“Voluntary Health Plan Subsidies and Public Expenditure”

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Abstract

Countries that seek to provide universal health coverage deal with considerable publicly funded expenses. This article discusses if a private health insurance subsidy policy can reduce the expenses covered by the public system. A theoretical model is developed in which individuals are characterized by two dimensions: inherited risk of illness and preferences for prevention activities. It is shown that when beneficiaries of a voluntary plan have lower risk, i.e. advantageous selection scenario, a subsidy raises health expenses if articulation between coverage is complementary. On the contrary, in adverse selection scenarios a subsidy reduces expenditure if articulation is supplementary. Intermediate scenarios are also considered where articulations between coverages have both complementary and supplementary components, which is apparently the case for the Colombian health system. Calibrated numerical simulations are provided using the Colombian system data. The calibration strategy employed reveals that selection is adverse in the Colombian voluntary health insurance market. Furthermore, we identify the level of subsidy and changes in articulation (towards supplementarity) that could lead to a reduction in public spending.

JEL Codes: I13, I18, G22.

Key words: Health insurance, Regulation, Subsidies.

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1. Introduction

Government intervention in health service markets and social security insurance schemes (health, pension, etc.) is varied: it can go from direct provision of health services to regulation of private markets. Public intervention in the health sector can have a double purpose. On the one hand, State intervention tries to correct market failures, which are mainly consequence of the presence of asymmetric information in health insurance markets. On the other hand, a lot of countries aim to provide universal health coverage, but at the same time aim to hold back or diminish public health expenses, due to inefficiencies that could cause an excessive raise of public expenses. In many health systems where public and private coverages coexist, it has been proposed to subsidize private insurance as a tool that allows i) to enlarge coverage against the financial risks caused by expenditures in health ii) to reduce expenses made by the public system because of the migration of individuals from public to private coverage plan.

The interaction between two factors seems to explain whether subsidies to voluntary plans increase or decrease public expenditure: 1) the nature of selection, that is, if individuals that purchase voluntary/private plans have a higher risk (adverse selection) or lower risk (advantageous selection) relative to average risk of public system beneficiaries; 2) the nature of the relationship, or articulation between public coverage and coverage provided by private health insurance contracts, that is, if private contracts complement or substitute public coverage. When the articulation is complementary, coverage by the private sector allows access to higher quality services, but also pays off out‐of‐pocket expenses not covered by the public system. Conversely, a supplementary articulation means that individuals choose to purchase private health coverage and exit the public health system which does not generate any more expenses for this sector. There is also an alphabet soup of mid‐way scenarios between these two systems, which are characterized by mixed articulations that have both complementary and supplementary components.

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3 The goal of universal health coverage is to ensure that all people have access to health services they need, without having to go through financial hardship in order to pay for them. On December 12 2012, the General Assembly of the United Nations unanimously adopted a resolution that recognizes the role of health in achieving international development goals. It urges governments to move towards providing all people with access to affordable, quality health‐care services. http://www.who.int/universal_health_coverage/un_resolution/en/

4 Inefficiencies brought about by distortions that health financing can bring into the labor market, especially in Bismarckian systems.

5 For example Medicaid program in the United States does not admit individuals who have additional coverages of private insurances.

6 In the national health systems in Spain and United Kingdom, there is a supply of private insurances that are used by individuals to gain access to providers that usually are not in the public system supply. Individuals are always beneficiaries of the public system, nevertheless they can use services provided by the private insurance which can replace or substitute for public coverage. On the other hand, some plans offer access to health technology that are not covered by the public system, which complements the coverage offered by the public system.
The discussion regarding the first factor has evolved around the presence of asymmetric information, which has a particular resonance in health insurance markets, particularly because its consequence is that markets will not have efficient allocations, at least in a first-best perspective (Rothschild and Stiglitz, 1976; Harris and Townsend, 1981). Asymmetric information leads to scenarios of over or underinsurance that affects the possible impact of the subsidy (Pauly, 1974; Einav et al., 2010). In other words, the subsidy may decrease the underinsurance caused by adverse selection, or it can worsen the overinsurance caused in the event of advantageous selection.

A theoretical model is developed in order to analyze how the interaction between the nature of the selection of individuals who purchase voluntary health insurances and the articulation of public and private coverage affect public expenditure when voluntary insurances are subsidized. Basically, individuals can choose to have public coverage only, or to buy a private coverage that offers a better service. The articulation between public and private coverage is parametrized in a way such that situations of complementary and supplementary coverages, as well as intermediate scenarios can be considered.

Agents are heterogeneous in two dimensions: On the one hand, they differ by their inherited health risk; on the other hand, they have different preferences for self-protection activities, which decrease the probability of suffering from an illness. Even though these two dimensions have countervailing effects on the probability of illness, they both have a positive effect on the willingness to pay for a private health coverage plan, which offers a better quality. Then, scenarios of both adverse and advantageous selection are recreated, depending on the relative weight of each dimension on the probability of illness and the willingness to pay for the insurance, which determines the private insurance demand function. In the scenario of advantageous selection (respectively adverse selection), average probability of illness of those individuals who purchase a private coverage plan is less (resp. more) than the average probability of those who remain with public coverage.

Analytical results show that in the advantageous selection scenario, a subsidy increases expenses in health care if coverage articulation is complementary. On the contrary, in the adverse selection scenario, a subsidy decreases expenses if articulation is sufficiently supplementary. In the intermediate scenarios, subsidies can have ambiguous consequences on the amount of public expenditure. Therefore, we calibrate our model using data from the Colombian health system, which seems to have components of both complementary and supplementary in its articulation between private and public health coverage. The calibration strategy applied allows us to determine a unique level of articulation for each selection scenario (adverse or advantageous). Also, our calibration exercise reveals that it is the adverse selection scenario that is the closest to the observed total public expenditure.

Based on this calibration, numerical simulations are performed. They reveal that a scenario of adverse selection with a sufficiently supplementary articulation is the most promising for a subsidy program to contribute to diminish public expenditure. Being the articulation between public and private coverage a political
decision, this simulation describes how that articulation can be used as an intervention tool, which together with a subsidy can lead to a decline of public expenses. If the selection is adverse, small changes in the level of complementarity (toward a more supplementary scenarios) and subsidies of a moderate magnitude can lower public expenditure. On the contrary, if selection is advantageous, even under a completely supplementary articulation, subsidies would increase public spending regardless of its amount.

This paper makes a contribution by proposing a bi-dimensional model that leads to a very simple way of recreating adverse and advantageous selection scenarios. Leaving out a few exceptions (Finkelstein and McGarry, 2006; Olivella and Schroyen, 2011; Einav et al., 2013; Veiga and Weil, 2014), economic theory has mainly focused on unidimensional insurance models (Rothschild and Stiglitz, 1976; De Meza and Webb, 2001; Einav et al., 2010), which do not allow to recreate adverse and advantageous selection within a unified approach. This work proposes a model that recreates both of the selection scenarios through a parameter that represents the relative weight of the two previously mentioned dimensions which intervene in opposite directions in order to determine health risk but that increase willingness to pay for voluntary plans.

