Endogenous Uncertainty and Credit Crunches*

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Abstract

We develop a theory of endogenous uncertainty where the ability of investors to learn about firm-level fundamentals declines during financial crises. At the same time, higher uncertainty reinforces financial distress of firms, giving rise to "belief traps"—a persistent cycle of uncertainty, pessimistic expectations, and financial constraints, through which a temporary shortage of funds can develop into a long-lasting funding problem for firms. At the macro-level, belief traps can explain why financial crises can result in long-lasting recessions. In our model, financial crises are characterized by high levels of credit misallocation, an increased cross-sectional dispersion of growth rates, endogenously increased pessimism, uncertainty and disagreement among investors, highly volatile asset prices, and high risk premia. A calibration of our model to U.S. micro data on investor beliefs explains a considerable fraction of the slow recovery after the 08/09 crisis.

Keywords: Belief traps, credit crunches, dispersed information, endogenous uncertainty, internal persistence of financial shocks, resource misallocation.

JEL Classification: D83, E32, E44, G01.

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1 Introduction

Financial crises often entail deep and long-lasting recessions (Reinhart and Rogoff, 2009; Hall, 2014; Ball, 2014). A common view gives a central role to uncertainty as both an amplifier of financial pressure and a source for the persistence in the decline of economic activity following the post-crisis recession.[1] This paper develops a theory that formalizes the interaction between financial crises and uncertainty expressed in these views.

The theory provides a narrative on how a temporary disturbance of the financial sector is reinforced and amplified by endogenously increasing uncertainty, developing into a long-lasting crisis of the real economy. The theory is consistent with a number of stylized facts regarding the 2008/09 recession: (i) increased resource misallocation, accounted for by an increase in the labor wedge and a drop in measured productivity; (ii) highly volatile asset prices and high risk-premia; (iii) an increased cross-sectional dispersion (and skewness) of firm growth-rates (e.g., Bloom et al. 2014; Salgado, Guvenen and Bloom 2015); (iv) the contemporaneous increase in measured uncertainty[2]; and (v) forecasts and expectations marked by high levels of pessimism (relative to the true state of nature) as well as high levels of disagreement among forecasters (Senga, 2015).

In the theory, firms are financed by investors subject to a maximal line of credit that is based on how investors assess a firm's business conditions. If investors find it likely that a firm is productive, credit to that firm will be more generous than when investors are pessimistic and uncertain regarding a firm's business potential. The key friction is that investors have only limited information about which firms are productive in the cross-section. The cross-sectional allocation of credit is hence governed by the ability of investors to learn the productivities of firms. Their learning is based on various statistics that include information relating to the production and investment of firms.

In this environment, a temporary disturbance of a firm's funding can trigger a persistent (and perfectly rational) spiral of uncertainty, coupled with pessimistic expectations and tight credit, that can persistently disrupt the firm's access to funds. This is because, when a firm's production becomes dictated by financial constraints, it is less revealing of the underlying productivities, impeding the ability of investors to learn the firm's productivity. As a result, investors' views remain uncertain and previous forecast errors persist into the future, opening the door to "belief traps"—a self-reinforcing disruption of external funding driven by pessimistic and uncertain investors' beliefs.

The persistence of financial disturbances at the firm-level implies a similar persistence of aggregate

[1] For example, Olivier Blanchard (2009) speculated at the height of the recent financial crisis that "the crisis would largely go away" if it were not for uncertainty, whereas Bloom et al. (2014) document how uncertainty was repeatedly recognized by the Federal Open Market Committee as a key driver of both, the recession that followed the dot-com bubble in 2001, and the recent Great Recession. An increasing number of empirical studies further substantiates these ideas, pointing to the Great Recession being likely "an acute manifestation of the toxic interaction between uncertainty and financial shocks" (Caldara et al., 2016; see also Stein and Stone, 2013, Stock and Watson, 2012, Caldara et al., 2013, and Gilchrist, Sim and Zakrajšek, 2014).

[2] Unusually high levels of uncertainty during the recent financial crisis have been documented using a variety of different approaches (see, e.g., Jurado, Ludvigson and Ng, 2014, Born, Breuer and Elstner, 2014, and the studies cited in Footnote 1). Most closely related to the concept of uncertainty explored in this paper is the forecast-based evidence given in Senga (2015), which documents a sharply increasing uncertainty among financial analysts regarding firm-level business conditions.

financial shocks. In particular, an aggregate shock to firms' funding that increases the fraction of firms that are financially constrained reduces the average quality of information among investors, leading to a rise in credit misallocation. The misallocation manifests itself through an increase in the economy's efficiency and labor wedges that may persist even after financial stress has subsided.[3] At the same time, the interaction between investors' uncertainty and firms' funding can account for an increase in investors' average disagreement and pessimism (even though all signals are unbiased), an increase in asset price volatility, and a divergence (and skewness) in firm growth-rates.

We explore the quantitative potential of our model in a simple calibration to the U.S. economy. A novel aspect of our calibration is the explicit use of micro data on analyst forecasts made at the firm-level that mirrors the learning problem of investors in our model. Equipped with these data we use a similar approach as in David, Hopenhayn and Venkateswaran (2015) and construct a number of target moments that pin down the information parameters in our model.

We conduct two experiments in the calibrated model. First, we explore how a temporary credit shock (with a half-life of four quarters) propagates through the economy, and then compare it with counterfactual responses, where we keep investor uncertainty constant. While in the constant-uncertainty counterfactual such a credit shock produces a short-lived recession with a half-life of 2 quarters, the same shock produces a long-lasting recession with a half-life of 10 quarters in the economy with endogenous uncertainty.[4] We interpret the difference between these two responses as the contribution of belief traps to the internal persistence of financial shocks.

Second, we compare the quantitative predictions of our calibrated model with U.S. data from the 2008/09 financial crisis. To do this, we feed our model data from the St. Louis Fed Financial Stress Index to capture the relatively short-lived distress within the financial sector. We scale the magnitude of the shock to match a fraction of 20 percent of firms that has reported to be constrained by financial factors in the third quarter of 2008 (Campello, Graham and Harvey, 2010). Comparing the resulting model responses to U.S. data, our model matches the observed series quite well. In particular, the model-implied efficiency wedge is able to account for 73 percent of the observed drop in TFP, and, in combination with a countercyclical rise in the labor wedge, explains 67 percent of the observed drop in output. Qualitatively, the model also matches the dynamics of analysts' disagreement and expectations, asset price volatility, and the cross-sectional dispersion of growth rates.

At a methodological level, uncertainty is state-dependent in this paper because learning from financially constrained firms gives rise to a *nonlinear* signal structure, where, all else equal, signals about more constrained firms' fundamentals are less informative. In a related contribution (Straub

[3] For simplicity, the model abstracts from physical capital and instead works with a working-capital requirement. In a richer version of our model, credit constraints would manifest themselves as investment wedges as well.

[4] The half-life in the counterfactual economy is smaller than that of the exogenous shock since in our model the propagation of financial shocks is generally convex; that is, small "day-to-day" fluctuations in financial markets have only little impact on the real economy, whereas rare "tail" events cause pronounced recessions. Accordingly, absent an additional source for persistence, a given rate of recovery in the financial market translates to a *faster* rate of recovery of the real economy. This effect is offset in the model with endogenous uncertainty due to the additional internal persistence generated by belief traps.

and Ulbricht, 2015), we show that signal nonlinearities generally imply state-dependent posterior uncertainty. One technical challenge in analyzing the dynamic properties of our model is that nonlinear Gaussian signal structures do not pair with any (reasonable) conjugate prior distribution. In this paper, we address this issue by developing a simple approximative approach, which captures the key features of the nonlinear learning problem while preserving tractability.

Related literature Our paper is related to a large and growing literature that introduces dispersed information into macroeconomics (e.g., Lorenzoni, 2009; Angeletos and La'O, 2010, 2013; Amador and Weill, 2010, 2012; Maćkowiak and Wiederholt, 2015; Hassan and Mertens, 2014*a*,*b*; Acharya, 2013; Hellwig and Venkateswaran, 2014; Chahrour and Gaballo, 2015). La'O (2010) shares with us the combination of information heterogeneities with financial frictions, but considers a static model with a constant level of uncertainty. David, Hopenhayn and Venkateswaran (2015) also analyze information frictions as a source for factor misallocation, but focus on long-run consequences rather than fluctuations driven by financial shocks.

Our paper also contributes to a recent literature that explores the role of endogenous fluctuations in uncertainty for business cycles, including van Nieuwerburgh and Veldkamp (2006), Ordoñez (2013), and Fajgelbaum, Schaal and Taschereau-Dumouchel (2015).[5] In these papers, the level of aggregate investment determines the amount of information and hence aggregate uncertainty. An important distinction relative to these papers is this paper's focus on uncertainty regarding firm-specific fundamentals rather than economy aggregates (see Senga 2015 for a similar approach).[6] On the one hand, this allows us to explain the above-mentioned stylized facts relating to the cross-sectional distribution of firms and investor beliefs. On the other hand, this also helps overcoming an important challenge of the endogenous uncertainty literature; namely that often unrealistically large fluctuations in uncertainty are needed to generate a significant amplification. In our model, learning breaks down when *a firm* is constrained, not when aggregate economic activity comes to a stand-still, implying that the aggregate economy scales approximately proportional with the fraction of firms being constrained. Accordingly, even small variations in average uncertainty, can have severe consequences.

A second difference to the existing endogenous uncertainty literature is that this paper links *financial* crises and uncertainty through a novel mechanism, explaining why high levels of uncertainty are particularly prevalent during financial crises.[7] In our model, it is not the level of economic activity that determines how much information about firms' fundamentals is revealed; rather it is

[5] Studies of endogenous uncertainty in financial market settings include Veldkamp (2005), Yuan (2005), Albagli (2011), and Sockin (2015). However, none of these papers considers spillovers from financial distress on the real economy that are at the core of this paper.

[6] In a previous version of this paper, we have explored a version of our model where investors are learning about aggregate business conditions instead (Straub and Ulbricht, 2012). While the two versions of the model are qualitatively similar, we argue here that learning at a firm-level is more likely to be quantitatively important.

[7] For instance, measuring uncertainty using the average disagreement among analysts over firm-specific earnings forecasts (using data from the IBES database, see Appendix B for details), the contemporaneous correlation of HP-filtered ($\lambda = 1600$) output and uncertainty is +0.37 between 1985 and 2006, which stands in stark contrast to the sharp increase of uncertainty at the wake of the Great recession to +88 percent compared to its pre-crisis average.

the degree to which firms' actions (investments, employment, production, etc) actually reflect these fundamentals. This insight naturally implies that actions from financially constrained firms carry less information than actions from unconstrained firms.

In our model, the emergence of uncertainty from financial distress interacts with a propagation of uncertainty through the financial sector. In support of such a financial transmission channel, Caldara et al. (2013) and Gilchrist, Sim and Zakrajšek (2014) present evidence that uncertainty strongly affects investments *via* increasing credit spreads, but has virtually no impact on investments when controlling for credit spreads. The financial transmission of uncertainty relates our model to a recent literature around Christiano, Motto and Rostagno (2014), Arellano, Bai and Kehoe (2012), and Gilchrist, Sim and Zakrajšek (2014), which stresses the importance of uncertainty (or risk) shocks in the financial sector, but treats these shocks as exogenous.[8]

The predictions of our model are also broadly consistent with a recent empirical literature that measures the effects of financial constraints. Giroud and Mueller (2015) show that establishments of firms that are more likely to be financially constrained were heavily affected by falling collateral values (house prices). In fact, they show that the entire correlation of employment loss and house prices is explained by these arguably financially constrained firms. Similar in spirit, Chodorow-Reich (2013) documents that firms borrowing from less healthy lenders experience relatively steeper declines in employment during the financial crisis, consistent with the interpretation that these firms faced tighter financial constraints (see also Chaney, Sraer and Thesmar, 2012). Our model clarifies how an intense but relatively short-lived financial crisis can still translate into *persistent* financial constraints for firms, making it much harder for them to weather such periods and retain their employment and capital.

Outline The plan for the rest of the paper is as follows. The next section introduces the model economy. Section 3 characterizes the equilibrium. Section 4 studies the workings of belief traps. Section 5 analyzes the model's response to aggregate shocks and compares it to data on the 2008/09 financial crisis. Section 6 concludes and offers a few policy insights.

2 Model

Our model is built on a standard real business cycle model without capital in which there is a continuum of monopolistically competitive firms producing consumption varieties. Firms are organized into a continuum of islands where each island is home to its own continuum of firms. The model deviates from this standard setup in two respects: First, firms produce subject to a working capital constraint, where the tightness of the constraint depends on how "local" (island-specific)

[8] Two other related strands of the literature study the propagation of exogenous uncertainty through real options as in Bloom (2009), Bloom et al. (2014), and Bachmann and Bayer (2009, 2013), and through risk premia as in the time-varying (disaster) risk literature (e.g., Gabaix, 2012; Gourio, 2012). Related to the latter, Kozlowski, Veldkamp and Venkateswaran (2015) explore a model where agents learn about tail-risks and where belief revisions after short-lived financial shocks can have long-lasting effects. Similar, Nimark (2014) presents a mechanism that increases uncertainty after rare events, if news selectively focus on outliers.

fundamentals are perceived by investors. Second, in order to form these expectations, investors have access to only limited information about each island's business fundamentals. Time is discrete and indexed by $t \in \{0, 1, 2, \dots\}$. Islands are indexed by $i \in I$. Firms are indexed by $(i, j) \in I \times J$, where both islands and firms have a unit mass.

Households The preferences of the representative household are given by

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t U(C_t, N_t), \tag{1}$$

with separable isoelastic preferences over consumption C_t and labor supply N_t,

$$U(C_t, N_t) = \log C_t - \frac{1}{1+\zeta} N_t^{1+\zeta},$$

where $\zeta \geq 0$ is the inverse of the Frisch elasticity of labor supply and $\beta \in (0, 1)$ is the discount factor. C_t is a composite consumption good given by

$$C_t = \left[\int_{I \times J} C_{ij,t}^{\frac{\xi-1}{\xi}} \, \mathrm{d}(i,j) \right]^{\frac{\xi}{\xi-1}},$$

where $C_{ij,t}$ is the consumption of good (i, j) at time t, and $\xi > 1$.

The representative household provides firms with riskless loans $L_{ij,t}$ subject to a working capital constraint (further described below). For simplicity, these loans are assumed to be made within periods, implying an infinitely elastic credit supply to unconstrained firms and a risk-free rate of $R_t = 1$. The budget constraint of the household is

$$\int_{I \times J} P_{ij,t} C_{ij,t} \, \mathrm{d}(i,j) \leq W_t N_t + (R_t - 1) \int_{I \times J} L_{ij,t} \, \mathrm{d}(i,j) + \int_{I \times J} \Pi_{ij,t} \, \mathrm{d}(i,j), \tag{2}$$

where $P_{ij,t}$ is the price of good (i, j), W_t is the nominal wage rate, and $\Pi_{ij,t}$ are profits of firm (i, j).

In equilibrium households choose consumption, loans, and hours worked to maximize expected utility subject to their budget constraint. From the household's optimization problem it follows that demand for good (i, j) is

$$C_{ij,t} = \left(\frac{P_{ij,t}}{P_t} \right)^{-\xi} C_t, \tag{3}$$

where

$$P_t = \left[\int_{I \times J} P_{ij,t}^{1-\xi} \, \mathrm{d}(i,j) \right]^{1/(1-\xi)}$$

is the economy-wide price index. Throughout, we normalize $P_t = 1$, defining the composite consumption good to be the numeraire.

Firms Each good $(i,j) \in I \times J$ is produced by a monopolistically competitive firm which has access to a linear production technology

$$Y_{ij,t} = A_{ij,t} N_{ij,t}, \tag{4}$$

where $A_{ij,t}$ is the firm's productivity, and $N_{ij,t}$ is the firm's employment. Wages must be paid before production takes place and are financed by within-period working capital loans of size $L_{ij,t} = W_t N_{ij,t}$.[9] We assume that similar to e.g. Jermann and Quadrini (2012) the total amount of loans is limited by the market value of pledgable revenues. Keeping with the static nature of the firms' decision problem, we restrict pledgability of revenues to the current period, yielding the constraint

$$L_{ij,t} \leq \chi_t Q_{ij,t}, \tag{5}$$

where $Q_{ij,t}$ is the market valuation of firm (i,j)'s revenues in period t and χ_t is the pledgable fraction of current period revenues. Profits $\Pi_{ij,t} = P_{ij,t} Y_{ij,t} - W_t N_{ij,t}$ are distributed to households at the end of each time period.

In our model, we abstract from physical capital as a factor of production. Together with assuming that lending takes place *within* periods, this improves tractability and further sharpens our predictions by ruling out any source of persistence beyond the information channel explored in this paper.

Productivities Productivities have an island-specific component $A_{i,t}$ and a firm-specific component $\epsilon_{ij,t}$. There is no uncertainty about the economy-wide aggregate productivity distribution.

