Forthcoming *Review of Economic Studies*

TOO DOMESTIC TO FAIL
LIQUIDITY PROVISION AND NATIONAL CHAMPIONS*

Emmanuel Farhi and Jean Tirole†

February 15, 2024

Abstract

Authorities' support policies shape the location and continuation of industrial and banking activity on their soil. Firms' locus of activity depends on their prospect of receiving financial assistance in distress and therefore on factors such as countries' relative resilience. We predict that global firms are global in life and national in death; and that they become less global when competition is more intense, times are turbulent, and international risk sharing (say, through swap lines) weak. We analyze the competitive benefits of industrial and banking policies as well as their limitations, such as currency appreciation.

Keywords: Economic geography, national champions, cross-border banking, liquidity support, too domestic to fail, home bias, exhorbitant duty, hegemon.

JEL numbers: D43, E58, F33, G15, G21.

*Emmanuel Farhi passed away in July 2020, after a first draft of this paper had been completed. While the impetus for, and many insights of this paper can be credited to him, the shortcomings in its realization are due to no fault of his own. The paper, previously titled "Industrial Monetary Policy", benefitted from helpful comments from the participants at the BCL-TSE conference on "The Future of the International Monetary System", the Bank of England 7^{th} Macro-finance workshop, the Bank of Israel, SUFE's finance conference, and the NBER Summer Institute, and from our discussants Arvind Krishnamurthy, Pablo Kurlat, and Xavier Vives. Kourosh Khounsari, Paul-Henri Moisson, Elias Papaioannou, Goetz von Peter, Andreas Schaab, Philipp Wangner and four referees provided outstanding comments on an earlier draft. This project received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation program (grant agreement no. 669217 - ERC MARKLIM). Jean Tirole acknowledges funding from ANR under grant ANR-17-EURE-0010 (Investissements d'Avenir program).

†Toulouse School of Economics (TSE) and Institute for Advanced Study in Toulouse (IAST).

"*Global banks are global in life and national in death*" (Mervyn King)

1 Introduction

Countries endeavor to attract and maintain economic activity on their soil. A large literature accordingly studies how this desire shapes trade agreements and protectionist policies (from export and plant-creation subsidies to import tariffs). It however focuses on one-shot attempts at boosting domestic industries' competitiveness in the global arena and ignores the benefits of offering the long-term supporting environment that is conducive to corporate location on the territory. Airbus and Boeing received large financial supports from their respective governments during Covid-19, as did many other industrial companies that could demonstrate a clear national identity in their production or strategic importance more broadly. Offering a stable, supportive environment however is costly and requires financial muscle. The support may be spontaneous, that is decided after shocks accrue, or partially planned. This paper aims at understanding which countries will bring or plan such backing (and when), and how this affects corporate decisions with regards to location and international diversification.

The literature's neglect of corporate entities' concern about being able to rely on governmental support in dire straits also explains why the literature on industrial policy has by and large ignored the banking system. While some insights gleaned in this literature also apply to finance, there are also cross-border-banking specificities, such as the key role of public liquidity provision and the presence of supervisors. Both features have played a significant role during the recent financial, European, Covid and Ukrainian crises and the reforms that resulted from these events. This paper thus also aims at extending the trade literature to the banking system and thereby at shedding light on some ongoing debates.

Section 2 first describes the bare-bones model, featuring two identical countries, a single bank, and three dates (we use the banking terminology when building our framework, which also applies to the industrial context): A date 0 at which the bank chooses how much and where to invest, a date 1 at which it may require liquidity support, and a date 2 at which long-term (private and social) benefits of these investments are reaped (date 2 stands for the "future"). The bank is initially not credit constrained when it "acquires" (say, SME or retail) customers or "projects" at date 0. Acquiring customers or projects is a costly activity. This investment need not be one-shot, as at date 1 the bank may have to reinvest in the funded projects so as to enable them to continue.[1]

To capture public liquidity provision in a stark and simple way, we assume that there is no store of value (more generally, there is an insufficient amount of stores of value to cover at least some states of nature). That is, we abstract from the standard liquidity-underinvestment, or investment-in-the-wrong-kind-of-liquidity market failure that has received much attention in

[1] Interpreting projects as SME borrowers, one may have in mind the standard learning-by-lending argument. The incumbent lender over time knows more about the borrower than alternative lenders and will underbid the new lender whenever profitable, leaving the latter with an adversely selected sample of negative-value customers, with the outcome being an ex-post monopoly position for the historical lender (see Section 9.4 in Tirole 2006 for an exposition of the standard argument).

the economics literature. Because there are no stores of value at date 0 in the model, only the state can supply the required date-1 liquidity. Countries supply liquidity because they care about the continuation of activity at home. Concerns about sovereignty, economic spillovers (fire sales and other domino effects), fragile or politically influential groups (SMEs, depositors), or the media visibility of the distress of big companies all concur to raising their governments' interest in continuation; in contrast, resentment towards bankers, the establishment, or the elites makes it harder to provide bailouts. In Section 2, we assume that the states do not commit to liquidity provision; rather they supply the liquidity in a time-consistent manner.

Our first key assumption is that money is (at least in part) fungible: The countries cannot ensure that all the money they supply at date 1 will be used domestically. Under perfect fungibility for instance, banks allocate their date-1 funds internationally as is optimal for them. This creates a leakage (some of the funds are channeled abroad) and an associated free-riding problem (at date 1, a country benefits from the liquidity provided by the other country). Section 2 shows that, despite the symmetry of the model, the bank specializes geographically: The bank wants to be the national champion of one country so as to be able to count on its support in case of distress.[2] And that country bears the brunt of the rescue. The degree of geographic specialization may vary from partial to complete. In standard bailout models, banks pile up on one type of risk so as to maximize the put on taxpayer or central bank money; here the put is maximized not by taking risk per se, but by minimizing free riding by governments through limiting international diversification.

Because countries are ex-ante identical, the bare-bones model does not make a prediction as to which country the bank favors. Section 2 therefore relaxes the symmetry assumption in order to derive predictions as to which country will attract banking activity. We identify three determinants. A larger room for fiscal or monetary manoeuver offers a comparative advantage to the country. So does a higher eagerness to rescue banks. Differences in the distributions of domestic shocks are relevant as well. In particular, the support may now come from the low-presence, but high-shocks country.

Our second key assumption is that a world of countries does not behave as a borderless one. In our context, international date-1 coordination of liquidity provision would lead to better support for the firm. In practice though, and despite numerous attempts at policy reforms in the banking context (e.g., resolution planning, MoUs requiring information sharing), international coordination has mostly failed in 2008 and during other episodes of cross-border bank distress. Cross-border bailouts end up in disaster because of urgency, or of a lack of mutual knowledge of countries' willingness to rescue the bank or exposures, or else of a differential availability of funds across countries. In Mervyn King's quip cited above: "*Global banks are global in life and national in death*".

Section 2 implies that at date 0 a country can boost its attractiveness in several ways. Because there is higher trust in a country's liquidity support when the country is resilient, the country

[2]We focus on banks' attention to future liquidity-support programs when they choose their location. Needless to say, banks have other considerations when choosing their location, such as the available skills and the wages they command (as in the comparative advantage literature) or the distance to headquarters and the ease of monitoring remote operations (as in the gravity literature). As these are by now well-established factors, we ignore them and focus on the novel factor.

can keep room for manoeuver on the fiscal side (keeping spare borrowing capacity) or the monetary side (being able to supply liquidity without risking inflation). Another important determinant is the country's eagerness, rather than ability, to bring liquidity support; in this respect, turning a blind eye on a bank being systemic or opting for a funded pension system act as commitments to bring liquidity support.

Section 3 studies another form of liquidity commitment: pledging to act as a lender of last resort (LOLR) in the banking context, or state guarantee schemes in the industrial one. LOLR differs from the provision of liquidity considered hitherto, in that it requires a commitment, perhaps sustained by reputation, while the provision of liquidity studied in Section 2 is ex-post voluntary; put differently, LOLR involves going beyond what the country would naturally do at date 1. We show how a conditional form of LOLR enables countries to attract investment onto their own soil, while an unconditional one tends to backfire. We also show that exchange rate appreciations are an unavoidable byproduct of liquidity commitments. Exchange rate appreciations, in turn, are a limiting factor for liquidity commitments by endogenously making them costlier to operate.

Section 4 studies competition in banking with ex-ante identical countries. At date 0, identical banks compete à la Cournot and pick their footprint in each country; so, they choose their overall scale and their degree of specialization. At date 1, each bank can receive liquidity support from the countries. We show that, under reasonable conditions, the only equilibria in this otherwise symmetric environment are asymmetric equilibria in which the banks specialize in different countries. We then demonstrate that Coasian bargains would lead not only to a better allocation of support to economies at date 1, but also, by limiting the incentive to become a national champion, to more banking competition at date 0. We also show that the more competitive the banking system (as captured by the number of banks divided by the number of countries), the more banks specialize. They always operate in a single country for a large enough bank/country ratio.

Section 5 relaxes the perfect fungibility assumption. We look at the impact of ringfencing (allowing the home country to bias the allocation of liquidity in its favor), with a ringfencing parameter going from 0 (perfect fungibility) to 1 (perfect ringfencing). Ringfencing has a benefit and a cost: It makes the home country more eager to exert its responsibility; but it also creates a misallocation of liquidity within the bank. Section 6 reviews the relevant literature and Section 7 concludes. Missing proofs can be found in the Appendix.

Banking application. As we already noted, state support in dire straits is particularly prominent in the banking sector.[3] Countries are eager to preserve the integrity of their financial system with regard to the "core functions": lending to SMEs and serving retail depositors. Rescues are

[3] Liquidity support to banks often operates through the central bank. This support can be targeted to a specific bank, which can use the discount window and emergency liquidity assistance (ELA). The Fed's or ECB's supervisory arm can also exercise supervisory leniency. Alternatively, in the presence of macroeconomic shocks, the support may aim at the banking sector as a whole: asset purchase programs (corporate bonds, public debt, ABS, covered bonds), monetary bailouts (consisting in lowering the interest rate to facilitate the banks' refinancing; see Farhi-Tirole 2012), and swaps with other central banks. While liquidity support is prominent in the banking world, there are alternative strategies for attracting banking activity such as issuing stores of value and granting fiscal subsidies to investment. These strategies are more outside the Central Bank's remit, but also relevant. We later comment on their limits and stress the benefits of liquidity support.

also likely when there are externalities on the payment system or more generally onto other regulated banks and insurance companies; the externality may be generated by counterparty exposures or by the threat of fire sales.[4] A contrario, the location of a hedge fund in Bermuda is likely to be driven by other considerations than the prospect of state support, which is much lower than for the above-mentioned actors.

The raw data for international banking fit well with our framework (unless stated otherwise, what follows was communicated to me by Goetz von Peter, using BIS data). First, banks' home bias is very strong, even among the largest banks. Most banking systems book (substantially) more than 50% of their global assets and liabilities in their respective home country, which BIS statistics define as the country where the headquarters are located. Yu and Wangner (2023) also document the home bias in banking.[5] A decomposition of the assets of large cross-border banks into home country/rest of region/rest of the world generally reveals limited diversification. Even a European banking passport does not guarantee a diversified European presence. Of course, inertia and the dominance of home banks on their home turf prior to banking liberalization partly account for this strong geographic specialization. Yet, our paper focuses on another source of geographic specialization: the greater ability to access liquidity when having a strong presence in a resilient country. In that regard, the observed structure of international banking makes plausible the link we emphasize in the paper between the locus of banking activity and the country of supervision: the country of a bank's main operations provides support and has the biggest stake in effective supervision.

Second, we argued that home countries have specificities, such as resilience, that limit their club to a small membership. The "home country status" is indeed highly concentrated, certainly among the largest banks. The 30 banks designated as global systemically important (G-SIBs) have banking operations in more than 160 countries and jurisdictions ; but they are headquartered in only 11 countries. These countries, G7 plus China, Spain, the Netherlands and Switzerland, coincide with the IMF's SDR five (USD, EUR, JPY, GBP, RMB) plus two other trusted currencies (CHF, CAD). This is not surprising given that the characteristics that make a country appealing for establishing activities also are likely to make it a reserve currency issuer.

Third, leakage is viewed as constraining the effectiveness of monetary policy (Correa et al 2022, Granja et al 2022) and by extension liquidity support more broadly. One may conjecture, though, that liquidity spillovers are limited by a widespread adoption of the subsidiary form, as a subsidiary is supervised both by the host and home countries. However, monitoring where the money goes is difficult. First, money can be made fungible across countries for example by billing a common purchase (IT, licenses...) to a specific establishment of the banking group, by locating within a specific country an experiment or PR campaign that is costly but benefits the entire group, or by arranging with financial counterparties terms and reimbursements that de facto transfer money from an establishment in one country to another establishment in a different country. Second, straight transfers between the home bank and its foreign affiliate

[4]Such concerns may also induce authorities to bail out non-bank actors (like in 2008 AIG, whose bankruptcy might have generated domino effects onto regulated banks and insurance companies).

[5]They also show that the home bias, unlike the equity bias, rises during recessions, which aligns with our conclusion that turbulent times reduce international diversification. For more evidence on the banking home bias, see Coeurdacier-Rey (2013) and Schoenmaker (2013).

may take place ahead of the official announcement of distress. Last, bank affiliates take different legal forms, depending on operational reasons and regulatory constraints.[6] Among foreign banks in the 35 countries that report branches and subsidiaries separately to the BIS, local activity is dominated by subsidiaries (66%), which makes sense given their ability to collect local deposits; foreign branches have a slightly larger share (41%) for cross-border activity. We thus see that branches are an important legal form among bank affiliates.

2 A bank's international diversification strategy

We start with a single bank in a given strategic environment (monopoly, oligopoly or competitive), as summarized by its marginal cost of acquiring a customer. This will allow us to study its diversification trade-off when liquidity provision is time-consistent (Section 2), as well as the country's optimal liquidity commitment (Section 3). Sections 4 and 5 will extend the basic model to analyze how banking rivalry and ringfencing impact behaviors.

2.1 Model Description

There are two identical countries, A and B, and one cross-border bank which acquires "clients" or "projects" in the two countries. There are three dates $\{0,1,2\}$, and players do not discount the future.

Date 0

The bank picks a mass q^k of clients (a continuum of them) in each country $k \in \{A,B\}$. The unit cost of customer acquisition in country k is $c(q^k)$. This unit cost function satisfies $c(0) = 0$, $c' > 0$, and $qc'' + c' > 0$, where the latter condition is sufficient to guarantee the convexity of the acquisition cost $c(q)q$ and (when we introduce banking competition) strategic substituability.[7]

This assumption thus implies that $c(q^A)q^A + c(q^B)q^B \geq 2c(\frac{q^A+q^B}{2})(\frac{q^A+q^B}{2})$ for all (q^A,q^B), that is diversification is cost efficient. Let

$$\hat{c}(q^k) \equiv c(q^k) + q^k c'(q^k)$$

denote the marginal acquisition cost in country k. From our previous assumption, $\hat{c}$ is increasing.

Let $q \equiv \sum_k q^k$ stand for total bank size, and $\sigma^k \equiv \frac{q^k}{q}$ denote the bank's relative presence in country k. The parameter

$$\sigma \equiv \max_{k\in\{A,B\}} \sigma^k$$

[6] Subsidiaries require more capital and liquidity than branches as the bank holding cannot pool resources and use excess cash in one country to meet obligations in another (Cerutti et al 2010).

[7] As an illustration, suppose that there is a mass 1 of borrowers and that each borrower at date 0 requires an investment of 1. A borrower has endowment y that is distributed according to $G(y)$ on $[0,1]$. For simplicity assume that this endowment is not observable by the bank. The supply of loans q^k in country k determines a cutoff $y^k = G^{-1}(1-q^k)$ and a cost $c(q^k) \equiv 1-y^k$. For instance, for a uniform distribution of borrower endowment on $[0,1]$, $c(q^k) \equiv q^k$.

measures the bank's geographic specialization. The bank is (fully) diversified if $\sigma = 1/2$. For conciseness, country k such that $\sigma^k = \sigma > 1/2$ will be called the "home country" or "high-presence country", while the other country is the "foreign country" or "low-presence country".

The bank has resources only at date 0. There are no stores of value at that date, and so the acquisitions' cost is the bank's only expense at date 0. Unless otherwise specified, we will assume that the bank is not credit constrained. That is, for its optimal size q^* and specialization σ^*, the bank's date-0 resources exceed $C(q^*, \sigma^*)$, where C is the bank's investment cost function:

$$C(q,\sigma) \equiv c(\sigma q)\sigma q + c((1-\sigma)q)(1-\sigma)q.$$

Our assumptions imply that $C_q > 0$ and $C_{qq} > 0$, and (in the relevant range $\sigma \geq 1/2$) $C_\sigma \geq 0$, (with strict inequality unless $\sigma = 1/2$), $C_{\sigma\sigma} > 0$, and $C_{\sigma q} > 0$, where $C_q = \partial C/\partial q$, etc. For example, for a linear unit cost $c(q) = q$ (see footnote 7 for foundations), the acquisition cost is quadratic:

$$C(q,\sigma) = [\sigma^2 + (1-\sigma)^2]q^2.$$

The bank's date-0 establishment choices are commonly observable.

Date 1

In each country, projects are hit by liquidity shocks at date 1. The distribution of the bank's projects' idiosyncratic liquidity shocks on its various projects, $F(\rho) \sim [0, +\infty)$, is the same in both countries and is common knowledge. So these are no macroeconomic shocks, only project-specific shocks (Subsection 2.6 develops an alternative interpretation involving a macroeconomic shock). These shocks are to be understood as net of any date-1 revenue; we do not allow net date-1 revenues to be strictly positive only for notational simplicity (strictly positive net revenues for some projects can easily be accommodated as they are used to cover other projects' liquidity shocks. See Subsection 2.6).

The cumulative distribution F is twice continuously differentiable. For conciseness, its density f satisfies $f' \leq 0$ (as is the case for uniform, exponential and Pareto distributions); this condition guarantees the concavity of the objective functions. The weak concavity of F is also sufficient for the hazard rate $f(\rho)/F(\rho)$ to be monotonic, and indeed for the stronger property that the elasticity of the cumulative distribution, $f(\rho)\rho/F(\rho)$, be decreasing in ρ and always less than or equal to 1. Finally, the truncated mean $E[\rho|\rho \leq \rho^*]$ is smaller than $\rho^*/2$; and $\lim_{\rho\to\infty} f(\rho)\rho = 0$ provided the distribution has a mean (see Appendix OA for the proof of the latter property).

Countries, which have observed the bank's investments $\{q^k\}$ in the two countries, face costs of public funds, $\{\lambda^k\}$ for country k, and simultaneously select their liquidity support[8] $\{T^k \geq 0\}$. So the bank receives $T = T^A + T^B$. We will let $\kappa^k \in [0, q^k]$ denote the continuation scale in country k.[9] That is, a fraction κ^k/q^k of initial investments is safeguarded thanks to the bank's date-1

[8] In the model these are pure bailouts as we have assumed that there is no pledgeable income that the bank can return at date 2. With (say, a random) date-2 income, this liquidity support takes the more familiar form of a risky collateralized loan to the banks.

[9] Here, the goal is to bring existing projects to completion. More generally, one could allow for both expansions and contractions of firm size, as is described in Appendix OB, which is set in the industrial context (the two frameworks being mathematically equivalent).

reinvestment in country k. As discussed in the introduction, λ^k stands for country k's shadow cost of raising resources- whether fiscal or monetary- to support the bank.

Date 2

The bank enjoys its private benefit, $[\sum_m \kappa^m]b$, and country k receives social benefit $\kappa^k\beta$ (for micro-foundations of b and β, see e.g., Farhi-Tirole 2021). Note that there is no pledgeable income at date 2, as there was no revenue at date 1. This is just for notational simplicity, as we will later note that the model can accommodate verifiable date-1 and date-2 revenues.

Objective functions

Whether transfers are ringfenced (as in Section 5) or not (as will be the case until then), the bank allocates the available money within a given country to those projects that require the smallest reinvestment. Letting ρ^k denote the bank's cutoff in country k,[10] $\kappa^k = F(\rho^k)q^k$. The bank's utility is then equal to the expected continuation benefit minus the cost of acquisition:

$$U = [\sum_k F(\rho^k)q^k]b - [\sum_k c(q^k)q^k].$$

Countries do not necessarily care about the bank itself, but they internalize the continuation of economic activity on their soil. Country k's welfare comprises two terms: The benefit of the economic activity net of the transfers needed to maintain it; and the rent S of date-0 project owners. The rent S satisfies $dS/dq = qc'(q)$ (the product of the number of inframarginal units times the necessary price increase brought about by the marginal one). It is increasing and convex and is given by:[11]

$$S(q^k) = \int_0^{q^k} xc'(x)dx.$$

The intertemporal welfare of country k is the sum of the date-0, date-1 and date-2 welfares:

$$W^k = S(q^k) + [F(\rho^k)q^k\beta - \lambda^k T^k].$$

Absence of ringfencing

We assume that money is fungible: Countries cannot target their liquidity assistance to reinvestment within the country. This implies that the bank allocates funds as it sees optimal, i.e. to the projects that have the lowest cost of reinvestment and so the cutoffs in the two countries coincide:

$$\rho^A = \rho^B = \rho^*.$$

[10]That is, projects that require reinvestment $\rho \leq \rho^k$ are continued.