This paper also makes part of literature that studies the role of private health insurance subsidies in systems where private and public insurances coexist. Some of the work made try to evaluate if subsidies can generate enough savings for public expenses to cover the costs of the subsidies, nevertheless in countries like the United Kingdom, Australia, or Spain, it has been found that the cost of a subsidy exceeds the benefit for the public sector (Emmerson et al., 2001; Frech and Hopkins, 2004; López and Vera-Hernández, 2008). It has been shown recently that reductions in the health insurance subsidy generates a net savings for the Australian government (Cheng, 2014) or that in the United States of America, for each dollar of tax subsidies for care of the elderly, Medicaid saves approximately $0.84 (Goda, 2011). Still, some of the literature has used dynamic models to show that complete removal of the tax subsidies can lead to a partial collapse of insurance markets, reduction of the coverage and less welfare (Jeske and Kitao, 2009). Our analysis contributes to the understanding of the factors that affect the effect of a private health insurance subsidy on public expenditure and identifies a political tool that leads to obtain the preferred effects on public expenses.

The plan of this article is as follows. The second section presents the model. The third section is devoted to the calibration strategy and the numerical simulation results. The fourth and last section presents the conclusions.

2. The Model

In this section a theoretical model is developed in order to determine how the effect of a subsidy on public expense is determined by the nature of the selection and how coverage offered by private insurance articulates with public coverage. Three
subsections are described: supply of the coverages and its articulation, characteristics of the policyholders, and public expense and subsidy function.

2.1 Insurance supply and articulation between coverages

In this model two types of insurance policies are offered to cover health expenses:

- **Coverage 1**: those enrolled in this coverage benefit from $X_1\%$ coverage (which means that beneficiaries assume out-of-pocket expenses of $1 - X_1$).
- **Coverage 2**: those enrolled in this coverage benefit from $X_2\%$ coverage (out-of-pocket expenses of $1 - X_2$).

It is assumed that $0 < X_1 \leq 1$ and $0 < X_2 \leq 1$, and that $X_2 > X_1$, which means that coverage 2 is more generous than coverage 1. Every policyholder must be enrolled in one of these two coverages.

The expenses made by beneficiaries of coverage 1 are completely covered by the public insurance. The proportion of the expenses made by beneficiaries of coverage 2 that are covered by public insurance is $\alpha$ with $0 \leq \alpha < 1$ while a proportion of $1 - \alpha$ is covered by the private insurance. This means that $\alpha$ is the parameter that represents the articulation between the two coverages. The types of articulation that there can be depend on $\alpha$:

- **Purely supplementary**: This case corresponds to $\alpha = 0$. In other words, the only expenses covered by the public systems are the ones generated by beneficiaries of coverage 1.

- **Purely complementary**: It corresponds to $\alpha = X_1/X_2$. In such a case, public insurance covers a proportion of the expenses made by beneficiaries of coverage 2 when $\alpha X_2 = X_1$.

- **Complementary-supplementary**: It corresponds to values of $\alpha$ such that $0 < \alpha < X_1/X_2$. In this intermediate scenario, public insurance covers a proportion of the expenses generated by beneficiaries of coverage 2 when $0 < \alpha X_2 < X_1$.

2.2 Policyholders’ heterogeneity

Policyholders are heterogeneous in two dimensions which are their private information. $\theta$ captures the inherited risk of having an illness and it is known by each individual. For example, an individual may know her family background, which may increase her willingness to pay for a higher level of coverage, while this information is

\[\text{This restriction on the value of } \alpha \text{ is imposed due to the fact that it is inequitable (and politically unlikely) for the public insurance to take on a larger coverage for a fraction of the people that can purchase a private insurance, which corresponds to the higher-income fraction. Therefore, } \alpha \text{ should be subject to } \alpha X_2 = X_1.\]
unknown by the insurance company. \( \varphi \) represents preventive attitudes or self-protection activities that individuals may carry out. These may be healthy habits or self-care interventions that decrease probabilities of future illness.\(^8\) \(^9\) Even though these two dimensions that differentiate individuals have a positive effect on the individual’s willingness to pay for coverage \( X^2 \), they have opposite effects on the probability of illness.

The population is defined by a joint distribution \( F(\theta, \varphi) \) with \( \theta, \varphi \in [0,1] \). We define \( \omega = \omega(\theta, \varphi) \) as the function that represents individual’s preferences for insurance, where \( \partial \omega / \partial \theta > 0 \) and \( \partial \omega / \partial \varphi > 0 \). In addition, we define \( h = h(\theta, \varphi) \) as the function that represents the probability of illness where \( \partial h / \partial \theta > 0, \partial h / \partial \varphi < 0 \) and \( h \in [0,1] \).

While coverage 1 is funded by public resources, individuals have to pay a price \( P^2 \) to benefit from coverage 2. \( u^1(\theta, \varphi_i) \) and \( u^2(\theta, \varphi_i; P^2) \) denote individual \( i \)'s utility level from having coverage 1 or 2 respectively, with \( u^2(\theta, \varphi_i; P^2) \) strictly decreasing on \( P^2 \). Taking into account what has been previously stated, individuals choose coverage 2 if and only if \( u^2(\theta, \varphi_i; P^2) \geq u^1(\theta, \varphi_i) \), i.e. if they obtain a greater level of utility with coverage 2 than with coverage 1. We define \( w(\theta_i, \varphi_i) \equiv \max\{P^2 | u^2(\theta_i, \varphi_i; P^2) \geq u^1(\theta_i, \varphi_i)\} \), the maximum price that individual \( i \) is willing to pay in order to benefit from coverage 2. \( w(\theta_i, \varphi_i) \in [\underline{w}, \overline{w}] \) where \( \underline{w} \) and \( \overline{w} \) are the lower and upper boundaries, respectively, of the distribution of \( w(\theta_i, \varphi_i) \), which is represented in the following expression:

\[
\begin{equation}
\begin{aligned}
w(\theta, \varphi) &= \omega(\theta, \varphi)CU^2(X^2 - X^1), \\
\end{aligned}
\end{equation}
\]

where \( CU^2 \) denotes the unitary cost of health attention for each coverage 2 beneficiary.

Assuming that there exists a price \( P^2 \in [\underline{w}, \overline{w}] \), we define \( \bar{w}(\theta, \varphi, P^2) \equiv w|w(\theta_i, \varphi_i) = P^2 \) as the willingness to pay that makes policyholders indifferent between purchasing or not purchasing coverage 2. For the above reasons, individuals that purchase coverage 2 are those for which \( w(\theta_i, \varphi_i) \geq \bar{w}(\theta, \varphi, P^2) \).

Enrollment in one of the two coverages is mandatory and without loss of generality, we assume that total population is of mass 1. Therefore, coverage 1 and 2 demand functions are given by:

\(^8\) This is a reduced form of a more sophisticated model in which individuals choose directly a level of prevention. However, because the goal is not to model the prevention but to understand its consequences, a simpler form has been chosen.

\(^9\) The two dimensions in our model are compatible with empiric and theoretical literature of multidimensional individuals in selection scenarios (Finkelstein and McGarry, 2006; Veiga and Weil, 2014; Einav et al., 2013).
\[ D^2(\theta, \varphi) = \int_{-\infty}^{\infty} 1 \left( w(\theta, \varphi) \geq \bar{w}(\theta, \varphi, P^2) \right) \, dF(\theta, \varphi) = \Pr[w(\theta_i, \varphi_i) \geq \bar{w}(\theta, \varphi, P^2)] \]  

and

\[ D^1(\theta, \varphi) = 1 - D^2(\theta, \varphi). \]  

\[ \bar{h}(\theta, \varphi) \equiv \mathbb{E}\left[h(\theta, \varphi) \right] \] is the average probability of illness that characterizes the general population. Policyholders who benefit from to coverage 1 have an average probability of illness of \( \bar{h}^1(\theta, \varphi) \equiv \mathbb{E}[h(\theta_i, \varphi_i) | w(\theta_i, \varphi_i) \leq \bar{w}(\theta, \varphi, P^2)] \); and people who benefit from to coverage 2 have an average probability of illness of \( \bar{h}^2(\theta, \varphi) \equiv \mathbb{E}[h(\theta, \varphi) | w(\theta, \varphi) \geq \bar{w}(\theta, \varphi, P^2)] \).