Conditional on $A_{i,t}$, within-island productivities are i.i.d. and log-normally distributed:

$$\log A_{ij,t} = \log A_{i,t} + \epsilon_{ij,t},$$

where $\epsilon_{ij,t} \sim \mathcal{N}(0, \sigma_\epsilon^2)$. As becomes clear below, this within-island dispersion is introduced for technical reasons only and should be thought of as being small. With this in mind, our preferred interpretation is that one island corresponds to one firm in the data. The remainder of the setup is tailored towards this interpretation, placing the island-specific component of firms' productivities at the center of our analysis. It is also worth noting that variations in local productivities $A_{i,t}$ can be equivalently interpreted as shifts in relative demand across islands and are more generally meant as a stand-in for a variety of shocks that shape local business conditions. Log-productivities follow an AR(1) process with

$$\log A_{i,t} \sim \mathcal{N}(\rho_a \log a_{i,t-1}, \sigma_a^2).$$

[9] While we refer to the funding of firms as credit throughout this paper, the story is also consistent with other means of finance such as corporate bonds or equity finance (see also our discussion below). Since our model does not allow for firm entry and exit, we do abstract, however, from the possibility of firms saving their way out of constraints. See Cooley and Quadrini (2001), Khan and Thomas (2013), Moll (2014), and Siemer (2014) for studies exploring firm dynamics in the presence of financial constraints.

Household and firms' information For simplicity, we assume that the realization of current and past aggregate variables in the economy are common knowledge across all agents. Moreover, we further assume that $A_{ij,t}$ is known by firm (i,j) at date t. In consequence, both the household and firms have access to all relevant information, so that they follow the same decision rules as they would under full information. The only information friction in our model concerns the information available to financial markets regarding the *local* productivities of firms.[10]

Financial market To complete the description of the model, we need to specify the pricing of firms' revenues that determine the credit limits $\bar{A}_{i,t}$. To this end, we introduce a continuum of one-period-lived investors of mass m that trade a fraction m of each firms' current period revenues in an island-specific financial market. To improve the tractability of our model, we assume that m is small, $m \to 0$.

Specifically, an investor born into the financial market on island i is endowed with a claim on $1/2$ of the revenues of each firm on island i. Investors choose their portfolios to maximize $\mathbb{E}\{c_{ik,t}^{1-\gamma}|\mathcal{I}_{ik,t}\}$ where

$$c_{ik,t} = \int_J \left(\frac{1}{2}Q_{ij,t} + x_{ik,j,t}\left[P_{ij,t}Y_{ij,t} - Q_{ij,t}\right]\right) \mathrm{d}j \tag{6}$$

is investor (i,k)'s end-of-period consumption, $x_{ik,j,t}$ is her asset position in firm (i,j), and $\mathcal{I}_{ik,t}$ is her information set. For simplicity, we abstract from firms' strategically selling claims, limiting the supply of assets to $(1-\eta_{i,t})m$, where

$$\eta_{i,t} \sim \mathcal{N}(\rho_\eta \eta_{i,t-1}, \sigma_\eta^2)$$

is an island-specific noise term that prevents asset prices from perfectly revealing the private information of investors. In equilibrium, $\eta_{i,t}$ will introduce an exogenous variation in the working capital constraint across islands, which we interpret more generally as heterogeneity in firms' dependence on external funds.

We are left to specify the investors' information sets. In line with our interpretation of islands as firms, we rule out any means to differentiate between firms within an island. The critical assumption is that investors do not directly observe the local (average) productivities $\{A_{i,t}\}$, but form expectations on the basis of three distinct signals regarding that island's productivity. First, for each island, investors observe the public history of working capital investments perturbed by some noise. In particular, letting $L_{i,t} = \int_J L_{ij,t}\,\mathrm{d}j$ denote investments in island i at date t, investors observe the history of signals $\{s_{i,s}^1\}_{s\leq t}$, where

$$s_{i,t}^1 = \log L_{i,t-1} + \nu_{i,t}^1$$

[10]The implicit assumption here is that firms cannot *credibly* signal their private information to investors. This friction is commonplace in many models of financial markets, e.g., Townsend (1979) or Stiglitz and Weiss (1981). In an earlier version of this paper, Straub and Ulbricht (2012), we develop a similar model without this kind of asymmetric information, where learning is instead with respect to aggregate, rather than island-specific, business conditions.

with $\nu^1_{i,t} \sim \mathcal{N}\left(0, \sigma^2_{s,1}\right)$. This is meant as a stand-in for observing news about investments and other production-related signals, which are often thought to be valuable indicators about productivities and other fundamentals.

Second, investor (i,k) has access to a private signal $s^2_{ik,t}$ that directly communicates $A_{i,t}$ perturbed by some noise,

$$s^2_{ik,t} = \log A_{i,t} + \nu^2_{ik,t}$$

with $\nu^2_{ik,t} \sim \mathcal{N}(0, \sigma^2_{s,2})$. This signal is meant as a stand-in for all direct information about $A_{i,t}$ as well as endogenous sources (e.g., information gained from observing output, employment, etc) that have a precision that is unaffected by the credit channel that we highlight in this paper. Since investors are short-lived, their private information does not persist from one period to the next, allowing us to avoid dealing with Townsend's (1983) infinite regress problem.

Finally, investors observe the history of asset prices $Q_{i,t}$ on island i up to date t, which endogenously aggregates some of the information dispersed across investors. Here we anticipated that in equilibrium asset prices will be identical across firms within any given island, allowing us to drop the j-subscript from $Q_{ij,t}$.

In sum, the information set available to investor (i,k) is given by

$$\mathcal{I}_{ik,t} = \{s^1_{i,s}, Q_{i,s}, \chi_s\}_{s \leq t} \cup \{s^2_{ik,t}\},$$

where the history of aggregate credit supply shocks $\{\chi_s\}^t_{s=0}$ is a sufficient statistic for the aggregate state of the economy (so we can w.l.o.g. abstract from information of investor (i,k) regarding other islands than i.)

Discussion We wish to discuss three crucial assumptions in our model. First, we do not allow investors to trade assets across island borders, even though they would want to do so for insurance purposes. This assumption is a way of implementing a kind of "expert system" where traders are required to have enough skin in the game, e.g. for monitoring purposes (see the recent survey by Brunnermeier, Eisenbach and Sannikov (2012) for other kinds of skin-in-the-game constraints). The assumption ensures that in equilibrium there is a risk-premium on island-specific risks.

Second, firms in our model face a limited pledgability constraint that prevents them from taking full advantage of positive productivity shocks. We share this assumption with a long literature on explicit credit constraints (see Moll (2014) or Buera and Moll (2015) for recent examples). While this assumption is commonly used to describe firms in developing countries, we think it is crucial for developed countries' firms as well, in the spirit of the recent empirical evidence by Campello, Graham and Harvey, 2010; Giroud and Mueller, 2015; and Chaney, Sraer and Thesmar, 2012.

Finally, firms in our model are financed each period by pledging a fraction of their revenues that is then traded in a financial market. While this is, of course, overly simplistic, what ultimately matters for our model is that a firm's funding supply depends on market beliefs about firm fundamentals. Our model achieves this in an admittedly crude but practical way, without having to specify a full-blown banking sector.

Definition (competitive equilibrium) Given a stochastic process for $\{A_{ij,t}, \chi_t, \eta_{i,t}, \nu^1_{i,t}, \nu^2_{ik,t}\}$ a *competitive equilibrium* consists of stochastic processes for $\{N_t, C_{ij,t}, N_{ij,t}, Y_{ij,t}, L_{ij,t}, x_{ik,j,t}\}$ together with the associate price processes $\{W_t, R_t, P_{ij,t}, Q_{ij,t}\}$, such that:

1. the process for $\{N_t, C_{ij,t} L_{ij,t}\}$ maximizes (1) subject to (2) and (5);
2. the process for $\{N_{ij,t}\}$ maximizes firms' profits subject to (3)–(5);
3. the process for $\{x_{ik,j,t}\}$ maximizes $\mathbb{E}\{c^{1-\gamma}_{ik,t} | \mathcal{I}_{ik,t}\}$ subject to (6);
4. all product, labor, loan, and asset markets clear.

3 Equilibrium characterization

In this section, we provide a recursive characterization of equilibrium. We first characterize allocations in the product markets conditionally on working capital constraints. Then, we characterize working capital constraints as a function of investors' beliefs, and further describe the equilibrium law of motion for investors' beliefs. The interaction between working capital constraints and investors' beliefs is subsequently explored in Section 4.

3.1 Product markets

The characterization of the product markets is facilitated by the purely backward-looking structure of our model. In particular, from the firms' optimization problem it follows that firms access working capital loans

$$L_{ij,t} = \min\{A_{ij,t}, \bar{A}_{ij,t}\}^{\xi-1}\, \Omega_t, \tag{7}$$

where

$$\Omega_t = \left(\frac{\xi - 1}{\xi}\right)^{\xi} \frac{C_t}{W_t^{\xi-1}}$$

summarizes the aggregate state of the economy, and where

$$\bar{A}_{ij,t} = \left(\frac{\chi_t Q_{ij,t}}{\Omega_t}\right)^{1/(\xi-1)} \tag{8}$$

denotes the working capital constraint in terms of "productivity-units". Let $R_{ij,t}$ denote the shadow rate at which firms value additional funds[11],

$$R_{ij,t} = \max\left\{1, \left(\frac{A_{ij,t}}{\bar{A}_{i,t}}\right)^{(\xi-1)/\xi}\right\}. \tag{9}$$

[11]That is, $R_{ij,t} = 1 + \lambda_{ij,t}$ were $\lambda_{ij,t}$ is the multiplier on the credit limit $\chi Q_{ij,t}$. Equivalently, $R_{ij,t}$ is precisely the rate at which a firm would borrow if there were a competitive, firm-specific credit market with a limited supply of $\chi Q_{ij,t}$.

Given these definitions, economy-wide output and employment can be expressed purely in terms of the cross-sectional distribution over $(A_{ij,t}, R_{ij,t})$.

Proposition 1. *In equilibrium, economy-wide hours and output are given by*

$$N_t = (1 - \tau_t^N)^{1/(1+\zeta)}$$
$$Y_t = (1 - \tau_t^A) A^{\text{eff}} N_t,$$

with labor and efficiency wedge,

$$1 - \tau_t^N = \frac{\xi - 1}{\xi} \frac{\int_{I \times J} A_{ij,t}^{\xi-1} R_{ij,t}^{-\xi} \,\mathrm{d}(i,j)}{\int_{I \times J} A_{ij,t}^{\xi-1} R_{ij,t}^{1-\xi} \,\mathrm{d}(i,j)}$$
$$1 - \tau_t^A = \frac{1}{A^{\text{eff}}} \frac{\left(\int_{I \times J} A_{ij,t}^{\xi-1} R_{ij,t}^{1-\xi} \,\mathrm{d}(i,j)\right)^{\xi/(\xi-1)}}{\int_{I \times J} A_{ij,t}^{\xi-1} R_{ij,t}^{-\xi} \,\mathrm{d}(i,j)},$$

where

$$A^{\text{eff}} = \left[\int_{I \times J} A_{ij,t}^{\xi-1} \,\mathrm{d}(i,j)\right]^{1/(\xi-1)}$$

is the efficient productivity level that would obtain if the marginal products of labor were equalized across all firms.

Binding working capital constraints (or, equivalently, $R_{ij,t} > R_t = 1$) manifest themselves through a labor wedge τ_t^N and efficiency wedge τ_t^A.[12] A positive labor wedge reflects an inefficiently low labor demand by firms whose working capital constraint is binding. The efficiency wedge in turn reflects that in the presence of credit constraints marginal productivities are not equalized across firms, decreasing the effective (Solow) productivity in the economy through credit misallocation. If all firms were unconstrained (i.e., $R_{ij,t} = R_t$ for all firms), then the economy would only face the usual labor wedge due to monopolistic competition ($\tau_t^N = 1/\xi$) and no efficiency wedge ($\tau_t^A = 0$). However, with heterogeneity within islands, $\sigma_\epsilon > 0$, there are always some firms that are constrained in the cross-section, so we generally have that $\tau_t^A > 0$ and $\tau_t^N > 1/\xi$.

Substituting out $R_{ij,t}$, economy-wide output and employment can be written as a function of $\{A_{ij,t}, \bar{A}_{ij,t}\}$. While the distribution over $A_{ij,t}$ is exogenous, the distribution over $\bar{A}_{ij,t}$ is endogenous (it depends on $Q_{ij,t}$). The remainder of this section characterizes $\bar{A}_{ij,t}$, completing the description of equilibrium.

[12] As usual, the labor wedge, $1 - \tau_t^N$, amounts to the household's marginal rate of substitution divided by the economy's marginal product of labor; and the efficiency wedge, $1 - \tau_t^A$, amounts to the economy's marginal product of labor divided by the efficient productivity A^{eff}. See Chari, Kehoe and McGrattan (2007) for details.

3.2 Financial markets

One technical challenge in analyzing the investors' portfolio problem is that their equilibrium beliefs generally do not conjugate with any reasonable prior distribution, making it hard to track investors' equilibrium beliefs in a dynamic model. This is because $s^1_{i,t}$ and $\log Q_{i,t}$ are nonlinear statistics in $\log A_{i,t}$. In keeping the analysis tractability, we take a linear approximation (further detailed below) to both signals that ensures that posterior beliefs are always Gaussian.

Credit limits Following Campbell and Viceira (2002), we use the usual discrete time approximation to derive investors' asset demands for CRRA utility and log-normal beliefs. Leaving the details to the appendix, we get the following expression for the market clearing asset price:

$$\log Q_{i,t} = \int \mathbb{E}\{\overline{\text{rev}}_{i,t}|\mathcal{I}_{ik,t}\}\,\mathrm{d}k - 2\gamma\left(1 - \eta_{i,t} - \frac{1}{4\gamma}\right)\text{Var}\{\overline{\text{rev}}_{i,t}|\mathcal{I}_{ik,t}\}, \tag{10}$$

where

$$\begin{aligned}\overline{\text{rev}}_{i,t} &= \log \int P_{ij,t}Y_{ij,t}\,\mathrm{d}j \\ &= \log\left(\frac{\xi}{\xi-1}\right) + \log\Omega_t + f(A_{i,t}, \bar{A}_{i,t})\end{aligned} \tag{11}$$

are the log-linearized revenues of the average firm on island i, with a log-linear function f.[13] The equilibrium asset price is given by the average belief across investors regarding island-revenues $\overline{\text{rev}}_{i,t}$ and a risk-premium that depends on their uncertainty. Note that the price is constant within-islands and the risk-premium only depends on investors' uncertainty regarding the island-wide component $A_{i,t}$. This is because investors treat firms symmetrically within islands based on their information, so optimal portfolios will be fully diversified against all within-island risks.

Substituting (10) into (8), the equilibrium credit limit is then given by the fixed point between (8) and (11). This yields the following relationship between investors beliefs and the working capital constraint $\log \bar{A}_{i,t}$.

Proposition 2. *The working-capital constraint* $\log \bar{A}_{i,t}$ *is given by*

$$\log \bar{A}_{i,t} = \underbrace{\bar{\mathbb{E}}_t\{\log A_{i,t}\}}_{\text{market belief}} - \underbrace{\pi_\sigma \hat{\sigma}^2_{i,t}}_{\text{uncertainty}} + \underbrace{\pi_\chi \log\chi_t + \pi_\eta \hat{\sigma}^2_{i,t}\eta_{i,t}}_{\text{financial shocks}} + \pi_0, \tag{12}$$

where $\bar{\mathbb{E}}_t\{\,\cdot\,\}$ *denotes the average expectation of island-i investors,* $\hat{\sigma}^2_{i,t}$ *denotes their uncertainty regarding* $\log A_{i,t}$*, and* π_σ*,* π_χ*,* π_η *and* π_0 *are positive constants.*

The credit limit (12) is increasing in investors' expectations and decreasing in their uncertainty, reflecting that credit is more readily available when financial markets believe firm fundamentals

[13] Keeping with the approximation approach in Campbell and Viceira (2002), log-linearizing $\overline{\text{rev}}_{i,t}$ ensures that investors' equilibrium beliefs about firms' revenues are indeed log-normal so long as their beliefs regarding $\log A_{i,t}$ are Gaussian. With $\log Q_{i,t}$ given by (10), this will be the case if $s^1_{i,t}$ is Gaussian (see below).

to be more favorable. While (12) results from a particular formulation of the financial market, this feature is, implicitly or explicitly, present in much of the previous macro-finance literature as borrowers' funding position naturally depend on lenders' beliefs about the expected repayment.[14] In this sense, we view our particular formulation of the financial market mainly as a convenient way to capture the belief-dependence of funding conditions generally present in both debt and equity arrangements.

In addition to investors' beliefs, the working capital constraint also depends on financial factors that shape credit limits for reasons that are orthogonal to the productive potential given by $A_{i,t}$. For instance, one can think of the island-specific factor $\eta_{i,t}$ as the model's way of generating exogenous heterogeneity across firms in their ability or dependence to raise external funding. In contrast, the aggregate factor χ_t is the model's way of generating a financial crisis that represents, e.g., changes in the financial sector's ability to absorb risk, to lend, or to refinance.

Equilibrium beliefs To complete the characterization of equilibrium, we need to describe the equilibrium law of motion for investors' beliefs that feed into the working capital constraint (12).