[11]In the example given in footnote 7,

$$S(q^k) = \int_{G^{-1}(1-q^k)}^{\infty} (y - y^k)dG(y);$$

so for example with a quadratic acquisition cost,

$$S(q^k) = (q^k)^2/2.$$

2.2 Date-1 liquidity provision, and date-0 bank size and diversification

Let us assume for the moment that the two countries face identical costs of public funds: For all k, $\lambda^k = \lambda$.

Countries' incentives under full fungibility

At date 1, the bank receives transfers from both countries, in total amount $T \equiv T^A + T^B$. It then efficiently allocates this liquidity to the least-continuation-cost projects, so as to maximize the date-2 private (and social) value. Let ρ^* denote the bank's cutoff for continuation ($\rho^A = \rho^B = \rho^*$). The date-1 budget constraint given aggregate size q writes:

$$\left[\int_0^{\rho^*} \rho dF(\rho)\right] q = T. \tag{1}$$

Furthermore

$$\kappa^k = F(\rho^*) q^k.$$

Country k chooses its support level at date 1 so as to solve:

$$\max_{\{T^k\}} \{\beta \kappa^k - \lambda T^k\} = \max\{\beta F(\rho^*) q^k - \lambda T^k\}$$

with $\rho^*(T^A + T^B)$ given by (1), and so:

$$\frac{\partial(\beta\kappa^k - \lambda T^k)}{\partial T^k} = \frac{\beta\sigma^k}{\rho^*} - \lambda,$$

and $\frac{\partial^2(\beta F(\rho^*(T^A+T^B))q^k - \lambda T^k)}{\partial (T^k)^2} < 0$. This implies that, unless $\sigma = 1/2$, only the home country (country k such that $\sigma^k \equiv \sigma > 1/2$) brings liquidity to the bank. The cutoff is given by [12]

$$\rho^* = \hat{\beta}\sigma, \tag{2}$$

where $\hat{\beta}$ is the countries' cost-adjusted willingness to pay for continuation:

$$\hat{\beta} \equiv \frac{\beta}{\lambda}.$$

If $\sigma = 1/2$, then $\rho^* = \hat{\beta}/2$; we can assume for example that each country contributes for half of the liquidity provision. There are actually a continuum of equilibria, all yielding the same date-1 continuation scale, and therefore the same bank behavior at date 0: Any $\{T^A, T^B\}$ such that $T^A + T^B = [\int_0^{\hat{\beta}/2} \rho dF(\rho)]q$ is an equilibrium of the liquidity provision stage. Because the bank is not affected by who is bringing the overall support T, this indeterminacy has no impact on the date-1 equilibrium outcome beyond the redistributive aspect. Besides, we will shortly show that $\sigma = 1/2$ is never an optimal choice for the bank.

[12] We assumed that the distribution F has support on $[0, +\infty)$ so there is no issue with an upper bound. When we use the uniform distribution for illustrative purposes, though, we must assume that $\hat{\beta}\sigma$ does not exceed the upper bound.

The bank's continuation scale is therefore $F(\hat{\beta}\sigma)$ where, recall, $\sigma \in [1/2, 1]$ denotes the bank's degree of specialization. The continuation scale is higher, the lower the probability of leakage, where leakage can be measured by the fraction, $1-\sigma$, that benefits the other country. Because the efficient level of support in a union of the two countries would be $\rho^* = \hat{\beta}$, there is free riding between the two countries unless the bank is fully specialized, i.e. not cross-border ($\sigma = 1$). The leakage associated with the inability to perfectly ringfence induces the home country to bring too little liquidity to the bank, from the point of view of the other country (and of course from the point of view of the bank).

Finally, let us check that the high-presence country indeed wants to bring the liquidity level corresponding to the first-order condition (i.e., its level of support is a global optimum). Its net utility is given by $\lambda q\Big[F(\hat{\beta}\sigma)\hat{\beta}\sigma - \int_0^{\hat{\beta}\sigma} \rho dF(\rho)\Big]$. The term in brackets is equal to 0 for $\hat{\beta} = 0$, is increasing in $\hat{\beta}$ (from the envelope theorem), and therefore is always positive. Put differently, if the country is willing to bring liquidity to withstand a marginal shock, it is a fortiori willing to bring liquidity that will serve to withstand (lower) inframarginal shocks. As we will see, this property need not hold when the shock distributions are asymmetric: The extent of leakage of liquidity may differ between inframarginal and marginal shocks.

Optimal size and diversification

At date 0, the bank chooses an overall presence, q, and a specialization $\sigma \in [\frac{1}{2}, 1]$, so as to maximize:

$$U(q,\sigma) \equiv F(\hat{\beta}\sigma)bq - C(q,\sigma)$$

or

$$\max_{\{q,\sigma\}} \Big\{[F(\hat{\beta}\sigma)q]b - c(\sigma q)\sigma q - c((1-\sigma)q)(1-\sigma)q\Big\}.$$

For simplicity, we assume that this program is concave.[13] Recalling that $\hat{c}(q) \equiv c(q) + c'(q)q$, the first-order conditions with respect to q and σ, respectively, are for an interior solution[14]

$$F(\hat{\beta}\sigma)b = \sigma\hat{c}(\sigma q) + (1-\sigma)\hat{c}((1-\sigma)q) \tag{3}$$

and

$$f(\hat{\beta}\sigma)\hat{\beta}b = \hat{c}(\sigma q) - \hat{c}((1-\sigma)q) \tag{4}$$

(the corner solution $\sigma = 1$ in contrast requires that $f(\hat{\beta})\hat{\beta}b \geq \hat{c}(q)$). Let (q^{nc}, σ^{nc}) denote the bank's optimal choices under no-commitment (i.e. when the countries act in a time-consistent manner). The first observation comes from looking at condition (4): Necessarily $\sigma^{nc} > 1/2$.

[13] Under our assumptions, $C_{qq} > 0$ and $C_{\sigma\sigma} > 0$. However $C_{qq}C_{\sigma\sigma} - (C_{q\sigma})^2$ is not always positive. On the other hand, we only need $F(\hat{\beta}\sigma)qb - C(q,\sigma)$ to be concave, with $\partial^2(F(\hat{\beta}\sigma)qb)/\partial q\partial\sigma = f(\hat{\beta}\sigma)\hat{\beta}b$, so that for $\hat{\beta}b$ large enough concavity is guaranteed.

[14] Our assumption that the bank's date-0 assets, Ω, be large enough can be written $\Omega \geq C(q,\sigma)$. For example, in the quadratic-acquisition-cost case $\big(C(q,\sigma) \equiv [\sigma^2 + (1-\sigma)^2]q^2\big)$ and denoting σ^{nc} the bank-optimal specialization (obtained from (3) and (4)), this condition writes:

$$\Omega \geq \frac{F^2(\hat{\beta}\sigma^{nc})b^2}{4[(\sigma^{nc})^2 + (1-\sigma^{nc})^2]}.$$

Starting with equal presence in the two countries, the marginal cost $\hat{c}$ for the bank of investing is the same in both, and so a slight increase in σ has no effect on the investment cost (the RHS of (4) is 0): The cost in terms of diversification is second order. But the geographic specialization brings in more liquidity support and increases bank payoff to the first order (this is the term of $f(\hat{\beta}\sigma)\hat{\beta}b$ on the LHS of (4)). Condition (3) says the marginal benefit of overall size expansion, $F(\hat{\beta}\sigma)b$, is equal to its average marginal cost.

Proposition 1 *(country specialization). Under fungibility, symmetric countries, and time-consistent support policies:*

(i) Date-1 liquidity support is brought only by the home country, increasingly so as the bank specializes more, i.e. as σ increases.

(ii) At date 0, the bank chooses to specialize partly or fully ($\sigma^{nc} \in (\frac{1}{2}, 1]$). For any $\hat{\beta}$, the bank fully specializes ($\sigma = 1$) for a uniform distribution ($f' = 0$) and diversifies ($\sigma < 1$) otherwise ($f' < 0$). In the latter case, the bank diversifies maximally when support is expected to be generous ($\lim_{\hat{\beta}\to+\infty} \sigma = 1/2$). *Conversely, when expected support is meager, the bank specializes* ($\lim_{\hat{\beta}\to 0} \sigma = 1$).

(iii) More turbulent times (in the sense of a uniform shift θ toward higher shocks: $F(\rho-\theta)$) lead to more specialization, as the bank is keen on securing liquidity from the home country: Full specialization ($\sigma^{nc} = 1$) obtains for a wider set of parameters; also, with linear cost, σ^{nc} weakly increases with θ.

Proof. Part (i) was proved earlier.

(ii) Consider now the bank's choice of specialization. The condition for full specialization is simply obtained by substituting $\sigma = 1$ into equation (3), using the fact that $\hat{c}(0) = c(0) = 0$, and checking that the LHS of (4) exceeds its RHS. Next, when $\hat{\beta} \to +\infty$, (nearly) all shocks are covered by the home country and so cost minimization is the bank's overriding consideration: because the distribution of shocks satisfies $\lim_{\rho\to+\infty} f(\rho)\rho = 0$, condition (4) shows that the bank must diversify maximally when $\hat{\beta}$ is large. Conversely, $\lim_{\hat{\beta}\to 0} F(\hat{\beta})/f(\hat{\beta})\hat{\beta} = 1$, and so (almost) full specialization is expected for low liquidity support ($\hat{\beta}$ small).

(iii) Consider a uniform shift θ in the distribution of shocks, which becomes $F(\rho-\theta)$. A larger θ means larger shocks. Simple computations show that (a) full specialization ($\sigma^{nc} = 1$) obtains for a larger set of parameters as θ increases, and (b) in the quadratic acquisition cost case ($c(q) = q$, and so $\hat{c}(q) = 2q$), σ^{nc} is a weakly increasing function of θ. This is easily understood: The bank faces a trade-off between specializing more to secure more liquidity support from its home country and conquering the marginally more lucrative foreign market (i.e., benefiting from diversification). The first concern looms larger when shocks become more likely. ■

Quadratic acquisition cost example. Suppose for the rest of the subsection that $c(q) = q$ (and so $\hat{c}(q) = 2q$). Combining (3) and (4) yields

$$\frac{f(\hat{\beta}\sigma)(\hat{\beta}\sigma)}{F(\hat{\beta}\sigma)} = \frac{2\sigma^2 - \sigma}{2\sigma^2 - 2\sigma + 1}. \tag{5}$$

The RHS of (5) is increasing in σ and is equal to 1 for $\sigma = 1$. The LHS of (5) represents the elasticity of the distribution function $F(\rho)$. Because this elasticity is decreasing in ρ, then the equilibrium specialization σ^{nc} is a decreasing function of the eagerness $\hat{\beta}$ to support banking activity. Intuitively, when the countries are very eager to provide liquidity support, a high specialization is no longer needed to obtain this support.

We can complete this analysis by looking at the case of two specific distributions.

(a) *Exponential distribution.* Suppose that $F(\rho) = 1 - e^{-\rho}$ on $[0, +\infty)$. Then

$$\frac{f(\rho)\rho}{F(\rho)} = \frac{\rho}{e^{\rho} - 1}$$

is smaller than 1 and decreasing in ρ. So there is never full specialization ($1/2 < \sigma^{nc} < 1$), in conformity with Proposition 1 (ii). And σ^{nc} is decreasing in β and increasing in λ. Furthermore, we verify the more general properties:

$$\lim_{\hat{\beta} \to +\infty} \sigma^{nc} = \frac{1}{2} \quad \text{and} \quad \lim_{\hat{\beta} \to 0} \sigma^{nc} = 1.$$

(b) *Uniform distribution.* For $F(\rho) = \rho$ on $[0, 1]$, $f(\rho)\rho/F(\rho) \equiv 1$. So we check Proposition 1 (ii) for the special case of a quadratic cost: $\sigma^{nc} = 1$ regardless of $\hat{\beta}$.

2.3 Does the home country benefit from being the home country?

The answer to this question is obviously yes if $\sigma^{nc} = 1$. When the bank does not fully specialize however, the home country bears the brunt of the liquidity support but receives only part of the countries' liquidity benefits; so it is not obvious that it is better off than the foreign country. Let V^h and V^f denote their payoffs. Then

$$V^h - V^f = \left[S(\sigma q) + \left[F(\hat{\beta}\sigma)\beta\sigma - \lambda \int_0^{\hat{\beta}\sigma} \rho dF(\rho) \right] q \right] - [S((1-\sigma)q) + F(\hat{\beta}\sigma)\beta(1-\sigma)q].$$

or

$$V^h - V^\ell = \left[F(\hat{\beta}\sigma)\beta(2\sigma - 1) - \lambda \int_0^{\hat{\beta\sigma}} \rho dF(\rho) \right] q + [S(\sigma q) - S((1-\sigma)q)]$$

Appendix A obtains the following proposition:

Proposition 2 *(benefit from being the home country) There exists* $\underline{\sigma} \in (1/2, 2/3)$ *such that* $V^h > V^f$ *iff* $\sigma > \underline{\sigma}$.

Of course, the extent σ of home bias is endogenous. We saw that for quadratic acquisition costs for example, σ increases with the cost of public funds λ and decreases with the government's eagerness to rescue banks β. It depends also on the shape of the distribution (e.g. $\sigma \equiv 1$ for a uniform distribution), as well as on the magnitude of shocks: A corollary of Proposition 2 (combined with part (iii) of Proposition 1) is that as times become more turbulent, it is more likely that $V^h > V^f$.[15]

Capital controls. Suppose now that countries can limit the investment on their soil, perhaps through a date-0 capital control or through prudential requirements. Country k sets a cap $\bar{q}^k \in [0,+\infty]$ on the bank's domestic investment: $q^k \leq \bar{q}^k$. By convention, $\bar{q}^k = +\infty$ means the absence of capital control. Consider the game in which at date −1, countries simultaneously set ceilings $\bar{q}^k$ on the bank's investment. Such a ceiling captures in a stylized way capital controls or strict solvency requirements. In order to avoid a multiplicity of equilibria in the absence of capital controls, suppose that when indifferent, the bank picks country B as the home country.[16] The equilibrium in the absence of capital control is then unique and has the bank invest $q^A = (1-\sigma^{nc})q^{nc}$ in country A and $q^B = \sigma^{nc}q^{nc}$ in country B. Countries A and B obtain no-capital-control welfares V^f and V^h, respectively.

Proposition 3 *(capital controls) The following is an equilibrium behavior of the capital-control game:*

(i) When $V^h \geq V^f$ at the unconstrained date-0 investment policy $\{q^{nc},\sigma^{nc}\}$, countries do not impose capital controls and so the outcome is the same as in the absence of capital controls.

(ii) When $V^{ff} = V^f - \left[F(\hat{\beta}\sigma^{nc}) - F\left(\frac{\hat{\beta}}{2}\right)\right]\beta(1-\sigma^{nc})q^{nc} \leq V^h < V^f$ at the unconstrained date-0 investment policy $\{q^{nc},\sigma^{nc}\}$, country A imposes capital control $\bar{q}^A = (1-\sigma^{nc})q^{nc}$ and country B imposes no capital control. The outcome is again the same as in the absence of capital control.[17]

When being the home country is beneficial despite the associated duty, country A would like to challenge country B's dominance; however, country A cannot make itself more attractive by imposing a capital control on the bank's investment there (part (i) of Proposition 3, proved in Appendix B). When being the home country is penalizing (part (ii) of the proposition), country B would like to switch roles with country A, and can do so in the absence of capital control by country A: it suffices that country B make itself less attractive through a capital control a bit below $q^B = \sigma^{nc}q^{nc}$. Country A can however protect itself against such passing the buck by imposing a capital control at level q^A, which in equilibrium has no effect, but deters country B from switching roles: To make country A the home country, country B must then choose a lower capital control; but that implies that the liquidity support will be meager- an effect captured by the difference, $\left[F(\hat{\beta}\sigma^{nc}) - F(\hat{\beta}/2)\right]\beta q^A$, between V^f and V^{ff}.

2.4 Asymmetric countries and the bank's choice of home country

With symmetric countries, the bank is indifferent regarding its choice of home country. This is no longer so if we introduce asymmetries.

[15]Of course turbulent times also reduce the overall scale, but here we discuss only relative payoffs.

[16]One may justify this lexicographic choice by imagining that the bank has a (vanishingly small) preference for country B. For example, the marginal investment cost in country B might be slightly smaller than in country A, with the difference going to 0. Or country B be might be slightly more eager to rescue the bank than country A.

[17]We conjecture that there exists a mixed-strategy equilibrium of the capital control game when $V^h < V^{ff}$.

2.4.1 Asymmetric eagerness to bring liquidity support

Suppose that countries are symmetrical except that $\hat{\beta}^A > \hat{\beta}^B$. That is, the stability of the banking sector is more important to country A ($\beta^A > \beta^B$) or/and country A has more financial muscle than country B ($\lambda^A < \lambda^B$). Then for any degree of specialization $\sigma > \frac{1}{2}$, there is more liquidity support if the bank makes country A its home country:

$$\hat{\beta}^A \sigma > \max\left\{\hat{\beta}^B \sigma, \hat{\beta}^A (1-\sigma)\right\}.$$

This simple result captures the idea that dominance is facilitated by healthy public finances, as the latter allow the country to bring liquidity support in dire straits. Similarly, a country which is eager to maintain activity on its soil is more attractive to the bank.

Proposition 4 *(asymmetric eagerness to bring liquidity support). Suppose that countries differ only in their resilience ($\lambda^A \neq \lambda^B$) or/and their willingness to keep activity on their soil ($\beta^A \neq \beta^B$). Then the country with the highest index $\hat{\beta}^k \sigma^k$ provides the liquidity. The bank specializes in the highest-$\hat{\beta}^k$ country. The bank is therefore more likely to specialize in country A if (i) country A is more resilient ($\lambda^A < \lambda^B$) and/or (ii) country A is more eager to keep activity on its soil ($\beta^A > \beta^B$).*

The parameter β has multiple drivers. In the context of banking, we emphasized the supply of core functions to fragile or politically sensitive customers (small depositors, SMEs) and financial stability implications as two keys to determining whether a financial institutions is likely to receive support. In the industrial context, a sovereignty concern (defense industry, chips…) is conducive to rescues. Finally, in both contexts, size matters, either because distress would be intensely covered in the media, or by dint of a stronger political influence, or else in virtue of large spillover effects of a distress on subcontractors: this is the standard “too big to fail” argument.

2.4.2 Asymmetric shock distributions

Alternatively, the asymmetry may refer to a differential in the distributions of shocks. The projects in the two countries may exhibit different profitability (see below our discussion of date-1 revenue or date-2 pledgeable income) or require different reinvestment. For example, a country’s indebtedness may translate into higher taxes instead of a lower willingness to spend public funds (as in Section 2.4.1). Or the country may have a carbon-intensive industry or a green electorate (or both) and will have to decarbonize in the near future.

Appendix OC assumes that countries differ solely according to their distribution of shocks, $F^A(\rho)$ and $F^B(\rho) = F^A(\rho - \delta)$. That is, country B is the high-shocks country, as depicted in Figure 1. Country A may receive a small fraction of the benefits at the margin, but still be the main beneficiary of the rescue. Proposition 12 in Appendix OC shows that that there is a unique equilibrium of the date-1 rescue game between the two countries; in this equilibrium, either country A supplies liquidity (with country B either not supplying any liquidity or topping up that supplied by country A) or country B supplies the entire liquidity.

The country with less severe shocks arguably has stronger fundamentals. Indeed, ceteris paribus, lower shocks give a comparative advantage to the country. In that sense, it is related to Proposition 4, that points out that, for a given business friendliness, the resilient country will be the home country. However, one cannot conclude that the low-shocks country will be the home country because the bank's choice of home country is strategic. Indeed it may be the case that the bank picks the high-shocks country as the home country if this increases the put on public money.