Comparing values taken on by \( \bar{h}^1(\theta, \varphi) \) and \( \bar{h}^2(\theta, \varphi) \) indicates the nature of the selection that prevails. More precisely, if policyholders who benefit from coverage 2 have a greater average probability of illness than policyholders in coverage 1, then the selection that prevails is adverse (adverse selection if \( \bar{h}^1(\theta, \varphi) < \bar{h}^2(\theta, \varphi) \)). On the contrary, if policyholders who benefit from coverage 2 have a smaller average probability of illness (than individuals in coverage 1), selection is advantageous (advantageous selection if \( \bar{h}^1(\theta, \varphi) \geq \bar{h}^2(\theta, \varphi) \)).

The average probability of illness for people who benefit from coverages 1 and 2 are respectively:

\[ \bar{h}^1(\theta, \varphi) = \mathbb{E}\left[h(\theta_i, \varphi_i) | w(\theta_i, \varphi_i) < \bar{w}(\theta, \varphi, P^2) \right] = \frac{1}{F(\bar{w}(\theta, \varphi, P^2))} \int_{\bar{w}(\theta, \varphi, P^2)}^{\varphi(\theta, \varphi, P^2)} h(\theta, \varphi) \, dF \]  

and

\[ \bar{h}^2(\theta, \varphi) = \mathbb{E}\left[h(\theta_i, \varphi_i) | w(\theta_i, \varphi_i) \geq \bar{w}(\theta, \varphi, P^2) \right] = \frac{1}{1 - F(\bar{w}(\theta, \varphi, P^2))} \int_{\bar{w}(\theta, \varphi, P^2)}^{\varphi(\theta, \varphi, P^2)} h(\theta, \varphi) \, dF. \]

2.3 Subsidy and public expenditure

A subsidy is offered by the government to policyholders who opt for coverage 2. The total public expenditure function \( G(\theta, \varphi, s) \) is defined as the function of all of the costs generated by policyholders who benefit from each of the coverages funded by public resources.

The effects of the subsidy on the average probability of sickness of coverages 1 or 2 beneficiaries are respectively determined by the partial derivatives of the average probabilities as a function of the subsidy, which are (see appendix 1 for more details):
The following Lemma summarizes the consequences of a subsidy upon the average probabilities of sickness depending on the chosen coverage.

**Lemma 1:** A subsidy increases policyholders’ average probability of sickness in both coverages if selection is advantageous. Conversely, this average probability decreases with a subsidy if selection is adverse.

It is assumed that the premium associated with the coverage 2 is equal to its actuarial value, which corresponds to the expected value of the cost that the insurer must assume for the claims that beneficiaries to that level of coverage have.\(^{10}\) We assume that the price offered corresponds only to claim-related expenses and not any other (for example administrative, legal representation expenses, etc.). Therefore, the price that an individual has to pay in order to benefit from coverage 2 is equal to the actuarial premium minus the amount offered the public subsidy. Thus, the price of enrollment in coverage 2 is:

\[ P^2(\theta, \varphi, s) = (1 - \alpha)X^2C^U^2\widehat{h}^2(\theta, \varphi) - s. \]  

(6)

Because a positive subsidy reduces the price paid, this implies that, *ceteris paribus*, more individuals will purchase coverage 2. If the selection is advantageous, individuals that change coverages because of the subsidy are those coverage 1 beneficiaries who have lesser probability of illness (these also have a greater illness probability than those under coverage 2), because of this the transfer of the marginal individual implies an increase of the average probabilities under both coverages. The opposite happens in the adverse selection scenario.

The total public expenditure function is defined as:

\[ G(\theta, \varphi; s) = X^1CT^1(\theta, \varphi, s) + \alpha X^2CT^2(\theta, \varphi, s) + s[D^2(\theta, \varphi, s)], \]

(7)

where \( CT^i(\theta, \varphi, s) = C^U^i\widehat{h}^i(\theta, \varphi, s)D^i(\theta, \varphi, s) \) is the total cost generated by the beneficiaries of coverage level \( i \) with \( i \in \{1, 2\} \). Therefore, the demand for coverages 1

\(^{10}\) A fair actuarial premium is chosen for simplification purposes (assuming competitive market) therefore, strategic behavior of firms who offer coverage 2, considering the government subsidy, is not considered. If we assume a utility margin that insurance companies charge in the premium, our results do not change qualitatively.
and 2 (equations (2) and (3)) and the average probabilities of coverage 1 and 2 beneficiaries (equations (4) and (5)) also depend on the government subsidy. We suppose that $C_{U1} < C_{U2}$, which captures the *ex post* moral risk phenomenon due to the fact that coverage $X_1 < X_2$.

Replacing (2), (3), (4) and (5) in (7) one obtains:

$$G(\theta, \phi; s) = X_1 C_{U1} \bar{l}^1(\theta, \phi, s)[1 - D^2(\theta, \phi, s)] + \alpha X_2 C_{U2} \bar{l}^2(\theta, \phi, s)D^2(\theta, \phi, s) + s[D^2(\theta, \phi, s)].$$

(8)

Expression (8) is the objective function that determines the effect of the subsidy, depending on the sign of $\partial G(s)/\partial (s)$. The expected results of the effect of the subsidy on the expenditure function for each of the possible scenario (each nature of the selection and articulation between the two coverages) are presented below.

The effect of a subsidy on the expenditure function is given by equation (9) (see appendix 1 for details on this derivation):

$$\frac{dG(s)}{ds} = X_1 C_{U1} \frac{d\bar{l}^1(s)}{ds} \left[(1 - f[\bar{\omega}(s)]) \left(\bar{l}^2(s) - \bar{l}^1(s)\right)\right]$$

(9.1)

$$+ \frac{d\bar{\omega}(s)}{ds} \left[h^2(s)(1 - f[\bar{\omega}(s)]) - h[\bar{\omega}(s)]\right] \left[\alpha X_2 C_{U2} - X_1 C_{U1}\right]$$

(9.2)

$$+ 1 - F[\bar{\omega}(s)] - f[\bar{\omega}(s)] \frac{d\bar{\omega}(s)}{ds} s.$$  

(9.3)

The three components of expression (9) allow us to predict the effect of a subsidy. It is known that $\partial \bar{\omega}(s)/\partial s < 0$ due to the fact that a subsidy initially lowers the price of coverage 2 as seen in (6). In an adverse selection scenario, $\bar{l}^2(s) > \bar{l}^1(s)$ which means that (9.1) will be negative when $1 > f[\bar{\omega}(s)]$. In an advantageous selection scenario (9.1) will be positive when $1 > f[\bar{\omega}(s)]$. The sign of (9.2) depends on the articulation between coverages ($\alpha$) and the sign of the expression $\bar{l}^2(s)(1 - f[\bar{\omega}(s)]) - h[\bar{\omega}(s)]$. Finally, (9.3) is always positive. The following proposition summarizes the consequences of a subsidy on public expenditure.

**Proposition 1.** The effect of a subsidy on public expenses are the following:

i) **When selection is advantageous**, public expense increases with the subsidy if articulation between the two coverages is sufficiently complementary.

11 Except for continuous distributions, most probability distributions take on values lower than 1.
When selection is adverse and elasticity of demand is sufficiently low, public expenditure decreases with a subsidy when:

a) Adverse selection is sufficiently strong and articulation is sufficiently complementary;

b) Adverse selection is sufficiently weak and articulation is sufficiently supplementary.