Consider first the ability of investors to learn from firms' working capital investments via $s^1_{i,t}$. Integrating over (7), we have

$$s^1_{i,t} = \log \Omega_{t-1} + \log \int_J \min\{A_{ij,t-1}, \bar{A}_{i,t-1}\}^{\xi-1}\,\mathrm{d}j + \nu^1_{i,t}. \tag{13}$$

The informativeness of the signal is declining in the fraction of firms that investors suspect to be constrained on a given island. Let $a_{i,t} \equiv \log A_{i,t}$ and $\bar{a}_{i,t} = \log \bar{A}_{i,t}$. The following lemma provides the basic intuition.

Lemma 1. *The sensitivity of $s^1_{i,t}$ in $a_{i,t-1}$ is given by*

$$\frac{\partial s^1_{i,t}}{\partial a_{i,t-1}} = h(a_{i,t-1} - \bar{a}_{i,t-1}),$$

with $h > 0$, $h' < 0$, $\lim_{x\to-\infty} h(x) = \xi - 1$ and $\lim_{x\to\infty} h(x) = 0$.

When $a_{i,t} \ll \bar{a}_{i,t}$, the majority of firms on island i will operate away from the working capital constraint, so (7) implies that their investments respond to changes in their productivity at a rate $\xi - 1$. Accordingly, in the limit where no firm is constrained, also the average investment $\log L_{i,t}$ will change with $\log A_{i,t}$ at a rate $\xi - 1$. By contrast, as $a_{i,t}$ increases relative to $\bar{a}_{i,t}$, the investment choices of an increasing number of firms will be dictated by the financial constraint

[14] In the case of unsecured debt, fundamentals affect the repayment amount via the *expected* repayment probability, for secured debt that repayment amount further depends on fundamentals via the *expected* collateral value. Seminal frameworks where beliefs (implicitly) shape the supply of funds are, e.g., the perfect information models by Kiyotaki and Moore (1997) and Bernanke, Gertler and Gilchrist (1999), where borrowers refinance themselves by writing debt contracts. The debt contracts are then valued according to *lenders' beliefs*, which in these perfect information models coincide with borrowers' beliefs. In models with imperfect information or heterogeneous priors, this dependence on lenders' beliefs is more explicit, for example in the models by Simsek (2013*a*,*b*) or Fostel and Geanakoplos (2008, 2015).

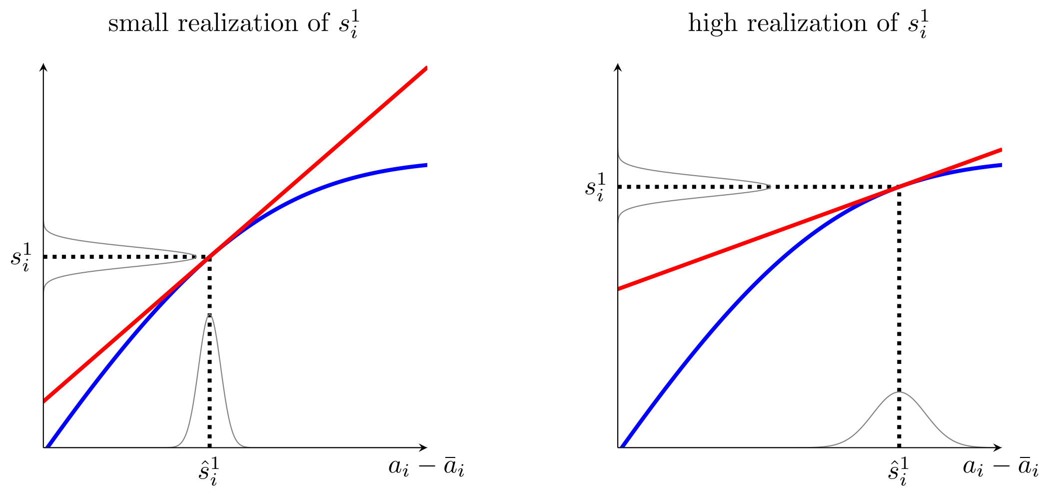

Figure 1: Illustration of the approximate Gaussian updating procedure. The blue line shows the relationship between $s_{i,t}^1$ and $a_{i,t-1} - \bar{a}_{i,t-1}$ (up to an observed term contained in $\mathcal{I}_{ik,t}$). Conditional on $s_{i,t}^1$, investors linearize the true relationship by the red line, preserving the signal-dependent uncertainty that is generated by the nonlinearity.

$\bar{a}_{i,t}$. Accordingly, working capital investments will respond on average less to variations in the productivity $a_{i,t}$, reducing the sensitivity of the working capital signal. Since a reduced sensitivity implies a smaller signal-to-noise ratio, this implies that the speed at which investors learn from $s_{i,t}^1$ declines as well when they suspect a large fraction of firms to be constrained at $t-1$.

In a related contribution (Straub and Ulbricht, 2015), we show that with very few assumptions on the distribution of $\nu_{i,t}^1$ such a "concave" signal structure generally implies an investors' uncertainty that is increasing in $a_{i,t-1}$ and decreasing in $\bar{a}_{i,t-1}$. Here we instead use a specific linear approximation to the signal extraction from $s_{i,t}^1$ that ensures that investors' posteriors are Gaussian while preserving the state-dependency in investors' ability to learn from $s_{i,t}^1$. Specifically, we assume that conditional on a realization of $s_{i,t}^1$, agents linearize (13) around the "face value" of $s_{i,t}^1$, where the face value $\hat{s}_{i,t}^1$ is defined as the value for $a_{i,t-1} - \bar{a}_{i,t-1}$ that generates $s_{i,t}^1$ if there were no noise in the signal (i.e., for $\nu_{i,t}^1 = 0$). Figure 1 provides a graphical illustration of the approximation procedure. The resulting signal extraction from $s_{i,t}^1$ is stated in the following proposition.[15]

Proposition 3. *Suppose investors linearize* (13) *around the signal's "face-value"* $\hat{s}_{i,t}^1$ *to assess the likelihood of observing* $s_{i,t}^1 | a_{i,t-1}$. *Then, agents update as if* $\hat{s}_{i,t}$ *was a fictitious Gaussian signal, distributed according to* $\mathcal{N}(\mu_{i,t}, \varsigma_{i,t}^2)$ *with*

$$\mu_{i,t} = a_{i,t-1} - \bar{a}_{i,t-1}$$
$$\varsigma_{i,t} = \frac{\sigma_{s,1}}{h(\hat{s}_{i,t}^1)},$$

and where agents update as if $\varsigma_{i,t}$ *were exogenous.*

We are now ready to describe the evolution of investors' equilibrium beliefs. Our approximation

[15] Notice that this approximate Gaussian updating requires island-wide investments $L_{i,t}$ be differentiable in $A_{i,t}$, which is the reason for having a small but positive within-island dispersion of productivities in the model.

approach ensures that all signals contained in the information sets of investors are linear statistics in $a_{i,t}$, allowing us to track investors' beliefs using the Kalman filter. Specifically, since investors were assumed to be one-period lived, their private information $s^2_{ik,t}$ dies after one period as well, so each generation of investors has prior beliefs based on the *public* history of signals $\tilde{\mathcal{I}}_{i,t} \equiv \{s^1_{i,s}, Q_{i,s}, \chi_s\}_{s \leq t}$. Concordantly, we denote with a "tilde" the expectation and variance of public beliefs $\tilde{\mathbb{E}}_t\{\cdot\} \equiv \mathbb{E}\{\cdot|\tilde{\mathcal{I}}_{i,t}\}$ and $\tilde{\sigma}^2_{i,t} \equiv \text{Var}\{a_{i,t}|\tilde{\mathcal{I}}_{i,t}\}$. In conjunction with $s^1_{i,t}$ (which is a signal about $a_{i,t-1}$), these public beliefs define the prior at date t. Projecting $\mathbb{E}\{a_{i,t-1}|\tilde{\mathcal{I}}_{i,t-1}, s^1_{i,t}\}$ forward (in time) to obtain a prior estimate of $a_{i,t}$, it can be shown that the relevant precisions of the prior and the working capital signal are given by $\delta_{i,t}\tilde{\sigma}^{-2}_{i,t-1}$ and $\delta_{i,t}\varsigma^{-2}_{i,t}$, where

$$\delta_{i,t} = \left(\rho^2_a + (\tilde{\sigma}^{-2}_{i,t-1} + \varsigma^{-2}_{i,t})\sigma^2_a\right)^{-1}$$

measures the decay in past information due to the stochastic progression in $a_{i,t}$ (see the proof to Proposition 4 for details).

Regarding the information contained in $Q_{i,t}$, applying techniques similar to those used when solving a standard CARA-Normal asset pricing equilibrium (e.g., Hellwig, 1980) one finds that observing the asset price $Q_{i,t}$ is informationally equivalent to receiving the signal

$$s^Q_{i,t} = a_{i,t} + \sigma^2_{s,2}\pi_\eta \left(\eta_{i,t} - \rho_\eta \tilde{\mathbb{E}}_{t-1}\{\eta_{i,t-1}\}\right),$$

which has time-varying noisiness $\sigma^2_{Q,i,t} = \rho^2_\eta \tilde{\sigma}^2_{i,t-1} + \sigma^2_\eta(\sigma^2_{s,2}\pi_\eta)^2$.

After these preparations, we can now use the Kalman filter to recursively filter through all public information up to period $t-1$, and then use the filter one last time taking into account the private information available in period t. The following proposition summarizes the result.

Proposition 4. *In equilibrium, average financial market beliefs are given by*

$$\bar{\mathbb{E}}_t\{a_{i,t}\} = \frac{\hat{\sigma}^2_{i,t}}{\sigma^2_{s,2}} a_{i,t} + \frac{\hat{\sigma}^2_{i,t}}{\tilde{\sigma}^2_{i,t}} \tilde{\mathbb{E}}_t\{a_{i,t}\}$$
$$\hat{\sigma}^2_{i,t} = \left(\sigma^{-2}_{s,2} + \tilde{\sigma}^{-2}_{i,t}\right)^{-1},$$

with public beliefs given by

$$\tilde{\mathbb{E}}_t\{a_{i,t}\} = \tilde{\sigma}^2_{i,t} \begin{bmatrix} \delta_{i,t}\varsigma^{-2}_{i,t} & \sigma^{-2}_{Q,i,t} & \delta_{i,t}\tilde{\sigma}^{-2}_{i,t-1} \end{bmatrix} \times \begin{bmatrix} \rho_a(\hat{s}^1_{i,t} + \bar{a}_{i,t-1}) \\ s^Q_{i,t} \\ \rho_a \bar{\mathbb{E}}_{t-1}\{a_{i,t-1}\} \end{bmatrix}$$
$$\tilde{\sigma}^2_{i,t} = \left(\delta_{i,t}\varsigma^{-2}_{i,t} + \sigma^{-2}_{Q,i,t} + \delta_{i,t}\tilde{\sigma}^{-2}_{i,t-1}\right)^{-1}$$
$$\tilde{\mathbb{E}}_t\{\eta_{i,t}\} = \frac{1}{\sigma^2_{s,2}\pi_\eta}\left(s^Q_{i,t} - \tilde{\mathbb{E}}_t\{a_{i,t}\}\right) + \rho_\eta \tilde{\mathbb{E}}_{t-1}\{\eta_{i,t-1}\}.$$

The intuition for the equations in Proposition 4 is as follows. Since the signal structure is

Gaussian in our framework, expectations are convex combinations of signals. In particular, public expectations are a convex combination of the working capital signal $\hat{s}^1_{i,t}$, the information contained in asset prices $s^Q_{i,t}$, and the prior expectation. Investors' private expectations are very similar, except that they also include a term $(\hat{\sigma}^2_{i,t}/\sigma^2_{s,2})a_{i,t}$ coming from private signals.

The key ingredient in this otherwise standard Kalman filtering problem is that the noise in the working capital signal, $\varsigma^2_{i,t}$, is endogenous. When this noise increases, investors optimally shift weight away from the working capital signal towards the three other signals. This affects their posterior expectations, as well as their posterior uncertainty: Expectations $\bar{\mathbb{E}}_t\{a_{i,t}\}$ become "sticky" in that now more weight is on the prior expectation. And posterior uncertainty $\hat{\sigma}^2_{i,t}$ naturally increases since one of the signals, the working capital signal, loses some of its precision.

3.3 Uniqueness and computation

The previous two subsections provide a complete characterization of the equilibrium in the model economy. As established in Proposition 1, the economy's aggregate quantities only depend on the cross-sectional distribution of $(A_{ij,t}, \bar{A}_{i,t})$. By Proposition 2, the latter is pinned down by exogenous shocks and investors' average beliefs, $\bar{\mathbb{E}}_t\{a_{i,t}\}$ and $\hat{\sigma}^2_{i,t}$, which by Propositions 3 and 4 can be in turn computed recursively as functions of exogenous shocks and productivities. Combining Propositions 1–4, it follows that the equilibrium in our economy is unique and entirely backward looking.

4 Belief traps

We are now ready to explore the interaction between working capital constraints and investors' beliefs. In this section, we begin by exploring the dynamics for a single island in isolation. The behavior of the aggregate economy will be explored in Section 5.

In the last section we have seen how the ability to learn about an island's productivity deteriorates in "credit tightness" $a_{i,t} - \bar{a}_{i,t}$. Using (12) to substitute for $\bar{a}_{i,t}$, we have

$$\underbrace{a_{i,t} - \bar{a}_{i,t}}_{\text{credit tightness}} = \underbrace{a_{i,t} - \bar{\mathbb{E}}_{i,t}\{a_{i,t}\}}_{\text{optimism/pessimism}} + \underbrace{\pi_\sigma \hat{\sigma}^2_{i,t}}_{\text{uncertainty}} - \underbrace{\pi_\chi \log \chi_t - \pi_\eta \hat{\sigma}^2_{i,t}\eta_{i,t}}_{\text{financial shocks}} - \pi_0. \tag{14}$$

Equation (14) shows how adverse investors' beliefs, marked by pessimism and uncertainty, result in tight working capital constraints and hence reduce the informativeness of the working capital signal. From Proposition 4 it then follows that past pessimism and uncertainty get reinforced into future periods, creating persistently tight working capital constraints.

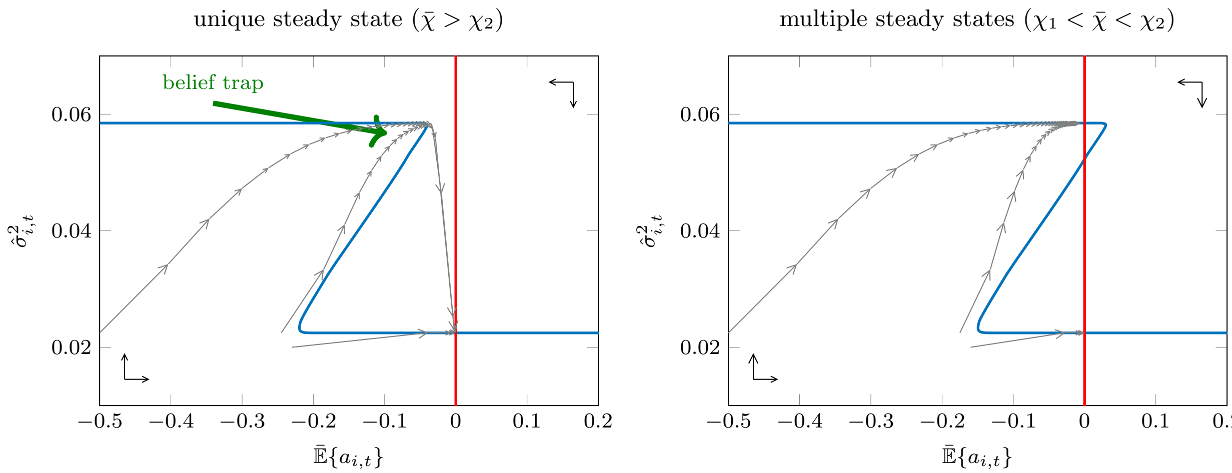

Figure 2: Phase diagrams for island level beliefs in the absence of shocks. Z-shaped blue lines are $(\Delta\hat{\sigma}_{i,t} = 0)$-loci; vertical red lines are $(\Delta\bar{\mathbb{E}}\{a_{i,t}\} = 0)$-loci. Arrowheads represent distinct points in time along the plotted trajectories, so that the distance between two consecutive arrows can be viewed as an inverse measure of the speed at which the state is moving. *Left:* Case with a unique steady state. *Right:* Case with multiple steady states.

