Heterogeneous switching costs[1]

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October 26, 2015

[1]We would like to thank Andrew Clausen, Philipp Kircher, George Mailath, Curt Taylor, Paul Klemperer, an anonymous referee, editor Yongmin Chen and the participants at seminars where this paper has been presented, especially those of the Economics department at the University of Cambridge.

Abstract

We consider a two period model where consumers have different switching costs. Before the market opens an incumbent sells to all consumers; after the market opens competitors appear. We identify the equilibrium both with Stackelberg and Bertrand competition and show how the presence of low switching cost consumers benefits the incumbent, despite the fact that it never sells to any of them. Furthermore, we identify a free rider effect among consumers.

JEL: D43, L13
keywords: switching cost, competition, Stackelberg, Bertrand.

1 Introduction

A firm which, in the past, has acquired a clientele composed of consumers with low switching costs will be forced, in the presence of entry, to charge low prices for fear of losing its consumers. By contrast, a firm which in the past has acquired a clientele composed of high switching cost consumers will be able to charge a high price. Foreseeing this, high switching costs consumers will have incentives to "follow" low switching costs consumers — *i.e.*, to purchase from the same firms as they do. This implies that heterogeneity of switching costs has complex strategic consequences which have largely been ignored in the literature. It will influence the strategies of firms, the equilibrium distribution of clients, and the value of incumbency. In this paper we explore these links.

In order to do so, we build a simple two period model. There is a continuum of consumers who need to buy one unit of a good in each of two periods. Changing supplier induces a switching cost and consumers try to minimize the total discounted sum of the prices which they pay and the switching costs that they incur. Consumers have heterogenous switching costs. There are also two types of firms: at the beginning of period 1, there is an Incumbent firm which has sold to all consumers in the past and both in periods 1 and 2 there are at least two potential entrants.

If consumers all had the same switching costs, they could make their purchasing decisions without taking into account the choices of other consumers. We showed in Biglaiser, Crémer, and Dobos (2013) (see also section 3) that in this case the discounted profit of the incumbent over any finite number of periods is equal to its profit in a one period model: in any period the entrants compete away any future rent.

On the other hand, when switching costs differ, a firm's price depends on the types of its past customers. Therefore, when making their purchasing decisions rational consumers take into account not only the price that firms charge, but also the types of their other clients. This makes the strategies of the consumers interdependent; as we will see, for a large range of parameters, some high switching cost consumers, expecting a lower price in period 2, choose in period 1 to purchase from an entrant who has attracted a sufficient number of low switching cost consumers despite the fact that by doing so they incur a higher current total cost. The high switching cost consumers will free ride on the presence of the low switching cost consumers.

This interdependence has a significant effect on the pricing decisions and on the profits of the firms as well as on the equilibrium market shares. We show this, first, in a model where consumers are of two different types: they either have a low switching cost, σ_L or a high switching costs, σ_H. If σ_L/σ_H

is small enough the incumbent firm will price so that it will *always* lose some of the high switch cost consumers in equilibrium. Furthermore, while the incumbent does not sell to any low switching cost consumer their presence affects its profits which increase as their number increases and as their switching cost decrease. We are even able to identify circumstances where, for intermediate values of σ_L/σ_H, the profits of the incumbent are smaller in the two period model than in the one period model (see Corollary 4). Finally, we demonstrate that for a large class of parameters the negative consequence for the incumbent of a decrease in the switching costs of high switching cost consumers can be overwhelmed by an equal decrease in the switching cost of the low type consumers.

We also find that the free rider effect is present in the second case, when there is a continuum of consumer switching costs. In this setting, a period 1 entrant attracts the lowest type consumers up to some cutoff switching cost. It then prices in period 2 so as to lose its period 1 consumers with the smallest switching costs. Nearly all its period 2 consumers pay a price smaller than their switching costs and profit from the presence of consumers with lower switching costs. We show that the profits of the Incumbent in the two period model are larger than in the one period model, and smaller than the discounted value of one period profit.

In a companion paper, Biglaiser et al. (2013), we studied an infinite horizon model where some consumers have (the same) positive switching cost and others have zero switching cost. Although these hypotheses lead to clean results, the main externality which we identify in the current paper is not present. We discuss this more precisely in section 6, where we also present a detailed discussion of the links between this paper and the literature.

We present the model in section 2. Then, as a benchmark, in section 3, we briefly discuss the equilibrium when all consumers have the same switching cost. In section 4 we characterize the equilibrium when there are two types of consumers, before turning to the case with a continuum of types in section 5. We discuss the literature in section 6 and present some concluding remarks in section 7.

2 The model

We consider a two period model, with initially one Incumbent, and free entry in every period. The firms produce identical goods at zero cost.[1] There is a

[1] In order to attract consumers, firms will charge negative prices. Our model yields exactly the same results with strictly positive marginal cost: the negative price must simply be interpreted as a discount below marginal cost. In markets where negative prices

continuum of consumers with mass normalized to 1. They have a perfectly inelastic demand for one unit of the good, and therefore buy one unit in each period — their aim is to minimize the cost of acquiring these units. In this section and in section 4, there are two types of consumers: high switching cost (HSC) consumers, who represent a fraction $\alpha \in (0,1)$ of the population, have a switching cost equal to σ_H, while low switching cost (LSC) consumers, who form a fraction $1-\alpha$ of the population, have a switching cost equal to σ_L, with $0 < \sigma_L < \sigma_H$. The switching cost is incurred every time a consumer changes from one supplier to another. It reflects industry wide similarities or compatibilities between products, rather than idiosyncrasies of specific sellers.

In period 0, and, maybe, in previous periods, the consumers have bought from the Incumbent, firm I. We take I's position as given and do not examine how it became the incumbent, but only the continuation game once other firms can enter. Thus, we do not analyze whether some of the incumbency rents that we identify were exhausted in the competition to become the Incumbent (see Klemperer (1987)).

For simplicity, in the main text we assume that there is free entry, *i.e.*, that there is a large number[2] of potential entrants which can enter the market at zero cost, in periods 1 and in period 2 — it is easy to see that the proofs hold with minor changes if there are only two potential entrants in each period.[3] In period 2 the incumbent(s) are all the firms that sold to a strictly positive mass of consumers in each period. These incumbents include the Incumbent, if it "keeps" some of its consumers, and the *successful* or *active* period 1 entrants, that is those who attracted consumers. The main focus of our study is the following "Bertrand" game:

Stage 1: The incumbent(s) and the entrants set prices;

Stage 2: The consumers choose from which firm to buy.

All of our qualitative results also hold true, and are sometimes easier to establish, in the "Stackelberg" version of this game:

are impossible, for instance because of free disposal, we would need to assume that the marginal cost is large enough compared to the switching cost (alternately, firms could add free features, such as video content, which would play the same role as negative prices).

[2]Technically, a denumerable set. This is equivalent to a zero profit condition for entrants.

[3]As we will prove, consumers always buy from one of the lowest price entrants. Therefore Bertrand competition among the entrants forces them to use strategies which yield zero expected profits: in any equilibrium in which this would not be the case, at least one of the entrants would have incentives to undercut the other(s). The free entry assumption simplifies the exposition by making the discussion of this undercutting unnecessary.

Stage 1: The incumbent(s) sets price(s);

Stage 2: The entrants set their prices;

Stage 3: The consumers choose from which firm to buy.

The discount factor, common to all the firms and all the consumers, is $\delta \in (0, 1]$.

Note that, following much of the literature, we assume that only short term contracts are used and that consumers' switching costs do not depend on the firm from which they purchase.

We solve for the subgame perfect equilibrium, assuming that the equilibrium strategy of each firm depends only on the number (measure) of consumers of each type which purchased its product in the previous period. This constraint on the strategies only binds in the second period, and implies that a consumer can change supplier in the first period without affecting second period prices.

3 All consumers have the same switching cost

Before turning to the analysis of the model in section 2, it is worthwhile to sketch the analysis of the case where all consumers have the same switching cost: $\sigma_H = \sigma_L \stackrel{\text{def}}{=} \sigma$ (see Biglaiser et al. (2013) for a more detailed description).

If there is only one period, at equilibrium the Incumbent would clearly charge σ and "keep" all the consumers. Because there is a mass 1 of consumers, its profit would also be equal to σ.

Turning to the two period model, in equilibrium, whether in the Bertrand or Stackelberg models, all second period incumbents (that is all firms that have sold a positive amount of the good in the first period) charge σ, and make profits equal to σ times the mass of their first period customers. Therefore, the lower bound of the prices that entrants can charge in the first period without making negative profits is $-\delta\sigma$. Consumers know that all incumbents will charge σ in the second period. Hence, firm I will be able to "keep" its customers only by charging a price less than or equal to $-\delta\sigma + \sigma$. It is straightforward to show that it indeed charges this price and "keeps" all its customers, under both Bertrand or Stackelberg competition. Hence its discounted profit is $-\delta\sigma + \sigma + \delta\sigma = \sigma$. The profits are the same in the intertemporal model as in the static model: a firm gets only one bite of the switching costs! This logic holds for all finite of periods and for an infinite horizon, subject to stationarity conditions (see Biglaiser and Crémer (2011)).

