Risk Aversion and Bandwagon Effect in the Pivotal Voter Model

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Abstract

The empirical literature on the effects of opinion polls on election outcomes has recently found substantial evidence of a bandwagon effect, defined as the phenomenon according to which the publication of opinion polls is advantageous to the candidate with the greatest support. This result is driven, in the lab experiments, by a higher turnout rate among the majority than among the minority. Such evidence is however in stark contrast with the main theoretical model of electoral participation in public choice, the pivotal voter model, which predicts that the supporters of an underdog candidate participate at a higher rate, given the higher probability of casting a pivotal vote. This paper tries to reconcile this discrepancy by showing that a bandwagon effect can be generated within the pivotal voter model by concavity in the voters' utility function, which makes electoral participation more costly for the expected loser supporters. Given the strict relationship between concavity and risk aversion, the paper also establishes the role of risk aversion as a determinant of bandwagon.

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1 Introduction

The effects of opinion polls on election outcomes are, despite considerable research, still unclear. In particular, an open question in the study of electoral participation is whether the knowledge of voters' preferences over the candidates in an election is advantageous or disadvantageous for the candidate with the greatest support. The political science literature has defined the bandwagon effect as the phenomenon according to which the publication of opinion polls gives further advantage to the favorite candidate, who ends up with a larger than expected lead. The underdog effect refers instead to the case in which the polls' publication undermines the expected win margin by favoring a trailing candidate. These effects may be driven by changes both in the vote choices and in the turnout choices of voters and may have many different underlying causes, e.g. a psychological desire to be on the winner's side (which would induce bandwagon) or free-riding behavior within the largest group (which would induce underdog). The fear that these effects may distort electoral competition is the reason that several countries ban the release of polling results in the last days of the electoral campaigns.

For many decades scholars in political science and public choice have tried to provide empirical evidence of the existence and sizes of both bandwagon and underdog effects, facing considerable trouble in disentangling the causal effect of polls on electoral outcomes. With opinion polling data, isolating these effects from a shift in the electorate's preferences for some other reason can be very challenging. Laboratory experiments, in which preferences and information can be controlled, have often been preferred to cope with these difficulties (Marsh 1985) . Although the early literature has frequently generated inconclusive results (Irwin and Van Holsteyn 2000), many recent experimental studies have provided substantial evidence of bandwagon effects caused by higher turnout rates among the majority candidate's supporters (Klor and Winter 2007; Grosser and Schram 2010; Agranov et al 2015; Duffy and Tavits 2008). Some recent work also confirms bandwagon effects exploiting district-level electoral data (Kiss and Simonovits 2014; Morton et al 2015). These results suggest the prevalence of bandwagon effects in electoral contests and are in line with the widespread perception that people are more likely to vote for a candidate if his or her chances of success are thought to be favorable. The overconfidence with which all candidates in any election tend to claim that they will win also illustrates the extent to which politicians generally expect a bandwagon effect.

Such diverse evidence of bandwagon effects is, however, in stark contrast to the prediction of the main theoretical model of electoral participation in public choice: the pivotal voter model with costly voting (see section 3 for a review of the model and its main results). The literature on the pivotal voter model has indeed converged to an apparently very robust underdog result, which predicts, contrary to the evidence mentioned previously, that the members of a minority group participate at a higher rate than those of a majority group, given the greater probability of casting a pivotal vote for the underdog (Goeree and Grosser 2007; Taylor and Yildirim 2010a,b; Kartal

2015). Accordingly, previous attempts to provide a theoretical model of the bandwagon effect relied on additional elements with respect to the pivotal voter model, such as the introduction of a psychological desire to win (Callander 2007) or informational asymmetries on candidates' qualities (Cukierman 1991; Banerjee 1992).

This paper tries to reconcile this discrepancy by showing that a turnout-driven bandwagon effect can be generated within the pivotal voter model by concavity in the voters' utility function. This hypothesis requires relaxing an assumption at the core of the model, whose discussion has been surprisingly neglected by the existing literature: the assumption of a voter's utility that is linear in the election outcome benefit and in the cost of voting. Concavity in the voters' utility function makes electoral participation more costly for the supporters of the likely loser, raising the relative value of abstention. The hypothesis of concavity is especially suited to the analysis of experimental voting games, in which both the electoral benefit and the cost of voting are usually defined in monetary terms.

I first present a simple decision-theoretic model to study the turnout decision of an agent, who might support either an expected winning or losing candidate. I define the bandwagon effect as the case in which the agent would be more likely to vote if he supports the likely winner than if he supports the likely loser: the result is that the bandwagon effect occurs if the voter's utility is concave enough. Given the strict link between concavity and risk aversion, this result also reveals the non-intuitive role of risk aversion as a determinant of bandwagon. The interpretation is nonetheless easier if we look at concavity not in terms of risk aversion but in terms of decreasing marginal benefits. Let's consider the cost and benefit of voting in the pivotal voter model. The benefit is the increase in the probability of electing the preferred candidate, which is equal to the probability of casting a pivotal vote. The cost is an opportunity cost, which is subtracted from the election outcome benefit if the agent votes. The decreasing marginal benefits property of a concave utility function implies, then, that the cost of voting has a stronger negative effect on the voter's utility if it is more likely subtracted from a small election outcome benefit (as that of a supporter of the likely loser) than from a large one (as that of a supporter of the likely winner). Hence, for any fixed increase in the probability of electing the preferred candidate generated by the act of voting, the supporters of the expected loser will find electoral participation more costly.

In reality, in the most common electoral systems[1], the increase in the probability of electing the preferred candidate from the act of voting is larger for a supporter of the expected losing candidate than for a supporter of the expected winner. This higher probability of being pivotal is what drives the underdog effect in the standard pivotal voter model, by making abstention, as opposed to the effect of concavity, more attractive to a supporter of the expected winner. The total effect then depends on which of the two forces prevails: if the utility function is concave

[1]Such as the majority rule system that I consider in the following sections, but also a proportional representation system (Kartal 2015).

enough, the agent would turn out more frequently when he supports the likely winner, despite the lower probability of being pivotal.

I then extend the foregoing model to a game-theoretic framework. Building on the costly voting literature (Börgers 2004; Goeree and Grosser 2007; Taylor and Yildirim 2010b), I present a model in which n risk-averse agents can vote for two candidates, who are defined relatively to the expected sizes of their supporters' groups. In this case, the expected winner and loser are determined in equilibrium by the turnout rates of the two groups and the bandwagon and underdog effects are defined, more appropriately, as a higher turnout rate by the (stochastic) majority and minority respectively. I show that an equilibrium of such a model always exists and that, as the number of agents grows without bounds, the turnout rates of both groups tend to zero. In a simpler version of the model in which there are only two agents ($n = 2$), any equilibrium displays the same property of the decision-theoretic model: the underdog effect prevails for low degrees of risk aversion and the bandwagon effect for high ones. The general model with n agents is technically cumbersome but, solving the model numerically, I find that the presence of an equilibrium with the previous property is robust to the increase in n. However, as it turns out, the general model can also have equilibria in which the underdog effect emerges even if agents are very risk-averse. The reason is that in the game-theoretic model both the majority and the minority candidates can end up being the expected winner depending on the turnout rates and, according to the logic of the decision-theoretic model, concavity can thus make participation more costly for either group. Nonetheless, I argue that bandwagon should be expected for sufficiently high degrees of risk aversion, since coordination would be easier on an equilibrium in which the candidate supported by the majority is the expected winner. Overall, both the decision-theoretic and the game-theoretic frameworks make clear that the underdog result of the standard pivotal voter model is not robust to the introduction of concavity.

The remainder of the paper is organized as follows. Section 2 discusses the previous literature on bandwagon. Section 3 recalls the standard pivotal voter model and its main results. The decision-theoretic and the game-theoretic models with risk aversion are presented in sections 4 and 5, respectively. Section 6 offers a concluding discussion on the relevance of risk aversion in voting experiments.

2 Previous literature on bandwagon

The bandwagon effect has been the object of a large literature in political science and public choice. The literature prior to 2000 is surveyed carefully in Irwin and Van Holsteyn (2000), who analyze 79 articles on the effects of public opinion polls. Even though the total evidence is far from being definitive, the authors show how, after 1980, the literature has found bandwagon effects more frequently than underdog effects. The literature reviews in Marsh (1985), McAllister and Studlar (1991), and Nadeau et al (1993) also provide valuable summaries of the previous research and offer new evidence of bandwagon. Because of the difficulty in isolating the causal effects of polls on election outcomes, experiments, in which preferences and information can be controlled for, often have been preferred to studies based on opinion polls. Many recent experimental studies have found new evidence of bandwagon effects: Klor and Winter (2007), Duffy and Tavits (2008), Grosser and Schram (2010), Agranov et al (2015), and Morton and Ou (2015). In these studies, the bandwagon effect is generated by higher participation rates among the majority's supporters than among the supporters of the minority,[2] even though Morton and Ou (2015) also find evidence of bandwagon vote choices (i.e. vote switching). Some recent papers also exploit district-level electoral data: Hodgson and Maloney (2013) find evidence of bandwagon in British elections over the 1885-1910 period, while Kiss and Simonovits (2014) describe strong bandwagon effects in the 2002 and 2006 general elections in Hungary. Morton et al (2015) examine France's 2005 voting reform, which changed the order of voting between the mainland and the western overseas territories, showing that the knowledge of exit polls increased bandwagon voting. During the 2000 US presidential election, the national media mistakenly called the Florida election as being over and assigned the victory to the Democratic Party, while the polling stations were still open in the western Panhandle counties. In these counties, Lott (2005) documented a large drop-off in the turnout rate of the republican supporters relative to that of the democratic supporters, following the wrong announcement. These results make the debate on polls' effects lean in favor of bandwagon and align the empirical research with the widespread perception that the bandwagon effect exists. Given the current status of the theoretical research, they also call for further work in trying to identify the underlying mechanisms of bandwagon.