4.1 Steady state dynamics

Let $a_{i,s} = \eta_{i,s} = \nu^1_{i,s} = 0$ and $\chi_s = \bar{\chi}$ for all s, and some constant $\bar{\chi}$. In this case, the laws of motion for investors' expectations and uncertainty can be expressed as

$$\bar{\mathbb{E}}_{t+1}\{a_{i,t+1}\} - \bar{\mathbb{E}}_t\{a_{i,t}\} = g_{\mathbb{E}}(\bar{\mathbb{E}}\{a_{i,t}\}, \hat{\sigma}_{i,t}) \tag{15}$$

and

$$\hat{\sigma}^2_{i,t+1} - \hat{\sigma}^2_{i,t} = g_{\hat{\sigma}}(\bar{\mathbb{E}}\{a_{i,t}\}, \hat{\sigma}_{i,t}). \tag{16}$$

Figure 2 shows the resulting phase diagram for two different values of $\bar{\chi}$. The red line corresponds to the constant expectations locus ($g_{\mathbb{E}} = 0$), which is given by $\bar{\mathbb{E}}\{a_{i,t}\} = 0$ since $a_{i,t}$ is mean-reverting. The blue line corresponds to the constant uncertainty locus ($g_{\hat{\sigma}} = 0$). The latter is "Z"-shaped, because higher levels of uncertainty $\hat{\sigma}^2_{i,t}$ not only directly increase uncertainty at $t+1$ but also indirectly by reducing the informativeness of $s^1_{i,t}$ via (14): For sufficiently optimistic (or pessimistic) expectations this feedback has no effect as firms' access to funds will be secured (or denied) regardless of $\hat{\sigma}^2_{i,t}$, uniquely pinning down uncertainty at time $t+1$. For moderate levels of $\bar{\mathbb{E}}\{a_{i,t}\}$, however, uncertainty becomes pivotal to the availability of funds, hence generating multiple stationary values of uncertainty for a given level of $\bar{\mathbb{E}}\{a_{i,t}\}$. Specifically, for high levels of uncertainty, credit is tight and information is scarce, reinforcing a high level of uncertainty, and vice versa. Intersecting the two loci, we can have a unique or multiple steady states, depending on the value of $\bar{\chi}$. (In either case, the logic given in Section 3.3 applies and the equilibrium is unique.)

Proposition 5. *There exist two thresholds $\chi_1 < \chi_2$, such that for all $\chi_1 \leq \bar{\chi} \leq \chi_2$ there exist 3 steady states at the island-level (2 if the condition holds with equality), and otherwise there is a unique steady state.*

For intermediate levels of $\bar{\chi}$, belief traps are infinitely persistent in the absence of shocks. Accordingly, a one-time disruption in a firms' funds (e.g., through a shock to $\eta_{i,t}$) can indefinitely cut off the firm from future funding.

However, even when the steady state is unique, belief traps may emerge as a persistent (though not indefinite) disruption in a firms' access to funding. This is illustrated by the gray trajectories in the left panel of Figure 2. Along these trajectories, each arrowhead represents a distinct point of time, so that the distance between two consecutive arrowheads can be viewed as an inverse measure of the speed at which the state is evolving.

The three trajectories differ in the persistence of pessimism and the amount of uncertainty induced along the path. The path starting with the (relatively) most optimistic expectations rapidly converges back to the unique steady state. The two other paths, however, behave distinctly different: By starting to the left of the kink of the blue locus, investors are sufficiently pessimistic to induce a critical level of credit tightness so that learning breaks down. This implies that along the two paths investors accumulate higher and higher levels of uncertainty (since the informativeness of past information about the current state of $a_{i,t}$ decays over time) and, accordingly, pessimism starts fading out slower and slower—two tendencies that jointly reinforce each other through tighter and tighter working capital constraints. Caused by the decreasing velocity of expectation dynamics in the neighborhood of such a "belief trap", its effects can be very persistent (more than 20 periods along the left-most trajectory)—even though the steady state is unique.

4.2 Impulse response to an island-specific credit shock

We next look at the response to an island-specific credit shock. The difference to the steady-state analysis above is that in general $(\bar{\mathbb{E}}_t\{a_{i,t}\}, \hat{\sigma}_{i,t})$ will be pinned down by the history of shocks $\{a_{i,s}, \eta_{i,s}, \nu^1_{i,s}, \chi_s\}_{s\leq t}$, effectively pinning down a particular starting point in Figure 2. In addition, shocks may also have a direct impact on $(a_{i,t} - \bar{a}_{i,t})$ amounting to a horizontal shift of the uncertainty locus. Otherwise the insights developed for the steady-state case directly apply to the stochastic model.

In particular, an adverse shock to $\eta_{i,t}$ reduces $\bar{a}_{i,t}$ (a temporary shift of the uncertainty locus to the right), which for sufficiently large shocks can trigger a belief trap spiral similar to the ones depicted in Figure 2. The adverse impact of $\eta_{i,t}$ on $\bar{a}_{i,t}$ is further reinforced by its impact on $\bar{\mathbb{E}}_t\{a_{i,t}\}$ (recall that $\eta_{i,t}$ enters as signal noise in the asset price signal $s^Q_{i,t}$), which as seen above further ignites belief traps.[16]

Comparison with exogenous-uncertainty counterfactual Figure 3 displays the model's response of a single island (solid black lines) to a $-3\sigma_\eta$ innovation in $\eta_{i,t}$, while the aggregate

[16]This mutually reinforcing impact is specific to financial shocks, making them the most likely trigger of belief traps. Innovations in $a_{i,t}$ or $\nu^1_{i,t}$ instead induce variations in $\bar{\mathbb{E}}\{a_{i,t}\}$ that are partially offsetting their direct impact on the location of the uncertainty locus (i.e., expectations and the uncertainty-locus both move into the *same* direction).

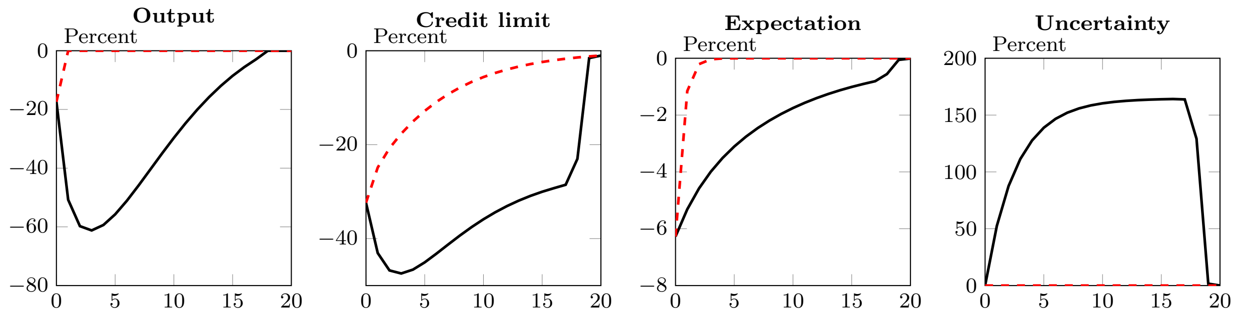

Figure 3: Impulse responses to island-specific financial shock. Solid black lines are impulse responses to a -3σ shock to $\eta_{i,t}$. Dashed red lines are counterfactual responses to the same shock where the signal quality $\varsigma^2_{i,t}$ is exogenously fixed at its pre-shock level. All responses are in percentage deviations.

financial state χ_t is set to a constant and the economy is in its stochastic steady state.[17] To identify the contribution of the endogenous-uncertainty channel, the model's response is contrasted with a counterfactual response that isolates the pure impact of the financial shock by fixing the signal quality $\varsigma^2_{i,t}$ at its pre-shock level (dashed red lines).

Upon impact the financial shock affects the model economy in the same way as it affects the counterfactual response. This is because investors observe *lagged* working capital, so that an increase in uncertainty affects the dynamics with a delay of 1 period. In both simulations the financial shock reduces the credit limit $\bar{a}_{i,t}$ and leads to more pessimistic expectations $\mathbb{E}_t\{a_{i,t}\}$ as investors learn from $Q_{i,t}$.

The difference between model and counterfactual emerges in the second period, when learning slows down in the model economy, triggering the aforementioned self-reinforcing cycle of uncertainty and pessimism. In the first panel, we can see the persistent impact on output, which stands in stark contrast to the small impact in the counterfactual case.[18] Similar to Figure 2, uncertainty *accumulates* over time when reliable information ceases to arrive, explaining the hump-shaped response of uncertainty, credit limits, and output.

Since the simulated model parameters imply a unique steady state, beliefs and credit limits eventually recover. Once expectations cross to the right of the uncertainty locus (see Figure 2) and the island becomes sufficiently unconstrained, there is a sharp drop in uncertainty and a rise in expectations. At this point, the island has essentially left the "belief trap", and further recovery proceeds quickly.

[17] The simulation uses the parametrization presented in Section 5.1, which features a unique steady state at the island-level. (In either case, uniqueness of equilibrium always implies a unique stochastic steady state of the aggregate economy.)

[18] Underlying the discrepancy in the output between model and counterfactual is an inherent nonlinearity in the propagation of credit limits to output. Intuitively, for small variations in the credit limit, only firms in the right tail of the productivity distribution are limited in their production, which under standard assumptions on the productivity distribution are few in numbers. For larger variations in the credit limit, however, the marginally constrained firm within an island moves closer to the median where the probability density is larger. Accordingly, the marginal impact of a decline in an island's credit limit on its output is necessarily increasing until output is significantly affected.

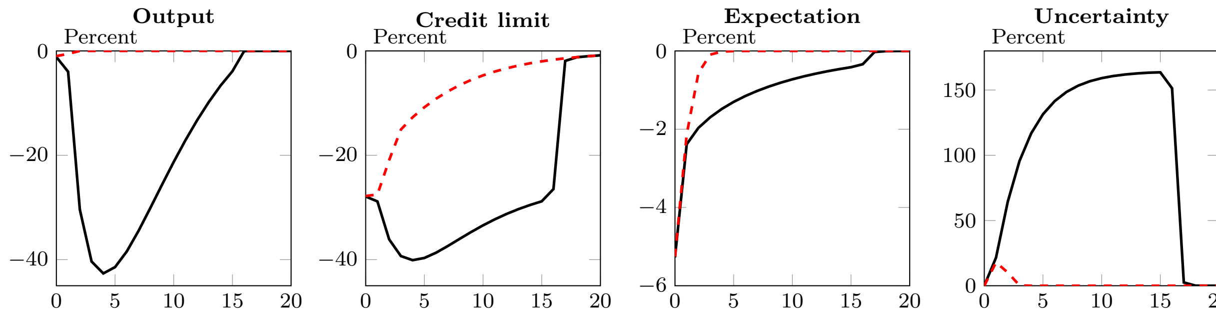

Figure 4: Impulse responses to island-specific financial shock so that island is "marginally constrained" . Solid black lines and dashed red lines are impulse responses to $-2.51\sigma_\eta$ and $-2.50\sigma_\eta$ shocks to $\eta_{i,t}$, respectively. All responses are in percentage deviations.

Marginally constrained versus marginally unconstrained islands Before turning to the behavior of the aggregate economy, it is useful to compare the dynamics of an island that is "marginally constrained" with the ones of an island that is "marginally unconstrained". Based on our previous discussion, we can define a threshold $\bar{\eta}_{i,t}$ such that for all $\eta_{i,t} \leq \bar{\eta}_{i,t}$ the majority of firms in island i is constrained ($\bar{a}_{i,t} < a_{i,t}$), whereas for all $\eta_{i,t} > \bar{\eta}_{i,t}$ the majority of firms is unconstrained.[19] In particular, letting $\mathcal{S}_{i,t} \equiv (a_{i,t}, \bar{\mathbb{E}}_t\{a_{i,t}\}, \hat{\sigma}^2_{i,t})$, we define $\bar{\eta}_{i,t} = \bar{\eta}(\mathcal{S}_{i,t}, \chi_t)$ such that for all $\eta_{i,t} \leq \bar{\eta}_{i,t}$ at least a fraction $u = 1/2$ of firms in island i is constrained.[20] The number of constrained firms on island i is given by $u = \Phi((a_{i,t} - \bar{a}_{i,t})/\sigma_\epsilon)$, so

$$\bar{\eta}(\mathcal{S}_{i,t}, \chi_t) = \frac{1}{\pi_\eta \hat{\sigma}^2_{i,t}} \left(a_{i,t} - \bar{\mathbb{E}}_{i,t}\{a_{i,t}\} - \pi_\chi \log \chi_t - \pi_0 \right) + \frac{\pi_\sigma}{\pi_\eta}. \tag{17}$$

Figure 4 displays impulse response functions for two ex-ante identical islands, but where one is hit by a shock to $\eta_{i,t}$ slightly above $\bar{\eta}_{i,t}$, whereas the other is hit by a shock to $\eta_{i,t}$ slightly below $\bar{\eta}_{i,t}$. By design, the responses for the marginally constrained island (solid black lines) closely resemble the belief trap dynamics seen in Figure 3. In contrast, the responses for the marginally unconstrained island (dashed red lines) show little sign of an information breakdown, similar to the counterfactual case discussed above. Even though the described discontinuity is not a model prediction that should be taken to the data at face value, it does illustrate that even a possibly small amount of constrainedness can fundamentally change the trajectory of a firm. This is consistent with recent evidence on the impact of credit constraints on firm performance (Giroud and Mueller, 2015), and with evidence on the highly persistent effects that recessions more generally have on young firms (Moreira, 2015).

[19] In general, learning breaks down when most firms on an island are constrained ($\bar{a}_{i,t} \ll a_{i,t}$). Since σ_ϵ is small in our parametrization, $\bar{a}_{i,t}$ essentially defines a threshold where for slightly smaller realizations in $a_{i,t}$ most firms are unconstrained and for slightly larger realizations most firms are constrained.

[20] Here the precise value for u is not crucial as long as it is not too close to 0 since in our calibration within-island firm heterogeneity is small.

Table 1: Parameter values used in simulation

Parameter	ζ	ξ	γ	ρ_η	ρ_a	σ_a	σ_ϵ	$\sigma_{s,1}$	$\sigma_{s,2}$	σ_η
Value	0.50	4.00	1.50	0.84	0.90	0.15	0.01	0.23	0.47	0.52

5 Aggregate financial shocks

We are now ready to explore how a temporary decline in the economy's financial capacity χ_t propagates through the model. In the model, χ_t matters through its effect on the threshold $\bar{\eta}(\mathcal{S}_{i,t}, \chi_t)$ that determines the likelihood of an island to fall into a belief trap. From (17), $\bar{\eta}$ is strictly decreasing in χ_t. During a financial crisis when χ_t is small, firms are therefore more prone to idiosyncratic financial shocks in the sense that it takes smaller realizations of $\eta_{i,t}$ for a firm to become constrained. Due to the discontinuity in the dynamics of islands around $\bar{\eta}(\mathcal{S}_{i,t}, \chi_t)$, even a temporary such shift in $\bar{\eta}$ can have severe and long-lasting consequences for aggregate efficiency and output. In this section, we explore these consequences using numerical simulations.

5.1 Numerical example

We interpret one period as a quarter. The inverse Frisch elasticity of labor supply ζ is set to 0.5, the elasticity of substitution between consumption goods ξ is set to 4, and the relative risk-aversion of investors γ is set to 1.5. We set $\rho_\eta = 0.84$, reflecting a four quarter half-life of the island-specific working capital shocks.[21] The productivity parameters are set to $\rho_a = 0.9$ and $\sigma_a = 0.15$, so that islands can be interpreted as firms. These parameters are consistent with the existing literature on firm-level dynamics.[22] To interpret islands as firms, we further keep the within-island dispersion low, setting σ_ϵ to 0.01.

It remains to specify the learning-related parameters. To this end, we use forecasts about earnings per share (EPS) by financial analysts from the IBES database to construct calibration targets that mirror the investors' problem to learn about firm-level fundamentals in our model. We

[21] Introducing persistence in $\eta_{i,t}$ ensures that adverse financial shocks do not fully recover by the time investors learn from the previous period's lending signal so to allow for the complementary presence of uncertainty and financial distress that lays the grounds for belief traps. In the sense that $\eta_{i,t}$ proxies for firm-specific factors determining the access to funds (e.g., the availability of internal funding), it seems plausible that shocks to $\eta_{i,t}$ are persistent in reality.

[22] See, for example, Gilchrist, Sim and Zakrajšek (2014, Appendix 4).

exploit data on the following 4 panel variables (see Appendix B for details)[23]:

$$\begin{aligned}
\bar{\mu}_{i,t} &\equiv \text{average cross-analyst belief about firm } i\text{'s end-of-quarter EPS} \\
\sigma_{i,t}^{\text{cross}} &\equiv \text{cross-analyst belief dispersion about firm } i\text{'s end-of-quarter EPS} \\
\text{EPS}_{i,t} &\equiv \text{end-of-quarter realization of EPS} \\
\Delta p_{i,t} &\equiv \text{log returns of firm } i\text{'s stock (adjusted for splits and dividends)}
\end{aligned}$$

To isolate the firm-specific components in these series, we extract time-fixed effects from each of them, with the exception of $\sigma_{i,t}^{\text{cross}}$ (for which we target the sample mean). We choose $\sigma_{s,1}$, $\sigma_{s,2}$ and σ_η to jointly match (i) the average belief dispersion $\mathbb{E}\{(\sigma_{i,t}^{\text{cross}})^2\}/\mathbb{V}\{\text{EPS}_{i,t}\}$, where $\mathbb{E}$ and $\mathbb{V}$ denote the sample mean and variance;[24] (ii) the signal-to-noise ratio of stock prices regarding future earnings $\mathbb{V}\{\Delta p_{i,t}\}/\mathbb{V}\{\text{noise}_{i,t}\}$, where $\text{noise}_{i,t}$ are the residuals from regressing $\Delta p_{i,t-1}$ on $\text{EPS}_{i,t}$ and firm-level fixed effects; and (iii) the correlation between average cross-analysts beliefs and actual realizations $\text{Corr}\{\bar{\mu}_{i,t}, \text{EPS}_{i,t}\}$.