4 High and low switching costs: analysis

Except in subsection 4.5, we assume that σ_L is small enough that the condition

$$\sigma_L < \frac{\alpha\delta}{1+\delta}\sigma_H, \tag{1}$$

holds. This implies

$$\sigma_L < \alpha\sigma_H. \tag{2}$$

In a one period model, entrants will always charge a price of 0. Thus, (2) implies that the Incumbent would charge σ_H, sell to all the HSC consumers and to no LSC consumer, and make a profit equal to $\alpha\sigma_H$ instead of selling to all consumers at a price of σ_L for a profit of σ_L. (In subsection 4.5, we study environments where inequality (1) does not hold.)

Turning to the two period model, we present the profits of the Incumbent and some comparative statics in Proposition 1 and in Corollary 1. After some comments and interpretation, we prove Proposition 1 for the Stackelberg model in 4.1 and for the Bertrand model in 4.2. It turns out that, despite the fact that profits are the same in both models, consumer surplus and welfare differ; we discuss these differences in 4.4.

Proposition 1. *If condition (1) is satisfied, the equilibrium profit of the Incumbent is*

$$\Pi = \sigma_H \left[\frac{\alpha\sigma_H - \sigma_L}{\sigma_H - \sigma_L}(1+\delta-\alpha\delta)\right] \tag{3}$$

under either Stackelberg or Bertrand competition. Π *is greater than the one period profit,* $\alpha\sigma_H$*, and smaller than the discounted value of a flow of one period profit,* $\alpha\sigma_H(1+\delta)$*.*

The presence of LSC buyers enables the Incumbent to generate higher profits than it would receive in the one period model, $\alpha\sigma_H$. Furthermore, as α converges to 1, Π converges to σ_H, the one period profit. This is also the case if σ_L converges to $\alpha\delta\sigma_H/(1+\delta)$, the right hand side of (1).[4]

Proposition 1 implies the following corollary.

Corollary 1. *Under the hypotheses of Proposition 1:*

1. The profit Π *of the Incumbent is increasing in* α *and* σ_H *and decreasing in* σ_L*;*

2. If $\alpha < (\sigma_L + \sigma_H)/2\sigma_H$*, which is always satisfied if* $\alpha < 1/2$*, then an equal increase in* σ_H *and* σ_L *leads to a decrease in* Π *(*$\partial\Pi/\partial\sigma_L + \partial\Pi/\partial\sigma_H < 0$*).*

[4] As detailed in Corollary 4, with $\sigma_L > \alpha\delta\sigma_H/(1+\delta)$ the profit in the one period model is greater than the profit in the two period model.

3. If $\sigma_L < \alpha^2\delta\sigma_H/(1+\delta)$, then a small increase in the number of LSC *consumers increases the profits of the Incumbent. For small enough η, if σ_H is increased by η and σ_L reduced by $\eta(1-\alpha)/\alpha$, so that the market average switching cost stays constant, then Π increases.*

Without surprise, when α or σ_H increase, the profit of the Incumbent increases. To understand why an increase in σ_L decreases profits, we take as granted what we will show below: the Incumbent will always price in such a way that it sells to no LSC consumer. Let us assume, only for expository purposes, that only one entrant attracted customers in the first period — it has attracted all the LSC consumers and a proportion $\gamma' > 0$ of the HSC customers. Therefore, its second period profit is $\alpha\gamma'\sigma_H$ if it charges σ_H, and $(\alpha\gamma' + (1-\alpha))\sigma_L$ if it charges σ_L. It is indifferent between charging σ_L and σ_H if $\gamma' = \gamma$, where γ is defined by

$$\alpha\gamma\sigma_H = (\alpha\gamma + (1-\alpha))\sigma_L \iff \gamma = \frac{1-\alpha}{\alpha}\frac{\sigma_L}{\sigma_H - \sigma_L} = \frac{1-\alpha}{\alpha}\left(\frac{\sigma_H}{\sigma_H - \sigma_L} - 1\right). \quad (4)$$

From (4), it is straightforward that an increase in σ_L leads to an increase in γ: the benefits for the entrant of 'keeping' the LSC customers in the second period increases, thus the number of HSC consumers attracted in the first period must increase if the entrant is to be kept indifferent between its two plausible second period strategies.[5] In equilibrium, in the first period a proportion γ of HSC consumers purchase from the entrant: if fewer than this proportion did so, the entrant would charge a low price in the second period, and be very attractive to HSC customers.[6] Therefore when σ_L increases, the Incumbent loses more customers, which explains the result.

Whether an equal increase in both σ_H and σ_L will increase or decrease the profit of the Incumbent will therefore depend on the relative strengths of two opposing effects, which, by (3), can be determined by evaluating the change in $\sigma_H(\alpha\sigma_H - \sigma_L)$. Adding η to both σ_H and σ_L and taking the derivative for $\eta = 0$, we obtain result ii) in Corollary 1: the negative consequences for the Incumbent of an increase in σ_L swamps the positive consequences of an equal increase in σ_H when α is small enough.[7] (As explained in section 6,

[5] In our companion paper with $\sigma_L = 0$, there is no positive γ that makes an entrant indifferent between pricing to keep only the HSC and pricing to keep all the consumers: in the latter case its profit is 0!

[6] As we will see shortly, the entrant mixes between σ_H and σ_L in the second period.

[7] It is easy to prove by computing the value of the derivative of

$$(1+\eta)\sigma_H\left[\frac{\frac{\alpha}{1+\eta}\sigma_H - \sigma_L}{\sigma_H - \sigma_L}(1+\delta - \frac{\alpha}{1+\eta}\delta)\right] = \frac{\sigma_H}{\sigma_H - \sigma_L}(\alpha\sigma_H - (1+\eta)\sigma_L)(1+\delta - \frac{\alpha}{1+\eta}\delta).$$

we obtain these "counterintuitive" comparative statics for reasons different than the rest of the literature.)

Note that part *3* of the corollary requires a σ_L smaller than the upper bound authorized by equation (1). Indeed, when σ_L is small entrants do not want to attract LSC customers, and an increase in their number makes the entrants less aggressive. On the other hand, when σ_L is larger, LSC customers become valuable enough to entrants that an increase in their number makes the entrants more aggressive.

Much empirical research is focussed on changes of the average switching cost, but this is not the relevant statistic to evaluate the consequences of switching costs on market outcomes. Indeed, routine calculations demonstrate that if σ_H is increased by η and σ_L is reduced by $\eta\alpha/(1-\alpha)$, so as to maintain the average switching cost but increase the variance of the switching costs, then the Incumbent's profit is increased. *An increase in the variance of the switching cost increases the profits of the Incumbent.*

We now turn to the proof of the Proposition 1.

4.1 Proof of Proposition 1 for the Stackelberg model

In period 2 of the Stackelberg model, all the firms which sold strictly positive amounts in period 1 (the "period 2 incumbents") announce their prices first, followed by the entrants. Because period 2 entrants charge 0, the following lemma is straightforward.

Lemma 1. *In period 2, incumbents charge*

(i) σ_H *if the proportion of* HSC *consumers in their first period clientele is strictly greater than* σ_L/σ_H*;*

(ii) σ_L *if this proportion is strictly smaller than* σ_L/σ_H*;*

(iii) σ_H *or* σ_L*, maybe mixing between the two, if it is equal to* σ_L/σ_H*.*

If the firm from which it purchased in period 1 charges σ_H in period 2, a LSC consumer will choose to purchase from a period 2 entrant at a price of 0. Hence, his total (price + switching cost) period 2 cost will always be equal to σ_L, no matter which firm he purchased from in period 1. This proves the following lemma where we call p_I the price charged by the Incumbent in period 1 and p_E the *lowest* price charged by any entrant in that period.

Lemma 2. *In the first period,* LSC *consumers purchase from one of the lowest price entrants if* $p_E + \sigma_L < p_I$ *and from the Incumbent if* $p_E + \sigma_L > p_I$*. If* $p_E + \sigma_L = p_I$ *they are indifferent between purchasing from one of the lowest priced entrants and purchasing from the Incumbent.*

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with respect to η for $\eta = 0$.

Lemma 2 shows that LSC customers minimize their cost in each period.

Because it is strictly more expensive for HSC than for LSC consumers to switch firms, no HSC consumer switches suppliers in the first period unless all the LSC consumers switch. This implies that the proportion of HSC consumers in the first period clientele of the Incumbent will be at least equal to α, and therefore proves part *(i)* of Lemma 3.