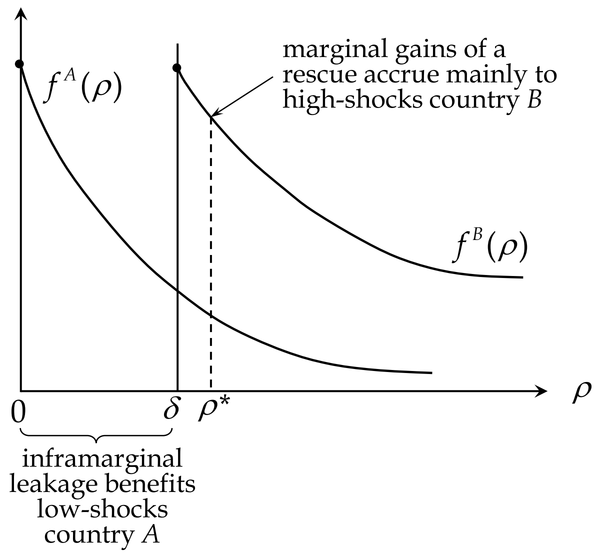

Figure 1: Discrepancy between marginal and inframarginal leakage when shocks are asymmetric (exponential distribution).

2.5 Coasian bargains and their breakdowns

Suppose that the two countries face the same shadow cost λ and bargain efficiently at date 1 (they share the gains from trade). The countries then jointly provide the socially efficient level of support to the bank, that satisfies

$$\rho^* = \hat{\beta}.$$

Anticipating a Coasian bargain, the bank chooses $\{q, \sigma\}$ so as to solve:

$$\max_{\{q,\sigma\}} F(\hat{\beta})bq - C(q, \sigma).$$

For any size q, the continuation scale is independent of specialization, and so the bank just minimizes cost:

$$\sigma = \frac{1}{2}. \tag{6}$$

Scale q is given by

$$F(\hat{\beta})b = \hat{c}\left(\frac{q}{2}\right). \tag{7}$$

For $c(q^k) = q^k$ for example, $q = F(\hat{\beta})b$. In contrast, in the absence of Coasian bargain, $q = \frac{F(\hat{\beta}\sigma)b}{2[\sigma^2+(1-\sigma)^2]}$. So there is more investment under a Coasian bargain for two reasons: The prospect of a stronger liquidity support ($F(\hat{\beta}) \geq F(\hat{\beta}\sigma)$) and a cost benefit from diversification ($\frac{1}{2} < \sigma^2+(1-\sigma)^2$). More generally, the cost benefit from country diversification implies a higher investment under Coasian bargaining.

Proposition 5 *(higher investment under Coasian bargaining). The bank invests more if it expects a Coasian bargain: It receives a stronger liquidity support; and because this liquidity support does not require geographic specialization, it enjoys the cost benefit of diversification.*

Swaps. One of the impediments to Coasian bargains is that a country may not have the financial (or political) ability to compensate the other ex post. That suggests that countries that are roughly similar at date 0 should design a mutual-insurance scheme, allowing each other to have access to funds.[18]

2.6 Discussion of modeling choices and simple extensions

Correlation of shocks. Due to risk neutrality, the model admits two equivalent interpretations. The one developed above has a continuum of independent projects whose realized shock distribution is, by the law of large numbers, identical to the distribution of shocks $F(\rho)$ for an arbitrary project. Alternatively, the projects sponsored by the bank all face the same shock ρ drawn from $F(\rho)$. In this correlated-shocks interpretation, $F(\rho^*)$ is to be interpreted as the probability that all projects continue, rather than as the fraction of surviving projects. In this case, the bank may need no cash with an arbitrary probability (then $F(0) > 0$). The level of λ^k thus stands more generally for the availability of public funds in times of global stress, that is in those states of nature in which the banks need liquidity support.

More than two countries. The analysis is unchanged when the cross-border bank is potentially present in n countries. Specialization in a single country can still arise, actually under the exact same condition as when there are only two countries. In particular, specialization occurs iff $f(\hat{\beta})\hat{\beta}/F(\hat{\beta}) \geq 1$ (actually equal to 1 –the case of the uniform distribution of shocks– given our simplifying assumption that F is weakly concave). If this condition is violated, then $\sigma^{nc} < 1$, and there is more diversification, the larger the number of countries; intuitively, there are more opportunities and so the cost of not diversifying is higher while the gain from not diversifying (the increased access to financial support) is independent of the number of countries.[19] The home country may no longer host the majority of the bank's activity. Indeed, (the generalization of) condition (4) implies only that it must shelter more than $1/n$ of the activity.

[18] See Appendix OD for a start on this. See also Tirole (2015) for a mechanism design analysis of risk sharing agreements under cross-country externalities and various degrees of ex-ante asymmetries among countries.

[19] Let $C^n(q,\sigma) \equiv c(\sigma q)\sigma q + (n-1)c\left(\frac{1-\sigma}{n-1}q\right)\left(\frac{1-\sigma}{n-1}\right)q$, so that $C^2(q,\sigma) = C(q,\sigma)$. It is optimal for the bank to select a specialization σ in a country to solve $\max_{\{q,\sigma\}} F(\hat{\beta}\sigma)bq - C^n(q,\sigma)$. For quadratic acquisition costs, condition (5) becomes:

$$\frac{f(\hat{\beta}\sigma)\hat{\beta}\sigma}{F(\hat{\beta}\sigma)} = \frac{\sigma(n\sigma-1)}{n\sigma^2-2\sigma+1}.$$

Date-1 revenue and date-2 pledgeable income, and prudential supervision. Little would be altered if the bank produced income at date 1 and/or pledgeable income at date 2 in case of project continuation. To the extent that the date-2 pledgeable income is bailinable,[20] it plays the same role as a date-1 revenue, except that the revenue is contingent on continuation. The analysis directly carries through, replacing "liquidity shocks" by "net liquidity shocks". We now show how to incorporate date-1 income into the analysis.

(i) Let the bank receive date-1 revenue rq where $r \geq 0$ is the per-unit revenue. Assume in a first step that the bank cannot divert the date-1 revenue. In the absence of public liquidity provision, the bank faces cutoff ρ_0^* such that $[\int_0^{\rho_0^*} \rho dF(\rho)]q = rq$ provided that the bank is eager enough to continue ($\rho_0^* \leq b$, the bank does not produce excess liquidity by itself), which we will assume. Assuming also that the bank is not liquidity self-sufficient ($\rho_0^* < \hat{\beta}\sigma$, otherwise the home country brings in no liquidity support), the home country offers $\left[\int_{\rho_0^*}^{\hat{\beta}\sigma} \rho dF(\rho)\right]q$. The first-order condition again defines a global optimum, as $\left[F(\hat{\beta}\sigma) - F(\rho_0^*)\right]\hat{\beta}\sigma \geq \int_{\rho_0^*}^{\hat{\beta}\sigma} \rho dF(\rho)$. The equilibrium is then as described in Proposition 1. The only difference is that the home country liquidity injection is smaller as the bank already generates inside liquidity.

(ii) Suppose next that, when left unsupervised/monitored, the bank can abscond with the liquidity[21]; say, it can grab ξrq where $\xi \in (0,1]$, where $1-\xi$ is the deadweight loss associated with diverting money. Introduce the possibility that countries can, at a cost and in a non-contractible fashion, take steps to prevent liquidity diversion by the bank (see Appendix OE for more details). The equilibrium allocation of supervisory activities is described in part (ii) of the following proposition.

Proposition 6 *(inside liquidity and supervision by the home country). Assume that the bank receives revenue rq at date 1.*

(i) For $r \leq r_0 \equiv \int_0^{\hat{\beta}\sigma} \rho dF(\rho)$, then $\sigma = \sigma^{nc}$, where σ^{nc} is the level of specialization in the absence of inside liquidity. For $r > r_0$, then the bank's continuation is self-financed.

(ii) Suppose that the bank, if left unmonitored, can divert cash to its own benefit and that the countries can engage in costly supervision to prevent such diversion. If $r \leq r_0$, then only the home country, which has to cover any increase in the liquidity shortfall, has an incentive to engage in costly supervision. If $r > r_0$, the home country has more incentives to supervise the bank (both because of the larger scale of its operations in the country and because it will supply the liquidity); in equilibrium, either the home country is the only supervisor or the foreign country may join in.

Support vs. expropriation. While the paper stresses country support to the bank in dire straits,

[20]That is, the corresponding claims can be transferred or sold, letting the firms' contribute to the liquidity support and thereby reducing the public funds commitment. See Dewatripont-Tirole (2019) and Clayton-Schaab (2021) for analyses of optimal bailinability.

[21]It is then optimal for the bank to divert the entire liquidity, as any shortfall to cover shocks below $\hat{\beta}\sigma$ will be covered by the home country. Note also that diversion is optimal for the bank whenever ρ_0^* is smaller or even slightly bigger than $\hat{\beta}\sigma$.

the mirror-image of the model is the possibility that the country might at date 1 "expropriate" the bank's investment, for example through new tax, regulatory, lending, prudential, labor, data sharing or other requirements or by failing to fulfill its public infrastructure promises. In that case too, a future willingness to not hinder its activity (captured by the parameter β) as well as the good health of public finances (captured by λ), which reduces the incentive to expropriate, reassure the bank and induce it to invest in the country. The leakage comes from the fact that the resulting need for cash in one country sucks resources available in the other country. To complete the analogy, the "home country" is less eager to tax the bank as it partly shoots itself in the foot.

More formally, suppose that the bank receives verifiable revenue rq at date 1, and that the home country taxes this revenue at level τq ($\tau < 0$ corresponds to liquidity support). Thus, the continuation decision is given by $\left[\int_0^{\rho^*} \rho dF(\rho)\right]q \leq (r-\tau)q$. At date 1, the home country selects the tax rate so as to maximize $[\lambda\tau + F(\rho^*)\sigma\beta]q$, and so $\rho^* = \hat{\beta}\sigma$. Taxes $r - \hat{\beta}\sigma$ (if $r > \hat{\beta}\sigma$) are smaller, the higher the geographic specialization (σ) and the more bank-friendly ($\hat{\beta}$) the country is.

How important is the prospect of liquidity support in determining location? Is this consideration at the top of mind for banks and firms when they choose whether to diversify? Location and investment decisions are long-term decisions. While the standard factors (such as comparative advantage, or the existence of a home market when production exhibits returns to scale and there are transportation costs) are important, the country's attractiveness (captured in the model by business friendliness parameter β and by resilience parameter λ) also plays a meaningful role looking ahead. Indeed, countries do not only subsidize investment on their soil; they also try to reassure investors as to their financial resilience and their business friendliness. And hegemons strive to facilitate liquidity provision by assuming their exorbitant duty as the flip side of their exorbitant privilege. Our focus on this new consideration of course does not invalidate the significance of the more traditional ones. Finally, we just noted that the mirror-image of the model is the possibility that the countries might at date 1 "expropriate" the bank's investment, a preoccupation that is clearly in the mind of managers when choosing their location decisions.

Limited equity and debt caps. To focus on liquidity shortages in the simplest manner, we assumed that the bank faces no date-0 solvency constraint. When debt is issued to finance the initial investment, the liquidity shock can be interpreted as a solvency shock. It is straightforward to relax the assumption of an abundant date-0 endowment. The insights remain the same, unsurprisingly given that our focus is on liquidity provision, not on ex-ante solvency: See Appendix OF.

3 Liquidity commitments

Liquidity support interventions studied so far have been ex-post (i.e. time-consistent) interventions. We now study whether countries would want to attract banks at date 0 by promising domestic LOLR services.[22] We add a prior stage ("stage −1") at which countries may commit to a level of liquidity support. They do this conditionally (contingent on the absolute or relative presence in the country) or unconditionally. Suppose again a single bank and two countries. In a first step, only country A has cash to bring liquidity: $\lambda^A = \lambda$ and $\lambda^B = +\infty$ (in that sense, country A is an "hegemon"); in the following, $\hat{\beta}$ will therefore stand for $\beta/\lambda^A = \beta/\lambda$. Regardless of any ex-ante commitment, the bank will necessarily specialize in country A, as it can count on no support from country B.

3.1 Unconditional commitment

Let us first assume that the hegemon commits to a level of support $T \geq 0$ that is not conditioned on the bank's footprints in the two countries. If $T < [\int_0^{\hat{\beta}\sigma} \rho dF(\rho)]q$, not topping up the committed liquidity T to the level that would prevail in the absence of a committed line, is not time-consistent: The mutually beneficial renegotiation to the higher liquidity support enabling $\rho^* = \hat{\beta}\sigma$ takes place. In contrast, if T is larger than what sustains $\rho^* = \hat{\beta}\sigma$, renegotiation cannot occur as long as there are no gains from trade (i.e. unless T is extremely high), which never occurs for the ex-ante optimal T. When facing a promised liquidity support T, the bank therefore solves at date 0:

$$\max_{\{q,\sigma,\rho^*\}} F(\rho^*)bq - C(q,\sigma)$$

s.t.

$$\left[\int_0^{\rho^*} \rho dF(\rho)\right] q \leq \max\left\{T, \left[\int_0^{\hat{\beta}\sigma} \rho dF(\rho)\right] q\right\}$$

Given that the bank's liquidity constraint is always binding, the optimal choice $\{q,\sigma\}$ may exhibit one of two configurations:

Proposition 7 *(unconditional liquidity support). Suppose that the hegemon commits to a liquidity support $T \geq 0$ at date 0. There exists $T_1 \in (0, T^{nc})$ such that*

- *for $T \leq T_1$, the commitment is irrelevant and the bank chooses the time-consistent outcome $\{q^{nc}, \sigma^{nc}\}$,*
- *for $T > T_1$, the bank fully diversifies* ($\sigma = 1/2$).

[22]There is no contradiction with the fact that financial regulators pursue "constructive ambiguity" and often warn their industry that they will not bail it out. This strategy is meant to reduce bank-level moral hazard and to give regulators a mandate to not bail out banks whose failure has little impact on SMEs, wholesale markets or domestic employment. But it does not prevent bailouts and generous liquidity support. The model does not have but could add banking moral hazard (see Proposition 6(ii)). Moral hazard, if not properly supervised, would make it harder to promise lavish support.

The ability to commit to assist entities in distress presumably is reputation-based for the banking industry. Explicit state guarantees may offer an alternative commitment device in the industrial context. Last, and referring to the model's mirror-image (expropriation), the presence of an independent court system may set a cap on the magnitude of expropriation. Overall, a country attracts activity on its soil both by altering its business friendliness (β) and its ability to intervene (λ) in a way that is consistent with time consistency and by using various means to commit to go the extra mile.

Proposition 7, proved in Appendix C, shows that an unconditional support either is irrelevant or induces the bank to diversify at $\sigma = 1/2$ rather than specialize at $\sigma^{nc} > 1/2$. For a given size q, the commitment necessarily reduces country A's welfare. So, if for instance the bank has an overall managerial capacity constraint $\bar{q}$ that lies below or exceeds slightly q^{nc}, the unconditional support can only reduce country A's welfare. In the absence of such a capacity constraint (our paradigm so far), a higher commitment T leads to a larger bank size (as shown by a revealed preference argument), and so country A might benefit from the expanded scale even though it captures only half of the global benefit.

Overall, a commitment to an unconditional liquidity support is hindered by the non-appropriation by country A of two externalities, on country B and on the bank: Country B benefits from the diversification and from an increased scale. And, unless country A is able to capture the increase in its rent, the bank gains from the liquidity support policy.[23]

3.2 Conditional commitment

Suppose now that country A can condition its liquidity support T on (a) a presence $\{q^k\}_{k\in\{A,B\}}$ in the two countries and (b) possibly a date-0 transfer $T_0 (\gtrless 0)$. The contract is thus as complete as it can be given the fungibility constraint. We assume (in this subsection and the next) that the shadow price of public funds at date 0 is equal to λ. Nothing changes qualitatively if it is an arbitrary $\lambda_0 \in [1, \lambda]$.

A key question is whether the hegemon can charge the bank at date 0 for the enhanced liquidity support. This in turn hinges on whether the bank has free cash flow to pay for the additional liquidity support (TU: utility is then transferable) or not (NTU: utility is non transferable). At one extreme, the bank had just enough net worth to finance its date-0 investment cost $C(q,\sigma)$ in the absence of liquidity commitments; this NTU paradigm is also a good characterization for the situation of credit rationing familiar in corporate finance. At the other extreme, the bank's date-0 endowment is very large, so charging for the additional liquidity is not an issue. Accordingly, we will consider both the cases of "no date-0 transfer" and "date-0 transfer".

NTU. The worst-case scenario for liquidity commitments corresponds to limited instruments (no date-0 transfer). Appendix D shows that even in this case, the hegemon deviates from the time-consistent solution. Intuitively, increasing the liquidity support T slightly above T^{nc} imposes only a second-order loss on the hegemon and leads to a first-order gain for the bank. However, the hegemon must (a) secure itself some quid-pro-quo; and (b) protect itself from the incentive for diversification induced by the commitment to some level of liquidity support (see Section 3.1). Objective (b) can be achieved through the specification of presences $\{q^k\}_{k\in\{A,B\}}$, while objective (a) results from the bank's willingness to increase q in reaction to enhanced liquidity support (or else from a slight contractual increase in q^A).

TU. Let us consider now (the simpler case of) transferable utility. The hegemon can offer a transfer ($T_0 \gtrless 0$) at date 0. The shadow price of the constraint $U \geq U^{nc}$ is then equal to λ. The

[23] Whether an unconditional support can benefit country A hinges on whether the bank is credit constrained (unless its endowment much exceeds the investment cost, country A will not be able to capture much of the bank's extra rent). We will study date-0 transfers in the next subsection.

maximization of hegemon's surplus subject to the bank's utility being no less than U^{nc} writes

$$\max_{\{q,\sigma,\rho^*\}} S(\sigma q) + \left[F(\rho^*)[\beta\sigma + \lambda b] - \lambda \int_0^{\rho^*} \rho dF(\rho)\right] q - \lambda C(q,\sigma)$$

yielding, letting $\hat{S} \equiv S/\lambda$,

$$\begin{aligned} \rho^* &= \hat{\beta}\sigma + b && (8) \\ C_\sigma(q,\sigma) &\leq F(\rho^*)\hat{\beta}q + \hat{S}'(\sigma q)q \quad \text{(with equality if } \sigma < 1\text{)} && (9) \\ C_q(q,\sigma) &= F(\rho^*)[\hat{\beta}\sigma + b] - \int_0^{\rho^*} \rho dF(\rho) + \hat{S}'(\sigma q)\sigma && (10) \end{aligned}$$

For a given specialization σ, liquidity is supplied in greater amount than in the time consistent case ($\rho^{nc} = \hat{\beta}\sigma^{nc}$) because of the internalization of the bank's benefit from continuation under TU.[24]

Note that the choice of specialization still satisfies $\sigma > 1/2$, and obeys two opposite considerations. First, a higher specialization directly benefits the hegemon. Second, the investment cost is raised by specialization directly at the time-consistent outcome; this second consideration calls for less specialization, combined with a promise of sufficient liquidity support (which itself calls for a commitment not to diversify too much). Proposition 8 below is proved in Appendix D.

Proposition 8 *(Hegemon's exhorbitant duty). The bank specializes in the hegemon because of its resilience, and this regardless of the hegemon's ability to commit to a support policy. The hegemon always provides for more continuation*[25] *under a contingent liquidity commitment (but may accept less specialization) than under non commitment:*

$$\rho^* = \hat{\beta}\sigma + \frac{\mu}{\lambda} b > \rho^{nc} = \hat{\beta}\sigma$$

where $\mu = \lambda$ under transferable utility, and $0 \leq \mu \leq \lambda$ under non-transferable utility.

The optimal policy comes with an "exorbitant duty" (Gourinchas-Rey 2022): The hegemon must provide more liquidity than it would wish ex post in exchange of attracting more investment into the country.

3.3 Exchange rate appreciation

We now introduce an endogenous date-1 exchange rate. We show that exchange rate appreciations are both an unavoidable byproduct of liquidity policies, and a limiting factor for liquidity provision by endogenously making it costlier to operate. Intuitively, the activity on the home

[24] For a date-0 shadow price $\lambda_0 \in [1, \lambda]$ more generally, the same analysis reveals that $\rho^* = \hat{\beta}\sigma + \frac{\lambda_0}{\lambda} b$ under TU and $\rho^* \in \left[\hat{\beta}\sigma, \hat{\beta}\sigma + \frac{\lambda_0}{\lambda} b\right]$ under NTU.

[25] So we here compare the cutoffs ρ rather than the liquidity support T, as the scale q need not be the same in both environments.

country's soil generates a future demand for reinvestment and therefore for the country's currency.

Appendix OG analyzes the two-way interaction between liquidity support and exchange rate appreciation, both for a time-consistent support policy and for a contingent liquidity commitment. We make our simple point through a real-exchange-rate model. Formally, we assume that consumers in each country k have identical preferences $E[c_0^{k,A}+c_0^{k,B}+u(c_1^{k,A},c_1^{k,B})+c_2^{k,A}+c_2^{k,B}]$, where $c_t^{k,m}$ is the consumption of country k of good produced in country m at date t. Suppose that investment in country k requires reinvestment in the goods produced in this country. Denote by ω the date-1 endowment of home goods in country k, which we assume are owned by consumers of country k. This endowment is non-storable. Also it cannot be pledged at date 0 to create date-0 stores of value (think of ω as date-1 labor income).

