcomes. This form of nominal rigidity underlies the ubiquitous New Keynesian model of monetary policy (Woodford, 2003) and constitutes one of the building blocks of the growing literature on quantitative dynamic stochastic general equilibrium (DSGE) models (Christiano, Eichenbaum, and Evans, 2005; Smets and Wouters, 2007). It has proven important to understand the general equilibrium effects of shocks to monetary or fiscal policy, as well as to technology. Eventually, it is supported by microeconomic evidence on the behavior of individual prices, which suggests that aggregate prices can be sticky even though micro-level prices change frequently (Kehoe and Midrigan, 2010).

Guided by the widespread use of one-sector models, the literature has mostly focused on price rigidity in the consumption sector. Even benchmark two-sector DSGE models, for instance Justiniano, Primiceri, and Tambalotti (2010, 2011), feature sticky consumption prices but flexible investment prices. While convenient for aggregation, ruling out nominal frictions in the investment sector imposes strong limitations on the model’s internal mechanisms. For instance, Basu, Fernald, and Liu (2013) demonstrate that the propagation of technology shocks is highly sensitive to the presence of investment price stickiness and Barsky, House, and Kimball (2007) show that this is also true of monetary policy shocks. Additionally, there is ample empirical evidence that investment prices are indeed sluggish. Bils and Klenow (2004) report that the monthly frequency of price changes for durable goods, typically classified as investment in DSGE models, is virtually the same as that for nondurable goods, close to 30 percent. Moreover, Basu, Fernald, Fisher, and Kimball (2011) find that the pass-through of technology shocks to prices takes several years in the investment sector, again suggestive of strong rigidities. Eventually, price sluggishness is a well-known characteristic of the housing market (Case and Shiller, 1989; Iacoviello, 2010).

In this context, my contribution in this paper is threefold. First, I use standard Bayesian methods to confirm the empirical relevance of investment price rigidity within a monetary DSGE model.\footnote{To my knowledge, Bouakez, Cardia, and Ruge-Murcia (2014) is the only alternative paper formally estimating a multisector DSGE model with sector-specific pricing frictions. However, their perspective is fundamentally different from mine as they use a much more disaggregated structure and base their estimation on price microdata. This is not comparable to the DSGE literature I address in this paper.} I consider a two-sector economy, where the sectors produce respectively consumption and investment goods. Building on the RBC literature, the model includes real reallocation frictions in production factors through imperfect substitution of hours worked and capital services across sectors. Following Barsky, House, and Kimball (2007) and Basu, Fernald, and Liu (2013), it also incorporates sector-specific
nominal rigidities, with different frequencies of price and wage adjustments across sectors. Finally, on top of the usual economy-wide shocks to preferences or monetary policy, the model includes a rich array of sectoral disturbances affecting technology, price and wage markups, and government purchases.

I estimate the model using quarterly U.S. time series. To sharpen identification, I include both aggregate and sectoral variables among observables. The estimated model captures the salient features of the data and, in particular, it correctly reproduces aggregate and sectoral macro comovements. Both real reallocation frictions and sector-specific nominal rigidities are needed to obtain a good fit, but the latter are significantly more important. Remarkably, price stickiness in the investment sector constitutes the single most important friction to fit the data, even though it is typically ignored by the DSGE literature.

Second, I analyze the role of investment price rigidity in business-cycle dynamics. Regarding the sources of business cycles, the model confirms findings from earlier research, for instance Justiniano, Primiceri, and Tambalotti (2011): shocks to the marginal efficiency of investment (MEI) are the most important drivers of U.S. economic fluctuations. These disturbances affect the transformation of investment goods into installed capital and leave the productivity of investment-producing firms unchanged, thus constituting pure investment demand shifters. My results show that their predominant role is robust to the introduction of pricing frictions in the investment sector.

On the other hand, investment price stickiness constitutes a key mechanism to understand the dynamic effects of technology shocks. The model implies that technology improvements are expansionary in the consumption sector and instead contractionary in the investment sector. These patterns, consistent with Basu, Fernald, Fisher, and Kimball’s (2011) growth-accounting results, have not been previously documented within the empirical DSGE literature. The underlying economic intuition, developed in Barsky, House, and Kimball (2007) and Basu, Fernald, and Liu (2013), is straightforward. With sluggish prices, an improvement in investment technology makes current investment expensive relative to the future since firms adjust only gradually their prices. Investment demand being highly elastic, current demand falls and triggers a generalized recession. Symmetrically, an improvement in consumption technology makes current investment relatively cheaper and generates an expansion.

Third, I examine the link between relative technology shocks and the relative price of investment goods. While much of the DSGE literature imposes flexible investment prices, extended nominal rigidities break the usual identity between relative technology and the relative price. Notably, only one-fifth of the cyclical variance of the relative price of investment is due to technology shocks in the estimated model, while the contribution of
markup shocks exceeds 50 percent. This result calls into question the validity of the usual empirical approach imposing a period-by-period equality between relative technology in the investment and consumption sectors and the relative price of investment.

The paper is organized as follows: Section 2 sets up the DSGE model, while Section 3 describes the estimation procedure and the data. Section 4 reports estimation results, including a discussion of the model fit. Section 5 examines the implications of investment price stickiness for the sources of business cycles, the effects of technology and monetary shocks, and the properties of the relative price of investment. Eventually, Section 6 concludes.

2. A Two-Sector DSGE Model

The model builds on Basu, Fernald, and Liu (2013), who extend the medium-scale sticky-price economies from Smets and Wouters (2007) and Justiniano, Primiceri, and Tambalotti (2010, 2011) to an explicit two-sector structure. I add to their framework frictions affecting the sectoral allocation of production factors. The economy is populated by seven classes of agents: a final retail sector producing homogeneous consumption and investment goods, two intermediate sectors specializing in producing inputs for the consumption and investment retailers, households, competitive labor packers, monopolistic labor unions, a central bank, and a government. Their decisions are described in turn.

2.1. Final retail sector. There are two competitive retailers, one for each sector. They purchase a continuum of differentiated sector-specific intermediate inputs and produce the final consumption and investment goods in quantities $Y^c_t$ and $Y^i_t$ according to

$$Y^c_t = \left( \int_0^1 Y^c_t(j) \frac{1}{1+\eta^c_t} dj \right)^{1+\eta^c_t}, \quad Y^i_t = \left( \int_0^1 Y^i_t(j) \frac{1}{1+\eta^i_t} dj \right)^{1+\eta^i_t}.$$

The elasticities $\eta^c_t$ and $\eta^i_t$ correspond to sector-specific price markup shocks and evolve according to

$$\ln(1 + \eta^c_t) = (1 - \rho_{\eta^c}) \ln(1 + \eta^c - \theta) + \rho_{\eta^c} \ln(1 + \eta^c_{t-1}) + \epsilon^c_{t-1},$$

$$\ln(1 + \eta^i_t) = (1 - \rho_{\eta^i}) \ln(1 + \eta^i) + \rho_{\eta^i} \ln(1 + \eta^i_{t-1}) + \epsilon^i_{t-1},$$

with $\epsilon^c_{t-1} \sim iidN(0, \sigma^2_{\eta^c})$ and $\epsilon^i_{t-1} \sim iidN(0, \sigma^2_{\eta^i})$. Standard manipulations yield the expressions of the aggregate consumption and investment prices:

$$P^c_t = \left( \int_0^1 P^c_t(j) \frac{1}{1+\eta^c_t} dj \right)^{-\eta^c_t}, \quad P^i_t = \left( \int_0^1 P^i_t(j) \frac{1}{1+\eta^i_t} dj \right)^{-\eta^i_t}. $$
2.2. Intermediate sector. Monopolistically competitive firms produce intermediate consumption and investment inputs using capital and labor services, according to

\[ Y^c_t(j) = K^c_t(j)^{\alpha_c} [\Phi^c_t L^c_t(j)]^{1-\alpha_c} - \Omega^c_t \Phi^c_t, \quad Y^i_t(j) = K^i_t(j)^{\alpha_i} [\Phi^i_t L^i_t(j)]^{1-\alpha_i} - \Omega^i_t \Phi^i_t. \]

Here, \( K^c_t(j) \) and \( L^c_t(j) \) denote the amounts of capital and labor services employed by firm \( j \) in sector \( x \), while \( \alpha_x \) and \( \Omega^x_t \Phi^x_t \) measure the capital share and the fixed production cost. Factor shares may differ across sectors. \( \Omega^x_t \) is a sector-specific stochastic trend included to ensure proper scaling of the fixed cost along the balanced growth path of the model. \( \Gamma^c_t \) and \( \Gamma^i_t \) are two-sector-specific stochastic productivity trends that evolve according to

\[
\begin{align*}
\ln \mu^c_t &= (1 - \rho_{mc}) \ln \mu^c_t + \rho_{mc} \ln \mu^c_{t-1} + \epsilon^mc_t, \\
\ln \mu^i_t &= (1 - \rho_{mi}) \ln \mu^i_t + \rho_{mi} \ln \mu^i_{t-1} + \epsilon^mi_t,
\end{align*}
\]

with \( \mu^c_t = \Gamma^c_t / \Gamma^c_{t-1} \) and \( \mu^i_t = \Gamma^i_t / \Gamma^i_{t-1} \).