Lemma 3. *In period 2, the first period Incumbent charges* σ_H.

Clearly, any entrant who attracted consumers in period 1, will set prices in period 2 according to Lemma 1. We leave the easy proof of the following lemma to the reader.

Lemma 4. *If some* HSC *consumers purchase from an entrant in the first period, they purchase from one of the lowest price entrants and so do all the* LSC *consumers. Furthermore, in the second period successful first period entrants either all charge* σ_L*, all charge* σ_H *or all mix between* σ_L *and* σ_H *with the same probabilities.*

Going back to period 1, these three lemmas enable us to prove the following lemma, which describes the continuation payoff of the Incumbent as a function of the price it charges in the first period (see the appendix).

Lemma 5. *For a given price,* p_I*, charged by the Incumbent in the first period:*

(i) if $p_I < (1-\delta)\sigma_L$*, the Incumbent sells to all consumers in period 1 and to all* HSC *consumers (at price* σ_H*) in period 2. Its profit is* $p_I + \delta\alpha\sigma_H$.

(ii) if $(1-\delta)\sigma_L < p_I < (1-\delta)\sigma_H$*, the Incumbent sells to all* HSC *consumers in both periods and to no* LSC *consumer in either period. Its profit is* $\alpha(p_I + \delta\sigma_H)$.

(iii) if $(1-\delta)\sigma_H < p_I < (1-\alpha\delta)\sigma_H$*, there exists* $\gamma \in (0,1)$ *such that the Incumbent sells to* $\alpha(1-\gamma)$ HSC *consumers at price* p_I *in period 1 and at price* σ_H *in period 2, while its sales to* LSC *consumers are equal to 0 in both periods. Its profit is* $\alpha(1-\gamma)(p_I + \delta\sigma_H)$.

(iv) if $(1-\alpha\delta)\sigma_H < p_I$*, the Incumbent has zero sales in both periods.*

From Lemma 5, the profits of the Incumbent are increasing on the intervals $(-\infty, (1-\delta)\sigma_L)$, $((1-\delta)\sigma_L, (1-\delta)\sigma_H)$ and $\big((1-\delta)\sigma_H, (1-\alpha\delta)\sigma_H\big)$. Given the restrictions that we have imposed on σ_L/σ_H, it is easy to check that it is maximized on the union of these intervals for p_I smaller than and 'very close to' $(1-\alpha\delta)\sigma_H$. Therefore, the only equilibrium of the game has the Incumbent charging $(1-\alpha\delta)\sigma_H$ in the first period with the continuation equilibrium described in point *iii*, yielding the profits described by equation (3). For the HSC consumers to be indifferent between purchasing from the Incumbent and an entrant in period 1, it must be the case that any entrant who attracts consumers in period 1 charges σ_H in period 2 with probability

$$q \stackrel{\text{def}}{=} \frac{\alpha\sigma_H - \sigma_L}{\sigma_H - \sigma_L}. \tag{5}$$

This proves Proposition 1 for the Stackelberg model.

4.2 The proof of Proposition 1 in the Bertrand model

In this subsection, we prove that in any Bertrand equilibrium equation (3), and therefore Proposition 1, hold. In subsection 4.3 we prove that there exists a Bertrand equilibrium.

Notice first that Lemmas 1 to 4 of section 4.1 also hold in the case of Bertrand equilibria — the proofs are exactly the same. Because of free entry, period 2 entrants choose a price equal to 0. By Lemma 3, as in the Stackelberg case, a period 2 incumbent charges σ_L or σ_H depending on whether the proportion of its HSC customers in period 1 was less or greater than σ_L/σ_H, and, clearly, the Incumbent charges σ_H in period 2. Furthermore, this implies that, as in the Stackelberg case, in period 1 LSC consumers will optimally behave as if they were myopic, switching to one of the lowest price entrants if the difference between its price and the Incumbent's price is greater than σ_L and not switching if this difference is smaller than σ_L. It also implies that any HSC consumer who does not buy from the Incumbent in period 1 also buys from one of the lowest priced entrants. Indeed, any other entrant would attract only HSC customers, and therefore charge σ_H in period 2.

We are now ready to study the pricing behavior of the firms in period 1. Claims 1 to 6 hold for any set of parameters and are not restricted to the case when $\sigma_L < \alpha\delta\sigma_H/(1+\delta)$. We begin by Claim 1 which describes the behavior of entrants.

Claim 1. *In period* 1, *any active entrant charges a price in* $[-\delta\alpha\sigma_H, -\delta\sigma_L]$.

Proof. Any entrant who has attracted consumers in period 1 will charge at least σ_L in period 2. Therefore, competition and free entry will ensure that in period 1 no entrant which charges more than $-\delta\sigma_L$ attracts a positive measure of customers with positive probability. If the lower priced entrants charge prices strictly smaller than $-\delta\alpha\sigma_H$ and attract consumers their aggregate profit is negative: they attract all the LSC consumers; therefore the proportion of HSC consumers in their clientele is at most α and their aggregate second period profit at most $\alpha\sigma_H$. □

The next two claims describe properties of the Incumbent's first period demand function.

Claim 2. *If the Incumbent charges a price strictly greater than $\sigma_H(1-\delta)$ in period 1, then it sells to at most $(1-\gamma)\alpha$ HSC customers.*

Proof. Assume $p_I > (1-\delta)\sigma_H$. Because $-\delta\sigma_L + \sigma_L < (1-\delta)\sigma_H$, Claim 1 implies that all the LSC consumers, who optimally act myopically in the first period, purchase from entrants. If in the aggregate the entrants attract a proportion of HSC customers smaller than γ, at least one of them will have a proportion of period 1 HSC customers strictly smaller than σ_L/σ_H and therefore charge σ_L with probability 1 in period 2. HSC customers would find this entrant more attractive than the Incumbent as $(-\delta\sigma_L + \sigma_H) + \delta\sigma_L < \sigma_H(1-\delta) + \delta\sigma_H$, which establishes the contradiction. □

Claim 3. *If in period 1 the Incumbent charges a price strictly smaller than $\sigma_H(1-\alpha\delta)$, then it sells to at least $(1-\gamma)\alpha$ HSC customers.*

Proof. If the Incumbent sold to fewer that $(1-\gamma)\alpha$ consumers, at least one of the successful entrants would attract enough HSC customers in the first period to charge σ_H in the second period; by Claim 1, these HSC customers would incur total discounted costs equal to at least $-\delta\alpha\sigma_H+\sigma_H+\delta\sigma_H$, which is strictly larger than the total discounted costs that they would incur from buying from the Incumbent in both periods. □

Claims 2 and 3 show that for $p_I \in (\sigma_H(1-\delta), \sigma_H(1-\alpha\delta))$, the Incumbent sells to exactly $(1-\gamma)\alpha$ customers. This implies the following claim.

Claim 4. *The Incumbent will never choose a first period price in $(\sigma_H(1-\delta), \sigma_H(1-\alpha\delta))$.*

Furthermore, this allows us to put a lower bound on the profit of the Incumbent.

Claim 5. *By choosing p_I below but 'close to' $\sigma_H(1-\alpha\delta)$, the Incumbent can guarantee itself discounted profits arbitrarily close to $(1-\gamma)\alpha\sigma_H(1-\alpha\delta+\delta)$.*

Proof. It sells to at least $(1-\gamma)\alpha$ HSC consumers at price $(1-\alpha\delta)\sigma_H$ in period 1 and at price σ_H in period 2. □

Letting $\underline{b}_I$ denote the lower bound on the Incumbent's price, we find

Claim 6. $\underline{b}_I \leq \sigma_H(1-\alpha\delta)$.

Proof. The Incumbent makes strictly positive profits. This implies that p_E is not strictly smaller than $p_I - \sigma_H$ with probability 1. However, if $\underline{b}_I > \sigma_H(1-\alpha\delta)$ an entrant could charge a price in $(-\alpha\delta\sigma_H, \underline{b}_I + \sigma_H)$ and obtain strictly positive expected profits. In the states of nature where it is not the lowest priced entrant, it would attract no consumers and make a profit equal to 0. When it is the lowest price entrant, which would happen with strictly positive probability by Claim 1, it would undercut the other entrants and also undercut the Incumbent by more than σ_H; its discounted profit would be strictly positive, which establishes the contradiction. □

Up to now, the only restriction on σ_L/σ_H which we have used is that it is smaller than α. We now restrict the analysis to the cases where condition (1) ($\sigma_L < \alpha\delta\sigma_H/(1+\delta)$) holds. This enables us to prove the following claim and by implication Proposition 1.