The formal theory of bandwagon started with Simon (1954), who was concerned about the possibility of forecasting in the social sciences. Theoretical models of bandwagon have mainly fallen into two groups: one that explains bandwagon with a psychological desire to stand on the winner's side and one that provides a rational explanation according to which following the polls is the best strategy for voters who are uninformed about candidates' qualities.[3] Social psychologists have long since observed a preference for conformity in many domains. Callander (2007) analyzed rigorously the consequences of introducing a desire to conform with the majority in a sequential voting game, proving the existence of an equilibrium in which bandwagon effects start with probability 1. The informational hypothesis has been developed by McKelvey and Ordeshook (1985) and Cukierman (1991) in the context of voting, and by Banerjee (1992) in a more general framework of herd behavior. The idea is that, in the presence of uncertainty concerning candidates' qualities, it is rational for citizens who are uninformed to take candidates' support in the polls as a signal of quality and thus vote for the likely winner because he is the

[2] The studies by Levine and Palfrey (2007) and Herrera et al (2014) find, however, higher turnout rates among the minority than the majority.

[3] Two other possible explanations have been proposed: Hong and Konrad (1998) argue that bandwagon can be caused by aversion to the uncertainty associated with an election; Morton and Ou (2015) study the possible role of other-regarding motives.

best candidate in expectation.

This paper proposes another explanation for bandwagon, based on the different turnout rates of the majority and minority supporters often observed in the experimental literature. I argue that, if agents are risk-averse, the underdog's supporters will find voting more costly and thus will participate at a lower rate. The relationship between risk attitudes and turnout choices has received very little attention. Kam (2012) finds a positive relationship between risk-acceptance and general political participation, but no effect of risk attitudes on turnout in elections. However, the author does not look at whether the interaction between risk-attitudes and preferences for a favorite or underdog candidate affects the turnout decision.

3 The pivotal voter model

The pivotal voter model studies citizens' participation decisions in elections contested by two candidates (where participation is always followed by sincere voting for the preferred candidate) and assumes that the citizens' aim is to influence the outcome of the election (instrumental voting). According to the model, every citizen decides whether or not to vote based on a comparison between the benefit of electing the preferred candidate, discounted by the probability of casting the pivotal vote, and the cost of voting. The cost of voting captures the opportunity cost in terms of time to go to the polls and vote as well as the cost of getting information to identify the preferred candidate.

This formulation dates back to Downs (1957) and Riker and Ordeshook (1968). Ledyard (1984) and Palfrey and Rosenthal (1983, 1985) adapted it to a game-theoretic framework, in which the probability of being pivotal is not exogenous, but depends on the electorate's participation decisions, so that it is jointly determined with turnout rates in equilibrium. A recent strand of literature refined Palfrey and Rosenthal results by proving the uniqueness of a symmetric Bayesian equilibrium (in which all supporters of the same candidate use the same strategy) in a game-theoretic model of costly voting with privately known political preferences (Börgers 2004; Goeree and Grosser 2007; Krasa and Polborn 2009; Taylor and Yildirim 2010a,b; Kartal 2015).

The empirical validity of the pivotal voter model is debated. Such a model is definitely not able to give a complete account of voters' behavior: the main drawback is the strong prediction, even in a game-theoretic framework, of very small turnout in large elections, because the probability of being pivotal quickly goes to zero. This result is clearly contradicted by reality.[4] On the other hand, it is hard to reject the model as useless, given the evidence that citizens do take the costs and benefits of participation into account when they have to decide whether to vote or abstain (Shachar and Nalebuff 1999; Blais 2000; Levine and Palfrey 2007). In particular, the empirical

[4]Solutions to the so-called paradox of voting include introducing ethical (or group-utilitarian) voters (Coate and Conlin 2004; Feddersen and Sandroni 2006; Evren 2012), assuming that mandate matters for the implementation of the winning platform after the election (Castanheira 2003), or introducing aggregate uncertainty about candidates' support (Myatt 2015). For a survey on voter turnout models and a comparison between the pivotal voter and the ethical voter models, see Merlo (2006).

relationship between turnout and elections' closeness gives strong confirmation of the model's prediction that citizens will participate more if the election is tight, given the larger probability of being pivotal. Such strategic voting behavior calls for caution in discarding the pivotal voter model, at least with respect to its comparative statics results.

The recent theoretical literature has agreed on a further result: in the standard pivotal voter model the turnout rate of the minority is greater than that of the majority; hence, the model yields the underdog effect (Goeree and Grosser 2007; Taylor and Yildirim 2010b; Herrera et al 2014; Kartal 2015). The underdog effect follows from the fact that, in the most common electoral systems, the probability of being pivotal is larger for the underdog's supporters than for those backing the leading candidate, which implies that in equilibrium the threshold cost that makes a citizen indifferent between voting and abstaining has as well to be larger for the underdog's supporters. Hence, in the presence of the same cost distribution in the two groups, minority supporters participate at a higher rate in equilibrium. The underdog effect is only partial in the presence of heterogeneous costs and complete (i.e. toss-up election) if the costs of voting are homogeneous (Taylor and Yildirim 2010b; Kartal 2015). As I already wrote, the robustness of this theoretical result is at odds with the empirical and experimental evidence on the presence of bandwagon effects in elections, generated by the majority's higher turnout rates.[5] In the next sections, I show how the pivotal voter model can be reconciled with the empirical relevance of bandwagon by modifying the assumption of a linear voter's utility function in favor of a concave one.

4 Decision-theoretic model

An agent in the standard pivotal voter model decides to vote if the expected relative benefit from the victory of his preferred candidate, discounted by the probability of being pivotal, is greater than his cost of voting, i.e. if

$$pB > c \tag{1}$$

However, this formulation follows, quite implicitly in all the previous literature, from an assumption of linearity of the voter's utility function in the election outcome benefit and in the cost of voting. In fact, the decision of whether to vote or not in an election gives the agent a choice between two lotteries. Suppose two candidates only and consider the choice of the agent. If he abstains, then he gets a lottery in which his preferred candidate wins with some probability and the other candidate wins with the complementary probability. By voting (sincerely), the agent

[5] However, it should be noticed that an important assumption for the underdog result is the presence of uncertainty concerning the preferences of the electorate. Indeed, a bandwagon effect may in some cases be compatible with the standard pivotal voter model under the assumption that the supporters' group size is known with certainty. This is often the case in lab experiments, where the majority and minority group sizes are held fixed in order to make the experiment easier to understand for the participants. The evidence of bandwagon is however much greater than the one that could be explained by such cases.

gets instead a lottery in which the probability of victory for his preferred candidate increases by some amount and the payoff from either election outcome is reduced by the cost of voting. The amount by which the probability of victory for his preferred candidate increases is the probability that the agent will be pivotal. I denote by $B > 0$ the expected benefit from the victory of the preferred candidate and I normalize to 0 the expected benefit from the victory of the other candidate. The cost of voting is $c \geq 0$. Finally, I denote by q the probability that the preferred candidate wins if the agent abstains and by p the probability of being pivotal.[6] I assume that the agent's preferences over lotteries have an expected utility form, whose corresponding utility function u is differentiable and strictly increasing. Then if the agent abstains, he gets the following lottery

$$(u(B), q; u(0), 1-q)$$

which gives as payoff the benefit B from the victory of his preferred candidate with probability q and the 0 benefit from the victory of the other candidate with probability $1-q$. By voting, he gets instead the following lottery

$$(u(B-c), q+p; u(-c), 1-q-p)$$

where the probability of his preferred candidate's victory is increased by p, but the benefits from both election outcomes are reduced by the cost of voting. By comparing the two lotteries we see that the agent will vote if

$$(q+p)u(B-c) + (1-q-p)u(-c) > qu(B) + (1-q)u(0) \tag{2}$$

This condition[7] is different from the one derived in the standard pivotal voter model because it does not assume linearity of the agent's utility function. If linearity is assumed, then the two conditions are equivalent.[8] In the presence of non-linearity, the equation that defines the agent's decision depends on the probability q assigned to the victory of the preferred candidate in the case of personal abstention.

An important observation is that the agent should reasonably perceive the probability of being pivotal differently depending on the value of q, in particular higher if the election is expected to be close ($q \approx \frac{1}{2}$). In what follows, I take this observation into account by letting p be itself a function of q.