In intermediate cases, the effect of subsidies is ambiguous.

The first statement in Proposition 1 is due to the fact that when the selection is advantageous (9.1) is positive, while the sign of (9.2) depends on the articulation between coverages. More precisely, when the articulation is sufficiently complementary then $\alpha X^2 C U^2 > X^1 C U^1$ and (9.2) is positive. In such a case, the subsidy increases public expenses unambiguously. It is due to the fact that when there is advantageous selection, subsidy raises the average probability of illness of beneficiaries of both coverages (Lemma 1). If on top of that, the articulation is sufficiently complementary, the increase of the average probability of illness of people that purchase coverage 2 produces an externality that contributes to increase public expenses.

The second statement (ii) is particularly interesting because it points out the complex interaction between the articulation nature and the adverse selection. It can be seen that (9.1) is always negative when there is adverse selection, while (9.3) is always positive. Consider the case where the demand is sufficiently inelastic, such that (9.3) does not generate a first-order effect and that adverse selection is strong enough to ensure that $\bar{h}^2(s)(1 - f[\bar{\omega}(s)]) > h[\bar{\omega}(s)]$. In such a case, it yields that (9.2) is negative when the articulation is sufficiently complementary, that is, if $\alpha X^2 C U^2 > X^1 C U^1$. As the subsidy decreases the average probabilities of individuals in both levels of coverage in the adverse selection scenario, then a greater complementarity implies a reduction of public expense because it causes a positive externality, that is, this decrease of average probability of illness of coverage 2 beneficiaries lower the amount of expenditure funded by public resources. On the contrary, when adverse selection is not so strong, (9.2) is only negative if the articulation between both coverages is sufficiently supplementary.

The third statement (iii) in Proposition 1 shows that there are other scenarios in which the effect of a subsidy on public expenditure is ambiguous. Next section proposes various numerical simulations in order to shed light on the effect of subsidies in these cases.

3. Numerical Exercise

The Colombian health system is an ideal scenario to simulate our theoretical model. In Colombia, a public health insurance system coexists with a voluntary/private insurance market. According to official figures, in 2012, 9.65% of
the public health system beneficiaries had a private insurance policy. The public system fee (Unit of Payment for Capitation-UPC) is determined according to the gender, age, and location of the beneficiary. It is 568,944 COP on average. For the private system, there are different prices offered by the market, but the average amount is 2,270,000 COP per year.

3.1 Calibration Strategy

Flowchart 1 shows all of the steps taken as a calibration strategy. A random number sequence was generated to simulate a uniform distribution of parameters θ and φ with θ,φ ∈ [0,1]. Based on this information the following functions are estimated: h(θ, φ) = 1/(1 + exp(kφ − θ)) and ω(θ, φ) = θ + φ, with k > 0. k is the parameter that stand for the nature of the selection, where k < 1 stands for the scenarios of adverse selection while k > 1 stands for scenarios of advantageous selection. Finally, when k = 1 no selection type prevails. The data is calibrated according to the information observed for the Colombian health system on 2012.

At this stage, we do not make any assumptions regarding the value of k and the functional forms of h(θ, φ) and ω(θ, φ) are used to stand for the adverse, advantageous, and neutral scenarios. Based on the observed demand for voluntary insurance in the Colombian market in 2002 (D^2 = 0.096), we obtain the demands for coverages 1 and 2. With this information we find the average probability of illness for every level of coverage in each selection scenario. By setting (1) and (6) to be equal and assuming that initially there is no subsidy, we find the value of α° = 1 − [(X^2 - X^1)ω(θ, φ)/X^2h^2(θ, φ, 0)] for each of the selection scenarios that solves the equilibrium price observed in Colombia in 2012 (P^2 = 2,270,000). With this value for α° we infer the value of CU^2, which is assumed to be the same in all selection scenarios. Table 1 summarizes all of the simulated data. It can be seen that CU^2 > CU^1, to capture the presence of ex post moral risk. α° decreases as selection type changes from adverse to advantageous. Nevertheless, for simulated selection scenarios, there is no α° that allows for the articulation between coverages to be purely supplementary or complementary. Figure 1 shows the probability distribution of illness for individuals that purchase, or do not, private insurance for different values of k. This figure illustrates how that parameter represents the nature of the selection that prevails in markets.

12 Fasecolda (www.fasecolda.org), and the Superintendencia Nacional (http://www.supersalud.gov.co).
13 Information taken from a study made by the Ministry of Health and Social Protection: Estudio de suficiencia y de los mecanismos de ajuste de riesgo de la Unidad de Pago por Capitación para garantizar el Plan Obligatorio de Salud en el año 2013 (Study of sufficiency and of risk adjustment mechanisms of the Unit of Payment for Capitation to guarantee the Obligatory Plan of Health in the year 2013)
14 In addition to the simulation of a uniform distribution for the values of θ and φ, simulations were made using exponential, normal, beta and gamma distributions. Results are shown in appendix 2.
15 A logit function is chosen to model the probability of illness as a function of the parameters θ and φ. ω(θ, φ) follows a triangular distribution.
With the data presented in Table 1, the willingness to pay function \( w(\theta, \varphi, 0) \), was estimated. Figure 2 presents that function’s distribution and its relationship with the probability of illness. It is seen how this relationship is represented by a rhombus (or rhomboid, depending on the selection type) and how \( \widehat{w}(\theta, \varphi, 0) \) (black line) divides the distribution between those who choose coverage 1 (under the black line) or coverage 2 (above the black line).

Two simulation exercises are shown below. The first one reveals the effects of a subsidy on public expenses without any changes in \( \alpha^* \), i.e. taking as given the articulation between coverages that emanated from the calibration strategy. The second one points out the effect that a change in the level of articulation between coverages and the subsidy would have on public expenses.

3.2 Simulation 1: Effect of the subsidy on public expenditure without changes in \( \alpha^* \)

Flowchart 2 exhibits all of the steps taken in the simulation exercise. In the first one, the implementation of a subsidy is simulated as a policy tool that generates migration of individuals from coverage 1 to coverage 2, without any changes on the degree of articulation between coverages. Figure 3 depicts the effect of the subsidy on public expenditure according to the nature of the prevailing selection type. Each colored line represents the government's expense function relative to the level of public subsidy. The two uppermost (blue and red) are the adverse selection scenarios; the middle line (green) is the neutral selection scenario; and the lowermost lines (yellow and grey) represent the advantageous selection scenarios. For each selection scenario proposed there is a unique degree of complementarity (\( \alpha^* \)) between the two coverages that allows a simulation of the observed equilibrium prices in the Colombian market in 2012 (presented in Table 1). It can be noticed that the more adverse the selection scenario is, the greater the value of alpha; however, no value is equal to zero or to the highest possible value for alpha (\( \alpha = X^1/X^2 \)). In other words, there is no scenario of a completely complementary or completely supplementary bond, but rather a mixed scenario (complementary-supplementary). On the other hand, it can be seen that advantageous selection scenarios have more supplementary levels of articulation than adverse selection scenarios.

**Result 1.** In the Colombian scenario, when the nature of the selection is more adverse (respectively advantageous), articulation between coverages is more complementary (resp. supplementary). With the parameters chosen for calibration, it

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16 In order to better describe the effect of the subsidy on public expenditure, simulation includes negative values of the subsidy. However, we are interested in describing the effect of a positive value for the subsidy.