Claim 7. *If condition* (1) *holds, then at equilibrium* $\underline{b}_I > \sigma_H(1-\delta)$*.*

Proof. The Incumbent never sells to any LSC consumers. Indeed, to do so it would have to choose a price smaller than or equal to $(1-\delta)\sigma_L$ and its profit would be smaller than or equal to $(1-\delta)\sigma_L + \delta\alpha\sigma_H$. When (1) holds this profit is less than the profit which it can guarantee by selling to $(1-\gamma)\alpha$ consumers (see Claim 5). Indeed, because

$$1-\gamma = \frac{1}{\alpha}\left(1 - \frac{(1-\alpha)\sigma_H}{\sigma_H - \sigma_L}\right)$$

we have

$$\begin{aligned}
&(1-\delta)\sigma_L + \delta\alpha\sigma_H - [(1-\gamma)\alpha\sigma_H(1-\alpha\delta+\delta)] \\
&\qquad = (1-\delta)\sigma_L + \delta\alpha\sigma_H - \sigma_H(1-\alpha\delta+\delta)\left(1 - \frac{(1-\alpha)\sigma_H}{\sigma_H-\sigma_L}\right) \qquad (6)\\
&\qquad < \sigma_H\left[\frac{(1-\delta)\alpha\delta}{1+\delta} + \delta\alpha - (1-\alpha\delta+\delta)\left(1 - \frac{1-\alpha}{1-\frac{\alpha\delta}{1+\delta}}\right)\right] \qquad (7)\\
&\qquad = \sigma_H\left[\frac{(1-\delta)\alpha\delta}{1+\delta} + \delta\alpha - (1-\alpha\delta+\delta)\frac{\alpha}{1+\delta-\alpha\delta}\right]\\
&\qquad = \frac{\alpha\sigma_H}{1+\delta}(-1+\delta) < 0,
\end{aligned}$$

where the inequality in (7) stems from (1) and from the fact that the right hand side of (6) is decreasing in σ_L

By Claim 2, if the Incumbent chooses $p_I > \sigma_H(1-\delta)$, at least $\alpha\gamma$ HSC consumers buy from a period 1 entrant. Thus, the highest profit the Incumbent could make while selling to all HSC consumers in period 1 is $\sigma_H(1-\delta) + \delta\sigma_H = \alpha\sigma_H$. Using Claim 5, the Incumbent can improve its profit by charging a price larger than $\sigma_H(1-\delta)$, since equation (1) is equivalent to $\alpha\sigma_H < (1-\gamma)\alpha\sigma_H(1-\alpha\delta+\delta)$. □

Claims 4, 6 and 7 imply $\underline{b}_I = \sigma_H(1-\alpha\delta)$ whenever (1) holds. By Claim 2 this implies that the discounted profit of the Incumbent is bounded above by

$$(1-\gamma)\alpha\sigma_H(1-\alpha\delta+\delta\sigma_H) = (1-\gamma)\alpha(\underline{b}_I+\delta\sigma_H).$$

By Claim 5, this quantity is also a lower bound on the profit, and this proves Proposition 1.

4.3 What do Bertrand equilibria look like?

The reasoning of 4.2 is sufficient to prove equation (3), but a) leaves open the question of existence of equilibrium and b) does not provide much intuition about the equilibrium strategies of the agents. In this subsection, we tackle both of these issues by describing explicitly one equilibrium of the Bertrand game.

In all equilibria the Incumbent and the entrants use mixed strategies in period 1. For simplicity, we present an equilibrium where there is only one[8] active entrant, who chooses its price p_E in $[-\alpha\delta\sigma_H, -\delta\sigma_L]$, while the Incumbent chooses p_I in $[\sigma_H(1-\alpha\delta), -\delta\sigma_L+\sigma_H)$ and at least one other entrant charges $-\delta\sigma_L$ with probability 1. Then, all LSC customers buy from the active entrant, and, depending on the difference between p_I and p_E, either all or a fraction γ of HSC customers purchase from the entrant:

- if $p_I - p_E \geq \sigma_H$, then all HSC consumers buy from the entrant, who therefore charges σ_H in the second period — its second period profit is $\alpha\sigma_H$;
- if $p_I - p_E < \sigma_H$, a proportion γ purchases from the entrant, who in the second period uses a mixed strategy: he chooses prices σ_L and σ_H with probabilities such that the HSC customers are indifferent between switching and not switching suppliers in period 1 — its second period profit is $(\alpha\gamma+(1-\alpha))\sigma_L = \alpha\gamma\sigma_H$ (in the states of nature where its second period price is σ_H, all the LSC customers switch to a period 2 entrant).

Therefore, in equilibrium, a proportion at least equal to γ of the HSC consumers purchase from the entrant in period 1.

The entrant chooses p_E according to the following distribution G_E, which

[8]Our equilibrium is also an equilibrium if there are several active entrants and they each choose a mixed strategy such that the distribution of the minimum of the prices they charge is the function G_E defined below. In other words, there are many different payoff equivalent equilibria. The key to all equilibria is that there is free entry and that all entrants make zero expected profits in equilibrium.

has a mass point at $-\delta\sigma_L$:

$$G_E(p_E) = \begin{cases} \dfrac{p_E + \alpha\delta\sigma_H}{p_E + (1+\delta)\sigma_H} & \text{if } p_E \in [-\alpha\delta\sigma_H, -\delta\sigma_L), \\ 1 & \text{if } p_E = -\delta\sigma_L. \end{cases} \tag{8}$$

Then, for any $p_I \in [\sigma_H(1-\alpha\delta), \sigma_H - \delta\sigma_L)$, the Incumbent's expected discounted profit is

$$G_E(p_I - \sigma_H) \times 0 + (1 - G_E(p_I - \sigma_H)) \times (1-\gamma)(p_I + \delta\sigma_H) \\ = (1-\gamma)\sigma_H(1 + \delta - \alpha\delta).$$

To understand why the Incumbent will not find it profitable to choose a price outside of the interval $[\sigma_H(1-\alpha\delta), \sigma_H - \delta\sigma_L)$, we check for possible deviations. It is not profitable for the Incumbent to choose $p_I \geq \sigma_H - \delta\sigma_L$, as this implies $p_I - p_E \geq \sigma_H$ with probability 1, and no sales! To show that it is not profitable to choose $p_I < \sigma_H(1-\alpha\delta)$, we proceed in two steps.

a) By charging $\sigma_H(1-\alpha\delta)$, the Incumbent sells to a proportion $1-\gamma$ of HSC customers. By Claim 2, to sell more, the Incumbent must choose $p_I \leq \sigma_H(1-\delta)$, which implies that as long as it does not sell to LSC customers, its profit, $\alpha(p_I + \delta\sigma_H)$, is at most $\alpha\sigma_H$ and therefore smaller than $(1-\gamma)\sigma_H(1+\delta-\alpha\delta)$ by (1).

b) In order to sell to LSC customers, the Incumbent needs to make their total costs, over both periods, less than σ_L, which is the upper bound of their cost if they switch to the entrant in the first period. Given that they will switch in period 2 when it charges σ_H, this necessitates $p_I \leq (1-\delta)\sigma_L$, which leads to profits $p_I + \delta\alpha\sigma_H$ smaller than the profits when using the equilibrium strategy.

In our equilibrium the Incumbent chooses p_I according to the distribution

$$G_I(p_I) = \frac{p_I - \sigma_H(1-\alpha\delta)}{p_I - \sigma_H(1-\alpha\delta) + ((1-\alpha) + \alpha\gamma)(\sigma_H - p_I - \delta\sigma_L)}.$$

Then, the profit of the active entrant is

$$G_I(p_E + \sigma_H) \times (p_E + \delta\sigma_L)(1 - \alpha + \alpha\gamma) \\ + (1 - G_I(p_E + \sigma_H)) \times (p_E + \delta\alpha\sigma_H) = 0$$

when it chooses a price in $[-\alpha\delta\sigma_H, -\delta\sigma_L]$, and smaller than or equal to 0 when it chooses a price outside of this interval (the presence of another "inactive" entrant who charges $-\delta\sigma_L$ is crucial for this last point).