In studying the agent's decision between voting and abstaining, it is instructive to look at the cost of voting that makes him indifferent between the two options and at how this value of the cost c varies with respect to the parameters B, q and $p(q)$. A higher voting cost necessary for

[6] Clearly, I assume $q + p \leq 1$.

[7] In Appendix C, I propose a graphical representation of the decision-theoretic model through the analysis of equation (2).

[8] Indeed, if $u(x) = x$, then $(q+p)(B-c) + (1-q-p)(-c) > q(B) + (1-q)0 \Rightarrow pB > c$. Riker and Ordeshook (1968) derive (1) from (2) but do not discuss the underlying hypothesis of linearity.

indifference implies, if the voting cost is a random variable, a greater likelihood of voting rather than abstaining. The value of c that makes the agent indifferent between voting and abstaining solves equation (2) with equality, which can be rewritten as

$$qu(B) + (1-q)u(0) - (q+p)u(B-c) - (1-q-p)u(-c) = 0 \tag{3}$$

where p is a function $p(q)$. Equation (3) has a unique solution.

Proposition 1. *For any values of B, q and $p(q) > 0$, there exists a unique $c \in (0, B)$ such that equation (3) holds.*

The proofs of this and the following propositions are in Appendix B. The next result describes how the cost that yields indifference varies with B and p, when q is fixed and $p(q)$ can thus be seen as a parameter.

Proposition 2. ***(i)*** *For any fixed B and q, equation (3) implicitly defines a unique continuous function $c(p)$ which gives the cost of voting that yields indifference as a function of the probability of being pivotal p. Such $c(p)$ is increasing in p.*
(ii) *For any fixed q and p, equation (3) implicitly defines a unique continuous function, $c(B)$, which gives the cost of voting that yields indifference as a function of the benefit from electing the preferred candidate B. If u is (weakly) concave, such $c(B)$ is increasing in B.*

The first part of proposition 2 confirms the result of the standard pivotal voter model that the agent will be more likely to vote if the probability of being pivotal is higher. The second part of the proposition claims that if the utility function is not linear, then the result that the agent will be more likely to vote in higher-stakes elections is confirmed under the assumption of concavity of u, but is not guaranteed if u is strictly convex.[9] The intuition for this second result is that, if u is convex, the negative effect of the cost of voting on the voter's utility is stronger, the greater is the election outcome benefit B, for any $q \neq 0$. If this effect of convexity is strong enough to offset the positive effect given by the increase by p in the probability of getting a larger B, the agent would find participation more costly and thus will be less likely to vote when elections are high-stakes.
The relationship between the cost that yields indifference c and q will give results concerning the bandwagon or underdog effect. Indeed, if such cost is higher when $q > \frac{1}{2}$ than when $q < \frac{1}{2}$, the model predicts bandwagon; if the opposite is true, the model predicts underdog. However, how the solution c in equation (3) varies with q depends on the assumptions on $p(q)$ and this brings out the issue concerning how the agent forms estimates of q and $p(q)$. Since specifying a functional form for $p(q)$ in such a decision-theoretic model would be difficult and arbitrary, I

[9]The fact that people are more likely to vote in elections that they deem more important is well-established. This is not a peculiar prediction of the pivotal voter model, which is characterized by the claim that the effect of B on the agent's decision is discounted by the probability of being pivotal.

simply assume that the agent has some given subjective estimates of these probabilities. It is then useful to fix the probabilities of victory for the two candidates in case of agent's abstention, q and $1-q$, and the two associated probabilities of being pivotal $p(q)$ and $p(1-q)$, and to look at the cost that makes the agent indifferent both in the case he supports the likely winner and in the case he supports the likely loser. Without loss of generality, I fix $q < \frac{1}{2}$, so that the candidate with probability of victory equal to q is the expected loser. Then $p(q)$ is the subjective probability of being pivotal if the agent supports the likely loser and $p(1-q)$ is the subjective probability of being pivotal if the agent supports the likely winner. In this framework, I only need an assumption on the two values $p(q)$ and $p(1-q)$: in order to be consistent with the previous literature, which identified a driver of the underdog effect in the higher probability of being pivotal for the expected loser's supporters, I assume

$$p(1-q) \leq p(q)$$

and, since q is fixed, I relabel the two as

$$p(q) = p$$
$$p(1-q) = p - \delta$$

with $\delta \geq 0$. The equations that define the cost that yields indifference can then be rewritten as follows. If the agent supports the likely loser, then the cost of voting that yields indifference solves the same equation as before

$$qu(B) + (1-q)u(0) - (q+p)u(B-c) - (1-q-p)u(-c) = 0 \tag{4}$$

If he instead supports the likely winner, then the cost that yields indifference solves

$$(1-q)u(B) + qu(0) - (1-q+p-\delta)u(B-c) - (q-p+\delta)u(-c) = 0 \tag{5}$$

Proposition 1 applies to both equations; hence, they both have a unique solution in $(0, B)$. Let's denote c_l the solution of equation (4) and c_w the solution of equation (5). It is useful to study first what happens if $\delta = 0$ and then to let δ be strictly positive.

Proposition 3. *Assume that $\delta = 0$. Then $\forall q \in (0, \frac{1}{2})$, $c_l < c_w$ if and only if u is strictly concave.*

Hence, under the assumption of concavity of u, if the agent has the same subjective probability of being pivotal both when he supports the likely winner and when he supports the likely loser, the cost of voting that makes him indifferent between voting and abstaining is higher when he supports the likely winner. This result implies that, if the voting cost is a random variable, the agent is more likely to vote if he supports the likely winner. The interpretation is again

straightforward if we consider the decreasing marginal benefits property of a concave function. The benefit of voting given by the larger probability of electing the preferred candidate is the same in both cases if $\delta = 0$, but decreasing marginal benefits imply that the cost of voting has a stronger negative effect on utility if it is subtracted from an election outcome benefit that is likely low (as that of a supporter of the probable loser) than if it is subtracted from a likely high election outcome benefit (as that of a supporter of the probable winner). Concavity thus makes voting relatively more costly for the underdog's supporters.
Let's assume now that $\delta > 0$. The following result follows from Proposition 2(i).

Proposition 4. *For any fixed B, q and p, equation (5) implicitly defines a unique continuous function $c_w(\delta)$ which gives the cost of voting that yields indifference to a supporter of the likely winner as a function of δ. Such $c_w(\delta)$ is strictly decreasing in δ (and it reaches 0 when $\delta = p$).*

Hence, a strictly positive δ pushes the cost that yields indifference for a supporter of the likely winner down. Since c_l does not depend on δ, this result has two implications. First, there exists a unique value δ^* of δ for which the agent has the same cost of voting that yields indifference both when he supports the likely winner and when he supports the likely loser. Second, if δ is small enough (i.e. $\delta < \delta^*$) then the cost that yields indifference is still higher when the agent supports the likely winner.
How low is low enough? Can we be sure that the agent will have a larger cost that yields indifference if he supports the likely winner? The next proposition shows that this is the case if the utility function is concave enough (i.e. if the agent is risk-averse enough).

Proposition 5. *Consider the agent in two cases: when his utility function is u and when it is h, a concave transformation of u (i.e. $h = f \circ u$, with $f' > 0$, $f'' < 0$). The resulting δ^* when utility is u is strictly smaller than the resulting δ^* when utility is h.*

The result implies that, no matter what the subjective probabilities of being pivotal are, if the agent is risk-averse enough, the cost that yields indifference will be higher if the agent supports the likely winner than if he supports the likely loser.

5 Game-theoretic model

In this section, I shall verify whether the result of the previous model can be preserved in a game-theoretic framework. Game-theoretic voting models are more compelling than decision-theoretic ones: the outcomes of elections are indeed determined by the turnout rates of the competing groups rather than by their sizes and it is reasonable to assume that citizens take expectations about the electorate's participation into account when they decide whether to vote or abstain (Ledyard 1984; Palfrey and Rosenthal 1983, 1985). In a game-theoretic model the electorate's turnout decisions, candidates' probability of victory and voters' probability of being pivotal are jointly determined in equilibrium.

I present a model that follows closely some of the recent costly voting literature (Börgers 2004; Goeree and Grosser 2007; Taylor and Yildirim 2010a,b; Kartal 2015). The original contribution is that I assume the agents' utility function to be concave instead of linear. The model features an electorate composed of n agents, who can vote in a majority-rule election contested by two candidates (1 and 2). In case of a tie, each candidate wins with probability $\frac{1}{2}$. The agents' preferences follow expected utility, with a continuous, strictly increasing, and strictly concave utility function u. It is common knowledge that each agent prefers candidate 1 with probability π and candidate 2 with probability $1 - \pi$ and that each agent has a cost of voting drawn independently from a uniform distribution on $[0, 1]$. The agents, however, observe only their own realizations of the preferred candidate and the cost of voting.[10] Without loss of generality, I assume $\pi > \frac{1}{2}$, so that the supporters of candidate 1 are the majority group in expectation. These assumptions allow one to calculate the probabilities of victory for the candidates in the case of an agent's abstention as well as the probability of being pivotal as functions of the electorate's participation decisions. Following the previous literature, I look for a symmetric Bayesian equilibrium, in which all supporters of the same candidate use the same strategy.