Then the exchange rate e of country A at date 1 is the relative price of good B vs. good A. With Cobb-Douglas preferences $u(c_1^{k,A},c_1^{k,B}) = 2(c_1^{k,A}c_1^{k,B})^{\frac{1}{2}}$, the date-1 exchange rate is given by equating the demand for good A with the supply of good A net of (lump-sum) taxes levied to finance the bank's continuation in both countries. The bank's geographic specialization in country A increases the demand for date-1 A-goods and appreciates the exchange rate. This appreciation in turn increases country A's opportunity cost of providing date-1 liquidity.

Proposition 9 *(currency appreciation). The country that attracts the most investment (the hegemon) sees its date-1 exchange rate appreciate when it follows its time-consistent support policy, and a fortiori when it engages in contingent liquidity commitment. This appreciation makes it more costly to provide liquidity and limits the liquidity support; in particular it dampens the benefit of a contingent liquidity commitment.*

The introduction of an exchange rate, combined with the exhorbitant duty studied previously, allows us to develop another interpretation of what was called "projects" or "clients" in the model. Prior to 2008, foreigners purchased a substantial amount of American assets and took on dollar liabilities; with the global crisis, they needed dollars to honor their liabilities. The swap lines granted by the Fed to other Central Banks allowed them to overcome the initial dollar shortage. In the framework of the model, one can replace "projects" by "US assets", which foreigners invested in at "date 0" (i.e. prior to 2008), creating a rent (S) for their issuers. At "date 1" (the financial crisis), US financial institutions would have been hurt by a fire sale of these assets. That gave the US a stake (β) in the continuation of activities on their soil (limited sales by foreigners). Furthermore, the US may well have gone beyond what they would have done in a time-consistent manner; their liquidity provision may have reflected the desire to keep a reputation for providing liquidity to the world in times of stress and therefore comfort a position as a hegemon. This obedience to the "exorbitant duty" may perhaps be illustrated by the rescue of AIG;[26] this rescue as well as the other bailouts underline the special capability of the US to operate large-scale liquidity provision.

[26]This was not the only motivation for the rescue. The US saved AIG also because they worried about a possible contagion as the world was falling apart (it was right after Lehman). The Congressional Oversight Panel (2010) estimated that about $62 billion of TARP money targeted to AIG went to European banks and other foreign institutions.

3.4 Multiple hegemon wannabes

Suppose now that $\lambda^A = \lambda^B = \lambda$, and so each country has the ability to attract investment on its soil through a commitment to future support. Each country competes à la Nash to attract the bank onto its territory. Assuming transferable utility (TU), country k's liquidity commitment offer is a vector $(T_0^k, T^k, q^k, \sigma^k)$, where T_0^k is a (positive or negative) date-0 transfer to the bank, and T^k, the date-1 liquidity support, without loss of generality satisfies the credibility constraint $R(T^k/q^k) \geq \hat{\beta}\sigma^k$. More formally, the timing goes as follows: (1) The two countries select their offers. (2) The bank either picks one of these offers or proceeds without commitment (and then receives utility U^{nc}). Letting the bank's, the home country's and the foreign country's utilities be denoted by U, V^h and V^f:

$$\begin{aligned} U(T_0, T, q, \sigma) &\equiv T_0 + F(R(T/q))bq - C(q, \sigma), \\ V^h(T_0, T, q, \sigma) &\equiv S(\sigma q) + \left[F(R(T/q))\beta\sigma - \lambda \int_0^{R(T/q)} \rho dF(\rho)\right]q - \lambda T_0, \end{aligned}$$

and for the foreign country (whose payoff is determined by the home country's policy (T^*, q^*, σ^*)).

$$V^f(T^*, q^*, \sigma^*) \equiv F(R(T^*/q^*))\beta(1-\sigma^*)q^*,$$

then a symmetric equilibrium $(T_0^*, T^*, q^*, \sigma^*)$ solves

$$\max_{\{T_0, T, q, \sigma\}} U(T_0, T, q, \sigma)$$

s.t.

$$V^h(T_0, T, q, \sigma) \geq V^f(T^*, q^*, \sigma^*)$$

and

$$U(T_0, T, q, \sigma) \geq U^{nc}.$$

The latter constraint, the bank's participation constraint, is non-binding. Furthermore, the contract between a country and the bank must be bilaterally efficient, and so must satisfy conditions (8) through (10). In the Bertrand-like outcome, each country is indifferent between engaging in liquidity commitments with the bank and so being the home country, and passing on this opportunity and being the foreign country.

Proposition 10 *(competition in liquidity commitments to attract the bank). Suppose that countries are symmetrical, and in particular have the same cost of funds λ. Their competition to attract the bank under TU delivers the same $\{T, q, \sigma\}$ as under an hegemon (where country A had cost of funds λ, and country B was drained of funds). The only difference relates to the date-0 transfer T_0, which is more favorable to the bank than under an hegemon.*

4 Banking competition

So far, we have looked at a single bank in isolation. Returning to the model of Section 2 ($\lambda^k = \lambda$ for all k and no liquidity commitment), we now investigate the consequences of our analysis for the intensity of banking competition. Consider I cross-border banks (indexed by subscripts i, j) and K countries (indexed by superscript k). We assume that there are more banks than countries: $I = nK$ with $n \geq 1$; we take n to be an integer to avoid technicalities. Later on, we state the (similar) results when there are more countries than banks. At date 0, banks compete à la Cournot. Let q_i^k denote the size of bank i in country k. Letting $q^k \equiv \Sigma_i q_i^k$, the unit cost of customer acquisition is $c(q^k)$ in country k, with again $c'(q^k) > 0$ and $q^k c''(q^k) + c'(q^k) > 0$ (which will guarantee that investments are strategic substitutes). As we said, the countries face the same shadow cost λ of public funds at date 1.

Let $\sigma_i^k \equiv q_i^k / q_i$ denote the relative presence of bank i in country k, and $\sigma_i \equiv \max_{k \in \{1,\ldots,K\}} \sigma_i^k$ denote its degree of specialization. As in Section 2 liquidity support is determined by σ_i: The cutoff for bank i is

$$\rho_i^* = \hat{\beta}\sigma_i.$$

The following proposition is proved in Appendix OH.

Proposition 11 *(banking competition). Consider an ex-ante symmetric oligopoly of I banks choosing their footprints in $K = I/n$ symmetric countries, where $n \geq 1$. In the absence of commitment,*

(i) Banks specialize and they do so in different countries.

(ii) Full specialization obtains for a wider range of parameters than in the single-bank case. More generally, an increase in banking competition (as captured by the number of banks divided by the number of countries) makes it more likely that the banks fully specialize.

(iii) More turbulent times lead to banking renationalization.

(iv) A Coasian bargain would increase competition by reducing specialization.

When countries are symmetrical, banks' home countries are spread evenly, as entry into a relatively unserved market is more profitable than entry into a competitive market. Banks also tend to specialize more when the banks-over-countries ratio increases. To understand why, take the case two countries. Under monopoly, a given level of specialization $\sigma > 1/2$ induces different marginal costs in the two countries, with a higher marginal cost in the home country. With two banks and the same degree of specialization, the marginal costs in the two countries $c\left(q^k\right)$ are equalized (although the marginal costs of acquisition $c(q^k) + q_i^k c'(q^k)$ are not - they are equalized only in a perfectly competitive market), as the banks select different home countries. So the gains from diversification are smaller, and champions become even more national. The intuition for parts (iii) and (iv) in the proposition is the same as in the single-bank case.

When there are fewer multinational banks than countries ($K = mI$, where $m \geq 1$ is an integer to avoid technicalities), some countries are not home countries for any bank. In equilibrium, banks do not enter each other's turf. Either they fully specialize, or they invest some amount in their home country and some smaller amount in each of the countries without home banks. Home-bank countries have the highest overall banking activity: See Appendix OH.

5 Ringfencing

Finally, we briefly investigate the impact of the home country's ability to reduce the leakage of its liquidity injection to the other country. Thus, our interest here lies with the implications of supervision on the allocation of liquidity within the cross-border bank. Assume that $\lambda^k = \lambda$ for both countries. We return to the focus on a single bank with the same shock distribution is both countries (for expositional simplicity only). We posit that the home country can retain more liquidity in the country than under fungibility:

Assumption (regulation and ringfencing). When the home country k regulates the bank (case on which we focus for expositional convenience), the fraction of its total liquidity T that is used by the bank in country k is equal to $s^k T$, where $s^k \equiv \sigma^k + \alpha(1-\sigma^k)$. Similarly, we define $s^l \equiv (1-\alpha)\sigma^l$ as the fraction of the liquidity that is employed in the bank's foreign country. The parameter α in $[0,1]$ is the ringfencing parameter; the case $\alpha = 0$ corresponds to perfect fungibility, the case $\alpha = 1$ describes perfect ringfencing.[27]

Suppose that the home country k (and only country k) brings liquidity to the bank; its ex-post utility is then $\beta F(\rho^k)q^k - \lambda T^k$, where $[\int_0^{\rho^k} \rho dF(\rho)]q^k = s^k T$. The threshold ρ^k in country k is then given by:

$$\rho^k = \hat{\beta} s^k.$$

The cutoff in the host country is then $\mu^l \le \rho^k$ where

$$\frac{s^k}{s^l} = \frac{\int_0^{\rho^k} \rho dF(\rho)}{\int_0^{\mu^l} \rho dF(\rho)} \frac{\sigma^k}{\sigma^l} \Longleftrightarrow 1 + \frac{\alpha}{(1-\alpha)\sigma^k} = \frac{\int_0^{\rho^k} \rho dF(\rho)}{\int_0^{\mu^l} \rho dF(\rho)}.$$

Because projects in the branch's host country l are sacrificed, that would cost little to rescue, we must check that country l does not want to supply liquidity as well. Only country k brings liquidity if and only if $\mu^l > \hat{\beta} s^l = \hat{\beta}(1-\alpha)\sigma^l$, or

$$1 + \frac{\alpha}{(1-\alpha)\sigma^k} \le \frac{\int_0^{\hat{\beta} s^k} \rho dF(\rho)}{\int_0^{\hat{\beta} s^l} \rho dF(\rho)}. \tag{11}$$

As ringfencing becomes more potent (α increases), the LHS of this inequality (which is equal $\left[s^k/\sigma^k\right]/\left[s^l/\sigma^l\right]$) increases, suggesting that the increased leakage makes country l less eager to bring liquidity; however the RHS also increases with α, as few projects continue in country l and so the marginal cost of rescuing projects in that country is small.

Example (power law distribution). Let $F(\rho) = \rho^v$ on $[0,1]$ with $v > 0$. Inequality (11) then writes (assuming interior solutions for the cutoffs):

$$\left(\frac{(1-\alpha)\sigma^l}{\sigma^k + \alpha\sigma^l}\right)^{v+1} \left[1 + \frac{\alpha}{(1-\alpha)\sigma^k}\right] \le 1.$$

[27] This formulation of ringfencing is only one of several assumptions with similar consequences that we could entertain. For example, the ringfencing parameter could apply only to the transfer made by country k and to revenues earned in country k.

This inequality is satisfied for any $\alpha \in [0,1]$ provided that $\sigma^k \geq 1/2$.

Assume that only country k (the home country, with regulates the bank) brings liquidity -as is indeed consistent with equilibrium for the power law distribution.

Observation. Ringfencing has two opposite consequences: First, it implies a *misallocation of the bank's resources* among the projects in the two countries; valuable projects are stopped in the host-country/cash-poor establishment (where "poor" is relative to presence), that would have been pursued had they been located in the home-country/cash-rich one. Second, ringfencing *alters liquidity support*; it unambiguously increases liquidity support if regulation takes place in the country with the highest bank presence.[28]

This first investigation only points at some costs and benefits of ringfencing. It says little about the difference between the subsidiary and branch forms in the foreign country. Presumably, if, taking advantage of the subsidiary form, the foreign country is able to engage in ringfencing, it is also more likely to provide liquidity. We conjecture that the foreign country still will not supply any liquidity if there is much specialization and ringfencing has limited efficiency (α is small enough). In contrast, if α is large, the foreign country is incentivized to supply liquidity as a) much of it will remain in the country, and b) the bank diversifies less, as access to liquidity no longer hinges on being a national champion (indeed for $\alpha = 1$, the bank fully diversifies).

6 Related literature

This paper connects to numerous literatures.

Trade. Relative to the trade literature, this paper offers a new driver of the choice of countries in which to be active, that does not result from comparative advantage (e.g., Costinot et al 2015), home-market effects associated with increasing returns in production and transportation costs (e.g., Krugman 1980), or informational limitations. And its theory of imperfect international diversification, unlike the international portfolio literature, does not rely on geopolitical risk.

Starting with Brander-Spencer (1985), Eaton-Grossman (1986), and Krugman (1986), a rich strategic-trade literature has analyzed how production subsidies, export tax-cum-subsidies, and other trade-impacting instruments can benefit (or to the contrary hurt) a country's domestic industry. It assumes that countries aim at offering a competitive advantage to their oligopolistic firms, or at softening the competition they face. In contrast, our work does not rest on imperfect competition, as its insights apply to the full range from monopoly to perfect competition. Furthermore, new insights on geographic specialization and on what makes a country a plausible host for one's industrial or banking activity (looking ahead, as we study the impact of future rather than current support) are derived even in the absence of policy commitment effects: See Sections 2 and 4. Finally, our focus is as much on the financial sector as on industry, and, when studying policy commitments, we point at important differences in the effectiveness of conditional and unconditional policies.

Hegemons. A large literature on dominant currencies is motivated inter alia by the observation

[28] By contrast, it would tend to reduce liquidity support if regulation takes place in the low-presence country, unless the establishments in the two countries have similar sizes.

that the dollar is still dominant today in international bond issuance, in the currency denomination of invoices (dollar invoicing is much larger than the share of the US in foreign trade: e.g. Amiti et al 2022), and in the choice of foreign exchange reserves. Complementarities between currency used for working capital and currency used for invoicing sales in Bahaj-Reis (2022), and strategic complementarities in pricing in Gopinath et al (2020) are drivers of such dominance. Our paper abstracts from the important nominal issues (the section on exchange rates indeed focused on real exchange rates) and looks at policies that are directly targeted at making the country attractive for banking investment.[29] A favorable business environment and a resilient economy is conducive to dominance as the country's ability to supply liquidity in difficult times is crucial for becoming an hegemon.

Farhi et al (2011) focus on the role of stores of value and argue that the United States may be facing a modern version of Triffin's dilemma, with a growing demand for U.S. issued safe assets but a limited fiscal capacity; they discuss ways to increase the global supply of safe assets by pooling resources, using swap lines more systematically or encouraging the emergence of alternative safe assets from large and fiscally sound economies. Farhi and Maggiori (2018) analyze the international demand for reserve assets and argue that a multipolar world may be more unstable than one with a hegemon.[30] More generally, Gourinchas and Rey (2022) argue that the exorbitant privilege of a country that lies at the center of the international monetary system comes with an exorbitant duty, and document that the US provides insurance to the rest of the world, especially in times of global stress.[31] Finally, a different dimension of hegemony relates to a country's ability to form coalitions and exert pressure on other countries to obtain economic or national security concessions. Clayton et al (2023), in the more informal tradition of Baldwin (1985) and Hirschman (1945), construct a trade framework in which an hegemon uses multi-market contact to obtain gains from (endogenously defined) enemies while rewarding (also endogenously defined) friends. The structure of input-output linkages and contract enforceability are central to their analysis.

Regulatory competition. Our paper naturally shares with the international trade and international finance literatures the existence of cross-border externalities. In particular, several papers were inspired by the observation that, starting in 1988, Basel Accords aim at reducing cross-border financial externalities by establishing a level-playing field in terms of capital and liquidity requirements. Morrison and White (2009) revisit the costs and benefits of a level-playing field.[32] Clayton and Schaab (2022) analyse regulatory externalities in a world of multi-

[29] There is a debate as to whether network externalities are sufficiently strong (and the cost of obtaining protection from future exchange rate changes sufficiently high) so as to predict that international currency status is a natural monopoly: see Eichengreen et al (2018) and the literature reviewed next.

[30] In their model, a reserve country issues a "safe" bond, for which there is a demand by risk-averse investors in the rest of the world. The country may later on default at a cost, possibly destroying the reserve value of the bond. A higher level of borrowing triggers a higher risk of (endogenous) default: this is Triffin's dilemma. A monopoly issuer may produce too much of the reserve asset: it tends to produce too little because of the standard monopoly distortion, but also may produce too much of the reserve asset in the absence of commitment.

[31] An analogy can be drawn between a commitment to supply liquidity and the traditional commitment by suppliers to supply an add-on at a low price in the future (e.g. Shepard 1987, Farrell-Gallini 1988).

[32] They assume that regulators in different countries are differentially apt at curbing bank risk taking. An inexperienced regulator must offset his lack of competency by asking for higher capital charges, even for healthy banks which would be safe even with lower capital requirements. When banks are mobile, talented-regulator

national banks, each with a domestic base and regulator. Multinational banks exert fire sale externalities in their various countries of operation. Clayton and Schaab show that countries' setting of quantity regulations, such as solvency and liquidity requirements, do not achieve the coordinated optimum, but that price-based regulation does as individual regulators adopt a Pigovian approach that directly addresses the externality at stake.

Countries' lack of internalization of externalities of cross-border banks led Schoenmaker (2013) to suggest the existence of a "financial trilemma"; any two of three following three objectives can be combined, but one has to give: (1) a stable financial system; (2) international banking; (3) national financial policies for supervision and resolution. Our paper also connects to the broader literature on the impediments of the free flow of capital. While Feldstein-Horioka (1980) and the contributions that followed it stressed impediments originating with the government or mere costs of doing business abroad (taxes, FX and regulatory risks, or transaction costs), we emphasize a novel channel that originates with the banks' desire to count on support in times of stress; relatedly, the government can create impediments through new routes (liquidity commitments, ring fencing).

Competition in banking. In a closed economy framework, the literature has highlighted other industrial organization trade-offs than those emphasized here. Most prominent here is the debate (reviewed in Vives 2011) concerning a possible trade-off between competition and financial stability. The standard "charter value" argument states that a bank has less incentive to take the risk of failure if it expects to be profitable in the future; a reduction in competition therefore reduces risk. This charter value argument suggests either adjusting regulation (more supervision, higher capital and liquidity requirements) when the bank is in a more competitive environment or loosening the competition policy rules. There are other effects too: First, high banking concentration may make the banks too-big-to-fail, inducing them to take on more risk. Second, for wholesale deposits, the higher interest rates generated by competition may exacerbate bank runs. We refer to Vives' paper for the policy implications of these theories.

Strategic choice of balance sheet. The paper belongs to the line of contributions emphasizing how public funds can be brought into play by banking choices.[33] These choices may be a shortage of liquid assets or an excess of short-term debt, creating the prospect of bankruptcy or fire sales in case of a subsequent macroeconomic shock. It may also involve an excessive investment in

countries exert negative externalities on untalented-regulator ones: Healthy banks want to migrate to the former (non-insured depositors still trust the banks located there despite low capital requirements), leaving an adversely selected sample to the latter. While a level-playing field eliminates this cherry-picking externality, it also penalizes countries with a talented regulator, as capital requirements must adjust to the lowest common denominator (i.e. high capital requirements, not the lowest common denominator envisioned in the Basel debate). Overall, the level-playing field has ambiguous welfare effects. While Morrison and White study regulatory competition among countries, in Carletti et al (2016)'s principal-agent problem, a centralized supervisor (e.g., the ECB) has full control rights over banks, but relies on local supervisors to collect the information necessary to act. Local supervisors are assumed to be less inclined to intervene relative to a centralized supervisor. Because the local supervisors do not control the use of information under a centralized structure, they collect less information than in a decentralized structure in which they would have the control rights. The centralization of supervision then has an ambiguous welfare impact despite the improvement in the objective function of the decision-maker.

[33] Examples of papers in this literature are Clayton-Schaab (2021, 2022), Farhi-Tirole (2012, 2018, 2021). Papers emphasizing fire sales externalities include Shleifer-Vishny (1992), Allen-Gale (1994), Lorenzoni (2008), Stein (2012), Kurlat (2016) and Stavrakeva (2020).

the domestic sovereign bond, generating a doom loop. Our paper differs from the literature in that it shows how a home bias emerges endogenously from the banks' desire to garner liquidity support in distress and looks at the consequences of this choice for public policy. The paper shows that another and so far neglected negative side effect of the too-big-to-fail conundrum is that it also perpetuates home bias.

Transmission of shocks through a cross-border banking system. Beyond the general literature on the cross-country propagation of shocks through the financial system (e.g. Giannetti-Laeven 2012), the banking literature points at a stronger propagation of shocks in the home country to the host country when parent banks operate through branches in the host country and they are short of tier-1 capital (Danisewicz et al 2017).