Intuitively, the first of these moments determines the contribution of investors' private signals $s_{ik,t}^2$ relative to all other signals, the second moment pins down the predictive power of the information contained in asset prices $s_{i,t}^Q$,[25] and the third moment parametrizes the overall information available to investors. Tables 1 and 2 summarize the target moments and the calibrated variance parameters. In line with the asset pricing literature, the signal-to-noise ratio of prices is close to unity, reflecting a low correlation between prices and fundamentals.[26] The dispersion of beliefs and the correlation of beliefs and actuals, however, suggests that learning from the other sources is significantly more efficient. Taken together the learning parameters imply a moderate posterior uncertainty that averages to about one fifth of the unconditional uncertainty at the steady state. Appendix C contains various robustness specifications.

5.2 Simulation of an aggregate credit shock

We conduct two numerical experiments. In this subsection, we illustrate the model's implication by simulating its response to a temporary shock to $\log \chi_t$ that decays at a geometric rate. The next

[23]In the calibration, we compare pre-crisis model moments to monthly data prior to the financial crisis (1984Q2–2006Q4). To reduce the sensitivity of our calibration to outliers, we trim for each month the 2% tails of all variables. The resulting panel contains on average 2053 firms per month. Price data is adjusted for dividends and splits and is obtained from the CRSP database. The model moments are computed at the stochastic steady state with a constant $\chi_t = \bar{\chi}$ set so that 2.5 percent of firms are constrained.

[24]We normalize the average dispersion relative to $\mathbb{V}\{\text{EPS}_{i,t}\}$ to make it unitless, allowing us to directly compare it to the dispersion of investors' beliefs in our model without relying on further structural assumptions.

[25]In the model, we interpret $s_{i,t}^Q$ as a natural counterpart to stock prices, since similar to stock prices it is a public signal aggregating information through its dependence on financial market expectations. Paralleling the treatment of the data, we do not assume any prior knowledge when computing the signal-to-noise ratio of $s_{i,t}^Q$; i.e., we match the (average) unconditional ratio, $\text{Var}\{s_{i,t}^Q\}/\text{Var}\{s_{i,t}^Q|a_{i,t}\} = \text{Var}\{a_{i,t}\}/\sigma_{Q,i,t}^2 + 1$, to the data.

[26]The calibration implies a relative contribution of the credit supply signal to investors' learning between 2 and 5 percent. This is broadly consistent with David, Hopenhayn and Venkateswaran (2015) who find that the information contained in stock market prices contributes between 2 and 8 percent to learning about firm fundamentals at a 3-year horizon.

Table 2: Calibration targets for learning parameters (based on IBES data, see text)

Moment		Value
Average of firm-specific cross-analyst dispersion	$\mathbb{E}\{(\sigma_{i,t}^{\text{cross}})^2\}/\mathbb{V}\{\text{EPS}_{i,t}\}$	0.0206
Signal-to-noise ratio of stock returns	$\mathbb{V}\{\Delta p_{i,t}\}/\mathbb{V}\{\text{noise}_{i,t}\}$	1.1059
Correlation of firm-specific average belief and realized EPS	$\text{Corr}\{\bar{\mu}_{i,t}, \text{EPS}_{i,t}\}$	0.8697

Note: All target moments are exactly matched by the calibrated learning parameters. The model moments are computed at the stochastic steady state for a constant value of the aggregate credit shock $\chi_0 = \chi_1 = \cdots = \bar{\chi}$, chosen so that 2.5% of firms are constrained.

subsection then chooses a different process for χ_t that is aimed to replicate the distress within the financial sector during the 2008/09 crisis.

We let the initial aggregate financial state $\chi_0 = \cdots = \chi_{t-1} = \bar{\chi}$ be such that 2.5 percent of firms are constrained at the stochastic steady state. At date t the economy is hit by an aggregate shock Δ that reduces χ_t and decays with a half-life of 4 quarters:

$$\log \chi_{t+s} = \log \bar{\chi} - (0.5)^{s/4}\, \Delta.$$

The size of the initial impact Δ is chosen, so that 20 percent of firms are constrained at the peak of the crisis, consistent with the number of firms that reported to be "very affected" by difficulties in accessing the credit market during the recent financial crisis (Campello, Graham and Harvey, 2010).[27]

Figure 5 depicts the responses of aggregate output, employment, the efficiency wedge $(1 - \tau_t^A)$, the labor wedge $(1 - \tau_t^N)$, average expectations $\int_I \bar{\mathbb{E}}_t\{a_{i,t}\}\,\mathrm{d}i$ and average uncertainty $\int_I \hat{\sigma}_{i,t}^2\,\mathrm{d}i$, the economy-wide fraction of constrained firms, and the economy-wide average "credit spread" between shadow and risk-free rates $\int_{I\times J}(R_{ij,t} - R_t)\,\mathrm{d}(i,j)$.[28] All responses are reported in percentage deviations from the steady state, except for the fraction of constrained firms (which is reported in percentage points) and the credit spread (which is reported in percentage points relative to the steady state).

The solid black lines show the responses in the endogenous uncertainty economy. The model dynamics are contrasted with counterfactual responses (dashed red lines) where we exogenously fix $\varsigma_{i,t}$ at their unconstrained level. The counterfactual responses reflect the direct impact of tighter

[27] Campello, Graham and Harvey (2010) conduct a survey among CFOs of public companies, finding that that 35% of firms report that they experience less access to credit in the 3rd quarter of 2008, 27% report that they experience higher costs of funds, and 18% state that they have difficulties in accessing a credit line. Asked about how much credit constraints affected their operations, 56% of firms report to be "somewhat affected" by difficulties in accessing the credit market in the third quarter of 2008, whereas 20% of firms report to be "very affected". It is worth stressing that these 20% are by no means small firms: The survey in Campello, Graham and Harvey (2010) focuses on public firms, and even oversamples large public firms, compared to Compustat. Moreover, even among large public firms (those with more than 1$ bn in sales in their paper), a significant fraction of 16% still reports to be "very affected" (see Table 2 in their paper). The fact that large firms were also financially constrained is consistent with the recent evidence in Giroud and Mueller (2015).

[28] Credit spreads define the difference between a firm's marginal value of funds (i.e., the interest rate it would be willing to pay on a competitive market) and the risk-free rate. This would be precisely the observed credit spread in an isomorphic version of the model where credits are in fixed supply $\chi Q_{i,t}$ and are priced competitively at the firm level.

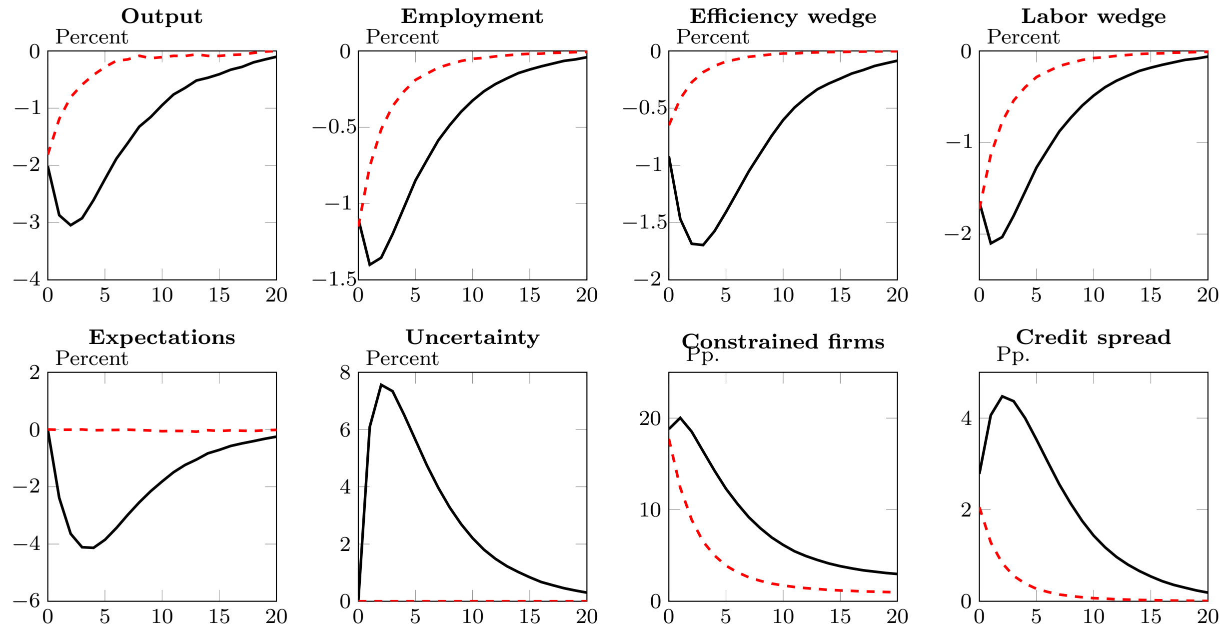

Figure 5: Impulse responses to aggregate financial shock. Solid black lines are impulse responses of the aggregated (or averaged) endogenous uncertainty economy; dashed red lines are counterfactual responses where the signal precision is exogenously fixed at its unconstrained level. All responses, except the last two plots, are in percentage deviations.

credit limits on the economy (shutting down any amplification and persistence stemming from belief traps), so that the difference between the counterfactual and the endogenous uncertainty economy can be attributed to the belief trap mechanism.

By construction, the simulated shock increases the fraction of constrained firms upon impact, resulting in an increase in the average credit spread. Tighter constraints then lead to credit and resource misallocation, reflected in increased efficiency and labor wedges, and further causing aggregate output and employment to fall.

Notice that upon impact, there is no conceptional difference between the counterfactual responses and the ones in the endogenous uncertainty economy—all visible differences are due to variations in the steady state distributions between the two economies. Starting with the first period after the initial impact, however, the responses between the model and the counterfactual diverge as in the endogenous uncertainty economy islands with binding working capital constraints are pushed into belief traps.

Internal persistence On an aggregate level, the disproportionately long-lasting contraction of firms that are out of funds results in a discrepancy between the underlying aggregate financial shock, which was set to a half-life of 4 quarters, and the persistence in the endogenous responses of the economy. This is documented in Table 3, which lists the half-lives of the simulated responses in output and employment. It is evident that the endogenous uncertainty model has a high degree of internal persistence, meaning that the half-life of output (10 quarters) and hours (8 quarters) significantly outlasts the financial shock that caused the crisis. The small internal persistence of the

Table 3: Half-life of output and employment to an aggregate financial shock with a half-life of 4 quarters. The size of the shock is calibrated so that 10 or 20 percent of firms are constrained at the peak of the crisis.

	10% constrained		20% constrained	
	model	counterfactual	model	counterfactual
financial shock	4	4	4	4
output	11	2	10	2
hours	10	3	8	2

same shocks in the counterfactual economy (2 quarters) illustrates how in the absence of belief traps the fundamental impact of financial constraints dissipates quickly.[29] For a comparison, the table also reports the half-lives for a simulation where only 10 percent of firms are constrained at the peak of the crisis, which is similar in magnitudes, reflecting that the economy scales approximately proportionally in the fraction of firms being constrained (see below).

Endogenous pessimism An interesting implication of the endogenous uncertainty channel is that the average pessimism in the economy increases as χ_t falls, even though χ_t is common knowledge and does not directly affect learning. The reason for this is statistical selection: When investors are pessimistic about an island, that island is more likely to become constrained. At the same time, expectations about constrained islands are endogenously persistent due to the belief trap mechanism. When investors are, by contrast, optimistic about an island, credit constraints become lax so that the signals become more informative and investors are more likely to learn about their too optimistic views. This asymmetry between optimism and pessimism causes the economy-wide average expectation to fall when credit limits tighten.

Cross-sectional dispersion A fact often stressed about the recent financial crisis is that the cross-sectional dispersion of growth rates has drastically increased compared to the pre-crisis level (e.g., Bloom et al. 2014). This stands in contrast to what a simple model of the financial crisis based on financial constraints would predict: Financially constrained firms can only respond less to productivity innovations, which increases comovement. The belief trap mechanism opposes this effect by increasing the discrepancy between constrained and unconstrained firms. Depending on the parametrization of our model, either of the two effects can dominate (see Appendix C). The first panel of Figure 6 shows the response for our baseline calibration. Unlike some of the alternative parametrizations explored in the appendix, the "constraints"-effect still dominates here at the beginning of the crisis, causing the dispersion to initially fall, before it is overturned by increasing dispersion in the aftermath of the crisis. In contrast, dispersion is unambiguously reduced in the counterfactual due to the absence of belief traps.

[29] In the counterfactual economy, a given rate of recovery in the financial market translates to a *faster* rate of recovery in the real economy, due to a general convexity of our model. That is, large and adverse shocks have a disproportionately adverse effect on the economy (see below), so that recovery is generally faster than exponential. In the endogenous uncertainty economy, this effect is partially offset by the emergence of belief traps. (Since the decay is initially still slightly faster than exponential, the *half-life* (by definition) is slightly lower for the case where 20% of firms are constrained on impact.)

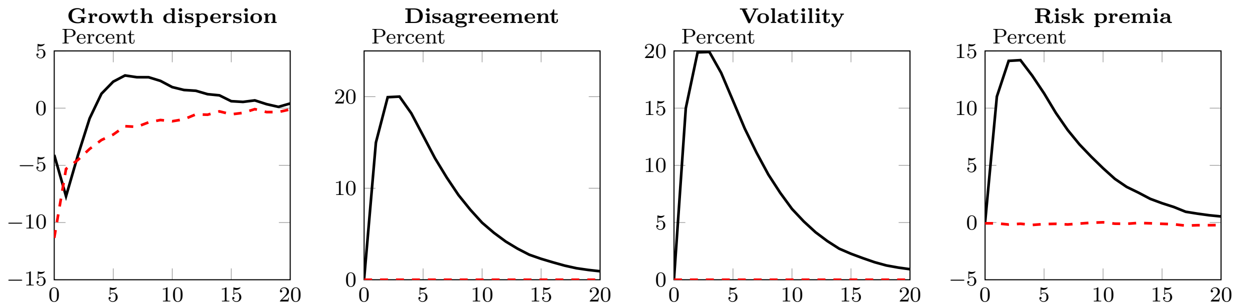

Figure 6: Further impulse responses to an aggregate financial shock. Solid black lines are impulse responses in the endogenous uncertainty economy; dashed red lines are counterfactual responses where the signal precision is exogenously fixed at its unconstrained level. All responses are in percentage deviations.

The asymmetric responses of firms in belief traps compared to non-constrained firms also helps explaining the skewness of output growth documented in recent empirical studies (e.g., Salgado, Guvenen and Bloom, 2015). Specifically, while output growth is essentially symmetric at the steady state ($\gamma_1 = -0.01$), it increases to $\gamma_1 = -0.22$ at the peak of the crisis. In comparison, Bloom et al. (2014) report a skewness of $\gamma_1 = -0.33$ in sales growth at the establishment level in a pooled sample over the years 2008-2009.

Disagreement, volatility, and risk premia The endogenous uncertainty channel in our model also helps explaining the increase in disagreement among financial analysts as well as the increases in volatility and risk premia of stock prices that are typically observed during financial crisis.[30]

An increase in disagreement (defined by $\text{Var}\{\mathbb{E}\{a_{i,t}|\mathcal{I}_{ik,t}\}\} = \hat{\sigma}^4_{i,t}/\sigma^2_{s,2}$ where the variance is with respect to k) is a direct corollary of Proposition 4. Intuitively, when $\varsigma_{i,t}$ increases, investors respond by increasing the weight on their private information $s^2_{ik,t}$, implying an increase in the cross-investor dispersion of beliefs. A similar effect also explains the increase in volatility (defined as the variance in $\log Q_{i,t}$ conditional on the complete history up to date $t-1$ and conditional on χ_t). Specifically, when $\varsigma_{i,t}$ increases, investors also increase the weight on $s^Q_{i,t}$, making their expectations more exposed to financially noise $\eta_{i,t}$. Finally, risk premia in our model are proportional to the term $\pi_\eta(1-\eta_{i,t})\hat{\sigma}^2_{i,t}$ in (12), which again increases when investors' uncertainty increases.[31]

The second to last panel in Figure 6 illustrates these effects, plotting disagreement, volatility and risk-premia averaged across all islands along the simulated impulse response path. Notably, none of the three series increases in the counterfactual where uncertainty is fixed at the pre-crisis level.

Asymmetric impact of financial shocks While the model's response to an aggregate financial shock scales almost linearly with the *fraction* of firms that are constrained during the crisis[32], the

[30] See, e.g., Senga (2015) and the data provided below on disagreement, and the VIX index for volatility. More generally, the implied co-movement between disagreement, volatility and risk-premia is consistent with the evidence presented by Carlin, Longstaff and Matoba (2013), documenting a strong positive relationship between these variables.