A strategy is defined by a cutoff cost that makes the agent indifferent between voting and abstaining: the agent votes if his voting cost is below the cutoff and abstains if it is above. The symmetric Bayesian equilibrium is then defined by the two cutoff costs for the two candidates' groups of supporters. Given the assumption of a uniform voting cost distribution over $[0, 1]$, the cutoffs also identify the ex-ante probability of voting for an agent in the two groups and thus the expected turnout rates of the two groups. Each agent's decision problem and the equation that defines indifference between voting and abstention are the same as in the previous section, but now the key probabilities are functions of the participation decisions of the two groups. I denote the two cutoff costs for the supporters of candidate 1 and 2 respectively c_b and c_s.[11] These are then solutions of the following system of equations

$$\begin{cases} qu(B) + (1-q)u(0) - (q+p_1)u(B-c_b) - (1-q-p_1)u(-c_b) = 0 \\ (1-q)u(B) + qu(0) - (1-q+p_2)u(B-c_s) - (q-p_2)u(-c_s) = 0 \end{cases} \tag{6}$$

where $q = q(c_b, c_s)$ denotes the probability of victory for candidate 1 in case of agent's abstention, while $p_1 = p_1(c_b, c_s)$ and $p_2 = p_2(c_b, c_s)$ are the probabilities of being pivotal respectively for a supporter of candidate 1 and 2. As already highlighted by the previous literature, note that each agent is pivotal only in two cases: either when the rest of the voting agents is split equally

[10] The assumption of heterogeneity in voting costs turns out to be a crucial one for a bandwagon result. In Appendix D, I show that in the presence of homogeneous costs of voting the model would yield the same full underdog result as in the standard linear pivotal voter model.

[11] With respect to the previous section, I change the subscripts from w (winner) and l (loser) to b (bigger) and s (smaller) because in the game-theoretic model, even if candidate 1 has a larger group of supporters, the expected winning candidate is determined by the turnout rates.

between the two candidates (turning a draw into a victory) or when the opposing candidate would win by one vote if the agent abstains (turning a loss into a draw). The expressions of the probabilities p_1, p_2 and q are derived from the model's assumptions in Appendix A, as a function of the turnout rates c_b and c_s and of the parameters n and π.
Any solution (c_b, c_s) for system (6) then represents a couple of equilibrium cutoffs for the two groups of supporters. The following proposition guarantees that a solution to the system exists.

Proposition 6. *If $B \leq 1$, system (6) has a solution (c_b^*, c_s^*) such that $\forall n, \pi$ both $c_b^* \in (0,1)$ and $c_s^* \in (0,1)$.*

The condition on B is needed only to make sure that $c_b, c_s < 1$. A cost that yields indifference greater than 1 for a group of supporters would imply full turnout by the members of that group, which is not an interesting case for the interpretation. The uniqueness of the solution is, however, not guaranteed and, unfortunately, even for simple specifications of the function u, the system does not have analytical solutions.
Before turning to the bandwagon or underdog effect, I prove that the well-known zero-asymptotic-turnout result of the pivotal voter model in large elections, first shown by Palfrey and Rosenthal (1985), holds in the risk-aversion framework as well.

Proposition 7. *In equilibrium $\lim_{n\to\infty} c_b^* = \lim_{n\to\infty} c_s^* = 0$*

As proposition 7 establishes, if the number of agents becomes arbitrarily large, the turnout rates of both groups tend to zero, since the probability of being pivotal becomes negligible.
In order to have results concerning the bandwagon or underdog effects, I specify u as a constant absolute risk aversion (CARA) utility function, with a coefficient of absolute risk aversion equal to $\gamma > 0$, which allows one to study the relationship between the two cutoffs for different degrees of risk aversion:

$$u(x) = -e^{-\gamma x} \tag{7}$$

Moreover, in light of proposition 6, I assume from now on that

$$B = 1 \tag{8}$$

The simplest case of only two agents ($n = 2$) is more tractable and allows me to derive a result in line with the decision-theoretic model. The general case $n > 2$ is more complicated and I have to rely on some graphical results obtained by solving the system numerically. I discuss these two cases separately.

5.1 Case $n = 2$

If one assumes only two agents, the system becomes simpler (yet not solvable analytically). Indeed the key probabilities simplify to

$$q = \frac{1}{2}\pi c_b - \frac{1}{2}(1-\pi)c_s + \frac{1}{2}$$
$$p_1 = \frac{1}{2} - \frac{1}{2}\pi c_b$$
$$p_2 = \frac{1}{2} - \frac{1}{2}(1-\pi)c_s$$

and thus the system (6) - (8) becomes

$$\begin{cases} [\frac{\pi}{2}c_b - \frac{1-\pi}{2}c_s + \frac{1}{2}]e^{-\gamma} + [\frac{1-\pi}{2}c_s - \frac{\pi}{2}c_b + \frac{1}{2}] - [1 - \frac{1-\pi}{2}c_s]e^{-\gamma(1-c_b)} - \frac{1-\pi}{2}c_s e^{\gamma c_b} = 0 \\ [\frac{1-\pi}{2}c_s - \frac{\pi}{2}c_b + \frac{1}{2}]e^{-\gamma} + [\frac{\pi}{2}c_b - \frac{1-\pi}{2}c_s + \frac{1}{2}] - [1 - \frac{\pi}{2}c_b]e^{-\gamma(1-c_s)} - \frac{\pi}{2}c_b e^{\gamma c_s} = 0 \end{cases} \tag{9}$$

The following result shows a characterization of any solution[12] of system (9) which replicates the result of the decision-theoretic model.

Proposition 8. ***(i)*** *There exists a threshold $\bar{\gamma}$ such that any solution (c_b^*, c_s^*) of system (9) satisfies $c_b^* = c_s^*$ if $\gamma = \bar{\gamma}$, $c_b^* < c_s^*$ if $\gamma < \bar{\gamma}$, and $c_b^* > c_s^*$ if $\gamma > \bar{\gamma}$.*
(ii) *$\bar{\gamma} \approx 1.54$. It is the unique solution of*

$$e^{\gamma(\frac{3e^{-\gamma}-1}{e^{-\gamma}-1})} = 2 \tag{10}$$

Note that the threshold $\bar{\gamma}$ does not depend on π, that is, on how more likely the two agents support candidate 1 rather than candidate 2.[13]
The logic of the decision-theoretic model thus extends to the game-theoretic model in the simple case of two agents: in the symmetric Bayesian equilibrium, the members of the expected majority turn out at a lower rate than those of the expected minority for low degrees of risk aversion, and at a higher rate for high degrees of risk aversion.[14] Hence, also in a game-theoretic framework, the underdog result of the standard pivotal voter model is not robust to the introduction of risk aversion.

[12]In fact, the solution of system (9) is unique in all my numerical computations.

[13]The value 1.54 might seem very high compared to the usual estimates of an absolute risk-aversion parameter, but it comes from the fact that I only evaluate the utility function in the small range (-c,1) with $c \in (0,1)$. My numerical computations show that proposition 8(i) would still be true if I specify a constant relative risk aversion (CRRA) utility function (bound by 1 to the left to make sure that the numerator is positive, i.e. $u(x) = \frac{(x+1)^{1-\gamma}}{1-\gamma}$), instead of a CARA function. In this case, the threshold for the relative risk-aversion parameter would be approximately equal to 1.77, which is lower than its usual estimates. For an agent with an absolute risk-aversion parameter equal to 1.54, the certainty equivalent of a lottery that yields a payoff of either 1 or 0 both with probability one-half is 0.32.

[14]With two agents, the expected number of candidate 1 supporters is 2π (> 1) and the expected number of candidate 2 supporters is $2(1-\pi)$ (< 1).

5.2 Case $n > 2$

The technical difficulty of the model prevents me from deriving a result similar to proposition 8 for the general case with $n > 2$ agents; I thus rely on numerical solutions and graphical results to study the equilibria. I have run simulations for different choices of the parameters n, π and γ. Concerning n, I have restricted the analysis to small electorates, which are the ones the literature on the pivotal voter model usually refers to, given the zero-asymptotic-turnout result and the quickly increasing computational time-cost.

From the graphical results, two main observations emerge. First, there is always one equilibrium that shows the same pattern as the one in the $n = 2$ case. In particular, for any n there exists a threshold $\bar{\gamma}$ such that there exists an equilibrium for which $c_s = c_b$ if $\gamma = \bar{\gamma}$, $c_s > c_b$ if $\gamma < \bar{\gamma}$, and $c_b > c_s$ if $\gamma > \bar{\gamma}$. Second, multiple solutions, which correspond to multiple equilibria, exist in some regions of the parameter space when $n > 2$.

As an example, Figure 1 shows the implicit plot of system (6) - (8) in the (c_b, c_s) plane for $n = 50$, $\pi = 0.55$ and for three values of the risk aversion parameter γ ($\gamma = 0.01, 1, 5$), which allow one to compare the two relevant cases in terms of uniqueness or multiplicity of equilibria. The solutions to the system, and thus the equilibria of the model, are given by the intersections of the two curves. The graphs also display the 45° line: any intersection above that line implies a higher turnout rate by the smaller group ($c_s > c_b$) and any intersection below implies a higher turnout rate by the larger group ($c_b > c_s$).