Unlike much of the literature, I allow technology innovations to be correlated across sectors. This is a natural assumption, as new technologies or management practices may prove relevant for both sectors and trigger simultaneous adoption, or instead embed some specificity and prompt adoption in a single sector. Theoretically, Basu, Fernald, Fisher, and Kimball (2011) also show that in an economy where the final sectors use different combinations of intermediate technologies, measured sector-specific technology processes feature correlated innovations. Therefore, I assume that \([\epsilon^mc_t \; \epsilon^mi_t] \) is iid \( N(m, \Sigma) \) with \( m = [0 \; 0]' \) and

\[
\Sigma = \begin{bmatrix}
\sigma^2_{mc} & \sigma_{mc}\sigma_{mi} \\
\sigma_{mc}\sigma_{mi} & \sigma^2_{mi}
\end{bmatrix}.
\]

In the following, I call \( \epsilon^mc_t \) the C-shock, and \( \epsilon^mi_t \) the I-shock.

In both sectors, firms are subject to nominal pricing frictions à la Calvo (1983). Each period, an intermediate firm in the C-sector can reoptimize its price with probability \( 1 - \xi_{pc} \). Those that cannot do so index their prices to lagged consumption inflation according to

\[
P^c_t(j) = \pi^c_{c,t-1} \pi^{1-\epsilon_{pc}} P^c_{t-1}(j),
\]

where \( \pi_{c,t} = P^c_t / P^c_{t-1} \). Letting \( \tilde{P}^c_t \) denote the optimal price chosen by reoptimizing C-firms, the Calvo assumption ensures that the consumption price index evolves according to

\[
(P^c_t)^{-\frac{1}{\xi_{pc}}} = (1 - \xi_{pc})(\tilde{P}^c_t)^{-\frac{1}{\xi_{pc}}} + \xi_{pc} (\pi^c_{c,t-1} \pi^{1-\epsilon_{pc}} P^c_{t-1})^{-\frac{1}{\xi_{pc}}}.
\]

Symmetrically, the law of motion for the investment price index writes

\[
(P^i_t)^{-\frac{1}{\xi_{pi}}} = (1 - \xi_{pi})(\tilde{P}^i_t)^{-\frac{1}{\xi_{pi}}} + \xi_{pi} (\pi^i_{i,t-1} \pi^{1-\epsilon_{pi}} P^i_{t-1})^{-\frac{1}{\xi_{pi}}}.
\]
where $\tilde{P}^i_t$ denote the optimal price for a reoptimizing I-firm, $\xi_{pi}$ and $\iota_{pi}$ are the Calvo and indexation parameters in the I-sector, and $\pi_{i,t} = P^i_t/P^i_{t-1}$.

Consolidating the last two equations yields an expression for the relative price of investment goods, $RPI_t = P^i_t/P^c_t$. Absent Calvo frictions, the nominal price in each sector is equal to the product of the exogenous sector-specific markup with the nominal marginal cost. In that case, $RPI_t$ takes the simple form

$$
\frac{P^i_t}{P^c_t} \propto 1 + \eta^i_t (\Gamma^c_t)^{1-\alpha_c} (W^i_t)^{1-\alpha_i} (R^k_t)^{\alpha_i},
$$

where $W^x_t$ and $R^x_t$ denote the nominal wage and rental rate of capital for firms in sector $x$. This expression shows that fluctuations in the relative price of investment originate from three different sources: (i) shifts in relative markups across sectors, (ii) shifts in relative technology across sectors, and (iii) shifts in the unit production cost across sectors. By assumption, points (i) and (ii) relate to exogenous factors. On the other hand, point (iii) implies that all shocks hitting the economy will be endogenously passed to the relative price in presence of limited factor mobility or differences in factor shares. If nominal rigidities pop in, the same logic applies but the pass-through of non-markup shocks to the relative price of investment may be considerably slowed down.

The implication that all shocks affect the equilibrium path of the relative price of investment is in sharp contrast with the assumption of a direct mapping between $RPI_t$ and relative technology across sectors typically embedded in estimated DSGE models. Within the economy at hand, such a tight link would arise as a knife-edge case in a restricted specification with price flexibility, perfectly competitive good markets, full factor mobility, and identical factor shares across sectors. I show in the estimation exercise below that such restrictions are strongly rejected by U.S. data.

2.3. Households. The economy is populated by a measure one of households. The representative household’s lifetime utility function writes

$$
E_0 \sum_{t=0}^{\infty} \beta^t \zeta_t \left[ \frac{(C_t - h\overline{C}_t)^{1-\sigma}}{1 - \sigma} \exp \left( \frac{\sigma - 1}{1 + \kappa} \left[ ((L^c_t)^{1+\omega} + (L^i_t)^{1+\omega}) \frac{1+\omega}{1-\omega} \right] \right) \right],
$$

where $C_t$, $L^c_t$, and $L^i_t$ respectively denote individual consumption and hours worked in the C- and I-sectors, $\overline{C}_t = \int_0^1 C_t(l)dl$ is the average level of consumption in the economy, $\beta \in (0, 1)$ is the discount factor, $\sigma$ is the risk-aversion coefficient, and $h \in (0, 1)$ measures external habit formation. As in Horvath (2000), the specification of the disutility of working implies imperfect labor mobility across sectors when $\omega > 0$, allowing for sectoral heterogeneity in wages and hours worked. $\kappa \geq 0$ measures the aggregate elasticity of
labor supply, while $\zeta_t$ is an intertemporal preference shock that evolves according to

$$\ln \zeta_t = \rho \ln \zeta_{t-1} + \epsilon^\zeta_t,$$

with $\epsilon_t^\zeta \sim iidN(0, \sigma^2_{\zeta_t})$. These preferences are consistent with the existence of a stochastic balanced growth path.

The real flow budget constraint of the representative household is

$$C_t + RPI_t \left[ I_t + \Psi(u_t)K_{t-1} \right] + T_t + \frac{B_t}{P_t^c} \leq \frac{W^c_t L^c_t + W^i_t L^i_t}{P_t^c} + RPI_t (r^{kc}_t K^c_t + r^{ki}_t K^i_t) + \frac{\Pi_t + R_{t-1} B_{t-1}}{P_t^c}.$$

On the expenditure side, $I_t$ denotes purchases of new investment goods, $\Psi(u_t)K_{t-1}$ is the cost of capital utilization, $T_t$ is a lump-sum tax from the government, and $B_t$ is holdings of nominal riskless one-period bonds with rate of return $R_t$. On the income side, $W^x_t L^x_t / P_t^c$ is real labor income from sector $x$, $RPI_t r^{kx}_t K^x_t$ is income from renting capital services to firms in sector $x$, and $\Pi_t / P_t^c$ are real profits rebated by firms and labor unions.

The economy-wide stock of physical capital, $K_t$, accumulates according to

$$K_t = (1 - \delta) K_{t-1} + \upsilon_t \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right] I_t,$$

where $\delta \in [0, 1]$ is the depreciation rate. The adjustment cost function $S(.)$ verifies $S(\mu^i) = S'(\mu^i) = 0$ and $S''(\mu^i) = s$. As in Justiniano, Primiceri, and Tambalotti (2011), $\upsilon_t$ is a shock to the marginal efficiency of investment that captures disturbances to the process by which investment goods are transformed into installed capital. This shock acts as a demand shifter in the investment market and evolves according to

$$\ln \upsilon_t = \rho \ln \upsilon_{t-1} + \epsilon^\upsilon_t,$$

with $\epsilon_t^\upsilon \sim iidN(0, \sigma^2_{\upsilon_t})$.

To capture frictions in the sectoral allocation of capital, I use a specification similar to that of hours worked.\(^2\) Namely, letting $K_t = u_t K_{t-1}$ denote the amount of capital services available at date $t$, I assume that

$$K_t = u_t K_{t-1} = \left[ (K^c_t)^{1+\nu} + (K^i_t)^{1+\nu} \right]^{\frac{1}{1+\nu}},$$

with $\nu \geq 0$. The cost of capital utilization is of $\Psi(u_t)$ units of investment goods per unit of physical capital. The cost function $\Psi(.)$ is normalized so that in steady state,

\(^2\)This specification of intersectoral frictions also eschews the identification problem pointed by Kim (2003) in presence of both inter- and intratemporal adjustment costs.
$u = 1$ and $\Psi(1) = 0$. As usual, I parametrize the function $\Psi$ by $\psi \in (0, 1)$ such that $\Psi''(1)/\Psi'(1) = \psi/(1 - \psi)$.

2.4. Labor market. Households supply hours worked to sector-specific unions, which differentiate labor services and set nominal wages subject to Calvo frictions. Competitive labor packers purchase those differentiated services and produce the final labor input usable by firms.