[31] As $\eta_{i,t}$ and $\hat{\sigma}^2_{i,t}$ are positively correlated, average risk-premia increase more (in percentage terms) than average uncertainty does (even though $\eta_{i,t}$ averages to zero in the cross-section).

[32] To see what drives the linearity, observe that absent general equilibrium effects, the output loss is approximately given by the fraction of island in a belief trap times the average output loss among those islands. The linearity then

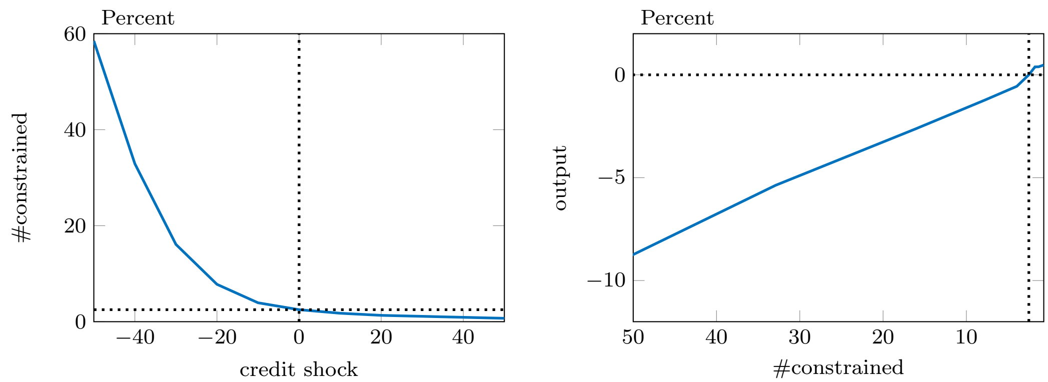

Figure 7: Output loss at the peak of the crisis in relation to the magnitude of the underlying credit supply shock to χ_t (in percent) and the corresponding fraction of firms that was constrained on impact. Black dotted lines indicate the stochastic steady state (where 2.5 percent of all firms are constrained).

model's response is highly nonlinear and asymmetric in the *magnitude* of the exogenous shock to χ_t. Figure 7 illustrates this, relating the output loss at the peak of a crisis to the magnitude of the initial shock (measured in percentage deviations) and the corresponding fraction of firms that becomes constrained on impact. As a reference, the black dotted lines indicate the stochastic steady state with a fraction of 2.5 percent of constrained firms. Two things can be noted. First, even very large positive shocks have only a muted impact on the economy. This is because there is no "over-borrowing" in our model, so that positive shocks to χ_t merely ensure that almost all islands are unconstrained, but do not lead to "credit booms". Second, negative but small credit supply shocks are less severe than the large shocks simulated in this section, since they map into disproportionately less firms that become constrained. Similar to, e.g., Brunnermeier and Sannikov (2014), this nonlinearity generates a discrepancy between high-frequency day-to-day fluctuations in financial markets, which have little impact on the real economy, and rare tail events, which cause pronounced recessions.

5.3 Application to the 2008/09 financial crisis

While the model is too stylized for a full quantitative exploration, we wish to highlight its potential as a contributing factor to the 2008/09 financial crisis. To this end, we study a second simulation similar to the one in the previous subsection, but where we replace the geometrically decaying shock to $\log \chi_t$ with one that is aimed to replicate the disturbances in the financial system seen during the recent crisis. Specifically, starting from the stochastic steady state (see above), we now simulate a sequence $\{\chi_{t+s}\}$,

$$\log \chi_{t+s} = \log \bar{\chi} - \Delta_s,$$

follows because the endogenous tightening of credit limits caused by pessimism and uncertainty for islands in a belief trap is large compared to the direct effect of the financial shock.

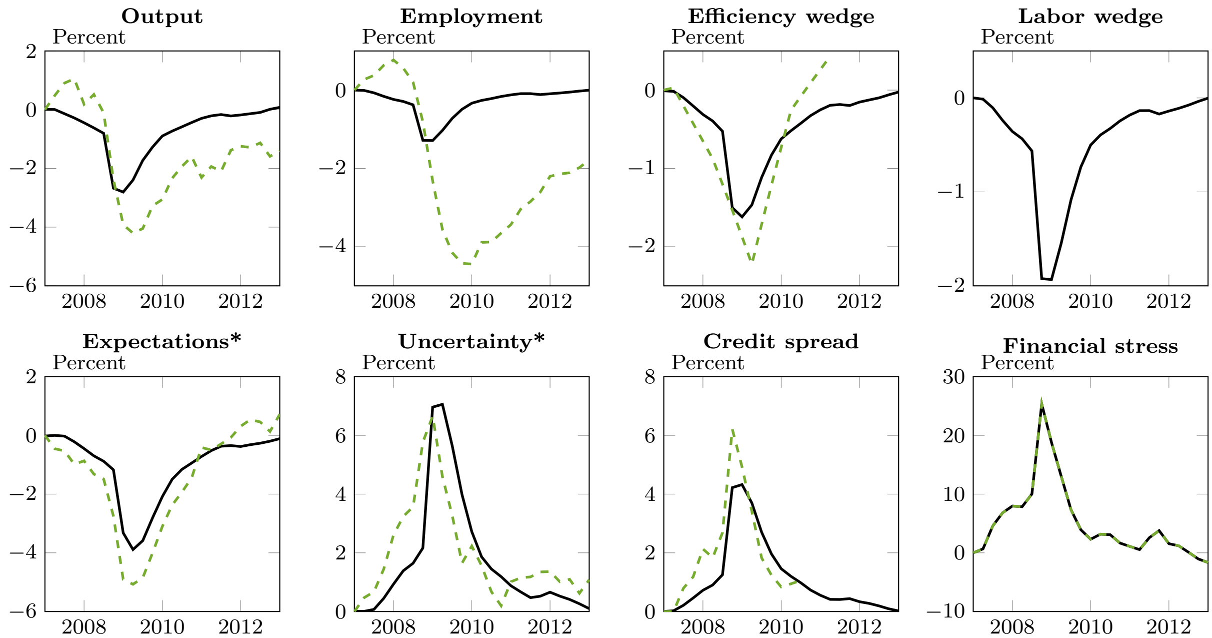

Figure 8: Simulation of 2008/09 financial crisis. Solid black lines are simulated responses of the aggregated (or averaged) endogenous uncertainty economy; dashed green lines are corresponding data series. The number of constrained firms and the credit spread are in percentage points, all other responses are in percentage deviations. Data series marked with * proxy their respective model counterparts without sharing the same units. In order to improve readability, the data series were scaled down by a factor 10 to retain the same scale.

where Δ_s is set proportional to the St. Louis Fed's financial stress index (STLFSI), measuring the "financial stress" component underlying the performance of the financial sector.[33] In our simulation, we treat the STLFSI as a disturbance in credit supply that is intrinsic to the financial sector, but we do not take a stand on the original cause or the propagation of that cause within the financial sector. The bottom right panel in Figure 8 plots the evolution of the STLFSI between 2007 and 2013. Since the series has no natural unit, we scale $\{\Delta_s\}$ so that at the peak of the crisis 20 percent of firms in the economy are constrained. This is broadly consistent with empirical evidence on the fraction of firms affected by the recent financial crisis (c.f,. Footnote 27).[34]

Figure 8 shows the resulting model responses (black solid lines) along with the corresponding data moments in the U.S. (green dashed lines).[35] The bottom right panel displays the St. Louis financial

[33]The STLFSI is based on a principal components analysis, where "financial stress" defines the most important factor explaining the comovement of several financial stress indicators, including 6 interest rate series, 5 yield spreads and 2 volatility indices. See https://www.stlouisfed.org/On-The-Economy/2014/June/What-Is-the-St-Louis-Fed-Financial-Stress-Index for details.

[34]Since our economy scales approximately linear in the fraction of firms constrained at the peak, variations in the scaling of $\{\Delta_s\}$ affect mainly the magnitude but not the persistence of the crisis.

[35]See Appendix B for a detailed description of the data. The data on output, employment, and efficiency wedge (measured using TFP data) are detrended using the HP-filter ($\lambda = 1600$). Data on credit spreads is obtained from Gilchrist and Zakrajšek (2012), defining the average spread between corporate bonds and a hypothetical Treasure security that mirrors the cash flow of the corporate bond. Expectations and uncertainty are computed using IBES data. Both series are an order of magnitude larger in the data than in the model, likely due to EPS being an imperfect proxy for fundamentals. To make the graphs readable we therefore scaled down the expectations and uncertainty series by a factor 10.

stress index to which we calibrated the exogenous shock χ_t. The other panels display the endogenous responses. First, supporting the main mechanism, it can be seen that firm-level uncertainty in the data (proxied by the average cross-investor standard deviation of firm-specific forecasts in the IBES database) roughly matches the shape of the predicted uncertainty. Similarly, average expectations in the data (measured by the average firm-specific forecast in the IBES database) show a similar shape to the one predicted by the model.

In regard to the transmission to the real economy, the model predicts an increase in credit spreads by 433 basis points, compared to an increase in the credit spread by 621 measured in the data. The increased spreads then translate into an increase in the efficiency and labor wedges. Comparing the predicted decline in the model's efficiency wedge to the measured drop in Solow residuals in the data, the model accounts for 73 percent of the observed fall in aggregate productivity at the peak of the crisis. In comparison, the counterfactual without belief traps (not plotted here) only accounts for 27 percent. Similar to standard RBC models, the model underpredicts the fall in employment, due to the decline in the wage rate that leads unconstrained firms to increase hours throughout the crisis.[36] Nevertheless, the model explains 67 percent in the drop in output at the peak (37 percent in the counterfactual). At the same time, the model contributes to explaining the persistent decline in output after financial stress has faded out in 2009Q3 even though it cannot account for the significant drop in employment.

6 Concluding remarks

This paper explores a novel mechanism that endogenously links financial crises to uncertainty about the economy's fundamentals and explores the implications for the real economy. When firms see their financial constraints tighten during times of financial distress, they are forced to respond by cutting hiring—and more generally investment—even for projects they deem profitable according to their own, private information. This makes information about firm fundamentals *endogenously scarce*, generating uncertainty.

We mention a number of consequences uncertainty has within our model—e.g. dispersion of output and beliefs, pessimism, and asset market volatility—but the most important is the feedback effect through the financial sector's belief about firm profitability: Financial markets see higher credit risks associated with lending to (or more generally investing into) distressed firms, worsening the shortage of funds even further. We illustrate that after aggregate shocks to the financial sector this vicious circle entails not only significant losses for distressed firms, but also for the whole economy.

There are two key externalities in our model. Both pertain to the way in which agents fail to internalize the effects of their actions on information generation and hence future financial constraints. First, in our model, constrained firms find it optimal to use up their whole credit limit—with the consequence that their actions become flat in their private information about productivity. In the

[36] By design the empirical labor wedge matches the model's labor wedge exactly when viewed through the lens of our model.

aggregate, this implies a greater loss of information than if agents were pursuing actions that varied according to their private information. Of course, given credit limits, this alternative might entail a short-run reduction of an island's productivity but might greatly help to reduce the amplification and persistence of financial shocks.

Second, investors do not internalize their effect on information generation. Given that information plays the role of a public good in this economy, investors provide inefficiently little credit: More credit would relax firms' financial constraints and improve the quality of information, akin to an investment into a public good. This externality is particularly strong when the marginal effect of credit on information aggregation is large. Financial intermediaries underinvest *precisely* when it would be needed most to support the economy.

There are three policies that our analysis speaks to. First, a bank recapitalization is not helpful to restore lending in our economy. The reason for this is simple: As long as it does not change the amount of credit granted to firms, it only represents a transfer from households to investors, which in our setup share one budget constraint with households anyway. What *is* helpful, however, is direct public lending to firms.[37] While in normal times such policies crowd out bank lending, in distressed times, they crowd *in* bank lending: Public lending relaxes firms' borrowing constraints and hence provides information as to which firms are worth investing more in. Finally, the most obvious policy implication of our setup is about information policy. While many firms commonly attempt to obfuscate their true performance during recessions, our setup highlights that actually policymakers might want to push for the opposite: Increasing transparency always helps, but *especially so* during crises when other sources of information, such as information from investment or business statistics, dry up. Given the significance of our positive results we speculate that the real effects of the recent financial crisis could have been mitigated if a combination of the latter two policies had been implemented more forcefully.

We believe there to be various interesting extensions and other applications of our mechanism. Two especially promising examples include applications to financial constraints on households, rather than firms; and to borrowing constraints for sovereigns. We leave both examples for future research.

[37] Public guarantees of firm debt are similar in that they incentivize lenders to lend despite their own private beliefs.

A Mathematical appendix

A.1 Proof of Proposition 1

Firms' optimality implies that

$$P_{ij,t} = \frac{\xi}{\xi - 1} \frac{W_t R_{ij,t}}{A_{ij,t}} \tag{18}$$

$$Y_{ij,t} = P_{ij,t}^{-\xi} C_t = \left(\frac{\xi - 1}{\xi}\right)^{\xi} \left(\frac{A_{ij,t}}{W_t R_{ij,t}}\right)^{\xi} C_t \tag{19}$$

$$L_{ij,t} = W_t N_{ij,t} = \left(\frac{\xi - 1}{\xi}\right)^{\xi} \left(\frac{A_{ij,t}}{W_t}\right)^{\xi - 1} R_{ij,t}^{-\xi} C_t, \tag{20}$$

where $R_{ij,t} \equiv \max\left\{1, (A_{ij,t}/\bar{A}_{ij,t})^{1-1/\xi}\right\}$ is the multiplier on the working capital constraint.

Aggregating over $N_{ij,t} = Y_{ij,t}/A_{ij,t}$ and $Y_{ij,t}$, yields

$$N_t = \left[(1 - \tau_t^A) A^{\text{eff}}\right]^{\xi - 1} \left[(1 - \tau_t^N) W_t^{-1}\right]^{\xi} C_t \tag{21}$$

$$Y_t = \left[(1 - \tau_t^A) A^{\text{eff}}\right] N_t, \tag{22}$$

where

$$A^{\text{eff}} = \left[\int_{I \times J} A_{ij,t}^{\xi - 1} \, \mathrm{d}(i,j)\right]^{1/(\xi - 1)}$$

is the economy-wide efficient productivity, and where

$$1 - \tau_t^A = \frac{\text{MPN}_t}{\text{MPN}_t^{\text{opt}}} = \frac{Y_t/N_t}{A^{\text{eff}}} = \frac{1}{A^{\text{eff}}} \frac{\left(\int_{I \times J} A_{ij,t}^{\xi-1} R_{ij,t}^{1-\xi} \, \mathrm{d}(i,j)\right)^{\xi/(\xi-1)}}{\int_{I \times J} A_{ij,t}^{\xi-1} R_{ij,t}^{-\xi} \, \mathrm{d}(i,j)}$$

and

$$1 - \tau_t^N = \frac{\text{MRS}_t}{\text{MPN}_t} = \frac{W_t}{(1 - \tau_t^A) A^{\text{eff}}} = \frac{\xi - 1}{\xi} \frac{\int_{I \times J} A_{ij,t}^{\xi-1} R_{ij,t}^{-\xi} \, \mathrm{d}(i,j)}{\int_{I \times J} A_{ij,t}^{\xi-1} R_{ij,t}^{1-\xi} \, \mathrm{d}(i,j)},$$

define the aggregate efficiency and labor wedge. Because $A_{ij,t}$ is normally distributed around $A_{i,t}$ there always exists a positive measure of firms (in each island) with $R_{ij,t} > 1$, and thus $1 - \tau_t^N < (\xi - 1)/\xi$. To see that $(1 - \tau_t^A) < 1$, rearrange to obtain

$$\int_{I \times J} A_{ij,t}^{\xi-1} R_{ij,t}^{1-\xi} \, \mathrm{d}(i,j) < \left[\int_{I \times J} A_{ij,t}^{\xi-1} R_{ij,t}^{-\xi} \, \mathrm{d}(i,j)\right]^{(\xi-1)/\xi} \left[\int_{I \times J} A_{ij,t}^{\xi-1} \, \mathrm{d}(i,j)\right]^{1/\xi}$$

Defining $x_{ij} \equiv A_{ij,t}^{(\xi-1)^2/\xi} R_{ij,t}^{-(\xi-1)}$, $y_{ij} \equiv A_{ij,t}^{(\xi-1)/\xi}$, $p = \xi/(\xi - 1)$, and $q = \xi$ this can be rewritten as

$$\int x_{ij} y_{ij} \, \mathrm{d}(i,j) < \left[\int x_{ij}^p \, \mathrm{d}(i,j)\right]^{1/p} \left[\int y_{ij}^q \, \mathrm{d}(i,j)\right]^{1/q}.$$

Since $1/p + 1/q = 1$, this is an immediate consequence of Hölder's inequality. The strictness follows since $R_{ij,t} > 1$ for a positive measure of indices (i, j).