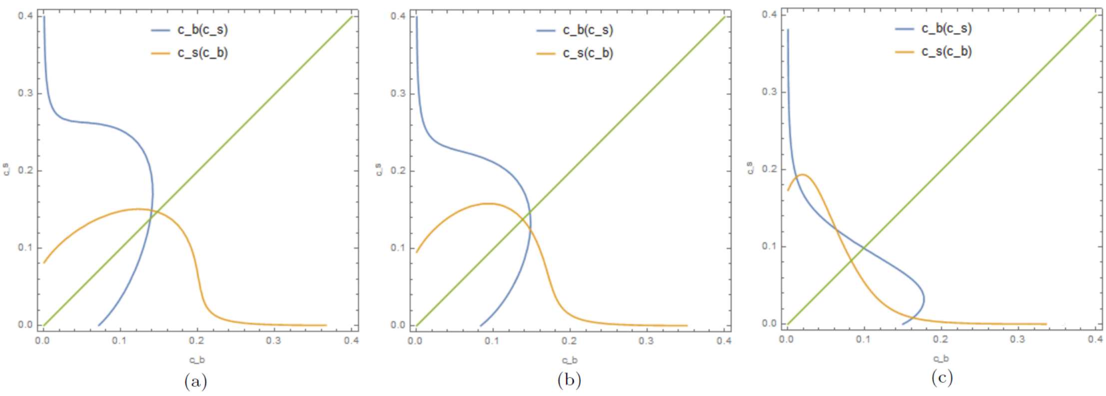

Figure 1: Implicit plot of system (6) - (8): $n = 50$, $\pi = 0.55$
γ: (a) = 0.01; (b) = 1; (c) = 5

As it turns out, for some values of γ the equilibrium is unique ($\gamma \in \{0.01, 1\}$), while for some others multiple (three) equilibria exist ($\gamma = 5$). The intersection of the two curves in graph (a) moves continuously to that in graph (b) and then to the one below the 45° line in graph (c) as γ increases continuously from 0.01 to 1 and then to 5. Hence, the equilibria in (a), in (b), and the one below the 45° line in (c) can be seen as representing an equilibrium that, as a function of γ, moves continuously from above the 45° line to below it, when γ increases. Such equilibrium features the supporters of the smaller group turning out at a higher rate for low degrees of risk aversion and at a lower rate for high degrees of risk aversion, thus replicating the result of the $n = 2$ case. Its existence and property are robust to other specifications of the parameters n and π.[15]

Graph (c) of Figure 1, however, shows a case in which multiple (three) equilibria exist. Interestingly, these yield opposite predictions in terms of which group of supporters turns out at a higher rate. As we can see, for some (high) degrees of risk aversion, there can be equilibria both for which $c_b > c_s$ and equilibria for which $c_s > c_b$. Even though multiple equilibria seem to exist only in small regions of the parameter space[16], this result may look very unsatisfactory because it prevents unambiguous predictions and seems to contradict the logic of the decision-theoretic model. As I discuss in the next section, however, such a conclusion would be fallacious: the equilibria with opposing predictions are all consistent with the results of the decision-theoretic model.

To conclude this section, it is interesting to look at the value of the threshold $\bar{\gamma}$, which determines the minimum degree of the agents' risk aversion in order for the model to have an equilibrium displaying a bandwagon effect. For the general case of $n > 2$ agents, this value $\bar{\gamma}$ is also a function of π, i.e. of the expected sizes of the two groups. Figure 2 shows the computations of $\bar{\gamma}$ as a function of $n \in [2, 250]$ for two specifications of $\pi = 0.51, 0.75$.

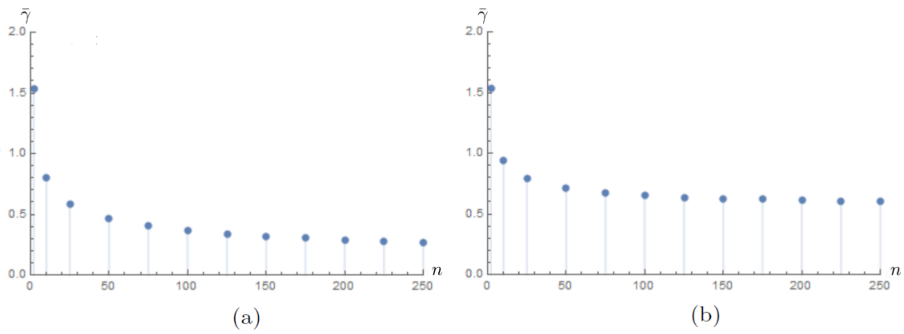

Figure 2: Discrete plot of $\bar{\gamma}(n)$, π: (a) $= 0.51$; (b) $= 0.75$

According to Figure 2, the threshold $\bar{\gamma}$ is generally declining in n in small electorates.[17] This suggests that, as n grows, the model can yield a bandwagon effect in equilibrium even for a mild degree of agents' risk aversion. For example, in a close election where $\pi = 0.51$, while $\bar{\gamma} \approx 1.54$ for $n = 2$, the value falls to $\bar{\gamma} \approx 0.27$ if $n = 250$.[18]

[15] Owing to computational difficulties, I have run simulations only for $n \leq 250$.

[16] They disappear for higher values of π, lower values of n and higher and lower values of γ.

[17] However, for extreme values of π (e.g. $\pi = 0.95$), my computations show that $\bar{\gamma}(n)$ is first decreasing and then again increasing in n.

[18] For an agent with a constant absolute risk aversion parameter equal to 0.27, the certainty equivalent of a lottery that yields a payoff of either 1 or 0 both with probability one-half is 0.47.

5.3 On the multiplicity of equilibria

The existence of different equilibria in the game-theoretic model is explained by the way in which the majority and minority groups are defined and by the fact that the key variable for predictions in the decision-theoretic model is determined endogenously in equilibrium in the game-theoretic model. This key variable is q, which identifies the expected winner: the decision-theoretic model predicts that for sufficiently high degrees of risk aversion the agent turns out more likely if he supports the expected winner, while for low degrees of risk aversion the agent turns out more likely if he supports the expected loser. Now, in the decision-theoretic model, which candidate is the expected winner is given exogenously and the result can be interpreted unambiguously in terms of the majority and minority groups if we reasonably define the majority group as the expected winner's supporters. In the game-theoretic model, instead, the majority and minority groups are defined in terms of their expected sizes (i.e. relatively to π) and the expected winning candidate is determined in equilibrium by the turnout rates of the two groups. In particular, candidate 1 is the expected winner if c_b is greater than c_s or if c_s is greater than c_b, but not enough to offset the size advantage, while candidate 2 is the expected winner if c_s is enough greater than c_b to offset the size advantage. These two possibilities create scope for different equilibria, all consistent with the decision-theoretic model. Consider, indeed, the case of high degrees of risk aversion: both options $c_b > c_s$ and $c_s > c_b$ are in principle compatible with a higher turnout rate by the expected winner's supporters as long as the cutoffs make candidate 1 the expected winner in the first case and candidate 2 the expected winner in the second case. Thus, the game-theoretic model embeds a coordination game that makes both groups able to end up being the expected majority of the voting agents, no matter their expected sizes.

Note that the existence of different equilibria is instead not possible for low degrees of risk aversion. In this case, following the logic of the decision-theoretic model, we expect the supporters of the expected loser to participate at a higher rate. That is not compatible with $c_b > c_s$: candidate 1, indeed, could not be the expected loser, since his group of supporters is bigger in expectation and would turn out at a higher rate. Hence, for low degrees of risk aversion, we expect only equilibria in which $c_s > c_b$.

How many and which equilibria effectively exist depends on the specification of the utility function and the model's parameters. The presence of an equilibrium in which $c_b > c_s$ when agents are risk-averse implies that the bandwagon prediction of the decision-theoretic model also can hold in a general game-theoretic framework. Furthermore, whenever both kinds of equilibria are possible, I also find it reasonable to expect easier coordination to the equilibrium in which $c_b > c_s$: indeed, if the agents were to coordinate on the equilibrium in which $c_s > c_b$, the members of the smaller group would not only have to participate at a higher rate than the members of the bigger group, but also to do it to the extent necessary to offset the expected difference in sizes and thereby make their candidate the expected winner. The empirical and experimental evidence in favor of bandwagon that I reviewed in the previous sections also suggests much easier coordination on the equilibrium in which the larger group votes at a higher rate.

6 Conclusion

The paper presents a theoretical mechanism able to produce a bandwagon effect in line with what is observed in many empirical studies. The key assumption for the result is concavity of voters' utility function, i.e. risk-averse voters. I have presented both a decision-theoretic model and a game-theoretic model. In the decision-theoretic model, I have shown that an agent who is risk-averse enough would participate more likely when he supports the expected winning candidate, despite the smaller probability of casting a pivotal vote. In the game-theoretic model, I have found the existence of an equilibrium in which the members of the majority participate at a higher rate than those of the minority for high degrees of risk aversion and at a lower rate for low degrees of risk aversion. Hence, in the presence of risk-averse agents, both the decision-theoretic and the game-theoretic models yield a bandwagon effect. These results show how the apparently robust underdog effect highlighted by the previous theoretical literature depends on an assumption of linearity. Finally, I have shown the possibility of multiple equilibria in the game-theoretic model, which can also display an underdog effect for high degrees of risk aversion, due to the model's dual endogeneity of turnout rates and the identity of the expected winning candidate.