2.4.1. Labor packers. There are two competitive labor packers in the economy, one for each sector. They purchase a continuum of differentiated sector-specific labor services and produce usable labor inputs according to

$$L^c_t = \left(\int_0^1 L^c_t(u)^{1+\eta^{wc}_t} du\right)^{1+\eta^{wc}_t}, \quad L^i_t = \left(\int_0^1 L^i_t(u)^{1+\eta^{wi}_t} du\right)^{1+\eta^{wi}_t}.$$  

The two wage markup shocks $\eta^{wc}_t$ and $\eta^{wi}_t$ evolve according to

$$\ln(1 + \eta^{wc}_t) = (1 - \rho \eta^{wc}_t) \ln(1 + \eta^{wc}_{t-1}) + \epsilon^{wc}_t - \theta_{wc} \epsilon^{wc}_{t-1},$$

$$\ln(1 + \eta^{wi}_t) = (1 - \rho \eta^{wi}_t) \ln(1 + \eta^{wi}_{t-1}) + \epsilon^{wi}_t - \theta_{wi} \epsilon^{wi}_{t-1},$$  

with $\epsilon^{wc}_t \sim iidN(0, \sigma^2_{\eta^{wc}_t})$ and $\epsilon^{wi}_t \sim iidN(0, \sigma^2_{\eta^{wi}_t})$.

2.4.2. Labor unions. In each sector, labor unions intermediate between households and the labor packer by differentiating homogeneous hours worked and setting nominal wages. The probability that a particular union in the C-sector can reset its nominal wage at period $t$ is constant and equal to $1 - \xi_{wc}$, and nominal wages that are not reoptimized are partially indexed according to

$$W^c_t(u) = (\pi_{c,t-1} \mu^{wc}_{t-1})^{1-\eta^{wc}_t} W^c_{t-1}(u),$$

where $\mu^{wc}_t$ is the equilibrium growth rate in the real sectoral wage $W^c_t/P^c_t$, with steady-state level $\mu = (\mu^c)^{1-\alpha}(\mu^i)^{\alpha}$. Letting $\hat{W}^c_t$ denote the optimal wage chosen by reoptimizing C-unions, the law of motion of the aggregate wage index in the C-sector is then

$$\left(W^c_t\right)^{-\frac{1}{\eta^{wc}_t}} = (1 - \xi_{wc})(\hat{W}^c_t)^{-\frac{1}{\eta^{wc}_t}} + \xi_{wc} \left[(\pi_{c,t-1} \mu^{wc}_{t-1})^{1-\eta^{wc}_t} W^c_{t-1}\right]^{-\frac{1}{\eta^{wc}_t}}.$$  

Similar computations deliver the wage equation for the I-sector:

$$\left(W^i_t\right)^{-\frac{1}{\eta^{wi}_t}} = (1 - \xi_{wi})(\hat{W}^i_t)^{-\frac{1}{\eta^{wi}_t}} + \xi_{wi} \left[(\pi_{c,t-1} \mu^{wi}_{t-1})^{1-\eta^{wi}_t} W^i_{t-1}\right]^{-\frac{1}{\eta^{wi}_t}},$$

where $\hat{W}^i_t$ denotes the optimal wage for a reoptimizing I-union and $\xi_{wi}$ and $\eta^{wi}_t$ are the Calvo and indexation parameters in the I-sector.
2.5. **Central bank.** The monetary authority sets the nominal interest rate according to a Taylor-like rule:

\[
\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\left(\frac{\pi_{c,t}}{\pi_c}\right)^{\phi_{\pi}} \left(\frac{X_t}{\mu X_{t-1}}\right)^{\phi_x}\right]^{1-\rho_R} \gamma_t^m,
\]

where \(X_t\) is real GDP in consumption units, defined below.\(^3\) The policy rule is shifted by a disturbance \(\gamma_t^m\), capturing both persistent movements in the central bank’s inflation target and discretionary monetary shocks. It evolves according to

\[
\ln \gamma_t^m = \rho_m \ln \gamma_{t-1}^m + \epsilon_t^m,
\]

with \(\epsilon_t^m \sim iidN(0, \sigma_{\epsilon_t}^2)\).

2.6. **Government.** Fiscal policy is Ricardian. The government purchases exogenous amounts of consumption and investment goods, respectively denoted \(G_{c,t}\) and \(G_{i,t}\), whose final use is not specified. In particular, I do not allow for a productive feedback from the unmodeled stock of public capital. Letting \(g_{c,t} = G_{c,t}/\Omega_{c,t}\) and \(g_{i,t} = G_{i,t}/\Omega_{i,t}\) denote detrended expenditures, I assume that

\[
\ln g_{c,t} = (1 - \rho_{gc}) \ln g_{c,t-1} + \epsilon_{gc,t},
\]

\[
\ln g_{i,t} = (1 - \rho_{gi}) \ln g_{i,t-1} + \epsilon_{gi,t},
\]

with \(\epsilon_{gc,t} \sim iidN(0, \sigma_{\epsilon_{gc}}^2)\) and \(\epsilon_{gi,t} \sim iidN(0, \sigma_{\epsilon_{gi}}^2)\). Lump-sum taxes \(T_t\) adjust to balance the government budget constraint at each date:

\[
T_t = G_{c,t} + RPI_t G_{i,t}.
\]

2.7. **Market clearing.** Market clearing requires that \(B_t = 0\) in the bond market, that

\[
C_t + G_{c,t} = Y_{c,t},
\]

\[
I_t + G_{i,t} + \Psi(u_t) K_{t-1} = Y_{i,t}
\]

in the consumption and investment good markets, and that

\[
\int_0^1 K_{c,t}^e(j)dj = K_{c,t}^e,
\]

\[
\int_0^1 K_{i,t}^i(j)dj = K_{i,t}^i,
\]

\[
\int_0^1 L_{c,t}^e(j)dj = L_{c,t}^e,
\]

\[
\int_0^1 L_{i,t}^i(j)dj = L_{i,t}^i.
\]

\(^3\)In theory, the policy rule could allow for different responses to C-inflation, I-inflation, growth in the C-sector, and growth in the I-sector. From an empirical perspective however, this richer policy rule only marginally improves the fit and leaves the main results unchanged. I have thus opted for the simplest specification here.
in the factor markets. Because price dispersion does not matter at the first order, aggregate output in each sector relates to production factors according to

\[ Y^c_t = (K^c_t)^{\alpha_c} \left[ \Gamma^c_t L^c_t \right]^{1-\alpha_c} - \Omega^c_t \Phi^c_t, \quad Y^i_t = (K^i_t)^{\alpha_i} \left[ \Gamma^i_t L^i_t \right]^{1-\alpha_i} - \Omega^i_t \Phi^i_t. \]

In this economy, nominal GDP is defined as \( P^c_t (C_t + G^c_t) + P^i_t (I_t + G^i_t) \). As usual, capital utilization costs are accounted for as intermediate consumption and do not show up in this expression. Real GDP in consumption units is then given by

\[ X_t = C_t + G^c_t + RPI_t (I_t + G^i_t). \]

2.8. **Identifying investment shocks.** Abstracting from government investment and utilization costs, I can rewrite the physical capital accumulation equation as

\[ K_{t+1} = (1 - \delta) K_t + (1 - S_t) v_t \Gamma^i_t \left[ (k^i_t)^{\alpha_i} (L^i_t)^{1-\alpha_i} - \Phi^i_t \right], \]

where \( k^i_t = K^i_t / \Gamma^i_t \) and \( S_t = S(I_t / I_{t-1}) \). As in Justiniano, Primiceri, and Tambalotti (2011), this formulation emphasizes that capital accumulation is directly affected by two investment shocks: the I-shock \( \Gamma^i_t \) and the MEI shock \( \Phi^i_t \). This raises the question of their respective identification, that the literature has addressed in various ways. For instance, Smets and Wouters (2007) and Justiniano, Primiceri, and Tambalotti (2010) treat the two shocks as a single unobserved disturbance, while Justiniano, Primiceri, and Tambalotti (2011) restrict the behavior of the I-shock by imposing a direct mapping between relative technology and the relative price of investment.

Within this paper’s model, none of these approaches would work. As discussed at the end of Section 2.2, introducing price rigidities in both the consumption and investment markets breaks the link between relative technology and relative price, so Justiniano, Primiceri, and Tambalotti’s identification strategy would not be appropriate. Another possible scheme, exploiting long-run restrictions, is plagued by arbitrariness because there is no compelling reason to attribute permanent effects to a specific investment shock only. In particular, remark that while the above model assumes such a clear-cut decomposition, with a permanent I-shock and a transitory MEI shock, the persistence of the latter can be estimated arbitrarily close to one from the data.

More fundamentally, the difference between I-shocks and MEI shocks relates to the supply-demand decomposition of investment fluctuations. Even if it affects demand through general-equilibrium mechanics, the I-shock is primarily a supply shock. As such, and following well-known arguments exposed for instance in Galí (1999), one expects I-shocks to trigger negative comovements between investment production and I-hours in this sticky-price economy. On the other hand, the MEI shock affects investment demand but leaves I-firms’ technology unchanged, thereby triggering positive comovements between investment production and I-hours. As shown below, the estimated model supports
these intuitions, so the I- and MEI shocks are effectively identified by the different conditional comovements they imply. Practically, inclusion of sectoral hours series among observables will be key to separate out the two investment shocks during estimation.