17 Theoretically, the voluntary insurance services should offer a supplementary coverage, however there are two reasons why this does not happen. The first one is that most insurance companies in the voluntary insurance market also offer coverage in the public insurance market, which means they have incentives to report accounting information within the coverage of the public insurance. The second reason is that the government doesn’t monitor the voluntary insurance market. Government’s information systems are not effective and are subject to the information reported by the private insurance companies.
can be seen that the degree of articulation in Colombia for 2012 is mixed (complementary-supplementary).

In Figure 3, the dotted vertical line found in the value of 0 subsidy represents the simulated amount of public expenses for each selection scenario for the observed parameters in Colombia for 2012. The dotted horizontal line is the estimated public expense reported by Barón (2014) for 2012 in the contributive system. The point where those two lines cross represents the scenario that best describes the observed data in Colombia in 2012. As can be observed, the intersection of these lines is located above the no selection scenario, closer to the simulated adverse selection scenarios. Therefore, this figure tends to show that the scenario that comes closest to what is observed in Colombia by the parameters of this simulation (previously described) is adverse selection.

**Result 2.** In the Colombian scenario, the nature of selection that is closest to the official health system figures is adverse.

The slope of the expenditure function relative to the subsidy can be seen in figure 3. The higher the level of subsidy is, without changes in the optimum relation between coverages for the observed data for 2012, the higher public expense is, both for adverse and advantageous selection scenarios. Finally, the slope is steeper in the scenarios of adverse selection, which explains why the degree of complementarity is greater than it is other selection scenarios.

3.3 Simulation 2: Effect of changes in articulation between coverages and the subsidy on public expenses

Scenarios are considered in which a change in the level of articulation between coverages is made, from $\alpha^*$ to $\alpha$, greater or less than $\alpha^*$. This works proposes $\alpha$ as a policy tool because it can be modified through health system legislation. In Colombia, for instance, there was a proposal made by the Ministry of Health and Social Protection to modify the articulation between public insurance and voluntary (private) coverages towards a purely supplementary articulation.

Figures 4 and 5 reveal the effect of the subsidy on the public expenditure function for different values of alpha, each proposed for different selection scenarios.

---

18 The official observed estimation is Barón’s for 2004 and 2011. It is published in Cifras financieras del Sector Salud. Gasto en Salud de Colombia: 2004-2011. Boletín bimestral no 2. Enero – Febrero del 2014. Ministerio de Salud y Protección Social. (Financial Figures of the Health sector. Health Expenses in Colombia: 2004-2011). For 2011 this figure was $17,372,703. However, Baron made a presentation in Medellín on November 7, 2014, where the progress in health accounting in Colombia from 2004 to 2013 is shown and proposes an estimate of $19,607,686 for 2012. ([http://www.udea.edu.co/portal/page/portal/bibliotecaSedesDependencias/unidadesAcademicas/FacultadCienciasEconomicas/ElementosDiseno/Documentos/Memorias/general/Cuentas%20de%20Salud%20de%20Colombia_%20Gilberto%20Bar%C3%B3n.pdf](http://www.udea.edu.co/portal/page/portal/bibliotecaSedesDependencias/unidadesAcademicas/FacultadCienciasEconomicas/ElementosDiseno/Documentos/Memorias/general/Cuentas%20de%20Salud%20de%20Colombia_%20Gilberto%20Bar%C3%B3n.pdf) 16 de marzo de 2015). This last figure is used in our simulation exercise and corresponds to the dotted horizontal line in Figure 3.
Each selection scenario has a unique $\alpha^*$ which solves the observed equilibrium price in Colombia; these are shown in Table 1. Each figure has different changes in the level of articulation between coverages through different values of alpha. The maximum corresponds to 0.89 ($\alpha = X^1/X^2$), in this case articulation is purely complementary. It corresponds to the fuchsia line in Figures 4 and 5. The minimum value for $\alpha$ is 0, which happens when articulation is purely supplementary; it corresponds to the blue line in figures 4 and 5. For mixed articulation, other values of $\alpha$ between these two limits are simulated. Each figure has a dotted black line that corresponds to basal public expense amount in the scenario of no subsidy and without changes in $\alpha^*$ so that the area below the dotted line corresponds to the different values of subsidy and changes in coverage articulation that induce a reduction in public expenditure. The area above the dotted line corresponds to subsidy and articulation combinations that lead to public expense increase.

Figures 4.A and 4.B correspond to simulated adverse selection scenarios. 4.A is the scenario where the highest intensity of adverse selection is simulated ($k=0.33$). For this scenario $\alpha^* = 0.73$ because the only value of $\alpha$ that comprises a rise in complementarity of coverages is $\alpha = 0.89$, which corresponds to a situation of perfect complementarity. The other simulated values of $\alpha$ correspond to increases in supplementarity of articulation. This figure (4.A) corresponds to the scenario where there is a greater quantity of subsidy amounts that allow for public expenses to reduce (area under the dotted line). Figure 4.B describes a scenario with less intensity of adverse selection that also has less complementarity relative to the previous scenario. In this scenario it can also be seen that plenty subsidy amounts can reduce public expenditure. The simulation results presented in figures 4.A and 4.B confirm the findings in the theoretical model, specifically as described in section (ii) of Proposition 1. That is, that in scenarios of adverse selection there are different options for reducing public expenses through a subsidy policy together with a change in coverage articulation, towards a more supplementary articulation (lower $\alpha$).

**Result 3.** In the Colombian scenario, there are different options in which a subsidy can reduce public expenditure, these correspond to a combination of changes in articulation of coverages towards more supplementary scenarios, together with a certain subsidy level. It is more likely for a subsidy to reduce expenses in adverse selection scenarios.

Figures 5.A and 5.B correspond to advantageous selection scenarios. Figure 5.A is the scenario where the highest intensity of advantageous selection is simulated ($k=2$). For this scenario $\alpha^* = 0.41$ so that three of the simulated values of change in $\alpha$ correspond to increases in complementarity of coverages ($\alpha = 0.89, \alpha = 0.71, \alpha = 0.53$). It can be seen that for this scenario there are less values of the subsidy that reduce public expenses. In Figure 5.B there are more values of the subsidy that are able to reduce public expenditure, however, the difference is not very significant
compared to the previous figure. The results from this simulation correlate with those found in the theoretical model in section (i) of Proposition 1.19

3.4 Decomposition of the effect of the subsidy

The only goal of the last numerical exercise is to identify the three terms of equation (9), which correspond to the decomposition of the effect of the subsidy on public expense.20 Initially, values of \( f[\hat{w}(s)] \) and \( F[\hat{w}(s)] \) were determined for each level of subsidy and for each selection scenario. As the value of \( h[\hat{w}(s)] \) corresponds to an interval that varies for each selection scenario, which is represented in Figure 2 as the intersection between \( \hat{w}(s) \) (the red horizontal line) and the probability of illness distribution for each selection scenario, it is estimated based on the total effect of the subsidy and equations (9.1) and (9.3). Table 3 shows the calculated values of \( f[\hat{w}(s)] \) and \( F[\hat{w}(s)] \) for each subsidy amount and Figure 6 presents the distribution of \( h[\hat{w}(s)] \) used in each selection scenario.

The global effect of the subsidy on the public expenditure function is determined for each possible subsidy value. As can be seen in Figure 7, as the subsidy increases and selection becomes more adverse, the global effect increases. The scale used for the vertical axes is hundreds of Colombian pesos (COP), so we can see that the effect of a subsidy of 1 COP is 32,804,915,07 COP in the adverse selection scenario and 16,164,296,9 COP in the advantageous selection scenario.