Whether citizens' utility functions are concave in a voting context is an interesting question. Any answer is complicated by the fact that the benefit from the election outcome and the cost of voting in real elections are supposed to be measured in different units. It is a central assumption of economic modeling that the utils derived from these two variables can be summed, but any assumption on the shape of the utility function over the sum of these utils may well seem arbitrary. Nonetheless, concavity identifies regular properties (risk aversion and decreasing marginal benefits) that are supposed to hold in many different contexts and, in the voting setting, it conveys the reasonable idea that people might perceive electoral participation to be more costly if they are likely to be let down by the outcome.

On the other hand, the mechanism that I have described gives a clear justification for the result of higher turnout rates for the members of the majority group observed in the experimental studies. In these experiments, both the benefit from the election outcome and the cost of voting are usually given in monetary terms: hence, the assumption of a concave utility over money is a very natural one.

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A Expressions for p_1, p_2 and q

I derive here the expressions for p_1, p_2 and q in system (6). For the calculation of p_1 and p_2, I draw on the analysis in Goeree and Grosser (2007). An agent is pivotal only when either the rest of the voting agents is equally split between the two candidates (turning a draw into a victory) or when the opposing candidate would win by one vote if the agent abstains (turning a loss into a draw). Given the tie-breaking rule in the case of a draw, in both cases the probability of being pivotal is divided by one-half. Clearly the first scenario can happen if the number of other voters is even, while the second if such number is odd. The two cases can be combined using the floor operator $\lfloor \cdot \rfloor$. Consider first a supporter of candidate 1, supposing that k other agents also prefer candidate 1, $n-k-1$ prefer candidate 2 and a total of l agents vote. Assuming that the cutoff costs c_b and c_s are both in $(0,1)$, the probability of being pivotal p_1 is equal to

$$\begin{aligned} p_1(c_b, c_s) = \frac{1}{2} \sum_{l=0}^{n-1} \sum_{k=\lfloor \frac{l}{2} \rfloor}^{n-1-\lfloor \frac{l+1}{2} \rfloor} \binom{n-1}{k} \pi^k (1-\pi)^{n-k-1} \binom{k}{\lfloor \frac{l}{2} \rfloor} \binom{n-k-1}{\lfloor \frac{l+1}{2} \rfloor} \times \\ \times (c_b)^{\lfloor \frac{l}{2} \rfloor} (1-c_b)^{k-\lfloor \frac{l}{2} \rfloor} (c_s)^{\lfloor \frac{l+1}{2} \rfloor} (1-c_s)^{n-k-1-\lfloor \frac{l+1}{2} \rfloor} \end{aligned} \tag{11}$$

Analogously for a supporter of candidate 2, the probability of being pivotal p_2 is equal to

$$\begin{aligned} p_2(c_b, c_s) = \frac{1}{2} \sum_{l=0}^{n-1} \sum_{k=\lfloor \frac{l}{2} \rfloor}^{n-1-\lfloor \frac{l+1}{2} \rfloor} \binom{n-1}{k} (1-\pi)^k \pi^{n-k-1} \binom{k}{\lfloor \frac{l}{2} \rfloor} \binom{n-k-1}{\lfloor \frac{l+1}{2} \rfloor} \times \\ \times (c_s)^{\lfloor \frac{l}{2} \rfloor} (1-c_s)^{k-\lfloor \frac{l}{2} \rfloor} (c_b)^{\lfloor \frac{l+1}{2} \rfloor} (1-c_b)^{n-k-1-\lfloor \frac{l+1}{2} \rfloor} \end{aligned} \tag{12}$$

In addition to p_1 and p_2, I need to calculate the probability q, which emerges in the agent's decision problem if the assumption of linearity in the utility function is relaxed. To do so, notice that when l other agents vote, candidate 1 wins if at least $\frac{l}{2}+1$ agents vote for him for l even, or at least $\frac{l+1}{2}$ for l odd. The two cases can again be combined using the floor operator: if l agents vote, candidate 1 wins if at least $\lfloor \frac{l}{2}+1 \rfloor$ vote for him. Moreover, by the assumption on the tie-breaking, candidate 1 also wins with probability one-half if the voters are equally split between the two candidates (l must be even). Hence, assuming c_b and c_s both in $(0,1)$, q is equal to

$$\begin{aligned} q(c_b, c_s) = \sum_{l=0}^{n-1} \sum_{k=v}^{n-1-l+v} \sum_{v=\lfloor \frac{l}{2}+1 \rfloor}^{l} \binom{n-1}{k} \pi^k (1-\pi)^{n-k-1} \binom{k}{v} \binom{n-k-1}{l-v} \times \\ \times (c_b)^v (1-c_b)^{k-v} (c_s)^{l-v} (1-c_s)^{n-k-1-l+v} \\ + \frac{1}{2} \sum_{l \text{ even},\, l=0}^{n-1} \sum_{k=\frac{l}{2}}^{n-1-\frac{l}{2}} \binom{n-1}{k} \pi^k (1-\pi)^{n-k-1} \binom{k}{\frac{l}{2}} \binom{n-k-1}{\frac{l}{2}} \times \\ \times (c_b)^{\frac{l}{2}} (1-c_b)^{k-\frac{l}{2}} (c_s)^{\frac{l}{2}} (1-c_s)^{n-k-1-\frac{l}{2}} \end{aligned} \tag{13}$$

The probabilities p_1, p_2 and q are strictly positive also in the case that c_b or c_s are equal to zero (see the proof of proposition 6 in Appendix B). In particular, $p_1 = p_2 = q = \frac{1}{2}$ if $c_b = c_s = 0$.

B Proofs

Proof of proposition 1. Since u is continuous and strictly increasing in its argument, the left-hand side of equation (3) is continuous and strictly increasing in c. Moreover, for any values of B, q and $p(q) > 0$, if $c = 0$ the left-hand side of (3) is negative, while if $c = B$ it is positive. Hence, by the intermediate value theorem, for any values of B, q and $p(q) > 0$, there exists a unique $c \in (0, B)$ such that equation (3) holds.

In proposition (2), (3) and (4) I use the following version of the implicit function theorem, whose hypotheses are verified in the proof of proposition 1.
Implicit Function Theorem. *Given the equation $g(x, y) = 0$, if (i) g is continuous and (ii) strictly monotone in y for any fixed x and if (iii) g changes sign as y varies for any fixed x, then there exists a unique continuous function $f(x)$ such that $g(x, f(x)) = 0$.*

Proof of proposition 2. (i) Existence and uniqueness of the implicit function are guaranteed by the implicit function theorem (see above). By implicit differentiation of equation (3) with respect to p, we have

$$(q+p)u'(B-c)\frac{\partial c}{\partial p} + (1-q-p)u'(c)\frac{\partial c}{\partial p} - (u(B-c) - u(-c)) = 0$$

$$\Rightarrow \frac{\partial c}{\partial p} = \frac{u(B-c) - u(-c)}{(q+p)u'(B+c) + (1-q-p)u'(c)} > 0$$

(ii) Existence and uniqueness of the implicit function are guaranteed by the implicit function theorem. By implicit differentiation with respect to B, we have

$$qu'(B) - (q+p)u'(B-c)(1 - \frac{\partial c}{\partial B}) + (1-q-p)u'(-c)\frac{\partial c}{\partial B} = 0$$

$$\Rightarrow \frac{\partial c}{\partial B} = \frac{q(u'(B-c) - u'(B)) + pu'(B-c)}{(q+p)u'(B-c) + (1-q-p)u'(-c)}$$

which is (strictly) positive if u is weakly concave.

Proof of proposition 3. If $\delta = 0$, equations (4) and (5) are both of the form

$$xu(B) + (1-x)u(0) - (x+p)u(B-c) - (1-x-p)u(-c) = 0 \tag{14}$$

where $x = q < \frac{1}{2}$ in equation (4) and $x = 1 - q > \frac{1}{2}$ in equation (5). By the implicit function theorem (see above), equation (14) implicitly defines a unique continuous function $c(x)$. It suffices then to show that $c(x)$ is strictly increasing in x if u is strictly concave and weakly decreasing if u is weakly convex. By implicit differentiation with respect to x, we have

$$u(B) - u(0) - u(B-c) + (x+p)u'(B-c)\frac{\partial c}{\partial x} + u(-c) + (1-x-p)u'(-c)\frac{\partial c}{\partial x} = 0$$

$$\Rightarrow \frac{\partial c}{\partial x} = -\frac{u(B) - u(0) - (u(B-c) - u(-c))}{(x+p)u'(B-c) + (1-x-p)u'(-c)}$$

which is strictly positive if u is strictly concave and weakly negative if u is weakly convex.