To compute the aggregates, note that household optimization yields

$$C_t N_t^{\zeta} = W_t. \tag{23}$$

Collecting equations and setting $C_t = Y_t$, aggregate employment, output, and the wage rate are pinned down by the solution to (21), (22) and (23), yielding

$$\begin{aligned} W_t &= (1 - \tau_t^N)(1 - \tau_t^A)A^{\text{eff}} \\ N_t &= (1 - \tau_t^N)^{1/(1+\zeta)} \\ Y_t &= (1 - \tau_t^A)A^{\text{eff}} N_t. \end{aligned}$$

A.2 Derivation of Equation (10)

Since investors have no means to differentiate between firms within a given island i, optimality implies a perfectly diversified portfolio across firms within a given island. Accordingly, we focus on investor (i, k)'s portfolio allocation between riskless storage and island-i's market portfolio. Under the assumption of log-normality of $c_{ik,t}$ we have that

$$\mathbb{E}\{c_{ik,t}^{1-\gamma} | \mathcal{I}_{ik,t}\} = (1 - \gamma)\mathbb{E}\{\log c_{ik,t} | \mathcal{I}_{ik,t}\} + \frac{1}{2}(1 - \gamma)^2 \text{Var}\{\log c_{ik,t} | \mathcal{I}_{ik,t}\}.$$

Dividing by $1 - \gamma$ and subtracting the logarithm of investors' initial wealth $\log(Q_{i,t}/2)$, investors' objective is given by

$$\mathbb{E}\{r_{ik,t}^p | \mathcal{I}_{ik,t}\} + \frac{1}{2}(1 - \gamma)\text{Var}\{r_{ik,t}^p | \mathcal{I}_{ik,t}\}, \tag{24}$$

where $r_{ik,t}^p = \log(2c_{ik,t}/Q_{i,t})$ is the return on investor (i, k)'s portfolio. Following Campbell and Viceira (2002, Section 2.1.3), we use a Taylor expansion to approximate $r_{ik,t}^p$ as

$$r_{ik,t}^p \approx 2x_{ik,t} r_{i,t}^{\text{rev}} + x_{ik,t}(1 - 2x_{ik,t})\text{Var}\{\overline{\text{rev}}_{i,t}\}.$$

where $r_{i,t}^{\text{rev}} = \overline{\text{rev}}_{i,t} - \log Q_{i,t}$.[38] Substituting into (24), the first-order condition to investor (i, k)'s portfolio problem is

$$\mathbb{E}\{r_{i,t}^{\text{rev}} | \mathcal{I}_{ik,t}\} + \left(\frac{1}{2} - 2x_{ik,t}\gamma\right)\text{Var}\{\overline{\text{rev}}_{i,t}\} = 0$$

or

$$x_{ik,t} = \frac{\mathbb{E}\{\overline{\text{rev}}_{i,t} | \mathcal{I}_{ik,t}\} - \log Q_{i,t} + \frac{1}{2}\text{Var}\{\overline{\text{rev}}_{i,t}\}}{2\gamma \text{Var}\{\overline{\text{rev}}_{i,t}\}}. \tag{25}$$

Market clearing requires

[38] Notice that the log-return on risk-free storage is 0 in our case, dropping from the approximation.

$$m = m \int x_{ik,t} \,\mathrm{d}k + m\eta_{i,t}.$$

Substituting (25) and rearranging yields (10).

A.3 Derivation of Equation (11)

Combining (9), (18) and (19), we have that non-log-linearized revenues are given by

$$\log \int \frac{\xi}{\xi - 1} \left(A_{ij,t}^{\xi-1} \min\left\{1, A_{ij,t}/\bar{A}_{i,t}\right\}^{\frac{(\xi-1)^2}{\xi}} \right) \Omega_t \,\mathrm{d}j.$$

We define f as the log-linearized version of

$$\hat{f}(A_{i,t}, \bar{A}_{i,t}) \equiv \log \int \left(A_{ij,t}^{\xi-1} \min\left\{1, A_{ij,t}/\bar{A}_{i,t}\right\}^{\frac{(\xi-1)^2}{\xi}} \right) \mathrm{d}j, \tag{26}$$

which is well-defined since $A_{ij,t}$ is normally distributed across j with mean $A_{i,t}$ and standard deviation σ_ϵ. This immediately implies (11). Since σ_ϵ turns out to be very small in our calibration, we also log-linearize $\hat{f}$ around $\sigma_\epsilon = 0$. Noticing that $\bar{A}_{i,t}^{-(\xi-1)} \hat{f}(A_{i,t}\bar{A}_{i,t})$ only depends on $\log\left(A_{i,t}/\overline{A}_{i,t}\right)$ and denoting the linearization point as a^* we then find for the linearization

$$f(A_{i,t}, \overline{A}_{i,t}) = (\xi - 1)\log \overline{A}_{i,t} + \kappa_0(a^*) + \kappa_1(a^*)\left(\log A_{i,t} - \log \overline{A}_{i,t}\right)$$

where

$$\kappa_0(a^*) = (\xi - 1)a^* + 1_{\{a^* > 0\}} \frac{(\xi - 1)^2}{\xi} a^*$$

$$\kappa_1(a^*) = 1_{\{a^* < 0\}}(\xi - 1) + 1_{\{a^* > 0\}} \frac{(\xi - 1)^2}{\xi}.$$

Both expressions depend on whether that island is mainly unconstrained ($a^* < 0$) or mainly constrained ($a^* > 0$) at the linearization point. Our calibration—where only 2.5% of firms are constrained at the steady state—will imply that $a^* < 0$, which is the case we use from hereon. Therefore,

$$f(A_{i,t}, \overline{A}_{i,t}) = (\xi - 1)(a^* + \log A_{i,t}). \tag{27}$$

A.4 Proof of Proposition 2

Substituting (10) and (11) into (8), we have

$$\log \bar{A}_{i,t} = \frac{1}{\xi - 1} \log\left(\frac{\xi\chi_t}{\xi - 1}\right) + \frac{1}{\xi - 1} \int \mathbb{E}\{f(A_{i,t}, \bar{A}_{i,t}) | \mathcal{I}_{ik,t}\} \,\mathrm{d}k \\ - \frac{2\gamma}{\xi - 1}\left(1 - \eta_{i,t} - \frac{1}{4\gamma}\right) \mathrm{Var}\{f(A_{i,t}, \bar{A}_{i,t}) | \mathcal{I}_{ik,t}\} \tag{28}$$

with f defined by (27). To simplify the exposition, we introduce the following shortcut notations. We denote beliefs of investor (i,k) at date t by $\mu_k \equiv \mathbb{E}\{\log A_{i,t}|\mathcal{I}_{ik,t}\}$ and $\sigma^2 \equiv \mathrm{Var}\{\log A_{i,t}|\mathcal{I}_{ik,t}\}$ (where we dropped the time and island subscripts). Further, we drop the subscripts from $\bar{A}_{i,t}$, $A_{ij,t}$, $Q_{i,t}$ and denote their logs with lower case $\bar{a}$, a_j, q. In this notation, (28) implicitly defines a function $\bar{a} = \bar{a}(\{\mu_k\}_k, \sigma^2, \log\chi_t)$. We now simplify the second and third terms in (28).

Substituting in (27), the second term can be expressed as

$$\frac{1}{\xi-1}\int \mathbb{E}\{f(A_{i,t}, \bar{A}_{i,t})|\mathcal{I}_{ik,t}\}\,\mathrm{d}k = a^* + \int \mu_k\,\mathrm{d}k$$

and the third term becomes

$$\frac{2\gamma}{\xi-1}\left(1-\eta_{i,t}-\frac{1}{4\gamma}\right)\mathrm{Var}\{f(A_{i,t}, \bar{A}_{i,t})|\mathcal{I}_{ik,t}\} = 2\gamma(\xi-1)\left(1-\eta_{i,t}-\frac{1}{4\gamma}\right)\sigma^2.$$

Putting the two back together into (28) yields

$$\bar{a} = \frac{1}{\xi-1}\log\left(\frac{\xi\chi_t}{\xi-1}\right) + a^* + \int \mu_k\,\mathrm{d}k - 2\gamma(\xi-1)\left(1-\eta_{i,t}-\frac{1}{4\gamma}\right)\sigma^2,$$

which, after collecting terms, can be rearranged into

$$\bar{a} = \int \mu_k\,\mathrm{d}k - \pi_\sigma\sigma^2 + \pi_\chi\log\chi_t + \pi_\eta\sigma^2\eta_{i,t} + \pi_0,$$

where $\pi_0 = a^* + \frac{1}{\xi-1}\log\left(\frac{\xi}{\xi-1}\right)$, $\pi_\chi = \frac{1}{\xi-1}$, $\pi_\sigma = 2\gamma(\xi-1)\left(1-\frac{1}{4\gamma}\right)$, $\pi_\eta = 2\gamma(\xi-1)$. This proves Proposition 2.

A.5 Proof of Lemma 1

Since $A_{ij,t}$ is log-normal, we can rewrite (13) as

$$s^1_{i,t} = \log\Omega_{t-1} + H(a_{i,t-1}, \bar{a}_{i,t-1}) + \nu^1_{i,t}$$

with

$$H(a,\bar{a}) \equiv \log\int_{-\infty}^{\infty} e^{(\xi-1)\min\{u,\bar{a}\}}\,\mathrm{d}\Phi\left(\frac{u-a}{\sigma_\epsilon}\right).$$

The integral can be explicitly solved to give

$$H(a,\bar{a}) = (\xi-1)\bar{a} + \mathcal{L}(a-\bar{a})$$

where

$$\mathcal{L}(x) \equiv \log\left[e^{(\xi-1)x+\frac{1}{2}(\xi-1)^2\sigma_\epsilon^2}\Phi\left(-\frac{x}{\sigma_\epsilon}-(\xi-1)\sigma_\epsilon\right)+\Phi\left(\frac{x}{\sigma_\epsilon}\right)\right].$$

To prove the lemma, we thus need to show that $h(x) \equiv \mathcal{L}'(x)$ is positive and strictly decreasing, with limits $\xi-1$ as $x\to-\infty$ and 0 as $x\to\infty$. It is easy to see that $\mathcal{L}$ is smooth. Next we prove that any function of the form

$$\mathcal{F}(x) = \log\left[e^{x+\frac{1}{2}\sigma^2}\Phi\left(-\frac{x}{\sigma}-\sigma\right)+\Phi\left(\frac{x}{\sigma}\right)\right]$$

for $\sigma > 0$ is strictly increasing and strictly concave. Notice that any result about $\mathcal{F}$ can easily be translated into one of $\mathcal{L}$ since $\mathcal{L}(x) = \mathcal{F}((\xi-1)x)$ with $\sigma = (\xi-1)\sigma_\epsilon$.

The first derivative of $\mathcal{F}(x)$ is given by

$$\mathcal{F}'(x) = \left(1 + \frac{\Phi\left(\frac{x}{\sigma}\right)}{e^{x+\frac{1}{2}\sigma^2}\Phi\left(-\frac{x}{\sigma}-\sigma\right)}\right)^{-1}$$

which is clearly positive—and thus so is $h(x)$— and strictly between 0 and 1. Using L'Hospital's rule it is straightforward to see that $\lim_{x\to+\infty}\mathcal{F}'(x) = 0$ and $\lim_{x\to-\infty}\mathcal{F}'(x) = 1$, hence $\lim_{x\to+\infty} h(x) = 0$ and $\lim_{x\to-\infty} h(x) = \xi - 1$.

The second derivative of $\mathcal{F}(x)$ is given by

$$\mathcal{F}''(x) = -\mathcal{F}'(x)\left(1-\mathcal{F}'(x)\right)\left[\frac{1}{\sigma}\frac{\phi(\frac{x}{\sigma})}{\Phi(\frac{x}{\sigma})} + \frac{1}{\sigma}\frac{\phi(-\frac{x}{\sigma}-\sigma)}{\Phi(-\frac{x}{\sigma}-\sigma)} - 1\right]$$

which, using the fact[39] that $\frac{\phi(z)}{\Phi(z)} > -z$ for any z proves that $\mathcal{F}''(x) < 0$, so h' is negative.

A.6 Proof of Proposition 3

Suppose the working capital signal $s^1_{i,t}$ realizes at some s. If agents linearize the function $\mathcal{L}$ (defined in the proof to Lemma 1) around the face value $s^{\text{face}} = \mathcal{L}^{-1}(s)$, this means that they replace $\mathcal{L}$ by the following linearized function in their information updating problem,

$$\mathcal{L}_{\text{linear}}(x) = \mathcal{L}(s^{\text{face}}) + h(s^{\text{face}})(x - s^{\text{face}}).$$

This then implies that agents perceive the nonlinear signal $s^1_{i,t}$ *as if* it came from the corresponding "fictitious" linearized signal,

$$\mathcal{L}_{\text{linear}}(a_{i,t-1} - \bar{a}_{i,t-1}) + \nu^1_{i,t},$$

or informationally equivalent to this, they update as if they saw the signal

$$\begin{aligned}\hat{s}^1_{i,t} &= (\mathcal{L}_{\text{linear}})^{-1}(\mathcal{L}_{\text{linear}}(a_{i,t-1} - \bar{a}_{i,t-1}) + \nu^1_{i,t}) \\ &= a_{i,t-1} - \bar{a}_{i,t-1} + \frac{1}{h(s^{\text{face}})}\nu^1_{i,t},\end{aligned}$$

realizing at $\hat{s}^1_{i,t} = s^{\text{face}}$. This proves the lemma.

[39] One way to prove this fact is using a continued fraction expansion of $\Phi(x)$, or equivalently, the complimentary error function. See https://en.wikipedia.org/wiki/Error_function#Approximation_with_elementary_functions and Cuyt et al. (2008).

A.7 Proof of Proposition 4

Signals Consider the information set of trader (i,k) at time t, $\mathcal{I}_{ik,t} = \{s^1_{i,s}, Q_{i,s}, \chi_s\}_{s\leq t} \cup \{s^2_{ik,t}\}$. By definition, χ_t is orthogonal to $a_{i,t}$ and can thus be ignored for the purpose of learning about $a_{i,t}$. Given our approximation approach, the remaining elements of $\mathcal{I}_{ik,t}$ are Gaussian signals so that we can characterize $\mathbb{E}\{a_{i,t}|\mathcal{I}_{ik,t}\}$ using a standard Kalman filter. In particular, since $\tilde{\mathcal{I}}_{i,t} = \{s^1_{i,s}, Q_{i,s}, \chi_s\}_{s\leq t}$ is common knowledge, we can characterize beliefs recursively by first filtering through the publicly observable history $\tilde{\mathcal{I}}_{i,t}$, and then apply the filter one last time to process the information contained in $s^2_{ik,t}$.

From Proposition 3, $s^1_{i,t}$ is informational equivalent to observing $\hat{s}^1_{i,t} \sim \mathcal{N}(a_{i,t-1} - \bar{a}_{i,t-1}, \varsigma^2_{i,t})$. Asset prices $\{Q_{i,s}\}$ are thus the only endogenous signals that remain to be characterized. From (8), observing $Q_{i,t}$ is equivalent to observing $\bar{A}_{i,t}$. Further stripping away informationally irrelevant quantities from (12), the information in $\bar{A}_{i,t}$ is equivalent to the one in

$$\bar{\mathbb{E}}_{i,t}\{a_{i,t}\} + \pi_\eta \hat{\sigma}^2_{i,t}\eta_{i,t}. \tag{29}$$

The information content in (29) is endogenous and depends on the average expectation. To solve this fixed point, we postulate (and verify below) that (29) is informationally equivalent to a normal signal $s^Q_{i,t}$ with yet to be determined noise $\sigma_{Q,i,t}$.