Contribution games. Liquidity support granted to a cross-border bank is in part a public good. As such, the paper is related to the sizeable literature on the non-cooperative provision of public goods. Within that literature, the most closely related paper is Compte-Jehiel (2003), which shows that an exogenous asymmetry in payoffs can substantially alleviate the free-riding problem by "designating" a natural provider of the public good. While the applications differ, a similar effect is at work here. Compte and Jehiel do not investigate the consequences for the endogenous emergence of asymmetries.

7 Summary and alleys for future research

This paper is a first foray into the two-way interaction between corporate investment and location decisions and countries' readiness to support their national champions in dire straits. It confirms the conventional wisdom that global firms are global in life and national in death. This gives them an incentive to have a strong presence in one country when international diversification would be cheaper. Such geographic specialization is more extensive (a) when product market competition is stronger, (b) when times are turbulent, and (c) when international risk sharing (say, through swap lines) is weaker.

We identified three determinants of a high presence of financial activity in a country: (a) room for fiscal and monetary manoeuver, (b) eagerness to rescue national champions, and (c) the distribution of shocks. While these determinants were studied in the context of time-consistent policies, they also bear implications for liquidity commitments to the extent that they are in part endogenous; for instance, a country may enhance its attractiveness to firms by building resilient public finances or by making its polity less hostile to finance (for example through funded pension schemes).

Liquidity commitments can alternatively manifest through an "exorbitant duty", namely an explicit promise for support beyond the time-consistent level. When unconditional, such commitments often backfire as they encourage a lower relative presence in the country. When conditional however, they always benefit the home country through either a higher presence or- when feasible- an ex-ante payment by the beneficiary of the liquidity commitment. Except perhaps for the equilibrium payment, the liquidity commitment allocation is the same whether there is a hegemon or competition among hegemon wannabes. Finally, a liquidity commitment comes with an exchange rate appreciation, a limiting factor for the policy.

A number of extensions seem desirable. In particular, while liquidity provision will always be part of the broader picture due to the limitations on alternative instruments,[34] its interaction with these instruments is worth studying. One, directly in line with the model, would be a relaxation of prudential standards, which would increase the risk of resorting to public funds. Conversely, the regulatory trend towards more bailinability (assuming it is enforced, which was not the case for Credit Suisse) should indirectly benefit cross-border diversification. Another is the issuance of government bonds that can serve as cheap stores of value as long as they remain safe (as in He et al 2019). In particular, American sovereign bonds are a handy instrument to hedge risk in a world in which many investments and transactions are in dollars. But the bond-issuing country then is faced with Triffin's dilemma. Finally, countries can try to attract activity onto their soil by subsidizing investment. This third alternative also faces limits in its effectiveness. As the literature has shown, capital subsidies do not obliterate the need for liquidity support. Finally, while we identified the continuation and competitive benefits of international policy coordination at a global or regional scale, we just scratched the surface. In view of the observed difficulties in reaching such agreements both prior to and during crises, we feel that this issue should be high on our research agenda.

References

ALLEN, F. and D. GALE (1994), "Limited Market Participation and Volatility of Asset Prices," *American Economic Review*, 84, 933–55.

AMITI, M., ITSKHOKI, O. and J. KONINGS (2022), "Dominant Currencies: How Firms Choose Currency Invoicing and Why it Matters," *Quarterly Journal of Economics*, 137 (3), 1435–1493.

BAHAJ, S. and R. REIS (2022), "Jumpstarting an International Currency," mimeo.

BALDWIN, D. (1985), *Economic Statecraft*, Princeton University Press.

BRANDER, J. and B. SPENCER (1985), "Export Subsidies and International Market Share Rivalry," *Journal of International Economics*, 18 (1-2), 83-100.

CARLETTI, E., DELL'ARICCIA, G. and R. MARQUEZ (2016), "Supervisory Incentives in a Banking Union," IMF Working Papers WP/16/186.

CERUTTI, E., DELL'ARICCIA, G. and S. MARTINEZ PERIA (2007), "How Banks Go Abroad: Branches or Subsidiaries?" *Journal of Banking and Finance*, 31, 1669–92.

[34]For example, ex-ante interventions such as protection or subsidies, which have been much studied in the literature, may be too indiscriminate, while ex-post ones are limited to a subset of firms, which allows more scrutiny and also economizes on liquidity provision. Liquidity requirements aim at reducing liquidity support and its cost, but have two drawbacks; the first, well-studied in the literature, comes from the scarcity of stores of value and therefore from the cost (low yields) of self-insurance against liquidity needs (besides, the literature has also emphasized that the government is the liquidity provider of last resort when a large shock hits the economy). More novel is the observation that hoarded liquidity can be employed by the bank to fund continuation in the various countries in which it operates, and therefore does not encourage the bank to specialize, making liquidity requirements even less attractive to a home country.

CERUTTI, E., ILYINA A., MAKAROVA, Y. and C. SCHMIEDER (2010), "Bankers Without Borders? Implications of Ring-Fencing for European Cross-Border Banks," International Monetary Fund, WP/10/247.

CLAYTON, C. and A. SCHAAB (2021), "Bail-Ins, Optimal Regulation, and Crisis Resolution," R & R, *Review of Financial Studies*.

CLAYTON, C. and A. SCHAAB (2022), "Multinational Banks and Financial Stability," *Quarterly Journal of Economics*, 137 (3), 1681–1736.

CLAYTON, C., MAGGIORI, M. and J. SCHREGER (2023), "A Framework for Geoeconomics," mimeo.

COEURDACIER, N. and H. REY (2013), "Home bias in Open Economy Financial Macroeconomics," *Journal of Economic Literature*, 51 (1), 63-115.

COMPTE, O. and P. JEHIEL (2003), "Voluntary Contributions to a Joint Project with Asymmetric Agents," *Journal of Economic Theory*, 112, 334–342.

Congressional Oversight Panel (2010), *The Global Context and International Effects of the TARP*, Washington.

CORBAE, D. and P. D'ERASMO (2021), "Capital Buffers in a Quantitative Model of Banking Industry Dynamics," *Econometrica*, 89 (6), 2975–3023.

CORREA, R., PALIGOROVA, T., SAPRIZA, H., and A. ZLATE (2022), "Cross-border Bank Flows and Monetary policy," *Review of Financial Studies*, 35, 438–481.

CORSETTI, G., GUIMARAES, B. and N. ROUBINI (2006), "International Lending of Last Resort and Moral Hazard: a Model of IMF's Catalytic Finance," *Journal of Monetary Economics*, 53, 441–471.

COSTINOT, A., DONALDSON, D., VOGEL, J. and I. WERNING (2015), "Comparative Advantage and Optimal Trade Policy," *Quarterly Journal of Economics*, 130 (2), 659–702.

DIAMOND, D. (1984), "Financial Intermediation and Delegated Monitoring," *Review of Economic Studies*, 51 (3), 393-414.

EATON, J., and G. GROSSMAN (1986), "Optimal Trade and Industrial Policy Under Oligopoly," *Quarterly Journal of Economics*, 101 (2), 383–406.

EICHENGREEN, B., MEHL, A. and L. CHITU (2018), *How Global Currencies Work: Past, Present and Future*, Princeton University Press.

FARHI, E., GOURINCHAS, P-O. and H. REY (2011), *Reforming the International Monetary System*, Center for Economic Policy Research, London.

FARHI, E. and M. MAGGIORI (2018), "A Model of the International Monetary System," *Quarterly Journal of Economics*, 133 (1), 295–355.

FARHI, E. and J. TIROLE (2012), "Collective Moral Hazard, Maturity Mismatch and Systemic Bailouts," *American Economic Review*, 102, 60–93.

FARHI, E. and J. TIROLE (2018), "Deadly Embrace: Sovereign and Financial Balance Sheets Doom Loops," *The Review of Economic Studies*, 85 (3), 1781–1823.

FARHI, E. and J. TIROLE (2021), "Shadow Banking and the Four Pillars of Traditional Financial Intermediation," *The Review of Economic Studies*, 88 (6), 2622–2653.

FARREL, J. and N. GALLINI (1988), "Second-Sourcing as a Commitment: Monopoly Incentives to Attract Competition," *Quarterly Journal of Economics*, 103, 673–694.

FELDSTEIN, M. and C. HORIOKA (1980), "Domestic Saving and International Capital Flows," *Economic Journal*, 90 (358), 314–329.

GIANNETTI, M. and L. LAEVEN (2012), "The Flight Home Effect: Evidence from the Syndicated Loan Market during Financial Crises," *Journal of Financial Economics*, 104, 23–43.

GOPINATH, G. and J. STEIN (2021), "Banking, Trade, and the Making of a Dominant Currency," *Quarterly Journal of Economics*, 136 (2), 783–830.

GOPINATH, G., BOZ, E., CASAS, C., DIEZ, F., GOURINCHAS, P-O. and M. PLAGBORG-MØLLER (2020), "Dominant Currency Paradigm," *American Economic Review*, 110 (3), 677–719.

GOURINCHAS, P.O. and H. REY (2022), "Exorbitant Privilege and Exorbitant Duty," CEPR Discussion Paper No. DP16944.

GRANJA, J., LEUZ, C. and R. RAJAN (2022), "Going the Extra Mile: Distant Lending and Credit Cycles," *Journal of Finance*, 77, 1259–1324.

HE, Z., KRISHNAMURTHY, A. and K. MILBRADT (2019), "A Model of Safe Asset Determination," *American Economic Review*, 109 (4), 1230–62.

HIRSCHMAN, A. (1945), *National Power and the Structure of Foreign Trade*, Vol. 105, University of California Press.

HOLMSTRÖM, B. and J. TIROLE (1998), "Private and Public Supply of Liquidity," *Journal of Political Economy*, 106 (1), 1–40.

KRUGMAN, P. (1980), "Scale Economies, Product Differentiation, and the Pattern of Trade," *American Economic Review*, 70 (5), 950–959.

KRUGMAN, P. (1986), *Strategic Trade Policy and the New International Economics*, MIT Press.

KURLAT, P. (2016), "Asset Markets with Heterogeneous Information," *Econometrica*, 84 (1), 33–85.

LORENZONI, G. (2008), "Inefficient Credit Booms," *Review of Economic Studies*, 75, 809–833.

MATSCHKE, J. (2021), "National Interests, Spillovers and Macroprudential Coordination," Federal Reserve Bank of Kansas City, Working Paper 21-13.

MORRISON, A. and L. WHITE (2009), "Level Playing Fields in International Financial Regulation," *Journal of Finance*, 64, 1099–1142.

ROCHET, J.C. and X. VIVES (2004), "Coordination Failures and the Lender of Last Resort: Was Bagehot Right after All?" *Journal of the European Economic Association*, 2, 1116–1147.

SCHOENMAKER, D. (2015), "The New Banking Union Landscape in Europe: Consolidation Ahead?" *The Journal of Financial Perspectives*, 3 (2), 189–201.

SCHOENMAKER, D. (2018), "Resolution of International Banks: Can Smaller Countries Cope?"

International Finance, 21 (1), 39–54.

SCHOENMAKER, D. and W.B. WAGNER (2013), "Cross-border Banking in Europe and Financial Stability," *International Finance*, 16 (1), 1–22.

SHEPARD, A. (1987), "Licensing to Enhance Demand for New Technologies," *Rand Journal of Economics*, 18, 360–368.

SHLEIFER, A. and R. VISHNY (1992), "Liquidation Values and Debt Capacity: A Market Equilibrium," *Journal of Finance*, 47, 1343–1365.

STAVRAKEVA, V. (2020), "Optimal Bank Regulation and Fiscal Capacity," *Review of Economic Studies*, 87 (2), 1034–1089.

STEIN, J. (2012), "Monetary Policy as Financial Stability Regulation," *Quarterly Journal of Economics*, 127, 57–95.

TIROLE, J. (2006), *The Theory of Corporate Finance*, Princeton University Press.

TIROLE, J. (2015), "Country Solidarity in Sovereign Crises," *American Economic Review*, 105 (8), 2333–2363.

TOOZE, A. (2018), "The Forgotten History of the Financial Crisis," *Foreign Affairs*, September/October, 199–210.

VALIANTE, D. (2016), "Europe's Untapped Capital Market," *Vox EU*, March 13.

VIVES, X. (2011), "Competition Policy in Banking," *Oxford Review of Economic Policy*, 27, 479–497.

VIVES, X. (2016), *Competition and Stability in Banking: The Role of Regulation and Competition Policy*, Princeton University Press

YU, L. and P. WANGNER (2023), "Monetary Policy and Global Bank Lending: A Reversal Interest Rate Approach", mimeo.

Appendix

A Proof of Proposition 2 (benefit from being the home country)

For $\sigma = 1/2$, one can see that $V^h - V^f = -\lambda\left[\int_0^{\hat{\beta}/2} \rho dF(\rho)\right]q < 0$. More generally, ignoring the positive rent differential term $S(\sigma q) - S((1-\sigma)q)$, a sufficient condition for the home country to be better off in relative terms is:

$$A \equiv F(\hat{\beta}\sigma)\beta(2\sigma - 1) - \lambda \int_0^{\hat{\beta}\sigma} \rho dF(\rho) > 0 \ \Rightarrow \ V^h > V^f.$$

From the concavity of $F(\cdot)$:

$$\frac{\int_0^{\hat{\beta}\sigma} \rho dF(\rho)}{F(\hat{\beta}\sigma)} = \mathbb{E}[\rho \mid \rho \leq \hat{\beta}\sigma] \leq \frac{\hat{\beta}\sigma}{2} \Longrightarrow A \geq F(\hat{\beta}\sigma)\beta(\frac{3}{2}\sigma - 1).$$

Since $F(\cdot)$ is concave, the elasticity is less than or equal to 1. Thus, $\partial A/\partial\sigma \geq 0$. Therefore,

$$\frac{\partial}{\partial\sigma}(V^h - V^f) = q\Big[\frac{\partial A}{\partial\sigma} + S'(\sigma q) + S'((1-\sigma)q)\Big] > 0.$$

■

B Proof of Proposition 3 (capital controls)

(i) Suppose that $V^h \geq V^f$. First, note that if country B imposes a capital control in $[q^A, q^B)$, the bank is de facto unconstrained and the roles are reversed; and so country A becomes the home country with investment q^B and country B becomes the foreign country with investment q^A; because $V^h \geq V^f$, country B does not gain from doing this. Country B choosing a capital control in $[0, q^A)$ inflicts an even worse loss on country B, using the forthcoming characterization showing that the foreign country does not benefit from imposing a capital control (except to prevent role switching).

So, let the binding constraint be $q^A \leq \bar{q}^A$. Note that $\bar{q}^A \geq q^B$ does nothing to change the banks' behavior. When it is binding ($q^A = \bar{q}^A$), the bank solves over q^B or equivalently $\sigma = q^B/(q^B + q^A)$:

$$\max_{\sigma}\Big[F(\hat{\beta}\sigma)\frac{q^A}{1-\sigma}\Big]b - c(\frac{\sigma q^A}{1-\sigma})\frac{\sigma q^A}{1-\sigma} + c(q^A)q^A,$$

which results in:

$$\Big[(1-\sigma)\hat{\beta}f(\hat{\beta}\sigma) + F(\hat{\beta}\sigma)\Big]b = \hat{c}\Big(\frac{\sigma q^A}{1-\sigma}\Big). \tag{A1}$$

From here one can see that the left-hand side is weakly decreasing in σ (as $f' \leq 0$), while $\frac{1}{1-\sigma}$ is increasing in σ. Therefore, σq^A decreases as σ increases, which means that the elasticity of σ with respect to q^A, is greater than -1:

$$\frac{d}{d\sigma}(\sigma q^A) = q^A + \sigma\frac{dq^A}{d\sigma} \leq 0 \Longrightarrow \frac{q^A}{\sigma}\frac{d\sigma}{dq^A} \geq -1.$$

On the other hand, the utility of country A is:

$$U^A(\sigma, q^A) = F(\hat{\beta}\sigma)\beta q^A + S(q^A),$$

and so

$$\begin{aligned}\frac{d}{dq^A}U^A &= \frac{d\sigma}{dq^A}\hat{\beta}f(\hat{\beta}\sigma)\beta q^A + F(\hat{\beta}\sigma)\beta + S'(q^A) \\ &= F(\hat{\beta}\sigma)\beta\Big[1 + \frac{q^A}{\sigma}\frac{d\sigma}{dq^A}\cdot\frac{\hat{\beta}\sigma f(\hat{\beta}\sigma)}{F(\hat{\beta}\sigma)}\Big] + S'(q^A) \geq 0.\end{aligned}$$

Increasing q^A benefits directly country A, but it might reduce the size of the bailout granted by country B. This result nonetheless shows that the direct effect dominates.

(ii) Suppose now that $V^h < V^f$. Provided that country B imposes no capital control, country A incurs no cost in setting $\bar{q}^A = q^A = (1-\sigma^{nc})q^{nc}$ (setting $\bar{q}^A > q^A$ does not alter the banks' behavior and $\bar{q}^A < q^A$ is dominated by $\bar{q}^A = q^A$ by the result obtained in part (i)). Can country B make more than V^h? If it sets control $\bar{q}^B \in [q^A, q^B)$, then it remains the home country, and, from the envelope theorem, reduces its welfare. From the characterization in (i), the best choice of B when the foreign country in $[0, q^A]$ is q^A. This yields B utility

$$V^{ff} \equiv F(\hat{\beta}/2)\beta q^A + S(q^A) = V^f - \left[F(\hat{\beta}\sigma^{nc}) - F(\hat{\beta}/2)\right]\beta q^A.$$

So, if $V^{ff} \leq V^h$, country A does not want to select $q^A - \varepsilon$ (or a fortiori a smaller cap) and selects q^B. So, country B does not gain from becoming the foreign country either. ■

C Proof of Proposition 7 (unconditional liquidity support)

Low commitment: Suppose that, in equilibrium, the bank chooses $\{q, \sigma\}$ such that $T \leq \left[\int_0^{\hat{\beta}\sigma} \rho dF(\rho)\right] q$. Then $\rho^* = \hat{\beta}\sigma$ and the corresponding allocation, which therefore maximizes $F(\hat{\beta}\sigma)bq - C(q, \sigma)$, could be equally achieved in the absence of commitment ($T = 0$).

High commitment: Assume, next, that, in equilibrium, $T > \left[\int_0^{\hat{\beta}\sigma} \rho dF(\rho)\right] q$. Let $R(T/q) > \hat{\beta}\sigma$ be defined by

$$\int_0^{R(T/q)} \rho dF(\rho) \equiv \frac{T}{q}.$$

The bank's optimum then solves

$$\max_{\{q,\sigma\}} F(R(T/q))bq - C(q, \sigma)$$

and so

$$\sigma = \frac{1}{2}.$$

In words, the country's commitment is either irrelevant (low commitment) or induces a diversification that is unwanted by country A (high commitment). Because the concepts of "low commitment" and "high commitment" are endogenous, let us look at which prevails. The difference of the bank's utility between diversification and specialization is

$$\Delta(T) \equiv \max_{\{q\}} \left\{F\left(R\left(\frac{T}{q}\right)\right)q - C\left(q, \frac{1}{2}\right)\right\} - \left[F(\hat{\beta}\sigma^{nc})bq^{nc} - C(q^{nc}, \sigma^{nc})\right].$$

Let T^{nc} denote the level of liquidity support in the absence of commitment ($R(\frac{T^{nc}}{q^{nc}}) = \hat{\beta}\sigma^{nc}$). Because $q = q^{nc}$ is an option for the bank when it diversifies

$$\Delta(T^{nc}) \geq C(q^{nc}, \sigma^{nc}) - C\left(q^{nc}, \frac{1}{2}\right) > 0.$$

Because Δ is increasing in T and $\Delta(0) < 0$, there exists $T_1 < T^{nc}$ such that the bank diversifies iff $T \geq T_1$. ■

D Proof of Proposition 8 (liquidity commitment under NTU and TU)

Let us investigate the possibility of a counterfavor by the bank to the hegemon's increased liquidity provision.

Under NTU, this amounts to finding $\{q, \sigma, \rho^*\}$ such that

$$V^h = S(\sigma q) + \left[F(\rho^*)\beta\sigma - \lambda \int_0^{\rho^*} \rho dF(\rho)\right] q > V^{nc} = S(\sigma^{nc} q^{nc}) + \left[F(\hat{\beta}\sigma^{nc})\beta\sigma^{nc} - \lambda \int_0^{\hat{\beta}\sigma^{nc}} \rho dF(\rho)\right] q^{nc}$$

and

$$U = F(\rho^*) bq - C(q, \sigma) \geq U^{nc} = F(\hat{\beta}\sigma^{nc}) bq^{nc} - C(q^{nc}, \hat{\beta}\sigma^{nc}).$$

Considering the maximization of V^k and letting μ denote the shadow cost of the constraint, the FOC with respect to ρ^* is $\rho^* = \hat{\beta}\sigma + \frac{\mu}{\lambda} b$. Thus, more liquidity is supplied than in the absence of commitment.[35] Finally, $\mu \leq \lambda$ results from the fact that the hegemon is constrained by the bank's inability to transfer money at date 0 (NTU).