Proof of proposition 4. Existence and uniqueness of the implicit function are guaranteed by the implicit function theorem (see above). By implicit differentiation of equation (5) with respect to δ, we have

$$(1-q+p-\delta)u'(B-c)\frac{\partial c}{\partial \delta}+(q-p+\delta)u'(-c)\frac{\partial c}{\partial \delta}+(u(B-c)-u(-c))=0$$

$$\Rightarrow \frac{\partial c}{\partial \delta}=-\frac{u(B-c)-u(-c)}{(1-q+p-\delta)u'(B-c)+(q-p+\delta)u'(-c)}<0$$

Proof of proposition 5. The value δ^* is determined, together with the associated value of c, by the following system of equations

$$\begin{cases} qu(B)+(1-q)u(0)-(q+p)u(B-c)-(1-q-p)u(-c)=0 \\ (1-q)u(B)+qu(0)-(1-q+p-\delta^*)u(B-c)-(q-p+\delta^*)u(-c)=0 \end{cases} \tag{15}$$

which yields

$$\delta^*=(2q-1)\left(\frac{u(B)-u(0)}{u(B-c)-u(-c)}-1\right) \tag{16}$$

Clearly $\forall\ q<\frac{1}{2}$, δ^* is positive if u is concave and negative if u is convex. Moreover $\frac{u(B)-u(0)}{u(B-c)-u(-c)}>\frac{h(B)-h(0)}{h(B-c)-h(-c)}$, so δ^* is greater if the utility is h.

Proof of proposition 6. The proposition is true by the Poincaré-Miranda theorem.

Theorem (Poincaré-Miranda). *Let $I^n:=[a,b]^n$ and let $f=(f_1,...,f_n):I^n\to\mathbb{R}^n$ be a continuous map such that $\forall i\leq n, f_i(I_i^-)\subset(-\infty,0]$ and $f_i(I_i^+)\subset[0,\infty)$, where $I_i^-:=\{x\in I^n:x(i)=a\}$ and $I_i^+:=\{x\in I^n:x(i)=b\}$. Then there exists a point $c\in I^n$ such that $f(c)=0$.*

In our case $f=(f_1,f_2)$ is the left-hand side of system (6), $I^n=[0,B]^2$ and $\forall B,n,\pi$

$$f_1(c_b=0)<0 \text{ and } f_1(c_b=B)>0 \quad \forall c_s$$

$$f_2(c_s=0)<0 \text{ and } f_2(c_s=B)>0 \quad \forall c_b$$

Hence if $B\leq 1$, there exists a solution $(c_b,c_s)\in(0,1)\times(0,1)$.

Note that the theorem requires to evaluate system (6) at $c_b=0$ and $c_s=0$, where equations (11)-(13) are not defined. Hence, I briefly sketch the calculation of p_1, p_2 and q in these cases. If $c_b=0$ (i.e. the other supporters of candidate 1 don't vote), a voting supporter of candidate 1 is pivotal when either no supporters of candidate 2 vote or only one supporter of candidate 2 votes (in both cases with probability $\frac{1}{2}$, given the tie-breaking rule). Hence

$$p_1(c_b=0,c_s)=\frac{1}{2}\sum_{k=0}^{n-1}\binom{n-1}{k}(1-\pi)^k\pi^{n-k-1}[(1-c_s)^k+kc_s(1-c_s)^{k-1}]$$

Analogously, we have

$$p_2(c_b,c_s=0)=\frac{1}{2}\sum_{k=0}^{n-1}\binom{n-1}{k}\pi^k(1-\pi)^{n-k-1}[(1-c_b)^k+kc_b(1-c_b)^{k-1}]$$

Concerning q, the probability that candidate 1 wins when $c_b = 0$ (i.e. his supporters don't vote) is the probability that also no supporter of candidate 2 votes, times one half because of the tie-breaking rule. That is,

$$q(c_b = 0, c_s) = \frac{1}{2}\sum_{k=0}^{n-1}\binom{n-1}{k}(1-\pi)^k\pi^{n-k-1}(1-c_s)^k$$

while the probability that candidate 1 wins when $c_s = 0$ (i.e. the supporters of candidate 2 don't vote) is the probability that at least one supporter of candidate 1 votes plus one half the probability that no supporter of candidate 1 votes, because of the tie-breaking rule. That is,

$$q(c_b, c_s = 0) = \sum_{k=0}^{n-1}\sum_{v=1}^{k}\binom{n-1}{k}\pi^k(1-\pi)^{n-k-1}\binom{n-1}{k}(c_b)^v(1-c_b)^{k-v}+ \\ +\frac{1}{2}\sum_{k=0}^{n-1}\binom{n-1}{k}\pi^k(1-\pi)^{n-k-1}(1-c_b)^k$$

By analogous reasoning, the probabilities p_1, p_2 and q are strictly positive for any combination of $c_b, c_s \in \{0, 1\}$.

Proof of proposition 7. The proof is based on the result that the probability of being pivotal goes to zero in large elections, that I report below as a lemma and for which I refer to Taylor and Yildirim (2010b).

Lemma: Fix $(c_b, c_s) \in [0, B]\times[0, B]$ such that $(c_b, c_s) \neq (0, 0)$. Then $\lim_{n\to\infty} p_1(c_b, c_s, n) = \lim_{n\to\infty} p_2(c_b, c_s, n) = 0$

Proof. See Lemma A1 in Taylor and Yildirim (2010b).

Then suppose that $\lim_{n\to\infty} c_b^* \neq 0$. This means that there exists a subsequence of $c_b^*(n)$ that is bounded away from zero. Since $c_b^*(n) \in [0, B]$, by Bolzano-Weierstrass theorem, there also exists a subsequence $\widehat{c_b^*}(n)$ that converges to some $l > 0$. This implies $\widehat{c_b^*}(n) > 0$ for n large enough and thus, by the lemma, $\lim_{n\to\infty} p_1(\widehat{c_b^*}(n), c_s^*, n) = 0$. Then, at the limit the first line of system (6) becomes

$$q[u(B) - u(B - c_b^*)] + (1 - q)[u(0) - u(-c_b^*)] = 0$$

which is a contradiction since the left-hand side is strictly positive, for any limit value of q in $[0, 1]$. Analogously, suppose that $\lim_{n\to\infty} c_s^* \neq 0$. By the same reasoning, at the limit the second line of system (6) becomes

$$(1 - q)[u(B) - u(B - c_s^*)] + q[u(0) - u(-c_s^*)] = 0$$

which is a contradiction since the left-hand side is strictly positive, for any limit value of q in $[0, 1]$.

Proof of proposition 8. Summing and subtracting the two equations of system (9), we can rewrite it as

$$\begin{cases} (e^{-\gamma} + 1) - e^{-\gamma}(e^{\gamma c_b} + e^{\gamma c_s}) + (e^{-\gamma} - 1)[\frac{1-\pi}{2}c_s e^{\gamma c_b} + \frac{\pi}{2}c_b e^{\gamma c_s}] = 0 \\ (e^{-\gamma} - 1)[\pi c_b(1 - \frac{1}{2}e^{\gamma c_s}) - (1-\pi)c_s(1 - \frac{1}{2}e^{\gamma c_b})] - e^{-\gamma}(e^{\gamma c_b} - e^{\gamma c_s}) = 0 \end{cases} \tag{17}$$

Imposing $c_b = c_s$ in (17) yields after some algebra

$$e^{\gamma c_b} = 2 \quad \text{and} \quad c_b = \frac{3e^{-\gamma} - 1}{e^{-\gamma} - 1}$$

from which (10) follows. Imposing $c_b = c_s + \varepsilon$ with $\varepsilon > 0$, from the second equation of (17) we get

$$(e^{-\gamma} - 1)(2\pi - 1)c_s(1 - \frac{1}{2}e^{\gamma c_s}) + \psi = 0 \tag{18}$$

where $\psi = (e^{-\gamma} - 1)(1 - \pi)c_s \frac{1}{2} e^{\gamma c_s}(e^{\gamma\varepsilon} - 1) + (e^{-\gamma} - 1)\pi\varepsilon(1 - \frac{1}{2}e^{\gamma c_s}) - e^{-\gamma}(e^{\gamma c_s}(e^{\gamma\varepsilon} - 1)) < 0$ and $e^{\gamma c_s} < 2$. From (18) we get

$$e^{\gamma c_s} = 2 + \varphi \tag{19}$$

where $\varphi = \frac{2\psi}{(e^{-\gamma}-1)(2\pi-1)c_s} > 0$. From the first equation of (17) we get

$$(e^{-\gamma} + 1) - e^{-\gamma}(2e^{\gamma c_s}) + (e^{-\gamma} - 1)(\frac{1}{2}c_s e^{\gamma c_s}) + \zeta = 0 \tag{20}$$

where $\zeta = e^{-\gamma}e^{\gamma c_s}(1 - e^{\gamma\varepsilon}) + (e^{-\gamma} - 1)[\frac{1-\pi}{2}c_s e^{\gamma c_s}(e^{\gamma\varepsilon} - 1) + \frac{\pi}{2}\varepsilon e^{\gamma c_s}] < 0$. Substituting (19) in (20) we get

$$c_s = \frac{3e^{-\gamma} - 1}{e^{-\gamma} - 1} + \xi = 0 \tag{21}$$

where $\xi = -\frac{1}{2}c_s\varphi + \frac{2\varphi e^{-\gamma}}{e^{-\gamma}-1} - \frac{\zeta}{e^{-\gamma}-1} < 0$. Substituting (21) in (19) we get

$$e^{\gamma(\frac{3e^{-\gamma}-1}{e^{-\gamma}-1})} = 2 + \phi \tag{22}$$

where $\phi = \varphi - e^{\gamma(\frac{3e^{-\gamma}-1}{e^{-\gamma}-1})}(e^{\gamma\xi} - 1) > 0$.