In all equilibria, p_I will be distributed according to G_I and p_E, interpreted as the lower bound of the prices of the active entrants, will be distributed

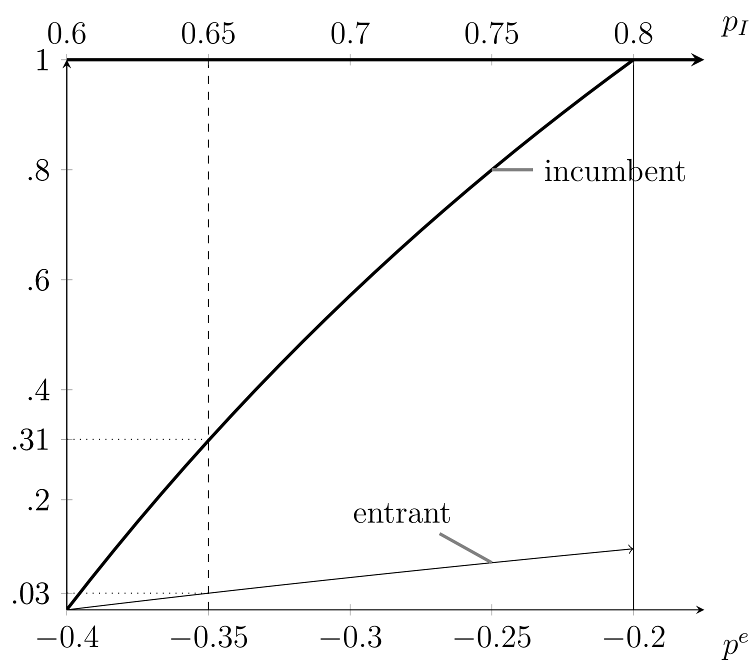

Figure 1: This figure represents the probability distributions in the mixed strategies of the Incumbent and the entrant with $\sigma_H = 1$, $\sigma_L = .2$, $\alpha = .4$ and $\delta = 1$, which implies $\gamma = 37.5\%$. For instance, reading along the vertical dashed line, if $p_E = -0.35$, we obtain $G(p_E) \approx 0.03$, which implies that if the Incumbent chooses $p_I = 0.65 = -0.35 + \sigma_H$, then it looses all its HSC customers with probability 3% and sells to a proportion $1 - \gamma$ of them with probability 97%. Similarly, if the entrant chooses $p_E = -0.35$, it sells to a proportion γ of HSC customers with a probability 31% and to all of them with probability 69%.

according to G_E. We will let the interested reader convince himself of this fact.

Figure 1 shows the equilibrium strategies with $\sigma_H = 1$, $\sigma_L = .2$, $\alpha = .4$ and $\delta = 1$.

4.4 Comparing Stackelberg and Bertrand equilibria

We found a pure strategy equilibrium in the Stackelberg model, but only mixed strategy equilibria in the Bertrand model. By Proposition 1, we saw that the equilibrium profits were the same. On the other hand, consumer utilities and welfare will differ across the two models. We first take up consumer utilities:

Corollary 2. *The expected utility of the* HSC *customers is lower in the Bertrand equilibrium than in the Stackelberg equilibrium, while the expected utility of the* LSC *customers is higher in the Bertrand than in the Stackelberg equilibrium.*

Proof. We first prove the results for the HSC consumers.

In period 1 of the Stackelberg equilibrium, some HSC consumers purchase from the Incumbent at price $p_I = \sigma_H(1 - \alpha\delta)$. These consumers pay σ_H in period 2, and their total discounted cost is $-\sigma_H(1 + \delta - \alpha\delta)$.

In the Bertrand equilibrium, there are only mixed strategy equilibria, where the Incumbent's first period price is strictly greater than $\sigma_H(1 - \alpha\delta)$, which is the price it charges in the Stackelberg equilibrium. In the states of nature when $p_I \leq p_E + \sigma_H$, the Incumbent sells to some HSC consumers. In period 2, these consumers will pay σ_H as in the Stackelberg equilibrium; therefore, in these states of nature, the utility of HSC consumers is smaller in the Bertrand model. In the states of nature where $p_I > p_E + \sigma_H$, none of the HSC consumers purchase from the Incumbent. In period 2, they pay σ_H to the entrant from which they purchased in period 1. Because this entrant charged at least $-\alpha\delta\sigma_H$ in period 1, the total cost of the HSC consumers is at least $\sigma_H(1 + \delta - \alpha\delta)$, which is also their utility in the Stackelberg equilibrium. Therefore, the HSC consumers are never better off and sometimes strictly worse off in the Bertrand equilibrium.

Now, we prove the result for the LSC consumers. In both equilibria, the period 2 cost of the LSC consumers is σ_L. In period 1 of both equilibria, the LSC consumers purchase from one of the lowest priced entrants. In the Stackelberg equilibrium, they pay $-\delta\sigma_L$ with probability 1. In the Bertrand equilibrium they never pay more than this amount, and pay strictly less with strictly positive probability, which proves the result. □

Now, we turn to the difference in welfare between the two equilibria. Since the firms offer identical products and all consumers make purchases each period, the only difference in welfare is due to switching costs.

Corollary 3. *Welfare is lower in the Bertrand than in the Stackelberg equilibrium.*

Proof. In the Stackelberg equilibrium all LSC customers switch in period 1 and a fraction γ of the HSC customers switch. In period 2, the LSC customers switch with probability q — see (5). Thus, the aggregate welfare loss due to switching is $\alpha\gamma\sigma_H + (1-\alpha)\sigma_L(1+\delta q)$.

In the Bertrand equilibrium, when $p_I > p_E + \sigma_H$ all the consumers switch in period 1 and all the LSC customers switch in period 2, for an aggregate welfare loss equal to $\alpha\sigma_H + (1-\alpha)\sigma_L(1+\delta)$, which is strictly greater than the loss in the Stackelberg equilibrium.

When $p_I \leq p_E + \sigma_H$ a fraction γ of the HSC customers and all the LSC customers switch in period 1, as in the Stackelberg equilibrium. In period 2, none of the HSC customers switch, again as in the Stackelberg equilibrium, and the LSC customers all switch with probability

$$1 - \frac{\sigma_H + p_E - p_I}{\delta(\sigma_H - \sigma_L)}.$$

Since this probability is strictly greater than q, the LSC customers switch more in period 2 in the Bertrand equilibrium. Hence, there is always at least as much switching in the Bertrand as in the Stackelberg equilibrium. □

Corollary 3 is a consequence of the fact that in the first period, in some states of nature, all the HSC customers will purchase from an entrant in the Bertrand equilibrium, which is not possible in the Stackelberg equilibrium. Furthermore, when all HSC consumers switch suppliers in period 1, the LSC customers will switch in period 2 with probability 1, to avoid paying σ_H.

4.5 Equilibrium with large σ_L

For completeness of the two type switching cost model, we now turn to a discussion of the equilibrium when equation (1) does not hold. Proofs and more details can be found in the web appendix of this paper.

If σ_L is very large, *i.e.,* $\sigma_L \geq \alpha\sigma_H$, then everything happens as if all the consumers were LSC consumers: the Incumbent charges $\sigma_L(1-\delta)$ in period 1, σ_L in period 2, and sells to all consumers in every period. Its total discounted profit is σ_L the same profit as in the one period model with free entry and the same profit as when all consumers are identical, see section 3.

This leaves open the question of the cases when $\sigma_L/\sigma_H \in (\delta\alpha/(1+\alpha), \alpha)$. If $\sigma_L/\sigma_H \in (x_C, \alpha)$, where x_C is the solution of (9) below, then there exists a pure strategy equilibrium where in period 1 the Incumbent sells to all the HSC consumers at a price $\sigma_H(1-\delta)$ and the entrants sell to all the LSC consumers at a price $-\delta\sigma_L$. In period 2, the Incumbent charges σ_H and 'keeps' all the HSC consumers — its profit over both periods is therefore $\alpha\sigma_H$. The best alternative strategy for the Incumbent would be to charge $\sigma_H - \delta\sigma_L$ in period 1, and sell to a proportion $1-\gamma$ of the HSC consumers. This strategy becomes more attractive as σ_L decreases, and dominates when $\sigma_L/\sigma_H < x_C$, where $x_C \in (\delta\alpha/(1+\alpha), \alpha)$ is the solution of

$$x_C\,(1+\delta+\alpha\delta-\alpha) = \delta\left(\alpha + {x_C}^2\right). \tag{9}$$

Claim A.3 in the web appendix shows that the mixed strategy equilibrium described in 4.3 for the case $\sigma_L < \alpha\delta\sigma_H/(1+\delta)$ also holds when $\sigma_L/\sigma_H \in (\delta\alpha/(1+\delta), x_C)$. The only difference stems from the fact that the profit of the Incumbent in the one period game is greater than the equilibrium profit in the two period game. As a consequence, we need to check that deviations where the Incumbent would retain all the HSC customers are not profitable. This yields the following corollary.

Corollary 4. *The equilibrium described in section 4.3 is also an equilibrium when σ_L/σ_H belongs to the interval $(\delta\alpha/(1+\delta), x_C)$. The profit of the Incumbent in this equilibrium is* smaller *than its profit in the equilibrium of the one period game.*

Before commenting on this result, it is worth emphasizing that we have shown that there exists *one* equilibrium of the dynamic game which yields profits smaller than static profit, *not* that all equilibria of the dynamic game satisfy this property.[9]

In the static model, all HSC consumers always purchase from the Incumbent at price σ_H; the LSC consumers impose no externality on the HSC consumers, since there is no future price to take into account. In the dynamic game, entrants are more aggressive than in the static game and also more attractive to the HSC consumers because they charge less than the Incumbent in the second period (at least if not too many HSC consumers change suppliers). When α is large enough, the entrants charge a sufficiently low price and attract sufficiently many HSC consumers that the Incumbent would be better off if it had only one period with less aggressive competitors to accumulate its profits. We believe that this is the first time that this phenomenon has been identified in the literature.