Under TU, the shadow price of the participation constraint is $\mu = \lambda$. To show that a liquidity commitment need not increase geographic specialization, suppose that the acquisition cost is quadratic ($c(q^k) = q^k$), that $F(\rho)$ is uniform on $[0, 1]$ and that the scale of operation is fixed at capacity $\bar{q} = 1$ for notational simplicity.

We know that for uniform distribution $\sigma^{nc} = 1$. So we only need to prove that there is not necessarily full specialization under a liquidity commitment. Using equations (8) and (9), we obtain

$$C_\sigma(1, \sigma) = F(\hat{\beta}\sigma + b)\hat{\beta} + \hat{S}'(\sigma).$$

Under quadratic acquisition cost, $C(1, \sigma) = [\sigma^2 + (1 - \sigma)^2]$ and $\hat{S}(\sigma) = \sigma^2/2\lambda$. Finally, for a uniform distribution $F(\rho) = \rho$. And so

$$2(2\sigma - 1) = (\hat{\beta}\sigma + b)\hat{\beta} + \frac{\sigma}{\lambda}.$$

So if $2 > (\hat{\beta} + b)\hat{\beta} + (1/\lambda)$, that is if $\hat{\beta}$ is small enough, $\sigma < 1$. ■

[35] To obtain some intuition, let us contemplate a small deviation $(dq, d\sigma, d\rho^*)$ from the no-commitment outcome along the bank's indifference curve. Provided that $d\rho^* = \hat{\beta} d\sigma$, the bank's optimization with respect to q and σ in the no-commitment case implies that the loss dU is second-order in dq and $d\sigma$. In contrast, the hegemon's gain is first-order if $V_q^h dq + V_\sigma^h d\sigma > 0$ (using the envelope theorem). So the hegemon can compensate for the bank's second-order loss (by increasing T slightly above $\hat{\beta} d\sigma$), and thus a Pareto-improvement can be achieved. If the cost C must meet a budget constraint, then one must pick $\{dq, d\sigma\}$ such that $V_q^h dq + V_\sigma^h d\sigma > 0$ and $C_q dq + C_\sigma d\sigma = 0$ (where V_q, V_σ, C_q, C_σ are all positive). The vectors of partial derivatives are generically non-colinear and so such a vector $\{dq, d\sigma\}$ can be found.

TOO DOMESTIC TO FAIL
LIQUIDITY PROVISION AND NATIONAL CHAMPIONS
ONLINE APPENDIX

Emmanuel Farhi and Jean Tirole

February 15, 2024

OA Tail property of the density $f(\rho)$

Suppose that $f(\rho)\rho$ does not converge to 0 as ρ goes to $+\infty$. Then $\exists\epsilon > 0$ *s.t.* $\forall\underline{\rho} : \exists\hat{\rho} \geq \underline{\rho}$: $f(\hat{\rho})\hat{\rho} \geq \epsilon$. Consider a sequence $\{\hat{\rho}_n\}_{n\in\mathbb{N}}$ satisfying $f(\hat{\rho}_n)\hat{\rho}_n \geq \epsilon$ such that $\hat{\rho}_n \geq 2\hat{\rho}_{n-1}$. Denote $\hat{\rho}_0 = 0$. Therefore,

$$\int_0^\infty f(\rho)d\rho = \sum_{n=1}^\infty \int_{\hat{\rho}_{n-1}}^{\hat{\rho}_n} f(\rho)d\rho \geq \sum_{n=1}^\infty (\hat{\rho}_n - \hat{\rho}_{n-1}) f(\hat{\rho}_n) \geq \sum_{n=1}^\infty (\hat{\rho}_n - \hat{\rho}_{n-1}) \frac{\epsilon}{\hat{\rho}_n} \epsilon \sum_{n=1}^\infty (1 - \frac{\hat{\rho}_{n-1}}{\hat{\rho}_n}) \geq \epsilon \sum_{n=1}^\infty \frac{1}{2} = \infty,$$

a contradiction. ■

OB Formal equivalence with industry support

Suppose a firm sets capacity q^k in country k at date 0. Let $X^k \in \mathbb{R}^+$ denote the scale expansion ($X^k \geq 1$) or reduction ($X^k \leq 1$) at date 1. Assume that the unit cost at date 1 is $L(X^k)$ with $L' > 0$ and $L'' > 0$. The liquidity constraint at date 1 is

$$\Sigma_k L(X^k)q^k \leq T^A + T^B = T.$$

The costs and benefits are otherwise as in the banking model. The unit cost at date 1, $c(q^k)$, and the surplus, $S(q^k)$, satisfy the properties stated in Section 2.1. The date-2 benefits are $[\Sigma_k X^k q^k]b$ for the firm and $[X^k q^k]\beta$ for country k.

The formal equivalence between the two models goes as follows: Let $X \equiv F(\rho^*)$ and $L(X) = \int_0^{\rho^*} \rho dF(\rho)$. And so $L'(X) = F^{-1}(X)$ and $L''(X) = \frac{1}{f(F^{-1}(X))}$.

The counterpart of condition (2) is given by

$$\max\left\{X(\Sigma_\ell T^\ell)q^k\beta - \lambda^k T^k\right\}, \quad \text{yielding } \; L'(X) = \hat{\beta}\sigma.$$

OC Asymmetric distributions of shocks

Suppose that the bank's distribution of shocks is country contingent (but countries are otherwise symmetrical). Let $F^k(\rho)$ denote the distribution in country k, with density $f^k(\rho)$. Generalizing the previous analysis, we have

$$\frac{1}{\lambda}\frac{\partial(\beta\kappa^k - \lambda T^k)}{\partial T^k} = \frac{\hat{\beta}}{\rho^*}\frac{f^k(\rho^*)q^k}{\sum_m f^m(\rho^*)q^m} - 1,$$

where the cutoff is given by:

$$\sum_m \int_0^{\rho^*} \rho q^m dF^m(\rho) = T.$$

A country's incentive to rescue the bank now depends not only on the relative presence of the bank in the country, but also on the relative densities around the cutoff shock (on the likelihood ratio $f^k(\rho^*)/f^l(\rho^*)$): It may now be the case that liquidity support be brought (solely) by country l even though $\sigma^l < 1/2$. All depends on where in the shock distribution the cutoff lies.

Intuitively, the bank wants to specialize in the country in which the rescue makes a big difference for the country at the margin, i.e. with a comparatively high density of shocks at the margin (i.e., country B if the likelihood ratio satisfies $f^B(\rho^*)/f^A(\rho^*) > 1$ at the cutoff ρ^*). However, inframarginal leakage may invalidate this conclusion, as illustrated in Figure 1 in the text: While country B (the high-shocks country) benefits more from 1 unit of extra liquidity at $\rho = \rho^*$, country A benefits more from liquidity provision overall and may actually be the rescuing country. Proposition 12 shows however that it is still often the case that a single country supplies liquidity. For tractability, we here specialize to exponential distributions of shocks, with country A being the low-shocks country: For some $\delta > 0$,

$$F^A(\rho) = 1 - e^{-\rho} \text{ on } [0, +\infty) \text{ and } F^B(\rho) = 1 - e^{-(\rho-\delta)} \text{ on } [\delta, +\infty).$$

Letting $\tilde{\sigma}^A \equiv \frac{\sigma^A}{\sigma^A + e^\delta \sigma^B}$ and $\tilde{\sigma}^B \equiv 1 - \tilde{\sigma}^A$ denote the "modified presences", country A benefits more than country B from marginal liquidity when $\sigma^* > \delta$ if and only if $\tilde{\sigma}^A > \tilde{\sigma}^B$ or $\sigma^A > e^\delta \sigma^B$.

Proposition 12 *(differential liquidity shocks). Suppose exponential shock distributions with country A the low-shocks country. Country k's willingness to bring liquidity support at the margin hinges on the product of two leakage-related coefficients: the bank's relative presence in the country, σ^k, and the likelihood ratio $f^k(\rho^*)/f^l(\rho^*)$, which measures the relative fraction of projects that will be supported at the margin in country k. There may be a region where the high-shocks country benefits more from liquidity support at the margin, and yet the low-shocks country supplies the liquidity. More specifically, there exists a unique equilibrium, satisfying:*

- *The low-shocks country, country A, supplies the entire liquidity if either $\hat{\beta}\sigma^B \le \delta$ (country B would not benefit from providing liquidity alone) or $\tilde{\sigma}^A \ge \tilde{\sigma}^B$ (country A has a higher stake at the margin when $\rho^* > \delta$).*
- *The high-shocks country, country B, supplies the entire liquidity if $\tilde{\sigma}^A < \tilde{\sigma}^B$ and $\hat{\beta} \ge \hat{\beta}^\dagger$ for some $\hat{\beta}^\dagger$.*
- *Otherwise, country A supplies some deterministic liquidity T^A and country B randomizes between not supplying any liquidity and topping up with liquidity T^B to bring the cutoff ρ^* to $\hat{\beta}\tilde{\sigma}^B$.*

The choice of specialization with asymmetric shock distributions is complex in general. One can however obtain a few partial results: a) for low willingnesses to support (technically: for $\hat{\beta} \le \delta$), the bank specializes fully in country A (that is $\sigma^A = 1$), as shocks in country B will never be covered; b) for high willingnesses to support ($\hat{\beta} \to +\infty$), the bank gets its liquidity from country B and $\tilde{\sigma}^B > 1/2$.

Proof of Proposition 12. Let $\tilde{\sigma}^A \equiv \frac{\sigma^A}{\sigma^A + e^\delta \sigma^B}$ and $\tilde{\sigma}^B \equiv 1 - \tilde{\sigma}^A$. The variables $\tilde{\sigma}_A$ and $\tilde{\sigma}_B$ are the density-adjusted shares in countries A and B. For exponential distributions, the FOC are:

$$\frac{1}{\lambda}\frac{\partial}{\partial T^A}(\beta\kappa^A - \lambda T^A) = \frac{\hat{\beta}\tilde{\sigma}^A}{\rho^*} - 1$$

and

$$\frac{1}{\lambda}\frac{\partial}{\partial T^B}(\beta\kappa^B - \lambda T^B) = \frac{\hat{\beta}\tilde{\sigma}^B}{\rho^*} - 1 \quad \text{(for } \rho^* \geq \delta; = -1 \text{ for } \rho^* < \delta).$$

As anticipated and depicted in Figure 1, these first-order conditions imply that at the margin the "bang-for-the-buck" is higher in the high-shocks country, country B, whenever $\rho^* \geq \delta$.[1] But this ignores the individual rationality constraint for country B; suppose country B (and not country A) supplies the liquidity. Then country B must prefer doing so rather than not supplying any liquidity:

$$\beta F^B(\rho^*)\sigma^B q \geq \lambda\left[\sum_k \sigma^k \int_0^{\rho^*} \rho dF^k(\rho)\right] q \tag{OA1}$$

where $\rho^* \equiv \hat{\beta}\tilde{\sigma}^B$. That (OA1) need not be satisfied is also illustrated in Figure 1. There, $\hat{\beta}$ is chosen such that ρ^* barely exceeds δ. While marginal liquidity goes mainly to country B, almost no inframarginal liquidity does. So the LHS of (OA1) is close to 0, which is not the case for the RHS.

One must thus distinguish between the marginal leakage for country k (raising the cutoff from ρ^* to $\rho^* + d\rho^*$ involves the same marginal cost for both countries, and a marginal benefit $\beta(\sigma^k q f^k(\rho^*) d\rho^*)$ and the inframarginal leakage (the total benefit is $\beta(\sigma^k q F^k(\rho^*)))$.

(a) *Only country A provides liquidity*

If country A provides the liquidity,

$$\rho^* = \begin{cases} \hat{\beta} \text{ if } \hat{\beta} \leq \delta \\ \delta \text{ if } \delta \leq \hat{\beta} \leq \delta/\tilde{\sigma}^A \\ \hat{\beta}\tilde{\sigma}^A \text{ if } \hat{\beta} \geq \delta/\tilde{\sigma}^A \end{cases}$$

To see this, note that country A's incentive per unit of output (normalized by the shadow cost of funds) is at most $\hat{\beta}$, the incentive that prevails when all the liquidity serves to sustain projects in country A. This is indeed the case if $\hat{\beta} = \rho^* \leq \delta$. For slightly higher $\hat{\beta}$, the liquidity is shared, actually in the favor of country B at the margin. And so ρ^* remains at δ until $\hat{\beta} \geq \delta/\tilde{\sigma}^A$, at which point country A is willing to provide more liquidity even though it mostly benefits country B at the margin.

[1] More formally, fix some $\{q, \sigma \geq 1/2\}$. Then $\hat{\beta}\sigma/[\sigma + e^\delta(1-\sigma)] < \hat{\beta}e^\delta\sigma/[e^\delta\sigma + (1-\sigma)]$, and so there more continuation if the bank specializes in country B than if it specializes in country A.

A necessary condition for the absence of top-up by country B in the region in which $\rho^* \geq \delta$ is given by the FOCs:

$$\tilde{\sigma}^A \geq \tilde{\sigma}^B.$$

Conversely, if $\tilde{\sigma}^A \geq \tilde{\sigma}^B$, it is easy to check that the IR constraint for country A,

$$\hat{\beta} F^A(\rho^*)\sigma^A \geq \Sigma_k \sigma^k \int_0^{\rho^*} dF^k(\rho),$$

is satisfied: the country's objective function is concave and the cutoff ρ^* is optimal for country A.

So country A supplies the entire liquidity if and only if either $\hat{\beta}\sigma^B \leq \delta$ or $\{\hat{\beta}\sigma^B > \delta$ and $\tilde{\sigma}^A \geq \tilde{\sigma}^B\}$

(b) *Only country B provides liquidity*

For country B to exclusively supply the liquidity it must be the case that

$$\tilde{\sigma}^B > \tilde{\sigma}^A,$$

Using the envelope theorem, (OA1) is satisfied iff

$$\hat{\beta} \geq \hat{\beta}^\dagger$$

for some $\hat{\beta}^\dagger < +\infty$ (since the difference between the LHS and the RHS goes to $+\infty$ as $\hat{\beta}$ goes to $+\infty$).

(c) *Both may provide liquidity*

Let us show that for $\tilde{\sigma}^B > \tilde{\sigma}^A$ but $\hat{\beta} < \hat{\beta}^\dagger$, there is a unique equilibrium . In this equilibrium, country A supplies T^A for sure and country B mixes between $T^B = 0$ and a top up of T^A, T^B, that increases ρ^* from some $\tilde{\rho} < \delta$ to $\hat{\beta}\tilde{\sigma}^B$.

Given the (mixed) strategy of country B, the country A, solves:

$$\max_{T^K} \mathbb{E}_{T^B}\Big[\hat{\beta} q \sigma^A F^A(\rho^*) - T^A\Big], \text{ where}$$

$$q \int_0^{\rho^*} \rho\Big(\sigma^A f^A(\rho) + \sigma^B f^B(\rho)\Big) d\rho = T^A + T^B.$$

The FOC is:

$$\mathbb{E}_{T^B}\left[\frac{1}{h(\rho^*)}\right] = \frac{1}{\hat{\beta}},$$

where

$$h(\rho^*) = \begin{cases} \rho^* & \text{if } \rho^* < \delta \\ \frac{\rho^*}{\tilde{\sigma}^A} & \text{if } \rho^* > \delta \end{cases}$$

Note that $h(\rho^*)$ is strictly increasing in ρ^* and ρ^* is strictly increasing in T^A. Thus, regardless of the distribution of T^B, the left-hand side of the FOC is decreasing in T^A and has a unique solution. As a result, in any equilibrium, country A plays a pure strategy. This similarly holds for country B, with the difference that it may be indifferent between $T^B = 0$ or:

$$T^B = -T^A + \int_0^{\hat{\beta}\tilde{\sigma}^B} \rho\Big(\sigma^A f^A(\rho) + \sigma^B f^B(\rho)\Big) d\rho.$$

Therefore, the only non-pure equilibrium is when country B mixes between the above T^B with probability p and $T^B = 0$ with probability $1-p$.

Denote by $\tilde{\rho}$ as the cutoff ρ^* induced by T^A alone:

$$T^A = \int_0^{\tilde{\rho}} \rho\Big(\sigma^A f^A(\rho) + \sigma^B f^B(\rho)\Big) d\rho.$$

Country B is indifferent iff:

$$\begin{aligned}
\hat{\beta}\sigma^B F^B(\tilde{\rho}) &= \hat{\beta}\sigma^B F^B(\hat{\beta}\tilde{\sigma}^B) - T^B \\
&= \hat{\beta}\sigma^B F^B(\hat{\beta}\tilde{\sigma}^B) + \int_0^{\tilde{\rho}} \rho\Big(\sigma^A f^A(\rho) + \sigma^B f^B(\rho)\Big) d\rho - \int_0^{\hat{\beta}\tilde{\sigma}^B} \rho\Big(\sigma^A f^A(\rho) + \sigma^B f^B(\rho)\Big) d\rho.
\end{aligned}$$

If $\tilde{\rho} > \delta$, the only solution for that is $\tilde{\rho} = \hat{\beta}\tilde{\sigma}^B$, a contradiction. Therefore, country B could be indifferent only when

$$\tilde{\rho} \le \delta \le \hat{\beta}\tilde{\sigma}^B \le \hat{\beta}.$$

Thus, the indifference condition is:

$$0 = \hat{\beta}\sigma^B F^B(\hat{\beta}\tilde{\sigma}^B) + \int_0^{\tilde{\rho}} \rho\sigma^A f^A(\rho) d\rho - \int_0^{\hat{\beta}\tilde{\sigma}^B} \rho\Big(\sigma^A f^A(\rho) + \sigma^B f^B(\rho)\Big) d\rho.$$

Simplifying this:

$$\hat{\beta}\tilde{\sigma}^B e^{-\delta} + e^{-\hat{\beta}\tilde{\sigma}^B} = \tilde{\sigma}^A(1+\tilde{\rho})e^{-\tilde{\rho}} + \tilde{\sigma}^B(1+\delta)e^{-\delta}.$$

The right-hand side is decreasing in $\tilde{\rho}$ and $(1+\tilde{\rho})e^{-\tilde{\rho}}$ goes from 1 to 0 as $\tilde{\rho}$ goes from 0 to ∞. Therefore, $\tilde{\rho}$ exists iff

$$\tilde{\sigma}^B(1+\delta)e^{-\delta} \le e^{-\hat{\beta}\tilde{\sigma}^B} + \hat{\beta}\tilde{\sigma}^B e^{-\delta} \le \tilde{\sigma}^A + \tilde{\sigma}^B(1+\delta)e^{-\delta},$$

where the second inequality is equivalent to $\hat{\beta} \le \hat{\beta}^\dagger$. Moreover, the left-hand side is convex in $\hat{\beta}$ and minimized at $\hat{\beta}\tilde{\sigma}^B = \delta$. So, the first inequality always holds. At this point $\tilde{\rho}$ is equal to δ. Thus, $\tilde{\rho}$ is less than or equal to δ when it exists: There is no leakage with probability $1-p$.