3. **Bayesian Inference**

I solve the model with standard linearization techniques and use Bayesian methods to estimate its parameters. This section discusses the data used to build the likelihood function, the calibration of some parameters, and the specification of prior distributions for the remaining ones.

3.1. **Data.** I estimate the model using eleven observables: real private consumption growth, real private investment growth, real public consumption growth, real public investment growth, hours worked in the C-sector, hours worked in the I-sector, real wage growth in the C-sector, real wage growth in the I-sector, inflation in the C-sector, the relative price of investment growth rate, and a nominal interest rate. I define private consumption as personal consumption expenditures on nondurable goods and services, while private investment includes both expenditures on durable goods and fixed investment. I use standard chain aggregation methods to construct the relevant quantity and price series. All quantities are expressed in per-capita terms. Appendix A provides data sources and describes the linkage to observables.

My selection of observables differs from that typically used in the DSGE literature in that I include substantial information about the sectoral structure of the economy. Two objectives underlie this choice. First, sectoral observables provide a useful source of identification for sectoral shocks and frictions. For instance, I argued in Section 2.8 that observations on I-hours were needed to separate out the two investment shocks. Likewise, consolidating the representative consumer’s two first-order conditions for labor supply yields

\[
\frac{\overline{W}_c}{\overline{W}_i} = \left(\frac{L_c}{L_i}\right)^\omega,
\]

an equation that shows it would be difficult to identify \(\omega\), the parameter capturing reallocation frictions in labor, without sectoral data on hours and wages.\(^4\) Second, matching sectoral variables helps pushing the model toward capturing both aggregate and sectoral

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\(^4\)The equilibrium allocation of capital services is characterized by \(r_k^{kc}/r_k^{ki} = (K_i^c/K_i^i)\nu\). Given the absence of data on the return to capital or the sectoral allocation of capital, identification of \(\nu\) appears somewhat more fragile.
comovements. Eventually, there are as many structural shocks in the model economy as observables used in estimation.\footnote{Some authors (Sullivan, 1997; Iacoviello and Neri, 2010) argue that the BLS series for sectoral hours and wages suffer from measurement error, especially regarding long-run trends. The demeaning procedure described below is a way to cope with this issue. Additionally, I have tried estimating the model allowing for independent measurement errors on observables. However, in that case the estimated model missed the positive intersectoral comovement of hours worked.}

I demean all series prior to estimation. This procedure ensures that potential discrepancies between the model’s implied balanced growth path and the data will not distort inference at the business-cycle frequencies of interest. However, the approach also implies that steady-state information will not be used for identification. The calibration of specific parameters reflect this choice. Additionally, I remove independent quadratic trends from the two hours series. This is required by hours worked displaying different long-run behavior in the two sectors, with C-hours rising significantly more than I-hours over the sample. This detrending procedure also ensures that estimation focuses on business-cycle comovements rather than on low-frequency patterns the model is not designed to capture.

Eventually, the estimation sample runs from 1965Q1 to 2008Q3, which is the first quarter in which the nominal interest rate hit the zero lower bound in the U.S. economy.

3.2. \textbf{Calibrated parameters}. I keep thirteen model parameters fixed during estimation: the subjective discount factor $\beta$; the steady-state depreciation rate $\delta$; the four steady-state markup parameters $\eta^c$, $\eta^i$, $\eta^{wc}$, and $\eta^{wi}$; steady-state inflation in the C-sector $\pi^c$; the steady-state growth rates in sector-specific technologies $\mu^c$ and $\mu^i$; the factor shares $\alpha_c$ and $\alpha_i$; and the two steady-state government spending ratios $G^c/Y^c$ and $G^i/Y^i$. These parameters are difficult to identify without steady-state information as they have little effect on model dynamics.

Table 1 reports the chosen values. Consistent with the estimates reported in Smets and Wouters (2007) and Justiniano, Primiceri, and Tambalotti (2010, 2011), I set $\beta = 0.998$. Together with the calibrated values for $\pi^c$, $\mu^c$, and $\mu^i$ and with the point estimate for the risk aversion coefficient $\sigma$, this choice implies a steady-state annual nominal interest rate of 7.7 percent, somewhat above the sample average of 6.4 percent. I fix the depreciation rate of capital $\delta$ at 0.025, a standard choice for quarterly models, and assume 10 percent markups in both good and labor markets.

I calibrate $\pi^c$, $\mu^c$, and $\mu^i$ by matching the sample averages for inflation in the C-sector, growth in private consumption, and growth in private investment. In particular, there is faster technological progress in the I-sector relative to the C-sector, as $\mu^i > \mu^c$. The implied steady-state gross inflation rate in the I-sector is 1.007, in line with its sample
Table 1. Calibrated parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.998</td>
<td>Subjective discount factor</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
<td>Steady-state depreciation rate</td>
</tr>
<tr>
<td>$\eta^c, \eta^i, \eta^{wc}, \eta^{wi}$</td>
<td>0.10</td>
<td>Steady-state net good- and labor-market markups</td>
</tr>
<tr>
<td>$\pi^c$</td>
<td>1.011</td>
<td>Steady-state gross C-inflation</td>
</tr>
<tr>
<td>$\mu^c$</td>
<td>1.003</td>
<td>Steady-state gross growth rate in C-technology</td>
</tr>
<tr>
<td>$\mu^i$</td>
<td>1.008</td>
<td>Steady-state gross growth rate in I-technology</td>
</tr>
<tr>
<td>$\alpha_c$</td>
<td>0.35</td>
<td>Capital share in the C-sector</td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>0.30</td>
<td>Capital share in the I-sector</td>
</tr>
<tr>
<td>$G^c/Y^c$</td>
<td>0.23</td>
<td>Steady-state share of public consumption</td>
</tr>
<tr>
<td>$G^i/Y^i$</td>
<td>0.15</td>
<td>Steady-state share of public investment</td>
</tr>
</tbody>
</table>

counterpart. Thus, the model matches the steady-state gross growth rate in the relative price of investment as well. I use Basu, Fernald, Fisher, and Kimball’s (2011) growth-accounting estimates of sectoral capital shares to fix $\alpha_c$ and $\alpha_i$. They report final-use capital shares equal to 0.36 for consumption-producing firms and to 0.35 for government consumption, so I set $\alpha_c = 0.35$, as well as capital shares ranging from 0.26 to 0.31 for investment-producing firms, which I aggregate into $\alpha_i = 0.30$. Eventually, I fix the steady-state ratios of public to private consumption and public to private investment by matching their sample averages.

3.3. Prior distributions. I estimate all remaining parameters. The first columns in Tables 2 and 3 display the chosen prior distributions. Most are in line with the previous DSGE literature.

Starting the representative household’s preferences, the risk aversion coefficient $\sigma$ has a prior mean of 1.5, the habit parameter $h$ is centered around 0.6, and the inverse elasticity of labor supply $\kappa$ fluctuates around 2. The prior distribution for $\omega$, the parameter capturing the elasticity of substitution across hours in the two sectors, has a mean of 2, somewhat above the value of one estimated by Horvath (2000) in a more disaggregated model. Indeed, a prior predictive analysis conducted before estimation emphasized the role of large $\omega$ values in generating sectoral comovements. Yet, to let the data speak as much as possible, I adopt a fairly diffuse gamma prior with a standard deviation of 0.75. I use an identical prior for $\nu$, the parameter quantifying sectoral frictions in capital reallocation.

Prior distributions for other friction parameters are quite standard. In particular, I choose beta distributions centered at 0.65 for the four Calvo coefficients. Regarding
Table 2. Prior and posterior distributions of structural parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Distribution</td>
<td>Mean</td>
</tr>
<tr>
<td>Preferences</td>
<td>Normal</td>
<td>1.50</td>
</tr>
<tr>
<td>σ</td>
<td>Beta</td>
<td>0.60</td>
</tr>
<tr>
<td>h</td>
<td>Gamma</td>
<td>2.00</td>
</tr>
<tr>
<td>κ</td>
<td>Gamma</td>
<td>2.00</td>
</tr>
<tr>
<td>ω</td>
<td>Gamma</td>
<td>2.00</td>
</tr>
<tr>
<td>Frictions</td>
<td>Gamma</td>
<td>5.00</td>
</tr>
<tr>
<td>s</td>
<td>Beta</td>
<td>0.50</td>
</tr>
<tr>
<td>ψ</td>
<td>Beta</td>
<td>0.65</td>
</tr>
<tr>
<td>ξ_{pc}</td>
<td>Beta</td>
<td>0.50</td>
</tr>
<tr>
<td>ξ_{pi}</td>
<td>Beta</td>
<td>0.65</td>
</tr>
<tr>
<td>ξ_{pc}</td>
<td>Beta</td>
<td>0.50</td>
</tr>
<tr>
<td>ξ_{pi}</td>
<td>Beta</td>
<td>0.65</td>
</tr>
<tr>
<td>ξ_{wc}</td>
<td>Beta</td>
<td>0.50</td>
</tr>
<tr>
<td>ξ_{wi}</td>
<td>Beta</td>
<td>0.65</td>
</tr>
<tr>
<td>ξ_{wc}</td>
<td>Beta</td>
<td>0.50</td>
</tr>
<tr>
<td>Monetary policy</td>
<td>Normal</td>
<td>1.70</td>
</tr>
<tr>
<td>ρ_{r}</td>
<td>Beta</td>
<td>0.40</td>
</tr>
</tbody>
</table>

monetary policy, I assume that the three parameters of the Taylor rule, ρ_{r}, ϕ_{π}, and ϕ_{x}, respectively fluctuate around 0.7, 1.7, and 0.4.