[9] We conjecture, but have been unable to prove, that all equilibria of the dynamic game do satisfy the property.

5 A continuum of types

In this section, we reexamine our model when there is an continuum of consumers of mass 1 with switching costs distributed according to the distribution function $F(\sigma)$ over[10] $[\underline{\sigma}, \overline{\sigma}]$. We will say that F is *spread out* if $\underline{\sigma} = 0$, $\overline{\sigma} > 0$, and F has no atom — this hypothesis ensures that the model is sufficiently different from the one switching cost model and that the discounted two period profit is not equal to the one period profit.[11]

In the one period case with a continuum of consumers, the free rider effect manifests itself in the following way. The monopolist charges the price

$$p^m = \arg\max_p p(1 - F(p)).$$

Consumers with $\sigma \geq p^m$ incur a cost of p^m rather than the cost σ that they would incur if they were the only consumer. (Consumers with $\sigma < p^m$ incur no cost because of the presence of the other consumers.) If F is spread out, there will be some consumers who do not purchase from the Incumbent.

We can show the following proposition.

Proposition 2. *In the continuum case, with $\delta \in (0, 1)$ the profit of the Incumbent is greater than the one period profit and smaller than the discounted value of a flow of one period profits, both with Stackelberg and Bertrand timing. When the distribution F is spread out, both inequalities are strict.*

As is obvious from Proposition 1, the fact that F is spread out is not necessary for the fact the profit of the Incumbent is strictly smaller than the value of a flow of one period profit. For instance, Proposition 2 also holds if there are many "small" atoms or if $F(\sigma) = 0$ for $\sigma \in [0, \varepsilon]$ with ε small enough.

For simplicity, in the formal development that follows, we focus on equilibria where in period 1, a) there are at least two low price entrants and b) all consumers who do not purchase an incumbent purchase from the same firm. This does not affect equilibrium profits or choices.

For both the Stackelberg and the Bertrand case, it is easy to prove that there exists a σ_I such that in the first period all the consumers with switching costs greater than σ_I purchase from the Incumbent and all the consumers

[10] The assumption that the lower bound of the distribution is 0 is not essential, but simplifies the discussion by ruling out boundary effects.

[11] For example, if we have $F(\sigma) = \sigma - 100$ for $\sigma \in [100, 101]$ and 0 otherwise, or $F(\sigma) = \sigma/100000$ for $\sigma \in [0, 100)$ and $F(100) = 1$, all the economics are essentially the same as if the switching cost of every consumer is equal to 100.

with switching costs smaller than σ_I purchase from the entrant. It is straightforward to show than for any σ_I in the second period the Incumbent will charge[12]

$$p_2^I(\sigma_I) \geq \max\{p^m, \sigma_I\}.$$

5.1 Stackelberg competition

Under Stackelberg competition, after p_1^I has been chosen, σ_I and the lowest entrant price p_1^E satisfy (we are simplifying the notation by not writing explicitly their dependence on p_1^I)

- a zero profit condition for entry:

$$F(\sigma_I)p_1^E + \delta[F(\sigma_I) - F(p_2^E(\sigma_I))]p_2^E(\sigma_I) = 0; \tag{10}$$

- a condition that states that the consumer with switching cost σ_I is indifferent between purchasing from the entrant or from the Incumbent (notice that if he purchases from the Incumbent, his second period cost will necessarily be σ_I as the Incumbent will never charge less in the second period and if it charges more the consumer can purchase from a second period entrant at price 0):

$$p_1^I + \delta\sigma_I = \sigma_I + p_1^E + \delta p_2^E(\sigma_I) \Longrightarrow p_1^I - (1-\delta)\sigma_I = p_1^E + \delta p_2^E(\sigma_I). \tag{11}$$

Therefore, in the first period, the Incumbent chooses p_1^I to solve

$$\begin{aligned} &\max_{p_1^I, \sigma_I, p_1^E} \Pi = [1 - F(\sigma_I)]\, p_1^I + \delta p_2^I(\sigma_I) \left[1 - F(p_2^I(\sigma_I))\right], \\ &\text{subject to (10) and (11).} \end{aligned} \tag{12}$$

Multiplying (11) by $F(\sigma_I)$ and adding to (10), we obtain

$$p_1^I = (1-\delta)\sigma_I + \delta\frac{F(p_2^E(\sigma_I))}{F(\sigma_I)}p_2^E(\sigma_I). \tag{13}$$

Substituting (13) into (12), we obtain the profit as a function of σ_I:

$$\begin{aligned} \Pi = \delta\left[1 - F(\sigma_I)\right] &\frac{F(p_2^E(\sigma_I))}{F(\sigma_I)}p_2^E(\sigma_I) \\ &+ (1-\delta)[1 - F(\sigma_I)]\sigma_I + \delta p_2^I(\sigma_I)[1 - F(p_2^I(\sigma_I))]. \end{aligned} \tag{14}$$

This yields the following proof.

[12] In the second period, the effective competition for the Incumbent is coming from the second period entrants, not from the first period entrant who is trying to extract profits from the consumers it attracted in the first period.

Proof of Proposition 2 with Stackelberg timing. The second and third terms of (14) obviously reach their maxima for $\sigma_I = p^m$. Because the first term is positive (and strictly positive whenever F does not have an atom at σ_I), the maximum value of Π is (strictly) greater than

$$(1-\delta)[1-F(\sigma^m)]\sigma^m + \delta\sigma^m[1-F(\sigma^m)] = \sigma^m,$$

which is the one period profit of the Incumbent.

To show that the profit is less than the value of a two period flow of one period profits, notice that $p_2^E(\sigma_I) \le \sigma_I$ and therefore $F(p_2^E(\sigma_I)) \le F(\sigma_I)$, with both inequalities strict if F is spread out. Therefore, the first term of (14) is smaller that $\delta[1-F(\sigma_I)]\sigma_I$ (and strictly smaller if F is spread out). This implies that Π is less (or strictly less) than

$$[1-F(\sigma_I)]\sigma_I + \delta p_2^I(\sigma_I)[1-F(p_2^I(\sigma_I))],$$

whose maximum is $(1+\delta)(1-F(\sigma^m)\sigma^m$, which proves the result. □

5.2 Bertrand competition

Now, we turn to Bertrand timing.

In the first period, the equilibrium conditions are the following:

- the Incumbent takes p_1^E as given and chooses p_1^I to maximize

  $$\Pi = [1-F(\sigma_I)]p_1^I + \delta p_2^I(\sigma_I)[1-F(p_2^I(\sigma_I))],$$

  subject to (11).

- Given p_1^I, the price chosen by the lower price entrant p_1^E and σ_I satisfy conditions (10) and (11).

We can now finish the proof of Proposition 2.

Proof of Proposition 2 with Bertrand timing. We first prove that the profit of the Incumbent is greater than the one period profit. Using (11) to substitute in for p_1^I and thinking of the Incumbent as choosing σ_I given p_1^E, we have

$$\Pi = (1-\delta)[1-F(\sigma_I)]\sigma_I + \delta p_2^I(\sigma_I)[1-F(p_2^I(\sigma_I))] + [1-F(\sigma_I)]\big(p_1^E + \delta p_2^E(\sigma_I)\big) \quad (15)$$

The profit of the entrant is

$$0 = p_1^E \times F(\sigma_I) + \delta p_2^E(\sigma_I)\big(F(\sigma_I) - F(p_2^E(\sigma_I))\big) \le \big(p_1^E + \delta p_2^E(\sigma_I)\big)F(\sigma_I)$$

and therefore the last term of (15) is positive (strictly positive if F is spread out). This implies that the profit of the Incumbent is greater (strictly when F is spread out) than

$$\max_{\sigma_I}(1-\delta)[1-F(\sigma_I)]\sigma_I + \delta p_2^I(\sigma_I)[1-F(p_2^I(\sigma_I))],$$

which is the one period profit.

We now prove that the profit of the Incumbent is smaller than a discounted flow of one period profits. Competition between the first period entrants imply $p_1^E \leq 0$, with a strict inequality when F has no atom. Whatever the σ_I chosen by the Incumbent, $p_2^E(\sigma_I) \leq \sigma_I$ (with a strict inequality when F is spread out). Therefore, the last term of (15) is (strictly when F is spread out) smaller than $\delta[1-F(\sigma_I)]\sigma_I$, which implies the result by the same reasoning as in the Stackelberg case. □

5.3 Example: the uniform distribution

Now, we illustrate the equilibrium in the continuum of type case with σ uniformly distributed on $[0,1]$. This will allow us to show that the Incumbent's equilibrium profits can be different in the Stackelberg and the Bertrand case.