Furthermore, country A chooses T^A such that:

$$\frac{p}{\hat{\beta}\tilde{\sigma}^B/\tilde{\sigma}^A} + \frac{1-p}{\tilde{\rho}} = \frac{1}{\hat{\beta}},$$

which means:

$$p = \frac{\hat{\beta} - \tilde{\rho}}{\hat{\beta} - \frac{\tilde{\sigma}^A}{\tilde{\sigma}^B}\tilde{\rho}}.$$

Since $\tilde{\rho} \leq \hat{\beta}$, for the probability p to be well-defined, it is necessary to have: $\tilde{\sigma}^A \leq \tilde{\sigma}^B$. Because:

$$0 \leq \frac{\hat{\beta} - \tilde{\rho}}{\hat{\beta} - \frac{\tilde{\sigma}^A}{\tilde{\sigma}^B}\tilde{\rho}} \Rightarrow 0 \leq \hat{\beta} - \frac{\tilde{\sigma}^A}{\tilde{\sigma}^B}\tilde{\rho},$$

and

$$\frac{\hat{\beta} - \tilde{\rho}}{\hat{\beta} - \frac{\tilde{\sigma}^A}{\tilde{\sigma}^B}\tilde{\rho}} \leq 1 \Rightarrow \hat{\beta} - \tilde{\rho} \leq \hat{\beta} - \frac{\tilde{\sigma}^A}{\tilde{\sigma}^B}\tilde{\rho} \Rightarrow \tilde{\sigma}^A \leq \tilde{\sigma}^B.$$

In summary, the mentioned mixed equilibrium exists iff

$$\tilde{\sigma}^A \leq \tilde{\sigma}^B \quad \text{and} \quad \frac{\delta}{\tilde{\sigma}^B} \leq \hat{\beta} \leq \hat{\beta}^{\dagger}.$$

■

OD Breakdowns of the Coasian bargain and their implications

As noted in the introduction, Coasian bargains require coordination, informational commonality (say, about the willingness of each country, β^k, to rescue the banks) as well as the availability of public funds in both countries. If either condition is violated, Coasian bargaining breaks down. We here content ourselves with the case of different shadow costs of public funds. We will do so in a stark manner by considering negatively correlated shocks and by assuming that at date 1, one country has cash ($\lambda^k = \lambda$) while the other is broke or has reached its indebtedness limit imposed by a treaty or by financial markets ($\lambda^l = \infty$). Which country will have money at date 1 is not known at date 0, and each country is equally likely to be that country. The implicit assumption of negative correlation of course makes ex-post Coasian bargains impossible. The allocation of activity between the two countries solves:

$$\max_{\{\sigma,q\}}\Big[\frac{F(\hat{\beta}\sigma) + F(\hat{\beta}(1-\sigma))}{2}b - c(\sigma q)\sigma - c((1-\sigma)q)(1-\sigma)\Big]q$$

yielding first-order conditions

$$\frac{F(\hat{\beta}\sigma) + F(\hat{\beta}(1-\sigma))}{2}b = \sigma\hat{c}(\sigma q) + (1-\sigma)\hat{c}((1-\sigma)q)$$

$$\frac{f(\hat{\beta}\sigma) - f(\hat{\beta}(1-\sigma))}{2}b\beta = \hat{c}(\sigma q) - \hat{c}((1-\sigma)q).$$

Whether or not the bank specializes now depends finely on the shock distribution. In particular, if $f' \leq 0$ as we have assumed, then $\sigma = 1/2$. The next example exhibits specialization over a range of parameters.

Binary shock distribution: Suppose that at date 1, within each country a fraction x of (distressed) projects face shock ρ while the remaining fraction of (healthy) projects face shock 0. Then the bank will be able to continue full scale under full diversification if and only if $\hat{\beta}/2 \geq \rho$; the equilibrium is then the same as under a Coasian bargain. By contrast, in the interesting case in which $\hat{\beta}/2 < \rho < \hat{\beta}$,

- Either the bank specializes and picks $\sigma = \rho/\hat{\beta}$ (is insured against liquidity shocks) and total scale $q \equiv q^I(x)$ given by $\sigma\hat{c}(\sigma q) + (1-\sigma)\hat{c}((1-\sigma)q) = (1-\frac{x}{2})b$.

- Or the bank diversifies maximally ($\sigma = 1/2$ for all i) and total scale is given by $q \equiv q^D(x)$ given by $\hat{c}(\frac{q}{2}) = (1-x)b$.

Let

$$C(\sigma, q) \equiv \sigma\hat{c}(\sigma q) + (1-\sigma)\hat{c}((1-\sigma)q),$$

and U^I and U^D denote the bank's utilities when $\sigma = \rho/\hat{\beta}$ (insurance) and when $\sigma = 1/2$ (diversification). We have

$$\frac{\partial U^I}{\partial x} = \frac{\partial}{\partial x}\left[\max_q\left\{\left(1-\frac{x}{2}\right)bq - C\left(\frac{\rho}{\hat{\beta}}, q\right)\right\}\right] = -b\frac{q^I(x)}{2}$$

and

$$\frac{\partial U^D}{\partial x} = \frac{\partial}{\partial x}\left[\max_q\left\{(1-x)bq - C\left(\frac{1}{2}, q\right)\right\}\right] = -bq^D(x).$$

Assume that $\partial(q^D(x) - (q^I(x)/2))/\partial x \geq 0$, as is the case for a linear cost ($c(q) = q \Rightarrow \hat{c}(q) = 2q$, and so $q^I(x) = \frac{(1-\frac{x}{2})b}{2[\sigma^2+(1-\sigma)^2]}$ and $q^D(x) = (1-x)b$) or more generally for power functions ($\hat{c}(q) = \gamma q^\eta$ with $\gamma > 0$ and $\eta \geq 1$). Because $U^I(0) < U^D(0)$ and $U^I(1) > U^D(1)$, there exists $x^* \in (0,1)$ such that for $x < x^*$, the bank diversifies ($\sigma = 1/2$) and for $x > x^*$, it specializes ($\sigma = \rho/\hat{\beta} > 1/2$).

OE Proof of Proposition 6 (supervision by the home country)

(i) First, fix the degree of specialization σ. Two cases are then possible:

1. *Low revenue*. If $\rho_0^* \leq \hat{\beta}\sigma$, the continuation scale is unchanged (at $F(\hat{\beta}\sigma)q^k$ in country k) relative to the $r = 0$ benchmark of Proposition 1: The home country provides additional liquidity $[\int_0^{\hat{\beta}\sigma} \rho dF(\rho) - r]q$, conditional on the bank's revenue being reinvested. Note that the date-1 revenue does not benefit the bank.[2] Rather, it serves to reduce the public outlay.

2. *High revenue*. If $\rho_0^* > \hat{\beta}\sigma$ (which requires $\rho_0^* > \hat{\beta}/2$), there is no public liquidity provision, and also no specialization as specialization is costly for the bank and motivated only by the desire to garner liquidity support.

[2]This would not be the case if either the bank had some bargaining power or there were states of nature without liquidity needs (or with minor ones).

Next, consider the choice of specialization. If $\rho_0^* \leq \hat{\beta}/2$, then $\sigma = \sigma^{nc}$, while if $\rho_0^* \geq \hat{\beta}$, then $\sigma = 1/2$. More generally, either the bank sets σ high enough that is resorts to public liquidity ($\rho_0^* < \hat{\beta}\sigma$), and then the optimal solution does not depend on the unit revenue r; as we noted, this revenue does not benefit the bank. The specialization is then the same, σ^{nc}, as when $r = 0$. Or $\rho_0^* \geq \hat{\beta}\sigma$, so continuation is entirely self-financed. Necessarily $\sigma = 1/2$ as diversification minimizes the cost of acquisition. Because the utility $\max_q F(\rho_0^*(r))bq - C(q, \frac{1}{2})$ is increasing in r, there exists a cut-off r such that the bank does without public liquidity iff $r > r_0$.

(ii) This analysis suggests that (a) were the date-1 revenue subject to moral hazard, careful monitoring of activities that determine this revenue should be undertaken; and (b) the natural supervisor is the country with the largest presence, as this country will end up footing the bill. Indeed, when $\rho_0^* \leq \hat{\beta}\sigma$, only the home country has an incentive to supervise as the continuation scale does not depend on whether the bank absconds with the liquidity or not. When $\rho_0^* > \hat{\beta}\sigma$ however, the bank might reduce its continuation scale by diverting some of the liquidity, and so the two countries might monitor simultaneously. Suppose however the following monitoring technology: In the absence of monitoring, the bank can consume the date-1 cash (as a private benefit) before the country can harness it as a contribution toward the desired liquidity provision. At cost $\psi(z)q$, (where $\psi' > 0$, $\psi'' \geq 0$), a prudential supervisor can prevent this diversion of money with probability z. To rule out supervision by the two countries, let us further assume that if both countries monitor, then the probability of preventing cash diversion is $z = \max\{z^A, z^B\}$; the idea is that monitoring involves going first for the easy sources of diversion. [We could of course assume that a fresh pair of eyes per se improves monitoring, but in practice having multiple supervisors need not be the most efficient approach.] Then only the high-presence country monitors.

■

OF Capital requirements

For conciseness, we use the hegemon model that will be employed in Section 3 (capital requirements may here not be feasible when countries compete; they would be feasible in richer models such as Farhi-Tirole 2012): only country A has cash (at date 0 and at date 1). A single bank selects its scale and its location in the two countries, A and B. The bank has no endowment at date 0 and borrows to finance its investment. The resulting debt is senior (non bailinable); otherwise, the bank would not be able to raise debt in this environment.

The bank specializes in country A, as the latter is the only one that can be trusted to bring funds at date 1. Country A is also the regulator (see Appendix OE). Regulation amounts here to setting a per-unit-of-investment maximal indebtedness, $\bar{d}$, per unit of investment. Assume a linear cost ($c(q) = q$) for simplicity. For a given specialization σ, the date-0 solvency constraint yields the maximal scale of investment:

$$[\sigma^2 + (1-\sigma)^2]q^2 = dq$$

where $d \leq \bar{d}$ is the debt ratio.

At date 1, the bank has no income and no pledgeable income (some could be added to the model along the lines of Section 2.6). Because the debt is senior, country A either brings no liquidity and its welfare is $W^A = 0$ (there is no continuation; we will later show that at the optimal regulation, the country does not pick this option); or it covers the debt obligation and brings extra liquidity $\left[d + \int_0^{\rho^*} \rho dF(\rho)\right]q$, yielding country welfare

$$W^A = \left[F(\rho^*)\beta\sigma - \lambda\left[\int_0^{\rho^*} \rho dF(\rho) + d\right]\right]q.$$

If so, the optimal liquidity provision is still given by

$$\rho^* = \hat{\beta}\sigma.$$

Next, we investigate the choice of specialization by the bank. The bank solves

$$\max_{\{q,\sigma\}} F(\hat{\beta}\sigma)bq$$

s.t.

$$C(q,\sigma) \leq \bar{d}q$$

yielding $\sigma = \sigma(\hat{\beta})$ where (5) holds:

$$\frac{f(\hat{\beta}\sigma)(\hat{\beta}\sigma)}{F(\hat{\beta}\sigma)} = \frac{2\sigma^2 - \sigma}{2\sigma^2 - 2\sigma + 1}. \tag{5}$$

Note that the specialization is scale independent. So choosing $\bar{d}$ is for the principal equivalent to choosing q. The latter solves

$$\max_{\{q\}} \frac{W^A}{\lambda} = \max_{\{q\}} \left[\left[F(\rho^*)\rho^* - \int_0^{\rho^*} \rho dF(\rho)\right]q - [\sigma^2 + (1-\sigma)^2]q^2\right]$$

where $\rho^* \equiv \hat{\beta}\sigma(\hat{\beta})$. This yields

$$q = \bar{q} = \frac{\int_0^{\rho^*}(\rho^* - \rho)dF(\rho)}{2[\sigma^2 + (1-\sigma)^2]}.$$

And so $W^A > 0$ at the optimal $\bar{d}$. Finally, noting that $d\rho^*/d\hat{\beta} > 0$,[3] the debt limit increases as β increases and λ decreases.

OG Exchange rate appreciation in the binary case

For conciseness, we obtain the proposition in the binary-shock case (see Appendix OJ). A fraction x of projects face shock ρ while the remaining fraction face shock 0. We assume that

[3] From the analysis in the text, Condition (5) implies that $d\sigma/d\hat{\beta} < 0$. But from (5) as well, $\frac{d\rho^*}{d\hat{\beta}} = \frac{d(\hat{\beta}\sigma(\hat{\beta}))}{d\hat{\beta}}$ has the same sign as $d\sigma/d\hat{\beta}$.

$\hat{\beta}/2 < \rho < \hat{\beta}$, so that the bank will have to downsize by a factor x at date 1 if it fully diversifies. Let $\sigma \in (\frac{1}{2}, 1)$ be defined by $\sigma\hat{\beta} = \rho$.

Exchange rate without commitment. We first examine the equilibrium in the time-consistent case when ringfencing is impossible. We then extend the analysis to accommodate a liquidity commitment and ringfencing. How does the bank split liquidity support at date 1? For simplicity, we assume that the liquidity is given to the bank in a way that neutralizes changes in the exchange rate brought about by changes in the split (this can be done by making the liquidity support contingent on the exchange rate, for example via an appropriate currency composition). This ensures that the bank does not seek to manipulate the exchange rate when it decides its split, and chooses the same cutoff for all countries.[4] There are two possible equilibria: I (insured) and D (diversified). The equilibrium depends on which of these equilibria yields higher utility to the bank.

In the first possible equilibrium, the bank is specialized in country A, country A's exchange rate is appreciated $e < 1$, and country A provides liquidity support. Country A provides liquidity (and using the notation of Appendix OJ, then fungibility implies that $\xi^A = \xi^B = \xi$ because the bank does not manipulate the exchange rate) up to the point where

$$\beta q^A - \frac{\rho}{e^{\frac{1}{2}}} q^A - \rho e^{\frac{1}{2}} q^B - \frac{1}{2e} \frac{\omega - [\xi - (1-x)]\rho q^A - e[\xi - (1-x)]\rho q^B}{e^{\frac{1}{2}}} \frac{\partial e}{\partial \xi} = 0$$

or at a corner ($\xi = 1$ or $\xi = 1 - x$) if the condition holds as an inequality ($\geq$ or $\leq$), where

$$e = \frac{\omega - [\xi - (1-x)]\rho q^A}{\omega - [\xi - (1-x)]\rho q^B} < 1$$

and so

$$\frac{\partial e}{\partial \xi} = -\frac{\rho(q^A - q^B)}{[\omega - [\xi - (1-x)]\rho q^B]^2} < 0.$$

Let $\xi(q^A, q^B)$ be the solution. At date 0, the bank solves[5]

$$U^I = \max_{\{q^A, q^B\}} \{b\xi(q^A, q^B)(q^A + q^B) - c(q^A)q^A - c(q^B)q^B\},$$

where "I" stands for "insured". Specializing increases liquidity support, but appreciates the exchange rate, which triggers the two offsetting forces described above. The solution $\{q^A, q^B, e, \xi\}$ is then given by the solution to this system of four equations.

[4] Alternatively, we could assume a large number of (non-competing) banks without market power on the exchange rate.

[5] For example, suppose that the equilibrium is such that $\xi = 1$. This means that

$$U^I = \max_{\{q^A, q^B\}} \{b(q^A + q^B) - c(q^A)q^A - c(q^B)q^B\}$$

s.t.

$$\beta \frac{q^A}{q^A + q^B} - \frac{\rho}{e^{\frac{1}{2}}} \frac{q^A}{q^A + q^B} - \rho e^{\frac{1}{2}} \frac{q^B}{q^A + q^B} + \frac{1}{2e} \frac{\omega - x\rho q^A - ex\rho q^B}{e^{\frac{1}{2}}} \frac{\rho(q^A - q^B)l}{(\omega - x\rho q^B)^2} = 0$$

where

$$e = \frac{\omega - x\rho q^A}{\omega - x\rho q^B}$$

In the second possible equilibrium, the bank diversifies, and $e = 1$. Its utility under diversification ("D") is

$$U^D = \max_q\{2[(1-x)b\frac{q}{2} - c(\frac{q}{2})\frac{q}{2}]\}$$

and

$$q^D = (1-x)b$$

exactly as above.

Liquidity commitment and ringfencing. With Cobb-Douglas preferences and a binary shock, the exchange rate is given by:

$$\frac{\omega - [\xi^A - (1-x)]\rho q^A + e[\omega - [\xi^B - (1-x)]\rho q^B]}{2} = \omega - [\xi^A - (1-x)]\rho q^A$$

which can be rewritten as:

$$e = \frac{\omega - [\xi^A - (1-x)]\rho q^A}{\omega - [\xi^B - (1-x)]\rho q^B}.$$

The exchange rate of country A is more appreciated (e lower), the higher is q^A, the lower is q^B, the higher is ξ^A, and the lower is ξ^B:

$$\frac{\partial e}{\partial \xi^A} = e\frac{-\rho q^A}{\omega - [\xi^B - (1-x)]\rho q^B} < 0 \qquad \frac{\partial e}{\partial \xi^B} = e\frac{\rho q^B}{\omega - [\xi^B - (1-x)]\rho q^B} > 0$$

$$\frac{\partial e}{\partial q^A} = \frac{-[\xi^A - (1-x)]\rho}{\omega - [\xi^B - (1-x)]\rho q^B} < 0 \qquad \frac{\partial e}{\partial q^B} = e\frac{[\xi^B - (1-x)]\rho}{\omega - [\xi^B - (1-x)]\rho q^B} > 0.$$

The joint surplus of the bank and country A is now given by:

$$U + W^A = S(q^A) + \xi^A \beta q^A + \sum_k [\xi^k b - c(q^k)]q^k + \frac{\omega - [\xi^A - (1-x)]\rho q^A - e[\xi^B - (1-x)]\rho q^B}{e^{\frac{1}{2}}}.$$

This expression is decreasing in e:

$$\frac{\partial(U + W^A)}{\partial e} = -\frac{1}{2e}\frac{\omega - [\xi^A - (1-x)]\rho q^A + e[\xi^B - (1-x)]\rho q^B}{e^{\frac{1}{2}}}.$$

Assuming ringfencing (ξ^A and ξ^B can be selected independently), the first-order conditions are

$$\hat{\beta} + b = \frac{\rho}{e^{\frac{1}{2}}} - \frac{1}{q^A}\frac{\partial(U + W^A)}{\partial e}\frac{\partial e}{\partial \xi^A}$$

$$b = \rho e^{\frac{1}{2}} - \frac{1}{q^B}\frac{\partial(U + W^A)}{\partial e}\frac{\partial e}{\partial \xi^B}$$

$$(b + \hat{\beta})\xi^A = c(q^A) + [\xi^A - (1-x)]\frac{\rho}{e^{\frac{1}{2}}} - \frac{\partial(U + W^A)}{\partial e}\frac{\partial e}{\partial q^A}$$

$$b\xi^B = c(q^B) + c'(q^B)q^B + [\xi^B - (1-x)]\rho e^{\frac{1}{2}} - \frac{\partial(U + W^A)}{\partial e}\frac{\partial e}{\partial q^B}.$$

The desire of country A to expand bank activities more in country A than in country B leads to more liquidity injections in country A at date 1, which in turn increases the demand for country A's currency and appreciates country A's exchange rate. As for banking activities, there are new forces: Liquidity injections in A (B) are more (less) costly because the exchange rate of country A is appreciated at date 1 (this pushes towards lower q^A, higher q^B, lower ξ^A, higher ξ^B); liquidity injections in A (B) appreciate (depreciate) the country A exchange rate and help (hurt) country A's terms of trade manipulation (this pushes towards higher q^A, lower q^B, higher ξ^A, lower ξ^B). The first force captures exchange rate appreciations as a limiting factor for liquidity support policies.

OH Proof of Proposition 11 (arbitrary number of countries and banks)

Two-bank, two-country case. Let us start with the case $K = I = 2$; we will later generalize the analysis. Bank i solves:

$$\max_{\{q_i^A, q_i^B\}} \{F(\hat{\beta}(\max_{k\in\{A,B\}}(\frac{q_i^k}{q_i^A + q_i^B}))(q_i^A + q_i^B)b - \sum_{k\in\{A,B\}} c(q_i^k + q_j^k)q_i^k\}.$$

This yields the following first-order condition with respect to q_i^k:

If $0 < q_i^k < q_i^l$ (the LHS of (OA2) must be weakly negative if $q_i^k = 0$ and weakly positive if $q_i^k = q_i^l$)

$$F(\hat{\beta}(\frac{q_i^l}{q_i^A + q_i^B}))b - c(q^k) - c'(q^k)q_i^k - f(\hat{\beta}(\frac{q_i^l}{q_i^A + q_i^B}))\frac{q_i^l}{q_i^A + q_i^B}\hat{\beta}b = 0. \tag{OA2}$$

If $q_i^k > q_i^l \geq 0$ (the LHS of (OA3) is weakly negative if $q_i^k = q_i^l$)

$$F(\hat{\beta}(\frac{q_i^k}{q_i^A + q_i^B}))b - c(q^k) - c'(q^k)q_i^k + f(\hat{\beta}(\frac{q_i^k}{q_i^A + q_i^B}))\frac{q_i^l}{q_i^A + q_i^B}\hat{\beta}b = 0. \tag{OA3}$$

Conditions (OA2) and (OA3) are the standard Cournot conditions except for the last terms on the RHSs (those proportional to the density f), which reflect the concern about receiving liquidity support. An increased scale in the low-presence country dilutes the benefit for the high-presence country to supply liquidity, increasing the cost for the bank of diversifying across countries. And conversely for the choice of scale in the high-presence country.

Thus a symmetric, balanced outcome, in which banks invest equally in each country cannot exist: The LHS of (OA2) would be non-negative and the LHS of (OA3) would be non-positive at $q_i^k = q_i^l$, which is impossible.

It can also be shown that, given that countries are symmetrical, banks do not pick the same home country. The reason for this is that entry into a relatively unserved market is more profitable than entry in a competitive market. We will therefore look for an equilibrium in which the banks both specialize, but in different countries.