If instead $\varepsilon < 0$, then $e^{\gamma c_s} > 2$, $\psi > 0$, $\varphi < 0$, $\zeta > 0$, $\xi > 0$ and $\phi < 0$. Hence $\phi > 0$ if $\varepsilon > 0$ and $\phi < 0$ if $\varepsilon < 0$, which, since the left-hand side of (22) is monotonically increasing in γ, yields the result.

C A graphical analysis of the decision-theoretic model

In this section, I present a graphical illustration of the decision-theoretic model of section 4 in the paper. According to the model (equation 2), the agent prefers voting over abstention if

$$(q + p)u(B - c) + (1 - q - p)u(-c) > qu(B) + (1 - q)u(0) \tag{23}$$

which can be rewritten as

$$q > \frac{1 - p\frac{u(B-c)-u(-c)}{u(0)-u(-c)}}{1 - \frac{u(B)-u(B-c)}{u(0)-u(-c)}} \tag{24}$$

Equation (24) defines a straight line in the (p, q) plane, which divides the region in which the agent votes and the one in which the agent abstains. The line has a slope equal to

$$-\frac{1}{1 - \frac{u(B)-u(0)}{u(B-c)-u(-c)}} \tag{25}$$

from which we can see that it is vertical if the agent is risk-neutral (u linear) and downward-sloping if the agent is risk-averse (u concave). In Figure 3, the dashed vertical line represents the case of risk neutrality, while the downward-sloping black line the case of a concave utility function u.

Note that, since the left-hand side of (23) is strictly decreasing in c, an increase in the agent's cost of voting c implies a shift of the line to the north-east of the graph.

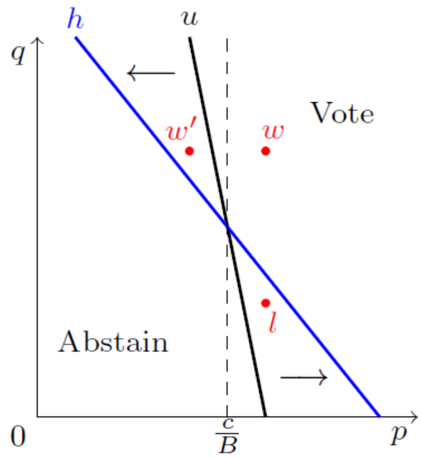

Figure 3: Graphical representation of the agent's decision in the (p, q) plane

Consider now the case $\delta = 0$. This corresponds to comparing two different points of the plane that have the same coordinate on the p-axis. Since the point that identifies the agent when he supports the likely winner is always above the point that identifies him when he supports the likely loser, the negative slope of the line makes it impossible for the agent to vote when he supports the likely loser, but abstain when he supports the likely winner. For some values of the cost of voting the opposite happens instead: the agent votes if he supports the likely winner, but abstains if he supports the likely loser. This case is shown in Figure 3 by the two red dots labeled l and w.

The case $\delta > 0$ corresponds instead to comparing two points in the plane that do not have the same coordinate on the p-axis anymore, because the point above (which identifies the expected winner's supporter) moves to the left. In this case, even if the line has a negative slope, it could be possible for the agent to vote if he supports the likely loser but abstain if he supports the likely winner. This possibility is shown in Figure 3 by comparing between the two red dots denoted l and w'. But, as we can see from (25), if the agent becomes more risk averse, the slope of the line declines in absolute value and the line thus rotates counterclockwise. Such rotation makes sure that, if the agent is risk-averse enough, he will vote if he supports the likely winner and abstain if he supports the likely loser. This case is shown in Figure 3 by the blue line, which corresponds to a utility function h that is a concave transformation of u.

D Homogeneous costs

In this section, I show that if agents are risk-averse but have homogeneous voting costs, the model of section 5 in the paper cannot overcome the full underdog result of the linear pivotal voter model. Suppose then that the agents in both groups have the same (deterministic) voting cost equal to c. In this case, the equilibrium (in which turnout is partial in both groups) is given by mixed strategies, according to which every agent in both groups is indifferent between voting and abstaining. Calling c_b and c_s the probability of voting in the equilibrium mixed strategies for an agent in the big group and an agent in the small group respectively (which now do not identify threshold costs anymore), these solve the following system of equations

$$\begin{cases} qu(B) + (1-q)u(0) - (q+p_1)u(B-c) - (1-q-p_1)u(-c) = 0 \\ (1-q)u(B) + qu(0) - (1-q+p_2)u(B-c) - (q-p_2)u(-c) = 0 \end{cases} \quad (26)$$

where $q(c_b, c_s)$, $p_1(c_b, c_s)$, and $p_2(c_b, c_s)$ are given by (13), (11), and (12) respectively and c is the cost of voting. Note that if voting costs are homogeneous, the agents' strategies only determine the probability of victory for the candidates and the probability of being pivotal but do not enter the agents' utility function u. In system (6) instead, in which voting costs are heterogeneous, the agent's strategies determine the threshold costs and thus enter also the utility of the indifferent agents in the two groups. I focus on the case of two agents ($n = 2$); in my numerical examples the result extends to the general case with n agents. If $n = 2$ we have

$$\begin{aligned} q &= \frac{1}{2}\pi c_b - \frac{1}{2}(1-\pi)c_s + \frac{1}{2} \\ p_1 &= \frac{1}{2} - \frac{1}{2}\pi c_b \\ p_2 &= \frac{1}{2} - \frac{1}{2}(1-\pi)c_s \end{aligned} \quad (27)$$

and system (26) simplifies to

$$\begin{cases} [\frac{\pi}{2}c_b - \frac{1-\pi}{2}c_s + \frac{1}{2}]u(B) + [\frac{1-\pi}{2}c_s - \frac{\pi}{2}c_b + \frac{1}{2}]u(0) - [1 - \frac{1-\pi}{2}c_s]u(B-c) - \frac{1-\pi}{2}c_s u(-c) = 0 \\ [\frac{1-\pi}{2}c_s - \frac{\pi}{2}c_b + \frac{1}{2}]u(B) + [\frac{\pi}{2}c_b - \frac{1-\pi}{2}c_s + \frac{1}{2}]u(0) - [1 - \frac{\pi}{2}c_b]u(B-c) - \frac{\pi}{2}c_b u(-c) = 0 \end{cases} \quad (28)$$

Now subtracting the second equation from the first in (28) we get

$$[\pi c_b - (1-\pi)c_s][u(B) - u(0) - \frac{1}{2}(u(B-c) - u(-c))] = 0 \quad (29)$$

from which we see that unless the term in the second brackets is zero (which is a very strong condition), in equilibrium we must have

$$\pi c_b = (1-\pi)c_s \quad (30)$$

i.e. full underdog (the turnout rates completely offset the size advantage and the election becomes toss-up). Hence if everyone has to be indifferent between voting and abstaining, the only possible equilibrium is that the system yields the underdog effect, while, as the model in the paper shows, if the equilibrium determines also who the indifferent agents are in the two groups, there can be a bandwagon effect in equilibrium.

However, note that in principle it would be possible to imagine an equilibrium in which bandwagon occurs but still all supporters of both groups are indifferent between voting and abstaining: the benefit of voting for the supporters of the big group would be smaller than for those of the small group, since the probability of being pivotal would be smaller, but also the effect of the

cost of voting on utility would be smaller because of concavity.
In order to see better why such equilibrium does not exist, we can look at the expression for δ^* in section 4, which gives the difference in the probabilities of being pivotal that would make the agent indifferent between voting and abstaining in both the case he supports candidate 1 and the case he supports candidate 2 for a given cost of voting c (see also equation (16), in the proofs' section):

$$\delta^* = p_1 - p_2 = (2q-1)\left(\frac{u(B)-u(0)}{u(B-c)-u(-c)} - 1\right) \tag{31}$$

The term in the last brackets in (31) is negative because of concavity. Now, from the expressions for p_1, p_2 and q in (27), we have that

$$p_1 - p_2 = -\frac{1}{2}(2q-1) \tag{32}$$

which implies that (31) can only hold if $p_1 = p_2$ and thus if $\pi c_b = (1-\pi)c_s$.
Hence, the existence of an equilibrium with bandwagon is prevented by the fact that in the model the difference in the probabilities of being pivotal is proportional to the difference in the probabilities of victory for the preferred candidate in case of abstention, i.e.

$$p_2 - p_1 \propto q - (1-q) \tag{33}$$

Note that equation (33) is true also in the model with heterogeneous costs; however, the additional degree of freedom given by the fact that the equilibrium determines also the threshold costs allows to get a bandwagon effect in equilibrium.