5.3.1 Stackelberg

With σ uniformly distributed on $[0,1]$, we have $p_2^E(\sigma_I) = \sigma_I/2$ and therefore, using (13), equation (14) becomes

$$\Pi = \delta\left(1-\sigma_I\right)\frac{\sigma_I}{4} + (1-\delta)\left(1-\sigma_I\right)\sigma_I + \delta\max\{\frac{1}{2},\sigma_I\}\left[1-\max\{\frac{1}{2},\sigma_I\}\right].$$

For any δ, this profit is maximized for $\sigma_I = 1/2$. The profit of the Incumbent, is $1+\delta/4$ times the static profit, 1/2.

Of course, $p_2^I = 1/2$ and, by (13), we obtain $p_1^I = (1-3\delta/4)\times(1/2)$ so that the total discounted cost for consumers with a switching cost greater than 1/2 is $(1+\delta/4)\times(1/2)$. Interestingly, whereas in the static case, consumers never lose from the presence of other consumers with different switching costs, in the dynamic case consumers whose σ belong to the interval $\left[1/2,(1+\delta/4)/2\right)$ have higher total cost than if all the consumers had the same switching cost than they have.

5.3.2 Bertrand

With σ uniform on $[0,1]$, (10) becomes

$$p_1^E = -\frac{\delta}{4}\sigma_I, \tag{10'}$$

whereas (11) becomes $p_1^I = (1 - \delta/2)\,\sigma_I + p_1^E$. Given the fact that there is a one to one correspondence between p_1^I and σ_I, it is possible, and turns out to be more convenient to think of the Incumbent as choosing σ_I.

We can show[13] $\sigma_I \geq 1/2$ and therefore $p_2^I = \sigma_I$ and by (15) $\Pi = [1 - \sigma_I]((1+\delta/2)\sigma_I + p_1^E)$. The first order condition of the Incumbent's problem yields $\sigma_I = 1/2 - p_1^E/(2(1+\delta/2))$, which combined with (10') yields the equilibrium[14] $\sigma_I = (4+2\delta)/(8+3\delta) > 1/2$. The equilibrium profit of the Incumbent is $(1+\delta/2)(1+\delta/4)^2/(2+3/4\delta)^2$.

This yields the following proposition.

Proposition 3. *If σ_I is distributed uniformly on $[0,1]$, at equilibrium in both periods the Incumbent charges more and has fewer customers in the Bertrand model than in the Stackelberg model. Its discounted profit is smaller in the Bertrand model.*

6 Literature

The main point of Biglaiser et al. (2013) is that the incumbent gains from having the zero switching cost consumers in the market because they get in the way of entrants trying to attract consumers with positive switching costs.[15] Since a firm will never make profits from the zero switching cost consumers their presence will not affect a firm's price that attracts zero switching cost consumers and the externality that we identify in the current paper is not present. That is, the fact that entrants can make profits from low switching cost consumers fundamentally changes the strategic opportunities of consumers and firms.[16]

Besides our companion paper, Taylor (2003) has the closest model to ours in the literature. The main aim of his paper is to analyze dynamic competition between firms in a subscription model where consumers draw

[13] With $\sigma_I < 1/2$ by (15), the profit of the Incumbent would be $(1-\sigma_I)p_1^I + \delta/4 = (1-\sigma_I)[(1-\delta/2)\sigma_I + p_1^E] + \delta/4$, whose derivative with respect to σ_I is $(1-\delta/2)\,(-2\sigma_I + 1) - p_1^E$. Because $-p_1^E > 0$, this derivative would be strictly positive, which establishes the contradiction.

[14] In the preceding paragraphs, we have only checked that the first order conditions holds. It is straightforward to check that this is indeed an equilibrium.

[15] We did not conduct the analysis where both types of consumers had positive switching costs in the infinite horizon model because of the difficulties in defining and proving that stationary equilibria exists in such a setting.

[16] The equilibrium construction is also different. Under Stackelberg competition, in the companion paper the Incumbent loses no HSC consumers, whereas in this paper it always loses a positive measure. Under Bertrand competition, in the companion paper the Incumbent loses either all or no HSC consumers whereas in this paper it loses some or all.

switching costs in each period from identical, independent distributions (in subscription models, introduced in the literature by Chen (1997), firms can discriminate between their current consumers and the consumers of other firms). With enough firms (in most of his analysis Taylor assumes free entry) competition drives the profits obtained from attracting consumers from other firms to zero.

In section 5 of his paper, Taylor examines a two period model where two types of consumers draw their switching cost (as before, independently in each period) from different distributions. His focus is on describing the equilibrium strategies of the firms and of the consumers. As in our models some HSC consumers want to "hide" among LSC consumers. He interprets this as an attempt by HSC consumers to acquire a reputation as LSC consumers. In our model consumers are anonymous and there is no way to discriminate between them; they have no way and no incentives to acquire a reputation. Yet, we obtain the same result. The explanation is simple: HSC consumers have incentives to mix with LSC consumers, as long as no too many of them do so. Our second contribution is that in the two type model we focus on the profit of the Incumbent and derive clean results comparing static and dynamic models. In particular, we are able to show how increasing the number of LSC consumers protects the Incumbent from the attempts by entrants to steal away some of the HSC consumers.

To the best of our knowledge, Taylor's section 5 and our companion paper are the only models in the literature where consumers have persistent differences between their switching costs.

For instance, Dubé, Hitsch, and Rossi (2009) have studied an infinite horizon model where consumers have random utility and firms sell differentiated products; their focus is on empirics, and, through the use of simulation methods, they provide numerical examples where prices fall when switching costs increase: as in our model, the increase in switching costs makes firms more aggressive as attracting consumers become more valuable. They assume that all consumers have the same switching costs.

A number of authors have constructed model versions of the model of Dubé et al. (2009) designed for theoretical exploration. For instance, Cabral (2013) analyzes an infinite horizon subscription model of competition between two producers. The relative value that the consumers attach to the goods produced by the two consumers is independent from period to period. He shows that an increase in switching cost from a small level leads to a decrease in the price, for the same reasons as in Dubé et al. (2009). All consumers have the same switching cost.[17,18] Somaini and Einav (2013) develop an overlapping generation model where consumes have identical switching costs that share some features of Cabral.[19]

It may also be worthwhile noting that most of the literature assumes a fixed number of firms, whereas our results hold true with free entry, and therefore more intense competition.[20] Hence, the Incumbent has no incentive to invest in the acquisition of new customers, on which it can only make zero profits — indeed, in equilibrium, the Incumbent does not try to "recover" the consumers that it has lost to other firms. Our comparative statics are entirely the consequence of the heterogeneity of switching costs, and of the fact that low switching cost customers protect the Incumbent from entry.

7 Conclusion

In this conclusion we provide some remarks about our modelling choices and about possible extensions.

Due to free entry, all our results hold if firms are allowed to discriminate on the basis of the past purchasing history of consumers — that is if we transformed our model in a subscription model. Indeed, an incumbent will compete for new consumers with entrants, and will therefore not be able to generate any profits on that market.

In our previous paper, Biglaiser et al. (2013) we studied a model similar to the model of this paper, with an infinite horizon, while assuming $\sigma_L = 0$. In that case, consumers allocate themselves in different firms (a firm will choose a "high" price as soon as its clientele contains one HSC consumer). Studying the case where σ_L is strictly positive generates a free rider effect as HSC consumers try to "follow" LSC consumers. Section 5 also showed that this effect holds with a continuum of types.

Network effects have sometimes been compared to social switching costs because they make it more difficult for groups of consumers to switch from

[17] Arie and Grieco (2013) also provide a theoretical analysis of Dubé et al. (2009), again which consumers who all have the same switching costs. Unlike Cabral (and like Dubé et al.), they assume that the consumers are myopic. Our consumers are forward looking.

[18] In a subscription model with forward looking consumers based on Chen (1997), Bouckaert, Degryse, and Provoost (2008) also show that higher switching costs can lead to lower profits. In their model, consumers are ex-ante identical and learn of their switching costs only after making their first purchase.

[19] Arie and Grieco (2013) also provide a theoretical analysis of Dubé et al. (2009) but, unlike Cabral (and like Dubé et al.), they assume that the consumers are myopic. (Our consumers are forward looking.)

[20] As mentioned above, Taylor (2003) also analyses free entry.

one platform to another. In order to test this parallelism Biglaiser and Crémer (2015) study dynamic competition in a model with an incumbent and free entry and with heterogenous network effects. They show that there are similarities but also substantial differences[21] between the two setups (see Crémer and Biglaiser (2012) for a preliminary discussion of some of these results and comparisons between the two setups).