We first look for an *partial-specialization equilibrium* in which the two banks invest in both countries and choose to specialize in different countries, with the same degree of specialization $\sigma \in (\frac{1}{2}, 1)$.

It is instructive to consider a bank's choice of specialization in say country k, given a fixed size q_i:

$$\max_{\{\sigma_i\}}[F(\hat{\beta}\sigma_i)b - c(\sigma_i q_i + q_j^k)\sigma_i - c((1-\sigma_i)q_i + q_j^l)(1-\sigma_i)]q_i.$$

In such a "symmetric"[6] equilibrium, for which we omit bank and country indices, the first-order condition with respect to σ_i yields:

$$f(\hat{\beta}\sigma)b\hat{\beta} = c'(q)q(2\sigma - 1).$$

Let q denote the total volume in a given country. Conditions (OA2) and (OA3) can be rewritten as

$$\begin{aligned} F(\hat{\beta}\sigma)b - c(q) - c'(q)(1-\sigma)q - f(\hat{\beta}\sigma)\sigma\hat{\beta}b &= 0 \\ F(\hat{\beta}\sigma)b - c(q) - c'(q)\sigma q + f(\hat{\beta}\sigma)(1-\sigma)\hat{\beta}b &= 0. \end{aligned}$$

Adding and subtracting these two conditions yields the condition obtained above together with a new condition:

$$f(\hat{\beta}\sigma)b\hat{\beta} = c'(q)q(2\sigma - 1) \tag{OA4}$$

$$F(\hat{\beta}\sigma)b = c(q) + c'(q)q[\sigma^2 + (1-\sigma)^2]. \tag{OA5}$$

For $\sigma > \frac{1}{2}$, $\sigma^2 + (1-\sigma)^2 > \frac{1}{2}$: As earlier, the marginal cost of investment is higher under specialization than when banks diversify their portfolio, because an increase in the bank's size has a larger inframarginal effect on the cost of acquisition in the market in which the bank is more present.[7]

This in turn implies that

$$F(\hat{\beta}\sigma)b = \left(\sigma - \frac{1}{2}\right)f(\hat{\beta}\sigma)\hat{\beta}b + H\left(\frac{f(\hat{\beta}\sigma)\hat{\beta}b}{2\left(\sigma - \frac{1}{2}\right)}\right) \tag{OA6}$$

where H is an increasing function defined implicitly by $H(X) = c(q) + c'(q)\frac{q}{2}$ when $X = c'(q)q$.[8]

A *full-specialization equilibrium* in which banks both stay in their home country and do not attempt to invade the other's territory has banks operate at scale q^M (where "M" stands for (local) monopoly) and requires that the following condition be satisfied:

$$\frac{f(\hat{\beta})\hat{\beta}}{F(\hat{\beta})} \geq \frac{c'(q^M)q^M}{c'(q^M)q^M + c(q^M)}.$$

[6]It is symmetric in magnitudes, but, as we have seen, banks specialize in different countries. A sufficient condition for the second-order condition with respect to σ_i to be satisfied is $f' \leq 0$, which we already assumed. The objective function is also concave in q.

[7]Note also that a small diversification has only a second-order effect on the marginal cost: $\left.\frac{d(\sigma^2+(1-\sigma)^2)}{d\sigma}\right|_{\sigma=1/2} = 0$. So the liquidity effect necessarily dominates starting from full diversification.

[8]For instance for $c(q) = q$ (see footnote 7), $H(X) = \frac{3}{2}X$; and more generally $H' \geq 1$ if and only if $c' \geq c''q$.

The RHS of this inequality decreases with the ratio of the average cost $c(q)/q$ over marginal cost $c'(q)$. Note that the condition for full specialization with multiple banks is weaker than that $(f(\hat{\beta})\hat{\beta}/F(\hat{\beta}) \geq 1)$ for a single bank: There is less incentive for bank i to diversify if bank j is strong in bank i's low-presence country.[9]

It is again interesting to investigate the consequences of a shift θ in the distribution (the distribution writes $F(\rho-\theta)$). A higher θ can be interpreted as more turbulent times. With this interpretation, more turbulent times lead to banking renationalization (a higher specialization).

To show that competition is stronger under a Coasian bargain, note that bank's size q in the absence of Coasian bargain is given by

$$c(q)+[\sigma^2+(1-\sigma)^2]qc'(q)=F(\hat{\beta}\sigma)b.$$

This is to be compared with a similar equation in the Coasian bargain case:

$$c(q)+\frac{1}{2}qc'(q)=F(\hat{\beta})b.$$

So overall, there is more banking activity under a Coasian bargain for two reasons: A stronger liquidity support and a smaller market power.

General case. There are K countries (indexed by $k \in \{1,...,K\}$) and I banks (indexed by $i \in \{1,\ldots,I\}$). For analytical simplicity, suppose that $I = nK$ (we had above $I = K = 2$ and $n = 1$). An increase in n, the banks-to-countries ratio, i.e. the intensity of competition, amounts to an increase in competition in the banking sector, while an increase in K leads, ceteris paribus, to an increase in banks' international diversification. We look for an equilibrium in which:

- All banks have the same scale q, which they allocate in proportion σ to one country, the home country, and $\frac{1-\sigma}{K-1}$ to each of the other countries.
- For any country k, exactly n banks select this country as their home country. Let $Q \equiv nq$ denote the size of the banking sector in any given country.

Following the steps of the two-bank, two-country case, bank i solves:

$$\max_{\{\sigma_i,q_i\}}\left\{F(\sigma_i\beta)q_ib-c(Q+\sigma_iq_i-\sigma q)\sigma_iq_i-(K-1)c(Q+\frac{(1-\sigma_i)q_i}{K-1}-\frac{(1-\sigma)q}{K-1})\frac{(1-\sigma_i)q_i}{K-1}\right\}$$

The first-order condition with respect to σ_i is:

[9] Consider a power function acquisition cost $c(q)=q^\eta$ with $\eta>0$ (which includes the linear case $c(q)=q$). Then $H(X)=(\frac{1}{2}+\frac{1}{\eta})X$ and (OA6) can be rewritten as

$$\frac{f(\beta\sigma)\beta\sigma}{F(\beta\sigma)}=\frac{\sigma^2-(\sigma/2)}{\sigma^2-\sigma+\frac{1}{2}(1+\frac{1}{\eta})}. \tag{OA7}$$

The LHS of (OA7) is non-decreasing in σ, while the RHS is (always) decreasing in σ on $[1/2,1]$. Thus condition (OA7) has a unique solution. And this solution is a corner solution of full specialization if and only if $\frac{f(\hat{\beta})\hat{\beta}}{F(\hat{\beta})}\geq\frac{\eta}{1+\eta}$.

Either $\sigma \leq 1$ and

$$f(\sigma\beta)b\beta = \frac{c'(Q)Q}{n}\left[\sigma - \frac{1-\sigma}{K-1}\right]$$

Or $\sigma = 1$ and

$$f(\sigma\beta)b\beta \geq \frac{c'(Q)Q}{n}.$$

As for the first-order condition with respect to q_i, we obtain:

$$F(\sigma\beta)b = c(Q) + \frac{c'(Q)Q}{n}\left[\sigma^2 + \frac{(1-\sigma)^2}{K-1}\right].$$

In particular, banks fully specialize (they are present in a single country: $\sigma = 1$) if and only if:

$$\frac{f(\beta)\beta}{F(\beta)} \geq \frac{c'(Q)Q}{c'(Q)Q + nc(Q)}.$$

For the power cost function ($c(Q) = Q^\eta$), this inequality writes:

$$\frac{f(\beta)\beta}{F(\beta)} \geq \frac{\eta}{\eta + n}.$$

So the stronger the competition, the more likely are banks to not be cross-border. More generally, whenever there exists $\varepsilon > 0$ such that $\frac{c(Q)}{c'(Q)Q} \geq \varepsilon$ for all Q, then as $n \to \infty$, banks specialize fully.

More countries than banks. Suppose that $K = mI$, where $m \geq 1$ is an integer. Consider a symmetric equilibrium in which

(i) Banks choose different home countries.

(ii) Each invests σq in its home country, $\sigma' q$ in each of the $(K-I)$ countries without home bank, and $\sigma'' q$ in the $(I-1)$ countries that have a different home bank, where $\sigma + (K-I)\sigma' + (I-1)\sigma'' = 1$ and $\sigma \geq \max\{\sigma', \sigma''\}$ (home country).

Let Q_H denote total output in each of the I countries with a home bank and Q_L the total output in each of the other $(K-I)$ countries (later, we will show that $Q_H > Q_L$).

Consider a deviation by bank i to $\{q_i, \sigma_i, \sigma_i', \sigma_i''\}$. The symmetric configuration: $\{q, \sigma, \sigma', \sigma''\}$, must solve:

$$\max_{\{q_i,\sigma_i,\sigma_i',\sigma_i''\}} \left\{F(\sigma_i\beta)bq_i - c(Q_H + \sigma_i q_i - \sigma q)\sigma_i q_i - (K-I)c(Q_L + \sigma_i' q_i - \sigma' q)\sigma_i' q_i - (I-1)c(Q_H + \sigma_i'' q_i - \sigma'' q)\sigma_i'' q_i\right\}$$

s.t.

$$\left[\sigma_i + (K-I)\sigma_i' + (I-1)\sigma_i''\right] q_i \leq q_i \qquad (\nu)$$

Letting $\mathcal{L}$ denote the Lagrangian, the FOC are (after dividing by q_i):

$$\frac{\partial \mathcal{L}}{\partial \sigma_i} = \nu + f(\sigma_i \beta)\beta b - \hat{c}'(Q_H)$$
$$\frac{\partial \mathcal{L}}{\partial \sigma_i'} = \nu - \hat{c}'(Q_L)$$
$$\frac{\partial \mathcal{L}}{\partial \sigma_i''} = \nu - \hat{c}'(Q_H).$$

So $\sigma'' = 0$. Furthermore, if $Q_L \geq Q_H$, then $\sigma' = 0$ and so $Q_L = 0$ after all.

■

OI Branches and subsidiaries

Section 5 relates to the large law and economics literature on the difference of regulatory and governance environments for subsidiaries and branches.[10] On the institutional front, a branch in the host country exposes the parent bank to liabilities in case of branch distress and therefore to higher[11] risk than a subsidiary, which has its own capital, board of directors and for which limited liability protects the parent bank. Modes of supervision also differ (the following is only the broad picture, as practice in the matter is somewhat heterogeneous). In all cases, the home supervisor supervises the entire banking group, including the affiliate (whether a subsidiary or a bank); the supervisor can limit the bank's range of activities and locations.[12] While the branch must obey the host country's regulations, it is most often not supervised by that country.[13] A subsidiary is by contrast supervised by both countries (indeed the host country is the lead supervisor for the locally incorporated subsidiary). There is a consensus that, whether for governance or regulatory reasons, the bank can more easily relocate liquidity across borders in the case of a branch. And subsidiaries require more capital and liquidity than a branch.[14]

[10]Useful reviews are provided in Danisewicz et al (2017), Fiechter et al (2011) and Schoenmaker (2013).

[11]Of course the parent bank has "its name on the door" and so may feel compelled to rescue its subsidiary despite the absence of legal obligation to do so. This reputation risk makes the distinction not as strong as it might appear.

[12]See Fiechter et al (2011).

[13]Note, though, that the host country may have responsibility for supervising the branch's liquidity position, as is the case within the European Union or under the Basel Concordat.

[14]They also facilitate the writing of living wills. Cross-border bank resolutions are known to be particularly tricky. Bolton and Oehmke (2019) look at the role of bailinable securities (loss-absorbing capital) to prevent bankruptcy in a multinational banking context (our model by contrast focuses on public liquidity support once these securities have been bailed in; adding bailinable securities- e.g. along the lines of Clayton-Schaab (2021) and Dewatripont-Tirole (2019)- to our model would be doable provided we added date-2 pledgeable income). A single point of entry for bank resolution is efficient when countries can commit to cooperate at the crisis stage (the equivalent of a date-1 Coasian bargain in our model). Bolton and Oehmke interpret this a supranational regulation. Multiple points of entry by contrast may be advantageous when no such commitment is feasible and resolution will involve large asymmetries in transfers. Calzolari and Loranth (2011) study the non-cooperative decisions of multiple regulators of a multinational bank. A bank at date 0 invests in projects in both countries using local deposits. The regulator (regulators in the case of a subsidiary) decide at date 1 whether to liquidate the bank's country operations/project or let it continue, on the basis of a signal on the likely date-2 return. Regulators stand for the interests of depositors. The paper stresses the externalities in liquidation decisions.

The branch vs. subsidiary choice is guided by various factors, some unrelated to the analysis of this paper. Differentials between corporate tax rates making a branch particularly appealing in a high-tax country is an obvious one. Another is the affiliate's source of funding –retail vs. wholesale-; retail deposit funding makes the subsidiary regime more likely, if only because the host country- which then will be in charge of deposit insurance- will often demand to supervise the affiliate.

The theoretical literature on the choice between the two is scant. Dell'Ariccia and Marquez (2010) show that this choice should be influenced by the type of host country risk; in case of (uninsurable) economic shock, the limited parent-affiliate liability afforded by the subsidiary structure pleads for the latter as it limits the propagation of shocks; by contrast the branch structure allows faster repatriation and protects the bank against expropriation by the host country.[15] Dell'Ariccia and Marquez predict that subsidiaries will take on more risk and will be larger. Cerutti et al (2007) find empirical evidence in support of this theory.

OJ Liquidity provision: The binary-shock case

Suppose that, in contrast with the model in the text, the shock structure is binary. A fraction x of projects face shock ρ while the remaining fraction face shock 0. We assume that $\hat{\beta}/2 < \rho < \hat{\beta}$, so that the bank will have to downsize by a factor x at date 1 if it fully diversifies. Let $\sigma \in (\frac{1}{2}, 1)$ be defined by $\sigma\hat{\beta} = \rho$. So specialization must exceed σ in order for all projects to continue.

Finally, let us assume a linear cost structure ($c(q^k) = q^k$). Let U^I (for "insured") and U^D (for "diversified") denote the bank's utility when it chooses to specialize at $\sigma = \frac{\rho}{\hat{\beta}}$ in country A and to diversify fully at $\sigma = \frac{1}{2}$:

$$U^I = \max_q \{bq - c(\sigma q)\sigma q - c((1-\sigma)q)(1-\sigma)q\}$$

and

$$U^D = \max_q \{2[(1-x)b\frac{q}{2} - c(\frac{q}{2})\frac{q}{2}]\}.$$

In the linear cost case ($c(q) = q$),

$$q^I = \frac{b}{2[(\frac{\rho}{\hat{\beta}})^2 + (1-\frac{\rho}{\hat{\beta}})^2]}$$

and

$$q^D = (1-x)b.$$

Suppose that the bank would want to diversify in the absence of LOLR ($U^D > U^I$):

$$(1-x)^2 > \frac{1}{2[(\frac{\rho}{\hat{\beta}})^2 + (1-\frac{\rho}{\hat{\beta}})^2]}.$$

[15] Through forced holdings of government bonds, forced lending to government-connected borrowers, capital controls, etc.

Country A's welfare is then equal to $S(\frac{q^D}{2})+\beta(1-x)\frac{q^D}{2}$. Will the country with cash want to grant LOLR to the bank?

Unconditional and free-of-charge liquidity support. Suppose first that country A commits to liquidity support T . The bank then keeps diversifying. Let $\xi = min\{1-x+\frac{T}{\rho q},1\}$ denote the fraction of activity that continues at date 1, where as earlier q denotes the bank's overall size.[16]

(i) For $T < x\rho[(1-x)b]$, the bank solves

$$\max_q\{(1-x+\frac{T}{\rho q})bq - c(\frac{q}{2})q\} = \max_q\{(1-x)bq - c(\frac{q}{2})q + \frac{Tb}{\rho}\}$$

yielding

$$q = q^D = (1-x)b.$$

Country A's welfare is now $S(\frac{q^D}{2}) + \beta[(1-x)+\frac{T}{\rho q^D}]\frac{q^D}{2} - \lambda T$. Country A's welfare is reduced:

$$\Delta W^A = T(\frac{\beta}{2\rho} - \lambda) < 0.$$

Such interventions only benefit the bank and reduce welfare: LOLR interventions that reduce date-1 downsizing without changing the date-0 scale cannot improve welfare since, if desirable for country A, they could be performed at date 1 anyway.

(ii) For $T \in [x\rho(1-x)b, x\rho b]$, the bank optimally chooses a size that leads to continuation with probability 1 and diversifies so as to reduce investment cost:

$$q = \frac{T}{\rho x}.$$

And country A welfare,

$$W = S\left(\frac{T}{2\rho x}\right) + \left[\frac{\beta}{2\rho x} - \lambda\right]T,$$

is convex in T. So, the optimum does not lie in the interior of this intermediate region.

(iii) For $T \geq x\rho b$, then

$$q = b.$$

The welfare gain (or loss) at $T = x\rho b$[17] relative to the absence of transfer is:

$$\Delta W^A = S(\frac{b}{2}) - S(\frac{(1-x)b}{2}) + \rho b x[\frac{\beta}{\rho}(1-\frac{x}{2}) - \lambda].$$

Therefore, there can be a welfare gain from large unconditional liquidity support if $\frac{\hat{\beta}}{\rho}(1-\frac{x}{2}) > 1$, i.e. if x is not too large. The source of the gains is two-fold: First, banks acquire more clients, which generates more inframarginal surplus; second, banks continue more often, which, given

[16] $\xi = \kappa/q$.

[17] Any transfer beyond this level is wasteful.

their larger scale, can also potentially generate more social benefits (note a potential complementarity here).

Optimal intervention. Now consider a contract between Country A and the bank specifying in the two countries a presence $\{q^k\}_{k\in\{A,B\}}$, a continuation scale $\{\xi^k\}_{k\in\{A,B\}} \in [1-x,1]$, a liquidity allocation $\{T^k\}_{k\in\{A,B\}}$, and a date-0 transfer between the bank and country A, so as to maximize the joint surplus of the two players:

$$U + W^A = S(q^A) + \xi^A \beta q^A + \sum_k [(\xi^k b - c(q^k) - (\xi^k - (1-x))\rho]q^k.$$

We can consider two cases

- fungibility: $\xi^A = \xi^B$;[18]
- ring-fencing: ξ^A and ξ^B can be chosen separately.

Optimal quantities satisfy:

$$(b+\hat{\beta})\xi^A = c(q^A) + [\xi^A - (1-x)]\rho$$

$$b\xi^B = c(q^B) + c'(q^B)q^B + [\xi^B - (1-x)]\rho.$$

As for continuation scales, under ringfencing, $\xi^B = 1$ if $b \geq \rho$ and $\xi^B = 1-x$ otherwise. Similarly, $\xi^A = 1$ if $b+\beta \geq \rho$ and $\xi^A = 1-x$ otherwise. Under fungibility, $\xi = \xi^A = \xi^B = 1$ if $(\beta + b - \rho)q^A + (b-\rho)q^B > 0$.

References

BOLTON, P. and M. OEHMKE (2019), "Bank Resolution and the Structure of Global Banks," *Review of Financial Studies*, 32 (6), 2384–2421.

CALZOLARI, G. and G. LORANTH (2011), "Regulation of Multinational Banks: A Theoretical Inquiry," *Journal of Financial Intermediation*, 20, 178–198.

CERUTTI, E., DELL'ARICCIA, G. and S. MARTINEZ PERIA (2007), "How Banks Go Abroad: Branches or Subsidiaries?" *Journal of Banking and Finance*, 31, 1669–92.

CLAYTON, C. and A. SCHAAB (2021), "Bail-Ins, Optimal Regulation, and Crisis Resolution," R & R, *Review of Financial Studies*.

DANISEWICZ, P., REINHARDT, D. and R. SOWERBUTTS (2017), "On a Tight Leash: Does Bank Organizational Structure Matter for Macroprudential Spillovers?" *Journal of International Economics*, 109, 174–194.

[18] Formally, the bank is indifferent between the two countries when it comes to covering shocks of magnitudes ρ. However, if the binary distribution approximates a smoother one, then the cutoff rule will be the same for both countries, as in Section 2.

DELL'ARICCIA, G. and R. MARQUEZ (2010), "Risk and the Corporate Structure of Banks," *Journal of Finance*, 65, 1075–96.

DEWATRIPONT, M. and J. TIROLE (2019), "Liquidity Regulation, Bail-ins and Bailouts," mimeo.

FIECHTER, J., OTKER-ROBE, I., ILYINA, A., HSU, M., SANTOS, A., and J. SURTI (2011), "Subsidiaries or Branches: Does One Size Fit All?" IMF Staff Discussion Note, March, SDN/11/04.

SCHOENMAKER, D. (2013), *Governance of International Banking: The Financial Trilemma*, New York: Oxford University Press.