Regarding the decomposition of the effect of the three terms (9.1), (9.2), (9.3), it can be seen that the first term is always negative in advantageous selection and positive in adverse selection scenarios. In the neutral selection scenario, this term is almost zero. Due to the fact that this term includes the difference between average probabilities of illness between individuals in the two different coverages, it captures the fact that as selection is more adverse, the effect of the subsidy on public expenditure is larger (Figure 8).

The second term (9.2) studies the complex relationship between the nature of selection and articulation of coverages (this was also mentioned in subsection (ii) of Proposition 1). Figure 9 represents the second term of the effect of a subsidy on public expenditure (equation 9.2). In advantageous selection scenarios there are some areas where a subsidy generates a reduction in public expense, these correspond to more supplementary scenarios.

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19 The results from the simulations with different distribution functions for (\( \theta, \phi \)) are presented in appendix 2. It can be seen that there are differences in magnitude, nevertheless conclusions drawn from these simulations are the same that those drawn from a uniform distribution.

20 For this exercise a uniform distribution was employed for \( \theta \) and \( \phi \), simulating a population 10 times bigger than the last one. This was done with the purpose of having individuals for which small variations in the amount of the subsidy would turn out as changes in her disposition to pay such that it would intersect with the value of \( \hat{w}(s) \).
Last of all, the third term of the decomposition of the equation of the effect of the subsidy on public expense does not represent a big share of the overall effect.

4. Conclusions

This paper analyses the impact of a private insurance subsidy on expenses made by the government in systems where there are both public and private insurance. Two key determinants of this impact are analyzed: the nature of the selection and the degree of articulation between coverages. Taking into account the heterogeneity of individuals in two dimensions, related to the inherited risk of illness and preferences for preventive activities, a theoretical model is developed which allows us to determine the conditions under which a subsidy decreases public spending. Furthermore a numerical simulation was made in order to recreate the Colombian scenario.

The theoretical model reveals that in advantageous selection scenarios, the subsidy can increase public expenditure if coverage articulation is complementary. On the other hand, in adverse selection scenarios, a subsidy can reduce spending if articulation is mainly supplementary. In intermediate scenarios, it is difficult to predict the outcome. The numerical exercise recreates these previous situations but also provides a unique articulation value $\alpha^*$ that solves the observed equilibrium price for each simulated selection scenario. According to this numerical exercise and under the previously described conditions, it can be seen that under this articulation level, there is no positive subsidy that can diminish public system expenditure, no matter what the scenario is. Nonetheless, a combination of subsidy and changes in coverage articulation could diminish public spending, mainly in adverse selection scenarios. This work leads to an interesting result which is the idea of using the articulation between public and private coverages as a public policy tool that can be used to reduce public expenditure.

This work does not take into account the oligopolistic structure in coverage supply. It would be interesting to take into account insurance companies’ strategic behavior which includes transaction costs and utilities.

References


Table 1. Description of the simulated data.

<table>
<thead>
<tr>
<th>Parameter k</th>
<th>0.33</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_0^2(\theta, \varphi, 0) )</td>
<td>0.637</td>
<td>0.603</td>
<td>0.498</td>
<td>0.393</td>
<td>0.298</td>
</tr>
<tr>
<td>( h_1^2(\theta, \varphi, 0) )</td>
<td>0.575</td>
<td>0.556</td>
<td>0.500</td>
<td>0.446</td>
<td>0.398</td>
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<tr>
<td>( \alpha^* )</td>
<td>0.728</td>
<td>0.712</td>
<td>0.651</td>
<td>0.558</td>
<td>0.418</td>
</tr>
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<td>$14,529,919,38</td>
<td>$14,529,919,38</td>
<td>$14,529,919,38</td>
<td>$14,529,919,38</td>
<td>$14,529,919,38</td>
</tr>
<tr>
<td>CU(^1)</td>
<td>$1,240,167,87</td>
<td>$1,282,590,58</td>
<td>$1,425,532,82</td>
<td>$1,595,760,73</td>
<td>$1,792,063,10</td>
</tr>
</tbody>
</table>

\( \alpha^* \): unique level of articulation, for each type of selection, which solves the observed equilibrium price in the Colombian market for 2012.

Table 2. Effect of the subsidy on demand for different coverage levels.

<table>
<thead>
<tr>
<th>Subsidy</th>
<th>Coverage 1</th>
<th>Coverage 2</th>
<th>Total Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>-</td>
<td>16.851</td>
<td>1.799</td>
</tr>
<tr>
<td>$</td>
<td>10.000</td>
<td>16.795</td>
<td>1.852</td>
</tr>
<tr>
<td>$</td>
<td>20.000</td>
<td>16.742</td>
<td>1.908</td>
</tr>
<tr>
<td>$</td>
<td>50.000</td>
<td>16.566</td>
<td>2.084</td>
</tr>
<tr>
<td>$</td>
<td>100.000</td>
<td>16.255</td>
<td>2.395</td>
</tr>
<tr>
<td>$</td>
<td>200.000</td>
<td>15.566</td>
<td>3.084</td>
</tr>
</tbody>
</table>

*Number of people in thousands.

Table 3. Values of \( f(\hat{w}(s)) \) and \( F(\hat{w}(s)) \) calculated for each subsidy level.

<table>
<thead>
<tr>
<th>Subsidy</th>
<th>( f(\hat{w}(s)) )</th>
<th>( F(\hat{w}(s)) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-$</td>
<td>200.000</td>
<td>2,05421E-07</td>
</tr>
<tr>
<td>-$</td>
<td>100.000</td>
<td>2,52895E-07</td>
</tr>
<tr>
<td>-$</td>
<td>50.000</td>
<td>2,76632E-07</td>
</tr>
<tr>
<td>-$</td>
<td>20.000</td>
<td>2,90874E-07</td>
</tr>
<tr>
<td>-$</td>
<td>10.000</td>
<td>3,05116E-07</td>
</tr>
<tr>
<td>$</td>
<td>-</td>
<td>3,00369E-07</td>
</tr>
<tr>
<td>$</td>
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<td>3,05116E-07</td>
</tr>
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<tr>
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<td>50.000</td>
<td>3,24106E-07</td>
</tr>
<tr>
<td>$</td>
<td>100.000</td>
<td>3,47843E-07</td>
</tr>
<tr>
<td>$</td>
<td>200.000</td>
<td>3,95317E-07</td>
</tr>
</tbody>
</table>
Figure 1. Distribution of the probability of illness for each type of coverage and selection scenario.

Figure 1 describes three scenarios. A) Adverse selection scenario, with $k<1$. The average probability of illness of individuals who benefit from coverage 2 is greater than that of the individuals who benefit from coverage 1. B) No selection scenario ($k=1$). The average probability of illness of individuals who benefit from both coverages is equal. C) Advantageous selection scenario, with $k>1$. 
Figure 2. Distribution of the willingness to pay according to the probability of illness for each selection scenario.

Three scenarios: A) Adverse selection scenario, with $k<1$. B) No selection scenario, with $k=1$. C) Advantageous selection scenario ($k>1$). The red line corresponds to the value $\hat{w} = 2.270.000$. 
Figure 3. Effect of the subsidy on public expenditure for different prevailing selection scenarios, leaving $\alpha^*$ unchanged.

One can see the effect of the subsidy on the expenditure function according to the nature of the selection without changes in the equilibrium articulation level. Each colored line represents the expenditure function for a different selection scenario. The two uppermost lines (blue and red) are the adverse selection scenarios; the middle line (green) represents the no selection scenario; and the lowermost lines (yellow and grey) represent advantageous selection scenarios. The vertical dotted line represents the value of zero subsidy. The horizontal dotted line is the estimated value of public expenditure for Colombia for 2012 ($19,607,686).
Figure 4. Effect of the subsidy on public expenditure for different levels of articulation between coverages in adverse selection scenarios.