Turning to parameters defining the shocks, I use beta distributions centered at 0.5 for most persistence coefficients. The autocorrelations of the technology processes are two exceptions: because Γ_{c}^{i} and Γ_{i}^{i} already feature unit roots, I use normal priors centered at zero for the autocorrelations of their growth rates. To ease estimation, I also use prior predictive checks to rescale the standard deviations of all shocks to be of similar order of magnitude. I then assume fairly diffuse inverse gamma distributions for these parameters. Eventually, I base the prior distribution for σ_{μ}, the correlation coefficient between sector-specific technology innovations, on Basu, Fernald, Fisher, and Kimball’s (2011) growth-accounting results. They report annual correlations between utilization-adjusted changes in C- and I-technologies ranging between 0.52 and 0.58, so I choose a beta prior with mean 0.5 and standard deviation 0.2 for σ_{μ}.
## Table 3. Prior and posterior distributions of shock parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior distribution</th>
<th>Mean</th>
<th>SD</th>
<th>Mode</th>
<th>Mean</th>
<th>5%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Persistence coefficients</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{gc}$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.91</td>
<td>0.91</td>
<td>0.87</td>
<td>0.95</td>
</tr>
<tr>
<td>$\rho_{gi}$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.81</td>
<td>0.79</td>
<td>0.68</td>
<td>0.89</td>
</tr>
<tr>
<td>$\rho_{uc}$</td>
<td>Normal</td>
<td>0.00</td>
<td>0.20</td>
<td>0.20</td>
<td>0.19</td>
<td>0.09</td>
<td>0.30</td>
</tr>
<tr>
<td>$\rho_{ui}$</td>
<td>Normal</td>
<td>0.00</td>
<td>0.20</td>
<td>0.02</td>
<td>0.02</td>
<td>−0.08</td>
<td>0.13</td>
</tr>
<tr>
<td>$\rho_{v}$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.54</td>
<td>0.54</td>
<td>0.45</td>
<td>0.64</td>
</tr>
<tr>
<td>$\rho_{\zeta}$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.93</td>
<td>0.92</td>
<td>0.88</td>
<td>0.96</td>
</tr>
<tr>
<td>$\rho_{\eta wc}$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.97</td>
<td>0.96</td>
<td>0.94</td>
<td>0.99</td>
</tr>
<tr>
<td>$\rho_{\eta wi}$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.93</td>
<td>0.92</td>
<td>0.87</td>
<td>0.97</td>
</tr>
<tr>
<td>$\rho_{m}$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.09</td>
<td>0.11</td>
<td>0.03</td>
<td>0.19</td>
</tr>
<tr>
<td>$\rho_{gc}$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.97</td>
<td>0.97</td>
<td>0.95</td>
<td>0.98</td>
</tr>
<tr>
<td>$\rho_{gi}$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.96</td>
<td>0.95</td>
<td>0.93</td>
<td>0.98</td>
</tr>
<tr>
<td><strong>MA coefficients for markup shocks</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_c$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.60</td>
<td>0.58</td>
<td>0.42</td>
<td>0.74</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.59</td>
<td>0.54</td>
<td>0.33</td>
<td>0.74</td>
</tr>
<tr>
<td>$\theta_{wc}$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.83</td>
<td>0.79</td>
<td>0.70</td>
<td>0.88</td>
</tr>
<tr>
<td>$\theta_{wi}$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.84</td>
<td>0.80</td>
<td>0.69</td>
<td>0.91</td>
</tr>
<tr>
<td><strong>SDs of innovations</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$100 \sigma_{\eta c}$</td>
<td>InvGamma</td>
<td>2.00</td>
<td>4.00</td>
<td>2.64</td>
<td>2.76</td>
<td>2.19</td>
<td>3.32</td>
</tr>
<tr>
<td>$100 \sigma_{\eta i}$</td>
<td>InvGamma</td>
<td>2.00</td>
<td>4.00</td>
<td>2.13</td>
<td>2.21</td>
<td>1.69</td>
<td>2.70</td>
</tr>
<tr>
<td>$100 \sigma_{\mu c}$</td>
<td>InvGamma</td>
<td>2.00</td>
<td>4.00</td>
<td>9.02</td>
<td>9.17</td>
<td>8.36</td>
<td>9.98</td>
</tr>
<tr>
<td>$100 \sigma_{\mu i}$</td>
<td>InvGamma</td>
<td>2.00</td>
<td>4.00</td>
<td>2.22</td>
<td>2.25</td>
<td>2.05</td>
<td>2.45</td>
</tr>
<tr>
<td>$100 \sigma_{v}$</td>
<td>InvGamma</td>
<td>2.00</td>
<td>4.00</td>
<td>5.77</td>
<td>6.44</td>
<td>4.70</td>
<td>8.15</td>
</tr>
<tr>
<td>$100 \sigma_{\zeta}$</td>
<td>InvGamma</td>
<td>2.00</td>
<td>4.00</td>
<td>2.19</td>
<td>2.31</td>
<td>1.88</td>
<td>2.74</td>
</tr>
<tr>
<td>$100 \sigma_{\eta wc}$</td>
<td>InvGamma</td>
<td>2.00</td>
<td>4.00</td>
<td>3.08</td>
<td>3.19</td>
<td>2.65</td>
<td>3.71</td>
</tr>
<tr>
<td>$100 \sigma_{\eta wi}$</td>
<td>InvGamma</td>
<td>2.00</td>
<td>4.00</td>
<td>1.79</td>
<td>1.83</td>
<td>1.34</td>
<td>2.30</td>
</tr>
<tr>
<td>$100 \sigma_{\mu m}$</td>
<td>InvGamma</td>
<td>2.00</td>
<td>4.00</td>
<td>2.53</td>
<td>2.58</td>
<td>2.31</td>
<td>2.84</td>
</tr>
<tr>
<td>$100 \sigma_{\mu g}$</td>
<td>InvGamma</td>
<td>2.00</td>
<td>4.00</td>
<td>1.25</td>
<td>1.27</td>
<td>1.15</td>
<td>1.38</td>
</tr>
<tr>
<td>$100 \sigma_{gi}$</td>
<td>InvGamma</td>
<td>2.00</td>
<td>4.00</td>
<td>2.62</td>
<td>2.64</td>
<td>2.41</td>
<td>2.86</td>
</tr>
</tbody>
</table>

**Correlation of technology innovations**

| $\sigma_{\mu}$  | Beta              | 0.50         | 0.20         | 0.30         | 0.30 | 0.19 | 0.41 |

### 4. Estimation Results

This section presents the estimation results. I report parameter estimates and posterior distributions. I also discuss the ability of the model to capture the salient properties of the data.
4.1. **Posterior distributions.** The last columns in Tables 2 and 3 report the posterior modes, means, and 90% probability intervals for the estimated parameters. All parameters seem well identified from the data.

On the preference side, the point estimate of the risk aversion coefficient is equal to 1.26, above the value of one that would correspond to a logarithmic specification. The representative household also displays a moderate degree of consumption habits, with a point estimate of $h$ close to its prior mean at 0.64. The estimated Frisch elasticity of labor supply is close to 0.8, in the range of the microestimates reviewed in Rios-Rull, Schorfheide, Fuentes-Albero, Kryshko, and Santaeulália-Llopis (2012). The point estimate of $\omega$ is equal to 2.77, well above its prior mean. This is suggestive that the model needs large labor adjustment costs to fit the data. On the other hand, reallocation frictions in capital services seem unimportant, as the estimated value of $\nu$ is close to zero. The data are strongly informative about both $\omega$ and $\nu$, whose posterior distributions are much tighter than the priors.

Turning to the Calvo coefficients, prices are reoptimized on average once every four quarters in C-sector, and once every fourteen quarters in the I-sector. Although the estimate of $\xi_{pi}$ may appear implausibly high, remark that all prices change every period in the model due to indexation. Thus, the low frequency of price optimization does not translate into extreme observed price sluggishness. Also, the model abstracts from strategic complementarities in price setting, which offer a mechanical way to lower estimates of Calvo coefficients in linearized DSGE models (Eichenbaum and Fisher, 2007). Overall, it is interesting that the data point toward higher price rigidities in the I-sector since the DSGE literature usually assumes that $\xi_{pi} = 0$. Turning to wages, there is also more rigidity in the I-sector than in the C-sector, so the usual assumption of an aggregate labor market again hides substantial sectoral heterogeneity. Eventually, all estimated indexation coefficients are quite low.