[21] From a modelling viewpoint, the main difference is probably that with network effects the migration from one platform to the other is the outcome of a coordination game between consumers. In order to develop the model one needs to take a stance on the coordination failures that might arise. See Biglaiser, Crémer, and Veiga (2015) for a more detailed study of these failures.

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Appendix

Proof of Lemma 5

In this Appendix, we prove Lemma 5. We begin by establishing three claims; the first one is part *iv* of the lemma.

Claim A.1. *If $p_I > (1-\alpha\delta)\sigma_H$, the Incumbent has zero sales in both periods.*

Proof. If $p_I > (1-\alpha\delta)\sigma_H$, a unique lowest price entrant who would charge $p_E \in (-\alpha\delta\sigma_H, p_I - \sigma_H)$ would make strictly positive profits equal to $p_E + \delta\alpha\sigma_H$, as it would attract all the consumers in period 1. Free entry prevents this, and therefore in the continuation game, one or several entrants must charge $-\alpha\delta\sigma_H$, and attract all the consumers while making zero profits. □

Claim A.2. *If $p_I < (1-\alpha\delta)\sigma_H$, no entrant attracts enough* HSC *consumers in period 1 that it finds it optimal to charge σ_H with probability 1 in period 2.*

Proof. Assume that entrant $\tilde{e}$ attracted a large enough proportion of HSC customers that it found it optimal to charge σ_H in period 2. Because LSC consumers always find it strictly more profitable to switch suppliers than do HSC consumers, the Incumbent would have no LSC customers and, therefore, HSC customers can guarantee themselves a second price of σ_H by "staying with" the Incumbent. Therefore, entrant $\tilde{e}$ must have chosen a period 1 price $p_{\tilde{e}} \leq p_I - \sigma_H < -\delta\alpha\sigma_H < -\delta\sigma_L$ (the last inequality is a consequence of equation (2)). By lemma 2, all the entrants that attract consumers in the first period also charge $p_{\tilde{e}}$. They attract all the LSC consumers and, at best, some HSC consumers. The sum of their profits is therefore smaller than $\max\{p_{\tilde{e}} + \delta\sigma_L, p_{\tilde{e}} + \delta\alpha\sigma_H\} < 0$, which establishes the contradiction. □

Claim A.3. *If $p_I < (1-\delta)\sigma_H$, all* HSC *consumers purchase from the Incumbent in period 1.*

Proof. By Claim A.2, any period 1 entrant who has attracted consumers in period 1 charges σ_L with positive probability in period 2. Therefore, its second period profit will be σ_L times the mass of consumers it attracted in the first period and, by free entry, its period 1 price must be $-\delta\sigma_L$. The total discounted cost for a HSC consumer who would purchase from a period 1 entrant would therefore be at least $(-\delta\sigma_L + \sigma_H) + \delta\sigma_L = \sigma_H$ (it would be greater if in period 2 the entrant charged σ_H with a strictly positive probability). If the consumer purchases from the Incumbent, his total cost is $p_I + \delta\sigma_H < \sigma_H$, which establishes the claim. □

Parts *i* and *ii* of the lemma follow immediately from Claim A.3.

If $p_I \in \big((1-\delta)\sigma_H, (1-\alpha\delta)\sigma_H\big)$, HSC consumers prefer to purchase from an entrant if its period 2 price is σ_L and from the Incumbent if the entrant's period 2 price is σ_H. Therefore, there can be an equilibrium only if the entrants play a mixed strategy in period 2, which is feasible only if in period 1 they attract a proportion $\gamma > 0$ of the HSC consumers. This establishes part *iii* of the lemma and completes the proof.

Web Appendix

In this Appendix, which is not intended for publication, we prove the results discussed in subsection 4.5. Note that we are less ambitious than when (1) holds: we are only trying to identify one equilibrium for each value of σ_L/σ_H, not to characterize all the equilibria. We present the results under the form of three claims, starting with the largest value of σ_L/σ_H.

Claim WA.1. *If $\sigma_L/\sigma_H > \alpha$, then the two period Bertrand game has a unique equilibrium in which the Incumbent charges $\sigma_L(1-\delta)$ in period 1 and σ_L in period 2. It sells to all consumers and its profits are σ_L.*

As in the one period model, when $\sigma_L > \alpha\sigma_H$ the Incumbent and the entrant act as if there were only LSC customers in the economy. We leave the proof of the claim to the reader.

For $\sigma_L/\sigma_H \in (x_C, \alpha)$, with x_C defined by (9), we establish the following claim:

Claim WA.2. *If $\sigma_L/\sigma_H \in [x_C, \alpha]$, there exists a pure strategy equilibrium in which the Incumbent, whose profits are $\alpha\sigma_H$, sells to the* HSC *consumers in both periods, at prices respectively equal to $\sigma_H(1-\delta)$ and σ_H. All* LSC *customers purchase from entrants at price $-\delta\sigma_L$ in period 1 and at price σ_L in period 2.*

Proof. We show that the strategies described in the claim form an equilibrium. The LSC customers are clearly better off switching in period 1. The strategy of the HSC customers is a best response to the strategy of the other agents as they are indifferent between purchasing from the Incumbent in both periods and switching to an entrant in the first period — in both cases their total discounted costs are equal to σ_H.

This indifference of HSC consumers implies that the Incumbent would loose at least a proportion γ of its customers if it increased its period 1 price. It is straightforward to see that, under these circumstances, its most profitable increase in price is to $\sigma_H - \sigma_L$. This deviation is unprofitable as long as

$$\alpha\sigma_H \geq (1-\gamma)(\sigma_H - \delta\sigma_L + \delta\sigma_H) \iff \frac{\sigma_L}{\sigma_H}\left[1+\delta+\alpha\delta-\alpha\right] \geq \delta\left[\alpha + (\sigma_L/\sigma_H)^2\right],$$

and therefore, as long as $\sigma_L/\sigma_H \in [x_C, \alpha]$. A small decrease in period 1 price obviously decreases the profits of the Incumbent. A decrease to $\sigma_L(1-\delta)$ allows it to sell to all consumers in period 1, but decreases its profits. Finally, it is easy to show that the entrants strategy is indeed a best response to the strategies of the other agents. □

For the remaining set of parameters, we can prove the following claim.

Claim WA.3. *The equilibrium described in 4.3 is also an equilibrium when* $\sigma_L/\sigma_H \in [\alpha\delta/(1+\delta), x_C]$.

Proof. The proof is exactly the same as when $\sigma_L/\sigma_H \leq \alpha\delta/(1+\delta)$, except that we need to be a bit more careful when showing that the Incumbent does not gain by deviating to p_I in $(\sigma_L(1-\delta), \sigma_H(1-\delta))$ and possibly selling to all the HSC consumers. The Incumbent sells to all the HSC customers if

$$p_I + \delta\sigma_H \leq p_E + \sigma_H + \delta\sigma_L \Longleftrightarrow p_I \leq p_E + (1-\delta)\sigma_H + \delta\sigma_L.$$

This implies that if $p_I \in [(1-\alpha\delta-\delta)\sigma_H + \delta\sigma_L, (1-\delta)\sigma_H]$, the Incumbent sells to all the HSC consumers with probability strictly between $1-\gamma$ and 1 — in the other states of nature, it sells to a proportion $1-\gamma$ of them. Using the mixing probability of the entrant, see equation (8), the Incumbent's profit for prices in $[(1-\alpha\delta-\delta)\sigma_H + \delta\sigma_L, (1-\delta)\sigma_H]$ is

$$\begin{aligned}
&(p_I + \delta\sigma_H)\alpha\left[G_E(p_E) + (1-G_E(p_E))(1-\gamma)\right] \\
&\quad = (p_I + \delta\sigma_H)\alpha\Big[G_E(p_I - (1-\delta)\sigma_H - \delta\sigma_L)) \\
&\qquad\qquad + (1 - G_E(p_I - (1-\delta)\sigma_H - \delta\sigma_L))(1-\gamma)\Big] \\
&\quad = \frac{(p_I + \delta\sigma_H)\alpha}{p_I + 2\delta\sigma_H - \delta\sigma_L} \\
&\qquad\qquad \times \Big[p_I - \sigma_H(1-\delta-\alpha\delta) - \delta\sigma_L + (1-\gamma)\sigma_H(1+\delta-\alpha\delta)\Big].
\end{aligned}$$

The first and the second terms are both increasing in p_I. So, the maximal profit from this deviation is at a price of $p_I = \sigma_H(1-\delta)$. At this price the Incumbent only keeps all the HSC customers when the entrant is pricing at $-\delta\sigma_L$, where there is an atom in the distribution of its prices. It is straightforward to see that the profit is less than the putative equilibrium profit. Finally, it is straightforward to show that a deviation that keeps the LSC customers cannot improve profits. □