4.A. $k = 0.33$

4.B. $k = 0.5$

The effect of the subsidy on public spending can be seen for different levels of articulation between coverages (values of alpha) in adverse selection scenarios. The dotted line represents the simulated value of public expenditure without a subsidy and with $\alpha^*$ that is unique for each selection scenario. 4.A. Scenario with the highest intensity of adverse selection ($k = 0.33$). $\alpha^* = 0.73$. The basal value of public expenditure is $20,518,706,720.64$. 4.B. Scenario with less intensity of adverse selection ($k = 0.5$). $\alpha^* = 0.71$. The basal value of public expenditure is $19,709,817,294.31$. 
Figure 5. Effect of the subsidy on public expenditure with changes in the level of articulation between coverages in advantageous selection scenarios.

5.A. $k=2$

5.B. $k=1.5$

Figure 5. One can see the effect of the subsidy on public expenditure for different levels of articulation between coverages (values of alpha) in scenarios of advantageous selection. The dotted line represents the simulated value of public expenditure without a subsidy and with $\alpha^*$ that is unique for each selection scenario; under this line there are savings in public spending. 5.A. Scenario with the highest intensity of advantageous selection ($k=2$). $\alpha^* = 0.41$. The basal value of public expenditure is $12.540.415.534.67$. 5.B. Scenario with less intensity of advantageous selection ($k=1.5$). $\alpha^* = 0.55$. The basal value of public expenditure is $14.767.419.231.73$. 
Figure 6. Distribution of the values of $h[\tilde{w}(s)]$ used in each selection scenario.

Figure 7. Overall effect of the subsidy on public expenditure
Figure 8. First effect of the subsidy on public expenditure (equation 9.1)

Figure 9. Second effect of the subsidy on public expenditure (equation 9.2)
Flowchart 1: Calibration strategy

1. Simulation of a uniform distribution for $y \phi_0$, with $0, \phi_0 \in [0,1]$.

2. Calculating functions $h(\theta, \phi)$ and $\omega(\theta, \phi)$
   - Note: $k$ represents the nature of the selection, where $k < 1$ represents adverse selection, $k = 1$ represents no selection and $k > 1$ represents advantageous selection.
   - Functions $h(\theta, \phi)$ and $\omega(\theta, \phi)$
     - $\omega(\theta, \phi) = \theta + \phi$
     - $h(\theta, \phi) = \frac{1}{1 + \lambda \omega - \theta}$
     - $\lambda = [0.33, 0.5, 1, 1.5, 2]$

3. The demand for contracts 1 and 2 are obtained from $D^1$.

4. Calculating $\tilde{D}^1(\theta, \phi, 0)$ and $\tilde{D}^2(\theta, \phi, 0)$ for each $k$ (selection scenario).

5. Setting equations (1) and (6) from the theoretical model to be equal and identifying $\alpha^*$.
   - Equation (1)
     - $\hat{\omega}(\theta, \phi, 0) = \omega(\theta, \phi) + \mu(\theta, \phi)$
   - Equation (6)
     - $\hat{p}^2(\theta, \phi, 0) = (1 - \alpha)\hat{p}^2(\theta, \phi, 0)$

6. Calculating $\alpha^*$ for each $k$
   - $\alpha^* = 1 - \left(\frac{\tilde{D}^1(\theta, \phi)}{\tilde{D}^2(\theta, \phi, 0)}\right)$

7. Calculating $\tilde{D}^1$ and $\tilde{D}^2$ for $P^1(\theta, \phi, 0)$ and $P^2(\theta, \phi, 0)$.

8. Calculating the willingness to pay for each $i$ and each $k$
   - Equation of the willingness to pay
     - $w(\theta, \phi, \psi) = \omega(\theta, \phi, \psi) + \mu(\theta, \phi, \psi) + \eta(\theta, \phi, \psi)$

9. Simulation of the effect of the subsidy on $P^2(\theta, \phi, 0)$.

Coverages from contracts
- $\tilde{D}^1 = 0.8$
- $\tilde{D}^2 = 0.9$

Observed prices
- $P^1 = 560,944$
- $P^2 = 2,270,000$

Eq. (1) $\hat{\omega}(\theta, \phi, 0) = \omega(\theta, \phi) + \mu(\theta, \phi)$
Eq. (6) $\hat{p}^2(\theta, \phi, 0) = (1 - \alpha)\hat{p}^2(\theta, \phi, 0)$
Flowchart 2: Simulation exercise

1. Simulation of the effect of the subsidy $s$ on $P^2(0,\varphi,s)$

2. Calculating $w(0,\varphi)$ for each $s$ and for each $k$, partitioning from the equality $w(0,\varphi) = P^2(0,\varphi,s)$

3. Simulation of the public expenditure function for each $s$ and each $k$.

4. Identifying the selection scenario and type of articulation for Colombia in 2012.

   Note: under these conditions, the nature of the selection is adverse and there is mixed articulation in Colombia.

5. Simulation of the effect of the subsidy and changes in the level of articulation on $G(0,\varphi,s)$

6. Simulation of the public expenditure function for each value of $\varphi$, $s$ and $k$.

7. Identifying scenarios where a subsidy and a change in articulation between coverages reduces public expenditure.

   Note: there are different options in which a subsidy can reduce public expenditure, these correspond to a combination of changes in the level of articulation, towards more supplementary scenarios, together with a subsidy level. It is more likely for subsidy to reduce expenditure in adverse selection scenarios.

Equation $P^2(0,\varphi,s)$

$$P^2(0,\varphi,s) = (1 - \alpha)(\varphi^2 + P^2(0,\varphi,0)) - s$$

Con $s = \{-20000, -10000, -50000, -20000, -10000, 0, 10000, 20000, 50000, 100000, 200000\}$

Equation $G(0,\varphi,s)$

$$G(0,\varphi,s) = X^T C^T U^T I(0,\varphi,s)[1 - D^2(0,\varphi,s)] + \alpha X^T C^T U^T D^2(0,\varphi,s) + sD^2(0,\varphi,s)$$

With $\alpha = \{0.018, 0.36, 0.53, 0.71, 0.89\}$

Total observed expenditure $G(0,\varphi,0) = 19,607,606$
Appendix 1: Derivative of the expenditure function with respect to the subsidy

The expenditure function is:

\[ G(\theta, \varphi, s) = X^1 \cdot C \cdot u^1(\theta, \varphi, s)[1 - D^2(\theta, \varphi, s)] + \alpha \cdot X^2 \cdot C^2 \cdot u^2(\theta, \varphi, s) \cdot D^2(\theta, \varphi, s) + s \cdot [D^2(\theta, \varphi, s)]. \]

As in the previous case, we write the function in terms of \( s \), therefore:

\[
\frac{dG(s)}{ds} = X^1 \cdot C^1 \left[ F[\hat{\omega}(s)] \frac{d\tilde{h}^1(s)}{ds} + f[\tilde{\omega}(s)] \frac{d\tilde{h}^1(s)}{ds} \right] \\
+ \alpha X^2 \cdot C^2 \left[ (1 - F[\hat{\omega}(s)]) \frac{d\tilde{h}^2(s)}{ds} - f[\tilde{\omega}(s)] \frac{d\tilde{h}^2(s)}{ds} \right] \\
+ 1 - F[\tilde{\omega}(s)] - f[\tilde{\omega}(s)] \frac{d\tilde{\omega}(s)}{ds} \cdot s
\]