The estimated Taylor rule is consistent with a large empirical literature, as the central bank reacts strongly to both C-inflation and output growth. There is some interest rate smoothing and it is interesting to note that, given the estimated policy rule, the model does not need a persistent monetary policy disturbance. Other forcing processes, for instance the four markup shocks, the preference shock, and the two government spending shocks, display strong autocorrelations. Finally, the estimated correlation between quarterly sectoral technology disturbances is equal to 0.30, only about half the value reported by Basu, Fernald, Fisher, and Kimball (2011). While differences in datasets

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6I use the random-walk Metropolis-Hastings algorithm with a single chain to construct the posterior distribution, keeping 500,000 draws after a burn-in period of 1,000,000 draws. I set the step size to ensure an acceptance rate close to 0.32 and use standard tests to confirm convergence.
4.2. Model fit. To assess the ability of the model to fit the data, Figure 1 compares the theoretical and empirical cross-correlation functions for observables. To increase readability, I omit the two government spending series from the Figure. Their cross-correlations functions with other variables are essentially zeros at all leads and lags, a fact correctly captured by the model.
Recall that a likelihood-based estimator tries to match the entire autocovariance function of the data. It is therefore not surprising that the estimated model cannot simultaneously fit all moments. The general picture is, however, satisfactory and suggests that the model captures salient properties of the U.S. economy. Plots on the diagonal show that the own correlation structures of most variables are accurately reproduced. The biggest discrepancies between the data and the model are the overestimated persistence of I-hours and the underestimated persistence of C-inflation. All other model autocorrelations fall within the empirical confidence bands.

In terms of macro comovements, the correlation patterns between consumption and investment on the one hand, and C-hours and I-hours on the other, are matched well. Notably, the growth rates of consumption and investment are positively correlated, as are equilibrium hours in the two sectors. The only disparity relates to investment growth: while it leads consumption growth by one quarter in the model, it does not in the data. Also, in each sector, the dynamic correlations between physical output and labor input are reproduced well. The model thus does a good job at accounting for business-cycle comovements at the sectoral level. In the aggregate, the main theoretical cross-correlations between quantities and prices lie within their empirical confidence bands. Eventually, the model accounts well for the empirical properties of the relative price of investment goods.

5. Macroeconomic Effects of Investment Price Stickiness

This section demonstrates the importance of investment price stickiness for business-cycle analysis. First, I show that nominal rigidity in the investment sector is the single most important friction in terms of fitting the data, suggesting its constitutes a powerful propagation mechanism. I confirm this idea by studying how the introduction of investment price sluggishness affects inference about the sources of macro fluctuations and the effects of structural economic shocks in the model. Eventually, I examine the drivers of the relative price of investment in presence of extended nominal rigidities and conclude against the common view that supply shocks predominate.

5.1. The empirical role of investment price rigidity. I start by assessing formally the empirical role of investment price stickiness in terms of fitting the data. Indeed, the model includes many different frictions and one may be worried that rigid investment prices are not important to capture the dynamics of U.S. time series. To show that they do matter, I reestimate the model shutting off once at a time specific channels and use Bayes factor to evaluate the relative fit of the restricted specifications. This is a stringent way of checking the relevance of individual frictions: since it allows other parameters to
adjust to compensate as much as possible the loss of fit resulting from the restriction, only mechanisms which cannot be replaced by others will be singled out as important.

Table 4 reports the log-marginal data densities and Bayes factors comparing the baseline model with several restricted alternatives. With one exception, richer models are always preferred, suggesting that most frictions are useful to fit the data. Also, Bayes factors especially emphasize the empirical relevance of nominal frictions. Among them, investment price rigidity is associated with the highest factor, thus standing as the single most important model mechanism. Again, it is a remarkable result that price stickiness in investment is more useful to fit the data than consumption price rigidity, as only the latter is typically considered in quantitative macroeconomic models.

As expected, removing nominal rigidities deteriorates the ability of the model to fit the behavior of prices and wages. Without I-price rigidity, the model is not able to capture the own comovements of the relative price of investment, nor its correlation patterns with other variables. Compared to the benchmark, it also do worse at reproducing the comovements between consumption and investment growth, as the latter is now predicted to lead consumption growth by two quarters. Without C-price stickiness, the model underestimates the persistence of C-inflation and also misses the autocorrelation structures of the two sectoral real wage series. Eventually, without nominal wage inertia, the model has difficulties matching the persistences of wages. In addition, a model without wage stickiness in the I-sector generates a near zero correlation between C- and I-hours worked, whereas these are strongly positively correlated in the data.

It is also interesting to look at real rigidities, and I focus on the role of reallocation frictions. As clear from the estimate of $\nu$, capital frictions are not important according to the model and, indeed, removing them improves the marginal data density. Again, one caveat to this finding is the lack of information about the sectoral allocation of capital in the data. On the other hand, labor reallocation frictions matter and imposing

### Table 4. Model fit comparisons.

<table>
<thead>
<tr>
<th>Model specification</th>
<th>Restriction</th>
<th>Log-marginal data density</th>
<th>Bayes factor relative to baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>—</td>
<td>6,788</td>
<td>1.0</td>
</tr>
<tr>
<td>No investment price stickiness $\xi_{pi} = \epsilon_{pi} = 0$</td>
<td>6,558</td>
<td>exp(230)</td>
<td></td>
</tr>
<tr>
<td>No consumption price stickiness $\xi_{pc} = \epsilon_{pc} = 0$</td>
<td>6,666</td>
<td>exp(122)</td>
<td></td>
</tr>
<tr>
<td>No investment wage stickiness $\xi_{wi} = \epsilon_{wi} = 0$</td>
<td>6,579</td>
<td>exp(209)</td>
<td></td>
</tr>
<tr>
<td>No consumption wage stickiness $\xi_{wc} = \epsilon_{wc} = 0$</td>
<td>6,699</td>
<td>exp(89)</td>
<td></td>
</tr>
<tr>
<td>No reallocation friction in capital $\nu = 0$</td>
<td>6,805</td>
<td>exp(-15)</td>
<td></td>
</tr>
<tr>
<td>No reallocation friction in labor $\omega = 0$</td>
<td>6,770</td>
<td>exp(18)</td>
<td></td>
</tr>
</tbody>
</table>

*Notes. Log-marginal data densities computed using the Laplace approximation.*
\( \kappa = 0 \) generates a significant loss of fit. In particular, the model without labor frictions counterfactually predicts a negative correlation between C- and I-hours, as households can now easily substitute the workforce between sectors. Therefore, labor adjustment costs are needed to capture the positive sectoral comovement of hours worked in the data.

5.2. The economics of investment price rigidity. Having shown that price rigidity in the investment sector is crucial to fit the data, I examine in more details the economic mechanisms through which it affects the model dynamics.

5.2.1. Sources of business cycles. I first ask whether inference about the sources of aggregate fluctuations is sensitive to the inclusion of investment price stickiness in the model. With this objective in mind, Table 5 provides the variance decomposition for seven key variables: output (in consumption units), consumption, investment, total hours, hours in the C-sector, hours in the I-sector, and the relative price of investment. I include sectoral hours to shed light on the sectoral dimension of the data, and the relative price of investment to assess the common view that its movements reflect relative technology shocks. I focus on business-cycle frequencies, as obtained from the HP filter with smoothing parameter 1,600.

Two results stand out. First, shocks to investment efficiency explain the bulk of short-run fluctuations in investment and hours worked: the MEI shock accounts for 64 percent of the cyclical variance of private investment and about 50 percent of that of total hours. It also represents one-third of business-cycle movements in aggregate output. These statistics thus confirm Justiniano, Primiceri, and Tambalotti’s (2011) conclusion that shocks to the efficiency of investment have been the key drivers of macro fluctuations in the postwar U.S. economy. Second, the restricted model without investment price stickiness attributes the same predominant role to MEI shocks. Thus, inclusion of pricing frictions in the investment sector does not affect much inference about the main driver of business cycles.

To understand the prevalence of MEI shocks, Figure 2 reports the dynamic responses of consumption, investment, and hours worked to a positive innovation to the marginal efficiency of investment. The shock induces an economy-wide expansion, as hours worked in both sectors positively comove with the produced quantities of C- and I-goods. The economic logic is simple. In this sticky-price model, output and employment are mostly demand determined. By stimulating investment demand, the MEI shock triggers a rise in investment and I-hours, and the resulting increase in household income boosts consumption and C-hours. Also, remark that the positive comovement between investment and I-hours after the shock is consistent with the argument developed in Section 2.8.
While investment price rigidity has little effect on the estimated role of MEI shocks, it matters more for assessing the contributions of technology shocks. According to the complete model, they account for a moderate share of business-cycle movements but are not negligible: together, they represent about 30 percent of the fluctuations in output and 20 percent for consumption and hours worked. However, they do not explain much of investment movements. Interestingly, these contributions are reversed when investment pricing frictions are excluded from the model, as technology shocks then account for 26 percent of investment fluctuations but for only 10 percent of hours movements. As discussed in Section 5.2.2 below, these divergent patterns originate from the strikingly different effects of technology shocks when the model includes or excludes investment price rigidity.