Law of motion of public beliefs Letting $\tilde{\mathbb{E}}_{t-1}\{a_{i,t-1}\}$ and $\tilde{\sigma}^2_{i,t-1}$ denote the (public) prior mean and variance given $\tilde{\mathcal{I}}_{i,t-1}$, we are now ready to update beliefs given $\hat{s}^1_{i,t}$ and $s^Q_{i,t}$. Since $\hat{s}^1_{i,t}$ is a signal about $a_{i,t-1}$, we split the updating into two steps, first forming expectations about $a_{i,t-1}$ using only $\hat{s}^1_{i,t}$ and the prior. Standard Bayesian updating yields

$$\begin{aligned}
\mathbb{E}\{a_{i,t-1}|\tilde{\mathcal{I}}_{i,t-1}, \hat{s}^1_{i,t}\} &= \frac{\tilde{\sigma}^{-2}_{i,t-1}\tilde{\mathbb{E}}_{t-1}\{a_{i,t-1}\} + \varsigma^{-2}_{i,t}(\hat{s}^1_{i,t} + \bar{a}_{i,t-1})}{\tilde{\sigma}^{-2}_{i,t-1} + \varsigma^{-2}_{i,t}}, \\
\mathrm{Var}\{a_{i,t-1}|\tilde{\mathcal{I}}_{i,t-1}, \hat{s}^1_{i,t}\}^{-1} &= \tilde{\sigma}^{-2}_{i,t-1} + \varsigma^{-2}_{i,t}
\end{aligned}$$

Projecting forward, we get

$$\begin{aligned}
\mathbb{E}\{a_{i,t}|\tilde{\mathcal{I}}_{i,t-1}, \hat{s}^1_{i,t}\} &= \rho_a \frac{\tilde{\sigma}^{-2}_{i,t-1}\tilde{\mathbb{E}}_{t-1}\{a_{i,t-1}\} + \varsigma^{-2}_{i,t}(\hat{s}^1_{i,t} + \bar{a}_{i,t-1})}{\tilde{\sigma}^{-2}_{i,t-1} + \varsigma^{-2}_{i,t}} \\
\mathrm{Var}\{a_{i,t}|\tilde{\mathcal{I}}_{i,t-1}, \hat{s}^1_{i,t}\}^{-1} &= \left(\tilde{\sigma}^{-2}_{i,t-1} + \varsigma^{-2}_{i,t}\right)\delta_{i,t},
\end{aligned}$$

where

$$\delta_{i,t} = \left(\rho_a^2 + \left(\tilde{\sigma}^{-2}_{i,t-1} + \varsigma^{-2}_{i,t}\right)\sigma_a^2\right)^{-1}.$$

Now treating $\mathbb{E}\{a_{i,t}|\tilde{\mathcal{I}}_{i,t-1}, \hat{s}^1_{i,t}\}$ and $\mathrm{Var}\{a_{i,t}|\tilde{\mathcal{I}}_{i,t-1}, \hat{s}^1_{i,t}\}$ as prior, updating with respect to $s^Q_{i,t}$

yields

$$\tilde{\mathbb{E}}_t\{a_{i,t}\} = \tilde{\sigma}_{i,t}^2 \begin{bmatrix} \delta_{i,t}\varsigma_{i,t}^{-2} & \sigma_{Q,i,t}^{-2} & \delta_{i,t}\tilde{\sigma}_{i,t-1}^{-2} \end{bmatrix} \times \begin{bmatrix} \rho_a(\hat{s}_{i,t}^1 + \bar{a}_{i,t-1}) \\ s_{i,t}^Q \\ \rho_a \tilde{\mathbb{E}}_{t-1}\{a_{i,t-1}\} \end{bmatrix}$$

and

$$\tilde{\sigma}_{i,t}^2 = \left(\delta_{i,t}\varsigma_{i,t}^{-2} + \sigma_{Q,i,t}^{-2} + \delta_{i,t}\tilde{\sigma}_{i,t-1}^{-2}\right)^{-1}.$$

Private and average beliefs If in addition the private signal $s_{ik,t}^2$ is observed, straightforward updating yields the following posterior, given by

$$\mathbb{E}\{a_{i,t}|\tilde{\mathcal{I}}_{i,t}, s_{ik,t}^2\} = \frac{\hat{\sigma}_{i,t}^2}{\sigma_{s,2}^2} s_{ik,t}^2 + \frac{\hat{\sigma}_{i,t}^2}{\tilde{\sigma}_{i,t}^2} \tilde{\mathbb{E}}_t\{a_{i,t}\} \tag{30}$$

and

$$\hat{\sigma}_{i,t}^2 = \left(\sigma_{s,2}^{-2} + \tilde{\sigma}_{i,t}^{-2}\right)^{-1}. \tag{31}$$

Aggregating across agents, we have that

$$\bar{\mathbb{E}}_t\{a_{i,t}\} = \frac{\hat{\sigma}_{i,t}^2}{\sigma_{s,2}^2} a_{i,t} + \frac{\hat{\sigma}_{i,t}^2}{\tilde{\sigma}_{i,t}^2} \tilde{\mathbb{E}}_t\{a_{i,t}\}. \tag{32}$$

Characterizing $\sigma_{Q,i,t}$ To complete the characterization, we still have to determine $\sigma_{Q,i,t}$. For this, substitute (32) back into (29). Note, however, that the last term in (32) is common knowledge among investors, so that (29) is informationally equivalent to observing $\frac{\hat{\sigma}_{i,t}^2}{\sigma_{s,2}^2} a_{i,t} + \pi_\eta \hat{\sigma}_{i,t}^2 \eta_{i,t}$, or, equivalently, $a_{i,t} + \sigma_{s,2}^2 \pi_\eta \eta_{i,t}$. Finally, subtracting the common knowledge term $\rho_\eta \tilde{\mathbb{E}}_{t-1}\{\eta_{i,t-1}\}$ from $\eta_{i,t}$, the $Q_{i,t}$ signal is equivalent to

$$s_{i,t}^Q \equiv a_{i,t} + \sigma_{s,2}^2 \pi_\eta \left(\eta_{i,t} - \rho_\eta \tilde{\mathbb{E}}_{t-1}\{\eta_{i,t-1}\}\right). \tag{33}$$

Notice that the belief over $\eta_{i,t}$ evolves according to

$$\tilde{\mathbb{E}}_t\{\eta_{i,t}\} = \frac{1}{\sigma_{s,2}^2 \pi_\eta} \left(s_{i,t}^Q - \tilde{\mathbb{E}}_t\{a_{i,t}\}\right) + \rho_\eta \tilde{\mathbb{E}}_{t-1}\{\eta_{i,t-1}\}.$$

We subtracted the prior belief over $\eta_{i,t}$ in (33) since

$$\left(\eta_{i,t} - \rho_\eta \tilde{\mathbb{E}}_t\{\eta_{i,t-1}\}\right) \Big| \tilde{\mathcal{I}}_{i,t-1} \sim \mathcal{N}(0, \sigma_{\eta,i,t}^2)$$

where

$$\sigma_{\eta,i,t}^2 = \sigma_\eta^2 + \rho_\eta^2 \mathrm{Var}\{\eta_{i,t-1}|\tilde{\mathcal{I}}_{i,t-1}\},$$

and

$$\mathrm{Var}\{\eta_{i,t-1}|\tilde{\mathcal{I}}_{i,t-1}\} = \mathrm{Var}\left\{ \frac{s_{i,t-1}^Q - a_{i,t-1}}{\sigma_{s,2}^2 \pi_\eta} \middle| \tilde{\mathcal{I}}_{i,t-1} \right\} = \left(\sigma_{s,2}^2 \pi_\eta\right)^{-2} \tilde{\sigma}_{i,t-1}^2.$$

Hence,

$$s_{i,t}^{Q} \sim \mathcal{N}(a_{i,t}, \sigma_{Q,i,t}^{2}),$$

where

$$\sigma_{Q,i,t}^{2} = \rho_{\eta}^{2}\tilde{\sigma}_{i,t-1}^{2} + \sigma_{\eta}^{2}(\sigma_{s,2}^{2}\pi_{\eta})^{2}.$$

A.8 Proof of Proposition 5

To prove Proposition 5, we show that for $\sigma_\epsilon = 0$, the $g_{\hat{\sigma}} = 0$ locus is Z-shaped. First, notice that $\hat{\sigma}_{i,t} = \hat{\sigma}_{i,t-1}$ is equivalent to $\tilde{\sigma}_{i,t} = \tilde{\sigma}_{i,t-1}$. Because it turns out to be slightly easier, we are going to express the law of motion of $\tilde{\sigma}_{i,t}$ in terms of $\tilde{\sigma}_{i,t-1}$ and look for the Z shape of the $\tilde{\sigma}_{i,t} = \tilde{\sigma}_{i,t-1}$ fixed point.

Using the equations in Proposition 4 and the ones right before it, and setting $a_{i,s} = \eta_{i,s} = \nu_{i,s}^{1} = 0$ and $\chi_s = \bar{\chi}$, after some algebra we find that

$$\tilde{\sigma}_{i,t}^{-2} = \frac{\varsigma_{i,t}^{-2} + \tilde{\sigma}_{i,t-1}^{-2}}{a + b(\varsigma_{i,t}^{-2} + \tilde{\sigma}_{i,t-1}^{-2})} + \frac{\tilde{\sigma}_{i,t-1}^{-2}}{c + d\tilde{\sigma}_{i,t-1}^{-2}},$$

where $a, b, c, d > 0$ are positive constants. Notice that the right hand side is strictly increasing in $\tilde{\sigma}_{i,t-1}^{-2}$ and strictly positive and finite for any $\tilde{\sigma}_{i,t-1}^{2} \in [0, \infty]$. For constant $\varsigma_{i,t}^{-2}$, the right hand side is also strictly concave in $\tilde{\sigma}_{i,t-1}^{-2}$.

With $\sigma_\epsilon = 0$, $\varsigma_{i,t}^{-2}$ is equal to 0 for $\overline{a}_{i,t-1} < 0$, jumping up to $\frac{\sigma_{s,1}}{\xi - 1}$ for $\overline{a}_{i,t-1} > 0$ (see Lemma 1 and Proposition 3). This means, the right hand side jumps up as $\tilde{\sigma}_{i,t-1}^{-2}$ increases enough to push $\overline{a}_{i,t-1} = \overline{\mathbb{E}}_{i,t-1}a_{i,t-1} - \pi_\sigma \hat{\sigma}_{i,t-1}^{2} + \pi_\chi \log \overline{\chi} + \pi_0$ above zero. This occurs once

$$\frac{\pi_\sigma}{\hat{\sigma}_{i,t-1}^{-2}} < \overline{\mathbb{E}}_{i,t-1}a_{i,t-1} + +\pi_\chi \log \overline{\chi} + \pi_0.$$

Moreover, notice that an increase in $\overline{\mathbb{E}}_{i,t-1}a_{i,t-1}$ moves the threshold of the jump to the left.

More abstractly, we can therefore express the law of motion of $\tilde{\sigma}_{i,t}^{-2}$ as

$$\tilde{\sigma}_{i,t}^{-2} = \mathcal{G}(\tilde{\sigma}_{i,t-1}^{-2}, \overline{\mathbb{E}}_{i,t-1}a_{i,t-1})$$

where $\mathcal{G}$ is strictly increasing in $\tilde{\sigma}_{i,t-1}^{-2}$, with an upwards jump at a threshold that decreases in $\overline{\mathbb{E}}_{i,t-1}a_{i,t-1}$. Since $\mathcal{G}$ is also strictly concave in $\tilde{\sigma}_{i,t-1}^{-2}$ to the left and to the right of the threshold, there can be up to two fixed points $\tilde{\sigma}_{i,t} = \tilde{\sigma}_{i,t-1}$, depending on the value of $\overline{\mathbb{E}}_{i,t-1}a_{i,t-1}$. This naturally creates the Z-shape.[40]

[40] To be more precise, the middle section of the Z shape (the third possible fixed point) comes about if $\sigma_\epsilon > 0$ since then the jump is "smoothed out" somewhat and no longer a jump.

B Data

B.1 Data used for calibration

Our calibration of the learning parameters is based on the "Summary History" for US firms from the Institutional Brokers Estimate System (IBES). We use forecasts about current quarter earnings per share (EPS), which are available starting in the third quarter of 1984. From the original sample, we exclude all forecasts that are recorded prior to the beginning of the current fiscal period and that are recorded less than 1 week before the forecast period ends. To reduce the sensitivity to outliers, we trim the 2% tails for each month and variable. Returns are obtained from the CRSP database and are adjusted for splits and dividends. To assess the predictive power of prices towards future earnings, we merge the two data sets so that returns at month $t-1$ are matched to EPS realizations at month t.[41] The resulting panel contains on average 2053 firms per month.

For our calibration, we compare the pre-crisis model moments (computed at the stochastic steady state with $\bar{\chi}$ chosen so that 2.5 percent of all firms are constrained) to the available data prior to the financial crisis (1984Q3–2006Q4). To isolate the firm-specific components in our data series, we extract a time-fixed effects from each of them, except for $\sigma_{i,t}^{\text{cross}}$ (for which we exploit the sample mean in the calibration).[42]

B.2 Data used for Figure 8 and Footnote 7

Here we describe the data sources for the time series shown in Figure 8 and underlying Footnote 7. We measure output, employment, and efficiency using real GDP (GDPC96), total nonfarm employment (PAYEMS) and TFP (RTFPNAUSA632NRUG) data from the St. Louis Fed's FRED database. All three series are detrended using a (6,32) band-pass filter. Credit spreads data is obtained from Gilchrist and Zakrajšek (2012), defining the average spread between corporate bonds and a hypothetical treasure security that mirrors the cash flow of the corporate bond (available until 2011). The measure for volatility is the VIX. Output growth dispersion is measured using the dispersion in sales growth rates in the COMPUSTAT database.

Finally, for investors' expectations, uncertainty, and disagreement, we use the final data set prepared for our calibration (see above), looking at the cross-sectional sample average in a given quarter.[43] The resulting time series are adjusted using a stationary seasonal filter to get rid of a strong seasonal trend. Based on our model, disagreement $(\sigma_{i,t}^{\text{cross}})^2$ is proportional to $\hat{\sigma}_{i,t}^4$. Accordingly, we use $\sigma_{i,t}^{\text{cross}}$ to proxy for uncertainty, and use $(\sigma_{i,t}^{\text{cross}})^2$ for the disagreement time series.

[41] Due to small timing issues in the two data sets, the implemented lag varies by ± 3 days, resulting in a total lag between 28 to 33 days.

[42] Formally, we subtract from each data series the cross-sectional average for a given month (equivalent to regressing each variable on a time dummy and working with the residuals).

[43] Of course, here we cannot control for time fixed effects.

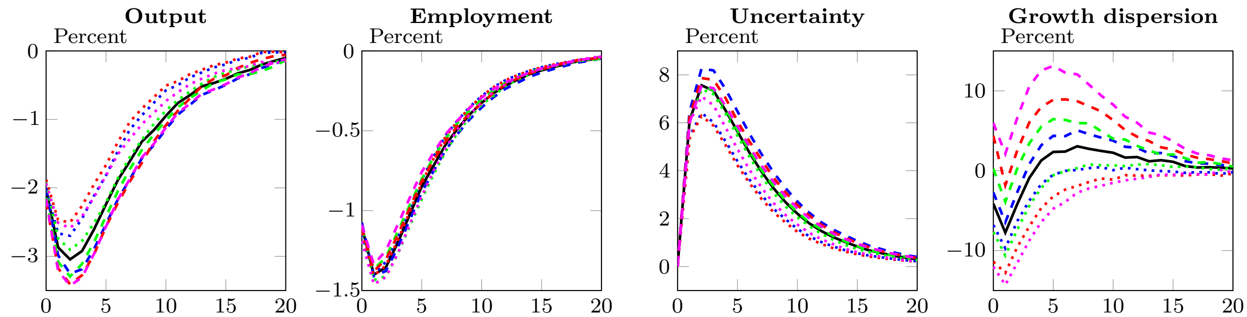

Figure 9: Impulse responses for output, employment, uncertainty, and the cross-sectional growth dispersion for alternative learning and credit rule parameters. The solid black lines correspond to the baseline parametrization. The dashed and dotted lines correspond to specifications with positive and negative changes in the parameters values, respectively. In particular, the blue lines correspond to a ± 20 percent change in σ_p; the green lines to a ± 20 percent change in σ_η; the red lines to a simultaneous ± 20 percent change in σ_ψ, σ_p and σ_η; and the magenta lines to a ± 40 percent change in π_σ. All impulse responses are in percentage deviations.

C Robustness specifications (for online publication)

In this appendix we explore the role of the learning parameters and the credit rule coefficient for the response of the model economy to an aggregate credit shock. For this purpose, we repeat our simulation in Section 5.2 of a geometrically decaying aggregate shock for various specifications. For each specification, we set $\bar{\chi}$ and Δ so that 2.5 percent of the firms are constrained in the steady state and 20 percent of the firms are constrained at the peak of the crisis akin to our baseline simulation. Figure 9 displays the resulting model responses for output, employment, uncertainty, and the cross-sectional growth dispersion.

Specifically, the figure shows four sets of specifications. First, the blue lines correspond to a ± 20 percent change in σ_p, reflecting a relative increase (dotted lines) or decrease (dashed lines) of the private investor signals relative to the benchmark case (solid black lines). Similarly, the green lines correspond to a ± 20 percent change in σ_η, whereas the red lines correspond to a simultaneous ± 20 percent change in all three learning parameters (σ_ψ, σ_p and σ_η). Finally, the magenta lines correspond to ± 40 percent changes in π_σ, reflecting values for the credit rule parameter based on our micro-foundation of the credit rule when investors have relative risk aversion of 1 and 2, respectively (see Appendix **??**).

It can be seen that variations in these parameters induce only small changes in the responses of output, employment, and uncertainty. The same holds true for all other model variables not depicted here. Specifically, looking at output, a variation in the relative signal precision (blue and green lines) results in responses that are within 0.5 and 0.25 percentage point bands around the benchmark, respectively. A simultaneous increase/decrease of the informativeness of all signals by 20 percent, leads to slightly larger deviations from the benchmark response of maximal 0.65 percentage points. Changes in the credit rule coefficient lead to deviations from the benchmark response of maximal 0.5 percentage points.

In contrast to the small impact of these parameter changes on cross-sectional *averages*, variations in the learning and credit rule parameters do have, however, important consequences for the cross-sectional *dispersion* of output growth. This can be seen in the forth panel of Figure 9. In particular, it can be seen that depending on the parametrization, the response in the growth dispersion may be both negative or positive throughout the whole impulse response path, or can be first decreased and then increased as is the case in our calibration.

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