Adding and subtracting the term \( X^1 \cdot C^1 \cdot \frac{d\hat{\omega}(s)}{ds} \left[ \tilde{h}^2(s)(1 - f[\tilde{\omega}(s)]) - h[\tilde{\omega}(s)] \right] \), one finds:

\[
\frac{dG(s)}{ds} = X^1 \cdot C^1 \cdot \frac{d\hat{\omega}(s)}{ds} \left[ (1 - f[\tilde{\omega}(s)]) \left( \tilde{h}^2(s) - \tilde{h}^1(s) \right) \right] \\
+ \frac{d\hat{\omega}(s)}{ds} \left[ \tilde{h}^2(s)(1 - f[\tilde{\omega}(s)]) - h[\tilde{\omega}(s)] \right] \left[ \alpha X^2 \cdot C^2 - X^1 \cdot C^1 \right] \\
+ 1 - F[\tilde{\omega}(s)] - f[\tilde{\omega}(s)] \frac{d\tilde{\omega}(s)}{ds} \cdot s.
\]
Appendix 2: Simulations of scenarios with different probability distributions

Inverse transform sampling is used to randomly generated data with a uniform distribution for θ and φ, with θ φ ∈ [0,1]. Based on this data, simulation is done in Excel for different distributions with parameters known for these two dimensions which characterize individuals.

The interpretation of the results of the simulation with different distributions does not differ very much from those of a uniform distribution. It can be summarized in the following way:

1. The degree of articulation between coverages is of mixed characteristics, which varies depending on the proposed scenario.
2. Regarding public expenditure, the scenario that is closest to the observed results for Colombia is the adverse selection scenario.
3. Only a change in the articulation policy between coverages, together with the presence of subsidies can reduce total public spending. The range of possibilities is wider in adverse selection scenarios.

The main results of these simulations are described below.
BETA DISTRIBUTION (Parameter \(a=0.4\) y parameter \(b=0.5\))

Description of the simulated data

<table>
<thead>
<tr>
<th>Parameter k</th>
<th>Adverse selection</th>
<th>No selection</th>
<th>Advantageous selection</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.33</td>
<td>0.5</td>
<td>1</td>
<td>1.5</td>
</tr>
</tbody>
</table>

\[ \begin{array}{cccc}
\bar{h}^2(\theta, \varphi, 0) & 0.646 & 0.611 & 0.500 & 0.390 & 0.291 \\
\bar{h}^3(\theta, \varphi, 0) & 0.563 & 0.546 & 0.499 & 0.456 & 0.417 \\
\alpha^* & 0.720 & 0.703 & 0.638 & 0.535 & 0.377 \\
\end{array} \]

\[ \begin{array}{cccc}
\text{CU}_2 & \$ 13,921,532.83 & \$ 13,921,532.83 & \$ 13,921,532.83 & \$ 13,921,532.83 & \$ 13,921,532.83 \\
\text{CU}_1 & \$ 1,265,796.51 & \$ 1,303,776.40 & \$ 1,427,075.58 & \$ 1,564,121.26 & \$ 1,709,350.23 \\
\end{array} \]

\(\alpha^*\): unique level of articulation, for each type of selection, which solves the observed equilibrium price in the Colombian market for 2012.

Effect of the subsidy on public expenditure for each prevailing selection scenario, leaving \(\alpha^*\) unchanged.
Effect of the subsidy on public expenditure with changes in the level of articulation between coverages in adverse selection scenarios
Effect of the subsidy on public expenditure with changes in the level of articulation between coverages in advantageous selection scenarios
EXPONENTIAL DISTRIBUTION (Parameter Lambda=1.1)

Description of the simulated data

<table>
<thead>
<tr>
<th>Parameter k</th>
<th>Adverse selection</th>
<th>No selection</th>
<th>Advantageous selection</th>
</tr>
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<td>0.33</td>
<td>0.33</td>
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<td>1</td>
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<td>h^2(θ, φ, 0)</td>
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<td>0.616</td>
<td>0.501</td>
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<td>h^1(θ, φ, 0)</td>
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<td>0.500</td>
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<tr>
<td>α^*</td>
<td>0.703</td>
<td>0.686</td>
<td>0.614</td>
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<td>CU^2</td>
<td>$13,033,152.93</td>
<td>$13,033,152.93</td>
<td>$13,033,152.93</td>
</tr>
<tr>
<td>CU^1</td>
<td>$1,180,691.21</td>
<td>$1,233,848.90</td>
<td>$1,425,722.33</td>
</tr>
</tbody>
</table>

α^*: unique level of articulation, for each type of selection, which solves the observed equilibrium price in the Colombian market for 2012.

Effect of the subsidy on public expenditure for each prevailing selection scenario, leaving α^* unchanged.
Effect of the subsidy on public expenditure with changes in the level of articulation between coverages in adverse selection scenarios
Effect of the subsidy on public expenditure with changes in the level of articulation between coverages in advantageous selection scenarios
NORMAL DISTRIBUTION (Average=1.1 and standard deviation=0.2)

Description of the simulated data

<table>
<thead>
<tr>
<th>Parameter k</th>
<th>Adverse selection</th>
<th>No selection</th>
<th>Advantageous selection</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>0.33</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>$\hat{h}_2(\theta, \varphi, 0)$</td>
<td>0.680</td>
<td>0.637</td>
<td>0.500</td>
</tr>
<tr>
<td>$\hat{h}_3(\theta, \varphi, 0)$</td>
<td>0.659</td>
<td>0.620</td>
<td>0.500</td>
</tr>
<tr>
<td>$\alpha^*$</td>
<td>0.643</td>
<td>0.619</td>
<td>0.515</td>
</tr>
<tr>
<td>$CU^1$</td>
<td>$1,081,147.30$</td>
<td>$1,148,461.42$</td>
<td>$1,425,824.54$</td>
</tr>
</tbody>
</table>

$\alpha^*$: unique level of articulation, for each type of selection, which solves the observed equilibrium price in the Colombian market for 2012.

Effect of the subsidy on public expenditure for each prevailing selection scenario, leaving $\alpha^*$ unchanged.
Effect of the subsidy on public expenditure with changes in the level of articulation between coverages in adverse selection scenarios
Effect of the subsidy on public expenditure with changes in the level of articulation between coverages in advantageous selection scenarios
GAMMA DISTRIBUTION (Parameter $a=0.5$ and parameter $b=1.5$)

Description of the simulated data

<table>
<thead>
<tr>
<th>Parameter $k$</th>
<th>Adverse selection</th>
<th>No selection</th>
<th>Advantageous selection</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0,33</td>
<td>0,5</td>
<td>1</td>
</tr>
<tr>
<td>$h^2(\theta,\varphi,0)$</td>
<td>0,670</td>
<td>0,614</td>
<td>0,507</td>
</tr>
<tr>
<td>$h^1(\theta,\varphi,0)$</td>
<td>0,563</td>
<td>0,546</td>
<td>0,502</td>
</tr>
<tr>
<td>$\alpha^*$</td>
<td>0,515</td>
<td>0,470</td>
<td>0,359</td>
</tr>
<tr>
<td>$CU^1$</td>
<td>$1.266.664,85$</td>
<td>$1.305.106,11$</td>
<td>$1.419.849,59$</td>
</tr>
</tbody>
</table>

$\alpha^*$: unique level of articulation, for each type of selection, which solves the observed equilibrium price in the Colombian market for 2012.

Effect of the subsidy on public expenditure for each prevailing selection scenario, leaving $\alpha^*$ unchanged.
Effect of the subsidy on public expenditure with changes in the level of articulation between coverages in adverse selection scenarios
Effect of the subsidy on public expenditure with changes in the level of articulation between coverages in advantageous selection scenarios