Finally, the last column in Table 5 shows that shocks to good-market markups account for 60 percent of the cyclical volatility of the relative price of investment in the model, while the contribution from technology shocks is much lower at 17 percent. This decomposition is another key result, because it goes strongly against the standard assumption.
that supply shocks explain all movements in the relative investment price. Instead, it is consistent with Beaudry, Moura, and Portier’s (2015) contention that the cyclical behavior of the investment price supports a leading role for demand shocks.\(^8\) Section 5.3 below elaborates on the economic intuition underlying this finding.

5.2.2. Effects of technology shocks. Basu, Fernald, Fisher, and Kimball’s (2011) growth-accounting results suggest that improvements in consumption technology have expansionary effects on output, consumption, investment, and aggregate hours, while improvements in investment technology instead trigger generalized contractions. In turn, Basu, Fernald, and Liu (2013) argue that these comovements, at odds with both flex-price and one-sector sticky-price models, can be explained by a two-sector economy featuring nominal rigidities in both the consumption and investment markets. My estimated model provides an ideal tool to evaluate these claims, which have the potential to bring back technology shocks at the forefront of business-cycle theory.

The top four panels in Figure 3 show the estimated impulse responses of consumption, investment, aggregate hours worked, and the relative price of investment to C- and I-shocks. To simplify the analysis, the responses correspond to orthogonal technology innovations. This is useful to isolate the specific mechanisms through which a change in one sector’s technology propagates through the economy, but of course provides little information about unconditional comovements given that the shocks are correlated.

\(^8\)See Bouakez, Cardia, and Ruge-Murcia (2014) and Gabler (2014) for related works concluding that relative prices do not reflect well relative technologies in multisector models with pricing frictions.
Remarkably, the estimated responses share important features with Basu, Fernald, Fisher, and Kimball’s (2011) estimates. Positive C-shocks trigger expansions in consumption and investment, while positive I-shocks push the economy into a severe recession. An important difference with Basu, Fernald, Fisher, and Kimball is that aggregate hours fall on impact after a C-shock, while they obtain an increase (although not statistically significant). Strikingly, both consumption and total hours worked stay depressed for more than five years after improvements in I-technology, while investment initially falls but recovers after about one year and a half. Overall, the correspondence with Basu, Fernald, Fisher, and Kimball’s results, based on an unrelated empirical strategy, bolsters confidence that C- and I-technology shocks as well as their propagation channels have been correctly identified by the Bayesian DSGE approach.

The bottom two panels display the responses of C- and I-hours, allowing to clarify the behavior of firms after technology shocks. Conditional on the responses of consumption and investment, those of sectoral hours are not surprising in this demand-driven economy. First, hours worked in the sector unaffected by the shock closely track the behavior of the corresponding output, as illustrated by I-hours after a positive C-shock. This is intuitive: if technology is unchanged, movements in output must be fully reflected in inputs. Second, hours in the sector affected by the shock also follow their output, but with a negative shift due to the less-than-proportional increase in demand after the productivity rise induced by price stickiness. This is especially visible in the response of I-hours to a positive I-shock: although investment increases steadily after about one year and a half, I-hours stay depressed at all horizons because the rise in productivity is sufficient to sustain higher production by itself.

Basu, Fernald, Fisher, and Kimball conclude from their results that C- and I-shocks may be a major source of fluctuations in the U.S. economy, given that they both generate business-cycle-like comovements between consumption, investment, and hours. As the variance decomposition from Table 5 shows, this contention is at odds with the estimated model, which instead favors MEI shocks. The intuition behind this prediction follows from the estimated responses just discussed. In the aggregate, C-shocks trigger negative short-run comovements between output and hours worked, while I-shocks generate negative medium-run comovements between investment and both consumption and hours. At the sectoral level, both shocks induce negative comovements between C- and I-hours. Given these patterns, the time for a dramatic reevaluation of technology shocks’ contribution to macro fluctuations may not have come yet.

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9The impact rise in C-hours after the I-shock seems puzzling given the simultaneous fall in consumption. It is in fact due to the one-shot jump in government expenditures on consumption goods induced by the stochastic trend.
Figure 3. Selected impulse responses to C- and I-specific technology shocks.

Notes. See the notes to Figure 2. The correlation between technology shocks is set to zero for the computation.

From the perspective of standard models, the conditional comovements displayed in Figure 3 are puzzling. Indeed, Kimball (1994) show that shocks to consumption technology have no effect on equilibrium labor or investment in frictionless real models, while Fisher (2006) and Justiniano, Primiceri, and Tambalotti (2010) emphasize the expansionary flavor of investment supply shocks in simple two-sector models. It is thus important to understand which frictions are responsible for the patterns of the responses shown in Figure 3. Because it is difficult to develop analytical insights from the large estimated model, I rely instead on comparisons between the baseline specification and the restricted versions discussed in Section 5.1.
Figure 4. Impulse responses to C- and I-shocks without nominal rigidity.

Notes. See the notes to Figure 2.

A priori, both real and nominal frictions may be relevant to explain the estimated responses to technology shocks. However, Figure 4, which plots the responses to C- and I-shocks in the flexible-price, flexible-wage version of the economy, suggests that only nominal rigidities matter here. With flexible prices, technology shocks are instantaneously passed to the relative price of investment and the responses of consumption, investment, and hours worked are very different from those in Figure 3. Consistent with Kimball’s (1994) argument, the C-shock is fully reflected in consumption but leaves investment and hours almost unaffected, while the I-shock generates a rise in investment and hours worked. Thus, real frictions alone cannot generate expansionary C-shocks, and even less recessionary I-shocks. It is in fact price sluggishness in the I-sector that is crucial in shaping these responses, especially for investment-specific technology shocks to trigger a strong economic downturn. Indeed, positive I-shocks induce an immediate jump in investment and output as well as a delayed rise in hours worked when I-prices are flexible. On the other hand, removing pricing frictions in the C-sector leaves most of the patterns displayed in Figure 3 unchanged, suggesting that it is not a key mechanism here.

The underlying economic logic is developed in Basu, Fernald, and Liu’s (2013), building on an intuition from Barsky, House, and Kimball (2007). The key observation is that
the shadow value of investment corresponds closely to the present value of expected utility flows from the stock of capital, which is stable over the cycle. It follows that this shadow value reacts little to shocks, so households are roughly indifferent to the timing of investment purchases. Equivalently, the intertemporal elasticity of substitution is very large for investment demand. Then, in presence of investment price rigidity, a positive I-technology shock triggers a large fall in investment demand as I-goods become relatively more expensive today since a fraction of I-firms are not able to lower instantaneously their prices. Because hours are largely demand driven in the short run, I-hours fall as well, and the corresponding reduction in household income depresses consumption in turn. A general recession thus follows. In contrast, a positive C-shock makes investment goods relatively cheaper today and a symmetric logic applies to generate an expansion.

5.2.3. Effects of monetary shocks. In the context of a stylized economy, Barsky, House, and Kimball (2007) demonstrate that investment price stickiness is key to the effectiveness of monetary policy. They show that a small durable sector with rigid prices within a flex-price model can make the economy react to monetary policy as if all prices were sticky, while in contrast flexibly-priced durables may make money neutral even when consumption prices are sticky. To add some empirical content to these results, I review here the model’s implications for the effects of monetary policy shocks.
Figure 5 reports selected estimated impulse responses to monetary policy shock lowering the nominal interest rate. The shock is clearly expansionary, as consumption, investment, and aggregate hours worked all increase together. At the sectoral level, both C- and I-hours rise simultaneously. Also, the relative price of investment falls for several periods, reflecting the ability of C-firms to increase their prices faster than I-firms in response to the increase in demand. Overall, the economy’s dynamics after a monetary shock resemble a lot those from a one-sector model with sticky prices.

In light of Barsky, House, and Kimball’s (2007) analysis, an interesting question is thus that of the relative role of consumption and investment price rigidities in shaping those dynamics. In fact, both constitute here quite equivalent mechanisms, probably because the estimated Calvo parameters are high in both sectors. Suppressing pricing frictions in one sector while leaving them in the other has little effects on the movements displayed in Figure 5. The only noticeable changes are a fall in the persistence of the responses of consumption, investment, and hours worked when prices are rigid in a single sector, and a switch in the sign of the response of the relative price of investment depending on which sector is able to instantaneously adjusts. On the other hand, suppressing nominal frictions in both sectors unsurprisingly makes monetary policy almost neutral.

5.3. **Shocks and the relative price of investment.** Following Greenwood, Hercowitz, and Krusell (2000), it is common to identify shocks to the relative technology between the C- and I-sectors using the relative price of investment. The literature has considered essentially two practical implementations, either based on a period-by-period mapping between the two series (Justiniano, Primiceri, and Tambalotti, 2011; Schmitt-Grohé and Uribe, 2012) or on long-run restrictions (Fisher, 2006). By allowing for investment price rigidity and relaxing the standard assumption of perfect pass-through of relative technology shocks to the relative price, this paper’s model allows to evaluate these empirical strategies.

As discussed in Section 5.2.1, C- and I-technology shocks account for only one fifth of the cyclical variance of the relative price of investment according to the model, while the contribution of price markup shocks is above 50 percent. These respective shares follow from the large estimated Calvo coefficients in both the consumption and investment markets. The inflation equation in the consumption sector may be written as

\[
\ln \pi_{c,t} - \ell_{pc} \ln \pi_{c,t-1} = \Theta_{pc} E_t \sum_{j=0}^{\infty} (\beta \mu^{1-\sigma})^j \ln mc^c_{t+j} + E_t \sum_{j=0}^{\infty} (\beta \mu^{1-\sigma})^j \ln \eta^c_{t+j},
\]

where \(\Theta_{pc} = (1 - \xi_{pc})(1 - \beta \mu^{1-\sigma} \xi_{pc})/\xi_{pc}\) is a function of structural model parameters — including the Calvo coefficient \(\xi_{pc}\) —, \(\mu\) denotes the average growth rate of the economy, \(mc^c_t\) is the real marginal cost in the C-sector, and \(\eta^c_t\) is the price markup shock in the
Figure 6. Historical contributions to the relative price of investment.

Notes. ‘Markup’ refers to the two price markup shocks $\epsilon^{pc}$ and $\epsilon^{mi}$, while ‘Technology’ refers to the C- and I-shocks $\epsilon^{uc}$ and $\epsilon^{ui}$. All series are demeaned. Shaded bands correspond to NBER recession dates.

C-sector. In the aggregate, the pass-through of marginal cost shocks to the consumption price index thus depends on two statistics: the value of $\Theta_{pc}$ and the persistence of the marginal shock response. Because the estimated value of $\xi_{pc}$ is close to unity, $\Theta_{pc}$ is close to zero so C-inflation responds little to shocks shifting only the marginal cost, including technology shocks. On the other hand, the Calvo specification implies that prices react quickly to markup shocks. A similar analysis holds for investment inflation.

The resulting slow pass-through of technology shocks to the relative price of investment is apparent in Figure 3: it takes about one year for C-shocks to be fully reflected in the price and the pass-through of I-shocks is even slower. Importantly, the small contribution of technology shocks in the estimated model is fully driven by the data: at the prior mean, a similar decomposition attributes 85 percent of the cyclical variance of the relative
Figure 7. Forecast error variance of the relative price of investment at different time horizons.

![Variance decomposition of RPI at different frequencies](image)

Notes. See the notes to Figure 6. The vertical dashed lines surround the frequencies between $\frac{2\pi}{32} = 0.19$ and $\frac{2\pi}{6} = 1.05$.

price to technology shocks, and only 2 percent to markup shocks. It is thus likelihood information that assigns a small weight to C- and I-shocks in driving price fluctuations.

Figure 6 provides a graphical representation of this decomposition. The solid line represents the actual time series for the relative price of investment, obtained by cumulating its demeaned growth rate over time. It can be interpreted as the model prediction conditional on estimated parameters, initial conditions, and smoothed shocks. On the other hand, dashed lines correspond the paths that obtain when only markup or technology shocks are fed into the model. The plots make clear that the behavior of the relative price is closely associated with markup shocks, whereas the contribution from technology shocks appears more disconnected. In particular, most high-frequency movements in the relative price originate from markup shocks. This is also reflected by simple statistics: the correlation between the growth rate of the relative investment price in the data and its estimated markup contribution is equal to 0.82, while it is only 0.11 with the counterfactual path driven only by technology shocks. These findings cast even more doubt on the identification approach assuming a period-by-period invertible mapping between relative technology and the relative price of investment.

Fisher’s (2006) alternative strategy is based on the long-run restriction that only relative technology shocks have permanent effects on the relative price of investment. It is straightforward to confirm that this restriction holds in the DSGE model, as the stochastic trend driving the relative price is a composite of the two technology processes. Its empirical relevance, however, largely depends on the actual frequency band in which
technology shocks are the leading contributors to the variance of the relative investment price. To take an extreme example, if technology disturbances dominate only in frequencies lower than 100 years, the long-run restriction would be of little practical use given the sample sizes typically available for macro series.

To shed light on this issue, Figure 7 plots the respective contributions of markup and technology shocks to the variance of the relative price of investment at different spectrum frequencies. The two vertical lines surround the frequency band commonly associated with business cycles, corresponding to 6 to 32 quarters. Echoing the statistics in Table 5, markup shocks are the leading sources of fluctuations in the relative price at business-cycle frequencies, and also at higher frequencies. On the other hand, technology shocks dominate at frequencies close to zero, reflecting the nonstationary behavior of the technological trend. The cutoff frequency for the lead of technology shocks is close to 36 quarters, or about 15 years. Given that available samples exceed by large such a time span, one could view this finding as providing some support in favor of long-run restrictions. However, Monte-Carlo experiments would be helpful to assess the robustness of this conclusion, for instance using the estimated DSGE model as data generating process in a simulation framework similar to Erceg, Guerrieri, and Gust (2005), Christiano, Eichenbaum, and Vigfusson (2007), or Chari, Kehoe, and McGrattan (2008).

6. Conclusion

This paper introduces sector-specific nominal rigidities and frictions in factor reallocation in a quantitative two-sector DSGE model. Bayesian estimation from quarterly U.S. data shows that such mechanisms are important to fit the data. In particular, I make an empirical contribution to the DSGE literature by showing the importance of price rigidities in the investment sector, which have been mostly ignored so far.

The model sheds new light on standard macroeconomics issues. For instance, I find that technology shocks account for only one third of the movements in the relative price of investment, calling into question the validity of a widespread identification approach. Also, consistent with the growth accounting literature, the model predicts that improvements in consumption technology generate an expansion while improvements in investment technology trigger deep recessions. Overall, a core message of the paper is that the DSGE literature has much to gain by paying more attention to the sectoral dimension of the data, which provides both new economic mechanisms and a relevant source of empirical information.
REFERENCES


This appendix provides data sources and describes the construction of observable variables used in estimation. All quantity series are converted to per-capita terms using the population series provided by the Bureau of Economic Analysis (BEA) in its National Income and Product Accounts (NIPA, Table 2.1, line 40).

**Consumption: Quantity and price series.** I define nominal consumption as nominal consumption expenditures on nondurable goods and services (BEA, NIPA Table 1.1.5, lines 5 and 6). The corresponding quantity series are provided by the BEA (NIPA Table 1.1.6, lines 5 and 6). I construct the aggregate consumption quantity and price series, $C_t$ and $P^c_t$, by chain aggregation.

**Investment: Quantity and price series.** Nominal investment is the sum of nominal consumption expenditures on durable goods and nominal fixed investment (BEA, NIPA Table 1.1.5, lines 4 and 8). The corresponding quantity series are provided by the BEA (NIPA Table 1.1.6, lines 4 and 8). I construct the aggregate investment quantity and price series, $I_t$ and $P^i_t$, by chain aggregation.

**Government consumption and investment.** Nominal government consumption expenditures and nominal gross government investment are provided by the BEA (NIPA Table 3.9.5, lines 2 and 3). I construct real government consumption and real government investment, $G^c_t$ and $G^i_t$, by deflating each series by the corresponding chain-aggregated price index.

**Hours worked.** The Bureau of Labor Statistics (BLS) provides series on employment and average hours worked for the nonfarm business sector (CES0500000007), construction (CES2000000007), durable manufacturing (CES3100000007), and professional and business services (CES6000000007). For each of these sectors, I compute total hours as the product of employment and average hours.

I define investment hours, $L^i_t$, as the sum of hours worked in construction, durable manufacturing, and professional and business services. I include the latter sector because more than 50 percent of its output is allocated to investment according to U.S. input-output tables. The paper’s findings are not sensitive to this inclusion. I then define consumption hours, $L^c_t$, as the difference between total hours in the nonfarm business sector and investment hours.

**Wages.** The BLS also provides series on nominal hourly compensation for each of the above sectors. To construct the relevant nominal wage rates, I first compute total wage
bills by multiplying total hours and hourly compensation. I then split the aggregate wage bill for the nonfarm business sector between consumption and investment, using the same classification as for hours worked. Eventually, I compute the nominal consumption and investment wage series, $W_t^c$ and $W_t^i$, by dividing the two sectoral wage bills by the corresponding hours series.

**Inflation and the relative price of investment.** Inflation in the consumption sector, $\pi_t^c$, is defined as the growth rate in the chain-aggregated consumption price index $P_t^c$. The relative price of investment goods, $RPI_t$, is defined as $P_t^i/P_t^c$.

**Interest rate.** The nominal interest rate, $R_t$, is measured as the quarterly average of the effective Federal Funds rate expressed in quarterly units.